# EXPLORING THE RELATIONSHIP BETWEEN TASK, TEACHER ACTIONS, AND STUDENT LEARNING

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*We are examining actions that teachers take to convert tasks into learning opportunities. In this paper, we contrast ways that three teachers convert the same task into lessons, and the way that their lessons reflect their intent. We found that the teachers did what they intended to do, that this was connected to their appreciation of the mathematics involved, and directly influenced the learning opportunities of the students. To the extent that the potential of the task was reduced, this seemed due to the lack of mathematical confidence in the case of two of the teachers.*

*Keywords*: Classroom research; Mathematics tasks; Teacher knowledge; Realistic mathematics

Exploración de las Relaciones entre Tarea, Acciones del Profesor y Aprendizaje del Estudiante

*Examinamos las acciones que los profesores llevan a cabo para convertir tareas en oportunidades de aprendizaje. En este artículo comparamos las maneras en las que tres profesores convirtieron la misma tarea en actividades para la clase y la manera en que sus actividades de clase reflejan sus intenciones. Encontramos que los profesores hicieron lo que pretendían hacer, que esto estaba relacionado con su percepción de las matemáticas que estaban implicadas y que esta relación influyó directamente en las oportunidades de aprendizaje de los estudiantes. En el caso de dos profesores, la reducción en el potencial de la tarea parece deberse a su falta de confianza matemática.*

*Términos clave*: Conocimiento del profesor; Investigación en el aula; Matemáticas realistas; Tareas matemáticas

We are investigating ways that particular types of mathematics classroom tasks create opportunities for students and challenges for teachers. Various authors

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have argued that classroom tasks are the medium through which teachers and students communicate, and that the type of task influences the nature of the learning (e.g., Christiansen & Walther, 1986; Hiebert & Wearne, 1997).

The data presented below are from the Task Type and Mathematics Learning[1] (TTML) project which focuses on four types of mathematical tasks as follows:

*Type 1*. Involves a model, example, or explanation that elaborates or exemplifies the mathematics.

*Type 2*. Situates mathematics within a contextualised practical problem to engage the students, but the motive is explicitly mathematics.

*Type 3*. Involves open-ended tasks that allow students to investigate specific mathematical content.

*Type 4*. Involves interdisciplinary investigations in which it is possible to assess learning in both mathematical and non mathematical domains.

The focus of our overall research is to describe how such tasks respectively contribute to mathematics learning, the features of successful exemplars of each type, constraints which might be experienced by teachers, and teacher actions which can best support students' learning.

The focus here is on actions that teachers take in implementing tasks in their class. We draw on the Stein, Grover and Henningsen (1996) model of task use. They described how the features of the mathematical task as set up in the classroom, and the cognitive demands it makes of students, are informed by the mathematical task as represented in curriculum materials, and influenced by the teacher's goals, subject-matter knowledge, and knowledge of students. One of the interesting results from Stein et al. was the tendency of teachers to reduce the level of potential demand of tasks. Doyle (1986) and Desforges and Cockburn (1987) attribute this phenomenon to complicity between teacher and students to reduce their risk of making errors. Tzur (2008) argued that there are substantial deviations between the ways that developers intend tasks to be used and the actions that teachers take. Tzur argued that there are two key ways that teachers modify tasks: (a) At the planning stage if they anticipate that the task cannot accomplish their goals; and (b) once they see student responses if they are not as intended. Charalambous (2008) argued that the mathematical knowledge of teachers is one factor determining whether they reduce the mathematical demand of tasks based on their expectations for the students. A related issue is the extent to which students are allowed to create their own solutions, as compared with following a method proposed by the teacher. It has been argued that students

[1] TTML is an Australian Research Council funded research partnership between the Victorian Department of Education and Early Childhood Development, the Catholic Education Office (Melbourne), Monash University, and Australian Catholic University.

choosing their own approaches, and their awareness of those choices, are key elements of mathematics learning (Watson & Sullivan, 2008).

The following comparison of three lessons based on the same task is intended to offer insights into the relationship between teachers' intentions, their actions, and the effect on the students, and especially on the relationship between the teachers' intentions, actions, and the task's potential.

## The Overall Project and Methods Used for this Phase

In a prior phase of the overall project, we worked with teachers to ensure that teachers have access to high-quality task exemplars. We led teacher development meetings focusing on the nature of the respective task types, the associated pedagogies, ways of addressing key constraints, such as diversity in culture, language background and readiness to learn, and student assessment.

At this current phase of the project, we worked with groups of teachers on coherent sequences of lessons, termed *teaching units*, drawing on a mix of the task types. The lessons reported below were from a teaching unit developed by a group of three combined grade 5-6 (11-12 years old) teachers from the same school serving a middle-class community in Melbourne, Australia. The first step was for the teachers, termed Ms A, B and C —although not all were women—, to identify the focus, which they proposed to be ratio and rates. The teachers met to plan the teaching unit, after which the researchers joined with the teachers to brainstorm possible activities from each of the task types. The teachers prepared a pre-test, including items such as "Write everything you know about fractions", and some specific content items. Each of the three teachers was observed in seven lessons, many of which were 90 minutes long. The observation schedule was developed from Sullivan, Mousley, and Zevenbergen (2005), and records details of classroom events, including the timing, teacher actions, some quotes, and the reactions of the observer. There were audio-recorded interviews with the teachers before and after the lessons, and the teachers completed a planning proforma before each lesson. We developed a content test in collaboration with the teachers for the conclusion of the unit, and we supervised its administration and its scoring.

The teaching unit was taught over a 3 weeks period. The teachers had developed a somewhat unusual format —unrelated to our project— in that they had arranged the class into like-achievement groups and created a set of up to nine tasks for each of the groups, although many of the tasks were similar across the four ability groups. The students could choose the order in which they worked, and this choice was emphasised by the teachers as having a pedagogical purpose. In the teachers' plan, one of the tasks for each of the groups was recorded, simply, as follows: "Usain Bolt ran the 100 m in 9.7 s. How fast is that in km/hr? How fast you can run in km/hr?"

The first part satisfies the definition of a Type 2 task: It is set in the context of the contemporary Olympic Games with potential to be interesting for the students; and it has an explicit mathematical purpose of conversion between comparable rates. The second part of the task could also be considered Type 3, with the openness being in the choice of the method, the choice of the mode of recording, the variety of correct answers, the possibility of interrogating the answers, and through the personal result.

Our specific questions in this phase were: How do teachers' actions relate to the task potential and to their intentions? and what is the impact of the teacher actions on student learning?

## Three Different Implementations of the One Lesson

All three teachers taught a lesson based on this task. The following are summaries of the lessons as derived from the teacher interviews and observations, with some interpretative comments. In the summaries we also draw on the responses to the following question on the student summative assessment that directly addressed the content involved in this task: "Usain Bolt's brother, Lightning Bolt, ran for one minute around the (school) track and covered 550 m. How fast did he run on average in kilometres per hour?"

We also presented students with a list of the 20 possible tasks they may have completed, and asked them to identify which one they liked the most, and from which one they had learned most.

### Teacher A

The written plan prior to the lesson indicated that Ms A intended to have an initial discussion linked to previous lessons, and a whole class discussion on km/hr, after which the students would work outside in pairs on the task, then a whole class debrief adding to an overall map of the concepts involved in the unit that the class was progressively and collaboratively developing.

As part of the 26 minute introductory discussion, Ms A, an early career teacher with confidence in her ability and mathematical knowledge, posed the following problem: "(The class turtle) escaped. He covered 10 metres in 30 seconds. How fast is this in km/hr?"

Note that the turtle question is of a different form from the Usain Bolt question. After working on this problem, one student wondered whether he could walk that fast. Ms A adjusted her plan to facilitate this incidental opportunity. Then, the students were asked to work out how fast they could run. There were detailed directions on organisational matters —e.g., use the stop watches—, but no instruction on how to do the running task.

The students then spent 30 minutes outside. The students worked in groups, with some choosing to measure how far they could run in a particular time. When asked, the students in those groups said that they chose their method deliberately since it would be easier to calculate. For example, one student said: "[Student name] and me chose to do 10 seconds because if you do 10 seconds, it needs to add to 1 hour but the distance doesn't really have to add to anything…"

Other groups measured how long it took them to run a particular distance.

When the students returned to class, they continued working in groups. Most of the eight or so students who had chosen the easier method calculated their answer readily. Many other students who had chosen the more difficult method struggled with the calculation. The teacher was extremely busy trying to help the students working on the difficult method, while the better students completed the work quickly —but pretended they had not yet finished—. This phase took 25 minutes. There was no concluding review, and therefore no discussion of the differing methods.

In the post lesson interview, Ms A recognised what had happened:

> *So those that had thought about time and a unit of time prior to it were able to do it more readily than those that had thought about a unit of distance. So if I was to do it again… I would try to make the specific ratio idea clearer… I always try to put it back on them.*

Of the 22 students in the class, 16 (73%) correctly answered the Lightning Bolt question on the test. In the survey of task preferences, five students chose as the one they most liked —none chose the Usain Bolt question—, giving the how fast can you run task comments like "it was fun and hard" and "we got to go outside I liked running around". A different five students chose the same task as the one from which they learned most, giving comments like "I learnt things I didn't know before".

We interpret this experience overall to suggest that Ms A had thought about the task and its pedagogical purpose, and gave the students ample opportunity to devise their solution path for themselves. The task was well introduced, with the turtle question being meaningful to the students, and at a lower level of difficulty. Ms A had not anticipated the way that the form of the calculation chosen determines the level of difficulty, although she realised this during the lesson. The task, and this lesson, clearly created opportunities for students and most students were able to respond to the assessment task. Nearly half of the class chose this as the task they either most liked or felt they learned most learned. The constraint was the lack of success by some other students, and the organisational difficulties created by having some students finished while others were struggling. This highlights that such contextualised and open-ended tasks are complex to implement. Even so the implementation of the task was as intended, and even when she realised the student difficulties, Ms A neither moderated the level of challenge for the students nor reduced the potential of the task, and certainly maintained a commitment to students choosing their own methods.

**Teacher B**

The lesson of Ms B was in two parts. In the pre-lesson interview, Ms B, a confident early career teacher, who was uncertain about aspects of her mathematics knowledge, described part of the lesson:

> *... Then we're going outside to calculate their kilometres per hour and that would be quite hard for some of them. So I'll just have to see how it goes with how far we get. The timing will be easy because we'll just time 100 metres and then convert it.*

In the introduction, which took 10 minutes, Ms B posed this task as "how fast can you run a kilometre?" She invited the students to suggest how they might do this. Various considerations such as the ability to maintain running speed were proposed. One student suggested "we could do 100 metres and times by 10" and this idea was adopted. Interestingly, other methods proposed by students were rejected quickly, as they appeared not to conform to Ms B's plan. One student asked whether they could do three sprints and find the average, and this was confirmed as a good idea by Ms B:

> *How you work it out is up to you, but you'll need to share the trundle wheel and stopwatches. Work out as a group how you're going to record results. When you've worked that out you can come and get a stopwatch and a trundle wheel off me. So groups of three or four would be best.*

The students then spent 25 minutes outside on the sports field working on their data collection in small groups. The last 5 minutes of this part of the lesson was back in the classroom, with the students together. There was a discussion on how they could find an average, with the teacher giving the instruction, "go back through your maths book and see if you can find how to do it". This part of the lesson concluded with the instruction: "Now think about how to change your time to how many km per hour you can run. Take it home and talk to your parents about it. See if you can work out how to do it."

In the post-lesson interview, Ms B was asked about the outcome of the lesson:

> *I think they'll all need a little bit of help but I think some of them will be able to work it out with a bit of help. A couple of them have already done the Bolt question where you convert his speed of how it takes him 9.7 seconds to run a hundred metres. I've already worked with a copy of students to help them convert that into kilometres per hour. So they'll be able to use that information to help them.*

In the following lesson, after an unrelated introduction, Ms B did a 6 minutes review of the "how fast can you run" lesson. After discussing strategies for calculating their average speed in km/hr, the students then worked individually or in pairs for 5 minutes on possible methods for calculating the speed. There was limited success. Ms B led a discussion about Usain Bolt's speed with questions like "how many times do you have to multiply the 9.7 to get to an hour?" She wrote on the board $9.7 \times ? = 60$ leading to a procedural presentation of a solution. She then repeated this with some of the students' times —e.g., 21 seconds— modelling the procedure and then asking them to work on their own answer. They worked on this for 25 minutes.

Ms B's students were less successful than Ms A's on the assessment item, with only 6 (35%) of the 17 students in this class correctly answering the Lightning Bolt question on the test. None of Ms B's students chose either task as the one the liked, but 5 reported learning most from the running task and another 3 said they had learned most from the Usain Bolt task. They wrote comments like "really didn't know how before" and "how to convert from… seconds into kilometres per hour."

We infer that Ms B posed the task —how long would it take you to run a kilometre?— this way to make the calculation easier, but it did not do this. The task she implemented was the task she intended. The orientation of the teacher towards allowing students to make their own choices was evident in her posing the home based continuation to the first part of the lesson. In contrast she did not allow students to choose their own method of solution to the task. In class, Ms B's attempt to simplify the task and the direct and the formal way that she presented a solution method was both planned and perhaps limited by her lack of confidence with the mathematics underlying the task. In other words, she attempted to reduce the potential challenge for the students, having anticipated student difficulties, and this seems connected to her own lack of confidence with the task itself. As it happens, her attempt to make the problem simpler for the students actually made it more procedural and more complex. Her students did not do well on the assessment item, but nearly half felt they had learned something from the experience.

**Teacher C**

The lesson of Ms C, in short, was similar to that of Ms B, but different in three major ways. She showed a video of the actual race, she spent time in the introduction and conclusion on calculating time differences —which was irrelevant to the ratio aspect of the lesson—, and she drew skilfully on students' suggestions. The observer's noted:

> *(Ms C) invited that same student to explain his method to the class, said: "I got 100 m and divided by 9.71, which gets me how much metres you got in a second, and then I multiplied by 3600." Ms C asked where the*

*3,600 came from, and the student replied "60 seconds by 60 minutes," and gave the answer as 37075.18. Ms C then restated the method suggested by the student, followed by a discussion of the need to divide by 1000.*

Of the 25 students in this class, 12 (52%) correctly answered the Lightning Bolt question on the test. This success rate is in between those of the other two teachers. Five students reported that they most liked the running task and a further 2 said they learned most from it.

In summary, Ms C had the intention of being explicit about the method the students should use, but drew the method from a student, before restating this method. After the lesson, she was aware that the method she chose was complex, and expressed a view that she would describe a simpler method another time. Ms C intended to restrict the student choice of method, and so reduced the potential of the task, and this seemed directly connected to her own lack of confidence in the mathematics needed to solve the task.

## Summary and Conclusion

The first part of the task that was the basis of these lessons is complex because of its real world nature —had Bolt run 100 m in 10 s it would have been easier—, and the other part of the task (the running) was complex because of its openness and the student choice involved. Between a third and a half of the students in each of the three classes claimed to most like or most learn from one or other of the parts of the task. This is significant given that there were a number of interesting tasks from which they could have chosen. Ms A did not moderate the demands of the task, although she did not use the Usain Bolt part. Both Ms B and Ms C intended to present a particular method of solution, apparently motivated by their lack of confidence with how to do the task themselves. At this level, knowing a formal method for rate conversions is of limited value, and the real potential of the task is the opportunity for students to work out a method for themselves. The reduction in the potential of the task by teachers B and C was mainly in the restriction of the students' choices of the methods of solution.

The three lessons confirm the applicability of the Stein et al. (1996) model, which asserts that the classroom implementation of a task is influenced by the teacher's goals and subject-matter knowledge. In each case, what the teachers intended was what they did. Anticipating student difficulties, two of the teachers (B and C) moderated the demands of the task before the lesson, and each of these teachers was explicit in the method they expected the students to use. Ms A's students were more successful on the assessment item and, paradoxically, some of her students discovered the easier method of solving the task in class. This highlights the complexity of converting tasks to lessons in that some challenges are difficult to anticipate, and must be dealt with as they arise. In this case, the confidence of the teacher who allowed her students more freedom to explore was rewarded with more interesting responses from the students and apparently better learning. All three teachers were willing and able to draw on the student ideas and were prepared to spend time and energy to facilitate this. All three teachers had designed the learning unit with an emphasis on student choice of task, but the choice of method for this task was only part of the lesson of one of the teachers. To the extent that the potential of the task was reduced by two of the teachers, it can be attributed, in a similar way to the teacher studied by Charalambous (2008), to their lack of mathematical confidence in solving the task themselves, and not to any lack of familiarity with, or confidence in, student enquiry or problem-based methods.

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