Extending Yackel and Cobb's sociomathematical norms to ill-structured problems in an inquiry-based classroom

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The aim of this paper is to extend and adapt Yackel and Cobb's (JRME, 1996) identification of sociomathematical norms in mathematical inquiry to problems that are ill-structured. The background theory influenced the design and local theory development of the research. This paper uses excerpts from an upper primary classroom to address the ill-structured mathematical inquiry question, Which bubble gum is the best? *Two norms are illustrated: (1) mathematising the ambiguity in an inquiry question, and (2) using the inquiry question to check progress towards a solution. Children demonstrated productive social norms and emergence of the sociomathematical inquiry norm of mathematising, but using the inquiry question was less prevalent. In both cases, children found it challenging to productively coordinate their everyday (relevant) and mathematical knowledge.*

Keywords: Sociomathematical norms; mathematical inquiry; mathematising; knowledge building

Problem statement and literature

Yackel and Cobb (1996; Yackel, Cobb, & Wood, 1991) explicate social and sociomathematical norms as central to learning mathematics in a classroom that practices inquiry. Social norms consist of expectations within the learning community, such as actively listening to peers and explaining and justifying one's solution (Makar, Bakker, & Ben-Zvi, 2015). Sociomathematical norms are distinct from social norms to highlight an epistemic focus unique to mathematical activity (Kazemi & Stipek, 2001). Sociomathematical norms overlap with but do not coincide with Brousseau's didactic contract (Laborde & Perrin-Glorian, 2005). For example, social and sociomathematical norms tend to place more emphasis on the social environment than a didactic contract (Allan, 2017).

Yackel and Cobb (1996) illustrated sociomathematical norms through examples in a second-grade classroom in which a teacher and his students discussed possible solutions and solution methods of mathematical problems. Yackel and Cobb emphasised an inquiry approach (distinct from a traditional approach), which allowed and encouraged students to contribute their ideas to create a collective learning environment. In their paper, Yackel and Cobb point out a taken-as-shared perspective of mathematical difference (including novelty and efficiency) and what constituted an acceptable mathematical explanation (justification, argument). The examples in their paper were closed-ended and well-structured in that the questions were clearly stated and had a single correct answer.

Although much of school mathematics is focused on closed-ended problems, there is a further set of problems utilised in inquiry-based learning. These are open-ended in nature and/or contain ambiguities in the problem statement or method of solution (ill-structured). In ill-structured problems, the problem statement, purpose, or method of solution contains ambiguities that must be (re)negotiated (Reitman, 1965). Well-structured problems contain complete information, limit the solution process to a set of known and organised principles and procedures, and knowable solutions, whereas ill-structured problems often require integration of knowledge (integrated, domain-specific and contextual), may require information beyond the problem, have multiple or no solutions or solution pathways, hold uncertainty around success criteria (determining if the problem has been solved), and require personal judgement and interpersonal negotiations (Jonassen, 2010). The importance of ill-structured problems has been highlighted in the literature as indicative of problems encountered outside of school, in everyday life and the professions (Jonassen, 2010; Yeo, 2017). For example, the question "How long does it take to get to the airport?" can be solved mathematically (as well as by experience or authority), but the answer may be dependent on the purpose (flying out or picking someone up, weighing the implications of being early or late, level of precision needed) and context (traffic, method of transport). In many applied problems, the initial question requires the solver to reformulate it into one that can be solved using a mathematical investigation (e.g., by clarifying ambiguities, being explicit about measures and assumptions, etc.). Research on ill-structured problems can be complex and highly context contingent. Tasks and solution pathways are less explicit, problem contexts and purposes often contain uncertainties or emergent constraints that may require responsive action, and success criteria can be implicit, subjective or absent.

This paper explores how Yackel and Cobb's (1996) sociomathematical norms could be extended to the solution of ill-structured problems in which the ambiguities of the problem are negotiated and the solution depends on an argument based on mathematical evidence (Makar et al., 2015). The research question under investigation is, *What sociomathematical norms were evident in an inquiry-based classroom addressing an ill-structured mathematical problem?* To address our research question, the paper analysed a lesson to identify sociomathematical norms specifically related to mathematical inquiry of an ill-structured problem driven by the inquiry question, *Which bubble gum is the best?*

The authors' interests are to develop research with close proximity to classroom practice in order to better understand and facilitate teaching and learning at the level of the classroom (Cobb & Yackel, 1996). We see theory as contributing to coherence across the diversity of mathematics education research by developing common languages and lenses to better share, build on, critique, improve and adapt research findings to practice. In this light, the contribution of this paper to TWG17 is three-fold: (1) re-emphasise the utility of Yackel and Cobb's (1996) foundational theory on sociomathematical norms by re-working and extending the theory to broader and more contemporary notions of mathematical inquiry, thus facilitating its application to practice; (2) re-imagine sociomathematical norms to theoretically unite the diversity of research on mathematical inquiry; that is, the paper invites mathematics education researchers to build on Yackel and Cobb's work to elaborate the practical and theoretical application of sociomathematical norms across a diverse set of contexts for learning mathematics; and (3) acknowledge and value the influence of researchers' background theory on epistemological and methodological decisions and insights from research. Elaborating and unifying the concepts and language of sociomathematical norms can provide more coherent body of research in mathematics education to improve its application to classroom practice. Better alignment and explicit discussion of researchers' epistemological lenses in relation to methodological decisions is essential to interpret, adapt and translate research findings across contexts.

Methodological approach

The study took knowledge building (Scardamalia & Bereiter, 2006) as its background theory, valuing how knowledge develops collectively through active discourse, the goal is idea improvement rather than "truth", and understanding is emergent. The methodological focus therefore did not isolate actions of the teacher or individual students, but analysed collective activities in context.

Setting and data

The data come from a multi-age classroom (children aged 10-12 years) of about 25 students in a low socioeconomic community in semi-rural Australia. We drew on video data from a class that conducted a mathematical inquiry around the question, *Which bubble gum is the best?* Students identified valued and measurable characteristics of bubble gum, developed appropriate measures, collected data on 3-4 brands of bubble gum and used their data to determine the best bubble gum.

Classroom videos were retrospectively analysed using an approach adapted from Powell, Francisco and Maher (2003). To respond to the research question, sections of video logs from the lessons were highlighted that provided insights into sociomathematical norms specifically related to ill-structured problems. Videos of these sections were reviewed again to be more selective in relation to the purpose. For example, highlighting was removed if episodes illustrated only social norms. Remaining highlighted episodes were transcribed and annotated to elaborate potential sociomathematical norms. From these episodes, a storyline was constructed to make sense of annotations by seeking connections between the inquiry problem, interpretations of students and teacher's actions, mathematical ideas that emerged and potential evidence of sociomathematical norms. This was a non-linear process supported by the background theory that often required returning to observe video, refine or rework interpretations, or enlarge or reduce contexts around episodes. Narrative was drafted to draw out local theory and further reflect on insights unique to each episode. In the process of writing, the selection of excerpts was narrowed to maximise insights and coherence in relation to the research question.

Classroom context

The teacher was experienced in teaching mathematics through inquiry after participating in research and professional development on mathematical inquiry by the first author for a number of years. The students were accustomed to working collaboratively, which was evident from the ease with which they worked with peers (freely generating and critiquing ideas, seeking approval from group before recording on a common page, keeping one another on task). Their collaborative social norms could be considered routine (Yackel et al., 1991), however their experience in solving ill-structured problems was limited. Therefore, the teacher was more explicit in co-constructing expectations with the class about acceptable activity when working in mathematical inquiry.

As argued in Yackel and Cobb (1996), an expectation within mathematical inquiry is for students to work towards autonomy. The teacher demonstrated this expectation in whole class discussions to have students generate ideas, with her explicitly privileging those that she wanted students to adopt as valued. Students worked collaboratively to devise and carry out a solution as the teacher rotated between groups to check and support their progress. She also paused the class regularly when she saw common issues to discuss as a whole class.

Illustrating norms of mathematical inquiry with ill-structured problems

We identified two sociomathematical inquiry norms in the lesson that specifically aimed to build student autonomy in solving ill-structured problems in mathematical inquiry. First, we illustrate the norm of mathematising the ambiguity in the inquiry question (*Which bubble gum is the best?*) so it could be investigated mathematically. Mathematising involves “translating a realistic problem into the symbolic mathematical world, and vice versa” (Jupri & Drijvers, 2015, p. 2483). Second, we illustrate the norm of using the inquiry question to drive and self-check progress. This second norm is different than mathematising. The use of the inquiry question during the solution process required students to shuttle back and forth between the real and the mathematical worlds, re-checking if their mathematical processes and evidence were still progressing the real world problem.

Mathematising

The teacher first oriented students towards mathematising the inquiry question. Her actions indicated that she was co-constructing the norm of mathematising as an expected activity in solving ill-structured inquiry problems, making explicit their obligations towards working autonomously. After introducing the inquiry question, the teacher led a whole class discussion to generate ideas to compare the brands of bubble gum. Students’ early suggestions were general, so the teacher pressed students to extend their ideas as a step towards productively mathematising by considering possible measures.

Oliver: Um, you could chew the bubble gum and then see which one is the best.

Teacher: So what would I be measuring, Oliver? [Oliver: Um, the taste, the texture.] Ok, so hang on a minute, there’s some good ideas! So I could measure the taste, texture. What do you mean by texture, Oliver?

Oliver: Uh, how it feels.

Teacher: Yes, ok. … What else would I be measuring for bubble gum to be the best? Imagine you are a supermarket and you decide I can only stock one bubble gum. Which one will I stock? How will I know it’s the best?

The teacher set a possible purpose for students to come up with relevant criteria that could be investigated mathematically. Not all ideas were relevant, which may be an artefact that school mathematics often does not often trigger sense-making (Schoenfeld, 1991). For example, students suggested examining which gum was healthiest, comparing smell, packaging, size, market, weight, dimensions (length, width, height), ingredients. The teacher recorded their ideas, emphasising ideas that were relevant and measurable in her intonation (“Yeah! The smell!”) and queried, but did not discount others ("Market? What do you mean by market?"). Yackel and Cobb (1996) point to this as a way that teachers signify productive ideas without explicitly labelling ideas as "good" or "bad".

After recording students' ideas, the teacher supported them to mathematise the problem by coming up with ways of measuring valued qualities. Students had no experience with non-conventional measures, so their suggestions (size, graphs, columns, packets) reflected familiar mathematics or personal knowledge, but not coordination of relevant and mathematical. The teacher was patient and continued to ask for ways they might compare taste (relevance) until she found an idea that she could build on to mathematise comparison of taste.

Adele: Vote out of 10?

Teacher: Ah, so you'd do a survey? Oh, thank you! And you said a vote out of 10, so in actual fact that's giving them a rating. … If I want to measure taste, I can't measure it with a ruler. … (Draws a number line labeled 0 to 5) Where you would place the bubble gum if this [one] is the best and this [other one] is the worst? What number would you give it (the best one)?

Oliver: I would give it a 4 ½.

Teacher: 4 ½? So how would you show that on there, Oliver (points to the number line)? (Oliver gets up and points to the space between 4 and 5.)

The teacher used the opportunity to explicitly teach students about rating scales as a tool to mathematise the problem. She left the students' ideas on the board (both productive and less so) and reiterated the task to start them off in their collaborative groups. Allowing students to wrestle initially demonstrated that the teacher expected them to generate and develop ideas to mathematise the problem. She didn't leave them unsupported, however, and used skilled questioning to press their ideas to be relevant and produce mathematical evidence. A group wanting to weigh the bubble gum were asked, "Does it have to be the heaviest to be the best bubble gum? … Which of those [ideas] is the most important, do you think?". She left them to decide whether to include weight. In other groups, she probed how they would record their measures and responded "Ooo! High five!" when a student suggested a table. She further probed, "Talk about what that table is going to look like, Jack. What it's going to have in it?" The guidance was moderated according to the progress of the group, but the expectation to work autonomously was consistent. For groups who struggled, the teacher spent more time pressing their thinking to deepen or refocus their ideas to make progress, then moved on.

Teacher: What are we measuring?

Charlotte: Taste.

Teacher: Rightio, what else?

Students: The smell. The width

Amelia: If you can smell it, you can see how strong it is.

Teacher: Ok, so you're going to smell how strong it is. … How would you show me you are measuring smell? So how will I know out of your 4 pieces of bubble gum, which one smells the best? …

Amelia: Well, we'll smell it first and then we'll write it down. [Teacher: What will you write down?] How, what we think, what, how strong it is in our opinion. …

William: We'll put 1, 2, 3, 4, 5 on it.

Teacher: … Draw it for me, William. … What does 1, 2, 3, 4, 5 stand for?

In this example, the teacher pressed students to be specific so that they could operationalise their ideas through mathematising the problem. Later, the teacher was giving instructions to a teacher aide and was overheard by a student.

Teacher: [Are] they up to tasting?

Teacher Aide: Yeah. … They've done a scale for smell. …

Teacher: [Ask] what else can they do with it? They can blow bubbles and stuff like that and try and see if they can measure and how they would go about doing that as well.

Noah: You know you just gave us the answers! We can blow bubbles!

Noah's comment indicated that the teacher violated a classroom norm, that in mathematising the problem, students should generate their own criteria and measures for what makes bubble gum "best". Because norms are not visible, they are often only detected when a member of the community breaks the norm (Yackel & Cobb, 1996). Noah's comment assisted us to identify mathematising as an emergent norm of mathematical inquiry with ill-structured problems.

Focus on the inquiry question

Students' ideas were often too vague or not relevant to the question. Therefore, another norm the teacher worked to instil was the value of the inquiry question to help students locate relevant measures and keep them on track towards a solution. In the example below, she noted that students were falling into patterns of mathematical activity that neglected sense-making. She modelled using the inquiry question as a way for them to self-check sense-making as they worked.

Teacher: Ok, just stop for a moment. Eyes up here. … (Talks about one group drawing a graph.) But if you are drawing a graph, you really need to think, "Well, what am I showing on the graph?" … You have to come back to the question *Which bubble gum is the best?* With all of your results, at the end of the day, you have to come back to that question. … You have to be able to tell me which one is the best and justify why our bubble gum is the best. … By *saying* it's the best is not proving anything. Your results have to prove it. … Jack's group is looking at a table of results and they're doing some graphing, other people are looking at scale ratings, fantastic idea! … If you need scales, how many will you need? How many lines will you draw? How many tables will I need? You need to look at what you are measuring, how I will record that information?

The teacher maintained an expectation of autonomy in using the inquiry question to check progress. She often stopped at groups, checked one element and moved on, indicating an expectation that students work autonomously. One group was weighing bubble gum when the teacher arrived.

Teacher: Ok, so if you are going to tell me that that bubble gum is the heaviest, does that mean it's the best?

Sam: No, I think that Hubba Bubba (brand of bubble gum), because it—

Teacher: So what is the point of telling me that that bubble gum is the heaviest if it doesn't tell me which one is the best? Because my question is *Which bubble gum is the best?*

Sam: Well, it's more popular—

Teacher: … I'm understanding what you're saying, but I'm just concerned that the testing you're doing is not going to show me anything. At the end of the day, your testing has to show me which bubble gum is the best.

Delia: Hubba Bubba because it's got the strongest smell, it's more popular and more people would buy it.

Teacher: So was there any point in weighing it? … Do we need to know which one is heavier when we want to know which one is the best?

The teacher continued to press students on connecting their activity to sense-making (Kazemi & Stipek, 2001) through the inquiry question. Sam and Delia's additional contributions (Hubba Bubba is more popular) connected their everyday knowledge to the inquiry question, but not to evidence the group had collected (weight). Using the inquiry question to guide their investigation was more abstract than mathematising in that students needed to step back and reflect on a larger purpose of the activity. We did not see evidence of students self-monitoring their progress with the inquiry question in this lesson and suspect it would take much more time to develop into a norm than mathematising.

Discussion

In this paper, we addressed the research question, *What sociomathematical norms were evident in an inquiry-based classroom addressing an ill-structured mathematical problem?* We considered mathematical inquiry in solving ill-structured problems using mathematical evidence (Makar et al., 2015). Two sociomathematical inquiry norms were illustrated during a lesson in which students addressed the mathematical inquiry question, *Which bubble gum is the best?*

- Ill-structured inquiry questions need to be mathematised through relevant and mathematical measures in order to be solved.
- The inquiry question is used to guide and check progress and mathematical evidence towards a solution.

These two were considered sociomathematical inquiry norms because they are central to mathematical inquiry in that (1) the question contains ambiguities that require negotiation in order to mathematise it into an operational investigation, and (2) a mathematical inquiry is driven by an inquiry question and its solution and evidence must respond to the inquiry question. Acknowledging these as useful norms to develop in an inquiry classroom can assist teachers in initiating, maintaining and extending student autonomy when addressing ill-structured inquiry problems in mathematics.

Theory adds coherence to a field by developing common language and lenses through which to gain insights and improve practice while acknowledging that researchers' background theory affects what is researched and how. Yackel and Cobb's (1996) research on sociomathematical norms has had a significant effect in progressing the mathematical reform agenda. Their work identified sociomathematical norms to ensure that teaching both improved social practices and focused on mathematics (Kazemi & Stipek, 2001). Given the nature of distinctive types of mathematical tasks (Yeo, 2017) and efforts to extend students' mathematical experiences to more open-ended problems, it would be useful for the field to extend the types of sociomathematical norms that would be useful in different forms of mathematical activity. We hope this paper makes some progress on this mission.

Acknowledgment

The authors acknowledge funding from the Australian Research Council (LP0990184; DP170101993) and thank the teacher and students for their contributions.

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