EXPLORING THE MYSTERY OF CHILDREN WHO READ, WRITE AND ORDER 2-DIGIT NUMBERS, BUT CANNOT LOCATE 50 ON A NUMBER LINE

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Two interpretive Place Value tasks were added to the Early Numeracy Interview *in 2010 to gain further insight about 761 Grade 2 and Grade 3 students' construction of conceptual knowledge associated with 2-digit numbers. Previously, the researchers had noticed that most students were successful at reading, writing and ordering 2-digit numbers, but that interpreting these numbers for problem solving remained a struggle for many. Analyses of students' responses showed that the new tasks distinguished students who previously were assessed as understanding 2-digit numbers, but who could not identify 50 on a number line or state the total of a collection (36) that was reduced by ten. The new tasks assist teachers to identify students who need further instruction to fully understand 2-digit numbers.*

INTRODUCTION

Research during the *Early Numeracy Research Project* in Australia (ENRP, Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002) found that being able to read, write, order and interpret 2-digit numbers was a difficult growth point for young children to reach. It seems that most children learn to read and write 2-digit numbers fairly easily, but that interpreting the cardinal value of these numbers is the greater challenge. In a later study involving over 7000 Australian primary students, Gervasoni, Turkenburg, & Hadden (2007) were also concerned by the number of students they identified in Grades 2-4 who were yet to fully understand 2-digit numbers. If we are to improve young children's whole number learning then it is important to understand the challenges children face in coming to understand 2-digit numbers. This is the key issue explored in this paper that reports on the refinement of the ENRP Early Numeracy Interview (ENI) and framework of growth points (Clarke et al., 2002) as part of the *Bridging the Numeracy Gap* Project (Gervasoni, Parish, Upton, Hadden, Turkenburg, Bevan, Livesey, Thompson, Croswell, & Southwell, 2010). The research team aimed to refine and extend the ENI and growth points, originally designed for use in the first three years of schooling, so that they were more appropriate for assessing students across the primary school. The aspect of the research reported here is the refinement of the assessment tasks for Growth Point 2 (GP2) – reading, writing, interpreting and ordering 2-digit numbers.

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THE CHALLENGE OF UNDERSTANDING 2-DIGIT NUMBERS

Many studies have provided insight about the challenges involved in understanding and using 2-digit numbers. One important finding is that children who have not constructed grouping and place value concepts often have difficulty working with multi-digit numbers (Baroody, 2004). Also, being able to interpret numerals to order them from smallest to largest is another difficulty for some children. Griffin, Case, and Siegler (1994) observed that this involves integrating the ability to generate number tags for collections, and make numerical judgments of quantity based on the construction of a mental number line (Griffin & Case, 1997; Griffin et al., 1994).

Other studies have found that successful problem solving with two-digit numbers depends on children's ability to construct a concept of ten that is both a collection of ones and a single unit of ten that can be counted, decomposed, traded and exchanged for units of different value (e.g., Cobb & Wheatley, 1988; Fuson et al., 1997; Ross, 1989; Steffe et al., 1988). Cobb and Wheatley (1988) found that some children develop a concept of ten that is a single unit that cannot be decomposed, and proposed that this type of concept is constructed when children learn by rote to recognise the number of tens and ones in a numeral, but do not recognise that the face value of a numeral represents the cardinal value of a group.

Fuson et al., (1997) identified five different correct conceptions of 2-digit numbers and one incorrect conception that children use, several of which may be available to a given child at a particular moment and used in different situations. These six conceptions provide researchers with a detailed model to analyse children's use of 2-digit numbers and was considered by researchers when developing the ENRP Place Value framework of growth points and the associated Early Numeracy Interview (ENI). However, for the ENRP, researchers opted for a less complex model than the Fuson et al. model that they hoped would be more user-friendly for teachers. Ten years on, in refining the ENRP assessment interview and framework of growth points as part of the research reported in this paper, it will be important to consider whether the Fuson et al. model better explains the difficulties that some children experience in coming to understand 2-digit numbers. The six conceptions of 2-digit numbers are explained in detail in Fuson et al. (1997). They are the: Unitary Multi-Digit Conception; Decade and Ones Multi-digit Conception (noticing word parts); Sequence of tens and ones conception (noticing the advantage of counting by tens associated with partitioning in tens); Separate Tens and Ones conception (noticing the number of tens and the number of ones); Integrated sequence-separate tens conception (noticing that the number of tens is linked to the number name); and the Incorrect Single-Digits Conception (viewing each digit as representing ones).

Fuson et al. (1997) contend that for full understanding of number words and their written symbols, children need to construct all five of the correct multi-digit conceptions, and that this requires considerable experience and time. Thus, we believe that the refinement of the Early Numeracy Interview needs to ensure that teachers can identify students who use the integrated sequence/separate tens conception of 2-digit numbers. Indeed, we are interested to learn whether or not students who previously reached GP2 in Place Value are successful with this task.

Constructing a Mental Number Line

Another important characteristic of number learning is forming a mental number line. This requires the ability to visualise and abstract a number line so that students can locate any given number, order numbers by quantity, and generate any portion of the number line that may be required for problem solving.

Griffin, Case and Siegler (1994) proposed that success in early arithmetic depends on the formation of a mental number line in association with understanding the generative rule that relates adjacent cardinal values (i.e., each adjacent number in the number line is one more or one less than its neighbour); and understanding the consequence of the previous idea: that each successive number represents a set which contains more objects, and thus has a greater value along any particular dimension.

One way to help children develop a mental number line for use in problem solving is to engage them in activities involving an empty number line. This is a strategy widely used in the Netherlands and aims to link early mathematics activities to children's own informal counting and structuring strategies. "The choice of the empty number line as a linear model of number representation up to 100 (instead of grouping models like arithmetic blocks) reflects the priority given to mental counting strategies as informal knowledge base" (Beishuizen & Anghileri, 1998, p. 525). This emphasis in the research literature on the importance of the mental number line and empty number line as a means of interpreting numbers is not reflected in the ENI until GP5. When refining the ENI it may be useful to include a 2-digit number line task earlier in the interview to determine whether, students who reach GP2 are able to interpret numbers on a number line.

ENRP Assessment and Growth Points

The Early Numeracy Interview developed as part of the Early Numeracy Research Project (Clarke, Sullivan, & McDonough, 2002), is a clinical interview with an associated research-based framework of growth points that describe key stages in the learning of nine mathematics domains. The data examined in this paper were drawn from the ENI and Growth Point Framework, so it needs to be understood.

The principles underlying the construction of the growth points were to: describe the development of mathematical knowledge and understanding in the first three years of school in a form and language that was useful for teachers; reflect the findings of relevant international and local research in mathematics (e.g., Steffe, von Glasersfeld, Richards, & Cobb, 1983; Wright, Martland, & Stafford, 2000); reflect, where possible, the structure of mathematics; allow the mathematical knowledge of individuals and groups to be described; and enable a consideration of students who may be mathematically vulnerable. The processes for validating the growth points, the interview items and the comparative achievement of students are described in full in Clarke et al. (2002). The following are the growth points for Place Value.

1. Reading, writing, interpreting and ordering single-digit numbers.
2. Reading, writing, interpreting and ordering two-digit numbers.
3. Reading, writing, interpreting and ordering three-digit numbers.
4. Reading, writing, interpreting and ordering numbers beyond 1000.
5. Extending and applying place value knowledge.

Each growth point represents substantial expansion in knowledge along paths to mathematical understanding (Clarke, 2001). The number tasks in the interview take about 20 minutes for each student and are administered by the classroom teacher. There are about 40 tasks in total, and given success with a task, the teacher continues in a domain (e.g., Place Value) for as long as a child is successful. Teachers report that the ENI provided insights that might otherwise remain hidden (Clarke, 2001).

Refining Assessment Tasks for 2-digit Numbers - Growth Point 2 (GP2)

This paper examines students' Place Value Knowledge and the effect of two new tasks designed to identify students who were assessed at GP2, but who may not interpret successfully the quantitative value of 2-digit numbers. The data examined is drawn from the 2010 assessment interviews of nearly 3000 Reception (R) to Grade 3 students (5-8 years old) from 42 low SES school communities in Victoria and Western Australia who are part of the *Bridging the Numeracy Gap Project* (Gervasoni, Parish et al., 2010). This is a Federal Government funded Project aiming to close the education gap for low SES and Aboriginal and Torres Strait Islander students, and is a collaboration between 42 school communities, Catholic Education Offices in the regions of Ballarat, Sandhurst, Sale, and Western Australia, and Australian Catholic University. The new tasks are shown in Figure 1 below.

Pop-Sticks Bundling Task

Ask the child to unpack the icy pole sticks. Here are some icy pole sticks in bundles of ten (*offer the chance to check a bundle if it seems appropriate).* Here are some more loose ones. *Show white card for 36.*

a) Get me this many (icy pol e) sticks. *(If the child starts to count all in ones, interrupt and ask them if they can do it a quicker way with the bundles. If they can't,* Tell me how you worked that out.

b) Please put one bundle back. How many sticks are there now? How do you kno w that?

2-Digit Number Line Task

Show the child the mauve 2 -digit number line card.

Look at this number line. Please tell me the largest number. *(100)*

Point to the little mark . What number would go here? *(acceptable number range is 45 -55).* b) Please explain.

Figure 1: New Growth Point 2 tasks. Students' Place Value Knowledge.

Part b of the Bundling Task was designed to distinguish those students who use the integrated sequence/separated tens strategy when interpreting a collection of 36 pop-sticks. Inclusion of the number line task reflects the emphasis in the research literature of the importance of students developing a mental number line to interpret quantities when problem solving.

A key issue for the research reported in this paper was to determine students' Place Value Growth Points, and whether the new GP2 tasks identified students who were not successfully interpreting the quantitative value of 2-digit numbers. Figure 2 shows the distributions of Growth Points at the beginning of the 2010 school year for nearly 3000 Reception-Grade 3 students.

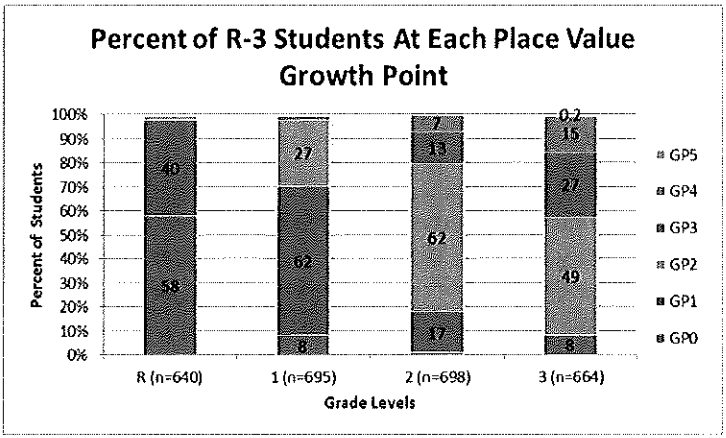

Figure 2: Place value growth point distribution for R-Gr 3 students.

Each student was assessed by their classroom teacher, and the growth points were calculated independently by trained coders to increase the trustworthiness of the data.

An issue highlighted in Figure 2 is the spread of growth points at each level, particularly from Grade 1 onwards. This has been noted elsewhere (e.g. Gervasoni & Sullivan, 2007; Bobis et al., 2005) and confirms the complexity of the teaching process and the importance of teachers identifying each student's current knowledge and knowing ways to customise learning to meet each student's needs.

These data indicate that about two-thirds of Grade 1 students have reached GP1, and therefore the initial focus for Place Value instruction is GP2 – 2-digit numbers. By the beginning of Grade 2, most students reach GP2. However, by Grade 3, half the students remain on GP2. Examination of the assessment tasks for GP3 and GP4 indicate that students cannot reach these growth points if they do not interpret the quantitative value of numbers. We also noted that students could reach GP2 successfully using only procedural knowledge to read, write and order numbers, and collect 36 pop-sticks. The original tasks did not actually require the interpretation of quantity, although conceptual knowledge was assumed.

Next we examined the data to determine the effect of the new GP2 tasks to determine whether these tasks identified any students who were not interpreting the quantitative value of numbers. The first new task required students to identify the value of a quantity that was reduced by ten. Students strategy for achieving this was observed and recorded by teachers on the assessment record sheet, and only students who were judged to be using Fuson et al.'s (1997) integrated sequence/separated tens strategy were deemed to be at GP2. This provided confidence that students were able to use all five correct conceptions of 2-digit numbers. The second task required students to interpret a number line. Students were asked to identify the number that was half way between 0 and 100 on the number line. Students who stated a number between 45 and 55 were deemed to be successful. As most students in Grades 2 and 3 had reached GP2, students in these grades who were assessed at GP2 were selected for further examination, and their responses to the two new tasks analysed.

The data presented in Figure 3 demonstrate that these tasks did identify some students who were assessed at GP2, but who did not successfully interpret 2-digit numbers in the Bundling and Number Line tasks. More than half of the Grade 2 students and one-third of the Grade 3 students on GP2 were not able to solve the two new tasks. This highlights that interpreting 2-digit quantities is an issue for many students. The number line task was the more difficult of the new tasks. The most common incorrect response was 10, with students counting by ones along the number line until they reached the half-way mark. Of the remaining students who were successful, analysis of their responses to the 3-digit assessment tasks showed that none of these students were successful with the interpretive tasks, although most could read, write and order 3-digit numbers. This inability to interpret quantities was the reason why all these students did not progress to GP3.

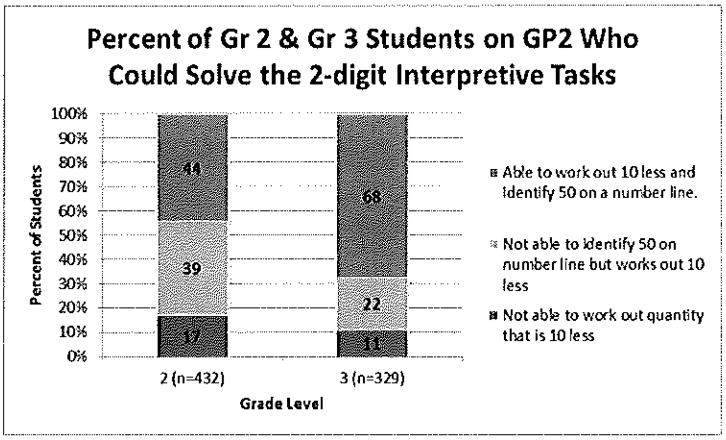

Figure 3: Percent of Gr 2 & 3 students on GP2 who could solve the 2-digit tasks.

CONCLUSION

Analysis of 761 Grade 2 and Grade 3 students' responses to new tasks in the ENI showed that these tasks distinguished students who were assessed as understanding 2-digit numbers, but who in fact could not identify 50 on a number line or state the total of a collection of bundled pop sticks (3 tens and 6 ones) that was reduced by ten. These additional tasks assist teachers to identify students who need further experience with 2-digit numbers to construct full conceptual understanding, and highlight the importance of teachers focusing instruction on interpreting quantities, and not simply reading, writing and ordering numerals. Most children learn to read and write 2-digit numbers quite easily, but interpreting the cardinal value of these numbers is the greater challenge. However, it is this interpretation of quantity that is essential for problem solving and conceptual understanding. Perhaps the fact that the ENI has not included tasks that identify students who do not fully interpret 2-digit quantities has given teachers an inflated impression of some GP2 students' understanding. We argue that some of these students need further instruction focused on their development of 2-digit number conceptions and a mental number line.

One implication of the findings is that learning trajectories associated with Place Value and the development of whole number concepts need to adequately account for students' interpretations of quantities. We believe that the ENRP Place Value growth points and the associated assessment interview needs to be modified accordingly. Such a refinement will give teachers more certainty about students' current knowledge and assist them to design more precise instruction.

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