Views of cognition: Different lenses for 'Looking-in' on classrooms

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Abstract The evolution of different theories of cognition over the years has given mathematics education researchers new tools for highlighting particular characteristics of classroom issues to enable more detailed investigation of how students learn mathematics. In the past there has been a strong dominance of the dualistic view separating body and mind. In recent times, however, the body has been given a more central role in shaping the mind. This has led to the situation where some long-standing conundrums of mathematics education have become more tractable to researchers. Similarly, by using older ideas in new ways, light is being shone on what is possible at different stages of development or different levels of schooling. In this issue different views of cognition have been mined by the authors of different articles to frame studies and analyses or invent and apply new tools. They bring new lenses for looking-in on classrooms, a fresh view with old lenses and new methodological tools to the fore. Using a small selection of the articles of the issue the empowerment that differing views of cognition have enabled for mathematics education research is demonstrated.

Keywords Cognition, sensuous cognition, schema, collective coherence, adjacency matrix

1 Introduction

Different views or theories of cognition (e.g., neurological approaches, Luria, 1966, 1973; information processing approaches, Neisser, 1967; Massaro and Cowan, 1993; connectionist approaches, Rumelhart and McCelland, 1986) that have evolved over the years have given researchers new tools for highlighting particular characteristics of classroom issues to enable more detailed investigation as different theories foreground some aspects of situations and background others (Ernest, 2010). In all these views of cognition there is a strong dominance of the dualistic view separating body and mind and the internal and external worlds and this is reflected in mathematics education research underpinned by these theories (e.g., Schoenfeld, 1987; Stillman and Galbraith, 1998). In recent times, however, the body has been given a more central role in shaping the mind (Wilson, 2002) spawning a different conceptualisation of cognition, namely embodied cognition. This has led to the situation where some long-standing conundrums of mathematics education have become more tractable to researchers. Similarly, by using older ideas in new ways, new light is being shone on what is possible at different stages of development or different levels of schooling. In this issue quite disparate views of cognition have been mined by the authors of different articles so as to bring together new insights and findings. These have been used to frame studies and analyses or invent and apply new tools enabling researchers to productively examine classroom activity in mathematics education. A selection of the articles in this issue will now be examined to illustrate this observation.

2 New lens for 'looking-in'on classrooms

Radford's sensuous cognition (2009) is typical of the newer conceptualisations of cognition which recognises that cognition is both culturally and historically constituted. According to Radford (2014), "mind, body, and world are conceived of as intertwined entities. Sensuous cognition stresses the idea that our thinking, feelings, deeds, and in fact all our relations to the world (hearing, perceiving, smelling, sensing etc.), are an entanglement of our body and material and ideational culture". Taking this view, development of cognition is related to cultural development of our senses and our multi-modal (e.g., visual-kinesthetic and visual-tactile) sensorial experiences of the world. Although Vygotsky and Luria (1994) much earlier contended that use of artefacts plays a pivotal role in the evolution of our ways of sensing, Radford extends their role from mere "mediators of human thinking and experience" to become "a constitutive part of thinking" where they have "a cognitive life".

Such a conceptualisation of cognition, allows other researchers a new lens for "looking-in" (Williams, 2006) on classrooms. Funahashi and Hino (2014), for example, investigate the learning culture set up by the teacher to focus students' attention on the important ideas that become the foundation for new mathematical content in a lesson. Links to Radford's sensuous cognition are obvious although the authors do not make them in the article.

These authors show that it is important for teachers to plan both the activities being used and the social interaction they will most likely elicit in accordance with their lesson objectives. Using the guided focussing pattern proposed by Funahashi and Hino, teachers elicit an object of examination that comes from important mathematical ideas proposed by the students. This becomes the *pronounced focus* of the lesson that is identified explicitly by the teacher. The *attended focus* is what is being attended to whilst someone is speaking and students' attention can be drawn to it by the teacher by gestures such as pointing to inscriptions or hand movement to draw attention to a possible relationship between parts of an inscription. Funahashi and Hino recommend that teachers use student thinking to "reveal an explicit attended focus that supports… development of new learning content". If the attended focus is not what the teacher intended he/she can rephrase, telegraph or extend the students' focus towards the *intended focus* intentionally using visual or bodily prompts as above. Students modify and make sense of the attended focus not only through hearing the words of their teacher and peers but also "by examining and attending to the inscriptions and gestures" the teacher uses.

Funahashi and Hino see these teaching practices as playing a critical role in controlling students' attention so as to guide students' mathematical thinking towards the new learning through classroom interaction. The teacher is thus productively focussing the students' mathematical noticing with a clear goal in mind. The students' hearing and perceiving has been deliberately guided by voice, privileging of particular student mathematical ideas rather than others, and gesture to allow a particular transformation of the knowledge presented and constructed to what would be commonly accepted and used, that is, the institutionalised knowledge of the mathematics classroom. The cognitive growth here is fostered by the cultural development of the students' senses and their multi-modal sensorial experiences of the mathematics classroom orchestrated by the teacher.

3 Taking a new look with an old lens

Over the years several mathematics education researchers have developed various local frameworks of conceptual growth theorising about how actions can be transformed into mental objects to achieve cognitive growth. Pegg and Tall (2010), for example, suggest this is achieved by the compression of information into thinkable concepts, that is, knowledge structures that are coherent enough to be conceived of holistically as a unit that can be named and thought about. The article by Nunes et al. (2014) dealing with the cognitive demands of understanding sample space in probability draws on traditional notions of cognition as this understanding is linked to the development of conceptual schemas (Anderson, 1995) which are based on schemas of thought. Such thought schemas are internalised schemas of action which result in the same sequential organization of thoughts that would accompany such external action but the person is able to do this, in the mind as it were, without observable actions.

In order to develop a sample space schema, Nunes et al. note development of a combinatorial schema is a necessary, but not sufficient, requirement. They point to a "significant role for instruction" in accelerating the development of such a schema in 10-11 year olds "using iconic representations of the processes" in the form of tree diagrams. They "hypothesize that the origin of the combinatorial schema is in the schemas of classification and logical multiplication". These schemas allow the construction of an inventory of all the equally likely outcomes in a probabilistic situation but for a problem solver to link them to their probability it is necessary to consider whether they should be treated singly or grouped depending on the probabilistic events that are the focus. This requires classification into favourable and unfavourable cases (in terms of the event being explored) and the ratio schema. Nunes et al. suggest that if these young students can "learn how to use their old logical multiplication schemas in the context of events…they could generate an inventory of cases for sample space problems". Further, they suggest that tree diagrams are a tool that "affords the systematicity" needed to facilitate young students' moving from simplistic to challenging problems in the context of probabilistic situations. The tree diagrams also facilitate the aggregation of cases that are equivalent in outcomes for the event that is in focus. Classification of outcomes into favourable and unfavourable cases via this aggregation links to identification of the ratio that is relevant to the solution.

The short teaching intervention used by the authors took these elements into consideration. The authors were then able to demonstrate that it is possible to help young students link cognitive schemas developed in other mathematical contexts to a new concept in another mathematical domain, namely, the concept of sample space in the domain of probability. General problem solving skills such as those involving problem interpretation and systematic search are insufficient to use the previously developed schemas, albeit combined in a new way, in the new domain. The importance of the contribution of the thoughtful design of a teaching program to ensure this "acceleration" cannot be under estimated in the authors' opinion.

4 Fresh eyes with new tools

Wawro (2014) takes a highly mathematical approach to investigating cognitive growth in the form of use of adjacency matrices as a means of capturing prevalent structures in reasoning and measuring the centrality of concepts over time in exploring how a classroom community reasons to establish meaning at the collective level. The context for this investigation was an undergraduate inquiry-oriented linear algebra class. The use of adjacency matrices as a methodological tool is not new to social science research although I have not seen it used before in mathematics education research; however, Wawro takes a novel approach to give a quite different picture of conceptual growth as the collective shifts in patterns of thinking and reasoning as understanding develops across the classroom community. The mathematics that the undergraduate students were collectively coming to terms with over a semester was the Invertible Matrix Theorem but we will concentrate on identifying the general rather than the particular in examining this article. The study contributes to the growing body of literature that focuses on the collective learning community rather than the individual learner within that community (e.g., Keene et al., 2012).

In terms of shifts in argumentation over the semester, when developing ways to reason about a new concept such as linear independence, initially there was a reliance on explanations as justifications for claims were considered necessary as the students grappled with the new idea during discussion. However, as the concept became a thinkable concept with a shared collective understanding of what it meant, there was less need to continue to unpack it so such explanations were soon discarded and the cycle began again for a new concept in the cluster of concepts entailed in the topic. The earlier developed concepts then underpinned the development of later concepts as the semester advanced. When an argument was the basis for reasoning about a new concept or implication between two concepts, a variety of interpretations of these concepts was observed particularly in justifying perceived implications. On the other hand, when arguments used already well-established concepts or connections between such concepts, definitive statements about the concept(s) were made rather than a variety of interpretations indicating that collective coherence in the knowledge structure underpinning the concept, in other words, a collectively shared concept image (Tall and Vinner, 1981), had developed. For the particular classroom community studied, reasoning about the new concept also involved reasoning about the negations of statements about the concept. Measures of centrality revealed the particular concepts that were most densely connected throughout the semester discussions.

The use of this research technique could prove useful in the future for identifying progression of conceptual growth in over-arching big-ideas through collective discussion for other curricular topics. Considering the extent to which collective discussion is encouraged these days in teacher education at both the class and group level, the technique seems a credible means for researching such collective activity. The use of a new tool has given us not only a new theoretical understanding of how students come to know through discussion in a collective but also a situated view of how students come to know the particular mathematical topic through discussion in a collective.

5 Moving forward

Even with this small sample of the articles of the issue it has been possible to demonstrate the empowerment that differing views of cognition have enabled for mathematics education research. At times the insistence by the gatekeepers of mathematics education research on the necessity of every reporting of a research study to have a theoretical framework has appeared to be a straight jacket that endangers mathematics education research to becoming an irrelevancy as all traces of creativity are quashed in the pursuit of conformity to the blinkered vision of others. Here, however, evolution of different views of cognition and their adoption by mathematics education researchers has facilitated the expansion of knowledge and the moving of the field forward in a productive manner.

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