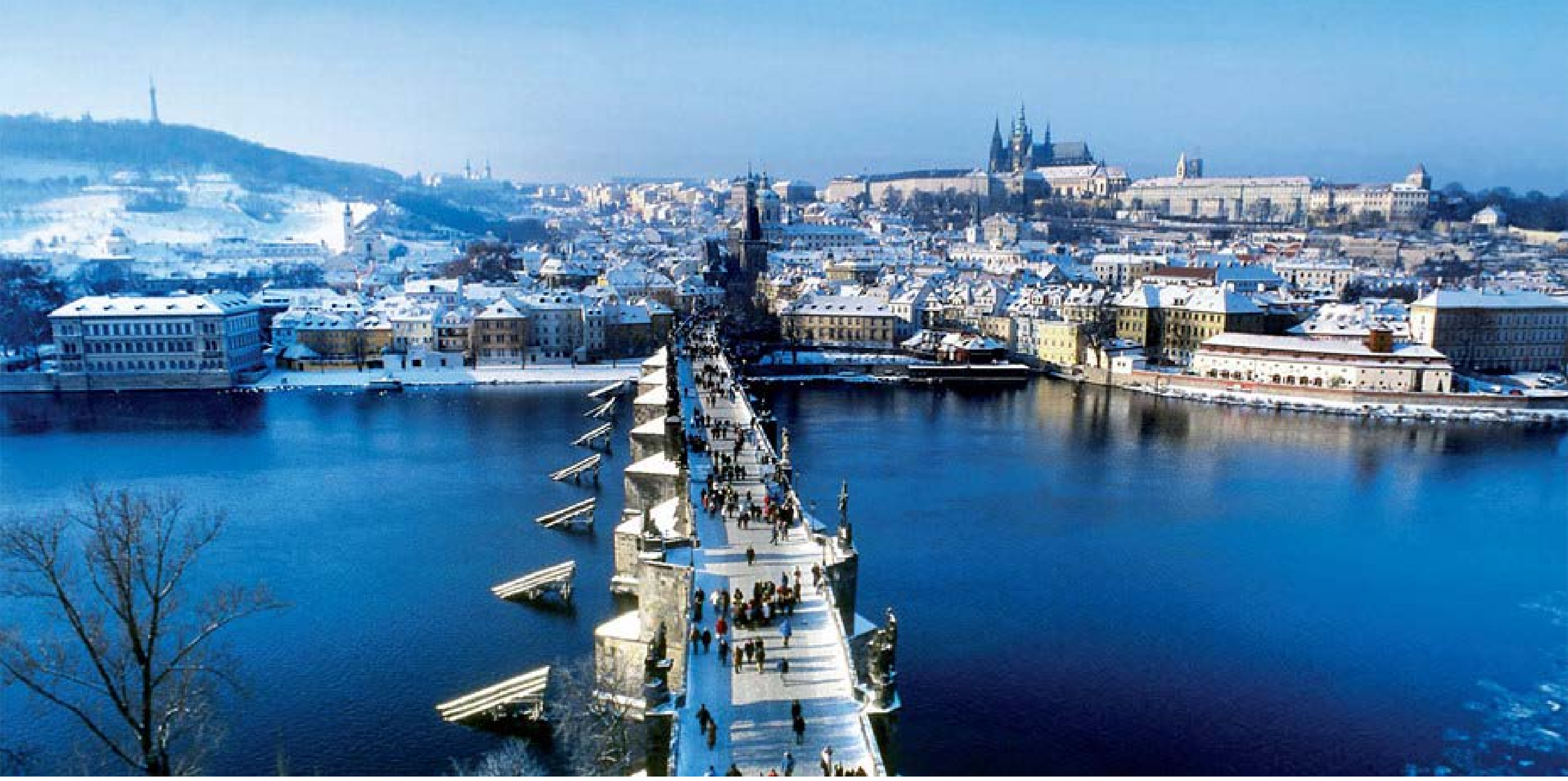

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Lessons we have (not) learned from past and current conceptualizations of mathematics teachers' knowledge

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This paper attempts to capture some of the breath of frameworks and models on mathematics teachers' knowledge in order to identify central lessons we have (not yet) learned from past and current approaches in theorizing and conceptualizing a knowledge base for teaching mathematics: there are accounts of the complex and multidimensional nature of teachers' knowledge but no accounts as to the reorganization of dimensions of teachers' knowledge in order to be more consistent with a constructivist view on learning and teaching; there are accounts of what teachers' knowledge is about but no accounts as to a structural description of teachers' knowledge. The paper highlights several unsettled issues of this research field and certain profitable directions for advancement.

Keywords: Teacher education/development, teacher knowledge.

MAPPING THE TERRAIN OF RESEARCH ON TEACHERS' KNOWLEDGE

With his influential construct of dimensions of teachers' knowledge in the 1980's, Lee S. Shulman (1986) at Stanford University has guided the research on teachers' knowledge in a new direction and, simultaneously, proposed an approach to educational reform that labelled teaching a *profession* (Shulman, 1987). Shulman (1986) promoted a *paradigm shift* in educational research by assuming the existence of a knowledge base that is special for the purposes of teaching. Since then, several interesting approaches, partly distinct and partly overlapping, have been developed that shape the current theoretical landscape in mathematics education research on teachers' knowledge. In the research literature, it is common to follow Shulman's (1987) conceptualization of a knowledge base for teaching including (1) content knowledge, (2) general pedagogical knowledge, (3) curriculum knowledge, (4) pedagogical content knowledge, (5) knowledge of learners, (6) knowledge of educational contexts, and (7) knowledge of educational ends, purposes, and values, and their philosophical and historical grounds. Several researchers have made attempts to identify features of mathematics teachers' knowledge that (may) matter in the work of teaching – in many ways, making Shulman's (1986, 1987) conceptualization of domains of teachers' knowledge, and, in particular, subject matter knowledge (SMK) and pedagogical content knowledge (PCK) specific to teaching mathematics.

The frameworks and models that shape the theoretical landscape in the conceptualization of and research on mathematics teachers' knowledge are within a broad spectrum of specificity, ranging along general, discipline-specific, domain-specific, and concept-specific frameworks and models. Various general frameworks contributed to the field, for instance, in (a) documenting teachers' resources (including knowledge), orientations (including beliefs), and goals as critically important determinants of what teachers do and why they do it (Schoenfeld, e.g., 2010), (b) highlighting that besides subject matter knowledge per se there is subject matter knowledge specific for teaching (Shulman, 1986, 1987), and (c) providing insights in teacher proficiency including the identification of various dimensions such as knowing students as thinkers and learners, reflecting on one's practice, among many others (Schoenfeld & Kilpatrick, 2008). Schoenfeld and Kilpatrick's (2008) contribution builds the bridge to discipline-specific frameworks since their work has been initially developed for identifying dimensions of mathematics teachers' proficiency but can and has been extended to a general (discipline-unspecific) framework.

A considerable number of research work is located in mathematics education research, providing both discipline-specific and domain-specific frameworks and models (e.g., Ball et al., 2008; Baumert et al., 2010; Blömeke et al., 2014; Fennema & Franke, 1992; Hill et al., 2008; Kilpatrick et al., 2006; Rowland et al., 2005; Tatto et al., 2008, 2012). These contributions, among others, are of interest in this paper since each contribution introduces and examines a particular approach in theorizing and conceptualizing the construct of mathematics teachers' knowledge. They are chosen because of their *complementary* power and their potential to study teachers' knowledge in a more comprehensive manner. Instead of reviewing each contribution in detail, the following section presents some central lessons we have (not yet) learned from these approaches.

Notice that, with few exceptions (e.g., Even, 1990), the mathematics education research community has almost neglected concept-specific frameworks. However, from the author's perspective, investigating teachers' knowledge at the level of specific concepts is an important issue that needs more attention in future research on teachers' knowledge.

THE RECENT DIVERSITY OF CONCEPTUALIZATIONS AS A RESOURCE FOR CURRENT AND FUTURE RESEARCH ATTEMPTS

The mathematics education research literature contains a broad range of approaches in theorizing and conceptualizing a knowledge base for teaching mathematics. The diversity of approaches is, of course, a reflection of the complexity of the research field that cannot be described, understood, or explained by only one theoretical framework. Different frameworks evolve for multiple purposes due to different needs in given contexts with different implications – some on a theoretical, methodological, and/or empirical level. The diversity of frameworks may provide a rich resource for future research attempts – the frameworks and models are important in their own right and may prove to be productive in some contexts.

The broad diversity of approaches starts with the versatile function of frameworks and models of teachers' knowledge: (a) as *tools* or (b) as *objects*. While most of the frameworks and models of teachers' knowledge are used as *tools* for guiding research practices, in particular for analysing data in empirical investigations, only a few function as an *object* of research – they are the aim of research practices. This distinction between 'tools for research' and 'objects of research' has already been made by Assude and colleagues (2008) with reference to theories in mathematics education. While the conceptualization by Ball and her colleagues (2008), for instance, can be understood as the result of an intensive 'job analysis', where 'conceptualizing a knowledge for teaching' was one of the goals, the conceptualizations by Blömeke and her colleagues (2014) and Tatto and her colleagues (2008, 2012) provide tools for empirical investigations in an international comparative, large-scale study. However, the distinction between frameworks as tools or as objects is rather inclusive (than exclusive) since the ways in which teachers' professional knowledge is understood and conceptualized impact on how teachers' knowledge is investigated, and vice versa.

THE KNOWLEDGE BASE FOR TEACHING MATHEMATICS IS COMPLEX AND MULTIDIMENSIONAL

The different approaches converge in an understanding that teachers' knowledge is complex and multidimensional. Although the discipline-specific models and frameworks mentioned above differ in detail, many of them converge in efforts to further *refine* the construct of subject matter knowledge (SMK) and pedagogical content knowledge (PCK). The following is an attempt to shed light on ways how Shulman's dimensions of SMK and PCK have been refined in the above mentioned contributions.

Subject Matter Knowledge (SMK)

The literature suggests that subject matter knowledge (SMK) can be further differentiated in terms of substantive and syntactic structures (Schwab, 1978), in terms of ways of understanding and ways of thinking (Harel, 2008), in terms of school mathematical knowledge and academic content knowledge (Bromme, 1994), among others. Each further distinction has shed light into important issues: Shulman (1986, 1987), for instance, emphasized Schwab's (1978) distinction between substantive and syntactic structures of a discipline. Substantive structures are the key principles, theories, and explanatory frameworks that guide inquiry in the discipline, while syntactic structures provide the procedures and mechanisms for the acquisition of knowledge, and include the canons of evidence and proof. As already noticed by Rowland and Turner (2008), the term 'syntactic' is mainly associated to the formal structure, thus, it seems that Schwab's (1978) choice of the word 'syntactic' is unfortunate since it does not capture the heart of the intended meaning that is, as argued by Rowland and Turner (2008), the heuristics of inquiry. However, Schwab's distinction has been an initial point to think about various dimensions of SMK.

In synthesis, it can be stated that several researchers have refocused on the centrality of SMK in teaching. However, crucial in the literature is the assumption that there is *unique* content knowledge for teaching mathematics and that having such knowledge is key to the enactment of rich mathematics. The notion of 'specialized content knowledge' introduced by Ball and her colleagues, described as pure content knowledge "that is tailored in particular for the specialized uses that come up in the work of teaching" (Hill et al., 2008, p. 436), is a key contribution in efforts to examine dimensions of mathematical knowledge considered as being crucial for the purposes of *teaching*. In contrast to the former refinements of SMK, the notion of 'specialized content knowledge' has the potential to go beyond just differentiating mathematical content knowledge in various (qualitatively different) sub-facets (such as to think about content knowledge in terms of procedural and conceptual knowledge, school mathematical and academic content knowledge, etc.). This 'specialized content knowledge' is not the kind that disciplinary experts would necessary possess. As a consequence, in contrast to Shulman (1986) treating 'subject matter knowledge *for* teaching' as equivalent to PCK, these considerations lead to the claim that there is pure mathematical knowledge specialized *for* teaching mathematics. Furthermore, it is argued that this kind of mathematical knowledge is not merely qualitatively but may be fundamentally different to SMK per se. This argument is rooted in the observation that SMK per se is primarily aimed at creating new knowledge, while SMK for teaching is essentially aimed at promoting students' mathematical thinking and learning. In this work, the former kind of knowledge is called *mathematical content knowledge per se* (MCK per se) and the latter kind of knowledge is called *mathematical content knowledge for teaching* (MCK for teaching).

Notice that 'mathematical content knowledge per se' is not equal to what Ball and her colleagues (e.g., 2008) described as 'common content knowledge' since it is not limited to the knowledge 'held or used by an average mathematically literate citizen' but may also include academic content knowledge, for instance. Mathematical content knowledge per se can be considered as not only including basic factual knowledge of mathematics but also the conceptual knowledge of structuring and organizing principles of mathematics as a discipline as described and operationalized in the TEDS-M framework (Tatto et al., 2008). Moreover, it can be described in terms of Kilpatrick, Blume, and Allen's (2006) *mathematical proficiency with content* including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition, and knowledge of structure and conventions, among others.

Pedagogical Content Knowledge (PCK)

Since Shulman's (1986) introduction of the construct of PCK, many researchers have added and further elaborated attributes and components of PCK. The above mentioned contributions provide various ways to refine the construct PCK, including, but not limited to, knowledge of cognitive requirements for learning, knowledge of students' conceptions, knowledge of epistemological obstacles of particular mathematical concepts, and knowledge of instructional strategies. Although even representing refinements of PCK, these subcategories are quite broad and often remain unspecified. Rowland and his colleagues' (e.g., 2005) work let to the identification of several subcategories that could be grouped into four units. Although their units are broad, the underlying subcategories provide some specificity. Another example is the work by Hill, Ball, and Schilling (2008) in making effort to conceptualize, develop, and test measures of teachers' knowledge of content and students (KCS). The same authors, although providing with KCS a subdivision of pedagogical content knowledge, state that even their subcategory is multidimensional (see Hill, Ball, & Schilling, 2008). Thus, various researchers assume that it is reasonable to further refine the various subcategories.

The subcategories of PCK identified in the above mentioned frameworks and models can be clustered into three dimensions, namely (1) an epistemological dimension, (2) a cognitive dimension, and (3) a didactical dimension. The *epistemological dimension* refers to knowledge about the epistemological foundations of mathematics and mathematics learning (see Bromme, 1994). For instance, Harel (e.g., 2008) calls for teachers' knowledge of epistemological issues involved in the learning of specific mathematical concepts, including knowledge of epistemological obstacles. The *cognitive dimension* refers to knowledge of students' cognitions (Fennema & Franke, 1992), in particular, knowledge of students' common conceptions (see Shulman, 1987), knowledge of students' cognitive difficulties involved in concept construction (Harel, 2008), and the interpretation of students' emerging thinking (Ball et al., 2008). In other words, it includes knowledge of how students think, learn, and acquire specific mathematical knowledge (Fennema & Franke, 1992). The *didactical dimension* refers to what Shulman (1986, p. 9) described as knowledge of "the most useful ways of representing and formulating the subject that make it comprehensible to others", including teachers' illustrations and alternative ways of representing concepts (and the awareness of the relative cognitive demands of different topics) (Rowland et al., 2005) and knowledge of the design of instruction (Ball et al., 2008).

In summary, it can be stated that the frameworks and models about teachers' knowledge mentioned above can be understood as elaborating rather than replacing Shulman's (1986; 1987) contribution within this field. The approaches taken and the conceptualizations of teachers' knowledge proposed are not inconsistent, nor are the identified dimensions of mathematics teachers' knowledge mutually exclusive. In contrast, the identified dimensions are *complementary* and provide, taken together, a more *refined* picture of conceptualizing the teachers' knowledge base.

The considerations proposed above demonstrate the multidimensional nature of mathematics teacher knowledge, in particular, the multidimensionality of SMK and PCK. Although the distinction between SMK and PCK is ambitious, several scholars take the view that the two categories, and, in particular, their corresponding subcategories, are useful tools in describing teachers' knowledge for research purposes and particularly in devising teachers' professional development programs.

MOVING AWAY FROM SHULMAN'S ORIGINAL CONCEPTUALIZATION

The above mentioned contributions present strong cases that progress can and has been made in the conceptualization of teachers' knowledge. As mentioned above, several scholars have particularly reformulated the concept of PCK, by refining sub-dimensions or identifying dimensions of teachers' knowledge and adding them to the construct of PCK. Thus, it can be seen that researchers have assimilated the notion of PCK and redefined it according to their beliefs or to findings from empirical studies. Although the mentioned studies represent reformulations of the concept of PCK, Shulman's conceptualization of PCK was still the theoretical starting point for these studies. In this process of further refinement and extension, however, researchers' understanding and interpretation of PCK have moved away from Shulman's original conceptualization. For instance, the concept of PCK has almost lost its most important characteristic, namely its topic specificity (Hashweh, 2005). PCK, according to Shulman's definition, is not only specifically related to topics within certain disciplines, but also research on PCK typically does not result in a description of 'expert teaching' as if there would be one optimal way to teach certain subject matter (see, Shulman, 1987). From the author's perspective, recent research on mathematics teachers' knowledge tend to ignore the complex nature of PCK as a form of teachers' professional knowledge that is highly topic, person, and situation specific (for overviews see, e.g., Abell, 2007; Van Driel & Berry, 2010).

A NARROW FOCUS ON THE DISCIPLINE

Many in the field of teacher education today take Shulman's conceptualization of the knowledge base for teaching for granted – accepting the view of pedagogical content knowledge (PCK) as an adaption of subject matter knowledge for the teaching enterprise, a process Shulman (1987) called *transformation*. However, with restricting PCK to the capacity to transform the subject matter of the discipline to subject matter of the school subject, Shulman places the subject matter content at the centre of conceptualizing the knowledge base for teaching. As a consequence, past and recent research on mathematics teachers' knowledge limited their focus on teachers' *unpacking of mathematics content* in ways accessible to their students. In doing so, the attention is focused entirely on the discipline. However, in being more consistent with a constructivist view of learning, the emphasis needs to be shifted from knowledge of the discipline to knowledge about how students' knowing and learning actually progresses. Thus, a reconceptualization of the knowledge base for teaching mathematics is needed toward a theory of teaching grounded in research on students' learning.

FROM REFINEMENT TO REORGANIZATION: TURNING THE REFINEMENTS ON THEIR HEADS

We have learned a great deal of the necessity for refining Shulman's initial work toward more specific descriptions of the knowledge base for teaching mathematics. Whereas it was important to initially identify and define various sub-dimensions of SMK and PCK and making progress in obtaining empirical evidence to support each piece of the puzzle, interpreting them in light of a model of cognition and learning certain subject matter may allow for the integration of the various pieces into one framework for mathematics teachers' knowledge. Thus, the time has come to move from further refining to reorganizing sub-dimensions of teachers' knowledge. As indicated above, the various refinements of PCK seem to converge in three domains, namely (1) *knowledge of students' understandings* (KSU), (2) *knowledge of learning mathematics* (KLM), and (3) *knowledge of teaching mathematics* (KTM). KSU refers to a cognitive perspective, KLM to an epistemological perspective, and KTM to a didactical perspective on this issue. In this work, knowledge of students' understanding (KSU), knowledge of learning mathematics (KLM), and knowledge of teaching mathematics (KTM), together with mathematical content knowledge per se (MCK per se) and mathematical content knowledge for teaching (MCK for teaching) *build* the knowledge bases that constitute the particular kind of knowledge that is considered as specialized for the purposes of teaching mathematics. In doing so, past and current approaches in research on mathematics teachers' knowledge are turned on their heads in the sense of taking the identified (and refined) knowledge dimensions as *building blocks* for the construct of 'knowledge for teaching mathematics'.

GOING BEYOND WHAT TEACHERS' KNOWLEDGE IS ABOUT: A WINDOW TO A STRUCTURAL DESCRIPTION OF TEACHERS' KNOWLEDGE

While the subcategories of mathematics teachers' knowledge identified in the above mentioned contributions are crucial pieces of the puzzle, we have not learned how these pieces fit together. In the past, the primarily focus was on what knowledge is held by teachers, and how that knowledge is used in practice. It seems that, with few exceptions, the literature has limited its focus on the content teachers do or should possess. However, a key theoretical concern arising in the realm of theorizing and conceptualizing mathematics teachers' knowledge is the question on how the knowledge is structured and organized. To put it in other words, what is missing in the current landscape of the conceptualization of mathematics teachers' knowledge are attempts to go beyond what the teachers' knowledge is about to include a *structural description* of teachers' professional knowledge. Drawing on the 'knowledge in pieces' framework developed by diSessa (e.g., 1993), Scheiner (2014) proposes to consider teachers' professional knowledge as a complex system of 'knowledge atoms'.

> 'Knowledge for teaching mathematics' is considered as the repertoire of 'knowledge atoms' that have been transformed along (1) knowledge of students' mathematical understandings (KSU), (2) knowledge of learning mathematics (KLM), and (3) knowledge of teaching mathematics (KTM), taking (4) mathematical content knowledge per se (MCK per se) and (5) mathematical content knowledge for teaching (MCK for teaching) as the cornerstones. (Scheiner, 2014, in press)

With this perspective, several angles for theoretical reflection on the *nature* and *form* of teachers' knowledge are presented, including those concerning the degree of integration, size, specificity, and source of teachers' knowledge. The notion of 'transformation', for instance, indicates that the constituent knowledge bases are inextricably combined into a new form of knowledge that is more powerful than the sum of its parts (concerning *degree of integration*), while the notion of 'knowledge atom' indicates that knowledge is of a microstructure, highly context-sensitive, and concept-specific and has to be considered as of a fine-grained size (concerning *size* and *specificity*). Notice that in contrast to Shulman and his proponents' work taking content knowledge and pedagogical knowledge as the constituent knowledge bases for teaching, it is KSU, KLM, and KTM, together with MCK per se and MCK for teaching that build the constituent knowledge bases for teaching mathematics (concerning *source*). A more detailed elaboration of first attempts towards a structural description of teachers' knowledge can be found in Scheiner (in press).

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