# Generating a task design and implementation framework for mathematical modelling tasks through researcher-teacher collaboration

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**Abstract** How to support student in applying the mathematical modelling (MM) process is an ongoing line of research enquiry. This chapter outlines interim findings from an Australian national project that aims to promote effective teaching and learning practices in MM through attention to implemented anticipation. This effort gained focus through attention to the generation of a Design and Implementation Framework for Modelling Tasks (DIFMT). The DIFMT was the result of collaboration between teachers and researchers aimed at the effective design and implementation of MM tasks in upper secondary classrooms. The study suggests that specific pedagogical practices can act as enablers of students' attempts to appropriate the process of MM.

## 1 Introduction

In keeping with a number of countries, Australia has been stressing the importance of equipping students to apply their mathematics in real-world settings (e.g., ACARA 2015). Such abilities are necessary for (1) successful participation in other school subjects where the use or interpretation of models is important; (2) gaining access to mathematics, science, technology and engineering (STEM) careers or other professions based on applied mathematics (e.g., economics); and (3) for informed participation in personal, civic and work life. In this chapter we outline our efforts to address these needs within a curriculum context in which mathematical modelling (MM) is a mandatory element of mathematics assessment within the final years of schooling (Years 11 and 12). Despite the requirement that Years 11 and 12 students engage with MM, experience and expertise in instruction for this element of the curriculum is varied – from very capable designers of MM tasks through to novices. Even among those that were capable task designers, we found a dearth of expertise in the implementation of MM activities. Our response to this theory/practice gap has been to work in collaboration with teachers to develop effective principles for instruction embedded in a Design and Implementation Framework for Modelling Tasks (DIFMT) within a nationally funded project. Central to the development of this framework was an understanding that the capacity to anticipate, is an essential meta-cognitive facility in both the deployment of the modelling process by students and teachers' capability with its instruction. Consequently, the aims of the project are to:

i) describe the nature of anticipatory metacognition and identify and describe the enablers necessary for students to translate real-world situations into successful mathematical models;
ii) design modelling tasks that support the development of students' anticipatory metacognition, and/or allow for the identification of issues that are problematic for that development;
iii) develop, trial, and refine teaching practices that support the growth of students' anticipatory metacognition while working on effective modelling tasks.

In the section which follow, we focus on the theoretical perspectives that underpin the DIFMT and describe other enablers of MM which emerged when teachers attempted to align their instructional practices with this framework. Evidence for the efficacy of these enablers are drawn from teachers' commentaries on their implementation of tasks.

## 2 The nature of mathematical modelling

Given the plethora of interpretations within the field of modelling in education we provide clarification of our meaning of the term. Consistent with statements in the opening paragraph, we are concerned to nurture qualities that enable students to apply mathematics to solve problems in domains outside itself (see Niss et al. 2007, p. 4). In the following we outline sequential stages in the modelling process; as an analytical reconstruction of a modelling/problem-solving process, remembering it is neither a lock/step approach, nor a detailing of moves made by individual modellers. In the diagrammatic representation below (Fig. 1a), the heavy clockwise arrows (1 to 7) depict the modelling process as a problem-solving activity, connecting stages (A to G). The double headed arrows indicate that in pursuing a solution there will be intermediate transitioning/revisiting, within and between any of the stages. This will include metacognitive and anticipatory activity. (These arrows are incomplete for clarity – they potentially connect any of the stages).

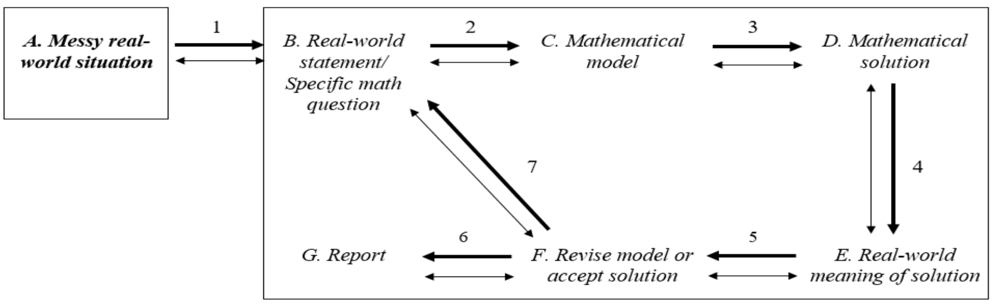


**Fig. 1a** Representation of the modelling cycle (Galbraith 2013)

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| 1. *Identify a (real-world) problem*<br>2. *Specify related mathematical question(s)*<br>3. *Formulate a mathematical model to address the question (involves making assumptions, choosing variables, estimating magnitudes of inputs etc.)*<br>4. *Solve the mathematics* | 5. *Interpret the mathematical results in terms of their real-world meanings*<br>6. *Make a judgment as to the adequacy of the solution to the original problem(s)*<br>7. *Either report success with recommendations, or make adjustments and try for a better solution* |

**Fig. 1b** Transitions between modelling phases (Galbraith 2013)

## 3 Anticipatory metacognition

*Implemented anticipation*, as formulated by Niss (2010), is a process by which students anticipate and carry out within the act of modelling: (a) actions that they perceive as potentially useful context-wise and mathematically in subsequent steps; and (b) decision making that bring those steps to fruition. *Implemented anticipation* is central to a modeller's ability to mathematise and to undertake the mathematical processes entailed, and then complete a modelling problem successfully.

The term *anticipatory metacognition* describes an associated construct that also includes the additional capabilities of 'modelling oriented noticing' and strategic planning, e.g. with regard to seeking and gathering information and data and deciding whether to involve statistical analyses of the data collected. This applies before and during a modelling experience. It represents the capacity to recognise possible avenues to pursue during the modelling process when engaging with an unstructured real-world problem by taking cues from progress made in other contexts and situations. Both require an ability to think forward and are applicable to learners and teachers.

For teachers it represents thinking along the lines "Where, in the modelling process, will this group of students be likely to encounter obstacles? And what can/should I do to help them move forward?" It involves reflecting on student thinking as intermediary to the problem itself. Resulting prompts direct students to use the modelling process to resolve an impasse, rather than giving direct hints as to the solution itself.

## 4 Anticipation and modelling

Because modelling proceeds through ideal-typical stages, an attribute for success is the ability to look forward and to anticipate what may be needed at a later point in the process, requiring that the modellers project themselves into subsequent modelling steps before taking them; and implement such anticipation throughout the modelling process (Niss 2010; Niss, Martin 2017; Jankvist and Niss 2019).

*Implemented anticipation* as an essential component of *anticipatory metacognition* pertains to all necessary steps in the modelling process: pre-mathematisation (e.g., posing questions, assumptions, simplifications), mathematisation, mathematical treatment, interpretation, and model evaluation. This capability is significant for individual modellers, but also for teachers and mentors, who seek to promote the development of modelling abilities in their students. Examples are listed below:

- Anticipating features that are essential in mathematising a feasible problem from the real situation being currently considered; anticipating mathematical representations and mathematical questions that, from previous experience, or present analysis, seem likely to be effective when forming a mathematical model
- Thinking forward about the utility of the selected mathematisation and the resulting model to provide a mathematical solution to the questions posed.
- Thinking forward to identify related problems and refinements that are suggested by progress. Some of these may not have been thought of at the outset of the problem.

## 5 Enablers of implemented anticipation

Enablers of implemented anticipation, developed previously (Niss 2010), were directed specifically at features central to developing individual modeller capabilities. Their European origins paid attention to contexts where the worth of modelling could not be taken for granted, for example, where only pure mathematics is considered an approved subject for study by the education system, or by students. Australia has a history within which applied mathematics has occupied an accepted role. However, ways in which respective preferences (e.g., pure versus applied) impact on teaching and learning remain a continuing influence. In theoretical terms these are impacted by considerations of socio-mathematical norms (e.g., Yackel and Cobb 1996) and didactical contracts (e.g., Brousseau 2002). Bearing in mind the Australian context, adaptations of Niss' original modelling enablers (ME) have been developed and an additional enabler, to do with knowledge of the modelling process, has been added to the original set of modelling enablers – ME3 (Table 1).

**Table 2** Niss' enablers adapted for Australian contexts

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| **ME1**: (adapted for Australia): Students believe that the inclusion of modelling activities is a valid component of mathematical coursework and assessment |
| **ME2**: Students possess mathematical knowledge able to support modelling activities (e.g., possess mathematical knowledge and skills, and ability to manage abstraction) |
| **ME3**: (additional): Students possess an understanding of a systematic modelling process that includes successive stages from problem question to model evaluation |
| **ME4**: Students are capable of using their mathematical knowledge when modelling. (This implies a core understanding of and engagement with the modelling process (Formulate, Solve, Interpret, Evaluate) so that the right questions can be asked and pursued systematically) |
| **ME5**: Students have perseverance and confidence in their mathematical capabilities (e.g., continue to follow through, or try new directions within a problem if necessary) |

**Table 1** Implementation enablers

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| **IE1**: The mathematical demand of problem tasks does not exceed the mathematical capabilities of the student group. |
| **IE2**: Problem tasks are introduced so as to engage the students fully with the task context, while ensuring that goal of the task is understood. |
| **IE3**: Assistance provided during modelling sessions (measured responsiveness) is geared to helping students use the modelling process to reach a solution, rather than treat a problem as an individual exercise. |
| **IE4**: Students are encouraged/required to organise and report their work using headings/sections consistent with the modelling process. |
| **IE5**: Productive forms of collaborative activity are used to enhance and hold to account the quality of on-task progress. Effective use of digital technologies. Students' interest in a problem. |

In terms of the project, the centrality of effective implementation means that *teaching (or implementation) enablers* (identification and description) have been added to the originals that were directed at enhancing the modelling process itself. See Table 2. In reviewing the developing enablers framework, after initial classroom observations, we became aware of factors, that while not exercising a gatekeeping role, could facilitate (or not) the success of modelling activities. We have designated them Catalytic Enablers (IE5).

## 6 Approach to developing the DIFMT

The project has been conducted over a three year-period. Data for this chapter are drawn from the engagement of three teachers from different schools and one class of their students per year (Years 9-11). The project coincided with a time of curriculum revision which included new course content and greater scrutiny of assessment practices, including a component devoted to MM. Two of the teachers had extensive prior experience in developing and implementing modelling tasks, while the third had only superficial familiarity.

The research design was based on an iterative process of design-implement-reflect as the basis for researcher/teacher collaboration in developing the DIFMT. This process was effected through three whole-day researcher/teacher meetings and two classroom observation visits per year. Classroom visits took place between researcher/teacher meetings. The purpose of researcher/teacher meetings was to: develop MM tasks; plan for their implementation in classroom; reflect upon the design of tasks and their implementation after each successive round of implementation; draft and refine the DIFMT. Classroom observation visits were conducted to generate data related to the effectiveness of: tasks, for specific classroom conditions; and teachers' approaches to task implementation. Initial tasks and advice on implementation was provided by researchers, with teachers becoming increasingly involved, moving toward autonomy, in the development of principles for the design of tasks and their implementation – leading to the drafting and successive refinement of the DIFMT as the project unfolded (for detail of this approach see Geiger et al. (2018)).

Data collection methods included video-recorded classroom observations of small groups of students during observation visits, teacher pre- and post-lesson interviews, student post-lesson interviews and student video-stimulated recall sessions following each visit. Students who were likely to articulate their approaches to a task clearly and without a sense of reserve were invited to participate in both video and interview sessions on the basis of teacher advice.

## 7 The DIFMT

In this section we provide an outline of the DIFMT. Word limit prevents a full discussion of its development; thus, the purpose of the following description is to provide the reader with sufficient background to link the DIFMT to implementation enablers for which we provided illustrative excerpts.

The DIFMT consists of three overarching structural dimensions – *Principles for modelling task design*, *Pedagogical architecture*, and *Completion* under which sit defining elements and their descriptions. While this chapter focuses on the Pedagogical Architecture dimension of the framework, a condensed version of the whole is presented in Table 3.

The dimensions and defining elements of the DIFMT are aligned with the *implementation enablers*. For example, IE1, which relates to the articulation of students' mathematical capabilities and the embedded challenge within a problem, is an important element of *task design*. The students' introduction to a problem (IE2) requires careful attention during the *pre-engagement/initial problem presentation* phase. The type of assistance students should receive when engaged with a problem (IE3) is captured in the *body of the lesson* descriptors. Responses to a problem will need to be reported in a structured manner (IE4), as outlined in the *completion* element of the DIFMT. Productive collaboration (IE5) is seen as a catalytic enabler and is also included in the *body of the lesson* descriptors.

**Table 3** Integrated Modelling Task and Pedagogy Framework

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| *Principles for modelling task design* | | |
| Nature of problem | Problems must be open-ended and involve both intra- and extra-mathematical information | |
| Relevance and motivation | There is some genuine link with the real world of the students | |
| Accessibility | It is possible to identify and specify mathematically tractable questions from a general problem statement | |
| Feasibility of approach | Formulation of a solution process is feasible, involving (a) the use of mathematics available to students, (b) the making of necessary assumptions, and (c) the assembly of necessary data | |
| Feasibility of outcome | Solution of the mathematics for a basic problem is possible for the students, together with interpretation | |
| Didactical flexibility | The problem may be structured into sequential questions that retain the integrity of the real situation | |
| *Pedagogical architecture* | | |
| Pre-engagement | Understand of the modelling process and its application - illustrate what the modelling process. Support materials include a modelling process diagram. | |
| Modelling process review | Reviewing pre-engagement as required | |
| Initial problem presentation | • Teacher provides brief general description of the problem scenario<br>• Students organised into small groups and provided with time to read the task description and ask questions of clarification<br>• Students in groups discuss how to approach the problem (including defining a mathematical question?) and report back to whole class via a group representative<br>• Teacher orchestrates discussion of mathematical question(s) towards consensus<br>• Students in groups consider assumptions and variables relevant to the agreed mathematical question. Outcomes reported back to whole class by a group representative<br>• Teacher synthesises/prioritises students' initial assumptions and variables sufficient to begin modelling process for an initial model (As students gain experience teacher scaffolding in this section can be greatly reduced and perhaps eliminated). | |
| Body of Lesson | Students<br>• Proceed in groups to create model, solve, interpret, etc. in terms of their mathematical question.<br>• Engage in productive student-student collaboration.<br>• Identify and make use | Teachers<br>• Help bring to student consciousness those things that are implicit<br>• Activate teacher meta-meta cognition: (a) How will the students be interpreting what I as a teacher am doing/saying at this point? (b) What should the students be asking themselves at this point in the modelling process?<br>• Structure mathematical questions that promote a viable solution pathway |

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| | of technology where applicable (e.g., source relevant information, check calculations and/or generate solutions)<br>• Develop a report of their progress in terms of the stages of the modelling process (e.g., formulate, solve, interpret, evaluate) | • Support students with making progress through the modelling process<br>• Anticipate where students might have problems, e.g., interpreting the problem<br>• Employ measured responsiveness – rather than providing specific advice about the problem, students should be prompted to think about where they are in the modelling process<br>• Encourage the use of tools (digital or other)<br>• Support student progressive development of a report (e.g., guidelines on report writing). |
| *Completion* | | |
| Present findings and summary | • A representative from each group shares their findings with justification. Findings should be reported in a succinct fashion (e.g., 3-4 minute video)<br>• Teachers/students ask questions of clarification or to test arguments. | |
| Report | • Students communicate their findings via a succinct, coherent, systematic report. The report must make use of appropriate mathematical language.<br>• Teacher checks for the validity of the solution and supporting justification | |

## 8 Emergent enablers

During task implementation, using the DIFMT as a guide, other aspects that promoted or constrained students' attempts at 'modelling problems' emerged. These included: actions related to teachers' personal engagement with a modelling task and its implementation; influences upon the teaching/learning environment (e.g., socio-mathematical norms and/or aspects of the didactical contract); and teachers' own anticipatory actions. We now present illustrative examples of such emergent enablers – supported through references to teachers' comments recorded during interviews that followed task implementation sessions.

### *8.1 Core teaching enabler: Utilising the modelling process*

It became apparent that teachers' thorough understanding of both the modelling process and the detail of any modelling problems they implemented was fundamental to their students' success in modelling. Teacher A was adamant that the modelling process must be understood by teachers themselves if instruction was to be effective.

Teacher A: [Teachers need to] go through the framework. Not just the problem but the process itself.

Teacher B comments on the importance they placed on developing a thorough personal understanding of a problem before implementing it in their classroom.

Teacher B: It was actually quite challenging for me to figure out exactly what I would do. I spent a fair bit of time researching.

### *8.2 Learning/Teaching Environment*:

The degree to which teachers took advantage of opportunities to engage their students with modelling tasks was influenced by their perception of factors that shaped classroom socio-mathematical mathematical norms and/or the didactical contract. For example, teachers perceived both opportunities and constraints related to their state-wide curriculum context. This perception inhibited or provided encouragement for how often they were prepared to implement tasks. Comments by Teacher C indicate he saw the demands of a new syllabus as limiting his opportunity to engage students with modelling activities because of expectations about developing student mastery of content objectives in a limited period of time. This was despite a strong emphasis in the syllabus on mathematical modelling.

Teacher C: We don't do [modelling] as much as we used to…because we just don't have time. The new syllabuses just don't allow that sort of stuff.

Teacher B, working within the same curriculum context, saw no such impediment.

Teacher B: I think it's a good task for Year Ten because we do all that volume and money exchange too, there's a little bit of that... It's good for Methods [Year 11] and General Maths [Year 11].

These differing commentaries on opportunities to implement modelling tasks point to in-school expectations about which aspects of mathematics should be prioritised – in this case, fluency with mathematical techniques versus open-ended mathematical learning experiences in the form of modelling tasks. How the influences of curriculum requirements are perceived can become manifest as school specific socio-mathematical norms and the didactical contract that, in turn, trickle down to student expectations of what should take place during mathematics instruction – their interpretation of the didactical contract. Thus, such influences can act as enablers or dis-enablers of student opportunity to engage with modelling tasks. Another interesting observation was that some of the teachers tended to scaffold students' work rather tightly by teaching them what to do and how to do it, thus extending traditional mathematics teacher behaviour to contexts where this is likely to impede students' independent modelling work – thus another potential dis-enabler.

## *8.3 Teacher anticipatory capability*

Also emergent from classroom observations was the importance of teachers’ own anticipatory capabilities as these related to looking forward into a lesson to where students might experience difficulties or blockages. This form of anticipation enabled teachers to plan for how to scaffold students’ modelling efforts in a measured but effective fashion. For example, Teacher A anticipated that some students might find challenge in the selection of essential information from a larger list.

Teacher A: It will be interesting to see if they can pick out that information from the table that's there. I think that will be a stumbling point for some of them …And they might be seeking a little bit of clarification there.

Teacher A did not see this challenge as a negative experience for students but rather an enabler of their development as modellers provided adequate support was in place – thus reinforcing the important role of their own anticipatory capability.

Teacher A: I think that students need a bit of struggle and challenge…but with bringing them back together and just getting that clarification before we go on, I think then they'll be right, and they'll run with it.

## 9 Conclusion

This chapter reports on interim findings from a national project, conducted in Australia, that aims to promote the effectiveness of both teaching and learning in mathematical modelling through a focus on teachers' and students' anticipatory capabilities. Both teacher and student practices, as syntheses of previous scholarly work or observed during initial implementation phases of the project, are represented in the form of the DIFMT – developed in an iterative fashion as a collaboration between teachers and researchers. Identifying other enablers or dis-enablers of students' opportunities to learn to model is ongoing. These include factors such as teachers' preparatory practices before engaging students with modelling, socio-mathematical norms and the didactical contract, and the development of teachers' own anticipatory capabilities. Our future work, within this study, will continue to focus on the identification of enabling factors, related to both students and teachers, that promote or inhibit students' efforts to employ mathematical modelling effectively when solving real-world problems and in particular those that impact on the pre-mathematisation and mathematisation phases of the modelling process.

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