An Investigation of Primary School Teachers' Problem-Solving Beliefs and Practices in Mathematics Classrooms

Submitted by

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Statement of Sources

This thesis contains no material published elsewhere or extracted in whole or in part from a thesis by which I have qualified for or been awarded another degree or diploma.

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Signature of Candidate: J A Anderson

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Abstract

Aims

Problem-solving approaches to teaching mathematics have been recommended in curriculum documents for some time but there is evidence to suggest that there has been limited classroom implementation both in Australia as well as overseas. This investigation explored the level of implementation of mathematical problem solving in primary classrooms in NSW. Teachers' beliefs about the role of problem solving in learning mathematics as well as their classroom practices were also investigated.

To explore what teachers believe and what they do in relation to problem solving, this investigation examined primary school teachers' reported beliefs about the role of problem solving in learning mathematics and their reported practices in classrooms. It aimed to discover how beliefs about mathematical problem solving influenced decision making in teachers' classrooms and what factors promoted and hindered the implementation of problem-solving approaches.

The results of this investigation could provide benefits to several different groups involved in mathematics education. Preservice teacher educators and presenters of inservice education courses could benefit from increased knowledge about the role of beliefs in determining practices as well as potential constraints on desirable practices. Associated with this may be the need to challenge teachers' beliefs that might not support the development of practices that promote problem-solving approaches. For practising teachers, professional development could focus on examining their beliefs and providing the necessary support for teachers to realise the aim of assisting their pupils to achieve problem-solving competence. Curriculum developers may benefit from an increased awareness of the difficulties associated with implementing recommended approaches. Finally, participating teachers may benefit from their involvement in the investigation through opportunities to reflect on their practice while completing the survey instrument as well as during interview discussions with the researcher.

Scope

Data collection focused on teachers' beliefs about mathematics, teaching mathematics and learning mathematics, as well as on their reported practices since all of these factors impact on what occurs in teachers' classrooms. A combination of methods was used to collect data so that there was increased confidence in the research findings. In this way, the results of one method could be tested against another for consistency, thus enhancing trustworthiness and dependability.

The data collection for this investigation was divided into two phases. The first phase involved the use of a questionnaire to seek data on teachers' reported problem-solving beliefs and practices as little is known overall in this area, particularly within the context of Australian teachers. Responses were received from 162 primary school teachers currently teaching in NSW. The instrument was designed with reference to similar instruments that had been used by other researchers in the field and incorporated a combination of closed and open questions.

The second phase of data collection incorporated interviews and observations that were conducted in the field. These methods were used to explore the problem-solving teaching approaches used by a small number of teachers in particular school settings. To collect data about teachers' planning for instruction, and opportunities that support or constrain innovative practices, it was more appropriate to explore particular contexts that would provide a rich set of data.

Conclusions

Analyses of data confirmed the spread of teachers' beliefs, the diversity of their practice, and revealed issues that could hinder their problem-solving efforts in classrooms. A small group of surveyed teachers reported holding *very traditional* views that were quite distinct from another group who reported support for *very contemporary* views. These differences were also apparent in relation to reported classroom practices and appeared to be linked to the current teaching grade level of the respondents. This was confirmed during the interviews and observations as it seems easier for teachers of the lower primary grades to implement practices identified as supporting problem-solving approaches. For teachers of upper primary grades, parents' and school expectations impinge on teachers' practices and potentially constrain their problem-solving efforts. For the two teachers who participated in the classroom observations, considerable energy was required to resist constraints and implement problem-solving approaches.

Recommendations for practice and future research include the need for an examination of constraints on practice, the role of reflective practice in implementing innovative practices, the viability of teaching *through* problem solving as a necessary and important teaching approach, and the use of a variety of problem types in preparing students to be successful problem solvers. In addition, teachers may need to be encouraged to continually reflect on practice and teacher educators may need to raise the awareness of preservice and inservice teachers to the issues involved in implementing problem-solving approaches in their classrooms.

Acknowledgments

This thesis is the product of several years of reading, researching, thinking, discussing, arguing and debating with many of my colleagues, family and friends. It has not been achieved alone but represents the culmination of considerable input from university lecturers, fellow research students, teachers I have worked with, and students, who form the focus of our efforts in classrooms. Most of these people will remain unnamed but their support and encouragement are recognised and appreciated.

There are others who have played an active and supportive role in the development of this work. First, my supervisor, Associate Professor Peter Sullivan from Patrick Campus of the Australian Catholic University, has been a mentor, a guide, a critic, and most of all, a friend over the past six years. Peter has shown amazing patience, incredible perseverance, and considerable insight as he has read my writing and challenged my thinking. In short, he has taught me how to research and write at this level. Second, my co-supervisor, Dr Paul White from Mount Saint Mary Campus of the Australian Catholic University has offered a great deal of support, advice and friendship. He has always been willing to react to my writing and his critical comments have added to the quality of this thesis.

Many teachers have responded to my plea to be involved in problem-solving research and have willingly provided the data for this investigation. However, the seven teachers who participated in the initial interviews were particularly keen to be involved and seemed to appreciate the opportunity to have someone listen to their views and concerns. All interviews were conducted during preparation time at school or after school hours; a generous contribution of precious time. The final two teachers who played the most signficant role in this investigation, Rose and Gaye (their pseudonyms), willingly showed me their classes at work, demonstrating problem solving in action. Their dedication and professionalism were always evident and they added considerably to my knowledge about teaching and learning, constantly reminding me of the challenges they face everyday.

Finally, my family has remained supportive, although at times they may have wished for a "normal" mum. Thank you Danii and Nicki for your tolerance and understanding. In addition, my husband Don, has listened to many hours of raving about the need to find ways to improve mathematics teaching and learning through the use of problem-solving approaches. At least as a mathematics teacher in his first career, he had some understanding and sympathy for my passion.

I dedicate this thesis to all of these people, to the teachers of mathematics who seek to inspire their students, and to the young problem solvers of tomorrow.

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CHAPTER 1

MAKING THE MESSAGE MEANINGFUL

1.1 The Researcher's Journey

This investigation into teachers' beliefs and approaches to problem solving arose as a result of my experiences as a mathematics teacher in schools in Australia and England and as a lecturer in mathematics education at several tertiary institutions. I regularly observed secondary school students who were bored and unmotivated in mathematics lessons. Poor attitudes were common, with frequent complaints being made about the repetitive nature of the subject and teachers' use of pen-and-paper tests to force students to "learn". While mathematics enjoyed a superior status because it was perceived to be a necessary subject for entrance to many career options, pupils' beliefs about the usefulness of the subject rarely went beyond the view that it was essential as a filter for future careers. Secondary teachers seemed committed to preparing students for high-stakes examinations and certainly tried to achieve this, but it is possible that many of their efforts reinforced negative attitudes towards, and poor perceptions of mathematics.

These experiences have not been unique to secondary schools. In primary schools, I have also observed students with negative attitudes towards mathematics and teachers focusing on basic skills with frequent use of repetitive exercises. In addition, the introduction of the externally prepared Basic Skills Tests in New South Wales (NSW) seems to have impacted on teachers' practice and potentially reinforced these observed approaches. It was also my experience that many primary teacher education students were concerned about teaching mathematics because they felt overwhelmed and somewhat intimidated by it thus projecting their negative attitudes and feelings to their students.

However, I have observed some teachers enthusing and motivating their students, usually by using innovative practices and incorporating problem-solving[1] experiences or realistic applications of mathematics into their lessons. Such approaches are recommended in current curriculum documents and can be found in readily available resource materials, yet many teachers do not seem to have assimilated these into their planning and teaching of mathematics.

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1 In the interests of clarity, and conventionality, the term *problem solving*, when used as an adjective, is hyphenated.

This led me to consider reasons for the continued emphasis on basic skills and algorithms for students in many of our mathematics classrooms. This is an approach that I believe reinforces negative attitudes to mathematics. The following is a discussion of some of the elements that informed my perspective in undertaking this research. It should be noted that this chapter is written to establish the perspective of the researcher from her personal experiences; each of the critical elements of this investigation is further discussed in relation to the relevant literature in Chapter 2.

1.2 Perspectives on Problem Solving and Mathematics

While current practices in mathematics are effective for some, they are certainly not appropriate for all, as there seem to be many bored, unmotivated and underachieving students in mathematics classrooms. It appears that much mathematics that is learnt is disconnected and memorised by students for the sole purpose of reproduction on examinations. This suggests that there is a need for teachers to address students' negative attitudes when planning for instruction and that there are innovative practices that might help in this regard. These include the use of problem-solving approaches that highlight the usefulness of mathematics and provide a purpose for learning much of the mathematical content contained in curriculum documents.

In addition, the investigation arose from a belief that education helps students function in, and contribute to society and that a sound education is needed for all students if they are to cope with the myriad of problems that will confront them in their future lives. The 1999 meeting of the Education Ministers, in the *Adelaide Declaration*, argued that education for diversity, complexity and problem solving is essential for Australia's future citizens. Workers have access to a great deal of information and need to be multi-skilled especially as an increasing number of people are becoming self-employed. It is anticipated that people may change careers several times during their working lives and so problem-solving skills that can be adapted to a variety of different contexts will be most useful. The process of problem solving is a vitally important life and career skill and therefore could potentially form a key element of the school curriculum.

This links directly to issues of the content of mathematics curriculums and the associated processes that students may need to experience if they are to develop such skills. Mathematics is used on a daily basis for the purposes of shopping, financial planning, travelling, understanding statistical information, and technological applications. However, it no longer seems necessary or sufficient for students to focus their learning on basic facts and algorithms given the ready access to technological tools. Mathematics is seen as a process that provides a powerful way of describing the world and in industry the ability to model situations is vital. Mathematics is also used in many other disciplines such as economics, science and engineering. There does not seem to be much point spending a large amount of time at school learning mathematics unless it can be used. It may be most appropriate to focus on developing the ability to solve unfamiliar problems as a vital skill to be nurtured from the early school years.

1.3 Teaching Problem Solving

A key issue is how best to incorporate a problem-solving, applications-focused approach into mathematics teaching. Many recommendations about problem-solving teaching approaches are provided in curriculum documents, in preservice and inservice education, and through teachers' professional journals. These recommendations suggest three important approaches including teaching *for*, *about* and *through* problem solving. Each of these is described in Chapter 2 but it is noted here that each has important implications for teaching problem solving. An additional important consideration is that problem solving can be viewed as both an object as well as a process of inquiry.

When teaching *for* and *about* problem solving, teachers focus on problems as objects of inquiry. Students need to learn about the problem-solving process as well as appropriate strategies such as planning, estimating, looking for patterns, conjecturing, generalising and evaluating. For these approaches, it is necessary to provide students with many problem-solving experiences so that they can develop strategies to solve unfamiliar problems. In addition, students need to know when to employ particular strategies, when to persevere, when to try a different approach, and when to consult others for advice.

Teaching *through* problem solving focuses on problem solving as a process of inquiry so, in this sense, problems are used as vehicles for learning mathematics. Problem solving and investigations provide opportunities for students to explore situations where they can use mathematical concepts and ideas. As students are challenged, it may be necessary for them to think about what they already know, consider strategies that may help them solve the problem, engage in dialogues with other students or the teacher in an effort to resolve conflicts, and ultimately reorganise existing knowledge to accommodate new ideas. All approaches have value in the mathematics classroom. However, it is critical that students have a belief in their ability to solve problems and a positive view of the value and usefulness of mathematics.

This then gives some direction for possible experiences for students in mathematics classrooms. For students to develop problem-solving skills it seems that they need to experience problem-solving situations within the context of an inquiry-based, interactive learning environment. This teaching approach contrasts with traditional, teacher-centred classrooms where students listen as the teacher transmits information and demonstrates algorithms.

The problem-solving focus to learning has been advocated for some time and yet many teachers do not seem to emphasise this approach. One possible explanation is that such teachers may not believe that problem-solving approaches are the best way to teach school mathematics. Alternatively, teachers may be committed to using problem-solving approaches but particular forces and circumstances might militate against their intentions. It may even be that a teacher's intentions and practices are different. The best plans of the perfect lesson can be unravelled in the reality of survival and coping with the hectic pace of organising and managing the class for learning. A planned lesson may be compatible with a problem-solving philosophy but this could be difficult to put into practice for a variety of reasons. Coming to understand teachers' beliefs and why it seems they are not adopting recommended approaches is a key to supporting them in their endeavours to enthuse, inspire and educate their students.

1.4 Teachers' Beliefs

It is proposed that teachers' classroom practices are directly influenced by their beliefs about teaching and learning. An investigation of teachers' mathematical problem-solving beliefs may illuminate why they choose to use particular classroom practices and reject others. Investigating teachers' beliefs is fraught with difficulty since beliefs cannot be directly accessed and when asking teachers about their beliefs, it is always possible that they will report what they consider to be the "correct" or "most desirable" responses. However, careful design of research methods can help to overcome some of the difficulties. It is acknowledged that there are other factors that might impact on teachers' actions in classrooms including their knowledge and understanding of such approaches, previous experiences, as well as the particular students present in their classrooms.

To investigate these influencing factors, a diagrammatic model is proposed in Chapter 2 that links factors and shows possible interrelationships between them. At the centre is teachers' beliefs and practices. The model suggests that teachers' reported beliefs are influenced by their actual beliefs, by their knowledge and interpretation of advice about teaching problem solving, and by their use and understanding of curriculum documents. It also proposes that reported classroom practices are influenced by reported beliefs, by actual practices in classrooms, and by the constraints and opportunities that are occurring within the school context. In addition, it suggests that constraints and opportunities are then mediated by teachers' previous experiences as learners of mathematics as well as by their experiences as teachers in classrooms.

1.5 Opportunities and Constraints

One of the key purposes of this investigation is to identify factors that support and those that constrain teachers' efforts. Opportunities that support teachers' attempts to use problem-solving approaches in their classrooms may include teachers' knowledge and confidence, school cultures that encourage the promotion of the processes of working mathematically, and support from other members of the school community. These factors are specific to particular contexts, varying greatly from school to school, and from classroom to classroom.

Factors that act as constraints as teachers attempt to put particular approaches into practice may include students' responses as well as the teacher's lack of confidence. School culture can constrain teacher planning and approaches because of the influence of prescribed programs, assessment and reporting practices, parents' expectations as well as the traditional beliefs of fellow staff members. Students may also constrain teachers' efforts since their responses to tasks in mathematics lessons may be influenced by their beliefs about what constitutes mathematical activity resulting in an active resistance of teachers' plans to adopt problem-solving approaches. In addition, teachers may abandon plans of using problem-solving tasks because of the preconceived ability of the students to cope with the demands of understanding and solving problems.

1.6 The Focus of the Investigation

To explore what teachers believe and what they do in relation to problem solving, this investigation examined primary school teachers' reported beliefs about the role of problem solving in learning mathematics and their reported practices in classrooms. It aimed to discover how beliefs about mathematical problem solving influenced decision making in teachers' classrooms and what factors promoted and hindered the implementation of problem-solving approaches.

The four main research questions for this study were

1. What do teachers believe is the role of problem solving in learning mathematics?
2. To what extent do teachers report that they incorporate problem-solving approaches in their planning and teaching of mathematics, and what specific practices do they report that they use?
3. In what ways do teachers incorporate problem-solving approaches into their planning and teaching of mathematics?
4. When teaching mathematics using problem-solving approaches, what factors can be identified that support or inhibit the implementation of such practices?

It should be noted that this investigation did not seek to evaluate whether problem-solving approaches are the best way to teach mathematics; nor did it attempt to critique the problem-solving literature. The relevant and associated aspects of problem solving are accepted as appropriate methods for teaching and learning mathematics although particular problem-solving approaches are described and further discussed in Chapter 2. It should also be noted that there is not a review of the literature on teacher change. Although teacher change and professional growth are clearly related to this research, the main aim was to examine teachers' current beliefs and practices, to report on these, and to identify salient influencing factors.

1.7 Benefits of the Investigation

The outcomes of this investigation should provide valuable information and advice to several different groups in the mathematics education community. There are benefits to be gained for the researcher and teacher participants, for preservice and inservice educators, for curriculum developers, and for classroom teachers.

One of the aims of this investigation was to discover some of the factors that inspire teachers to inspire students. This aim was born from the researcher's experiences in classrooms, as well as in working with classroom teachers in inservice courses and with trainee teachers in tertiary institutions. A clear benefit from this research would be to provide the researcher with knowledge and information to further develop her skills in working with these groups to enhance their implementation of problem-solving approaches.

The investigation involved many primary school teachers currently working in schools in NSW. These teachers may benefit from their participation in the project through opportunities to reflect on their practice while completing the survey instrument as well as during interview discussions with the researcher. Any opportunity to reconsider what one believes and why one chooses to use particular classroom practices may help to reaffirm current practice or to raise issues that could lead to a reexamination of one's professional practice. This would be particularly beneficial if it occurred in schools where participating teachers work.

Those involved in teacher preservice and inservice education may benefit from knowledge gained from this research. Preservice teacher educators, although aware of the lack of problem-solving experiences for many of their students, may also need to challenge students' beliefs that might not support the development of practices that promote problem-solving approaches. Presenters of inservice education courses could benefit from increased knowledge about the role of beliefs in determining practices as well as potential constraints on desirable practices.

Curriculum developers could benefit from knowledge about teachers' beliefs regarding problem solving so that these could be addressed in syllabus and support documents. Information about teachers' current use of such documents would also inform curriculum review processes and indicate what additional material is needed to support teachers. Curriculum developers need to consider the impact of documentation on teachers' beliefs and practices, particularly if recommended approaches are at odds with popular practice. A key issue in this regard is to state clearly the philosophical perspective of curriculum content and a rationale for recommended approaches.

For practicing teachers, professional development could focus on examining these beliefs where necessary and supporting teachers to realise the aim of assisting their pupils to achieve problem-solving competence. Alerting teachers to factors that might constrain their plans and actions in classrooms and promoting discussions about the issues would be an important step in providing support and enabling the promotion of problem solving in mathematics classrooms. It may assist the promotion of such practices if supportive factors can be determined and communicated to school teachers and principals.

1.8 Data Collection

Data collection focused on teachers' beliefs about mathematics, teaching mathematics and learning mathematics, as well as on their reported practices since all of these factors impact on what occurs in teachers' classrooms. A combination of methods was used to collect data so that there was increased confidence in the research findings. In this way, the results of one method could be tested against others for consistency thus enhancing trustworthiness and dependability.

The data collection for this investigation was divided into two phases. A questionnaire sought information about the first and second research questions. It was appropriate to use a questionnaire to seek this data as little is known overall about teachers' beliefs and practices in this area, particularly within the context of Australian teachers. Responses were received from 162 primary school teachers currently teaching in NSW. The instrument was designed with reference to similar instruments that had been used by other researchers in the field and incorporated a combination of closed and open questions.

The second phase of data collection incorporated interviews and observations that were conducted in the field. These methods were used to explore the third and fourth research questions for a small number of teachers in particular school settings. To collect data about teachers' planning for instruction, and opportunities to support or constrain innovative practices, it was more appropriate to explore particular contexts that would provide a rich set of data.

1.9 Structure of the Thesis

In this thesis, a proposed model is used to structure a review of relevant literature and to assist with the design of data collection. Chapter 2 incorporates an examination of a selection of studies from the mathematics education literature related to teachers' beliefs and practices, particularly in relation to problem solving. This includes a summary of the advice to teachers about problem-solving instruction and an overview of the emphasis of problem solving in school mathematics curriculums. Also, a summary of research about the adoption of problem-solving approaches is presented and other models representing the relationship between teachers' beliefs and practices are described.

Chapter 3 outlines the research approach including methodology and research methods followed by a description of the research tools and data gathering procedures. Chapter 4 presents the questionnaire analysis and Chapter 5 describes the fieldwork analysis. Finally, chapter 6 includes a general discussion of findings and the overall recommendations.

CHAPTER 2
THE RESEARCH IN CONTEXT

A model is used to guide the research and to provide structure for a discussion of relevant literature. The development of this model is elaborated later in the chapter. The model proposes interrelationships between factors that impact on teachers' reported beliefs and reported practices (Figure 1).

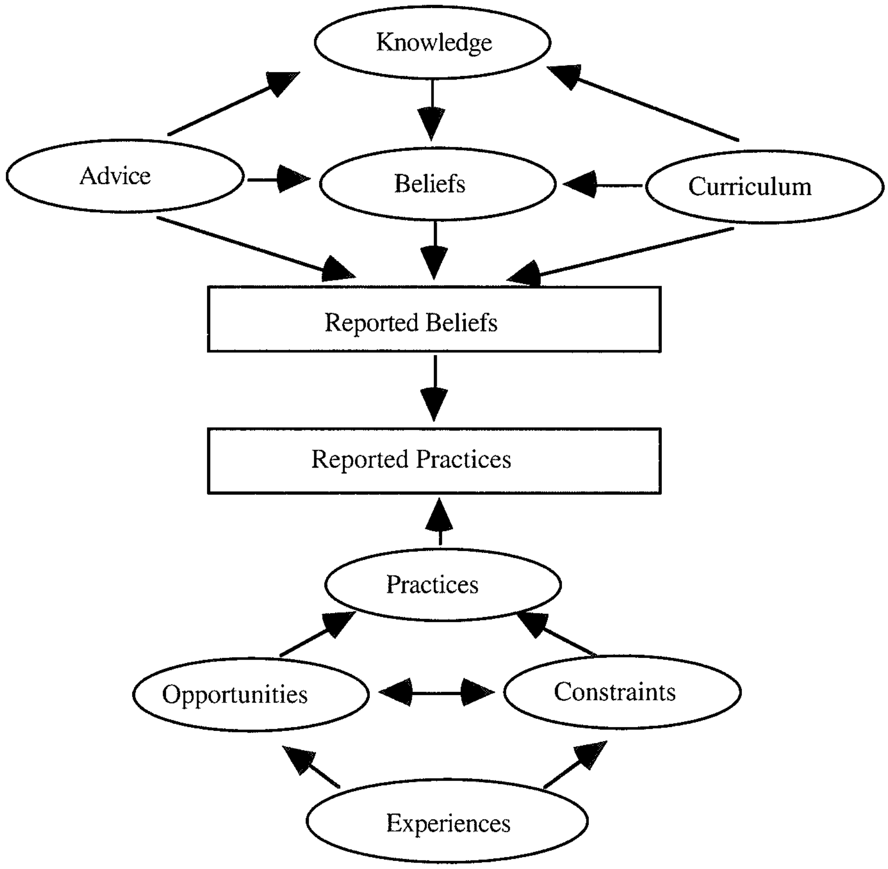


Figure 1. A model of the factors that impact on teachers' reported beliefs and practices.

The model suggests that advice to teachers, curriculum documents and teachers' knowledge influence teachers' beliefs and hence their reported beliefs. It also recognises that reported practices are influenced by actual classroom practices which in turn are either enhanced through opportunities or hindered by constraints in particular contexts. Each of these is also strongly influenced by teachers' previous experiences. Other models have informed the development of this model and are discussed in detail later in this chapter in Section 2.7.

Each of the key factors, in the above model, that impact on teachers' reported beliefs and practices have been described elsewhere. The discussion in this chapter focuses on problem solving in particular, and attempts to highlight issues for further clarification and investigation. These issues have informed this investigation and assisted in the development of the research questions.

To place the research in context, some significant studies and analyses are discussed in relation to the elements of the model. The links between the model and the following sections of this chapter are indicated in brackets. This discussion first explores changes in emphasis on problem solving in school mathematics curriculums (curriculum), followed by a clarification of the terms *problem*, *problem solving* and *problem-solving approaches* to teaching (curriculum). Recommendations involving approaches to the teaching of problem solving are examined including the structure of lessons, types of problem-solving tasks, and assessment practices that incorporate problem solving (advice). This is followed by a discussion of the role of the teacher in problem-solving classrooms (advice). Teachers' knowledge is discussed briefly in relation to associated beliefs and practices (knowledge).

Comments on the level of adoption of problem-solving teaching approaches are presented (practices) and research into teachers' problem-solving beliefs and practices in mathematics classrooms is outlined (beliefs and practices). This is followed by a discussion of factors that support (opportunities) teachers' efforts and hinder (constraints) teachers' attempts to incorporate problem-solving approaches in their classrooms. Both of these factors are linked to teachers' previous experiences (experiences). Finally, models representing the relationship between beliefs and practice are presented (beliefs, reported beliefs, practices, reported practices) with a description of the model used in this investigation. The chapter concludes with a more detailed explanation of the research questions.

2.1 Emphasis on Problem Solving in School Mathematics Curriculums

Fundamentally this research is about teachers' beliefs and practices but it is important to consider what the curriculum says about problem solving and problem-solving teaching approaches. Figure 2 is a section of the model presented in Figure 1 that provides structure to the following discussion. It suggests that curriculum documents support teachers' knowledge about appropriate content and teaching approaches and that these also influence teachers' beliefs and hence their reported beliefs.

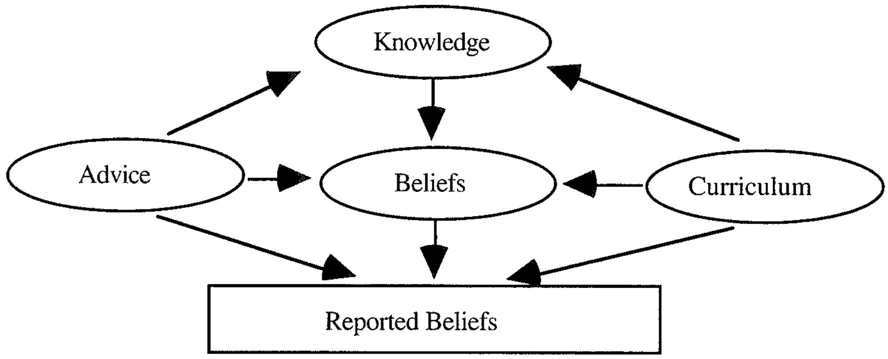


Figure 2. The influence of curriculum documentation on teachers' beliefs and knowledge.

The current emphasis on problem solving and the important role it plays in teaching and learning mathematics represents a significant change from the mathematics curriculum before the 1980's. Problems have been part of school curriculums for many years but problem solving has not (Stanic & Kilpatrick, 1989). One impetus for change was that national testing programs in many countries in the late 1970's found that basic skills and concepts, although mastered by many students, could not be readily applied in unfamiliar, problem contexts (Siemon, 1986). There was also a belief that problem-solving approaches better prepare students to function in society, as the mathematics that they need relies little on algorithms and procedures but requires solution of problems, often unrelated to traditional school mathematics (Stacey, 1990). In addition, it has been argued that problem-solving approaches promote a deeper and more meaningful understanding of mathematics (Schoenfeld, 1992).

Including problem solving as an integral part of mathematics curriculums represents a change in focus from earlier school programs that promoted the development of basic skills, the memorisation of formulae, and the practice of routine procedures (Grimison, 1995). Increased emphasis on problem solving is evident in curriculum documents from a range of English speaking countries including the United States of America (USA), the United Kingdom (UK), and Australia. Calls for reform have continued and are evidenced in all Australian curriculum documents and, in particular, in NSW syllabuses (e.g. New South Wales Department of Education, 1989).

The importance of the acquisition of mathematical problem-solving skills was highlighted in international publications on mathematics education. Materials published from the late Seventies and early Eighties described problem solving as central to the school curriculum. In the USA, the publication of *An Agenda for Action* (National Council of Teachers of Mathematics, 1980) stated that "problem solving must be the focus of school mathematics" (p. 2).

In the UK, *Mathematics Counts* (Cockcroft, 1982) was a report of an extensive investigation into the teaching and learning of mathematics. The often-quoted key recommendation was

> Mathematics teaching at all levels should include opportunities for:
> - exposition by the teacher;
> - discussion between teacher and pupils and between pupils themselves;
> - appropriate practical work;
> - consolidation and practice of fundamental skills and routines;
> - problem solving, including the application of mathematics to everyday situations; [and]
> - investigational work (p. 71).

Each of these elements of good practice is discussed in the report and the suggestion is made that "the ability to solve problems is at the heart of mathematics" (p. 73). In addition, the report stated

> At each stage of the mathematics course the teacher needs to help pupils to understand how to apply the concepts and skills which are being learned and how to make use of them to solve problems. These problems should relate both to the application of mathematics to everyday situations within the pupils' experience, and also to situations which are unfamiliar. For many pupils this will require a great deal of discussion and oral work before even very simple problems can be tackled in written form (p. 73).

Investigations that are responses to students' enquiries are also recommended as a valuable part of the learning process. The authors recognised that good teachers already incorporated these strategies into their repertoire but there was concern that there were classrooms that did not include some of these elements of good teaching.

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) described five new goals for students, one of which was that all students "become mathematical problem solvers" (p. 5). In addition to this, *mathematics as problem solving* is the first standard across all grade levels (K-4, 5-8, 9-12). For K-4, this standard states

> The study of mathematics should emphasise problem solving so that students can -
> - use problem-solving approaches to investigate and understand mathematical content;
> - formulate problems from everyday and mathematical situations;
> - develop and apply strategies to solve a wide variety of problems;
> - verify and interpret results with respect to the original problem;
> - acquire confidence in using mathematics meaningfully (p. 23).

A similar reform was proposed in Australia with the publication of several national documents supporting changes to school curriculums. The *Australian Mathematics Education Program* (AMEP) (Curriculum Development Centre, 1982) listed problem solving as an important and necessary part of mathematics education. The authors of the AMEP position paper on problem solving in school mathematics, Carss and Osborne

(1981), described the rationale for problem solving in mathematics and suggested that it is necessary for the following reasons

1. to improve student understanding of the application and power of mathematics;
2. to facilitate the retention of basic mathematical knowledge and skills;
3. to provide access to a wide range of embodiments of mathematical principles and examples of fundamental mathematical processes; and
4. to demonstrate and confirm the use and appreciation of general problem solving heuristics applicable in mathematics and more generally in all areas of the school curriculum (p. 53).

A National Statement on Mathematics for Australian Schools (Australian Education Council (AEC), 1991) included content as well as process strands, one of which was Mathematical Inquiry. This strand involved the use of strategies for problem posing and problem solving. A subsequent national publication, *Mathematics* - A *Curriculum Profile for Australian Schools* (AEC, 1994), included a strand entitled Working Mathematically that encompassed investigations and problem solving at all of eight levels. The *Employment-Related Key Competencies* (Mayer, 1992) listed eight key competencies, one of which was Solving Problems. These documents provided advice to both curriculum developers and teachers. It is clear that in each of them, problem solving was promoted as an important and necessary element of learning generally, and learning mathematics in particular.

A different indication of the interest in problem solving, and its influence on curriculum and teaching, was the number of presentations at mathematics education research conferences, and the number of workshops at teachers' conferences that focused on problem solving. To consider this, an analysis of conference proceedings was undertaken. Table 1 shows the proportion of papers that included the terms *problem* or *problem solving* in the title. This does not include those papers that used problem in the sense of student difficulties or misunderstandings.

Table 1
Percentage of papers that include problem or problem solving in the title for four conference groups over a period of 21 years (na = not available)

Organisation	1977	1978	1980	1982	1988	1992	1997
Mathematics Education Research Group of Australasia (MERGA)	9%	na	10%	10%	na	13%	9%
Australian Association of Mathematics Teachers (AAMT)	na	na	na	15%	4%	8%	3%
Mathematical Association of NSW (MANSW)	na	0%	4%	0%	na	4%	10%
Mathematical Association of Victoria (MAV)	0%	4%	4%	8%	5%	2%	2%

The data suggest that problem solving, as a focus for research and public discussion, has continued although there have been fluctuations. Papers from the MERGA conferences have maintained a consistently high rate of interest in problem solving and indeed it may be the single most important topic of investigation. For the teachers' conferences there is much more fluctuation in the figures, and they have been generally lower than at the MERGA conferences. This analysis was possibly not fine grained enough, so in order to consider teachers' conferences more closely, papers from the MANSW conferences were reexamined. Papers that either explicitly referred to problem solving as an important part of the school curriculum, included advice to teachers about teaching problem solving, or described suitable problem-solving tasks were included in the new analysis (Table 2).

Table 2
Proportion of papers about problem solving in MANSW conference proceedings (expressed as a percentage)

Year	'78	'80	'81	'82	'90	'92	'93	'94	'95	'96	'97	'98
Proportion of Papers (%)	0	33	46	50	53	29	48	35	64	42	38	28

The data suggest that the focus on problem solving has fluctuated but has maintained a high profile in this professional community. It is noted that there were three conference proceedings available for MANSW conferences before 1980. Each of these focused around a theme, for example the 1978 proceedings was entitled "Mathematics for the Atypical Child", and none of the papers included either problem or problem solving in the title. Not all of the proceedings were available for MANSW conferences since 1980 but for those that were available, the data show a continued high level of interest in papers about problem-solving approaches.

It is also noteworthy that the references to problem-solving approaches include a diverse set of terms. A closer examination of the papers in the MANSW conference proceedings held in 1997 shows that authors have referred to student explorations, real-life applications, working mathematically, problem posing, non-directive questions, inquiry-based learning, problem-solving tasks, mathematical modelling, and making generalisations (Anderson, Bobis, & Southwell, 1998). This is in contrast to papers from the early 1980's that used fewer terms including activities, discovery or inquiry learning, real-life problems, applications and mathematical investigations (Ford, Kumar, & Grimison, 1981; Ford & Grimison, 1982). It seems that authors now use a wider set of terms to discuss the kinds of problem-solving tasks that might be used in classrooms. This reflects the terminology used in relation to problem-solving teaching approaches in current curriculum documents (Board of Studies NSW, 1996; NSWDE, 1989).

Stacey and Groves (1989) noted that every Australian state and territory included problem solving as part of their curriculum aims by 1988. Current mathematics curriculums in NSW organise material under content headings such as Number, Geometry, Algebra, Measurement, and Statistics however, the importance of developing processes as well as problem-solving competence are still discussed. For example, in the *Mathematics Syllabus for Years 7-8* (NSWDE, 1986), Problem Solving is one of six strands and it is clearly stated at the beginning of the document that

> problem solving should be integrated into the total program and be incorporated into the teaching and learning of other topics continuously throughout the two years. It should not be taught as a single topic at one time only (p. 3).

In the *Mathematics K-6* (NSWDE, 1989) syllabus, teachers are provided with advice about what constitutes a problem, sources of problems, approaches to teaching problem solving, and a variety of suitable teaching strategies. The mathematical content is described in three strands - Space, Measurement and Number. Table 3 shows how each strand is further subdivided into sub-strands. Teaching and learning units for each sub-strand are organised under content objectives.

Table 3
Strands and Sub-Strands in the NSW K-6 Syllabus document

Strand	Space	Measurement	Number
Sub-Strand	3D 2D Position Graphs	Length Area Volume Mass Temperature Time	Numeration Addition Subtraction Multiplication Division Fractions and Decimals Money

A more recent publication, *Mathematics K-6 Outcomes and Indicators* (BOSNSW, 1998) has introduced a fourth strand, Working Mathematically. Table 4 lists the sequences for each of the four strands contained in this document.

Table 4
Strands and Sequences in the K-6 Outcomes and Indicators document

	Working Mathematically	Space	Measurement	Number
Sequences	Questioning	3D	Measurement Attributes, Units and Tools	Whole Numbers
	Problem Solving	2D	Length	Fractions
	Communicating	Spatial Patterns	Area	Number Facts
	Verifying	Position	Capacity and Volume	Number Operations
	Reflecting	Data Representation	Mass	Applying Number
	Using Technology		Temperature	
			Time	

This new strand includes Problem Solving as one of six sequences and it includes outcomes for problem solving at each of the four stages for K-6 mathematics education - Early Stage 1, Stage 1, Stage 2 and Stage 3. This seems to be an attempt by the curriculum developers to make explicit the desired outcomes related to Working Mathematically as well as those related to Problem Solving.

In summary, school mathematics curriculums over the last twenty-five years have moved towards the inclusion of skills related to problem solving and working mathematically. This has occurred in many countries with the publication of materials supporting this approach. In Australia, this call for reform has been embraced in curriculum documents in all states and territories with new syllabuses in NSW including problem solving at all levels. Mathematics teaching curriculums now have explicit statements about problem solving including advice to teachers about problem-solving approaches.

2.2 Clarification of the Terms *Problem, Problem Solving,* and *Problem-Solving Approaches*

There is some evidence that teachers may not have responded to these recommendations and there has been ongoing speculation about the reasons for this (Blane, 1992; Holton, Spicer & Thomas, 1995; Lester, 1994; Pegg, 1997). A possible factor is the lack of consistent and clear definitions for the terms *problem* and *problem solving*. A common criticism of much of the problem-solving research literature has been the lack of consistency of the use of these terms (Schoenfeld, 1992). As this study focused on the use of problem-solving approaches in primary school classrooms, it is appropriate to define each of the terms *problem, problem solving*, and *problem-solving approaches*.

This section examines the information presented to teachers about problem solving and related teaching approaches. It includes advice to teachers as well as information provided in curriculum documents in relation to each of the terms *problem, problem solving*, and *problem-solving approaches*. This information relates to the advice and curriculum factors suggested in the model at the beginning of this chapter.

A key issue relates to the meaning of these terms. Kilpatrick (1981) argued that problem solving could become a catch cry or "an empty vessel that we can fill with our own meanings" (p. 2). This concern over definitions was recognised by Carss and Osborne (1981) who suggested

> Confusion arises because of the many different goals, processes, skills and concepts that can be considered and the many different educational functions that can be encompassed under the phrase 'problem solving'. Problem solving possesses a wide variety of meanings for various people ... (p. 53).

These sentiments have been echoed by several other authors. Stanic and Kilpatrick (1989) stated that

> the term problem solving has become a slogan encompassing different views of what education is, of what schooling is, of what mathematics is, and of why we should teach mathematics in general and problem solving in particular (p. 1).

Southwell (1980) suggested that problem solving could be considered a goal, a process, or a basic skill. Siemon (1993) argued that there are two relatively distinct views of problem solving. One describes problem solving as a process in that students are presented with a situation, or a given state, and they need to find a solution, or a goal state, without having an obvious solution at hand. The other view suggests that problem solving is a form of learning; a view compatible with the more recent constructivist model of learning.

Ernest (1991a) suggested that problems can be the "object of inquiry" or the "process of inquiry". The focus is either on particular problems, or, it is on the activity of seeking paths to solutions. He also described an "inquiry-based pedagogy" that incorporates problem-solving approaches along with guided discovery and investigatory approaches. The variety of terms that are used in the literature adds to the dilemma for teachers as they attempt to interpret and understand the implications for classroom practice.

Even though attempts have been made to define the terms *problem* and *problem solving* there has been little agreement and much of the problem-solving research from the 1980's has been criticised because of the diverse and conflicting definitions that were given (Schoenfeld, 1992). Hembree's (1992) examination of a large number of studies revealed definitions of a problem that ranged from traditional word problems to non-standard problems. Earlier definitions of a problem tended to include mathematics questions that were phrased in words but that only required the recall of a fact or the application of an algorithm. More recent definitions require the presence of a blockage as a necessary prerequisite (Hembree & Marsh, 1993). In other works, students do not have a ready procedure to answer the question but they need to plan, reflect, try a variety of possible procedures, and evaluate the outcomes. Schoenfeld (1992) suggested that "*real* problem solving" involved working on problems that were unfamiliar, and not necessarily related to the particular mathematics topic currently being studied.

A commonly cited definition of a problem is that from Charles and Lester (1982) who suggested that a problem is a task for which

> 1. The person confronting it wants or needs to find a solution.
> 2. The person has no readily available procedure for finding the solution.
> 3. The person must make an attempt to find a solution (p. 5).

Problem solving is then the process of attempting to find a solution to such a task. Another useful definition from House, Wallace, and Johnson (1983) stated that

> ... a mathematical problem is a situation that involves a goal to be achieved, has obstacles to reaching that goal, and requires deliberation, since no known algorithm is available to solve it [and] problem solving is the process of attacking such a problem ... (p. 10).

While no definition of a problem was offered in the lower secondary syllabus in NSW (NSWDE, 1986), the following definition was provided in the primary syllabus (NSWDE, 1989).

> A problem has three characteristics:
> • there is a goal to be reached
> • an obstacle prevents ready solution
> • the solver is motivated to reach a solution (p. 22).

All of these definitions have common elements that include the presence of a goal, an obstacle that prevents ready solution, and the need for some effort or motivation on the part of the solver. For the purposes of this investigation, these three characteristics are used to define a problem.

Problem solving can be considered in two ways. In one sense, it involves finding solutions to problems and can be referred to as an *object of inquiry*. In another sense, it can be considered to be a critical element in the learning process since it is through problem solving that students reflect on current knowledge and reorganise existing mental structures to accommodate new ideas. In this way, problem solving is referred to as being a *process of inquiry*. As a consequence, problems need to engage students in higher levels of thinking so that their understandings are confronted and either confirmed or challenged.

Another term that is commonly used in the literature is *problem-solving approaches*. In this investigation, this term is used to refer to the teaching practices that are recommended in advice to teachers regarding the appropriate implementation of problem solving. This advice includes the use of:
- a variety of problem tasks;
- individual, small group, and whole class activities;
- hands-on, concrete experiences to support problem-solution attempts; and

- the acceptance of a wide variety of strategies and approaches to solving problems (e.g. AEC, 1991; Callahan & Garafalo, 1987; Charles & Lester, 1982; Frank, 1988; NCTM, 1989; NSWDE, 1989; Stacey & Groves, 1985; Sullivan, 1994).

It also includes use of teaching strategies that are referred to as *for*, *about* and *through* or *via* problem solving (eg. NSWDE, 1989; Schroeder & Lester, 1989; Siemon & Booker, 1990). These aspects of problem-solving teaching approaches are discussed in more detail in the next section of this chapter.

2.3 Problem-Solving Instruction

Approaches to teaching problem solving are many and varied. Advice in curriculum documentation recommends that problem solving should be integrated into all content areas in the mathematics curriculum (NSWDE, 1986) and that problem solving could be a regular component of each teacher's program (NSWDE, 1989). Research about problem-solving instruction suggests that there are many elements to be considered. These include suitable problem-solving tasks, a clear view of the role the teacher should play, and guidelines for organisation and implementation (Kroll & Miller, 1993).

A consideration of factors that influence students' problem-solving abilities has indicated the importance of students' knowledge, control strategies, beliefs and affects, and the social and cultural conditions of the classroom. A detailed explanation of the importance of each of these factors is presented in Appendix 1. In addition, an examination of the role of problem solving in developing thinking and supporting learning suggests that problem-solving approaches should be an integral part of mathematics lessons. Appendix 2 presents an overview of the implications for classroom practice.

There are some clear recommendations that teachers need to consider when teaching mathematical problem solving. Lester's (1994) reflections on twenty-five years of problem-solving research suggested that the literature on problem-solving instruction provided five important results

> 1. Students must solve many problems in order to improve their problem-solving ability.
> 2. Problem-solving ability develops slowly over a prolonged period of time.
> 3. In order for students to benefit from instruction, they must believe that their teacher thinks problem solving is important.
> 4. Most students benefit greatly from systematically planned problem-solving instruction.
> 5. Teaching students *about* problem-solving strategies and heuristics and phases of problem solving ... does little to improve students' ability to solve mathematics problems in general (p. 666).

From these results, teachers need to be aware that problem solving can be incorporated into their programs on a regular basis and that problems ought to be an integral part of mathematical activity. There are several ways that this can be achieved.

One particular set of terminology that helps to clarify problem-solving approaches is teaching *for*, *about*, and *through* problem solving. Each of these approaches has implications for the structure of mathematics lessons and the kinds of problems that might be presented to students. In the following discussion, strategies used to teach problem solving are outlined, structures of lessons are described, and the variety of problem-solving tasks that have been used in classrooms is presented. The impact of assessment practices on teaching problem solving is examined followed by a summary of the recommended role of the teacher.

2.3.1 Problem-Solving Teaching Approaches

Three approaches to teaching problem solving include teaching *for*, *about* and *through* problem solving (Schroeder & Lester, 1989; Siemon & Booker, 1990). All three approaches are recommended in curriculum documentation in NSW (NSWDE, 1986 & 1989). Each of these approaches has implications for the types of activities that might be presented to students in mathematics lessons.

2.3.1.1 Teaching *For* Problem Solving

Teaching *for* problem solving most closely resembles a traditional approach to teaching mathematics where students learn mathematical content so that they can apply it to problems related to that content area. In this approach, teachers provide students with the necessary skills and knowledge needed to solve mathematics problems. Problems are usually related to the mathematical content just studied and students are provided with a variety of applications in which the mathematics may be used. This is an approach that may be adopted in classrooms where there is a reliance on a prescribed text that presents problems at the end of the chapter.

One issue for teachers when using this approach is that the notion that a problem must have a *blockage* may be removed if students are able to follow a rehearsed procedure. However, teachers intend that students are able to transfer knowledge to a variety of problem contexts as this is the main reason for learning the mathematics in the first place (Schroeder & Lester, 1989). Unfortunately, students seem to experience difficulty transferring knowledge to unfamiliar situations, particularly when the curriculum is presented in discrete topic areas (Schoenfeld, 1992).

2.3.1.2 Teaching *About* Problem Solving

Teaching *about* problem solving includes guidance about the problem-solving process and instruction about a variety of problem-solving strategies. It often includes the recommendations of Polya (1957), or adaptations of this approach, with students guided through the phases or stages of the problem-solving process (Charles & Lester, 1982). These phases can include understand the problem, plan a solution, carry out the plan, and evaluate the solution.

When teaching *about* problem solving, students also learn to use a variety of problem-solving strategies, or heuristics, including make a list, draw a diagram, act it out, solve a simpler example, and guess and check. Some textbooks have exercises on each of these strategies so that students can practise using them (see for example Jones, Couchman, & Carroll, 1986). This approach can reduce problem solving to an activity rather than a process. It can also become another topic to be studied in mathematics or the focus of a lesson at the end of each week. It can reduce problem-solving strategies to algorithms where each is rehearsed after the teacher has demonstrated the technique.

In this approach it appears that even though students have the requisite background knowledge and are aware of the heuristics, these are not readily transferred to new problem situations (Flavel, 1976; Schoenfeld, 1985; Stacey, 1990). Again, the issue is that students may not be able to apply the appropriate strategy when confronted with an unfamiliar problem out of context. Students need to be able to recognise the problem type and a suitable strategy without necessarily receiving cues from what they have just learned.

2.3.1.3 Teaching *Through* Problem Solving

Teaching *through*, or, *via* problem solving is different from the two previous approaches in that it treats problem solving as a process of inquiry rather than as an object of inquiry. This approach implies that, rather than students being taught new ideas and skills with some applications and problems added on at the end, problems are used as the vehicle for learning, or "as the context in which the learning of mathematical ideas takes place" (Stacey, 1990, p. 72).

This approach focuses more on student understanding as an attempt must be made to make sense of the mathematical procedures needed to solve the problem, and it engages students in doing mathematics. Problem solving becomes a *means* rather than an *end*. Problems can be posed so that student knowledge is challenged thus providing a need for the student to reorganise his understanding in order to resolve the problem.

Schroeder and Lester (1989) argued that *understanding* in mathematics assists the development of problem-solving abilities. It was suggested that this occurs in four ways

> 1. understanding increases the richness of the types of representations that the problem solver can construct;
> 2. understanding assists the problem solver in monitoring the selection and execution of procedures (eg. strategies, algorithms);
> 3. understanding aids the problem solver in judging the reasonableness of results; and
> 4. understanding promotes the transfer of knowledge to related problems and its generalisability to other situations (p. 41).

These are certainly powerful reasons to consider students' understanding as the main goal in mathematics classrooms. This understanding can be supported through the use of problems as a focus of discussion and to promote thinking and reflection.

It has been argued that there is a place for all three approaches although the third approach is considered most appropriate. For example, Siemon and Booker (1990) suggested that teaching *for* problem solving provides knowledge, skills and strategies, teaching *about* problem solving provides "the means to access, monitor and direct what is known and what can be done" (p. 26), and teaching *through* problem solving provides a context for further learning. They also argued that problem solving should not be just a topic in the program or an isolated activity for Friday's lesson but should be integrated into all aspects of the program as problem-solving behaviour "is acquired by conscious, critical reflection on past and immediate experience, the foci of which is self, task, strategy and process" (p. 23).

Schroeder and Lester (1989) also suggested that all three approaches have value but that teachers should be aware of the shortcomings of the first two approaches if used in isolation. They argued that when teaching *for* problem solving, problems can just become applications of recently learned concepts and may not require very much mathematical thinking by students. Also, they indicated that teaching *about* problem solving can lead to problem solving being treated as another topic in the program. Finally, they recommended that teaching *through* problem solving is most likely to promote understanding and is the approach that supports the goals and standards outlined in USA curriculum documentation (NCTM, 1989). They stated

> Instead of making problem solving the focus of mathematics instruction, teachers, textbook authors, curriculum developers, and evaluators should make *understanding* the focus and their goal. By doing so they will shift from the narrow view that mathematics is simply a tool for solving problems to the broader conception that mathematics is a way of thinking about and organising one's experiences (p. 39).

These approaches, *for*, *about* and *through* problem solving, were key elements of the data collection in this investigation and were used as a way to describe teaching approaches. The structure of lessons also needs to be considered since they can by quite different according to the problem-solving approach that teachers adopt.

2.3.2 The Structure of Lessons

Lesson structures can vary considerably depending on the purpose of the lesson and its role in the overall program. In mathematics, lessons can be teacher-centred and focus on exposition of a particular skill or procedure followed by students practising with teacher support, or they can be student-centred with practical activities where students apply mathematical ideas. Lessons can involve considerable individual seat work, or students can work on tasks in small cooperative groups to achieve a common goal. A predominance has been noted in mathematics classrooms of traditional, teacher-centred approaches that incorporate teacher exposition with student practice (Clarke, 1984; Schoenfeld, 1989). The problem-solving approaches outlined in the previous section have implications for the lesson structure and the types of tasks that might be set for students to attempt.

A traditional mathematics lesson may consist of exposition by the teacher, some practice by the students, refinement of the strategies by the teacher, and further practice by the students that might be finished for homework. This structure has been referred to by Burkhardt (1988) as an "exposition, examples, exercises" mode of teaching. This is the structure that is most likely to be used when teaching *for* problem solving as lessons usually focus on mathematical content with students practising problems that relate to that content. It may also be employed when teaching *about* problem solving, particularly if the practice of particular strategies is the focus of the lesson. In these lessons, it is usual for students to attempt a large number of repetitive exercises that can usually be completed rather quickly (Schoenfeld, 1989).

When teaching *through* problem solving, a new structure is recommended. Lessons tend to begin with a problem and the key mathematical ideas are developed as students require mathematical techniques to solve the problem (Schroeder & Lester, 1989). Using this problem-solving approach, students may be presented with fewer problems during each lesson. Lampert (1986) used between one and three problems per lesson with her class of Year 4 students as they invented and justified solutions to problems involving multiplication. In subsequent research, Lampert (1990) planned lessons for a Year 5 class that focused on one main problem and the processes used to find a solution. Through whole class discussion, students negotiated appropriate methods and possible solutions rather than relying on the teacher to suggest approaches and confirm ideas.

Stigler and Stevenson (1991) conducted extensive research comparing Asian and American classrooms with respect to several variables including lesson structure and type of tasks. Their research involved observations of mathematics lessons in 20 classes in each of Years 1 and 5 in Taipei (Taiwan), Sendai (Japan) and Minneapolis (USA). Japanese lessons were described as coherent and were compared to a good story with a beginning, a middle and an ending. The students were active participants who were engaged in problem-solving activities and discovering new ideas. The lesson usually began with a practical or word problem and time was always spent ensuring that all students understood the problem. The teachers acted as "knowledgeable guides" spending considerable time in each lesson asking provocative questions and instructing the whole class. An equally important part of each lesson was the summarisation of the main ideas that had arisen including clarification of rules and definitions of terms.

Stigler and Stevenson (1991) reported that in most Japanese lessons, students spent the whole period working on one problem whereas in American classrooms a far more traditional approach was observed. The comparison of lesson structure in Table 5 shows that in American classrooms, the typical approach resembles Burkhardt's (1988) description of "exposition, examples, exercises". The typical Japanese lesson more closely resembles the approach of teaching *through* problem solving.

Table 5
Comparison of the structure of typical Japanese and American Elementary School lessons in mathematics

Japanese Lesson	American Lesson
• teacher poses rich problem	• teacher instructs students about concepts or skills and demonstrates how to solve example problems
• students struggle with problem on their own	• students practise on their own while teacher helps individuals
• students present their ideas and discuss these	
• teacher concludes lesson	

In contrast, lessons in American classrooms had little coherence with teachers regularly changing topics. It was suggested by teachers that regular change was used to add variety to lessons so that the students would remain more motivated. However, much time seemed to be wasted on inefficient transitions between activities and on the many disruptions that typified American lessons. Mathematical concepts and definitions were usually introduced at the beginning of lessons with the remainder of lessons spent on learning and practising the rules. Only about 50% of class time was spent on direct instruction since after work had been set, most teachers wandered between students assisting them with individual concerns. Little time, if any, was spent at the end of lessons summarising the main ideas and forming the lesson into a coherent whole.

Desirable lesson structures for mathematics instruction have been proposed by other authors. Sullivan (1994) recommends a lesson structure with four stages that can be used when teachers use problems as a focus of instruction. This includes

Stage 1: Teacher poses the task.
Stage 2: Students work on the task, individually or in groups.
Stage 3: Whole-class review.
Stage 4: Teacher summary (p. 51).

As in Japanese classrooms, whole-class review of students' ideas and a teacher summary are important parts of the lesson. Sullivan emphasises that teacher input should occur at the end of the lesson rather than at the beginning as in conventional, or traditional classrooms. In this way, students are given the opportunity to think and struggle with the mathematical ideas on their own and in groups before substantial guidance occurs.

In order to engage students for an extended period of time, problems need to be rich and provide scope for exploration on several levels. Japanese teachers spend large amounts of out-of-class time preparing their lessons and thinking of good questions. This focus on inventing a good question does not seem to occur in American schools and Stigler and Stevenson (1991) argued that it is not a skill taught in teacher training courses. Lampert (1990) also emphasised the importance of choosing problems that engage students; problems that were referred to by Kilpatrick (1987) as "structured problems requiring productive thinking" (p. 134).

There are many types of mathematical problems that can be used in classrooms to engage students in productive thinking. Suitable choice of a variety of rich problem-solving tasks is another important role for the mathematics teacher.

2.3.3 Types of Problems

Several attempts have been made to classify problems into groups. Each type of problem has a purpose in supporting students' learning of mathematics. Some of the early classifications included questions that were merely exercises but more recent listings seem to focus on questions that require higher-order thinking and that may present a blockage for students. A variety of studies have explored the usefulness of problem types in promoting learning although most research has focused on the effectiveness of open-ended or goal-free problems. There is evidence to suggest that these problems elicit better responses from students since they reduce cognitive load.

Early classifications of problem types can be found in Krulik (1980) and Charles and Lester (1982). Krulik classified problems into five types - recognition exercises, algorithmic exercises, application problems, open search problems, and problem situations. However, it is possible that if students have the prerequisite knowledge, they will readily answer questions relating to recognition and algorithm exercises. Charles and Lester listed six different types of problems. These included:

- drill exercises;
- simple translation or word problems;
- complex translation or word problems that involve at least two steps;
- process problems that are less familiar and require the use of thinking processes such as planning, guessing, conjecturing, testing and looking for patterns;
- applied problems that refer to real-world situations where mathematics can be used to organise and represent data; and
- puzzle problems that can be solved by knowing the 'trick' and may not require the use of any mathematics.

The first two categories may well not contain a blockage for many students and indeed more recent writers in the field would not classify drill exercises as problems as this implies routine practice (Schoenfeld, 1992; Siemon, 1986; Silver, 1990; Stacey, 1990; Sullivan & Clarke, 1991).

An alternative classification was suggested by Clarke and McDonough (1989) who categorised problems as puzzles and posters, packaged tasks, open-ended questions, situation problems, 'active-involvement' problems, Fermi problems, and extended investigations. While these categories are not discrete, the problem types can be grouped according to problem context and possible methods of solution. All of these problem types can be useful in the classroom and may provide learning opportunities for students.

Based on the above classifications, it seems that most mathematics questions can be classified as either exercises, application problems, open-ended problems, or unfamiliar problems. Exercises are those questions that require the application of a known fact or mathematical procedure, and since there is no blockage, would not be classified as problems using the earlier definition. Application problems are those that are typically found in textbooks and that provide examples of where the topic currently being studied is applied. Some application problems may be routine while others may be non-routine and therefore would be classified as problems. Open-ended problems typically have several solutions and there are usually different ways of finding and recording solutions. Unfamiliar problems are those problems that are not open-ended and that the students do not meet on a regular basis in their classrooms. Open-ended and unfamiliar problems offer real challenges to students and provide rich tasks to be used in problem-solving contexts. This classification of problem types was used to assist the data collection for this investigation.

A variety of different problems have been used in studies and reported in the research literature. Several researchers have used problems related to specific content areas to investigate students' understanding and learning in that particular topic; for example, multiplication and division (Mulligan, 1992); spatial relationships (Owens, 1994); chance and data (English, 1993); geometrical proof (Schoenfeld, 1985); and algebraic concepts (Sweller & Cooper, 1985).

Another rich area of research has been the investigations of the use of open-ended, or goal-free problems (Ayres, 1993; Sullivan & Clarke, 1991; Sweller, 1993). Open-ended questions have considerable value as they reduce cognitive load and can be used to promote higher levels of thinking, challenge student understanding, encourage students to discuss mathematical ideas with each other, and make generalisations. Using these questions in the classroom challenges beliefs about how mathematics questions should be answered and provides opportunities for the development of appropriate problem-solving behaviours.

Sullivan and his colleagues (1991, 1992) have conducted several studies that explored the effectiveness of "good questions" both for instruction and assessment. Good questions were defined as those that required more than the recall of a fact or replication of a procedure, and were both educative and open. Sullivan, Clarke and Wallbridge (1991) sought responses to a variety of good questions from students of various ages and from a range of classes. The study indicated that the inclination to provide multiple responses increased with age although prompting with younger students did result in more responses. There were no differences in the quality of the responses from individual efforts compared to students working in pairs. They also noted that some tasks were more likely to result in multiple responses than others and this seemed to be influenced by the presence of "multi-concepts" that tended to make questions more difficult and less accessible.

Sullivan, Clarke, Spandel and Wallbridge (1992) implemented a unit of seven lessons that consisted entirely of good questions involving length, area and perimeter and compared the learning outcomes to a similar group of Year 6 students who were given explicit instruction with the aid of textbook questions. Data collection included lesson observations, questionnaires, pen-and-paper tests, and interviews and aimed to explore "whether pupils learn substantive mathematical content through the use of content-specific open questions with no direct teaching" (p. 9). Pupils responded well to the innovative unit and they were actively engaged in the lessons. However on post test questions, they rarely responded to open questions with multiple solutions. The researchers proposed possible explanations including the difficulty level of the questions, the need for skill practice for consolidation, the brevity of the program, and the need for teachers to spend more time at the end of lessons on review.

Several studies have investigated the use of goal-free problems with secondary mathematics students (Sweller, 1993). Owen and Sweller (1985) found that if students are confronted with problems that have reduced goal specificity, or goal-free problems, they acquire knowledge and skills more effectively since more of working memory is available to focus on the task. It had been observed in earlier studies by Sweller and his colleagues that novices often use the relatively ineffective means-ends strategies to solve problems. These strategies retard the development of schemata that can then be used in future problem-solving efforts. Ayres (1993) investigated the errors made by students as they solve two-move problems. Many errors were made at the subgoal stage and this was attributed to the increase in cognitive load as students used means-ends strategies. Ayres found that presenting the same mathematical content to students using goal-free problems reduced the stage effect and students were more likely to find the correct solution path.

Pegg (1997) laments the lack of adoption of problem solving in Australian classrooms and suggests that frequent use of textbook style questions might be constraining teachers' efforts. He recommends an "evolution rather than a revolution" in that teachers should consider relatively simple adaptations of the problems they currently use. These include:

- problems with insufficient information so that students need to think about what extra information is required to solve the problem;
- problems with too much information so that students need to really think about what information they need to use; and
- problems that have been reversed so that answers are presented and students need to determine the question.

The last class of problems includes open-ended or goal-free problems. Pegg argues that these questions will promote higher-order thinking, that they should be used on a regular basis, and that they support the development of *insight* which is the ability to act *adequately* with *intention* in a *new situation* (van Hiele, 1986).

The notion of problem types was fundamental to this investigation and formed a critical part of the data collection. Student problem types were used to describe problem-solving tasks and as the basis of potential teaching approaches. In addition to a consideration of the types of tasks teachers use in classrooms, it is of interest to consider their use of similar tasks for assessment purposes.

2.3.4 Problem Solving as a Part of Assessment Practices

Regardless of the problem-solving tasks and approaches that teachers use in classrooms, there is evidence to suggest that unless problem solving forms part of assessment procedures, students will not value problem-solving tasks and therefore will not make a commitment to such tasks. It is possible that in settings where basic skills are externally examined, there is a focus on recall of facts and computation rather than on developing problem-solving abilities.

As teachers adopt problem-solving teaching approaches in classrooms, assessment may need to mirror these changes so that data can be collected about students' problem-solving abilities and their use of the processes involved. Also, if problem solving is to be valued by students as well as teachers, it may need to form part of classroom assessment practices as well as more formal school and state assessment procedures (Clarke, Clarke, & Lovitt, 1990). There is evidence to suggest that there have been changes in classroom practice where problem solving has been incorporated into "high stakes" assessment. A result that suggests that if real change is to occur in classrooms, it is necessary to introduce it through compulsory, external testing procedures (Blane, 1992). However, assessing problem-solving competence is not easy and has received little attention in problem-solving research.

Calls for the use of a variety of assessment approaches in mathematics classrooms have been widespread. The *Standards* document in the USA (NCTM, 1989) advocated assessing mathematical power, problem solving, communication, reasoning, as well as mathematical concepts, procedures, and dispositions. It would be difficult to design a pen-and-paper test that would be able to adequately measure all of these elements. Clarke et al. (1990) advocate a wide variety of assessment practices that include student self assessment and the adaptation of traditional test questions, but emphasise that assessment should be linked to instruction.

There are several examples where alternative assessment procedures including questions that focus on problem-solving skills have been incorporated into high stakes external assessment. In California, open-ended questions that required explanations were first used in Grade Twelve in 1987-88 to assess students in the state-wide achievement test (California Assessment Program (CAP), 1989). These questions were introduced so that assessment reflected a curriculum that emphasised problem solving and the development of mathematical processes. Students were required to answer a bank of multiple choice questions as well as one of five open-ended questions. The following is an example of one of the questions used.

> James knows that half of the students from his school are accepted at the public university nearby. Also, half are accepted at the local private college. James thinks that this adds up to 100 percent, so he will surely be accepted at one or the other institution. Explain why James may be wrong. If possible, use a diagram in your explanation (CAP, 1989, p. 21).

A large number of students provided inadequate responses. It was suggested at the time that students were not familiar with answering such questions and that performance would improve with practice.

In England, attempts to implement more problem solving in external assessment examinations were initiated through the inclusion of problem-solving questions in the non-compulsory part of the General Certificate of Education (GCE) 'O' Level examination in mathematics in the mid-Eighties. Blane (1992) reports that

> these questions formed an increasingly large proportion of the examination over successive years, thus initiating a real possibility for change in the mathematics curricula of the participating schools (p. 295).

The new 1989 National Curriculum has embraced recommendations from the Cockcroft Report (Cockcroft, 1982) with "a strong emphasis on an investigative and problem-solving approach to mathematics teaching and learning" (Joffe, 1992, p. 199). Both Standard Assessment Tasks (SATs) and school-based teacher assessment were used to assess students with teachers encouraged to evaluate all of the Attainment Targets including those that focus on problem solving (Joffe, 1992).

In 1990, Victoria adopted the use of problem solving as part of final high school assessment in the Victorian Certificate of Education (VCE) when investigative projects were introduced. These projects provided opportunities for students to explore mathematical ideas and to explain, justify and evaluate their findings. A report of 1500 words was required after 15 to 20 hours exploring centrally set themes. An interesting consequence of this has been the impact on school-based assessment practices as well as changes to the way that mathematics is taught in secondary classrooms (Clarke, Stephens, & Wallbridge, 1993; Stacey, 1994). This has been referred to as the 'ripple effect'. Clarke et al. (1993) reported in the first stage of their investigations that considerable classroom time was now being spent on problem-solving activities and developing appropriate ways to communicate solutions to problems in lower secondary grades.

In this set of studies by Clarke and his colleagues, the second stage of the research surveyed teachers with results supporting early findings although experience with teaching at the VCE level was not necessary for these alternative practices to be adopted. Rather, it appears that the new assessment procedures at the high-stakes level had created a "climate of change" (Stephens, Clarke, & Pavlou, 1994). At the latest stage of this research, Victorian results were compared with preliminary results from NSW schools where a more traditional form of assessment is used. There were significant differences between the two states in the use of problem solving in classrooms (Barnes, Clarke, & Stephens, 1995). This research raises the issue of the impact of external factors on teachers' practices and the way that they may bring about change in classrooms.

The evidence presented at this stage suggests that problem solving may need to be an integral part of instruction and assessment if it is to be valued by teachers, students and parents. Incorporating problem-solving tasks into high-stakes assessment gives it even more legitimacy and lifts the status of such abilities considerably. Unfortunately this is evidence of a top-down approach to curriculum development but it has certainly changed practices in mathematics classrooms in many Victorian schools. The use of problem-solving tasks in assessment practices is another critical consideration of this investigation and informed collection of data.

2.3.5 The Role of the Teacher in a Problem-Solving Classroom

There is a need to describe the role of the teacher in a problem-solving classroom as this is an important and necessary component of advice to teachers (Clarke, 1997; Lester, 1994). The teacher's role is critical in this process as she needs to decide what tasks to use, when and how to use them, and what assessment procedures are appropriate. Lester suggests that any problem-centred mathematics curriculum will not be successful until the teacher's role has been clearly delineated and problem-solving classrooms have been described so that teachers know what they look like.

The teacher's role in schools is complex and multifaceted and can be interpreted at three levels. Teachers have responsibilities to the system, the school and the students in their class. At the system level they need to interpret advice provided in curriculum documentation and implement appropriate recommendations. At the primary school level in NSW, this documentation includes the *Mathematics, K-6* syllabus (NSWDE, 1989) and the *Mathematics K-6: Outcomes and Indicators* (BOSNSW, 1998). At the school level, teachers are usually required to follow pre-determined scope and sequence charts, write appropriate programs, and implement assessment procedures to enable them to report to parents. In the classroom, teachers plan lessons, determine good tasks that aim to achieve mathematical outcomes, motivate and engage students in mathematical discourse, assess understanding and provide support to students when required (Sullivan et al., 1991).

To fulfil all of these responsibilities, teachers need support and guidance both at the preservice and inservice stages of their training and professional growth. Advice is also available through curriculum documentation and guidelines and in teacher professional journals. For example, advice for teachers in the USA is provided in the *Professional Teaching Standards* (NCTM, 1991). This publication recommends four actions for teachers in order for them to implement the *Curriculum and Evaluation Standards* (NCTM, 1989). These are:

(1) choose worthwhile mathematical tasks;
(2) orchestrate classroom discourse;
(3) create a safe and constructive environment for learning; and
(4) analyse students' understanding and the contribution of classroom practices to students' learning.

Each of these aspects is an important component in problem-solving classrooms but this advice is rather general and needs to be more explicit.

Many other sources offer advice about the role of the teacher in classrooms that have problems as the focus of learning. Teaching *through* problem solving implies a significant change in the teachers' role as teachers assume the roles of guide, coach, questioner and co-solver (Lester, Masingila, Mau, Lambin, Pereira dos Santos, & Raymond, 1994), or of role model, questioner, moderator and interlocutor (Lappan, 1993). Tobin and Imwold (1993) describe the role as one of a mediator where the focus is on learners rather than on the discipline itself. In this, teachers value students' thinking and encourage them to take responsibility for justifying and evaluating solutions thus negotiating meaning between themselves (Davis, Maher, & Noddings, 1990). This implies that teachers relinquish some of their power in classrooms and become "knowledgeable participants" without always having the answers. This is a real challenge to achieve as most teachers would be unfamiliar with this approach since they would not have experienced such approaches when they were learners of mathematics.

Clarke (1997) worked with middle-school teachers as they implemented innovative mathematics materials that used non-routine problems as the focus of instruction. He developed a framework that consisted of a six-component categorisation of the role of teachers in reformed classrooms. In Clarke's research, the term *role* was interpreted as:

> both what the teacher did in the classroom in terms of interaction, organisation, and decision making, but also those choices, and commitments made in preparing for instruction (p. 279).

The six components of the teacher's role were determined after Clarke analysed the reports of seven studies that focused on reform of classrooms. In this, the role of the teacher incorporated the following components

1. The use of non-routine problems as the starting point and focus of instruction, without the provision of procedures for their solution
2. The adaptation of materials and instruction according to local contexts and the teacher's knowledge of students' interests and needs
3. The use of a variety of classroom organisational styles (individual, small-group, whole-class)
4. The development of a "mathematical discourse community", with the teacher as "fellow player" who values and builds on students' solutions and methods
5. The identification and focus on the big ideas of mathematics
6. The use of informal assessment methods to inform instructional decisions (p. 280).

Clarke's (1997) case study research enabled him to examine the changes in each teacher's role as they taught an innovative unit of work and reflected on their decisions, as well as their own professional growth through conversations with the researcher.

A consequence of Clarke's (1997) investigation resulted in the addition of a seventh component to the role of the teacher. This was the need to facilitate students' reflections on their activities and understandings; a component that became a critical element to ensure connections between mathematical concepts and to relate content to personal experiences. This set of components provides a useful framework to explore implications for classroom practice and is further examined in Section 2.4.3.2 in relation to teachers' beliefs.

One attempt to describe the classroom and teaching actions in a problem-solving classroom is provided by House et al. (1983). The following is a brief summary of a teacher who taught with a problem-solving focus. This teacher displayed problems on the walls of the classroom that usually reflected students' interests. Her students were encouraged to find a range of solutions and to create similar problems for others to try. Discussion in groups was encouraged with whole-class sharing of solutions and methods. She used a textbook but often began in the middle or at the end of the chapter so that the students did not know the "applicable algorithms" and so the questions became problems rather than exercises. Students were encouraged to do as many as they could and then to record what information was necessary to solve those they could not do. The teacher focused on the processes involved in solving problems rather than answers and the students were confident, motivated and enthusiastic; qualities that were observed in the teacher as well.

In addition to a clear description of the teacher's role, appropriate resource materials may support the implementation of problem-solving approaches in classrooms. In this regard, a substantial number of resource materials have been published; for example, RIME (Lowe & Lovitt, 1984) and MCTP (Lovitt & Clarke, 1988). Clarke (1997) suggests that the provision of innovative materials was one of the factors that supported the implementation of problem-solving approaches, but there was concern that this was not sufficient to encourage teachers to adopt such approaches. Lovitt's reservations about this were cited in Holton et al. (1995)

> all the early problem-solving efforts were mostly devoted to the creation of suitable problems in the belief that teachers could present these in classrooms and generate effective learning with the same maths they used for expository teaching. It has taken some time to recognise that this is not the case ... (p. 346).

In his research, Clarke (1997) concluded that another critical enabling factor in the process of changing practice was the "daily opportunity to reflect on classroom events in conversations and interviews with the researcher" (p. 278). This is a process that could possibly be achieved in school settings if teachers worked together to prepare and evaluate their programs and teaching practices.

It is noted here that reviews of teaching practices (Koehler & Grouws, 1992) and strategies that promote mathematical thinking (Schoenfeld, 1992) reveal a variety of approaches to the teaching of mathematics. Koehler and Grouws (1992) outline five paradigms or different research programs that address approaches to teaching and learning mathematics. While these were varied the authors noted that they all "... accept the premise that students are not passive absorbers of information ... [and] ... view the teacher as an informed and reflective decision maker" (p. 123).

Advice to teachers about their role in a problem-solving classroom recommends a variety of teaching strategies. In summary, the teacher's role is to plan for instruction, to develop appropriate tasks or problems, to encourage discussion and listen to responses, and to modify instruction to meet the needs of individual students. All of this needs to be achieved within an appropriate learning environment. An environment that encourages mathematical thinking (Schoenfeld, 1992) and promotes students' mathematical communication (Weissglass, Mumme & Cronin, 1990).

While there have been some attempts to define the role of teachers in problem-centred classrooms as well as to describe the environment in those classes, there is still considerable evidence that calls for reform have not been heeded by teachers. There are many possibilities as to why this is the case. It could be teachers' lack of confidence with this approach or lack of time to find appropriate tasks when preparing innovative units of work that incorporate the content from curriculum documents. Another possibility may be the use of assessment procedures that limit teachers to assessing basic skills and understandings rather than problem-solving abilities. These possibilities, and others, are examined in Section 2.6. Another possibility is that teachers' knowledge is incomplete or that their beliefs are not congruent with such reforms. The importance of knowledge and beliefs in determining instructional practices is explored in the next section.

2.4 Teachers' Knowledge, Beliefs and Practices in Mathematics

Teaching is a demanding activity in that teachers are constantly making decisions based on their beliefs, knowledge, judgements and thoughts. It has been argued that teachers' decisions about how to teach mathematics are made on the basis of their knowledge and beliefs about how students learn mathematics. Such decisions are also influenced by beliefs about mathematics and beliefs about appropriate teaching approaches. This suggests that different beliefs may result in different instructional practices. Figure 3 is a part of the model used in this investigation to highlight the relationship between knowledge and beliefs and their influence on reported beliefs and practices.

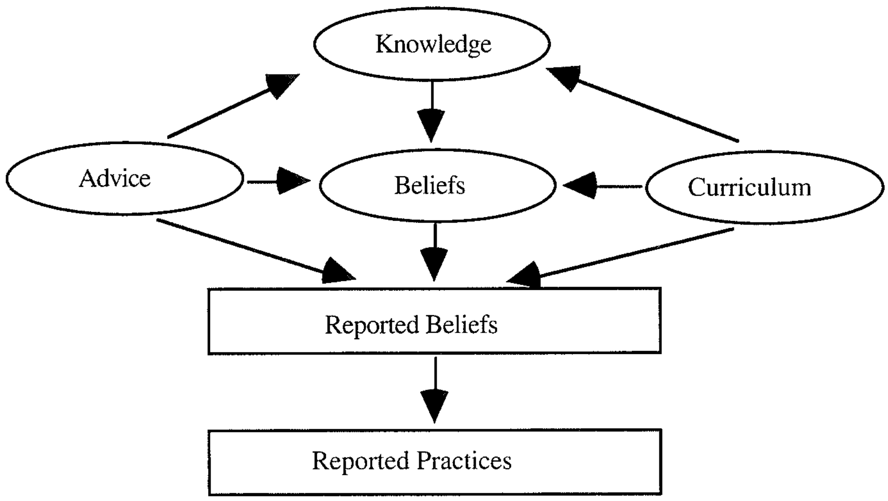


Figure 3. The impact of knowledge and beliefs on teachers' reported beliefs and practices.

In this section, there is a discussion of teachers' knowledge and beliefs about mathematics. Ways of categorising knowledge and beliefs are examined and categorisations of mathematics teaching based on different approaches are considered. For the purposes of this investigation, possible perspectives about mathematics teaching and learning are presented that include beliefs about the place of problem solving in school mathematics curriculums.

2.4.1 Teachers' Knowledge and Beliefs

Teachers' knowledge and beliefs are closely linked but much research has focused on one or the other. Thompson (1992) was critical of the usefulness of studies that tried to separate the two and suggested that it was more fruitful to examine teachers' conceptions, or,

> mental structures, encompassing both beliefs and any aspect of the teachers' knowledge that bears on their experience, such as meanings, concepts, propositions, rules, mental images, and the like ... (p. 141).

In spite of this criticism, Thompson (1992) does distinguish between knowledge and beliefs in her review of the literature on teachers' beliefs and conceptions.

Defining both knowledge and beliefs is not an easy task but it is claimed that it is important to "... consider seriously where teachers' knowledge ends and beliefs take over" (Peterson, Fennema, & Carpenter, 1991, p. 61). A possible distinction described by Cobb, Yackel and Wood (1988) is

> ... knowledge has traditionally been defined as true belief ... From the anthropological perspective, knowledge can be defined as institutionalised belief ... the distinction between knowledge and belief is relative to the practices of a community (p. 106).

In addition, beliefs are not consensual and can be held with varying degrees of conviction whereas knowledge is associated with truth and certainty and must meet particular criteria of evidence (Thompson, 1992).

Pajares (1992) suggested that "all beliefs have a cognitive component representing knowledge, an affective component capable of arousing emotion, and a behavioural component activated when action is required" (p. 314). He argued that belief systems comprised values, attitudes and beliefs and stated that

> When clusters of beliefs are organised around an object or situation and predisposed to action, this holistic organisation becomes an attitude. Beliefs may also become values, which house the evaluative and judgemental functions of beliefs and replace predisposition with an imperative to action (p. 314).

Belief systems are dynamic and loosely-bound, and may be used to describe how an individual's beliefs are organised. According to Nespor (1987)

> Belief systems often include affective feelings and evaluations, vivid memories of personal experiences, and assumptions about the existence of entities and alternative worlds, all of which are simply not open to outside evaluations or critical examination in the same sense that the components of knowledge systems are (p. 321).

Ernest (1989) suggested that two teachers can have similar knowledge and yet adopt quite different teaching approaches in classrooms. He claimed that this is a consequence of different views about the nature of mathematics, the nature of mathematics teaching and the process of learning mathematics. Further, Ernest argued that these views form the three components of beliefs about teaching mathematics.

It has also been argued that beliefs about mathematics are directly influenced by early mathematical learning and experiences in classrooms (Morine-Dershimer & Corrigan, 1996). A view supported by Pajares (1992) who stated that beliefs come from "a process of enculturation and social construction" (p. 316). This suggested that beliefs are formed through interaction with the cultural environment, through schooling, and possibly through experiences at home.

For further consideration, Morine-Dershimer and Corrigan (1996) listed several important characteristics of teachers' beliefs. These included

> beliefs are deeply personal, and have a strong emotional component;
> beliefs serve as a filter or screen, influencing teachers' interpretations of events, and they may distort information processing;
> the longer a belief is held, the more difficult it is to change; and
> beliefs about teaching are initially formed from the perspective of a pupil, and thus fail to account for the full complexity of classroom interactions (p. 2).

Each of these characteristics has implications for this investigation and were considered during data collection.

In summary, beliefs and knowledge are important to consider when examining teachers' actions in classrooms. Although there are arguments suggesting that they should not, and perhaps cannot, be separated, knowledge and beliefs about mathematics teaching and learning are important components that were considered in this investigation. It is also necessary to acknowledge that there are many different aspects to teachers' knowledge and beliefs.

2.4.1.1 Developing a Categorisation of Knowledge and Beliefs

Several different types of knowledge have been described in the literature. Shulman (1986) distinguished between different kinds of teacher's knowledge including subject-matter knowledge, pedagogical-content knowledge, and curriculum knowledge. Thus, he suggested that it is important for teachers to know about mathematics, about learners' thinking in relation to mathematical topics, and about appropriate instructional materials. Others have described practical, personal knowledge (Elbaz, 1983) and situated knowledge (Resnick, 1987).

Peterson (1988a) expanded upon Shulman's (1986) framework of teacher's knowledge and included knowledge about students' thinking, knowledge about the growth of students' thinking, and self-awareness of the teachers' own cognitive processes. She argued that mathematical knowledge is only useful if teachers know how it relates to students' cognitions and their own metacognition. It is recognised that all of these types are important components of teachers' knowledge but it is of particular relevance to this investigation to explore the links between knowledge and beliefs.

On the basis of their review of the literature, Fennema and Franke (1992) proposed a model of teachers' knowledge (Figure 4) that emphasised the context of the classroom as well as the importance of teachers' beliefs. They stated

> Teacher knowledge cannot be separated from the subject matter being investigated, from how that subject matter can be represented for learners, from what we know about students' thinking in specific domains, or from teacher beliefs (p. 161).

This suggests that knowledge and beliefs are closely linked and that both play a critical role in teachers' decision making in classrooms.

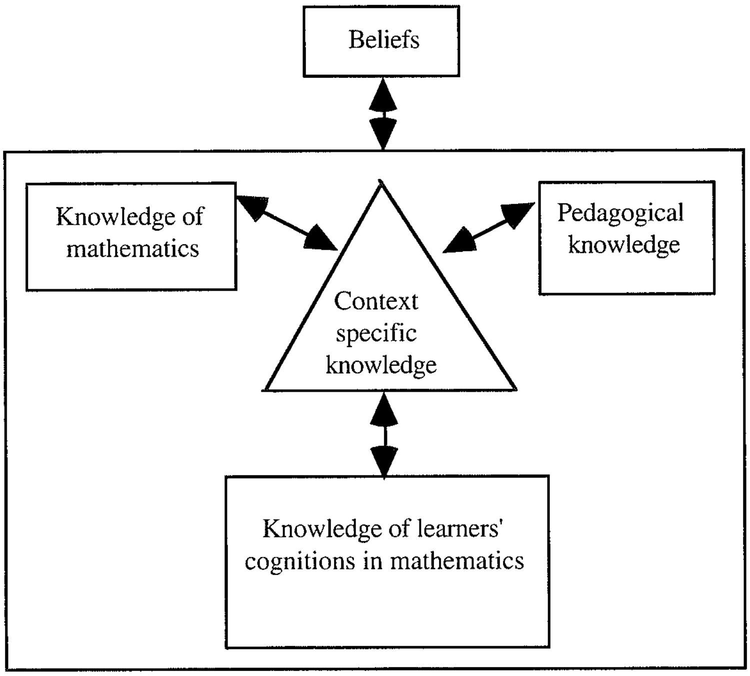


Figure 4. Teachers' knowledge: Developing a context (Fennema & Franke, 1992, p. 162).

Any investigation of teachers' beliefs about problem solving in mathematics needed to encompass several components related to mathematics, teaching mathematics, and learning mathematics. The following typology of knowledge and beliefs can be constructed in relation to the teaching and learning of mathematical problem solving (Table 6). To investigate teachers' beliefs about mathematical problem solving, all of these elements are important and needed to be considered. These aspects of knowledge and beliefs were used to assist with the design of data collection procedures for this investigation.

Table 6
A typology of knowledge and beliefs for the teaching and learning of mathematical problem solving

Mathematics	• knowledge of mathematical content • knowledge of problem solving • conceptions of the nature of mathematics • beliefs and knowledge about what constitutes a mathematical problem
Teaching mathematics	• beliefs and knowledge about teaching mathematics • beliefs and knowledge about teaching problem solving • beliefs and knowledge about problem-solving approaches to teaching mathematics • beliefs about their own classroom practices • knowledge of the curriculum and resources
Learning mathematics	• beliefs about the role of problem solving in learning mathematics • beliefs and knowledge about how students learn mathematics

Most of the research that has examined teachers' knowledge in mathematics has focused on content knowledge, knowledge of mathematical representations, knowledge of learning, and pedagogical knowledge (Fennema & Franke, 1992). Some have expressed concerns about elementary school teachers' knowledge of mathematics (Ball, 1988; Brown, Cooney, & Jones, 1990). Few have explored teachers' knowledge and beliefs about mathematical problem solving and how this might impact on classroom practices.

2.4.2 The Impact of Teachers' Knowledge and Beliefs on Classroom Practices

It has been suggested that teachers' knowledge and beliefs about the discipline of mathematics, teaching mathematics, and learning mathematics impact on classroom practices. Case study research has provided evidence indicating that when teachers have a sound knowledge of the content to be taught, they use a wider variety of problems, allow more discussion, and respond more readily to student questions (Fennema, Carpenter, & Peterson, 1989). Also, teachers who were not confident with the subject matter relied on the text, directed the instruction, and allowed less discussion. Leinhardt and Smith (1985) argued that teachers whose mathematical knowledge was more connected and conceptual were more conceptual in their approach to teaching. Also, those whose knowledge was less integrated were more rule-based in teaching mathematics.

Attempts have been made to categorise mathematics teaching using a variety of approaches. Ammon and Hutcheson (1989) proposed a hierarchy of levels of pedagogical thinking with associated learning experiences for students and instructional practices for teachers. Kuhs and Ball (1986) identified four views of mathematics teaching based on the focus of instruction. Ernest (1989, 1991a, 1991b) described several different, yet related categorisations based on teachers' conceptions of the nature of mathematics. Each of these categorisations has implications for problem-solving instruction and are described in the following sections.

2.4.2.1 A Categorisation of Mathematics Teaching Based on Pedagogical Thinking

One categorisation was proposed by Ammon and Hutcheson (1989) and was based on the development of teachers' pedagogical thinking. It suggested that problem-solving approaches were more likely to occur in classrooms where teachers had an integrated view of student learning. This developmental structure proposed five levels related to pedagogical conceptions that ranged from a traditional, teacher-centred approach based on "showing and telling" to more innovative, student-centred approaches based on problem solving by students. This categorisation was summarised by Levin and Ammon (1992) and is presented in Table 7.

Table 7
Levels of pedagogical conception as summarised by Levin and Ammon (1992, p. 21)

	Qualitative Level	Learning comes from:	Teaching is essentially:
1.	Naive Empiricism	Experiencing	Showing and telling
2.	Everyday Behaviourism	Doing (ie. Practicing)	Modelling and reinforcing
3.	Global Constructivism	Exploring	Providing hands-on experience
4.	Differentiated Constructivism	Sense making	Guiding thinking within domains
5.	Integrated	Problem solving	Guiding thinking across domains

In this categorisation, Levin and Ammon (1992) argued that "more advanced understandings are assumed to evolve from earlier ones in an invariant sequence" (p. 21). This suggested that for teachers to incorporate problem-solving approaches into their teaching, they would have developed appropriate pedagogical conceptions by progressively considering and necessarily rejecting other approaches at lower levels. A consequence is that teachers would only embrace such approaches after thinking about alternatives. It also suggested that to move to higher levels, teachers must think about their practices, reconsider their approaches, and move from lower level unidimensional thinking to higher level multidimensional thinking. This categorisation provides a framework for considering teachers' views about teaching and learning and the role of problem solving. However, teachers are likely to use a combination of these teaching actions depending on current teaching context.

2.4.2.2 A Categorisation of Mathematics Teaching Based on the Focus of Instruction

Another categorisation was proposed by Kuhs and Ball (1986). This suggested that there are four different views of mathematics teaching based on the focus of instruction. The authors stated that there are

> at least four dominant and distinctive views of how mathematics should be taught:
> 1. *Learner-focused:* mathematics teaching that focuses on the learner's personal construction of mathematical knowledge;
> 2. *Content-focused with an emphasis on conceptual understanding:* mathematics teaching that is driven by the content itself but emphasises conceptual understanding;
> 3. *Content-focused with an emphasis on performance:* mathematics teaching that emphasises student performance and mastery of mathematical rules and procedures; and
> 4. *Classroom-focused:* mathematics teaching based on knowledge about effective classrooms (p. 2).

It was proposed that the learner-focused teaching approach emphasised student engagement and involvement and was more likely to include problem-solving approaches than any of the other views. These four views provided another useful framework for discussing possible teacher actions but it is more likely that particular teachers will not fit neatly into one view as they may incorporate elements from different views into their teaching.

2.4.2.3 A Categorisation of Mathematics Teaching Based on Conceptions of Mathematics

Other related categorisations of mathematics teaching were proposed by Lerman (1983) and Ernest (1989, 1991a, 1991b). These were based on teachers' conceptions of mathematics, that is teachers' views about the nature of mathematics. Teachers' conceptions were also described by Thompson (1992) who suggested that they include "teacher's conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics" (p. 132).

Lerman (1983) proposed two teaching perspectives based on philosophies of mathematics. He named these the "knowledge-centred" and the "problem solving" perspectives. These were based on the "Euclidean program" which was "an attempt to base mathematics on firm foundations" and the "quasi-empirical program, the recognition that mathematics progresses heuristically, and through the re-transmission of falsity" (p. 59).

According to Lerman, the knowledge-centred perspective first focuses on learning methods with understanding followed by a consideration of relevance and applications.

The problem solving perspective focuses on "searching out solutions to problems at all levels from pre-school to research" (p. 63). Based on Piaget's theories of development, Lerman argued that the problem-solving approach "reflects both the conceptual growth view of mathematical knowledge and also the nature of the learning process" (p. 64).

One of Ernest's (1991a) categorisations is based on three commonly held philosophies of mathematics with accompanying theories of teaching and learning (Table 8). This suggested that the more traditional approaches to teaching and learning, that is explanation and acquisition, were associated with an absolutist conception of mathematics. More innovative practices involving discussion, negotiation and collaboration were associated with a fallibilist conception of mathematics.

Table 8
Three contrasting teachers' mathematics-related belief systems (Ernest, 1991a, p. 296)

PHILOSOPHY OF MATHEMATICS	THEORY OF TEACHING MATHEMATICS	THEORY OF LEARNING MATHEMATICS
ABSOLUTIST	TRANSMISSION	UNDERSTANDING
A fixed structured body of applicable pure knowledge	Clear explanation	Acquisition of meaning
	Illustration	Application
	Motivation	
PROGRESSIVE-ABSOLUTIST	LEARNER-CENTRED	DISCOVERY
An absolute body of knowledge uncovered by human activity	Facilitation of exploration	Active search for pattern
	Management of inquiry	Creativity
	Process emphasis	Experience
FALLIBILIST	NEGOTIATION	EMPOWERMENT
A social construction:	Discussion	Problem-posing
challenged, changing and reformulated	Negotiation	Discussion
	Overt discussion of teacher's role	Collaboration
		Development of Autonomy

Another categorisation proposed by Ernest (1989) suggested a hierarchy of teachers' conceptions about mathematics. These began with what Ernest called the "instrumentalist" view "that mathematics is an accumulation of facts, rules and skills to be used in the pursuance of some external end" (p. 250). Next there was the "Platonist" view of mathematics as "a static but unified body of certain knowledge", followed by the "problem-solving" view of mathematics "as a dynamic, continually expanding field of human creation and invention, a cultural product" (p. 250).

This categorisation also incorporated teaching approaches and the nature of learning and included additional information about the possible use of curriculum materials. Table 9 presents a summary of the categorisation including teacher's role, use of materials, and nature of learning.

Table 9
A categorisation of teachers' conceptions of the nature of mathematics, associated teachers' roles, use of curriculum materials, and the nature of learning (Ernest, 1989)

Conceptions of Mathematics	Teacher's Role	Use of Curriculum Materials	Nature of Learning
instrumentalist view	instructor	strict following of text or scheme	learning as mastery of skills
Platonist view	explainer	modification of textbook, supplemented by additional problems and activities	learning as the reception of knowledge
problem-solving view	facilitator	teacher or school construction of the mathematics curriculum	learning as the active construction of understanding, possibly even as autonomous problem-posing and problem-solving

The problem-solving view that is at the highest level in the hierarchy, suggested that the teacher acts as a facilitator as students actively construct understanding while engaging in problem posing and problem-solving tasks. In contrast to this, the other conceptions suggested a more traditional approach with the teacher as instructor or explainer and students mastering skills or receiving knowledge.

A further categorisation by Ernest (1991b) proposed five ideologies of mathematics education based on particular social groups and their aims of mathematics education. Table 10 summarises for each group the associated philosophy of mathematics, focus of the curriculum in relation to mathematical aims, roles of teachers, and approaches to problem solving. These views about problem solving have particular relevance for this investigation.

Table 10
Ideologies of mathematics education for particular social groups (Ernest, 1991b)

Social Group	Ideology	Mathematical Aims	Teacher's Role	Approach to Problem Solving
Industrial Trainer	dualist/absolutist	back-to-basics	authoritarian transmission	rejected as frivolous
Technological Pragmatist	multiplicist/absolutist	useful maths to appropriate level	skill instructor	also an adjunct - applied problem solving and mathematical modelling
Old Humanist	relativist/absolutist (separated)	transmit body of knowledge	explainer	an adjunct to enrich content - non-routine applications
Progressive Educator	relativist/absolutist (connected)	creativity, self-realisation through mathematics	facilitator of individual learning	a pedagogical approach
Public Educator	relativist/fallibilist	critical awareness through mathematics	facilitator within the social setting	a pedagogical approach - based on authentic materials and social problems

Based on these ideologies, Ernest (1991b) argued that there are three distinct views about the role of problem solving in the mathematics curriculum. The Industrial Trainers rejected problem solving as frivolous and a waste of time. Both the Technological Pragmatists and the Old Humanists viewed problem solving as additional content in the curriculum. They both valued problems as important applications of mathematical content and processes and so problems were treated as objects of inquiry. The third view was held by Progressive as well as Public Educators who saw problem solving as a pedagogical approach and not an adjunct to the curriculum. As Ernest stated "the full incorporation of these processes into the curriculum, including problem posing, leads to a problem solving and investigational pedagogy" (p. 288).

In summary, several attempts have been made to categorise mathematics teaching according to a variety of criteria. These include pedagogical thinking, the focus of instruction, and conceptions of mathematics. Regardless of the number of categories proposed, there appears to be a considerable difference in teaching approaches at either end of each categorisation. All suggest that at one extreme, there is an emphasis on explanation by the teacher and mastery of content by students. If problems are used in these settings, they are more likely to be applications of content, and viewed as additional to the curriculum or as another topic of study. An approach that suggests that the problems are viewed as objects of inquiry.

At the other extreme, there is likely to be teacher facilitation of approaches that support active construction of understanding by students. Students tend to be more autonomous, possibly engaging in problem posing as well as problem solving. In this case, problem solving is more likely to be viewed as a pedagogical approach, or as a process of inquiry. These categorisations of mathematics teaching suggest the possibility of a continuum of perspectives that is explored in the next section.

2.4.3 Perspectives about Mathematics Teaching and Learning

To facilitate a discussion about problem-solving beliefs and practices, an artificial continuum of teaching and learning, described here as perspectives about the role of problem solving, is used. In this investigation, perspectives refers to a teacher's overall view of mathematics teaching and learning incorporating knowledge, beliefs and preferences. One end of this continuum is the belief that mathematics is a fixed body of facts to be delivered by teachers and internalised by students referred to in this study as a *traditional* teaching approach. At the other end, teachers believe that mathematics is a dynamic subject to be explored and investigated and is referred to in this investigation as a *contemporary* teaching approach.

Each of these perspectives has associated with it a set of beliefs related to the teaching and learning of mathematics and how problem solving can be incorporated into the mathematics curriculum. Recognising the oversimplification and generalisation of this continuum, a characterisation of beliefs and practices helps to support the design of and data collection for this investigation. Each of these perspectives is discussed in greater detail in the following sections.

2.4.3.1 A *Traditional* Perspective

The *traditional* perspective is described as being associated with classroom practices involving individual student work with rehearsal of routine questions and reliance on textbooks or worksheets. The teaching approach is one of imposition whereby teachers may attempt to constrain students' efforts by insisting on prescribed methods (Cobb, 1988). This view may be accompanied by a belief that problem solving is an *end* and that problems should be presented to students after they have mastered basic facts and skills. This perspective is compatible with Ammon and Hutcheson's (1989) naive empiricist level of pedagogical thinking, Kuhs and Ball's (1986) content-focused view, Lerman's (1983) knowledge-centred perspective, and Ernest's (1989) instrumentalist view of mathematics teaching.

This perspective is also compatible with a view of teaching and learning that was reinforced by earlier mathematics curriculums. In these, computation was emphasised along with "rules governed by standards of accuracy, speed and memory" (National Research Council, 1989, p. 44) with less emphasis on thinking and understanding. To learn school mathematics, students were trained "by progressing through carefully scripted schedules of skill acquisition" (Battista, 1994, p. 463). The teacher was a manager of learners (Desforges & Cockburn, 1987) whose role was to demonstrate procedures and tell students how to perform similar tasks. The teacher, with the possible exception of the textbook, was the authority who knew all of the answers and was the source of mathematical truth and correctness (Garofalo, 1989). Certainly there was emphasis on answers rather than processes.

Teachers who support this perspective do not need to know a great deal about how children learn mathematics (Battista, 1994; Ernest, 1989). It is sufficient to accept that students passively receive information in a submissive and compliant manner and if they are able to reproduce procedures then this is evidence that it has been learned. The teacher instructs, often from a prescribed text, in which all of the necessary subskills are sequenced with sets of exercises to practise. There is little need for reflection or even an awareness that there may be viable alternatives to this method of teaching (Ernest, 1989).

As this investigation aimed to explore teachers' problem-solving beliefs and practices, it was appropriate to consider possible beliefs and practices associated with this perspective. Cobb (1988) suggested indicators of the extremes of this approach. In this, teachers were

> 1. believing that elementary school mathematics is basically arithmetic and that arithmetic consists of learning basic facts and standard computational procedures;
> 2. regarding specific facts and skills as isolated instructional goals;
> 3. inflexibly relying on the textbook;
> 4. teaching direct explanation or demonstration and then assigning individual seatwork on paper-and-pencil exercises;
> 5. dealing with instructional failures by repeating the demonstration-seatwork cycle one or more times; and
> 6. regarding students' alternative methods as undesirable behaviours to be eliminated (p. 97).

Beliefs that might be associated with a *traditional* perspective have been listed by Weissglass (1994) who determined these by discussing with teachers the beliefs that would support traditional approaches to teaching mathematics. These include

> you must master the content before you can use your brain to think mathematically;
> people learn mathematics by listening to someone talk about it and from doing homework problems;
> practice makes perfect;
> mathematics is best developed linearly; and
> students are incapable of deciding what to learn (p. 68).

This is one possible characterisation of a view of teaching and learning that was used to ascertain teachers' beliefs on a questionnaire designed for this investigation.

2.4.3.2 A *Contemporary* Perspective

At the other end of the continuum is a different characterisation described as a *contemporary* perspective. This perspective has been described as representing a reformed classroom (Clarke, 1995). Classroom practices associated with this perspective usually involve more group work and the use of non-routine questions that promote mathematical thinking and the development of problem-solving skills. The teaching approach may be described as negotiation (Cobb, 1988), and may be accompanied by beliefs that problem solving is a *means* and that problem solving can be the basis of a pedagogical approach to teaching mathematics. This perspective is compatible with Ammon and Hutcheson's (1989) integrated level of pedagogical thinking, Kuhs and Ball's (1986) learner-focused view of mathematics teaching, Lerman's (1983) problem-solving perspective, and Ernest's (1989) problem-solving view.

A curriculum based on this perspective assumes that mathematics is a "dynamically organised structure located in a social and cultural context" (Ernest, 1989, p. 250) where problems are posed so that students need to reorganise and rethink their current ideas. The teacher's role is that of a facilitator and requires considerable reflection as the teacher must observe student responses, challenge student thinking, and encourage risk taking within a supportive classroom environment. Teachers are also learners (Rowland, 1984) as they develop a sense of inquiry and critically analyse classroom practice. It is clear that this approach requires far more of the teacher than the *traditional* perspective.

Teachers need to know about how children learn mathematics (Battista, 1994) and to review constantly student understanding in order to "guide children's constructive activities until they eventually 'find' viable techniques" (Steffe & Cobb, 1988, p. vii). Steffe and Cobb also suggested that it is necessary to understand what the child already knows and what tasks are appropriate in order to accommodate growth. This knowledge can be gained by listening to exchanges between students, by asking students questions and by communicating ideas in whole-class discussions. In this way meaning is shared and negotiated between members of the social setting.

This approach to teaching seems to be most compatible with a view of learning that embraces constructivist principles (Cobb, 1988). However, Cobb argues that constructivism cannot provide an unambiguous set of teaching approaches since we do not know how all students construct concepts. He states

> The best that can be done is to propose general instructional heuristics compatible with teaching by negotiation and to suggest a variety of specific activities and interventions that might work with some children ... it is clear that the constructivist view of instruction implies that the teacher must be a reflective pedagogical problem solver who, in effect, conducts an informal research program (p. 101).

Possible beliefs and practices associated with this perspective have been suggested by Clarke (1997). In Section 2.3.5 of this chapter, Clarke's seven key components describing the teacher's role in a reformed classroom were listed. Associated with this, is a set of related beliefs about the teaching and learning of mathematics. These include

> Students can solve non-routine problems without first being taught a procedure.
> Mathematics needs to be studied in living contexts which are meaningful and relevant to students ...
> Differences in mathematical tasks and preferred learning styles of individuals demand variety in classroom organisation.
> An atmosphere of conjecture and justification of mathematical ideas enhances learning ...
> Important mathematical ideas are not confined to specific procedures in isolated content areas, but rather mathematics is seen as an integrated whole ...
> Observing and listening to students provides a 'window' into their thinking which can be used to plan further instruction (p. 280) [and]
> Reflection provides the opportunity to revisit recently encountered concepts and procedures and to identify connections ... (p. 295).

The implication here is that particular teachers' actions will be strongly influenced by a related set of beliefs. This set of beliefs helped to inform the design and data collection for this investigation.

2.4.3.3 Implications for this Investigation

For this investigation, it is proposed that the *traditional* and *contemporary* perspectives described above represent end-points of a continuum of beliefs. It is also assumed that the perspectives adopted by teachers represent their belief systems about the nature of mathematics and influence decisions made when teaching mathematics in classrooms. These perspectives were used to describe two imaginary teachers in the questionnaire designed for this investigation. One teacher held beliefs congruent with a *traditional* perspective and the other teacher held beliefs congruent with a *contemporary* perspective. It was anticipated that questionnaire respondents' agreement or disagreement with the beliefs of each imaginary teacher would provide an indication of their reported beliefs.

It is possible that many teachers hold beliefs situated somewhere between these end-points, or, that teachers' beliefs vary according to context. Hoyles (1992) proposed that beliefs may be embedded in particular situations and could vary according to the type of school, students in classrooms, and particular mathematics topics. It is also possible that since beliefs are fluid and that several different, and potentially contradictory, beliefs can be held at the same time, that teachers may fluctuate between different sets of belief systems. Hoyles argued that if beliefs are situated, then it is "self-evident that any individual can hold multiple (even contradictory) beliefs" (p. 40).

To address the issue of beliefs varying according to context, the beliefs of the two imaginary teachers were expressed in terms of teaching two digit addition to a Year 3 class. In this way, all questionnaire respondents would be considering the same situation when they indicated their level of agreement with each of the belief statements. However, it was also recognised that in making decisions when responding to questionnaire items, teachers could be strongly influenced by their experiences and possibly their current school situations. In this way, questionnaire responses provided a snapshot of beliefs and practices at a particular point in time.

In summary, teachers' knowledge, beliefs and practices are closely linked. Teachers make decisions based on their understanding and knowledge of students' thinking, mathematics topics in the curriculum, and the best ways to teach such topics. It is recognised that teachers teach in different ways and this has led to attempts to categorise mathematics teaching according to a variety of criteria. These criteria recognise the importance of conceptions and beliefs about mathematics teaching and learning. It has also been argued that beliefs filter knowledge and advice and play a critical role in the level of adoption of recommended approaches. The following section examines research into teachers' problem-solving beliefs and practices including evidence to suggest that many teachers have not responded to the advice to adopt problem-solving approaches.

2.5 Research into Teachers' Problem-Solving Beliefs and Practices

Thompson (1992) suggested that while a great deal of the literature has explored teachers' beliefs and practices in relation to the teaching and learning of mathematics, few studies have focused on teachers' problem-solving beliefs and practices. Given the increased attention to problem-solving approaches in curriculum documents and advice to teachers, this appears to be a fruitful area for investigation. Any study in this area would need to consider several components in the model that guides this investigation (Figure 5).

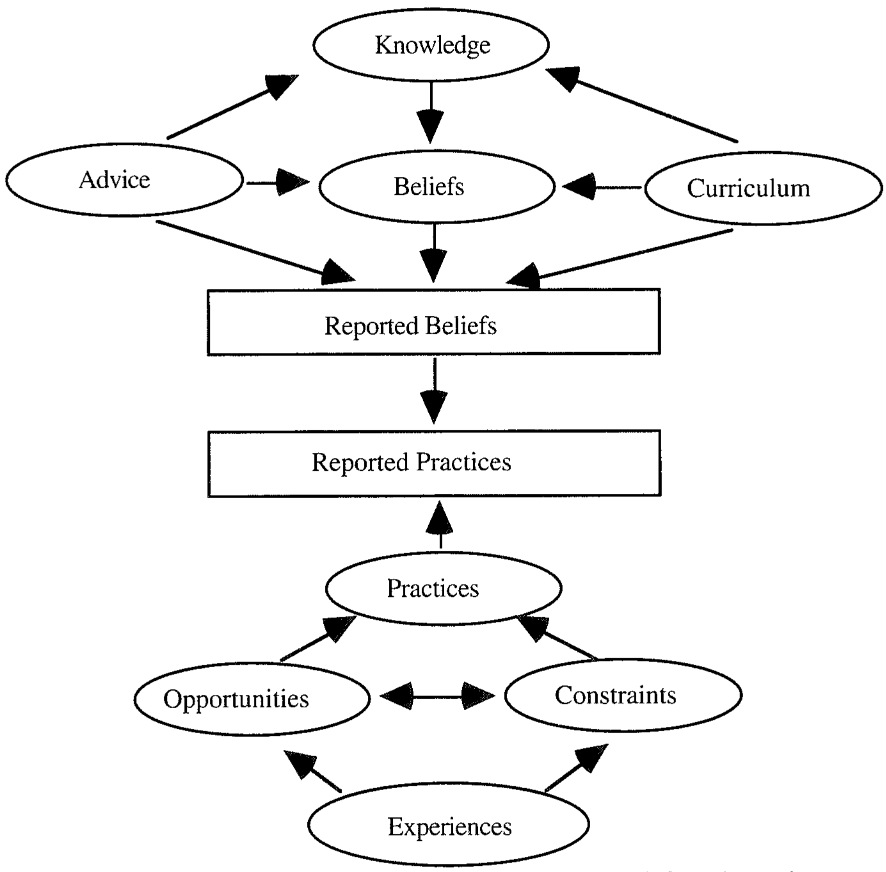


Figure 5. The factors that impact on teachers' reported beliefs and practices.

Considerable research on problem solving has been conducted since the early eighties. Most of this research has explored strategies for teaching problem solving and methods for improving learning outcomes for students (Schoenfeld, 1992). There have been recommendations to investigate teachers' beliefs about problem solving and teachers' attempts to implement problem-solving approaches in their classrooms (Thompson, 1992). The following sections describe studies that have examined teachers' problem-solving beliefs and practices and possible inconsistencies between beliefs and practices.

2.5.1 Teachers' Problem-Solving Beliefs and Practices

It appears that teachers' problem-solving beliefs and practices vary considerably. Teachers hold a variety of beliefs about the terms *problem* and *problem solving* and also about problem-solving instructional practices. For many teachers, there is a consistency between reported beliefs and practices and also between reported practices and what actually occurs in their classrooms. However, for some teachers, reported practices are not readily observed; a situation that suggests either lack of reflection or the possibility of constraining factors that might be adversely influencing teachers' plans.

One study examined teachers' views about problem solving and found that teachers hold a variety of views about what constitutes problem solving. Funkhouser (1993) asked teachers attending inservices on mathematical problem solving to describe what was meant by the term *problem solving* as it was used in a realistic school-based scenario. Two-thirds of the responses were categorised as vague, for example

> Problem solving is finding a solution to a problem.
> Problem solving is using thinking skills (p. 82).

He found that only one-third of teachers were able to give a precise definition that involved either reference to strategies or skills such as "problem solving is identifying the problem issue, determining the steps, then solving the problem" (p. 83) and suggested that teachers need more advice. The variety of responses in this study suggested that teachers who have an interest in the area of problem solving still display a range of knowledge and understandings about key definitions.

Teachers' conceptions of problem solving and its instruction were explored by Grouws, Good and Dougherty (1990). Twenty-five junior high school teachers were interviewed and their conceptions were categorised as:

- problem solving is word problems;
- problem solving is solving problems;
- problem solving is solving practical problems; and
- problem solving is solving thinking problems.

Many of the reported instructional practices matched teachers' conceptions of problem solving. For example, those who believed that problem solving involved solving practical problems usually used examples of these in their classrooms. However, the researchers concluded that while there was evidence of relationships between beliefs and practices, external factors seemed to impact on instructional practices. These external factors included "textbooks, district expectations, and standardised testing" (p. 135).

In another study, Dougherty (1990) examined the relationship between teachers' cognitive levels and problem-solving instruction. Eleven upper primary teachers were involved in case study research that incorporated written assessment, interviews, and classroom observations. A cognitive level for each teacher was determined by the administration of a written assessment that required participants to respond to open-ended statements encouraging reflection on beliefs. These responses were then independently evaluated to ascertain the appropriate cognitive level, from the four identified levels, for each teacher. Dougherty stated that

> The four cognitive systems lie on a concrete-abstract continuum. The most rigid of the systems, A, is identified as concrete formalism ... System D, abstract constructivism, is the most abstract or flexible of the four systems (p. 120).

The teachers were also classified according to their agreement with the following views of mathematics as a discipline.

> (1) Mathematics is a set of rules and procedures,
> (2) Mathematics is a tool for everyday life,
> (3) Mathematics is an application of logical thinking and/or step-by-step methods,
> (4) Mathematics is experiential and not a static body of knowledge (p. 123).

In addition, conceptions about problem solving were identified for each teacher. These were the same four conceptions that were used in the Grouws et al. (1990) study discussed above.

Dougherty (1990) reported that classroom observations revealed interesting differences between teachers depending on their cognitive levels and also that there were consistencies between cognitive level qualities and instructional practices in relation to problem solving. For example, for those teachers who were classified as cognitive level A (the rigid concrete formalism level) representing a majority of 8 of the 11 teachers, lessons were teacher directed, inflexible, and focused on "an algorithmic or mechanistic style" (p. 125). In contrast, the single teacher who was classified as level D (abstract constructivism) supported student participation, encouraged students to take responsibility for their contributions to lessons, accepted differences in solution strategies, and acknowledged creative thinking.

In another study, Thompson (1989) argued that one of the difficulties in encouraging teachers to teach problem solving relates to their views about what constitutes a problem in mathematics. She conducted a study

> to document changes in the conceptions of mathematical problem solving of 16 elementary school teachers over a three-week summer course on problem solving and after a year of teaching problem solving in their classrooms (p. 234).

She found that five of the teachers believed that

1. It is the answer that counts and when an answer is found the problem is finished.
2. There is a right way to find the answer.
3. The answer is usually a number.
4. Every context is associated with a unique procedure for getting answers.
5. It is important to know and remember what to do.

She argued that such beliefs would impact on classroom practices and could potentially reinforce similar students' beliefs. The remaining teachers in the study held views that "reflected a more generalised view of what constituted a mathematical problem" (p. 235).

Thompson (1989) also documented changes in conceptions about problem solving and appropriate problem-solving teaching approaches during the year. Not all teachers successfully implemented problem-solving approaches and regularly expressed insecurity with their attempts. In addition, Thompson described ongoing obstacles to implementation such as time constraints, resistance to change by students, and the spread of abilities of students in classrooms. These issues will be discussed in more detail in Section 2.6 of this chapter.

Earlier, Thompson (1984) had conducted case-study research with three junior high school teachers. Classroom observations over a four-week period combined with interviews and written tasks revealed that the teachers held quite different views of mathematics and demonstrated contrasting practices in classrooms. In relation to problem-solving practices, Thompson (1985) described two of the teachers, Jeanne and Kay. Jeanne held conceptions about mathematics that were more absolutist in character supporting rather traditional teaching practices. Thompson stated

> Jeanne's discomfort with unanticipated events in the course of a lesson, which was a result of her strong concern for control, appeared to rule out the use of a problem-solving approach to teaching (p. 287)

The only effort to incorporate problem solving was to issue an assignment sheet with 12 story problems. Jeanne professed to support active student participation but observations suggested that there was little evidence of discussions between students or even between students and teacher.

In contrast to Jeanne, Kay held a view of mathematics that was more fallibilist, her practices were more contemporary in nature, and there was a greater consistency between her beliefs and practices. She engaged students in discussions, used a variety of enquiry-based activities, and frequently incorporated problem-solving sessions into her program. She stated that she wanted to develop her students' reasoning skills and was concerned that they had positive attitudes to mathematics. In this study, there was a high level of consistency between Kay's beliefs and her instructional practices.

Thompson (1985) suggested that this was a consequence of Kay's frequent reflections about instructional actions. The role of reflection seems to be a critical factor in the consistency of implementation of beliefs into practice (Clarke, 1997; Ernest, 1989; Levin & Ammon, 1992). It is possible that inconsistency may lead to a potential mismatch between beliefs and practices.

2.5.2 The Existence of a Mismatch Between Beliefs and Practice

Several studies have found a gap, or a mismatch, between teachers' beliefs and practices. Thompson (1985) reported that Jeanne did not implement student discussions even though she believed that this approach supported learning in mathematics. In addition, studies by Cooney (1985), Levin and Ammon (1992), and Van Zoest, Jones and Thornton (1994) documented other cases of inconsistency.

Cooney (1985) examined the beliefs about problem solving of Joe, a beginning mathematics teacher. Joe professed to support a problem-solving view of mathematics and yet his classroom practice relied heavily on a textbook with a "cookbook" style of presenting information. Problems were presented as separate to content and seemed to be used for interest and motivation rather than as the basis of instruction. Joe blamed lack of time for preparation as well as students' lack of interest and ability for the discrepancy. Joe also realised that the approach he wanted to adopt conflicted with student expectations. Joe suggested that he was unable to put his beliefs into practice and Cooney argued that this was a result of Joe's inexperience and his beliefs about what the role of problem solving really was in relation to the learning of mathematics.

Some studies have found differences between teachers within the same study as far as the extent of the gap or mismatch between beliefs and practices, or between reported practices and actual practices. Levin and Ammon (1992) used four case studies to explore the development of beginning teachers' pedagogical thinking and found that there was not a "one-to-one correspondence" between teachers' thoughts in interviews and actions in the classroom. One of the teachers in their study, Sally, described much frustration at trying to integrate computation and problem solving. However, during lessons, Sally was observed successfully integrating the two. Another teacher, Julie, described higher level pedagogical concepts but was observed using a teacher-centred, non-interactive approach. The researchers suggested that Julie's practice was a result of her need to be liked by the students as well as her use of the students' desire to please her as a management strategy. The final two case studies in this research had a much closer match between pedagogical thinking and pedagogical practice.

Van Zoest et al. (1994) compared the beliefs about mathematics teaching of four pre-service elementary teachers who were involved in an intervention program, with non-involved peers. The intervention group were involved in small-group teaching experiences and workshops. This group professed stronger beliefs in socio-constructivist instructional practices by the completion of the intervention but they were not uniformly successful in putting these beliefs into practice.

These researchers used a structured approach, consisting of four distinct phases, to act as a guide to the implementation of problem-solving approaches. They argued that this structure promoted discussion between students and hence encouraged students to share, refine and defend possible solutions. The structure included:

(a) a problem statement and clarification phase;

(b) a solution exploration phase;

(c) an impasse relief phase; and

(d) a solution presentation and interpretation phase.

These phases were then used to assess whether pre-service teachers were able to implement a socio-constructivist perspective into their teaching of mathematics.

The researchers found that there were differences in the ability of the four student teachers to implement these phases. After the first phase, or problem statement and clarification, the implementation of stated beliefs became more difficult and student teachers' actions diverged to a more traditional perspective. Van Zoest et al. (1994) suggested that this may have been due to the teachers' feelings of unease as pupils struggled with problems combined with their doubts about the ability of some pupils to solve particular problems. There also appeared to be concerns about the amount of time being taken to solve each task.

There are inconsistencies in the research findings in relation to the existence of a gap between reported beliefs and classroom practices. It is possible that this has occurred as a consequence of the different data collection methods, or because teachers may hold beliefs about teaching and learning that could be different to beliefs about instruction. Some researchers have found a high level of agreement between teachers' beliefs about teaching and instructional practice (Thompson, 1992) while others have reported significant gaps (Thompson, 1984; Cooney, 1985). These inconsistencies highlight the complexity of the relationship.

Thompson (1992) suggested that one explanation for inconsistent research findings is the way that beliefs are measured. She cautioned against reliance on verbal responses as the only source of data since

> some of the beliefs professed by teachers are more a manifestation of a verbal commitment to abstract ideas about teaching than of an operative theory of instruction (p. 138).

Lerman (1998) also argued that the issue may be one of different data collection methods, or that researchers may be trying to compare different things. He proposed that teachers may have beliefs about teaching and learning that could be different to beliefs about instruction.

Further support for inconsistent teachers' beliefs occurred when Sosniak, Ethington and Varelas (1991) used data from the Second International Mathematics study to explore teachers' beliefs. The authors expressed concern that some teachers in the USA seem to

> ... teach their subject matter *without* a theoretically coherent point of view. They hold positions about the aims of instruction in mathematics, the role of the teacher, the nature of learning, and the nature of the subject matter itself which would seem to be logically incompatible (p. 127).

In an attempt to explain these discrepancies, Sosniak et al. suggested that it could be the method of data collection. Another possibility was the "distance" between the variables in the study and the act of teaching in one's classroom. They argued that

> We have a set of findings regarding teachers' curricular orientations which shift systematically from "progressive" to "traditional" as the teachers move from considering the issue most distant from schooling and classroom instruction to the issue most central to schooling and classroom instruction (p. 129).

To account for this mismatch, Hoyles (1992) discussed the possibility of a distinction between *beliefs* and *beliefs in practice* and argued that beliefs may be context specific. If this is the case, then the notion of a gap or a mismatch may not be relevant and it may be more profitable if research sought to determine the factors that might have contributed to the differences. Another important aspect worthy of exploration is the factors that link and impact on beliefs and practices.

The studies described in this section highlight the range of teachers' problem-solving beliefs and practices and confirm the difficulties inherent in using recommended problem-solving approaches in classrooms. This investigation aimed to determine factors that might support or constrain teachers' efforts to implement problem-solving approaches. In earlier sections of this chapter, considerable advice has been offered that could support teachers' efforts to implement problem-solving approaches. The next section focuses on the level of adoption of problem-solving approaches and the factors that might constrain teachers' efforts.

2.6 The Adoption of Problem-Solving Approaches

This section examines the level of adoption of problem-solving approaches presented to teachers in advice and curriculum materials. Figure 6, a section of the model that guides this investigation, indicates that such practices may be influenced by opportunities, constraints and teachers' experiences.

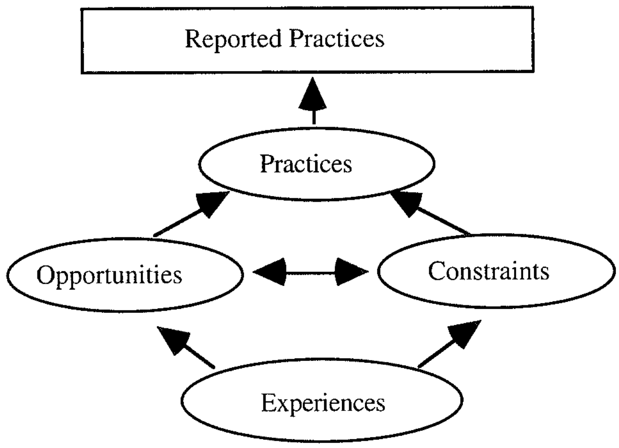


Figure 6. Factors that impact on teachers' classroom practices.

Opportunities are those factors that support teachers' efforts and could include advice from preservice and inservice courses, curriculum materials and professional journals, that promote problem-solving approaches. Constraints are those factors that interfere with teachers' efforts and could include students', parents' and other teachers' negative beliefs about problem solving, rigid school structures and programs that do not incorporate problem-solving approaches, and external assessment procedures that focus on basic skills. Teachers' previous problem-solving experiences as learners and teachers of mathematics may also either support or constrain their efforts depending on whether those experiences were successful or unsuccessful.

In this section, prevailing classroom practices are discussed and some reasons for lack of implementation of calls for reform are considered. Constraining factors that impact on teachers' practices and hence their reported practices are examined. Also, it is recognised that constraints may be influenced by opportunities as well as by the previous experiences of teachers.

2.6.1 Issues that Impact on the Adoption of New Approaches

Observations about the lack of implementation of problem solving in classrooms have occurred internationally as well as in Australia. In the UK, research by Desforges & Cockburn (1987) indicated that teachers had not responded even though they were aware of current curriculum advice, and in the USA, similar claims were made in a report from the National Research Council (1989). The *Mathematics Report Card* (Dossey, Mullis, Lindquist, & Chambers, 1988) summarised American students' performance on the 1986 National Assessment and concluded that

> the rarity of innovative practices is a matter of true concern. Students need to learn to apply their newly acquired mathematics skills by involvement in investigative situations, and their responses indicate very few opportunities to engage in such activities (p. 76).

In Australia, *The Discipline Review of Teacher Education in Mathematics and Science* (Speedy, 1989) suggested that the teaching of mathematics in classrooms was "little different from what it was 20 years ago" (p. 16). Similar observations and concerns that teachers have not adopted new approaches have continued to appear in the literature since then (e.g. Blane, 1992; Pegg, 1997; Sullivan et al., 1992). It would seem that even though problem-solving approaches have been widely advocated for at least twenty-five years, these recommendations have not been realised in a large number of mathematics classrooms.

Many authors have argued that in spite of international and national calls for reform in mathematics classrooms, there has been limited change in teaching practices (Acquarelli & Mumme, 1996; Brown et al., 1990; Prawat, Remillard, Putnam, & Heaton, 1992; Schoenfeld, 1992; Stigler & Stevenson, 1991; Weissglass, 1994). Several reasons are postulated that may account for the lack of implementation although it should be noted that there are teachers who have embraced these reforms (Lampert, 1990).

In brief, the reasons include the suggestion that teachers may not understand new approaches and how to implement them successfully in classrooms. Alternatively, if teachers are aware of innovative practices, they may not be confident about implementing such ideas particularly if they believe that their current practices are successful (Desforges & Cockburn, 1987). Teachers may be cautious about embracing another "fad" that may not really work (Mousley, Clements, & Ellerton, 1992), or that may just disappear like earlier fads (Sperry Smith, 1997). Tensions may exist between aiming to promote investigative approaches and concern about fulfilling all of the curriculum requirements (Jaworski, 1991).

All of these reasons for lack of implementation relate to teachers' knowledge, understanding and experiences of such approaches but there may be other factors that impact on the level of adoption. Such factors could include the attitudes and beliefs of students, aspects of school culture such as beliefs of members of the school community, and education system requirements that might include standardised testing. Such factors may support implementation but there appears to be evidence to suggest that each of these can act as powerful constraints.

2.6.2 Factors that Constrain Teachers' Efforts to Implement Problem-Solving Approaches

Given the advice, power of the argument, and resource development, there are still concerns about the limited implementation of problem-solving approaches. Teachers may be reluctant to change existing practices, or they may want to implement problem-solving approaches but are constrained by a variety of factors. Most constraints discussed in the literature can be grouped into four broad categories; those relating to the teachers themselves, to the students, to school culture, and to the education system. Each of these is discussed in more detail in the following sections.

2.6.2.1 Teachers as a Constraining Factor

Some constraints on the adoption of problem-solving approaches relate to the personal characteristics of teachers including knowledge, beliefs, confidence, past experiences, and the level of teacher thinking, particularly in relation to reflection. Teachers may have good intentions and even believe that they should use problem-solving approaches in classrooms but have difficulty with implementation. It may be the case that inconsistencies between beliefs and practices are a consequence of the lack of skills and knowledge that are necessary to implement teaching ideals.

The key factor of teachers' knowledge has been discussed in a previous section but it is pertinent to consider that it can act as a powerful constraint. Thompson (1989) suggested that for teachers to be able to implement problem-solving approaches they must be "knowledgeable about and feel comfortable 'doing' mathematics" (p. 234) but it is also important that teachers have knowledge about how children learn as well as specific knowledge about what their students know (Fennema et al., 1989; Levin & Ammon, 1992; Putnam, Heaton, Prawat, & Remillard, 1992). It has also been argued that teachers' knowledge and beliefs influence how they interpret and implement curriculum documents (Clark & Peterson, 1986; Romberg & Carpenter, 1986).

Teachers' beliefs have also been discussed earlier but it is also clear that the existence of particular beliefs can influence teaching actions and constrain problem-solving attempts. In a case study of the adoption of a new curriculum that contained innovative practice including problem solving, Putnam et al. (1992) observed three beliefs that seemed to impact on the level of adoption of the reforms advocated in the *Californian Mathematics Framework* (California State Department of Education, 1985). These beliefs included

- teaching as telling;
- learning should be fun, engaging and relevant; and
- understanding may have to wait.

The researchers observed that these beliefs reinforced the teaching of basic facts and computational skills rather than problem-solving competence. A more detailed description of the case study is included in Appendix 3.

Other constraints include teachers' interpretation of advice and their ability to implement such recommendations. Ernest (1991a) argued that these were the two main obstacles to the curriculum reform proposed by Cockroft (1982) and the NCTM (1980, 1989) publications. The concepts of problem solving and investigations are interpreted through the perspectives of teachers based on their beliefs about mathematics. Ernest suggested that implementation depends on the relationship between teachers' theories of teaching and learning and classroom practices. He states

> On the large scale, this is the difference between the planned and the taught curriculum. On the small scale, this is the difference between the teacher's espoused theories of teaching and learning, and the enacted versions of these theories (p. 289).

The tension that exists between theories about mathematics teaching and learning and putting those theories into practice is clearly another reason why teachers struggle to implement problem-solving approaches (Confrey, 1993; Mousley et al., 1992). Cobb (1986) identified tensions between maximising student achievement and promoting students' positive attitudes, as well as the demands of coverage of the curriculum and developing students' understanding. Mason (1986) described the dilemma of the didactic contract between teachers and students. He argued that the teacher must foster learning and the student must learn although students usually "wish to be told what they need to know, and often they wish to invest a minimum of energy in order to succeed" (p. 20). Mason also proposed that the dilemma for teachers is

> The more the teacher is explicit about what behaviour is wanted, the less opportunity the pupil has to come to it for themself and make underlying knowledge or understanding, their own (p. 20).

Tensions between theory and practice were also described by Jaworski (1991). She identified the existence of a didactic/constructivist tension in her case study research and argued that this tension occurs when teachers recognise that their students will make their own constructions in lessons while the curriculum requires "certain formal constructions to be made by pupils and tested by the establishment" (p. 214). Mousley et al. (1992) summarised the situation regarding tensions as follows

> There appears to be an inherent tension between the idea of creating teaching/learning environments in which children own the mathematics they learn and the expectation that school mathematics will assist children to acquire an understanding of a more or less fixed set of socially valued mathematical concepts, skills and principles (p. 116).

Coping with these tensions is difficult for teachers and this impinges on decisions about what to teach and how to teach it. Reflecting on such tensions and reconsidering differences between theories and practice seems to be necessary if they are to be resolved.

This suggests that the role of reflection may be a critical element in narrowing the gap between beliefs and practice (Clarke, 1993; Ernest, 1989; Levin & Ammon, 1992; Richardson, 1990; Thompson, 1984). It is through high levels of thought that the teacher is able to consider inconsistencies and develop strategies that might close this gap (Ernest, 1989). Critical self-evaluation can highlight personal strengths and weaknesses as well as alert the teacher to the constraints that are impinging on practice. It may not be possible to change readily some of the constraints but an awareness of them may encourage the teacher to discuss concerns with other members of staff. In Clarke's (1993) study, one of the teacher participants, Mr Martin, identified inconsistencies between his beliefs and practice with regard to basic skills and assessment. Although these were not resolved in the course of the study, the fact that Mr Martin was aware of this may lead him to a resolution in the future.

Another possibility is that teachers need to rethink their role and to be prepared for the unexpected; a situation that may make many teachers feel uncomfortable. Burkhardt (1988) suggested that there are three important factors that make it more difficult for teachers to implement problem-solving approaches in their classrooms. Students can often invent and create different solution methods and procedures than those the teacher anticipated. It then becomes necessary to evaluate these methods and provide meaningful feedback. Also, the teacher needs to make instant decisions about when to intervene and what advice to provide without telling the students too much. Finally, teachers may find themselves in a position of not knowing all of the possible answers or outcomes for a particular task and this can be rather threatening.

It does not seem to be the case that teachers are not aware of the recommendations, or that they do not understand the implications. Crawford (1982) argued that even though teachers know about theory, most continue to teach as they were taught and she suggests that translating theory into practice may be too complex for many teachers. Desforges and Cockburn (1987) reported that traditional practices endure as "they are the result of teachers' efforts to manage a situation which stretches their resources to the limits" (p. 22). It is possible that if trying out innovative practice means giving up control, teachers will hesitate as control will always take precedence (Brown & Cooney, 1991).

Others have commented on the reality of teachers' daily lives and the lack of support to assist them in achieving such goals (Pegg, 1997). Some have proposed that statements abound regarding the principles of good mathematics teaching but there are insufficient guidelines about appropriate teaching practices to facilitate the attainment of these goals (Sullivan et al., 1992). Price and Lowenberg Ball (1997) suggested that a lack of resources has meant that any observable changes in classroom practices are cosmetic and that real change "would require substantial intellectual resources - ideas, images, material, time - to provide opportunities to learn about mathematics, students and pedagogy" (p. 638).

In an effort to justify their lack of response, teachers readily provide reasons that incorporate difficulties with implementation and insufficient resources. Wilson, Fernandez and Hadaway (1993) listed some reasons that teachers gave to explain why they did not use more problem-solving approaches in their classrooms. These included

> problem solving is too difficult;
> it takes too much time;
> the school curriculum is very full, and so there is no room for problem solving;
> problem solving will not be measured and tested;
> because mathematics is sequential, students must master facts, procedures and algorithms;
> appropriate mathematics problems are not available;
> problem solving is not in the textbooks; and
> basic facts must be mastered through drill and practice before attempting the use of problem solving (p. 66).

To satisfy curriculum requirements, some teachers attempt to implement problem solving by having one-off lessons that do not integrate with the current curriculum; this leads to a "discontinuity between teaching problem solving and teaching traditional mathematics content" (Pegg, 1997, p. 33). Teachers' good intentions mean that they will try to respond to advice in the way that best suits their circumstances.

In summary, constraining factors that relate to the teachers themselves include their knowledge and beliefs about problem solving, level of interpretation of advice, resolution of tensions between theories and practice, ability and willingness to reflect on their practice, level of discomfort with less control in the classroom, and availability of resources that include ideas, images and time. Students in particular class settings can also interfere with teachers' attempts to incorporate problem solving into lessons or to use problem-solving approaches.

2.6.2.2 Students as a Constraining Factor

Constraints relating to the students can include their beliefs about mathematics and what constitutes mathematical activity, their resistance to new approaches, and behaviours associated with these reactions. Students' beliefs about mathematics in general, and problem solving in particular, can determine their reactions to teachers' attempts to use problem-solving approaches in classrooms. Teachers' perceptions of the abilities of students in their classes impacts on the types of tasks they choose to use in mathematics and the kinds of problem-solving activities that are incorporated into programs.

Students can develop narrow beliefs about mathematics as a result of the way they have been taught, and these views in turn can have an effect on the way they respond to subsequent teaching. Cobb (1986) stated

> students who have constructed instrumental beliefs about mathematics [Skemp, 1978] anticipate that future classroom mathematical experiences will "fit" these beliefs. They intend to rely on an authority as a source of knowledge, they expect to solve tasks by employing procedures that have been explicitly taught, they expect to identify superficial cues when they read problem statements, and so forth (p. 4).

Frank (1988) also indicated that students who hold such beliefs do not accept problem solving as mathematics. This is supported by Stephens and Romberg (1985) who found that students did not take mathematics lessons seriously if they were not associated with the textbook.

For problem solving to be a focus of mathematics classrooms, students' beliefs about the nature of mathematics may need to be challenged, yet this can be frustrating for teachers. For example, Joe, the beginning teacher in Cooney's research (1985), encountered considerable resistance from the students in his classroom as he attempted to introduce problem-solving tasks. This also happened to Imwold (Tobin & Imwold, 1993) and to the teachers in the Thompson study (1989). Garofalo (1989) observed an eighth grade teacher who tried to encourage his students to invent and justify their own methods to solve problem situations. His attempts were met with comments such as

> You're not a math teacher;
> This is not how you are supposed to teach math; and
> You should show us what to do, not ask us how to do it (p. 453).

This resistance can challenge classroom order (Tobin & Imwold, 1993) and make teachers feel rather uncomfortable.

Another issue that can threaten classroom order relates to the use of problem-solving tasks that are open-ended since these tasks present students with high levels of risk and ambiguity (Desforges & Cockburn, 1987). This can disrupt the accountability system in the classroom (Doyle, 1986). Students may also respond by passive dismissal and a lack of interest (Cooney, 1985). An easy solution for teachers can be to abandon attempts to use problem-solving approaches even though they may believe that this is the best way for students to learn mathematics.

A wide spread of abilities of students in classrooms also impacts on the implementation of problem-solving approaches (Thompson, 1989). Romberg (1984) reported that teachers tend to choose tasks on the basis of the perceived ability of students. He discovered that more able students tend to be given problem-solving and higher-order tasks while less able students are more frequently presented with drill-and-practice exercises. It seems that some teachers believe that students need to know basic skills and computational strategies before they can solve problems. It is also possible that such decisions may be made on the basis of other abilities such as language and social skills. Children need to be able to read and interpret problems and those pupils who have not developed the necessary skills may have difficulty (Noddings, 1989).

Problem-solving classrooms require students to work together, share understandings, support and encourage collaborative efforts, all of which require well developed social skills. If teachers perceive that their students will not be able to work cooperatively (Tobin & Imwold, 1993) or cope with this approach, particularly in classes with a wide range of ability (Thompson, 1989), then they may not adopt practices congruent with their beliefs.

In summary, students can constrain teachers' efforts by withdrawing participation because chosen tasks and activities do not match their beliefs about legitimate mathematical activity. Also, the increased risk associated with many problem-solving tasks can worry students, threaten classroom order, and unsettle teachers. Students' mathematical ability as well as language and social skills all impact on the types of tasks teachers choose and hence may limit tasks to lower-order questions.

2.6.2.3 School Culture as a Constraining Factor

Another constraining factor relates to school culture. Weissglass (1994) defines culture as follows

> culture is the attitudes, beliefs, values, and practices shared by a community of people which they often do not state or question and of which they may not be consciously aware (p. 68).

Culture is taken here to include school programs and policy statements, assessment procedures and reporting practices, parents' beliefs and expectations, school structures and available resources, as well as the beliefs and practices of executive members and fellow teachers in the school setting.

From these considerations, it is clear that accessing school culture could be a real challenge but it is necessary to consider the above elements of school life as these may impact on teachers' efforts to implement problem-solving approaches. Elements of school culture can be so powerful that teachers working in the same school who have quite different beliefs have been observed to adopt similar classroom practices (Ernest, 1989; Lerman, 1983). Teachers may have less flexibility in their planning because school programs have been restricted by curriculum decisions already made by the Principal (Clark & Peterson, 1986) or other school Executive members.

Parents are influenced by the way they learnt mathematics and can have a constraining influence on teachers practices (Clarke, 1994). If the school community values the learning of number facts by rote and sees mathematics as practising algorithms then teachers' attempts to focus on problem solving may be interpreted as not "real" mathematics (Tobin & Imwold, 1993). Also, school assessment practices and reporting procedures to parents may focus on skills other than problem-solving competence.

Other aspects of school structures such as available resources, including textbooks and commercial schemes, can also inhibit the realisation of teachers' aims (Desforges & Cockburn, 1987). In an intensive investigation in the United Kingdom of the teaching practices of seven experienced first (primary) school teachers, Desforges and Cockburn reported that all of the teachers were "perfectly aware of the aspirations held for mathematics education and that they fully endorsed these aims" (p. 124). Also, these teachers had

> an elaborate view of learning which saw a place for inquiry, practice and problem solving and they articulated the range of teaching behaviours necessary to make the most of such a multifaceted theory of the acquisition of knowledge and skill (p. 124).

However, the teachers were unable to implement these practices even though they were "teachers of considerable quality" (p. 124).

Desforges and Cockburn (1987) argued that a number of factors impacted on teachers' attempts and these factors seemed to push them towards direct instruction and drill-and-practice and away from inquiry methods. Two of these factors included the program of work and working conditions. Time limitations to cover the amount of material in the program as well as to prepare good teaching materials were of major concern. Task demands created by commercial schemes were often unpredictable resulting in a practice of explaining every task to protect the children from confusion. The children generally worked alone, were less interested in discussion, and treated task completion as a race. Because classes usually had thirty pupils, the teachers found it difficult to diagnose and cater for individual needs and so approximately one-third of the students in each of the classes could not really cope with the level of difficulty of the assigned work.

Teachers can also be constrained by what their peers believe should be taught and how it should be taught at each year level (Tobin & Imwold, 1993; Wilcox, Schram, Lappan, & Lanier, 1991; Wright, 1994). This is of real concern in the first year of schooling when children are frequently under challenged given the informal knowledge that many of them bring to school (Desforges & Cockburn, 1987; English, 1990; Fennema et al., 1989; Groves & Stacey, 1990). These early experiences with learning mathematics can have consequences in later years "because it is in the first three years that the child first experiences success or failure, interest or boredom, challenge or frustration" (Wright, 1994, p. 23).

In the Wilcox et al. (1991) study, Alison, a first year teacher of fourth graders, felt considerable pressure from the Principal and the Fifth Grade teachers. She was told "that they expect the students who leave her class to have mastered computational facts" (p. 37). Alison wanted to teach in non-traditional ways but resorted to spending much of her time with children working individually on drill-and-practice and timed tests. This pressure can be compelling for young, or inexperienced teachers.

The impact of school culture should not be underestimated. The influence of particular classrooms and the social structure and practices within schools can strongly influence teachers' actions. Hoyles (1992) argued that

> It seems that actions and beliefs are shaped by the conditions of classrooms and teacher decisions stem more from the social practices which frame teaching than the cognitive structures and beliefs of individual teachers (p. 37).

In summary, constraints on teachers arising from the school culture include pressure from authority figures and parents, internal assessment and reporting procedures, prescribed texts and commercial schemes, class size, and the beliefs of other teachers. Teachers' intentions to implement problem-solving approaches can be severely curtailed by the expectations of members of the school community and pressures to conform. In addition to these, there are education system requirements that frequently place teachers under considerable pressure to focus on particular aspects of curriculums and assessment rather than on implementing desired instructional practices.

2.6.2.4 Education Systems as a Constraining Factor

Curriculum documents, external assessment procedures, and programs and projects that place emphasis on particular teaching approaches, can be imposed on schools and teachers by education systems. These requirements can interfere with teachers' intentions to implement innovative practices. Grouws et al. (1990) found that even though the teachers in their study held beliefs that supported problem-solving approaches, district expectations and standardised testing had a constraining effect on implementation.

It should be noted that education system requirements can both support and constrain teachers' efforts to implement new approaches. On the one hand, curriculum documents may promote and encourage innovative practices; for example, in NSW the new *Mathematics K-6: Outcomes and Indicators* (BOSNSW, 1998) support document has introduced a Working Mathematically strand that includes Problem Solving outcomes. On the other hand, a mandated curriculum can put pressure on teachers to cover the content in a fixed time period, regardless of students' needs and abilities.

Time is critical for many teachers who complain about disruptions and lack of time when they feel pressure to present all of the material to all of their students. Several studies (Cooney, 1985; Desforges & Cockburn, 1987; Thompson, 1989; Tobin & Imwold, 1993; Van Zoest et al., 1991; Wilcox et al., 1991; Wilson et al., 1993) found that teachers were concerned about completing the program in a given time and felt they had to abandon notions of students learning at their own pace. It can seem easier to just "tell" students rather than wait for them to understand even when teachers are aware that this is not the best approach (Clarke, 1993; Putnam et al., 1992).

In addition to curriculum requirements and associated time constraints, external assessment procedures that emphasise basic skills and computation may force teachers to focus on these (Clarke, 1994; Desforges & Cockburn, 1987; Tobin & Imwold, 1993). All of these factors can place teachers in a position of needing to meet system requirements rather than concentrating on students' needs and aspects of their practice that they value as important.

In summary, there seem to be many constraints that are preventing well-intentioned teachers from incorporating problem-solving approaches into their teaching. These factors can be grouped into four main categories that relate to the teachers themselves, to students, to school cultures, and to education systems.

There are other factors that impact on the belief systems teachers hold as well as the practices they adopt in mathematics classrooms. The model presented at the beginning of this chapter suggested that key factors include teachers' knowledge, the advice teachers receive, and the curriculum materials that are to be implemented. In addition to this, there are associated opportunities and constraints that either support or hinder teachers' attempts to implement particular approaches and each of these is influenced by their experiences as learners and teachers of mathematics. Several other models that incorporate a variety of factors connecting beliefs and practices have been proposed in the literature and are reviewed in the following section.

2.7 Models Representing the Relationship Between Teachers' Beliefs and Practices

In order to capture the complexity of the interrelationships between the various factors, it was relevant to develop an illustrative model. Models have been used in several studies that have examined relationships between beliefs and practice. This section reviews the factors used in these models, describes related studies and their impact on model development, and outlines the influences on the model developed for *this* investigation.

Models have been used in educational research to portray the interrelations between factors that relate to the focus of study. Although they can result in oversimplifications, they can nevertheless be useful for the purposes of prediction or explanation (Keeves, 1997). They can also provide structure that aids discussion and investigation. Keeves argued that

> the use of a symbolic or diagrammatic form can often serve to make explicit and definite the structure of the model that would otherwise remain hidden in an excess of words (p. 387).

Romberg (1992) suggested that models are often used as a "heuristic device to help clarify a complex phenomenon" (p. 51).

Several models have been used to represent the relationship between beliefs and practices (eg. Ernest, 1991b; Fennema et al., 1989; Flexer, Cumbo, Borko, Mayfield, & Marion, 1994; Raymond, 1997; Romberg, 1984). As evidence of the complexity of the relationship, there are some similarities but many differences between key factors in the models.

Common elements include knowledge, beliefs, decisions or actions, and learning outcomes. Less frequently used factors include intervention, assessment, textbooks, prior experiences, social norms, teachers' personality, and students' and teachers' lives outside of school.

Table 11 summarises the key factors used in each of these models. In the following sections, each model is described in relation to its impact on the development of the model used in this investigation.

Table 11
A summary of models used to investigate beliefs and practices

Researchers	Summary of Key Factors
Romberg (1984)	A model of mathematics pedagogy that includes teachers' beliefs and mathematics content as determining factors in teachers' plans, classroom actions and student performance.
Fennema, Carpenter, & Peterson (1989)	A model for curriculum development that connects teaching and learning and includes teachers' knowledge, beliefs and decisions as influencing factors on instruction and students' learning.
Flexer, Cumbo, Borko, Mayfield & Marion (1994)	A model of the belief system of teachers that includes beliefs about children's learning and appropriate mathematics content with beliefs about instruction and assessment as factors influencing practice.
Ernest (1991b)	A model of espoused and enacted beliefs recognising the influence of teachers' conceptions of knowledge and mathematics, their views about mathematics teaching and learning, and acknowledges the constraints and opportunities of the classroom and school setting.
Raymond (1997)	A model of the relationships between teachers' beliefs and practices that includes the influence of teacher education programs, experiences, teachers' and students' lives outside of school, teachers' personality traits and social teaching norms.

2.7.1 The Elements of a Model of Mathematics Pedagogy (Romberg, 1984)

One model that influenced this research was proposed by Romberg (1984). This model was developed to explore the differences between classrooms where the same mathematics content was taught. The model aided the investigation of teachers' intentions or plans, teachers' classroom actions, and students' performances. The model proposed that teachers' beliefs, and the mathematical content to be taught, impacted on teachers' plans and actions and ultimately, on students' performances (see Figure 7).

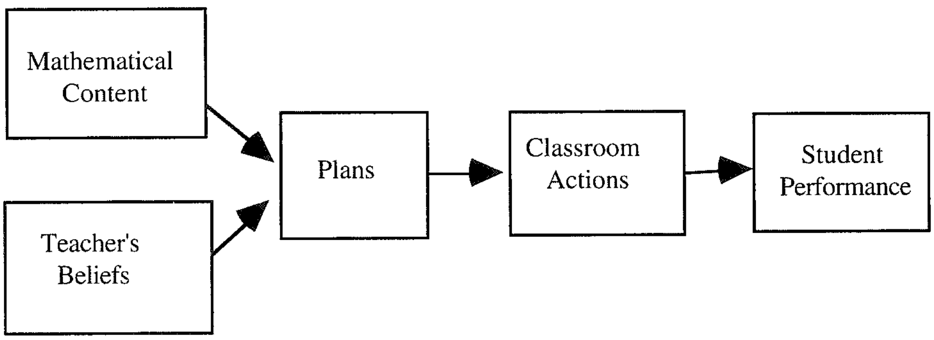


Figure 7. The elements of a model of mathematics pedagogy (Romberg, 1984, p. 125).

In a three-year longitudinal study, Romberg collected time-on-task data in elementary school classrooms as teachers taught addition and subtraction. It was postulated that the time teachers spent on particular aspects of the mathematics content would indicate the perceived importance of that component of the curriculum. One important outcome of the study indicated that teachers differentiated tasks according to the abilities of students. Drill-and-practice exercises tended to be set for lower ability students whereas explorations were set for more able students. Romberg's model recognised that teachers may teach different mathematics content in different ways.

The model developed for *this* investigation incorporated mathematics content into teachers' knowledge, and teachers' plans and actions were recognised as a part of practices and reported practices. Student performance was not considered in this investigation as the focus was on teachers' beliefs and practices.

2.7.2 A Model for Curriculum Development (Fennema, Carpenter, & Peterson, 1989)

Other significant factors are presented in the curriculum development model of Fennema et al. (1989) that emphasised the role of the teacher and the influence of teachers' knowledge and beliefs on students' learning (Figure 8). It was proposed that classroom instruction is determined by teachers' decisions which in turn are influenced by interaction between knowledge and beliefs.

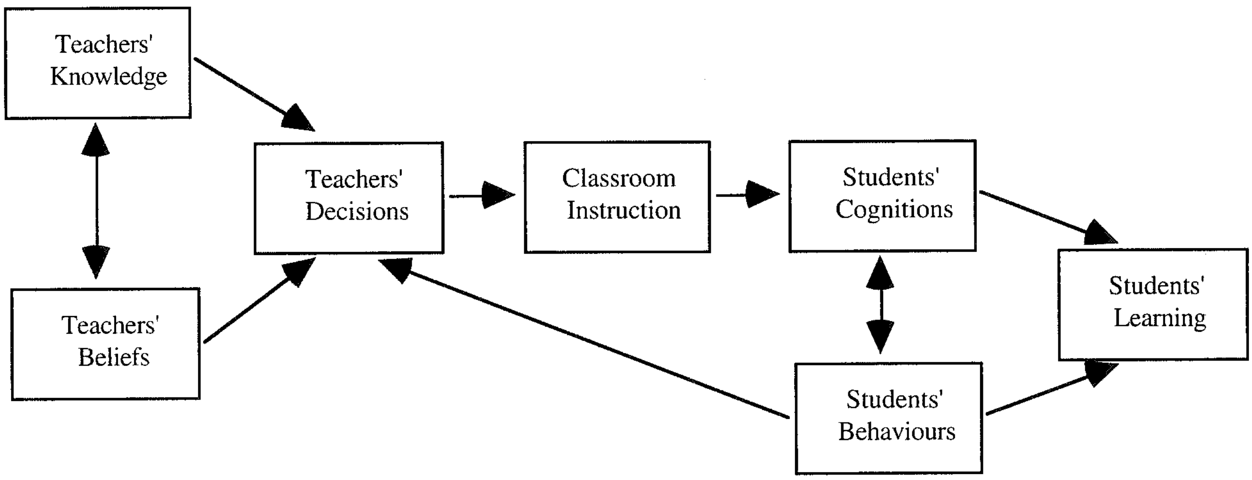


Figure 8. A model for curriculum development (Fennema et al., 1989, p. 180).

Fennema et al. (1989) used the model to investigate whether teachers' knowledge and beliefs influenced instructional decisions and if those decisions then influenced students' learning. The researchers had proposed a particular research informed approach to teaching, termed Cognitively Guided Instruction (CGI). They argued that CGI should be the focus of instruction where the child's thinking and learning in mathematics is central. The goal for teachers "is to facilitate children's active mental involvement in important mathematical tasks" (p. 181) with an emphasis on cognitive as well as affective dimensions of learning.

For teachers to implement CGI it was proposed that they needed to hold certain beliefs. To determine teachers' beliefs, four scales or continua were used to represent beliefs about:

- how children learn;
- how skills are taught in mathematics;
- how the content of mathematics should be sequenced; and
- the role of the teacher.

On each of these continua, beliefs ranged from a behaviourist or traditional viewpoint to a more constructivist viewpoint. The traditional viewpoint was based on beliefs that children receive knowledge, skills are taught in isolation, mathematics has a formal structure with a fixed sequence, and teachers present knowledge to students. The constructivist viewpoint was based on beliefs that children construct knowledge, skills are taught in relationship to understanding and problem solving, children's natural development provides the sequence for topics, and instruction facilitates learning. The researchers proposed that for teachers to embrace CGI, they would need to hold beliefs that were more consistent with this latter perspective.

To explore the effects of teachers' knowledge, beliefs and instructional practices on children's learning, first-grade teachers were involved in workshops that provided instruction about children's cognition. This experimental (or CGI) group participated in a four-week workshop before the beginning of the school year and were then supported in their classrooms during the year. The workshops focused on knowledge about students' thinking so that teachers could then make decisions to facilitate each individual student's learning. The content involved addition and subtraction word problems and the focus for discussion was on types of problems and possible solution strategies. Teachers were not given a planned curriculum or a set of materials to use in classrooms, rather they were provided with structured knowledge about this particular content area of the curriculum. The CGI teachers were compared to a control group of teachers to determine the effects of providing this knowledge.

Initially, teachers' knowledge was fragmented and disorganised and their beliefs varied widely. After the workshop, knowledge and beliefs seemed to have changed in that the CGI teachers knew more about the mental processes used by children and they

> believed more strongly than did the control teachers that skills should be based on understanding and problem solving. Further, in contrast to control teachers, CGI teachers believed that instruction should facilitate children's construction of knowledge (p. 184).

Fennema et al. (1989) concluded that "children learned more and better mathematics when teachers were given access to knowledge that enabled them to act as professionals" (p. 185). Further, they suggested that

> Perhaps the long-term emphasis on changing curriculum which has dictated to teachers what should be done and has created new and better curriculum materials needs to be closely examined. We believe that the emphasis should be on helping teachers gain access to knowledge that will empower them to make instructional decisions that change curriculum and schooling (p. 186).

This investigation did not examine students' learning nor did it explore changes in teachers' beliefs and practices. However, it is clear from the Fennema et al. model that important factors that needed to be considered when looking at classroom practice should include teachers' beliefs, decisions and instructional practices, as well as students' reactions.

2.7.3 Belief System of Teachers (Flexer, Cumbo, Borko, Mayfield, & Marion, 1994)

Another model relevant to this investigation was described by Flexer et al. (1994) and is presented in Figure 9.

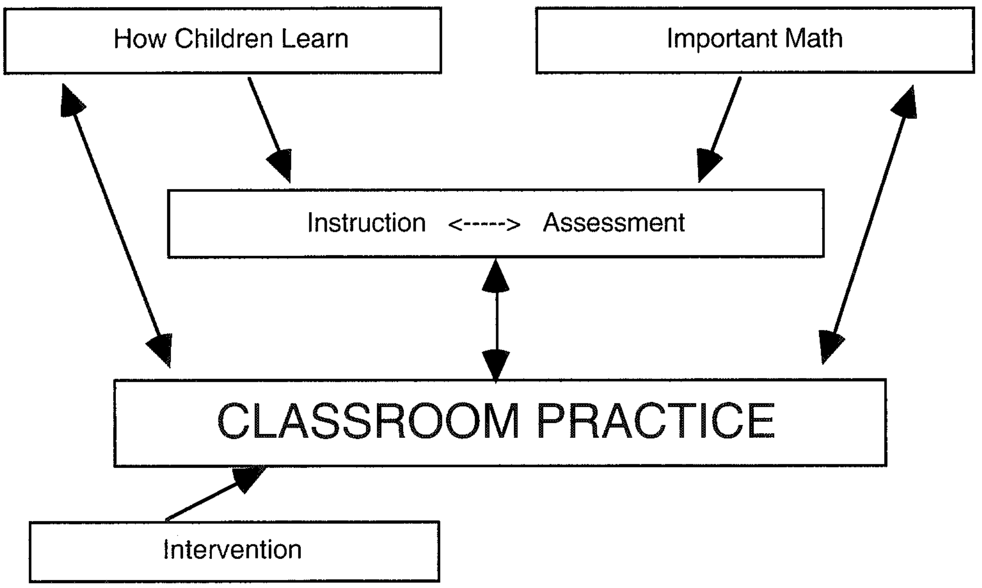


Figure 9. Belief system of teachers (Flexer et al., 1994, p. 3).

Flexer et al. proposed that three key sets of beliefs that have an impact on classroom practice. These were beliefs about:
- how children learn;
- what mathematics is important to teach children; and
- instruction and assessment.

Their research examined the impact of a professional development program that focused on alternative assessment procedures. They recognised the importance of beliefs on practices but also acknowledged that changes in practices may alter beliefs. They stated

> We believe that belief and practice can be causally related in both directions, and that it is not only the case that a change in belief causes a change in practice. A shift in practice may lead to a shift in belief which can lead to further shifts in practice (p. 3).

In this study, fourteen Third Grade teachers in three schools participated in a professional development intervention program that aimed to improve classroom-based performance assessment in reading and mathematics. Most participating teachers held traditional views about mathematics and instruction that included

> children learn by being told; school math is about facts and computation; instruction is through the text; assessment is through tests of facts and computation (p.7).

A series of workshops was conducted to help teachers expand their assessment practices but teachers were keen to be provided with instructional materials. As a result of these requests, tasks were provided for teachers to use; they usually involved problem solving with opportunities for students to reason and explain their thinking.

A representative group of six teachers was chosen to participate in further data collection that involved interviews and participation in biweekly workshops. Analysis of data revealed themes related to beliefs and practices about learning, instruction and assessment in mathematics. Two beliefs about how children learn are of interest to this investigation. The first belief that "some children are more capable of doing mathematics that others" (p. 8) led to selective practices involving tasks that required higher-order thinking. Children who were perceived to be less able were less likely to be given reasoning tasks or questions that required applications of problem-solving processes.

The second belief that "children learn mathematics by having mathematical concepts and procedures explained to them in small steps" (p. 9) was connected to beliefs that "children learn by being told and shown and then practising exercises" (p. 9). Practices associated with these beliefs suggested that teachers were concerned about students feeling comfortable and not frustrated when doing mathematics, and also that teachers were keen to develop a positive self-esteem in relation to each student's ability.

Themes that related to school mathematics and what was of importance to teachers involved the roles of computation and problem solving. All teachers in the study described "the importance of knowing and understanding facts, skills, and computation" (p. 11). Initially, teachers discussed the importance of problem solving as an instructional goal but a heavy reliance on textbooks suggested to the researchers that problems focused on the story problems contained therein. As the program progressed throughout the year, there was a marked change in such beliefs. It was noted that

> teachers gave more importance to strategies for problem solving and being able to explain how problems are solved and how procedures are done ... [and also] ... not only did their comments broaden to include more higher-order thinking, problem solving, and explaining,

> but they showed a keener awareness of the evidence they can collect as proof of these processes (p. 11).

At the conclusion of the study, it was reported that teachers adopted changes in instructional practice moving towards a perspective of mathematics compatible with the NCTM *Standards* (1989). These practices included providing more problem-solving experiences, relying less on a textbook, using more hands-on small group activities, and employing a wider variety of assessment strategies. Flexer et al. (1994) concluded that changes in beliefs alter practice, that changes in practice may lead to shifts in belief and hence that these changes appeared to be mutually reinforcing. They cautioned that it is not clear how long these changes may last. They also noted that:

> each teacher will adopt different changes that match her or his existing beliefs and practices ... [and] ... that teachers can comfortably hold inconsistent views and engage in inconsistent practices for a very long time (p. 27).

Like the Fennema et al. (1989) study, the Flexer et al. (1994) study involved an intervention program to inform teachers' practices. The investigators highlighted the importance of assessment practices on teachers' decision making.

This investigation aimed to ascertain teachers' current beliefs and practices and how they were created and did not attempt to change practice. A snapshot of beliefs and practices were sought from teachers at a particular point of time in their teaching careers.

2.7.4 The Relationship Between Espoused and Enacted Beliefs of the Mathematics Teacher (Ernest, 1991b)

Another relevant model was proposed by Ernest (1991b) who identified factors that may account for the disparity between espoused theories of teaching and learning and enacted practices. It distinguished between models (a term used by Ernest that is used differently in this investigation) of teaching and learning mathematics as well as between espoused models and enacted models. He argued that

> the constraints and opportunities afforded by the social context of teaching cause teachers to shift their pedagogical intentions and practices away from their espoused theories ... The socialisation effect of the context is sufficiently powerful that despite having differing beliefs about mathematics and its teaching, teachers in the same school are observed to adopt similar classroom practices (p. 289).

The model suggested that a primary component of a teacher's perspective is their personal philosophy of mathematics. In addition, the model proposed that espoused models of teaching and learning impact on enacted models of teaching and learning but these were "mediated by the constraints and opportunities provided by the social context of teaching" (p. 290). Further, espoused models of teaching also impacted on the use of textbooks since they

> embody an epistemology, and the extent to which their presentation and sequencing of school mathematics is followed is a crucial determinant of the nature of the implemented curriculum (p. 290).

The model is presented in Figure 10.

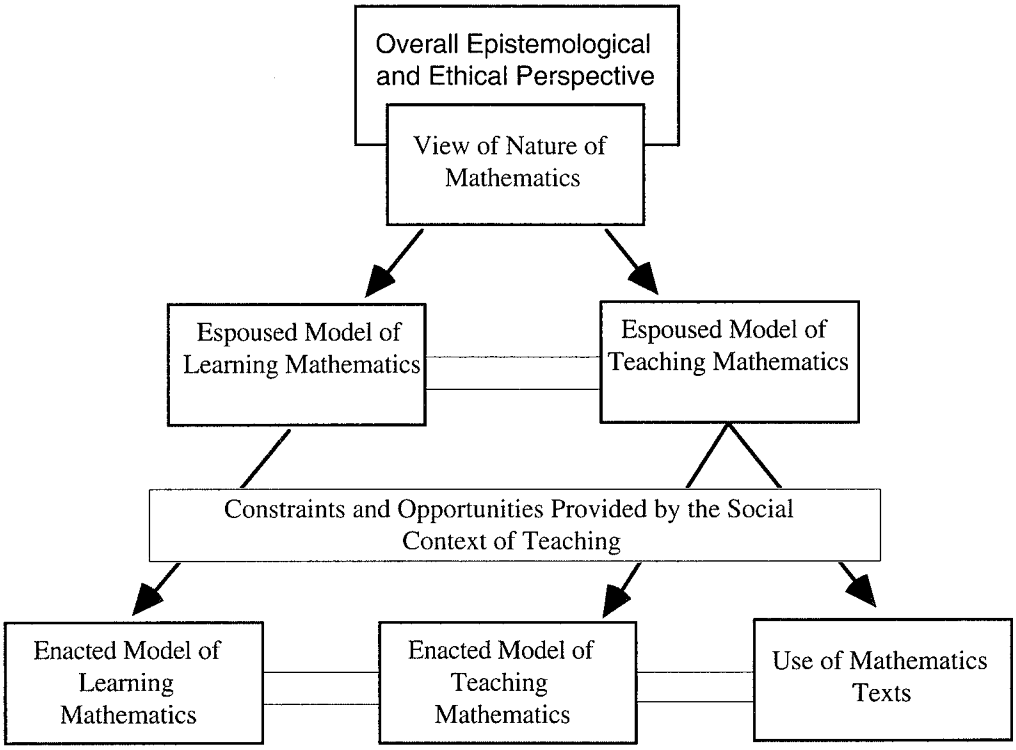


Figure 10. The relationship between espoused and enacted beliefs of the mathematics teacher (Ernest, 1991b, p. 290).

Ernest suggested that the mismatch between beliefs and practices may be the result of many influences. He proposed that the influence of social context as well as teachers' level of thought can account for this disparity. In his model, social context included the expectations of other people such as students, parents, peers and superiors. It also included the "institutionalised curriculum" that encompasses the mandated curriculum, adopted texts and assessment procedures. Ernest argued that the extent to which a teacher:

- is aware of having adopted and is able to justify particular views;
- is aware of alternative perspectives; and
- is able to reflect on practice and reconcile conflicts between beliefs and practice

may also account for the level of mismatch.

This investigation aimed to identify constraints and opportunities that operated in particular contexts. While Ernest (1989) described a mismatch between espoused and enacted models of teaching and learning, Hoyles (1992) argued that the models may be context specific. In other words, beliefs may be situation specific and as a consequence, teachers may have beliefs about teaching and learning that they describe to a researcher and yet they may have different beliefs that operate in the classroom.

Lerman (1998) also expressed concern about the notion of a mismatch and argued that the context of data collection may in fact account for such differences. He stated

> whilst there is a "family resemblance" between concepts, beliefs and actions in one context and another they are qualitatively different by virtue of those contexts. One cannot speak of decontextualised opinions or actions; the setting in which the questions are asked constitutes the conversation and is not separable from it. The activities of answering a questionnaire, talking with a researcher, or teaching, are not, in essence, the same. Essentialism is not viable as an underlying supposition of human social behaviour (p. 36).

2.7.5 A Model of the Relationships Between Mathematics Beliefs and Teaching Practices (Raymond, 1997)

From another perspective, Raymond (1997) acknowledged the existence of inconsistencies between beliefs and practice and sought to investigate the level of inconsistencies as well as the reasons why this occurs. The model presented in Figure 11 provided the conceptual framework for the investigation as earlier studies had identified several possibilities. In this model, social teaching norms included the school setting, programs, other teachers and parents of students. The immediate classroom situation represented the current mathematical topic of study, the students and time constraints.

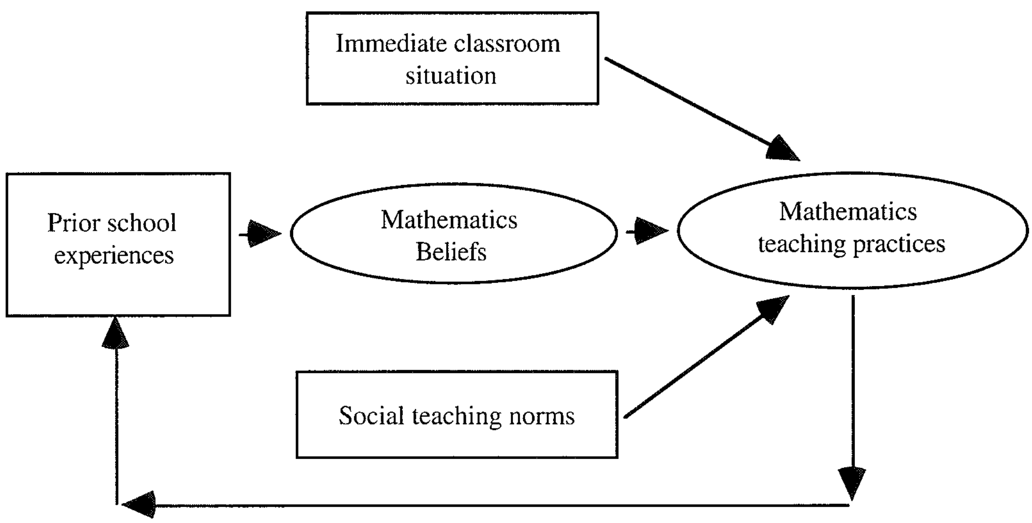


Figure 11. A model of the relationships between mathematics beliefs and teaching practices (Raymond, 1997, p. 551).

Raymond (1997) used multiple-case study research methods to explore teachers' beliefs and practices for six first- and second-year teachers. Data were collected over a ten-month period and included interviews, observations, analysis of lesson plans, concept maps, and questionnaires. Analysis placed each of the cases on a five-level scale for the nature of mathematics, mathematics teaching, and mathematics learning. The levels were traditional, primarily traditional, even mix of traditional and nontraditional, primarily nontraditional, and nontraditional. Table 12 lists two criteria for each of the levels of beliefs about teaching mathematics; criteria that were initially determined with reference to the relevant literature. There were between five and nine criteria for each level on the scale; the listed criteria were selected on the basis that they include elements that relate to problem solving which is the focus of the current investigation.

Table 12
Two criteria for each level on Raymond's teachers' beliefs about teaching mathematics scale

Level	A Selection of Two Criteria
Traditional	• the teacher emphasises mastery and memorisation of skills and facts
	• the teacher instructs solely from the textbook
Primarily Traditional	• the teacher emphasises memorisation over understanding
	• the teacher includes a limited number of opportunities for problem solving
Even mix of Traditional and Nontraditional	• the teacher has students work in groups and individually in equal amounts
	• the teacher uses textbook and problem-solving activities equally
Primarily Nontraditional	• the teacher emphasises understanding over memorisation
	• the teacher makes problem solving an integral part of class
Nontraditional	• the teacher's role is to guide learning and pose challenging questions
	• the teacher provides only problem-solving, manipulative-driven activities

One of the cases, Joanna, held traditional beliefs about mathematics but primarily nontraditional views about the teaching and learning of mathematics. Raymond indicated that

> the only perspective Joanna exhibited that was not traditional was her belief that problem solving was a big part of mathematics. However, she specifically referred to problem solving as a "topic" in mathematics rather than as the nature of mathematics (p. 561).

Also, her beliefs about learning and teaching were very inconsistent with her practices. Discussions with Joanna revealed that the factors that most influenced her beliefs were prior school experiences and teaching practice. The influencing factors on Joanna's practice were beliefs, students and mathematics topic. Identified reasons for inconsistencies between beliefs and practice were time constraints, scarcity of resources, classroom management issues, and standardised testing.

Raymond (1997) argued that Joanna thought she implemented some of her stated beliefs and practices but there was little evidence that this was the case. Joanna believed that she did incorporate problem solving into her teaching as she included a "problem of the day" in each mathematics lesson. She believed that she used manipulatives to support learning and yet the observations indicated that she rarely did. Raymond stated

> I do not believe that Joanna saw the distinction between intentions and actual practice. She seemed to view her mathematics teaching practice in terms of what she wanted to do, or thought she should do, rather than by what she actually accomplished ... It was as if she thought that believing in good mathematics teaching practices was a way of practicing good mathematics teaching (p. 569).

Raymond speculated that Joanna's beliefs about the nature of mathematics were more deeply held and therefore exerted a greater influence over her practices. She suggested that Joanna's beliefs about teaching and learning mathematics were more superficial and therefore did not influence her practices to the same extent. All of this resulted in classroom practices that were primarily traditional.

Raymond's research led to a revised model of relationships between beliefs and practices (see Figure 12). She stated

> although the model cannot be applied universally without amendment, it suggests complex relationships between beliefs and practice and builds towards an understanding of factors that contribute to the inconsistency between them (p. 570).

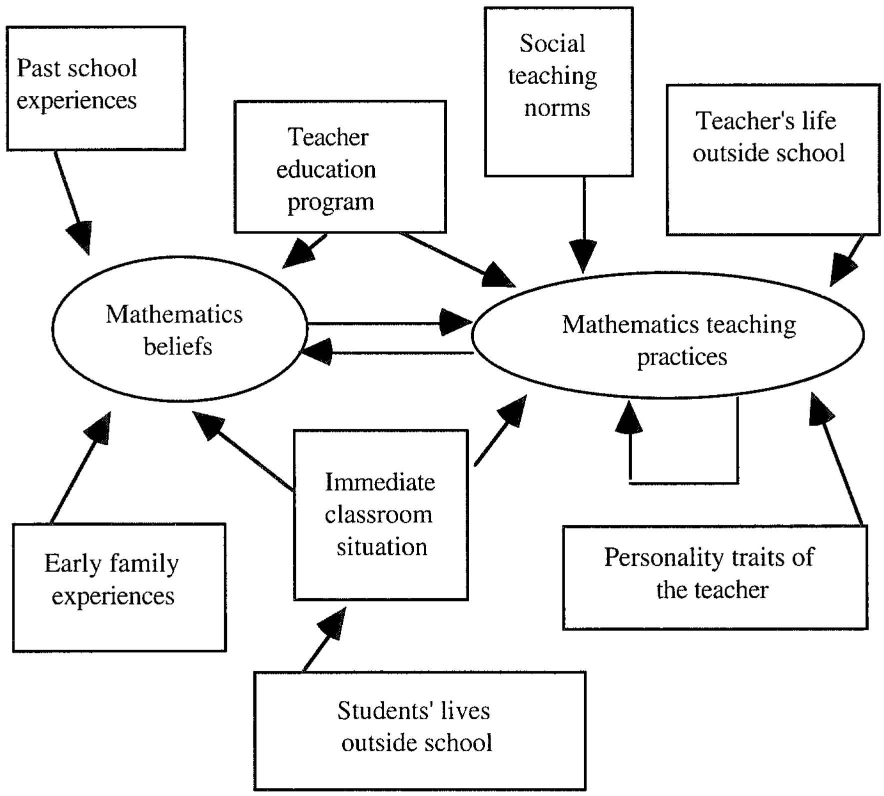


Figure 12. A revised model of relationships between mathematics beliefs and practice (Raymond, 1997, p. 571).

Raymond concluded that social teaching norms and the immediate classroom situation play key roles in influencing practice. She argued that it is these elements that are most likely to create inconsistencies between beliefs and practices as they change depending on the school the teacher is working in and the class that she teaches each year. In addition, she stated

> ... in many instances, teaching style is governed more by the sum of these other factors despite the teachers' perception that beliefs should play a major role in determining practice (p. 570).

It is also of interest that, like many other studies, she suggested

> ... deeply held, traditional beliefs about the nature of mathematics have the potential to perpetuate mathematics teaching that is more traditional, even when teachers hold nontraditional beliefs about mathematics pedagogy (p. 574).

This investigation did not seek to identify teachers' beliefs about mathematics but incorporated these into beliefs about teaching and learning. Raymond's (1997) study provided powerful evidence of the role of external factors such as school context in determining classroom practices. The current study aimed to investigate these factors but also aimed to explore opportunities that might support and enhance teachers' efforts as they attempt to incorporate problem-solving teaching approaches into their mathematics lessons.

2.7.6 A Model of the Factors that Impact on Teachers' Reported Beliefs and Practices

Based on the above models, reading of some of the research findings and analyses, and early investigations in classrooms, a model was proposed for this investigation. The models described in the previous sections indicate the complexity of the relationship between teachers' beliefs and practices and suggest a variety of contributing factors. Key elements appear to be teachers' knowledge that could incorporate mathematical content knowledge, advice from teacher education programs, prior experiences, and constraints and opportunities that might include assessment practices, the immediate classroom situation including students' behaviours, and social teaching norms.

Essentially *this* investigation was about what teachers believed and how they viewed their own practices. Therefore, the proposed factors that impact on teachers' problem-solving beliefs and practices included advice, teachers' knowledge, curriculum advice, constraints and opportunities as well as teachers' experiences in classrooms (Figure 13).

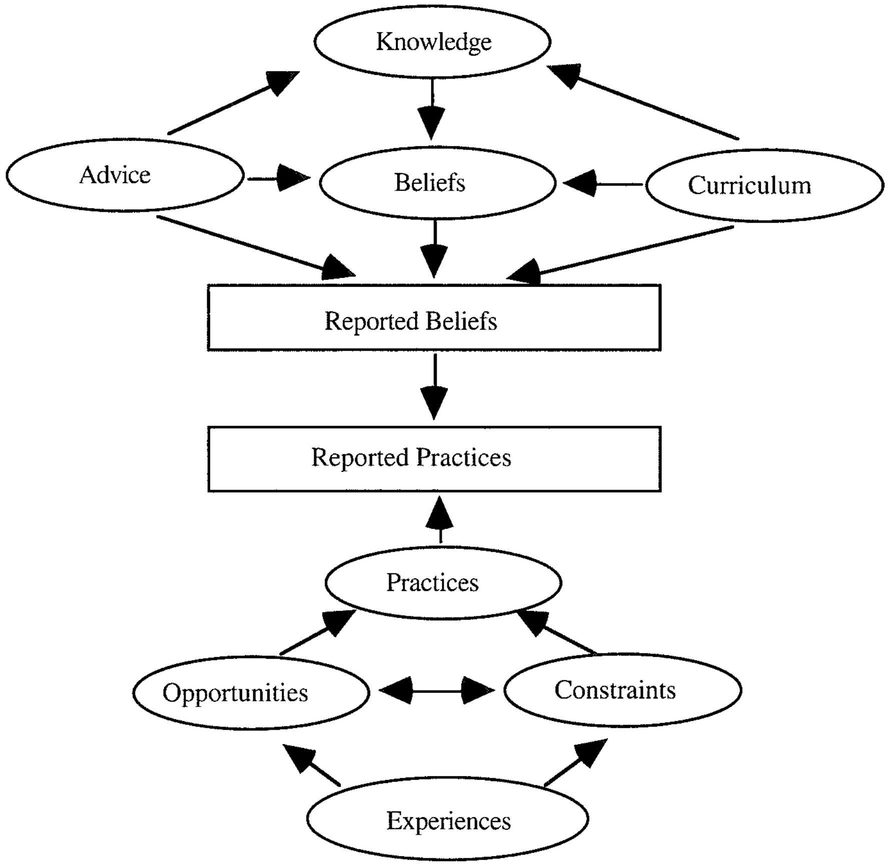


Figure 13. A model of the factors that impact on teachers' reported beliefs and practices.

The relevant literature suggests that teachers' reported beliefs are influenced by their actual beliefs (Thompson, 1992), by their knowledge and interpretation of advice about teaching problem solving (Fennema et al., 1989), by their use and understanding of curriculum documents (Morine-Dershimer & Corrigan, 1996), and by their own experiences as learners of mathematics as well as by their experiences in classrooms (Ball, 1988). Reported classroom practices are influenced by reported beliefs, by actual practices in classrooms as well as by the constraints and opportunities that occur within the school context (Tobin & Imwold, 1993).

It is acknowledged that this model may represent an oversimplification of the relationship between beliefs and practices. However, it appears to be the case that each of the factors in the model acts as a filter that impacts on teachers' decision making and cannot be easily separated as is suggested in the diagrammatic representation. In fact there are complex interrelationships between factors that may change depending on the context that teachers are operating within (Hoyles, 1992). Because this investigation does not aim to explore teacher change, the model does not incorporate the possibility of beliefs changing as a consequence of practice. It should also be noted that the model is a static one to illustrate a possible relationship that helps to explain existing beliefs and practices and not the development of beliefs.

The links between beliefs and practices are subtle and complex. In *this* investigation, these links between teachers' reported beliefs and reported practices were examined by investigating teachers' beliefs about problem solving and their use of problem-solving teaching approaches. Further, relationships between factors were examined to determine what factors operated as constraints in particular contexts and what factors operated as opportunities for those teachers who were keen to implement innovative practices supporting contemporary approaches. These ideas were the basis of the research questions for this investigation.

This review of the relevant studies and analyses has highlighted the extent of the advice to teachers of the importance of developing students' problem-solving skills in mathematics classrooms. To date, research has focused on developing students' problem-solving abilities and on programs to enhance teachers' understanding of students' thinking. There has also been considerable research about teachers' beliefs and practices in mathematics. Few studies have examined teachers' problem-solving beliefs and practices in detail. There is also evidence to suggest that few teachers have responded to the advice and this lack of response may be a consequence of a variety of factors including beliefs and previous experiences as well as constraints in particular contexts. This seems to be a rich area for investigation if teachers are to be supported in the future.

2.8 The Research Questions

There were four main research questions in this investigation. The first two were global and aimed to discover teachers' beliefs about the role of problem solving in relation to how children learn mathematics and what teaching practices are used in this endeavour. The second two aimed to identify practices in particular classrooms with respect to teachers' efforts to incorporate problem-solving approaches. Also, opportunities and constraints in those contexts were examined. Each of the four main research questions is accompanied by a set of associated questions.

The first main research question addressed teachers' beliefs about problem solving and how problem solving helps students learn mathematics.

1. What do teachers believe is the role of problem solving in learning mathematics?

In particular, what are teachers' beliefs about the relationship between problem solving and:

- mathematics; and
- students' learning of mathematics?

This question investigated teachers' stated beliefs about mathematics and the connection between school mathematics and problem solving. It considered beliefs about how students learn mathematics and whether teachers believe that basic skills and computation can be taught using problem-solving approaches.

The second main research question dealt with the way that teachers reported that they implemented problem-solving approaches in their mathematics classrooms.

2. To what extent do teachers report that they incorporate problem-solving approaches in their planning and teaching of mathematics, and what specific practices do they report that they use?

In particular, when incorporating problem-solving approaches in their mathematics teaching:

- what specific classroom strategies do teachers claim to use;
- what types of problem-solving tasks do teachers claim to prefer and what tasks do they claim to use;
- what do teachers report is the form of their planning; and
- what resources and materials do teachers report they use?

This question explored what teachers believed they were doing in their classrooms in relation to the teaching of problem solving. The question aimed to investigate teachers' planning and the resources that were used both in preparation as well as in implementation. This included curriculum documentation, reference books, inservice materials and support from fellow staff members. The teaching strategies focused on lesson structure that included use of individual, group and whole-class activities as well as questioning techniques and the use of discussions about problem-solving strategies and solutions to problems.

The third main research question involved an examination of some teachers' actions in particular classrooms.

3. In what ways do teachers incorporate problem-solving approaches into their planning and teaching of mathematics?

In particular, when incorporating problem-solving approaches in their mathematics teaching:

- what specific classroom strategies do teachers use;
- what types of problem-solving tasks do teachers use;
- how do teachers plan; and
- what resources and materials do teachers use?

This question involved observation of particular classrooms and identified practices that related to planning, use of materials, strategies and the types of questions that teachers posed.

The fourth main research question related to the third as it explored particular classroom situations and focussed on factors that supported or interfered with teachers' attempts to carry out a plan of action.

4. When teaching mathematics using problem-solving approaches, what factors can be identified that support or inhibit the implementation of such practices?

In particular, what impact do each of the following aspects have on teachers' planning as well as implementation of problem-solving approaches:

- the teachers themselves:- including personal aspects such as confidence, past experiences that incorporate successes and failures, and opportunities for teacher development;
- students:- including the influence of their beliefs, resistance, attitudes; the perceived ability of the students with respect to: learning mathematics; proficiency with language; participation in group work;
- school culture:- including the influence of prescribed programs and textbooks, assessment and reporting practices, parents' expectations, principals' and other teachers' influence; and
- the education system:- including curriculum documents, particular programs, external assessment procedures.

This final main question aimed to explore the factors that support or constrain teachers' efforts in particular contexts. There are many factors to be considered that may either support or interfere with the ability of teachers to put their ideals into practice. These have been grouped into four broad categories of the classroom teacher, students, school culture and education system requirements.

This chapter has summarised advice on problem solving, teachers' knowledge, beliefs and practices, issues regarding the implementation of problem-solving approaches, and models that have been used proposing relationships between relevant variables in related studies. Each of these aspects informed this investigation and provided a focus for each of the research questions.

To explore these research questions, it was clear that a combination of data collection methods was needed. An examination of earlier studies into teachers' beliefs and practices in mathematics revealed a diversity of possible data collection procedures. The next chapter outlines the research approach for this investigation including methodology and research methods as well as the development of the research tools and data gathering procedures.

CHAPTER 3
THE RESEARCH APPROACH

The four main research questions were grouped in pairs; the first two addressed general issues and the second two focused on specific issues. This had implications for the data collection procedures. The first two questions explored issues related to teachers' beliefs about the role of problem solving in learning mathematics and the use of problem-solving teaching practices. It was appropriate to use survey research methods to seek these data as little is known overall about teachers' problem-solving beliefs and practices, particularly for Australian teachers.

In order to enrich these global data and because many of the issues involved in this investigation were complex, more specific information in particular contexts was sought to explore and elaborate relationships. The second two research questions involved a description of relevant situations and so it was necessary to seek more in-depth information from some teachers who had been identified as those who were likely to reveal this information. This was done through interviews and observations in particular school settings.

In summary, the methods for this investigation combine survey research that explored beliefs held by a larger population of teachers, with fieldwork conducted in a small number of natural social settings. There are two sections to this chapter; the first describes general approaches and methods used in related research and the second outlines the development of the specific methods used and the data gathering procedures employed in this investigation.

3.1 Methodology and Research Methods

This investigation combined survey research methods and fieldwork in an attempt to describe and compare teachers' problem-solving beliefs and related problem-solving practices, as well as to explore important aspects of the relationships between teachers' beliefs and practices in particular school settings. In this way, the study incorporated a combination of quantitative and qualitative research methods. This section examines general approaches to research, summarises methods used in related problem-solving research, describes the use of multiple research methods, and outlines the research approach for this investigation.

3.1.1 Situating the Research in the Context of Common Paradigms

The following discussion explains fundamental assumptions that underpin the nature of this research. These can be considered in the context of discussions about particular research paradigms. Paradigm is taken here to mean

> overarching philosophical systems denoting particular ontologies, epistemologies, and methodologies ... belief systems that attach the user to a particular worldview (Denzin & Lincoln, 1994, p. 2).

There is considerable variation regarding the naming of paradigms discussed in the literature as well as the number of categories of paradigms that have been described.

Some authors have sought to delineate paradigms as bipolars or continuums. Examples from the literature include scientific and naturalistic (Guba & Lincoln, 1987), scientific and field based (Romberg & Carpenter, 1986), positivist and phenomenological (Firestone, 1987), positive and interpretive (Geertz, 1973), or analytic and humanistic (Brown et al., 1990).

Others have used more than two categories to classify approaches to inquiry. For example, Romberg (1992) described empirical/analytical, symbolic and critical paradigms while Connole, Smith and Wiseman (1993) summarised empiricist, interpretive, critical and deconstructive/poststructural approaches. Guba and Lincoln (1994) expanded on their earlier classification of two distinct paradigms and described four approaches to inquiry that included positivism, postpositivism, critical theory and associated approaches, and constructivism.

While many different terms are used in the literature, it seems that each paradigm is based on a set of assumptions about knowledge and about the relationships between the researcher and the objects of research. Various assumptions that elaborate differences between, for example, scientific and naturalistic paradigms (Guba & Lincoln, 1987) are summarised in Table 13.

Table 13
The five axioms that distinguish the scientific and naturalistic paradigms (Guba & Lincoln, 1987)

Axiom	Scientific Paradigm	Naturalistic Paradigm
Reality	"There exists a single, tangible reality fragmented into independent variables and processes" (p. 147).	"... there exist multiple realities which are, in the main, constructions existing in the minds of people" (p. 147).
Inquirer-object relationship	"... the inquirer is able to maintain a discrete and inviolable distance from the "object" of inquiry ..." (p. 147).	"... the inquirer and the respondent in any human inquiry inevitably interact to influence one another" (p. 147).
Generalisability	"... the aim of inquiry is to develop a nomothetic body of knowledge; this knowledge is best encapsulated in generalisations ..." (p. 148).	"... the aim of inquiry is to develop an idiographic body of knowledge; this knowledge is best encapsulated in a series of "working hypotheses" that describe the individual case" (p. 148).
Causality	"... the determination of cause-effect relationships is of the highest importance" (p. 148).	"... the determination of cause-effect relationships is a search for the Holy Grail [since] ... factors, events, and processes are constantly exerting influences on one another, mutually impinging, changing, and being changed simultaneously" (p. 148).
Value freedom	"... assume that inquiry is value free and can be guaranteed to be so by virtue of the "objective" methodology" (p. 148).	"... asserts that values impinge upon an inquiry in at least five ways ... Of particular interest is the possibility of resonance or dissonance between the substantive and methodological assumptions which can produce quite misleading results" (p. 148).

These basic but important differences between two main research paradigms indicate key issues that were considered in the design of this investigation. In each case, this investigation is more closely aligned with the naturalistic perspective. There are also elements of this approach to research that are close to Connole et al.'s (1993) interpretive paradigm which

> ... places a priority on searching for and interpreting what is happening and being done, according to the interpretations of the participants in the social activities being studied (p. 105).

Similarly, there are common elements between Romberg's (1992) description of the symbolic paradigm and the aims of this investigation. Romberg suggested that the goal of the symbolic paradigm, that includes hermeneutic, interpretive, or phenomenological perspectives, is to understand "how humans relate to the social world they have created" (p. 55). Of the terms used to describe paradigms of research, the interpretive approach is the closest to describe this investigation.

There has been some criticism of the use of the term "interpretive" to describe a research paradigm. For example, Denzin and Lincoln (1994) suggested that all research is interpretive, claiming that it is "guided by a set of beliefs and feelings about the world and how it should be understood and studied" (p. 13). They argued that there are four major interpretive paradigms including positivist and postpositivist, constructivist-interpretive, critical, and feminist-poststructural. Using Denzin and Lincoln's classification, this investigation is most closely related to the constructivist-interpretive paradigm; a paradigm that recognises multiple constructed realities, accepts that the "knower and subject create understandings" (p. 13), and uses a naturalistic set of methodological procedures. However, in the same volume, Schwandt (1994) frequently used the expression "constructivist or interpretivist" (p. 118) suggesting that there is substantial overlap between the two approaches.

Another perspective on research that helps to clarify the approach taken here is the distinction made by Shulman (1986) in his review of research on teaching. In this, he distinguished between the process-product tradition and ecological research. In comparing these two research programs, Shulman stated

> The emphasis of process-product research on the essential role of achievement outcomes, on the relative decontextualisation of analyses, on the objectification of data in the search for positive laws, is typically missing in this family of [ecological] research (p. 18).

Further, he suggested that the classroom ecology tradition was "an extended family of inquiries, not a simple, tightly knit one" (p. 18) and included investigations by ethnographers, sociologists, psychologists, and curriculum and teaching specialists. Another distinguishing feature of these two traditions resided in the unit of investigation. Shulman argued

> While process-product researchers view classrooms as reducible to discrete events and behaviours which can be noted, counted, and aggregated for purposes of generalisation across settings and individuals, interpretive scholars view classrooms as socially and culturally organised environments (p. 20).

In the same volume edited by Wittrock (1986), Romberg and Carpenter summarised the contributions made by both scientific and field-based studies in mathematics education. At that stage in the history of research in mathematics classrooms, they suggested that

> ... "field-based studies" of mathematics teaching, while considerably fewer than the "scientific" studies, are interesting in that through different conceptual lenses, different aspects of teaching are illuminated ... these studies add to our knowledge about the traditions of schooling. In particular, they illuminate aspects of the environment of classrooms overlooked in scientific studies (p. 868).

Since the review by Romberg and Carpenter, many more studies have used field-based approaches since the importance of the interactions between *all* participants in classrooms has been recognised. This is evident in the large number of studies reviewed by Fennema and Franke (1992), Schoenfeld (1992), and Thompson (1992).

In another review, Brown et al. (1990) described research into mathematics teacher education and categorised studies as based on either analytic or humanistic paradigms.

Each approach views research, mathematics and schooling in different ways as presented in Table 14.

Table 14
Distinctions between analytic and humanistic approaches (Brown, Cooney, & Jones, 1990, p. 652)

Paradigm	Research	Mathematics	Schooling
Analytic	detached	received	value free
Humanistic	connected	constructed	value laden

From this classification, this investigation most closely matches the humanistic paradigm, viewing classroom knowledge as constructed by teachers from their experiences. Brown et al. (1990) acknowledged that this is a simplified conception and that much research crosses the boundaries between the two approaches. They recommended greater collaboration between researchers from each group.

The literature on research methodologies uses an abundance of terms to describe paradigms and suggests that some authors use different terms to describe the same paradigm. In her introduction, Ely (1991) confessed that she uses naturalistic inquiry, ethnographic methodologies, qualitative research, and interpretive research "in a roughly synonymous way" (p. 2). She suggested that

> The field is shot through with a variety of labels and proponents of those labels. These derive from a number of theoretical models and a range of modifications and variations upon these models that guide why and how to do research ... Underlying this collection of competing labels are certain commonalities that link them together - a network of underlying principles and philosophical beliefs that constitute a paradigm or world view (p. 2).

This brief overview of commonly referred to paradigms of research suggests that this investigation most closely fits the world view of the interpretive paradigm. This approach recognises that there are multiple realities, that the research aimed to describe the reality of the participants at a particular point in time, and that this reality is not static but is constantly being influenced by a variety of interacting factors. Also, it is accepted that the researcher influenced the research outcomes and that the research was not value free.

An interpretive perspective suggests that classrooms are recognised as "socially and culturally organised environments" (Shulman, 1986, p. 20) and that such investigations aim to illuminate the highly complex nature of specific contexts. It was appropriate to ensure that the chosen research methods reflected the complexity of the task. These decisions were informed by reviewing approaches used in studies focusing on teachers' beliefs and practices and aspects of the problem-solving literature.

3.1.2 Summarising Investigative Methods used in Research about Teachers' Beliefs and Practices

In order to determine the most appropriate research methods, it was relevant to explore methods used in other studies of teachers' beliefs and practices as well as to examine changes in research focus and methods used in problem-solving research. This section summarises the methods used in studies that have informed this investigation and discusses the use of the term *case study* to describe some of this research.

3.1.2.1 Commonly Used Research Methods

A variety of approaches have been used to determine teachers' beliefs and practices including questionnaires, interviews and classroom observations. Some studies have used one main data collection method while others have used a combination of approaches. Problem-solving studies have also used a variety of research methods but a change in focus has been noted during the last three decades.

An examination of trends in problem-solving research indicates a change from more quantitative methods to qualitative methods. This is evidenced in Lester's (1994) review of problem-solving research as he reflected on developments over the past twenty-five years. His summary table is reproduced here in Table 15 and highlights the changes in research approaches from 1970 to 1994. There has been a change from scientific methods that used statistical analysis and "teaching experiments", to more field-based, ethnographic investigations. This has been accompanied by a change from focusing on isolated skills in problem solving to examining the influence of the social aspects of classrooms on students' problem-solving efforts. It has been recognised that it is more appropriate to consider teachers' decisions and influencing factors within the school context.

Table 15
An Overview of Problem-Solving Research Emphases and Methodologies: 1970-1994 (Lester, 1994, p. 664)

Dates *	Problem-solving research emphases	Research methodologies used
1970-1982	Isolation of key determinants of problem difficulty; identification of characteristics of successful problem solvers; heuristics training.	Statistical regression analysis; early "teaching experiments"
1978-1985	Comparison of successful and unsuccessful problem solvers (experts vs. novices); strategy training	Case studies; "think aloud" protocol analysis
1982-1990	Metacognition; relation of affects/beliefs to problem solving; metacognition training	Case studies; "think aloud" protocol analysis
1990-1994	Social influences; problem solving in context (situated problem solving)	Ethnographic methods

* Of course, the dates shown are only approximate. However, the chronology is reasonably accurate.

Teachers' beliefs and practices have been explored by researchers using a range of investigative methods. In her review, Thompson (1992) reported that "most research on teachers' beliefs and conceptions is interpretive in nature and employs qualitative methods of analysis" (p. 131). A close examination of these studies suggests that extensive use of multiple methods, or combinations of research methods, is used depending on the main focus of the study.

Typically, research about teachers' beliefs has used survey research methods to collect this information. In mathematics education research, a popular method of collecting data about teachers' beliefs has been the use of questionnaires. Examples include *The Beliefs Survey* used by Peterson, Fennema and Carpenter (1987) and more recently, questionnaires have been used by Perry, Howard and Conroy (1996) and Howard, Perry and Lindsay (1997). Funkhouser (1993) used a survey instrument with open-ended questions to ascertain teachers' perceptions of *problem solving*.

A popular method for collecting data about teaching practices has been the use of a variety of classroom observation techniques. Classroom observations have been used in many studies including for example, Clarke's (1984) investigation of secondary mathematics teachers' lesson structures and preferred practices, Stigler and Stevenson's (1991) cross-cultural investigations of teaching practices in Japan, Taiwan and the United States of America, and Sullivan and Leder's (1990) examination of the use of question types by beginning primary school teachers. In contrast to the above, Barnes, Clarke and Stephens (1995) did not use observations to collect data about teaching practices that incorporated assessment and reporting practices from teachers in NSW and Victoria. Instead, they used a combination of document analysis, questionnaires and interviews.

Incorporating an examination of teachers' beliefs *and* practices in the same investigation has typically resulted in the use of a combination of methods (Thompson, 1992). Favoured data collection procedures have included a variety of combinations of observations, interviews, questionnaires and writing tasks. Open-ended questions, scenarios, writing prompts, journals, lesson plans and concept maps have been used to obtain written data from teachers. Table 16 lists studies relevant to this investigation under the main data collection methods used.

Table 16
A categorisation of studies about teachers' beliefs and practices based on use of particular data collection methods

Research Methods	Researcher(s)
Questionnaire	Sosniak, Ethington, & Varelas (1991)
Observations	Tobin & Imwold (1993)
Questionnaire and Observations	Fennema, Carpenter, & Peterson (1989)
Observations and Documentary Evidence	Lubinski (1993)
Interviews	Grouws, Good, & Dougherty (1990)
	Flexer, Cumbo, Borko, Mayfield, & Marion (1994)
Questionnaires and Interviews	Lerman (1990)
Interviews and Observations	Clarke (1993, 1995, 1997)
	Cooney (1985)
	Desforges & Cockburn (1987)
	Jaworski (1991)
	Levin & Ammon (1992)
	Prawat, Remillard, Putnam, & Heaton (1992)
	Wilcox, Schram, Lappan, & Lanier (1991)
Interviews, Observations and Written Tasks	Dougherty (1990)
	Hoyles (1992)
	Thompson (1984, 1985)
Questionnaires, Interviews and Observations	Van Zoest, Jones, & Thornton (1994)
Questionnaires, Interviews, Observations and Written Tasks	Raymond (1997)
	Thompson (1989)

The above table indicates the variety of methods used and the range of combinations employed to gain information about teachers' beliefs and practices. A majority have used combinations of methods to enrich the data source and to check for consistency. Few studies have attempted to collect data from a large number of teachers. Most report the results of in-depth investigations of a small number of teachers using case-study methods. Clearly, researchers in education choose methods appropriate to the particular research questions.

3.1.2.2 Case-Study Approaches

Case-study approaches are being used increasingly in educational research. These studies have tended to investigate a small group of teachers as they work in their schools and classrooms. In these, data are collected from the teachers and possibly from their students or other teachers from the school. Data gathering usually takes considerable time resulting in large amounts of data to be analysed so that a detailed description of the setting can be obtained. Several of the studies listed in Table 16 have been described as case studies in the review of relevant literature in Chapter 2 of this thesis but it seems that they could be better described as rich descriptions using multiple-method approaches.

In order to gain detailed information about a particular social setting, it has been suggested that large amounts of time are required to uncover cultural features as well as factors that impact on that particular environment. Stake (1994) argued that

> Qualitative case study is characterised by the main researcher spending substantial time, on site, personally in contact with activities and operations of the case, reflecting, revising meanings of what is going on (p. 242).

In contrast to this view, Yin (1989) suggested that case studies need not take considerable time. His definition does not include a statement about time and indicated that

> A case study is an empirical inquiry that:
> - investigates a contemporary phenomenon within its real-life context; when
> - the boundaries between phenomenon and context are not clearly evident; and in which
> - multiple sources of evidence are used (p. 23).

Not all of the studies described earlier as case studies report that the researcher spent "substantial time, on site, personally in contact with activities and operations of the case" (p. 242) as Stake (1994) suggested. In some instances where the investigation lasted several months, visits to sites were limited to two or three observational periods, each lasting only a few hours (e.g. Prawat et al., 1992). Other studies involved many hours over many weeks incorporating observations in classrooms and interviews with teacher participants (e.g. Clarke, 1997).

Hoyles (1992) argued that case study has become the dominant methodology in mathematics education research and that this "fashion" needs to be carefully reviewed with full consideration of all of the associated issues. She states

> ... a case study can all too easily degenerate into a mere description of a specific setting, providing little in terms of critical insight or theoretical illumination ... it must seek to throw light on the general, through the particular - whilst keeping intact the complexity of the setting ... it is also very apparent that case studies are hard to write and harder still to read - their very richness mitigates against their communicability (p. 41).

Most of the studies listed in Table 16 involved the use of fieldwork. This recognises that to explore teachers' beliefs and practices means examining teachers' actions in the context of their classrooms and daily activities since beliefs can be situation specific. However, there are clear differences between the studies with respect to the amount of time spent in the field. It would seem from Hoyles' (1992) comments and concerns that collecting data in the field does not automatically mean that the researcher is using a case-study approach. Both case studies and fieldwork typically employ multimethod approaches.

Rather than describing some investigative approaches as case studies, it seems more appropriate to suggest that some investigations of particular teachers are detailed descriptions that have been obtained using multiple research methods. For some of the above studies, substantial data have been collected, although not always on site. It is usual that classroom observations are supplemented with interviews and written tasks suggesting that the individual teacher description has been compiled from several data sources, or by multiple-method approaches.

3.1.3 Using Multiple Research Methods

The previous section highlights the extensive use of qualitative research methods to collect data about teachers' beliefs and practices. Some studies have used combinations of qualitative and quantitative data collection methods. This section outlines some of the concerns surrounding the use of combining the two approaches and argues that the advantages of multiple research methods outweigh the suggested disadvantages.

In this discussion, qualitative methods are taken to mean "any kind of research that produces findings not arrived at by means of statistical procedures or other means of quantification" (Strauss & Corbin, 1990, p. 17). Such procedures could include interviews and classroom observations that avoid quantifying teaching actions. Some of the studies discussed in the previous section have incorporated questionnaires with interviews and observations thus combining quantitative and qualitative approaches.

For paradigmatic reasons, researchers have differed in their views on the use of combinations of quantitative and qualitative approaches in the same study. On the one hand, some have suggested that the different perspectives are irreconcilable since quantitative methods are based on a positivist paradigm and qualitative methods on a phenomenological paradigm (e.g. Smith, 1983). On the other hand, Tesch (1990) stated that

> the use of the two different types of data does not coincide with the two research approaches which scholars commonly distinguish today: traditional (or positivistic), and 'new paradigm' (post-positivistic or human science) research (p. 56).

Tesch (1990), Ely (1991) and others argued that qualitative researchers *can*, and when appropriate, *should* use numerical data.

Guba and Lincoln (1987, 1994) argued *against* the use of a combination of the two methods, or "mix and match" strategies. They stated

> ... there exists a synergism among methodological elements such that commitment to one paradigm's preference on any one methodological element requires, if one is to be true to the axiomatic system, commitment to counterpart positions on the others as well (p. 149).

However, this argument is rejected by Brewer and Hunter (1989) who suggested that

> Rather than being wedded to a particular theoretical style, its pet problems and questions, and its most compatible method, one might instead combine methods that would encourage or even require the integration of different theoretical perspectives to interpret the data. If hypotheses and variables that have been previously isolated each within their own theoretical systems are instead empirically interrelated in the same study, then conceptual linkages between different theoretical systems are likely to follow (p. 74).

Commenting on the differences between the two points of view, Firestone (1987) stated that

> The purists believe that the two method types are incompatible because they are based on paradigms that make different assumptions about the world and what constitutes valid research ... The pragmatists see a more instrumental relationship between paradigm and methods. To them methods are more collections of techniques (p. 16).

Further, he argued that each method lends itself to different kinds of rhetoric hence using "different techniques of presentation to project divergent assumptions about the world and different means to persuade the reader of its conclusions" (p. 16).

Others suggested that there are good reasons for combining the two approaches. For example, Miles and Huberman (1984) recognised that "this is a nontrivial battle" but observed that "few working researchers are *not* blending the two perspectives" (p. 20). Strauss and Corbin (1990) argued that the approaches can be used effectively in combination. Shulman (1986) described the use of "hybrid designs" as an "exciting new development in the study of teaching" (p. 4). Also, Hammersley (1990) argued that "one should use data that are available, of whatever type, if they allow one to develop and test one's theory effectively" (p. 113).

Firestone (1987) suggested that quantitative data and qualitative data have different descriptive strengths. He indicated that quantitative methods assess the magnitude of relationships more precisely and can show a pattern that extends across a large number of situations. Further, he stated that qualitative methods can provide "concrete depiction of detail, portrayal of process in an active mode, and attention to the perspectives of those studied" (p. 20). In summary, each can provide the researcher with different kinds of information about the same phenomenon.

Salomon (1991) also supported the notion that each approach can offer different modes and different levels of understandings. He argued for the transcendence of the qualitative-quantitative debate and recommended the use of analytic and systemic approaches. He outlined three challenges that are faced by both approaches including the need for validation, standards of quality, and some means of facilitating generalisability. Further, he suggested that since classrooms are complex and therefore cannot be easily studied as sets of single events and single variables, it is appropriate to use a combination of approaches. He stated

> ... because the two paradigms under consideration address different issues, yielding different kinds of knowledge, they ought to be seen as complementing and enriching each other, rather than ruling each other out (p. 11).

He described the analytic and systemic approaches to the study of classrooms and demonstrated the applicability of both with an example of one of his research projects. In the same study, the analytic approach was used to highlight key variables which then guided and informed the more in-depth systemic study that followed.

In addition to this, combining research methods adds to the reliability of data by a process of triangulation (Brewer & Hunter, 1989; Firestone, 1987; LeCompte & Goetz, 1982; Yin, 1989). Triangulation of data, especially using more than one method of data collection, or mixed-method approaches, enables the researcher to identify continuity of meanings throughout the study so that categories and themes can emerge. Brewer and Hunter (1989) stated that

> Triangulation requires multiple sets of data speaking to the same research question from different viewpoints ... data must be collected with truly different methods that are employed independently of one another but that are focused as tightly as possible upon the particular question being investigated (p. 83).

Triangulation also tests one source of information against another to check quality, to develop a more complete understanding, and to put the whole situation into perspective (Fetterman, 1989). Denzin and Lincoln (1994) cautioned that "triangulation is not a tool or a strategy of validation, but an alternative to validation" (p. 2). Further, they suggested that

> The combination of multiple methods, empirical materials, perspectives and observers in a single study is best understood, then, as a strategy that adds rigour, breadth, and depth to any investigation (p. 2).

Another reason for using a variety of data collection methods is that it can help to overcome weaknesses in any one method and strengthen the findings overall. Brewer and Hunter (1989) argued that

> ... individual methods may be flawed, but fortunately the flaws in each are not identical. A diversity of imperfection allows us to combine methods not only to gain their individual strengths but also to compensate for their particular faults and limitations (p. 17).

One example of this is the use of a combination of fieldwork approaches and survey research methods to overcome different weaknesses in each approach. As Brewer and Hunter (1989) suggested

> Fieldwork, for example, gives access to variables and hypotheses that pertain to relatively confined natural social settings ... but fails to give access to those that pertain either to past eras, or to large contemporary populations ... Surveys can provide information about such populations but only about topics on which respondents are able and willing to report (p. 45).

There are additional issues that concern the use of questionnaires and interviews. On the one hand, use of questionnaires for data collection has been criticised because of the distance between the researcher and the respondent even though it is an efficient method of collecting a large amount of data. This creates concerns about interpretation of items in the sense that

> Knowing whether the researcher and the respondent are on the same wavelength, sharing common assumptions and understandings about the questions, is difficult - perhaps impossible (Fetterman, 1989, p. 64).

On the other hand, interviews enable the researcher and respondent to negotiate common understandings but time limitations would preclude interviewing the large number of respondents usually involved in questionnaire procedures.

The notion of collecting detailed information using a variety of approaches is supported by Brown et al. (1990) who advocated a holistic approach when gathering data about teachers' beliefs. Commenting on the reported inconsistencies between teachers' beliefs and practices, Brown et al. suggested that "... teachers' beliefs are often incompatible with the context within which they are supposedly realised" (p. 652). In recognising the importance of all of the factors that can impact on beliefs, they recommended that it is necessary to try to understand the culture within which teachers work. They were critical of some of the attempts of mathematics education researchers who have used ethnographic approaches. They stated that these investigators

> ... have tended to be more piecemeal (a legacy of the analytic tradition) than holistic in their efforts to understand the culture and the evaluation of its beliefs. Any attempt to study beliefs in isolation from the whole will fail to reveal the significance of problems associated with potentially conflicting beliefs (p. 652).

This view of incorporating a holistic perspective into research is supported by Fetterman (1989). He suggested that in order to describe a culture or social group in detail requires

> ... a great deal of time in the field to gather the many kinds of data that together create a picture of the social whole. It also requires multiple methods and multiple hypotheses to ensure that the researcher covers all angles (p. 29).

Data collection methods, including interviews, observations and questionnaires, are recommended by Fetterman. He argued that the latter have their place in ethnographic research since they "are an efficient means of large-scale data collection" (p. 65).

It seems from the literature that the use of numerical data in qualitative research is certainly accepted by many researchers. Ely (1991) argued against the notion that qualitative research must be number-free. Further, she stated that

> ... every category, every theme, every finding, whatever its form, arises from the fact that it exists in the data and as such can be counted even though the researcher may choose not to do so ... The responsibility of a qualitative researcher in the final report is to bring public spotlight on her/his decision-making process in establishing findings. When this holds, readers have the information with which to judge for themselves whether the findings are reasonable (p. 156).

In summary, using data collection methods that combine qualitative and quantitative data collection procedures adds strength to results in several important ways. Each can complement the other by providing different information about the same phenomenon. Overall patterns from larger populations can inform in-depth examinations of particular contexts. Data from different sources can be compared for consistency and they can also be used to overcome potential weaknesses for particular methods. In addition to this, using combinations of approaches enriches the results and is more likely to depict the whole picture. On balance, it seems that a multiple research method approach is appropriate for this investigation of primary school teachers' problem-solving beliefs and practices in mathematics.

3.1.4 Describing the Research Approach for this Investigation

Multiple research methods involving a combination of questionnaire, interviews and observations were used in this investigation to gain information about teachers' problem-solving beliefs and practices. This section presents an overview of the research methods and their relationship to the proposed model. Strategies for developing confidence in the research are described. This is followed by separate discussions about issues related to the use of questionnaires, interviews and classroom observations as data collection procedures.

3.1.4.1 Research Methods and Their Relationship to the Proposed Model for This Investigation

To ascertain the beliefs and practices of teachers in NSW primary schools, data were collected from a large group of teachers before interviews and observations were conducted. Such large-scale data are typically collected using survey methods such as questionnaires. The data from the questionnaire informed the fieldwork as it highlighted key issues for further exploration.

In Chapter 2, a model was presented that was derived from other models connecting teachers' beliefs and practices. Essentially this research is about what teachers believe and how they view their own practices. Proposed factors that impact on teachers' problem-solving beliefs and practices include advice, teachers' knowledge, curriculum advice, constraints and opportunities as well as teachers' experiences in classrooms.

A questionnaire was developed for the purpose of collecting data from a group of primary school teachers in NSW regarding their problem-solving beliefs and practices. The questionnaire sought to illuminate factors that impact on beliefs and practices through the use of a combination of closed and open questions. Specific questions addressed issues relating to advice, curriculum and knowledge. The closed questions were designed with reference to previously used questionnaires and their analyses. It was anticipated that the open questions might reveal other issues related to the implementation of problem-solving approaches in schools for this group of teachers.

The fieldwork involved interviews with, and observations of, a small number of teachers to obtain richer data about their beliefs and practices, to examine the factors that impact on each teachers' problem-solving beliefs and practices, and to determine the constraints and opportunities operating in those contexts. By entering these classrooms, observing interactions and recording discourse, data about the teaching/learning process and the use of problem-solving teaching approaches were collected. In this way the two data collection methods complemented each other.

Each aspect of data collection addressed particular elements of the proposed model enabling the researcher to consider the suitability of the model as a pictorial representation of teachers' reported beliefs and practices. It should be noted that this investigation is not described as a case study since substantial time was not spent with the teachers in the school settings that were the focus of the fieldwork. Rather, a combination of data collection methods were used to provide a rich, or detailed, description of some teachers' problem-solving beliefs and reported practices and to ascertain constraints and opportunities operating in their classroom contexts. The combination of data collection methods also enabled triangulation procedures to be used to confirm findings thus strengthening the research and adding to confidence in the results.

3.1.4.2 Developing Confidence in the Research

As explained above, this investigation most closely aligns with the interpretive paradigm of inquiry described by Shulman (1986), Romberg (1992) and Connole et al. (1993). Use of the interpretive approach in social research is not without criticism. These criticisms include suggestions that it is based on unreliable and possibly naive accounts of everyday life and that it places too much emphasis on participants' subjective meanings of reality. It has already been acknowledged that this research is not value free but confidence in the research can be supported through a consideration of strategies that can enhance trustworthiness, authenticity, dependability and confirmability.

Regardless of the validity of participants' interpretations of reality, or the appropriateness of their beliefs, this investigation aimed to reveal those beliefs in order to understand subsequent actions. In fact, the investigation is based on the assumption that people act on their beliefs. It is therefore appropriate to ask for participants' interpretations of what they believe and what they do in their classrooms. However, several issues needed to be considered to enhance confidence in this research. These include notions of validity of subjective accounts, replicability of context specific data, influence of the researcher on the social setting and participants, reporting of data by the researcher, and reliability of the researcher's interpretation of results.

Use of the terms *reliability* and *validity* stem from scientific approaches to research and involve the notions of replicability of results and accuracy of findings. Guba and Lincoln (1994) advocated the use of different terms for reliability and validity for interpretive research. They proposed the use of *trustworthiness* for internal validity, *authenticity* for external validity, *dependability* for reliability, and *confirmability* for objectivity.

The use of a new set of terms is supported by others including Ely (1991) and Lather (1986). Ely stated

> For many of us, the use of different terms for "validity" and "reliability" is a deliberate and liberating act. To speak of trustworthiness and its components of credibility, transferability, dependability, and confirmability ... or of authenticity criteria ... is to remind ourselves of the issues and processes that must weave their way through and beyond our qualitative research to keep it and us honest and believable ... we have found that the language of positivist research is not congruent with or adequate to qualitative work, and its use is often a defensive measure that muddies the waters (p. 95).

Ely recommended several strategies that can aid trustworthiness. These included going back to the field to confirm findings, peer involvement in the research process, peer checking such as inter-rater reliability checks, member checking by confirming interpretations with participants in the study, and triangulation.

Trustworthiness can be established through triangulation as data obtained by a variety of methods are checked against each other. Ely (1991) suggested that triangulation can also involve the same data collection method but used over different time periods, or, by comparing different reports written by two different researchers studying the same phenomenon. Triangulation can result in convergence, inconsistency and contradiction. Ely indicated that

> inconsistencies and contradictions may help us to refine and revise our framework and findings; but they may be just what they seem: inconsistent and contradictory findings that must stand as they are and be reported as such (p. 98).

Lather (1986) proposed a different set of terms including negotiation, reciprocity and empowerment which she described as "research as praxis" (p. 257). Lather stated that "reciprocity implies give-and-take, a mutual negotiation of meaning and power" (p. 263). She recommended strategies to attain full reciprocity including "self-disclosure on the part of the researcher" in interviews, sequential interviews, negotiation of meaning, and discussions of false consciousness. The latter strategy is described as

> ... a dialectic between people's self-understandings and researcher efforts to create a context which enables a questioning of both taken-for-granted beliefs and the authority that culture has over us ... There ... lies the opportunity to create reciprocal, dialogic research designs which not only lead to self-reflection but also provide a forum in which to test the usefulness, the resonance of conceptual and theoretical formulations (p. 266).

To address issues of trustworthiness of data, Lather argued for a combination of triangulation, reflexivity, and member checks. She suggested that it is necessary to "formulate self-corrective techniques that check the credibility of data and minimise the distorting effect of personal bias upon the logic of evidence" (p. 270) and that it is critical to use multiple data sources. She also recommended continual negotiation of findings with at least some participants.

Another set of advice is provided by Lincoln and Guba (1985). They argued that

> certain measures be employed *during* the implementation of the inquiry that either increase the probability that a judgement of trustworthiness will eventually be achieved or that provide the data that will subsequently be needed to reach that judgement (p. 281).

These measures included maintaining field journals, mounting safeguards against distortions, triangulating and doing debriefing with a non-involved professional peer. Distortions can arise through the researcher's presence and involvement since some participants want to be "helpful". Awareness of these strategies and careful auditing of all steps in the research process help to maintain confidence in the research.

In summary, it has been argued that the terms *reliability* and *validity* have been replaced with other more appropriate terms for interpretive research. These include *trustworthiness* and *dependability*. Regardless of terminology, the issue of confidence in research findings needed to be addressed by using strategies to assist in confirming the accuracy of data gathering and analysis techniques. Several useful strategies have been recommended and were considered in relation to each of the data collection procedures for this investigation. The next section considers issues that relate to the use of questionnaires, interviews and classroom observations as data collection procedures.

3.1.4.3 Specific Issues Related to Each of the Main Data Collection Methods

This investigation combined a questionnaire with interviews and observations as the main data collection methods. Each of these procedures had particular issues to be addressed if data were to be trustworthy and dependable, and these are considered in this section for each of the three methods.

Issues Related to the Use of a Questionnaire

The first phase of the investigation involved the use of a questionnaire to collect large-scale data about teachers' problem-solving beliefs and practices. Questionnaires have been used to collect data about teachers' beliefs in many studies relating to education and, in particular, mathematics education. Procedures are recommended that assist in ensuring that the data obtained from questionnaires are reliable even though there is a distance between the researcher and respondents.

Questionnaires have been used extensively to gather data from large populations in educational research. Studies conducted by the International Association for the Evaluation of Educational Achievement (IEA) and by the National Assessment of Educational Progress (NAEP) in the USA employed questionnaires to collect information about factors that might impact on student achievement in particular subject areas (Rosier, 1997). For example, the recent Third International Mathematics and Science Study (TIMSS) used questionnaires to gather data from Education Department officers, school principals, teachers of sampled classes, and participating students (Lokan, Ford, & Greenwood, 1997). The teachers' questionnaire consisted of items related to classroom practices, including planning and teaching strategies, and questions that explored teachers' beliefs about teaching as a career.

In mathematics education research, a popular method of collecting data about teachers' beliefs and practices has been the use of questionnaires. For example, *The Beliefs Survey* used by Peterson et al. (1987) was designed with four subscales including

- the role of the learner;
- the relationship between skills and understanding;
- the sequencing of topics; and
- the role of the teacher.

Another example is the *Beliefs About Teaching Mathematics* questionnaire designed by Van Zoest et al. (1994) that incorporated a range of views adapted from Kuhs and Ball (1986). More recently, questionnaires have been used by Perry et al. (1996), Sullivan and Leder (1990), Sullivan and Mousley (1994), Howard et al. (1997) and Raymond (1997).

For this investigation, a questionnaire was used to gather a wide variety of data from the target population of primary school teachers in NSW. It was used to both describe teachers' reported beliefs and practices and to examine the relationship between particular variables that represented some of the factors in the proposed model presented earlier. The questionnaire was considered to be a less intrusive method of data collection that could be administered in a relatively short period of time in a selection of country and city schools. As all teachers would be responding to the same set of questions, it was anticipated that the responses could provide a reliable set of data for that group of teachers.

When using a questionnaire to gather data, particular assumptions are made. Wolf (1997) suggested that the assumptions include notions that

> (a) the respondent can read and understand the questions or items;
> (b) the respondent possesses the information to answer the questions or items;
> (c) the respondent is willing to answer the questions or items honestly (p. 422).

Keats (1993) described other issues related to the use of questionnaires. He stated

> It must rely entirely on the effectiveness of its format to cope with a wide variety of responses and respondents. In many cases respondents have difficulty fitting their 'true' answers into the response categories which have been provided. Many respondents to questionnaires also have difficulty encapsulating their thoughts into precise, brief wording, so they do not use the additional few lines sometimes set aside for qualifying answers or making comments (p. 12).

Each of these assumptions and issues needed to be addressed in the design of the questionnaire and in the planning and administration of this phase of the study.

In addition, survey research methods can be fraught with many problems unless the design of the questionnaire is carefully considered. Issues of readability, layout and length are important but the choice and structure of items is critical. Fetterman (1989) suggested that while questionnaires are an efficient means for large-scale data collection, there can be problems with misinterpretation and misrepresentation caused by the distance between researcher and respondent. Items needed to be developed that would provide useful and reasonably accurate information about teachers' beliefs and practices. Selection of the types of items was informed by several earlier studies including those conducted by Fennema et al. (1989), Perry et al. (1996), Peterson et al. (1987) and Van Zoest et al. (1994).

Another issue is that using a questionnaire to ask teachers about their beliefs can create several difficulties. It is possible that some teachers have never really thought about what beliefs they hold and whether there may be different ways of looking at the same thing. Beliefs are fluid and can be subject to change based on the experiences and influences that may be taking place (Thompson, 1992), or they may be context specific (Hoyles, 1992). Hence it is acknowledged that the completion of questionnaires will only represent a snapshot of each teachers' beliefs at a particular point in time, providing the items are appropriate to elicit such beliefs.

Strategies can be employed in order to overcome concerns about the trustworthiness and dependability of questionnaire data. Confidence in research results can be developed by adopting advice offered in the literature about survey research methods. For example, Fowler (1988) discussed issues relating to sampling, item design and total survey design. Random sampling of a particular population can assist generalisation of results to the total population. Confidence can also be enhanced by ensuring that all respondents answer the same set of questions, interpret questions in similar ways, and understand meanings of terms that might have multiple interpretations. Providing definitions of such terms can support mutual understanding.

Item design includes consideration of aspects such as careful construction of questions to meet the objectives of the study, use of closed and open questions, and selection of the appropriate types of response scales. In this regard, Converse and Presser (1986) recommended the use of "simple language, common concepts, manageable tasks, and widespread information" (p. 10). Fowler (1988) argued that open-ended questions can be more difficult to answer than closed questions but they are valuable as they can reveal information not anticipated by the researcher.

Many different types of scales can be used in questionnaires including varieties of nominal and ordinal scales but there is also advice about the use of some of these scales. Converse and Presser (1986) discussed the criticisms of using agree-disagree responses because of the "acquiescence response set" (p. 38). This suggests that some respondents will generally agree with statements regardless of their meaning. Checks within the questionnaire can be created by designing multiple probes such as several items relating to the same phenomenon, and wording some of these in a negative way. Converse and Presser also recommended that scales should not include an "unsure" category as this is an easy option to avoid taking a particular position on an issue. Fowler (1988) discussed the importance of minimising "a sense of judgement" (p. 94) in questionnaire items as this may cause respondents to answer according to the way they think the researcher would prefer, or to adopt the most desirable position regardless of their true beliefs.

Overall design was also an important aspect of constructing a useful questionnaire and needed to be carefully checked using trialing procedures. The overall design included a consideration of the type of background information to request from respondents, ordering of items, layout and length. Trialing of initial questionnaire drafts was a critical step in the process. During the trialing, Converse and Presser (1986) recommended checking for variation, meaning, task difficulty, respondent interest and attention, flow, order of questions, and timing. This could be done by administration of the questionnaire to a small sample of the population with follow-up discussions and interviews. Fowler (1988) also argued that guarantees of confidentiality and anonymity are more likely to result in honest responses to questionnaire items.

Another important issue that establishes confidence in the findings is the response rate to the questionnaire. Insufficient responses may suggest problems with the design or administration procedures and may result in a biased, or non representative, set of results. These concerns were expressed by Brewer and Hunter (1989) as follows

> The generalisability of survey results depends not only upon the sampling techniques but also upon respondents' willingness to cooperate. If respondents fail to respond, then the researchers must question the survey's findings, because those respondents who fail to answer may be significantly different from those who do. Informant interviews can check the inferences made from low response surveys ... (p. 51)

In summary, questionnaires have been used extensively in studies that have investigated teachers' beliefs and can be used as useful sources of information. In designing the questionnaire, issues to be addressed include sampling of the target population, item design, and overall questionnaire design. Substantial advice is available for each of these aspects that helps to support the trustworthiness and dependability of the results. This advice was heeded in the questionnaire design for this investigation and its adoption is explained in Section 3.2.1 of this chapter.

One method of checking to confirm interpretation of questionnaire items by respondents and interpretation of responses by the researcher is to conduct interviews with a small number of respondents. This strategy was used in this investigation. The initial interviews were also used to probe for further information about teachers' beliefs and practices. There are associated issues with the use of interviews to collect data and these are discussed in the next section.

Issues Related to the Use of Interviews

The second phase of this investigation was conducted in the field and involved interviews and classroom observations of a small number of teachers who had responded to the questionnaires. This section examines issues related to the use of interviews as data collection procedures while the following section outlines issues related to the use of classroom observations.

Fieldwork is context specific and as a consequence its unique features are interpreted and reported through the eyes of the researcher. This has implications for establishing confidence in results from research. To overcome possible concerns, careful and thorough description of the choice and use of settings, the social conditions, the role and status of the researcher, the theoretical and analytical constructs, and data collection and analysis, enables the reader to analyse the research for trustworthiness and dependability.

Interviews are a commonly used data collection procedure in qualitative research (Ely, 1991; Fetterman, 1989; Fontana & Frey, 1994). Fetterman suggested that they help to "classify and organise an individual's perception of reality" (p. 50). Ely proposed that a major purpose of interviews "is to learn to see the world from the eyes of the person being interviewed" (p. 58). Lincoln and Guba (1985) suggested that "a major advantage of the interview is that it permits the respondent to move back and forth in time - to reconstruct the past, interpret the present, and predict the future" (p. 273). Keats (1993) stated that

> Interviews give information immediately which can be qualified or added to if necessary, they allow on-the-spot interpretations and reactions, they have flexibility ... they can be used to supplement other means of gaining information (p. 13).

Using interviews to collect data from participants in studies is a powerful procedure enabling clarification of understanding and discussion of more personal, or sensitive, issues. Audio tapes provide accurate recordings of responses, however, issues to be considered when using interviews include selection of participants, type of interview to be used, and analysis of interview data. Trustworthiness and dependability of data can be enhanced by careful consideration of these issues as well as recognising the influence of the researcher on data collection and analysis.

Careful choice of participants is critical to yielding useful data that addresses research issues and provides data that focuses on the research questions for the study. For this investigation, participants were chosen on the basis of their responses to the questionnaire. This method of selection has been described by Fetterman (1989) as "judgemental sampling" and involves "judgement to select the most appropriate members of the subculture or unit, based on the research question" (p. 43).

Interviews have been classified in various ways. One example is Fetterman's (1989) classification of interviews as structured, semistructured, informal, and retrospective. Fetterman described the first two as "verbal approximations of a questionnaire with explicit research goals" (p. 48) enabling comparison between participants. Structured interviews "tend to shape responses to the researcher's conception of how the world works" (p. 48). Informal interviews are typically used in ethnographic investigations and could be described as casual conversations. Retrospective interviews can be either structured, semistructured or informal and encourage the participant to reconstruct past events. Fontana and Frey (1994) also described group interviews; frequently referred to as "focus group" interviews. They argued that

> The group interview has the advantages of being inexpensive, data rich, flexible, stimulating to respondents, recall aiding, and cumulative and elaborative, over and above individual responses (p. 365).

However, disadvantages of using group interviews include the possibility of domination by one or a few participants, group culture may interfere with individual participation, group dynamics can stress interviewer skills, and sensitive topics may be less likely to be discussed honestly and openly.

All interview types are useful at particular times during an investigation. Structured interviews are appropriate when the researcher has information about particular issues and therefore wishes to address specific questions to each of the participants. Less structured interviews can provide information about situations where the researcher has fewer notions of the phenomenon of consideration, or the possible interrelationships between contributing factors in the investigation.

Analysis of interview data can take several forms. Tesch (1990) described the processes of segmenting and categorising interview data. Ely (1991) suggested that

> establishing categories from qualitative data seems rather like a simultaneous left-brain right-brain exercise. That is, one job is to distil categories and the other is to keep hold of the large picture so that the categories are true to it (p. 87) ... [also] ... making categories means reading, thinking, trying out tentative categories, changing them when others do a better job, checking them until the very last piece of meaningful information is categorised and, even at that point, being open to revising the categories (p. 145).

Final analysis often involves a search for themes. Ely suggested that

> it can be thought of as the researcher's inferred statement that highlights explicit or implied attitudes toward life, behaviour, or understandings of a person, persons, or culture (p. 150).

Another form of analysis described by Fetterman (1989) is the use of maps, or visual representations of the main ideas that arise from interviews. He stated that "the process of drawing also crystallises images, networks, and understandings and suggests new paths to explore" (p. 95).

Critical to the process of data collection when entering the field is building and maintaining trust (Lincoln & Guba, 1985) and establishing a rapport (Fontana & Frey, 1994). Before conducting interviews, it is appropriate to inform participants fully about their involvement in the study, to affirm confidentiality, and to describe the final distribution of all results. During the interview, it is necessary for researchers to be prepared to answer questions from participants, to keep an open mind, and not to make judgemental comments about responses (Keats, 1993). After interviews, reassurance is still necessary.

The role of the researcher and her subsequent influence on the research must be recognised. Issues include power relations between participants and researcher, acquiescence and attempted helpfulness by participants, expectancy and halo effects by researchers, and the overall social relationships between the researcher and those in the social setting (Connole et al., 1993).

In summary, interviews are useful and worthwhile procedures to collect detailed information from a small number of participants. They enable negotiation of meaning, clarification of ideas, and creation of new issues for investigation. Confidence in the research can be enhanced by appropriate choice of participants, use of suitable interview types, and clear analysis of data. Detailed descriptions of all procedures are also necessary in the final report of the research. This advice was used to plan and conduct the interviews for this investigation and is described in Section 3.2.2 of this chapter. Some of the issues discussed in this section are also applicable to the use of classroom observations.

Issues Related to the Use of Classroom Observations

In this investigation, classroom observations were used as the other main data collection method during the fieldwork phase of the study. Combining interviews and observations strengthens the research results in important ways. This section describes issues related to the use of classroom observations including strategies to ensure the trustworthiness and dependability of the data.

Observations of human behaviour have formed the basis of much social science research, often in conjunction with other data collection methods (Adler & Adler, 1994). Used in combination with other data collection procedures, observations enable researchers to confirm data reported on questionnaires or in interviews. If observations reveal inconsistencies in data, further interviews and observations can alert the researcher to alternative influencing factors, or indicate that participants may even be unaware of the existence of such contradictions.

There are several forms of observation that depend on the level of involvement of the researcher in the culture or group being investigated. Much of the literature distinguishes between participant and non-participant observations (e.g. Woods, 1986). Atkinson and Hammersley (1994) suggested that this classification may not be appropriate since all observations impact on those being observed in the sense that "we cannot study the social world without being part of it" (p. 249). Adler and Adler (1994) used categories that included "the complete-member-researcher, the active-member-researcher, and the peripheral-member-researcher" (p. 379). Peripheral membership recognises that the presence of the researcher does impact on participants and that

> ... they observe and interact closely enough with members to establish an insider's identity without participating in those activities constituting the core of group membership (Adler & Adler, 1994, p. 380).

The research questions for any study frame the focus of observations, guide the choice of appropriate data collection and analysis procedures, and determine the style of data reporting. Systematic classroom observations, usually involving coding protocols, have been used to address rather specific questions relating to particular aspects of teaching and learning in classrooms. An example is the investigation of particular kinds of teachers' questions as in the Sullivan and Leder (1990) study. Other more general investigations may lead to field notes or videotaping of lessons to provide descriptions of teaching actions and behaviours. An example is the investigation of teacher change conducted by Clarke (1997). Fine-grained investigations are more likely to result in the use of some quantitative data to report findings, whereas a more general exploration of aspects of teaching actions may result in a "thick description" as described by Fetterman (1989, p. 114). Such differences in analysis could also be described as analytic and holistic (Koehler & Grouws, 1992).

Systematic observation procedures have been used widely in educational research. According to McIntyre and Macleod (1986) this includes

> ... those procedures in which the observer, deliberately refraining from participation in classroom activities, analyses aspects of these activities through the use of a predetermined set of categories or signs (p. 10).

They suggested that this approach means that "the researcher focuses attention on some things to the neglect of others" (p. 11), however, researchers "make quite explicit the aspects of teaching on which they are focusing attention, and make any ideological commitment quite transparent" (p. 11).

There have been many criticisms regarding the use of such schemes including the use of predetermined categories, loss of other potentially valuable data, overuse of quantification of data, and the suggestion that observations do not take into account meanings for actions. In addition to these concerns, McIntyre and Macleod (1986) suggested that there is an assumption that the observer has no influence on what occurs in the classroom being observed. Delamont and Hamilton (1986) expressed concern that "systematic observation schemes typically ignore the temporal and spatial context in which the data are collected" (p. 29) and that such schemes are only concerned with overt, observable behaviour focusing "on small bits of action or behaviour rather than global concepts" (p. 29).

More global, or holistic, investigations of classrooms have examined teachers' and students' behaviours using a variety of data collection procedures. Delamont and Hamilton (1986) argued for an ethnographic approach to classroom observation and suggested

> Ethnographies involve the presence of an observer for prolonged periods in a single or a small number of classrooms. During that time the observer not only observes, but also talks with participants ... In addition to observing classroom life, the researcher may conduct formal interviews with the participants and ask them to complete questionnaires ... The ethnographer uses a holistic framework (pp. 35-36).

Further, Delamont and Hamilton suggested that "an adequate classroom study must acknowledge and account for both the internal and external aspects of classroom life" (p. 38). It is possible that ethnographic approaches could be used in combination with systematic procedures so that data from both sources complement each other.

A useful form of data collection is to use video tapes to record classroom events although decisions must be made beforehand regarding the focus of the recording. Such recordings enable ongoing analysis as they can be viewed repeatedly by the same researcher, or by several different researchers. Fetterman (1989) suggested that repeated analysis can yield "new layers of meaning or nonverbal signals from teacher to student, from student to teacher, and from student to student" (p. 85). Ely (1991) also mentioned the value in "repeated studying". However, use of a video recording may cause tunnel vision as the researcher focuses on particular aspects of events to the exclusion of others. In addition, field notes can be used to record general observations of classroom activities thus enhancing material collected on the video recordings.

Additional issues to be addressed when conducting observations in the field relate to establishing trustworthiness and dependability. Trustworthiness of fieldwork requires confirmation that analyses are accurate interpretations of what has been observed. Ongoing discussions with participants and sharing of written accounts enables the researcher to verify and confirm findings. Fetterman (1989) argued that there is a need to confirm strong impressions whenever possible although Ely (1991) reported that credibility can be established through "prolonged and persistent observation" (p. 96). This view is supported by Adler and Adler (1994) who recommended that observations need to be "conducted systematically and repeatedly over varying conditions" (p. 381).

It has also been recognised that research changes those being researched (Guba & Lincoln, 1994). These issues can be overcome if the researcher recognises the possible impact of the research on the data, produces a careful description of those who provided the data, and attempts to keep an open mind to all that occurs. Ely (1991) argued that

> ... the very act of observing can alter what is being observed. It follows, then, that even at our most unintrusive, we influence the very phenomenon we are studying ... for qualitative researchers the important issues are: (1) that we participate as closely as possible in line with the needs of our study; (2) that we make ourselves as aware as possible of the ripples caused by our participation; (3) that we attempt to counter those ripples that might hinder the participant-observer relationship and, hence, the study; and (4) that we describe in the report both what worked and what did not (p. 47).

Also, Ely noted that observations are never "judgment-free" since "observation comes out of what the observer selects to see and chooses to note" (p. 53).

Reporting the findings from observations requires careful consideration of the above issues. Ely (1991) likened the report to "the creation of a narrative" (p. 169) and suggested the use of possible narrative devices including descriptions of "critical incidents" and presentations of "snapshots" of events. Regardless of style, advice suggests a clear description of settings, participants, data collection and analysis procedures, and methods used to address issues of trustworthiness and dependability.

The language used by the researcher to tell the story of a participant must also be considered according to Brown et al. (1990). In criticising some studies into teaching, they stated

> For the most part these stories were told by investigators who were themselves professional educators and who assumed that they had a common language with their informants ... researchers operated on occasion as if they understood informants who used words and expressions like *problem solving*, *tests*, and *student ability* (p. 651).

The notion of shared understandings, or perceptions about crucial concepts, is critical to accurate reporting of data. These ideas are of considerable importance to this investigation as many of the terms used can have multiple meanings. Considerable attempts were made in the design of the questionnaire to overcome different interpretations of terms such as *problem* and *problem-solving approaches* to teaching. In addition to this, there were ongoing discussions during the interviews to establish common interpretation of ideas. These are discussed in detail in the next section on the development of the instrument.

In summary, classroom observations provide valuable data that can be used in combination with other data collection procedures such as interviews. When using observations, it is important to consider issues relating to the participation level of the researcher, focus of the observations, appropriate data collection procedures, checking of data interpretation with participants, and suitable reporting style. All of these factors were considered during the planning, data collection and analysis, and reporting of the results for this investigation and are described in Section 3.2.2.

It has been noted that there are issues of consideration relating to *all* of the data collection procedures that were used for this investigation. Particular strategies have been suggested in the literature that can be used to enhance confidence in the research, particularly in relation to trustworthiness and dependability. This advice informed this investigation in important ways including the use of ongoing negotiation of meaning during interviews and observations, triangulation of results, and careful reporting of all aspects of the investigation.

This investigation was based on a multiple methods approach to data collection. In the first phase, a questionnaire was used to collect considerable data about teachers' problem-solving beliefs and practices. In the second phase, interviews and observations were conducted in the field to collect detailed information about some teachers' beliefs and practices and to determine possible constraints and opportunities that impact on teachers' efforts to implement problem-solving approaches.

It has been argued that the data collection methods used in this investigation complement each other and therefore strengthen the results. Questionnaires are high on dependability but low on trustworthiness since even though respondents may answer questions that are asked in exactly the same way, it is impossible to determine if they have interpreted the questions in the same way. Interviews and observations in the field are high on trustworthiness but low on dependability since meaning can be negotiated with the participants but each context is different and is influenced by different factors thus rendering comparability difficult.

The use of similar mixed-method approaches is common in the literature that has explored and reported on teachers' beliefs and practices, and these studies have informed the development of the research tools for this investigation. The next section of this chapter outlines the development of the research tools and describes the data gathering procedures in detail.

3.2 Development of the Research Tools and Data Gathering Procedures

The data collection in this investigation involved two phases. The first phase of the research investigated the first two research questions and the primary data source was a questionnaire that was distributed to groups of primary school teachers. The second phase of data collection involved interviews with, and observations of a small number of teachers. This section describes the development of the questionnaire, including preliminary investigations and trialing of the instrument, and then outlines the fieldwork procedures used in the second phase of the investigation.

3.2.1 The Questionnaire

The purpose of the questionnaire was to determine teachers' beliefs about the role of problem solving in learning mathematics, to gauge the extent of teachers' problem-solving practices, and to discover other issues that may be impacting on the implementation of problem-solving approaches. The first two research questions, as listed in Table 17, formed the focus of the questionnaire.

Table 17
The first two research questions for this investigation

Number	Question
1	What do teachers believe is the role of problem solving in learning mathematics? In particular, what are teachers' beliefs about the relationship between problem solving and: • mathematics; and • students' learning of mathematics?
2	To what extent do teachers report that they incorporate problem-solving approaches in their planning and teaching of mathematics, and what specific practices do they report that they use? In particular, when incorporating problem-solving approaches in their mathematics teaching: • what specific classroom strategies do teachers claim to use; • what types of problem-solving tasks do teachers claim to prefer and what tasks do they claim to use; • what do teachers report is the form of their planning; and • what resources and materials do teachers report they use?

There are four parts to this section, each of which describes a stage in the development and administration of the questionnaire. The first provides a summary of the initial investigations that were conducted to assist in the development of the questionnaire. The second and third parts describe the development of the instrument and outline the trialing procedures. The final part includes a description of the teachers who were targeted to respond to the questionnaire, an overview of procedures used to collect data, and a discussion of issues relating to the collection of questionnaire responses.

3.2.1.1 Initial Investigations of Questionnaire Items

There were several stages in the development of the instrument. Preliminary investigations were used to discover the meaning teachers give to the term *problem*, to investigate factors that they describe as being important in the learning of mathematics including the role of problem solving, and to determine the kinds of problems teachers prefer to use.

This section summarises the first stage in the development of the questionnaire with a more detailed description provided in Appendix 4. A *Preliminary Survey* (Appendix 5) was designed to obtain information about teachers' background, beliefs and favourite problems. The background information included gender, number of years of teaching experience, grade currently being taught, role in the school, number of days inservice attended in the last two years as well as the number of these that were devoted to mathematics. It was anticipated that there may be some link between the amount of mathematics focused professional development and knowledge about problem-solving teaching approaches. It was also anticipated that the types of problems that teachers recorded would provide insights into their understanding of the term *problem*. Each teacher was asked to record their two favourite problems and to describe briefly why these were chosen. This question was designed to be an unobtrusive measure of teachers' interpretation of *problem*.

The *Preliminary Survey* provided useful information about teachers' beliefs, preferred problems, and reasons for problem choices. The types of problems chosen provided some indication of each teacher's classification of what constitutes a problem and what characteristics of these chosen problems make them particularly useful. However, the open-ended nature of the questions meant that responses were often difficult to interpret and compare. It was evident from this preliminary investigation that a more structured approach was required in the final questionnaire. It was determined that items needed to be clearer and Likert items should be used in order to force a choice about particular aspects of problem-solving instruction.

It was also revealed that most teachers support the use of problem solving as an *end* in teaching mathematics but further issues for exploration were identified. Teachers do have different beliefs about the role of problem solving in learning mathematics but in order to decipher exactly what these are requires clarification of what each teacher understands by the terms *problem* and a *problem-solving approach* to teaching. A decision was made that providing examples of problem types and avoiding the use of the term *problem-solving approach* may help to overcome these issues.

The data about favourite problems highlighted the need to have teachers describe how the problem was used in the classroom. Teachers make decisions about appropriate tasks based on their beliefs, the grade and individual students, as well as the skills and concepts they are trying to develop. These were not revealed in the surveys and it would have been more useful if teachers had been asked to describe a recent problem-solving lesson. Also, it was determined that placing items in a context that teachers could relate to may help them to interpret the appropriateness of problems and teaching actions. In addition to these ideas, it may have been more useful to present teachers with a set of problems and to have them rank these according to their usefulness with a particular grade, as well as their value for promoting understanding of mathematical concepts.

The next stage in the initial investigations involved another survey instrument entitled *Favourite Problems for Year 3* (Appendix 6). This survey required teachers to rank a set of problems that might typically be used in a Year 3 class. The problems were taken from the set of favourite problems that had been recorded by participants in the earlier investigation and were chosen on the basis of coverage of content areas and representation of the variety of problem types. Respondents were required to rank the seven problems by allocating "1" for the problem they would most likely use, to "7" for the problem they would least likely use. They were also asked to record their reasons for choosing a particular problem first and another last.

There was a wide spread of selections of favoured problems from the set presented. Every problem was chosen by at least one respondent as most favoured and every problem was chosen by at least one respondent as least favoured. This suggests that teachers choose different problems for different purposes. While the responses were certainly of interest, because of the range of selections for each problem this item was not included in the final questionnaire. In addition, the problems covered all of the content strands and represented a variety of closed and open questions. This may have influenced teacher choices based on their confidence with particular topic areas or their preference for either closed or open questions. It was determined that problems based on the same content area that would be given to a typical year group may remove some of this variability between problems.

These initial investigations demonstrated that it was appropriate to use a combination of closed and open questions in the final survey instrument. This would more readily allow comparisons between teachers on closed items and yet allow for diversity of issues on the open items. To determine teachers' problem-solving beliefs and practices, a decision was made to use structured Likert items that suggested either a *traditional* or a *contemporary* perspective to teaching mathematics. In addition, placing such items within the context of teaching a particular topic in the K-6 mathematics curriculum may enable teachers to more easily make decisions.

Definitions of problem types needed to be provided in order to overcome the variety of interpretations of the term *problem.* Also, by providing an example of each problem type, teachers could use that as a reference point for the beliefs statements. These examples could then be used to invite a response about frequency of use rather than a question involving ranking of preferred questions as it was postulated that all types would be used at some stage by each teacher. It was deemed appropriate to request a recently used problem from each respondent but to include a request for purpose of use. These initial investigations provided useful information to aid the development of the final questionnaire.

3.2.1.2 The Development of the Questionnaire

The survey instrument, or questionnaire, was designed to gather data for the first phase of the study. Items needed to address the first two research questions that focused on teachers' reported beliefs and reported practices. The questionnaire was designed with a number of components including Likert items, scales and free-response questions. Each of these components was used to explore several aspects of the first two research questions. The Likert questions were used to find the level of agreement with statements which might reflect teachers' views about problem solving and learning mathematics. Scales were used to ascertain planning procedures, frequency of use of resource materials, particular teaching strategies, and preferred problem types. Free-response, or open-ended questions, were used to invite comments on all aspects of both research questions. Table 18 matches the questionnaire components with the research questions.

Table 18
Questionnaire components and research questions

	Question 1		Question 2			
Components	Maths	Learning	Planning	Resources	Strategies	Problems
Likert items	√	√				
Scales			√	√	√	√
Free-response	√	√	√	√	√	√

The design of the instrument is critical to its usefulness as a measurement tool. Care was taken to ensure that items were simply worded, unambiguous, specific and conceptually clear. This was achieved by consulting a variety of questionnaires that had been developed by other researchers in the field (e.g. Fennema et al., 1989; Howard et al., 1997; Peterson et al., 1987; Van Zoest et al., 1994), by discussing items with post-graduate research students at the employing institution of the researcher, and by sharing ideas and obtaining feedback from fellow researchers at research conferences (Anderson, 1996, 1997). Many changes were made at this stage of the questionnaire development.

Open-ended questions, such as those used in free-response items, were designed by the researcher and used to allow respondents the freedom to provide information that the researcher had not anticipated. The responses to these open questions were intended to "describe more closely the real views of the respondent" (Fowler, 1988, p. 87) and hopefully provided a more realistic description of teachers' beliefs about problem solving and mathematical learning.

The final questionnaire included a title page, an initial section to gain information about respondents, a set of background information for participants to read, nine questions to be answered including two with Likert items, three with scales and four open-ended responses. Data were sought that would provide information about respondents, including school, gender, experience, current role, and time spent on professional development in the last two years. Participants were invited to include their name only if they were willing to be contacted by the researcher for a subsequent interview. In the design of the questionnaire it was clear that one of the most important issues to address was teachers' interpretation of the term *problem.* To overcome this, a set of student question types with examples was presented at the beginning (see Figure 14).

Background Information:

For the purposes of this survey, the following definitions are given to assist understanding of the terms which are used.

After teaching 2 digit addition students could be asked to answer the following:

Type of Student Question:	**Example:**
Exercise (we are not calling this a problem)	35 + 27
Application problem	If there are 32 oranges in one box, 37 in a second box, and 35 in a third box, how many oranges are there altogether?
Unfamiliar problem (ie. a problem type students haven't seen before)	There are pigs and chickens in a farmyard and altogether there are 23 heads and 68 legs. How many pigs and how many chickens are there?
Open-ended problem	□ □ + □ □ 1 3 4 What might the missing numbers be?

Figure 14. Background information about problem types presented at the beginning of the questionnaire.

The student question types were chosen on the basis of typical questions or problems discussed in the literature (e.g. Clarke & McDonough, 1989) as well as in resource materials and textbooks in NSW. The example for each problem type related to two digit addition since this was the topic chosen for the basis of items in the first two questions on the questionnaire.

The first and second questions in the questionnaire sought to obtain information about each teachers' beliefs about problem solving. To encourage teachers to respond as honestly as possible, questions about beliefs were presented as representing the views of two imaginary teachers, Naomi and Gwendolin. Naomi's statements represented a belief system that suggested she uses problem solving as an *end* in learning mathematics (Table 19). This perspective is based on a *traditional* approach to teaching and learning mathematics that was described in Section 2.4.3.1. Gwendolin's statements suggested that she uses problem solving as a focus for learning mathematics, or as a *means* (Table 20). This perspective is based on a more *contemporary* view that was described in Section 2.4.3.2.

Table 19
Naomi's (traditional)statements about problem solving in relation to the teaching and learning of mathematics

Naomi's statements
students should learn all of the basic number facts before they do *application* and *unfamiliar problems*
students should learn algorithms before they do *application* and *unfamiliar problems*
students cannot solve problems until they know how to perform the four operations
the best problems are those which relate directly to the number facts and algorithms the students have been practising
application and *unfamiliar problems* are best left to the end of the topic in mathematics
most mathematics lessons should focus on practising skills
students have trouble solving problems unless they know how to do the mathematics before they begin
some students find problem solving difficult because of the language involved in the problems

Table 20
Gwendolin's (contemporary) statements about problem solving in relation to the teaching and learning of mathematics

Gwendolin's Statements
students should be regularly presented with problems which they cannot do automatically
mathematics lessons should focus on problems rather than on repetitive practice of routine procedures
students are more motivated to want to learn basic facts and algorithms if they can see a reason for learning them
mathematics problems should challenge students to think about what mathematics they know and how they can use it
many students can learn most mathematical concepts by working out for themselves how to solve *unfamiliar* or *open-ended problems*
it is important for students to explore their own ways of doing mathematics questions before being shown efficient methods
most students forget mathematics procedures and so it is best to let them work out their own methods first

The statements were presented in the context of teaching two digit addition to a Year 3 class and with reference to the student question types described in the background information. Respondents were required to record their level of agreement with each of the statements. There were four levels for selection including "strongly agree", "agree", "disagree", and "strongly disagree". A middle position of "unsure" was not used in an effort to encourage respondents to make a definite decision.

Choice of statements was made on the basis of reference to earlier instruments (e.g. Peterson et al., 1987; Van Zoest et al., 1994), to the studies reviewed in Chapter 2, and to comments made by teachers in response to the *Preliminary Survey*. Instead of using the general term *problem* in the items, one of the more specific student question types from the background information was used. Initially, reference was going to be made to the *Mathematics K-6* curriculum document (NSWDE, 1989) and its use of the terms *for*, *about* and *through* problem solving. After consultation with some teachers, it became apparent that teachers may not be aware of these terms and would not be clear about their meanings.

The third question in the questionnaire required teachers to record an example of a mathematical problem that they had presented in the classroom recently and to describe its purpose. This question aimed to collect information about the types of problems teachers prefer and to explore the uses of such problems. The earlier investigation suggested that it was appropriate to have teachers describe the context of use in more detail but this would have taken a great deal more time for respondents to record.

The fourth question listed twenty items that related to teaching strategies. This question aimed to explore teachers' reported practices when teaching mathematics. The items were chosen on the basis of strategies mentioned in the literature (e.g. Clarke, 1995; Koehler & Grouws, 1992; Van Zoest et al., 1994) as well as in curriculum documentation (NSWDE, 1989).

Teachers were requested to rate the frequency of use of each of the strategies. Categories for rating included "hardly ever", "about once a month", "about once a week", and "all of the time". Early discussions suggested changing this last category to "almost always" as there would be few strategies that teachers would use all of the time. It was anticipated that the frequency with which teachers reported that they use these strategies would provide an indication of the perceived importance of each of them as well as supporting other evidence about individual teachers' beliefs about problem solving. The strategies are listed in Table 21.

Table 21
Teaching strategies in mathematics

Teaching Strategies
you ensure that the students work alone
you explain in detail what the students have to do to solve problems
at the end of a problem solving lesson you lead a whole class discussion so that students can share solutions and strategies
you have calculators available for students to use
you encourage the students to work in small, cooperative groups
you present *unfamiliar* and *open-ended problems* to the class with very little indication of how to solve them
you encourage students to record their own procedures and methods of solving problems
you encourage students to pose their own problems
you provide a set of problems and the students are allowed to choose a problem they would like to work on
you allow the class or individual students to spend several lessons on the same problem
you use problems to show students that there are mathematical skills and procedures which they need to know
you present *application problems* which allow students to practise the skills they have just learnt
you provide concrete materials for those students who need them
you model the problem solving process to the class
you discuss useful problem solving strategies (eg. make a list, draw a diagram, work backwards)
you discuss the problem solving process (ie. make a plan, carry out the plan, check the calculations)
you use problems which arise from the school context or which relate to the students' experiences
you pose *open-ended problems* to allow students to explore mathematical situations for themselves
you set *exercises* to allow the students to practise their skills
you pose *unfamiliar problems*

The fifth and sixth questions in the questionnaire explored the frequency of use of each of the student question types and reasons for those choices. Question 5 listed the four question types and the same examples that had been presented in the background information at the beginning of the questionnaire. Teachers were required to indicate how frequently they use each in their teaching. The categories were listed as "often", "sometimes" and "never". This item was intended to gauge the types of questions teachers prefer. In Question 6, teachers were requested to explain why they choose to present those particular types of questions to their students.

Initially, the same set of four questions were to be presented to teachers to rank as most favoured to least favoured. Given the results of the earlier investigation it was decided that this item would not offer more information about teachers' use of these questions. Many teachers would access most of these types of questions at some stage in their teaching, and so it was decided to seek the frequency of use of these types as this may provide a better indication of the importance teachers place on the role of problem solving in learning mathematics.

Question 7 asked for an indication of the frequency of use of particular resource materials. Earlier discussions with teachers suggested that the main sources of problems were the *Mathematics K-6* curriculum document (NSWDE, 1989), textbooks, reference books, inservice courses, and sharing ideas with fellow staff members. Teachers were requested to indicate frequency of use as either "hardly ever", "about once a month", "about once a week", or "almost always".

The final two questions were open-ended and invited respondents to describe the professional development needs of the teachers at their school as well as to comment on a negative statement that had been made about problem solving. The first of these questions aimed to discover teachers' concerns about the level of implementation of problem solving. The second gave teachers an opportunity to agree with a negative comment about problem solving in a potentially non threatening way. The item was

> The following statement was made recently at a teacher inservice course:-
> "People who push problem solving in mathematics obviously don't work in classrooms. It is a waste of time."
> How do you react to this statement?

These nine questions formed the basis of an instrument that was designed with several purposes. These included ascertaining teachers' reported beliefs and practices in relation to problem solving, discovering favoured problems, determining frequency of use of particular problem types and reasons for their use, and illuminating other issues that might relate to teachers' attempts to implement problem-solving approaches. To determine whether this instrument would provide worthwhile information about these aspects of this investigation, it was necessary to trial the questionnaire with a small sample of teachers from the target population.

3.2.1.3 Trialing and Refining the Questionnaire

The draft questionnaire was trialed to review whether the above purposes were achievable and to check other important aspects related to design. Converse and Presser (1986) suggest that trialing needs to check for understanding of items, variation in responses, task difficulty, respondent interest and attention, flow, order of questions, and timing. Trialing of the instrument was undertaken with 12 practising teachers; a group of five of these teachers were interviewed to gather further information. The questionnaire was also discussed with five mathematics education lecturers from tertiary institutions. Several issues were raised that were addressed.

The twelve teachers involved in the trialing were part-time, post-graduate students studying at the institution of the researcher. All were full-time practising primary school teachers and were enrolled in research methods courses at the post-graduate level. They completed the questionnaire during class time so that the researcher could time completion and answer any questions. After completion, a voluntary group of five were interviewed together about their interpretation of each item on the questionnaire.

Each of these teachers was clear about the definitions of the types of questions in the Background Information. It seemed that they were familiar with the examples given and the information presented was adequate to communicate the meanings of terms used in the questionnaire. Minor changes were made to the numbers in each of the examples. To enable easier comparison between the student question types, it was recommended that the numbers used in each of the student questions be altered so that they were related. This also removed another difference as it could have been perceived that the different sets of numbers added to the difficulty of some of the student question types. In the final questionnaire, the questions used the numbers 37 and 34 or the sum given was 71 (see Figure 15).

Background Information:

For the purposes of this survey, the following definitions are given to assist understanding of the terms which are used.

After teaching 2 digit addition students could be asked to answer the following:

Type of Student Question:	**Example:**
Exercise (we are not calling this a problem)	37 + <u>34</u>
Application problem	If there are 34 oranges in one box and 37 in another box, how many oranges are there altogether?
Unfamiliar problem (ie. a problem type students haven't seen before)	The sum of my mother's age and my father's age is 71. My father is 3 years older than my mother. How old is my mother and how old is my father?
Open-ended problem	□ □ + □ □ 7 1 What might the missing numbers be?

Figure 15. Background information about student problem types presented at the beginning of the final questionnaire.

Responses recorded on the questionnaires and discussions about wording of belief statements in the first two questions revealed considerable variation of beliefs. The discussions about these items involved differences of opinion as to how some of them should be worded and resulted in some rewording of the statements. It became clear that teachers hold different views about the role of problem solving in learning mathematics and that the items were going to be able to elicit these views.

Two examples where changes were made include the following. Gwendolin's first statement was changed from "students should be regularly presented with problems which they cannot do automatically" to "it is a good strategy to begin the topic of 2 digit addition with unfamiliar problems involving 2 digit addition". In Gwendolin's sixth statement, "efficient methods" was changed to "the teacher's methods". The meaning of the second phrase was clearer to the teachers in the trial group.

Changes were also made to the categories for the frequency of use of teaching strategies and resources. The original questionnaire used the categories "hardly ever", "about once a month", "about once a week" and "almost always" but these were considered too restrictive by the respondents. Several teachers found that they were unable to portray accurately what their practices were and wanted to have other descriptors for the categories. After much discussion, there was agreement that it would be better to change them to "hardly ever", "sometimes", "often" and "almost always". For frequency of use of the different types of questions, the categories of "often", "sometimes" and "never" were changed to "often", "sometimes" and "rarely" so that they were more balanced.

Issues related to task difficulty, maintenance of respondent interest, flow and order of questions were also considered during the trialing. The only question that was potentially difficult to answer was recording a recently used problem and describing its use in the classroom. Teachers found it difficult to think of an example while completing the questionnaire but thought it was still a good question and should be included in the final instrument. Respondents had no difficulty completing the rest of the questionnaire and maintained interest throughout. Flow and order of questions was considered appropriate, particularly the use of alternating the open-ended and closed questions.

There was initially some concern that the questionnaire was too long and would therefore be onerous to complete. Most teachers took from 20 to 25 minutes to complete the instrument. When asked about this aspect, the teachers in the trial group thought that it was reasonable and did not recommend a reduction in length. In fact it was suggested that a final question be added to ask for any further comments. Some of the teachers indicated that there might be issues about mathematical problem solving that respondents would like to comment on.

Overall purposes of the survey instrument were also reconsidered during the trialing stage of development of the questionnaire. It was evident from responses and discussions that the items would reveal diverse information about teachers' reported beliefs and practices. It was anticipated that most teachers would be able to record a favoured problem. Use of particular problem types should also be revealed and there was potential for the instrument to illuminate other issues.

In summary, the trialing was necessary for assessment of overall purposes and for confirmation of the questionnaire design. This included checks for understanding of items and interpretation of instructions. The main recommendations from the trialing involved changing the student question type examples to the same set of numbers, some rewording of items, a new classification of some of the categories for frequency of use, and the addition of an open-ended question inviting further comment. On the basis of this trialing, the questionnaire entitled *Problem Solving in Mathematics Teaching: Teachers' Views and Teaching Practices*, was finalised and printed (Appendix 7).

This questionnaire was then administered to a sample of primary school teachers in NSW. Initially, after discussions with mathematics consultants who worked for the Department of School Education, a list of school principals who may have been interested in the research was compiled. These principals were contacted in writing and invited to participate in the study with the researcher discussing the study with the whole school staff and encouraging all teachers to complete the questionnaire. A description of the data collection procedures follows.

3.2.1.4 Data Collection using the Questionnaire

The final questionnaire was used to collect data from primary school teachers in NSW schools. In addition, a group of mathematics education lecturers were asked to respond to the teaching strategies listed in Question 4 in relation to the advice given in the problem-solving literature. In the analysis, the responses for the two groups for this question were compared to determine whether this group of teachers had responded to relevant advice about problem-solving teaching approaches. The data collection methods for both groups is described in this section.

Data From Teachers

Selection of teacher participants for this investigation required careful consideration. If random sampling procedures had been used and the questionnaires mailed to teachers throughout NSW, it could have been perceived by teachers as a process of checking on what they were doing. It has been argued earlier that many primary school teachers do not feel confident about mathematics, teaching mathematics, or teaching problem solving and could have found this questionnaire threatening. To gain meaningful information from this instrument, it was considered necessary to discuss its purposes with teachers before they responded. This section outlines the procedures used to collect data using this questionnaire.

As a consequence of the above considerations, a decision was made to target principals and to seek their support in investigating issues surrounding the teaching and learning of mathematical problem solving. Before principals were contacted, permission to conduct the study in departmental schools was sought from the Department of School Education in NSW. This was granted on the basis that a list of participating schools was compiled and submitted for approval, and a report on the study would be provided on completion.

Mathematics curriculum consultants in eight of the 40 districts in NSW were contacted during 1996. The chosen districts were spread across NSW; four were in the Sydney metropolitan area, one each in Newcastle and Wollongong and two in country areas in the south of the state. Consultants were asked to provide a list of three or four names of principals who would be open-minded about such a study. Each of the consultants agreed to the request and in two cases approached the principals themselves to gain initial approval.

A total of 29 principals were contacted during October, 1996 and details of the study were provided. The letter included an offer to deliver a school report at the completion of the study as well as to present the results to the staff. Three principals declined to participate on the grounds that it was too late in the year and teachers were busy planning for the following year. They also suggested that teachers were not interested in completing surveys. Two of the principals did not respond or return messages and twelve requested that they be contacted in the following year. Of the remaining twelve schools, five principals invited the researcher to discuss the study with them and to leave a set of questionnaires that they would distribute to interested staff members. In seven schools, principals suggested that the researcher speak to the whole staff during a scheduled meeting. Only one of these provided time for staff to complete the questionnaire.

It was recognised that many teachers feel uncomfortable and possibly even threatened by the notion of describing their teaching practices, particularly with regard to mathematics. In an effort to overcome the potentially threatening nature of completing a questionnaire about mathematics and problem solving, the researcher spent considerable time talking to teachers about the value of providing honest information. It was made clear to potential respondents that all views would be valued and that participation was voluntary. Respondents were encouraged to contact the researcher if further information was required and in addition, the researcher volunteered to speak to participating teachers about the results when they were analysed. Although few contacts were made, it seems that most teachers who responded had an interest in teaching mathematics or in issues surrounding the use of problem-solving approaches.

A total of 220 questionnaires were distributed during this period at the end of 1996, with a response of 79 completed questionnaires. This response rate was considered to be low given the positive reaction that had been received at the time of the school visits. As it was late in the school year, it is possible that teachers were busy and preoccupied with report writing and programming for the following year.

At the beginning of 1997, the remaining twelve principals were contacted. Two of these declined to participate on the basis that other priorities had arisen. Five principals from country schools were willing to discuss the study by telephone and then agreed to distribute the questionnaires to their staff members. The remaining five principals invited the researcher to speak to the whole staff during scheduled meeting times and one of these made time available for teachers to complete the questionnaire. During this period early in 1997, a further 220 surveys were distributed and an additional 53 responses were received.

The low response rate was still of concern. This issue was discussed with two of the principals in an effort to determine why the response rate was so low. They indicated that they were reluctant to put teachers under pressure to be involved in such activities as morale in departmental schools was low. Also, teachers seemed to view completing questionnaires as not providing ultimate support for their teaching since they rarely see results or feel that they benefit in any way. Several teachers from participating schools were asked for their reactions to the low response rate and most were not surprised. It was suggested that teachers who completed the questionnaires were probably those who had a definite view about mathematical problem solving or who had a personal interest in the area.

Other reasons for the low response rate were communicated directly to the researcher by teachers. At the time of school visits, the researcher was approached by several teachers who were not currently teaching a class of their own and who indicated that they felt that they would not be able to complete the questionnaire in a meaningful way. Other lower primary teachers, particularly those of kindergarten, indicated that they did not do problem solving with such young students and therefore decided not to respond.

At this stage in the data collection, a total of 132 questionnaires had been completed by teachers from 21 schools. One of the targeted schools had returned no questionnaires even though the Principal had been contacted on three occasions after the school visit. The participating schools represented a range of school sizes and settings throughout the state. Table 22 outlines the distribution and number of teachers on the staff for this group of schools.

Table 22
Location and number of teachers in participating schools

	Number of Teachers			
Location	1-10	11-20	21-30	>30
Sydney	0	5	2	4
Wollongong and Newcastle	1	1	3	0
Country	2	2	1	0

As the analysis would be better completed on a larger set of questionnaires, further groups were invited to participate during 1997. Twelve responses were received from a group of teachers who were studying a mathematics education course while completing the Bachelor of Education degree on a part-time basis. Six beginning teachers who had recently been appointed to schools in NSW also responded. A further group of 12 questionnaires were received from teachers who attended inservice courses presented by the researcher.

The final set of 162 questionnaires was not obtained from a random sample of primary school teachers in NSW and is therefore not presented as a representative sample. However, responses were obtained from teachers in about 36 schools. These included departmental, Catholic and private schools of varying sizes from country and metropolitan settings. Respondents included beginning to experienced practitioners, teaching Kindergarten to Year Six classes, who were participating in a variety of roles within schools. As this group represented a range of teachers in NSW, it was anticipated that the analysis of this set of questionnaire responses would provide valuable information about teachers' problem-solving beliefs and practices.

After this data collection, the responses to some of the questions were analysed using *Statview* (Feldman, Gagnon, Hoffman & Simpson, 1988), a computer-based statistical analysis tool that provided a predominantly descriptive summary of the range of responses relying on contingency tables and box plot comparisons. The open-ended questions were analysed by sorting and categorising data until themes emerged. This analysis provided valuable global data as well as information required for the second phase of the research. Analysis of questionnaire data is presented in Chapter 4.

Data From Mathematics Education Lecturers

To consider whether teachers had responded to advice about teaching strategies presented in the problem-solving literature, mathematics education lecturers were asked to respond to the same set of twenty teaching strategies in terms of their interpretation of this advice. To enable a comparison to be made between the responses of the teachers to those of an informed community, a group of 21 tertiary mathematics education lecturers from NSW and Victoria were invited to complete the same set of survey items relating to the frequency of use of teaching strategies that were presented in Question 4 of the questionnaire. The prompt for lecturers required them to indicate the desirable frequency of use of each of these teaching strategies according to their understanding of advice given in the problem-solving literature. The results from this comparison are summarised in Section 4.3.3 with a more detailed description in Appendix 8.

From the questionnaire analysis, a small group of teachers was identified to participate in the second phase of the investigation. Teachers were chosen using a variety of criteria including their level of agreement or disagreement with each of the teaching perspectives that were used to compile the items in the first and second questions on the questionnaire. Also, teachers were identified who described important aspects of their beliefs and practices in the open-ended questions. The second phase of the investigation involved initial interviews, classroom observations, and reflective interviews. A description of the methods and procedures used in the fieldwork is presented in the next section.

3.2.2 The Fieldwork

Data collection for this investigation occurred in two phases. The questionnaire was used in the first phase and was designed to obtain information about primary school teachers' problem-solving beliefs and practices. The second phase of data collection involving interviews and observations was informed by analysis of questionnaire data to the extent that it highlighted key issues and factors for further consideration. On the basis of questionnaire responses, particular teachers were identified for participation in the fieldwork.

The fieldwork was used to explore the third and fourth research questions (Table 23). These questions focused on teachers' beliefs and practices in particular contexts and attempted to ascertain constraints and opportunities that might operate in those situations. To allow intensive exploration, a small group of teachers participated in this phase of the investigation.

Table 23
The second two research questions for this investigation

Number	Question
3	In what ways do teachers incorporate problem-solving approaches into their planning and teaching of mathematics? In particular, when incorporating problem-solving approaches in their mathematics teaching: • what specific classroom strategies do teachers use; • what types of problem-solving tasks do teachers use; • how do teachers plan; and • what resources and materials do teachers use?
4	When teaching mathematics using problem-solving approaches, what factors can be identified that support or inhibit the implementation of such practices? In particular, what impact do each of the following aspects have on teachers' planning as well as implementation of problem-solving approaches: • the teachers themselves:- including personal aspects such as confidence, past experiences that incorporate successes and failures, and opportunities for teacher development; • students:- including the influence of their beliefs, resistance, attitudes; the perceived ability of the students with respect to: learning mathematics; proficiency with language; participation in group work; • school culture:- including the influence of prescribed programs and textbooks, assessment and reporting practices, parents' expectations, principals' and other teachers' influence; and • the education system:- including curriculum documents, particular programs, external assessment procedures.

This section has two parts. The first part outlines the semi-structured interview procedures that were used to confirm questionnaire interpretation by the researcher and to explore issues that had arisen from questionnaire responses. The second part describes the classroom observation and reflective interview procedures that were used in particular school settings to enrich questionnaire data, investigate problem-solving constraints and opportunities operating in those contexts, and explore the usefulness of the proposed model for this investigation.

3.2.2.1 Initial Interviews

The initial interviews had several purposes. These included the need to clarify teacher interpretation and confirm researcher analysis of questionnaire responses, explore and elaborate teachers' comments from the questionnaire, reveal opportunities and constraints that might impact on teachers' efforts to implement problem-solving approaches, and to select participants for the classroom observations. This section describes the selection of participants for the initial interviews and discusses the questions used in the semi-structured interview.

Initial Interview Participants

Interviewees were chosen from those respondents who recorded their names on the questionnaire. Of the 162 completed questionnaires, 45 respondents included their name indicating that they were prepared to participate further in the study. This group was diverse and included classroom teachers, administrators and specialist teachers. In addition they were from a wide geographical distribution, including new and experienced teachers, as well as those teaching lower, middle and upper primary.

Selection for participation in this stage of the data collection was based on several criteria. These included current, full-time responsibility for an infants or primary class, proximity to the Sydney metropolitan area, and informative responses to at least some of the questionnaire items. Examples of such responses are included in Chapter 5. As some of the initial interview questions focused on current teaching practices, it was deemed appropriate to interview practising teachers so that discussions could involve issues associated with current classroom situations. Proximity to the metropolitan area was necessary to enable the researcher to visit the schools at a convenient time for each of the participants.

Responses to questionnaire items were considered in three ways. The first was the level of agreement with either the *traditional* perspective that was portrayed by Naomi, the imaginary teacher in the first question on the questionnaire, or the *contemporary* perspective that was portrayed by Gwendolin in the second question. A second consideration was the amount of additional information that had been provided including comments made in the open-ended questions. It was possible that this was an indication that these teachers had reflected on their practice, considered other perspectives, and held definite views about the role of problem solving in learning mathematics. A third consideration was the inclusion of additional factors or issues that were unique or offered interesting discussion points.

Nine teachers were selected to participate in this stage of data collection. Four teachers were chosen who agreed more with Naomi's views than Gwendolin's; two were chosen on the basis that they agreed more with Gwendolin than Naomi; and a final three were chosen who seemed to have mixed levels of agreement with the two teachers in the questionnaire. The last three open-ended questions on the questionnaire provided teachers with opportunity to discuss professional development needs of staff at their schools, to comment on one teacher's negative statement about problem solving in mathematics, and to add any further comments. The selected participants' responses to at least some of these questions were informative, reflective, and frequently emotive.

The group of nine teachers, all of whom were female, were contacted and all agreed to be interviewed in June, 1997. It is acknowledged that a set of nine male teachers may have responded differently and this could provide a possible area for further intestigation. The teachers were from six different schools and those who were from the same school were invited to be interviewed as a group as it was felt that this would be less threatening and might facilitate further discussion and debate. Two pairs of teachers agreed to be interviewed together and so seven interviews were conducted, each taking about thirty minutes. In each case, more time was available if needed. The researcher continued each interview until she was convinced that she had a clear understanding of each participant's perspective on problem-solving approaches to teaching mathematics. Interviews were undertaken in quiet locations at each school so that each could be audio taped.

A summary of some of the background data about the teachers who participated in the initial interviews is presented in Table 24. This includes pseudonyms, years of teaching experience and grade being taught in 1997. Further details about the schools involved, teachers' experiences and classes currently being taught are included in Chapter 5 where the analysis of these interviews is presented.

Table 24
Background information for teachers involved in the initial interviews

Name (Pseudonym)	School	Experience (Years)	Teaching Grade Level	Views
Lois	A*	20	6	Tended to agree with Naomi
Janice	A*	20	4	Tended to agree with Naomi
Jane	A	12	3	Tended to agree with Naomi
Susan	B*	8	3/4	Strongly agreed with Naomi
Gaye**	B*	6	6	Mixed agreement
Elise	C	25	5	Mixed agreement
Faye	D	28	5/6	Mixed agreement
Rose**	E	15	2	Strongly agreed with Gwendolin
May	F	4	2 then 4	Strongly agreed with Gwendolin

* These teachers were interviewed in pairs at their respective schools.
** These teachers were chosen to participate in the classroom observations and reflective interviews.

Initial Interview Questions

Even though the questionnaire was trialed before administration, it was still considered appropriate to check understanding of items and to clarify that responses were being correctly interpreted by the researcher. Questionnaires can only provide a limited amount of information as they need to be kept to a reasonable length and items are limited to those that are non-threatening and easily answered.

The initial interview questions were designed by the researcher to obtain general background information about each teacher, descriptions of recent mathematics lessons, comments about the questionnaires in general and specific questions relating to individual responses. The interview sessions were semi-structured to allow for deviation from this list of questions depending on the responses and reactions of the teachers. After obtaining additional background information, questions related to recent lessons to encourage teachers to relax and talk about their classes. This then led to discussions about recent problem-solving lessons, the frequency of problem-solving lessons, the desire to implement more problem solving, and possible factors that might be inhibiting teachers' efforts to increase the level of problem solving in their classrooms. The questions that formed the basis of discussions are listed in Figure 16.

Introduction:-You may recall that I surveyed the staff at your school some time ago. The next stage of the research is to interview a group of teachers so that I can verify that we both have the same understanding of the questions from the questionnaire and to gather more in-depth information about your beliefs and practices.

You should be aware that you own all of this data and that you can ask to withdraw any of it at any time. You can also withdraw from the study at any time.

Are you happy for me to record our conversation? If so, I would like to gather some background information.

How many years have you been teaching? How many years have you been at your current school? What class are you currently teaching? What previous experience have you had on this grade?

1. Describe your most recent mathematics lesson (did it have problem solving in it?).

2. Describe your most recent problem solving lesson in mathematics (if problem solving did not form a substantial part of the recent maths lesson).

3. How were these lessons the same/different?

4. Do you consider that you have done much problem solving in the last 5 mathematics lessons? Do you wish you could do more? Why/why not?

5. What difficulties have you encountered when you have tried to do problem solving in mathematics? Perhaps consider the influence of each of the following:-

- students/classes?
- school culture (other teachers, parents, school programs, resources)?
- self (experience, mathematics background, inservice courses)?

6. (Read the background information.) Can you explain what you think this means. OR

Was it clear to you what was meant by the different types of problems in the survey? Did you have trouble answering the questions about types of problems?

From the survey results, most teachers reported using exercises and application questions most frequently because these are easier to do and to find. How do you feel about this?

7. Did you have any trouble answering the question on frequency of use of problem solving teaching strategies? Are there other strategies which you like to use?

Many teachers seem to use group work, concrete materials, discussion of problem solving strategies and modelling of the problem solving process. What do you believe is meant by the last strategy?

Few teachers report using calculators. Would you use calculators in problem solving lessons? Why/why not?

8. In your survey, you mention ....

Could you explain or elaborate?

9. Would you be prepared to participate in the second phase of my research? This involves observations of your class as they participate in problem solving as well as interviews about the lessons.

Figure 16. Initial interview questions.

Questions focused on the understanding of questionnaire items and sought reactions to some results from the questionnaire analysis. Interpretation of background information and, in particular, student question types and related examples were discussed with each of the participants. Reactions to the reported frequent use of exercises and application problems and the reported infrequent use of calculators were sought. Teachers were encouraged to discuss their preferred teaching strategies and whether there were other strategies that had not been listed in Question 4 on the questionnaire.

For each participant, specific questions about their responses to the questionnaire were addressed. The teachers were each presented with their completed questionnaire and were invited to comment on particular statements or to expand on issues that they may have noted. An invitation was extended to all interviewees to participate in the subsequent classroom observations and reflective interviews.

Time constraints prohibited the observation and further interviewing of more than two participants in the next stage of this investigation and so it was necessary to choose two teachers from this group to be involved in the remainder of the fieldwork. Interview data were transcribed and analysed and then final selections were made. The main considerations for selection were the depth and richness of responses to the initial interview questions, the potential of particular school and classroom situations to yield informative data and yet still represent typical contexts, and the sense of the existence of a rapport between the researcher and the participants. A more detailed description of the selection process with examples is presented in Chapter 5.

3.2.2.2 Classroom Observations and Reflective Interviews

After analysis of initial interview transcripts, two teachers were selected to participate in the remainder of the fieldwork. This stage of the fieldwork involved a combination of observations of three consecutive lessons and several unstructured reflective interviews to discuss lessons and other related issues. It was anticipated that this stage of data collection would enrich data obtained from questionnaires and initial interviews as well as reveal possible supporting and constraining factors that operate in particular school contexts to enhance or inhibit teachers' efforts to implement problem-solving approaches.

The two chosen teachers, Rose and Gaye, agreed to be involved in the remainder of the study. Each was in the process of completing postgraduate study at different institutions and both had shown a keen interest in this investigation through their responses to the questionnaire. While there were some similarities between their reported beliefs and practices in mathematics, each had different experiences as teachers and learners of mathematics and the school contexts that they taught in were also quite different. It was anticipated that these teachers would provide rich data to further explore issues of interest for this investigation. A detailed description of the data collected for each teacher and subsequent analyses are presented in Chapter 5.

Before the school visits, transcripts of initial interviews were analysed for each of the two teacher participants. After reading the transcripts, the researcher used a process of segmenting and classifying comments into categories as described by Tesch (1990). Fetterman's (1989) advice to use a map, or visual representation, of key comments to assist analysis was taken and this yielded themes from the data for each teacher. A report was written for each participant describing their beliefs and practices as determined by the researcher from both the questionnaire and initial interviews. This was then sent to be read before observations and reflective interviews. It was considered critical to this investigation to seek each teacher's interpretation of their beliefs and practices as these may be quite different from the researcher's perceptions. It was anticipated that this would provide opportunities to confirm analyses of data, highlight issues that had been overlooked by the researcher, and promote further discussions during the school visits.

The purpose of the observations was to further validate questionnaire data as well as to clarify and confirm initial interview comments, compare actual practice with reported practice, and to identify factors that seem to either support or constrain the implementation of problem-solving approaches. The researcher visited each of the schools on three consecutive days late in 1997 and observed a mathematics lesson on each occasion. The role assumed by the researcher was as a peripheral-member-researcher (Adler & Adler, 1994) since the researcher's presence was obvious to the students but she did not participate in lesson activities. Participants were requested to teach lessons that were part of their normal program but that incorporated problem-solving approaches.

Each lesson was videotaped for transcription and analysis at a later date as the researcher collected all data without assistance. Each teacher was fitted with a radio microphone so that the tape would yield a clear recording of all comments and instructions given by the teacher. Students' voices would only be heard if the teacher was talking with a group or individual at close range. This was considered to be adequate as the main focus of attention was teaching actions including instructions and question types.

Unstructured, or informal, reflective interviews were used to provide the opportunity for teachers to discuss actions and decisions that were made during the course of each lesson and to illuminate factors that might be inhibiting desirable practices. This enabled teachers to identify reasons for those decisions and provided opportunities to discuss beliefs that might impact on such decisions. These interviews were deliberately unstructured to allow for a relatively informal discussion between the researcher and each participant. Such conversations at this stage in the research were fairly personal to encourage depth of feelings and revelation of true beliefs compared to the potentially superficial beliefs that may be revealed on questionnaires and in initial interviews before a trusting relationship had been established. These discussions included potentially sensitive conversations about fellow teachers, school executive members, and school practices that may be viewed as restrictive and undesirable.

As suggested by Brown et al. (1990), it was decided to investigate beliefs in context so that any observed inconsistencies could be revealed and discussed with participants. It was of interest to explore what each teacher did not do but would have liked to do and to discuss what factors restricted their practice. Many lessons do not go to plan and so it was of considerable interest to determine what factors impacted on teaching and learning outcomes of each of the lessons observed. This helped to reveal some factors that impacted on planned teaching actions.

After the school visits, the videotapes of lessons were viewed and transcribed to assist further analysis. Analysis of lessons was both holistic and analytic (Koehler & Grouws, 1992). The holistic analysis involved viewing all three lessons for each teacher and writing, from the perspective of the researcher, an overall description of the mathematics lessons including actions of the teacher and lesson outcomes. The analytic approach involved analysing the transcript seeking evidence of particular actions including use of particular student problem types, and teaching strategies that had been listed on the questionnaire. The combination of these approaches provided a rich description of each teachers' problem-solving practices during the time of the observations.

The reflective interviews were also transcribed for analysis. Data from these supported and enhanced data gathered from the initial interviews. It was also recognised that the researcher influenced the data because of the kinds of questions that were asked and the ways in which they were asked. This was particularly the case during the fieldwork phase of this investigation when the investigator spent time in classrooms, observing and discussing teaching actions with participants. While it was of interest to determine possible factors that might support or constrain teachers' problem-solving efforts, no cause-effect conclusions can be posited since no two school contexts are identical and no two teachers have experienced exactly the same influences.

The fieldwork represented the second phase of this investigation and included a combination of initial structured interviews, classroom observations and unstructured reflective interviews allowing for triangulation of data. By comparing responses to questions in the initial interview, observations of what teachers do in their classrooms, and comments made in the reflective interviews, it was intended to test one source of information against another. This process increased accuracy of results, improved quality of the data, and allowed the researcher to understand more completely the beliefs of each teacher by looking for patterns of thought.

In summary, this investigation of primary school teachers' problem-solving beliefs and practices was conducted in two phases. The first involved the use of a questionnaire to collect data from significant groups of teachers about their views on the role of problem solving in the teaching and learning of mathematics. It also aimed to gather data about their classroom practices in relation to problem-solving approaches and to illuminate issues that might support or inhibit teachers' efforts to act on the substantial advice that is included in curriculum documentation as well as in preservice and inservice courses. The second phase involved interviews and observations with a small number of the teachers who had responded to the questionnaire. This part of the investigation aimed to enrich the earlier data by confirming questionnaire analysis and raising other issues for consideration.

The overall results aimed to explore potential influencing factors in the relationship between beliefs and practices. To this end, a model was proposed that incorporated a variety of factors for consideration. This model was determined by a consideration of other models presented in the literature. The results of this investigation enabled an evaluation of this model and provided evidence to support some of the predicted influencing factors but challenged others.

This investigation used a multiple methods approach to research as it incorporated a combination of data gathering procedures. This approach is not unusual in research into teachers' beliefs and practices. It has been argued that the combination of approaches was important to provide a detailed picture of all aspects of the relationship between beliefs and practices as well as to support and enhance confidence in the research findings. Trustworthiness and dependability were confirmed by using a variety of strategies including negotiation of understanding between participants and the researcher, triangulation procedures where different sources of data were checked for consistency, and the provision of a detailed description of all participants and settings, as well as data gathering and analysis procedures. A full analysis of all data is presented in Chapters 4 and 5.

CHAPTER 4

QUESTIONNAIRE ANALYSIS

For this investigation, a questionnaire was used to gather data from the target population of primary school teachers in NSW. It sought information about teachers' reported beliefs and practices in relation to problem solving in mathematics. The questionnaire combined open and closed questions on several aspects of problem solving. Questions focused on teachers' beliefs about the role of problem solving in learning mathematics, frequency of use of a range of teaching practices, use of particular types of mathematical tasks and resource materials that teachers consult. In addition, information was sought from respondents about their concerns and perceived needs in relation to the implementation of problem-solving approaches.

As discussed in the previous chapter, several issues were considered when designing the questionnaire about teachers' problem-solving beliefs and practices. The willingness of teachers to report honestly about their beliefs and practices, the variety of meanings that are attributed to terms such as *problem* and *problem-solving approaches*, and the potentially threatening nature of a mathematics questionnaire for primary school teachers were issues that were addressed in the design and administration of this questionnaire. The strategies employed to overcome these difficulties helped to establish greater confidence in the results reported in this chapter.

Completed questionnaires were received from 162 primary school teachers in NSW. Questionnaire analyses revealed a spread of teachers' beliefs about problem solving, variation in preferred problem-solving tasks, and diversity in frequency of use of particular teaching strategies.

In summary, data indicated that open-ended and unfamiliar problems were less frequently presented to students than application problems and exercises and that less able students were less likely to be presented with open-ended and unfamiliar problems than their more able peers. Teachers reported that problems were used to support student learning, extend thinking, and to motivate and challenge more able students. The *Mathematics K-6* syllabus document (NSWDE, 1989) and textbooks were most frequently used by teachers as sources of mathematics problems.

Teachers reported the need for professional development that focused on appropriate teaching styles, problem-solving teaching approaches, and suitable resource materials. Additional issues raised by respondents included lack of time to implement problem-solving approaches properly and the constant need to meet an ever increasing list of priorities in schools. Respondents also mentioned the impact of parents' expectations, students' abilities and needs, and school as well as external assessment practices. Overall results indicated that there were teachers who had responded to advice and did report using many of the teaching strategies and problem-solving approaches that the literature claims promotes effective learning in mathematics. However, there were also teachers who rejected some of the recommendations, or who found implementation of such approaches difficult to achieve.

The results of questionnaire analyses are presented in this chapter beginning with a profile of respondents. Teachers' reported beliefs and teaching practices are examined and use of particular question types and recently used problems are described. The resource materials reportedly consulted by teachers are listed. Responses to open-ended questions on professional development needs and reactions to the negative comment about problem solving are categorised and described. The final section summarises key findings from questionnaire analyses.

4.1 Profile of Respondents

A profile of the respondents was compiled on the basis of responses relating to school, gender, number of years of teaching experience, current teaching role and inservice attendance in the last two years. Of the 162 responses received from teachers, 24 (15%) were male and 138 (85%) were female. The teachers were from 36 schools; these included Catholic, independent and departmental schools that were located in country and urban settings in NSW. Teaching experience was presented as three categories; results indicated that 31 (19%) teachers had been teaching from zero to four years, 27 (17 %) from five to nine years, and 104 (64%) for ten or more years.

There was a high proportion of female teachers and of teachers in the more experienced group who responded to this questionnaire. This is not surprising as most of the Sydney, Newcastle and Wollongong schools involved have older teaching staffs as they are located in areas that are perceived to be more desirable for teaching purposes. Table 25 groups respondents according to regional locations of their schools.

Table 25
NSW regional locations for respondents

Regional Location of Schools	Number (Percentage) of Respondents
Sydney	88 (54%)
Newcastle	50 (31%)
Wollongong	6 (4%)
Other Country Locations	18 (11%)
Total	162 (100%)

To ascertain current teaching roles, participants were invited to select a " ... category [that] best describes your role this year". Six categories were presented with the majority (75%) of respondents indicating that they were classroom teachers and of these, most of the respondents were teaching in the lower primary, or infants, grades (see Table 26). This is somewhat surprising since at the time of the school visits, several teachers in the infants grades did not complete surveys; they indicated to the researcher that they did not do problem solving in the early years of schooling. This is clearly not a belief held by all teachers in this category. It is noted that on three of the questionnaires, there was no indication of current role.

Table 26
Number of male and female respondents in each of the listed role categories

Role Category	Male	Female	Total
class teacher of years K, 1, or 2	3	44	47 (29%)
class teacher of years 3, or 4	4	33	37 (23%)
class teacher of years 5 or 6	10	28	38 (23%)
specialist teacher	3	14	17 (10%)
administrator	1	6	7 (4%)
other	3	10	13 (8%)
No role category given	0	3	3 (2%)
Total	24	138	162

This profile of the respondents highlights other features of this group including the proportion of females in the administrator category and the proportion and distribution of male teachers. The proportion of female administrators who completed the questionnaire was much higher than state figures. Data from the *Equal Employment Opportunity Annual Report* (NSWDE, 1998) indicated that the proportion of female principals in NSW was about 33%. Also, according to *Schools Australia* (Bureau of Statistics, 1998) the current proportion of male teachers in NSW primary schools was about 21% whereas for this survey, only 15% of respondents were male. Another feature is that more than half of the male teachers were teaching in the upper primary. Anecdotal evidence suggested that this may reflect a statewide trend.

Of the thirteen teachers who chose the "other" category, a variety of roles were described. Seven were teaching in the designated school on a regular casual teaching basis. One teacher described her role as a literacy/numeracy specialist, another described his role as a computer teacher in teacher relief time, and three others as a combination of teaching and administration. The final teacher described his role as teacher of a composite years 4/5 class. This highlighted a problem with the questionnaire design. For teachers who were teaching composite classes, some straddled the listed categories. To overcome this, several teachers chose more than one of the teaching categories. For the purposes of analysis, when this occurred, they were placed in the lower grade category since mathematics problems may be chosen to suit the lower grade students.

It is possible that teachers who have attended professional development courses that involve mathematics may be more informed about current thinking in the area of problem solving. For this reason, information about inservice days attended was included in the questionnaire. For participation in teacher inservice, most teachers who were employed by the Department of School Education in NSW would have had opportunity to participate in at least two professional development days each year. It is unclear whether teachers counted these days as teachers tend to view them as meeting days rather than as organised teacher inservice activities. Table 27 shows the reported number of days attended in the last two years for teacher inservice and mathematics inservice.

Table 27
Number of days attended for teacher inservice and for mathematics inservice in the last two years

Number of days of teacher inservice			Number of days of mathematics inservice		
0 to 1 days	2 to 6 days	more than 6 days	0 days	1 to 3 days	more than 3 days
7 (4%)	80 (50%)	74 (46%)	72 (45%)	77 (49%)	9 (6%)

Nearly all of the teachers in the study indicated that they had attended two or more teacher inservice days in the last two years. A total of 55% of respondents had attended at least one mathematics inservice day in the last two years. Forty-four percent of this group of teachers attended *no* mathematics inservice days in the last two years. It is possible that teachers may have attended after-school meetings or short courses and did not include these because the questionnaire referred to *days* rather than courses.

It should be noted that there has been considerable emphasis on literacy in NSW for the last five years and so teachers may have focused on courses related to the English Key Learning Area rather than mathematics. Both numeracy consultants and principals confirmed in subsequent discussions that in the last two years, the implementation of a new English syllabus in primary schools had been a priority and many schools had chosen to concentrate their inservice efforts in this area.

It is not suggested that this group of 162 respondents is a representative sample of primary school teachers in NSW. However, the group is diverse since respondents span levels of teaching experience, grades of classes, and other roles within schools. In addition, responses were received from teachers in small to large schools, country and urban settings, Catholic, independent and departmental schools. Teachers' reported beliefs are examined in the next section.

4.2 Teachers' Reported Beliefs

Teachers' reported beliefs were categorised by their responses to the first two questions on the questionnaire. Analysis of these questions took several forms and is presented in three sections. The first section describes an initial categorisation of responses according to level of agreement with either a *traditional* perspective or a *contemporary* perspective with comments related to problem-solving practices and issues of implementation. The second section presents an examination of responses to particular questionnaire items indicating the spread of beliefs and differences in reported beliefs for particular groups. Finally, findings from correlational analysis and factor analysis are outlined.

Before presenting results from each of these analyses, the first two questions from the questionnaire are reproduced here for reference. The first question presented a series of statements made by a fictional teacher, Naomi, who has a *traditional* view of teaching and learning and who believes in problem solving as an *end*. The question stated:

> Naomi teaches a Year 3 class and she uses *application problems* and some *unfamiliar problems* when she has finished teaching 2 digit addition.
>
> In staffroom discussions, she has made the following statements. Please indicate, **from your perspective**, the extent to which **you** agree with each of her statements:
>
> • students should learn basic number facts before they do *application* and *unfamiliar problems;*
> • students should learn algorithms before they do *application* and *unfamiliar problems;*
> • students cannot solve problems until they know how to perform the four operations;
> • the best problems are those which relate directly to the number facts and algorithms the students have been practising;
> • *application* and *unfamiliar problems* are best left to the end of the topic in mathematics;
> • mathematics lessons should focus on practising skills;
> • some students have trouble solving problems unless they know how to do the mathematics before they begin; and
> • some students find problem solving difficult because of the language involved in the problems.

The second question presented respondents with a set of statements made by another imaginary teacher, Gwendolin. This teacher has a *contemporary* view of teaching and learning and believes in teaching mathematics via problem solving so that problem solving becomes a *means* to learning. The question stated:

> Gwendolin is also a Year 3 teacher and she regularly uses lots of *open-ended problems* to teach her class 2 digit addition.
>
> In staffroom discussions, she has made the following statements. Please indicate, **from your perspective**, the extent to which **you** agree with each of her statements:

- it is a good strategy to begin the topic of 2 digit addition with unfamiliar problems involving 2 digit addition;
- mathematics lessons should focus on problems rather than on practice of algorithms;
- problems help most students to learn basic facts and algorithms because they can see a reason for learning them;
- all mathematics questions should challenge students to think about what mathematics they know and how they can use it;
- students can learn most mathematical concepts by working out for themselves how to solve *unfamiliar* or *open-ended problems;*
- it is essential for students to explore their own ways of doing mathematics questions before being shown the teacher's methods; and
- most students forget mathematics procedures and so it is best to let them work out their own methods first.

Respondents were requested to indicate their level of agreement with each of these statements using one of the categories: "strongly agree"; "agree"; "disagree"; or "strongly disagree". Results from analyses of the first two questions on the questionnaire are presented in the next three sections.

4.2.1 Sorting of Problem-Solving Beliefs into Broad Categories

The first stage of analysis involved sorting responses to the first two questions into categories according to their level of agreement with either a *traditional* perspective or a *contemporary* perspective. This was conducted so that the researcher could achieve an understanding of the range of beliefs for this group of teachers and to determine associated practices and general concerns before undertaking fine-grained analyses of each questionnaire question.

It became obvious that there were a large number of possible combinations of responses to these questions and therefore potentially many different categorisations could be used. However, five categories were formed and the questionnaires were sorted into each of these. For respondents who had been placed in the two extreme categories, their responses to *all* questions were read to gain an overall impression of commonalities between members of the group and to determine potential differences in practices and related issues between the two groups.

As the second stage of data collection required interviewing some of these respondents, it was desirable to identify teachers who represented the spread of problem-solving beliefs and practices. This required identifying teachers who represented the two extreme groups as well as those who could be placed on a continuum between these groups. Initial reading and sorting suggested that the best way to differentiate was according to level of agreement with the two teaching perspectives that were described in Chapter 2. These perspectives were presented in the questionnaire as statements made by

Naomi for the *traditional* perspective, and by Gwendolin for the *contemporary* perspective. The traditional categories were determined by a high level of agreement with the traditional perspective combined with considerable rejection of the contemporary position. The contemporary categories were determined by a high level of agreement with the contemporary perspective combined with considerable rejection of the traditional position.

It was noted that a large majority of respondents agreed with the last statement made by Naomi that suggested that "some students find problem solving difficult because of the language involved in the problems". The final tally indicated that 98% of respondents selected either "agree" or "strongly agree" and therefore this item did not distinguish between respondents who might hold different belief systems. As a consequence of this observation, responses to this item were ignored when sorting questionnaires. This left a total of seven statements in the traditional category and seven in the contemporary category.

For sorting purposes, responses that were either "strongly agree" or "agree" were grouped together, as were those for "disagree" and "strongly disagree". This was done to overcome the possibility that respondents may have used different interpretations of these descriptors. In addition to this, Fowler (1988) suggested that some respondents rarely choose extremes while others like to select extreme positions.

All questionnaires were sorted into five categories, referred to here as very traditional, traditional, mixed, contemporary, and very contemporary. A questionnaire was placed in the *very traditional* category if there was agreement with six or more of the traditional statements combined with disagreement with six or more of the contemporary statements. The reverse was the case for those placed in the *very contemporary* group. A questionnaire was placed in the *traditional* category if there was agreement with only five of the traditional statements and disagreement with five or more of the contemporary statements. The reverse was the case for those placed in the *contemporary* group. All other questionnaires were placed in the *mixed* group. Table 28 indicates the number of respondents placed in each of the five categories.

Table 28
The number of questionnaire responses in each of the five categories

Category	Number of Respondents (Percentage)
Very Traditional (Naomi)	6 (4%)
Traditional	17 (11%)
Mixed	112 (69%)
Contemporary	14 (9%)
Very Contemporary (Gwendolin)	13 (7%)
Total	162 (100%)

For those placed in the *mixed* category, it was noted that there were 13 respondents who agreed with eleven or more of the fourteen statements and three respondents who disagreed with eleven or more of all statements. It may be incorrect to label these *mixed* as it is possible that this group of sixteen respondents did not spend a great deal of time thinking about the fact that their choices were potentially contradictory. However, without further information, it is impossible to determine the extent of their beliefs.

4.2.1.1 A Summary of Questionnaire Responses from the *Very Traditional* Group

After the sorting process, each of the six questionnaires placed in the *very traditional* category were read and common elements noted. Background information indicated that the ratio of male to female respondents in this group was 1:5, four had been teaching for more than ten years, one for between five and nine years, and one for less than five years. Two were teaching in Years 5 or 6, two in Years 3 or 4, one described her role as a specialist teacher, and the last was employed as a full-time, casual teacher. There was a spread of inservice attendance, however all six had attended *no* mathematics inservice days in the last two years.

The recently used problems recorded by this group of respondents were all examples of application problems and these were reported as frequently used along with exercises. Reasons given for this choice included the need for practice, reinforcement, training, learning of the basics first, and opportunities for success. Teachers mentioned that choice of tasks depends on the ability of students and indicated that students struggle with other types of problems as they may also have reading or comprehension difficulties. Textbooks were used *almost always* by three respondents and *often* by the remaining three. The syllabus was also frequently consulted as a source of problems. The most frequently used teaching strategies for this group of teachers included students working alone, whole class discussions, and discussions with students about problem-solving strategies and processes.

Responses to the last three open-ended questions on the questionnaire revealed additional information about the focus for these teachers. There was a mixed reaction to professional development needs of staff members with comments that none was needed by three respondents and a suggestion that sharing ideas is desirable by the remaining three. Four disagreed with the negative comment about problem solving indicating that problem solving was relevant and useful in teaching mathematics. One indicated that she had no reaction. The final respondent agreed with the statement and stated that this was the case "where my present class is involved". Further reading of this questionnaire indicated that this teacher was currently teaching the lowest ability group in mathematics in the upper primary. Other issues related to implementation of problem-solving approaches raised by this group of teachers included time constraints to get through other work in the curriculum, struggling students, and the need for a structured approach in mathematics.

4.2.1.2 A Summary of Questionnaire Responses from the *Very Contemporary* Group

Each of the 13 questionnaires placed in the *very contemporary* category was also read to discover commonalities and potential differences between teachers in this group. The ratio of male to female teachers was 2:11. Four teachers had been teaching for zero to four years and nine had been teaching for ten or more years. Seven were teaching in the K-2 year groups, three in Years 3 or 4 and two in Years 5 or 6, the last teacher in this group indicated that she was casually employed at her school. All had attended a considerable number of inservice days however, seven had attended *no* mathematics inservices, five had attended between 1 and 3 mathematics days and one had attended four or more days.

Problem examples were provided by nine teachers from this group and these were all examples of either open-ended or unfamiliar problems. Frequently used problems covered all of the problem types with several teachers indicating that a mixture was appropriate. Three teachers indicated that they frequently present students with exercises and seven frequently use application problems as well as other problem types because they are contained in the class textbook or they are easy to find. Other reasons for choices included variety, real-life contexts, catering for varying learning styles, and meeting students' needs. The main sources of problems were the syllabus document, resource books, textbooks and ideas from other teachers.

It was reported by respondents that a wide variety of teaching strategies were frequently used. The most frequently used teaching strategies included whole class discussion, group work, concrete materials, encouraging children to record their own methods, teacher modelling and discussion of problem-solving strategies and processes, as well as use of problems related to the school context and students' interests.

Most teachers believed that more professional development was desirable. Comments suggested a need for ideas on integrating problem solving into teachers' programs, guidance regarding use of open-ended problems, modelling of problem-solving teaching approaches by expert teachers, and recommendations about useful resources. One teacher suggested that more teachers need to know that mathematics is more than textbooks and algorithms and another commented that teachers needed to be aware that Working Mathematically is more than just getting correct answers. Another respondent commented that problem solving is not regarded by some teachers as "serious mathematics" and as a consequence, it has an identity problem.

All rejected the negative comment about problem solving. Suggestions were made that the teacher who made this negative comment must be afraid of change and had a limited understanding, possibly a reflection of community views. Reasons given that supported problem solving in mathematics included the need to have children think and to apply their knowledge. It was noted that problem solving was a life skill and that it could be rewarding and fun. Also, teachers recognised that teaching problem solving was difficult and so some teachers probably avoid it, or rely on textbooks.

4.2.1.3 A Comparison of the *Very Traditional* Responses with the *Very Contemporary* Responses

A comparison of questionnaires from respondents that were placed in the *very traditional* category with those in the *very contemporary* category revealed similarities and differences. There were similarities with background information and some teaching practices. These included an almost equivalent male to female ratio and in both groups, about two thirds had been teaching for ten or more years. Both groups reported using exercises and application problems, the syllabus as a main source of problems, and whole class discussion and discussion with students about problem-solving strategies and processes as regular teaching strategies. Almost all members of both groups rejected the negative statement made about problem solving on the grounds that it *is* a relevant part of mathematics as well as a useful life skill. Both groups commented that choices of problems may depend on the ability or needs of students.

There were several overall differences between the two groups. In the *very contemporary* group, more teachers were working in lower primary and there was extensive use of concrete materials, encouraging students to use their own methods, and using problems that related to school contexts. In this group, there was also greater use of open-ended and unfamiliar problems with reported reasons indicating the need for variety, real-life applications and catering to the different learning styles of students. This group of respondents supported the need for professional development to address a variety of issues related to the teaching of problem solving including the need to focus on teachers' beliefs about the usefulness of problem-solving approaches. In contrast, the *very traditional* group more frequently mentioned struggling students, problems with lower ability students in mathematics, and the importance of students experiencing success in their learning.

The sorting of questionnaires enabled the researcher to identify participants for the fieldwork, or second phase, of data collection. It also confirmed that there were differences in belief systems that seemed to lead to at least some differences in teaching practices. Further analyses were undertaken to consider the full range of responses to the first two questions and to ascertain whether there was any significant correlation between items, or factors that seemed to link particular items. These two aspects of the analysis are considered in the next two sections.

4.2.2 Responses to *Traditional* and *Contemporary* Perspectives

In this section, the results from the first two questions are presented in two ways. First, the spread of beliefs is examined in terms of responses to each of the items. Second, particular statements are examined in relation to the experience of respondents and current teaching grade level.

4.2.2.1 Spread of Beliefs

The *Statview* software package (Feldman et al., 1988) was used to enter the data sets for each respondent. For the first two questions, responses were ranked on the basis of the level of agreement or disagreement with each of the statements: "strongly agree" was given a rank of three; "agree" was given a rank of two; "disagree" was given a rank of one; and "strongly disagree" was given a rank of zero. Table 29 indicates the frequency of responses in each category for each of the *traditional* items in Question 1.

Table 29
Teachers' responses to the traditional statements from Question 1 (%), n = 162

Statement	Strongly agree	Agree	Disagree	Strongly disagree
students should learn basic number facts before they do *application* and *unfamiliar problems*	36	43	18	3
students should learn algorithms before they do *application* and *unfamiliar problems*	15	30	51	4
students cannot solve problems until they know how to perform the four operations	9	24	55	12
the best problems are those which relate directly to the number facts and algorithms the students have been practising	8	46	41	5
application and *unfamiliar problems* are best left to the end of the topic in mathematics	4	23	63	10
mathematics lessons should focus on practising skills	7	42	44	7
some students have trouble solving problems unless they know how to do the mathematics before they begin	16	65	17	2
some students find problem solving difficult because of the language involved in the problems	58	40	2	0

As has been discussed in the previous section and for ease of discussion, for each item the values for the two agreement categories were combined, as were the two disagreement categories. In short, there was a high level of *agreement* with the view that students should learn basic facts before doing problems, that some students have trouble solving problems unless they know how to do the mathematics first, and that the language involved in problems can make problem solving more difficult for some students. In addition to this, there was quite a high level of *disagreement* with the view that students cannot solve problems until they can perform the four operations and that problems are best left until the end of the topic in mathematics.

A more even spread of responses was gained from the remaining three items. These included the belief that students should learn algorithms before solving problems, that the best problems are those that relate directly to the number facts and algorithms the students are practising, and that mathematics lessons should focus on practising skills. Further elaboration of the implications from these data appears after Table 30 which presents the frequency of responses in each category for the *contemporary* statements from Question 2 of the questionnaire.

Table 30
Teachers' responses to the contemporary statements from Question 2 (%), n = 162

Statement	Strongly Agree	Agree	Disagree	Strongly Disagree
it is a good strategy to begin the topic of 2 digit addition with *unfamiliar problems* involving 2 digit addition	4	33	54	9
mathematics lessons should focus on problems rather than on practice of algorithms	4	48	48	0
problems help motivate students to learn basic facts and algorithms because they can see a reason for learning them	20	72	7	1
all mathematics questions should challenge students to think about what mathematics they know and how they can use it	28	58	14	0
students can learn most mathematical concepts by working out for themselves how to solve *unfamiliar* or *open-ended problems*	3	33	58	6
it is essential for students to explore their own ways of doing mathematics questions before being shown the teacher's methods	9	47	41	3
most students forget mathematics procedures and so it is best to let them work out their own methods first	4	18	70	8

In short, there was a high level of *agreement* with the view that problems help motivate students to learn mathematics and that problems should challenge students to think about the mathematics they need to use. There was a high level of *disagreement* with the item that suggested that most students forget mathematics procedures and so it is best to let them work out their own methods first. Two items yielded a more even spread of responses. These included the notion that mathematics lessons should focus on problems rather than the practice of algorithms and that it is essential for students to explore their own ways of doing mathematics questions before being shown the teacher's methods.

These frequency tables reveal that there is a spread of beliefs for this sample of primary school teachers. To summarise the trends, Table 31 shows the items that yielded a high level of *agreement* and a high level of *disagreement* for statements made by both of the imaginary teachers described in the questionnaire. Naomi's statements suggested a more *traditional* teaching approach while Gwendolin's statements were more indicative of a *contemporary* teaching approach to mathematics. Items have been included in this table if more than two thirds of respondents either agreed or disagreed with each item.

Table 31
Statements that yielded a high level of agreement and a high level of disagreement with percentage of responses in brackets

Perspective	A High Level of Agreement	A High Level of Disagreement
Naomi's Statements (Traditional)	students should learn basic number facts before they do *application* and *unfamiliar problems* (79%)	students cannot solve problems until they know how to perform the four operations (67%)
	some students have trouble solving problems unless they know how to do the mathematics before they begin (81%)	*application* and *unfamiliar problems* are best left to the end of the topic in mathematics (73%)
	some students find problem solving difficult because of the language involved in the problems (98%)	
Gwendolin's Statements (Contemporary)	problems help motivate students to learn basic facts and algorithms because they can see a reason for learning them (92%)	most students forget mathematics procedures and so it is best to let them work out their own methods first (78%)
	all mathematics questions should challenge students to think about what mathematics they know and how they can use it (86%)	

The data suggest that, on the one hand, there is general agreement with particular aspects of teaching and learning mathematical problem solving. The majority of surveyed teachers believe that number facts and some basic mathematics are needed before students can tackle problems. Also, many teachers supported the notion that problems can motivate and challenge students. Given the overwhelming agreement with the item about language, it is clear that most teachers believe that problems can be made more difficult by the language that is often used.

On the other hand, there seems to be general disagreement with other aspects of the teaching and learning of mathematical problem solving. Most teachers rejected the view that to solve problems, students need to know the four mathematical operations and that problems should be left to the end of the topic. Encouraging students to invent their own methods was also rejected by a majority of respondents. It is possible that the rejection of the view that students should invent their own methods may reflect a rejection of the strategy of teaching *through* problem solving. Overall, the responses suggest that *traditional* approaches gained more support from this group of teachers than *contemporary* views.

It is noted that there appears to be a contradiction between the high level of agreement with the statement that "some students have trouble solving problems unless they know how to do the mathematics before they begin" and the high level of disagreement with the statement "students cannot solve problems until they know how to perform the four operations". It is possible that the negative wording of the second statement has led to misinterpretation. Alternatively, the use of the word "some" at the beginning of the first item may have been a significant prompt in responding to the item. Another possible explanation is that teachers view mathematics as more than just the four operations.

Several items yielded a mixed response from the teachers in this sample (see Table 32). These are the items that had between 40% and 60% agreement from respondents. From this, it is clear that there are some aspects of teaching and learning mathematical problem solving where this group of teachers adopted opposing views.

Table 32
Statements that provided a more even spread of responses (values in brackets are the percentage of agreement with each item)

Naomi's Statements (Traditional Perspective)	Gwendolin's Statements (Contemporary Perspective)
students should learn algorithms before they do *application* and *unfamiliar problems* (45%)	mathematics lessons should focus on problems rather than on practice of algorithms (52%)
the best problems are those that relate directly to the number facts and algorithms the students have just been practising (54%)	it is essential for students to explore their own ways of doing mathematics questions before being shown the teacher's methods (56%)
mathematics lessons should focus on practising skills (49%)	

Each of these five statements appeared to split the respondents into approximately equal groups. From these data it can be seen that about half of the respondents support the more traditional practices of learning algorithms before doing problems; relating problems to the specific content of lessons, and focusing on practising skills. This is balanced by the other half of respondents who clearly rejected these views. Also, about half of the surveyed teachers indicated support for the more contemporary practices of focusing on problems rather than exercises and exploring individual methods of doing mathematical questions. Similarly, approximately half of the group of surveyed teachers rejected these beliefs.

Further, it can be postulated that the focus of mathematics lessons is another issue that split the respondents. The data indicate that about half of these teachers believe that mathematics lessons should focus on practising skills while the other half believe lessons should focus on problems rather than practising algorithms.

The distribution of responses to these five items in particular identifies differences in beliefs. The spread of responses suggests that the instrument has been successful in discriminating beliefs about mathematical problem solving, thus establishing confidence in the instrument. It should be noted that, with the exception of the item about language, all other items provided a spread of responses suggesting that the questionnaire was a useful instrument to gain information about the range of teachers' problem-solving beliefs. Early analyses of some of these data were published in Anderson (1998a, 1998b).

4.2.2.2 Differences in Reported Beliefs for Particular Groups

Particular aspects of background information supplied by respondents were examined to explore possible differences between groups with respect to their reported problem-solving beliefs. These aspects included level of teaching experience, current teaching grade, and attendance at mathematics inservice days during the last two years. Differences between responses to belief statements of these groups within each of these categories is discussed in the following sections.

Belief Statements Compared to Level of Teaching Experience

One background characteristic was the number of whole years that each teacher had been teaching. Three categories were presented for respondents to select including 0 to 4 years, 5 to 9 years, and 10 or more years. These categories have been labelled "new", "moderate" and "experienced". To examine potential differences between these three groups, frequency tables were compiled for the belief statements presented to respondents in the first two questions (Tables 33 and 34).

Table 33
Proportions of respondents that agreed or strongly agreed with the traditional items from Question 1 for each of the three experience levels (as a percentage), n = 161

Experience Level	New	Moderate	Experienced
Statement	n=31	n=27	n=103
students should learn basic number facts before they do *application* and *unfamiliar problems*	71	80	81
students should learn algorithms before they do *application* and *unfamiliar problems*	39	38	49
students cannot solve problems until they know how to perform the four operations	29	44	31
the best problems are those which relate directly to the number facts and algorithms the students have been practising	48	54	56
application and *unfamiliar problems* are best left to the end of the topic in mathematics	19	12	33
mathematics lessons should focus on practising skills	60	56	43
some students have trouble solving problems unless they know how to do the mathematics before they begin	73	85	83
some students find problem solving difficult because of the language involved in the problems	97	100	97

The frequencies in Table 33 indicate that with additional teaching experience, there may be a slight increase in support for some aspects of a *traditional* approach to teaching mathematics. There was a slight increase in support for the view that students should learn basic facts and algorithms before they attempt application and unfamiliar problems, that it may be necessary for some students to be able to do the mathematics before they attempt to solve problems, and that problems are best left to the end of the topic.

In contrast to this, with an increase in teaching experience, there appears to be a *decrease* in support for the belief that mathematics lessons should focus on practising skills. Sixty percent of new teachers agreed that mathematics lessons should focus on practising skills compared to 56% for moderately experienced teachers and 43% for experienced teachers. This result was somewhat surprising since new teachers are more likely to have been recently exposed to innovative practices that would support more contemporary approaches.

Overall, these data suggest that experience does not seem to be an important predictor of teachers' problem-solving beliefs. A similar trend was observed for the *contemporary* statements listed in Table 34.

Table 34
Proportions of respondents that agreed or strongly agreed with the contemporary items from Question 2 for each of the three experience levels (as a percentage), n = 161

Experience Level Statement	New n=31	Moderate n=27	Experienced n=103
it is a good strategy to begin the topic of 2 digit addition with *unfamiliar problems* involving 2 digit addition	47	27	37
mathematics lessons should focus on problems rather than on practice of algorithms	57	54	51
problems help motivate students to learn basic facts and algorithms because they can see a reason for learning them	100	96	88
all mathematics questions should challenge students to think about what mathematics they know and how they can use it	84	89	84
students can learn most mathematical concepts by working out for themselves how to solve *unfamiliar* or *open-ended problems*	35	44	34
it is essential for students to explore their own ways of doing mathematics questions before being shown the teacher's methods	59	78	49
most students forget mathematics procedures and so it is best to let them work out their own methods first	29	20	20

The data in Table 34 indicate that with increase in teaching experience, there may be a slight decrease in support for some of the more *contemporary* aspects of teaching mathematics. This applies to the view that it is appropriate to begin a topic with unfamiliar problems, that problems help motivate students to learn basic facts and algorithms, that it is necessary for students to initially explore their own ways of doing mathematics questions, and that students are more likely to remember procedures if they are encouraged to work out their own methods. As this trend is slight and only applies to four of these items, it would be inappropriate to suggest that experience has a significant impact on teachers' beliefs.

Belief Statements Compared to Current Teaching Grade

Based on their current teaching role at school, there were three categories for class teachers including Kindergarten to Year 2, Years 3 or 4, and Years 5 or 6. These categories have been labelled "lower", "middle" and "upper". Respondents who indicated that they were not currently teaching one of the primary grades were not considered in this analysis thus reducing the number of questionnaires for analysis to 122.

To examine potential differences between these three groups, frequency tables were compiled for the belief statements presented to respondents. Table 35 summarises the proportion of responses that *agreed* with the *traditional* statements from Question 1.

Table 35
Proportions of respondents that agreed or strongly agreed with the traditional items from Question 1 for each of the three teaching grade levels (as a percentage), n = 122

Teaching Grades Statement	Lower n=47	Middle n=37	Upper n=38
students should learn basic number facts before they do *application* and *unfamiliar problems*	62	92	80
students should learn algorithms before they do *application* and *unfamiliar problems*	26	68	53
students cannot solve problems until they know how to perform the four operations	23	38	35
the best problems are those which relate directly to the number facts and algorithms the students have been practising	49	61	49
application and *unfamiliar problems* are best left to the end of the topic in mathematics	20	38	29
mathematics lessons should focus on practising skills	48	60	55
some students have trouble solving problems unless they know how to do the mathematics before they begin	77	89	78
some students find problem solving difficult because of the language involved in the problems	94	97	100

The data suggest that the grade level taught may have more of an influence on teachers' beliefs than teaching experience. Teachers who were currently teaching in the lower grades were less likely to agree with some of these traditional statements than teachers who were teaching higher grades. This was particularly evident for the belief that students should learn number facts and algorithms before doing problem solving, that students cannot solve problems until they know how to perform the four operations, and that problems are best left to the end of the topic.

The data in Table 35 also suggest that teachers who were currently teaching the middle grades were more likely to support most of these traditional beliefs than either of the other two groups of teachers. Middle grade teachers were more likely to support the view that students should learn basic facts and algorithms before they attempt problems. These teachers were also more likely to support the notion that the best problems relate directly to current content and that problems are best left to the end of a topic. Teachers who were currently teaching the upper primary grades revealed more support for some of these traditional beliefs than teachers of the lower primary grades but were not as strongly traditional as the teachers of the middle grades.

A possible explanation for the support of these *traditional* statements by the middle grade teachers is that algorithms are introduced and developed in these grades for most students. There is a focus in the curriculum on recall of facts, mental strategies, and written algorithms that might encourage teachers to focus on these skills and as a consequence, leave problem solving until later.

To continue the comparisons, Table 36 summarises the proportion of responses that *agreed* with the *contemporary* statements in Question 2.

Table 36
Proportions of respondents that agreed or strongly agreed with the contemporary items from Question 2 for each of the three teaching grade levels (as a percentage), n = 122

Teaching Grades Statement	Lower n=47	Middle n=37	Upper n=38
it is a good strategy to begin the topic of 2 digit addition with *unfamiliar problems* involving 2 digit addition	43	41	32
mathematics lessons should focus on problems rather than on practice of algorithms	70	40	44
problems help motivate students to learn basic facts and algorithms because they can see a reason for learning them	94	89	87
all mathematics questions should challenge students to think about what mathematics they know and how they can use it	85	86	82
students can learn most mathematical concepts by working out for themselves how to solve *unfamiliar* or *open-ended problems*	47	31	39
it is essential for students to explore their own ways of doing mathematics questions before being shown the teacher's methods	78	46	49
most students forget mathematics procedures and so it is best to let them work out their own methods first	39	19	11

The trend noted for the *traditional* items seemed to be reversed for the more *contemporary* views that were presented to teachers in the second question. The data in

Table 36 reveal that for all of the *contemporary* belief statements, teachers of the lower grades were more likely to agree than either middle grade or upper grade teachers. Support for three of these belief statements was much higher for lower grade teachers. These included the view that mathematics lessons should focus on problems rather than algorithms, that students should explore their own ways of doing mathematics questions before being shown the teacher's methods, and that as most students forget mathematics procedures, it is best to let them work out their own methods of doing mathematics. For some of the statements there was slightly more support from teachers of upper grades than middle grades and for other statements, the reverse was the case.

Again, this may reflect the focus of the curriculum in the lower grades. The syllabus recommends the use of real-life applications to introduce basic algorithm development and this is usually done through simple word problems. In addition, there is support for encouraging exploration of methods and individual recording of solutions rather than being introduced to formal methods at this early stage.

Belief Statements Compared to Attendance at Mathematics Inservice Days

Respondents were also asked to indicate the level of attendance at mathematics inservice days in the last two years. Three categories were presented for respondents to select including 0 days, 1 to 3 days, and 4 or more days. Of the 162 respondents to the questionnaire, only nine had attended 4 or more mathematics inservice days. As this group is very small, they have been eliminated from this part of the analysis.

To examine potential differences in problem-solving beliefs between the remaining two groups, frequency tables were compiled for the belief statements presented to respondents. Table 37 summarises the proportion of responses that *agreed* with the *traditional* statements from Question 1 and Table 38 summarises the proportion of responses that *agreed* with the *contemporary* statements from Question 2.

Table 37
Proportions of respondents that agreed or strongly agreed with the traditional items from Question 1 for two of the mathematics inservice levels (as a percentage), n = 149

Statement	Mathematics Inservice Days	0 Days n=72	1-3 Days n=77
students should learn basic number facts before they do *application* and *unfamiliar problems* (facts)		81	76
students should learn algorithms before they do *application* and *unfamiliar problems* (algorithms)		44	45
students cannot solve problems until they know how to perform the four operations (operations)		28	36
the best problems are those which relate directly to the number facts and algorithms the students have been practising (practice)		51	59
application and *unfamiliar problems* are best left to the end of the topic in mathematics (end of topic)		34	21
mathematics lessons should focus on practising skills (skills)		51	48
some students have trouble solving problems unless they know how to do the mathematics before they begin (maths first)		81	80
some students find problem solving difficult because of the language involved in the problems (language)		97	97

The level of attendance at mathematics inservice days seems to have little impact on the reported beliefs of teachers. There appears to be a slight decrease in support for the traditional view that problems are best left to the end of the topic in mathematics. For all other items in this question, there is little change in beliefs with increasing attendance at mathematics inservice days. Table 38 presents responses to the contemporary statements.

Table 38
Proportions of respondents that agreed or strongly agreed with the contemporary items from Question 2 for two of the mathematics inservice levels (as a percentage), n = 149

Statement	Mathematics Inservice Days	0 Days n=72	1-3 Days n=77
it is a good strategy to begin the topic of 2 digit addition with *unfamiliar problems* involving 2 digit addition (begin)		40	39
mathematics lessons should focus on problems rather than on practice of algorithms (problem focus)		49	60
problems help motivate students to learn basic facts and algorithms because they can see a reason for learning them (motivate)		90	91
all mathematics questions should challenge students to think about what mathematics they know and how they can use it (think)		81	91
students can learn most mathematical concepts by working out for themselves how to solve *unfamiliar* or *open-ended problems* (self strategy)		29	42
it is essential for students to explore their own ways of doing mathematics questions before being shown the teacher's methods (explore)		48	61
most students forget mathematics procedures and so it is best to let them work out their own methods first (own methods)		16	29

The data in Table 38 indicate that with increasing attendance at mathematics inservice days there may be a slight increase in support for some of the more contemporary belief statements. These statements include the view that lessons should focus on problems rather than on algorithms, that mathematics questions should challenge students to think about what mathematics they know, and that students can learn most concepts by working out for themselves how to solve problems. There was an increase in support for the view that students should explore their own methods of doing mathematics before being shown the teacher's methods and that most students forget mathematics procedures and so it is best to let them work out their own methods first.

In summary, a comparison of responses to the belief statements in Question 1 and Question 2 of the questionnaire revealed some differences between groups. A comparison of responses based on teaching grade level indicated that teachers of middle grades were more supportive of *traditional* approaches than the other two groups of teachers, and teachers of lower grades were more supportive of *contemporary* approaches than the other two groups of teachers. Reported attendance at mathematics inservice days may yield a slight increase in support for some of the more contemporary belief statements. Surprisingly, increased teaching experience seemed to have little impact on teachers' reported beliefs.

Analysis of the first two questions suggested that there was a spread of beliefs for this sample of teachers with some differences in beliefs based on teaching grade level. Further analyses were conducted to consider correlation between items and to determine the existence of factors within groups of items. The results of these are presented in the next section.

4.2.3 Results from Correlational Analysis and Factor Analysis

A correlational analysis was conducted to determine if there were relationships between variables for the set of *traditional* statements and the set of *contemporary* statements. In addition, a factor analysis was undertaken on the data for both questions using the *Statview* software package (Feldman et al., 1988) to examine whether there were identifiable underlying factors. This section summarises the main results from both correlational analysis and factor analysis for the first two questions on the questionnaire.

Strong positive correlations at the 95% confidence level were present between several of the *traditional* statements in Question 1. The correlation matrix in Table 39 presents the correlation coefficients for each of the *traditional* belief statements.

Statements are labelled according to the key idea for each as indicated in Table 37 in the previous section.

Table 39
Correlation coefficients for each of the traditional belief statements in Question 1

Item	Facts	Algorithms	Operations	Practice	End of Topic	Skills	Maths first	Language
Facts	1.000	.569 *	.449 *	.396 *	.364 *	.256 *	.429 *	.150**
Algorithms		1.000	.573 *	.379 *	.417 *	.345 *	.446 *	.176**
Operations			1.000	.436 *	.488 *	.292 *	.452 *	.148
Practice				1.000	.380 *	.433 *	.444 *	.205*
End of Topic					1.000	.296 *	.373 *	.105
Skills						1.000	.316 *	.030
Maths first							1.000	.199**
Language								1.000

** $p < .05$, $r = 0.15$
* $p < .01$, $r = 0.2$

The correlation coefficients indicate that many of these items are highly correlated. There appear to be strong links between teachers' responses to *facts*, *algorithms*, *operations*, *practice*, *end of topic*, *skills* and *maths first*. This means that if teachers agreed with one of these statements then they were likely to agree with the others. The item that has least correlation with the others is the statement about *language*. This is not surprising as 98% of respondents agreed with the statement and as a result it did not spread the respondents even though analysis of the other items suggested there were potential differences in belief systems.

There is clearly a strong relationship between these variables. This is confirmed by an unrotated factor analysis of all belief statements that identified the main underlying factor as being all of the *traditional* statements except language to which nearly all teachers responded similarly.

In order to elaborate the relationships identified in this table, a schematic representation of the relationships where $r > 0.4$ was drawn and is presented in Figure 17. The four most strongly related statements were those referring to *facts*, *algorithms*, *operations* and *mathematics first* where there appeared to be a clearly influencing trend. The strength of the relationships confirms the underlying structure of the questions and provides assurance of the trustworthiness of these items as well as confidence in the subsequent analyses.

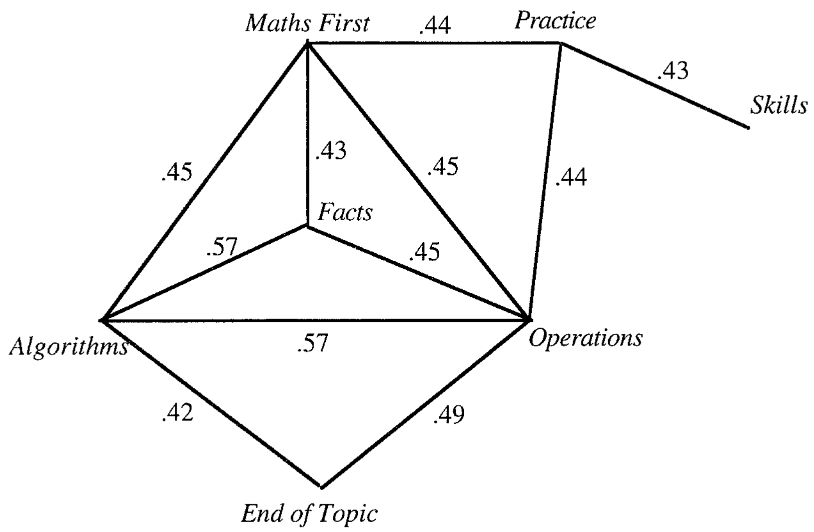


Figure 17. Schematic representation of interrelationships between strongly related *traditional* variables.

There were also strong positive correlations at the 95% confidence level between several of the *contemporary* statements about problem solving that were included in Question 2. Table 40 is the correlation matrix for these statements with statement labels relating to the key idea for each as indicated in Table 38.

Table 40
Correlation coefficients for each of the contemporary belief statements in Question 2

Item	Begin	Problem focus	Motivate	Think	Self strategy	Explore	Own methods
Begin	1.000	.461*	.244*	.171**	.360*	.273*	.246*
Problem focus		1.000	.227*	.229*	.487*	.442*	.475*
Motivate			1.000	.515*	.193**	.295*	.262*
Think				1.000	.329*	.331*	.326*
Self Strategy					1.000	.539*	.508*
Explore						1.000	.617*
Own methods							1.000

** $p < .05$, $r = 0.15$
* $p < .01$, $r = 0.2$

The correlation coefficients indicate that several of these items are highly correlated. In particular, teachers have responded to four of these variables in similar ways. For each of *own methods*, *problem focus*, *self strategy* and *explore*, those teachers who agreed with one statement tended to agree with them all, and similarly for those teachers who disagreed. This emphasis is confirmed by an unrotated factor analysis on all belief statements in Questions 1 and 2 in which the second of the selected factors recorded loadings of *own methods* (0.7), *explore* (0.64), *self strategy* (0.62) and *problem focus* (0.61) respectively.

In order to elaborate the relationships identified in this table, a schematic representation of the relationships where $r > 0.4$ was drawn and is presented in Figure 18.

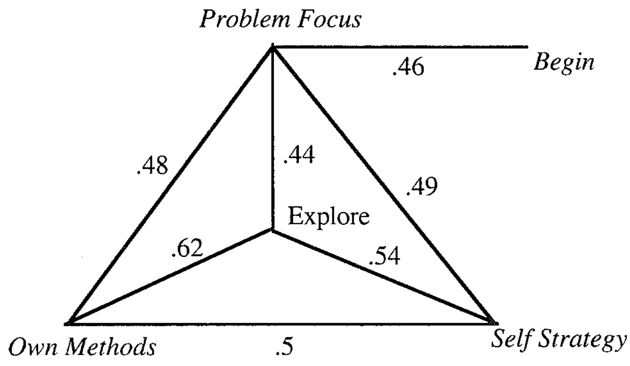


Figure 18. Schematic representation of interrelationships between strongly related *contemporary* variables.

For three of these strongly related items there is a theme of the students working out ways of doing problems for themselves. As seen earlier, these were the statements that tended to distinguish between the contemporary/traditional diad and supports the notion of different sets of beliefs for each of the teaching perspectives. This means that there is a consistency in teachers' responses, that the variables are clearly linked, and provides further confidence in subsequent analyses.

In addition to the above separate correlational analyses, another analysis was performed on all 15 items. The table is not presented here as there was only one strong negative correlation that occurred; this was between learning algorithms before solving problems and the view that mathematics lessons should focus on problems rather than on practising algorithms ($r = -.426$).

This section has described the analyses of Questions 1 and 2. To illuminate teachers' problem-solving beliefs, three separate analyses were undertaken. The first determined that the questionnaires could be sorted into five broad categories. Teachers placed in the extreme categories of *very traditional* and *very contemporary*, while reportedly using some similar practices, also reported clear differences in their approaches. The second revealed a spread of problem-solving beliefs for this group of teachers with some items clearly demonstrating the diversity of views about mathematical problem solving. In addition, there were slight differences between identified groups of teachers, particularly in relation to teaching grade level where the teachers of lower primary grades supported more contemporary approaches and the middle grade teachers supported more traditional approaches than the other groups.

The third used correlation and factor analyses to investigate relationships between variables. There was a strong relationship between the traditional statements except for *language* and a strong relationship between several of the contemporary statements. All of these analyses suggest that the instrument was successful in enabling the researcher to identify differences in beliefs thus enhancing confidence in the research findings.

4.3 Reported Use of Particular Teaching Strategies

Teachers plan and organise lessons using a variety of teaching practices and strategies. Some of these strategies have been referred to as problem-solving teaching approaches and are recommended to teachers in preservice and inservice courses, in professional journals, and in curriculum documentation. There were twenty items that related to problem-solving teaching strategies in Question 4 of the questionnaire. In addition, Question 5 sought to identify frequency of use of each of the four student question types. Question 3 asked teachers to record a recently used problem; analysis of this question is presented in Section 4.5.

This section consists of three parts. The first examines reported use of teaching strategies and the second explores differences in teachers' reported use of teaching strategies based on teaching experience and current teaching grade. The third part compares the responses of primary school teachers to those of mathematics education lecturers who responded to the set of teaching strategies according to their interpretation of advice given in the problem-solving literature.

4.3.1 Reported Use of Teaching Strategies by Teachers

In relation to the twenty listed teaching strategies, respondents were requested to rate the frequency of use of each of these as either "hardly ever", "sometimes", "often" or "almost always". For the frequency of use of each of the student question types, Question 5 required teachers to select "often", "sometimes" or "rarely". This section identifies strategies reported as frequently used and those reported as infrequently used by surveyed teachers. Also, reported use of each of the particular student question types described in the questionnaire will be examined.

The data indicate that there is a spread of frequency of use for each of the listed teaching strategies. Some strategies seem to be used far more frequently by a majority of teachers than other strategies. For example, whole class discussion to share problem solutions and strategies is *often* or *almost always* used by 79% of respondents. Also, concrete materials are used *often* or *almost always* by 93% of teachers.

In contrast to this, some strategies are used relatively infrequently by a majority of teachers. These include students working alone where 81% of teachers *hardly ever* or only *sometimes* use this approach. Also, 82% of surveyed teachers *hardly ever* or only sometimes present problems to students with little indication of how to proceed to a solution. The data for each respondent were entered into a *Statview* (Feldman et al., 1988) program for collation and to produce summary tables and graphs. For ease of entry, the "hardly ever" category was entered as 0, "sometimes" as 1, "often" as 2, and "almost always" as 3. Table 41 summarises the reported frequency of use of each of the teaching strategies from Question 4.

Table 41
Reported use of teaching strategies (%), n=162

Teaching Strategy	Hardly ever	Some times	Often	Almost always
you ensure that the students work alone (alone)	16	65	16	3
you explain in detail what the students have to do to solve problems (explain)	10	52	28	10
at the end of a problem solving lesson you lead a whole class discussion so that students can share solutions and strategies (discussion)	3	18	46	33
you have calculators available for students to use (calculators)	32	35	26	7
you encourage the students to work in small, cooperative groups (groups)	0	33	45	32
you present *unfamiliar* and *open-ended problems* to the class with very little indication of how to solve them (little help)	33	49	16	2
you encourage students to record their own procedures and methods of solving problems (record methods)	10	38	39	13
you encourage students to pose their own problems (pose problems)	19	49	26	6
you provide a set of problems and the students are allowed to choose a problem they would like to work on (choose problems)	40	44	15	1
you allow the class or individual students to spend several lessons on the same problem (spend more time)	47	35	13	5
you use problems to show students that there are mathematical skills and procedures which they need to know (need maths)	3	34	54	9
you present *application problems* which allow students to practise the skills they have just learnt (practise skills)	0	21	65	14
you provide concrete materials for those students who need them (concrete materials)	0	7	22	71
you model the problem solving process to the class (model)	3	21	45	31
you discuss useful problem solving strategies (eg. make a list, draw a diagram, work backwards) (strategies)	3	20	52	25
you discuss problem solving processes (ie. make a plan, carry out the plan, check the calculations) (process)	8	25	47	20
you use problems which arise from the school context or which relate to the students' experiences (experiences)	3	36	38	23
you pose *open-ended problems* to allow students to explore mathematical situations for themselves (open-ended)	11	54	28	7
you set *exercises* to allow the students to practise their skills (exercises)	3	26	56	15
you pose *unfamiliar problems* (unfamiliar)	21	55	23	1

Note: each strategy is accompanied by an abbreviated title in brackets for future reference.

In the following sections, box and whisker plots produced by *Statview* (Feldman et al., 1988) are used to examine the spread of responses for teaching strategies. Box plots provide a way of representing ordinal data indicating only the range and scope of scores. The box represents the middle 50% of the scores and the whiskers represent the top and bottom 25%. Given that just whole number responses were used for this analysis, in some places the bottom 25% corresponds with the lower end of the middle 50% and in such cases there is no bottom whisker. Generally, the spread of responses can be compared by considering the middle 50% of scores. The next two sections identify strategies that were *frequently* used as well as those that were *rarely* used.

4.3.1.1 Frequently Used Teaching Strategies

Strategies that were reported as used frequently by respondents can be divided into two broad groups that include *very frequently* used and *frequently* used strategies. These have been combined and displayed in Figure 19.

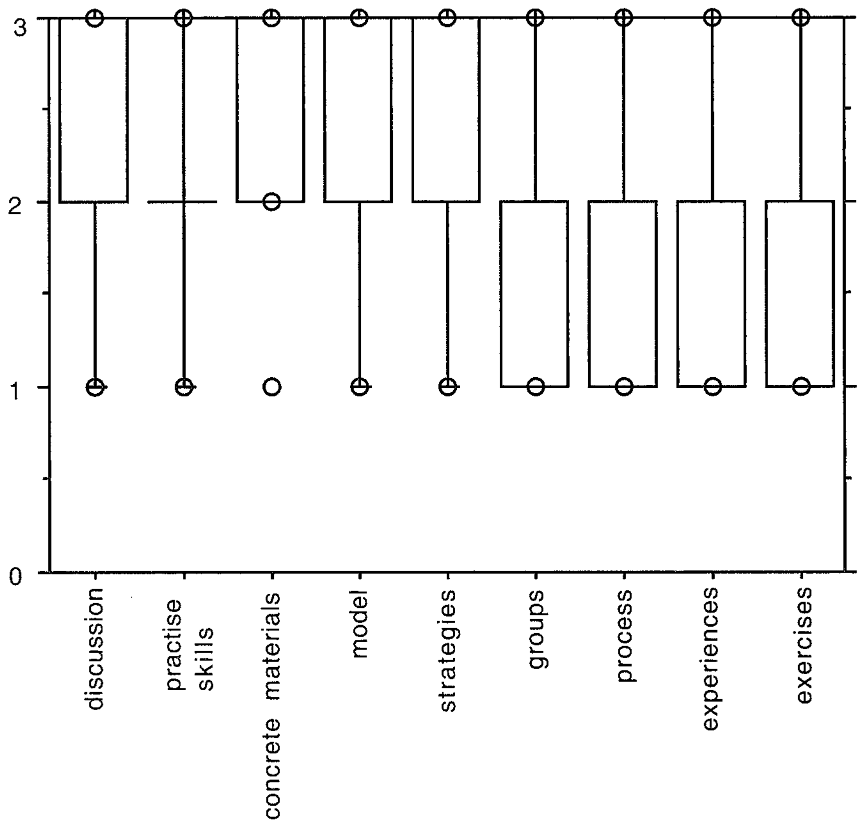


Figure 19. Teaching strategies that were reported as used frequently by teachers.

To clarify the reading of the above figure, consider the box and whisker plot for the item labelled "discussion". Up to 75% of teachers use discussion *often* (2) or *almost always* (3) and so there is no top whisker as it coincides with the middle 50% of responses. For the item labelled "practise skills", the box collapses to the median since more than 50% of the respondents indicated that they *often* (2) use this teaching strategy.

The *very frequently* used strategies included whole class discussion to share solutions and strategies, the use of application problems to practise recently learned skills, provision of concrete materials, modelling problem-solving processes, and whole class discussion about problem-solving strategies. Certainly the most frequently identified items are those recommended in advice to teachers but there is an underlying trend of a strong role of teacher direction and teacher control.

A second group of strategies were *frequently* used although they were not used as often as those already described. These included cooperative group work, class discussion of problem-solving processes, using problems that related to the school context or students' experiences, and setting exercises for students to practise skills.

4.3.1.2 Rarely Used Teaching Strategies

A further set of strategies were used infrequently or rarely by teacher respondents. These can also be divided into two groups based on the level of infrequency of use. Figure 20 indicates the spread of responses for these two groups of teaching strategies.

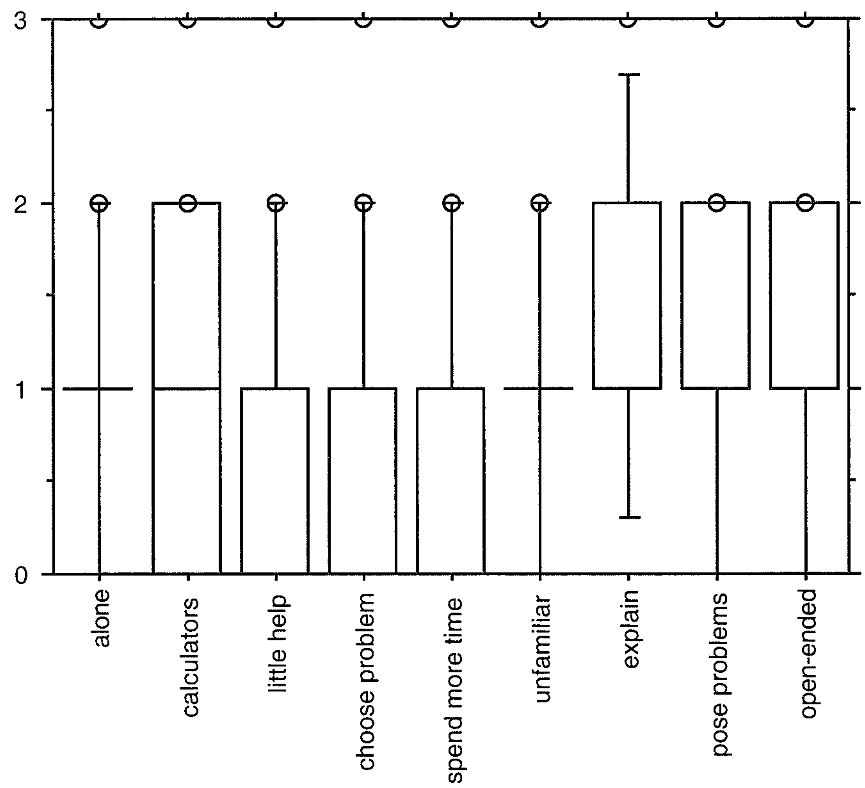


Figure 20. Teaching strategies that were reported as used rarely by teachers.

A group of six strategies were used *very rarely* including students working alone, availability of calculators, the presentation of unfamiliar and open-ended problems to the class with little indication of how to solve them, allowing students to choose their own problems, spending several lessons on the same problem, and posing unfamiliar problems. Another group of strategies were *rarely* used although teachers reported using these a little more frequently than the strategies already described. These included detailed explanations of how to solve problems, encouraging students to pose their own problems, and posing open-ended problems.

This suggests that for a large number of the surveyed teachers, several strategies are not viewed as necessary or appropriate for their students. It is clear that the use of unfamiliar and open-ended questions are not preferred strategies and allowing students choice and freedom to explore problems is not a commonly used approach by many of these teachers. A common aspect of these less frequently used items is the potential to allow students to take greater control of their own learning.

It should be noted that there was a spread of responses for all of these strategies with some teachers frequently using strategies that the majority of teachers rarely used. For example, 3% of respondents *almost always* have students working alone, 7% *almost always* have calculators available for student use, 2% *almost always* present problems to the class with little indication of how to proceed, and 5% of teachers *almost always* allow students to spend several lessons on the same problem.

4.3.1.3 Frequency of Use of Student Question Types

On the questionnaire, several items referred to the use of particular types of mathematics questions or problems. Four items in the list of teaching strategies in Question 4 related to the frequency of use of each of the student question types that were described in the Background Information. In addition to this, Question 5 asked respondents to indicate frequency of use for each type of mathematics question and included an example of each.

The items that explored frequency of use of different types of questions in Question 4 revealed a wide range of responses from teachers. Table 42 lists the four items and the frequencies for each category.

Table 42
Frequencies of the teaching strategies for teachers that relate to student question types in Question 4

Teaching Strategy	Hardly Ever	Sometimes	Often	Almost Always
you present *application problems* which allow students to practise the skills they have just learnt	0	21	65	14
you pose *open-ended problems* to allow students to explore mathematical situations for themselves	11	54	28	7
you set *exercises* to allow the students to practise their skills	3	26	56	15
you pose *unfamiliar problems*	21	55	23	1

Overall, open-ended problems and unfamiliar problems are less frequently presented to students than application problems and exercises. However, it should be noted that there are clearly individual differences in frequency of use since 35% of teachers pose open-ended problems *often* or *almost always* and 24% regularly pose unfamiliar problems. Also, 29% of teachers either *hardly ever* or only *sometimes* set exercises for their students.

The results for Question 5 are presented in Table 43. This question included an example of each of the student question types that may have made it easier for respondents to determine how frequently they would present such questions to their students.

Table 43
Frequency of use of student question types from Question 5 (%), n=162

Types of Questions	Rarely	Sometimes	Often
Application Problems	4	26	70
Open-ended Problems	22	58	20
Exercises	5	27	68
Unfamiliar Problems	37	52	11

Overall, the same trend is evident in Table 43 that was observed in Table 42; the majority of teachers use application problems and exercises more frequently than open-ended problems or unfamiliar problems. This may be a result of a reliance on textbooks where these questions are most common, or it may be that teachers are more confident with these particular student question types. These data will be further discussed in Section 4.3.3 when a comparison is made between teachers' reported use of each of the teaching strategies with mathematics education lecturers' recommendations of use for each of the teaching strategies.

4.3.2 Use of Teaching Strategies According to Experience and Teaching Grade Level

The earlier examination of teachers' problem-solving beliefs for particular groups of teacher respondents suggested that teaching experience had little effect on beliefs whereas teaching grade level seemed to result in slight differences in beliefs. A similar examination of teaching practices was conducted to ascertain the possible impact of experience and current grade level on teachers' reported use of particular strategies. This section examines differences in frequency of use of some teaching strategies based on level of experience and current teaching grade level.

Chi-Squared tests were not conducted as trends in the data are of interest to this investigation rather than the significance of differences between groups. There was no desire to find a causal link therefore statistical tests for significance of differences were not appropriate. For this reason, frequency tables are presented and comments are made about the trends in the data.

4.3.2.1 Use of Teaching Strategies According to Experience Level

For each of the twenty teaching strategies presented to teachers in Question 4 of the questionnaire, frequency tables were constructed and examined for trends and potential differences between "new", "moderate" and "experienced" teachers. Small differences in frequency of use for some teaching strategies were evident and are presented in this section.

Some teaching strategies were reported as used *slightly* more frequently by new teachers than either the moderately experienced or experienced teachers (Tables 44 and 45). These differences are small but are considered to be worthy of comment to provide an overall picture of potential differences between these three groups.

Table 44
Frequency of use of the concrete materials teaching strategy for each of the three experience levels (%), n=157

Teaching Strategy: you provide concrete materials for those students who need them (concrete materials)				
	Hardly Ever	Sometimes	Often	Almost Always
New (n=31)	0	3	16	81
Moderate (n=27)	0	4	20	78
Experienced (n=99)	0	9	24	66

Table 45
Frequency of use of the unfamiliar teaching strategy for each of the three experience levels (%), n=155

Teaching Strategy: you pose *unfamiliar problems* (unfamiliar)				
	Hardly Ever	Sometimes	Often	Almost Always
New (n=31)	19	39	42	0
Moderate (n=26)	15	62	23	0
Experienced (n=98)	23	58	17	2

Of the new teachers, 81% *almost always* have concrete materials available for students who need them compared to 78% of the moderately experienced teachers and 66% of the experienced teachers (Table 44). Also, 42% *often* pose unfamiliar problems compared to 23% of moderately experienced teachers and 17% of experienced teachers although 2% of experienced teachers indicated that they *almost always* pose unfamiliar problems (Table 45). It would appear that less experienced teachers are aware of the potential of unfamiliar problems and use them more frequently than more experienced teachers whereas there is a more universal, frequent use of concrete materials in primary classrooms.

Three other strategies were used *slightly* more frequently by this group of new teachers. Table 46 presents responses to the strategy that students ought to record their own methods and procedures, Table 47 presents responses to encouraging students to pose their own problems, and Table 48 presents responses to allowing students to choose their own problems.

Table 46
Frequency of use of the record methods teaching strategy for each of the three experience levels (%), n=155

Teaching Strategy: you encourage students to record their own procedures and methods of solving problems (record methods)				
	Hardly Ever	Sometimes	Often	Almost Always
New (n=31)	3	33	48	16
Moderate (n=26)	4	46	38	12
Experienced (n=98)	14	37	37	12

Table 47
Frequency of use of the pose problems teaching strategy for each of the three experience levels (%), n=158

Teaching Strategy: you encourage students to pose their own problems (pose problems)				
	Hardly Ever	Sometimes	Often	Almost Always
New (n=31)	10	45	32	13
Moderate (n=27)	19	56	14	11
Experienced (n=100)	22	48	27	3

Table 48
Frequency of use of the choose problems teaching strategy for each of the three experience levels (%), n=157

Teaching Strategy: you provide a set of problems and the students are allowed to choose a problem they would like to work on (choose problems)				
	Hardly Ever	Sometimes	Often	Almost Always
New (n=31)	26	45	23	6
Moderate (n=27)	48	30	22	0
Experienced (n=99)	42	47	11	0

From the above tables, responses made by the moderately experienced group of teachers and the experienced teachers are very similar. However, there are slight differences between these two groups and the new teachers. Of the new teachers, 64% reported that they *often* or *almost always* encourage students to record their own methods compared to 50% of moderately experienced teachers and 49% of experienced teachers (Table 46). Also, 45% percent of this group *often* or *almost always* encourage students to pose their own problems compared to 25% of moderately experienced teachers and 30% of experienced teachers (Table 47). Finally, 29% either *often* or *almost always* allow students to choose a problem from a set compared to 22% and 11% from the other two groups of teachers (Table 48).

Other teaching strategies were reported as *slightly* more frequently used by the moderately experienced teachers than either the new or experienced teachers. Table 49 presents the data related to the use of groups and Table 50 presents the data about the use of problems that arise from the school context or relate to students' experiences.

Table 49
Frequency of use of the groups teaching strategy for each of the three experience levels (%), n=158

Teaching Strategy: you encourage the students to work in small, cooperative groups (groups)				
	Hardly Ever	Sometimes	Often	Almost Always
New (n=30)	0	27	63	10
Moderate (n=27)	0	33	37	30
Experienced (n=101)	1	36	42	21

Table 50
Frequency of use of the experiences teaching strategy for each of the three experience levels (%), n=158

Teaching Strategy: you use problems which arise from the school context or which relate to the students' experiences (experiences)				
	Hardly Ever	Sometimes	Often	Almost Always
New (n=31)	3	35	39	23
Moderate (n=27)	0	30	30	40
Experienced (n=100)	3	37	41	19

Of the moderately experienced group of teachers, 30% *almost always* encourage students to work in cooperative groups compared to 10% of new teachers and 21% of experienced teachers (Table 49). Also, 40% of the moderately experienced group of teachers *almost always* use problems that arise from the school context or relate to students' experiences compared to 23% of new teachers and 19% of experienced teachers (Table 50). If the data for the *often* and *almost always* categories are combined for each of the three groups, the differences are very small. Generally, experience seems to have little influence on the teaching practices of using small groups or problems arising from the school context except that new teachers use small groups slightly less frequently than either of the other two groups of teachers. This suggests that experience may not be an important determinant of choice of teaching strategies.

In addition to this, the moderately experienced teachers report that they use application problems and exercises *slightly* more frequently than new or experienced teachers. Table 51 presents the data for the use of application problems to practise skills and Table 52 presents the data for the use of exercises.

Table 51
Frequency of use of the practise skills teaching strategy for each of the three experience levels (%), n=158

Teaching Strategy: you present *application problems* which allow students to practise the skills they have just learnt (practise skills)

	Hardly Ever	Sometimes	Often	Almost Always
New (n=31)	0	13	71	16
Moderate (n=27)	0	19	56	25
Experienced (n=100)	0	24	65	11

Table 52
Frequency of use of the exercises teaching strategy for each of the three experience levels (%), n=156

Teaching Strategy: you set *exercises* to allow the students to practise their skills (exercises)

	Hardly Ever	Sometimes	Often	Almost Always
New (n=31)	3	32	55	10
Moderate (n=27)	0	26	44	30
Experienced (n=98)	4	25	59	12

An examination of the responses in the *almost always* category indicate that 25% of the moderately experienced teachers *almost always* set application problems to allow students to practise skills compared to 16% of new teachers and 11% of experienced teachers (Table 51). Thirty percent of this group *almost always* set exercises to allow students to practise skills compared to 10% of new teachers and 12% of experienced teachers (Table 52). However, if the data for the *often* and *almost always* categories are combined, there is little difference between the three groups.

The experienced teachers reported that they more often use calculators than either of the other two groups of teachers. Table 53 presents the data for the frequency of use of calculators. Eleven percent of the experienced teachers *almost always* have calculators available for students to use compared to 0% of new teachers and 4% of moderately experienced teachers.

Table 53
Frequency of use of the calculators teaching strategy for each of the three experience levels (%), n=157

Teaching Strategy: you have calculators available for students to use (calculators)

	Hardly Ever	Sometimes	Often	Almost Always
New (n=30)	37	43	20	0
Moderate (n=27)	33	33	30	4
Experienced (n=100)	30	32	27	11

It should also be noted that experienced teachers were less likely to present students with open-ended problems compared to new and moderately experienced teachers (Table 54). Only 3% of experienced teachers *almost always* pose open-ended problems to allow students to explore mathematical situations for themselves compared to 13% of new teachers and 15% of moderately experienced teachers.

Table 54
Frequency of use of the open-ended teaching strategy for each of the three experience levels (%), n=158

Teaching Strategy: you pose *open-ended problems* to allow students to explore mathematical situations for themselves (open-ended)	Hardly Ever	Sometimes	Often	Almost Always
New (n=31)	7	45	35	13
Moderate (n=27)	11	52	22	15
Experienced (n=100)	12	57	28	3

In summary, experience level seems to have only a slight influence on the frequency of use of particular teaching strategies. This will be discussed in more detail later. Of the differences that were noted, new teachers reported that they slightly more frequently use concrete materials, pose unfamiliar problems, allow students to record their own methods, pose or choose their own problems than the other two groups of teachers. Also, moderately experienced teachers reported that they slightly more frequently use cooperative groups, pose problems arising from the school context, and set either exercises or application problems for skills practice than either of the other groups. Finally, experienced teachers more frequently have calculators available and less frequently pose open-ended problems than either of the others.

4.3.2.2 Use of Teaching Strategies According to Teaching Grade Level

For each of the twenty teaching strategies presented to teachers in Question 4, frequency tables were constructed and examined for trends or differences between "lower", "middle" and "upper" grade teachers. Differences in frequency of use of some of the teaching strategies are evident based on the teaching grade level. In this section, the smaller number of responses reported in each of the frequency tables is a consequence of only including those teachers who indicated that they were currently teaching a class from one of the teaching grade levels from Kindergarten to Year Six.

Some teaching strategies were reported to be used more frequently by teachers of lower grade classes compared to teachers of middle and upper grade classes. Table 55 presents the data for the frequency of use of explaining in detail what students should do to solve a problem and Table 56 presents the data for the use of cooperative groups.

Table 55
Frequency of use of the explain teaching strategy for each of the three teaching grade levels (%), n=120

Teaching Strategy: you explain in detail what the students have to do to solve problems (explain)	Hardly Ever	Sometimes	Often	Almost Always
Lower (n=47)	13	49	15	23
Middle (n=36)	3	61	31	5
Upper (n=37)	14	51	32	3

Table 56
Frequency of use of the groups teaching strategy for each of the three teaching grade levels (%), n=121

Teaching Strategy: you encourage the students to work in small, cooperative groups (groups)				
	Hardly Ever	Sometimes	Often	Almost Always
Lower (n=47)	0	26	40	34
Middle (n=36)	3	42	44	11
Upper (n=38)	0	37	50	13

The lower grade teachers indicated that 23% *almost always* explain in detail what students should do to solve a problem compared to 5% of middle grade teachers and 3% of upper grade teachers (Table 55). However, combining the categories of *often* and *almost always* suggests little difference between the three groups of teachers. In addition, 34% of lower grade teachers reported that they *almost always* encourage students to work in cooperative groups compared to 11% of middle grade teachers and 13% of upper grade teachers (Table 56). There is a more obvious difference between the three groups for this strategy since combining the two categories of *often* and *almost always* yields 74%, 55% and 63% for lower grade teachers, middle grade teachers and upper grade teachers respectively.

Three other strategies were reported by lower grade teachers to be used more frequently than for middle and upper grade teachers. Table 57 presents the data for encouraging students to pose their own problems. Table 58 presents the data for the use of spending several lessons on the same problem and Table 59 presents the data for the use of concrete materials.

Table 57
Frequency of use of the pose problems teaching strategy for each of the three teaching grade levels (%), n=121

Teaching Strategy: you encourage students to pose their own problems (pose problems)				
	Hardly Ever	Sometimes	Often	Almost Always
Lower (n=46)	20	50	13	17
Middle (n=37)	22	46	32	0
Upper (n=38)	18	50	29	3

Table 58
Frequency of use of the spend more time teaching strategy for each of the three teaching grade levels (%), n=119

Teaching Strategy: you allow the class or individual students to spend several lessons on the same problem (spend more time)				
	Hardly Ever	Sometimes	Often	Almost Always
Lower (n=45)	40	33	20	7
Middle (n=37)	59	30	8	3
Upper (n=37)	53	34	13	0

Table 59
Frequency of use of the concrete materials teaching strategy for each of the three teaching grade levels (%), n=120

Teaching Strategy: you provide concrete materials for those students who need them (concrete materials)				
	Hardly Ever	Sometimes	Often	Almost Always
Lower (n=46)	0	0	4	96
Middle (n=37)	0	11	24	65
Upper (n=37)	0	14	43	43

These tables indicate that 17% of lower grade teachers *almost always* encourage students to pose their own problems compared to 0% of middle grade teachers and 3% of upper grade teachers (Table 57). Also, 7% of lower grade teachers *almost always* allow their students to spend several lessons on the same problem compared to 3% of middle grade teachers and 0% of upper grade teachers (Table 58). Finally, 96% of lower grade teachers *almost always* have concrete materials available compared to 65% of middle grade teachers and 43% of upper grade teachers (Table 59).

Ensuring students work alone on mathematical tasks is a strategy that is used infrequently by lower grade teachers. Thirty-three percent of lower grade teachers *hardly ever* ensure that students work alone compared to 5% of middle grade teachers and 8% of upper grade teachers (Table 60). The data in this table suggest that many teachers, particularly lower grade teachers, rarely encourage students to work alone on mathematics activities. This may be a result of encouragement to have students working in groups so that they can discuss ideas and share mathematical thinking.

Table 60
Frequency of use of the alone teaching strategy for each of the three teaching grade levels (%), n=121

Teaching Strategy: you ensure that the students work alone (alone)				
	Hardly Ever	Sometimes	Often	Almost Always
Lower (n=46)	33	50	15	2
Middle (n=37)	5	68	24	3
Upper (n=38)	8	74	13	5

A strategy favoured by teachers of middle grades was the use of application problems to practise skills and procedures. Ninety-two percent of middle grade teachers either *often* of *almost always* use these student question types compared to 63% of lower grade teachers and 79% of upper grade teachers (Table 61).

Table 61
Frequency of use of the practise skills teaching strategy for each of the three teaching grade levels (%), n=121

Teaching Strategy: you present *application problems* which allow students to practise the skills they have just learnt (practise skills)				
	Hardly Ever	Sometimes	Often	Almost Always
Lower (n=46)	0	37	50	13
Middle (n=37)	0	8	73	19
Upper (n=38)	0	21	66	13

Strategies favoured by teachers of upper primary grades more frequently than teachers of lower or middle primary grades include the availability of calculators and allowing students to choose their own problems. Table 62 presents the data for the use of calculators and Table 63 presents the data for the use of allowing students to choose their own problems.

Table 62
Frequency of use of the calculators teaching strategy for each of the three teaching grade levels (%), n=120

Teaching Strategy: you have calculators available for students to use (calculators)				
	Hardly Ever	Sometimes	Often	Almost Always
Lower (n=46)	54	28	14	4
Middle (n=36)	42	31	19	8
Upper (n=38)	5	37	45	13

Table 63
Frequency of use of the choose problem teaching strategy for each of the three teaching grade levels (%), n=120

Teaching Strategy: you provide a set of problems and the students are allowed to choose a problem they would like to work on (choose problems)				
	Hardly Ever	Sometimes	Often	Almost Always
Lower (n=46)	43	43	14	0
Middle (n=37)	35	54	11	0
Upper (n=37)	46	30	23	3

Thirteen percent of upper grade teachers *almost always* have calculators available for student use compared to 4% of lower grade teachers and 8% of middle grade teachers (Table 62). Also, 26% of upper grade teachers either *often* or *almost always* allow students to choose their own problems compared to 14% for lower grade teachers and 11% of middle grade teachers (Table 63). The data indicate that upper primary teachers are more likely to have calculators available, a strategy that may recognise that some students have difficulty answering questions if they have not mastered basic facts and algorithms at this stage.

In summary, teaching grade level seems to have a slight influence on the frequency of use of some teaching strategies. Slightly more than teachers of middle and upper primary grades, teachers of lower grades seem to prefer to explain problem-solving methods, to use cooperative groups, to allow students to pose their own problems, to spend several lessons on the same problem, and to have concrete materials readily available. They less frequently encourage students to work alone. As was suggested in the beliefs analysis earlier in this chapter, this tendency may reflect curriculum recommendations regarding the focus of instruction in the early years of schooling.

Differences for the other two groups of teachers may also be explained by the curriculum focus. The above analysis suggests that middle grade teachers, slightly more frequently than teachers from the other two groups, present application problems to students for skills practice. A key focus in Years 3 and 4 is the development of basic facts and algorithms. Finally, upper grade teachers, slightly more frequently than either teachers from the lower or middle grades, have calculators readily available and allow students to choose their own problems. This could be accounted for by a curriculum focus on applying knowledge to problem situations. However, it should be noted that given the differences in students' ages and curriculum focus, the differences in choice of teaching strategies are only slight.

To consider whether teachers have responded to advice about teaching strategies that has been provided in the problem-solving literature, mathematics education lecturers were invited to respond to the same set of twenty teaching strategies in terms of their interpretation of this advice. This enabled a comparison to be made between the responses of the teachers and the responses of the lecturers. A summary of this comparison is presented in the next section.

4.3.3 Comparison of Teachers' Reported Strategy Use with Lecturer Recommended Usage

To compare teachers' responses to those of an informed community, a group of 21 tertiary mathematics education lecturers from NSW and Victoria were invited to complete the same set of survey items relating to the frequency of use of teaching strategies. The prompt for lecturers required them to indicate the desirable frequency of use of each of these teaching strategies according to their understanding of advice given in the problem-solving literature. This section compares the data from the mathematics education lecturers with that from the surveyed teachers. A more detailed report of the lecturers' data is presented in Appendix 8.

Responses from 21 mathematics education lecturers were received and entered into the *Statview* software package (Feldman et al., 1988) for analysis and comparison with the teacher respondents. Although there is a spread of responses from lecturers to the recommended frequency of use for these teaching strategies, it is evident that some strategies are advised as suitable for more frequent use than others. There is some agreement between teachers and lecturers regarding the frequency of use of particular teaching strategies. However, there are several differences, possibly indicating that teachers have not responded to some of the advice in the problem-solving literature. A comparison of the distributions of some of the teaching strategies for teachers and lecturers is presented here.

The strategies that lecturers rated as being appropriate for regular use are recommended in advice given to teachers about problem solving in the reformed classroom as well as in curriculum documentation. These included whole-class discussion for sharing solutions and strategies, small, cooperative group discussion, provision of concrete materials and calculators, recognition of the need to encourage individual student recording of methods and procedures, and encouragement of student posed problems as well as using problems that relate to student interests.

A comparison of the data from surveyed teachers with that from mathematics education lecturers was achieved by examining box and whisker plots for each of the teaching strategies as well as comparing the means for each distribution. The responses for both teachers and mathematics education lecturers were scored from zero (0) for "hardly ever" to three (3) for "almost always". This method was employed so that means could be calculated for each item and the responses of each group compared.

It is acknowledged that the differences between categories are not equivalent and therefore this process is used as a gross measure for comparison purposes only. The analysis enabled the construction of lists of rarely used teaching strategies and more frequently used teaching strategies for each group. Figure 21 presents box and whisker plots comparing the responses of both groups for those strategies where the means were *low*, or < 1.1. Also, Figure 22 presents similar plots comparing the responses for those strategies where the means were *high*, or > 1.9.

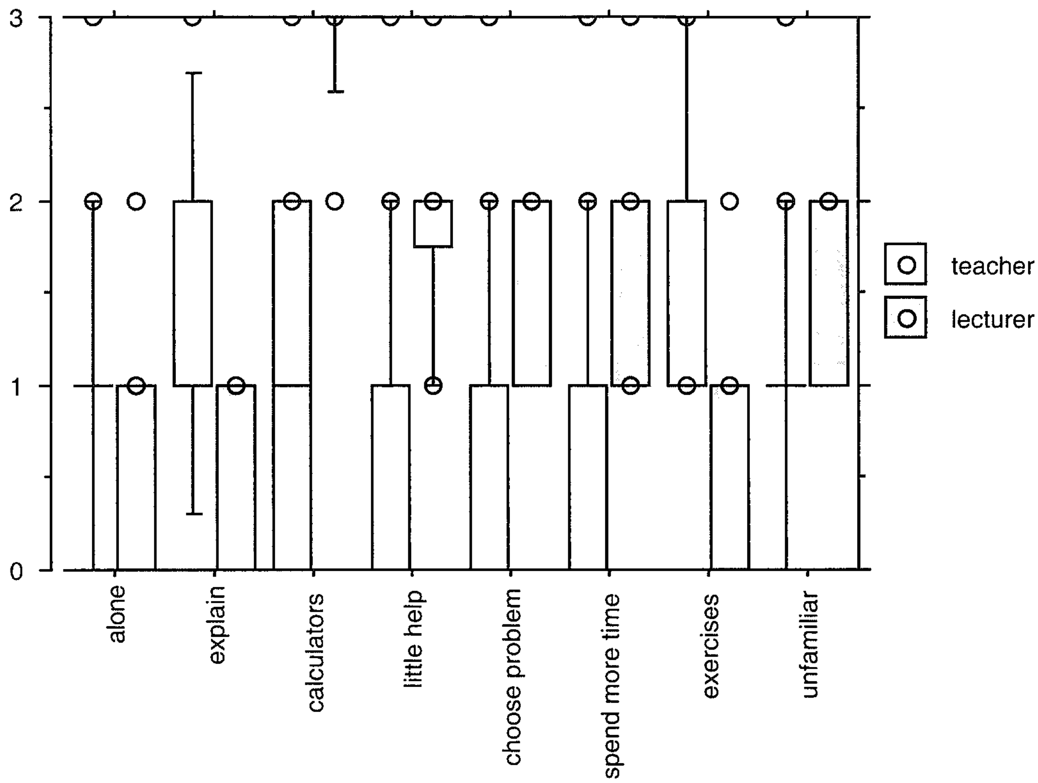


Figure 21. Box plots of items with *low* means for teachers and lecturers.

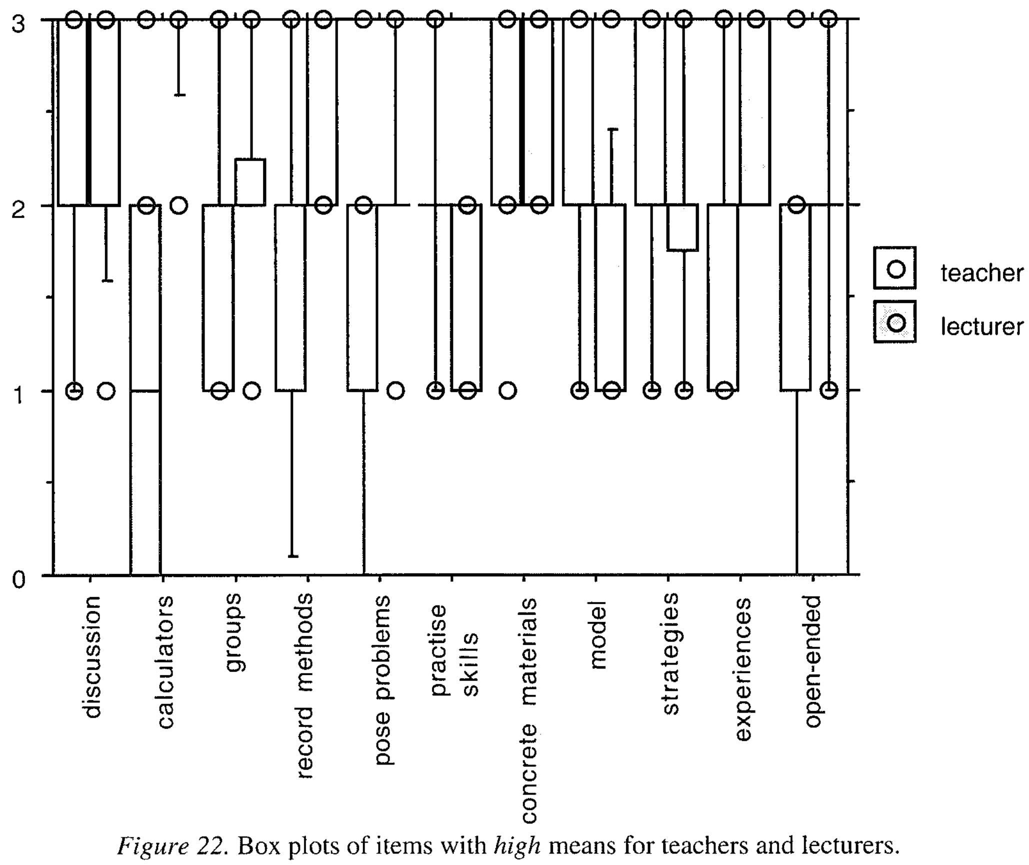


Figure 22. Box plots of items with *high* means for teachers and lecturers.

There appeared to be a large difference between lecturers and teachers in the distribution of responses for the use of calculators, exercises and the notion of presenting problems to students to explore with little indication of how to solve them (little help). Teachers reported that they rarely use calculators and frequently set exercises; a contrast to the advice that seems to be presented in curriculum documentation.

In addition, teachers reported a high frequency of use of modelling the problem-solving process to the class and setting application problems for skills practice. Encouraging students to record their own methods was reportedly used less frequently than lecturers believed to be desirable in mathematics classrooms. Encouraging students to pose their own problems was reported as a rarely used strategy by teachers as was the use of open-ended problems and problems relating to students' experiences.

It is clear from these results that most teachers see the value of whole-class discussion, teacher modelling and using concrete materials in classrooms. Teachers do not seem to have embraced the view that calculators can be an integral part of the primary mathematics classroom. Also, there is less frequent use of individual student methods of recording and student created problems.

A final comparison can be made between teachers and lecturers regarding the use of particular student question types. It was reported in Section 4.3.1.3 that teachers prefer to use application problems and exercises more frequently than either open-ended or unfamiliar problems. It should be noted that the prompts for each group were slightly different. Teachers were required to indicate how frequently they use each of the strategies whereas lecturers were asked to indicate how frequently teachers should use each of the strategies according to the problem-solving literature. These data were compared with those from the lecturers and are presented in Table 64.

Table 64
Frequencies of the teaching strategies for lecturers (and teachers) that relate to student question types in Question 4 - all values are percentages

Teaching Strategy	Hardly Ever	Sometimes	Often	Almost Always
teachers (you) present *application problems* which allow students to practise the skills they have just learnt	0 (0)	71 (21)	29 (65)	0 (14)
teachers (you) pose *open-ended problems* to allow students to explore mathematical situations for themselves	0 (11)	19 (54)	67 (28)	14 (7)
teachers (you) set *exercises* to allow the students to practise their skills	29 (3)	67 (26)	4 (56)	0 (15)
teachers (you) pose *unfamiliar problems*	5 (21)	33 (55)	62 (23)	0 (1)

In response to these four items, lecturers recommended a higher frequency of use of open-ended and unfamiliar problems than teachers report using. Eighty-one percent of lecturers suggested that open-ended problems should be used *often* or *almost always*. In contrast, only 35% of teachers reported that they *often* or *almost always* use open-ended problems. Also, 62% of lecturers recommended that unfamiliar problems should be used *often* while 24% of teachers reported that they *often* or *almost always* pose this type of student question.

Lecturers also recommended a lower frequency of use of application problems and a much lower frequency of use of exercises. Seventy-one percent of lecturers suggested that application problems should be used *sometimes* which contrasts with 21% of teachers. Also, 96% of lecturers recommended that exercises should be used *hardly ever* or only *sometimes* whereas 29% of teachers reported that they set exercises *hardly ever* or *sometimes*.

These differences between teachers and lecturers raise several questions about the discrepancies between their responses. Taking the lecturers' responses as a reflection of current advice, it appears that teachers either do not agree, or do not implement that advice. This is a critical aspect of this investigation. There are potentially many reasons for this including beliefs that are not congruent with these approaches, lack of knowledge and experience in using such approaches, or constraints that operate in particular school settings that might militate against implementation. Alternatively, these approaches may be too difficult to implement thus suggesting that lecturers may not be in touch with the practical aspects of classroom and school life.

To investigate these possibilities, and to reveal others, teachers were asked to provide reasons for their use of particular question types in Question 6 on the questionnaire. The responses made by teachers are discussed in the next section.

4.4 Reasons for Use of Particular Student Question Types

Several items on the questionnaire were designed to explore teachers' use of particular student question types and the reasons for those choices. Four student question types were described in the Background Information at the beginning of the questionnaire and these were then referred to throughout. Question 5 asked teachers "how often do you use each of these different types of questions in your teaching?" followed by Question 6 which stated "from your responses to the above question, briefly describe why you prefer to use those particular types of problems".

The data from Question 5 were presented in Section 4.3.1.3 but are reproduced here to enable discussion of the findings for Question 6. The data in Table 65 indicate that 68% of teacher respondents *often* used exercises and 70% *often* used application problems in their teaching of mathematics.

Table 65
Frequency of use of student question types from Question 5 (%), n=162

Types of Questions	Rarely	Sometimes	Often
Exercises	5	27	68
Open-ended Problems	22	58	20
Application Problems	4	26	70
Unfamiliar Problems	37	52	11

Also, a total of 80% of the surveyed teachers only *rarely* or *sometimes* used open-ended problems and 89% only *rarely* or *sometimes* use unfamiliar problems. As has already been discussed, these results were not consistent with mathematics education lecturers' interpretation of advice provided in the problem-solving literature. Lecturers' responses suggested that they believed that teachers should more frequently use open-ended and unfamiliar problems and they should less frequently use exercises and application problems.

It was anticipated that the data from Question 6 would provide information about several important aspects of this investigation. On the one hand, it was of interest to ascertain why exercises and application problems are considered by teachers to be the most appropriate student question types for regular classroom use. However, on the other hand, it was of interest to discover why some teachers have responded to the advice since 20% report that they *often* use open-ended problems and 11% *often* use unfamiliar problems. In addition, this investigation aimed to explore what issues or aspects of teachers' daily lives impact on their decision making in relation to the role of problem solving in the teaching and learning of mathematics. Teachers' responses to Question 6 were analysed to investigate these issues.

A variety of types of responses were received from the teachers who completed the questionnaire. Some teachers' comments focused on reasons for their selection of the most frequently used student question type. Others made comments that related to their reported frequency of use of each of the four student question types. Another group made general statements about the relevance of problem solving for their current class while nineteen of the 162 surveyed teachers made no comments about their selection of particular types of questions.

Analyses of data from this open-ended question were performed in two ways. First, teachers' comments were grouped according to their relevance to each of the four student question types. Second, responses to the first 25 questionnaires received were read and comments categorised according to the main issues discussed. As a result of this process, several categories emerged and these were then used to organise the remainder of the data. A summary of the main findings from each of these methods of analysis is presented here in separate sections.

4.4.1 Comments Relating to Each of the Student Question Types

Each of the responses to Question 6 was read and the comments were summarised according to their relevance to each of the four student question types. These question types were those presented in the Background Information and included exercises, application problems, open-ended problems, and unfamiliar problems. A summary of the main comments for each question type is presented in the following sections.

4.4.1.1 Exercises

It was clear from the results to Question 5 that many teachers use exercises with their students on a regular basis. From the data in Table 65, 68% of teachers *often* use exercises, 27% *sometimes* present exercises to their students, and only 5% *rarely* use exercises. Many of the teachers' comments referred to the use of exercises and as most teachers reported that they often use both exercises *and* application problems, their comments appeared to refer to both of these student question types. This section summarises the main comments made about the use of exercises.

Seventy-three, or 45%, of respondents made comments about their use of exercises in mathematics lessons. These comments were broadly grouped into nine categories. The first included comments about the role of exercises as a basic skill, for practise or as a building block to do other problems. The next category included comments about the ability of the students that incorporated remarks about poor language skills. The remaining categories included comments about affective factors such as feelings of success or anxiety, accessibility of exercises, the age or learning stage of the students, confidence of the teacher, curriculum requirements, for assessment purposes, and parents' expectations.

Table 66 indicates the number of comments made under each of these headings and provides an example of each. Note that there are more than 73 comments since some teachers made comments that related to more than one of the categories in the table.

Table 66
Categories of comments relating to the use of exercises with the number of teachers' who made comments about each category and an example, n=73

Category	Number of Teachers	Example of Comments
basic skill or practice	31	I prefer for children to have the basic skills of calculations
ability of students	27	I have the lowest group for maths and have to keep lessons fairly simple
affective factors	10	I try to ensure they have more success than frustration in maths lessons
ready accessibility of exercises	8	access to material and time to organise
age or stage of students	6	more appropriate to level of class and also to their age
confidence of the teacher	4	probably because that is what I am familiar with
curriculum requirements	3	treated as part of Syllabus where appropriate to grade requirements
assessment	1	easy to see if child has mastered desired skill
parents' expectations	1	parent expectations

This summary table provides useful data about the reasons teachers have provided for their use of exercises in mathematics lessons. Several of the reasons related to the students in the teacher's class with many comments beginning with a reference to her current class or mathematics group. For many of the surveyed teachers, exercises were used to practise skills and concepts to support student learning. One reaction is worth noting. A male teacher of an upper primary class who has been teaching for ten or more years reported that he is "obsessed with drills (tables etc)" although he indicated that he uses all other types of questions on a regular basis.

It was also reported that exercises were often chosen because of the ability of the students. It became clear when reading teachers' comments that they believed that exercises were most appropriate for lower ability students or for students who had poor language skills since exercises usually contain few words. Another consideration when choosing question types was the age or learning stage of the students in the class. Such comments were usually made by lower grade teachers who indicated that children in the beginning years of primary school needed to develop basic skills and concepts before they could do other types of problems. Several teachers commented on the need to provide success in mathematics lessons and exercises were considered to be more likely to provide that success.

Further considerations related to factors other than the students. One concern when planning was the need to find readily available material with several teachers suggesting that little time was available to search for mathematics questions. Another concern was the need to meet curriculum requirements. Teacher confidence was mentioned by a few teachers, with assessment issues referred to by one teacher, and parents' expectations also noted by one respondent.

An interesting comment was made by a teacher of an upper primary grade class who had been teaching for ten or more years that encapsulates many of the categories listed above. She reported that she often uses exercises, sometimes uses open-ended problems and application problems, and rarely uses unfamiliar problems. Her response to Question 6 was

> It's safer - children feel more comfortable if they're not made to think. I realise this is cynical - but for many children with low IQs and poor/non existent English language skills, the concept of problem solving is alien. Also it takes up too much time and there is great pressure to "get through" the curriculum. So whilst in theory I acknowledge the potential of problem solving, in reality with some clientele it's too hard.

This summary of teachers' comments highlights many issues that relate to the selection and frequency of use of exercises in mathematics lessons. Comments indicated that teachers believed that exercises provided practice in basic skills and procedures, particularly for lower ability students and for children in lower grades. Also, exercises were part of the curriculum, could be used to assess understanding, and enabled children to experience success. Teachers felt confident using exercises in their teaching and were able to readily access sets of exercises for student use thus saving valuable preparation time. Several of the comments about exercises related to the use of application problems as well but there were some comments that specifically referred to this type of mathematics question. This may contribute to an explanation for the apparent lack of adoption of problem solving in classrooms.

4.4.1.2 Application Problems

The reported frequency of use of application problems was similar to that for exercises. The data contained in Table 65 indicated that 70% of surveyed teachers *often* presented application problems to their class. Twenty-six percent *sometimes* used application problems and only 4% *rarely* used them. As a consequence, many of the teachers who regularly used exercises also regularly used application problems and as was mentioned earlier, many comments in the previous section also referred to application problems. This section summarises comments that were specifically made about application problems.

Thirty, or 19%, of respondents made comments about their use of application problems. These comments were broadly categorised into applications or real-life contexts, language issues, age of students, thinking skills, affective factors, teaching factors, and ability of the students. Table 67 indicates the number of teachers who made comments classified into each of these categories and provides an example of each. Again, there are more than 30 comments in the table since some teachers made comments related to more than one category.

Table 67
Categories of comments relating to the use of application problems with the number of teachers' who made comments about each category and an example, n=30

Category	Number of Teachers	Example of Comments
application or real-life contexts	15	apply basic skills to real-life situations
language issues	5	application problems focus on language as well as concepts - good for ESL children
age of students	4	the age of the children - Year 1
thinking strategies	4	it makes the children think more
affective factors	3	kids more excited, on task
teaching factors	2	application problems can be easily discussed and worked on with concrete materials
ability of students	1	application problems are more appropriate to my class as they can use known strategies

The main reason given for using application problems was the use of real-life contexts to show how mathematics can relate to students' experiences. An issue that was mentioned by quite a few teachers was the language difficulty that many students experienced when trying to interpret such problems. Interestingly, two teachers mentioned that they used such problems as a means of helping students come to terms with language use in mathematics. In common with the comments about the use of exercises, teachers also mentioned the age and ability of students as well as affective factors.

In contrast to the relatively common use of exercises and application problems, few teachers reported that they regularly use either open-ended or unfamiliar problems. Teachers' comments about these two problem types are presented in the next two sections.

4.4.1.3 Open-Ended Problems

According to the data in Table 65, 20% of respondents *often* use open-ended problems in their mathematics teaching. Fifty-eight percent of surveyed teachers *sometimes* use open-ended problems and 22% *rarely* use them. Most of the teachers' comments indicated why they do not use this type of student question in mathematics lessons.

Comments about the use of open-ended problems were made by 23 teachers and these were grouped into several categories. These included the ability of students, factors relating to learning or thinking, affective factors, the age of students, the accessibility of questions, variety of appropriate questions, confidence of the teacher, and the need to prepare students for mathematics competitions. Table 68 indicates the number of teachers who made comments classified into each of these categories and provides an example of each.

Table 68
Categories of comments relating to the use of open-ended problems with the number of teachers' who made comments about each category and an example, n=23

Category	Number of Teachers	Example of Comments
ability of students	10	open-ended used more so for GATS (gifted and talented) children.
learning/thinking factors	6	open-ended problems allow children to bring their own knowledge and strategies to the task as well as respond at their own level
affective factors	4	they experience frustration with open-ended problems. I try to ensure they have more success than frustration in maths lessons
age of students	3	tend to use more open-ended problems with older students
accessibility of questions	2	these types are not often in my math text that I use with my class - so occasionally I make some up
variety of questions	1	I like to vary what I use.
teacher confidence	1	feeling a little bit out of depth with some open-ended problem types even though I think that they have merit in the overall maths program.
preparation for competitions	1	we enter all competitions and do very well. The open type are common in competitions.

Teachers' comments indicated that open-ended problems were considered to be challenging and were therefore suitable for more able students or for the "advanced maths group". They were also considered to be more appropriate to extend the knowledge and experiences of older grade students, for use in cooperative groups, and to develop higher levels of thinking. Interestingly, three teachers commented that open-ended questions were suitable for *all* students since they can work at their own ability level to achieve at least some of the outcomes of the question. It was also reported that these questions were used for variety in lessons although they were not readily available in textbooks and teacher resource books.

4.4.1.4 Unfamiliar Problems

Fewer teachers reported using unfamiliar problems compared with open-ended problems. Table 65 indicates that only 11% of respondents *often* use unfamiliar problems while 52% *sometimes* use them and 37% *rarely* use them. Thirty teachers made comments that related to the use of unfamiliar problems. These comments were grouped into several categories including those that related to the ability of the students, learning and thinking factors, affective factors, the age of the students, factors relating to the language involved in the question, the confidence of the teacher, and the accessibility of such questions. Table 69 indicates the number of teachers who made comments classified into each of these categories and provides an example of each.

Table 69
Categories of comments relating to the use of unfamiliar problems with the number of teachers' who made comments about each category and an example, n=30

Category	Number of Teachers	Example of Comments
ability of students	12	extends the bright kids.
learning/thinking factors	10	provides an extension by calling for more lateral thinking skills
affective factors	9	children tend to give up, don't persevere to find answers
age of students	4	used more often when I've taught Years 5/6.
language factors	4	their reading abilities have an effect on a lot of students in terms of these questions.
teacher confidence	3	as a teacher, I'm not always sure how to explain how to solve these questions.
accessibility of questions	1	I do find they take a lot of work and ideas are hard to come by.

Reasons given for rare use of unfamiliar problems included the notion that these problems were challenging and therefore only suitable for the most able or gifted and talented students. Also, these questions were linguistically challenging because of the language involved and required perseverance that can often lead to frustration. In addition to these factors, three teachers mentioned their own lack of experience or confidence in using such problems.

For those teachers who reported using unfamiliar problems on a frequent basis, reasons included the desire to develop students' higher level and lateral thinking skills, to challenge and motivate more capable students, and the need to teach new problem-solving strategies. These teachers were usually teachers of upper primary grades or those who reported that they were responsible for the more able students in the grade.

4.4.1.5 General Comments About All Question Types

There were a group of 17 teachers who reported *sometimes* or *rarely* using all four question types and another group of 21 teachers who reported that they *often* or *sometimes* use all four question types. Each of these groups made general comments about the reasons for their selections and these are discussed in this section.

Reasons for Rare Use of All Question Types

Comments made by respondents who indicated that they *rarely* or only *sometimes* use each of the four question types related to the need to use a variety of questions, that their students had diverse abilities, or that they were teaching in the lower primary grades. One teacher suggested that it was important to use questions the students could confidently handle and several others reported the need to use "hands on" experiences with their students.

The following comment reflects the main ideas mentioned by this group of respondents. This female teacher reported that she was teaching in the Kindergarten to Year 2 grades and had been teaching for ten or more years.

> I feel children need a variety of questions to cater for different needs and styles. Being able to do exercises has little use if this knowledge cannot be applied to practical situations. The basics must be used and strengthened to gain a more complete understanding.

Reasons for Frequent Use of All Question Types

For those teachers who reported using all question types frequently, comments focused on the need for variety to cover the various problem-solving strategies and to provide an overall experience for students. It was also suggested that by providing a variety of questions, teachers were more likely to meet all students' needs and interests. Several respondents indicated that even though they frequently used all types of questions, the choice does depend on the students' abilities.

A small number of respondents mentioned specific purposes for the use of each type of question. It was suggested that exercises provide practice and are useful to start lessons, application problems can be used to apply skills and procedures that have been learned, and open-ended and unfamiliar problems provide opportunities for students to apply knowledge and are particularly useful for fast workers. One teacher suggested that a variety of question types was important as this

> ... allows the children to explore the variety of different answers. It encourages them to "figure things out" or find several solutions to one problem rather than just rote learning a procedure. It is much more relevant to their lives.

The need to be able to apply knowledge in a variety of problem contexts and the need to have a breadth of mathematical experiences were reported by teachers as critical elements in mathematics learning. The following two comments reflect these ideas. The first was made by an experienced female teacher of upper primary. She stated that

> Children need knowledge of and practise in the mechanics of maths. However, it is the opportunity and ability to transfer and use this knowledge in a wide variety of situations that gives maths its relevance and enjoyment. Some children will need more practice on basic exercises than others but all need the opportunity to explore maths fully.

The second comment was made by another female teacher of upper primary who had been teaching for five to nine years. She stated

I like to use a variety to add interest (sic) and cater for individual children's needs and interests. I believe all of the above strategies have a place in learning to become a competent and confident mathematician.

In summary, the analysis of the teachers' comments regarding reasons for choosing each of the four question types reveals several themes that seem to influence teachers' choice of problems. These themes are evident by examining the categories presented in Tables 66 to 69. To reveal similarities in reasons, the categories that appeared in more than one of these tables are reproduced in Table 70.

Table 70
Common categories of reasons for choice of each of exercises, application problems, open-ended problems and unfamiliar problems

Categories for Exercises	Categories for Application Problems	Categories for Open-Ended Problems	Categories for Unfamiliar Problems
ability of students	ability of students	ability of students	ability of students
age of students	age of students	age of students	age of students
affective factors	affective factors	affective factors	affective factors
	language issues		language issues
	thinking strategies	learning/thinking factors	learning/thinking factors
teacher confidence		teacher confidence	teacher confidence
accessibility		accessibility	accessibility

The common categories presented in Table 70 reveal issues or themes that determine teachers' choice of particular student question types. These issues relate to considerations of the students in relation to learning, ability, age and affective factors, characteristics of the question types including language and the promotion of student thinking, confidence of the teacher, and the availability of suitable resource materials that provide ready access to each of these student question types. These themes also emerged in the second approach to analysing Question 6. The findings from this analysis are discussed in the following section.

4.4.2 Issues Relating to the Use of Particular Student Question Types

The first 25 questionnaire responses to Question 6 were read to identify common themes or issues that teachers mentioned as determinants of frequency of use for the four student question types. Initially, the categories of students, questions, teachers, school and other were used to sort and organise teachers' comments. A further examination of responses led to a categorisation that included the five main areas of students, school, planning, teacher and question characteristics. Each of these main areas was further subdivided as indicated in Figure 23.

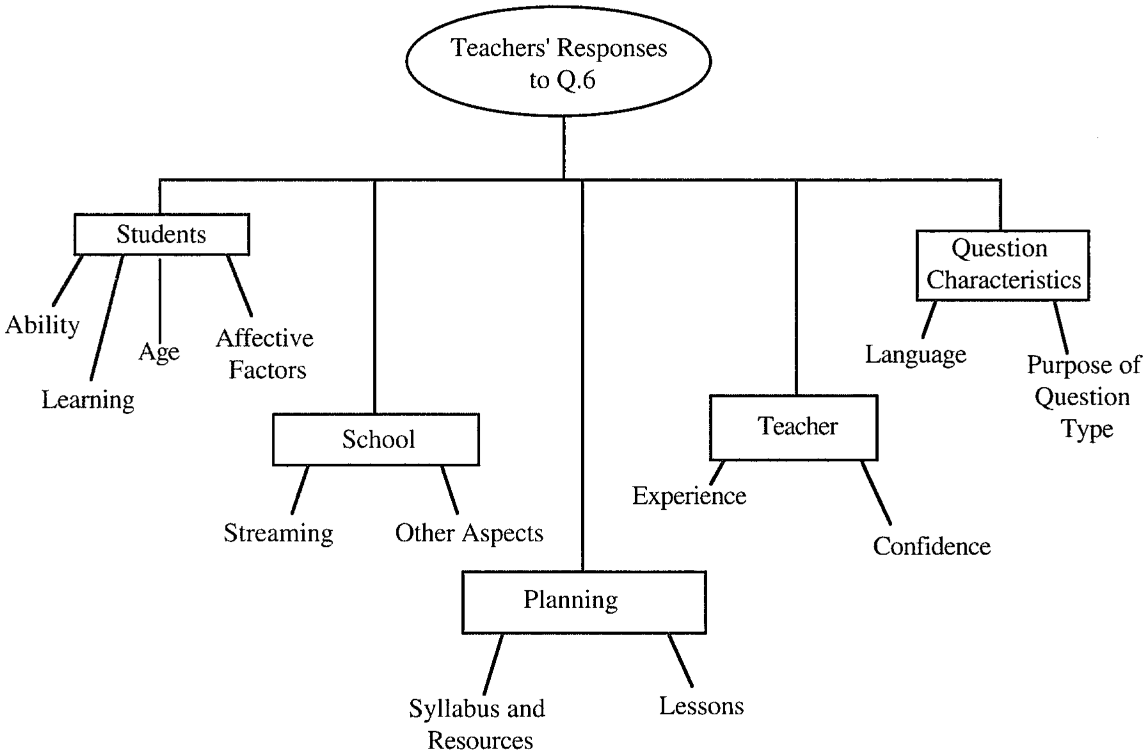


Figure 23. Classification of teachers' comments about reasons for use of particular student question types.

This classification enabled teachers' comments to be sorted into five broad categories with each subdivided into several focus areas. Comments about the students were organised into four categories of ability, aspects of learning, age and affective factors. School comments included those about streaming or ability grouping and other aspects of schools including assessment procedures, mathematics competition preparation, and parents' expectations. References to the syllabus, textbooks or other sources of problems, and the planning of lessons were grouped under the category of planning. Comments about the teacher usually related to experience or confidence. Finally, a large number of comments related to question characteristics including language and purpose of question types. Teachers' comments relating to each of these five broad categories are discussed in the following sections.

4.4.2.1 Comments About Students

A large number of reasons for use of particular question types in mathematics lessons related to the students that were being taught. These comments were grouped into four categories relating to the ability of the students, aspects of learning, students' ages, and affective factors such as confidence, enjoyment or frustration. Teachers' comments about each of these are discussed in the following sections.

Students' Ability

Fifty-seven, or 35%, of teacher respondents commented on the notion that the ability of students determines choice of particular question type. Many of these comments indicated that the structure of lessons and the type of questions used in mathematics lessons depends on the ability of students in the class. It was apparent from the comments that many teachers believed that it was important to develop students' knowledge of basic concepts and applications of these ideas before they considered using other types of problems. This confirms findings from the analyses of Questions 1 and 2 where 79% of teachers agreed that students should learn basic number facts before doing application and unfamiliar problems.

In teachers' responses, a clear distinction was made between the use of exercises and application problems compared with the use of unfamiliar and open-ended problems. It was suggested by teachers that exercises and application problems were more suitable for lower ability students as these students needed more practice on basics skills and responded better to more structured lessons with drill and practical application of mathematical ideas. Unfamiliar and open-ended problems were viewed as appropriate for more able students and were used for extension and to promote lateral thinking.

The following comment reflects many of the responses that related to students' abilities. This comment was made by an experienced female teacher of upper primary. She suggested that

> The composition of the group determines the approach. Children who do not have sound concepts and lack an understanding of number processes need a more basic approach first, then their knowledge can be channelled.

Aspects of Learning

Forty-eight, or 30%, of respondents made comments about using mathematics questions that promoted learning by focusing on understanding, thinking and applying knowledge. Mention was also made of the need for students to learn problem-solving strategies, develop skills, reinforce concepts, and to learn the language of mathematics. One teacher suggested the need to train students while two others wanted to develop speed and retention. Table 71 indicates the number of teachers who made comments relating to each of these aspects of student learning in mathematics lessons.

Table 71
Aspects of learning with the number of teachers who made comments about each aspect and an example, n=48

Aspect of Learning	Number of Teachers	Example of Comment
developing skills	13	basic skills are needed in maths
applying knowledge	8	to be able to apply that knowledge in contexts that are meaningful to them
understanding processes	7	to reinforce understanding of the 4 operations
thinking	7	it makes the children think more
needing problem-solving strategies	5	to develop strategies for solving new problems on their own
reinforcing concepts	4	to help reinforce concepts
learning the language of mathematics	2	unless children are taught the language of mathematics the process of completing an unfamiliar problem is too difficult
developing speed and retention	2	doing work with the development of speed and retention of facts
training	1	I prefer to train the children using a structured approach first

The data in this table indicate that teachers were concerned with the need to focus on basic skills, reinforce concepts, and apply this knowledge in problem contexts. Some respondents also recognised the need to develop understanding and thinking as well as to learn strategies to assist problem solving. According to many of the surveyed teachers, the student question types used in the classroom were chosen for particular purposes. This is a key result for this investigation. It is clear that teachers were concerned about using questions in mathematics lessons that promoted learning and they seemed to believe that particular aspects of the curriculum could be addressed by using different question types.

One teacher considered each of the question types and their potential role in the learning process. This new female teacher of lower primary grades suggested that

> My students complete exercises in the form of drill questions at the start of each maths lesson to develop and reinforce skills being studied. I prefer the core of the lesson to focus on open-ended and application problems to encourage students to apply their skills and develop and broaden their operational understandings. I am just beginning to introduce unfamiliar problems to broaden mathematical understandings, but students are still getting used to open-ended and applications as they have only ever encountered exercises in their schooling till this year.

This comment suggests that one constraint on teachers use of some question types is the previous experiences of the students.

Students' Age

Twenty-six, or 16%, of teachers suggested that their decision making is determined by the age or grade level of their students. Comments indicated that many teachers believed that younger children should experience more application problems whereas older students were more likely to be given open-ended and unfamiliar problems. It was suggested that older students were more able to think abstractly and solve such questions. These comments were usually tempered with suggestions that ability would also impact on the decision making, particularly for lower ability upper primary students.

Affective Factors

A total of 29 or 18% of respondents made comments about affective aspects of learning and problem solving in mathematics. Some comments related to negative feelings such as frustration, anxiety and the tendency of students to give up and not persevere when solving problems. Most comments were about positive aspects of problem solving and related to the need to develop students' confidence and to provide experiences that are less threatening. Several teachers wanted to challenge students but commented on the need to ensure success and enjoyment while problem solving. Two teachers suggested that it was important for students to value mathematics and to appreciate its relevance in their everyday lives.

These affective factors are encapsulated in the comment made by an experienced female teacher of lower primary grades. She stated

> Exercises, application problems and open-ended problems allow the child to experience success and a sense of achievement when they are just beginning to learn about number, it is still concrete and they still enjoy maths!

It is clear from the teachers' comments about students that this is a vital and important consideration when planning to teach mathematics. A combination of factors seemed to impact on teachers' decisions about choice of particular question types and some of these may act as constraints on their use of problems in mathematics lessons. It seemed that students' ability and age were critical factors in that lower ability students, as well as younger students, were more likely to be presented with exercises and application problems and were less likely to be presented with open-ended and application problems. In addition, teachers did not want students to experience frustration or anxiety and they generally wanted to ensure success and positive responses to problem solving.

4.4.2.2 Comments About the School

A small number of respondents made comments that were categorised under school aspects. These included comments about streaming or ability grouping, and other issues related to school including assessment, participation in mathematics competitions, and parents' expectations. These are described in the following sections.

Streaming or Ability Grouping

Ten teachers commented about the ability of their mathematics group. These comments indicated that streaming was used to organise mathematics classes in at least some schools. It was suggested by respondents that in these circumstances, decisions about the use of particular types of mathematics questions tended to be made according to the ability group allocated to each teacher.

This is evidenced by the comment made by an experienced female administrator. She stated that

> The group of students I take for Maths this year are well below grade average, have little ready recall of basic facts and/or mathematical concepts. Language development is also below average. We do a lot of practical activities in real-life situations as well as reinforcing basic facts and skills and becoming accurate in using a calculator.

These comments reinforced those made in Section 4.4.2.1 about the impact of the ability of students on choice of problems.

Other Aspects of Schools

Three responses included other aspects of schools. One teacher commented on the impact on her decision making of the curriculum with formal examinations twice each year. Another respondent indicated that the choice of problem types was also prompted by the school's participation in mathematics competitions. A final respondent indicated that her choice of regular use of exercises was influenced by parents' expectations. Clearly there are many influences on teachers' choice of problems.

4.4.2.3 Comments About Planning

Several comments were made about the planning of teaching experiences. This planning was clearly influenced by the syllabus, textbooks and other resource materials as well as decisions about lesson structure. Comments about each of these influences are described in the following sections.

References to the Syllabus, Textbooks and Other Resources

Two teachers indicated that the syllabus influenced their choice of student question types. It was suggested that the syllabus document recommends particular types of questions to be used with certain topics and grade levels. Five respondents made comments about the types of student questions found in the textbook or about the availability of such question types in resource materials. Teachers indicated that exercises and application problems were more commonly found in textbooks which meant that open-ended and unfamiliar problems needed to be sought from other sources. One teacher suggested that exercises and application problems were easier to construct. All of these responses seemed to suggest that teachers' decisions can be constrained by time to find suitable materials, an aspect that impacted on planning of lessons.

Planning of Lessons

Twenty-seven teachers mentioned lesson structure and planning implications as an influence on their choice of question types. Fifteen respondents suggested that questions were chosen to provide variety in lessons and the need to cater for individual learning styles. Three teachers discussed the need for hands-on approaches and two others indicated that questions were chosen so that group work could be used. One indicated that the content focus determined choice of question type and another described her need to plan lessons according to the class she was teaching at the time. Two teachers commented about the lack of time for proper planning and that this influenced what approach was adopted.

The following comment about the teaching approach of an experienced female teacher of lower primary indicated that her decisions were based on her belief about how children learn at this stage. It should be noted that this teacher sometimes used application problems but all other question types were used rarely. She stated

> Use hands on resources only. Children orally tell the class what they learned, demonstrating with resources. Children encouraged to record some of their learning. Children use the chalkboard and individual book. I agree with Constance Kamii that worksheets impede mathematical development in the early years of school. I have Kinder (5 years) telling me the cubic centimetre volume of empty muesli bar boxes and how they worked it out.

Another respondent indicated that issues of time, resource availability and the need to prepare students for high school determined his approach. This male teacher with moderate experience was teaching the upper primary grades. He stated

> Exercises and application problems are frequently used because they require least preparation are easy to mark, report on and compare. Not that I'm lazy, just busy (as all teachers are) and as an upper primary teacher I feel as though I have to prepare students for high school realities. I enjoy, as do my students using unfamiliar, open-ended, long problems. I do find they take a lot of work and ideas are hard to come by.

These reactions highlight important concerns for teachers as they prepare lessons for students. It is clear that the availability of exercises and application problems makes them easier to use on a regular basis. Even though teachers seemed to recognise the usefulness of open-ended and unfamiliar problems, their use significantly impacted on planning time. In this sense an important constraint on their use was the time available for preparation and the accessibility of suitable resource materials. This may have contributed to an explanation for the apparent lack of adoption of problem solving in classrooms.

4.4.2.4 Comments About the Teacher

Nine respondents made comments about themselves in relation to the choice of question type. These comments were grouped in two areas of experience and confidence and are discussed in the following sections.

Experience

Four teachers indicated that they were more inclined to use questions with which they were familiar. Interestingly, three of these respondents suggested that perhaps they should use other types but they were not familiar with how to use them. This confirms the issue of teachers' knowledge as a constraining factor.

Confidence

Another four respondents made comments suggesting that certain types of questions make them feel less confident in the classroom. These comments also suggested that teachers were not sure how they might use some of these question types with their students. One moderately experienced female teacher of middle primary grades stated that "I would like to have more inservice on problem solving and divergent thinking so that I can change the type of questioning I use".

The following comment reflected the sentiments expressed by this group. This experienced male teacher of upper primary grades stated

> 1. Personal familiarity with exercises and application problems and the ready availability of resources related to these.
> 2. Feeling a little bit 'out of depth' with some open-ended problem types even though I think that they have merit in the overall maths program.

This reaction is further confirmation of the impact of knowledge and experience as potential constraining factors on the use of a variety of problem types in mathematics lessons.

4.4.2.5 Comments About Question Characteristics

Several comments related to the particular question types and their characteristics. These have been grouped into comments about language and purpose of question type and are discussed in the following sections.

Language

Ten respondents indicated that the language contained in particular question types made them more difficult for some students to solve. Some of this group of teachers suggested that they deliberately chose to use problems with language so that the students learnt to understand and interpret mathematical language.

The following statement by an experienced female teacher of middle primary grades is indicative of the responses from this group of teachers. This comment indicated her concern about the language contained in some problems. She stated

> Language can often be a problem for our children, so lots of words often cloud the concept for them. We need to fully develop their ability to perform the skill then expand into more problem solving areas.

Purpose of the Question Type

Twenty-two respondents made specific comments about the purpose of each of the four question types. These have been discussed in detail in the previous analysis of Question 6 but in summary it was suggested by teachers that exercises were used to practise skills, application problems provided opportunities for students to apply their knowledge in real-life contexts, open-ended and unfamiliar problems were usually used to challenge and extend more able students. Several of the teachers in this group indicated that all types have their place in the teaching and learning of mathematics.

It was suggested earlier that differences between teachers' reported use and lecturers' recommended use of particular student question types may be a result of teachers' beliefs, teachers' lack of knowledge or experience, or constraints impacting on potential implementation. Teachers' responses indicated that they have quite powerful reasons for choosing to use particular question types. These reasons are based on many factors including students' characteristics such as age, ability, learning needs, and grade level as well as school organisation including streaming of students.

In addition, teachers seemed to have identified different purposes for each of the question types based on curriculum requirements and question characteristics. Teachers reported that exercises were necessary as they provided valuable practice in basic skills and procedures. Application problems enabled students to apply these basic skills and procedures in real-life contexts. Open-ended problems provided challenge, developed higher level thinking and allowed students to respond at their own level. Unfamiliar problems challenged and motivated more able students and provided opportunities for students to apply problem-solving strategies. The general view was that all problems were valuable but choice was made on the basis of accessibility so that students felt they could attempt the question.

These data also confirmed that there were constraints that impacted on teachers' use of these questions, particularly the use of open-ended and unfamiliar problems. These included age, ability and experiences of the students, but also, the time required to find suitable problems and resource materials as well as teachers' confidence, knowledge and previous experiences. Some mention was also made of formal assessment procedures in particular schools. An explanation of this for the proposed model is considered in Chapter 6.

4.5 Recently Used Problems and Their Purposes

It was anticipated that asking teachers to describe a recently used problem and its purpose would reveal useful data about the types of problems that teachers choose to use and the reasons those problems were chosen. In the questionnaire, Question 3 asked teachers to provide "an example of a mathematical problem you used recently in your class and describe what you used it for".

Of the 162 survey responses, 111 teachers, or 69% of respondents recorded a problem that they had recently used. Few of these teachers provided a purpose for the use of their problem. It is of some concern that 51 teachers, or 31%, did not record a problem. On one hand it may suggest that these teachers had not recently used a problem with their class. On the other hand, it was noted when scanning the questionnaires that several respondents had answered none of the open-ended questions, possibly because of lack of available time.

Responses to Question 3 were analysed in several ways. First, the problems were classified according to the syllabus content strand on which the problem focused. Second, the problems were categorised into particular student question types. Finally, teachers' recorded purposes were examined. These are each discussed in the following sections.

4.5.1 Classifying the Problems into Syllabus Content Strands

The first level of analysis was to classify each problem according to the content strand contained in the current K-6 mathematics curriculum in NSW (NSWDE, 1989). These strands are Number, Space and Measurement. Of the 111 problems provided by teachers, 109 could be placed into one of the three content strands.

The remaining two problems were too general to be able to determine the content focus of the question. An example of a problem that could not be classified was provided by an experienced male teacher of upper primary grades. He wrote that he used "mainly application problems of differing complexity".

Some problems contained a combination of content from two strands but were placed in the strand that was judged to be the major focus of the problem. Table 72 presents the number of problems classified as Number, Space and Measurement questions with an example of each.

Table 72
Number of problems that related to each of the content strands of Number, Space and Measurement with an example of each

Syllabus Strand	Number of Problems (%)	Problem Example
Number	79 (72%)	A box of 678 lollies was shared amongst Tim and six friends. How many did Tim get if he also got the remainder?
Space	17 (16%)	Using concrete material, students were asked to stack blocks and cylinders to see which shape stacked the best and to work out why.
Measurement	13 (12%)	Daily morning boardwork: name of day, date etc ... We work out the difference between today's date and Xmas day, discussing how we arrived at the numbers.
Total	109	

The high proportion of problems classified as Number questions suggested that the majority of teachers had been recently doing work with their classes from the Number strand of the syllabus. The data in the table indicate that only slightly more than one quarter of the problems were classified as Space or Measurement questions.

The syllabus recommends that teachers should spend approximately equal amounts of time on each of the three strands but anecdotal evidence suggests that teachers spend more time on Number than either of the other two strands. This may explain the greater proportion of Number problems in the teachers' responses. Another possible reason is linked to the high frequency of use of application problems reported earlier in this chapter. Many of the number problems were examples of application problems. This aspect of analysis is further examined in the next section.

4.5.2 Categorising the Problems into Particular Student Question Types

The second level of analysis involved the categorisation of each problem as one of the four student question types. In addition to this, problems allocated to each of the question types were examined for specific content focus and the teaching grade level with which each was used.

The description of the student question types that was contained in the Background Information in the questionnaire was used as a guide to classify the problems contained in teachers' responses. The four question types were described in Section 2.3.3 as a possible classification of problems drawn from the relevant problem-solving literature. These types are exercises, application problems, open-ended problems and unfamiliar problems.

Categorisation of each teacher's problem required judgements about the context or presentation of the problem to the class, previous experiences and ability of the students. These were determined by referring to other indicators on each teacher's questionnaire such as the grade of the class and reasons for use of particular student question types. Table 73 summarises the results from this categorisation.

Table 73
Number of problems classified in each of the student question types of Exercise, Application Problem, Unfamiliar Problem and Open-Ended Problem and an example of each

Student Question Type	Number of Problems (%)	Problem Example
Exercise	0 (0%)	-
Application Problem	59 (54%)	Mooen had 34 Tazos but needed 51 for the set. How many more did Mooen need?
Unfamiliar Problem	27 (25%)	There were 2 men and 8 boys on a river bank. There is only one canoe to get them to the other side and it can hold only 1 man or 2 boys at one time. How many trips will it take to get them all to the other side?
Open-Ended Problem	23 (21%)	The troll heard 14 footsteps going across the bridge. Whose footsteps might they have been?
Total	109	

Of the 111 problems recorded by teachers, two were too general to classify. One example of this was "I gave a pictorial spatial problem to see how well students could use their visual memories to solve the problem". No teachers recorded a question that was classified as an exercise. This is possibly because the examples in the Background Information clearly indicated that exercises were not considered to be problems.

The remaining 109 problems were placed in one of the three student question types of application problem, unfamiliar problem or open-ended problem. The data in Table 73 indicate that the majority of teachers recalled an application problem as a recently used problem. Since teachers reported a strong preference for the use of application problems in Question 5, it is not surprising that the majority of the surveyed teachers' problems were examples of application problems.

It is noteworthy that one-quarter of surveyed teachers recorded an unfamiliar problem and slightly more than one fifth recorded an open-ended problem. This suggests that many teachers choose to use unfamiliar and open-ended problems to promote mathematical learning.

A further examination of the problems categorised into each of these student question types reveals some differences between the content focus for each of these types. Table 74 indicates the proportion of problems that were classified into each of the strands for each of the student question types.

Table 74
Proportion of problem types classified in each of the syllabus content strands

Content Strand	Application Problems	Unfamiliar Problems	Open-Ended Problems
Number	87%	56%	57%
Space	3%	30%	30%
Measurement	10%	14%	13%

This table shows that the majority of the recorded application problems involved the use of number concepts. These data confirm the suggestion in the previous section that most of the application problems focused on number. The distributions of the problems into content strands for unfamiliar and open-ended problems were similar.

Another examination of the problems in each of the student question type categories indicates that there was a spread of recently used problems for each of the teaching grade levels. Table 75 summarises the proportion of problems in each student question type that was used with each of the teaching grade levels, K-2, 3-4, and 5-6. Several of the respondents were from other role categories including administrators and specialist teachers and so it was not clear what grade level used the particular problem that was recorded on these questionnaires. In these cases, the problems are included in the "other" category in Table 75.

Table 75
Proportion of problem types classified in each of the teaching grade levels

Teaching Grade Level	Application Problems	Unfamiliar Problems	Open-Ended Problems
K-2	36%	30%	30%
3-4	29%	15%	22%
5-6	22%	37%	26%
Other	13%	18%	22%
	100%	100%	100%

The differences between problem types in relation to the teaching grade level are small. There were slightly more application problems provided by teachers of lower grades and slightly more unfamiliar problems provided by teachers of upper primary grades. This may be because teachers reported in Question 6 that problems were often chosen to suit the needs of the grade being taught and it was suggested by several teachers that unfamiliar problems are more suitable for upper primary. It is noteworthy that teachers of middle primary grades reported in response to the teaching strategies that they preferred to use application problems more frequently than teachers of the other two grade groups. This was discussed in Section 4.3.2.2.

In summary, most of the teachers' problems were examples of application problems with several providing examples of unfamiliar and open-ended problems. Number concepts were the focus of most of the application problems and accounted for slightly more than half of the unfamiliar and open-ended problems. Sorting each of the problem types according to reported teaching grade level of respondents produced a spread across the grade levels with only slight differences in distributions for each of the problem types.

4.5.3 Examining Purposes for Problem Use

Few teachers recorded a purpose for the use of their particular problem. When purposes were recorded, most teachers indicated that the problem was used to practise, review or develop skills, to apply recently learned concepts, or to provide a context to give meaning to mathematical ideas.

One example of the application of concepts was recorded by an experienced female teacher of lower primary grades. She wrote

> My students enjoy doing 'Mentals' and I always give them 'Application Problems' that are relevant to their everyday lives. Eg. we had been counting by twos and we looked at the mathematical language involved which led us to look at pairs. I then gave them a problem to solve. Eg, I had 3 pairs of socks and I lost 1 sock, how many left?

Some teachers mentioned the need to develop specific problem-solving skills. The following examples demonstrate this. First, a new female teacher of lower primary grades presented the following problem with the accompanying purpose

> "Freddie the frog is at the bottom of an empty 10 metre well. If everyday he climbs up 3 metres but each night he slips back 2 metres, when will he be out of the well?"
> This problem was physically, then concretely, then symbolically completed by students as part of an addition/subtraction unit and integrated with the big book being studied.

Second, a new female teacher of upper primary grades stated

> "Someone offers you a job for 15 days. They also let you choose how you will be paid. You can start on 1c a day and double the new amount everyday. Or, you can start for $1 and add $1 to the amount each day. Which would you choose? Why?"

This was used in a problem-solving lesson on strategies used when problem solving, ie. draw a table etc.

These two examples provide support for the notion that teachers use problem contexts to support the development of student understanding as well as to develop specific problem-solving strategies.

It is possible that teachers recorded their problem in response to Question 3 but did not notice the request to describe what the problem was used for. The responses to Question 6 requesting reasons for use of particular problem types provided more information about purposes than the few responses to this particular question.

Overall, responses to the request to record a recently used problem provided additional information that supported earlier analyses of data. Most teachers recorded application problems that focused on number concepts. However, there were still many teachers who recorded either an unfamiliar or open-ended problem indicating that teachers were familiar with these types and choose to use them in mathematics lessons. In addition, teachers reported that problems were used for a variety of purposes thus supporting the comments reported in Section 4.4.

While open-ended and unfamiliar problems were recorded by several respondents in Question 3, these types are not typically found in student textbooks so it is apparent that other sources may be used to find useful problems. This was the focus of Question 7 on the questionnaire and these results are reported in the next section.

4.6 Resources

Earlier discussions with teachers about problem-solving approaches indicated that one of the issues surrounding the implementation of such approaches was the lack of appropriate resources. Another issue related to the use of textbooks in schools and the view held by teachers that these materials must be used when schools required parents to purchase them. If teachers are to use a variety of problem types in mathematics lessons, then this may require the use of a variety of resource materials as sources of problems.

Question 7 sought to explore the frequency of use by teachers of potential sources of problems. The question stated "please indicate how frequently you use each of the following as a source of mathematical problems". Table 76 indicates the responses to this question with the proportion of respondents for each of the frequency categories.

Table 76
Frequency of use of particular sources of mathematical problems (n=162)

Source	Hardly ever	Sometimes	Often	Almost always
K-6 Syllabus and support documents	6%	20%	36%	38%
textbooks written for the K-6 Syllabus	8%	27%	43%	22%
resource or reference books	8%	33%	47%	13%
inservice courses and notes	41%	44%	14%	1%
other teachers	18%	46%	31%	5%

The data in Table 76 highlight the preferred sources of mathematical problems. The syllabus and support documents were frequently used by many teachers. Seventy-four percent of respondents *often* or *almost always* used centrally developed materials. The syllabus contains many examples of problems in the teaching and learning units so it is possible that this source provided considerable support for the implementation of problem-solving approaches in teachers' classrooms.

These data indicate that another popular source of problems was textbooks written for the NSW K-6 syllabus. Sixty-five percent of surveyed teachers *often* or *almost always* used textbooks as a source of mathematical problems. Most textbooks available in NSW contain many examples of application problems but few examples of unfamiliar or open-ended problems. This may be another reason why teachers more frequently chose to use application problems with their students.

Resource and reference books were also consulted quite frequently. Sixty percent of teachers indicated that they *often* or *almost always* used these as a source of mathematical problems. Teachers were not asked to identify specific reference materials but many books are available that describe approaches to problem solving and that contain examples of good problems for students.

Other sources were used less frequently. Eighty-five percent of teachers *hardly ever* or only *sometimes* use inservice courses and notes as a source of mathematical problems. Also, 64% of respondents *hardly ever* or only *sometimes* used other teachers as a source of mathematical problems.

A final category of "other" sources was offered to teachers at the bottom of the table in Question 7. Twenty-three teachers recorded an additional source of mathematical problems. Those noted by more than one teacher included individual ideas, the use of situations that arise from school or student contexts, the internet, university notes, and school programs. It had been anticipated that more teachers would report using good ideas from teacher inservice courses or workshops since there is a substantial amount of such professional development in schools. It is possible that the focus of professional development sessions had not been on innovative ideas for mathematics classrooms. Teachers were invited to comment on the professional development needs of their staff in Question 8. The analysis of teachers' responses to this question is contained in the next section.

4.7 Perceived Professional Development Needs of School Staff Members

Teachers' beliefs about the professional development needs of their colleagues at school were sought in Question 8. It was anticipated that responses would highlight issues about the implementation of problem-solving approaches. The question stated "what do you see as the professional development needs of teachers in your school in relation to problem solving".

The 130 responses to this question were read and comments categorised into three broad areas. These included specific comments about:
- teachers' knowledge, beliefs or confidence;
- practical aspects of teaching problem solving; and
- issues related to students.

Comments placed in the first two categories were further subdivided as indicated in Figure 24.

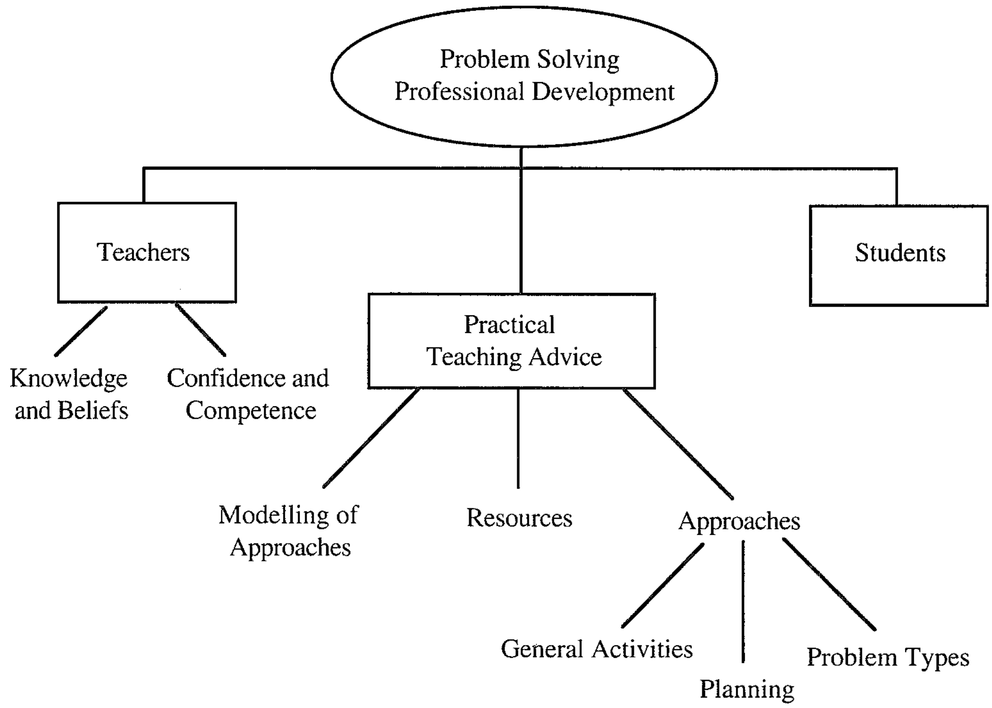


Figure 24. Classification of teachers' comments about the problem-solving professional development needs of school staff.

Comments about teachers generally referred to their knowledge and beliefs about mathematics and problem solving, or to their confidence and competence in teaching problem solving. Practical teaching advice focussed on the need for modelling of problem-solving approaches by competent or experienced teachers, issues about suitable resources, and approaches to teaching problem solving including useful activities, planning and use of specific problem types. A description of each of these areas as well as comments relating to students follows with examples of typical teachers' responses.

4.7.1 Comments About Teachers

A total of 26 comments were made about teachers' knowledge and beliefs or their confidence and competence. Of these, 18 referred to teachers' beliefs about mathematics or problem solving and the role of problem solving in learning mathematics. It was suggested by some respondents that teachers in schools needed to rethink their approaches to teaching mathematics and to move from more formal approaches to innovative, creative methods. Others indicated that teachers needed to be more aware of the benefits of problem solving in the mathematics curriculum. One suggested that there seemed to be some confusion as to what problem solving was really all about, especially for "older teachers" on the staff. These views are summarised by one teacher's statement that there was a need for "whole staff awareness raising".

It was clear that many respondents believed that problem solving should be discussed and its role in the teaching and learning of mathematics needed to be debated. The following comment, made by an experienced male teacher of lower primary grades suggested that problem solving may not be seen as legitimate mathematical activity by some teachers at his school. He stated that problem solving needs "credibility raising - it's not seen by teachers of Years 4 to 6 as *serious* maths".

While these comments generally supported the need for change towards a problem-solving curriculum, two teachers indicated that they believed that the approach should be more traditional. One stated that there was a need to "get back to the basics" while the other indicated that she didn't "have a problem with the traditional curriculum".

The remaining eight teachers in this group made comments that related to teachers' confidence and competence. Respondents suggested that teachers needed support and encouragement so that they would be more confident in their abilities to implement problem-solving approaches. The following comment by a new female teacher of middle grades suggested that many teachers needed support. She stated

> Be encouraged to do it in classrooms. I think so many teachers are very narrow in how they teach maths and their way of teaching maths reflects this (including myself). Slowly I would like to incorporate more and more problem solving in the classroom.

A moderately experienced female teacher of lower primary indicated that teachers may not be prepared to lose some of the control of students' learning that can happen in problem-solving classrooms. She stated

> A lot are scared to do it because they are not sure themselves of what they should be telling the children to do. They feel they are losing a "grasp" on exactly what the children are doing.

This notion of loss of control associated with a lack of confidence to allow students to explore and investigate freely is an important issue in being prepared to adopt problem-solving approaches, particularly if problem solving is considered to be a process of inquiry.

Finally, teachers' competence in relation to problem solving was considered to be a real issue for one respondent. This experienced female administrator suggested that

> Teachers themselves need to develop competence in problem solving processes and then through staff inservicing and interaction, devise appropriate problem solving strategies for their students. This needs to be a major thrust in 1997.

It is clear from these comments that several respondents believed that there was a need to consider teachers' beliefs and knowledge about problem solving. Also, teachers needed to be encouraged and supported in their efforts so that they could overcome a lack of confidence. These aspects were clearly some of the constraints that might have impacted on the implementation of problem-solving approaches.

4.7.2 Comments About Practical Teaching Advice

A large number of responses to this question included comments about the need for practical advice. Comments about advice were further grouped into three main areas. These included the need for modelling of problem-solving approaches, increasing school resources, and providing information about incorporating problem-solving approaches into their teaching of mathematics. Each of these areas will be further described with examples of comments made by respondents.

4.7.2.1 Modelling of Approaches

Eleven respondents made suggestions about the need to have problem-solving approaches demonstrated to staff by expert teachers. It was suggested that this modelling could take several forms including peer mentoring, external experts, or instructional videos. One teacher recognised that staff at her school had skills that should be shared with others. She stated

> We would like to see a few staff meetings incorporate peer mentor training, as we have some members of staff who have a lot to offer in regards to problem solving.

These comments seemed to suggest that while many teachers were aware of recommendations to teach problem solving, it may be difficult for teachers to know what this actually looks like, how to manage a class while students are doing problem solving, and what exactly the role of the teacher might be. This aspect supports comments made by Lester (1994) who suggested that advice to teachers needed to include a clear description of the teacher's role as well as an indication of what a problem-solving classroom looks like.

4.7.2.2 Resources

Another significant professional development requirement for many respondents was the availability and accessibility of good resource materials. Twenty-one teachers mentioned resources including books, posters, commercial products, software and concrete materials. Some suggested the need for demonstration of materials while others indicated the need for more equipment in classrooms. This is another interesting point since excellent materials have been written to support problem-solving approaches and yet according to Holton et al. (1995) and Clarke (1997), this is not sufficient to encourage adoption of such approaches.

Only four teachers made mention of the need for professional development in relation to the use of calculators or computers. One experienced female teacher of middle grades suggested two strategies to support teachers' problem-solving efforts and then made a comment about the lack of use of calculators at her school. She stated

> Maths days where people with a skill show others. Lots more equipment in classrooms. Haven't seen a calculator all year!

This was only one of two comments about calculators with two others mentioning the use of computers. This is somewhat surprising since much of the literature on problem solving describes the use of technology to support students' problem-solving efforts.

These teachers clearly believed that more resource materials or books about problem solving would solve issues in their school in relation to the implementation of problem solving. However, several of these respondents indicated that this was only one of the needs of staff. Respondents noted that more practical advice was necessary including workshops about teaching problem solving. These suggestions are examined in the next section.

4.7.2.3 Approaches to Teaching Problem Solving

A substantial number of comments were made about professional development in problem-solving teaching approaches. A total of 76 comments were further categorised into those relating to general classroom activities, planning and implementing the curriculum, and specific information about particular problem types. Each of these is discussed in the following sections.

General Activities

Thirty-nine comments were made about the need for practical, hands-on workshops demonstrating problem-solving approaches. Many teachers described sharing of ideas between teachers and discussing relevant approaches for different year levels. Others described the need for new ideas, fresh approaches and the latest trends.

The following comment by an experienced male teacher of upper primary grades encapsulated some of these ideas although his enthusiasm was somewhat unusual compared to other responses. He stated

> Providing fresh examples - displaying how problem solving works in groups - new ideas lead to a fresh outlook - lead to interest and positive gains - lead to great teachers.

Teachers seemed to be seeking ideas that could be readily implemented. In addition, most preferred workshops where they actually experienced the activities. There was also considerable recognition that teachers in schools have much to offer and should be encouraged to share ideas and examples of good practice. Teachers clearly recognised the skills of their peers and would value time to share this knowledge and experience. This may also support the use of reflection which, according to Thompson (1984), Clarke (1993) and other researchers, seems to be a necessary component for changing practice and adopting new approaches.

Planning

Nineteen teachers noted the need for support in integrating problem solving into daily mathematics lessons and incorporating problem solving into the curriculum. One teacher mentioned the need for thorough planning throughout the whole school so that scarce resources could be equitably shared among teachers and their classes.

Concerns were raised by several of these respondents in relation to adding problem solving into an overcrowded curriculum. One example was provided by an experienced female assistant principal who suggested that there was a need for

> Strategies which show how to fit problem solving activities into the demands of an already overcrowded maths syllabus and into an ever-expanding range of options in the KLAs.

It was also clear from these comments that problem solving was viewed as an added topic or an additional focus for teachers to incorporate into the curriculum. The following comment was made by an experienced female teacher of middle primary grades. She indicated that there was a "need to inservice them in terms of adding problem solving techniques and examples to our scope and sequence". This adds further support to the notion that many teachers believed that problem solving was an object of inquiry rather than a process of inquiry.

Five teachers indicated that time to implement new ideas was an issue and that schools needed to recognise this and to make more time available. This included the need for time to plan and implement as well as time to reflect on the success, or otherwise of potential changes.

Problem Types

The final group of 18 teachers indicated that there was a need for inservice activities that focused on the use of particular problem types and their role in promoting and developing students' problem-solving skills. In particular, there was a desire to learn more about the use of open-ended and unfamiliar problems and to be provided with more examples of these types of problems. The following two comments indicated the general views of this group of teachers.

The first example was recorded by a moderately experienced female teacher of upper primary grades. She indicted that for her staff

> Problems used need to be more open-ended. More variety and challenge in their level of questions. More use of complex problems relating to real-life situations.

The second example was provided by a new female teacher of upper primary grades. She indicated that at her school, teachers needed

> Access to unfamiliar problems [as] many view problems simply as application type problems, not even considering to give kids unfamiliar brain teaser type problems.

It is possible that the questionnaire design prompted some of these comments about problem types since items included reference to the four student question types as described in the Background Information. It seems that in some cases reference to these student question types raised teachers' awareness about the potential use of a variety of problems in teaching mathematics. While this may have influenced teachers' responses, it may have also provided an opportunity for them to reflect on their current practice, or it may have reminded them of other possibilities.

4.7.3 Comments About Students

Nineteen respondents recorded comments about professional development issues that focused on the diverse needs of students in the full range of classrooms from Kindergarten to Year 6. Most of these comments related to students' abilities and in particular about the need to develop the problem-solving abilities of lower achieving students. Another set of responses indicated that language was an issue for many students and this impacted on their ability to interpret and hence solve many problems. Additional needs of staff included management of students and appropriate grouping procedures.

The following set of responses encapsulate the comments made by these teachers. The first was made by an experienced female teacher of lower primary. She indicated that there was a need "to provide problem solving situations for *all* children in the class based on individual needs, not just the *smart* ones". The second was recorded by an experienced female teacher of upper primary grades. She stated that there was a need to be inserviced on "how to set up a framework of ongoing problem solving when faced with a group of children that have such diverse needs and abilities". The third was provided by an experienced female teacher of middle grades who stated

> Finding suitable problems for the age you need. There are plenty of application problems available and unfamiliar ones that are good for gifted children. How to use problem solving with the "mathematically challenging" children would be good.

The final example was written by a new female teacher of lower grades. She stated

> Recognition and understanding that problem solving can and should be completed by infants students.
> Education of ways around the language barrier involved in problem solving, particularly for NESB students.
> Education of the benefits of teaching mathematics through problem solving.

These four comments indicate that this group of teachers recognised the need to develop the problem-solving skills of *all* students in *all* classrooms. However, they indicated that teachers need support in this endeavour as many teachers believed that problem solving is only appropriate for more able or older students. The last comment reveals that this teacher understands the notion of teaching *through* problem solving and would like other teachers to understand the value in this approach.

Finally, of the 130 responses to this question, there was a group of 16 responses that could not be placed in one of the categories described above. Nine respondents provided little comment in response to the request to describe the professional development needs to their staff other than to write one of "more", "ok" or "none". The remaining seven indicated that they were either new to the school and therefore could not comment on the needs of the staff, or that it is up to individual members of staff to decide what professional development they need.

In summary, teacher respondents indicated that the problem solving professional development needs of school staff members are diverse. Identified needs included addressing teachers' knowledge and beliefs about problem solving as well as their confidence and competence, providing practical teaching advice in the form of modelling, additional resources and workshops about problem-solving approaches. These workshops needed to focus on suitable activities, advice about planning and incorporating problem solving into the existing curriculum, and examples of problem types and how these could be used to develop students' problem-solving abilities. Finally, there was an identified need to address the wide range of abilities and diverse needs of students in problem-solving classrooms.

4.8 Reactions to a Negative Statement about the Usefulness of Problem Solving

Another aspect of interest to the research was to seek to determine the extent of opposition to problem-solving approaches. To encourage teachers to reveal honestly such beliefs, an item was included in Question 9 that suggested that problem solving may be a waste of time and is promoted by people who haven't spent much time in classrooms. Teachers were invited to react to this particular view.

The question stated

> The following statement was made recently at a teacher inservice course:
> "People who push problem solving in mathematics obviously don't work in classrooms. It is a waste of time."
> What is your reaction to this statement?

There were 140 responses to this item although several of these were rather short responses such as "rubbish" or "a bit extreme". For those respondents who provided more detail, the comments were read and sorted into groups. The initial sorting yielded sets of comments that either agreed or disagreed with the statement in Question 9.

Comments that disagreed with the statement were further sorted into four broad groups. These included critical, personal comments about anyone who would support such a negative view, comments that described the importance of problem solving in demonstrating applications of mathematics, suggestions that problem solving enhances students' learning, or revelations of issues and concerns about teaching problem solving. Figure 25 summarises the overall categorisation of responses.

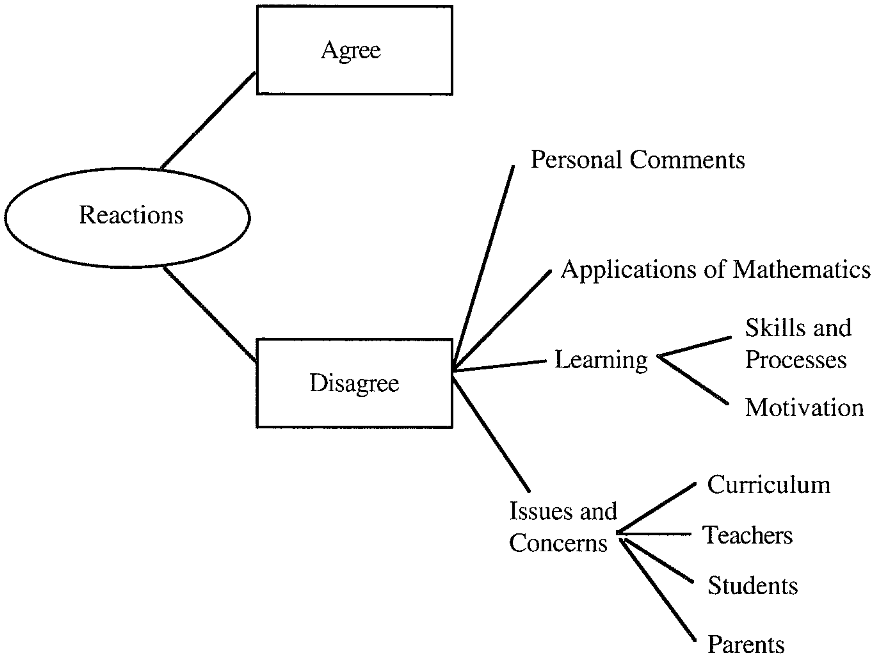


Figure 25. Classification of teachers' reactions to the negative comment about problem solving.

Two of these categories were further subdivided. Comments about students' learning were sorted into those that related to the development of problem-solving skills and those that described the motivational aspects of problem solving. Issues and concerns centred around curriculum requirements, teachers' confidence and professional development needs, students' abilities and needs, and parents' expectations. Typical respondents' comments that were categorised into all of these groups are described in the following sections.

4.8.1 Comments That Indicated Agreement with the Negative Statement

Only three respondents agreed with the statement presented in Question 9. Two of these described concerns about problem-solving approaches in relation to the range of students' abilities in classrooms. For example, the following was recorded by an experienced female teacher of middle primary grades.

> I agree with the statement as long as we keep in mind the great range of needs in a classroom. My students have a great love of maths but they have to be challenged at their level. I would use anything that gave them the power to be in control.

Another teacher who agreed with the statement described issues related to an overcrowded curriculum and finding time to do all that is required in teachers' lives. This experienced female teacher of upper primary stated

> It is time consuming and with all the other stuff in the curriculum that demands attention, I can concur. The pressure to do all things/subjects is at the heart of much teacher stress - hence the temptation to "do" the curriculum in the fastest, most time efficient manner.

It should be noted that even though many of the surveyed teachers *disagreed* with the negative statement about problem solving, this disagreement was conditional. They revealed reservations including concerns about catering for the needs of all students and meeting syllabus requirements. These reservations are described in the next section as Issues and Concerns.

4.8.2 Comments That Indicated Disagreement with the Negative Statement

A large majority of respondents disagreed with the negative statement about problem solving. All of these responses were read and sorted into four main groups. These groups included critical personal comments made about the person who made the statement, comments suggesting that problem solving provides a reason for learning mathematics, suggestions that problem solving supports learning through the acquisition of skills and processes as well as by motivating students. Finally, there was a group of teachers who disagreed with the statement but with reservations in the form of issues and concerns. Each of these groups, with examples of typical responses, will be presented in the following sections.

4.8.2.1 Personal Comments

Seventeen respondents made critical comments about the person who was said to have made the negative statement contained in Question 9. These comments were varied but most suggested that this teacher is either rather traditional, afraid of change, does not understand problem-solving approaches, or does not currently teach in schools.

The following three examples typify the reactions of this group of teachers. The first was provided by an experienced female teacher of upper primary. She suggested that

> They are not very effective, organised or confident teachers who are prepared to try different methods - too rigid in their ways. Need to be shown how effective cooperative learning can be in the maths area.

The second comment was recorded by an experienced female teacher of lower primary grades. She stated that

> My reaction is that this teacher is afraid of change, afraid that his/her knowledge of mathematics learning will be exposed - and he/she is expressing and reflecting community views about maths. He/she is probably into control in the classroom.

The final example was provided by a new female teacher of lower primary. She suggested

> Teachers who see problem solving as a waste of time are those who expect students to achieve instant success in problem solving without supporting students through the development of problem solving processes or strategies. They have not put in the time, effort or understanding themselves to see the wider mathematical understandings achieved by students through participation in problem solving.

Some of the comments in this group were rather impassioned with respondents suggesting that this teacher was "a waste of space", "an idiot", and even "a rash, ignorant person". It was clear that for this group of teachers, using problem-solving approaches was the best way to teach mathematics. However, it was also clear that these respondents recognised that teachers would need to be prepared to change, to reconsider their use of textbooks, to make an effort to understand recommended approaches, and to look at classrooms where these approaches were being implemented. The suggestions indicated that to incorporate problem-solving approaches into classrooms, teachers believed that reflection and careful examination of current practices was necessary.

4.8.2.2 Applications of Mathematics

A total of 44 respondents suggested a variety of reasons for studying problem solving that related to applications of mathematics. It was also suggested that problem solving is an important life skill that students must develop since "all of life is problem solving". Some indicated that it is through problem solving that students apply their mathematical skills and understandings. One example of this was provided by a new male specialist teacher. He stated

> Disagree - problem solving puts maths into a real world context, and makes children see that (and how) they use maths in their everyday lives (it relates it to their own situations, especially when they write their own [problems]).

Others suggested that problem solving provides a purpose for learning mathematics. An experienced female teacher of lower primary stated

> Problem solving to me means, or implies, that there is a reason for doing what we do. Without a reason, trying to convince people to participate in any activity is going to be difficult. I feel that children will see the point of learning mathematical concepts better by using this approach.

Some responses revealed quite different beliefs. Several teachers indicated that students should learn basic skills first and then use these in problem-solving situations. This supports a teaching approach that the syllabus refers to as teaching *for* problem solving. While others suggested that problems could be used as a focus of learning. An approach referred to as teaching *through* problem solving. This second view was demonstrated in the comment made by an experienced female administrator. She stated

> Horror! Problem solving is the core of maths. I think all maths work should stem from "a problem needs to be solved" situation so "thinking" is encouraged rather than just learning facts and how to apply them.

All of these responses indicated that for many teachers, mathematical skills were needed to solve problems, particularly in everyday situations. Also, it was through problem solving that mathematics became relevant and useful. It was also suggested that problem solving provided a purpose for learning basic skills. In addition, it was reported that problems provided meaningful contexts that supported student learning and aided motivation. These aspects of teachers' responses are discussed in the next section.

4.8.2.3 Learning

A large number of respondents made reference to student learning. These comments were further sorted into two main groups that referred to either the importance of acquiring skills and processes related to problem solving or to the increased motivation that can occur when problems are used in mathematics lessons. Each of these categories is described and supported by examples of teachers' responses.

Skills and Processes

Forty-seven respondents mentioned the importance of students gaining problem-solving skills and processes. In addition, it was suggested that problem solving promotes thinking, enhances understanding, extends and challenges students, and enables assessment of the use of processes.

Some of these ideas are evidenced in the following comments made by two respondents. The first is from an experienced male teacher of middle grades who suggested that

> Problem solving goes hand in hand with the learning, practise, mastery of mathematical skills. Oral and written, simple application and unfamiliar problems should be used regularly, applying them to the needs and abilities of your children.

The second was provided by a sympathetic, experienced female teacher of upper primary. She stated

> I can understand this type of comment. The necessity to ensure the basic concepts are understood is of paramount concern, but problem solving can be a useful tool to assess students' skills and enhance their mathematical understanding - practical application.

As well as the important acquisition of skills, development of processes, and enhancement of understanding, problem solving was seen by some teachers as a useful vehicle to integrate learning in a variety of Key Learning Areas in the curriculum. Thus, allowing integration of ideas and less compartmentalisation of knowledge.

Motivation

Fifteen teachers mentioned the role of problem solving in providing motivation to learn. It was suggested that problems added variety to lessons, could be fun to solve, and made mathematics more enjoyable and relevant. Typical comments included the view that problems were a "great motivator", that they could "stimulate learning" and "provide fun situations".

The following comment by a new female teacher of upper primary indicated that in her classroom, problems were the only way that the students would engage with mathematics. She stated

> No way! My kids have gotten to the point where they will only do maths if it's posed as a problem and if they do happen to receive an algorithm etc, they then pose their own problems. They love problem solving as it's real world mathematics.

Some respondents mentioned the important need to build confidence so that students were not overwhelmed by problems that were beyond their grasp. The following response was provided by an experienced female teacher of upper primary grades. She stated

> I do not agree with the statement, but a lot of confidence has to be established so that children will take risks. They are often put off by the fact that more than one solution is correct. Being right is more important than being investigative or adventurous.

This comment related to student learning and the need for support but it could also be considered an issue or concern. Many respondents reported disagreement with the negative statement but raised issues or concerns in teaching problem solving. These are discussed in the next section.

4.8.2.4 Issues and Concerns

Fifty respondents disagreed with the negative statement about problem solving but then provided reservations, or issues and concerns about its successful implementation. These reservations were based on curriculum requirements, teachers' confidence and professional development needs, students' abilities and needs, and parents' expectations. Teachers' comments were sorted into these four categories and are discussed in this section.

Curriculum

Comments that related to the curriculum included the need to teach basic skills before presenting problems to students, the need for a balanced curriculum, the belief that the curriculum is already overcrowded which then raises issues of time pressures to complete required work. An experienced female specialist teacher described her concerns in relation to basic skill acquisition. She stated

> There is a need to have basic calculation skills in place. The use of problem solving has its place but there are many other factors to consider and learning skills that need to be in place, including language skills.

The following comment typifies comments about the demands of the curriculum and was made by an experienced female teacher of middle grades. She stated

> The curriculum is extremely crowded. There isn't enough time to spend, allowing children to sit in groups and work out problems which may take 2-3 days. Lunch times are given to "gifted children" who problem solve and brain storm, but this is a luxury. It's not a waste of time but we're already trying to do the impossible.

The issue of lack of time to do all that is required of teachers was highlighted by many of the surveyed teachers. These comments are discussed next.

Teachers

Comments relating to teachers included those concerned with having little time and few resources, feeling threatened, needing support, and the difficulties associated with assessing problem solving. Time was clearly a critical and important consideration for many respondents. An experienced female teacher of lower primary stated "I disagree but often sources of problems are scarce and they take heaps of preparation time". Another experienced female teacher who teaches upper primary reported

> I understand the response as the amount of work we are to cover is extensive but problem solving is a valid way of teaching which generally needs more time.

Additional issues of support for teachers and difficulties in classrooms are evident in the following comments. A new female teacher of lower primary stated "we just need help in how to use it, set it up, implement it into the different syllabus areas". Also, an experienced male teacher of upper primary stated

> I disagree strongly. I actually like to see kids involved in problem solving, but through lack of personal knowledge and resources and a perception that I wouldn't properly be covering the strands, I have restricted my number of this type of lesson.

It is clear from these responses that teachers supported the notion of teaching problem solving but felt considerable pressure to meet curriculum requirements and believed that to incorporate problem solving into their programs would require substantial time. Time is a precious resource when teachers are already struggling to cope with all that they must do. Another factor is the need for support through professional development so that teachers gain confidence and experience. In addition, teachers were concerned about students' abilities to cope with problem-solving approaches. All of these aspects support earlier findings and some of these concerns are further described below.

Students

Comments about students focused on their abilities, language difficulties, special needs, and the added burden of large and diverse classes. The following comment made by an experienced female teacher of lower primary typifies these concerns. She stated

> It is harder to organise and implement, particularly with a big number of children in the class and small classrooms. Also difficult if a large number of children have poor literacy skills. Classes with ADHD and integrated children also add to the difficulty. Smaller classes essential. Preparation time needs to be longer.

The following lengthy response was made by a new female specialist teacher. She described additional issues relating to assessment of student learning and the need to provide the necessary skills for *all* students. She stated

> I certainly don't think it is a waste of time, but I feel that there are "pros" and "cons". My primary fear is that children finding their own ways to solve problems is difficult for teachers to monitor for errorless learning. I feel that basic facts/skills must be taught to enable students to have the correct tools to solve problems. Trying to build a house on wrong foundations will inevitably fail! GAT kids will obviously thrive on "open-ended" approaches but this cannot be guaranteed for all students, especially kids with special needs. I fear that if a skill is learnt or applied incorrectly it will be difficult to re-teach this successfully, similarly it cannot be left to chance that kids will just pick it up because we have facilitated the opportunity to learn.

Many responses suggested that teachers believed that more able students could cope with problem solving far better than less able students. Also, teachers believed that basis skills and number facts must be in place before problem-solving situations were presented to students. Other issues related to the diversity of needs in the one classroom and the difficulty teachers face when trying to cater for all of the different needs of these students. These were recurring themes and clearly impacted on what teachers believed, and what they did in their classrooms.

Interestingly, there were no references to students' beliefs about the role of problem solving in learning mathematics and whether problem solving constituted legitimate mathematical activity. However, there was one comment about parents' perceptions.

Parents

One teacher commented about parents' perceptions and the belief that exercises were desirable as evidence of mathematical learning. This experienced female administrator stated

> My experience with children "turned on" to mathematics is that problem solving is a fantastic broadening skill. Parent perceptions of problem solving as a useful activity is a concern. Exercises are often perceived as more time-efficient and "proof" of learning.

It was somewhat surprising that this was the only comment about parents' beliefs and what should be presented to students in mathematics lessons.

In summary, Question 9 provided a rich source of data about teachers' beliefs as well as their concerns about teaching problem solving. Almost all of the teachers who responded to this question disagreed that problem solving was a waste of time. Many indicated that problems were real-life applications of mathematics and as such provided an important purpose for learning mathematics. Others suggested that problem solving aided learning and provided motivation in mathematics lessons.

Although there was much support for this approach, there were many reservations about implementing problem-solving approaches. These reservations related to satisfying curriculum requirements, coping with all of the pressures on teachers as well as need for more professional development, meeting the diverse needs of students in classrooms, and confronting parents' expectations about mathematical activity. These aspects provided evidence of the constraints that teachers believed were preventing them from implementing problem-solving approaches in their classrooms. Additional evidence was obtained from the last question on the questionnaire.

4.9 Additional Comments Offered by Surveyed Teachers

The last question was added to provide teachers with an opportunity to make any additional comments. The question stated: "are there any other comments that you would like to make"? Fifty-five teachers provided comments that were read and sorted into categories. Many of the responses to this question were long, and involved reference to several different aspects of teaching problem solving. In these cases, comments to different aspects were separated and placed in the appropriate category.

All comments were grouped into six categories including those related to the curriculum, students, teachers, parents, resources and the questionnaire itself. Most comments described issues and concerns for teachers, particularly in relation to the need to establish basic facts and skills before problem solving, students' needs, and approaches to teaching mathematics and problem solving. Each of the categories of responses is discussed in the following sections.

4.9.1 Curriculum

There were 20 comments about the curriculum that discussed pressure to complete curriculum requirements, the place of learning basic facts and algorithms in relation to problem solving, and the desire to teach mathematics using an integrated approach. Examples of each of these aspects are presented below.

Pressure to complete curriculum requirements as well as do all of the other tasks teachers are expected to perform were discussed by several respondents. The first example was part of the response of a moderately experienced female teacher of lower primary. She also raised the issue of assessment procedures that she clearly believed were taking up valuable time. She stated

> ... Today there is pressure from all KLA areas, and I believe the organisation, collection of assessment data etc almost requires a separate worker (secretary) in the room!! Smaller schools are sometimes limited in people resources and this means they have many extra tasks to perform that take them away from more time consuming methods.

A second response was provided by an experienced female teacher of upper primary. She stated

> In an ideal world ... without all the other "compulsory" subjects/activities we have to fit into a busy day, I would attempt much more problem solving in maths ...

Ten respondents described the importance of teaching basic facts and algorithms before problem solving should be introduced. Teachers commented on the desire to spend more time on basic facts or tables, that number facts were essential, but that this knowledge then needed to be applied to problems, particularly application problems. Reinforcement of basic facts was seen to be particularly important for lower ability students. Additional comments about students' needs are presented in the next section.

One exception to the view that learning basic skills should precede problem solving was presented by a moderately experienced female teacher of upper primary. This teacher indicated that she believed that both algorithms and problems should be presented together. She stated

> In my past experience I have found that problems are needed to understand what maths is all about. I also believe that an algorithm is an essential recording method. I therefore believe that the two methods should be taught along side each other.

Several teachers indicated that they preferred to use an integrated approach so that connections were made between all three strands in the mathematics syllabus and between Key Learning Areas. The following comment was provided by an experienced female teacher of lower primary. She stated

> Maths is one of our KLAs. I believe in an integrated approach and that maths is part of a learning process. I set up learning centres where children are encouraged to record and describe/explain orally what they learn. In this way all subjects are integrated and learning is meaningful. The children need to learn to take responsibility for their learning - how to learn as well as what.

It is clear from many of these comments that teachers were concerned about the time to do problem solving with their students. It would appear that problem solving was viewed as an added extra to the curriculum rather than part of it, or as a teaching approach that may enhance learning. Another issue was the need to establish basic skills before problem solving was presented to students and this could mean that for lower ability students, problem solving may never be a part of their mathematical experiences. The diverse needs of students was an issue of real concern for many respondents. All of these aspects were raised in responses to earlier questions but teachers obviously felt the need to add these comments to reinforce their concerns.

4.9.2 Students

A total of 23 separate comments were made about students. The main focus of these comments was on students who have difficulty learning mathematics and the need to cater for these students in mixed and diverse classes. Issues that were discussed by respondents included the availability of appropriate resources, the desire to have graded mathematics groups, the need to use different problem types with students of different abilities, and the need to use different teaching approaches with different students.

One teacher cited the need for problem-solving resource materials that match syllabus substrands and that cater for mixed ability groups by providing graded problem sets. She wrote

> I'd like to see a support document that links in with the syllabus ... that listed appropriate problems that would cater for a variety of levels within a typical class, ie. basic, simple non-threatening for kids with very poor language skills up to extension for gifted kids.

Two teachers reported that they supported streamed or graded mathematics groups as this enabled teachers to better prepare material that was more suitable for the particular group of students. A moderately experienced teacher of upper primary wrote

> I feel strongly that maths should be a streamed subject. I have the slowest children in Year 6 and have had this type of class several times. The beauty of this is that I can spend time with

> the very basic concepts without having to have already set work for others. The children do not mind asking questions and even making mistakes in this class. Problem solving can be much more specific to the children in this setting.

Several teachers indicated that students with different abilities should be presented with different problem types. It was suggested that lower ability students need more exercises and application problems whereas more able students should be presented with open-ended and unfamiliar problems. This is particularly evident in the comment from an experienced female administrator who wrote

> Problems, both application and unfamiliar, are used more extensively with our capable students because they acquire the basic skills and concepts very easily. They have little difficulty in using these basic skills and concepts to solve problems. The difficulty arises when the two areas ... are not taught concurrently - brilliant adding machines but unable to apply ... With weaker students the difficulty is to find the balance - if they do not have some skill development they can be easily discouraged and overwhelmed by "problems". Similarly, with children who are not of a "risk taking" personality - they just want to know "how to do it".

Some teachers suggested that it is critical to start with the needs of the student and design an approach that suits them as no one approach will suit all students. One experienced female teacher of middle grades stated

> We should go beyond thinking that only one approach is suitable. You take each child at whatever level they are and go from there. Children have different learning styles.

A similar comment was presented by an experienced female specialist teacher who suggested that

> No one method should be employed in teaching a subject. Different children succeed using different methods. The main focus for me is to ensure that each student is given enough support mechanisms to achieve success. This may involve peer tutor, concrete materials, teacher/student demonstration. No matter what method is employed the basics need to be enforced so that the speed and accuracy levels are improved.

Some reference was made to students' language difficulties and their impact on problem solving. Particular mention was made about students' difficulties with reading, comprehension and interpretation of problems. This was viewed by teachers as another issue that impacted on students' abilities to engage successfully in problem-solving lessons and in completing problem-solving tasks.

In summary, it seemed to be the view of many surveyed teachers that students with different abilities needed different problem-solving experiences and this could be a challenge in classes that have students with diverse needs. Teachers found it difficult to meet all students' needs in normal mathematics lessons. In addition, they believed that problem-solving approaches to teaching mathematics were difficult to plan and implement because of the wide range of abilities in most classrooms, the impact of the extra language demands in many problems, and a lack of suitable resources.

4.9.3 Teachers

Comments relating to teachers included suggestions that a variety of teaching approaches were needed in all grades, a change in focus from more traditional approaches was desirable, and an increase in professional development was needed to support teachers' efforts.

One teacher criticised the lack of use of concrete experiences in higher grades and suggested that teachers move to more abstract work too soon. This experienced female teacher of lower grades stated

> As a Kinder/Year 1 teacher, I use problem solving all of the time using concrete materials to sort, classify, share, divide, measure, add, take away, fill etc. These provide the basis for problem solving in later years when using numerals and more difficult language. I think too many teachers in higher grades throw out concrete/hands on problem solving for solving equations and number stories.

Three examples of comments are presented that demonstrate that some teachers are aware of the need to reflect on their beliefs and practices and to consider alternative views and approaches. The first was provided by an experienced male specialist teacher. He suggested that "most people grow up fearing maths because of how it is taught, especially in high school. We need to break the cycle". The second teacher revealed a change in her beliefs. She stated

> I always thought in my early years of teaching that problem solving was just a separate section of maths but now I realise it covers all aspects of maths whether it be Space, Number or Measurement.

The third was provided by an experienced male teacher of upper primary who admitted that he would like to do more problem solving with his students. He stated

> I don't use problem solving as much as I would like to as I just don't have the time to get organised. I feel strongly that each child still needs to have a grasp of concepts of +, -, x, and / before they can begin to understand fully and explore the problem solving concept especially unfamiliar problems. Open-ended problems, I really haven't used that much (maybe I need to reassess). Application problems are essential for purpose and reinforcement of WHY? we learn mathematics.

Overall, teachers viewed problem-solving approaches as more innovative and supported the need for professional development to assist teachers in their efforts. Others indicated that staff do not understand the approach and lack confidence in teaching mathematics. Again, the issue of time to implement changes was discussed by several respondents.

4.9.4 Parents

Four respondents made reference to parents including their expectations and perceptions of mathematics and problem solving. One teacher believed that parents must be educated about the value of problem solving while another, who indicated that she preferred traditional approaches, suggested that many of the parents of her students agreed with her views.

One experienced female teacher of lower primary grades included parent issues as well as student issues in her response. She stated

> As a classroom teacher I found it sometimes difficult to implement my views when dealing with parent expectations, non-English speaking children, children with language disorders and ADHD children - all in the same class!!

In general, comments about parents indicated that some teachers were aware that parents' expectations may not match the use of problem-solving approaches in classrooms. This was considered an issue that needed to be addressed if change was to take place.

4.9.5 Resources

Five teachers mentioned the need for more resource materials to support teachers' efforts. These issues were described earlier in relation to students' needs but it is clear that teachers would value ready prepared materials for use in classrooms. This issue seemed to relate to the lack of time to plan, prepare and implement innovative practices in classrooms with students with diverse needs.

4.9.6 The Questionnaire

Eight respondents made a comment about the questionnaire itself. These included the view that responses to some questions would depend on the particular age and grade level being considered, a request for the results, and acknowledgment that completing the instrument provided "food for thought".

Illustrative examples include the following three responses. An experienced female teacher of lower grades suggested that "some of these questions have been difficult to answer when you take into account children's ages and stages of development". An experienced female teacher of upper primary commented that

> Responses to this survey would be different if our school did not have maths ability groups. What I do with a higher level group is quite different to what I'd do with a low ability group.

Finally, an experienced male administrator suggested that "this survey has been stimulating and probably will generate some professional debate".

These additional comments have provided further evidence about many of the issues that teachers described in their responses to earlier questions. These issues related to curriculum requirements and the view by many respondents that problem solving added to an already overcrowded curriculum. Also, catering for the needs of all students in classrooms was a real challenge for teachers in mathematics lessons but it seemed to become an even bigger challenge in relation to problem solving. Issues were also discussed in relation to teachers' beliefs and practices and the view that many teachers lacked confidence and seemed to be willing to continue with traditional practices. Further comments were made about parents' expectations and the lack of suitable resource materials.

A few respondents commented about the questionnaire itself and highlighted the issue that responses may be dependent on the particular group of students being considered. This added further support to the notion that students' needs vary and must be considered in problem-solving classrooms. It was interesting to have some surveyed teachers acknowledge the usefulness of completing such an instrument as providing "food for thought" and potentially enhancing their views about mathematical problem solving.

4.10 Summary of Questionnaire Analyses

The purpose of the questionnaire was to seek insights into teachers' beliefs about the role of problem solving in learning mathematics, to gauge the extent of teachers' problem-solving practices, and to discover other issues that may be impacting on the implementation of problem-solving approaches. The analysis of 162 questionnaires from primary school teachers in NSW confirmed the spread of teachers' beliefs and the diversity of their practice. In addition, the analysis of each of the open-ended questions revealed issues for teachers through recurring themes, many of which could hinder teachers' problem-solving efforts in classrooms.

The first two research questions, as listed in Table 77, formed the focus of the questionnaire. The questionnaire analysis has provided considerable information about each of these questions. This section considers each of these research questions and summarises the main findings from the questionnaire analysis.

Table 77
The first two research questions for this investigation

Number	Question
1	What do teachers believe is the role of problem solving in learning mathematics? In particular, what are teachers' beliefs about the relationship between problem solving and: • mathematics; and • students' learning of mathematics?
2	To what extent do teachers report that they incorporate problem-solving approaches in their planning and teaching of mathematics, and what specific practices do they report that they use? In particular, when incorporating problem-solving approaches in their mathematics teaching: • what specific classroom strategies do teachers claim to use; • what types of problem-solving tasks do teachers claim to prefer and what tasks do they claim to use; • what do teachers report is the form of their planning; and • what resources and materials do teachers report they use?

The first research question sought to identify teachers' beliefs. This was addressed by creating an artificial continuum about teaching and learning, with a *traditional* teaching approach at one end and a *contemporary* teaching approach at the other. It was acknowledged that teachers may hold beliefs situated somewhere between these end-points, or that teachers' beliefs may vary according to context. Responses to Questions 1 and 2 confirmed the spread of beliefs. In addition, questionnaires were sorted into five categories based on the level of agreement with each of these perspectives. While the majority of responses were placed in a middle category that suggested a mixed set of beliefs incorporating both *traditional* and *contemporary* views, there appeared to be notable differences between the questionnaire responses of teachers who were placed in the extreme categories.

In summary, many more teachers who were placed in the *very contemporary* group were working in lower primary and there was extensive use of concrete materials, encouraging students to use their own methods, and using problems that related to school contexts. In this group, there was also greater use of open-ended and unfamiliar problems with reported reasons indicating the need for variety, real-life applications and catering to the different learning styles of students. In contrast, the *very traditional* group more frequently mentioned struggling students, problems with lower ability students in mathematics, and the importance of students experiencing success in their learning. This seems to reflect a difference between the two groups of a focus on a child-centred approach to learning compared to an approach that is based on concern that students are comfortable in their learning environment.

While there was a spread of responses across 14 of the 15 items included in the first two questions, the distribution of five items in particular clearly identified differences in beliefs. This spread of responses suggests that the instrument has been successful in discriminating beliefs about mathematical problem solving, thus establishing confidence in the instrument. Statistical analyses of these first two questions also yielded support for the use of the instrument. There was a high correlation between seven of the eight items in Question 1 and between five of the seven items in Question 2. In addition, two strong factors were revealed in a factor analysis; the first linked most of the traditional items while the second linked several of the contemporary items.

A comparison of responses to the beliefs statements in Question 1 and Question 2 of the questionnaire reveals some differences between particular groups of teachers. A consideration of teaching grade level indicated that teachers of middle grades were more supportive of *traditional* approaches than either teachers of lower or upper grades, and teachers of lower grades were more supportive of *contemporary* approaches than the other two groups of teachers. In addition, reported attendance at mathematics inservice days may yield a slight increase in support for some of the more *contemporary* belief statements. Surprisingly, increased teaching experience seems to have little impact on teachers' reported beliefs.

A possible explanation for these differences in preferred approaches may be the influence of syllabus documents and the appropriate recommendations made at each of the grade levels. Support for the *traditional* statements by the middle grade teachers could occur since algorithms are introduced and developed in these grades for most students. In addition, there is a focus in the curriculum on recall of facts, mental strategies, and written algorithms that might encourage teachers to focus on these skills and as a consequence, leave problem solving until later. In contrast, in the lower grades, the syllabus recommends the use of real-life applications to introduce basic algorithm development and this is usually done through simple word problems. Also, there is support for encouraging exploration of methods and individual recording of solutions rather than being introduced to formal methods at this early stage.

The open-ended questions provided further information in relation to teachers' beliefs about the relationship between problem solving and mathematics, and between problem solving and students' learning of mathematics. A common belief that was reported in many of the responses was the view that the attainment of basic skills and procedures should precede the development of problem-solving skills. It was acknowledged by teachers that mathematical skills are needed to solve problems, particularly in everyday situations. It was revealed that many teachers believe that problem solving highlights the relevance of mathematics and provides a purpose for learning mathematics. Also, it was reported that problem solving can promote learning by focusing on understanding, thinking and applying knowledge. Mention was also made of the need for students to learn problem-solving strategies and to learn the language of mathematics through problems. In addition, teachers indicated that problem solving can motivate and challenge more able students. However, many teachers noted that the role of problem solving differs according to the age and ability of the students. This aspect is further discussed later in this summary.

In relation to the second main research question, considerable data were collected about preferred teaching strategies and problem-solving tasks, as well as the use of resources and planning of problem-solving lessons. The most frequently used strategies included whole class discussion to share solutions and strategies, the use of application problems to practise recently learned skills, provision of concrete materials, modelling problem-solving processes, and whole class discussion about problem-solving strategies. Rarely used strategies included students working alone, availability of calculators, the presentation of unfamiliar and open-ended problems to the class with little indication of how to solve them, allowing students to choose their own problems, spending several lessons on the same problem, and posing unfamiliar problems. It should be noted that there was a spread of responses for all of these strategies with some teachers frequently using strategies that the majority of teachers rarely used. These data suggest that it is appropriate to use many of the listed strategies and also, that many teachers have heeded advice to use a variety of strategies on a regular basis.

Further data were sought about teachers' use of each of the student question types including a comparison with views of mathematics education lecturers. Teachers reported a preference for using exercises and application problems than either of the other two problem types. Lecturers recommended a higher frequency of use of open-ended and unfamiliar problems than teachers report using. They also recommended a lower frequency of use of application problems and a much lower frequency of use of exercises than teachers reported using. These differences between teachers and lecturers raises several questions about why all teachers have not responded to advice from the problem-solving literature as well as from syllabus and support documents. However, teachers' responses indicated that they have quite powerful reasons for their choice of particular question types.

There appeared to be several considerations in teachers' choice of particular student question types. First, each type was considered to have different purposes based on curriculum requirements and question characteristics. Teachers reported that exercises were necessary as they provided valuable practice in basic skills and procedures. Application problems enabled students to apply these basic skills and procedures in real-life contexts. Open-ended problems provided challenge, developed higher level thinking and allowed students to respond at their own level. Unfamiliar problems challenged and motivated more able students and provided opportunities for students to apply problem-solving strategies. The general view was that all problems were valuable but choice was made on the basis of accessibility so that students were not overwhelmed by task demand thus avoiding anxiety and negative attitudes.

Second, there were other factors that impacted on choice of questions including considerations of the students in relation to learning, ability, age and affective factors, confidence of the teacher, and the availability of suitable resource materials that provide ready access to each of these student question types. Some of these aspects are further discussed in relation to issues for teachers that might constrain the implementation of problem-solving approaches.

In teachers' responses, a clear distinction was made between the use of exercises and application problems compared with the use of unfamiliar and open-ended problems, mainly in relation to the age and ability level of the students. It was suggested by teachers that exercises and application problems are more suitable for students in lower primary grades as well as for lower ability students since these students need more practice on basics skills and respond better to more structured lessons with drill and practical application of mathematical ideas. Unfamiliar and open-ended problems were viewed as more appropriate for students in upper primary grades as well as for more able students since they can be used for extension and to promote lateral thinking.

A further indication of problem preferences was provided when teachers recorded a recently used problem. The majority of teachers' problems were examples of application problems with some providing examples of unfamiliar and open-ended problems. Number concepts were the focus of most of the application problems and accounted for slightly more than half of the unfamiliar and open-ended problems. Sorting each of the problem types according to reported teaching grade level of respondents produced a spread across the grade levels with only slight differences in distributions for each of the problem types.

Important concerns for teachers were revealed in relation to planning lessons using problem-solving approaches. It is clear that the availability of exercises and application problems makes them easier to use on a regular basis. Even though teachers seem to recognise the usefulness of open-ended and unfamiliar problems, their use significantly impacts on planning time. In this sense an important constraint on their use is the time available for preparation and the accessibility of suitable resource materials. In addition, meeting curriculum requirements was mentioned by many teachers as a necessary factor in planning for instruction.

Preferred sources of problems included the curriculum documents and textbooks written for the NSW syllabus. These teachers clearly believe that more resource materials or books about problem solving would solve issues in their schools in relation to the implementation of problem solving. However, several indicated that this was only one of the needs of staff since more practical advice was necessary including workshops about teaching problem solving.

Further issues were raised in response to the professional development needs of staff in schools. In summary, teacher respondents indicated that the problem-solving professional development needs of school staff members were diverse. Identified needs included addressing teachers' knowledge and beliefs about problem solving as well as their confidence and competence, providing practical teaching advice in the form of modelling, additional resources, and workshops about problem-solving approaches. These workshops would need to focus on suitable activities, advice about planning and incorporating problem solving into the existing curriculum, and examples of problem types and how these can be used to develop students' problem-solving abilities. Finally, there was an identified need to address the wide range of abilities and diverse needs of students in problem-solving classrooms.

It was clear that for this group of teachers, problem solving was considered to be a desirable way to teach mathematics. However, it was also clear that these respondents recognised that teachers would need to be prepared to change, to reconsider their use of textbooks, to make an effort to understand such approaches, and to look at classrooms where these approaches are being implemented. The suggestions indicated that to incorporate problem-solving approaches into classrooms, teachers believe that reflection and careful examination of current practices is necessary.

To confirm questionnaire analyses and to further investigate identified issues, additional data were sought from a small group of teachers during the fieldwork phase of data collection. These teachers were chosen on the basis of their responses to the questionnaire so that they represented the spread of problem-solving beliefs and practices. The fieldwork analysis is presented in the next chapter.

CHAPTER 5
FIELDWORK ANALYSIS

Data collection for this investigation occurred in two phases. The questionnaire was used in the first phase and was designed to obtain information about primary school teachers' problem-solving beliefs and practices. The second phase of data collection, involving interviews and observations, was informed by analysis of questionnaire data to the extent that it highlighted key issues and factors for further consideration. On the basis of questionnaire responses, particular teachers were identified for participation in the fieldwork.

The fieldwork data collection incorporated initial interviews, classroom observations and reflective interviews with a small group of classroom teachers. The initial interviews were used to clarify teacher interpretation and confirm researcher analysis of questionnaire responses, explore and elaborate teachers' comments from the questionnaire, reveal opportunities and constraints that might impact on teachers' efforts to implement problem-solving approaches, and to select participants for the classroom observations. The purpose of the observations was to further validate questionnaire data as well as to clarify and confirm initial interview comments, compare actual practice with reported practice, and to identify factors that seem to either support or constrain the implementation of problem-solving approaches. Finally, the reflective interviews provided an opportunity to discuss and clarify observed practice as well as to confirm influencing factors with each of the participants. This data provided a rich picture of each teacher's problem-solving beliefs and practices and enabled the researcher to explore the usefulness of the proposed model for this investigation.

Participants in the fieldwork were chosen from those respondents who recorded their names on the questionnaire. Of the 162 completed questionnaires, 45 respondents included their name indicating that they were prepared to participate further in the study. Based on the questionnaire analysis, nine teachers were chosen who represented a spread of problem-solving beliefs and practices. In addition, each of these teachers had current, full-time responsibility for an infants or primary class. Semi-structured interviews were conducted with each of these teachers focusing on their responses to the questionnaire. These initial interviews also served to identify participants for classroom observations and reflective interviews. In the final stage of data collection, two teachers were observed teaching three consecutive problem-solving lessons and were interviewed during the observation period.

The fieldwork yielded data supporting findings from the questionnaire and revealed possible factors that operate in particular contexts to support or constrain teachers' efforts to implement problem-solving approaches. Initial interviews confirmed the spread of problem-solving beliefs and the perceived desirability of implementing problem-solving approaches. However, they can be difficult to implement because of the diversity of students' needs, assessment and reporting procedures in particular schools, parents' perceptions, teachers' confidence and previous experiences, pressure on teachers to successfully achieve all that is expected of them, and the availability of suitable resource materials. When teachers were able to implement such approaches, they were enthusiastic about student learning outcomes and believed that mathematics was more meaningful and enjoyable for students.

In this chapter, the first section describes the teachers who participated in the fieldwork. The second section outlines the initial interview procedures and presents a summary of data collected from seven of the participants including their questionnaire responses and initial interview comments. The third section presents an analysis of questionnaire responses, initial interview data, classroom observations and reflective interviews for two of the participants. Finally, a summary of data analysis from the fieldwork is presented.

5.1 Fieldwork Participants

Selection for participation in this stage of the data collection was based on several criteria that were described in Section 3.2.2. However it is noted that it was desirable that this group of teachers represented a spread of problem-solving beliefs and practices so that the diversity of factors impacting on use of problem-solving approaches could be gauged. After questionnaire analyses, nine teachers were selected to participate in this stage of data collection. Four teachers were chosen who agreed more with Naomi's views than Gwendolin's; two were chosen on the basis that they agreed more with Gwendolin than Naomi; and a final three were chosen whose responses were neither clearly *contemporary* nor *traditional* since they agreed with some comments from each perspective. In addition, teachers' responses to the last three open-ended questions on the questionnaire were considered.

The group of nine female teachers were from six different schools. Two pairs of teachers agreed to be interviewed together resulting in seven separate interviews of approximately thirty minutes each. A summary of some of the background data about the teachers who participated in the initial interviews is presented in Table 78. This table includes years of teaching experience and current teaching grade level for each of the participating teachers. In addition, the category of views about teaching and learning mathematics that was determined from the questionnaire analysis is included.

Table 78
Background information for teachers involved in the initial interviews

Name	School	Experience (Years)	Teaching Grade Level	Views
Lois	A*	20	6	Traditional
Janice	A*	20	4	Traditional
Jane	A	12	3	Traditional
Susan	B*	8	3/4	Very Traditional
Gaye**	B*	6	6	Mixed
Elise	C	25	5	Mixed
Faye	D	28	5/6	Mixed
Rose**	E	15	2	Very Contemporary
May	F	4	2 then 4	Very Contemporary

* These teachers were interviewed in pairs at their respective schools.
** These teachers were chosen to participate in the classroom observations and reflective interviews.

Further information about these teachers is presented in the sections that follow. Details of interview procedures are provided in the next section as well as the initial interview analysis and questionnaire data for seven of the participants.

5.2 Initial Interviews

The initial interviews were used to collect detailed information from a small group of teachers who represented the diversity of views about teaching and learning mathematical problem solving. This section describes the interview procedures and presents the data obtained from the seven teachers who only participated in the initial interviews. A subsequent section of this chapter presents a detailed analysis of all of the data collected from the two remaining teachers who also participated in classroom observations and reflective interviews.

5.2.1 Initial Interview Procedures

A semi-structured procedure was used for the initial interviews. Interview questions included general background information about each teacher, descriptions of recent mathematics lessons, comments about the questionnaires in general and specific questions relating to individual questionnaire responses. After obtaining additional background information, questions focused on recent lessons to encourage teachers to relax and talk about their classes. This then led to discussions about problem-solving lessons, frequency of problem-solving lessons, desire to implement more problem solving, and possible factors that might be inhibiting teachers' efforts to increase the level of problem solving in their classrooms. Further detail about these interviews including the interview questions was presented in Section 3.2.2.1.

Additional details about the schools involved, teachers' experiences and classes currently being taught are included in each of the analyses that follow. For seven of the teachers, there is a summary of questionnaire responses and interview discussions as well as an examination of the questionnaire as a suitable instrument for collecting data about these teachers' beliefs and practices. In addition, each teacher's problem-solving focus is described along with the opportunities and constraints that have impacted on her implementation of problem-solving approaches.

As Rose and Gaye participated in the observations and reflective interviews, in-depth descriptions of their questionnaire responses and interviews is included in Section 5.3 of this chapter. These two teachers were selected because their responses were in some ways unique and because they were able to articulate their views in an interesting and informative manner.

5.2.2 Initial Interview Analysis

The interview data were analysed by reading each of the transcripts and comparing comments with questionnaire responses to determine more clearly each teacher's problem-solving beliefs and teaching practices. In addition, issues were identified that related to the use of problem-solving approaches. Every effort was made to have categories emerge from the data rather than to bring predetermined factors to the analysis.

Each report includes slightly different information since the semi-structured interview approach led to discussions about issues that related to particular contexts. In some cases, some of the initial interview questions were not addressed. As a whole the reports provide a rich picture of issues that impacted on this group of teachers as they taught mathematics and mathematical problem solving.

5.2.2.1 Lois and Janice

Lois and Janice taught at the same school and were interviewed together. Since they held similar beliefs, their questionnaire responses and initial interviews were analysed simultaneously and are presented together. Both had been teaching for about 20 years and both had been at their present school for six years. Lois was teaching Year Six and Janice was teaching Year Four at the time of the interviews. The school had about 500 students and was located in the Sydney metropolitan area. Many different cultural groups were represented although there was a large number of Asian students. Several public selective high schools and private schools were located nearby and many parents were keen for their students to attend these for secondary education.

Data collected from Lois and Janice are presented in the following sections. The first section describes their problem-solving beliefs and teaching practices. The second section reports issues about the use of problem-solving approaches that were described by these teachers.

Problem-Solving Beliefs and Teaching Practices

The preliminary analysis of questionnaire responses placed both Lois and Janice in the *traditional* category that was described in the questionnaire analysis presented in Chapter 4. This indicates that these teachers agreed more with a *traditional* perspective of mathematics teaching and learning and they tended to reject the more *contemporary* perspective. In particular, they agreed with the statements that proposed that students should learn basic facts before solving problems and that problems should be left to the end of topics in mathematics. However, both disagreed with the proposal that mathematics lessons should focus on practising skills. From the more *contemporary* perspective, both disagreed with the statements that it is a good strategy to begin topics with unfamiliar problems, and that students should be encouraged to explore their own ways of doing mathematical questions before being shown the teacher's methods.

These beliefs were confirmed during the interviews. Basic skills were viewed as necessary knowledge before students could do problem solving. Lois stated

> it seems to me that students who aren't particularly capable in maths will only become even amenable to sitting down and trying to solve a problem if they know some way of doing that. And there's really not much point in saying well you could read any of these if you can't add or subtract or multiply, then it's like learning something in Dutch.

Rather than just focusing on skills, both teachers indicated that variety was needed in mathematics lessons. A focus on problems alone was viewed as boring. To support this, Janice described asking her class how they prefer to learn mathematics. The students indicated that they liked to use equipment and different textbooks.

While both teachers advocated a variety of activities and experiences in mathematics lessons, they reported frequent use of exercises and application problems. Less frequent use was made of open-ended and unfamiliar problems since these types of problems were viewed as more difficult for students. Lois stated

> I don't really think that's particularly fair ... you respond much better if you go from something you know to something you don't know. From something you feel comfortable with to something new, and I think the students learning maths are in exactly the same position.

Lois continued this line of thinking and indicated that using unfamiliar and open-ended problems did not work, particularly with her current class. She stated

> They really do need, they need focusing and they need to feel what they're doing is important and I have discovered that even with very bright students unless you indicate that there is a way to solve this and it's important that you find that way, lots of students don't respond to airy fairy do this anyway you like.

Lois and Janice frequently used a variety of teaching strategies. These included whole class discussion at the end of lessons, cooperative group work, and concrete materials. Less frequently used strategies included presenting problems with little indication of how to solve them, allowing students free choice of problems, and spending several lessons on the same problem. Discussion during the interviews supported this data and in particular, Lois commented that the latter group of strategies did not "work with the kind of class I had last year and the class I've got this year. That would be a recipe for disaster". It became clear from these discussions that in both teachers' classrooms, students were given short tasks with guidelines as to the process required for task completion. Both teachers believed that a structured approach was desirable in mathematics lessons.

During the interviews, information was sought about current classes and teaching approaches. Lois was teaching a mixed ability Year Six class of 22 students although the more able students in Year Six were in a separate A-streamed class. She noted that the size of her class was most unusual as several of her students had left the school during the year. Recent mathematics lessons focused on revision for half-yearly examinations and recent problem-solving lessons incorporated Olympiad-type questions to be solved in small groups. She indicated that when forming groups, more capable students were placed in groups with less able students so that they could help with difficult questions. It was confirmed that for Lois, problem solving meant solving Olympiad-type questions.

Lois' decisions and planning centred around the needs of the students in her class. Typical lessons involved offering individual support, particularly for the less able students. She normally does little problem solving since the students need practice on basic skills and generally find problem solving difficult. Interestingly, in response to questions about students' needs she added "if I had an A class or when I've had A classes then the answer would be different", thus suggesting that problem solving was considered appropriate for the more able students.

Janice's class contrasted with Lois' in several ways. She was teaching an A-streamed Year Four class of 35 students. This meant that the students were much more capable. Also, larger class group crammed into a small classroom restricted the use of group work and other practical activities.

Recent mathematics lessons also focused on revision for the half-yearly examinations and recent problem-solving lessons incorporated questions from The University of New South Wales Mathematics Competition papers. She indicated that this year's competition was imminent and the students would need the practice. In addition, she stated

> They are a clever class and instead of giving them the Fourth Grade [paper] I gave them the Sixth Grade paper. And some of them flew through it easily in the time. There is a group of about eight who are really quite advanced and they do a lot of outside maths as well.

Extra tuition seemed to be common for more able students at the school and reflected parents' views that their children needed to be provided with every opportunity to be placed in selective high schools after completing primary education. This was typically the case for many of the Asian students.

Janice's decision making about mathematics lessons and her use of problem solving were heavily influenced by the school program and class textbook. She mentioned a "fairly tight program within the grade" that meant she focused on program completion and considered it necessary for students to complete all tasks presented in the textbook. Janice indicated that she would have to use time allocated to other subjects in order to do more problem solving with her students. Most students worked alone but extension was provided on an individual basis as assigned work was completed. Throughout the interview, mention was make of the constant need to get through the work with students successfully completing a large number of assigned tasks.

Janice's beliefs about appropriate mathematical activity were evidenced in several of her comments. She indicated that the students liked to achieve and did not object to doing many similar tasks provided they scored full marks. She encouraged this approach and stated "the goal is to get 100 percent and I do reward that because they are all totally capable". In addition, she stated "I think they're fairly highly trained at this school to zoom through things". This view of mathematical activity is further discussed in the next section as one of the issues about the use of problem-solving approaches.

Issues About the Use of Problem-Solving Approaches

Several factors or issues arose that seemed to impact on the use of problem-solving approaches at this school and in Lois' and Janice's classrooms. Some were identified in questionnaire responses while others became apparent during interview discussions. The issues could be broadly grouped as those relating to students including affective factors, beliefs about mathematics, and particular student needs. Another group of issues related to school factors including the use of textbooks, a formal program, and assessment procedures. Finally, there were issues related to parents' expectations, out of school tuition, and goals of future attendance at local selective and private high schools.

Lois noted on her questionnaire response that one issue relating to the use of problem solving was the potential to increase student anxiety. She considered it to be more important to increase students' confidence in mathematics. This issue also arose during interview discussions and was clearly of concern to Lois, particularly with her current, lower-ability class. She also suggested that her students were "often put off by the fact that more than one solution is correct. Being right is more important than being investigative or adventurous". To reinforce this view, Janice stated during the interview that

> The teacher of 3A was saying that she was doing this sort of thing with them, problem solving in groups, but kids kept coming up and saying "can't we just get on with our work?"

Both Lois and Janice believed that problem solving meant doing difficult questions and was therefore an appropriate approach for more able students. However, they also felt that most students did not really enjoy problem solving and preferred to do textbook-type tasks that were viewed to be of more value by the students.

On her questionnaire, Janice noted the use of textbooks, the highly structured curriculum and formal assessment procedures at the school, and the focus on entering Mathematics Competitions. She also described the pressure to complete an extremely crowded curriculum and admonished the researcher for focusing on problem solving rather that on considering *all* aspects of the mathematics curriculum as well as other subjects. She commented that students at the school were very successful in competitions and "do very well at high school without a great deal of time spent on problem solving".

When these issues were raised during the interview, both teachers indicated that there was much more to mathematics than just solving problems. It also became clear that Lois' and Janice's beliefs about learning mathematics and the role of problem solving in learning mathematics were reinforced by the approach taken at local high schools and at after-school tuition classes. Lois noted from her daughter's experiences of mathematics at a local selective high school, that problem solving was non existent. She described her daughter's mathematics lessons as

> all they did was slog, slog, slog and more slog. Practice, practice and more practice. Algorithms, algorithms and more algorithms. Algebra, algebra and more algebra.

This view that mathematics involved practising skills, doing repetitive exercises and completing tests was also the approach adopted in the after-school hours tuition classes attended by many of the students from this school.

In this particular school, streaming seemed to be one of the factors that influenced teachers' decision making. Streaming occurred on the basis of results in end-of-year examinations whereby marks were used to determine placement in classes. Four hundred marks were allocated to English and 300 to mathematics. These marks incorporated reading, writing, language and spelling as well as Space, Measurement and Number. Streaming occurred in Years Three to Six although a decision had been made to stream in Years Four to Six from the following year.

Parents' expectations were not discussed in detail but it was clear that parents wanted their students to achieve well in mathematics. To this end, extra tuition was common and the desire to have students attend local selective high schools for secondary education was high. Success in mathematics competitions was highly valued and the school community was proud of these achievements.

The questionnaire analysis placed both teachers in the *traditional* category for problem-solving beliefs. Interview comments confirmed this interpretation of their responses as it was evident that they both emphasised basic facts and skills acquisition in mathematics lessons before problem solving was considered. Mathematics was presented to students using a highly structured approach typically involving individual work from prescribed textbooks.

Problem solving appeared to be viewed as an added extra to the curriculum with problems tending to focus on those presented in the Mathematics Olympiad or other competition-type questions. As a consequence, for Lois and Janice, problem solving was viewed as an object of inquiry and as a suitable extension activity for more able students. Thus contrasting with the view that problem solving can be a process of inquiry, or a teaching approach suitable for *all* students. In fact, both teachers rejected this view and were highly critical of this approach during the interviews.

Both teachers were clearly happy with their approach and their efforts in teaching mathematics. The researcher experienced a sense of hostility during the interview when they described the pressure to cover all aspects of the curriculum and the lack of resources and time to do all that was required. Elaboration of questionnaire comments confirmed the potential constraints in this particular context of a school culture that focused on a high level of success for its students based on parents' and school community expectations. In addition, the school curriculum was highly structured with an expectation that programs would be completed in association with prescribed mathematics textbooks.

Lois' and Janice's beliefs seemed to have been reinforced by the students' success in mathematics competitions and in gaining entry to selective schools, by the students' views, and by the way mathematics was presented to students at local high schools and in tuition classes held in the local area. Several of these issues also arose when the third teacher from this school was interviewed.

5.2.2.2 Jane

Jane was teaching at the same school as Lois and Janice but she requested an individual interview. She had been teaching for 12 years including two years at her current school. Jane had taught in several different schools during her teaching career and she had spent several years working with hearing impaired children spanning the Kindergarten to Year Six range. At the time of the interviews, she was teaching a mixed ability Year Three class of 31 students that did not include the most able students as these had been placed in an A-stream class. The following sections outline Jane's problem-solving beliefs and teaching practices as well as the issues she identified as impacting on the implementation of problem-solving approaches.

Problem-Solving Beliefs and Teaching Practices

In the questionnaire analysis, Jane was identified as having a *traditional* belief system since she agreed with most of the traditional statements and disagreed with most of the contemporary statements. In particular, she indicated that she believed that students should learn basic facts and algorithms before they did problems and that students could not solve problems until they knew how to perform the four operations. She also believed that mathematics lessons should focus on practising skills and that students had trouble solving problems unless they knew how to do the mathematics before they began. She rejected the view that it was a good strategy to begin topics with unfamiliar problems and that mathematics lessons should focus on problems. She also disagreed with the statements that students could learn most mathematical concepts by working them out for themselves and that it was essential for students to explore their own ways of doing mathematics questions before being shown the teacher's methods.

Jane indicated that she often used exercises and application problems with her class and only sometimes used open-ended or unfamiliar problems. The recently used problem that she recorded was an example of an application problem. She believed that these were more suitable for her students as she stated

> Only the top 10 or so in my class can handle unfamiliar problems. The whole class need strategies to help them solve problems, therefore application problems are more appropriate to my class as they can use known strategies ...

It became clear from Jane's interview responses that she believed that only the more able students could cope with open-ended or unfamiliar problems. She felt that exercises and application problems were more suitable for the lower ability students. In addition to this, she indicated that the high number of English as a Second Language (ESL) students would have difficulty solving many problems because of the language they contained.

Jane's questionnaire responses indicated that frequently used teaching strategies included students working alone, whole class discussion, concrete materials, and discussion about problem-solving processes and strategies. Less frequently used strategies included small cooperative groups, encouraging students to record their own procedures, posing their own problems, or choosing their own problems.

The reported infrequent use of small cooperative groups was unusual compared to the majority of questionnaire responses. In fact Jane was the only respondent to indicate that she hardly ever encourages students to work in small groups. Jane provided further comment about this in response to the open-ended question that requested a reaction to a negative comment about problem solving. She stated

> I don't believe it is a waste of time. The children develop skills that help them in life, not only in mathematics. The children however should be given strategies to help them solve the problems. I also believe that children should work alone to try and solve the problem then get together as a small group to discuss possibilities as in my experience the more able ones always take over a group. If the child can work it out themselves (or partly work it out) then the child would be more confident to contribute in a group situation.

During the interview, Jane commented that it is difficult in her class to use small groups because of the wide range of abilities. She indicated that she had a student who had just come first in the grade and another who was placed 97th so it was easier to teach the class as a whole. She stated

> I find that a much easier way to teach maths is to teach up the front and stand up the front and explain to the whole class and then go around individually and make sure they've understood. It is a much easier way for me to teach.

Jane's most recent mathematics lesson involved feedback to students from a diagnostic test but she indicated that the lesson before had involved addition and subtraction using trading with base ten materials. Students were using the materials to answer questions from their textbook. She indicated that she did very little problem solving with her class but tended to do more "drilling how to add and subtract".

Jane reported that the students had three different textbooks. These included an ordinary mathematics textbook, a mentals book and a problems book. The latter included word problems that were mainly application problems. Jane stated that

> I'll give them 10 problems to do and they have to do the key phrase, the number sentence and the answer sentence and working out.

She agreed that this was a very structured approach to problem solving. She also stated that even though she frequently used concrete materials, these were used to answer textbook questions rather than to investigate or to do other types of questions.

Jane agreed that this highly structured approach to problem solving was very limiting and restricted the kinds of things that she could do with her students. This was one of the issues that influenced her use of problem-solving approaches although others were also identified.

Issues About the Use of Problem-Solving Approaches

Jane indicated that she would prefer to do more problem solving but there were several factors inhibiting this approach. Issues impacting on the implementation of problem-solving approaches were identified as a lack of time associated with a full program, use of set textbooks, streaming of students, parents' expectations, and formal examinations. Jane believed that she would be confident to do problem solving with a class if she had the opportunity.

The reasons Jane wanted to do more problem solving included notions of developing students' thinking skills and preparing them for future needs. She stated "I think problem solving is great for the children's thinking skills" and later she added

> ... well I really think it develops critical thinking skills and I think that's very important for children to develop higher level thinking skills and that goes through all their work. Not only mathematical problem solving but just problem solving in general. I would like to do more, like they do in OC classes, a lot of the problem solving, critical thinking skills, the six hats and all of that sort of stuff, yeah, I would love to do that but we just haven't got the time ...

Jane suggested that the approach at her current school was too structured and narrow and did not lead to the development of critical thinking. However, she indicated that the A-streamed classes may have more opportunity to do problem solving.

Jane commented that she felt considerable parental pressure to have students succeed, particularly in mathematics. She believed that the Asian parents were keen for their students to be placed in the A-streamed class, and to be ultimately chosen for selective high schools. She stated

> so we've got that sort of pressure and that sort of pressure necessitates that we do have to do a lot of drill ... we do have to do tables and do the old fashioned method of, and it works I mean I'm not saying anything against old fashioned methods ...

This statement provided further evidence that Jane was quite comfortable with a more traditional approach to teaching mathematics and believed that it was a successful method of teaching.

Jane's questionnaire responses indicated that she held somewhat *traditional* beliefs about problem-solving in mathematics. Her interview responses revealed some inconsistencies in that she supported problem-solving approaches and yet her practices appeared to be very traditional. However, Jane's comments did at least suggest that she recognised that problem solving was a worthwhile learning approach and could provide students with the opportunity to learn thinking strategies. In this way, she seemed to view problem solving as both an object of inquiry as well as a potential process of inquiry.

Jane was obviously aware that she was not implementing problem-solving approaches in her current classroom and was willing to suggest reasons for this. She readily presented a list of constraints and suggested that in a different context she would teach quite differently. In addition, it appeared that her desire to do more problem solving was tempered with a view that it was more appropriate for more able students and she would need to have a less structured program with more time available.

The main issues that impacted on Jane's approach to teaching mathematics in this school were similar to those outlined by Lois and Janice. The highly rigid program of work, use of mathematics textbooks, formal assessment procedures, streaming of students, and parents' expectations all impacted on Jane's practices. She clearly felt considerable pressure from the parent community as well as pressure to get through the curriculum. There was a sense of preparing students for future high school experiences, even at the Grade Three level. Parents wanted their students to be in the A-streamed classes and were very keen to see them succeed, particularly in mathematics. It was possible that these strong parent views were culturally based as this was not evident in other settings with fewer Asian students.

5.2.2.3 Elise

Elise was teaching in a medium sized primary school with about 15 classroom teachers that was located in a semi-rural area on the outskirts of Sydney. The school population was mostly Celtic with few students of non-English speaking background. The Principal was supportive of the research and was keen for his teachers to participate in the study. During interview discussions with Elise it was revealed that the teachers were encouraged by the Principal to use innovative practices and there was considerable freedom in choosing appropriate teaching approaches.

Elise had been teaching over a 25 year period with some breaks from teaching during that time, and had been at her current school for five years. She had a variety of teaching experiences including teaching English and history in secondary schools and she had taught all primary grades from Kindergarten to Year Six. At the time of the interviews she was teaching a mixed ability Year Five class although the more able and better behaved Year Five students had been placed in a composite Year Five and Year Six class. Elise's problem-solving beliefs and practices are presented in the next section followed by issues relating to the implementation of problem-solving approaches.

Problem-Solving Beliefs and Teaching Practices

Elise's questionnaire responses suggested that she may have held a mixed set of problem-solving beliefs. Her views, however, were unclear since she had disagreed with many of the beliefs statements in both of the first two questions thus seemingly disagreeing with *traditional* as well as *contemporary* perspectives of mathematics teaching and learning. Although this suggested some contradictions in her beliefs, her responses to the open-ended questions seemed to indicate that she held more *contemporary* beliefs about problem solving and problem-solving teaching approaches. This was confirmed during interview discussions as it became apparent that Elise held strong views about the value of problem solving and the need to ensure that students understand what they are doing in mathematics. These views had been reinforced by Elise's experiences as a learner of mathematics.

Elise indicated that it was important for students to understand what they are doing and that the teacher's role was critical in this process. She reflected that she had been disappointed with her mathematical experiences at school and so she returned to evening classes at a technical college to repeat a senior secondary mathematics course. She stated

> ... [we] went over all of the things that I hadn't really understood. I was doing it but didn't know why and I had this most brilliant teacher who just opened the whole door for me and from then on it went just from being a bogey to one of my favourites and you know if you don't get someone who inspires you with that confidence and then teaches you how, I think it's often going to be a bogey for a lot of children.

Discussions during the interview confirmed that there may have been some misunderstandings when Elise recorded her questionnaire responses. These were clarified and tended to be based on which grade level Elise was considering or what the mathematical ability of the students may have been. While she had disagreed with the statement that students should learn basic number facts before they do problems, she acknowledged that it is certainly helpful for students to know their basic facts. However, she described a problem-solving situation where Kindergarten children could solve a problem without knowing formal procedures and symbolism. She stated

> ... if there are three children there and there are four children there and we push them all in together, how many have we got? So, I haven't formally taught the process but they are aware that we're putting them together or we're making more so that is the concept of addition ...

On the questionnaire, she had agreed with the statement that some students have trouble solving problems unless they know how to do the mathematics before they begin. When this was raised, Elise commented that her lower ability students did have considerable trouble solving problems in mathematics and did need to know their basic skills in order to be able to attempt problems.

A variety of teaching strategies were employed in Elise's lessons as she had indicated on the questionnaire that almost all strategies were used *often* or *sometimes.* She reported that she often used all four of the student question types and stated that "all are equally worthwhile when providing an overall experience". The problem recorded on Elise's questionnaire was an example of an open-ended investigation. She wrote

> How could we show the variance in heights [of students] from the beginning of the year to the end of the year?
>
> Children solved in their own way and we discussed various methods used to solve and ... which were most effective.

She explained that the class had been studying graphs and the question was useful because "there isn't necessarily one way to solve it". She also indicated that she liked using open-ended problems that provided the answer at the beginning as students tended to feel more confident with the problem.

Elise reported that she sometimes used calculators. Further discussions revealed that these were used for checking work rather than for investigations. She did not want students to rely on calculators and preferred that they work things out using basic facts or algorithms.

When asked to describe a recent mathematics lesson, Elise outlined the previous day's lesson involving decimals, fractions and percentages. She indicated that she had begun the topic with problem solving by talking about dividing pizzas into fractions and discussing the percentage each piece represented. She indicated that she liked to start with a concrete representation before moving to more formal methods, particularly for less able students.

Elise believed that it was part of her role to teach the problem-solving process and she likes to teach the students to unpack the question in order to decide what the question is really asking. She suggested that the number of words in unfamiliar problems can make them look difficult but she encouraged the students to sift through the jargon for the key ideas. These discussions revealed a misinterpretation of a questionnaire item. Elise had interpreted the statement "you model the problem solving process to the class" as using concrete materials to make a model to help solve a particular problem rather than working through problems as a role model for the students.

Interview discussions revealed Elise's belief that problem solving was an important part of the teaching and learning process as was the need for students to be confident in mathematics lessons. She stated

> I do problem solving all of the time. At least twice a day even if it's just lateral thinking, puzzles, group work. Now this is what we're going to do, what do you think is the best way to do it so you have to plan which skills will we use. I use it all of the time because I think it is one of the most important things to teach.

In addition to this, Elise was concerned that less able students maintained interest and confidence in their mathematics. She mentioned the importance of the students knowing that they don't have to understand the work the first time they meet the ideas. She stated

> ... my teaching of maths says that you don't have to understand it straight away at all, don't worry if you don't. If you understand a little bit you'll lose it tomorrow and you'll get it back the next day and it's that gradual process of relaxing.

Textbooks were used as the main source of problems although Elise indicated that one particular textbook provided a focus for her teaching. She stated that

> ... I use it also for programming because it covers everything so I tend to work through it fairly well ...

She described her daily approach as using a particular page as a focus for the lesson and then, when necessary, supplementing the problems with her own. She did not feel constrained by the use of the textbook in her class since teachers at the school were free to choose whether they use it. In fact few issues arose that seemed to have impacted on Elise's use of problem-solving approaches.

Issues About the Use of Problem-Solving Approaches

Time was raised as the key issue preventing Elise from doing more problem solving. She indicated that she tried to balance teaching basic facts and skills with creative activities but she often needed to set skills practice for homework as there was not enough time at school to do all that was required.

Another issue was the students' ability and the need to cater for individuals. She believed that some students did have trouble with problem solving and she suggested that there appeared to be two particular issues involved here. One was the low general ability level of some students and another was that there were some more able students who "just have a problem with maths". She believed that the latter group of students often lacked confidence in mathematics and just needed to experience success and to be encouraged. She also indicated that for these students, she reinforced the belief that mathematics was a set of unchangeable rules. She stated

> ... I say to them that in English you can learn a rule but that rule changes five or six times for spelling but in maths if you take the time to learn the rules it's never going to change and that's the most comforting security blanket thing about maths.

Elise's response to the final question on the questionnaire seemed to summarise her beliefs about teaching mathematics. She stated

> Providing children with as many and varied mathematical experiences as possible to make maths enjoyable, is, I feel essential. Plus giving them the confidence to try, and making them realise that understanding maths is for most a gradual thing and to learn little by little is acceptable, and is in fact admirable. Basic learning of number facts is essential to further learning success.

Even though Elise's questionnaire responses suggested that she may have held a mixed set of views incorporating both *traditional* and *contemporary* perspectives, the interview seemed to present an impression of a teacher who used problem-solving approaches daily and was committed to this approach. At the same time, she emphasised skills and structure in mathematics and believed that mathematics was a fixed set of rules and procedures. This was compatible with the other aspects of the questionnaire. It seemed that experiences related to her own learning, and particularly to understanding, have had an impact. Certainly the supportive school seemed to help.

Even though problem solving was a key part of her teaching, Elise was concerned that some lower ability students had difficulty. This, and time, seem to be constraining features. In her case, some questionnaire responses were potentially misleading, but not the questionnaire as a whole. Elise had coherent, well articulated views, and was able to illustrate and elaborate her approach using examples. Personal experiences, and her beliefs seemed to have an impact on her practice. She had views orientated to problem solving and her practice seemed to validate these.

5.2.2.4 May

May had changed schools between completion of her questionnaire and the initial interview. In the questionnaire, her responses indicated that she held *very contemporary* beliefs about problem solving in mathematics. However, during interview discussions, May suggested that her attempts at problem solving in her first school may have been overambitious and that she had neglected the basic skills of her students thus indicating that there had been a tempering of May's reported problem-solving beliefs. Further discussions revealed that she still held *contemporary* views but she now believed in a more balanced approach to teaching mathematics.

At the time of questionnaire completion, May was teaching a Year Two class in a small public school located in a lower socio-economic area in Sydney. This was May's first appointment as a mature-age student from university and she described the school as difficult to teach in. There was a large staff turnover which led to many inexperienced teachers working in the school. May was in her fourth year of teaching and was Mathematics Coordinator, responsible for organising equipment and supporting fellow teachers in planning mathematics programs.

May received a transfer early in her fifth year of teaching to a slightly larger school in a working- to middle-class area that had a far more experienced staff with many teachers who had been at the school for more than ten years. This was probably related to the belief that this was a good school located in a more affluent area. May noted that the approach to teaching mathematics in her new school was far more traditional. At the time of the interviews she was teaching a mixed ability Year Four class. May's problem-solving beliefs and practices and issues related to teaching problem solving are outlined in the following sections.

Problem-Solving Beliefs and Teaching Practices

In the questionnaire, May disagreed with all of the *traditional* statements with the exception of the last item involving problem solving difficulty being based on the language involved. She agreed with all of the *contemporary* statements thus placing her in the *very contemporary* category in the questionnaire analysis.

Her reported teaching strategies were also typical of those used frequently by this group of teachers. She indicated that she *hardly ever* explained in detail to the students what to do to solve problems. She *almost always* presented students with problems with little indication of how to solve them, and frequently encouraged students to record their own procedures and pose their own problems. She *often* had calculators available for students to use and she indicated that she regularly used a variety of problem types. May supported the use of calculators from Kindergarten upwards because they helped students develop confidence and "some of them will learn by using them".

May indicated that she *often* used open-ended and application problems, *sometimes* used unfamiliar problems and *rarely* presented exercises to her students. Her response to why she preferred to use such problems was that

> Open-ended problems allow children to bring their own knowledge and strategies to the task as well as respond at their own level. My children write their own problems which are often in the form of application problems.

The example of a recently used problem was also indicative of May's preference for investigative activities. She stated

> Students were asked to investigate which numbers were the most common when 2 dice were rolled and totalled. They were then questioned as to why the "7" was the most frequent.

> [This was] used to illustrate the concept of chance, develop data skills and further develop addition skills.

During the interviews, information was sought to support the views outlined in the questionnaire. May indicated that she would now probably qualify some to the statements she had rejected or supported in the questionnaire. While she still agreed that "problem solving can actually teach you some of the basic number facts", she now felt that this was not the case for all students. To support her change in views she said "I think I was going too far into the problem solving and not giving them enough of the traditional stuff". However, she still supported the use of problem-solving approaches since she stated

> I like the idea of using problem solving because a lot of the problems you use relate maths to the real world and I know that every time I mention mathematics in the real world with my class they immediately attend much better than they normally would ... it does motivate them because they like a challenge. Any child likes a challenge.

In addition to motivational aspects, May also liked problem solving since

> It allows them to use strategies that they may normally not be given the opportunity to use and to respond at the level at which they feel comfortable. You can give the same problem to a wide group of children and each will respond in their own way and you learn a lot more about the way they're thinking and the way they're solving things by giving them such an open-ended problem.

One aspect of May's views that she had not changed involved the practice of algorithms. She stated she was not convinced that the students needed copious sets of algorithms to solve since

> ... we're taught to subtract a certain way now and with all the trading and all that, a lot of children don't use that. They compute mentally in their heads, they're not going to use all that formal way of doing it. I think some kids need to know how to do it that way because some of them don't have that number sense. Um, so it's a strategy for them but they need to be shown more than one strategy.

May's recent mathematics lesson involved the students estimating the mass of a variety of items and comparing these to masses of one kilogram. The students then recorded their results. She indicated that there was little problem solving in the lesson as most students had little difficulty with the task and demonstrated a good understanding of the kilogram. Recent problem-solving lessons had involved searching for patterns in the hundreds chart and on the multiplication chart.

One problem-solving lesson she described in detail involved the students exploring the circumference of various items. She wanted the students to find objects with a circumference of about 30 centimetres. To assist them she discussed a problem-solving process that she described as "see, plan, do, check" that she indicated was based on

Polya's model of the problem-solving process. The lesson highlighted the students' lack of knowledge of the meaning of *circumference* thus providing May with valuable feedback for future planning. She also described students' unsuccessful attempts to measure circumference with a 30 centimetre ruler leading them to the choice of a more suitable strategy.

May was pleased with this problem-solving lesson even though the students struggled with many of the ideas. She indicated that the students were not used to the approach she had taken but that she tried to use a problem-solving approach to introduce new ideas. This was countered with the comment that she believed there is a need for a balance of problem solving with skills practice. However, May also indicated that she would like to do a thematic unit with her class later in the year when they become more familiar with problem-solving approaches. She thought she would try a unit on dinosaurs and

> ... there's another one I've always wanted to do with houses where they plan the house and they do all of the costing and furnishings and all that sort of thing, draw up floor plans and actually physically plan it in the playground.

This slight change in views from May's questionnaire responses to her interview comments seemed to be influenced by the new school setting with its different culture as well as a change from teaching in the lower primary to middle primary grades. This latter factor was acknowledged by May when she said "the older the child the more formal you tend to become". She agreed that there was more of a place for algorithm practice in Years Three to Six and that this meant teachers were more inclined to set exercises in these grades than in the lower primary grades.

May indicated that there was more freedom to develop ideas in the early years of primary school. In her previous school she had spent two years teaching Kindergarten classes where she encouraged the students to make up stories and write them down for others to solve. She also conducted whole-class discussions about application problems she had created and taught the students the problem-solving process of "see, plan, do check". In her new setting, there seemed to be more constraints influencing her attempts to implement problem-solving approaches.

Issues About the Use of Problem-Solving Approaches

The key influencing factor preventing May from implementing more problem solving was time. She spoke quite passionately about the restrictions on her teaching time in her new school. She stated

> The way the timetable is structured, I can teach four lessons of maths properly, or not even properly a week. We're looking at half hour or 40 minute lessons plus I can do a mentals

> lesson really quickly on a Friday and it's just not enough maths for me. I just have too many disruptions. Time is a big factor.

The disruptions May was referring to included library lessons, classes in languages other than English, sport on Friday, choir rehearsals and assemblies. She indicated that the disruptions at this school were far worse than at her old school, mainly because of the compulsory language lessons.

May indicated that she believed the Principal in her current school would support her problem-solving approaches but that the staff were far more cynical as they had seen many changes introduced during their teaching experiences. May commented that she could understand why these teachers had such views.

From her questionnaire responses, May suggested that teachers needed more inservicing on the definition of a problem, problem-solving strategies for the classroom, and "modelling in the classroom on how to teach problem solving effectively". It was also apparent that she believed that teachers who did not support problem-solving approaches did not understand what it was all about, and in particular, the theory that problem solving was based on. In her new school, May believed that problem solving would have to be "sold" to the teachers and they would need to be convinced that it was worthwhile if they were to incorporate problem-solving approaches into their teaching. Also, it would need to be presented in such a way that their current practices were not criticised.

May clearly believed in problem solving as a process of inquiry. It appeared that her initial enthusiasm in using problem-solving approaches in her first school was strongly influenced by her experiences in preservice education courses. She had been a student of the researcher and had embraced much of the advice from the problem-solving literature. In practice, May had found it easier to implement such approaches in the Kindergarten to Grade Two classes than in middle primary grades. In addition, her first school had a younger staff, possibly with similar enthusiasm to such ideas.

In her second school, May was encountering a more conservative teaching staff whose influence seemed to be causing her to reflect on her practice and to question her views about mathematics teaching and learning. It appeared that she was still convinced that there was a place for problem-solving approaches but she now considered the need to present students with more skills practice and a more structured learning approach to algorithms. May voiced some discomfort in expressing this change in views to the researcher but she had clearly considered both perspectives and was prepared to discuss reasons for her change in views. She confided that another factor that was impacting on her current approach was the lack of time both in the length of mathematics lessons and the number of mathematics lessons available each week.

While there appeared to be some inconsistency in May's current views and her questionnaire responses, she recognised this and acknowledged that she was in the process of reconsidering her approach. She was doing little problem solving with her current class but she wanted to do more and was already planning future problem-solving lessons. It was evident that if May had been required to complete the questionnaire at the time of the interviews, her views may have been classified as *contemporary* rather than *very contemporary.*

However, the questionnaire had still been a useful tool for collecting information about May's beliefs and practices and in identifying key issues. It is recognised that questionnaire responses will only provide a snapshot of information gathered at a particular time and place and it is also accepted that beliefs are fluid and subject to change.

5.2.2.5 Faye

Faye was an experienced Assistant Principal of a medium sized public school in a low to medium socio-economic area in Sydney. She had been teaching for 28 years that included four years of maternity leave. She had taught in several different schools in the same area and most of her teaching had been in the lower primary grades although more recently she had been teaching upper primary. At the time of the interview she was teaching a mixed ability composite Year Five and Six class.

On the questionnaire, Faye's responses placed her in the *mixed* category for views about problem solving in mathematics. During the interview, evidence was sought to further identify Faye's beliefs and practices in mathematics. These are presented in the next section followed by issues that were discussed in relation to the implementation of problem-solving approaches.

Problem-Solving Beliefs and Teaching Practices

Questionnaire responses suggested that Faye had a *mixed* set of views about mathematical problem solving although she seemed to favour a more *contemporary* approach. She disagreed with five of the eight *traditional* statements and agreed with three of the *contemporary* statements. Interview discussions supported the notion that Faye held a more *contemporary* view as she was clearly in favour of using problems in mathematics lessons. In addition, it appeared that Faye viewed problem solving as both an object of inquiry as well as a process of inquiry.

Favoured teaching strategies included those more frequently used by the *contemporary* teachers in the study. These included whole class discussion, cooperative group work, presenting problems with little indication of how to do them, encouraging students to record their own methods and pose their own problems. Calculators were readily available but were mainly used to check solutions. Faye suggested that as calculators were part of our world, we should teach students to use them confidently as another tool.

Faye reported that she frequently used a variety of problem types including "mind benders" that seemed to resemble puzzle type problems. She believed that variety was important as

> ... students need as many strategies for solving problems as possible. I also like to challenge students' thinking to see if they can apply knowledge and numeric understandings in a variety of situations.

Interview discussions indicated that Faye liked to use problems after students had learned mathematical skills and procedures. She stated

> what I usually do is I teach the concept first and then we give it a bit of realism because there's no point in the kids learning a concept unless it is put into some sort of real context.

Faye believed that problems provided a purpose for learning mathematics and made mathematics more enjoyable for the students. She commented that problem solving was an important teaching strategy since problem solving skills "are applicable to all sorts of life situations". Faye was also concerned about students' attitudes to mathematics and the impact of negative views on future mathematical learning. She believed that problem solving was motivational as her students enjoyed problem solving. She stated

> I think it enhances maths actually because it makes it more enjoyable for the kids and I think if kids leave primary school hating maths they're going to hate maths for the rest of their life. And one of the things I really aim to do is to get kids loving maths and discovering it's easy and there are lots of short cuts and it's not this big burden thing that they have to carry off to high school ... if they go off enjoying maths they're going to do maths and they're going to do it well. If they go off hating it, it's going to be the first thing they drop and it's going to be the bane of their lives.

To promote positive attitudes, Faye was keen to see students attempt the variety of problems she presented to the class so she focused on the processes involved rather than the correct answer. In addition, Faye suggested that another reason her students enjoyed problem solving may have been her own positive attitude to it. She stated "we leap around and if we make a mistake we all have a good laugh and we try again". The students were encouraged to support each other's efforts and to assist where possible.

For mathematics, the students in Years Five and Six were graded into mathematics ability groups. Faye was teaching the more able group although she believed that all students were capable of doing problem solving if the right questions are chosen. To support less able students or students who were having difficulty understanding the work, Faye was available during most lunch times to offer extra explanations. She indicated that a group of five students had been to see her that day for individual lessons. This commitment to the students was further evidence of her enthusiasm and interest in supporting their learning and encouraging them in mathematics.

When describing a recent mathematics lesson, Faye indicated that the students were learning about perimeter of common shapes. Her most recent problem solving lesson involved finding discounts as percentages using application problems. She was very positive about teaching mathematics and using problem solving as a part of that process. There seemed to be few issues that impacted on her efforts to use problem-solving approaches with her students.

Issues About the Use of Problem-Solving Approaches

Faye indicated on the questionnaire and during the interview that there were no real factors operating that constrained her use of problem-solving approaches. This was not surprising as the school had a reputation for innovative practices in mathematics, particularly in the early grades where all students in Grades One and Two spent three of their five mathematics lessons each week in the hall participating in practical activities. The students were placed in mixed groups and were encouraged to work cooperatively on the tasks by the teacher facilitators. Most of these activities focused on the Space and Measurement strands so Number was usually the focus of the remaining lessons that were conducted in the classroom with the students' normal teacher. This approach was acknowledged as very successful by teachers and was enjoyed by students.

In addition to a focus on innovative practices in the early grades, textbooks were not used for mathematics and programs incorporated problem solving into the planning of lessons. However, Faye acknowledged that her ESL students often found the language of problems difficult but she stated that "my class enjoy problem solving". To overcome this difficulty, less able students were frequently grouped with more able students during problem-solving lessons.

She noted that parents do not always understand the approach she takes in mathematics, particularly those from different cultural backgrounds. Parents were inclined to think that the class is "playing games and kids shouldn't be allowed to use calculators" so a parent evening was organised annually, with interpreters, to explain the approach used by the teachers in the school.

Even though Faye's responses to the first two questions on the questionnaire suggested she might support some *contemporary* approaches and some *traditional* approaches, her comments in the open-ended questions indicated that she might be more supportive of a *contemporary* approach. Interview discussions confirmed that her problem-solving beliefs and practices were more *contemporary* than *traditional.* In fact, Faye was able to articulate her views and she provided considerable justification for her beliefs. She was most concerned that students were prepared for high school with positive attitudes and a sense of problem solving as an important life skill. Few factors interfered with her attempts to incorporate problem-solving approaches into her mathematics lessons and her practice appeared to be consistent with *contemporary* approaches.

5.2.2.6 Susan

At the time of questionnaire completion, Susan had been teaching for a total of eight years. This involved an initial period of two years when she found teaching uninspiring and so had a break of 12 years working in a variety of retail positions. She then returned to the classroom and was currently teaching a composite Year 3 and 4 class in a large public school in a middle class area in Sydney. Susan's questionnaire responses placed her in the *very traditional* category. Answers to the open-ended questions suggested that she was dismissive of approaches that were non-traditional.

Susan and Gaye were interviewed together as they were teaching at the same school. As Gaye held views that were a mixture of *contemporary* and *traditional* approaches, there were interesting exchanges between the two teachers. However, since Gaye was one of the teachers involved in classroom observations and reflective interviews, her responses during the initial interview are contained in Section 5.3.3. The following sections describe Susan's problem-solving beliefs and practices as well as issues that related to her potential use of problem-solving approaches.

Problem-Solving Beliefs and Teaching Practices

Susan was one of six teachers placed in the *very traditional* category during questionnaire analysis. She agreed with all of the *traditional* statements and disagreed with all of the *contemporary* statements. These views were confirmed during the interview as Susan was outspoken about her dislike for many of the new approaches to teaching mathematics as well as for those advocated in other curriculum areas. She strongly believed that students needed to spend considerable time practising tables, other number facts and algorithms.

Reported teaching strategies indicated that Susan *sometimes* used most of those listed in Question 4 on the questionnaire. She *hardly ever* had calculators available, presented unfamiliar and open-ended problems with little indication of how to solve them, or allowed the students to spend several lessons on the same problem. She *often* modelled the problem-solving process, discussed problem-solving strategies and processes, and used problems that arose from the school context. Further discussion confirmed that these problems were usually application problems.

On the questionnaire, Susan indicated that she *often* used exercises and *sometimes* used each of the other problem types. Her preference for exercises was explained as

> I prefer to train the children using a structured approach first, then enable the more capable children to attempt the higher level thinking skills.

She commented that she does problem solving as the need arises and that she does not "do it for the sake of doing it to make them all good problem solvers". Textbooks and resource books were used as the main source of problems and Susan indicated that for most mathematics lessons she followed the textbook as it outlines a procedure for her to follow. Her example of a recently used problem required the students to write a word problem to match an algorithm. This was a favoured approach of Susan's since she suggested that her students have difficulty interpreting word problems and recording the correct algorithm to match the problem. Susan's focus for problem solving was clearly as an object of inquiry.

During interview discussions Susan asked the researcher to clarify the meaning of problem solving. This led to further discussion of the types of problems that were described on the questionnaire. Susan revealed that she did not like open-ended problems because they have more than one answer so it is difficult to know if you have achieved a correct answer. She reinforced her view that problem solving involved "high level thinking" and was therefore more appropriate for the "brighter children". She had found that most students in her class were not able to decipher word problems and write the appropriate number sentence. If problems were to be used in her classroom, it was clear that these would be application problems.

While Susan indicated that she sometimes encouraged the students to work in small cooperative groups, it became apparent during the interview that this was not a favoured teaching strategy. She believed that students do not normally work well in small groups. She stated that "kids need guidance" and further

> ... it's like cooperative learning. You know putting them all in groups. It's a wonderful idea, it's a wonderful project but hey, you know who's standing there with each group, has each group got a teacher to stand there to guide, to focus ... if you've got your 5 or 6 groups, you

> know to make sure that they're each taking turns, to make sure that the dominant person isn't over-riding and treading on everybody else.

Susan commented during the interview that she did not really like the approach presented in the current syllabus for primary mathematics. She believed that not enough time was available to practise the algorithms and that there were too many different concepts to teach the students. She stated

> I still think that there are life skills that are not necessarily important to the classroom, the facts are important. To know your times tables, that's important because you've got to apply that ... the curriculum had no mentals in it. Kids need mentals because they've got to practise computing skills.

In addition, Susan disagreed with the recommendation to spend equal amounts of time on each of Number, Space and Measurement as she believed that Number was far more important and that students could learn about Space and Measurement at home. She stated that she supported the teaching of problem solving although this was viewed as an extension of Number and was particularly relevant for more able students. She wrote on her questionnaire

> I don't have a problem with the traditional curriculum or the traditional approach to problem solving. There needs to always be a balance and I see Number as an essential element in mathematics ... There is too much weight put on Space and Measurement in the K-6 Curriculum and not enough time given to Number, eg. Tables and Algorithms. I see problem solving as an extension of Number, ie higher level thinking skills. Children need structure! Many, many parents would agree with me!

To support her last statement, Susan surveyed the parents of students in her class and tabled these during the interview. Eight parents responded and of these, only two agreed that there should be more time spent on Number thus seemingly contradicting the above statement. Susan was also critical of advice to use base ten materials to teach place value and support algorithm development but in her survey only three parents indicated that these materials were not necessary for teaching such processes. In response to Susan's question asking if there "should be more emphasis on basic facts and operations", five parents agreed.

Overall, these parent responses were somewhat surprising. Susan's questionnaire comment was probably correct in that most parents seem to want teachers to ensure that students know their tables and can perform algorithms. It is possible that the eight parents who responded to Susan's survey were those who understood the questions, disagreed with some of her statements, and so felt inclined to comment. The lack of support for her views on these surveys did not worry Susan at all. She still firmly believed that she was correct and would continue to focus on Number and skills practice.

Learning tables was viewed by Susan as essential for mathematics learning. She believed that knowing tables gave students confidence. She reported that she had suggested to parents to send their children to Kumon Mathematics lessons after class as she did not have enough time to spend on tables practice during lessons. In addition, Susan believed that to learn mathematics effectively a structured approach was desirable. The Kumon lessons were highly structured with students individually completing pages of repetitive exercises. As the student's speed and accuracy improved, more difficult exercises were presented.

When asked to describe a recent mathematics lesson, Susan indicated that her class had recently completed a test and she was reviewing concepts that were not understood. She suggested that even though the students could perform the algorithms with the base ten materials they could not always repeat the process in a test when the materials were not available. This was one reason she was not in favour of using concrete materials so the students were practising "pages of algorithms" without them.

Issues About the Use of Problem-Solving Approaches

Several factors impacted on Susan's use of problem-solving approaches. Her *very traditional* beliefs about mathematics teaching and learning were reflected in her questionnaire responses and supported by interview comments. It was clear that she did not view problem solving as a necessary skill for all students to achieve. These strong beliefs meant that Susan did not implement problem-solving approaches in her classroom as she did not see their value.

Susan was generally critical of current curriculum documents and the teaching approaches that they recommended. She believed that a traditional approach was desirable and that the best way to learn mathematics was through a highly structured approach that involved demonstrating algorithms followed by students doing copious practice. She was highly critical of new theories and the people who devised new teaching methods. At this stage in the interview, Susan became quite emotional and in referring to the new changes in the English curriculum where functional grammar had been recommended she suggested that this had been "absolutely scandalous because millions of children have suffered because of this academic blunder" and that "there should be a Royal Commission into education".

Susan held *very traditional* views about mathematics teaching and learning and did not view problem solving as an important part of learning mathematics. These views were clearly articulated during interview discussions thus confirming her questionnaire responses. Her practices were consistent with a *traditional* approach and she readily rejected the mathematics syllabus focus on *contemporary* approaches. She felt strongly enough about her beliefs that she sought parent support through a survey and condemned academics for their folly in promoting some of the more *contemporary* practices. While some problem solving was presented to students in Susan's class, these were application problems focusing on algorithm practice. It is her belief that problem solving involves higher level thinking and is therefore appropriate for more able students.

The initial interviews with this group of seven teachers provided support for the data recorded on each of their questionnaires and have further revealed key issues that had impacted on teachers' efforts to implement problem-solving approaches. The questionnaire analysis placed each of these teachers in one of five categories for problem-solving beliefs. Interview discussions sought to confirm this categorisation, to discuss reported practice for consistency in approaches, and to examine influencing factors in relation to current practice. Further discussion of these overall aims is presented in Section 5.4 at the conclusion of this chapter. The next section outlines all of the data collected from the remaining two teachers who participated in the fieldwork phase of data collection.

5.3 Classroom Observations and Reflective Interviews

A more intensive part of the fieldwork involved classroom observations and reflective interviews with two teacher participants. This stage of the fieldwork required collection of data through a combination of observations of three consecutive mathematics lessons and unstructured reflective interviews to discuss lessons and other related issues. It was anticipated that this stage of data collection would further validate questionnaire data as well as clarify and confirm initial interview comments, enable comparison of actual practice with reported practice, and illuminate factors that seem to either support or constrain the implementation of problem-solving approaches.

Rose and Gaye were selected to participate in this stage of data collection from the nine teachers who were involved in the initial interviews. The main considerations for selection were the depth and richness of responses to the initial interview questions, the potential of particular school and class situations to yield informative data and yet still be somewhat typical or representative contexts, and the sense of the existence of a rapport between the researcher and the participants. Rose was considered to be representative of the group of teachers who were categorised as *very contemporary* and appeared to have a consistent set of beliefs. Gaye held *mixed* views and there appeared to be some inconsistencies in her responses that needed to be further explored.

While each teacher was in the process of completing a masters degree and had shown a keen interest in the investigation, there were several differences between the two teachers. These differences included experience level, teaching grade level, role in the school, as well as differences in reported beliefs and practices. It was anticipated that they would provide rich data to further explore issues of interest to this investigation.

This section presents the data analysis from the remainder of the fieldwork. First, a summary of the procedures used to collect data from the classroom observations and reflective interviews is presented. This is followed by a detailed analysis of data collected from Rose and then Gaye, including their questionnaire responses, comments made during the initial interview, summaries of data from the classroom observations, and analyses of the reflective interviews.

5.3.1 Summary of Procedures for Classroom Observations and Reflective Interviews

Before the school visits, transcripts of initial interviews were analysed for each of the two teacher participants for this stage of the study. This process was described in Section 3.2.2.2 but in summary, a map or visual representation of key comments was constructed yielding themes from the data for each teacher. Then, a description of each teacher's beliefs and practices was written and sent to each participant to read and comment on before the observations and reflective interviews.

The observations were used to further validate questionnaire data as well as to clarify and confirm initial interview comments, compare actual practice with reported practice, and to identify factors that seemed to either support or constrain the implementation of problem-solving approaches. The researcher visited each of the schools on three consecutive days and observed a mathematics lesson on each occasion. Rose and Gaye were asked to teach lessons that were part of their normal program but that incorporated problem-solving approaches. Each lesson was videotaped for transcription and analysis.

Unstructured, or informal reflective interviews were used to provide an opportunity for teachers to discuss actions and decisions made during the course of each lesson and to illuminate factors that might be inhibiting desirable practices. This enabled teachers to identify reasons for decisions and provided opportunities to discuss beliefs that might impact on such decisions.

After the school visits, the videotapes of lessons were viewed and transcribed to enable both holistic and analytic analyses. The holistic analysis involved viewing all three lessons for each teacher and writing an overall description of the mathematics lessons including the actions of the teacher and lesson outcomes. The analytic analysis involved examining the transcript seeking evidence of particular actions including use of particular student problem types, and teaching strategies that had been listed on the questionnaire. The combination of these aspects of analysis provided a rich description of each teacher's practice and the use of problem-solving approaches during the observation period.

The reflective interviews were also transcribed for analysis. Data from these supported and enhanced data gathered from the questionnaire, the initial interviews and the classroom observations. Rose's problem-solving beliefs and practices are presented in the next section with a discussion of issues impacting on the use of problem-solving approaches. This is followed by a description of the classroom observations and the associated data analysis as well as final comments obtained from the reflective interviews. Data collected from Gaye and their analyses are presented in Section 5.3.3.

5.3.2 Rose

At the time of the initial interviews, Rose had been teaching for a total of 15 years although her experiences had spanned thirty years. Most of her teaching had been in the lower primary grades and she was currently teaching a Year Two class that she described as a "challenge". She had been at her current school for three years and was one of two Assistant Principals. The school was a large public school with 20 teachers and about 440 students and was located in a low socio-economic area in the Wollongong region of NSW. It was an old school but well maintained with signs on the walls in the playground encouraging students to try hard at all times.

In this section, Rose's problem-solving beliefs and teaching practices are presented. This is followed by a description of the factors that have influenced her beliefs and practices as well as the issues that were raised in relation to the use of problem-solving approaches in classrooms. The results of the classroom observations are then described with the final section revealing key factors that were identified during the reflective interviews. Early analysis of Rose's problem-solving beliefs and practices were published in Anderson (1998c).

5.3.2.1. Problem-Solving Beliefs and Teaching Practices

Rose was one of thirteen teachers who were placed in the *very contemporary* category during the questionnaire analysis. In her questionnaire responses, she disagreed with all of the *traditional* statements except for the final item about language use in problems and she agreed with all of the *contemporary* statements. Her responses to the questionnaire items indicated that she was a good representative for this group of teachers.

During interview discussions, Rose was asked to comment on her responses to the *traditional* statements presented in the first question. After reading the first statement about the need to learn number facts before solving problems she said

> Well I think that's the wrong way around because I think the understanding of basic number facts comes from actually doing things ... That if you do things and if you've actually had a lot of hands on practice then the basic number facts are going to come so that's why I disagree with that.

She continued to read several statements and commented that this *traditional* approach did not work when she was learning mathematics and so she does not believe that it is the best way to teach. She then agreed that the *contemporary* statements presented in the second question more closely resembled her preferred teaching approach.

Rose reported that she used a variety of teaching approaches in her classroom. Rose reported that she *almost always* used whole class discussion after problem solving, small cooperative groups, and concrete materials. She *often* presented unfamiliar and open-ended problems with little indication of how to solve them, encouraged students to record their own procedures, used problems to show the students that there are mathematical skills and procedures that they needed to know, and modelled the problem solving process to the class. The two strategies that she *hardly ever* used were ensuring students work alone and explaining in detail what students needed to do to solve problems. All other strategies were used *sometimes.*

Rose's very frequent use of group work and infrequent use of calculators were discussed during the initial interview. She indicated that group work provided opportunities for the students to share their thinking when solving problems and also, her students much preferred to work in groups. Since most of the other teachers in the *very contemporary* category reported that they more frequently had calculators available for students to use, Rose was asked about her less frequent use of calculators and she said that she would "love to have a set in the room but I don't have one". She said she would use them more frequently if they were readily available as calculators are "another learning tool".

Use of particular question types indicated that Rose *sometimes* used each of exercises, open-ended problems and application problems while she *rarely* used unfamiliar problems. Her reasons for this preference indicated that she believed that "it is important for the children to experience a variety of questions. In this way I hoped to appeal to each child's preferred learning method". Rose's recently used problem was an example of an open-ended question. She had recorded

> I gave each child a number between 5 and 19. They had to write as many questions as they could with that answer.

This example was a favourite open-ended task that supported Rose's belief that these problems were good teaching and learning vehicles. She also indicated that open-ended tasks allowed the more able students to be challenged while less able students could still attempt the question at their own level. Rose's responses to the fourth question from the questionnaire are presented in Table 79.

Table 79
Rose's responses to teaching strategy frequency

Teaching Strategy	Hardly ever	Sometimes	Often	Almost always
you ensure that the students work alone	√			
you explain in detail what the students have to do to solve problems	√			
at the end of a problem solving lesson you lead a whole class discussion so that students can share solutions and strategies				√
you have calculators available for students to use		√		
you encourage the students to work in small, cooperative groups				√
you present *unfamiliar* and *open-ended problems* to the class with very little indication of how to solve them			√	
you encourage students to record their own procedures and methods of solving problems			√	
you encourage students to pose their own problems		√		
you provide a set of problems and the students are allowed to choose a problem they would like to work on		√		
you allow the class or individual students to spend several lessons on the same problem		√		
you use problems to show students that there are mathematical skills and procedures which they need to know			√	
you present *application problems* which allow students to practise the skills they have just learnt		√		
you provide concrete materials for those students who need them				√
you model the problem solving process to the class			√	
you discuss useful problem solving strategies (eg. make a list, draw a diagram, work backwards)		√		
you discuss problem solving processes (ie. make a plan, carry out the plan, check the calculations)		√		
you use problems which arise from the school context or which relate to the students' experiences		√		
you pose *open-ended problems* to allow students to explore mathematical situations for themselves		√		
you set *exercises* to allow the students to practise their skills		√		
you pose *unfamiliar problems*		√		

The sources of mathematical problems that Rose reported using included all of those listed on the questionnaire. She *often* used the syllabus and resource and reference books. She *sometimes* used textbooks and ideas from other teachers but *rarely* used ideas from inservice courses. As she has attended no inservice courses that focused on mathematics in the last two years, it was not surprising that she rarely used notes from such courses.

Rose intended to provide students with meaningful yet challenging experiences and believed that problem solving was a good way to do this thus suggesting that her problem solving focus was as a process of inquiry. She liked to teach her students how to think and she believed that they learnt by being actively involved in lessons. She described the previous lesson where students were examining number patterns and were required to determine the next three numbers in each pattern. She felt that it represented a good problem-solving lesson because they were "buzzing" and helping each other. She said "they got really enthusiastic about it and they loved it". She observed that some of the students found the number patterns difficult to do and she suggested that this would help to inform the focus of future lessons. She would provide a "scaffold of some counters and things" in the next lesson for the students she had identified as needing this support.

Rose does not like to "tell" the students how to do problems but described her approach as "the guide on the side" rather than "the sage on the stage". She described another problem-solving lesson she had embarked upon recently where the students were required to consider how they might construct three-dimensional shapes. She gave no instructions but left them to think about the idea. She reported that some students tried to work this out whereas others were less inclined to engage in the task. She listened to the suggestions and guided their ideas over the next ten lessons until they all had constructed reasonable solids. She commented that these were successful lessons because she did not tell the students how to do the task. On the questionnaire, Rose had indicated that she *hardly ever* explained in detail what the students have to do to solve problems.

Interview discussions clearly confirmed that Rose's approach to problem solving in mathematics was from a *contemporary* perspective. Her beliefs were well articulated as she was able to justify her approach and at the same time she rejected more *traditional* beliefs and practices. Descriptions of recent lessons suggested that her practices were consistent with her beliefs. In this case, the questionnaire had been a useful instrument to gauge her problem-solving beliefs and practices and had provided a vehicle for identifying key issues and factors that have impacted on her beliefs and practices.

On the questionnaire, Rose had described several factors that appeared to have impacted on her beliefs about problem solving and on her use of problem-solving approaches in the classroom. These were further discussed during the interview thus leading to the identification of several other key issues. All of these key factors have been categorised into three main areas and are presented in the next section.

5.3.2.2 Factors that have Influenced Rose's Problem-Solving Beliefs and Practices

Several factors were raised on the questionnaire and during the initial interview that seemed to have influenced Rose's problem-solving beliefs and practices. To further analyse Rose's initial interview comments, a diagram or map was drawn to note key comments and to record connections between ideas (see Appendix 9). From this map a simplified diagram was constructed that connected the three main categories of comments (see Figure 26).

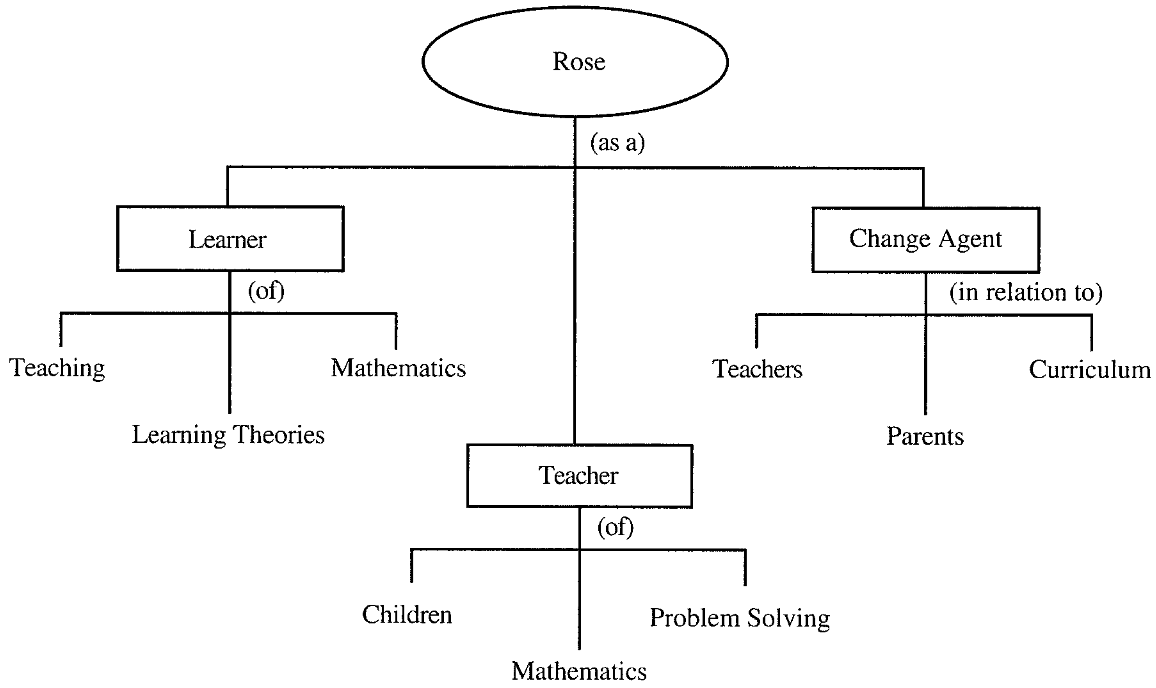


Figure 26. Factors that have influenced Rose's beliefs and practices.

The identified factors that have had a profound influence on Rose's beliefs and practices could be grouped under the three main categories of Rose as a learner, as a teacher, and as a change agent. Her experiences in learning to teach, as a postgraduate student, and her own learning of mathematics have influenced her beliefs and teaching practices. In addition, her experiences in classrooms coping with students who have diverse needs, teaching mathematics and attempting to implement problem solving have impacted on her current perspectives. Other influences have included other teachers' beliefs and practices, parents' expectations, and changing curriculum requirements. All of these influences are elaborated in the following sections.

Rose as a Learner

Rose began teaching in the mid-sixties and taught for seven years before a long break for child rearing. In describing her history as a teacher, she spoke at length about the level of change she experienced when she returned to teaching as well as the high rate of change that continues in education. A new *Mathematics Syllabus K-6* (Department of Education, 1989) was about to be introduced when she re-entered the teaching profession. This syllabus included Number from the previous syllabus but introduced Space and Measurement as two additional strands of study; two strands Rose described as "only ever incidental before". She also mentioned change in other curriculum areas including the "process writing approach" to teaching English.

Rose reflected on the level of change she had confronted and described herself as feeling like "a beginning teacher". Not only did she have to cope with new content and new teaching approaches but she also had to cope with teaching in a difficult school before a transfer to the Wollongong area. She described this period as "an enormous learning curve" although it was apparent that she was still concerned about her knowledge of teaching. She stated "I always worry about my own knowledge and confidence and understanding". Because of these worries she continued to study even though she struggled to find enough time to do all of the things she would like to do.

Since returning to teaching in 1988, Rose had completed a Bachelor of Education degree and, at the time when data were being collected, was enrolled in a Master of Education in Special Education program. Rose had a strong belief in life-long learning and a keen desire to know more about teaching and learning. Her studies reflected her commitment to meeting the needs of *all* students although she stated: "it's the children at the top and the bottom that really concern me". She had studied three subjects that focused on Gifted and Talented students as well as courses that focused on children with Learning Difficulties. She expressed disappointment that none of the subjects in the course focused on mathematics as she sees it as "an area of deficit that I have to work on". She intended seeking further study in mathematics education at a later date.

Further study, and in particular, her knowledge of learning theories had offered Rose supporting reasons for her classroom efforts. She mentioned Vygotsky's notion of scaffolding and the zone of proximal development. In discussing her belief of challenging the children to think about new ideas she suggested that it was important to extend them but cautioned that the challenge must not be beyond their reach. It was clear that she wanted to extend the more able students while still allowing less able students to work at their own pace and to enjoy some success.

Rose's own learning of mathematics had played an important role in her views about teaching mathematics. She described herself as "a person who failed maths" and that because of this she was a different kind of teacher of mathematics from those she had experienced. She stated

> Maths is a bogey in my life ... but I maintain that actually because of that I'm probably a more sensitive teacher in the area because I remember we had a teacher who couldn't understand why we had problems when I went to school and I hated maths and she used to just dismiss our problems as being stupid therefore you know I had a problem and I do think that maths is taught fairly poorly ... in the school system.

She claimed that she learnt things in mathematics but she did not understand them. She believed that this approach to teaching mathematics was poor since students up to Year Ten should have hands-on experiences instead of just a set of rules to learn.

The teaching approach that Rose's daughter was experiencing at secondary school was also criticised. She described the approach as rule driven and believed that this contributed to poor understanding, a lack of success and ultimately poor attitudes. She described her daughter as "a kid who I think is actually bright and very badly under achieving and with a terrible attitude to maths that I didn't want her to have". Rose was adamant that she did not want other children to have the same experiences as herself and her daughter. Her experiences as a learner had greatly influenced her beliefs and practices in mathematics.

Rose as a Teacher

Rose believed that her own negative experiences as a learner of mathematics had enabled her to develop a more sympathetic approach to teaching mathematics. She was more aware of how the children felt when they could not do the work. She described her concerns about their individual needs and said "I do try to look at every child as if they are mine and say well now what do you need, what is it that I can give to you to solve this problem for you". She would spend considerable time with a few students who needed extra support although she expressed frustration because she felt that other children were being ignored while this is occurring.

The Year Two class Rose was teaching had been described as a real challenge because of the diverse needs of the pupils. Her class was a middle set in a streamed Year Two with 20 boys and five girls. Five were quite bright and another five needed considerable extra support and individual help. One had just arrived from another school and had special needs in relation to behaviour and another was described as "very depressed". A teachers' aide was available to help during some of the week. It was clear in the initial interview that Rose was keen to help all students but she admitted that this was quite difficult.

Rose was concerned that her students understood the mathematical ideas in the curriculum and at the same time enjoyed their experiences. In her efforts to teach mathematics well, she realised that many of her students needed considerable support as well as challenges. These challenges were usually presented in the form of problems, some of which were too difficult for some of the class. It was clearly an issue for Rose to find tasks that engaged most students without being too overwhelming that the students felt frustrated and gave up. Rose was not discouraged by her efforts but felt strongly about the need to engage students in problem-solving experiences and continued to try to implement problem-solving approaches.

According to Rose, mathematics was not being given enough of a focus for staff discussions. She raised the results of the recently released Third International Mathematics and Science Study (TIMSS) and described a concern about the Australian results in mathematics. She believed that these should inform teaching practices in mathematics classrooms and she wanted mathematics to have more attention in staff development at her school. To this end she viewed herself as a potential change agent.

Rose as a Change Agent

Rose wanted to support other teachers in their efforts to confront change and expressed a concern about the level of understanding of teachers in relation to problem solving. When asked about the professional development needs of teachers at her school, Rose wrote "the teachers - me included - need a lot of PD re problem solving. We are all still locked into old style concepts of maths". During interview discussions she added "I still think as teachers we're really only at the beginning of this. I don't think we have enough, um understanding of it".

Rose was positively orientated to seeing herself as a learner. She suggested that "because I feel I'm still learning ... I can show other teachers that learning continues". She revealed that she had ambitions to be a Principal so that she could have a greater influence on teachers and their approach to change. She believed that teachers could be afraid of change and so they often used excuses of being stressed or too busy.

Further evidence of Rose's beliefs about change was identified in the questionnaire. In response to the negative comment about problem solving, Rose wrote

> My reaction is that this teacher is afraid of change, afraid that his/her knowledge of mathematics learning will be exposed - and he/she is expressing and reflecting community views about maths. He/she is probably into control in the classroom.

When questioned about this response, Rose indicated that she believed that a lot of teachers in the South Coast Region were older and used perceived community views as an excuse not to change. In addition, she acknowledged that some teachers were not confident about teaching mathematics.

Rose had also organised signs that were displayed on walls around the school. Teachers, parents and students had voted on the twenty best signs to display. Rose said

> And the one right in the back paddock I actually fudged and I had that put up even though it wasn't in the top 20 signs that went up and that is "change is a conscious choice" and I deliberately put it so that every teacher who walks into this school every morning has to see "change is a conscious choice".

She indicated that she could understand why teachers found it difficult to change but she would like "to drag teachers kicking and screaming into this next level of understanding".

Rose was also prepared to take a role in influencing parents' views as well as having input to curriculum development. She indicated that she always informed parents about her approach to teaching mathematics and that she had not experienced problems with parents' views. She believed that this is partly because of her communication to parents but also because she was teaching in the infants department and so hands-on experiences were acceptable. She suggested that if she were teaching in a higher grade, parents might not be as sympathetic because they may "see maths as adding up and subtracting and dividing and multiplying". She indicated that it would be more of a challenge to change the views of such parents. Another factor that she felt may have assisted with parents' views was that the Principal emphasised "thinking skills and problem solving" and he "encourages anyone who's doing it".

Rose had recently reviewed a draft of the new *Outcome Statements* distributed to schools by the Board of Studies. She was quite concerned about the document because she believed that there was a gap between the syllabus and teachers' knowledge and understanding. She felt quite passionate about it and stated "I found myself really reacting to that".

Discussing these factors with Rose revealed a teacher who had concerns about her own knowledge and understanding of mathematics and problem-solving approaches, but who was still totally committed to this approach of teaching mathematics. Even though she acknowledged that there might be difficulties with meeting the needs of all students, she clearly believed that the more commonly used *traditional* approaches were unsuitable and did not lead to good understanding of mathematics, or to positive attitudes. To this end she was determined to change other teachers' beliefs and practices but realised that to do this she would probably need to seek a Principal's position in another school. Rose was so passionate about this that there was a sense of missionary zeal in relation to her wanting to change the world.

These discussions had raised key issues that seemed to have impacted on Rose's current use of problem-solving approaches. Identified issues encompassed previous experiences including postgraduate study and her own mathematical learning, factors relating to the classroom including the students' needs and the role of problem solving in learning mathematics, and a variety of aspects of the culture of her current school. These are discussed in the next section.

5.3.2.3 Issues About the Use of Problem-Solving Approaches

As Rose recorded her beliefs on the questionnaire and described further experiences and influences during the preliminary interview, it became apparent that there were significant issues impacting on her use of problem-solving approaches. These can be organised under the categories of previous experiences, classroom factors, and school culture as in Figure 27. This is an adaptation of the diagram presented in Figure 26 that summarised the factors impacting on Rose's beliefs and practices.

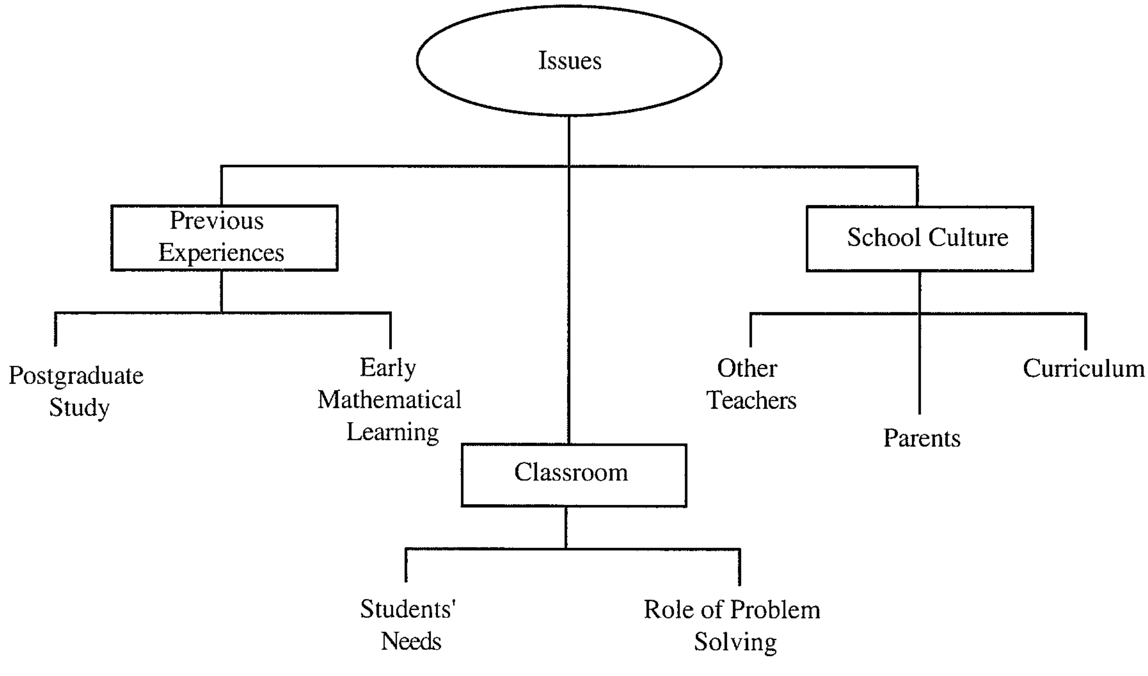


Figure 27. Issues related to Rose's use of problem-solving approaches.

The main issues relating to previous experiences seemed to include Rose's own learning of mathematics and her more recent postgraduate study. She was clearly unhappy with the way she was taught mathematics and she believed that there was a better way to teach students. A similar, poor experience for her daughter convinced her that students needed to understand mathematical concepts and that this could be achieved if they were actively involved in investigating and discovering mathematical ideas. In addition, postgraduate study had further convinced Rose that there were other approaches to teaching mathematics that included discussion between students and modelling by the teacher.

Classroom factors that raised important issues included students' needs and the role of problem solving in learning mathematics. Rose was concerned that she was able to meet the diverse needs of the students in her class. She believed that providing opportunities for students to work in groups enabled more of them to experience some success. She also liked to provide them with varied experiences through the use of a variety of problem types. She believed that open-ended questions allowed all students to engage in the task and find at least one solution to the problem. Problem-solving opportunities were used to engage students in mathematical thinking as Rose endeavoured to encourage students to think through the ideas without assistance and to try to find solutions before whole class discussion and sharing of ideas.

The school culture raised additional issues regarding other teachers, parents and the curriculum. Rose identified a need for more professional development in mathematics, particularly in relation to problem-solving approaches. She indicated that the Principal was supportive but there was a real need for teachers to reflect on their practice and to be supported in changing to new approaches. While the parents of students in Rose's class were satisfied with her approach, she was certain that other parents held different expectations, particularly in middle and upper primary grades. In addition, the requirements of the curriculum were worrying for Rose as she identified recent changes as out of step with teachers' knowledge and experiences.

These issues assisted in the identification of possible constraints and opportunities impacting on Rose's use of problem-solving approaches. While there appeared to be few factors interfering with Rose's attempts to do problem solving with her students, she was clearly concerned about meeting the diverse needs of *all* students in her class, confronting other teachers' more *traditional* practices, and challenging parents' views. Opportunities had been provided through the knowledge gained in postgraduate education and a supportive Principal.

The data from Rose's questionnaire and initial interview provided substantial information about her problem-solving beliefs and practices. Further information was sought through classroom observations and reflective interviews. Data analysis from these sources is presented in the following sections.

5.3.2.4 Classroom Observations

Rose was observed teaching three consecutive mathematics lessons to her Year Two class. Analysis of the lessons was undertaken in two ways. The first level of analysis yielded an overall description of the lessons reporting general observations and including the actions of the teacher and lesson outcomes. The second level of analysis sought to identify particular actions including use of a variety of student problem types and teaching strategies. These are presented in separate sections.

General Observations of Rose's Lessons (Holistic Analysis)

At the time of the school visits, the students were engaged in a unit of work entitled "money". This unit was integrated throughout most of the key learning areas in the curriculum. Mathematics lessons involved the students using a "bank" of paper coins and notes to count and solve money related questions and problems. Lesson structure generally involved an introductory session of review and demonstration followed by students working on their tasks and a final session of sharing of results. In this section, each of the observed lessons is briefly described with a general description of Rose's problem-solving teaching approaches and strategies. In addition, factors are noted that seemed to support or constrain Rose's planned or intended use of problem-solving approaches in teaching mathematics.

During the first lesson the students were required to answer a set of money-related problems. They were encouraged to work with a partner and to record their solutions on paper. The lesson began with Rose reminding the students about the tasks they had undertaken to date and encouraging them to reflect on previous experiences. She reminded them that in the previous lesson they had been using calculators to add the price of toys to a total of $50.

Rose then introduced the day's lesson by indicating that the students would be solving a set of problems without the use of their paper money. The students were required to make different combinations of coins for a particular money value. They were instructed to record answers by drawing a circle for each coin with the amount written inside.

Students had been allocated to one of four groups for mathematics lessons. Groups 1 and 2 were to attempt more difficult questions than groups 3 and 4. The questions were recorded as follows. The harder questions stated

> Use 5c, 10c, 20c, and 50c. Draw circles with the amount inside:
>
> 1. There are 18 ways of making 60c. How many ways can you find using 3 coins?
> 2. Can you find three ways without using a 5c coin?
> 3. Can you find three ways without using a 20c coin?

4. Try making other totals.
5. Try using six ways of making each total.

The easier set of questions stated

Use 5c, 10c and 20c coins. Use the sheet with coins down the side.

1. Make 30c with different numbers of coins.
2. What amounts can be made with one coin?
3. Can you find two different ways of making 30c with three coins?
4. How can you make 50c with four coins?

Rose spent time ensuring that the students understood what had to be done before they attempted the questions. Students were encouraged to work in pairs but many completed the task alone. Several students experienced difficulty with the questions and Rose spent much of the lesson supporting these students by repeating the questions and confirming correct responses. The students experiencing most difficulty were eventually allowed to use their paper money to assist task completion.

The majority of Rose's comments during the lesson involved keeping students on task and encouraging others to find multiple solutions. The questions seemed to be quite difficult for most students and they rarely found more than one combination. Students were content to record one possibility and then move on to the next question. Several students needed constant supervision and were regularly off task. It became clear that Rose's greatest challenge was to engage students with a task they could understand and do while at the same time keeping them focused and attentive. Rose's earlier description of the diverse needs of the students in her class was not an exaggeration. Several students were noisy, easily distracted and completed little work during the lesson.

In the second lesson, the students were required to use a local store's toy catalogue and calculators to select items they would like to buy with $70. The lesson began with a whole class discussion about using the calculator for money questions. There was considerable discussion about entering $1.25 on the calculator using the decimal point key. Students recalled the correct procedure and were instructed to select a catalogue and work with a partner on the task.

Worksheets were distributed to encourage students to record the item from the catalogue with its price and then to keep a running total of the money remaining from $70. Rose tried to encourage the students to begin with $70 at the top of the page and to gradually subtract the price of selected toys until they were close to $0. She continually asked students "how much do you have left now". This process seemed difficult for most students as they just made a list of desirable items and added them to see how close they were to $70. When asked how much money they had left, most students told Rose how much they had spent. Their approach to the task seemed to reflect an earlier lesson where they had to make a shopping list totalling $50.

The students were encouraged to make different lists and to do this in pairs or at least to discuss ideas. They enjoyed the lesson and were more focused than the previous day. They talked about what they would like to buy and some pairs made several lists of items. The lesson concluded with sharing of ideas and suggestions from several students.

In the third lesson, the students were required to use their paper money to display a variety of totals. Rose had deposited interest into students' envelopes before the lesson so they were initially required to sort their money into each denomination to check how much they now had. Students worked individually and were instructed to display 75c using their coins. This was followed by a request for alternative combinations. This process continued with other amounts including $1.45 and $1.95.

Rose then asked students a variety of open-ended questions. Students were required to make 85c with exactly 6 coins. Next, they needed to use exactly one 50c coin, one 20c coin, one 10c coin and one 5c coin to suggest purchases that could be made from the school canteen where no change would need to be given. After several suggestions, Rose asked students to consider the possibility of making exactly 5c, 10c, 15c, 20c, 25c and so on to 90c using only these four coins. This was an interesting exercise although not all students engaged in the task or kept up with faster workers. All of these questions were presented orally with no recording of solutions.

This lesson was very teacher directed with Rose giving money amounts and then asking for responses from students. Larger amounts of money were also given such as $35.75, $10.35 and $114.55. After some time one of the students was given the opportunity to "be the teacher" and say the money amounts. They clearly enjoyed this aspect of the lesson as well and many volunteered to participate. More students remained on task throughout the lesson and enjoyed using the money to answer questions. Rose referred to this as a "much more formal lesson", an approach she used about once a week for mathematics.

These general observations of the three mathematics lessons indicated that Rose did attempt to implement problem-solving approaches in her classroom thus suggesting that her practice reflected her beliefs. Also, her actual practice seemed to reflect reported practice as indicated on the questionnaire. She used a variety of question types including open-ended problems, encouraged students to work together and share their thinking, provided concrete materials to support student learning, and used whole-class discussion to demonstrate, consolidate and share ideas. Further analysis of the use of these strategies is presented in the next section.

Classroom observations also suggested that there were factors that both supported and constrained Rose's efforts. Supporting factors included the classroom layout as desks were positioned in groups to enable discussion between students, the availability of concrete materials and calculators, and the general attitude of the students towards the activities as they responded well to the tasks and willingly attempted each activity. In addition, Rose had spent considerable time and effort planning and creating the ideas and problems for the students that included several real-life applications and teaching ideas that were highly motivational. The use of toy catalogues was clearly a source of enjoyment and interest and the discussions about canteen purchases were both relevant and pertinent to the students' everyday world.

However, there appeared to be several constraining factors that were impacting on Rose's efforts. The diverse needs of the students in her class were apparent. There were several students who had considerable difficulty remaining on task even though they were willing to start each activity. These students required constant supervision that then interfered with Rose's efforts to support several other students who needed additional explanations and assistance in order to understand and attempt tasks. There were a few students who worked quickly to achieve solutions and then requested extra work. Rose needed to constantly encourage these students to find additional solutions to the open-ended questions. It should also be noted that the presence of the researcher was an added distraction that clearly unsettled some students. The impact of this additional factor appeared to decrease over the duration of the classroom observation period.

Specific Observations of Rose's Lessons (Analytic Analysis)

Lesson transcriptions were read and evidence was sought to support Rose's reported use of particular student problem types and problem-solving teaching strategies. Each of these is considered in this section. The questions that were the focus of each of the mathematics tasks presented to the students in Rose's lessons were categorised and the results are presented in Table 80.

Table 80
Use of student question types for Rose's three mathematics lessons

Types of Questions	Lesson 1	Lesson 2	Lesson 3	Questionnaire Response
Exercises				sometimes
Open-ended Problems	√	√	√	sometimes
Application Problems			√	sometimes
Unfamiliar Problems				rarely

The videotape for each lesson was viewed to find evidence of use of each of the teaching strategies listed on the questionnaire. These are noted in Table 81 with the questionnaire response provided by Rose that indicated her reported frequency of use of each strategy.

Table 81
Rose's use of each of the teaching strategies in each of the observed lessons compared to her reported use of each strategy as recorded on her questionnaire

Teaching Strategy	Lesson 1	Lesson 2	Lesson 3	Questionnaire Response
you ensure that the students work alone			√	hardly ever
you explain in detail what the students have to do to solve problems			√	hardly ever
at the end of a problem solving lesson you lead a whole class discussion so that students can share solutions and strategies	√	√		almost always
you have calculators available for students to use	√	√		sometimes
you encourage the students to work in small, cooperative groups	√	√		almost always
you present *unfamiliar* and *open-ended problems* to the class with very little indication of how to solve them	√			often
you encourage students to record their own procedures and methods of solving problems				often
you encourage students to pose their own problems			√	sometimes
you provide a set of problems and the students are allowed to choose a problem they would like to work on				sometimes
you allow the class or individual students to spend several lessons on the same problem				sometimes
you use problems to show students that there are mathematical skills and procedures which they need to know		√		often
you present *application problems* which allow students to practise the skills they have just learnt			√	sometimes
you provide concrete materials for those students who need them	√		√	almost always
you model the problem solving process to the class				often
you discuss useful problem solving strategies (eg. make a list, draw a diagram, work backwards)				sometimes
you discuss problem solving processes (ie. make a plan, carry out the plan, check the calculations)				sometimes
you use problems which arise from the school context or which relate to the students' experiences			√	sometimes
you pose *open-ended problems* to allow students to explore mathematical situations for themselves	√	√	√	sometimes
you set *exercises* to allow the students to practise their skills				sometimes
you pose *unfamiliar problems*				sometimes

Open-ended questions were used in all three lessons and clearly formed the basis of most of the tasks presented to the students during the observation period. No exercises were used during the three lessons and there was no evidence of the use of unfamiliar problems. Application questions were used during the last lesson but these were still somewhat open-ended as there was potentially more than one answer to each question.

Rose had reported on her questionnaire that she *sometimes* used each of exercises, open-ended and application problems but *rarely* used unfamiliar problems. During the initial interview she indicated that she had a preference for open-ended problems and this was evidenced during the observations.

As the observations were only conducted over three consecutive lessons and, as Rose was requested to use problem-solving approaches, it cannot be suggested that these three lessons were representative of Rose's usual practice. However, it did provide some basis for ascertaining the strategies that she felt were appropriate for the particular content and focus of the unit of work on "money" for her class. During these lessons, whole class discussion was frequently used at the beginning, during and at the end of lessons, students were frequently encouraged to share their responses and to listen to the responses of other students, concrete materials and calculators were available and students were encouraged to use them, and open-ended questions were also frequently used.

The lessons supported Rose's responses to the questionnaire and comments made in the initial interview that she wanted the students to think and that she did not like to "tell". At the same time she offered considerable support for students by providing them with paper coins, calculators, regular feedback, and further questioning. Problem types were open-ended and the activity in the second lesson was placed in a real-life context. Her efforts were certainly made more difficult by the range of abilities of the students, the lack of sufficient calculators, and the inability of some students to stay on task. Additional information about the lessons, the reasons for making particular decisions, and the possibility that there were other factors operating that might support or constrain Rose's efforts were sought in the reflective interview.

Time constraints prevented more than one reflective interview with Rose. As a member of the executive team, Rose had many responsibilities and was unable to provide more time for interviews. This 30 minute interview was conducted just before the last lesson. A transcript of the interview was used to analyse the data that is presented in the next section.

5.3.2.5 Reflective Interview

The reflective interview confirmed the earlier analysis written by the researcher and then focused on issues related to the school context as well as Rose's approach to teaching mathematics. This conversation confirmed previously described factors and revealed new factors that seemed to constrain or support Rose's efforts in her classroom. In particular, streaming of classes and some aspects of the school culture were viewed as constraining factors while Rose's experiences in postgraduate courses supported her use of problem-solving approaches.

Streaming of classes created difficulties for Rose. The lowest ability Year Two students had been placed in a composite Year One and Year Two class. Specialist teachers focused most of their attention on this class of students so that the lower ability students in Rose's class did not receive the additional support they required. This frustrated Rose as she felt that she was not meeting the needs of these students. In addition to this concern, Rose stated that there were "no really good role models here for these kids to push them up". This meant that her students were not exposed to the mathematical thinking of more able students as well as models of good behaviour. Rose mentioned that there was staff resentment to this approach but the appointment of a new Principal for the following year had encouraged staff to lobby for changes.

Rose described a school culture that was "very tense" with a leadership approach that did not support all staff and was not in the best interests of the students. Interview comments mentioned a coercive Principal who readily suspended problem students, was more concerned with superficial outcomes so that the school was perceived to be successful, and who constantly put pressure on staff rather than offering support. In addition, he focused on achieving good results in the annual, externally moderated Basic Skills Tests in Years Three and Five. These revelations contrasted with Rose's earlier positive comments about her Principal.

As a consequence of this Principal's approach, Rose believed there had been negative outcomes exhibited by some students and also by some staff members. She stated

> The culture and climate of the school is exhibited by these children and [there is a feeling] of being watched and doing the right thing and getting away with it if they're not being watched ... so, in a sense in this classroom I'm fighting an entire culture

Rose suggested that some teachers responded by doing only what was necessary. She stated

> ... the culture and climate of the school does need quite a revamp to get other teachers to reflect on their practice rather than tick boxes and say "I've done that".

This notion of reflecting on practice and changing teaching approaches had been described in Rose's questionnaire and discussed during the initial interview. These aspects were raised again during this reflective interview with Rose indicating that she believed there was a better way to teach classes in the infants grades than the approach adopted in her current school. She suggested that she would like to "knock walls out and change the whole place around"; an approach that had been successfully achieved in Rose's previous school.

Rose spoke positively about most teachers at the school and reported that there were several very capable and talented teachers. Others were referred to as being more *traditional* in their approach. Rose stated

> but there's a solid hard core at the top of the primary that by the time [the students] get to them they're, you know, a lot of their basic learning is done so it doesn't do as much damage as it would otherwise.

This comment indicated that Rose believed that the most important learning was achieved in lower and middle primary grades.

Considerable discussion centred around Rose's approach to teaching mathematics. She noted that this approach could be frustrating and was harder to implement than more *traditional* approaches but she was convinced that this was the correct method to teach students in mathematics as well as other subjects. She stated

> I think by going around the room and by watching them, I know it's a very hard way to teach and I do it in English too ... I make them find words, I make them engage with the work that they're doing, and it's much harder for me and it means that I haven't got an ordered classroom, but I think I've got kids who are having to think about what they are doing.

She noted that the success of this approach was confirmed by other teachers.

> I don't know how successful I am at that, although um the teachers who have my kids in future years seem to be happy enough with what they've learnt and I guess that's the only way I can gauge what's happening.

Rather than teaching the students to sit and be quiet, as other teachers were inclined to do, Rose indicated that she was teaching them to think and to take some responsibility for their behaviour. She believed that they responded to this approach and suggested

> ... whether it's English, or maths, or science, it seems to me that the only way to go is a hands-on approach, a child-centred approach, and a problem-solving approach where you throw the responsibility back on the kids.

Rose was asked why she had used a more open-ended approach to the unit of work on money rather than using the textbook or worksheets. She stated

> Because I don't know what they're doing if they're doing that. Because I don't know. I wouldn't know that Sian couldn't understand something because what she would do is look at the person next to her and copy it down and I wouldn't know how she had done it.

This response suggested that Rose used an open-ended approach to encourage the students to tackle the problem at their own level. This assisted assessment of students' knowledge and understanding of the topic. Encouraging the students to work in groups also assisted assessment as Rose could listen to group discussions and focus on individual responses.

Another key issue for Rose was attitudes to mathematics, particularly those of parents. She stated

> In terms of mathematics, I think I'm fighting lots of attitudes here. I'm fighting attitudes from parents, I'm fighting that I have to explain what I'm doing, and that's why I'm giving that talk that brings in the games approach because I feel that parents need that kind of support.
>
> Most of them have got tapes at home where they learn their tables, and stuff like that, so I'm fighting that kind of approach. I'm lucky that I'm an Assistant Principal and they tend not to question me as much as they probably question young teachers doing the same thing, so I get away with it in a sense.

Further, she indicated that coaching clinics were entering the community and she expected parents would start sending their children for extra tuition.

Finally, Rose was asked about role models in her own career that might have influenced her beliefs. She indicated that lecturers during postgraduate study had provided support and confirmation of her views. The knowledge had empowered her and encouraged her to defend her position by providing her with arguments to support her views in discussions with the Principal. She stated

> I have all the confidence in the world to back up what I think is the right approach ... [and] ... that has all empowered me because everything I read, everything that I delve into, tells me that I'm on the right track.

At each stage of data collection, new layers of knowledge about Rose were added to provide a detailed picture of her problem-solving beliefs and practices as well as the factors that impacted on her implementation of problem-solving approaches. The questionnaire enabled the researcher to identify her beliefs and practices and to compare these to a large group of other teacher respondents. She was categorised as holding *very contemporary* beliefs and responses to the open-ended questions suggested that she had reflected on her approach to teaching mathematics. She was also concerned that other teachers had not responded to the challenge to promote mathematical thinking through problem solving.

These views were confirmed during the initial interview and new issues were identified. Rose clearly articulated her views and the reasons she felt this way. She was concerned that students were actively involved in their learning and did not experience a diet of skills and meaningless practice, as she had done. Her views were reinforced by postgraduate study and enabled her to confidently reject more *traditional* approaches as inappropriate, unsuccessful and the cause of negative attitudes. She was aware that other teachers also needed to reflect on practice if they were to reconsider their approach to teaching mathematics.

Classroom observations enabled the researcher to compare Rose's reported practice with actual practice and to witness her problem-solving attempts in the context of her classroom. It was clear that she attempted to put her beliefs into practice but it was also clear that her class was as challenging as she had described. In spite of the difficulties encountered in catering for a variety of students' needs, the students were motivated by Rose's attempts to engage them in meaningful activities. She also attempted to use a variety of teaching strategies even though she confessed that each week she had lessons that were teacher-centred and focused on skills practice.

The reflective interview revealed a new set of issues. Staff disagreement over streaming of classes, an unhappy school culture, and a coercive Principal were raised in this last opportunity to discuss Rose's experiences in her current school. This honesty seemed to be an outcome of Rose's confidence that the researcher would not reveal any details of these conversations. Rose had previously presented a more positive attitude towards the school and her Principal but she now confided that things would not change while he remained. She did not really feel constrained by his views and practices but her passion for other teachers to adopt similar practices would not be met until a new, more collaborative Principal was appointed.

Another consideration to support the findings from the fieldwork was the level of coherence of beliefs and practices across a variety of data collection methods. In this case, Rose consistently and coherently reported the same beliefs and practices on the questionnaire and during interviews. Because she had strongly held views about mathematics teaching and learning, she had been able to resist constraints on using this approach. Rose believed that her position as a member of the school executive as well as her increased knowledge through further study had empowered her to resist the constraints that were operating in her school.

The next section presents the data collected from the final teacher who participated in the fieldwork. Gaye's initial interview was conducted with Susan since they taught at the same school. Their beliefs and practices were quite different and so, Gaye's responses were reported separately. In the following sections of this chapter, Gaye's questionnaire responses and initial interview comments are presented, the results from the classroom observations are outlined, and analysis of the reflective interviews is described.

5.3.3 Gaye

At the time of data collection, Gaye was teaching in a large school in a middle-class suburb of Sydney with approximately 700 students and 40 staff. She had taught at the same school for six years after training as a mature-age student. Her current class was a mixed ability Grade Six group although for mathematics lessons she taught a lower ability group of Grade Six students that she had volunteered to teach. She was undertaking postgraduate study, was very enthusiastic about mathematics teaching and learning, and was keen to participate in this investigation.

Gaye appeared to have definite beliefs about particular aspects of teaching and learning mathematics as well as of problem-solving approaches, however her beliefs about some other aspects seemed to be inconsistent. These were explored during all stages of data collection and are reported in the following sections that include her problem-solving beliefs and teaching practices, the factors that have influenced her beliefs and practices, issues that have impacted on her use of problem-solving approaches, data from classroom observations, and analysis of reflective interviews.

5.3.3.1 Problem-Solving Beliefs and Teaching Practices

The questionnaire analysis placed Gaye in the *mixed* category since she agreed with some of the *traditional* statements as well as some of the *contemporary* statements. Her responses to the open-ended questions suggested that she may hold more *traditional* views than *contemporary*, particularly in relation to her support for streaming in mathematics and a preference for exercises and application problems. However, she appeared to be in favour of the use of problem-solving approaches and stated that it was important for teachers to remain informed of new ideas and current thinking in mathematics education. Gaye's questionnaire responses provided many worthwhile discussion points for the initial interview. An examination of her problem-solving beliefs and practices is presented in this section.

Gaye's responses to the first two questions on the questionnaire gave some indication of her beliefs about the role of problem solving in learning mathematics. However, her mixed responses suggested possible inconsistencies in her beliefs. One example of this was her agreement with the statement that problems should be done after the acquisition of number facts and algorithms coupled with a strong disagreement that problems should be left to the end of a topic. During interview discussions, Gaye indicated that she does like to integrate problems into her lessons to support the learning of particular mathematical ideas.

Gaye also gave mixed reactions to the *contemporary* statements. She *agreed* that lessons should focus on problems rather than on practising algorithms and also, that problems help to motivate and challenge students. She *strongly disagreed* that it is a good idea to start a topic with an unfamiliar problem or that students can learn mathematics by working out for themselves how to solve unfamiliar or open-ended problems. In addition, she *disagreed* that students should explore their own methods of doing mathematical questions before being shown the teacher's methods.

The questionnaire responses and initial interview discussions confirmed that Gaye held a mixed set of problem-solving beliefs and practices. She believed that problem solving played a central role in learning mathematics, a potentially more *contemporary* view, but she had qualified support for this approach. She agreed that problems motivate and challenge students and that lessons should not just focus on practising skills and algorithms. She also believed that students needed prior knowledge as well as considerable guidance when they attempted problems. She mentioned the need to "train" students and to "spend time with the very basic concepts" thus suggesting a more traditional approach. Gaye's beliefs seemed to have been strongly influenced by her experiences with teaching the lower ability students in Year Six. This is further discussed in the next section.

Her responses indicated that Gaye used a variety of teaching strategies on a fairly regular basis with her mathematics group. Strategies that were *often* used included group work, whole-class discussion, and individual recording of procedures and methods. She also encouraged regular use of concrete materials and calculators, where appropriate. She regularly modelled problem solving and discussed problem-solving strategies and problem-solving processes. In addition, Gaye often used problems to show students the usefulness of mathematical skills and procedures, and frequently presented application problems so students could practise skills.

Another group of strategies were used less frequently. Gaye *sometimes* encouraged students to work alone and sometimes provided detailed explanation of how to do a problem. Also, students *sometimes* posed their own problems, chose a problem to do, or did problems relating to school context. Gaye's reported strategy use on the questionnaire indicated that she preferred to use a wide variety of strategies suggesting a preference for innovative practice even though she was teaching lower ability students.

In relation to problem solving types, Gaye preferred to use exercises and application problems with her mathematics group. She less frequently used open-ended or unfamiliar problems although she believed that these problem types could be useful. During the interview, understanding of the problem types was discussed and it was clear that Gaye interpreted the terms as was intended. The reasons for her preference for these particular types of problems were

> I have a very slow Year 6 class and feel they need much reinforcement by having success, so before giving problems which they need to explore and decide the method, I try to make sure they have *all* the tools necessary.

A statement that further reinforced her view that students needed facts, algorithms and basic skills before being presented with problems.

Frequency of use of particular teaching strategies was sought in Question 4 of the questionnaire. Gaye's responses to these are listed in Table 82.

Table 82
Gaye's responses to teaching strategy frequency

Teaching Strategy	Hardly ever	Sometimes	Often	Almost always
you ensure that the students work alone		√		
you explain in detail what the students have to do to solve problems		√		
at the end of a problem solving lesson you lead a whole class discussion so that students can share solutions and strategies			√	
you have calculators available for students to use			√	
you encourage the students to work in small, cooperative groups			√	
you present *unfamiliar* and *open-ended problems* to the class with very little indication of how to solve them		√		
you encourage students to record their own procedures and methods of solving problems			√	
you encourage students to pose their own problems		√		
you provide a set of problems and the students are allowed to choose a problem they would like to work on		√		
you allow the class or individual students to spend several lessons on the same problem	√			
you use problems to show students that there are mathematical skills and procedures which they need to know			√	
you present *application problems* which allow students to practise the skills they have just learnt			√	
you provide concrete materials for those students who need them			√	
you model the problem solving process to the class			√	
you discuss useful problem solving strategies (eg. make a list, draw a diagram, work backwards)			√	
you discuss problem solving processes (ie. make a plan, carry out the plan, check the calculations)			√	
you use problems which arise from the school context or which relate to the students' experiences		√		
you pose *open-ended problems* to allow students to explore mathematical situations for themselves		√		
you set *exercises* to allow the students to practise their skills			√	
you pose *unfamiliar problems*		√		

For sources of mathematics problems, Gaye *often* used the syllabus, textbooks and resource and reference books. She *sometimes* used inservice notes but hardly ever used ideas from other teachers. Gaye positively described her undergraduate and postgraduate courses and indicated that she had an excellent set of notes and materials to use when preparing lessons and writing programs. These clearly provided additional support material for her approach even though she did not indicate this on the questionnaire.

Several factors were pursued during the interview that seemed to have influenced Gaye's problem-solving beliefs and practices. These included her views about learning mathematics, in particular the need for streaming and the role of learning styles, her views about teaching mathematics in relation to mathematical activities and problems, and her views about appropriate mathematical knowledge including number facts, applications and the need for a balanced curriculum. These are presented in the next section.

5.3.3.2 Factors that had Influenced Gaye's Problem-Solving Beliefs and Practices

Gaye's teaching approach was strongly influenced by the group of lower ability students that she was teaching for mathematics. This appeared to have also influenced her problem-solving beliefs and practices. Linked with this were Gaye's beliefs about the importance of using practical mathematical activities and problems combined with the development of number facts and algorithms in her teaching. She was concerned that other teachers in her school needed to review their knowledge and understanding of current trends and useful resources. She viewed several of them, including Susan, as having *very traditional* beliefs and practices. All of these issues are discussed in this section.

To assist initial interview analysis, the transcript was read and a map of key issues was drawn (see Appendix 10). Most of Gaye's comments seemed to relate to either learning mathematics, teaching mathematics, or the mathematical knowledge required by students. These headings were used to summarise the main points Gaye made in her responses to questions and discussions during the interview. The original map connecting these categories was adapted to produce Figure 28.

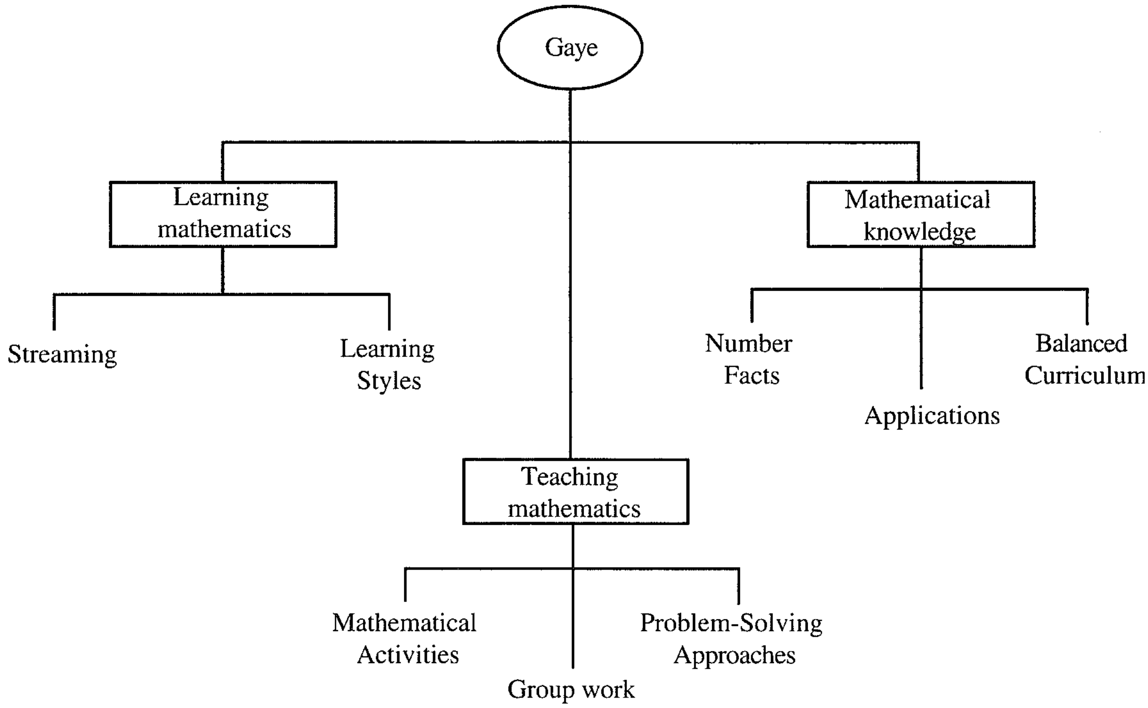


Figure 28. Factors that have influenced Gaye's beliefs and practices.

Categories emerged under each of the three main headings in the diagram. Issues related to learning mathematics included the need for streaming of students and catering for a variety of learning styles. Teaching mathematics focused on the activities used, classroom organisation including the use of group work, and problem-solving approaches. Mathematical knowledge included knowing number facts or tables, applying these to practical or real-life situations, and experiencing a balanced curriculum that encompassed Number, Space and Measurement. Each of these aspects is considered in the following sections.

Learning Mathematics

Gaye's views about how students best learn mathematics were based on a belief that streaming supports learning. When asked why streaming was adopted at the school, Gaye said that "the Year 6 teachers decided as a group that that's how we wanted it because we felt we would be able to give better training". This clearly supported responses she gave on the questionnaire where her further comments included

> I feel strongly that Maths should be a streamed subject. I have the slowest children in Year 6 and have had this type of class several times. The beauty of this is that I can spend time with the very basic concepts without having to have already set work for others.

She added that if mathematics is presented at the appropriate level of difficulty, then students tended to stay on task and enjoy success. She also believed that students felt less threatened since they "do not mind asking questions and even making mistakes in this class".

Combined with this view of streaming was the notion of adequate support to meet the needs of these students. Gaye indicated that the students were not independent learners and they tended not to "think things through". They often said that they did not understand what they had to do but Gaye made them "sit down and look at it". She believed that their efforts are improving and that they were learning how to approach unfamiliar situations.

Learning styles were mentioned as another factor that could influence how students learn. Gaye suggested that an awareness of each student's preferred learning style could assist learners and she suggested that it was helpful to use a variety of teaching approaches to support students' needs. Catering for the needs of her lower ability group and considering the potential variety of learning styles in the class had impacted on Gaye's teaching approach in mathematics.

Teaching Mathematics

In Gaye's school, mathematics lessons were held on Monday to Thursday between 11 am and 12 pm. This enabled the students to leave their regular class to join a mathematics group. There were three mathematics groups in Year Six with the less able students going to Gaye's group.

To teach mathematics, Gaye used a variety of approaches. One approach was to use activities as a focus for learning. Activities were chosen from the syllabus and support documents and were recorded on cards for the students to attempt in groups. In planning, she tried to have 6 to 8 different activities for students to complete. When asked to describe a recent mathematics lesson, she said that she had used activities involving 2D and 3D space. One example was an activity requiring students in small groups to cut squares out of graph paper and to use them to make pentominoes. The challenge was for students to discover the total number of different pentominoes and to record the results in their own way.

This lesson demonstrated the use of several of the teaching strategies that Gaye recorded on her questionnaire. These included group work and the use of concrete materials. The activity represented an example of an unfamiliar problem that was still accessible to all students and encouraged individual recording of results. Gaye believed that it was a very successful lesson as there was "a lot of thinking [and] a lot of talking".

Gaye frequently used group work with her mathematics class. She stated that as the children were working "I just supervise and see how they go". This allowed her to observe and support students having difficulties. During the interview, Gaye described a strong belief in the usefulness of group work to support student learning although she conceded that children needed to be taught how to work in groups. There was some concern expressed by Gaye that many new approaches to teaching, including group work, were not implemented as they were intended and that better inservicing of staff would assist in the proper embracing of innovative practice.

Gaye had expressed some concerns about the professional development needs of the staff at her school. In response to the open-ended question on professional development, Gaye wrote

> I feel the staff needs full inservicing in the Mathematics Document and Support Documents from the DSE. Up-to-date ideas need to be presented regularly to engender enthusiasm.

She also made the comment that she would like the opportunity to share the resource materials that she had collected from her university courses. She had been involved with considerable professional development in other curriculum areas, particularly English, but she would like more of a focus on the Mathematics Key Learning Area.

Problem solving was perceived by Gaye as providing the purpose for learning mathematics. She liked to use realistic and practical examples to support the students' learning of mathematical ideas. She gave one example where the students were discussing how fast they could walk. They talked about how long it would take them to walk ten metres and then they tested their conjectures. Following this the students were invited to estimate how long it would take them to walk 100 metres. Gaye made an error on the board as she recorded 44 minutes instead of 44 seconds. The discussion continued until one student decided something was wrong with what they had recorded. Gaye's reaction to this in the interview was

> one of the kids said you put minutes instead of seconds and now that's real problem solving in my opinion ... if he hadn't estimated he would never have found that ... and that was a real break through I think. It was really really something.

Gaye was thrilled with this because they had been thinking about the mathematics and what was realistic and she believed that it was a good example of problem solving. Another aspect of this lesson that Gaye was particularly pleased with was the use of estimation to check if solutions to problems were realistic.

Mathematical Knowledge

Gaye frequently talked about the mathematics that the students needed to know. She believed that they needed to know number facts as it made mathematics easier for them. During the interview, Gaye stated that "poor ability children are usually very good with their tables". Although Gaye would prefer that students knew their basic number facts, she liked to have calculators available. She stated that calculators provided support since if "they get bogged down working out the tables [then] they forget where they're up to in the problem". This also applied to students' knowledge of algorithms. Gaye wanted them to know how to perform the necessary calculations but calculators could be used to assist solution of problems. Because Gaye believed that lower ability students should do problem solving, she was willing to provide the necessary support to assist them with this process.

Gaye preferred to use application problems with her students since they were more realistic and showed the relevance and importance of mathematics in everyday life. This was evidenced with her example of a recently used mathematics problem. She described the problem as follows

> We spoke about the pocket money the children receive and how it would benefit them to save 10% each pay. They individually worked out the amount they would have when they left school at 18.

This suggested that Gaye viewed the solution of such problems as an important life skill for students and a necessary element of mathematical knowledge.

Further confirmation of Gaye's view that mathematics needed to be learnt for a purpose was presented on the questionnaire in her reaction to the negative statement about problem solving. She wrote

> I feel this person may need to see the benefit of allowing children to apply their knowledge to problems in the everyday world. Knowledge and proficiency in concepts needs to be applied. I cannot see any benefit unless this is done. Children need problems to help them remember the concepts.

In this sense, she believed that mathematical knowledge needed to be used to solve problems, however, she also believed that problems could be a powerful tool to help students remember mathematical ideas. This seemed to suggest that Gaye viewed problem solving as both an object of inquiry as well as a process of inquiry.

Finally, Gaye believed that a balanced curriculum including equal emphasis on Number, Space and Measurement was desirable. The skills associated with all three strands were considered important and in addition, the Measurement and Space strands provided many opportunities to apply mathematics to real-world situations.

While there were some inconsistencies in Gaye's questionnaire responses, discussions during the initial interview clarified her beliefs and practices and further revealed factors that seemed to have influenced her problem-solving beliefs and practices. She was able to articulate quite definite views related to the role of problem solving, particularly as a necessary life skill although she firmly believed that encouraging students to learn basic facts and skills in mathematics was desirable. Gaye appeared to be very comfortable with her approach to teaching her students even though she was aware that some of her views may have seemed more *traditional*. She had no difficulty explaining her views, justifying this approach, and supporting her claims with examples of classroom experiences.

She was clearly aware of more *contemporary* teaching approaches even though her practice did not always reflect this. Her reasons for adopting particular teaching approaches were based on her view of the necessity to stream students and a decision to teach the lower ability group in Year Six. This was clearly an issue that influenced her use of problem-solving approaches in the classroom. This and other issues are discussed in the next section.

5.3.3.3 Issues About the Use of Problem-Solving Approaches

The key issues relating to Gaye's use of problem-solving approaches in her classroom involved the needs of her students, the desire to use a hands-on, activities-based approach when teaching mathematics, resistance to the use of a prescribed textbook, and a belief in teaching basic skills and procedures before problem solving. It should be noted that the only factor Gaye believed was interfering with her desire to teach in a particular way was the prescribed textbook.

When asked if anything prevented her from implementing a problem-solving approach in her classroom, Gaye stated

> ... the textbook that we've got to use and it drives me insane. We are tied to this particular textbook and I use it now for homework.

She has adapted her use of the textbook to fit in with the approach that she believed best met the needs of her students. The homework was chosen from the textbook so that it closely matched the work the students were doing in class during that day's lesson. The homework content reflected the activities Gaye had planned for the students to do. She believed that this was a better approach than just working from a textbook each lesson as the students "do a good job because they've learnt it".

There appeared to be few constraints operating that impacted on Gaye's use of problem-solving approaches in her classroom. Even though the prescribed textbook was a potential constraint, Gaye had adapted its use to suit her program and planning. She indicated that parents of students in her class had not complained about her use of the textbook and so she was convinced that they must be satisfied with her approach. At this stage she had not mentioned other aspects of the school culture including the views of other teachers or the Principal. This was one element of the school context that was investigated in the reflective interviews. It was also of interest to examine Gaye's classroom practice and to compare this with her reported practice. These aspects of the investigation are discussed in the following sections.

5.3.3.4 Classroom Observations

Three consecutive mathematics lessons were observed to further validate questionnaire data as well as to clarify and confirm initial interview comments, compare actual practice with reported practice, and to identify other factors that might have supported or constrained the implementation of problem-solving approaches. Analysis of the lessons involved both holistic and analytic processes. These are presented in separate sections.

General Observations of Gaye's Lessons (Holistic Analysis)

Gaye's classroom was set up in a traditional layout with four rows of tables grouped in pairs. There were displays of students' project work on the back wall and the six *de Bono* thinking hats were positioned at the front of the room above the chalk board. The students were excited to have a visitor in the room and were particularly enthusiastic about being videotaped. They were rather unsettled until Gaye explained the reason for the researcher's presence and the need to make a videotape recording of the next three lessons. She assured the students that the main purpose of the investigation was to observe her teaching the class rather than to watch the students' reactions or responses.

As the video camera was being set up Gaye made several comments suggesting that she was somewhat apprehensive about the visit, that she wanted to please, and that she was concerned that her reported practice may not match actual practice. She stated "I don't want you to think that I am faking it. I really do do what I say". She reminded the researcher that this was a lower ability group and that she constantly tried to address students' self esteem in mathematics lessons.

One teaching approach that Gaye used was to plan several different activities that related to a particular concept. Six different activities had been recorded on separate cards for students to complete with each including the instruction to "work in pairs" and to "keep records and make a display to present to the class". Students were to attempt at least two activities in the four mathematics lessons for the week. Gaye indicated that some students did many more of the activities than others. They were encouraged to choose an activity that interested them and to work on it in small groups of between two to four students.

At the time of the school visits, the students were investigating the concept of "scale" that is contained in the Measurement strand of the syllabus. Gaye indicated that the chosen activities were from the syllabus and support documents. As the observations were conducted from Tuesday to Thursday, the students had been presented with the activities during the Monday lesson and were engaged in the tasks when the observations began. The cards are presented in Figure 29.

Models	**House Plans**
1. Using 1 cm grid paper, construct top views, front views and side views for simple buildings. 2. A scale of 1 cm representing 20 or 25 m is suitable for the buildings. 3. Exchange plans and use 1 cm cubes to model each of the plans. 4. Calculate the number of cubes used in each model. 5. Make a display for the display table.	1. Collect and discuss different house plans. These are often published in newspapers. 2. Determine the scale on one or two plans. 3. What is the area of the floor plan? 4. How wide are the doorways? 5. What is the ratio of the bedroom area to the living area? 6. Which is the best design you have?
Projections	**Maps**
1. Place a flat object on the overhead projector. 2. Trace the image on some paper. 3. Compare the tracing with the original. 4. Estimate the difference in size. 5. Now measure both for a direct comparison and determine the scale.	1. Examine maps and street directories to determine different scales. 2. List each scale and talk about what it means. 3. Calculate the distances between places shown on the maps by referring to the scale. 4. What are the commonly found scales on the maps you have looked at? 5. Why do you think these are the most often used scales?
Scale	**Area and Volume**
Create a project using cardboard showing the following information: 1. Discuss the meaning of the word "scale" and write a short summary of the different meanings. 2. Collect pictures and photos and discuss whether the size of the objects is the same or different to the actual size of the object. 3. Why are the pictures smaller or larger than the object? 4. Why is it necessary in real life to be able to represent objects to scale? 5. Make rubbings of coins, blocks or environmental materials. This scale is 1:1.	1. Draw a regular shape on grid or dot paper. 2. Double the side lengths and draw the new shape. 3. Predict the area of both shapes. 4. Check by counting the squares inside the shapes. 5. Build a rectangular prism using blocks. 6. Build another prism with sides double the length. 7. Compare the number of cubes in the two prisms. 8. Build other prisms and double their sides as well. 9. Make a table and record the results for each prism. 10. Can you generalise about the relationship between the number of cubes in each prism?

Figure 29. The six activity cards designed by Gaye for students to complete during the observation period.

Each lesson began with a drill activity involving basic multiplication facts. The students were to complete as many questions as possible during a one minute interval. The aim for each student was to increase speed and accuracy since they were encouraged to compete against themselves. After this activity, students formed groups of two to four by joining tables together. The remainder of the lesson involved Gaye moving amongst groups, checking their work, encouraging their efforts, and ensuring students remained on task. In two of the three lessons there was some whole class discussion.

The first lesson was busy with most students actively engaged in the chosen activity. A parent helper was also present, working with two groups during the lesson on the necessary tasks. Gaye encouraged students to use equipment, share resources, and support each other's efforts. In several groups, students tended to work alone and Gaye constantly encouraged cooperative work. A group was considering the scale of a map of the local area and Gaye suggested they try to estimate the distance to a nearby shopping centre. This was a challenge so she suggested they pace out the distance to the front of the school and then use this to assist the estimation process. There was no whole class sharing of ideas during this first lesson.

The second lesson was even busier with students being constantly reminded to produce summaries or posters of their work for sharing with the class and for displaying on the back wall. In this lesson, there was a brief period of whole class discussion with a report of the estimated distance to the nearby shopping centre. Gaye spent much of this lesson encouraging slower groups, asking questions to support their efforts, and offering suggestions of how they might complete particular tasks. It was evident that she did try not to provide answers but made comments about students findings and asked questions to encourage them to think about what they were doing.

The third lesson followed a similar pattern except that Gaye stopped the class with about twenty minutes remaining for a discussion about the Area and Volume task. She was trying to encourage students to discover a pattern but her efforts were not very successful. When asked how the results could be recorded, one student suggested a graph but this was not helpful. In addition, results were not recorded systematically so students' estimates of a possible pattern seemed to be guesses with little thinking about what the pattern might be.

Overall, there was a sense of interest and enthusiasm from the students. They appeared to enjoy the activities, were willing to participate, and most remained on task throughout the three lessons. Some students were reluctant to record their findings even though they had engaged in the activities listed on the cards. Some groups worked more cooperatively than others with sharing of tasks and resources, and recording of solutions. Gaye seemed to be very aware of this and made regular comments like "you are supposed to be working together" or "you are to work as one person, helping each other".

Gaye was constantly moving among groups, observing, asking questions, encouraging, and offering support. It was clear that she was aware of individual student's needs and what each group was doing. Some groups received considerable support while others were praised for their efforts and encouraged to continue. An example of Gaye's comments made to a group of boys working on the Models task, and who seemed to require substantial assistance, is

> What are you drawing? The side view of this. Right, you have to draw this, so he can make it, he knows what it looks like, so maybe you need to adjust it so that he doesn't know what it looks like, okay. So maybe you can draw a side view that is a little bit different to what you see there, okay. That's the side view that you can see, so if you can draw something a little bit different because I want him to be able to make it up from your plan, okay.
>
> Well that's the top view, so yes one of those has to be the side. Now would a building be built like that? What sort of building would be built like that? What sort of factory? What would be built very, very wide like that, and very, very long like that? I'm thinking of an aeroplane. I think an aeroplane hanger. So that could well be a building. Think about the shape of the building. It's an unusual shape for a building isn't it to be like that? Right, buildings today are a bit taller than that, because we don't have the room do we to build a building like that?

Gaye was trying to encourage the boys to think about building types and their shapes and to consider realistic representations.

The following is an example of comments made by Gaye to a group of girls who had made good progress on the Scale task. She stated

> Larger than the object? Let's have a talk about it now? Why do you think that might be? Why would the picture be bigger in the book or smaller in the book? Let's sit down over there so we can all hear. Okay, here we are. So that makes it smaller. How's that? Huh huh, excellent – I don't even have to ask any questions you've got it all under control haven't you? Good.

In summary, the three lessons gave an impression of a teacher who was endeavouring to engage her students in practical activities that were relevant to their experiences. She encouraged them to work together, sharing ideas and talking about their mathematics. She was clearly concerned that they were on task and at the same time she wanted them to achieve some success. The students appeared to be motivated and interested in what they were doing, they readily responded to Gaye's comments and prompts, and most remained on task for the three observed lessons. Gaye constantly asked a variety of questions to support students' efforts and seemed to implement a variety of teaching strategies during this observation period. A more specific examination of use of student question types and teaching strategies follows.

Specific Observations of Gaye's Lessons (Analytic Analysis)

Lesson transcriptions were read and evidence was sought to support Gaye's reported use of particular student question types and problem-solving teaching strategies. Each of these is considered in this section.

Each of the six mathematical activities that were designed for the students to complete during the observations included instructions and questions. Some of these questions were closed while others were more open-ended. One example of a closed question appeared on the House Plans card, where the students were asked "what is the area of the floor plan"? An example of an open-ended question occurred on the Maps card, where students were asked "what are the commonly found scales on the maps you have looked at? [and] why do you think these are the most often used scales"?

Gaye had reported that she *often* used exercises and application problems and *sometimes* used unfamiliar and open-ended questions. During these three lessons, there was no evidence of use of either application or unfamiliar problems whereas there was use of exercises to begin each lesson and open-ended questions appeared on some of the activity cards. Use of student question types is summarised in Table 83.

Table 83
Use of student question types for Gaye's three mathematics lessons compared to her reported use from the questionnaire

Types of Questions	Lesson 1	Lesson 2	Lesson 3	Questionnaire Response
Exercises	√	√	√	often
Open-ended Problems	√	√	√	sometimes
Application Problems				often
Unfamiliar Problems				sometimes

The videotape for each lesson was viewed to find evidence of use of each of the teaching strategies listed on the questionnaire. Strategies that were frequently used in all three lessons included providing concrete materials, having calculators available, encouraging the students to work in groups and to record their own procedures and methods of doing the activities, allowing students to choose from a set of problems, and allowing students to spend several lessons on the same problem. In addition, Gaye's questioning provided a good model of the problem-solving process that also included suggestions about possible problem-solving strategies. Gaye endeavoured to relate the problems to real-life contexts thus showing the students that there were mathematical skills they needed to know and several of the problems related to students' experiences, or Gaye was able to do this during discussions. A summary of use of each of the teaching strategies is presented in Table 84 with the questionnaire response provided by Gaye that indicated her reported frequency of use.

Table 84
Gaye's use of each of the teaching strategies in each of the observed lessons compared to her reported use of each strategy as recorded on her questionnaire

Teaching Strategy	Lesson 1	Lesson 2	Lesson 3	Questionnaire Response
you ensure that the students work alone				sometimes
you explain in detail what the students have to do to solve problems				sometimes
at the end of a problem solving lesson you lead a whole class discussion so that students can share solutions and strategies			√	often
you have calculators available for students to use	√	√	√	often
you encourage the students to work in small, cooperative groups	√	√	√	often
you present *unfamiliar* and *open-ended problems* to the class with very little indication of how to solve them				sometimes
you encourage students to record their own procedures and methods of solving problems	√	√	√	often
you encourage students to pose their own problems				sometimes
you provide a set of problems and the students are allowed to choose a problem they would like to work on	√	√	√	sometimes
you allow the class or individual students to spend several lessons on the same problem	√	√	√	hardly ever
you use problems to show students that there are mathematical skills and procedures which they need to know	√	√	√	often
you present *application problems* which allow students to practise the skills they have just learnt				often
you provide concrete materials for those students who need them	√	√	√	often
you model the problem solving process to the class	√	√	√	often
you discuss useful problem solving strategies (eg. make a list, draw a diagram, work backwards)	√	√	√	often
you discuss problem solving processes (ie. make a plan, carry out the plan, check the calculations)	√	√	√	often
you use problems which arise from the school context or which relate to the students' experiences	√	√	√	sometimes
you pose *open-ended problems* to allow students to explore mathematical situations for themselves	√	√	√	sometimes
you set *exercises* to allow the students to practise their skills	√	√	√	often
you pose *unfamiliar problems*				sometimes

Gaye had reported that she *often* leads whole class discussions at the end of lessons so that students can share solutions and strategies. This did not occur in the first two lessons although it was Gaye's plan to do so. She seemed to misjudge the time available and was intent on students answering many questions rather than sharing with the whole class. In the last lesson, she spent almost one third of the available time discussing the

Area and Volume task and trying to encourage the students to think about a relationship between areas of similar rectangles. Most students did not really follow what she was discussing as the concept seemed too difficult.

The main discrepancies between Gaye's reported strategy use and observed use included less frequent use of whole class discussion and application problems. There appeared to be more frequent use of student choice of problems, presenting problems that related to students' experiences, and open-ended problems. Gaye had already commented that she did allow students to spend more than one lesson on the same problem.

During the classroom observation period, there seemed to be no real constraints operating in Gaye's classroom that prevented her from implementing the problem-solving approaches that she wanted to use. There appeared to be a reasonable correlation between her reported strategy use and her actual practice which in turn was consistent with her beliefs. However, there were also identifiable differences in that she reported *hardly ever* allowing students to spend several lessons on the same problem and only *sometimes* using problems that arose from the school context or open-ended problems. During the observations, it was apparent that the approach taken did incorporate each of these three strategies. It is possible that during the observation period Gaye made a real effort to incorporate many of the strategies that she believed were appropriate and desirable. Potential issues in the larger school context needed to be explored during the reflective interviews and are described in the next section.

5.3.3.5 Reflective Interviews

Three reflective interviews of approximately 20 minutes each were conducted at the end of each of the observation lessons. Time was spent reflecting on the three lessons, confirming Gaye's beliefs and practices, and further clarifying interpretation of data collected on the questionnaire, during the initial interview and classroom observations. In addition, these conversations confirmed previously described factors and revealed new issues that had impacted on Gaye's approach to teaching mathematics, potentially constraining or supporting her efforts in the classroom. The next two sections present a clarification of Gaye's beliefs and practices followed by the identification of additional issues.

Clarification of Beliefs and Practices

Gaye indicated that she was disappointed with the first observation lesson because of the amount of time spent on classroom management. Normally she had little trouble with her students and it was agreed that the presence of the researcher and the video camera was probably unsettling for the students. She was also critical of the parent helper's efforts since she provided little support to some of the groups in most need. In the second and third observation lessons, the students were noticeably more focused on their activities.

Discussions about the work Gaye's students were currently doing indicated that Gaye viewed these as investigations or activities rather than as problems. Although she did suggest that "there are problems within the activities". This discussion led to the following comments about her approach and her view of a problem

> The idea is to get them to learn what they're talking about when they're talking about scale, so it's a concept development more than problems. Um, but they are problems. I mean anything that they can't understand, if they've stopped and can't do it, it's a problem.

She indicated that she focused on problem solving at other times and described lessons where she presents "word problems" to the class. The students needed to use strategies that were listed at the front of their workbooks. She stated

> For instance they can make a table, or they can guess and check, or they can work backwards. They've got all these strategies in the front, and we go through and we decide, or they go through, I don't always do it as a class, but I say don't forget to decide the strategy you've got to use. Which one do you think would be most appropriate for this sort of thing? That also comes into problem solving ... this sort of problem solving is all hands on and it's not always that, because you've got to have these different learning things.

Gaye was asked why her current approach to teaching the concept of "scale" was better than a textbook, teacher-centred approach. She responded that it was not necessarily better but it was a different approach that worked for her students. She stated

> Some kids won't listen when you do that. I could point out the children who will not listen and they will probably disrupt the class, so that's one reason why you wouldn't use that terribly much in this room with some of the children, they'll just tune out completely ... but then who's to say that somebody won't miss it completely with this [approach], so I do a mix. Sometimes I'll come out and I'll teach and the discipline isn't terribly hard in this school even with those children that are dreadful, it's not terribly hard so that's not the reason they you would do it that way.

Reasons for the current approach seemed to be to engage the students in practical, hands-on activities and for variety in lessons. However, Gaye indicated that she does use other teaching approaches as well. Some of these included more *traditional* strategies that seemed to be related to the more *traditional* views Gaye supported in her questionnaire responses.

There were also discussions about Gaye's questionnaire responses, initial interview comments and the analysis written by the researcher. Before the observation period, the researcher had sent a copy of the questionnaire and initial interview analysis to Gaye to confirm interpretation of her responses. The possibility that Gaye was unclear of her views about some of the traditional statements presented in the first question on the questionnaire was confirmed. Gaye commented that she had been hesitant about her level of agreement with some of the questionnaire statements. She had *strongly disagreed* with the statement that "students cannot solve problems until they know how to perform the four operations". In the interview she stated "I may have changed my mind here" and indicated that she actually *agreed* with the statement. Gaye had clearly reflected on her responses after receiving the analysis from the researcher and realised that her response to that particular item may not have represented her true beliefs. It is possible that the opportunity to reflect may have helped her to clarify her thinking.

Gaye had also reflected on her reported frequency of use of the set of listed teaching strategies. She stated

> I have read what you have written and I realise that I do let them work on some problems for an extended period.

She also indicated that she "sometimes tells the kids what to do but I try not to". This last statement raised a potential dilemma for Gaye. She clearly believed that it was a good idea for students to investigate mathematical ideas and explore their own methods of doing mathematics, however in relation to lower ability students it was difficult for them to do this without substantial guidance and support.

It was apparent that Gaye was aware of innovative, or *contemporary*, practices and that she would like to be able to implement many of these ideas but she knew that strategies needed to be adapted to meet the needs of the students. She revealed that she was currently learning more about learning styles and was interested in incorporating some of the recommended approaches into her lessons. In relation to the "thinking hats" that were displayed in her room, she indicated that she did not usually use them in mathematics but she tried to use them for other Key Learning Areas. Discussions in these interviews led to the identification of particular issues related to her students, her current school, and her own experiences that seemed to have impacted on Gaye's practice.

Identification of Additional Issues

Particular issues for Gaye related to the needs of the less able students in her mathematics group, the school culture that included expectations of staff members, and her experiences as a learner of mathematics as well as a trainee teacher. Each of these three aspects is discussed in this section.

In relation to the mathematical needs of the students, Gaye was particularly concerned about students' attitudes towards mathematics and their belief in their ability to do mathematics. It was clear that for Gaye, one of her first tasks was to ensure that the students believed they were capable of doing mathematics and that they were comfortable asking questions and working together. She stated that

> the main thing is to make sure that the children understand they can and do do maths ... they don't think that they are capable of doing maths.

A related issue was that these students could find mathematics abstract and difficult to understand. Gaye indicated that she did not teach them formulas because she wanted them to work out the processes for themselves and it was more meaningful if they could then use their own rules. She stated

> But if you teach them formulas they won't remember these children, so you've got to teach them absolute understanding. I think all children have to do that, but particularly these children, because they can't remember the formulas.

According to Gaye, she spent considerable time working on the students' social skills because without the ability to work cooperatively together, she believed that the students would not develop the understanding they needed. A critical element in this was students' confidence. She described one student who

> doesn't think she can do maths, so she looks off into space all the time ... I think she's been told somewhere along the line that she can't do maths, so she doesn't do maths.

This was a real challenge for Gaye to encourage and support this student throughout the year. She reported that progress had been made and this student would now attempt some activities, particularly those that were perceived to be easier and more "fun".

Gaye believed that streaming of students enabled better planning and ensured that the students' needs were more readily met. She revealed that she thought all subjects should be streamed. However, she understood that this would be impossible in the primary school situation since she acknowledged that it was better for the students to spend the majority of their time with one teacher. Gaye chose to take the lower ability group because she believed that other Grade Six teachers "won't do it properly because they don't want to do it".

This comment led to the identification of issues related to the school culture including assessment procedures, the use of the textbook, and programming. Gaye believed that one of the main reasons other teachers did not like to teach lower ability students was because of the subsequent lower assessment results. She suggested that some teachers believed this would reflect on their teaching ability from the parents' perspective. Gaye suggested that

> they don't like to teach the low children because they don't get the results, and this school is very result oriented. If they don't get the results people don't think they're good teachers.

She indicated that she was unconcerned with parents' views and since she had no complaints in six years of teaching at the school, she believed that the parents must be satisfied with her efforts. In particular, she noted

> ... I get some nice results in the end, but it doesn't look like it in the beginning. It looks as if my kids are coasting because I approach it in a different way. When I've got the low children I don't make them start working straight away. First thing we do in the first couple of weeks is to make sure that they understand that they can do what they're doing, and other teachers won't do that because they know they've got to stick to the text book, they've got to follow it through and they've got to get the work done, and it's really sad, it's not fair.

Gaye was critical of other teachers' use of the textbook in the school. She suggested that the Principal and other members of the school Executive believed that the textbook was "just one of the many tools you can use" but she indicated that "if you go into the classroom and you look what they're using, they're using the textbook and nothing else". She agreed that, particularly in Year Six classrooms, reliance on the textbook may reflect a lack of confidence in teaching the content of the course. Further support for this view was the refusal by many staff to teach Year Six classes because they did not wish to teach the mathematics.

The assessment approach used was a formal assessment period three times per year. Each of these involved a pen-and-paper test in mathematics even though Gaye reported that the Principal's position on assessment was that it was ongoing. Gaye was particularly upset that her students had completed an assessment at the end of Term Three and were required to do another early in Term Four. She described a conversation with the Principal where she had objected to this situation. She stated

> I'm really uptight about this. I said why are we doing this assessment, we've just finished one. Oh, we need it for the next year's teacher. I said we don't have next year teachers, we're year 6, why are we having these tests again, and he said just to make you feel good so that you can see how well you have done. I said this is garbage, I want to teach my children, I don't want to be assessing them again. I said if you need an assessment we've just done one, we don't need another one.

In addition to the frequency of assessments, Gaye was concerned that her class were required to do the same examination as the more able students in the other two Year Six mathematics groups. She considered this to be particularly unfair as her group did more practical activities rather than textbook style questions that were reflected on the examination paper.

School programming procedures were another issue for Gaye. She revealed that in mathematics, the scope and sequence followed the content in the prescribed textbook. Staff were required to use a specified proforma for programming that Gaye had found particularly unhelpful. After several discussions with Executive members, she completed the required proforma and then rewrote much of it to suit her own needs. This was another issue that Gaye was rather emotional about. She described herself as being "really browned off" about the school's programming approach.

Another issue that was mentioned in relation to the observed lessons was lack of time to complete prescribed work. Gaye admitted that she had the luxury of more time to spend on topics at the end of the year and that if it was earlier in the year, "I would have had to finish scale today. I couldn't carry if over to next week, we just don't have the time".

Finally Gaye was asked why she had decided to train to become a primary school teacher in her late thirties. She said there had been several reasons but she had been particularly impressed by the Opening Ceremony at the Commonwealth Games in Brisbane several years ago where primary school teachers had helped to plan and present several aspects of the ceremony. In addition to this, the primary education received by her own children had been outstanding and she felt she "just had to give something back". Her own education had finished with little success at the end of Year Nine. She became a secretary although her interest in working with young children was fostered by the Boy Scout movement where she led groups for many years. She also spent much time at her children's primary school where she assisted with reading and other activities.

With failures in English and Mathematics at school, she was not confident and decided to return to high school education before tackling university studies. She was very successful and so commenced university education, keen to pursue her interest in English. Her academic record was very good and she was successful in majoring in English and had almost completed a Masters degree at the time of the observation period. Gaye believed that her lack of success at school was related to being a female and not being encouraged to achieve. She added that the university lecturers had inspired her to teach the way she did.

At each stage of data collection, new knowledge about Gaye's problem-solving beliefs and practices was obtained. Questionnaire analysis place Gaye in the *mixed* category because she had agreed with some *traditional* statements and some *contemporary* statements. Her responses to the open-ended questions suggested that her beliefs may be more traditional than contemporary while her reported practices suggested the implementation of more contemporary approaches. These potential inconsistencies needed to be explored in subsequent data collection stages.

Interview discussions revealed a strong belief in the streaming of students in mathematics. This was based on a view that student learning would be better supported since teacher planning could specifically address the needs of the group. In addition, Gaye was concerned about students' attitudes towards mathematics and the need for all students to experience success. Her reported and actual practices appeared to be more *contemporary* with the use of a combination of small group, student activities and drill-and-practice exercises. However, when asked to explain why this approach was better than more *traditional* approaches, Gaye indicated that she did not believe it was and that her lower ability students needed a combination of both approaches. It was apparent that Gaye had considerable knowledge and understanding of *contemporary* approaches and she believed that they were worthwhile. However, her experiences with the lower ability students had led her to reconsider this and to adapt her practice to better meet the needs of this group.

Another consideration to support the findings from the fieldwork was the level of coherence of beliefs and practices across a variety of data collection methods. After reflection, Gaye indicated that she may have responded incorrectly to some of the questionnaire items. In addition, she revealed that her responses to some of the items was based on her experiences with the lower ability students in Year Six. While she may have agreed with them in general, she believed that this was not the case with her current group of students. In this way, her belief in more *contemporary* approaches was tempered by the need to support these lower ability students and to provide substantial guidance. This suggested that the inconsistencies in Gaye's responses were related to the conflict between answering in relation to notions of "good practice", or responding in relation to the needs of the lower ability students she was teaching thus highlighting the issue of the difference between "what I believe and what I do". This may be a case of *beliefs* as potentially different to *beliefs in practice* as was described by Hoyles (1992).

For Gaye, potential constraints included particular aspects of the school culture that included a prescribed textbook, formal assessment procedures, and rigid programming. Gaye seemed to be able to resist these constraints by adapting her practice to accommodate the requirements of the school Executive. For example, the prescribed textbook was used as a source of homework exercises to support the activities planned for class time. As Gaye had trained as a mature-age student and had a strong character, she seemed to be able to resist the imposition of these potential constraints. The next section presents a summary of the findings from the fieldwork phase of data collection including the data for Gaye, Rose and the other seven teachers who participated in the initial interviews.

5.4 Summary of Fieldwork Analyses

The fieldwork represented the second phase of this investigation and included a combination of initial structured interviews, classroom observations, and unstructured reflective interviews allowing for triangulation of data. By comparing responses to questions in the initial interview, observations of what teachers do in their classrooms, and comments made in the reflective interviews, it was intended to test one source of information against another. This process increased accuracy of results, improved quality of the data, and allowed the researcher to understand more completely the beliefs of each teacher by looking for patterns of thought.

The fieldwork was used to explore the third and fourth research questions (Table 85). These questions focused on teachers' beliefs and practices in particular contexts and attempted to ascertain constraints and opportunities that might operate in those situations. To allow intensive exploration, nine teachers participated in this phase of the investigation. This section summarises the fieldwork analyses in relation to the third and fourth research questions. It includes a discussion of the dependability of the questionnaire for collecting data about teachers' beliefs and practices and explores the coherence and consistency of reported beliefs and practices for two of the teacher participants.

Table 85
The second two research questions for this investigation

Number	Question
3	In what ways do teachers incorporate problem-solving approaches into their planning and teaching of mathematics? In particular, when incorporating problem-solving approaches in their mathematics teaching: • what specific classroom strategies do teachers use; • what types of problem-solving tasks do teachers use; • how do teachers plan; and • what resources and materials do teachers use?
4	When teaching mathematics using problem-solving approaches, what factors can be identified that support or inhibit the implementation of such practices? In particular, what impact do each of the following factors have on teachers' planning as well as implementation of problem-solving approaches: • the teachers themselves:- including personal aspects such as confidence, past experiences that incorporate successes and failures, and opportunities for teacher development; • students:- including the influence of their beliefs, resistance, attitudes; the perceived ability of the students with respect to: learning mathematics; proficiency with language; participation in group work; • school culture:- including the influence of prescribed programs and textbooks, assessment and reporting practices, parents' expectations, principals' and other teachers' influence; and • the education system:- including curriculum documents, particular programs, external assessment procedures

In relation to the third research question, the questionnaire had provided substantial information about how teachers implement problem-solving approaches into their teaching. Two purposes of the initial interviews were to clarify teacher interpretation and confirm researcher analysis of questionnaire responses as well as to explore and elaborate teachers' comments from the questionnaire. These interviews confirmed that the questionnaire had provided a useful and dependable set of data about teachers' problem-solving beliefs and practices for this group of nine teachers.

A comparison of views as reported by teachers on the questionnaire and views described by teachers in the initial interview indicated that these were the same for six of the nine teachers interviewed. The interviews confirmed that the three teachers who were categorised as *traditional* did hold *traditional* views and used more *traditional* practices, and the one teacher who was categorised as *very traditional* held *very traditional* views and consistently used *traditional* classroom practices. For these four teachers, exercises and application problems were the main student question types used in mathematics lessons since there was a strong belief in the need to develop skills and procedures first. All mentioned a structured approach with a teacher-centred presentation of ideas, more individual student work, and a reliance on textbooks. Problems were usually used as extension questions for the more able members of the class.

Of the remaining five teachers, three were categorised as having a mixture of *traditional* and *contemporary* beliefs while two were *very contemporary*. For two of the *mixed* teachers, additional comments on the open-ended questions suggested that they may have more *contemporary* beliefs and used *contemporary* teaching approaches. This was confirmed in the interviews as both were supportive of problem-solving approaches and believed that problem solving was an important and necessary life skill for all students. The third teacher who had been placed in the mixed category appeared to have more *traditional* beliefs although interview discussions confirmed a *mixed* set of beliefs, particularly for her lower ability mathematics group. For the two teachers who were classified as *very contemporary*, interviews confirmed this for one while the other had tempered her beliefs and was now considered to be *contemporary*.

In relation to teaching approaches, all five of these teachers revealed clear differences in practice compared to the four teachers in the *traditional* group. They believed that *all* students could do problem solving and that it was an important life skill to be developed from the early years of schooling. They were more inclined to use all four student question types from a variety of sources although frequency of use was dependent on students' needs and abilities. In addition, all of these teachers mentioned the importance of developing student understanding and creating a supportive atmosphere so that students developed positive attitudes to mathematics.

Based on interview discussions, each teacher's problem-solving focus was determined by the researcher as either an object of inquiry, a process of inquiry or as a recognition of both. On one hand, those teachers with more *traditional* beliefs appeared to view problem solving as an object of inquiry, or as the solution of a variety of types of problems. This view was usually accompanied by a belief that problems were difficult mathematics questions that were appropriate for more able students. On the other hand, those teachers with more *contemporary* or *mixed* beliefs acknowledged that problem solving could be an important process that supports mathematics learning for *all* students. The problem-solving focus for each teacher participant is presented in Table 86 with their views from the questionnaire and views as determined in the initial interview.

Table 86
Each teacher's views from questionnaire analysis, views from initial interview, and problem-solving focus

Name	Views from Questionnaire Analysis	Views from Initial Interview	Problem-Solving Focus
Lois	*Traditional*	*Traditional*	object of inquiry
Janice	*Traditional*	*Traditional*	object of inquiry
Jane	*Traditional*	*Traditional*	object and process of inquiry
Susan	*Very Traditional*	*Very Traditional*	object of inquiry
Gaye	*Mixed*	*Mixed*	object and process of inquiry
Elise	*Mixed*	*Contemporary*	process of inquiry
Faye	*Mixed*	*Contemporary*	object and process of inquiry
Rose	*Very Contemporary*	*Very Contemporary*	process of inquiry
May	*Very Contemporary*	*Contemporary*	process of inquiry

The main factors identified as influencing this set of teachers' problem-solving practices were diverse. For the teachers who held more *traditional* views about mathematics teaching and learning, factors included the level of ability of the students in streamed classes, frequent use of set textbooks, formal assessment procedures, approaches used in local high schools and after-school tuition classes, students' success in mathematics competitions and in gaining entry to selective high schools, and parents' expectations. Some of these factors were also reported by Gaye who held a *mixed* set of views.

For the teachers who held more *contemporary* views, a common influencing factor was poor school experiences in mathematics that often seemed to convince teachers not to teach the way they had been taught. Other factors included enthusiasm for mathematics, a desire to promote positive attitudes, knowledge of more *contemporary* approaches, and other teachers' views.

In relation to the fourth research question, another purpose of the initial interviews was to identify factors that either supported or constrained teachers' problem-solving efforts in their current schools. These are listed in Table 87 as opportunities and constraints.

Table 87
Each teacher's initial interview views as well as opportunities and constraints

Name	Initial Interview Views	Opportunities	Constraints
Lois	*Traditional*	mathematics competitions	structured program, textbooks, students' beliefs, parents' expectations, lack of time
Janice	*Traditional*	A-streamed class and mathematics competitions	structured program, textbooks, students' beliefs, parents' expectations, lack of time
Jane	*Traditional*	-	structured program, steaming, formal assessment, textbooks, parents' expectations, lack of time
Susan	*Very Traditional*	-	lower ability students
Gaye	*Mixed*	knowledge of problem-solving approaches	potentially the textbook and some aspects of the school culture, lack of time
Elise	*Contemporary*	supportive staff and school culture	lack of time and lower ability students
Faye	*Contemporary*	supportive staff and school programs	potentially programs and ESL students
Rose	*Very Contemporary*	knowledge and supportive Principal	diverse student needs and school cultural issues
May	*Contemporary*	supportive Principal, interested students	conservative staff, middle primary class, lack of time

Opportunities were those factors that seemed to support or enhance the use of problem-solving approaches. The main identified opportunities were entry into mathematics competitions, knowledge of problem-solving approaches, a supportive school environment, and motivated students. It should be noted that for two of the teachers, no opportunities were identified that seemed to support problem-solving in their current schools. Regardless of the school context, Susan believed that problem solving was not necessary, however Jane believed that the current school culture in no way supported these approaches.

Constraints were those factors that might interfere with or prevent the implementation of problem-solving approaches. For this group of teachers, a constraint that was mentioned by five of the nine teachers was lack of time. Three of these teachers viewed problem solving as an object of inquiry thus suggesting that solving problems was an extra element to the program rather than as a process. Other identified constraints that related to the school culture included highly structured programs, use of textbooks, formal assessment procedures, parents' expectations, and a conservative or non-supportive staff. Constraints that related to the students included diverse needs, lower ability grouping, language difficulties, and students' beliefs that conflicted with problem-solving approaches. None of this group of teachers identified personal issues including a lack of knowledge or experience as a constraint although Rose had suggested that she was still learning about these approaches.

The final stage of data collection involved classroom observations and reflective interviews with two of the teachers who had participated in the initial interviews. This enabled validation of questionnaire data, clarification of initial interview comments, comparison of actual practice with reported practice, and further identification of factors that supported or constrained use of problem-solving approaches. Each level of data collection and analysis enhanced knowledge and understanding of each teacher's problem-solving beliefs and practices. In addition, the process of collecting four sets of data for these two teachers enabled triangulation of data thus increasing confidence in the research findings.

In summary, Rose was an experienced, dedicated teacher of a Year 2 class who believed that children needed to experience challenge and success in mathematics learning. She believed that the best way to achieve this was to use problem-solving approaches in the classroom since problems provided opportunities for students to think about mathematical ideas, to share their ideas with each other, to explore concepts using concrete materials, and to learn about mathematics in a meaningful way. From classroom observations, it was clear that Rose attempted to put her *very contemporary* beliefs into practice but it was also clear that her class was as challenging as she had described. In spite of the difficulties encountered in catering for a variety of students' needs, the students were quite motivated by Rose's attempts to engage them in meaningful activities.

Another consideration to support the findings from the fieldwork was the level of consistency and coherence of beliefs and practices across a variety of data collection methods. In this case, Rose consistently and coherently reported the same beliefs and practices on the questionnaire and during interviews. Because she has strongly held views about mathematics teaching and learning, she has been able to resist constraints on using this approach. In addition, Rose believed that her position as a member of the school Executive as well as her increased knowledge through further study have empowered her to resist the constraints operating in her school.

Gaye, the second teacher involved in all aspects of the fieldwork, believed that problems provided a purpose for learning about mathematical ideas since having learned basic facts and algorithms, students needed to be able to apply this knowledge to solve problems in the "everyday world". She also believed that mathematics was best learnt in "streamed" classes to better cater for students' needs and to enable the presentation of problems appropriate for that particular group of students.

There was considerable evidence that Gaye held a *mixed* set of beliefs. Her reported and actual practices appeared to be more *contemporary* with the use of a combination of small group, student activities, and drill-and-practice exercises. However, when asked to explain why this approach was better than more *traditional* approaches, Gaye indicated that she did not believe it was and that her lower ability students needed a combination of both approaches. It was apparent that Gaye had a sound knowledge and understanding of *contemporary* approaches and she believed that they were worthwhile. However, her experiences with the lower ability students had led her to reconsider this and to adapt her practice to better meet the needs of this group.

As a consequence, the level of consistency and coherence of Gaye's beliefs and practices across a variety of data collection methods seemed to be low. While she may have agreed with *contemporary* approaches in general, she believed that these were not the most appropriate for her current group of students. In this way, her belief in more *contemporary* approaches was tempered by the need to support these lower ability students and to provide substantial guidance. This suggested that the inconsistencies in Gaye's responses were related to the conflict between answering questions in relation to notions of "good practice", or responding in relation to the needs of the lower ability students she was teaching. Alternatively, it could highlight the possibility that for some teachers, there may by different sets of beliefs that depend on aspects of the current class or school context, a notion that could be referred to as *situated beliefs*.

The fieldwork phase of data collection supported and added to the rich data already collected on the questionnaire that was designed for this investigation. Also, it enabled a comparison of responses for a small group of teachers that ensured the results from the questionnaire were dependable thus increasing confidence in the research findings. In addition, the collection of several data sources for two teachers allowed for triangulation of data and the exploration of coherence and consistency of beliefs and practices. The data were also to be used to evaluate the usefulness of the model proposed in Chapter 2. This evaluation is presented in the following chapter with a summary of all results and suggestions for further investigation.

CHAPTER 6

DISCUSSION AND RECOMMENDATIONS

If problem solving is to be a focus of mathematics education and improving the problem-solving competence of our students is a desired outcome of schooling then it seems that teachers may need to examine their beliefs about the role of problem solving in learning mathematics as well as consider what practices are appropriate. This thesis describes an investigation of teachers' problem-solving beliefs and practices in mathematics teaching.

It was anticipated that this research would reveal information about teaching and learning of mathematical problem solving, illuminate factors that impact on teachers' problem-solving beliefs and practices, and raise questions for future investigation. A further aim was to identify and clarify particular teachers' beliefs to inform teacher education, curriculum planning, ongoing teacher development, and teaching practice. This chapter summarises the main results, evaluates the proposed model with respect to the results, evaluates the research methods employed, considers implications for practice and future research, and includes some recommendations for teacher education and development.

6.1 Results from this Investigation

There were four main research questions with data collected in two phases. The first phase involved the use of a questionnaire that explored teachers' beliefs and practices. Questionnaires were completed by 162 primary school teachers in NSW. The first two research questions related to the questionnaire, analysis of which was considered in detail in Chapter 4 with a summary of findings in Section 4.10. The second phase involved fieldwork methods that were used to collect additional data from nine of the teachers who completed the questionnaire. The last two research questions were addressed during the fieldwork phase with analysis presented in Chapter 5 and a summary provided in Section 5.4.

The following is a summary of the findings related to each of the main research questions. The first question was

> What do teachers believe is the role of problem solving in learning mathematics?

The results indicated that problems were considered by teachers to be an important component of mathematics. Problems were used in classrooms to motivate students and to demonstrate the usefulness of mathematics. Problem solving was viewed by many teachers as a necessary skill since it prepares students for life and future employment situations. However, many teachers tend to think that students find problems difficult to solve unless they are high achievers.

The second main research question dealt with the way that teachers reported that they implemented problem-solving approaches in their mathematics classrooms. It stated

> To what extent do teachers report that they incorporate problem-solving approaches in their planning and teaching of mathematics, and what specific practices do they report that they use?

To investigate this question, two perspectives were proposed that aimed to describe possible belief systems and related classroom practices. These perspectives described a *traditional* approach and a *contemporary* approach to the teaching and learning of mathematics that included the use of problem-solving approaches. Teachers' responses were categorised according to their support for each of these perspectives. The results supported the existence of these belief systems with 4% of respondents placed in the *very traditional* category, 11% in the *traditional* category, 9% in the *contemporary* category, and 7% in the *very contemporary* category. The remaining 69% of teachers could not be easily categorised since they supported some beliefs that were traditional and some that were contemporary.

Questionnaire analysis revealed slight differences in beliefs and practices between particular groups of teachers. Interestingly, years of teaching experience did not seem to play a key role in determining practices in classrooms. Either new teachers gain the necessary experience very quickly, or experience has little impact on practice. Teaching grade level did seem to impact on practice in relation to the types of tasks used and the focus of lessons. Teachers of the middle primary grades tended to use more traditional practices with a focus on the development of basic skills and algorithms. However, it should be noted that differences were still only slight with considerable similarity in practices for all groups.

In relation to problem-solving approaches, there appeared to be more of a focus on teaching *for* and *about* problem solving rather than teaching *through* problem solving. In this regard, problem solving was more frequently viewed as an *object* of inquiry rather than as a *process* of inquiry.

Data analysis revealed that teachers were more likely to employ application problems and exercises in their planning and teaching of mathematics than open-ended or unfamiliar problems. However, when a variety of problems were used, the problem choices were deliberate and for specific purposes. In general, exercises and application problems were used to teach basic skills and applications of algorithms, whereas open-ended and unfamiliar problems were more frequently used to challenge and motivate the more able students in classrooms. In addition, a comparison of teachers' reported usage of problem types with mathematics education lecturers' recommended usage revealed significant differences.

Teachers' questionnaire comments highlighted the issue that even though most teachers believe that problem solving is an important part of learning mathematics, problem-solving approaches can be difficult to implement for a variety of reasons. These issues, or constraints, were further explored in the fieldwork phase of data collection.

The third main research question involved an examination of some teachers' actions in particular classrooms. It stated

> In what ways do teachers incorporate problem-solving approaches into their planning and teaching of mathematics?

Initial interviews with nine teachers confirmed questionnaire analyses suggesting that the questionnaire was a useful instrument in gaining information about some primary school teachers' problem-solving beliefs and practices.

It became clear that for many teachers, a belief in using problem-solving approaches in classrooms was not always accompanied by related practices. There appeared to be powerful reasons why there were differences in beliefs and practices and these were potential constraints. The constraints can be grouped into those related to the teachers themselves, to the students, to the school culture, and to education systems. Data confirmed that constraints relating to the teachers included contradictory beliefs, lack of knowledge, and lack of confidence.

In relation to the students, a critical determining factor in the implementation of problem-solving approaches was the age and stage of development of the particular class. This was accompanied by the teacher's impression of the ability of the students, including their language proficiency, as well as their attitudes and beliefs. School cultural influences included programs, set mathematics textbooks, streaming, assessment practices, staff attitudes, and time constraints.

Time constraints were the most frequently referred to issues from the perspective of the teachers. These constraints were referred to in two quite distinct ways. First, time constraints on teachers' intentions seemed to be created because of system requirements that included a mandated curriculum and external assessment procedures. This was associated with the impression of a crowded curriculum. Second, time was a precious commodity because of the impact of teacher's daily work load.

The fourth main research question related to the third as it explored particular classroom situations and focussed on factors that supported or interfered with teachers' attempts to carry out a plan of action. It stated

> When teaching mathematics using problem-solving approaches, what factors can be identified that support or inhibit the implementation of such practices?

This final main question aimed to explore the factors that supported or constrained teachers' efforts in particular contexts. The two contexts explored at this stage of the investigation confirmed earlier data analyses and indicated that constraints are very real and can be difficult to confront. One factor that seemed to assist teachers in confronting such constraints was the notion of reflectiveness and a willingness to resist external pressures, if only within the classroom. It was at this stage of the investigation that it became clear that additional knowledge gained through postgraduate education provided considerable support for these teachers' actions, empowering them to confront constraining factors.

In addition to the above, some general findings from this investigation included the following. The research demonstrated that teachers:

- have clear, justifiable, well formed beliefs about problem solving and mathematics;
- as a group, hold the full range of beliefs about problem solving and mathematics;
- make active and deliberate decisions about classroom tasks and appropriate teaching actions; and
- plan lessons with their students' needs as the focus of their planning.

The results from this investigation suggested that teachers' beliefs influence their practice in relation to mathematical problem solving.

6.2 An Evaluation of the Proposed Model

The overall research aimed to illuminate potential influencing factors in the relationship between beliefs and practices. To this end, a model was proposed that incorporated interrelationships between factors that seemed to impact on teachers' reported beliefs and reported practices. It should be noted that the model was not intended to represent the dynamics of the situation but rather to identify key influencing factors and interrelationships between them at a particular point in time (Figure 30).

The results of this investigation have enabled an evaluation of this model by providing evidence to support some of the predicted influencing factors while other relationships have been challenged. These will be described in this section with the presentation of a revised model.

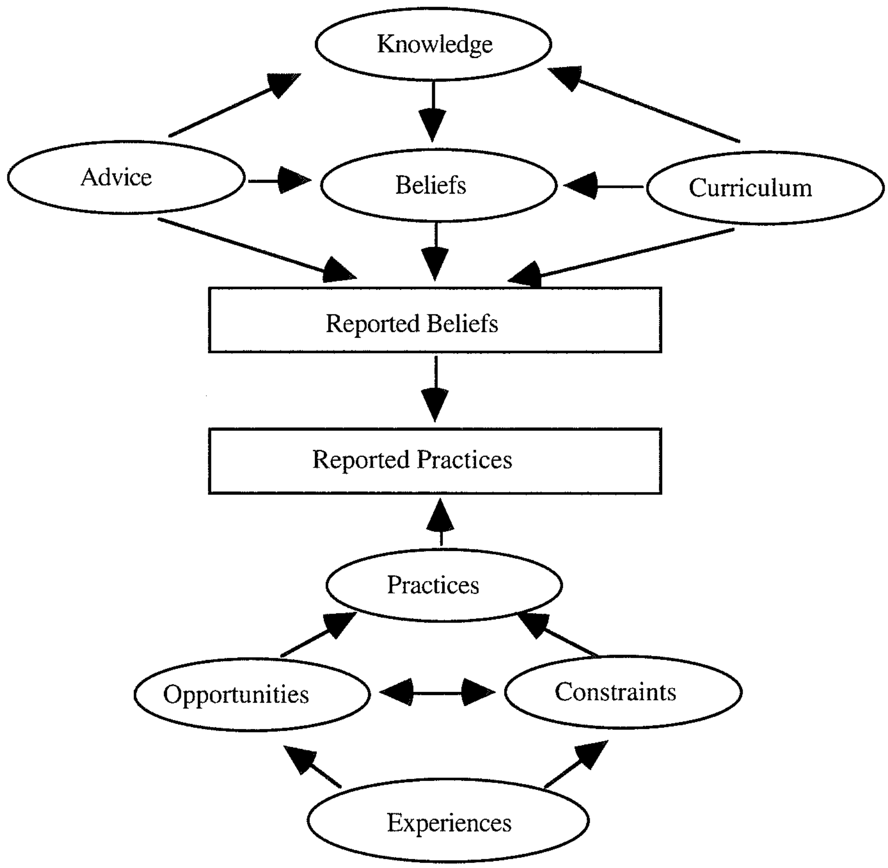


Figure 30. A model of the factors that impact on teachers' reported beliefs and practices.

The model suggests that *advice* to teachers, *curriculum* documents, and teachers' *knowledge* influence teachers' *beliefs* and hence their *reported beliefs*. The model also suggests that *reported practices* are influenced by actual classroom *practices* which in turn are either enhanced through *opportunities* or hindered by *constraints* in particular contexts. In addition, the model proposes that each of these is strongly influenced by teachers' previous *experiences*.

Results from this investigation confirmed the existence and influence of each of the factors presented in the proposed model. It was clear that many teachers were aware of the emphasis on problem solving in curriculum documentation, they had received advice in either preservice or inservice education, and they had at least some knowledge of problem types and problem-solving teaching strategies. However, it appeared that for at least some teachers, their previous experiences as both learners of mathematics and teachers of mathematics had acted as a powerful filter for these three factors and in turn influenced beliefs and practices. Interestingly, most of the surveyed teachers believed that teachers at their school would benefit from further problem-solving professional development opportunities.

It was also evident that the impact of the opportunities and constraints in particular contexts was strong. It is unfortunate that there appeared to have been few opportunities to enhance teachers' problem-solving efforts in each of the contexts for the nine teachers who participated in the fieldwork phase. However, there seem to be many more constraints that interfere with the implementation of problem-solving approaches than opportunities that might support such endeavours.

A revised, and potentially more useful model, is now proposed. This new model acknowledges the important influence of *experiences in classrooms* on beliefs and practices as well as the powerful impact of *previous mathematics learning* on teachers' knowledge and understanding of the teaching and learning of mathematics. In addition, a two-way arrow has been inserted between *knowledge* and *experiences in classrooms* to indicate that teaching aids teachers' knowledge of what approaches are appropriate for particular purposes. Also, two-way arrows have been inserted between *beliefs* and *reported beliefs* as well as between *practices* and *reported practices* since it appears that each of these relationships impact on each other in both directions. The revised model is presented in Figure 31.

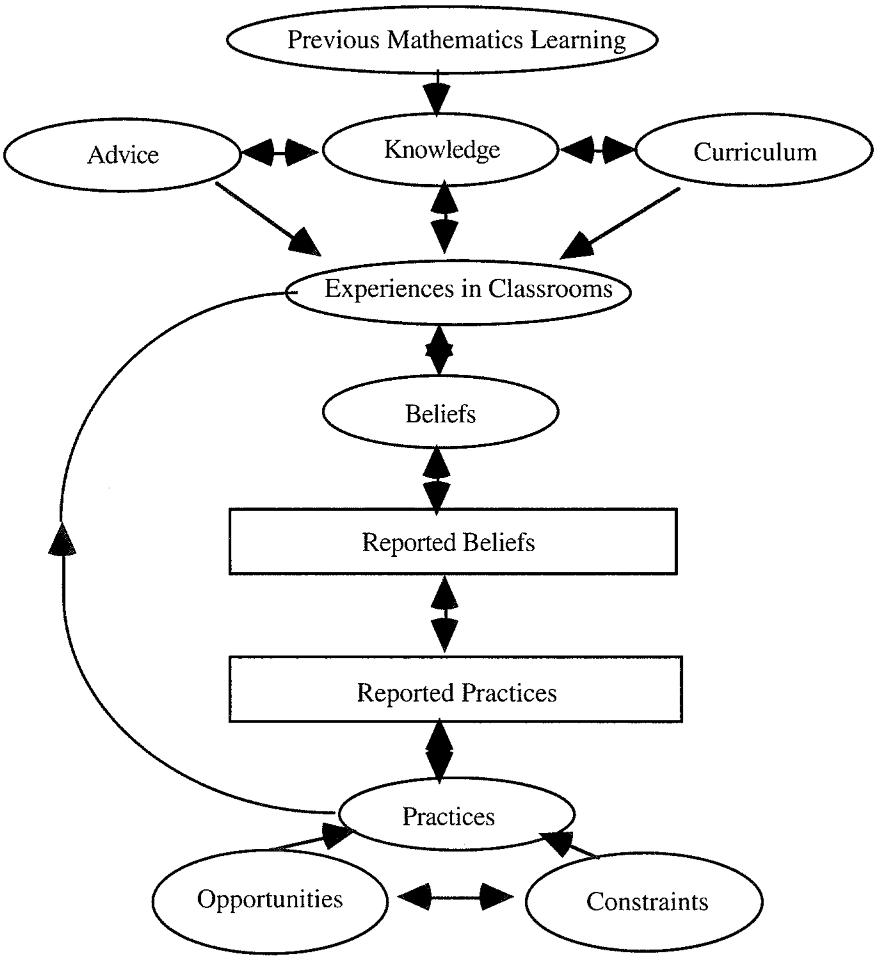


Figure 31. A revised model of the factors that impact on teachers' reported beliefs and practices.

It should be noted that for any particular teacher, some of these factors may have a stronger influence than others depending on previous experiences and current school context. Also, this model is a generalisation of a complex situation and is used to provide an illustrative guide to aid research into teachers' beliefs and practices.

6.3 An Evaluation of the Research Methods

This investigation was based on a multiple-methods approach to data collection. In the first phase, a questionnaire was used to collect considerable data about teachers' problem-solving beliefs and practices. In the second phase, interviews and observations were conducted in the field to collect detailed information about some teachers' beliefs and practices and to determine possible constraints and opportunities that impact on teachers' efforts to implement problem-solving approaches. The following brief comments present an evaluation of each of the research methods used in this investigation with some recommended adaptations for their future use in similar investigations.

6.3.1 The Questionnaire

The purpose of the questionnaire was to determine teachers' beliefs about the role of problem solving in learning mathematics, to gauge the extent of teachers' problem-solving practices, and to discover other issues that may be impacting on the implementation of problem-solving approaches. The questionnaire analysis, supported by data from the fieldwork phase of the investigation, suggested that each of these aims was realised. In particular, the questionnaire was a useful instrument in determining the spread of teachers' beliefs, in gauging the use of a variety of problem-solving teaching practices, and in revealing several issues that appear to impact on teachers' efforts to implement problem-solving approaches in their classrooms. Some factors arose during the investigation that could have impacted on the results and may need further consideration if the questionnaire were to be used in future research.

Extensive initial investigations and trialing were undertaken to achieve confidence in the questionnaire as an instrument for obtaining information about teachers' problem-solving beliefs and practices. During these early stages of the investigation, many adaptations and changes were made to the questionnaire that clearly improved its usefulness and aided the achievement of the aims stated above. In general, the items were understood by respondents and the use of a mixture of closed and open-ended questions was a good design feature. Also, the "Background Information" was helpful in informing teachers about the meaning of terms used in the questionnaire. However, there were a few items that may need to be reconsidered.

In the introductory section, teachers were requested to indicate their current role. Teaching grade levels were grouped as Years K-2, Years 3-4 and Years 5-6. For teachers who were teaching composite classes, some straddled the listed categories and so to overcome this several teachers chose more than one of the teaching categories. This created difficulties during the analysis when the above three categories were used to compare groups. Also, teachers were requested to complete the number of "teacher inservice days" they had attended in the last two years. Many inservice activities are less than one day and this may have lead to teachers not counting the total of all inservice activities thus not revealing the full extent of their participation in such professional development activities.

In Question 1, it was noted that many respondents agreed with the last statement made by Naomi that suggested that "some students find problem solving difficult because of the language involved in the problems". The final tally indicated that 98% of respondents selected either "agree" or "strongly agree" and therefore this item did not distinguish between respondents who might hold different belief systems. Because of the overwhelming positive response to this item, it was not used in the analysis of questionnaire responses, however, it is clear that further research into teachers' views about the impact of language on problem solving would be fruitful. To determine whether teachers' views are correct, this could be accompanied by an investigation of students' responses to problems with varying language difficulty levels.

The questionnaire analysis also revealed that there appeared to be a contradiction between the high level of agreement with the statement that "some students have trouble solving problems unless they know how to do the mathematics before they begin" and the high level of disagreement with the statement "students cannot solve problems until they know how to perform the four operations". It was possible that the negative wording of the second statement led to misinterpretation by respondents. Alternatively, the use of the word "some" at the beginning of the first item may have been a significant prompt in responding to the item. Another possible explanation is that teachers view mathematics as more than just the four operations.

During questionnaire administration, there was some concern with the small number of responses and the representativeness of the final sample. On the one hand, sending the questionnaires to a wider sample of schools in New South Wales may have yielded more responses and a more representative sample of primary school teachers. On the other hand, advice from other researchers indicated that seeking the support of Principals may yield more support from teachers. One suggestion that may have led to an increase in the number of responses was to send out the questionnaires earlier in the school year when teachers are potentially less busy with end of year commitments.

From the responses, it became clear that the teachers who did complete the questionnaire were those who had an interest in mathematics and/or problem solving. This led to rich data, particularly through the responses to the open-ended questions. The response rate for the open-ended questions ranged from 45% to 86% thus suggesting that most teachers took time to contemplate responses while completing the instrument. During the analysis, the responses to the open-ended questions often confirmed teachers' beliefs or raised important considerations.

It is recognised that questionnaires provide a snapshot of beliefs at a particular point in time and that teachers may respond differently depending on the particular class or students they are currently working with. This was revealed on several responses when teachers indicated that "it depends on the students". It was affirming for the researcher to receive positive feedback from some respondents who indicated in Question 10 that they appreciated the opportunity to be involved in the investigation and suggested that responding to the questionnaire had provided "food for thought".

The fieldwork phase of data collection confirmed teachers' interpretations of the questionnaire and provided considerable support for its use as an instrument to collect data about teachers' problem-solving beliefs and practices. Only minor adaptations are recommended, however, the instrument could be confidently used in future research.

6.3.2 The Initial Interviews

The initial interviews had several purposes. These included the need to clarify teacher interpretation and confirm researcher analysis of questionnaire responses, explore and elaborate teachers' comments from the questionnaire, reveal opportunities and constraints that might impact on teachers' efforts to implement problem-solving approaches, and to select participants for the classroom observations. Each of these aims was realised and the interviews were particularly useful for discussing and elaborating issues that had been revealed on questionnaire responses.

Clarification of interpretation of the questionnaire indicated that, for this group of teachers, the instrument had been a useful tool to gauge their problem-solving beliefs and practices. Only one of the nine participants seemed to have changed her views on some aspects of problem solving and this was associated with a change in school and a different teaching grade level. This highlighted the fluidity of beliefs and the possibility of the existence of "situated beliefs". This aspect of the investigation would be a fruitful area for further investigation and is discussed in more detail in Section 6.4.1.3 of this chapter.

The initial interviews were semi-structured with the questions providing a framework for discussions. Each interview quickly developed into a general discussion about school-based issues, revealing many key and important issues for each of the teachers interviewed. The researcher was confident that she was able to obtain a reasonably accurate depiction of each teacher's problem-solving beliefs and practices and the constraints that were operating on their potential implementation. All participants were relaxed and comfortable with the interview process and seemed genuinely interested in the research and its potential outcomes.

6.3.3 The Observations and Reflective Interviews

The purpose of the observations was to further validate questionnaire data as well as to clarify and confirm initial interview comments, compare actual practice with reported practice, and to identify factors that seem to either support or constrain the implementation of problem-solving approaches. Only two teachers were observed and it could be argued that they may not have been a representative sample of the group of teachers who completed the questionnaire. However, this stage of the investigation aimed to explore particular contexts and did not set out to reveal the full range of approaches with associated constraints on practice.

These teachers were requested to teach three consecutive lessons, incorporating problem-solving approaches where appropriate. It could also be argued that the observed lessons may not have been representative of each teacher's everyday practice. This is acknowledged and an improvement in the design of the investigation may have been to observe three or more randomly assigned lessons over a period of several weeks. However, this approach would have been far more intrusive to the teacher and her students. It was observed that by the third lesson, particularly in Rose's Year 2 class, the students paid little attention to the researcher and attended more to the teacher's instructions. The classroom observations were sufficient to clarify findings from earlier data collection and to reveal additional issues for consideration.

Unstructured, or informal, reflective interviews were used to provide the opportunity for teachers to discuss actions and decisions that were made during the course of each lesson and to illuminate factors that might be inhibiting desirable practices. This enabled teachers to identify reasons for those decisions and provided opportunities to discuss beliefs that might impact on such decisions. These interviews were very revealing and the researcher is confident that key issues were illuminated and discussed in the time available.

In summary, while some minor changes to the research methods may have enhanced the design, the analysis of the data collected provided a rich and useful source of information for future use by many involved in mathematics education. The use of a multiple-methods approach was a powerful way of confirming and ensuring confidence in the findings. In particular, at each stage of data collection, new layers of knowledge about Rose and Gaye were added to provide a detailed picture of each of their problem-solving beliefs and practices as well as the factors that impacted on their implementation of problem-solving approaches.

6.4 Recommendations

In this investigation, several factors have been identified that impact on teachers' problem-solving beliefs and practice. Some of these factors represent issues that could be further considered if teachers are to be encouraged to implement recommended teaching approaches in their classrooms. An acknowledgment of the impact of these issues represents a first step in the potential to support future change in preservice and inservice education. These issues are discussed in this section as implications for practice and future research followed by recommendations for teacher education and development.

6.4.1 Implications for Practice and Future Research

Five issues arose from this investigation that have implications for practice and may require further consideration in future research. First, there are constraints that are clearly present in many school contexts and that need to be recognised and addressed if teachers are to implement problem-solving teaching approaches. Second, some teachers appear to reflect actively on their practice enabling them to confront more readily potential inconsistencies between beliefs and practices. Third, for some teachers there is an inconsistency in reported beliefs and practices that may be accounted for by the existence of situated beliefs. Fourth, there is little evidence that teachers actively consider teaching *through* problem solving as a viable teaching approach. Finally, knowledge and regular use of a variety of problem types seems to be limited and may need to be specifically addressed.

6.4.1.1 Constraints on Practice

Many different constraints were identified in earlier studies that are confirmed in this investigation. A somewhat surprising outcome of this research is the seemingly overwhelming influence of a variety of issues for teachers as they attempt to implement problem-solving approaches. It is possible that some are used by teachers as an excuse not to change current, comfortable practices but interviews with teachers convinced the researcher that there is a real desire for teachers to find methods that improve mathematics teaching and learning. Many comments involved a plea to recognise teachers' busy lives and to provide time and increased opportunities to discuss such issues so that potential solutions can be found. This suggests that further research into school organisational policies and practices is necessary if teachers' efforts are to be supported.

The composition of classes and meeting the needs of all students was an ongoing issue for many teachers. Research into the dynamics of classrooms is clearly critical for supporting teachers in their ongoing decision making. These influences are clearly powerful since teachers' classroom experiences lead them to form coherent beliefs that are frequently at odds with expert advice.

It would appear that the role of the teacher in problem-solving classrooms is still unclear. In addition to this, the need for students to take more responsibility for their own learning in problem-solving classrooms may need further investigation. This could include research into strategies to improve students' awareness of their role. Also, students may still need considerable support in developing perseverance when solving problems. A further issue that seems to impact on teachers' implementation is the influence of students' and parents' negative attitudes to problem solving suggesting that these are not skills that are valued. These negative attitudes are potentially reinforced by the question types typically presented in external or high stakes examinations.

Suggestions by teacher respondents in relation to possible professional development initiatives in schools were interesting and worthwhile. They recognised the potential of teachers at their own schools and requested time to share ideas, reflect on their successes and failures in implementing innovative strategies, and to have support for such endeavours by school executive members and system personnel. This leads to the second issue of reflective practice.

6.4.1.2 Reflective Practice

Teachers' experiences in classrooms on determining their beliefs and practices seemed to have less influence on practices than might have been anticipated. This needs to be further explored, particularly in the light of interview comments that frequently referred to poor learning experiences in mathematics. It would appear that many teachers do not wish to present mathematics to their students in the same traditional way that they were taught but they are clearly not convinced that some of the recommended contemporary approaches are the solution. For some teachers, it seems easier, or more comfortable, to teach in traditional ways than to test new approaches or to spend time examining their current practice and the impact of adopted approaches.

It is clear that some teachers seem to engage actively in reflective practice and can readily and coherently articulate their beliefs and practices. It is possible that reflective experience may be a key variable in developing and implementing new approaches. Research examining whether opportunities for reflective practice support such changes may be useful.

In this investigation, those teachers who were more likely to embrace contemporary practices were those who had unsatisfactory school experiences, were highly influenced by successful undergraduate or postgraduate experiences, and were determined to change students' attitudes towards mathematics. One interesting finding from this investigation was the seemingly strong positive effect of postgraduate study on teachers' capacity to reflect and grow. Given the current context where postgraduate study is becoming more difficult due to the increasing cost and lack of system support, this is a potential concern. It would be useful to see whether this result is replicated elsewhere. If so, it may be that governments and employers could consider ways to support more teachers in postgraduate study.

Further investigations comparing the beliefs and practices of primary school teachers who were successful students of mathematics with those who were less successful may also be worthwhile. In addition, an exploration of the impact of postgraduate study in mathematics education on teachers' beliefs and practices is worthy of consideration.

6.4.1.3 Situated Beliefs

The level of consistency in reporting beliefs and practice was low for some participants in this investigation. This could be a result of lack of reflective practice or it could be an indication that potentially contradictory beliefs are a result of the existence of *situated* or *contextualised* beliefs. It may be the case that, for at least some teachers, they hold different sets of beliefs that could depend on the ability of the students, the grade currently being taught, or other aspects of the school context. It was noted during questionnaire analysis that one respondent had written "it depends on the students" and similar comments arose during initial interview conversations. In this sense, beliefs may vary according to who is being taught and what is the focus of particular lessons.

This is an area that needs further investigation. In addition, it is possible that some teachers' responses to belief statements could depend on whether the class of students is a mixed ability group or a streamed group. Hoyles (1992) discussed the existence of two sets of beliefs that she referred to as *decontextualised beliefs* and *beliefs-in-practice*. This possibility raises an important issue in relation to the dependability of using only one data collection method to gather data about teachers' beliefs. Also, Raymond (1997) referred to deep and surface beliefs that impact on practice in different ways. Clearly, these aspects of the influence of beliefs on practice have potential for fruitful future investigations.

6.4.1.4 Teaching *Through* Problem Solving

Even though advice suggests that child-centred approaches to learning with students engaged in exploring new ideas for themselves is more meaningful and ultimately more successful, teachers generally do not employ this approach. It would seem that teachers still believe that more traditional approaches to teaching and learning mathematics are more successful. They are more willing to teach *for* and *about* problem solving than *through* problem solving.

These findings seem to suggest that more contemporary views of learning mathematics have not been embraced by some teachers and that this may be a worthwhile area for future research. It is possible that contemporary approaches that provide students with opportunities to explore and investigate their own mathematical ideas are more difficult for teachers to implement successfully than traditional approaches. Associated with this approach is the use by teachers of particular problem types for specific purposes. Further research into the use of a variety of problem types to enhance students' understanding of mathematics could provide valuable advice for teachers.

6.4.1.5 Problem Types

The investigation found that teachers more frequently use exercises and application problems than open-ended or unfamiliar problems. This seemed to be accompanied by a more frequent focus on teaching *for* and *about* problem solving than teaching *through* problem solving. In addition, teachers were more likely to consider problem solving as an *object* of inquiry rather than as a potential *process* of inquiry. All of these aspects are clearly linked and need to be further explored in classrooms with teacher participants.

It seemed that for teachers, open-ended and unfamiliar problems were considered most suitable for more able students. Further research could explore whether this is a correct assumption, and if not, what strategies could be used to demonstrate to teachers that this is not necessarily the case. In this way, the use of the full range of problem types needs further investigation.

6.4.2 Recommendations for Teacher Education and Development

The five issues described in the previous section need to be further investigated to support teacher education and development. It is clear from this investigation that a focus is needed on teachers' practice and their awareness of the issues and factors that seem to impact on implementation of problem-solving approaches.

Particular classroom practices need to be further considered in preservice and inservice education. These include the use of the full variety of problem types with a careful consideration of their usefulness for *all* students. This could incorporate the notion of teaching *through* problem solving and the implications of this approach for teachers and learners. Another important aspect of this approach is the sense of not having control of the learning and teachers' willingness to allow students the freedom to explore and discover for themselves.

It is clear that teachers value collaborative opportunities within their schools. This may be a more powerful professional development activity than "expert" demonstrations at staff meetings or at inservice courses. All of these aspects of practice could provide important opportunities to develop teacher education students' as well as practicing teachers' knowledge and understanding of problem-solving approaches.

Focusing on teachers' beliefs and practices and encouraging them to reflect and describe what they believe may be the first stage in changing teachers' beliefs and practices. Identifying the sources of influence on beliefs provides a starting point for discussions in preservice and inservice education. As many teachers seem not to have responded to advice to incorporate problem solving into their teaching approaches, it is possible that prevailing beliefs held by many of these teachers militate against the implementation of *contemporary* approaches in classrooms. Confronting these beliefs seems to be a necessary step if change is to be embraced.

In this sense, there is a need to develop an awareness in teachers of the impact of beliefs on practice as well as the other factors that can interfere with their good intentions. This awareness would need to confront constraints and to encourage teachers and whole staffs to investigate ways of overcoming such restrictions on their practice. In addition, it would be desirable to have teachers consider how their beliefs and practices differ from expert advice and why this has occurred. There is a real need for all teachers to be aware that there are different ways of thinking about mathematics teaching and learning and the implications of this for classroom actions.

APPENDICES

Appendix 1 - Factors that Influence Students' Problem-Solving Abilities

Learning to solve problems can be a real challenge for many students as there are several factors that impact on students' abilities to solve problems. It is not sufficient to know facts and procedures. Nor is it sufficient to be aware of a range of problem-solving strategies. Students need to know how to manage this knowledge and how to coordinate their efforts. These aspects of developing problem-solving abilities are also influenced by the classroom environment, the teacher's actions, and other students in the class. All of this information impacts on teachers' knowledge and beliefs about teaching and learning mathematics and is encompassed in the advice factor as presented in the model that guides this investigation.

Problem solving is a complex activity that involves considerably more than just recalling facts and applying learned procedures. Lester (1987) expressed concern that many early studies attributed students' problem-solving difficulties almost exclusively to cognitive aspects and yet there was increasing evidence that there were other important factors that impact on the problem-solving process. Schoenfeld's review of the problem-solving literature led him to proclaim: "it's not just what you know; it's how, when, and whether you use it" (p. 355).

It may be that even when students have the necessary mathematical knowledge to solve problems, they are often unable to apply that knowledge to problems that are even slightly unfamiliar (Kroll & Miller, 1993; Schoenfeld, 1987a; Stacey, 1990). Several reasons for this difficulty have been discussed including comprehension difficulties (Noddings, 1989), insufficient experience with a variety of problem-solving tasks (Silver, 1987), continuing to apply incorrect procedures even when they are not really leading to a solution (Taplin, 1993), poor checking of solutions, and reflecting on productiveness of approaches (Garofalo & Lester, 1985; Schoenfeld, 1989).

The reasons for difficulties in problem solving have been grouped into broad categories by several researchers. Charles and Lester (1982) described three sets of interacting factors that included:

> 1. experience factors, both environmental and personal;
> 2. affective factors, such as interest, motivation, pressure, anxiety, and so on
> 3. cognitive factors, such as reading ability, reasoning ability, computational skills, and so on (p. 10).

Schoenfeld (1985) described four overlapping and interacting categories of mathematical behaviour that work together to impact on a student's ability to solve problems. These include resources, or knowledge; heuristics, or the "rules of thumb for effective problem solving" (p. 44); control, or management and allocation of resources; and belief systems, or the mathematical world view that is held by the student.

Kroll and Miller's (1993) review of problem-solving research led to a categorisation of the factors into four areas. Influencing factors were organised into the areas of knowledge, control, beliefs and affects, and social and cultural conditions. Knowledge included linguistic and factual, schematic, algorithmic, and strategic factors. Control factors included the decisions students make while they are attempting to solve problems. Beliefs and affects included emotions, preferences, attitudes and beliefs. These three areas were then contained within the social and cultural influences of the setting within which problem solving takes place. Kroll and Miller warn that "the social context of the classroom should not be overlooked as a potent influence on students' achievement" (p.65). This area is one that was not really recognised in the earlier discussions of Charles and Lester (1982). Kroll and Miller represented these factors diagrammatically and this is reproduced here in Figure 1. Each of these areas is discussed in the following sections, with reference to a selection of studies that highlight their impact on students' problem-solving efforts.

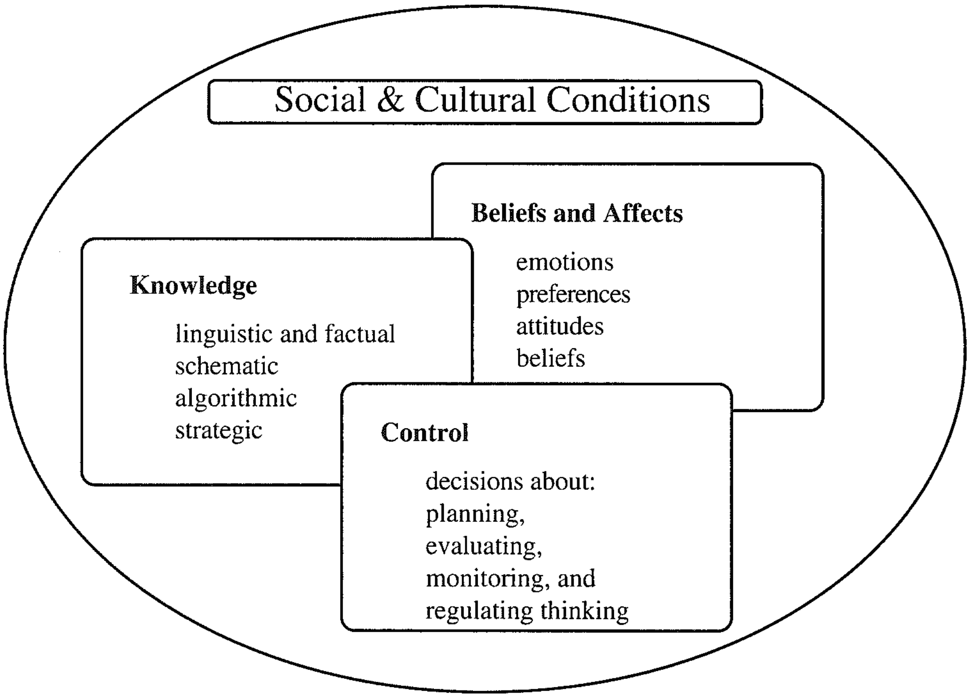


Figure 1. Factors involved in mathematical problem solving (Kroll & Miller, 1993, p. 62).

Knowledge

Several factors can impact on the knowledge that students require to enable them to solve problems. Poor reading skills and lack of familiarity with the language used in word problems can impact on students' abilities to understand problems but Kroll and Miller suggest that it is an "oversimplification to equate reading difficulties with problem-solving difficulties" (p. 63). Good problem solvers usually remember previous problems they have solved and can use them to inform current problem-solving efforts. Clearly computational and algorithmic skills are necessary to support problem solving but not all students who have this knowledge are able to use it to solve problems (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980; Siemon, 1986). Good problem solvers also have at their disposal a variety of problem-solving strategies that they can choose and use successfully. Kroll and Miller (1993) suggest that knowledge factors are necessary to support problem solving but they are not sufficient.

Schoenfeld (1992) describes a "knowledge base" that includes the contents of memory and its accessibility. Thinking is done in short-term memory that is relatively small and provides serious limitations to the kinds and amounts of mental processing that can be performed. Long-term memory enables people to access the necessary information that is stored in "chunks" or schemas. He suggests that

> People abstract and codify their experiences, and the codifications of those experiences shape what people see and how they behave when they encounter new situations related to the ones they have abstracted and codified (p. 352).

Research that investigated the expertise of grand masters at chess has supported the presence of schema (Sweller, 1993). The high level of competence of chess masters appears to be a result of playing many games over many years and relies on the fact that the best move for any particular board configuration has been learned from experience. It seems that chess experts are able to recognise patterns in the board configurations; patterns that have been built up over a considerable period of time (Silver, 1987). Sweller (1993) draws a parallel between chess playing and solving mathematical problems in that schema can be developed that enable students to develop expertise in solving problems. Schema guide the encoding and retrieval of problem-solving information and they also assist in shaping the representation of problems; a process that Silver (1987) describes as central to problem-solving activity.

Sweller (1993) argues against the presence of *general* problem-solving ability and believes that students need to solve a variety of problems to develop schema and also to develop automation that is necessary to reduce the load on working memory. Students need to be able to categorise problem types thus providing them with the necessary information to solve less familiar problems. In addition, he suggests that the development of appropriate schema can be enhanced through the use of goal-free problems and the examination of worked examples. Sweller is critical of current curriculum documents in that they emphasis *general* problem-solving ability which he believes cannot be taught. He suggests that "the major differences in problem-solving skill between experts and novices lie not in terms of general problem-solving strategies, but rather in terms of domain-specific knowledge" (p.11).

This research does not receive widespread acceptance in the mathematics education community. Schoenfeld (1992) argues that this is because it focuses on performance rather than on basic understanding and that it

> may produce surface manifestations of competent behaviour. However, that performance may, if not grounded in an understanding of the principles that led to the procedure, be error prone and easily forgotten (p. 352).

Specific knowledge is necessary but not sufficient for students to become good problem solvers. Sweller's research pays less attention to the impact of control, beliefs and affects although the Sweller and Cooper (1985) study note that individual differences in learning from worked-out examples might be accounted for by poor motivation. The potential influence of the sociocultural environment receives even less attention but students rarely solve problems in isolated situations. It is more often the case that they experience problem solving in the social context of classrooms, discussing ideas with peers and teachers.

Another aspect for consideration is that there appear to be significant differences in approaches taken by novice and expert problem solvers (Siemon, 1986; Silver, 1987; Romberg & Carpenter, 1986). Experts organise knowledge in larger chunks based on higher-order principles whereas novices organise in smaller chunks based on surface characteristics. Experts take an holistic approach and seem to possess control and managerial strategies that monitor their performance. Novices concentrate on smaller detail and waste time and effort on unfruitful calculations. The ability of experts to be able to control their efforts seems to be a significant enabling factor in the problem-solving process.

Control

Control, or the management of resources during problem-solving attempts has a major impact on the success or failure of students' problem-solving performances (Schoenfeld, 1985). As is discussed below, experts spend considerable time planning, reviewing and checking problem-solving efforts whereas novices waste time on unsuccessful trial-and-error strategies with little attention to an overall strategy or organisation of resources. Also considered are strategic behaviours that are critical to successful problem solving, as well as the important role of instruction.

Control, or metacognitive strategies involve decisions made about planning, evaluating, monitoring and being aware of one's thinking (Garafalo & Lester, 1985). An important difference between cognitive and metacognitive behaviour is that metacognitive strategies are planned and deliberate activities. There are several aspects to metacognition; one concerns the student's awareness of cognition, another concerns the ability to reflect on both knowledge and management processes, sometimes referred to as control (Campione, Brown, & Connell, 1989).

Strategic behaviours that are necessary in the problem-solving process can be organised into specific stages. In Schoenfeld's (1985) research, he identified the stages as read, analyse, explore, plan, implement, and verify. Schoenfeld (1985) argued that movement from one stage to another is evidence of control or executive decision making and it is this control that enhances successful problem-solving behaviour. Schoenfeld used a protocol parsing technique to analyse the transcripts of pairs of students solving unfamiliar problems. Each protocol was divided into episodes that represented periods of time for a related set of activities, and each episode was classified as belonging to one of the specific problem-solving stages.

Schoenfeld's research indicated that successful problem solvers moved freely backwards and forwards between these stages whereas unsuccessful problem-solving attempts usually resulted in students staying for long periods in one particular stage. For example, pairs of students were found to spend considerable periods of time at the exploration stage even though suitable solutions were not forthcoming. These students seemed to be unable to make decisions about going back to an earlier stage to re-examine their earlier efforts and assumptions. Experts were observed to spend more than half the allotted time trying to make sense of the problem and regularly making comments that related to the state of the problem solution.

Similar frameworks that have identified stages in the problem-solving process were used in the cognitive-metacognitive framework of Garafalo and Lester (1985) and the basic problem-solving skills of Clement and Konold (1989). Anderson (1992) adapted these two frameworks to generate a set of stage-specific, problem-solving skills and general, problem-solving skills as in Table 1.

Table 1
Basic problem-solving skills (Anderson, 1992, p. 12)

Stage-specific Skills:		
1	Orientation	• understanding and analysis of information
		• representing the information
2	Organisation	• planning - what will I do?
		• breaking the problem into parts - goals and sub goals
3	Execution	• trialing
		• checking
4	Verification	• evaluating, revising, justifying
General Skills:		
	• alternately generating and evaluating ideas	
	• striving for precision	
	• monitoring progress by:	- using written records
		- using confusion as a signal to rethink
		- proceeding slowly in the expectation of making and needing to correct errors

Anderson (1992) used this framework to explore the use of metacognitive strategies by Year 8 students as they solved unfamiliar problems in groups of four. She found evidence of use by students of all stage-specific and general skills during the problem-solving sessions, however when problem-solving attempts were not successful, the students spent a great deal of time on execution with not enough time spent on organisation and planning solution methods. Garafalo and Lester (1985) also argued that novice problem solvers are usually not aware of their thinking and fail to plan, evaluate and reflect on their problem-solving attempts.

Several studies have demonstrated that it is possible to use a variety of teaching approaches to improve the use of control strategies by students (Herrington, 1990; Schoenfeld, 1992; Siemon, 1993; Taplin, 1993). These strategies include the use of focus questions to encourage the development of reflection as well as models of strategy use that have been used to train students in the use of a variety of metacognitive strategies. It is clear that to improve students' problem-solving performance, strategies need to be made explicit and should be modelled by teachers or peers.

Herrington (1990, 1991) describes several successful techniques that he has implemented in primary classrooms including activities that support self-questioning and reflection. For example, a learning check was employed on a regular basis that includes questions such as

> Do I understand?
> What don't I understand?
> How can I find out?
> Can I as my own questions? (p.340).

Each of these key questions was accompanied by a series of related questions to encourage students to reflect and analyse their thinking. Herrington (1991) found that when students were involved in a learning strategies program that included activities focusing on control strategies, they were far more aware of these strategies and were more likely to use them.

Schoenfeld (1992) found that he could improve the problem-solving performance of college students by teaching metacognitive strategies based on a series of questions. Students were encouraged to work on problems in small groups from the early stages of his undergraduate course. He would move between groups asking them

> What (exactly) are you doing? (Can you describe it precisely?)
> Why are you doing it? (How does it fit into the solution?)
> How does it help you? (What will you do with the outcome when you obtain it?) (p. 356).

He reported that at first students were unable to answer the questions but as the semester progressed they were more comfortable with these questions and were more able to answer them. Many of the students' solution attempts also improved as they spent less time pursuing the first unsuccessful strategy and were far more likely to try different approaches.

Siemon's (1992) preliminary observations of students solving unfamiliar problems revealed a complex interaction between conceptual and procedural knowledge of both a cognitive and metacognitive kind. This led to the development of a model that provided a framework that could be used to identify students' problem-solving approaches. Siemon (1993) then conducted a year long teaching experiment in a Year 4 class that provided the students with a question and answer technique referred to as *ask-think-do* (Barry, Booker, Perry & Siemon, 1985). The questions encouraged discussion and reflection; an approach that was modelled by the teacher followed by discussion and recording of key questions and strategies. The intervention resulted in overall improvements in problem-solving performance, particularly for those students who had initially demonstrated the lowest use of metacognitive strategies.

From another perspective, Taplin (1993, 1994) investigated Year 6 and Year 10 students' perseverance strategies when solving unfamiliar number problems. She produced a model of a sequence of strategies that was then used in a small-scale training program. After working with six 13-year-old students in a clinical interview situation, preliminary results indicated that the problem-solving model was a useful tool. The teacher's role was necessary in the early stages to evaluate strategy use, choose the appropriate time to intervene, and to provide hints and suggestions without providing answers.

Callahan and Garafalo (1987) also highlight the importance of developing appropriate metacognitive processes. They argue that "mathematics instruction is focused too much on mathematical content and not enough on mathematical behaviour" (p. 22) and describe three different teaching strategies that can be employed to develop and reinforce appropriate behaviours when solving non-routine problems. These include:

- asking questions that encourage reflection on knowledge and behaviours;
- pointing out aspects of mathematical tasks that can influence performance; and
- demonstrating control decisions and actions to students during a problem-solving session and discussing why these are necessary and appropriate.

Silver (1987) argues that these strategies must be made explicit in all classrooms. He states

> If a major goal of mathematics instruction is the development of students' problem-solving abilities, metalevel processes need to become an important curricular focus. Until these processes receive explicit attention in the curriculum, we will continue to produce mathematics students who know fairly well what to do in routine and simple problem situations, but have little competence in handling unfamiliar or complex problems (p.55).

This advice for teachers is clear and suggests that they need to model the problem-solving process to their students.

It is also clear that the use of metacognitive, or control strategies supports students problem-solving efforts and that it is desirable to focus on these in classrooms. It is important to note that

> developing self-regulatory skills in complex subject-matter domains is difficult and often involves behaviour modification - "unlearning" inappropriate control behaviours developed through prior instruction (Schoenfeld, 1992, p. 357).

This takes considerable time and cannot be done without due consideration to the influence of the other factors highlighted by Kroll and Miller (1993) and detailed in Figure 1. These include students' beliefs and affects in relation to mathematical problem solving.

Beliefs and Affects

Students' experiences in mathematics classrooms lead them to have particular beliefs and attitudes about mathematics, about problem solving, and about themselves as learners of mathematics (Cobb, 1986). Tasks chosen by teachers in mathematics lessons can reinforce or challenge beliefs and attitudes as well as cause particular emotional responses. The impact of what transpires in mathematics lessons has been referred to as the "hidden mathematics curriculum" (Silver, 1987) since teachers and curriculum developers do not set out to create poor student attitudes or inappropriate beliefs, rather these develop as a consequence of some traditional pedagogical practices and unmotivating curriculums.

Beliefs and affects can have an overwhelming effect on problem-solving performance. Students can treat problem-solving activities as something different to mathematics, readily giving up problem-solving attempts after a short period of time, and believing that it does not matter if they cannot solve problems as only geniuses can do them anyway (Schoenfeld, 1985). If problem solving is to be valued by students as legitimate mathematical activity then teachers need to confront these inappropriate, or "dysfunctional", beliefs (Borasi, 1990).

The following brief definitions are provided to clarify the meanings of the terms *affects* and *beliefs*. *Affects* refers to students' attitudes and emotions. Attitudes that can impact on performance include "motivation, interest, confidence, perseverance, willingness to take risks, tolerance of ambiguity, and resistance to premature closure" (Lester, 1987). Emotions includes reactions to particular situations in mathematics classrooms and can include such feelings as frustration or joy. Lester (1987) distinguishes between these two terms by referring to attitudes as traits whereas emotions are situation-specific states. *Beliefs* about mathematics include "the perspectives with which one approaches mathematics and mathematical tasks" (Schoenfeld, 1985, p. 45). Beliefs shape attitudes and emotions and play a key role in the decisions students make when solving mathematical problems.

Students' beliefs about mathematics in general and problem solving in particular do not always assist the processes needed to become good problem solvers. For example, Frank (1988) found that typical beliefs held by middle school students included the view that

- mathematics is computation;
- mathematics problems should be quickly solvable in just a few steps;
- the goal of doing mathematics is to obtain "right answers";
- the role of the mathematics student is to receive mathematical knowledge and to demonstrate that it has been received; and
- the role of the mathematics teacher is to transmit mathematical knowledge and to verify that students have received this knowledge (p. 33).

If students hold these beliefs then it is likely that they will resist spending a long time on one problem and they will be more interested in getting correct answers. They may expect teachers to carefully explain how to do any challenging questions and to readily evaluate their answers. This does not encourage students to think about their mathematics, spend time reflecting on solutions and processes, and mitigates against independent learning.

Schoenfeld's (1988) year-long observations of geometry classes suggest that it is not surprising that students develop the beliefs they have. Students were observed solving many mathematical questions, that were referred to as *problems*, during each lesson; most taking 2 minutes or less to solve. Schoenfeld (1989) then used questionnaires to collect information from 230 Year 10 students about their beliefs and perceptions of mathematics. Students reported that questions in classrooms "were aimed at evoking quick recall [rather] than stimulating deep thought" (p. 345) and that most problems should take little time to solve, particularly if the work is understood. Schoenfeld (1989) concluded

> The rhetoric of problem solving has become familiar over the past decade. That rhetoric was frequently heard in the classes we observed - but the reality of those classrooms is that real problems were few and far between, if they were seen at all. Virtually all the problems the students were asked to solve were bite-size exercises designed to achieve subject matter mastery; the exceptions were clearly peripheral tasks that the students found enjoyable but that they considered to be recreations or rewards rather than the substance of what they were expected to learn (p. 348).

Teachers' decisions about what problem-solving tasks they will use and when and how they will be presented can have a significant impact on students' beliefs about problem solving. Stacey (1994) noted that teachers tend to choose very short problems that have a defined end-point and do not usually lead to other interesting mathematical ideas. Further, she suggested that many tasks tend to be "one-off" with little opportunity for generalisation or proof and do not seem to encourage reflection or student learning.

When this occurs, problem solving is often perceived as separate to the curriculum (Isaacs, 1990). This view may be reinforced by Task Centres and Mathematics Laboratories where students are presented with tasks once a week or at a particular timetable slot. These activities are then viewed as distinct from the normal mathematics curriculum (Siemon & Booker, 1990; Stacey, 1994). An additional concern is that skills or processes learned in the Mathematics Laboratory may not transfer to the mathematics classroom for application on other mathematical tasks.

Traditional approaches to teaching mathematics reinforce inappropriate beliefs. These beliefs need to be challenged if problem solving is to be valued as an integral part of the mathematics curriculum and problems are to be regarded as legitimate mathematical activity. If these beliefs persist, it will be difficult for teachers to use problem-solving approaches in their classrooms. Problem solving requires conjecture, investigation, exploration, creativity, persistence, discovery, analysis, monitoring, evaluation and justification among other things. Problem-solving approaches can challenge these beliefs and help to redefine mathematics and the role of problem solving in learning mathematics.

Students' beliefs about mathematics and learning mathematics were explored during a three-year study conducted in three secondary schools in California (Clarke, Wallbridge & Fraser, 1992). This study aimed to implement a problem-based mathematics program in senior classes (Interactive Mathematics Program, IMP) that changed mathematics instruction so that students were active learners and teachers became guides; aims that were consistent with the *Curriculum and Evaluation Standards* (NCTM, 1989). Questionnaires were used to collect data from 182 students participating in IMP as well as from 217 students from the same schools who were not involved in the program. After almost one year in the program, the IMP students held more positive attitudes towards mathematics, were more likely to perceive mathematics as a mental activity with applications to daily use, attached more value to interactive learning situations, and believed that writing and talking to other students assisted their learning. IMP students also averaged higher scores on the Mathematics Scholastic Aptitude Test (SAT). In conclusion, the authors state

> This research suggests that a problem-based curriculum is capable of developing traditional mathematical skills at least as successfully as conventional instruction, while simultaneously engendering measurably different belief systems in participating students (p. 17).

Borasi (1990) suggests that one of the first steps in changing students' conceptions is to make them aware of their beliefs and provide opportunities to discuss and become aware of alternatives. She believes that there needs to be ongoing discussions and selection of appropriate tasks so that "the activities ... do not remain isolated episodes within an essentially traditional mathematics curriculum" (p. 180). Borasi successfully conducted intervention studies to address dysfunctional beliefs and encourages teachers to seriously consider this approach if they are to implement problem-solving approaches in their classrooms. It is clear from these studies that the classroom environment also plays a key role in supporting students' problem-solving efforts.

Social and Cultural Conditions

The social context within which students experience problem solving can also influence their problem-solving abilities. Students need to observe teachers modelling effective thinking strategies and appropriate problem-solving behaviours. Resnick (1989) suggests that the social setting of the classroom provides opportunities for students to perform tasks while experiencing scaffolding from teachers and peers. She also argues that this learning environment is motivational, and encourages and supports a disposition in students for meaning construction.

Silver (1990) describes his vision of mathematics classrooms as

> places in which classroom activity is directed not solely toward the acquisition of the content of mathematics in the form of concepts and procedures but also toward the situated, collaborative practice of mathematical thinking (p. 9).

These are places where teachers and students engage in collaborative activity and where students are "*doing* mathematics rather than in having it done to them" (p. 8). The role of the teacher is critical in that they are charged with the important task of engaging students in worthwhile mathematical activity that includes shared problem solving.

In summary, the identified research and analyses suggest that to develop problem-solving skills, students need to experience a variety of problems on a regular basis, preferably within a curriculum that has problem solving integrated with mathematics. Discussion of solutions supports the process and enables students to clarify understanding, exchange ideas, defend their thinking and evaluate the ideas of other students. This also assists the development of metacognitive processes that substantially enhance problem-solving skills. An important consideration in developing students' problem-solving abilities is to confront their negative attitudes to mathematics. They also need to be aware of the place of problem solving in learning mathematics. A critical element in developing students' problem-solving abilities is the need to ask questions that make students think about what they know; this is best achieved in a supportive, collaborative environment.

Appendix 2 - The Role of Problem Solving in Developing Thinking and Supporting Learning in Mathematics

A consistent theme in the literature is that mathematical understanding develops as teachers provide appropriate learning experiences in classrooms. Opportunities to support learning can arise when teachers pose questions, promote discussion between students, encourage students to reflect on their understanding of ideas, and challenge individual constructions of mathematical concepts. Classroom environments can be structured to support these learning experiences and appropriate tasks can be developed by teachers to promote the learning of mathematical ideas.

Providing problem-solving experiences for students that require them to think and analyse situations can enrich their understanding of mathematics. This can be achieved through the use of good questions that require students to think about mathematical ideas, analyse the situation, propose possible solutions, and reflect on their knowledge and understanding. Such questions have been referred to as higher-order questions as they provide problem-solving opportunities and have the potential to enrich mathematical understanding (Sullivan & Clarke, 1991). Lower-order questions that only require the recall of basic facts often need little thinking by students and do not challenge students' understanding. This was recognised by Polya (1957) who suggested

> A teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of independent thinking (p. v).

Teaching practices that incorporate frequent use of short answer, lower-order questions and few problems until the end of a topic do not support the development of mathematical reasoning and problem-solving skills. Peterson (1988b) argues that some teachers teach basic skills to children before giving them problems to solve. The problems often relate to the skill or procedure that has just been learnt and children know that they only need to apply those new skills in order to solve the problems. Such lower-order thinking does not really engage students in reflective processes.

Higher-order thinking challenges students to consider a variety of procedures in order to solve unfamiliar problems. Peterson recommends that such problems should be used by teachers "from the very beginning of mathematics instruction" (p. 7) and could form the basis of the mathematics curriculum. This view is supported by Sullivan and Clarke (1991) who suggest that

> If our goals of education are for our students to think, to learn, to analyse, to criticise, and to be able to solve unfamiliar problems, then restricting teaching practice to the transmission of facts will not be productive (p. 13).

It seems that teachers use a high proportion of lower-order questions during mathematics lessons. Sullivan and Leder (1990) analysed forty-six lessons given by beginning teachers to determine the frequency of use of various kinds of questions. There was considerable variation between the ten teachers involved in the study but more than half of the questions required only the recall of information and as little as five per cent of the questions required pupils to think independently or to give more than one answer. Asking more higher-order questions, or posing more problems in classrooms, has the potential to develop students' mathematical thinking (Burns, 1985).

Another important role of problem solving is that problem-solving tasks can challenge students' understanding of mathematical ideas. Problem solving can be viewed both as central to the way students learn and as a means by which they learn as they are confronted with activities that are problematic (Wright, 1992a). Attempting to resolve these conflicts provides opportunities for students to reflect on current procedures and develop more sophisticated meanings and strategies. As Wright (1992b) states

> The paradox of teaching is that, for advancements to occur, the learner must be confronted with situations for which they do not have appropriate cognitive constructions. Problem solving is a necessary ingredient of learning. Thus new knowledge cannot be handed ready made by the knower to the learner. Rather, new knowledge is necessarily constructed through problem solving (p. 133).

This approach to teaching mathematics has implications for the role of the teacher and the classroom environment.

This advice seems to suggest to teachers that they could provide appropriate problem-solving tasks and then be prepared to allow the students to struggle with the ideas and talk about their thinking without telling them what to do. In this way, the classroom environment can be supportive of these struggles and challenges so that students are prepared to take risks and say what they think without fear of ridicule or judgements.

If students are to develop further understanding of mathematical ideas, current knowledge needs to be challenged. This can be achieved if teachers ask more higher-order questions and fewer lower-order questions. Higher-order questions provide opportunities for students to think, analyse, reflect and reorganise existing cognitive structures. This is often best achieved in group settings in classrooms so that students can discuss ideas using their own language.

Appendix 3 - Adoption of Problem-Solving Approaches: A Case Study

A study of the adoption of problem-solving approaches occurred in California to consider teachers' responses to the *California Mathematics Framework* (California State Department of Education, 1985). Recommended changes in this document included an increase in the amount of problem solving, a focus on estimation skills, and further use of manipulatives in elementary mathematics classrooms (Prawat, Remillard, Putnam, & Heaton, 1992). Also, the document had an impact on the content and teaching approaches used in textbooks in that state as it "served as a standard against which mathematics textbooks were to be judged" (p. 148). The study is particularly relevant to this investigation as it outlines key issues, connects beliefs with practices, and suggests possible methods for data collection.

The study conducted by Prawat and his colleagues involved two schools from each of three districts, and, within each school two teachers from each of Second Grade and Fifth Grade were observed and interviewed. Each teacher was visited twice during the 1988-1989 school year. Teachers were observed twice during each visit and interviewed on a range of issues. Questions focused on familiarity with the *Mathematics Framework*, use of new textbooks, reported goals for mathematics teaching, practices, assessment, beliefs about teaching and learning as well as mathematical content knowledge. There were also brief pre observation interviews and extensive post observation interviews to ascertain how and why they did what they did and what they believed students would learn from each lesson. The observations and field notes yielded lesson narratives and analyses that were discussed with members of the research team. The results revealed a complex interrelationship between knowledge, beliefs and practices.

From the study, the researchers determined that the key elements of influence on teachers' classroom practices included several aspects of teachers' knowledge and beliefs. These were:
- teachers' knowledge and beliefs about learners, learning and teaching;
- teachers' knowledge of mathematics; and
- teachers' knowledge and beliefs about mathematics.

In another report, Putnam et al. (1992) state

> We believe that what teachers know and believe guides how they construct lessons, interpret textbooks, and interact with students. Knowledge and beliefs also provide important lenses or filters through which teachers perceive and act on various messages to change the way they teach (p. 213).

Putnam et al. (1992) observed three beliefs that seemed to interfere with the adoption of the reforms advocated in the *Mathematics Framework.* The first belief they observed was "teaching as telling" (p. 216). Even when teachers presented problems to the class and used strategies like cooperative learning, they were still inclined to present the mathematical ideas and procedures to the students to help them achieve the necessary outcomes. The researchers cited evidence of the tension that teachers often feel between allowing students the time to make sense and construct meaning for themselves and the need to ensure that students learn the mathematics in the curriculum, or formal mathematical knowledge.

A second belief that was observed was the notion that "learning should be fun, engaging, and relevant" (p. 217). This belief seemed to take precedence over the mathematical ideas and concepts in some of the teachers' classrooms and became the focus of lessons. The third belief was "understanding may have to wait" (p. 218). This is premised on the notion that mathematics is hierarchical and that students must learn the basics first before they can do any real mathematics. Also, it is supported by the view that learning takes time and it is often necessary to revisit concepts many times before the students understand the ideas. Such teachers will often also believe that young children are not ready for abstract thinking or problem-solving tasks.

Each of these beliefs suggests a traditional focus in mathematics classrooms that reinforces basic facts and emphasises computational skills and may interfere with the adoption of new approaches. In addition to this, Putnam et al. (1992) found that the teachers in the study held narrow views of mathematics in that

> ... the computational algorithms that pervade the traditional elementary school curriculum constitute the core of mathematics (p. 223) [and] Problem solving in these classrooms involved applying well-practiced computational skills in particular situations, rather than opportunities to figure out what is reasonable and sensible in those situations (p. 224).

It was argued that in order to bring about change in classrooms, it would be necessary to significantly change teachers' beliefs. The study concluded that "these sorts of interrelated knowledge and beliefs serve as intellectual resources teachers bring to teaching. Our cases demonstrate how these resources both support and constrain mathematics instruction" (p. 225).

Commenting on the same study, Morine-Dershimer and Corrigan (1996) made suggestions as to the impact of beliefs on practice. They stated that

> the teachers had adapted the new concepts and procedures to conform to their own belief systems and techniques of practice, drastically transforming the mandated curriculum in the process (p. 1).

For example, Mark "skipped the lessons that didn't fit his belief that pupils should be learning mathematical rules and procedures step by step by step" (p. 1) and Cathy "conceived of manipulatives as a way to make number facts more concrete, so that her second graders would memorise the facts faster" (p. 1). They concluded that

> Each of these teachers willingly modified their practice as a result of the new curriculum policy, but the modifications were limited and shaped by the teachers' beliefs, knowledge, and experience. The result in each case was a strange amalgamation of traditional and innovative mathematics instruction. The outer trappings of the new framework were evident in their lessons, the central intentions were elusive (p. 2).

Further, they suggest that

> Teachers implementing mandated changes interpret those mandates through the screen of their prior beliefs, modifying and adulterating the desired reform strategies. New practices require new beliefs (p. 8).

This last statement by Morine-Dershimer and Corrigan suggests that the most powerful constraint preventing the implementation of new approaches may be teachers' beliefs.

In summary, the introduction of the *Mathematics Framework* in California resulted in a variety of responses from teachers. It is clear from this case study that teachers' beliefs played a key role in interpreting the advice and filtering information before the adoption of practices. This is further evidence that the influence of teachers' beliefs should not be underestimated and must form a key component of any investigation into teachers' practice.

Appendix 4 - Initial Investigations of Questionnaire Items

There were several stages in the development of the instrument. Preliminary investigations were used to discover the meaning teachers give to the term *problem*, to investigate factors that they describe as being important in the learning of mathematics including the role of problem solving, and to determine the kinds of problems teachers prefer to use. These investigations aided the development of the instrument that was trialed with a group of 12 practicing teachers. Guidance was also sought from mathematics education lecturers and other tertiary educators who have used survey research methods.

This section summarises the first stage in the development of the questionnaire. A *Preliminary Survey* (Appendix 5) was designed to obtain information about teachers' background, beliefs and favourite problems. The background information included gender, number of years of teaching experience, grade currently being taught, role in the school, number of days inservice attended in the last two years as well as the number of these that were devoted to mathematics. It was anticipated that there may be some link between the amount of mathematics focused professional development and knowledge about problem-solving approaches.

It was also anticipated that the types of problems that teachers recorded would provide insights into their understanding of the term *problem*. Each teacher was asked to record their two favourite problems and to describe briefly why these were chosen. This question was designed to be an unobtrusive measure of teachers' interpretation of *problem*. The types of problems chosen provided some indication of each teacher's classification of what constitutes a problem and what characteristics of these chosen problems make them particularly useful.

To reveal reported beliefs about mathematical problem solving and its role in the teaching and learning of mathematics, two vignettes or scenarios, were presented. Each scenario was structured so that two perspectives on the role of problem solving in learning mathematics were presented. Two teachers were described in each scenario; one had a *contemporary* perspective to the teaching and learning of mathematics while the other had a more *traditional* perspective, as described in Chapter Two. The contemporary teacher believed that problem solving could be used as the focus of learning mathematics while the traditional teacher believed that children primarily need basic skills practice with some problem solving when time permits.

Each scenario was placed in a context that might be relevant to teachers. The first scenario involved providing advice about the most appropriate teacher for a friend's child (see Figure 1).

A close friend tells you about two Year 5 teachers at her child's school. Ms Jones is concerned that too many children finish primary school without a solid background in the basics of mathematics. She ensures that plenty of basic skills in the 4 operations are practised in her class with the occasional use of problem solving. Ms Smith, on the other hand, does not practise these skills as much but prefers her children to develop their skills by drawing on them as they need them when solving problems. Your friend wants to know which teacher she should request to teach her child. What advice would you give her and why?

Figure 1. Scenario involving choice of appropriate teacher.

The second scenario involved a staffroom discussion about when to do problem solving in an already overcrowded curriculum (see Figure 2). Respondents were required to comment on each scenario giving reasons why they preferred particular approaches to teaching.

Two teachers are having a conversation in the staffroom. John is clearly frustrated by the number of things he has to fit into the curriculum and barely finds time to cover the essential basic skills in mathematics, let alone investigate problems. Bill responds by saying that you can teach all mathematics by using a problem-solving approach. They ask you for your opinion - what do you think?

Figure 2. Scenario involving integrating problem solving into the curriculum.

Responses were sought from practicing teachers, some of whom were part-time, post-graduate students in teacher education courses. A total of 39 teachers were surveyed; 23 responses were chosen for analysis on the basis that the respondents were responsible for teaching a primary grade class and that they had recorded at least one favourite problem. The NUD*IST software program (Qualitative Solutions and Research, 1994) was used to create an index system to manage and organise the data. Even though this was a small sample of teachers, the process of organising the data into categories highlighted emerging issues that informed the development of the final questionnaire. Early analysis of some of these data were published in Anderson (1996).

Responses to Favourite Problems

The favourite problems that teachers recorded were examined to identify content and problem type. Responses ranged from general statements, for example "money problems", to highly specific questions and were classified according to the main area of mathematical content or strand (Table 1). A total of 39 problems were provided by the 23 respondents and of these 15 involved number, 12 space, 9 measurement, 2 chance and data and 1 was very general and not content specific.

Table 1
Number of problems for each content area

Content Area	Number of Problems
Number	15
Space	12
Measurement	9
Chance and Data	2
Total	38

An initial classification of problem type included the categories of: real-life investigation; mathematical investigation; closed, content focused; and other but this classification failed to provide discrete categories. An improved classification involved an open versus closed division. Each of these was then further subdivided into contextualised or decontextualised. Table 2 indicates the number of each type and provides an example of each.

Table 2
Number of responses and an example for each problem type

Problem Classification	Number (Percentage)	Example
Open and contextualised	7 (18%)	Can a dinosaur fit into your classroom?
Open and decontextualised	10 (26%)	How many different combinations of coins can you find which total 50c.
Closed and contextualised	10 (26%)	There are 32 children in 4L, 31 in 4T and 30 in 4P, how many children are in year 4?
Closed and decontextualised	12 (30%)	Counting in twos via addition.

It was anticipated that teachers' favourite problems would reveal useful information about their definitions of a problem. The examples given did not provide this as problems need to be appropriate for the class or particular students that the teacher has in mind. One teacher recorded the problem:

> Kylie had 37 apples, she ate 12 apples, how many does she have left?

For many students this would not be a problem. However, in the lower grades, there may be students who would not have a readily available procedure for solving this problem. It is possible that this particular teacher may consider that a problem is a question stated in words. Other responses which did not reveal teacher's definitions of a problem were very vague, for example:

> Measurement - time.

From this information it is not possible to determine exactly what kinds of questions might be posed by this teacher.

It is worth noting that of the 16 survey responses not used in the analysis, seven of these were full-time classroom teachers who did not record a favourite problem. Two of these teachers commented that they "use many" whereas another two stated that they "can't think of any". In a follow-up interview with a voluntary group of eight of those surveyed, a comment was made that it is difficult to think of a problem when you are not in class with the children. Another teacher said that she thought of a textbook question but she did not want to write that down because these are not the best examples of problems. Other comments suggested that problem solving is not planned but just arises out of what happens in the classroom. If this is the case, then it is not surprising that so many of those surveyed were unable to record at least one favourite question.

Recording a favourite problem does not reveal how the problem was posed in the classroom or the way that it was explored or discussed with the students. The problem has been removed from the context of the classroom. A better question would have been to ask how the problem was used in the classroom or to describe a recent problem solving lesson. In an effort to explore what the characteristics of a favourite problem might be, teachers were asked to record why these particular problems were chosen.

Responses to this request were categorised into four groups - affective factors, problem characteristic factors, teaching factors and learning factors. Table 3 lists some of the factors provided by teachers for each of the four categories. Half of the teachers surveyed mentioned at least one learning factor which indicates that they are aware of the role that problem solving plays in the learning of mathematics. It is unclear whether the other teachers believe that problem solving does promote learning as their responses were placed in alternative categories.

Table 3
Categories of reasons for favoured problems and examples of factors for each category

Category	Factors
Affective	enthusing and motivating students, students enjoying a challenge
Problem Characteristics	open-ended, real-world contexts, in the context of a game, adaptable
Teaching Factors	group work, promotes use of mathematical language, hands on, incorporates several content strands, links with other KLAs
Learning Factors	estimation, remediation, visualisation, practise basic skills, introduce concepts, challenge, extension

Responses to the Scenarios

Responses to the scenarios as presented in Figures 1 and 2 were read to ascertain the level of support for either a *contemporary* perspective or a *traditional* perspective to teaching mathematics, and to discover issues that might lead teachers to support each of these perspectives. Issues that were raised in the comments made about the two scenarios were categorised according to four different criteria.

Analysis of responses revealed that the majority of respondents supported both perspectives to some extent, possibly indicating that they have mixed beliefs in relation to the role of problem solving in teaching and learning mathematics. Of the twenty-three responses, four clearly supported the contemporary perspective and three supported a traditional perspective.

Comments about the two scenarios were categorised into four groups - class, mathematics, learning and other. Many comments related to the class or individual students and included comments about ability, behaviour and previous experiences. Several related to the ability of students to cope with a problem-solving approach because of lack of basic skills knowledge. Comments were made about children with learning difficulties, backgrounds of children where English is not the first language, and the increased need to cope with language when solving problems. There was concern that children need to be cooperative when solving problems and that some children were uncooperative, antisocial and immature. One teacher suggested that previous experience with problem solving was necessary before a child would be comfortable with problem-solving approaches. Several comments referred to a consideration that children learn in different ways and hence respond to different approaches, this was summarised in the following comment:

> All children learn in different ways and I feel we should adopt a variety of approaches to cater for these.

One teacher noted that using problem-solving approaches gives students the opportunity to construct their own knowledge.

Comments about mathematics tended to indicate whether teachers believed that basic skills should come before problem solving or whether problem solving can be used to develop basic skills. Most teachers suggested that skills need to be developed before problem solving and seemed to support the problem solving as an *end* approach. This was reflected in the comment:

> Basic skills have to have time to be taught - has to be a priority! Teach these first, then apply problem solving to them.

Seven teachers wrote statements that supported problem solving as a *means*. This was reflected in the following:

> Because the children are learning skills through problem solving they have a need to use basic skills - a real need, rather than simply the completion of sums in a text.

Ten teachers wrote comments relating to the relevance and importance of children developing problem-solving skills and that problem solving provided an opportunity to learn mathematics in a real-life context. Other comments about mathematics included opportunities for discovery learning, investigation from open-ended questions, and integration with other areas of the curriculum.

Statements about learning were organised according to their support for problem-solving approaches, non problem-solving approaches, or a mixed position. Almost every teacher surveyed made a statement that supported the use of problems in the mathematics classroom. It was clear that teachers were aware that problems provide students with the opportunity to use valuable skills and that this was more interesting for them than practising algorithms from a textbook. Most teachers also recommended that skills must not be ignored and that extra practice may be needed.

It became clear that isolating comments into these categories did not really give an indication of each teacher's beliefs as many teachers provided comments that could have been placed in all three categories. There were only four cases where comments were placed in only one category and these teachers each wrote very little. The comments that teachers wrote in responding to these scenarios needed to be read as a whole.

For some teachers, overall beliefs became clear when the responses to both scenarios were read together. By selecting individual ideas or statements it seemed that the teacher supported a problem-solving approach and yet accompanying comments revealed reservations. One case is particularly noteworthy. In response to the first scenario the teacher wrote:

> Ms Smith would probably be the teacher most favoured by myself ... because the children are learning skills through problem solving.

For the second scenario she wrote:

> Yes I would probably agree that you can teach skills through a problem solving approach in some schools and with some students. I honestly would not adopt such an approach to teach maths.
> (1) The problem solving arena is very overwhelming.
> (2) Usually involves too much language.
> (3) I am yet to find suitable problems for all sub-strands.

This teacher was prepared to be very honest. Even though she was aware of current thinking involving the role of problem solving in learning mathematics she was not prepared to teach that way. She confessed to feeling overwhelmed and she raised two important issues that caused her concern. It is possible that other teachers may have held similar beliefs but these were not revealed on the surveys. Interviews may provide a better opportunity to reveal such beliefs.

Another issue relates to what teachers believe to be a *problem-solving approach* to teaching mathematics. Many respondents agreed with Bill in the second scenario and yet their comments suggested that their views of a problem-solving approach were different to what was intended. One teacher responded as follows:

> Agree with Bill - what does John do??? Short groups of problems or tasks can cover all mathematics areas - break them up into small manageable components.

This response seems to suggest sets of textbook word questions rather than using a problem as the focus of a lesson from which skills and understanding are developed. Another teacher cautioned that there may be different definitions of a problem-solving approach. She stated:

> It depends what's meant by a problem-solving approach. My maths uses plenty of problem solving with open-ended problems. This is the way to go I believe.

Given that the term was meant to suggest that problem solving was used as a *means* rather than an *end* in teaching mathematics, responses from a majority of teachers seemed to support the problem solving as an *end* approach. Eight teachers, or approximately one third, made direct statements indicating that they did not support the view that all mathematics could be taught *through* problem solving. Many teachers supported a middle position with their comments suggesting that there is a need to have children practising skills as well as doing problems in mathematics classrooms. This indicated that they interpreted the scenarios to mean that skills alone was one position, and problem solving alone was the other position.

Implications from the Preliminary Survey

The *Preliminary Survey* provided useful information about teachers' beliefs, preferred problems, and reasons for problem choices. However, the open-ended nature of the questions meant that responses were often difficult to interpret and compare. It was evident from this preliminary investigation that a more structured approach was required in the final questionnaire. Items needed to be clear and greater use of Likert items was needed in order to force a choice about particular aspects of problem-solving instruction.

It was also revealed that most teachers support the use of problem solving as an *end* in teaching mathematics but further issues for exploration were identified. Teachers do have different beliefs about the role of problem solving in learning mathematics but in order to decipher exactly what these are requires clarification of what each teacher understands by the terms *problem* and a *problem-solving approach* to teaching. Providing examples of problem types and avoiding the use of the term *problem-solving approach* may help to overcome these issues.

The data about favourite problems highlighted the need to have teachers describe how the problem was used in the classroom. Teachers make decisions about appropriate tasks based on their beliefs, the class and individual students, as well as the skills and concepts they are trying to develop. These were not revealed in the surveys and it would have been more useful if teachers had been asked to describe a recent problem-solving lesson. Also, placing items in a context that teachers could relate to may help them to interpret the appropriateness of problems and teaching actions. In addition to these ideas, it may have been more useful to present teachers with a set of problems and to have them rank these according to their usefulness with a particular grade, as well as their value for promoting understanding of mathematical concepts.

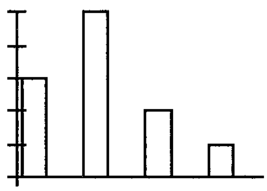
Teacher Ranking of Problem Examples

The next stage in the preliminary investigations involved another survey instrument entitled *Favourite Problems for Year 3* (Appendix 6). This survey required teachers to rank a set of problems that might typically be used in a Year 3 class. The problems were taken from the set of favourite problems that had been recorded by participants in the earlier investigation and were chosen on the basis of coverage of content areas and that they represented a variety of problem types. Respondents were required to rank the seven problems by allocating "1" for the problem they would most likely use, to "7" for the problem they would least likely use. They were also asked to record their reasons for choosing a particular problem first and another last.

To ascertain the usefulness of this survey item, this set of problems was presented to two different groups. The first was a group of 70 trainees in their third and final year of study in a Bachelor of Teaching program for primary teachers. The second was a group of 23 classroom teachers who were attending an inservice on problem solving for kindergarten to year three. Table 4 lists the mean rankings for each problem for each of the two groups. The lower the mean, the more likely teachers were to use the problem.

The overall order of mean rankings was quite different for the two groups. The greatest difference between the means occurred for the first, second and sixth problems. The trainee teachers did not favour the first problem because it was considered "too hard", "boring" or they indicated that they were "not confident" about using it. The second problem was most favoured by the practicing teachers because it was "hands-on" and "practical" whereas the sixth problem was least favoured by this group because it was considered "too vague" or "abstract".

Table 4
Mean rankings of favourite problems for Year 3 by Trainee Teachers and Practicing Teachers

No.	Problem	Trainee Teachers (n = 70)	Practicing Teachers (n = 23)
1	If I toss 2 coins, what is the chance that both will be heads?	5.1	4.2
2	What objects can you find which are the same length as your stride? What objects are longer? What objects are shorter?	3.7	2.8
3	If there are 32 oranges in one box, 31 in a second box and 30 in a third box, how many oranges are there altogether?	4.2	4.7
4	What combinations of coins can you find which have a total of 50c?	3.3	3.4
5	Could a dinosaur fit into our classroom? (List of dinosaur dimensions given.)	3.7	4.0
6	What do you think this might be the graph of? 	3.7	5.0
7	Construct a cubic metre using straws and masking tape.	4.3	4.1

There was a wide spread of selections by both groups of participants in this investigation. Every problem was chosen by at least one respondent as most favoured and every problem was chosen by at least one respondent as least favoured. This indicates that teachers choose different problems for different purposes. Reasons for choosing a problem as most favoured included comments such as "relevant", "creative", "concrete", "fun", "activity", "discovery" and "I like it". Reasons for least favoured problems included "only one answer", "too hard", "not real-life" and "I'm not confident".

While the responses were certainly of interest because of the range of selections for each problem, this item was not included in the final questionnaire. The problem examples covered all of the content strands and represented a variety of closed and open questions. This may have influenced teacher choices based on their confidence with particular topic areas or their preference for either closed or open questions. Problems based on the same content area that would be given to a typical year group would remove some of the variability between the individual problem.

These preliminary investigations demonstrated that it was appropriate to use a combination of closed and open questions in the questionnaire. This would more readily allow comparisons between teachers on closed items and yet allow for diversity of issues on the open items. To determine teachers' problem-solving beliefs and practices, it would be better to use structured Likert items that suggested either a *traditional* or a *contemporary* perspective to teaching mathematics. Placing such items within the context of teaching a particular topic in the K-6 mathematics curriculum would enable teachers to make judgements more easily.

Definitions of problem types needed to be provided in order to overcome the variety of interpretations of the term *problem.* Also, by providing an example of each problem type, teachers could use that as a reference point for the beliefs statements. These examples could then be used to invite a response about frequency of use rather than a question involving ranking of preferred questions as it was postulated that all types would be used at some stage by each teacher. It was deemed appropriate to request a recently used problem from each respondent but to include a request for purpose of use. These preliminary investigations provided useful information to aid the development of the final questionnaire.

Appendix 5 - Preliminary Survey

Please answer each of the following questions:

1. How many years have you been teaching?

   0 - 5 .... 6 - 15 .... more than 15 ....

2. What year are you currently teaching? ....

   If you do not have your own class, what is your role in the school?

   ....

3. How many teacher inservice days have you attended in the last two years (include whole school staff development)? ....

   How many of these have had a mathematics focus? ....

4. Are you male or female? ....

5. Write down 2 of your favourite mathematical problems which you have used in your classroom this year. Briefly describe why you like using these in particular.

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6. A close friend tells you about two Year 5 teachers at her child's school. Ms Jones is concerned that too many children finish primary school without a solid background in the basics of mathematics. She ensures that plenty of basic skills in the 4 operations are practised in her class with the occasional use of problem solving. Ms Smith, on the other hand, does not practise these skills as much but prefers her children to develop their skills by drawing on them as they need them when solving problems. Your friend wants to know which teacher she should request to teach her child. What advice would you give her and why?

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7. Two teachers are having a conversation in the staffroom. John is clearly frustrated by the number of things he has to fit into the curriculum and barely finds time to cover the essential basic skills in mathematics, let alone investigate problems. Bill responds by saying that you can teach all mathematics by using a problem solving approach. They ask you for your opinion - what do you think?

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Appendix 6 - Favourite Problems for Year 3

Classroom teachers were surveyed to find out their favourite mathematical problems. The following problems were supplied by teachers of year 3.

Would you please number these problems from the one you would be most likely to use, (1), down to the one you would be least likely to use, (7).

A If I toss 2 coins, what is the chance that both will be heads? ☐

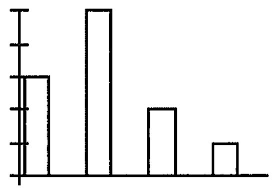
B What objects can you find which are the same length as your stride? What objects are longer? What objects are shorter? ☐

C If there are 32 oranges in one box, 31 in a second box and 30 in a third box, how many oranges are there altogether? ☐

D What combinations of coins can you find which have a total of 50c? ☐

E Could a dinosaur fit into our classroom? (List of dinosaur dimensions given.) ☐

F What do you think this might be the graph of? ☐



G Construct a cubic metre using straws and masking tape. ☐

Briefly describe why you would prefer the problem you chose as your first choice.

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Briefly describe why you would least prefer to use the problem you chose as your last choice.

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Appendix 7 - Problem Solving in Mathematics Teaching: Teachers' Views and Teaching Practices

We are interested in your views about problem solving and how you attempt to incorporate problem solving into your teaching of mathematics. There are no pre-determined "right answers" to the questions. We welcome and value your own opinions regarding these issues.

Your responses will remain confidential. The results of the study will be reported in a way which ensures that the responses from an individual or school cannot be identified. Thank you for completing the survey.

Further information can be obtained from:

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Oakleigh VIC 3166
(03) 9563 3671

Please answer each of the following questions or tick the appropriate category:

A Name (optional - only include your name if you are willing to be interviewed at a later date):

....

B School: ....

C gender:- male ☐ ; female ☐

D How many whole years have you been teaching ?:

0 to 4 ☐ ; 5 to 9 ☐ ; 10 or more ☐

E Which category best describes your role this year:-

class teacher of years K, 1 or 2 ☐ specialist teacher ☐

class teacher of years 3 or 4 ☐ administrator ☐

class teacher of years 5 or 6 ☐

other (specify)? ....

F How many teacher inservice days have you attended in the last two years (include whole school staff development)?

0-1 days ☐ ; 2-6 days ☐ ; more than 6 days ☐

How many of these inservice days have had a mathematics focus?

0 days ☐ ; 1-3 days ☐ ; 4 or more days ☐

Background Information:

For the purposes of this survey, the following definitions are given to assist understanding of the terms which are used.

After teaching 2 digit addition students could be asked to answer the following:

Type of Question:	**Example:**
Exercise (we are **not** calling this a problem)	37 + 34
Application problem	If there are 34 oranges in one box and 37 in another box, how many oranges are there altogether?
Unfamiliar problem (ie. a problem type students haven't seen before)	The sum of my mother's age and my father's age is 71. My father is 3 years older than my mother. How old is my mother and how old is my father?
Open-ended problem	□ □ + □ □ = 7 1 What might the missing numbers be?

1. Naomi teaches a Year 3 class and she uses *application problems* and some *unfamiliar problems* when she has finished teaching 2 digit addition.

   In staffroom discussions, she has made the following statements.

   Please indicate, **from your perspective**, the extent to which **you** agree with each of her statements.

Tick one box for each statement.

	Statement	Strongly Agree	Agree	Disagree	Strongly Disagree
a)	students should learn basic number facts before they do *application* and *unfamiliar problems*				
b)	students should learn algorithms before they do *application* and *unfamiliar problems*				
c)	students cannot solve problems until they know how to perform the four operations				
d)	the best problems are those which relate directly to the number facts and algorithms the students have been practising				
e)	*application* and *unfamiliar problems* are best left to the end of the topic in mathematics				
f)	mathematics lessons should focus on practising skills				
g)	some students have trouble solving problems unless they know how to do the mathematics before they begin				
h)	some students find problem solving difficult because of the language involved in the problems				

2. Gwendolin is also a Year 3 teacher and she regularly uses lots of *open-ended problems* to teach her class 2 digit addition.

In staffroom discussions, she has made the following statements.

Please indicate, **from your perspective**, the extent to which **you** agree with each of her statements.

Tick one box for each statement.

	Statement	Strongly Agree	Agree	Disagree	Strongly Disagree
a)	it is a good strategy to begin the topic of 2 digit addition with *unfamiliar problems* involving 2 digit addition				
b)	mathematics lessons should focus on problems rather than on practice of algorithms				
c)	problems help motivate students to learn basic facts and algorithms because they can see a reason for learning them				
d)	all mathematics questions should challenge students to think about what mathematics they know and how they can use it				
e)	students can learn most mathematical concepts by working out for themselves how to solve *unfamiliar* or *open-ended problems*				
f)	it is essential for students to explore their own ways of doing mathematics questions before being shown the teacher's methods				
g)	most students forget mathematics procedures and so it is best to let them work out their own methods first				

3. Please give an example of a mathematical problem you used recently in your class and describe what you used it for.

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4. Please indicate how frequently you use each of the following strategies in your mathematics teaching.

	Teaching Strategy	Hardly ever	Some times	Often	Almost always
a)	you ensure that the students work alone				
b)	you explain in detail what the students have to do to solve problems				
c)	at the end of a problem solving lesson you lead a whole class discussion so that students can share solutions and strategies				
d)	you have calculators available for students to use				
e)	you encourage the students to work in small, cooperative groups				
f)	you present *unfamiliar* and *open-ended problems* to the class with very little indication of how to solve them				
g)	you encourage students to record their own procedures and methods of solving problems				
h)	you encourage students to pose their own problems				
i)	you provide a set of problems and the students are allowed to choose a problem they would like to work on				
j)	you allow the class or individual students to spend several lessons on the same problem				
k)	you use problems to show students that there are mathematical skills and procedures which they need to know				
l)	you present *application problems* which allow students to practise the skills they have just learnt				
m)	you provide concrete materials for those students who need them				
n)	you model the problem solving process to the class				
o)	you discuss useful problem solving strategies (eg. make a list, draw a diagram, work backwards)				
p)	you discuss problem solving processes (ie. make a plan, carry out the plan, check the calculations)				
q)	you use problems which arise from the school context or which relate to the students' experiences				
r)	you pose *open-ended problems* to allow students to explore mathematical situations for themselves				
s)	you set *exercises* to allow the students to practise their skills				
t)	you pose *unfamiliar problems*				

5. How often do you use each of these different types of questions in your teaching?

Please circle the appropriate word.

a)	*Exercises* -for example:- 37 + 34	Often Sometimes Rarely
b)	*Open-ended problems* - for example:- □ □ + □ □ 7 1 What might the missing numbers be?	Often Sometimes Rarely
c)	*Application problems* - for example:- If there are 34 oranges in one box and 37 in another box, how many oranges are there altogether?	Often Sometimes Rarely
d)	*Unfamiliar problems*- for example:- The sum of my mother's age and my father's age is 71. My father is 3 years older than my mother. How old is my mother and how old is my father?	Often Sometimes Rarely

6. From your responses to the above question, briefly describe why you prefer to use those particular types of problems.

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7. Please indicate **how frequently** you use each of the following as a source of mathematical problems.

	Source	Hardly ever	Some times	Often	Almost always
a)	K-6 Syllabus and support documents				
b)	textbooks written for the K-6 Syllabus				
c)	resource or reference books				
d)	inservice courses and notes				
e)	other teachers				
f)	other? (please specify)				

8. What do you see as the professional development needs of teachers in your school in relation to problem solving?

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9. The following statement was made recently at a teacher inservice course:-

"People who push problem solving in mathematics obviously don't work in classrooms. It is a waste of time."

How do you react to this statement?

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10. Are there any other comments that you would like to make?

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Thank you for completing the survey.

Appendix 8 - Data from the Mathematics Education Lecturers

To compare teachers' responses to those of an informed community, a group of 21 tertiary mathematics education lecturers from NSW and Victoria were invited to complete the same set of survey items relating to the frequency of use of teaching strategies. The prompt for lecturers required them to indicate the desirable frequency of use of each of these teaching strategies according to their understanding of advice given in the problem-solving literature. Data from the mathematics education lecturers is presented followed by a comparison of data from the lecturers with that from the surveyed teachers.

Responses from 21 mathematics education lecturers were received and entered into the *Statview* software package (Feldman et al., 1988) for analysis and comparison with the teacher respondents. Although there is a spread of responses from lecturers to the recommended frequency of use for these teaching strategies, it is evident that some strategies are advised as suitable for more frequent use than others. There is some agreement between teachers and lecturers regarding the frequency of use of particular teaching strategies. However, there are several differences, possibly indicating that teachers have not responded to some of the advice in the problem-solving literature. A comparison of the distributions of some of the teaching strategies for teachers and lecturers is presented here.

The strategies that lecturers rated as being appropriate for regular use are recommended in advice given to teachers about problem solving in the reformed classroom as well as in curriculum documentation. These included whole-class discussion for sharing solutions and strategies, small, cooperative group discussion, provision of concrete materials and calculators, recognition of the need to encourage individual student recording of methods and procedures, and encouragement of student posed problems as well as using problems that relate to student interests.

Table 1 presents the data from the lecturers. It is noteworthy that members of the informed community were not in agreement regarding the appropriate frequency of use of each of these teaching strategies as there is a spread of responses for many items.

Table 1
Mathematics education lecturers interpretation of appropriate frequency of use of teaching strategies (as a percentage), n=21

Teaching Strategy	Hardly ever	Some times	Often	Almost always
teachers ensure that the students work alone (alone)	30	65	5	0
teachers explain in detail what the students have to do to solve problems (explain)	52	48	0	0
at the end of a problem solving lesson teachers lead a whole class discussion so that students can share solutions and strategies (discussion)	0	10	38	52
teachers have calculators available for students to use (calculators)	0	0	10	90
teachers encourage the students to work in small, cooperative groups (groups)	0	5	71	24
teachers present *unfamiliar* and *open-ended problems* to the class with very little indication of how to solve them (little help)	0	24	71	5
teachers encourage students to record their own procedures and methods of solving problems (record methods)	0	0	30	70
teachers encourage students to pose their own problems (pose problems)	0	5	80	15
teachers provide a set of problems and the students are allowed to choose a problem they would like to work on (choose problems)	5	43	52	0
teachers allow the class or individual students to spend several lessons on the same problem (spend more time)	0	48	48	4
teachers use problems to show students that there are mathematical skills and procedures which they need to know (need maths)	5	33	62	0
teachers present *application problems* which allow students to practise the skills they have just learnt (practise skills)	0	71	29	0
teachers provide concrete materials for those students who need them (concrete materials)	0	0	38	62
teachers model the problem solving process to the class (model)	0	52	38	10
teachers discuss useful problem solving strategies (eg. make a list, draw a diagram, work backwards) (strategies)	0	24	57	19
teachers discuss problem solving processes (ie. make a plan, carry out the plan, check the calculations) (process)	4	29	48	19
teachers use problems which arise from the school context or which relate to the students' experiences (experiences)	5	0	52	43
teachers pose *open-ended problems* to allow students to explore mathematical situations for themselves (open-ended)	0	19	67	14
teachers set *exercises* to allow the students to practise their skills (exercises)	29	67	4	0
teachers pose *unfamiliar problems* (unfamiliar)	5	33	62	0

Note: each strategy is accompanied by an abbreviated title in brackets for future reference.

Several teaching strategies were recommended by mathematics education lecturers as those teachers should use *very frequently*. Another set were recommended as suitable for frequent use. The distribution of responses for each of these groups is presented in Figure 1.

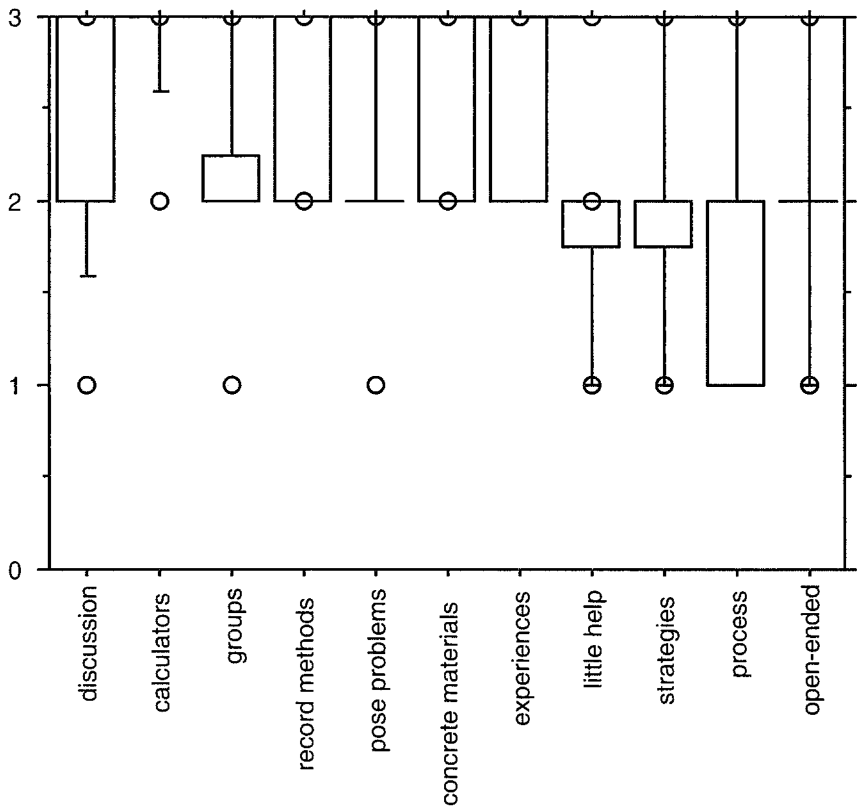


Figure 1. Teaching strategies that mathematics education lecturers believe should be used most frequently.

Strategies that mathematics education lecturers believed should be used *very frequently* included whole class discussion, the availability of calculators, cooperative group work, encouraging students to record their own procedures and methods as well as to pose their own problems, availability of concrete materials, and problems that focus on school contexts and students' experiences. An additional group of strategies that should be used *frequently* included setting problems with little guidance from the teacher, the discussion of problem-solving strategies and processes, and using open-ended problems.

For other teaching strategies, less frequent use was recommended. Lecturers indicated a group of strategies that should be used *very rarely* including students working alone, providing substantial explanation about how to solve particular problems, and using exercises to practise skills. Two further strategies were also given low levels of support and these included allowing students to choose their own problems and setting application problems to practise skills. The distributions for these strategies are presented in Figure 2.

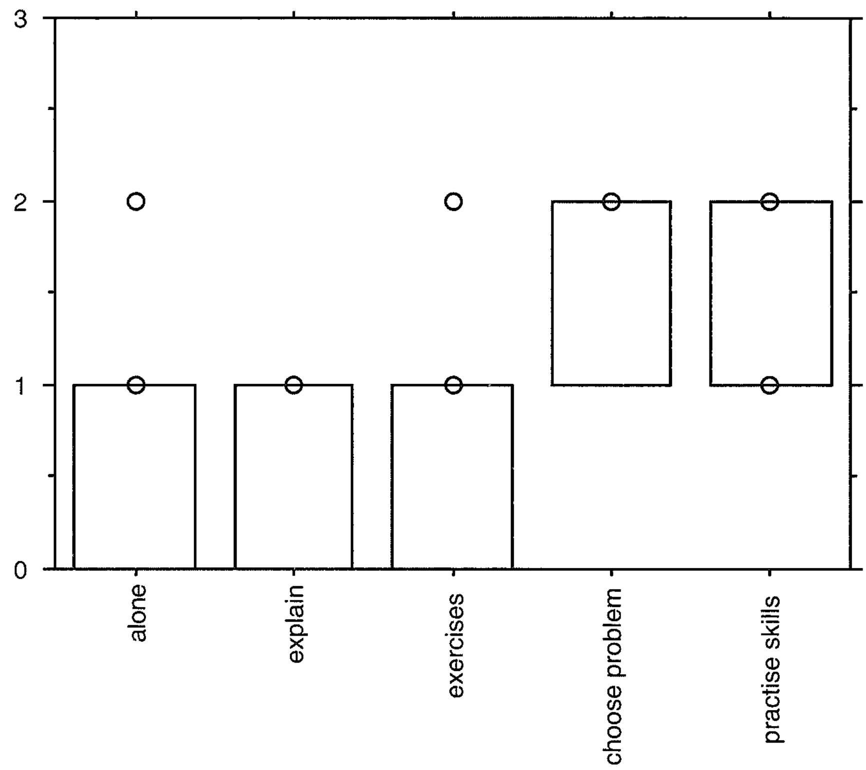


Figure 2. Teaching strategies that mathematics education lecturers believe should be used least frequently.

Comparison of Responses from Teachers and Lecturers.

A comparison of the data from surveyed teachers with that from mathematics education lecturers was achieved by examining box and whisker plots for each of the teaching strategies as well as comparing the means for each distribution. The responses for both teachers and mathematics education lecturers were scored from zero (0) for "hardly ever" to three (3) for "almost always". This method was employed so that means could be calculated for each item and the responses of each group compared. It is acknowledged that the differences between categories are not equivalent and therefore this process is used as a gross measure for comparison purposes only. The analysis enabled the construction of lists of less frequently used teaching strategies and more frequently used teaching strategies for each group.

Less Frequently Used Teaching Strategies

A mean of 1.1 was considered to indicate a *low* frequency of use of that particular strategy. This would suggest that the strategy was used *hardly ever*, or, slightly more than *sometimes*.

Items that had means less than 1.1, or *low* means, for the teachers included (with the variable name given in the brackets):

- you ensure that students work alone (alone);
- you have calculators available for students to use (calculators);
- you present *unfamiliar* and *open-ended* problems to the class with very little indication of how to solve them (little help);
- you provide a set of problems and the students are allowed to choose a problem they would like to work on (choose problem);
- you allow the class or individual students to spend several lessons on the same problem (spend more time); and
- you pose *unfamiliar problems*.

Items that had means less than 1.1 for the lecturers included:

- teachers ensure that the students work alone (alone);
- teachers explain in detail what the students have to do to solve problems (explain); and
- teachers set *exercises* to allow the students to practise their skills (exercises).

The first item is the only one in common between teachers and lecturers.

A comparison of the distributions of each of the eight distinct items listed above is presented in Figure 3.

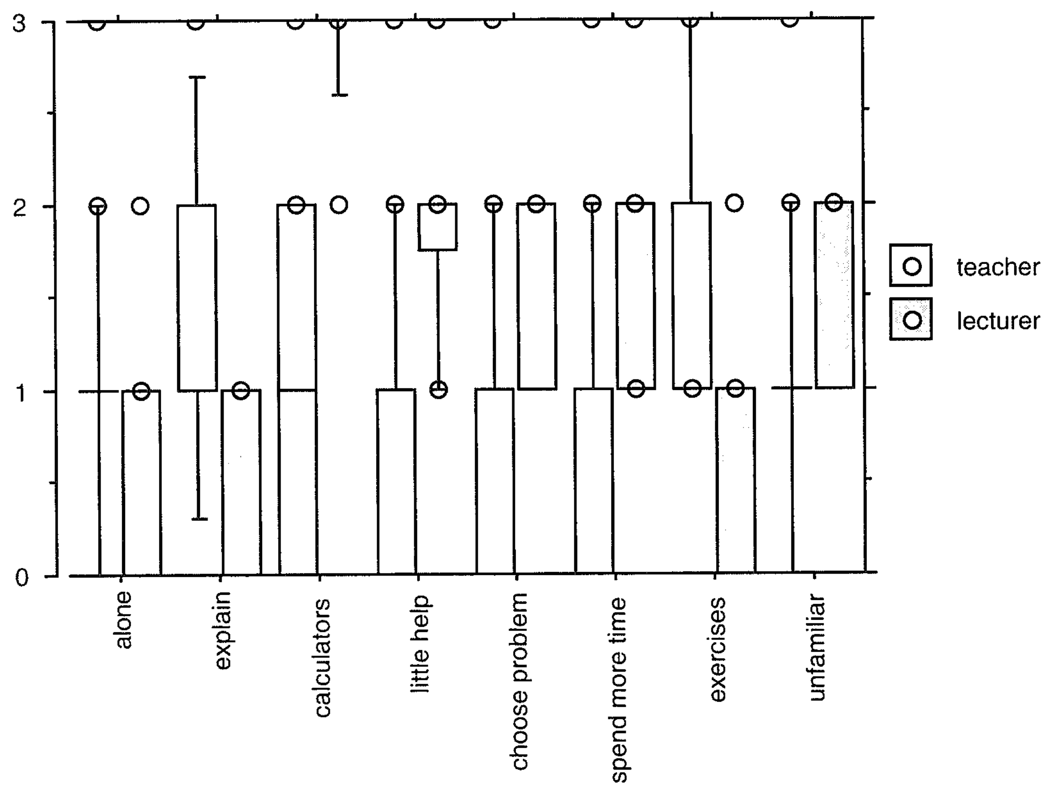


Figure 3. Box plots of items with *low* means for teachers and lecturers.

With only one item in common between the two groups, it is clear that there are considerable differences in beliefs about the appropriate use of each of these strategies. There appears to be a large difference between lecturers and teachers in the distribution of responses for the use of calculators, exercises and the notion of presenting problems to students to explore with little indication of how to solve them. Teachers report that they rarely use calculators and frequently set exercises; a contrast to the advice that seems to be presented in curriculum documentation.

More Frequently Used Teaching Strategies

A mean of 1.9 was considered to indicate a *high* frequency of use of that particular strategy. This would suggest that the strategy was used slightly less than *often*, or, that it was used *almost always*.

Items that had means greater than 1.9, or *high* means, for *both* teachers and lecturers included:

- at the end of a problem solving lesson you lead a whole class discussion so that students can share solutions and strategies (discussion);
- you provide concrete materials for those students who need them (concrete materials); and
- you discuss useful problem solving strategies (strategies).

The items that had a *high* mean for the teachers alone were:

- you present *application problems* which allow students to practise the skills they have just learnt (practise skills); and
- you model the problem solving process to the class (model).

Finally, items that had a *high* mean for the lecturers alone were:

- teachers have calculators available for students to use (calculators);
- teachers encourage the students to work in small, cooperative groups (groups);
- teachers encourage students to record their own procedures and methods of solving problems (record methods);
- teachers encourage students to pose their own problems (pose problems);
- teachers use problems which arise from the school context or which relate to the students' experiences (experiences); and
- teachers pose *open-ended problems* to allow students to explore mathematical situations for themselves.

The distributions of each of the eleven distinct items listed above, for both teachers and lecturers, is presented in Figure 4.

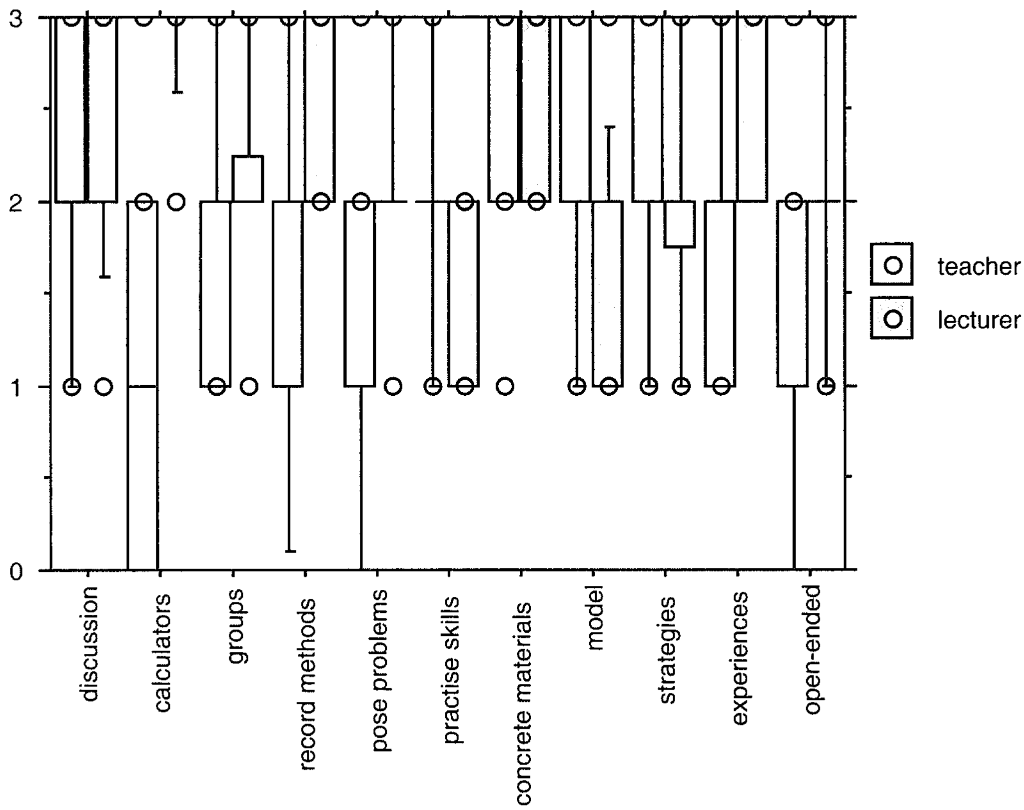


Figure 4. Box plots of items with *high* means for teachers and lecturers.

For this set of teaching strategies, three were rated of frequent use by both teachers and lecturers but for the remaining eight, there are differences in the distributions. Teachers reported a high frequency of use of modelling the problem-solving process to the class and setting application problems for skills practice. Encouraging students to record their own methods was reportedly used less frequently than lecturers believe to be desirable in mathematics classrooms. Encouraging students to pose their own problems was reported as a rarely used strategy by teachers as was the use of open-ended problems and problems relating to students' experiences.

The strategies that lecturers have rated as being appropriate for regular use are recommended in advice given to teachers about problem solving in the reformed classroom as well as in curriculum documentation. These include whole-class discussion for sharing solutions and strategies, small, cooperative group discussion, provision of concrete materials and calculators, recognition of the need to encourage individual student recording of methods and procedures, and encouragement of student posed problems as well as using problems that relate to student interests.

It is clear from the results that most teachers see the value of whole-class discussion, teacher modelling and using concrete materials in classrooms. Teachers do not seem to have embraced the view that calculators can be an integral part of the primary mathematics classroom. Also, there is less frequent use of individual student methods of recording and student created problems.

A final comparison can be made between teachers and lecturers regarding the use of particular student question types. Teachers prefer to use application problems and exercises more frequently than either open-ended or unfamiliar problems. It should be noted that the prompts for each group were slightly different. Teachers were required to indicate how frequently they use each of the strategies whereas lecturers were asked to indicate how frequently teachers should use each of the strategies according to the problem solving literature. These data were compared with those from the lecturers and are presented in Table 2

Table 2
Frequencies of the teaching strategies for lecturers (and teachers) that relate to student question types in Question 4 - all values are percentages

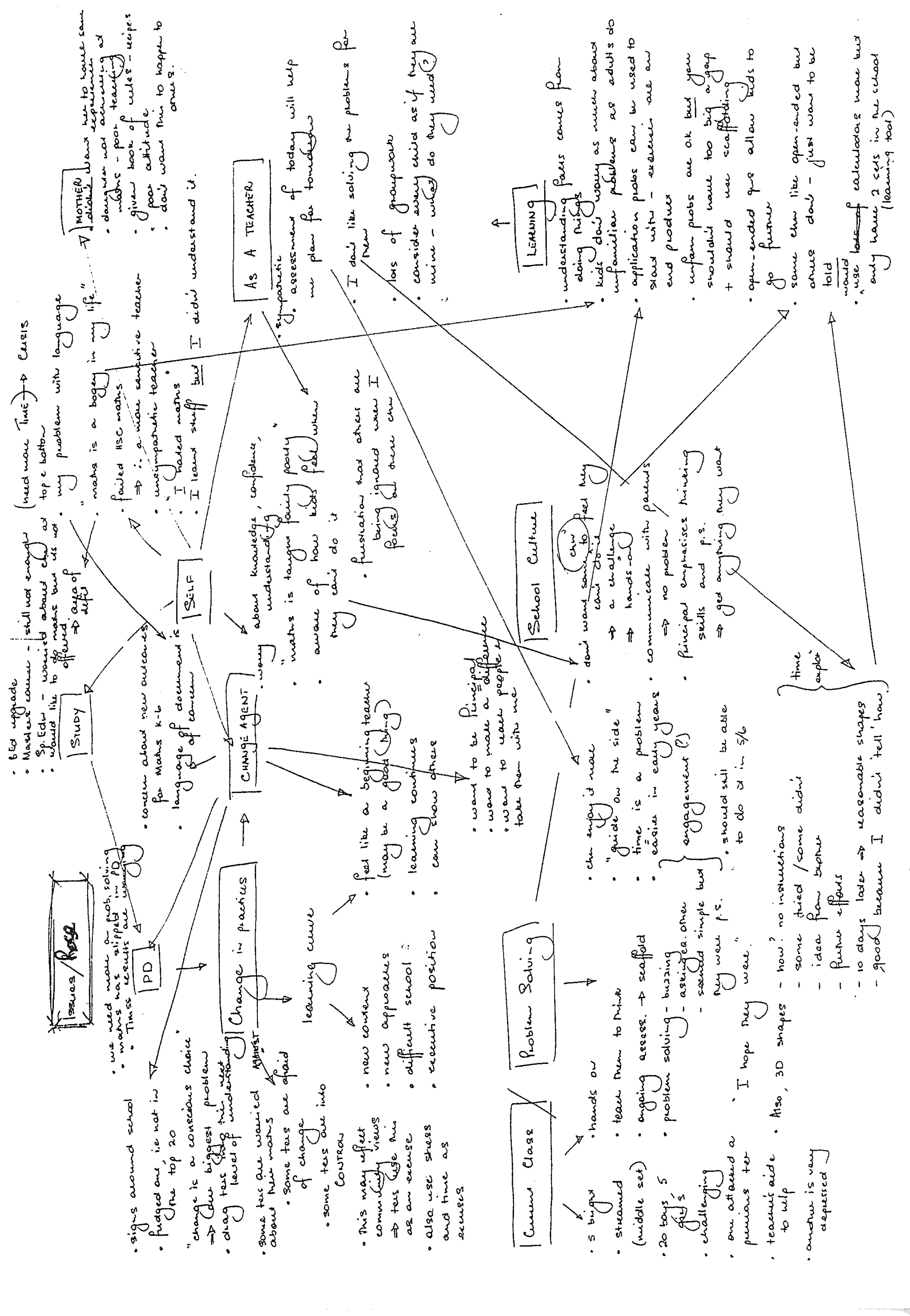
Teaching Strategy	Hardly Ever	Sometimes	Often	Almost Always
teachers (you) present *application problems* which allow students to practise the skills they have just learnt	0 (0)	71 (21)	29 (65)	0 (14)
teachers (you) pose *open-ended problems* to allow students to explore mathematical situations for themselves	0 (11)	19 (54)	67 (28)	14 (7)
teachers (you) set *exercises* to allow the students to practise their skills	29 (3)	67 (26)	4 (56)	0 (15)
teachers (you) pose *unfamiliar problems*	5 (21)	33 (55)	62 (23)	0 (1)

In response to these four items, lecturers recommended a higher frequency of use of open-ended and unfamiliar problems than teachers report using. Eighty-one percent of lecturers suggest that open-ended problems should be used *often* or *almost always.* In contrast, only 35% of teachers report that they *often* or *almost always* use open-ended problems. Also, 62% of lecturers recommend that unfamiliar problems should be used *often* while 24% of teachers report that they *often* or *almost always* pose this type of student question.

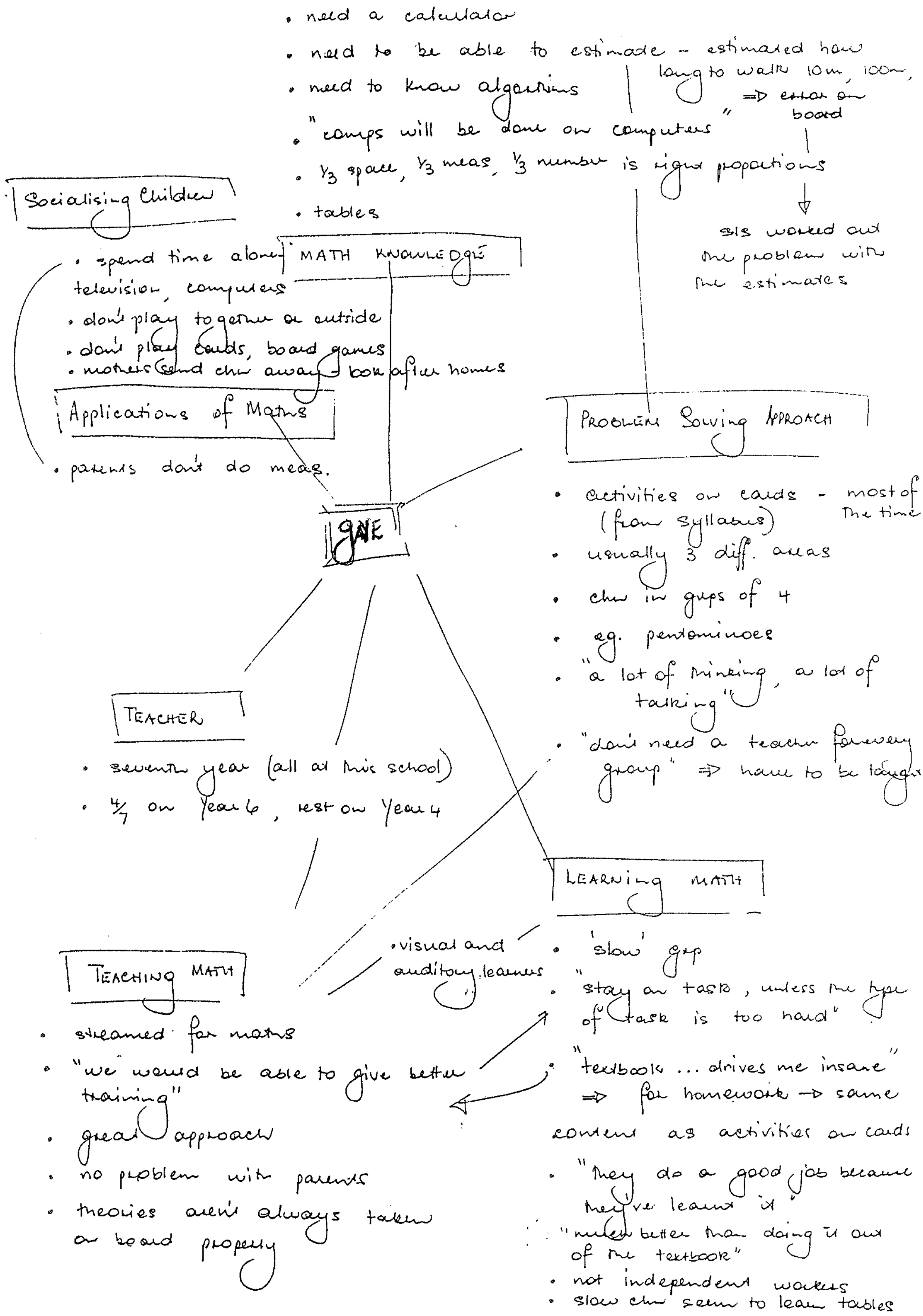
Lecturers also recommended a lower frequency of use of application problems and a much lower frequency of use of exercises. Seventy-one percent of lecturers suggest that application problems should be used *sometimes* which contrasts with 21% of teachers. Also, 96% of lecturers recommend that exercises should be used *hardly ever* or only *sometimes* whereas 29% of teachers report that they set exercises *hardly ever* or *sometimes.*

These differences between teachers and lecturers raise several questions about the discrepancies between their responses. Taking the lecturers responses as a reflection of current advice, it appears that teachers either do not agree, or do not implement that advice. This is a critical aspect of this investigation. There are potentially many reasons for this including beliefs that are not congruent with these approaches, lack of knowledge and experience in using such approaches, or constraints that operate in particular school settings that might militate against implementation. Alternatively, these approaches may be too difficult to implement thus suggesting that lecturers may not be in touch with the practical aspects of classroom and school life.

Appendix 9 - Diagrammatic Representation of the Issues Raised in Rose's Initial Interview



Appendix 10 - Diagrammatic Representation of the Issues Raised in Gaye's Initial Interview



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