Flipping the Learning of Mathematics: Different Enactments of Mathematics Instruction in Secondary Classrooms

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The concept of flipping the classroom and flipped learning is becoming increasingly more popular in secondary schools. Although more prevalent in tertiary teaching, flipped learning has a number of affordances that may address the challenging demands of teaching secondary mathematics. While enactments of the approach vary, flipped learning requires a reconceptualization of traditional secondary mathematics instruction in that instructional content is assigned as homework before class, providing for more targeted in-class teaching. This paper reports on a study that investigated the enactment of the flipped classroom in ten different secondary mathematics classes. Findings indicated that there were essentially three different ways in which the approach was enacted, yet all enactments appeared to offer similar affordances. The study adds to the limited research that documents flipped learning in secondary mathematics classes and contributes to current understandings of teachers' and students' perceptions of their experiences of such an approach and the affordances it offers. The study has implications for secondary students and teachers of mathematics, particularly those who are teaching within the constraints of prescribed textbooks and externally imposed assessment measures.

Teaching secondary mathematics can be a challenging domain, with teachers often having to deal with increasing curriculum demands and expectations, along with disengaged students who may not enjoy or understand the subject of mathematics. In order to address these challenges, some teachers have been turning to a flipped classroom approach in order to accommodate more targeted in-class teaching. The approach has been shown to have positive impacts upon teaching practice and learning, including more time for independent and student-led learning, practising and applying knowledge and skills, and individualized support (Straw, Quinland, Harland, & Walker, 2015). Increases in student achievement, success and engagement have also been reported (e.g., Hamdan, McKnight, McKnight, & Arfstrom, 2013), with some studies indicating students' preference for this approach, when compared with more traditional methods (e.g., Muir & Geiger, 2015; Muir, 2016). The findings are particularly encouraging as student disengagement in mathematics is of ongoing concern (Skilling, Bobis, & Martin, 2015), with negative attitudes towards mathematics often developed during secondary school (e.g., Beswick, Muir, Jade, Farrington, & Callingham, 2013; Nicolaidou & Philippou, 2003). Factors contributing to this disengagement include inappropriate curricula, insufficiently challenging tasks, ineffective teaching and societal changes related to technological and social developments (Luke et al., 2003). Flipped learning has the potential to enhance secondary mathematics practice, which has traditionally been dominated by textbook use and externally imposed assessment measures (e.g., Muir & Chick, 2014). It also provides an arguably more engaging alternative to traditional homework practices, which have been perceived by many middle school students as boring, too easy or too hard, or irrelevant (Xu & Wu, 2013).

Flipped learning is characterised as: a space where students take responsibility for their own learning, a classroom where students who are absent are not left behind, all students are engaged in their learning, class content is permanently archived for review or remediation, and students receive a personalised education (Bergmann, Overmyer, & Wilie, 2013). The terms "flipped classroom" and "flipped learning" are not synonymous; what is defined as

"school work at home and home work at school" is overly simplistic (Yarbro, Arfstrom, McKnight, & McKnight, 2014, p. 5) and does not capture the essence of what flipped learning entails. While a flipped classroom or 'inverted classroom' may lead to flipped learning, Bergmann and Sams, (2012) who are credited with conceptualising the approach, use the term 'flipped learning' to describe:

> … a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter. (Flipped Learning Network [FLN], 2014, para. 1)

According to Bergmann and Sams, there "is no single way to flip your classroom … no specific methodology to be replicated, no checklist to follow that leads to guaranteed results" (p. 34). Accounts of different enactments such as those contained in this paper, add to the limited research that report on enactments in secondary mathematics classrooms. The approach is particularly dominant in tertiary settings, but studies that have been conducted in high school settings, indicate that the approach is transferable to secondary contexts (e.g., Muir & Geiger, 2015; 2016; Straw, et al., 2015). Studies that have been reported in both secondary and tertiary settings tend to focus on results and outcomes, rather than comparing and contrasting approaches, which this paper aims to do. Specifically, this paper adds to previous research through describing how the flipped approach is enacted in a selection of secondary mathematics classes, and students and teachers' perceptions of the impact of the approach. In order to do this, the paper addresses the following research questions: How is flipped learning enacted in a selection of secondary mathematics classes? How do these enactments impact upon students' uptake of the approach? For the purpose of this paper, the term 'flipped learning approach' will be used to incorporate the various enactments as depicted in the literature, including flipped classroom, flipped learning and inverted classroom. The terms used by researchers when reporting on studies will be maintained.

Review of the Literature

Flipped Learning and the Flipped Classroom

The flipped learning approach has been distinguished from more traditional approaches through the following features:

- Homework time is typically used to deliver new content to prepare students for lessons, rather than being used for revision and/or consolidation
- Greater use is made of online learning, including the use of videos and presentations, rather than textbooks and worksheets
- Teachers spend more time in class coaching and facilitating learning, and less time on whole class instruction and/or demonstration

(Straw, et al., 2015).

While flipped learning typically involves the use of videos as a replacement for direct instruction, Bergmann and Sams (2012) emphasise that the greatest benefit is the additional in-class time that it provides. In order for authentic flipped learning to occur, there are Four Pillars of FLIP that should be incorporated into practice (Flipped Learning Network (FLN), 2014). These pillars are detailed further on in this paper and form the theoretical framework for interpreting the results. Through enacting these pillars, flipped mastery can occur, which is where the real potential of flipped learning is realized. In this approach, students typically access video resources when ready, work through resources at their own pace and

demonstrate mastery of the particular subject/topic through the completion of homework tasks. This enactment of flipped learning is described more fully in the next section, together with other enactments as referred in the literature.

Enactments of Flipped Learning

Accounts of flipped learning in the literature are dominated by studies undertaken in higher education. In a survey of the research conducted by Bishop and Verleger (2013), only one of the 24 studies cited occurred in a high school setting. Most of the studies cited examined student perceptions which were generally positive (Bishop & Verleger, 2013). More recently, a review by DeLozier and Rhodes' (2016) was dominated by accounts involving higher education contexts. Consistent with Bishop and Verleger's (2013) review, DeLozier and Rhodes (2016) also found that outcomes frequently reflected students' perceptions of their learning, rather than reported measures of student performance. Likewise, this paper reports on students' perceptions, rather than outcomes, due to the challenges of controlling for a range of variables.

In terms of describing how flipped learning is enacted in these higher education contexts, there are a number of studies where similar approaches are used. Common approaches required students to watch prerecorded video lectures or screencasts prior to attending class (Delozier & Rhodes, 2016), with more in-class time provided for active learning which was characterized by experiences such as group discussions, pair-and-share activities, and student presentations. As an illustrative example, Ford (2015) described how she created lessons using video cameras, storyboards or iPad apps for her pre-service teachers to view outside of class. Students were expected to watch the assigned videos before class, with the first 5-10 minutes of class time devoted to addressing questions from the students about the material. Students then worked in small groups to complete assigned exercises that covered the video lecture material.

Accounts of enactments in secondary mathematics classrooms are less evident in the literature (Bishop & Verleger, 2013; DeLozier & Rhodes, 2016). A study undertaken by Clark (2015) involved implementing a flipped classroom model of instruction with Grade 9 students who were enrolled in an Algebra course. The students prepared for class by watching videos, listening to podcasts, and reading articles. Class time was spent engaging in hands-on activities, participating in real-world applications and sometimes completing independent practice in the presence of the teacher. Similar to the higher education examples, Clark's participants reported on increased classroom participation and communication compared to passive class time as experienced through traditional lectures in the past. Using a similar approach whereby more routine aspects of instruction were removed from the classroom, an earlier study by Muir and Chick (2014), reported on the experiences of a senior secondary mathematics class. Through moving direct instruction out of the classroom, the teacher, Mr Smith, found he could cover the curriculum more efficiently and had more time to spend with assisting individuals. Classroom experiences, however, were still dominated by individual completion of textbook exercises, rather than active learning.

Straw et al. (2015) provide a rare example of flipped learning in a secondary mathematics context whereby teachers used existing instructional video resources, rather than creating their own video content. In the study, teachers were asked to use Khan Academy[1] mathematics resources to flip the learning in their classes. In terms of enactment, Straw et

[1] https://www.khanacademy.org/ An online learning resource that includes exercises, instructional videos and a personalized learning dashboard for a range of subjects, including mathematics

al. (2015) reported that flipped learning class time tended to be used mainly for offline learning, such as working through activities in a textbook, with students often encouraged to discuss their work with peers or in small groups. The teachers' roles involved facilitation of these activities and some whole-class discussions, with class work being dependent upon the students viewing the videos beforehand.

There are few examples in the literature of mathematics secondary classrooms that have adopted a flipped mastery approach. Muir (2016) provided an example of two cases whereby senior secondary mathematics teachers provided a bank of teacher-created video resources for students to access individually and work through at their own pace, thereby enacting a mastery approach not readily evident in the literature. Class time was spent on working independently through problems and examples primarily derived from a prescribed textbook. Students were working at different stages and on different topics and demonstrated mastery of the content through successful completion of regular assessment tasks. Students who experienced this approach reported increased satisfaction with the relevancy of materials provided, and greater engagement with, and autonomy over, their learning compared with that experienced in the past.

Affordances and Constraints

The term affordances is often used in mathematics education generally and in technology in mathematics education in particular (Brown, Stillman, & Herbert, 2004). For the purpose of this paper, the definition of affordance given by Tanner and Jones (2000) as a "potential for action, the capacity of an environment or object to enable the intentions of the student within a particular problem situation" (p. 78) has been adopted. In addition, the affordances have been considered along with constraints, in that the former describes potential for action, the latter the structure for action and that it is often "the structure imposed by the setting [that] may facilitate task progress" (Kennewell, 2000, p. 55, as cited in Brown et al., 2004). Affordances provide for allowable actions to occur between an 'object' and 'actor' (Gibson, 1977, p. 78) but the existence of an affordance does not necessarily imply that activity will occur.

A number of affordances of the flipped learning approach have been documented in the literature. Fulton (2012) for example, identified advantages such as students working at their own pace, more effective and creative use of classroom time, and increased levels of student achievement, interest, and engagement. Teachers in Straw et al.'s (2015) study reported quicker progression through class, increased opportunities for questioning and higher order level discussions, collaborative learning, student-led learning, individualized support, and development of independent learning skills. Muir (2016) identified similar affordances, along with accessibility, self-pacing, assessment preparation and optimization of class time. Muir (2016) also found that the uptake of the approach and the affordances it offered were related to students' motivations to engage with the approach.

In terms of constraints, access to technology or inadequate technology provision, has been identified (e.g., Herreid & Schiller, 2013; Straw et al., 2015), together with finding good quality videos or the time and resources to produce good quality videos (Herreid & Schiller, 2013). Students new to the method may be initially resistant or may not complete the assigned work at home (Herreid & Schiller, 2013), and hence come to class unprepared. Parents might also be skeptical of the approach due to its departure from the traditional schooling that they experienced (Fulton, 2012; Muir & Geiger, 2015).

Theoretical Framework

The theoretical framework that has been primarily adopted in this paper is derived from the Four Pillars of FLIP (FLN, 2014). These principles were devised by experienced Flipped Educators, who are key leaders of the FLN, and is widely cited in the literature (190; Google Scholar). According to the FLN (2014), which was established by Bergmann and Sams, (2012), teachers must incorporate the Four Pillars of FLIP into their practice in order to engage in flipped learning. Table 1 provides a summary of what is involved in each of the pillars.

Table 1
Overview of 4 Pillars of FLIP

Pillar	Characterised by
Flexible Environment	Establishment of spaces and time frames that permit students to interact and reflect on their learning as required; flexible spaces which allow students to choose when and where they learn
Learning culture	Giving students opportunities to engage in meaningful activities without the teacher being central; activities are accessible to all students; learning is personally meaningful
Intentional content	Concepts used in direct instruction are prioritised for learners to access on their own; relevant content is created or curated for students; content accessible and relevant to all students
Professional educator	Teacher available to all students for individual, small group, and class feedback in real time as required

The framework was used to interpret the enactments of the flipped learning approach as undertaken by the teachers in the study who were flipping their mathematics classes to different extents. In addition, students' and teachers' responses to open-ended survey and interview questions were analyzed to identify references to affordances or constraints. Details about the methodology employed, and the analysis of data in relation to the framework and affordances, are described in the next section.

Methodology

The research reported in this paper employed a mixed-methods approach (Creswell, 2003) to investigate students' and teachers' experiences of flipped learning in ten secondary mathematics classes. Online surveys containing a mix of Likert items and open-ended questions were completed by participating students. Classroom observations were undertaken and interviews were conducted with teachers, students and some parents. Sequential methods (Creswell, 2003) were used to inform the interview questions, allowing more detailed exploration with a few cases or individuals. The individuals that were selected for interviews included the class teachers and those students who indicated at the end of the online survey that they would be willing to participate in an interview. Topics covered in the interviews included describing a typical mathematics lesson, how students viewed and interacted with the videos at home, how or what motivated them to watch the videos and whether or not they would recommend the videos or approach to others. An exploratory case study approach was selected in order to bring new understandings to the fore (O'Leary, 2010), with the cases studied being the different secondary mathematics classes. The

selection of the classes was purposive in that the teachers were flipping their mathematics classes and agreed to participate in the research. Full ethical approval from all jurisdictions was granted for the research, and full consent, including student parental consent, was obtained prior to any data gathering.

School and Classroom Contexts

Table 2 provides an overview of each of the schools and classes involved in the research. The schools were large independent metropolitan schools and the classes studied ranged from Grade 8 to Grade 12. Two teachers, Mr Hill and Mr Smith, had been flipping their mathematics classes in varying degrees for three years, and their different classes were participants in the research conducted in 2014 and 2015. Previous reports of this research focused on students' perceptions of the affordances of each teacher's approach and the role of the teacher, rather than comparing enactments. All other classes were observed in 2015 and each teacher was in their first year of flipping, although some had used videos as a teaching resource in the past. It is likely, therefore, that Mr Smith and Mr Hill may have had better student uptake due to their experience in flipping. The Grade 8, 9 and 10 classes were all of mixed ability with the Grade 10 'maths extended' class containing students who were likely to study mathematics in Grade 11 or 12. Mathematics Methods is a senior secondary pre-tertiary course which covers topics such as functions, calculus and statistics and is externally examined. Specialist Mathematics is considered the most advanced mathematics course in Secondary School and includes topics such as conic sections, complex numbers, differential equations, kinematics, vector calculus and mechanics. All students had their own copies of the prescribed textbook and access to either laptops or iPads in class.

<insert Table 2 here>

Participants

The participants were seven teachers and their respective classes of students who ranged in Grades from 8-12. A total of 170 students participated in the study, 139 completed the online survey and 60 participated in 23 focus group interviews. Table 2 shows the grades and subjects taught by each teacher, along with the class size. Pseudonyms have been used throughout in the reporting of results.

Instruments and Procedure

Data collection instruments consisted of an online survey using Qualtrics[2], semi-structured student and teacher interviews, and classroom observations. Consistent with mixed-methods methodology, items and responses from the survey were used to inform the semi-structured interview schedule. Illustrative items included 'The tutorial helped me to understand a concept', 'I watched all of the tutorial from the beginning to the end' and 'I found the tutorial boring'. As the Likert-scale items from the survey were not validated, they are not reported on in this paper. The items are mentioned here as they formed part of the data that were referred to in the interviews. The survey also contained seven questions that provided for open-ended responses and are reported on in this paper. These included 'Which of the videos did you find most useful?' and 'Would you prefer watching the videos as

[2] An online survey construction tool: http://www.qualtrics.com/

homework or doing exercises from the textbook?' and 'Would you recommend the use of this approach to others?'

The student semi-structured interviews were designed to allow the researcher to probe more deeply into the students' experiences of the flipped classroom as reported through the survey. The students were given the option of either participating in a focus group interview or individually; all chose to be part of a focus group. Focus group participants varied from 2-3, and there was a total of 23 focus group interviews conducted. The interviews all occurred in a meeting room or empty classroom at the respective students' schools. The interviews were audio-recorded and transcribed and took approximately 20-30 minutes. The teacher interview schedule was designed to elicit information about the teachers' motivations for implementing the flipped classroom approach, how it was enacted in practice and their perceptions of the benefits and challenges of incorporating the approach. Teacher interviews were conducted individually either prior to or just after the classroom observations, and took approximately 40 minutes. All interviews were conducted by the researcher and transcribed by either the researcher or research assistant. A minimum of two classroom observations were undertaken with each class. The purpose of the observations was to provide the researcher with insight into how the approach was enacted in classrooms, and as a consistency 'check' to compare 'reality' with the teachers' and students' self-reports. Descriptive accounts are provided primarily when describing each classroom's context in the results.

Data Analysis

Qualitative data from the surveys and interviews were analyzed using reflexive iteration (Srivastava, 2009) whereby each sentence in the transcripts was coded using open themes that emerged from the data. This was done to ameliorate any researcher bias that may have occurred if only pre-conceived themes were used. In order to answer the first research question, the transcripts were then re-read to identify references to enactments of the approach. These references were then analyzed further to provide evidence of the four pillars of flipped learning. In order to answer the second research question, the open-ended survey and interview response codes were used to identify reference to affordances or constraints. As an example, the following student interview comment was coded as 'autonomy – affordance'- and 'flexible environment' and 'learning culture':

> You can always go back and view them, not like last year when you had to continuously ask for help.

The main themes that emerged were autonomy, competence, relevance, and relatedness. A number of codes, could be ascribed to each theme and each qualitative comment may have been attributable to one or more code – they were not mutually exclusive. A total of 11 codes were identified as affordances: Preparedness; Relevance; Goal attainment; Capacity to focus; Priming; Relatedness; Captivation; Autonomy; Convenience and Accessibility; Differentiation; Self-pacing. While some comments related to length of videos and access to technology could have been coded as constraints, students generally did not see them as issues, and offered suggestions on how they could be overcome. As this paper focuses on the enactments of the flipped learning approach, the data are primarily analyzed through the lens of the Four Pillars of FLIP framework (for a more detailed discussion of affordances, see Muir, 2016). Illustrative examples of students' references to affordances of the approach are used to highlight similarities and differences between their experiences, and to indicate students' uptake of the respective approaches.

Results

This section has been organised around the different enactments of the flipped learning approach as experienced by the participants in the study. As Table 2 showed, these enactments were: teacher paced curated, teacher paced created and student paced, teacher created. For each enactment an overview of the approach will be given, along with some illustrative examples of how it was enacted in practice. Data gathered from classroom observations, open-ended survey responses and student and teacher interviews are included in the results.

Teacher Paced Curated

Table 2 shows that Mr Shepherd was the only teacher in the study who curated, rather than created, his own videos. Some of the other teachers, however, did supplement their created videos with curated ones, and/or initially out-sourced videos when first implementing the flipped classroom approach. With both of his classes, Mr Shepherd expected his students to watch the prescribed videos before class, with lessons based on extending the content introduced in the videos. Students were also expected to take notes while watching the videos, using a framework of 'What I understand', 'What I need help with', and 'Questions'. Classroom observations and student accounts indicated that each lesson began with a review of the video content where the students had the opportunity to share their notes. Mr Shepherd would typically then expand on an example or contextualise the content for students. For example, the Grade 9 class were studying financial literacy whereby the pre-lesson video involved calculating percentages to work out commissions. Classroom observation showed that Mr Shepherd contextualised this through facilitating a class discussion on jobs that involved commissions, and the difference between commissions and royalties. This part of the lesson took approximately 15 minutes, with the remainder of the lesson spent on individually completing the allocated exercises (calculating percentages) from the text book. A similar approach was used in the Grade 8 class where the lessons observed and video tutorials assigned were based around identifying number patterns as part of an Algebra unit. Mr Shepherds' role was to monitor students' progress and assist when required; there was little further teacher demonstration or whole class facilitated discussion.

In his interview, Mr Shepherd indicated that he generally sourced his video tutorials from 8-10 YouTube channels or regular contributors, and acknowledged that it was sometimes time-consuming to identify appropriate material: "One video might only take about 12 minutes to watch, but it could have taken me an hour or more to find". The students who were interviewed following the lesson indicated that what was observed was typical in that "we review what we did, what we learnt from the videos and then if we have questions from the video, we ask and then we go over it … we then use the textbook to answer questions based on what we had just learnt for practice" [Chloe, Grade 8].

Students generally described the nature of the video tutorials as "a video of someone talking while drawing up the problem and how to solve it using a diagram" [open-ended survey response, Grade 8]. They identified a number of advantages of viewing content at home rather than in class, including the following affordances of *self-pacing* and *capacity to focus*:

> …You can go your own pace while learning. I'm a slow writer so last year I struggled with the understanding because I'd have to write down all the notes off the board and I was concentrating so hard on getting that done in time that I wouldn't take in anything that the teacher was saying. So then I would always have to go back again and ask questions. [Richard, interview, Grade 9]

> …Also don't have other kids, so some are like being distracting or, not helping your learning by talking or something, you don't have that whilst you're watching the videos, it's just pure concentration. [Stephen, interview, Grade 9]

Students also identified a number of advantages of online resources as compared to text books, but most viewed them as complementary to, rather than a replacement for, the prescribed text. Survey responses to this open-ended question included: 'you can search what you want to know', 'you can see people do it and it makes it easier to understand' and 'you don't have to carry a text book home' [Grade 8 responses]. These comments particularly relate to the affordances of *convenience and accessibility*, and *autonomy*. Richard (Grade 9) preferred the videos, stating that: "With the textbook you have to read all the stuff and go through all the questions properly again, whereas with the videos they only go for like 3 to 6 minutes. Short sharp and to the point."

When comparing the approach with mathematics teaching experienced in the past, interview responses showed that students compared it favourably and identified affordances such as *priming* and *preparedness*:

> I would probably prefer to do this … you have more time in class to really understand it … to do the questions and stuff. [James, Grade 8]

> You don't go into the class cold, so you know what you're going to be talking about and you know what you're going to be doing. [Albert Grade 8]

In terms of *relatedness*, interview comments revealed that for these group of students at least, it was not important that their teacher prepared the videos, but appreciated that Mr Shepherd "looks for the best ones so they're really good and show understanding of the topic" [James, Grade 8], indicating that *relevance* was perhaps more important. Some viewed it as an advantage: "You can get a variety of different points of view for your own learning … and if you didn't particularly relate to the teacher you're not going to be stuck the whole year because you can listen to other people's explanations" [Trudy, interview, Grade 9].

In terms of challenges, some students "wouldn't recommend it to everyone … I think it sort of depends on the person and your ability to do it" [Martha, interview, Grade 8]. Arthur (interview, Grade 8) stated that "basically you have to teach yourself for most of it" while Alison (interview, Grade 8) thought that "the harder the subject, the harder it might be teaching yourself".

Teacher Paced Created

Table 2 shows that this approach was taken by Mr Smith, Ms Brown, Mr Douglas and Mr Stephens. All four teachers expected their students to watch the allocated video tutorial/s that they had created before attending class and progress through the learning material was teacher directed. The interviews and survey data indicated that the students did watch the videos as required prior to attending class. Classroom observations showed that while the approach varied between classes, essentially lessons began with an 8-15 introduction/review/warm up with the majority of class time spent on working individually to complete exercises from the prescribed textbook. For example, Ms Brown's lessons typically began with an eight-minute 'warm-up' where students completed allocated exercises from the prescribed textbook. Ms Brown then facilitated students' oral responses to the problems, before briefly revising some of the content from the video tutorial that most students indicated they had watched. The remainder of the lessons (approximately 40 minutes) involved students working individually through allocated questions in the textbook,

which in the lessons observed for Ms Brown, involved the solving of simultaneous equations. Talk occurred between students but there was little whole class demonstration. This approach was similar to the ones observed for the other three teachers. The students in their interviews indicated that the lessons observed were typical.

The teachers varied in the way they produced their video tutorials. At the time of the study, for example, Ms Brown had recorded approximately 20 video tutorials, all based upon topics in the textbook and all about an hour in duration. Ms Brown preferred to create a video for each topic and then directed students to watch different parts of it, rather than break it up into shorter videos. She used Powerpoint with an OfficeMix add on to record her videos, which students accessed through an emailed link which was provided to students at least three days prior to class. Mr Smith used 'Doceri' (an iPad interactive whiteboard and screencast recorder app) to create his videos which were all about 10-15 minutes duration and were made available to students through the school's learning management system. All teachers indicated that the creation of the videos meant that they spent less time 'lecturing' in class, than in the past. For example, Mr Smith explained in his interview:

> I was shortening up my board work – I didn't eliminate it altogether - you still need to do board work, but … I was able to do more sophisticated examples and … I was saving 20-25 minutes being at the front of the class and I was able to use that time at the board then to be more targeted with what I needed to teach.

Students also appreciated that there was less time spent in class passively listening, and greater access to more individual time with their teachers, as the following illustrative comments show:

> It's better having the video and watching it at home and being able to come and ask the teacher if I am still unclear about how to do something or a particular concept … I think it's better than last year where we would go through the book and rather than have lengthy explanation in class, it's better to have an idea before you get to class. [Anna, interview, Grade 11, Ms Brown]

Anna's comment also includes reference to affordances such as *preparedness* and *priming*.

While Mr Smith created most of his videos at home, he occasionally videoed in-class 'demonstrations' as did Mr Douglas. This was usually done with topics that the teachers perceived as being particularly problematic for students or as a convenience when time did not allow for videos to be prepared beforehand. These were then uploaded for students to access and review and were a particularly good resource for students absent when the concepts were first covered.

In their interviews, students identified a number of affordances with the approach, including reference to *self-pacing, accessibility and convenience*. Helen (Grade 11, Ms Brown), for example, appreciated the autonomous nature of the approach, stating that: "You can always go back and view them, not like last year when you had to continuously ask for help".

With relation to the affordances of *relatedness* and *relevance*, students varied in their perceptions as to whether or not it was important that their teachers had prepared the videos. Abigail (Grade 11, Ms Brown) for example, stated that "You understand it better when it's someone you know … and they can explain it again in a similar way in class if they have to". Anna, however, commented that:

> I don't think it's really important who does it – whether one teacher does the video or the entire maths faculty … but what's good about a teacher from school doing it as opposed to Khan Academy is that they know what the curriculum is and know what's important to focus on … The few times I did that

> [looked up on Google] it was extremely lengthy and only a few relevant points so it is easier having Ms Brown give us the videos – it's a lot more concise and relevant to what we want.

As with feedback from Mr Shepherd's students, the above comment indicates that a sense of relevance may be more important than a sense of relatedness. The teachers, however, felt that it was important that the videos were created by them, with Mr Smith, for example, stating that:

> The students are used to my teaching style - it's different from somebody from Khan Academy – I'm sure they do it very well – probably far better than I do – but because they [students] relate to my board work and how I speak and how I explain things, it's just a continuation of me in class.

Student Paced, Teacher Created

Both Mr Hill and Mr Burns were incorporating a flipped mastery learning approach into their practice. While Mr Hill had gradually moved towards this over a period of three years, this was Mr Burns' first year with flipping and, as he stated in his interview: "I had to sort of make the decision whether I do a partial thing or maybe I just do a couple of videos or I do a number of videos and I decided, I'm going to do the whole thing and take the entire curriculum and I'm going to video it". At the time of the study, Mr Burns had created 193 video tutorials for his students to access that covered the requirements of the Grade 12 Specialist Mathematics course, and included topics such as conic sections, complex numbers, and differential equations. Mr Hill also had a bank of video tutorials that he had prepared over the past three years that were based on the topics covered in the prescribed textbook. In both cases, students would access the videos for each topic, attend class where they would individually work through associated exercises in the textbook, then sit a test to demonstrate mastery of the topic. Mr Hill tended to regulate this according to topic weeks, so mastery of a topic would typically occur over 2-3 weeks, then the next topic would be introduced to the whole class, with students then allowed to work at their own pace through that particular topic. Mr Burns' students, however, were encouraged to keep working through the topics; this resulted in at least one student finishing the required syllabus six weeks before the external exam was scheduled. While some class discussions and demonstrations were observed, students tended to 'opt in' according to relevance, and the majority of class time was spent on individual work, with the teachers providing personal assistance when required. In his interview Mr Hill described how the approach provided for more targeted instruction and interaction:

> … the kids are really into this system now – I have my whiteboard markers sitting next to me and kids will just generally walk up, and say do you mind if I just grab a marker, and they'll start working on the board and then when they're ready, like when they're at a point when they're stuck, they'll say Mr Herbert, I've done this – where do I go from here? And I'll come up and show them and work them through it … and they'll get it and then generally sit down, so there often is a board full of maths at the end of the lesson, but it's the students driving it [not me]

Interviews with Mr Hill and Mr Burns' students showed that the students particularly appreciated the effort that their teachers made to produce the videos. While again there were mixed responses about the importance of their teachers producing the videos, Mr Burns, like Mr Smith, believed that it was crucial:

> The students relate [to me] I think better than they do to somebody talking about a video that may contain 40 or 50% of what they are looking for; the videos they are looking at now contain 100% of what they are looking for so it's more important in that respect … I think it's very important … because you still need that teacher/student relationship and that works for the student and that works for the teacher. I can give them 20 or 30 videos to look at on a particular topic that are on YouTube

> but whether they'll get anything out of it compared to having me do the video and talking about them at their level … I know who they are and what they're doing, I think makes a big difference. I think it really is important that the teacher does the video, it really is.

As with the other classes in the study, both the video tutorials and the class work were based on exercises in a prescribed textbook. Similar to feedback received from other students, students from Mr Hill's and Mr Burns' classes viewed the approach as complementary rather than a substitute for the textbook. Open-ended survey responses, for example, included: "Online resources often give harder examples and more variety", and "Using online resources makes it possible to learn more information than what is possible in a textbook alone. A textbook can also be difficult to comprehend sometimes". Again, a number of affordances were identified, including *capacity to focus* and *accessibility*. Chris, for example, stated that:

> There's no interruptions [at home] whereas in class there are so many interruptions … he might be halfway through an explanation and then somebody interrupts … if you get distracted, [at home] you just pause the video and come back to it. [Chris, interview, Grade 12, Mr Burns]

The teachers also identified similar affordances, along with the autonomous nature of the approach as illustrated in the following comment from Mr Burns:

> A couple of months ago … one of the girls in the class came to me and she said … I've got to watch those videos 3 or 4 times before I understand what's going on and I thought to myself, gee I only teach it once and if I only taught it once, she wouldn't have got it.

Discussion and Conclusions

In order to answer the first research question, 'How is flipped learning enacted in a selection of secondary mathematic classes?', classroom observations and interview data showed that three different enactments were evident. While these enactments varied across the cases examined, all contained some elements of the pillars of flipped learning as depicted in Table 1 (FLN, 2014). The teachers that curated or created their own videos, but were operating within a teacher-controlled paradigm, were definitely providing students with *intentional content* through either selection or creation of relevant and appropriate videos, and through making themselves available as a *professional educator*. Aspects of the *learning culture* were present in that the teacher was not always central to the learning and all activities were accessible to students. The classroom observations showed, however, that students' experiences were centred on individual textbook exercises which did not seem to be especially personally meaningful. Similarly, while the homework environment may have been different, the classroom space was not a *flexible environment* in terms of students choosing where and when they learned. It seemed that in some enactments, flipped learning tended to reinforce established pedagogies rather than facilitating new and innovative mathematics teaching strategies.

Mr Hill's and Mr Burns' students experienced provision of *intentional content* through the bank of videos that they could access where and when it suited them, and access to a *professional educator*. A *flexible learning environment* was also evident in Mr Hill's and Mr Burns' classroom, through giving students autonomy over their learning and allowing them to self-pace their progress through the course.

Interview data were used to answer the second research question, 'How do these enactments impact upon students' uptake of the approach?' Despite the different enactments, it appears that students in all classes were willing to uptake the approach and compared it favourably with traditional forms of delivery as experienced in the past. Similar affordances

were identified by the students in this study as reported elsewhere in the literature. For example, students frequently mentioned working at own pace (e.g., Fulton, 2012; Muir, 2016), more effective use of class time (Fulton, 2012; Muir, 2016) and student-led learning (Straw et al., 2015). Interestingly, unlike the findings of Herreid and Schiller (2013), access to technology was not identified as a constraint in this study. Time and resources were mentioned by teachers as challenges, which were also found to be factors in Straw et al.'s (2015) study. Limitations related to the study include acknowledgement that the cohort of students taught in Grades 10, 11 and 12 were enrolled by choice in their classes and arguably strongly motivated to achieve. While Mr Shepherd's student cohort arguably represented a more 'typical' Grade 8 class, they were from an independent school and were used to completing assigned work out of class.

The results from the study have a number of implications for teachers, educational providers and students. In all classes, students were assessed through regular tests, with the Grade 11 and 12 students also required to sit external exams. A majority of the course content for the subjects in this study involved the use of worked examples that were likely to be examined. Preparing these examples through video tutorials enabled the teachers to prepare, review and revise material and scaffold students' learning in a considered manner that is not always so achievable in a busy classroom context.

The data from the study also showed that students were motivated to access the resources because they perceived them as being *relevant.* With research showing that students continue to become disengaged with mathematics (e.g., Skilling et al., 2015) and often view it as irrelevant (Beswick et al., 2013), the flipped learning approach may be one way to address this challenge. Furthermore, targeted resources that have been either selected or created by students' teachers may alleviate the time required to explore appropriate online resources that are consistent with teaching methods or the procedures required in specialist courses.

In conclusion, this study has demonstrated that the enactment of flipped learning can occur in a variety of ways and can be easily adaptable to different contexts and teachers' expertise. It has the potential for differentiating the learning for individual students, along with enabling them to take autonomy over their learning. The results show that while the flipped classroom may be enacted in various forms and to varying degrees, student and teacher perceptions indicate that the approach has merit, particularly in terms of complementing existing practices. The Four Pillars of FLIP (FLN, 2014) provided a useful framework for considering the enactments and could also be used by teachers as a guide for working towards full flipped learning mastery. While it is perhaps disappointing to find that the classroom environment for many of the students in the study was still dominated by textbook practice, this finding must be considered within the constraints experienced by the teachers. In most Australian senior secondary mathematics classrooms, the reality is that students are assessed through external measures, with the course content being mandated through curriculum bodies and recommended textbooks. While the flipped learning approach did not result in dynamic, interactive learning environments, it arguably did provide for greater satisfaction with homework tasks and reportedly increased engagement with mathematics classes when compared with past experiences. While certainly not advocating a transmission approach to teaching mathematics, it is hoped that the study reported in this paper has highlighted the potential of flipped learning for enhancing the mathematical experiences of students who are being taught within the constraints of prescribed textbooks and externally imposed assessment measures. Future research should focus on how the flipped learning approach can be utilized and structured to enhance students' conceptual understanding of mathematical concepts.

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