INVESTIGATING CHILDREN'S UNDERSTANDING OF THE MEASUREMENT OF MASS

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In this paper, we discuss the use of a framework of "growth points" in early mathematics learning and a related, task-based, one-to-one interview in assessing children's understanding of the measurement of mass. Data are presented from a sample of 1806 children in the first three years of school. An example of a child's responses is given to illustrate the kinds of thinking revealed by interviewing young children about their developing concepts of mass.

Background

The data discussed are from the Early Numeracy Research Project[1] (ENRP), where teachers and university researchers were seeking to find the most effective approaches to the teaching of mathematics in the first three years of school. At the beginning of the project, the research team identified the need for development of a comprehensive and appropriate learning and assessment framework for early mathematics, and a tool for assessing young children's mathematical thinking. The inappropriateness of pen and paper assessment at these grade levels (Clements & Ellerton, 1995) led to the development of a task-based, one-to-one interview schedule. The project team studied available research on the development of young children's mathematics learning in the mathematical domains of Counting, Place value, Addition and subtraction, and Multiplication and division (in Number), Time, Length, and Mass (in Measurement), and Properties of shape and Visualisation and orientation (in Geometry). In this paper, the focus is on the Measurement domain of Mass.

While much has been published about children's concept development in the measurement of Length (e.g., Clements & Sarama, 2009; Lehrer, Jenkins, & Osana, 1998) and Area (e.g., Outhred & Mitchelmore, 1992), little is published about Mass. However, research has provided some insights. For example, in researching the transitive nature of young children's ordinal ability, Brainerd (1974), found that 5 year-olds could arrange three balls of clay according to their mass and could arrange sticks according to their length.

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Brown, Blondel, Simon, and Black (1995) interviewed 48 Grade 2, 4, 6, and 8 children on four occasions on their understandings of Length and Weight measurement. Their work focused on what they termed underlying general concepts of measurement. Results suggested that "some aspects of competence seemed to progress more smoothly by age than did others" (p. 167) and showed variation in individual performances. The researchers believed that their data supported "a common model of progression in the form of a curriculum and assessment framework" (p. 168). They acknowledged that the results were indicative and tentative and their contention that the data led to a framework was seen as bold and exaggerated (van den Berg, 1995), owing in part to the small sample and idiosyncratic responses by the children.

Spinillo and Batista (2009) conducted a study with 40 children focused on 6 and 8 year olds' understandings of measurement, and found that children of both ages had an understanding of the relationship between the size of a unit and the number of units needed to measure an object, including for measurement of Mass. They found also that, while Distance and Volume were difficult for children to understand in terms of the relation between units of measure and objects being measured, Mass did not cause such problems. The researchers posited that this outcome was linked to children's experiences of weighing objects at home from an early age.

The general paucity of research on Mass is reflected in a recent publication on learning and teaching early mathematics (Clements & Sarama, 2009) where, in 325 pages, neither the word "mass" nor the word "weight" appear. Likewise, in the National Council of Teachers of Mathematics yearbook devoted to the learning and teaching of measurement, the place of Mass and Weight was clearly that of "other measurement domains" (Clarke, Cheeseman, McDonough, & Clarke, 2003, p. 75). Reference is sometimes made to Mass when giving an example of a measurement goal (e.g., Clements, Sarama, Spitler, Lange, & Wolfe, 2011) or describing a measurement investigation (e.g., Lehrer, Jaslow & Curtis, 2003), but with no further discussion of specifics related to the concept.

With the limited research on children's understandings of Mass, the research reported in this paper makes an important contribution to our understanding of this element of measurement. The framework developed for the attribute of Mass followed the same generic form used for each of the measurement domains (see Figure 1).

1. Awareness of the attribute and use of descriptive language
The child shows awareness of the attribute and its descriptive language.
2. Comparing, ordering, and matching with the attribute
The child compares, orders, and matches objects by the attribute.
3. Quantifying accurately, using units and attending to measurement principles
The child uses uniform units appropriately, assigning number and unit to the measure.
4. Choosing and using formal units for estimating and measuring, with accuracy
The child chooses and uses formal units for estimating and measuring, with accuracy.
5. Applying knowledge, skills and concepts
The child can solve a range of problems involving key concepts and skills.

Figure 1. ENRP Generic growth points for measurement.

The purposes of developing the framework for the learning of Mass as it applies to this paper included: to allow the description of the mathematical knowledge and understanding of individuals and groups; to provide a basis for task construction for interviews, and the recording and coding process that would follow; and to allow the identification and description of students' thinking.

Methodology: The interview

Assessment tasks were created to match the framework. The interview was very "hands-on", with considerable use of manipulative materials. Although the full text of the interview involved around 60 tasks in the various mathematical domains listed earlier, no child moved through all of these. The interview was of the form "choose your own adventure", in that given a child's success with the task, the interviewer continued with the next task in the given mathematical domain as far as the child can go with success; but given difficulty with the task, the interviewer abandons that section of the interview. The interview provided information about the growth points achieved by a child in each of the nine domains. It is important to stress that the growth points are "big mathematical concepts and skills", with many possible "interim" growth points between them. As a result, a child may have learned several important ideas or skills *necessary* for moving to the next growth point, but perhaps not of themselves *sufficient* to move there (Clarke et al., 2002; Sullivan et al, 2000).

Of course, decisions on assigning particular growth points to children are based on a *single* interview on a *single* day, and a teacher's knowledge of a child's learning is informed by a wider range of information, including observations during everyday interactions in classrooms (Clarke, 2001).

Interview tasks for Mass measurement

In each case, the instructions to the teacher are given in italics. The equipment needed for the interview questions is listed. The growth point(s) that the interview task addresses has been detailed before each task.

Equipment: tub of at least 20 teddies, 20 gram weight (2 x 20c pieces stuck together with masking tape), a collection of seven objects (a piece of foam, a rock, two plastic containers [short & fat and long & thin], a ball of string, a 1 kg mass or an object which weighs 1 kg [labelled 1 kg], and a tin of tomatoes in a shoe box), a set of balance scales, small film canister filled with water, at least eight ten-gram weights, a set of Salters' Slimmers kitchen scales, 120 g object, 1 kg of brown rice, small scoop.

The first interview task, *What Do You Notice?* was designed to investigate whether a child has an awareness of the attribute of mass, some of the descriptive language associated with weighing objects (growth point 1), and is able to compare masses by hefting and using the balance (growth point 2).

What do you notice?

Please take these things out of the box, and put them on the table.

a) What do you notice about them?

b) Which things are heavy and which things are light?

Push all items aside, except for the two yoghurt containers.
c) Take these two plastic containers *(place one plastic container in each hand for the child to feel).*
Which do you think is heavier?
d) How could you check?
e) Do you know about balances? *(allow some time for the child to become familiar with the balance)*
Use the balance to see which container is heavier.
f) Were you right? How did you know?

The second interview task, *Teddies and Coins* was designed to investigate whether a child could quantify mass accurately, using uniform units appropriately, assigning number and unit to the measure (growth point 3).

Teddies and coins

Place the balance and the tub of teddies in front of the child.
Show the two 20 cent coins wrapped together, and place in the child's hand.
How many teddies weigh the same as this?
(If the child estimates without using the balance, ask "Please use the balance to find out how many teddies weigh the same as this")
What did you find out?

The third interview task, *One Kilogram* was designed to investigate whether a child could use formal units for estimating (growth point 4).

One kilogram

Here is a 1 kilogram weight. I am going to put it in your hand. *(Please do so)*. Here is a tin of tomatoes for your other hand. *(Place the object in the child's other hand.)*
a) Do you think the tin of tomatoes is more than 1 kilogram or less than 1 kilogram weight?
b) Can you check? … What did you find?

The fourth interview task, *Using Standard Units* was designed to investigate whether a child could choose and use formal units for estimating and measuring, with accuracy (growth point 4).

Using standard units

Here is a container. Here are some 10 gram weights. Measure the weight of this container with these 10 g weights.
What did you find? *(To be judged as correct answer including units, the child must say "40 grams" as part of their response. If they say "4" ask "four what?", but even "four 10 gram weights" is not sufficient. We are looking for "40 grams".)*

The final mass interview task, *Using Kitchen Scales* was designed to investigate whether a child could apply their formal knowledge and skills of measurement of mass in context (growth point 5).

Using kitchen scales

Place the kitchen scales and the 120 g object on the table.
Have you seen scales like these before?
a) Please use the scales to weigh this object. What did you find?
If the child gives a number only (without units), ask, e.g., "120 what?"
b) Please use the scales and the scoop to measure out 135 grams of rice.
c) How do you know it is 135 grams?
d) How many more grams of rice would you need to have one kilogram? (865 g)

An example of a child's responses will be used here to illustrate the kinds of thinking revealed by interviewing young children about their developing concepts of mass.

The story of Jack

Jack was interviewed at the beginning of Grade 2. He hefted the plastic containers and could judge which was heavier, and appropriately used the terms heavier and lighter. However, he struggled to think of a way to check his estimate. When given a balance scale he showed interest and, although he said he had never used one, he promptly put a container in each pan and was convinced that his original estimate was correct, that is, that the shorter squat container weighed more. He appeared to interpret the balance tipping to the heavier side correctly. It could be said that Jack had an awareness of the attribute of mass, some of the descriptive language associated with weighing objects (growth point 1), and was able to compare masses by hefting and using the balance (growth point 2). The interviewer continued with the *Teddies and Coins task*. It was soon apparent that Jack was simply adding teddies to one pan of the scales and he did not have the concept of creating equal masses on the balance and using informal units. The Mass interview was concluded there and Jack was considered to have demonstrated growth point 2 in Mass.

Having described the framework and development of the interview protocol, and given an illustrative example of one child's responses, we will now examine the results of an entire cohort of children.

Results

In the domain of Mass, children were interviewed individually by teachers and proceeded through the interview as long as they continued to have success with tasks. Each child's response was recorded on a record sheet for later examination and analysis. Codes were assigned to the responses to reflect the growth point demonstrated by the child on that particular task.

Indicators of growth in Mass

To examine the way the growth points portray the nature of the increasing sophistication of the students' strategies, Table 1 presents a profile of students' achievement over three grade levels.

A random process for choosing students for whom to ask Mass interview questions was provided by the research team and used to provide a "snapshot" of the children's responses to the interview tasks. The data in Table 1 are from a single year of the project, using data from the start and end of the first year of formal schooling (called Prep in Victoria), and the end of Grades 1 and 2.

Table 1. Percentage of students achieving mass growth points over time.

	Prep Mar 2001 (*n* = 533)	Prep Nov 2001 (*n* = 538)	Grade 1 Nov 2001 (*n* = 479)	Grade 2 Nov 2001 (*n* = 256)
Not apparent	17	3	1	0
Awareness of attribute	15	7	2	0
Comparing masses	47	30	17	6
Quantifying masses	21	60	69	50
Using standard units	0	0	10	38
Applying	0	0	1	6

By the end of the Prep year, most students were able to compare masses, and three-fifths were able to use an informal unit to quantify a mass. By the end of Grade 1, virtually all students were able to compare masses, and 69% were able to quantify masses and were ready to move towards using standard units. By the end of Grade 2, over 40% were using standard units successfully, and the rest were ready to move towards that goal. No further growth points seem to be needed to describe growth at this level adequately.

It is noted that with 60% of Prep children being able to quantify masses at the end of the year, it might be expected that a greater number of Grade 1 children would be able to quantify masses or go beyond by this time. We suggest that this might have been due to insufficient experience with use of standard Mass units at the Grade 1 level. Perhaps some children were not being exposed to experiences for which they were ready. This becomes even more apparent when shown in visual form as in Figure 2.

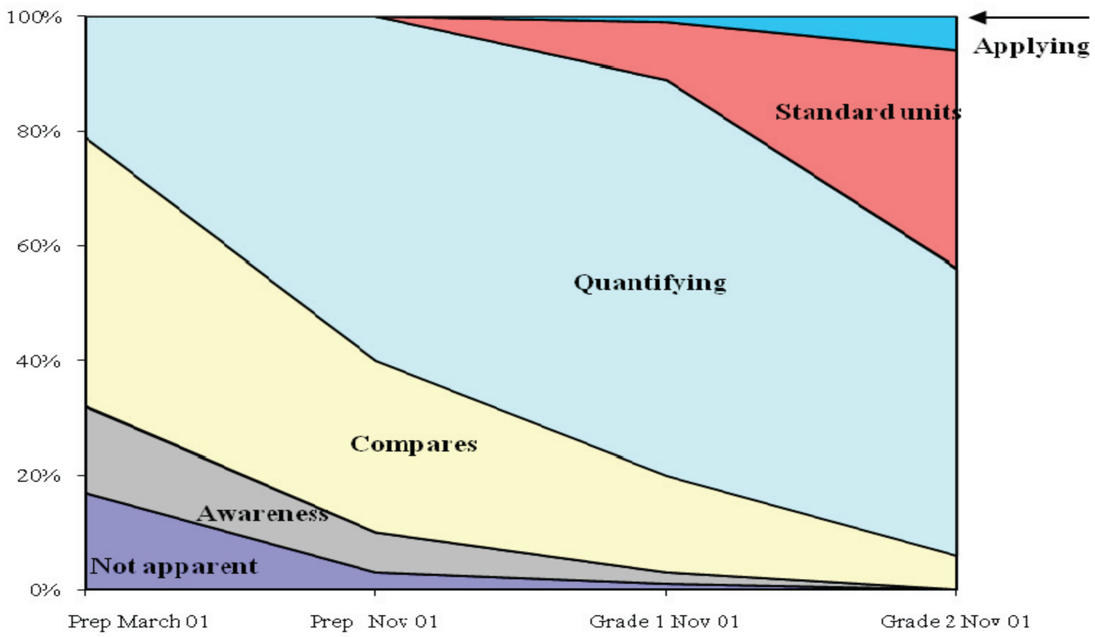

Figure 2. Students (%) achieving Mass growth points over time.

To give a sense of the progress of the students, the percentages of students at each Mass growth point over the four sets of data are shown in Figure 2. To assist with interpreting this representation of the data, it is worth moving the eye in two directions. First, by selecting a year level (Grade 1, Nov 2001), the reader can look vertically from that label, to ascertain the percentage of students achieving each growth point at that time. Second, by moving from the bottom left to the top right, we can see the relative time which students overall spend typically at a particular growth point.

As with Length (Clarke et al., 2002; McDonough & Sullivan, in press), students progress readily through the growth point, Awareness of the attribute. However, two transitions seem to take time: moving from comparing to quantifying; and moving from quantifying to using standard units. It is possible that quantifying Mass is dependent on particular experiences that are beyond the intended curriculum at this stage. The same may well be true for using standard units.

Curriculum expectations

It is interesting to compare the data reported here with the Mass outcomes and indicators in the relevant curriculum (Board of Studies, 2000) of the time. At the end of Prep, the outcomes referred to the attribute of mass, and estimating, measuring and comparing using informal methods. At the end of Grade 2, the outcomes referred to choosing an appropriate attribute, using everyday language, making comparisons, using informal units to estimate, comparing and ordering masses of objects, and measuring by comparing to formal and standard units.

Lately there has been a move towards a national curriculum which is written in broader terms. There are explicit curriculum statements about Mass in the Measurement and Geometry strand of the *Australian Curriculum* (Australian Curriculum Assessment and Reporting Authority, 2010) at Grades 2 to 6 (see Fig. 3).

Grade 2 level	Measure and compare length and capacity using uniform informal and familiar metric units and measure mass using balance scales with familiar metric units (p. 9)
Grade 3 level	Use direct and indirect comparison to order and compare objects by length and develop 'real life' benchmarks for familiar metric units of length, mass, and capacity including centimetre, metre, kilogram and litre (p.10).
Grade 4 level	Use metric units to estimate, measure, and compare the length, mass and capacity of familiar objects reading scales to the nearest graduation (p. 11).
Grade 5 level	Read and interpret scales using whole numbers of metric units for length, capacity, mass, and temperature (p.12).
Grade 6 level	Work fluently with the metric system to convert between metric units of length, capacity and mass, using whole numbers and commonly used decimals (p. 15)

Figure 3. National Curriculum statements concerning measurement of mass.

Clearly there are assumptions about prior learning and mathematical experiences underlying these statements. We hope that the reporting of the "snapshot" of young children's developing thinking about the measurement of Mass will serve to support teachers and mathematics educators as they consider what these prior learning opportunities might comprise.

Identifying targets for teaching Mass

Based on the data reported in this paper, teachers of children in the first year of formal schooling can reasonably aim that nearly all students be able to compare the mass of two objects with use of appropriate language (90%), and begin to move towards quantifying masses by the end of the school year.

Teachers of Grade 1 children could emphasise activities that move the thinking of all students toward the use of informal units to quantify masses, noting that four fifths are either at or moving towards using standard units.

Teachers of Grade 2 children could emphasise activities that stimulate and interest children in using standard units, as 44% were able to use standard units of kilograms and grams successfully. As in other domains, it seems that appropriately chosen activities and experiences can assist students in their development.

In conclusion

In telling the story of Jack, it was noted that his correct use of the balance beam for comparing masses, assuming his statement that he had not used such an instrument previously was correct, may have been learnt during the interview. This finding concurs with that of Brown et al. (1995) who found that

> It was apparent during the interviews themselves that the requirement for pupils to tackle practical problems that they had probably not met before was stimulating learning, since there were many cases where pupils refined their strategies as a result of being asked to explain that they were doing. (p. 165)

These findings point to the value of children having hands on experiences with Mass measurement situations. Indeed, in relation to measurement generally, Cross, Woods and Schweingruber (2009) wrote:

> Even preschoolers can be guided to learn important concepts if provided appropriate measurement experiences. They naturally encounter and discuss quantities (Seo and Ginsburg, 1994). They initially learn to use the words that represent quantity or magnitude of a certain attribute. Then they compare two objects directly and recognize equality or inequality (Boulton-Lewis, Wilss, and Mutch, 1996). At age 4-5, most children can learn to overcome perceptual cues and make progress in reasoning about and measuring quantities. They are ready to learn to measure, connecting number to the quantity, yet the average child in the United States, with limited measurement experience, exhibits limited understanding of measurement until the end of primary grades. (p.197)

From the Mass data reported in this paper, we argue that rich experiences involving measuring Mass are needed, particularly at the Grade 1 level where little progress appears to have been made. The Mass data from the Early Numeracy Research Project also suggest the importance of teachers assessing children's understandings of Mass measurement and structuring learning opportunities to build on and extend those understandings.

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