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Event-triggered output feedback containment control for a class of stochastic nonlinear multi-agent systems

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Abstract An output feedback-based containment control strategy is proposed with an event-triggered mechanism for a class of stochastic nonlinear multi-agent systems (MASs). For unavailable internal states of individual agent, a state observer is constructed for their estimations. The dynamic surface control (DSC) technology is employed to improve traditional backstepping, where a first-order filter is introduced for derivation calculation of virtual control laws. For the purpose of reducing communication resources, a fixed threshold-based triggered strategy is presented to reduce data transmission bits over communication channels. It is proven that all signals in the closed-loop system are bounded in probability and the containment control is achieved. Finally, the effectiveness of the proposed control strategy is verified via two examples.

Keywords Containment · Dynamic surface · Event-triggered · Neural networks · Stochastic multi-agent systems

1 Introduction

In the past decades, cooperative control of multi-agent systems (MASs) [1, 2, 3] has drawn large amounts of attentions. As one of critical issues in MASs' cooperative control [4, 5, 6], containment control maks followers remain in a convex hull spanned by leaders' information. It is applied in various fields in engineering, for instance, ship navigation, satellite attitude and unmanned aerial vehicles (UAVs) [7].

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Many efforts have been taken in containment control of MASs [8]. A linear scheme of containment control for MASs with switching communication topology problem is proposed [9]. MASs' dynamics in [10][11] are completely known. Owing to uncertain factors in many real applications, modeling agents with high accuracy is hard to achieve. The designed containment controllers should be equipped to adaptive algorithms to deal with uncertainties. In the existing works [12][13], random disturbances are ignored. Containment control for an MAS is discussed and a containment control scheme maintaining connectivity is presented [14]. In [15], authors simplify formation containment control problems of UAVs and put forward sufficient conditions to keep formation containment through state transformation and state space decomposition. Besides, [16] presents a singularity-free adaptive fuzzy fixed-time control scheme by establishing a new error conversion mechanism and designing a barrier Lyapunov function. By a reinforcement learning algorithm, authors in [17] report an adaptive fault-tolerant tracking control problem, and they effectively reduce the computational burden. In [18], Dong studies a containment control problem of MASs with time delays. From the viewpoint of the system feedback, the above results mainly focus on state feedback control. Nevertheless, in the real world, it is difficult to measure the internal states directly due to technological limitations of measurement and economical factors.

Output schemes are feasible solutions for containment control. For the distribute containment control of high-order nonlinear MASs with a directed topology, an observer is constructed to estimate unmeasurable state variables, and an output feedback containment control law is proposed to steer the followers' states converge to a convex hull spanned by leaders' trajectories [19]. A containment control law based on observer is proposed for nonlinear MASs with uncertainties [20]. In [21], an uncertain nonlinear strict feedback system is studied and a fuzzy containment control method with unmeasurable states is presented.

Apart from time-driven schemes, event-triggered mechanisms [22, 23, 24, 25] are alternative in MAS control. A containment control problem with a trigger is solved in a directed topology [26]. A kind of triggered mechanisms determines updated instants, and it effectively saves communication resources over networks [27, 28]. In the framework of backstepping control, an adaptive event-triggered consensus scheme [29] is developed for an MAS with an undirected graph. An event-triggered containment control strategy of MASs with external disturbances is studied [30]. For a consensus problem of MASs with general linear dynamics, a distributed and asynchronous event-triggered control strategy is proposed [31]. Finite time consensus control for MASs is reported [32], and a distributed event-triggered control strategy is presented. A polynomial event-triggered strategy is proposed to determine signal transmission, and a fault detection problem of nonlinear discrete time network system is addressed under event-triggered mechanism [33]. In [34], an aperiodic event-triggered strategy is proposed, and stochastic nonlinear systems with unmeasurable states are in an undirected graph. Recently, for a class of output-constrained uncertain nonlinear MASs, authors in [35] present a novel adaptive event-triggered control strategy on the basis of observers. In [36], an event-triggered strategy with a varying threshold is proposed for nonaffine pure-feedback MASs to reduce communication burdens.

In this paper, we focus attention on a class of stochastic nonlinear MASs [37, 38]. For unavailable internal states of individual agent, state observers are constructed, and a triggered strategy is proposed based on a fixed threshold. The main contributions in this paper are listed as follows.

1) An event-trigger-based containment control strategy is proposed for a stochastic nonlinear MASs with uncertainties. Different from most existing results [39][40][41], whose control plants are deterministic MASs, stochastic nonlinear ones are taken into account here. We are devoted to study the containment control of stochastic nonlinear MASs based on event-triggered mechanism, which is more applicable in practice. Only as a triggered condition is violated, control signal is transmitted via a communication channel. The control strategy in our paper reduces the resource occupation of network channels, and the number of data transmission bits over channels is reduced by this triggered mechanism.

2) A first-order filter is introduced for a stochastic nonlinear MAS with uncertain dynamics. Unlike [42], this paper studies DSC method for stochastic nonlinear MASs with completely uncertain dynamics. It avoids explosion of complexity in backstepping method [43], which needs large amount of calculation. In our paper, we can complete the complex derivation process by simple algebraic operation. Also, a compensator is designed for a class of stochastic nonlinear MASs with uncertain parameters to eliminate the influence of output error which produced by a first-order filter. Compared with the result in [44], the influence from a containment error is reduced, and the containment performance of the system is improved in our paper.

The rest of this paper is organized as below. Section 2 introduces problem formulation and some necessary preliminaries. Section 3 provides the design of local state observers to deal with unmeasured states and proposes an event-trigger-based containment control strategy as well as stability analysis. Section 4 gives two simulation examples for verifying the effectiveness of the proposed control strategy.

2 Problem Formulation and Preliminaries

2.1 System Dynamics

We consider a class of stochastic nonlinear MAS, the dynamic of the ith follower is given

$$\begin{cases} \mathrm{d}x_{i,k} = (x_{i,k+1} + f_{i,k}(\bar{x}_{i,k}))\,\mathrm{d}t + g_{i,k}^T(\bar{x}_i)\,\mathrm{d}w_i, \\ \mathrm{d}x_{i,n_i} = (u_i + f_{i,n_i}(\bar{x}_{i,n_i}))\,\mathrm{d}t + g_{i,n_i}^T(\bar{x}_i)\,\mathrm{d}w_i, \\ y_i = x_{i,1}, \end{cases} \tag{1}$$

where $i = 1, \cdots, N$, $k = 1, \cdots, n_i - 1$, n_i is the degree of the ith subsystem, $\bar{x}_{i,j} = [x_{i,1}, \cdots, x_{i,j}]^T \in \mathbb{R}^j$, $j = 1, \cdots, n_i$, $\bar{x}_i = [x_{i,1}, \cdots, x_{i,n_i}]^T \in \mathbb{R}^{n_i}$, and

$\bar{x}_{i,j}$ denotes the state variables of the ith agent, $u_i \in \mathbb{R}$ are the control inputs, $y_i \in \mathbb{R}$ is the output of the ith agent, w_i represents the r-dimensional standard Brownian motion defined on a complete probability space (Ω, F, P) with Ω being a sample space, F representing a σ-field, and P being a probability measure. $f_{i,j}(\cdot) : \mathbb{R}^j \to \mathbb{R}$ and $g_{i,j}(\cdot) : \mathbb{R}^{n_i} \to \mathbb{R}^r$ are unknown, smooth and bounded nonlinear functions.

Definition 1 Consider a stochastic system

$$\mathrm{d}x(t) = f[x(t)]\,\mathrm{d}t + h^T[x(t)]\,\mathrm{d}w. \tag{2}$$

For any given $V(x) \in \mathbb{C}^2$, the differential operator LV is defined as $LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2}\mathrm{Tr}\left\{h^T \frac{\partial^2 V}{\partial x^2} h\right\}$, where $\mathrm{Tr}(A)$ represents the trace of the matrix A.

Lemma 1 ([45, 46]) *For the stochastic system (2), if there exists a continuous differentiable Lyapunov function V such that*

$$LV \le -\alpha_0 V + \gamma_0, \tag{3}$$

where $\alpha_0 > 0$ and $\gamma_0 \ge 0$, the system is bounded in probability.

Lemma 2 ([47]) *For $\eta \in \mathbb{R}$ and constant $\text{ꝉ} > 0$, the inequality*

$$0 \le |\eta| - \eta \tanh\left(\frac{\eta}{\text{ꝉ}}\right) \le \tau \text{ꝉ} \tag{4}$$

holds, where $\tau = 0.2785$.

2.2 Graph Theory

There are N followers, labeled as ν_1 to ν_N, and M leaders, labeled as ν_{N+1} to ν_{N+M}. $\mathcal{G} = (V_{\mathcal{G}}, \varepsilon_{\mathcal{G}})$ represents a communication topology of a directed graph, where $V_{\mathcal{G}} = \{\nu_1, \nu_2, \cdots, \nu_{M+N}\}$ is denoted as a set of nodes, and $\varepsilon_{\mathcal{G}} \subseteq V_{\mathcal{G}} \times V_{\mathcal{G}}$ is a directed edge set. $\Lambda_{ij} = (\nu_i, \nu_j) \in \varepsilon_{\mathcal{G}}$ represents agent j can obtain information from i. The neighbors' set of ν_i is defined as $\mathcal{N}_i = \{\nu_j \,|\Lambda_{ji} \in \varepsilon_{\mathcal{G}}\}$. Communication topology of a digraph $\mathcal{G}$ is represented by an adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(N+M)\times(N+M)}$, if $\Lambda_{ji} \in \varepsilon_{\mathcal{G}}$ is satisfied, $a_{ij} > 0$, otherwise $a_{ij} = 0$. The value of a_{ij} denotes the communication weight from Agent j to Agent i. Laplacian matrix ℓ of the directed graph $\mathcal{G}$ is denoted by $\ell = \mathcal{D} - \mathcal{A}$, $\ell \in \mathbb{R}^{(N+M)\times(N+M)}$, where $\mathcal{D} = \mathrm{diag}(d_1, \cdots, d_{N+M})$ is the in-degree matrix of Agent i, and $d_i = \sum\limits_{j=1}^{N+M} a_{ij}$ is the weighted in-degree. The matrix ℓ can be partitioned as

$$\ell = \begin{bmatrix} \bar{\ell}_1 & \bar{\ell}_2 \\ 0_{M\times N} & 0_{M\times M} \end{bmatrix} \tag{5}$$

where $\bar{\ell}_1 \in \mathbb{R}^{N\times N}$ is a matrix related to the communication among N followers, and $\bar{\ell}_2 \in \mathbb{R}^{N\times M}$ denotes the communication relationship from M leaders to N followers.

Definition 2 The set $\Lambda \in \mathbb{R}^n$ is said to be convex if for any $x_1, x_2 \in \Lambda$, $\beta \in [0,1]$, and $\beta x_1 + (1-\beta)x_2$ is also in Λ. The convex envelope $Co(X)$ for a set of points $X = \{x_1, x_2, \cdots, x_n\}$ is defined as

$$Co(X) = \left\{ \sum_{i=1}^{n} \beta_i x_i \,|x_i \in X, \beta_i > 0\,, \sum_{i=1}^{n} \beta_i = 1 \right\}. \tag{6}$$

Assumption 1 *There exists at least one leader that holds a directed path to each follower.*

Assumption 2 *The leaders' output signals $y_{L,j} \in \mathbb{R}$ are smooth functions, where $y_{L,j}$ and $\dot{y}_{L,j}$ are bounded and can be obtained by ith follower, $j \in \mathcal{N}_i$.*

Lemma 3 ([48]) *Under Assumption 1, all eigenvalues of $\bar{\ell}_1$ in (5) have positive real parts. Each entry of $-\bar{\ell}_1^{-1}\bar{\ell}_2$ is nonnegative, and each row of $-\bar{\ell}_1^{-1}\bar{\ell}_2$ has a sum equal to 1.*

The control objective is to design an event-trigger-based containment control strategy u_i such that $\inf\limits_{h(t)\in Y_L(t)} E[\|y_i - h(t)\|] < \varepsilon, \forall \varepsilon > 0$, where $i = 1, 2, \cdots, N$, $Y_L(t) = Co\{y_{L,N+1}(t), \cdots, y_{L,N+M}(t)\}$, and all signals are bounded in probability. Also, there exists Zeno-free behavior.

Denoting $y_L = [y_{L,N+1}, \cdots, y_{L,N+M}]^T$, we obtain $y_d = [y_{d,1}, \cdots, y_{d,N}]^T = -\bar{\ell}_1^{-1}\bar{\ell}_2 y_L$. From Lemma 3, one has $y_{d,i}(t) \in h(t)$, $i = 1, \cdots, N$. Therefore, the containment control problem is transformed into a tracking issue with one leader $\inf\limits_{h(t)\in Y_L(t)} \mathrm{E}[\|y_i - y_{d,i}\|] < \varepsilon$, where $i = 1, \cdots, N$. The containment errors are defined as $\wp_i = y_i - y_{d,i}$. In vector, $\wp = [\wp_1, \cdots, \wp_N]^T$, $y = [y_1, y_2, \cdots, y_N]^T$, and it follows $\wp = y - y_d$.

Remark 1 In most existing results, an output signal from a controller is applied to the system often in a continuous manner [49, 50]. This manner is scheduled regardless of whether system performance is necessary or not. It is unreasonable as communication resources over a network are limited [51]. It is a challenge to design a triggered mechanism for saving network resources.

Remark 2 In most cases, system states are not able to measurable or even not obtained directly [52]. As for unavailable information, it is hard to develop control methods directly [53]. Thus, a local linear state observer is designed to deal with the unmeasured states.

3 Observer-based Containment Output Control

An output feedback containment control strategy, including observer and output controller, is presented.

3.1 Observer Design

A state observer is designed

$$\begin{cases} \dot{\hat{x}}_{i,k} = \hat{x}_{i,k+1} + l_{i,k}(y_i - \hat{x}_{i,1}), \\ \dot{\hat{x}}_{i,n_i} = u_i + l_{i,n_i}(y_i - \hat{x}_{i,1}), \end{cases} \tag{7}$$

where $i = 1, \cdots, N$, $k = 1, \cdots, n_i - 1$, $\hat{x}_{i,j}$ is estimation of $x_{i,j}$, and $l_{i,j}$ is a positive constant given later, $j = 1, \cdots, n_i$.

The error vector of the observer is defined as

$$e_i = [e_{i,1}, e_{i,2}, \cdots, e_{i,n_i}]^T = \bar{x}_i - \hat{\bar{x}}_i, \tag{8}$$

where $\hat{\bar{x}}_i$ is the estimation of $\bar{x}_i$.

The derivative of e_i, according to (1), (7) and (8), is

$$\mathrm{d}e_i = (M_i e_i + f_i(\bar{x}_i))\mathrm{d}t + \varphi_i^T(\bar{x}_i)\mathrm{d}w_i, \tag{9}$$

where $f_i(\bar{x}_i) = [f_{i,1}(\bar{x}_{i,1}), \cdots, f_{i,n_i}(\bar{x}_{i,n_i})]^T$, $M_i = \begin{bmatrix} -l_{i,1} & \\ -l_{i,2} & I_{n_i-1} \\ \vdots & \\ -l_{i,n_i} & 0 \cdots 0 \end{bmatrix}$ is a n_ith-order matrix, I_{n_i-1} is an identity matrix with (n_i-1)th-order, and $\varphi_i(\cdot)$ is the basis function vector. Due to the fact that M_i is a Hurwitz matrix, there exists a positive definite matrix $K_i = K_i^T$ satisfying

$$M_i^T K_i + K_i M_i = -Q_i \tag{10}$$

for a positive matrix $Q_i = Q_i^T$.

Remark 3 Indeed, not all choices of $l_{i,j}$ would automatically result in M_i being Hurwitz, and we have to choose $l_{i,j}$ appropriately such that the roots of a polynomial $s^n + l_{i,1}s^{n-1} + \ldots l_{i,n_i-1}s + l_{i,n_i}$ have negative real parts.

Choose a Lyapunov function $V_0 = \sum_{i=1}^{N} e_i^T K_i e_i$ for the observer. From (9) and (10), one has

$$LV_0 \leq \sum_{i=1}^{N} \left\{ -\lambda_{\min}(Q_i)\|e_i\|^2 + 2e_i^T K_i f_i(\bar{x}_i) \right\} + \sum_{i=1}^{N} \left\{ \mathrm{Tr}\left\{ \varphi_i^T K_i \varphi_i \right\} \right\}, \tag{11}$$

where $\lambda_{\min}(Q_i)$ is the minimum eigenvalue of Q_i.

From the Young's inequality and (11), we get

$$LV_0 \leq -\xi \|e_i\|^2 + \sum_{i=1}^{N} \left\{ \|K_i\|^2 \|\varepsilon_i\|^2 + \|K_i\|^2 W_i^* + \frac{1}{2}\|K_i\|^2 + \frac{1}{2}\|\varphi_i\|^2 \right\}, \tag{12}$$

where $\varepsilon_i = [\varepsilon_{i,1}, \cdots, \varepsilon_{i,n_i}]^T$, $\xi = \min_{1 \le i \le N} \{\xi_i\}$ with $\xi_i = \lambda_{\min}(Q_i) - 2$, and $W_i^* = \max\left\{N_{i,k} \left\|\vartheta_{i,k}^*\right\|, k = 1, 2, \cdots, n_i\right\}$ with $N_{i,k} = \varphi_{i,k}^T \varphi_{i,k}$.

From (12), we cannot obtain stability of the system, thus it is necessary to design a suitable controller for the MAS, which will be discussed in the following subsection.

3.2 Controller Design

With the observer in Section 3.1, we next make the following assumption for controller design.

Assumption 3 *The state $\hat{x}_{j,2}$ of the jth follower is available for its neighbor j satisfying $j \in \mathcal{N}_i$.*

Define the error surfaces

$$z_{i,1} = \sum_{j=1}^{N} a_{ij}(y_i - y_j) + \sum_{j=N+1}^{N+M} a_{ij}(y_i - y_{L,j}), \tag{13}$$

$$z_{i,k} = \hat{x}_{i,k} - \pi_{i,k}, \tag{14}$$

where $i = 1, \cdots, N$, $k = 2, \cdots, n_i$, $z_{i,1}$ is the distributed tracking error, and $z_{i,k}$ is the error surface. Let the virtual control law $\alpha_{i,k}$ pass through a first-order filter

$$\sigma_{i,k+1}\dot{\pi}_{i,k+1} + \pi_{i,k+1} = \alpha_{i,k} \tag{15}$$

with $\pi_{i,k+1}(0) = \alpha_{i,k}(0)$, where $i = 1, \cdots, N, k = 1, \cdots, n_i - 1$, $\sigma_{i,k+1}$ is a positive constant, and $\pi_{i,k}$ is the output of the first-order filter with virtual control function as the input.

In order to eliminate the influence of error from the filter, the dynamic equations of compensators are designed

$$\dot{\varpi}_{i,1} = -c_{i,1}\varpi_{i,1} + d_i\varpi_{i,2} + d_i(\pi_{i,2} - \alpha_{i,1}), \tag{16}$$

$$\dot{\varpi}_{i,k} = -c_{i,k}\varpi_{i,k} - \varpi_{i,k-1} + \varpi_{i,k+1} + \pi_{i,k+1} - \alpha_{i,k}, \tag{17}$$

$$\dot{\varpi}_{i,n_i} = -c_{i,n_i}\varpi_{i,n_i} - \varpi_{i,n_i-1}, \tag{18}$$

where $c_{i,1}$, $c_{i,k}$ and c_{i,n_i} are positive constants, $\varpi_{i,1}(0) = 0$, $\varpi_{i,k}(0) = 0$ and $\varpi_{i,n_i}(0) = 0$, $k = 2, \cdots, n_i - 1$, and $\alpha_{i,k}$ are virtual control laws designed in the following paper. The term for compensation of tracking error is defined as

$$v_{i,k} = z_{i,k} - \varpi_{i,k}, i = 1, \cdots, N, k = 1, \cdots, n_i. \tag{19}$$

Radial basis function NNs are for approximation of $\bar{f}_{i,k}(\bar{Z}_{i,k}) = \vartheta_{i,k}^{*T}\varphi_{i,k}(\bar{Z}_{i,k}) + \delta_{i,k}(\bar{Z}_{i,k})$, where $\bar{Z}_{i,k} = [\hat{x}_{j,2}, x_{i,1}, \hat{x}_{i,1}, v_{i,k}, \dot{y}_{L,j}]^T$, $j \in \mathcal{N}_i$, $i = 1, \cdots, N$, $k = 1, \cdots, n_i$. $\vartheta_{i,k}^*$ is the ideal constant weight vector defined by

$$\vartheta^* = \arg\min_{\vartheta \in \mathbb{R}^l} \left\{ \sup_{Z \in \Omega_Z} \left| f(Z) - \vartheta^T \varphi(Z) \right| \right\},$$

with $l > 1$ denoting the neural network number and δ being the minimum approximation error.

The virtual control laws are designed as

$$\alpha_{i,1} = -\frac{c_{i,1}}{d_i} z_{i,1} - \frac{1}{2r_{i,1}} \hat{W}_i v_{i,1}, \tag{20}$$

$$\alpha_{i,k} = -c_{i,k} z_{i,k} - z_{i,k-1} - \frac{1}{2r_{i,k}} \hat{W}_i v_{i,k} + \dot{\pi}_{i,k}, \tag{21}$$

with adaptive laws

$$\dot{\hat{W}}_i = \frac{\gamma_i}{2r_{i,1}} d_i v_{i,1}^2 + \sum_{k=2}^{n_i} \frac{\gamma_i}{2r_{i,k}} v_{i,k}^2 - \gamma_i \lambda_i \hat{W}_i, \tag{22}$$

where $i = 1, 2, \cdots, N$, $k = 2, \cdots, n_i$, $N_{i,k} = \phi_{i,k}^T \phi_{i,k}$, $r_{i,1}$, $r_{i,k}$, γ_i and λ_i are positive design parameters, $W_i^* = \max\left\{ N_{i,k} \left\| \vartheta_{i,k}^* \right\|^2, k = 1, 2, \cdots, n_i \right\}$, and $\hat{W}_i$ is estimation of W_i^*.

The control strategy based on fixed threshold

$$u_i(t) = \chi_i(t_p), \forall t \in [t_p, t_{p+1}) \tag{23}$$

is presented with

$$\chi_i(t) = \alpha_{i,n_i} - \bar{m}_i \tanh\left(\frac{\nu_{i,n_i} \bar{m}_i}{\varepsilon_i} \right) \tag{24}$$

and the following event-triggered mechanism

$$t_{i,p+1} = \inf\left\{ t > t_{i,p} \,\middle|\, |\breve{e}_i(t)| \geq m_i \right\}, t_{i,0} = 0, \tag{25}$$

where $\breve{e}_i(t) = \chi_i(t) - u_i(t)$ represents the error caused by the event-triggered mechanism (ETM), ε_i, m_i and $\bar{m}_i$ are design positive parameters, $\bar{m}_i > m_i$, $t_{i,p}$ is the triggered instant, $t_{i,p+1}$ is determined by (25), and $\chi_i(t_{i,p})$ holds as a constant during triggered interval $t \in [t_{i,p}, t_{i,p+1})$.

A diagram for control signal transmission is presented in Fig. 1. The initial value $u_i(t_{i,0})$ is transmitted through a network channel after encoding. For $t \in [t_{i,p}, t_{i,p+1})$, $p = 1, 2, \cdots$, the triggered condition, $|\breve{e}_i(t)| \geq m_i$, is checked. Then the digital signal 0/1 will be transmitted according to the event-triggered condition (ETC). The output of the encoder is transmitted to the decoder through the network channel. And the control signal changes on the basis of the digital signal received from the decoder accordingly.

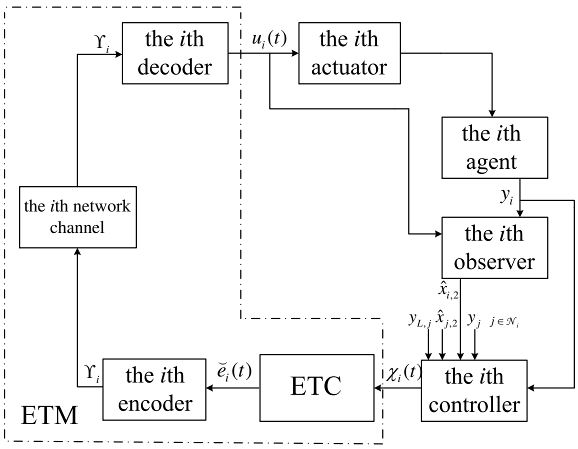

Fig. 1 A diagram for control signal transmission.

Remark 4 In this paper, the DSC method is used to design the control law. If the traditional backstepping method is used, the following derivation can be obtained.
Step $i,1$: Taking the derivative of (13) and with (7), we have

$$\begin{aligned} \mathrm{d}z_{i,1} = &\left(d_i \left(\hat{x}_{i,2} + e_{i,2} + f_{i,1} \right) - \sum_{j=N+1}^{N+M} a_{ij} \dot{y}_{L,j} - \sum_{j=1}^{N} a_{ij} \left(\hat{x}_{j,2} + e_{j,2} + f_{j,1} \right) \right) \mathrm{d}t \\ &+ d_i g_{i,1}^T \,\mathrm{d}w_i + \sum_{j=1}^{N} a_{ij} g_{i,1}^T \,\mathrm{d}w_j . \end{aligned} \tag{26}$$

Step $i,2$: Define the error surface

$$z_{i,2} = \hat{x}_{i,2} - \alpha_{i,1}, \tag{27}$$

and its derivative is

$$\mathrm{d}z_{i,2} = \mathrm{d}\hat{x}_{i,2} - \mathrm{d}\alpha_{i,1}. \tag{28}$$

Then, we can get

$$\mathrm{d}\alpha_{i,1} = \breve{\eta}_{i,1}\mathrm{d}t + g_{i,1}^T \frac{\partial \alpha_{i,1}}{\partial \hat{x}_{i,1}} \mathrm{d}w_i, \tag{29}$$

where $\breve{\eta}_{i,1} = \sum_{j=N+1}^{N+M} \frac{\partial \alpha_{i,1}}{\partial y_{L,j}} \dot{y}_{L,j} + \frac{\partial \alpha_{i,1}}{\partial \hat{W}_1} \dot{\hat{W}}_1 + \frac{1}{2} g_{i,1}^T g_{i,1} \frac{\partial^2 \alpha_{i,1}}{\partial \hat{x}_{i,1}^2} + \frac{\partial \alpha_{i,1}}{\partial \hat{x}_{i,1}} (x_{i,2} + f_{i,1})$. Thus, one has

$$\mathrm{d}z_{i,2} = \Big(\breve{\eta}_{i,1} + \hat{x}_{i,3} + l_{i,2}(y_i - \hat{x}_{i,1})\Big)\mathrm{d}t + g_{i,1}^T \frac{\partial \alpha_{i,1}}{\partial \hat{x}_{i,1}} \mathrm{d}w_i. \tag{30}$$

From (26) and (30), we can see that the traditional backstepping control approach essentially suffers from the main problem as explosion of complexity owing to repeated differentiations of virtual controllers. The strategy in our paper uses simple algebraic operation instead of complex derivation process. It introduces a first-order filtering of the virtual control law at each step of the backstepping design procedure. Also, a compensator is designed to eliminate the effect from a filter in containment control.

3.3 Stability analysis

Theorem 1 *The stochastic MAS is composed of M leaders and N followers. Under Assumption 1–Assumption 3, the event-triggered control strategy (23) with the state observer (8) and adaptive law (22) ensures the output of followers is in a convex hull formed by the leaders' trajectories. The containment error converges to the neighborhood around the origin in probability with appropriate parameters. All signals in the MAS are bounded in probability, and there exists Zeno-free behavior.*

Proof From (13) and (19), we obtain

$$\begin{aligned} v_{i,1} &= z_{i,1} - \varpi_{i,1} \\ &= \sum_{j=1}^{N} a_{ij}(y_i - y_j) + \sum_{j=N+1}^{N+M} a_{ij}(y_i - y_{L,j}) - \varpi_{i,1}. \end{aligned} \tag{31}$$

With (1), the derivative of (31) is

$$\begin{aligned} \mathrm{d}v_{i,1} = &\Bigg(d_i\left(v_{i,2} + e_{i,2} + \alpha_{i,1} + f_{i,1}\right) + c_{i,1}\varpi_{i,1} - \sum_{j=1}^{N} a_{ij}\left(\hat{x}_{j,2} + e_{j,2} + f_{j,1}\right) - \sum_{j=N+1}^{N+M} a_{ij}\dot{y}_{L,j}\Bigg)\mathrm{d}t \\ &+ d_i g_{i,1}^T \mathrm{d}w_i - \sum_{j=1}^{N} a_{ij} g_{i,1}^T \mathrm{d}w_j, \end{aligned}$$

where $i = 1, \cdots, N$.

For the first subsystem, the Lyapunov function is selected as

$$V_{i,1} = \frac{1}{2} v_{i,1}^2. \tag{32}$$

With (32) and Definition 1, one has

$$\begin{aligned} LV_{i,1} \leq & v_{i,1}\Big(d_i(v_{i,2}+e_{i,2}+\alpha_{i,1}+\bar{f}_{i,1}(\bar{Z}_{i,1})-v_{i,1})-\frac{1}{2}v_{i,1}\sum_{j=1}^{N}a_{ij}-\sum_{j=1}^{N}a_{ij}e_{j,2}+c_{i,1}\varpi_{i,1}\Big) \\ & +\frac{1}{2}{d_i}^2 g_{i,1}^T g_{i,1}+\frac{1}{2}\sum_{j=1}^{N}a_{ij}g_{j,1}^T\sum_{j=1}^{N}a_{ij}g_{j,1}, \end{aligned}$$

where $\bar{f}_{i,1}(\bar{Z}_{i,1}) = f_{i,1}+v_{i,1}+\frac{1}{2d_i}\sum_{j=1}^{N}a_{ij}v_{i,1}-\frac{1}{d_i}\sum_{j=1}^{N}a_{ij}(\hat{x}_{j,2}+f_{j,1})-\frac{1}{d_i}\sum_{j=N+1}^{N+M}a_{ij}\dot{y}_{L,j}$. From Young's inequality, the equation can be rewritten as

$$\begin{aligned} LV_{i,1} \leq & v_{i,1}\Big(d_i\big(v_{i,2}+\alpha_{i,1}+\bar{f}_{i,1}(\bar{Z}_{i,1})-\frac{1}{2}v_{i,1}\big)+c_{i,1}\varpi_{i,1}\Big) && (33)\\ & +\frac{1}{2}d_i\|e_i\|^2+\frac{1}{2}\|e_i\|^2\sum_{j=1}^{N}a_{ij}+\frac{1}{2}{d_i}^2 g_{i,1}^T g_{i,1}+\frac{N}{2}\sum_{j=1}^{N}a_{ij}^2 g_{j,1}^T g_{j,1}. && (34) \end{aligned}$$

For the kth subsystem, $k = 2,\cdots,n_i-1$, along with (14) and (19), one has

$$v_{i,k} = z_{i,k}-\varpi_{i,k} = \hat{x}_{i,k}-\pi_{i,k}-\varpi_{i,k}. \tag{35}$$

Choose a Lyapunov function $V_{i,k} = \frac{1}{2}v_{i,k}^2$. From the observer (7) and (35), we have

$$LV_{i,k} \leq v_{i,k}\Big(v_{i,k+1}+\alpha_{i,k}+\bar{f}_{i,k}(\bar{Z}_{i,k})-\frac{1}{2}v_{i,k}+c_{i,k}\varpi_{i,k}+\varpi_{i,k-1}-\dot{\pi}_{i,k}\Big), \tag{36}$$

where $\bar{f}_{i,k}(\bar{Z}_{i,k}) = l_{i,k}(y_i-\hat{x}_{i,1})+\frac{1}{2}v_{i,k}$, and $l_{i,k}$ is a positive constant.

Finally, for the n_ith subsystem, one has $v_{i,n_i} = z_{i,n_i}-\varpi_{i,n_i} = \hat{x}_{i,n_i}-\pi_{i,n_i}-\varpi_{i,n_i}$, and select a Lyapunov function

$$V_{i,n_i} = \frac{1}{2}v_{i,n_i}^2. \tag{37}$$

Within the event-triggered interval $t \in [t_k, t_{k+1})$, one has $|\chi_i(t)-u_i(t)| \leq m_i$ from (25). There exists a time-varying parameter λ_i such that $\lambda_i(t_k) = 0$, $\lambda_i(t_{k+1}) = \pm 1$, $|\lambda_i(t)| \leq 1$. Then, $\forall t \in [t_k, t_{k+1})$, we rewrite the control strategy

$$\chi_i(t) = u_i(t)+\lambda_i(t)m_i. \tag{38}$$

With (7), (14), (24), (37), (38), and Lemma 2, we further get

$$LV_{i,n_i} \leq v_{i,n_i}\Big(\alpha_{i,n_i}+\bar{f}_{i,n_i}(\bar{Z}_{i,n_i})-\frac{1}{2}v_{i,n_i}+c_{i,n_i}\varpi_{i,n_i}+\varpi_{i,n_i-1}-\dot{\pi}_{i,n_i}\Big), \tag{39}$$

where $\bar{f}_{i,n_i}(\bar{Z}_{i,n_i}) = l_{i,n_i}(y_i-\hat{x}_{i,1})+\frac{1}{2}v_{i,n_i}+0.2785\epsilon_i$.

We choose the following Lyapunov function $V = V_0 + \sum_{i=1}^{N} \left\{ V_{i,0} + \frac{1}{2\gamma_i}\tilde{W}_i^2 \right\}$, where $V_{i,0} = \frac{1}{2}\sum_{k=1}^{n_i} V_{i,k}$, and $\tilde{W}_i = W_i^* - \hat{W}_i$. Then, with (34), (36), (39), $0.5d_i + 0.5 + c_i \leq c_{i,1}$, $0.5d_i + 1.5 + c_i \leq c_{i,2}$, $2 + c_i \leq c_{i,k}$, $k = 3, \cdots, n_i - 1$, $1 + c_i \leq c_{i,n_i}$, where $c_i > 0$, $i = 1, \cdots, N$, and Young's inequality $v_{i,k}\bar{f}_{i,k}(\bar{Z}_{i,k}) \leq \frac{1}{2}v_{i,k}^2 + \frac{1}{2r_{i,k}}v_{i,k}^2 W_i^* + \frac{1}{2}r_{i,k} + \frac{1}{2}\varepsilon_{i,k}^2$, one has

$$
\begin{aligned}
LV \leq & -\left[\xi - \frac{1}{2}\sum_{i=1}^{N} a_{ij} - \frac{1}{2}\sum_{i=1}^{N} d_i\right]\|e_i\|^2 + \sum_{i=1}^{N}\left[-\sum_{k=1}^{n_i} c_i v_{i,k}^2 - \frac{1}{2}\lambda_i \tilde{W}_i^2 + \frac{1}{2}d_i r_{i,1}\right. \\
& + \frac{1}{2}\sum_{k=2}^{n_i} r_{i,k} + \frac{1}{2}d_i\varepsilon_{i,1}^2 + \frac{1}{2}\sum_{k=2}^{n_i}\varepsilon_{i,k}^2 + \|K_i\|^2\|\varepsilon_i\|^2 + \|K_i\|^2 W_i^* + \frac{1}{2}\|K_i\|^2 \\
& \left. + \frac{1}{2}d_i^2 g_{i,1}^T g_{i,1} + \frac{N}{2}\sum_{j=1}^{N} a_{ij}^2 g_{j,1}^T g_{j,1} + \frac{1}{2}\left|g_{i,1}^T g_{i,1}\right|^2 + \frac{1}{2}\lambda_i W_i^{*2}\right],
\end{aligned}
$$

where c_i is a design parameter. (40) can be rewritten as

$$
LV \leq -Q_1 V + Q_2, \tag{40}
$$

where $Q_1 = \min\left\{\xi - \frac{1}{2}\sum_{i=1}^{N}(a_{ij} + d_i), \min\{2c_i\}, \min\{\gamma_j\lambda_j\}\right\}$, $i = 1, \cdots, N$, $j = 1, \cdots, n_i$, $Q_2 = \sum_{i=1}^{N}\left\{\frac{1}{2}d_i\varepsilon_{i,1}^2 + \frac{1}{2}\sum_{k=2}^{n_i}\varepsilon_{i,k}^2 + \|K_i\|^2\|\varepsilon_i\|^2 + \|K_i\|^2 W_i^* + \frac{1}{2}\|K_i\|^2 + \frac{1}{2}d_i r_{i,1} + \frac{1}{2}\sum_{k=2}^{n_i} r_{i,k} + \frac{1}{2}d_i^2 g_{i,1}^T g_{i,1} + \frac{N}{2}\sum_{j=1}^{N} a_{ij}^2 g_{j,1}^T g_{j,1} + \frac{1}{2}|g_{i,1}^T g_{i,1}|^2 + \frac{1}{2}\lambda_i W_i^{*2}\right\}$.

Now, we prove that the Zeno behavior will be avoided, i.e., time interval $t^* > 0$ between any two trigger events that satisfies $\forall k \in z^+$, $t_{k+1} - t_k \geq t^*$.

From $e_i(t) = \chi_i(t) - u_i(t)$, $\forall t \in [t_k, t_{k+1})$, the time derivative of the trigger errors e_i is given, $\frac{\mathrm{d}}{\mathrm{d}t}|e_i| = \frac{\mathrm{d}}{\mathrm{d}t}(e_i * e_i)^{\frac{1}{2}} = \mathrm{sign}(e_i)\dot{e}_i \leq |\dot{\chi}_i|$. From (24), $\dot{\chi}_i$ is obtained $\dot{\chi}_i(t) = \dot{\alpha}_{i,n_i} - \frac{\bar{m}_i^2 \dot{\nu}_{i,n_i}}{\varepsilon_i \cosh^2\left(\frac{\bar{m}_i \nu_{i,n_i}}{\bar{\varepsilon}_i}\right)}$. Due to the fact that $f_i(\cdot)$ is continuously differentiable, $\dot{\chi}_i$ is a continuous function. x_i are global bounded, and there exists a constant $\kappa > 0$, such that $|\dot{\chi}_i| \leq \kappa$. From $e_i(t_k) = 0$ and $\lim_{t \to t_{k+1}} e_i(t) = m$, we can find a constant t^* satisfying $t^* \geq \frac{m}{\kappa}$, $t_{k+1} - t_k \geq t^*$. Thus, there exists Zeno-free behavior.

Integrating equation (40) yields

$$
0 \leq \mathrm{E}V(t) \leq \frac{Q_2}{Q_1}(1 - \exp(-Q_1 t)) + \mathrm{E}V(x(0), 0). \tag{41}
$$

Then, one has

$$
\mathrm{E}V(t) \leq \frac{Q_2}{Q_1}, \forall t > T_1, \tag{42}
$$

where $T_1 = \max\left\{0, \frac{1}{\mathcal{Q}_1}\ln\left(\frac{\mathcal{Q}_1}{\mathcal{Q}_2}\mathrm{E}V(0)\right)\right\}$. Then, from (42), the bounds of errors in the close-loop system are obtained

$$\sum_{i=1}^{N}\sum_{k=1}^{n_i}\mathrm{E}\left[\left|v_{i,k}\right|^2\right] \le 2\frac{\mathcal{Q}_2}{\mathcal{Q}_1}, \forall t > T_1, \tag{43}$$

$$\sum_{i=1}^{N}\frac{1}{\gamma_i}\mathrm{E}\left[\left|\tilde{W}_i\right|^2\right] \le 2\frac{\mathcal{Q}_2}{\mathcal{Q}_1}, \forall t > T_1, \tag{44}$$

and

$$\sum_{i=1}^{N}\lambda_{\min}(K_i)\mathrm{E}\left[\|e_i\|^2\right] \le \frac{\mathcal{Q}_2}{\mathcal{Q}_1}, \forall t > T_1. \tag{45}$$

The compensation signal ϖ_i eventually converges to a compact set $\phi_{\bar{\varpi}_{i,1}} = \left\{\bar{\varpi}_{i,1} = [|\bar{\varpi}_{1,1}|, \cdots, |\bar{\varpi}_{N,1}|]^T \in \mathbb{R}^N \middle| \|\bar{\varpi}_{i,1}\| \le \sum_{i=1}^{N}\frac{\xi_i}{\sigma_i}\right\}$, where $|\pi_{i,k+1} - \alpha_{i,k}| \le \xi_i$, and $\sigma_i = \min(c_{i,k})$. From (43), it can be seen that $v_{i,1}$ is bounded, and the bound of the error $z_{i,1}$ is $\phi_{\bar{z}_{i,1}} = \left\{\bar{z}_{i,1} = [|\bar{z}_{1,1}|, \cdots, |\bar{z}_{N,1}|]^T \in \mathbb{R}^N \middle| \|\bar{z}_{i,1}\| \le \sum_{i=1}^{N}\frac{\xi_i}{\sigma_i} + \sqrt{\frac{\mathcal{Q}_2}{\mathcal{Q}_1}}, \forall t > T_1\right\}$. From $\bar{z}_1 = \bar{\ell}_1 y + \bar{\ell}_2 y_L$, one has $\lim_{t\to\infty}\left\|y + \bar{\ell}_1^{-1}\bar{\ell}_2 y_L\right\| \le \frac{\|\bar{z}_1\|}{\|\bar{\ell}_1\|_F}$. With $y_d = -\bar{\ell}_1^{-1}\bar{\ell}_2 y_L$, the containment error satisfies $\|\mathcal{p}\| = \|y - y_d\| \le \frac{\|\bar{z}_1\|}{\|\bar{\ell}_1\|_F}$. Thus, as $t > T_1$, it reveals that

$$\phi_{\mathcal{p}} = \left\{\mathcal{p} \in \mathbb{R}^N \middle| \mathrm{E}\left[\|\mathcal{p}\|\right] \le \sum_{i=1}^{N}\frac{\xi_i}{\|\bar{\ell}_1\|_F\sigma_i} + \frac{1}{\|\bar{\ell}_1\|_F}\sqrt{\frac{\mathcal{Q}_2}{Q_1}} \le \delta\right\}. \tag{46}$$

The proof is completed.

Remark 5 In (10), the eigenvalues of the matrices K_i, Q_i and $M_i^T M_i$ are recorded as $0 \le \breve{\alpha}_1 \le \breve{\alpha}_2 \le \cdots \le \breve{\alpha}_n$, $0 \le \breve{\beta}_1 \le \breve{\beta}_2 \le \cdots \le \breve{\beta}_n$, and $0 \le \breve{\sigma}_1 \le \breve{\sigma}_2 \cdots \le \breve{\sigma}_n$, respectively. Here, $\breve{\alpha}_1 \ge \frac{\breve{\beta}_1}{2\sqrt{\breve{\sigma}_n}}$ and $\breve{\alpha}_n \ge \frac{\breve{\beta}_n}{2\sqrt{\breve{\sigma}_n}}$ hold. As $\breve{\sigma}_n$ increases and $\breve{\beta}_1$, $\breve{\beta}_n$ remains unchanged, the bounds of the maximum and minimum eigenvalues of K_i decrease. Accordingly, the value of $\mathcal{Q}_2$ tends to small resulting small size of the compact of the containment error.

Remark 6 To show the advantage of the proposed strategy, we calculate the bound of containment error via traditional DSC approach. $\mathrm{E}\left[\|\mathcal{p}\|\right] \le \sum_{i=1}^{N}\frac{\xi_i}{\|\bar{\ell}_1\|_F\sigma_i} + \frac{1}{\|\bar{\ell}_1\|_F}\sqrt{\frac{\mathcal{Q}_2+C}{Q_1}}$. A term $C = \frac{1}{2}||z_{i,1}||^2||s_{i,1}||^2 + \frac{1}{2}\sum_{k=2}^{n_i-1}||z_{i,k}||^2||s_{i,k}||^2$ is additional from filter errors in the traditional DSC, and it is not zero in most cases. In our paper, the bound of containment error converges to a smaller size of compact set without the additional term due to the compensator. Compensating signals leads to a small size of the compact set for closed-loop errors in an MAS. The advantage of this command filter is generating command signals to remove the influence from error vectors. In this manner, it indicates that the containment performance is improved in our paper.

4 Simulation Example

4.1 A Team of Robots

The dynamics of each robot are

$$\begin{cases} M_i\ddot{x}_i + B_{x_i}\dot{x}_i = f_{x_i} - k_{xd_i}\dot{x}_i, \\ M_i\ddot{y}_i + B_{y_i}\dot{y}_i = f_{y_i} - k_{yd_i}\dot{y}_i, \end{cases} \tag{47}$$

where M_i is the mass of a robot, B_{x_i} and B_{y_i} represent damper coefficients, and k_{xd_i} and k_{yd_i} are coefficients controlling the transient response.

Let $x_{i,1,1} = x_i$, $x_{i,1,2} = \dot{x}_i$, $u_{i,1} = u_{x_i}$, $x_{i,2,1} = y_i$, $x_{i,2,2} = \dot{y}_i$, and $u_{i,2} = u_{y_i}$. (47) is rewritten as

$$\begin{cases} \mathrm{d}x_{i,1,1} = f_{i,1,1}(x_{i,1,1}, x_{i,1,2})\mathrm{d}t + \sigma_{i,1,1}x_{i,1,1}\mathrm{d}w_i, \\ \mathrm{d}x_{i,1,2} = f_{i,1,2}(\bar{x}_{i,1,2}, u_{i,1})\mathrm{d}t + \sigma_{i,1,2}x_{i,1,2}\mathrm{d}w_i, \\ \mathrm{d}x_{i,2,1} = f_{i,2,1}(x_{i,2,1}, x_{i,2,2})\mathrm{d}t + \sigma_{i,2,1}x_{i,2,1}\mathrm{d}w_i, \\ \mathrm{d}x_{i,2,2} = f_{i,2,2}(\bar{x}_{i,2,2}, u_{i,2})\mathrm{d}t + \sigma_{i,2,2}x_{i,2,2}\mathrm{d}w_i, \\ y_{i,1} = x_{i,1,1}, \\ y_{i,2} = x_{i,2,1}, \end{cases} \tag{48}$$

where $\sigma_{i,1,1}, \sigma_{i,1,2}, \sigma_{i,2,1}, \sigma_{i,2,2}$ are design parameters, $y_{i,1}, y_{i,2}$ are coordinates of the followers, $\bar{x}_{i,1,2} = [x_{i,1,1}, x_{i,1,2}]^T$, $\bar{x}_{i,2,2} = [x_{i,2,1}, x_{i,2,2}]^T$, $u_{i,1}$ and $u_{i,2}$ are control inputs,

$$\begin{cases} f_{i,1,1}(x_{i,1,1}, x_{i,1,2}) = x_{i,1,2}, \\ f_{i,1,2}(\bar{x}_{i,1,2}, u_{i,1}) = [-(B_{x_i} + k_{xd_i})x_{i,1,2} + u_{i,1}]/M_i, \\ f_{i,2,1}(x_{i,2,1}, x_{i,2,2}) = x_{i,2,2}, \\ f_{i,2,2}(\bar{x}_{i,2,2}, u_{i,2}) = [-(B_{y_i} + k_{yd_i})y_{i,1,2} + u_{i,2}]/M_i, \end{cases} \tag{49}$$

$i = 1, 2, 3$, $M_i = 1$, $B_{x_i} = B_{y_i} = 1$, and $k_{xd_i} = k_{yd_i} = 9$.

The leaders' trajectories are chosen as $y_{L,4} = [t, \sin(t/2)]^T$, $y_{L,5} = [0.8t - 0.5, \sin(t/2)]^T$, $y_{L,6} = [0.8t - 0.5, \sin(t/3) - 0.5]^T$, $y_{L,7} = [t, \sin(t/3) - 0.5]^T$. Initial states are $x_{1,1}(0) = [0.1, 0]^T$, $x_{2,1}(0) = [0.1, 0]^T$, $x_{3,1}(0) = [0.1, 0]^T$, $x_{1,2}(0) = [0.2, 0]^T$, $x_{2,2}(0) = [0.05, 0]^T$, $x_{3,2}(0) = [0.3, 0]^T$. The triggered threshold of each agent is 0.5. The parameters of the first-order filter, observer, adaptive law and control law are selected as $l_{i,1,1} = l_{i,2,1} = 10$, $l_{i,1,2} = l_{i,2,2} = 150$, $\sigma_{i,1,2} = \sigma_{i,2,2} = 0.1$, $c_{1,1,1} = 10$, $c_{1,1,2} = c_{2,1,2} = c_{1,2,2} = c_{3,2,2} = 5$, $c_{2,1,1} = c_{1,2,1} = c_{3,2,1} = 15$, $c_{3,1,1} = 20$, $c_{3,1,2} = 25$, $c_{2,2,1} = 45$, $c_{2,2,2} = 35$, $r_{i,1} = r_{i,2} = 15$, $\gamma_{i,1} = \gamma_{i,2} = 1$, $\lambda_i = 1$, where $i = 1, 2, 3$.

The directed network topology for the MAS is shown in Fig.2, and the simulation results are shown in Fig.2–Fig.7. Table.1 records the followers' event-triggered number in the Y-axis direction. It can be seen that, with the event-triggered mechanism, the triggered number is much less than that without time-triggered one. The containment performance is shown in Fig.3, and, at three different times $t = 4s, 12s, 25s$, it can be seen that the output of the followers can be well maintained in the convex hull formed by the leaders. Fig.4 and Fig.5 show the event-triggered instants of three agents at X- and Y-axis directions respectively. In Fig.6 and Fig.7, the adaptive parameters of three followers are presented, respectively.

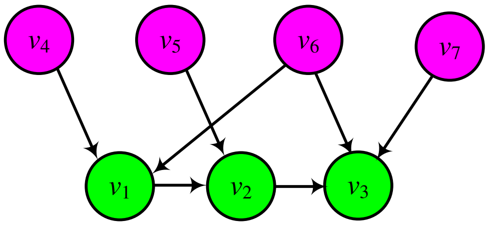

Fig. 2 Communication topology in Example A.

Table 1 Comparison of triggered number in Y-axis direction.

Strategy	Follower	Triggered Number
Event-triggered Strategy	1	215
	2	549
	3	255
Time-triggered Strategy	1&2&3	30005

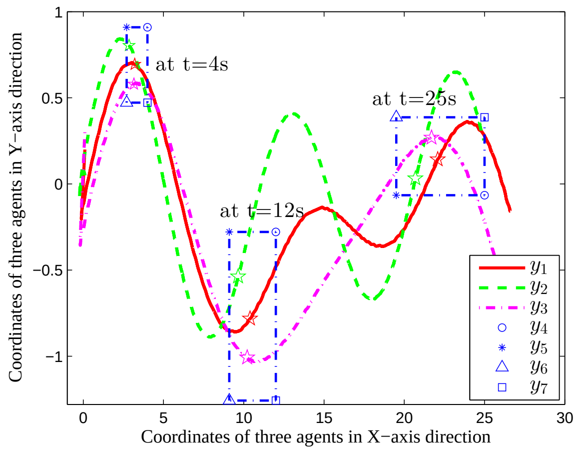

Fig. 3 Outputs of the three followers and the containment range at different instants $t = 4\text{s}, 12\text{s}, 25\text{s}$.

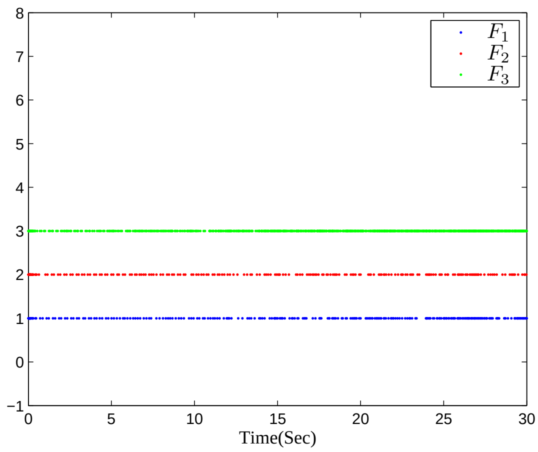

Fig. 4 Triggered instant at X-axis direction.

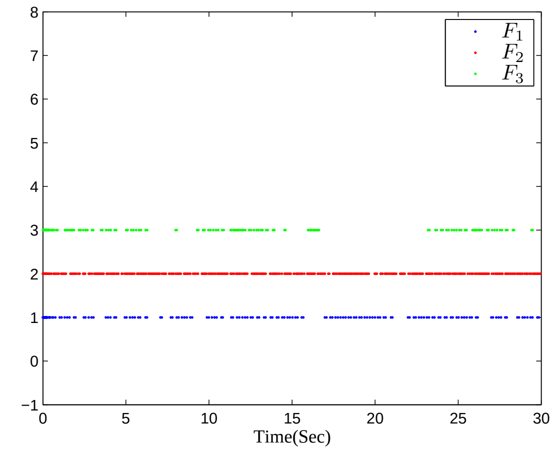

Fig. 5 Triggered instant at Y-axis direction.

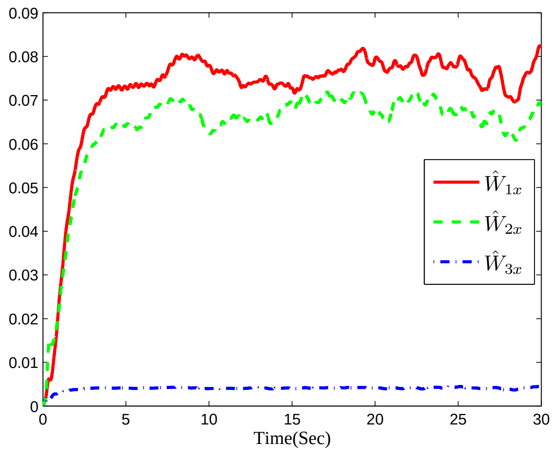

Fig. 6 The adaptive law at X-axis direction.

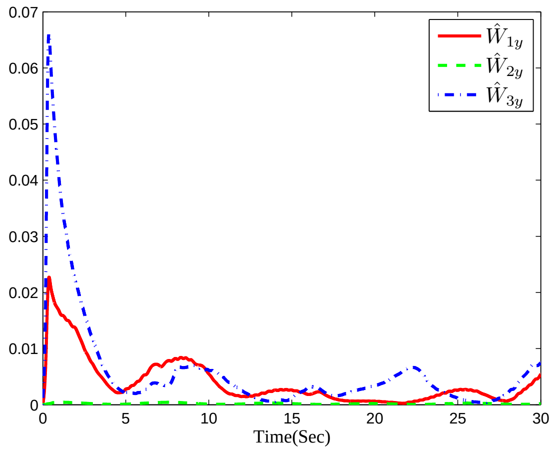

Fig. 7 The adaptive law at Y-axis direction.

4.2 A Numerical Example

Consider the following dynamics of the ith follower

$$\begin{cases} \mathrm{d}x_{i,1} = (x_{i,2} + x_{i,1}(1 - \sin(x_{i,1}))^2)\mathrm{d}t + 0.5\cos(x_{i,1})\mathrm{d}w_i, \\ \mathrm{d}x_{i,2} = (u_i - 3.5x_{i,2} + x_{i,1}x_{i,2}^2)\mathrm{d}t + 0.1x_{i,1}\sin(2x_{i,1}x_{i,2})\mathrm{d}w_i, \\ y_i = x_{i,1}, \end{cases} \tag{50}$$

where $i = 1, 2, 3, 4$, $x_{i,1}, x_{i,2}$ are state variables, and y_i is follower's output. A communication topology is shown in Fig.8. Two leaders' outputs are selected as $y_{L,4} = \sin(t)$, $y_{L,5} = 0.2(\sin(t)+\sin(0.5t))$. Parameters are chosen as $l_{i,1} = 10$, $l_{i,2} = 150$, $\sigma_{i,2} = 0.1$, $c_{1,1} = 35$, $c_{1,2} = c_{22} = 25$, $c_{21} = 45$, $c_{31} = 20$, $c_{32} = 35$, $r_{i,1} = r_{i,2} = 15$, $\gamma_{i,1} = \gamma_{i,2} = 1$, $\lambda_i = 1$, where $i = 1, 2, 3, 4$.

The simulation results are given in Fig.9–Fig.21. The red and orange lines in Fig.9 represent the outputs of the two leaders, respectively. It indicates that the control strategy governs the MAS with satisfied performance. It can be seen that the observers work well from Fig.11–Fig.13. Fig.14 represents the adaptive laws of three agents. Fig.15 shows the event-triggered instant of three agents. In Fig.16–Fig.18, the relationship between the trigger error and the threshold values is given, and the triggered thresholds are 0.2, 0.8 and 1.2, respectively. The bar chart shows the corresponding number of event-triggered in Fig.19. It can be concluded that, as the threshold scales down, this mechanism results in more triggered number. Fig.20 is the comparison of containment error with or without compensation signal. It can be seen that the containment error of the follower with compensator is smaller than the value without compensation signal. Table.2 lists three cases of parameters $c_{1,1}$ and $c_{1,2}$, and Fig.21 shows the comparison of containment error in the three cases. With the increase of parameters $c_{1,1}$ and $c_{1,2}$, $\mathcal{Q}_1$ in (46), the size of the compact of the containment error becomes smaller.

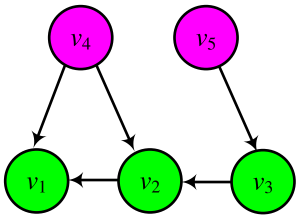

Fig. 8 Communication topology in Example B.

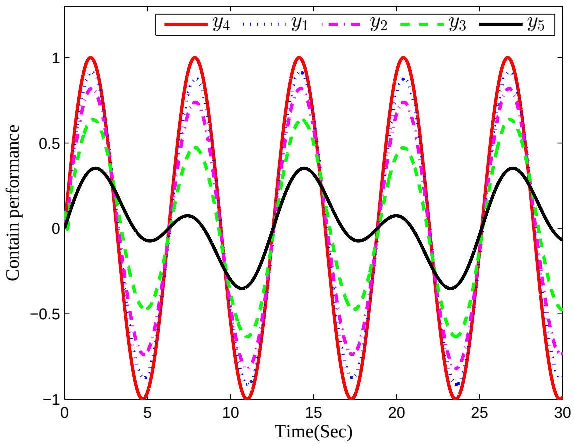

Fig. 9 Output of three followers and two leaders.

Table 2 Cases of $c_{1,1}$ and $c_{1,2}$.

Case	Parameters
1	$c_{1,1} = 25, c_{1,2} = 5$
2	$c_{1,1} = 45, c_{1,2} = 15$
3	$c_{1,1} = 65, c_{1,2} = 25$

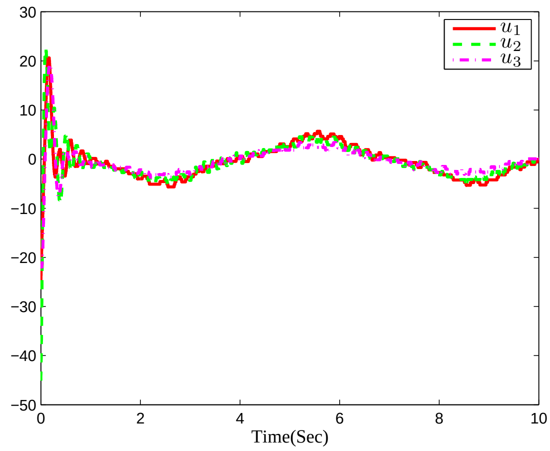

Fig. 10 Control inputs u_i, $i = 1, 2, 3$.

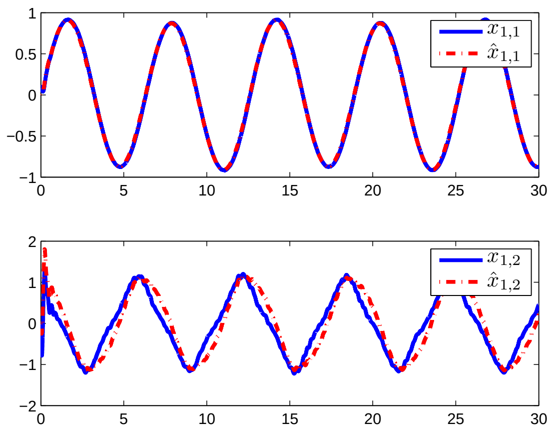

Fig. 11 States and their estimations for Follower 1.

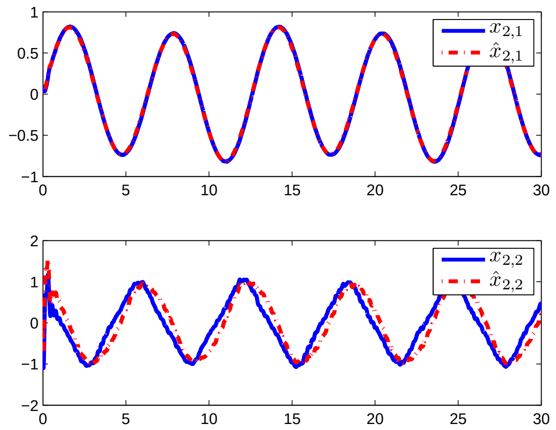

Fig. 12 States and their estimations for Follower 2.

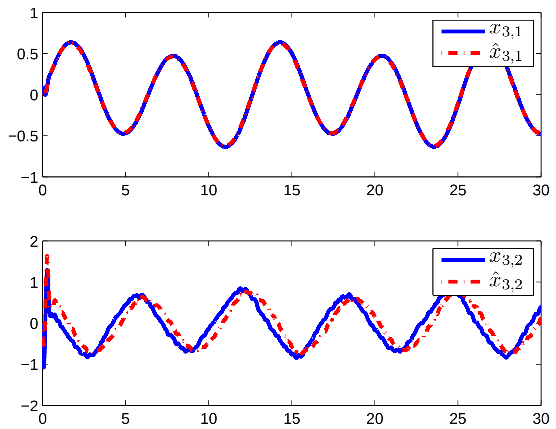

Fig. 13 States and their estimations for Follower 3.

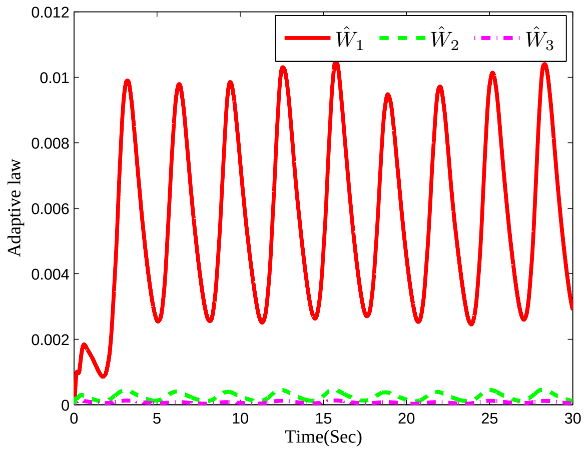

Fig. 14 Adaptive parameters.

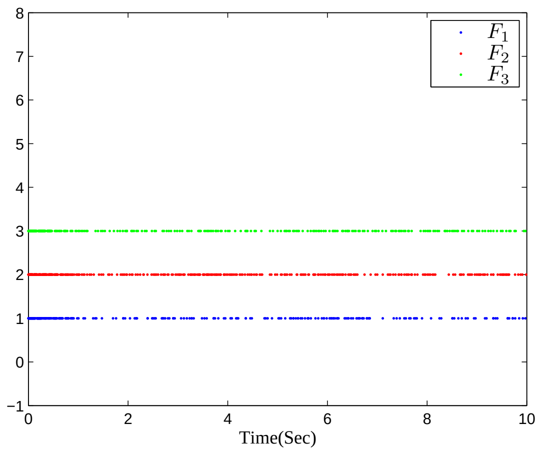

Fig. 15 Triggered instants of the three followers.

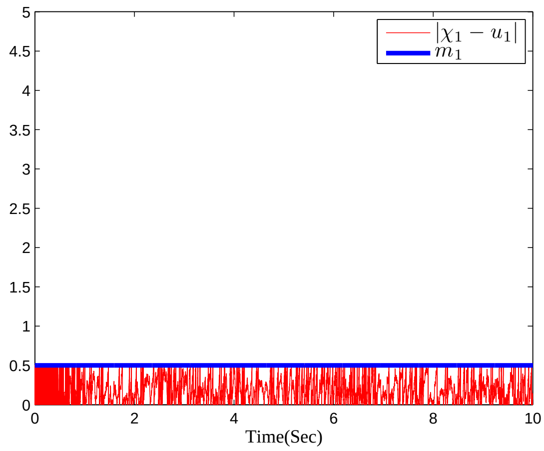

Fig. 16 Curves of the triggered error and the threshold value as m_2 is 0.5.

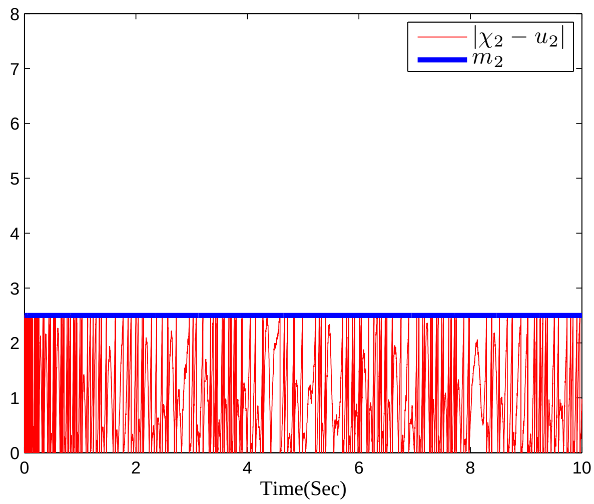

Fig. 17 Curves of the triggered error and the threshold value as m_2 is 2.5.

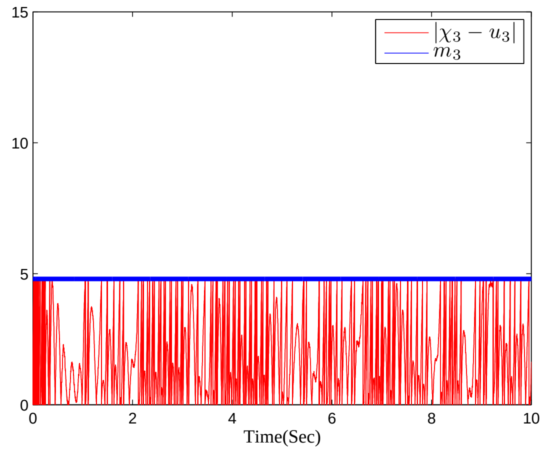

Fig. 18 Curves of the triggered error and the threshold value as m_2 is 4.8.

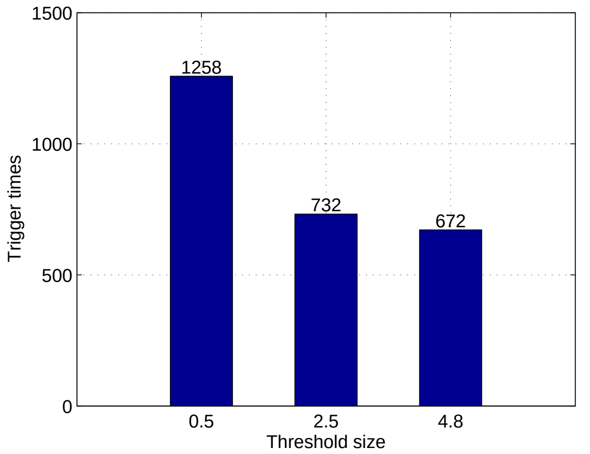

Fig. 19 Comparison of triggered number with different thresholds.

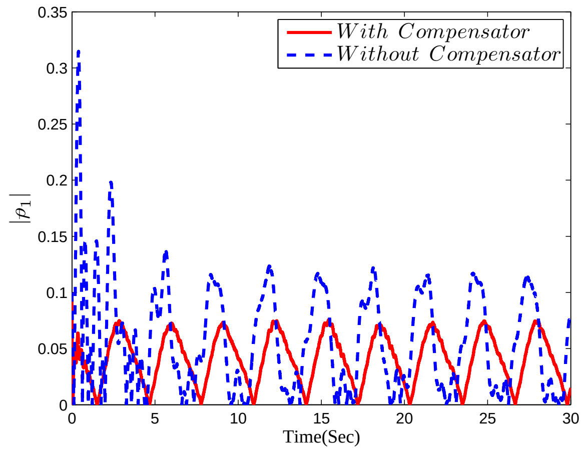

Fig. 20 Performance comparison of 1st follower with/without compensator.

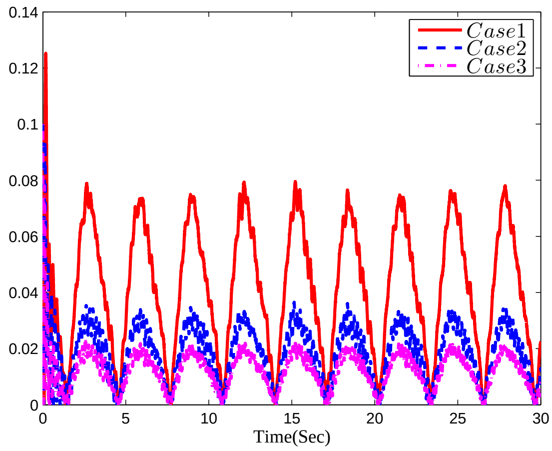

Fig. 21 Containment error with different parameters c_1.

5 Conclusion

A containment control strategy is proposed for a class of stochastic nonlinear MASs based on event-triggered with fixed threshold. For unknown states of the MAS, state observers are designed to reconstruct internal information. DSC technology is used to improve the strategy of backstepping, and NNs estimate unmodeled dynamics. For the purpose of reducing the resource occupation of the network channel, an event-triggered strategy with fixed threshold is designed to reduce the number of data transmission bits over the network channel. A Lyapunov function is developed to prove that all signals in the MASs are bounded in probability. The proposed control strategy is on the basis of fixed thresholds, where the threshold holds constant regardless of the magnitude of the control signal. This indicates that the result established in this paper is still far from a full investigation on the relation between triggered number and relative threshold as well as stochastic dynamics in individual followers, which is, therefore, worth our further research in the future.

Declaration of interest

The authors declare no conflict interest.

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References

1. Sung Jin Yoo. Distributed adaptive containment control of uncertain nonlinear multi-agent systems in strict-feedback form. *Automatica*, 49(7):2145–2153, 2013.
2. Qikun Shen and Peng Shi. Distributed command filtered backstepping consensus tracking control of nonlinear multiple-agent systems in strict-feedback form. *Automatica*, 53:120–124, 2015.
3. Yuanyuan Zhang, Renfu Li, and Xiaoming Huo. Stochastic consensus of discrete-time second-order multi-agent systems with measurement noises and time delays. *Journal of the Franklin Institute*, 355(5):2791–2807, 2018.
4. Engang Tian, Dong Yue, and Chen Peng. Reliable control for networked control systems with probabilistic actuator fault and random delays. *Journal of the Franklin Institute*, 347(10):1907–1926, 2010.
5. Zhaoping Du, Dong Yue, and Songlin Hu. H-infinity stabilization for singular networked cascade control systems with state delay and disturbance. *IEEE Transactions on Industrial Informatics*, 10(2):882–894, 2014.

6. Engang Tian, Dong Yue, and Zhou Gu. Robust H_∞ control for nonlinear systems over network: A piecewise analysis method. *Fuzzy Sets and Systems*, 161(21):2731–2745, 2010.
7. Fang Wang, Bing Chen, Chong Lin, and Xuehua Li. Distributed adaptive neural control for stochastic nonlinear multiagent systems. *IEEE Transactions on Cybernetics*, 47(7):1795–1803, 2016.
8. Dong Wang, Ning Zhang, Jianliang Wang, and Wei Wang. Cooperative containment control of multiagent systems based on follower observers with time delay. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 47(1):13–23, 2016.
9. Giuseppe Notarstefano, Magnus Egerstedt, and M Haque. Containment in leader–follower networks with switching communication topologies. *Automatica*, 47(5):1035–1040, 2011.
10. Yongcan Cao, Ren Wei, and Egerstedt Magnus. Distributed containment control with multiple stationary or dynamic leaders in fixed and switching directed networks. *Automatica*, 48(8):1586–1597, 2012.
11. Zhongkui Li, Wei Ren, Xiangdong Liu, and Mengyin Fu. Distributed containment control of multi-agent systems with general linear dynamics in the presence of multiple leaders. *International Journal of Robust and Nonlinear Control*, 23(5):534–547, 2013.
12. Zhouhua Peng, Dan Wang, Yang Shi, Hao Wang, and Wei Wang. Containment control of networked autonomous underwater vehicles with model uncertainty and ocean disturbances guided by multiple leaders. *Information Sciences*, 316:163–179, 2015.
13. Wei Wang, Dan Wang, Zhouhua Peng, and Hao Wang. Cooperative adaptive fuzzy output feedback control for synchronization of nonlinear multiagent systems in the presence of input saturation. *Asian Journal of Control*, 18(2):619–630, 2016.
14. Amir Ajorlou, Amir G Aghdam, and Ali Jadbabaie. A connectivity preserving containment control strategy for unicycles with static leaders. In *2012 American Control Conference (ACC)*, pages 4186–4191. IEEE, 2012.
15. Bocheng Yu, Xiwang Dong, Zongying Shi, and Yisheng Zhong. Formation-containment control for unmanned aerial vehicle swarm system. In *Proceedings of the 33rd Chinese Control Conference*, pages 1517–1523. IEEE, 2014.
16. Yingnan Pan, Peihao Du, Hong Xue, and Hak-Keung Lam. Singularity-free fixed-time fuzzy control for robotic systems with user-defined performance. *IEEE Transactions on Fuzzy Systems*, 2020. doi:10.1109/TFUZZ.2020.2999746.
17. Hongyi Li, Ying Wu, and Mou Chen. Adaptive fault-tolerant tracking control for discrete-time multiagent systems via reinforcement learning algorithm. *IEEE Transactions on Cybernetics*, 51(3):1163–1174, 2021.
18. Xiwang Dong, Qingdong Li, Zhang Ren, and Yisheng Zhong. Formation-containment control for high-order linear time-invariant multi-agent systems with time delays. *Journal of the Franklin Institute*, 352(9):3564–3584, 2015.

19. Yafeng Li, Changchun Hua, Shuangshuang Wu, and Xinping Guan. Output feedback distributed containment control for high-order nonlinear multiagent systems. *IEEE Transactions on Cybernetics*, 47(8):2032–2043, 2017.
20. Qing Wang, Jian Liu, and Yao Yu. Observer-based containment for a class of nonlinear multi-agent systems with time-delayed protocols. In *2016 31st Youth Academic Annual Conference of Chinese Association of Automation (YAC)*, pages 241–246. IEEE, 2016.
21. Wei Wang and Shaocheng Tong. Observer-based adaptive fuzzy containment control for multiple uncertain nonlinear systems. *IEEE Transactions on Fuzzy Systems*, 27(11):2079–2089, 2019.
22. Jianhui Wang, Zhi Liu, CL Philip Chen, and Yun Zhang. Fuzzy adaptive compensation control of uncertain stochastic nonlinear systems with actuator failures and input hysteresis. *IEEE Transactions on Cybernetics*, 49(1):2–13, 2017.
23. Xinfeng Shao and Dan Ye. Fuzzy adaptive event-triggered secure control for stochastic nonlinear high-order mass subject to dos attacks and actuator faults. *IEEE Transactions on Fuzzy Systems*, 2020. doi: 10.1109/TFUZZ.2020.3028657.
24. Xiuxia Yin, Dong Yue, and Songlin Hu. Adaptive periodic event-triggered consensus for multi-agent systems subject to input saturation. *International Journal of Control*, 89(4):653–667, 2016.
25. Tian-Yu Zhang, Dan Ye, and Xingang Zhao. Completely event-triggered consensus for multiagent systems with directed switching topologies. *IEEE Transactions on Cybernetics*, 2021. doi: 10.1109/TCYB.2021.3052988.
26. Kaien Liu, Zhijian Ji, Guangming Xie, and Ruiping Xu. Event-based broadcasting containment control for multi-agent systems under directed topology. *International Journal of Control*, 89(11):2360–2370, 2016.
27. Songlin Hu, Dong Yue, Chen Peng, Xiangpeng Xie, and Xiuxia Yin. Event-triggered controller design of nonlinear discrete-time networked control systems in T-S fuzzy model. *Applied Soft Computing*, 30:400–411, 2015.
28. Zhou Gu, Dong Yue, and Engang Tian. On designing of an adaptive event-triggered communication scheme for nonlinear networked interconnected control systems. *Information Sciences*, 422:257–270, 2018.
29. Changjiang Xi and Jiuxiang Dong. Event-triggered adaptive fuzzy distributed tracking control for uncertain nonlinear multi-agent systems. *Fuzzy Sets and Systems*, 402:35–50, 2021.
30. Zhaojun Tang and Chuandong Li. Distributed event-triggered containment control for discrete-time multi-agent systems. *International Journal of Control, Automation and Systems*, 16(6):2727–2732, 2018.
31. Wenfeng Hu, Lu Liu, and Gang Feng. Consensus of linear multi-agent systems by distributed event-triggered strategy. *IEEE Transactions on Cybernetics*, 46(1):148–157, 2015.
32. Zhengran Cao, Chuandong Li, Xin Wang, and Tingwen Huang. Finite-time consensus of linear multi-agent system via distributed event-triggered strategy. *Journal of the Franklin Institute*, 355(3):1338–1350, 2018.

33. Hongyi Li, Ziran Chen, Ligang Wu, Hak-Keung Lam, and Haiping Du. Event-triggered fault detection of nonlinear networked systems. *IEEE Transactions on Cybernetics*, 47(4):1041–1052, 2016.
34. Changjiang Xi and Jiuxiang Dong. Adaptive fuzzy guaranteed performance control for uncertain nonlinear systems with event-triggered input. *Applied Mathematics and Computation*, 363:124604, 2019.
35. Liqi Wang, Jiuxiang Dong, and Changjiang Xi. Event-triggered adaptive consensus for fuzzy output-constrained multi-agent systems with observers. *Journal of the Franklin Institute*, 357(1):82–105, 2020.
36. Hongjing Liang, Guangliang Liu, Huaguang Zhang, and Tingwen Huang. Neural-network-based event-triggered adaptive control of nonaffine nonlinear multiagent systems with dynamic uncertainties. *IEEE Transactions on Neural Networks and Learning Systems*, 32(5):2239–2250, 2021.
37. Pingsong Ming, Jianchang Liu, Shubin Tan, Gang Wang, Liangliang Shang, and Chunying Jia. Consensus stabilization of stochastic multi-agent system with markovian switching topologies and stochastic communication noise. *Journal of the Franklin Institute*, 352(9):3684–3700, 2015.
38. Zhou Gu, Peng Shi, and Dong Yue. An adaptive event-triggering scheme for networked interconnected control system with stochastic uncertainty. *International Journal of Robust and Nonlinear Control*, 27(2):236–251, 2016.
39. Fei Gao, Weisheng Chen, Zhiwu Li, Jing Li, and Bin Xu. Neural network-based distributed cooperative learning control for multiagent systems via event-triggered communication. *IEEE Transactions on Neural Networks and Learning Systems*, 31(2):407–419, 2019.
40. Qi Zhou, Wei Wang, Hui Ma, and Hongyi Li. Event-triggered fuzzy adaptive containment control for nonlinear multiagent systems with unknown bouccwen hysteresis input. *IEEE Transactions on Fuzzy Systems*, 29(4):731–741, 2021.
41. Yaxin Huang and Yungang Liu. Practical tracking via adaptive event-triggered feedback for uncertain nonlinear systems. *IEEE Transactions on Automatic Control*, 64(9):3920–3927, 2019.
42. Wei Wang and Shaocheng Tong. Adaptive fuzzy containment control of nonlinear strict-feedback systems with full state constraints. *IEEE Transactions on Fuzzy Systems*, 27(10):2024–2038, 2019.
43. Huanqing Wang, Bing Chen, Xiaoping Liu, Kefu Liu, and Chong Lin. Adaptive neural tracking control for stochastic nonlinear strict-feedback systems with unknown input saturation. *Information Sciences*, 269:300–315, 2014.
44. Shaobao Li, Meng Joo Er, and Jie Zhang. Distributed adaptive fuzzy control for output consensus of heterogeneous stochastic nonlinear multiagent systems. *IEEE Transactions on Fuzzy Systems*, 26(3):1138–1152, 2018.
45. Hai-Bo Ji and Hong-Sheng Xi. Adaptive output-feedback tracking of stochastic nonlinear systems. *IEEE Transactions on Automatic Control*, 51(2):355–360, 2006.

46. Shaocheng Tong, Yue Li, Yongming Li, and Yanjun Liu. Observer-based adaptive fuzzy backstepping control for a class of stochastic nonlinear strict-feedback systems. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 41(6):1693–1704, 2011.
47. Marios M Polycarpou and Petros A Ioannou. A robust adaptive nonlinear control design. In *1993 American Control Conference*, pages 1365–1369. IEEE, 1993.
48. Ziyang Meng, Wei Ren, and Zheng You. Distributed finite-time attitude containment control for multiple rigid bodies. *Automatica*, 46(12):2092–2099, 2010.
49. Songlin Hu and Dong Yue. $\mathcal{L}_2$-gain analysis of event-triggered networked control systems: a discontinuous lyapunov functional approach. *International Journal of Robust and Nonlinear Control*, 23(11):1277–1300, 2012.
50. Songlin Hu, Dong Yue, Xiuxia Yin, Xiangpeng Xie, and Yong Ma. Adaptive even-triggered control for nonlinear discrete-time systems. *International Journal of Robust and Nonlinear Control*, 26(18):4104–4125, 2016.
51. Chen Peng, Qing-Long Han, and Dong Yue. Communication-delay-distribution-dependent decentralized control for large-scale systems with ip-based communication networks. *Journal of the Franklin Institute*, 21(3):820–830, 2013.
52. Yang Yang, Dong Yue, and Yusheng Xue. Decentralized adaptive neural output feedback control of a class of large-scale time-delay systems with input saturation. *Journal of the Franklin Institute*, 352(5):2129–2151, 2015.
53. Chen Peng, Dong Yue, Engang Tian, and Zhou Gu. Observer-based fault detection for networked control systems with network quality of services. *Applied Mathematical Modelling*, 34(6):1653–1661, 2010.