Problem Solving, Thinking and Group Work in Mathematics: Developing an effective pedagogy

Gary R Thomas

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DECLARATION

This thesis contains no material published elsewhere or extracted in whole or in part from a thesis by which I have qualified for or been awarded another degree or diploma.

No parts of this thesis have been submitted towards the award of any other degree or diploma in any other tertiary institution.

No other person's work has been used without due acknowledgment in the main text of the thesis.

All research procedures reported in the thesis received the approval of the relevant Ethics/Safety Committees (where required).

Gary Raymond Thomas
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ABSTRACT

This research investigates the use of heuristics and thinking routines while problem solving in mathematics in a collaborative setting of small groups. The educational setting and context of this study is a year 2 classroom in a Victorian school. The study began in response to first; that a thinking classroom is an active, reflective and learning environment that promotes ideas and rich thoughts. Ron Ritchhart is his book, Intellectual Character (2002) discusses intelligence, being smart and developing intellectual character, secondly; that mathematical problem solving requires students to have a starting point. Polya's model of understand the problem, make a plan, carry out the plan and look back (Polya, 1957) does suggest a starting point, finally; through the idea of teamwork, cooperation and sharing of ideas and strategies this allows for learning to occur in small groups. Placed in this environment, the goal of the research became an investigation into how Year 2 primary students collaborate to use heuristics and thinking routines to complete mathematical problem solving tasks.

A multiple method, interpretive, case study methodology was used to explore the components of problem solving in a primary classroom. The research was conducted over four sessions that examined the use of the thinking routine *See Think Wonder*, the employment of appropriate heuristics and the collaboration within a small group setting while the students completed open-ended mathematical tasks. The students were audio taped and completed an initial concept map using the thinking routine *See Think Wonder* to assist them in planning their solution. Students completed a solutions page showing answers and working out. Four students were selected to participate a in one on one interview to assist in eliciting information on the use of heuristics, group learning and the use of the thinking routine *See Think Wonder*.

The findings were: (1) students were able to successfully use the *See* part of the thinking routine to assist them in solving the task; (2) students found the *Think* aspect of the thinking routine more challenging than the *See* part; (3) students were able to list some

Wonders for the mathematical task; (4) students were able to use a variety of heuristics assist them to solve the open-ended mathematical tasks; (5) a variety of categories were identified to illustrate group learning; (6) students were able to successfully provide solutions to all mathematical tasks.

Chapter One

Background and Rationale

1.1 Introduction

Any investigation can emerge out of the desire to find a solution to a problem, or curiosity about a situation that you believe warrants an exploration. In any case it is the nature of the problem or curiosity and the context in which it occurs in conjunction with the values and beliefs of the researcher that determine the nature of the research (Green & Browne, 2005). In education we have seen an untold number of changes in recent times. Schools have implemented; the whole–part-whole approach, problem solving approach, small group learning, the use of mathematical games, the implementation of information and communication technology (ICT) while differentiating the curriculum for all students. These changes have been guided through the Curriculum Standards Framework (CSF), to the Victorian Essential Learning (VELS) and the Australian Curriculum (AUSVELS) which all have a student centered, outcomes-based approach to learning and teaching.

A thinking classroom is an active, reflective and learning environment that promotes ideas and rich thoughts. Teachers should be teaching for understanding not just teaching skills that can be rote learnt. Understanding key concepts and exploring ideas to arrive at well informed answers should be the cornerstone and guiding fundamental principles of any classroom. The motivation for the research presented in this thesis came from asking how can collaborative learning, thinking routines and problem solving be explored in the mathematical curriculum?

How do thinking dispositions and thinking routines help in developing intellectual character and therefore a thinking culture within the classroom? How do the thinking dispositions and routines assist in understanding when learning? How does the use of problem solving heuristics assist in the solving of mathematical tasks? The significance of teaching thinking is important, as students place in context the understanding and knowledge they are gaining while learning. The importance of teaching thinking and valuing thinking in schools is paramount in creating students who years later, when confronted with an issue, use the same thinking dispositions to create a workable solution. Likewise with the problem solving heuristics, the objective for students is to employ the heuristics when attempting to solve unknown tasks. Thinking should not be taught in isolation and as an independent enterprise (Tishman, 1995). The following study allows for educators to gain insight into students' thinking, as they are confronted with various mathematical problems to contend with and attempt to solve.

With my interest in mathematics education, I was discovering that there was more to be done than just covering the curriculum. I wanted to be part of an educational movement where educational practice did change and reflect our time. With the increased use of computer technology within mathematics students were no longer just using paper, pen and concrete materials. Did this affect the amount of thinking happening? Was the thinking of a higher order: of analysis and synthesis? Was the thinking being shared with the whole class or was it still very much an individual pursuit? These questions led me to another broader question. Could I create a thinking culture within my classroom that is habitual and one of collegiality?

Therefore I felt the impetus to conduct my own research. I was in the fortunate position of being a full time teacher at an Independent School on the Mornington Peninsula in Victoria. The researcher's role as a Year 2 classroom teacher focused on both the academic endeavors' and the pastoral care of 8-year-old students.

1.2 Background to the Study

As the world we live in becomes more complex, schools are required to be places that will provide opportunities for their students to consider original ideas, be open minded to many perspectives, be strategic in planning and solving problems, be able to show their thinking in visible ways and to learn in collaborative groups. When reflecting on the daily pressures of the classroom, of teaching twenty-five students of varying ability, peer group and friendship issues and differing emotional maturity along with completing the curriculum, the quote from Ritchhart (2002) struck a chord. Ritchhart stated "When one considers the current emphasis on high-stakes testing and accountability, a more apt description of the mission of schools might be this: to promote the short-term retention of discrete and arcane bits of knowledge and skills" (p. 9). However when I entered the teaching profession seventeen years ago, fresh out of university, this teacher-researcher was under a different impression. I believed like others before me that thinking would be at the forefront of education. Thinkers such as the 17th Century philosopher Rene Descartes influenced me, when he stated 'I think, therefore I am', and David Perkins from Harvard University who stated, "Learning is a consequence of thinking" (Perkins, 1992, p. 8). Within the proficiency strand of *problem solving* in the mathematics curriculum by the Australian Curriculum, Assessment and Reporting Authority [ACARA] it was stated

> Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable. (2014)

It is in this higher level of thinking that formulating, interpreting, verifying, seeking answers and communicating that allows students to become proficient in the art of mathematics.

1.2.1 Positioning of Mathematics

Mathematics inhabits an important position in the program of all primary schools across Victoria and Australia. All classrooms are required to include the teaching of mathematics during each term and learning mathematics in the primary school setting is a precursor to students completing compulsory and elective mathematics courses during their secondary school education. Such common community notions are reflected in government policy. The Australian government, in the National Numeracy Review Report tells the electorate that mathematics is important as students need to learn when to apply their knowledge and interpret information to solve practical problems, when to use mathematical reasoning, to make assumptions and decide what is reasonable (Human Capital Working Group, 2008). The Department of Education in Victoria (2013) stated that students can be numerate in the Arts through exploring shape and scale, in English through interpreting information, in Economics through exploring budgets and finances and Geography through mapping, investigating populations and evaluating natural events.

There are three stated aims of the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2013): first, students are "confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens"; secondly, students "develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in *Number and Algebra, Measurement and Geometry, and Statistics and Probability*"; and finally, students "recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study". As can be noted from the above aims, problem solving within the primary mathematics classroom is seen as vitally important. Within the context of mathematical problem solving, thinking is an important aspect, which needs to be modeled, discussed and allowed to flourish. It gives the students the opportunity to engage, to investigate, to interpret and to gain an understanding of mathematical concepts.

Furthermore an exploration of the ACARA mathematics pages provides insight into the standards and content descriptors students at Year 2 should be progressing to and achieving by the end of the year. On the Year 2 mathematics curriculum page ACARA (2014) stated, "*Problem Solving* includes formulating problems from authentic situations, making models and using number sentences that represent problem situations". Within the content descriptors that assist teachers in formulating and organizing their teaching and sequencing of lessons, it is stated that students "solve problems by using number sentences for addition or subtraction" (ACARA, 2014). This study provided the opportunity for students to use addition and subtraction but also multiplication, division and investigating patterns.

Students are taught, the four operations; addition, subtraction, multiplication and division, how to measure and calculate time, volume, capacity, mass, area and length, how to recognize two-dimensional and three-dimensional shapes and how to read maps using compass points, grid references and coordinates. Many teachers get 'bogged down' in the curriculum content attempting to 'cover everything' while running out of time due to all the outside administrative demands and pressures exerted on the classroom teacher. Due to such time constraints and the need to complete the curriculum many teachers feel unable to delve deeper for understanding. Mathematics is about understanding. Without understanding the students are regurgitating and memorizing procedures, facts and skills.

1.2.2 My Introduction to Thinking

Within such a milieu, I still wanted my students to feel like Polya, a mathematician who is looked upon as the father of mathematics problem solving (Polya, 1957) and to be able to explain their problem solving strategies to others. I did not want to have a classroom in which I was 'teaching to the test'. Boaler (2002) exclaimed that in many homes, government departments of education, and schools in Britain and the US, the ultimate success is test success; that is students are able to produce through memory many unrelated procedures, skills and facts under test conditions. The concern for Boaler (2002) was students who fail assessment tasks composed in part at least of problems that require them to know when to use the different procedures, and sometimes to develop procedures of their own. So a tension is created between memory and understanding. Tishman, Perkins and Jay stated that "most efforts to teach thinking aim at cultivating thinking skills" (1995, p. 38) but in many classrooms students often are unable to put these into practice. Tishman, et al. (1995), further suggested, "One way is through an approach to teaching thinking that stresses the dispositional as well as the abilities side of thinking" (p. 38). Such an approach led me to the work of Ron Ritchhart from the Harvard Graduate School of Education.

Ritchhart (2002) discussed intelligence, being smart and developing intellectual character. He defined *intellectual character* as "an umbrella term to cover those dispositions associated with good and productive thinking" (p. 18). Tishman agreed with Ritchhart and stated, "a thinking disposition is a tendency, or leaning, towards a particular pattern of intellectual behavior" (Tishman, 1995, p. 2). For Harpaz (2007) it was about imparting good thinking skills and not large bodies of knowledge to students. He suggested that it is the recognition that people are not thinking as well as they could, and hence this becomes the foundation as to why we need to teach thinking (Harpaz, 2007).

Earlier Perkins (1992) suggested that schools learning processes should be thinking centred, that is putting the thinking as central within the whole curriculum. Others also claimed for this approach. Lazakidou, Paraskeva and Retalis, (2007) claimed that a priority for contemporary education systems is to provide a climate where independent and productive learners can grow. Students need to be given the opportunity to learn strategies and to develop thinking dispositions so that when they are confronted by unknown problems they are capable of solving the problem. In doing so, students years later when confronted with a novel issue, will be able to use the same thinking dispositions and routines to create a workable solution.

1.2.3 Moving to Mathematics

Within schooling, mathematics is a core element, and hence the general argument to promote thinking in schooling applies just as much to mathematics as to any other subject area. We have come to understand that mathematics is not just about teaching skills but it is essential to teach for understanding. That is not to suggest these are in opposition to each other and one should choose one and not the other. Both are important: "Such mathematical problems should enable the accessing of the deeper strategies, as well as the specific content" (Clarkson, 2008, p. 28). But such an approach is not always evident in Australian classrooms. Gunningham (2003) was dismayed when in discussions with teachers; she noted that getting through the textbook curriculum was hard when skills were poor. In reading this, two aspects immediately struck a chord with me: textbook curriculum and skills. Skills can be rote learnt with little or no understanding and therefore transferring that skill into unknown problems can present many issues. Wallace and McLoughlin (1979) suggested that more emphasis should be placed on application and comprehension rather than rote memorization of facts. Secondly, textbook curriculum sounds archaic and problematic for visual, kinetic learners.

Students struggle with mathematics for a variety of reasons. These include: teachers expecting all students to move from the concrete to the abstract thinking when moving into secondary education; the gradual, or sadly the more common sudden withdrawal of concrete materials; timetabling of short sessions that do not allow for deep inquiry; and an increase in drill and skill tasks in preparation for a higher frequency of tests (Gunningham, 2003). If Gunningham is correct, then teachers are not teaching for understanding, and therefore students will find it more difficult to solve unfamiliar problems. Nevertheless Gunningham detected some change when she noted, "A great deal of emphasis is now placed on students explaining and justifying why they chose to solve problems in a certain way and diversity of thinking is celebrated to a much greater degree than replication of learned processes" (p. 16). She was suggesting that it is through written or verbal explanations that students show their understanding of the problem. In progressing this approach, Noss, Healy and Hoyles (cited in Boaler 2002, p. 11) "argue that the act of making connections is important because mathematical meanings derive from mathematical connections". This suggests students need to argue at a deep level if they are to learn mathematics. As Polya stated, "Trying to find the solution, we may repeatedly change our point of view, our way of looking at the problem. We have to shift our position again and again" (Polya, 1957, p. 5).

1.2.4 Small Group Learning

Boaler in discussing the notion of mathematical practices suggested another nuance: the "main focus is not the learning of mathematics, but the doing of mathematics – the actions in which users of mathematics (as learners and problem solvers) engage" (Boaler, 2002, p. 16). She goes on to claim that one way this occurs is through the shared experience of listening to others explain how they arrived at their conclusion that assists others along the journey of understanding. The students who see past their frustrations of not capturing a complete answer and continue to use their thinking dispositions to seek the more complete answers will come to have a better understanding of the mathematical ideas and concepts before them. Through reasoning and questioning, modifying plans and in some cases abandoning one line of thought for another; the student contributes to their own intellectual character and the thinking culture of their peers.

In a further comment on encouraging students to think deeply, Boaler (2008) suggests "An approach commonly used by teachers across different subject areas to encourage communication and the sharing of ideas and respect is group work" (p. 171). Group work, in the best sense of that term, promotes collaborative learning where ideas and thoughts are shared in a safe environment, where opinions are respected and differing perspectives are valued. Boaler (2008) found that students through persistence and collaboration could solve complex problems and that through their collaboration, a deeper appreciation of how others think and learn developed.

Hence the general approach to situating thinking at the centre of schooling has implications for what mathematics teaching aims for and how it is carried on. It seems there are some strands in the mathematics education literature that echo this hope of giving pre-eminence to thinking in schooling. It was investigating this possibility that formed the central issue for this study.

1.3 Purpose of the Study

The purpose of the research study that forms the basis of this thesis was to investigate students' problem solving and thinking as they worked in small groups to solve open-ended mathematical tasks. The students to be investigated were Year 2 students aged approximately 8 years of age. The specific foci of the study were students' use and understanding of problem solving heuristics used during the solution process; the students' use and understanding of thinking routines to assist in solving the mathematical tasks; and the students' collaborative learning in small groups while engaged with the mathematical tasks. Of key interest were the mathematical, thinking (namely problem solving heuristics and thinking routines) and collaborative processes students used to discover solutions to open-ended mathematical tasks.

To focus the study the following research problem was formulated:

> How do students employ thinking routines and strategies, and mathematical problem solving heuristics when involved in mathematical problem solving in small groups?

1.4 Significance of the Study

Too often general educational literature is not interpreted for the specific sub categories of subjects into which schooling is divided. The significance of this study will be to examine whether the general, in this context general teaching about thinking strategies, does bridge into the specific context of the mathematics classroom, and further whether they can be used in conjunction with more specific mathematics based problem solving heuristics by the students as they solved the mathematical tasks. There seems to be little in the literature that has examined this conjunction of teaching approaches.

1.5 Overview of the Study

The thesis is presented in five chapters. Chapter one has detailed the background for the study and relates some events that raised this researcher's curiosity to ask some questions and begin my journey. Chapter two reviews the relevant literature associated with the three aspects of this research namely, thinking, heuristics and collaborative learning. The research questions that motivated the design and conduct of this study are presented at the conclusion of this chapter. Chapter three involves a discussion of the research methodology used to obtain and analyse the data. Chapter four summarises the findings of the research analyses. Chapter five presents these findings and contains the conclusions and implications the findings of the study have for teaching and learning about thinking, heuristics and collaborative learning in mathematics

Chapter Two

Literature Review

This chapter provides a review of the literature relevant to this investigation of how students use the taught problem solving heuristics and thinking routines within small groups to solve mathematical problems in a primary classroom. Set within the background of current education policy and practice, the fields of study included: (a) using problem solving heuristics to assist in solving mathematical tasks; (b) using set thinking routines and dispositions to assist in solving mathematical tasks; and (c) small group learning. This breadth was required to build an understanding of the changing mathematics primary classroom and the importance of discussion, collaboration, thinking and strategies within this context.

Through the asking of probing questions and the perseverance with a task, a student's mathematical character can be shaped (Darragh, 2013). The first section, Section 2.1, explores the literature relevant to *problem solving and problem solving heuristics* within the domain of mathematics. In this section I will explore the seminal work conducted by Polya and the influence this has had in problem solving in mathematics. Section 2.2 investigates the use of *thinking routines and dispositions* to promote thinking and intellectual character. In this section I will explore the work of Ron Ritchhart and the influence this had had on my own thinking. Finally, Section 2.3 explores *small group collaborative learning*.

2.1 Problem Solving and Heuristics in Mathematics

2.1.1 Early Learners in Mathematics

A shared characteristic of the various national documents is the acceptance that all students should learn mathematics with knowledge and comprehension, and that being able to use mathematics in everyday life and in the workplace is vital for life in the 21st century (NCTM, 2000). These conclusions imply that consolidating the understanding of mathematics of students in the early school years could be of vital benefit to students' mathematics learning in the long-term (Young-Loveridge, 2004). Bryant and Nunes (2002) suggested that rational and logical thinking, modelling and teaching of standard counting systems, and a meaningful context for learning mathematics form the basis for children's early mathematical understanding. Early numeracy skills, especially counting skills, have been found to be good predictors of later mathematics performance (Aubrey, Dahl, & Godfrey, 2006; Aubrey, & Godfrey, 2003), and counting skills in pre-school, such as number–word sequence skills, have shown to forecast basic mathematic skills in the early grades of primary schools (Desoete, Stock, Schepense, Baeyens, & Roeyers, 2009; Koponen, Aunola, Ahonen, & Nurmi, 2007). Both in the pre-school setting and the primary year levels, there can be a chronological age difference up to 12 months; this can be a factor for varying mathematical skill level. (Boardman, 2006; Dowker, 2008).

Mathematics has a crucial position in primary school curricula (Anthony & Walshaw, 2009) and mathematical understanding assists in shaping decision making and thinking for young people in their future lives. In recent years, the mathematics education of young children has received increased attention among Australian researchers (Clarke, Cheeseman, & Clarke, 2006; Perry, Young-Loveridge, Dolckett, & Doig, 2011). Australia like many other countries have identified the importance of teaching, modeling and promoting young children's mathematical thinking.

2.1.2 Notions of Problem Solving

In recent years, attention to the importance of teaching mathematics to young children has increased (Clements, Sarama, & DiBiase, 2004). Mathematics gives the opportunity for students to become absorbed in the complex and beauty of finding solutions to problems. Thinking about and understanding the mathematics involved in problem solving allows students to reflect on their own ideas. Boaler (2002, p. 14) suggested "in these different characterizations of the mathematical work employed by students and mathematicians we also gain an important sense of some mathematical traits that supported the work, including creativity, interest, and inquisitiveness”. Problem solving can be explained as a process of working through the details of a problem to reach a solution and is acknowledged as a very important task in mathematics learning (Lazakidou & Retalis, 2010). Hence for students to appreciate and indeed work with the complexity and beauty of mathematics, they need to learn how to think deeply mathematically.

Elshout (1987) identified problem solving as a cognitive function that made the problem solver recall and process the relevant information. Many argue, all students who learn mathematics should see themselves as mathematicians. Nelissen (1999) stated that if mathematicians are seen as detectives, looking at and exploring their own learning and the learning of others with a critical eye, learning to make and test conjectures, then mathematics teaching needs to also change. In gaining knowledge and understanding, students need to apply what they know in a strategically planned way to arrive at an answer that is possible and plausible. They need to learn how mathematicians think, to perform their own detective work, make mistakes and learn from them and develop their own approaches to problems (Nelissen, 1999). “This all means that students learn to think about their own mathematical thinking, their strategies, their mental operations and their solutions” (p. 191).

Problem solving is a “higher-order cognitive process” (Goldstein & Levin, 1987) that involves the ability to recognise, understand and analyse a problem. Then, all of the parts need to be assessed and included to produce a representation or solution. Gunningham (2003) discussed the Big Foot problem, where students were given a footprint and were required to determine the height of the person. It was noted that students attempted to work methodically with mathematical calculations, made connections between the size of the students’ feet and the size of other body parts, used ratios, averages and estimation, and calculated the area of the footprint in the vague notion this would assist them in calculating the height. Gunningham concluded that students needed to learn some general strategy for solving mathematical problems. When solving problems students need to go beyond memorized formulas and rote learning. Understanding what the problem is and having an entry point to a solution process, are the first steps to the beginning of solving the problem.

2.1.3 The Importance of Polya

Problem solving requires students to have a starting point. Polya's model; see, plan, do and check (Polya, 1957) is a pivotal model that has been used for over 50 years. This is an important element for successful problem solving. Polya maintained, "questions are of the greatest importance for the problem-solver. He checks his own understanding of the problem, he focuses his attention on this or that principal part of the problem" (1957, p. 214). For Polya, solving problems was: understanding the problem, struggling with and solving the problem and finally reflecting on the solution and thinking of a second solution (1957). Problem solving is producing solutions in the most efficient manner available and reflecting in a metacognitive way by asking "can I do this differently?"

Polya (1957) suggested that teachers should assist "to develop the student's ability so that he may solve future problems by himself' (p. 4) in mathematics. Polya continued that through attempting problems, observing and imitating other people and students during problem solving, you begin to learn to solve problems yourself (Polya, 1957). It is the opportunity to present robust problems to students and providing them the chance to discuss and discern possible solutions that allows students to gain a growing confidence in their problem solving abilities. As mentioned in the previous paragraph, Polya (1957) identified four phases a student should attend to when problem solving, understanding the problem; planning; carrying out; and finally looking back at the solution.

The model suggested that students needed to first understand what the question is asking and then begin to set a plan into motion that will allow for them to begin exploring possibilities. Polya's model suggested using strategies such as using concrete materials, drawing diagrams, creating tables, making a list, trial and error processes, working backwards, looking for patterns, writing algorithms or equations, acting out the scenario and solving a simpler related problem (Jones, 2003). Wong (2008) stated that the use of these heuristics allow the students to solve different types of problems. Primary school students are going to be moved towards drawing diagrams more often because they are visual learners, while others will use concrete materials because they need to use their hands or writing equations because they are more rational / logical learners. The use of the strategy draw a diagram is strongly advocated by mathematics educators as a tool for problem solving (Australian Education Council, 1991). For some students, drawing a diagram is the first step towards a successful solution (van Essen & Hamaker, 1990). Clarkson (2008) suggested that not only do students need to have a starting point to solve a problem they also need to know when they have discovered a solution. It is when students have recognized a possible solution that they have taken true responsibility for their own learning. This encourages students to not only recognize the solution but also test its adequacy by going back and checking it.

In this present study, I introduced students to the thinking routine *See Think Wonder* developed by the Harvard Graduate School of Education, to use when they were in the understanding and planning stage of solving the problem (this routine is discussed more fully in the next section). For Polya part of the planning stage was to “isolate the principal parts of your problem” (1957, p. 33). During the understanding stage of the problem, students need to consider the details and how they relate to each and in relation to the problem as a whole (Polya, 1957). The students used the problem solving heuristics when carrying out or ‘doing’ the mathematics. Finally the students employed their metacognitive thinking in reviewing and discussing the problem.

For Polya (1957) identifying links or connections and understanding the problem are vital if the student is to complete the problem successfully. “It is foolish to answer a question that you do not understand” (p. 6). Polya (1957) further stated that when understanding the problem the student should consider the problem from various angles and viewpoints.

Polya (1957) also indicated that students at the beginning will probably not know the solution but they can suggest a probable answer. This is when the heuristic "guess check and improve", a heuristic loosely based on Polya and advocated in various curriculum documents for students to estimate the solution and then later check the solution (ACARA, 2013) is a useful strategy because when used correctly it can provide a starting point to begin an investigation. Even though the guess is often incorrect, it is the beginning of the investigation where a student can continue and often use another heuristic to obtain the solution. Polya (1957) also stated "mere remembering is not enough for a good idea" (p. 9) but in fact a person may use past knowledge and understandings to assist them in solving the problem, now morphed into the 'guess' aspect of the above heuristic. Furthermore Ploya asked, "Do you know a related problem…Here is a problem related to yours and solved before. Could you use it?" (p. 9). The problem solving heuristic "solve a simpler related problem" could be employed by a student to gain an understanding of the problem; e.g. the farmer can count 112 legs in the paddock, some are ducks and some are cows. How many cows and ducks are there? The number maybe too large for some students, therefore the number could be reduced to 20 legs and the student is now managing to "solve a simpler related problem".

Like Polya, Romberg (1994) described how to do mathematics and solve problems: first, students need to make sense of the problem; secondly, articulate the problem and decide on the important parts and relationships between elements; thirdly, decide on a model; fourthly, find a solution; and finally think on the validity of the solution. A brief inspection shows how Romberg drew on Polya's earlier work. Alibali, Phillips and Fischer (2009) stated students might use inefficient strategies to solve problems because they have unsuccessfully identified the key information in the problem.

2.1.4 Teaching using problem solving

Brough and Calder (2012) discussed the importance of parts of the mathematics curriculum where some skills are not measurable through standardized tests. It is through the inquiry of a task or problem where students have the opportunity to hypothesize, check, abandon and begin again that illustrates mathematical reasoning and thinking. Brough and Calder argued "the ability to negotiate, create knowledge, think creatively and critically, and work together for the common good" (p. 139) should be viewed as the norm in all classrooms. Students need the time to analyze and synthesize the data that is before them so they can gain understandings. Draper (2002) is in agreement, he suggested there should be less focus on rote learning of isolated facts and skills and a greater importance towards problem solving and communication. In developing the communication skills of the students, the mathematics classroom becomes a hive of activity where language and ideas are discussed and shared among the whole populace of the classroom. For Boaler (2002) participating in mathematical discourse enables students to "develop identities as mathematics learners and a relationship with the discipline of mathematics" (p. 10). In doing so, the collective knowledge of the classroom is increased.

David Perkins suggested, "To think better, people need to develop general commitments and strategies toward giving thinking more time and thinking in more broad and adventurous, clear and organized ways" (1994, p. 4). Within the primary mathematics classroom, the learning of number facts can be easily achieved without total understanding. To 'do' mathematics is to be active. Problem solving tasks allow students to be active either as individuals or as group members. Schoenfeld (1992) and others maintained that problem solving is the goal of mathematics learning, while communication, that is teachers being aware of the students' thinking, is also and always has been an important part of the process (Marshman, 2012). Through discussion and 'thinking aloud' students "conjecture, test, and build arguments about a conjecture's validity ... and ... are encouraged to explore, guess and even make errors" (Battista, 1994, p. 463). It is through this investigation that students' learning and understanding can be viewed. For Polya even incomplete ideas should be investigated and if they begin to be viewed as advantageous then the idea should continue to be considered (Polya, 1957).

For Polya "if it looks reliable you should ascertain how far it leads you, and reconsider the situation" (p. 35).

As educators we want our students to become confident mathematicians. Pickering (1995) discussed the notion of initial thoughts and ideas or their extension of established ideas when a student is amidst the 'doing' of problem solving. As stated earlier citing Polya, it is important for students to have a starting point for their inquiry. In becoming confident mathematicians students also need to be taught how to think. When students are becoming confident mathematicians, teachers can continue to challenge them. "Even if we have succeeded in finding a satisfactory solution we may still be interested in finding another solution" (Polya, 1957, p. 61). It is in the 'beauty' of mathematics that students should strive to explore alternative ways of solving problems and to investigate efficient and clear solutions.

When unknown problems are encountered students need to be taught or learn how to work through the problem to arrive at a solution. Schoenfeld (1992) viewed high school and college students working with unknown problems and remarked that "roughly sixty percent of the solution attempts are of the 'read, make a decision quickly, and pursue that direction come hell or high water' variety" (p. 61). The students were not changing their method of 'attacking' the problem even though they were having no success. Alibali et al. (2009) found that students may use wrong or inefficient strategies to solve problems in that they neglect to correctly identify the main features of the problems. Wong (2008) also agreed "many pupils at all school levels have difficulty solving unfamiliar or so-called 'non-routine' problems" (p. 589). In contrast, mathematicians when solving an unfamiliar problem devote time making sense of the problem, then pursue leads, abandon attempts that are not getting anywhere and then solve the problem (Schoenfeld, 1992). To develop this ability Schoenfeld (1992) believed that students needed to be taught the "metacognitive aspects of mathematical thinking" (p. 63). In the next section of the literature review I will be discussing the importance of metacognition as a disposition students need to develop. Metacognitive ability takes time to develop and includes "assessing one's own knowledge, formulating a plan of attack, selecting strategies, and monitoring and evaluating progress" (Schoenfeld, 1985 cited in Yimer & Ellerton, 2006, p. 575) It is this metacognitive ability rather than mathematical knowledge that influences a student's ability to problem solve (Carlson, 1999).

Furthermore, when students begin to solve unfamiliar problems, they may attempt to use knowledge and understandings gained from previous lessons. The students in these circumstances may attempt to transfer knowledge, "in that the mathematical knowledge accessed in solving the problem is closely related to the mathematics recently learned" (Clark, Page & Thornton, 2013, p. 1). Clark et al. (2013) further stated that the Australian Curriculum: Mathematics, aims for students to respond to different types of familiar and unknown problems. ACARA (2013) indicated "these capabilities enable students to respond to familiar and unfamiliar situations by employing mathematical strategies to make informed decisions and solve problems efficiently". The capabilities in question are the knowledge and understandings that may be formed from previous lessons and applied in unrelated areas within mathematics and other curriculum areas. As previously stated, problem solving heuristics allow students to apply known heuristics to problems that they may at the beginning of the problem solving process be unclear, or unsure, of how to proceed. Therefore by providing both known and unknown situations the students are "developing increasingly sophisticated and refined mathematical understanding, fluency, logical reasoning, analytical thought and problem-solving skills" (ACARA, 2013).

Problem solving requires students to take the time and focus on the problem or task that is presented to them. The amount of time may vary depending on the situation of the teacher and the classroom. Problem solving should not be viewed as a five-minute activity. Students need the time to think, to hypothesize and to pursue their own line of thought. Burton (1998-1999) discovered that mathematicians collaboratively negotiate by exploring problems and actively seeking connections between the real world and mathematics. They also pursue links between the different areas of mathematics and the beauty of mathematical solutions. Sparrow (2008) stated that real mathematics is unlike the whiteboard, textbook and commercial black-line masters that many students are presented in the classroom. Students should be taught to emulate this approach and hence have “The ability to look at a problem from different angles (which she believed) is crucial.” (Burton, 2001, p. 597).

2.1.5 Resistance to using problem solving

Problem solving should not be thought of as an added extra to the mathematics curriculum. Nelissen (1999) stated the realistic approach to learning, of which solving problems should be seen as central, is a departure from just learning rules and formulas to working within contexts. Understanding the methods by which students discard old strategies in favor of newer strategies is one of the central challenges in the study of cognitive development (Siegler, 1996). Alibali et al. (2009) stated that students are often shown new problem-solving strategies in subjects such as mathematics. Nelissen (1999) further advocated that the problem situations can be real or fictional, since this allows the students to use the knowledge they have gained from their own musings and theorizing and makes the activity meaningful to them.

In Australia it has been reported that some teachers believe that schools needed to rethink their ideas on problem solving and become more creative and innovative (Anderson, 2005). Others didn’t see problem solving as a legitimate mathematical activity and wanted to be more aware of the benefits of problem solving (Anderson, 2005). Anderson stated that 130 primary school teachers from New South Wales were surveyed regarding their views on problem solving in mathematics. From this survey Anderson noted, “problem solving needs ‘credibility raising’ ” (p.92). Teachers not believing problem solving is a legitimate pursuit are not using a teaching strategy that clearly promotes understanding. Unfortunately, learning is too often afflicted by the difficulty of rigid knowledge that cannot be used to solve novel problems (National Research Council, 2000). True understanding can only be applied to problems that are unknown and unfamiliar. French (1992 cited in Boaler, 1996) believed that the harm in teaching set procedures could mean that students believe there is a 'proper way' to solve problems: that is there is only one correct method, and this will inhibit them from searching for alternative strategies. French argued trying alternatives and being flexible is central to effective mathematical thinking (Boaler, 1996).

In trying to understand the place of problem solving, Anderson, Sullivan and White (2004) identified two teaching styles: the traditional and contemporary approaches. They characterised the traditional approach as consisting of seeing mathematics as facts which were presented by the teacher and internalized by students. Hence teaching strategies were dominated by individual work, reliance on worksheets and textbooks, and rehearsal of routine questions. Sparrow (2008) discussed the notion that problems in textbooks are a 'dressed-up' calculation, which are then followed by another page of calculation exercises. Anderson et al. (2004) found that problem solving when it was engaged within this context was viewed as an end, and was to be engaged in when basic facts and skills had been mastered. The opposite style is what Anderson et al. (2004) called the contemporary approach. This approach sees mathematics as an opportunity to explore, to work in groups, and use non-routine problems to stimulate thinking. Problem solving in this contemporary approach is a means and not an end when learning mathematics (Anderson et al., 2004). The data from their study suggested that beliefs and practices are linked. Teachers using the traditional approach used practices that had students working alone, provided detailed explanations and set exercises for skill practice. In contrast, contemporary teachers used practices that promoted group work, gave ownership to students and encouraged students to investigate mathematical ideas. In their study, Anderson et al. (2004) found one of the traditional teachers viewed problem solving as an added extra. Problem solving was for able students rather than low achieving students who she felt needed additional practice on basic skills. This teacher did not see problem solving as an inquiry into thinking about mathematics but rather honing basic number skills.

Hollingsworth, Lokan and McCrae (2003) in their video study, analysed Year 8 mathematics lessons from seven countries. They found over three quarters of the problems presented by teachers were of low procedural complexity (2003). They also found that problems were repetitions of problems completed earlier in the lesson, most problems presented needed to have the correct procedures followed to be solved correctly, most problems were not discussed as a class but rather the solution was said ‘out loud’ and over 90% of problems had only one solution (2003). Hollingsworth et al. (2003) argued "Australian students would benefit from more exposure to less repetitive, higher-level problems, more discussion of alternative solutions, and more opportunity to explain their thinking" (p. xxi). This present study endeavours to illustrate that by allowing time for students to think and discuss their ideas, they will become actively involved and engaged in mathematical problem solving.

Anderson et al. (2004) found one of the contemporary teachers they worked with stated that she used open-ended problems because it gave the students the opportunity to use their understanding they had garnered from inside and outside the classroom to the problem being investigated. This present study will take the approach that mathematics should be viewed as a tool for understanding the world and engaging in it. By using open-ended tasks to promote discussion, justification of possible solutions, creative thinking and group work, students and teachers may gain insights into understanding (Gunningham, 2003).

2.1.6 Types of problems to use

Genuine rich problems are not one step exercises where the students just insert the correct operation, but rather such problems allow for students to investigate the many possibilities, apply mathematical thinking and communicate their ideas with others (Jones, 2003). “These situations can be used by creative teachers as wonderful opportunities for students to think deeply about mathematics and how it works” (Clarkson, 2008, p. 33). Engineering problems, which engage the student’s curiosity about the natural world (English, 2008) and thus should be promoted. English (2008) described engineering problems as authentic learning opportunities that provide a context for students to see the relationship between science, mathematics and the real world. Engineering contexts provide rich problems for primary students to engage in, as they need to hypothesize, plan and test their conjectures. Problems should be presented in a variety of ways and there should be links made to mathematics and everyday life (Bobis, Mulligan, Lowrie & Taplin, as cited in, Jones, 2003). Barr Goral and Gilderbloom (2008) initiated a mathematics project within their classrooms that involved measurement and the creation of a felt pencil case. They believed “it was an opportunity to engage students in an investigative and problem solving aesthetic experience” (p. 24). Furthermore Barr Goal and Gilderbloom (2008) observed that if the students are presented with activities they can engage in and be involved in, that also provide opportunities to use measuring skills in everyday life then the activities are then worthwhile. Often problems that have links to everyday living are complex, promote curiosity, are rich and substantial and require time (Jones, 2003). It is these types of problems that allow for growth and learning of the students.

Sparrow (2008) claimed, “Bringing a sense of relevance and realism to the primary classroom is not an easy task” (p. 5). He further suggested that the problems that are presented to the students maybe in context but for the bulk of students they see the problem as having little relevance to themselves. Hence not all tasks need to be real world problems but they do need to be relevant and interesting. Problem solving tasks, he noted, provide the opportunity for students to use the skills and number fact knowledge in an interesting and engaging way. For Sparrow real mathematics will engage the students because they will be able to make connections between what they experience and mathematics. Sparrow stated, “mathematics becomes real when children explore and solve problems that require them to use their mathematical knowledge and skills” (p. 8).

Another characteristic of such problems are when the answers are not immediate and require students to think of strategies they believe will assist them in learning a solution. Marshman noted,

> A mathematical investigation is real-life or life-like learning which is: open ended and provides opportunities for students to use multiple pathways to investigate the situation/problem. It may be framed as: a problem to be solved, a question to be answered, a significant task to be completed or an issue to be explored (Marshman, 2012, p. 492).

Opportunity for 'rich tasks' needs to be provided for students so they are engaged and curious from the beginning. Grootenboer (2009) described rich tasks as problems that "have intellectual depth and educational value, and it would require a significant amount of time to complete" (p. 696). Unfortunately, in Australia few complex problem-solving opportunities are being presented to students in Year 8 mathematics classes according to Anderson (2005). Stacey found through her analysis of lesson videos that the average mathematics lesson was shallow and a set of procedures for students to follow without reason (Stacey, as cited in Anderson, 2005). Although recalling number facts is important, it is not what makes a task rich and appealing to inquiring minds. Fredericks, Blumfield and Paris (2004) claimed authentic tasks promote engagement, which in turn promotes a students' ownership of their learning. Rich tasks can also be expressed as open ended tasks that students can become engaged in finding multiple answers often through making tables, lists, drawing diagrams and using concrete materials. "Open-ended tasks provided the vehicle for thinking, reflecting and communicating mathematically" (Anderson, 2005, p. 91).

Open-ended tasks provide an opportunity for students to showcase their knowledge, understanding and to produce as many solutions as possible. Sullivan, Griffioen, Gray and Powers (2009) through their work on task types stated, "open-ended tasks have multiple possible answers, they prompt insights into specific mathematics through students seeing and discussing the range of possible answers" (p. 5). Sparrow (2008) argued that open tasks also assist students in posing their own questions or queries.

Jaworski stated that investigative teaching, similar to Anderson et al. (2004) contemporary teaching, allowed for peer discussion and co-operation which allowed students to explore the questions for themselves with teachers talking with them and not lecturing at them (Jaworski, as cited in Anderson, 2005). This is an important point to consider. The best learning comes from grasping and thinking about the problem, and applying the knowledge and understanding so gained to other new questions. As the Big foot task (Gunningham, 2003) showed, learnt pieces of mathematics with no understanding cannot be applied successfully. Teachers want their students to transfer the understanding they have learnt into a new situation. Through student discourse and co-operation and a partnership with the teacher, students will have a better opportunity to create an answer that is plausible.

Clarkson discussed the classic game of tic-tac-toe and through playing the game, students should be encouraged to verbalise their strategies with others (Clarkson, 2008). Ben-Zvi and Arcavi (2001 cited in Boaler, 2002, p. 13) also highlighted the importance of 'habits' such as questioning, representing, concluding and communicating. Boaler (2006) discussing one particular group of students who had worked on open-ended problems noted that they developed many possible solutions. They became aware that various methods of problem solving could be valued. For these students, in the end, being successful meant asking good questions, explaining ideas, justifying and reasoning for their position and working logically. She found that students who were continually spoon fed the content of the curriculum with low order closed tasks without questioning, thinking and discussing were only able to reproduce what just had been previously learnt. When presented with more challenging material which was unknown, these students were either unable to complete the task or even willing to begin. Boaler (1996) stated

> "the teachers all seem to fracture problems in order to help the students get answers and do mathematics, as opposed to learn

> about mathematics or develop mathematical understanding. The teachers seem in many ways to be encouraging a kind of counter learning process that may be working to ensure the absence of higher level learning". (p. 23)

I now move to a more generalized approach to the promotion of thinking across the curriculum that is used in my school in contrast to the specific thinking strategies applied to mathematics just reviewed. Strangely however this generalized approach is often not seen as applying to mathematics.

2.2 Thinking Dispositions and Thinking Routines

Developing a teaching approach that prioritizes the thinking and reasoning of students for them to be creative and imaginative, to reflect and be metacognitive about their own learning should be an important aspect for all schools. Thinking in the classroom is more than brainstorming, and more than problem solving once a week in isolation with no connections to any of the 'normal' work. A thinking classroom is an active, reflective and learning environment that promotes ideas and rich thoughts. Understanding key concepts and exploring ideas to arrive at well informed answers should be the cornerstone of any classroom. In this section I probe relevant literature that speaks to this stance. Furthermore this section will be divided into two parts; *thinking dispositions* and the importance of these in promoting *intellectual character* and thinking; and secondly, *thinking routines* and the importance of these in promoting *intellectual character* and thinking. When appropriate I will link these general thinking dispositions to mathematics learning and teaching, and to problem solving in mathematics in particular.

2.2.1 Thinking Dispositions

To move forward sometimes you need to look back. Dewey (1938) on his writing on thinking and reflection wrote, “What [an individual] has learned in the way of knowledge and skill in one situation becomes an instrument of understanding and dealing effectively with the situations which follows. The process goes on as long as life and learning continue” (p. 44). Ryle (1949) discussed from a philosophical point of view dispositions as dispositional properties; Ennis (1986) offered a list of thinking abilities with dispositions; Facione and Facione (1992) along with Perkins, Tishman and Jay (1993) explored thinking dispositions.

For Ritchhart “the concept of *intellectual character* is an attempt to move out of the prevailing paradigm of abilities-based conceptions of intelligence” (2002, p. 18). The active use of knowledge to think with what you have learned should be one of the central goals of education (Perkins, 1992, p. 28). Hence Harpaz (2007) argued that the teaching of thinking in schools is important because it draws the student closer to understanding great ideas, not for the sole purpose of gaining knowledge but for the intrinsic value of these ideas. Ritchhart has borrowed from Shari Tishman (1995) a useful term to encapsulate this approach, *thinking dispositions*, which he argues will lead to better thinking (Ritchhart, 2002).

For the purpose of this research *thinking dispositions* will be defined following Ritchhart and Perkins as patterns of behaviour that motivate and guide our abilities (Ritchhart, 2002) and that are ongoing and cross many different circumstances (Perkins, 1992). Perkins (1992) elaborated “good thinkers are disposed to explore, to question, to probe new areas, to seek clarity, to think critically and carefully, to consider different perspectives, to organize their thinking, and so on” (p.40).

There is a tension involved with the word disposition, and this has in fact been the case since at least the time of Dewey (Ritchhart, 2002). The tension includes the notion of placing a label on a set of characteristics. For Ritchhart "*thinking dispositions* represent characteristics that animate, motivate, and direct our abilities toward good and productive thinking and are recognized in the patterns of our frequently exhibited, voluntary behavior" (p. 21). However what Ritchhart does want is a list of dispositions that promote thinking. In Table 2.1 is a list of the six *thinking dispositions* that Ritchhart has identified.

Table 2.1: *Ritchhart's Six Thinking Dispositions (2002)*

Thinking Dispositions
Open Minded
Curious
Metacognitive
Seeking truth and understanding
Strategic
Skeptical

Ritchhart defines these dispositions as follows:

- To be *open-minded* is to be flexible, to not just accept other's ideas, to create your own thoughts and options, to look beyond what is in front of you, to be active and not passive and to look from other perspectives (Ritchhart, 2002, p. 27). To be *open-minded* is to question the relevant data and information that you have at your disposal, and to ask to have the whole story. Questions to be used are: Are there any parts missing? If I do this what will be the outcome? To be *open-minded* is to understand existing information and then explore new ways. Within mathematics students need to remain *open-minded* when solving a problem and realize there could be more than one way to solve the task.
- To be *curious* is to wonder, to ask questions, to puzzle in the ordinary as well as the unexpected; it fuels our interest to think about things (Ritchhart, 2002, p. 28). Curiosity is the stepping-stone for the journey of discovery. Within mathematics the

curious student approaches the task with enthusiasm and optimism so that they can solve the task and gain insight and understanding.

- To be *metacognitive* is to be active in directing, regulating, monitoring and evaluating thinking (Ritchhart, 2002, p. 28). Goodrich saw "the importance of this kind of critical reflection in determining the differences between intelligence and intellectual character" (1995, p. 1). Within mathematics the student that displays metacognition is able to reflect the next time when they begin to solve a problem what worked and what didn't work.
- To *seek the truth and understanding* involves looking at the evidence in front of you, reflecting on its validity, investigating the links between theory and evidence, and exploring other possibilities and alternatives (Ritchhart, 2002, p. 29). Seeking the truth allows a student to delve deeper into the topic and truly understand what is unfolding in front of them. Brainstorming ideas and sharing thoughts only scratches the surface. But "by asking students why they think what they do or what is behind their beliefs or opinions, we can begin to engage them in a search for truth and understanding" (p. 29). Within mathematics seeking the truth and understanding is paramount in solving problems. Without understanding the problem, the student will find it difficult to engage the correct strategies to solve the problem.
- To be *strategic* is to plan, to be methodical, to set goals, to choose tactics, consider options, to assess and monitor existing plans and strategies, and hence to become more efficient (Ritchhart, 2002, p. 30). Strategic learners set plans in place but are able to reassess if their plans go awry. *Strategic* learners are flexible in their thinking. Even though strategic learners may be successful, they are still asking themselves is there a better way? Within mathematics *strategic* students apply their strategies to assist them in solving problems.
- To be *skeptical* is to look further, to probe, go beyond the obvious or the surface, search for proof, to become active and take a stance; it doesn't necessarily mean being suspicious and critical (Ritchhart, 2002, p. 30). In this context, being a *skeptical* thinker is not being negative. Within mathematics the *skeptical* thinker is assessing all the information available, and deciding to look further to uncover answers.

A crucial question for educators is how do we teach students to think using such *thinking dispositions* as these? For educators, this is the crux of the matter. What does it mean and look like from a practical point of view to look at a mathematical problem from a skeptical or open-minded disposition? For the students they need the opportunity to explore and investigate any topic under consideration. Some students will be more inclined towards certain dispositions, but by learning how to use all the *thinking dispositions*, the students are developing their 'intellectual character'; defined by Tishman (1995), as a way of referring to one's combined *thinking disposition*.

The *thinking dispositions* are evident in the student's actions, standards, motivations, attitudes and values (Tishman, 1995). The *thinking dispositions* pose multiple difficulties for educators. Ritchhart himself recognized that working with the dispositions 'won't be easy' (Ritchhart, 2002). The primary reason for this difficulty is that there is no definitive way to teach the dispositions. Perkins (1995) discussed that not all dispositions are positive, such as closed-mindedness, which is a negative thinking position. Perkins (1995) further stated that it is in the best interest of educators to pursue and cultivate positive thinking dispositions. Despite such a fundamental drawback, this avenue of education should still be pursued. In the study described later, by providing rich mathematical tasks in an environment that presupposes students will think creatively, it is hoped students will be able to illustrate their thinking with the dispositions.

The *thinking dispositions* are active in promoting *intellectual character*. A person is active when seeking the truth, being strategic and making plans, being skeptical and probing for more information, being curious about a topic and wanting to know more, being metacognitive and reflecting on one's own plans and being open-minded and asking questions. *Intellectual character* is an active pursuit. "An important difference between a *thinking disposition* and a cognitive ability is that an ability can lie dormant, while a disposition by definition has behavioral force" (Tishman, 1995, p. 4). Behavioral force implies action. There is action in seeking the truth. Tishman emphasizes that a person can have the ability to reason a point without engaging in the reasoning because of a lack of inclination (1995). Three parts must be present in high order thinking for dispositional behavior to occur: inclination, sensitivity and ability (1995).

Tishman (1995) defined these three terms: Inclination being how one feels towards behaviour, sensitivity being the appropriateness of the behaviour and ability being the capability to complete the behaviour. Tishman (1995) cites the example of reading a newspaper about a group of people who discover bones in North America dating back 25000 years. If a person has *intellectual character* they will be first curious of the report and want to find out all the information they can. They would be remaining open-minded as the story could be true but skeptical of the information presented and be seeking the truth to prove these claims correct or false. Before a person begins higher order thinking, three things must occur. The person has to be sensitive to the information presented and notice a poorly supported claim. Secondly a person must feel inclined to be active and ask what might be wrong, and alternatively what is correct about the information presented. Finally the person has to have the ability to seek the truth and find counter or corroboratory evidence (Tishman, 1995).

If Tishman is correct then the ability part of the equation will differ from student to student. Perkins (1994) stated, "Good thinking dispositions can thrive in any subject matter at any level" (p. 4). Students should be engaged and become truth seekers and understand the information they are uncovering. Dweck (1986) described a person possessing an incremental view of intelligence and focused on learning is likely to be inclined to analyse a challenging situation and use a variety of strategies. To notice when more investigation is needed, to discover the answer and to ask what might be wrong and be inclined to want to know will lead a student upon the road of discovery. Tishman et al., (1995) believed the environment created reinforces good thinking. The ability to decode and sift through relevant the information will be different from student to student. What educators can do is give all students the opportunity to discover answers by providing them with rich tasks and have a classroom that is rich in *intellectual character*.

2.2.2 Thinking Routines

For Ritchhart, Perkins, Tishman and Palmer understanding and developing understanding (Ritchhart, Church & Morrison, 2011) should be at the centre of all learning. With this in mind Ritchhart, Perkins, Tishman and Palmer identified six thinking moves vital for understanding; "observing closely and describing what's there; building explanations and interpretations; reasoning with evidence; making connections; considering different viewpoints and perspectives; and capturing the heart and forming conclusions" (p. 11). Ritchhart et al., (2011) stated that these "thinking moves directly support the development of understanding, (furthermore) this list can be useful to teachers in planning units" (p. 12).

"*Thinking routines* operate as tools for promoting thinking. Just like any tool, it is important to choose the right tool for the right job" (Ritchhart et al., 2011, p. 45). It is through the use of *thinking routines* that students thinking can be made visible. Hence they suggested that "When we make thinking visible, we get not only a window into what students understand but also how they are understanding it" (p. 27). To add "rather than just activities that help teachers engage their students more actively, thinking routines are tools that students can use to support their own thinking" (p. 46). It is through these *thinking routines* that students can first begin an exploration of an area of interest and then secondly delve deeper to learn and understand more. "The steps of the routine act as natural scaffolds that can lead students' thinking to higher and more sophisticated levels" (p. 47).

As previously stated in section 2.2.1 the language of the *thinking dispositions* becomes important for students to have a common language they understand and can use. The same can be said for the language and ideas of *thinking routines*. Ritchhart and his colleagues noted that the *thinking routines* are just that a routine that becomes part of the classroom fabric and make up (Ritchhart et al., 2011). *Thinking routines* are not activities. They make this crucial distinction as follows: "Whereas an instructional strategy may be used only on occasion, routines become part of the fabric of the classroom through their repeated use" (p. 48). It is through this repeated use, like the language of the thinking dispositions, which allow the students to become comfortable in applying them in their 'every day' learning.

These routines however should not be seen "as simple mundane patterns of behavior" (Ritchhart et al., 2011, p. 48) but rather active and evolving tools ready to be utilized at the fingertips of the students. They continued "with use, these tools become flexible rather than rigid, continuously evolving with use. Consequently, we observe that the teachers with whom we have worked are continually adapting the routines to better serve the learning at hand" (p. 48). The *thinking routines* when used repeatedly show a student's nascent thinking (Ritchhart et al., 2011) and that "learning then becomes about connecting new ideas to one's own thinking" (p. 49).

The *thinking routines* over time have been grouped in a variety of ways. Originally the Visible Thinking Project "grouped the routines around four key ideals: understanding, truth, fairness and creativity" (Ritchhart et al., 2011, p. 49). Ritchhart et al., grouped "the routines into three major categories: Introducing and Exploring, Synthesizing and Organizing, and Digging Deeper" (p. 49). For this study described later, the *thinking routine See Think Wonder* was employed and falls under the group of Introducing and Exploring.

The *See Think Wonder* routine emerged out of the power of looking closely (Ritchhart et al., 2011). "This routine was designed to draw on students' close looking and intent observation as the foundation for greater insights, grounded interpretations, evidence-based theory building, and broad-reaching curiosity... This seeing provides the opportunity to look carefully, to more fully observe, and to notice before interpreting" (p. 55). The wonder aspect of the routine allows the students to think of any new information that might arise, “think about and synthesize this information, and then identify additional wonderings” (p. 55). In this routine the students are sharing and discussing with each other their ideas and thinking at each stage (Ritchhart et al., 2011). This routine can be employed in a variety of ways. It can be used one step at a time using questions such as; what do you see? (See) What connections can you make? (Think) What questions do you have? (Wonder) In this usage the students spend time on the see aspect before moving onto the think part before finishing on the wonder element. The authors also note an alternative usage “by using the three prompts - See Think Wonder – together at the same time” (p. 57). For this study the students were directed to employ the routine one step at a time.

How is intellectual character being encouraged and the thinking dispositions and routines being used within schools? Fluellen (2007) described how high school students participated in some workshops on thinking run by Ron Ritchhart and through this experience students were able to engage in the language of thinking. It was noted that having the common language of thinking to communicate is important: “They learned that thinking dispositions could help them form habits of mind that support thinking, learning, and writing in the 21st century” (Fluellen, 2007, p. 1). Fluellen (2007) argued that for students good thinking takes time. It takes time to formulate questions from the initial wonderings. It takes time to order your thinking to form coherent ideas that can be placed within the scheme of an investigation. As mentioned earlier good thinking does take time and the answer might not be fully apparent at the beginning. But through being strategic, asking questions, planning, being curious and open-minded, a well reasoned and thought out answer can be provided. Opportunities are needed for students to engage in reflective thinking. Harpaz (2007) noted that by teaching the thinking dispositions a pattern of cultivation, thinking and intellectual character is being developed. Investing time in thinking skills and dispositions will create good thinking and this in turn creates understanding (Harpaz, 2007).

Making thinking visible (metaphorically) is of vital importance. As educators it is not enough for the students to say 'I did it all in my head'. It is the working out, the process, the thinking, teachers want to see and explore further. When the thinking is visible, more questions can be asked and the thought processes can be seen and acted upon to further enhance the research or to be changed because of new directions that have been discovered. Fluellen's (2007) students who engaged in the thinking workshops were also using *thinking routines* to make their thinking visible. These *thinking routines* can be used across all curriculum areas. Some *thinking routines* lend themselves better to certain subject areas but routines need to be used frequently and not as an isolated activity. As the name 'routine' suggests, it is a process where the thinking becomes a habit and can be applied in many settings. The routines can take time to introduce (Fluellen, 2007) and used with the *thinking dispositions*, allows for students to delve deep into research and understanding.

There is also some local evidence for the impact of teaching *thinking routines*. Alan Bliss at Melbourne Grammar and Lesley McLeod at Methodist Ladies College enthuse about the *thinking routines*, "we have found that when thinking routines are explored over time with a class, they yield very rich insights into the nature of thinking and learning" (McLeod & Bliss, 2007, p. 25). They further postulated, that the nature or idea of routines is that they become part of the method when thinking, investigating or exploring a given topic or part of topic (McLeod & Bliss, 2007). Bliss cited the example of his Year 8 History unit on Arthur where the *thinking routine* 'What Makes You Say That' was used. The routine has two parts; what do you notice and what do you think is going on with the crucial follow up, what makes you say that?

At another local school, Nellie Gibson at Bialik College, Hawthorn, believed that creating the thinking culture within the classroom takes time and a concerted effort (Gibson, 2006). For Gibson (2006) the use of thinking language in her year 2 classroom involved her deliberately using the language in front of her students in the hope it might spark their enquiring minds. "These routines should be easy to implement, consist of a few steps, be used over and over and across a variety of contexts and most of all encourage deeper thinking" (p. 42). She matched the thinking language and the routine, such as *See Think Wonder* when she introduced the topic and as the topic progressed she introduced other routines such as *Think Pair Share* (Gibson, 2006). As she reflected on the changes within her classroom, she noted that it was important to formally introduce the thinking words to her class, that planning and practice was important for the students in acquiring the thinking language (Gibson, 2006).

The *thinking dispositions* are active in promoting *intellectual character*. Teachers want students: to be curious and ask questions, to be skeptical and look at different perspectives, to be strategic and formulate plans, and to be open minded to others' ideas and opinions. The importance of the *thinking dispositions* is encapsulated by Perkins (1992) when he suggested "good thinkers are disposed to explore, to question, to probe new areas, to seek clarity, to think critically and carefully" (p. 40). I believe it is through developing the use of the *thinking dispositions* and *thinking routines* that allows students to approach mathematical problem solving in a well-balanced frame of mind.

Questions arising from this section of the literature review include:
(a) What role do thinking routines and dispositions have when solving mathematical problems?
(b) What understandings do students demonstrate of thinking dispositions when solving mathematical problems?

2.3 Collaboration and Small Group Learning

Classrooms are ecosystems of learners that are diverse in ability, personality and emotional intelligence. Some classrooms routines are set by school policy and hence outside the teacher's control, but within those constraints teachers need to be flexible. Teachers need to decide on how the curriculum is going to be delivered. Commonly teachers use a mixture of whole class instruction, while at other times they use small groups. For the purpose of this study I will define small group learning as between two-five students placed together.

Abrami, Lou, Chambers, Poulsen and Spence (2000) defined small group learning, as "a class of students is both physically placed in several small groups and taught accordingly" (p. 160). But the sitting arrangement is only the first characteristic. Abrami et al. (2000) stated several potential advantages for using small group learning in classrooms; students may engage in activities such as explanation of material to other group members, search for solutions to problems and discuss content, be motivated to learn as a cooperative unit rather than as competitors, and finally have the opportunities to develop communication skills through collaboration. Bruner (1996) expressed "there is a mutual sharing of knowledge and ideas, mutual aid in mastering material, division of labour and exchange of roles, opportunity to reflect on the group's activities" (p. xv). The mutual sharing of knowledge is an important aspect of any successful working group. Boaler (2006) found through explaining the task to each other, students gained a deeper understanding and had a movement in their thinking from individual and competitive to one of collaboration and a collective. In trying to understand the place of learning in small groups Abrami et al. (2000) discovered "students in all primary and secondary grades benefited from within-class groupings" (p. 162). Furthermore they found low, medium and high ability students all gained significantly when placed in small groups for learning. Kutnick and Berdondini (2009, p. 71) noted, "small-scale experimental studies show that positive relationships and dialogue among group members support cognitive-based learning". Another feature in small group learning is it is the students that can control their learning. The balance shifts from teacher-student to student-student as they take ownership of their ideas (Blatchford, Kutnick, Baines, & Galton, 2003) and the manner in which the group operates.

These notions have been taken up in the teaching of mathematics. Nelissen suggested, “Learning mathematics is not an individual, solitary activity, but rather an interactive one” (1999, p. 195). Abrami et al. (2000) also commented on this idea by stating that cooperative learning promotes interdependence with students taking ownership of their learning as well as contributing to the group task that has been assigned. “It is up to each group member to make sure that his or her actions help the group achieve its goal” (Abrami et al., 2000, p. 177). It is through this idea of team-work, cooperation and sharing of ideas and strategies that allows for learning to occur in small groups. Nelissen (1999) made the point that mathematics at its best should be an interactive pursuit. By then creating small groups that are collaborative in nature, the students may be motivated to assist their group members and direct their learning towards their intended goal. This idea of working in collaborative group work lends itself to the students participating in conversations about their thinking in mathematics. “Among researchers in didactics of mathematics interested in communication, there is a strong consensus that mathematics can and should, at least partly, be learned through conversation” (Ryve, 2004, p. 157). Boaler (2006) discovered in one of her studies on group work in a US high school that the students displayed concern for not only their own learning but also for their fellow group members. It is then of the utmost importance that each member of the group shares their thoughts and respects the opinions of all.

Hearing, seeing and discussing different viewpoints and perspectives can only be viewed as a positive. As stated earlier, one of Ritchhart’s *thinking dispositions* is to be *open-minded*. This *thinking disposition* is vital if a small group is to learn collaboratively. Boaler (2006, p. 77) found “the act of considering different mathematical ideas in the course of problem solving promotes a respect for and understanding of different viewpoints”. Boaler (2006) further stated that as students listened and contributed to mathematical discussions the students learned to value the contributions, the different methods, strategies and ideas from their classmates. “Many students at Railside talked about the ways in which they had become more open-minded as a result of the practices they learned in their mathematics classes” (p. 77).

Mathematics gives the opportunity for students to become absorbed in the complex and beauty of finding solutions to problems. Through investigation, thinking, discussing and collaborative group learning students can build on their understandings of mathematics. The active use of knowledge, to think with what you have learned should be one of the central goals of education (Perkins, 1992) and this clearly folds into this approach of learning mathematics. If all students who learn mathematics should see themselves as mathematicians then as Burton (1998-1999) noted being a mathematician also means participating in collaborative exploration. This suggested a clear link to the notion of using small group learning in mathematics teaching.

Collis and Romberg (1991 cited in Boaler, 1996) presented mathematics as an exciting field to pursue for students as it encourages them to reason, justify, think critically, solve problems and apply ideas in a creative manner. Thinking and understanding the mathematics involved in problem solving allows students to reflect on their own ideas and structures. Boaler (2002, p. 14) suggested “in these different characterizations of the mathematical work employed by students and mathematicians we also gain an important sense of some mathematical traits that supported the work, including creativity, interest, and inquisitiveness”.

Bearing this review of small group learning in teaching mathematics, the following questions arise when using this approach:
(a) Are students accepting of other’s ideas and thinking?
(b) Does this assist in his or her own understanding of mathematics?

2.4 Conclusion

In this chapter some of the pertinent research literature that deals with problem solving in mathematics, general thinking strategies that can be taught, and the use of small groups in teaching mathematics has been reviewed. Clearly the central overlapping ideas of these three literatures call for more focused teaching that promotes students' thinking deeply in their learning of mathematics: a distinct divergence from traditional ways of teaching mathematics.

The overall research question on which the following study is focused is:

> How do students employ thinking routines and strategies, and mathematical problem solving heuristics when involved in mathematical problem solving in small group learning?

During the literature review a number of questions, linked to the overall research question, surfaced which lead to the following research questions:

1. When solving mathematics problems do students use the taught thinking routines?
2. When solving mathematics problems do students use the taught mathematical problem solving heuristics?
3. When solving mathematics problems do students use ideas developed by other students?
4. How do students perform in problem solving?

These questions will be addressed in the research study, which is the basis of this thesis.

With these ideas, the next chapter presents the aims of the study, describes the students, the setting, and their learning experiences, and explains the research methods used. The research instrument used is presented and the methods by which the results were analysed.

Chapter Three
Research Methodology

3.1 Introduction

The preceding chapter examined the pertinent research literature related to the issues explored by this study and lead to key the research questions. The core issue for this study is:

> How do students employ thinking routines and strategies, and mathematical problem solving heuristics when involved in mathematical problem solving in small group learning?

Subsequent research questions, derived from the review of the research literature, which are the foci for the data collection, are:

1. When solving mathematics problems do students use the taught thinking routines?
2. When solving mathematics problems do students use the taught mathematical problem solving heuristics?
3. When solving mathematics problems do students use ideas developed by other students?
4. How do students perform in problem solving?

This chapter describes the research design and methods chosen for the research.

3.2 Research Methods

3.2.1 Qualitative Research Methods

The data collecting strategies selected for this study were steered by the research design. The data collection through the use of multiple methods was the adopted approach for this study. This is consistent with the theoretical perspective of the case study methodology. Further, multiple methods allows for methodological triangulation of findings, which helps ensure trustworthiness according to the data (Hitchcock and Hughes, 1995).

According to Merriam (1998) the number of participants in a study rests on the data being gathered, the questions being asked, the analysis in progress and the resources available to support the study. The selection of participants in qualitative research can be achieved through the method of non-probabilistic sampling. Qualitative methodology mostly describes, or tries to understand social phenomena. Non-probabilistic sampling allows researchers to explore what occurs, the implications of occurrences, and the relationship, which exists between occurrences (Merriam, 1998). The purposeful selection of students is the most common form of non-probabilistic sampling and was adopted for this study. Purposive sampling is "based on the assumption that the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned" (Merriam, 1998, p. 61) and hence involves the deliberate selection of participants (Creswell, 2008). Given that the purpose of this study was to investigate the mathematical thinking of students when working in small groups to solve various mathematical problem-solving tasks, participants were purposively selected based upon criteria established for the case under consideration.

3.2.2 Case Study

This is a case study of a group of learners in a primary mathematics classroom. The methodology, which seemed very appropriate for this study and sits within the above framework, is that of a case study. As described by Yin (2006) case study research method has the ability to examine in depth a "case" within its real life context: "The case study method is best applied when research addresses descriptive or explanatory questions and aims to produce a firsthand understanding of people and events" (Yin, 2006, p. 112). Hence, this methodology was appropriate in this qualitative research study given the research addresses a descriptive question: How do students employ thinking routines and strategies, and mathematical problem solving heuristics when involved in mathematical problem solving in small group learning?

In order to ensure adequate validity within this model, it is important that the researcher includes multiple sources of evidence. The "main idea is to triangulate or establish converging lines of evidence to make findings robust as possible" (Yin, 2006, p. 115). Convergence occurs when two or more independent sources point to same set of facts. This provided for triangulation and thus increased the credibility of the data being presented. For this reason the methodology includes multiple data sources to give insights into the research question including individual students ideas of thinking in mathematics using concept maps and a descriptive analysis of the thinking involved in small groups when collaborating on mathematical problem solving.

The case study followed a combination of single-case design, combined with comparative multiple-case study of participants and their responses to the teaching context. The single-case study will comprise a rich description of the program including the intended aims of the program and the implemented outcomes. The comparative multiple-case studies will describe the way different students develop and use their thinking strategies while learning mathematics.

3.3 Situational Context of the Study

3.3.1 The School

This study explored the thinking within a primary school mathematics classroom. The setting was an independent private school on the Mornington Peninsula, Victoria. Terry College* (pseudonym) teaches students from Early Learning Centre (ELC) to Year 12. Terry College is separated into two houses. Blue House* (pseudonym), the secondary school, follows the Victorian Curriculum from Years 7 to 12, is girls' only and offers boarding to both Australian and international students at secondary levels. White House* (pseudonym) is the junior branch of Terry College, is co-educational, and begins at the ELC with students aged 3 through to 12/13 when they are in year 6. Throughout this study the primary or junior school of Terry College will always be referred to as White House. In White House there are 260 students and 40 full time and part time staff.

White House is an accredited school of the International Baccalaureate (IB). The IB is a non-profit educational foundation (IBO, 2012). The IB mission statement is as follows; "The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect" (IBO, 2012). The IB offers three sequential programs for students aged 3 to 19. In particular they include the Primary Years Program (PYP) for students aged 3 to 12 and "focuses on the development of the whole child in the classroom and in the world outside" (IBO, 2012). It emphasizes the importance of transdisciplinary learning, recognising that learning should be authentic and students need opportunities to learn beyond the boundaries of separate subject areas. Hence when the opportunity arises, the individual subject area of mathematics is woven into and through the various transdisciplinary themes and units of inquiry taught through the year. Through this program of inquiry students explore challenging concepts, realising and respecting that the world they live and participate in, is enriched by the diversity of cultures and the multiple perspectives they bring.

Another key aspect of the IB to note in this context are the ten IB learner profiles. These are a set of ideals and traits that schools and teaches use to inspire and motivate learning within their school. IB learners strive to be: inquirers, knowledgeable, thinkers, communicators, principled, open-minded, caring, risk-takers, balanced, and reflective (IBO, 2012). Figure 3.1 shows the Learner Profiles displayed in the classroom. The Learner Profiles are displayed at the front of the classroom in a position the students are able to clearly see at all times.

Figure 3.1. PYP Learner Profiles

The importance to this study is that the learner profiles share some similarities with the thinking dispositions outlined earlier in chapter 2. Figure 3.2 displays the *thinking dispositions*. The *thinking dispositions*, discussed in chapter 2 but repeated here, are: metacognition, open-minded, sceptical, truth seeker, strategic, and curiosity. The overlap between attributes of the learner profiles and dispositions that are relevant to this study are reflective / metacognition, inquirers / curiosity, thinkers / strategic, open-minded, sceptical and truth seeker. Hence there is, or should be, a mutual reinforcing dynamic between these core aspects of the school. Hence it is not surprising that a variety of *thinking routines* (Ritchhart et al., 2011) have been implemented across many curriculum areas in the school to assist the students in their learner profiles and *thinking dispositions*.

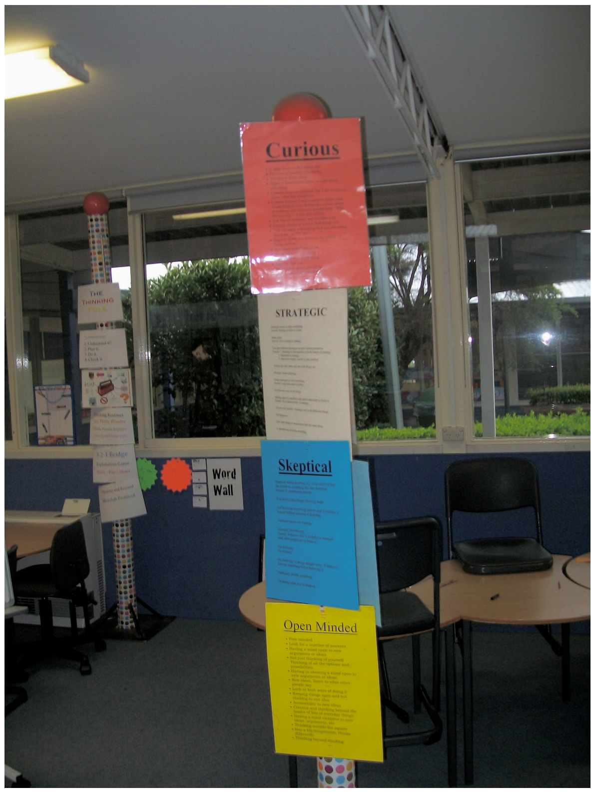

Figure 3.2. Thinking Dispositions.

Furthermore the mathematics problem solving heuristics, discussed in chapter 2, are displayed in a prominent space at the front of the classroom. At the beginning of the school year, mathematic tasks are delivered to the students so that they can apply the problem solving heuristics. Mathematic tasks are selected so a variety of problem solving heuristics could be used in completing the task. Students share their methods and the problem solving heuristics they employed through whole class discussions. Once again all the strategies are clearly visible for the students to see and refer to while attempting problem solving. As the classroom teacher I also refer to the strategies during lessons and feedback sessions with the students. Figure 3.3 displays the ten problem solving heuristics.

Figure 3.3. Problem Solving Heuristics

3.3.2 The Classroom Context

The seventeen students, 12 girls and 5 boys, who are the focus of this study, were in Year 2. The ages of the student range from 7 to 8 years of age. The teacher is the researcher and teaches the students across all areas of the curriculum.

As in most classrooms, visual displays are an important aspect in a primary school setting. The displays in this classroom acted as reminders and as a stimulus for the students, as well as presenting material related to the learning of the students at that particular moment. Hence thinking dispositions, thinking routines and problem solving heuristics were displayed, as well as the Primary Years Program (PYP) learner attributes. Figure 3.4 displays the PYP Learner attributes.

Figure 3.4. PYP Learner Attributes.

It was important for me as I was developing a thinking culture within the class that the displays were referred to and became an active rather than passive part of the class. Therefore any opportunity to insinuate the language of the routines, dispositions, problem solving heuristics and attributes into the normal course of my daily teaching was ensured as I often used the displays as a focus point for the discussion. At the beginning of the year this may have felt forced but it was important for the students to learn and understand the vocabulary and to be able to participate and verbalise their ideas and thinking in a common language. In Figure 3.1, 3.2, 3.3 and 3.4 the displays shown are static and are not added to. This is in contrast to the thinking poles, and mathematics and inquiry-learning wall in the classroom.

Two thinking poles were set up in the classroom. The thinking poles are display tools, employed to direct, remind and add new routines used in the classroom. I have designed both poles. Thinking pole A, (Figure 3.5) and thinking pole B, (Figure 3.6) are movable poles that allow me as the teacher to make them as visible as the need arises, either to the whole class, or to specific small groups of students.

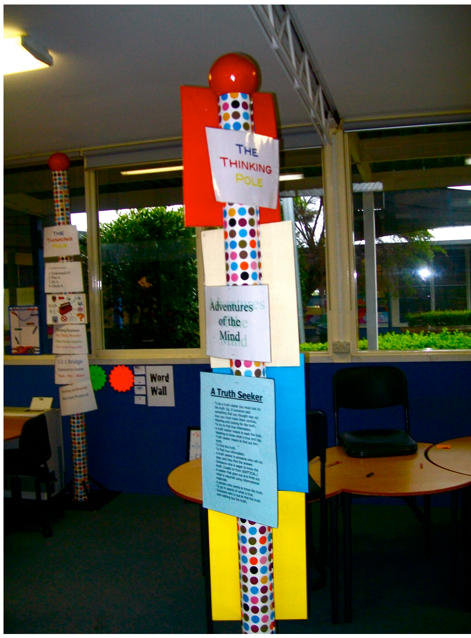

Figure 3.5. Thinking Pole A

Figure 3.6. Thinking Pole B

Each pole focuses on a different aspect of thinking. On Pole A, the *thinking dispositions* as outlined earlier in the study and the result of a brainstorm of ideas on these dispositions by students were displayed. Such a display, rather than showing just a textbook definition of the disposition, or just the names of the dispositions, was important because it was one way students gained ownership of their knowledge. It also enabled me as the teacher, and the students, to refer directly to how they thought about dispositions. Polya's model of problem solving see, plan, do and check (Polya, 1957) was shown on Pole B and provided a visual reminder of these four steps. A toolkit of the problem solving heuristics such as using concrete materials, drawing diagrams, creating tables, making a list, trial and error processes, working backwards, looking for patterns, writing algorithms or equations, acting out the scenario and solving a simpler related problem (Jones, 2003) was also displayed on this pole. Wong (2008) stated that the use of these heuristics allow the students to solve different types of problems. A variety of *Thinking*

Routines often employed within the classroom can be found also on Pole B. Figure 3.7 displays a close up shot of the toolkit and the *thinking routines*.

Figure 3.7. Thinking Pole B

The thinking poles were often referred to and were a visible reminder to the students of the thinking employed within the class. They allowed the students to use the vocabulary of the dispositions, routines and problem solving heuristics within class when discussing their ideas and thoughts. To some degree the thinking pole allowed the learning to be visible, to be shared through discussion.

The learning wall was a tool employed by myself to elicit questions and learning from the students during a unit of mathematics. The students were familiar with the learning wall, as they had been engaged with it during units of inquiry. The students had the opportunity to transfer this knowledge to the context in mathematics.

In general, mathematics lessons were of one-hour duration. Lessons were normally arranged so students had the opportunity to explore their thinking within the designated mathematical concept. Each lesson had three main components. First, with the purpose of tuning in and engaging the students, a starter activity was employed. The activity proceeded for between 5-15 minutes and frequently was directly related to the main teaching activity for the lesson. Activities range from skip counting, finding equations with the same answer, and listing characteristics of shapes, chance and data activities and operation games. The aim for this stage of the lesson was that all students were engaged and not 'sitting out' and becoming dis-interested. For example, the game tic-tac-toe was employed for students to grasp an understanding about tactics, to be reflective thinkers and to be open-minded in the ways they approach the game. Clarkson noted that through playing mathematical games, students can be encouraged to verbalise their strategies with others (Clarkson, 2008). "These situations can be used by creative teachers as wonderful opportunities for students to think deeply about mathematics and how it works" (Clarkson, 2008, p. 33).

The main teaching focus of a lesson was explored before students worked in small groups or individually. The main teaching focus could be; an explicitly taught concept, instructions or reminders on previously taught concepts, or a teacher led discussion to further build on prior understandings. The third component of each lesson was time for student discussion to take place. This gave students the opportunity to reflect on their own learning and listen to their peers. Both of these as this promoted the learning and implementation of the *thinking routines*, dispositions and problem solving heuristics. This discussion and reflection took place during the lesson at apt times, or at the end of the lesson as a summary.

Over the course of the year the students had the opportunity to work and learn in a variety of situations. The participants worked individually, in pairs, in small groups and as a whole class. As noted in chapter 2, Abrami et al. (2000) define small groups as "a class of students is both physically placed in several small groups and taught accordingly" (p. 160). Other aspects of small group learning more fully discussed in chapter 2 were also implemented. Importantly within the culture of my classroom, the ideas of collaboration, and collegiality were often referred to and modelled. The idea of collaboration and discussion enabled the students to learn from each other and gave all students the opportunity to express their mathematical thinking. My approach as the teacher was that "learning mathematics is not an individual, solitary activity, but rather an interactive one" (Nelissen, 1999, p. 195). Abrami et al. (2000) commented further on this idea that cooperative learning promoted interdependence with students taking ownership of their learning as well as contributing to the group task that has been assigned. This approach was central to my teaching.

3.3.3 Groupings

For this particular study the students remained in the same group for the four problem solving tasks (to be described later). This is not an unusual occurrence within the class. Students had been in like ability groups in other curriculum areas depending on their strengths and weaknesses. But I also used mixed ability groups depending on the activity or concept being taught. I decided from trial appropriate ways of grouping students earlier in the year that using mixed ability groups, and to not change the membership of the groups for the duration of this study would be the best. This meant time was not lost by students looking for a group to work with each lesson. I also hoped that by keeping a constant membership a positive learning environment where the participants became familiar with each other's learning styles would develop and they would be comfortable in sharing their ideas. This proved to be the case. The allocation of student groups is explained next.

Within this study the 17 participants were grouped in three groups of three and two groups of four. Group 1, 2 and 4 had three students whereas Groups 3 and 5 had four students. Due to the number of participants, an even number of students across all groups would not be possible. I decided not to organise a group with five students, therefore I decided to pursue the groupings I stated above.

Group 1 contained four students from the earlier trial period but was changed to three students. Two of the four students, John* (pseudonyms are used in referring to all students in the study) and Lucy were from the original group and they demonstrated from the earlier trial period that they would work well together. Anna was added to this group. The three students were very active in voicing their ideas verbally and I thought this would be a strong group both mathematically and also have good in-group dynamics. All three students had demonstrated very good mathematical ability, working with the thinking routines and employing various heuristics during problem solving.

Group 2 contained three students for the study. The three students, Steve, Cath and Ariel, chosen for this group were members of different groups in the earlier trial period. All three students were considered good group members from previous groups. They were very active in voicing their ideas verbally, and on the whole were good listeners. I therefore thought this would be a strong group both mathematically and also have good in-group dynamics. Two of the three students were above average in their mathematical knowledge and skills while the third student, although not as good mathematically was an active participant in mathematic activities. All three students had demonstrated a knowledge and understanding when employing the thinking routines and heuristics during problem solving. Steve and Cath participated in the one on one interview that will be discussed later in this chapter.

Group 3 consisted of Eddie, Val, Andrea and Mary. Three of the four students, Eddie, Andrea and Val had been in the same group for the earlier trial period. This was an interesting group as they more often than not displayed less collaboration as a group. Each member was often more concerned to hear their own voice and voicing their own thoughts rather than listening to each other and building on one another's ideas. I was therefore interested to observe (and hear) if the group was going to become more collaborative as the time went by. One of the students was above average in mathematical ability and employed the problem solving heuristics well, but wasn't flexible in listening to other member's ideas. Two students were of average mathematical ability but varied differently in their ability to verbalise their thinking. The last student found mathematics difficult but would always participate in all activities.

Group 4, like group 2, contained three students for the study. The three students, Terri, Grace and Victor, chosen for this group were members of different groups in the earlier trial period. Two students were quiet students but all three were very active in voicing their ideas verbally. All three students had demonstrated a good to very good mathematical ability, working with the thinking routines and employing various heuristics during problem solving. Two of the three students were expected to remain on track and stay focussed during the task while Terri would often become unfocussed. I was interested to observe the dynamics of the group and see how the group would remain focussed for the task. I knew the group had the potential to be a strong group and yield some excellent learning. But I also wondered if the group would be derailed. Terri participated in the one on one interview that will be discussed later in the chapter.

Group 5 contained four students for the study. Two of the four students, Henry and Claire were in the same group in the earlier trial period. Eva was added to the group and Chris was a new member of the class and had not participated in the earlier trial period. Henry was the strongest member of the group in mathematically ability. All four members of the group were strong verbally. Three of the students were confident using the thinking routines and heuristics when problem solving in mathematics. Chris was becoming more confident in the use of the *thinking routines* and heuristics as these were new concepts for her. I was interested in observing to see how the group would function because the group had the potential to produce some rich discussion and solutions to the tasks. Eva participated in the one on one interview that will be discussed later in this chapter.

3.3.4 The Layout of the Classroom

It was explained to the students, prior to the first activity, where and with whom they would be working with during the problem solving tasks. The groups worked in the same spaces each time. During the group sessions, each group sat on the floor in the class, so they could communicate with each other and all students could have access to all materials.

3.4 Data Collection Methods

The issue for this study and the research questions, which were to be the foci for this study, were presented at the beginning of this chapter. In order to gain some insight into these questions a variety of data was collected. This study included responses to a concept-mapping task, group solutions for problem tasks, and audio recordings of groups during the problem tasks and one-on-one interviews for four students at the completion of each problem-solving task. Before detailing each data collection method it is appropriate to give an overview of the teaching sequence and the timing of each data collection.

3.4.1 The Teaching Sessions

Data collection was undertaken over two weeks during term 4 of the year (four terms in a year). The data collection was focused on one mathematics session, twice a week for the two weeks. Each of these lessons used the same teaching sequence and focussed on one open ended problem that all the students in the class attempted. The students, as noted earlier always worked in the same small groups as they attempted to solve each problem.

Each student was assigned a number code and a colour for identification for later analysis. The students were reminded not to write names on any pieces of paper. When students were completing any written work as a group, they were asked to do so in the coloured pencil assigned to them. I decided to use the coloured pencil approach rather than each student writing in the same colour, so that I could distinguish each student from each other. The use of the same coloured pencil allowed me later to analyse what each student was writing and also how much each student was writing when contributing in the group problem-solving task.

It was important for the study that the directions given to the participants at the beginning of each of the four problem solving sessions were consistent each time. Each group was seated with the necessary equipment to begin the session. Once the groups were settled, I read out the problem twice (Table 3.1). I did this to make sure the groups were focussed and they heard clearly the problem posed. For this study, I told the students that unless each member of the group did not understand the task they were not to ask me any questions. It was important for the groups to work their way through understanding the problem in front of them.

Table 3.1: *Phases of the Task Solving Session.*

Order	Item
1	Collection of A3 paper for concept map and group solutions.
2	Presentation of problem (read twice)
3	Completion of 'See Think Wonder' thinking routines concept map.
4	Completion of group solutions
5	One on One Interviews – Four students.

The groups were presented with two A3 pieces of paper to complete their concept map and group solutions for the problem tasks at the beginning of each session (see later for details). Figure 3.8 *See Think Wonder* concept map and Figure 3.9 *Group Solutions* sheet that included a written version of the task were given to each group at the beginning of each session. A3 paper was decided upon, as it would give the students the best opportunity to all write on. The groups used the paper provided to work out as many solutions as possible in the time provided.

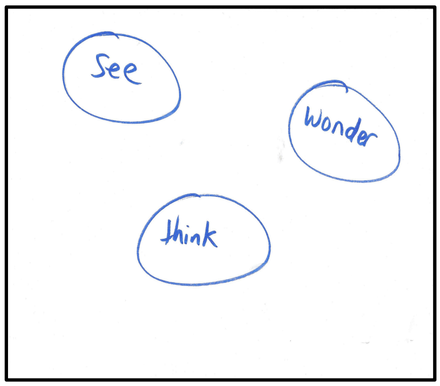

Figure 3.8. See Think Wonder Concept Map.

> Inside the bucket there are 20 balls. Some are red, some are blue and some are green. The task, is to work out how many might be red, how many might be blue and how many might be green?

Figure 3.9. Group solution sheet of the problem presented to the group for The Balls Task

In the following sections an outline of the different data collection strategies that were applied in each of the four sessions are given. In each session, the same consistent approach to the order of the session was employed. As noted in Table 3.1, one student from each group collected the A3 pieces of paper that students would be writing on. Once both papers were arranged in front of the groups, I read aloud the problem twice. I did this to be sure each group heard the problem. The students were then asked to complete the *See Think Wonder thinking routine* as a group. This was the concept map the students as a small group were asked to complete. The *See Think Wonder* routine was employed so the students would analyse the problem before attempting solutions. As outlined in chapter 2, the *See* part allows the students to locate important information within the problem and then by writing on the paper, making their thinking visible. The *Think* component allows the students to make connections to other relevant parts of mathematics and the *Wonder* invites the students to ask questions concerning the problem.

As shown in Table 3.1, the fourth part of the session involved the students completing solutions to the problem. The final component of each session were the One on One interviews conducted with four of the students. The students were Steve and Cath (Group 2), Terri (Group 3) and Eva (Group 5).

3.4.2 The Problem Tasks

As noted, one problem task was presented to the group in each of the four sessions. I decided for this study that it was important the tasks were open-ended since I wanted to observe if the students were able to identify patterns when working out solutions for the problems. This would suggest whether the students were working in a logical manner or in a random manner. I also wanted to see the number of solutions the students could arrive at. The tasks presented to the students were all number based. Throughout the year the students had completed open-ended problems in shape, time, and chance and data so they were use to completing this type of problem. The problems presented are shown in Table 3.2.

Table 3.2: *The Four Problem Solving Tasks*

Task Name	Task Question
The Balls Task	Inside the bucket there are 20 balls. Some are red, some are blue and some are green. The task, is to work out how many might be red, how many might be blue and how many might be green?
The Tyres Task	My brother Jack used to sell tyres for motorbikes and cars. One day he had a sale and sold 86 tyres. How many cars and how many motorbikes may have received new tyres?
The Legs Task	There are 36 legs in the field. There are some horses and ducks in the field. How many horses and ducks are there?
The Darts Task	List possible combinations you could get with 4 darts. The numbers on the targets are 7-5-3-1.

All four of the problems were modified from a chapter in, *Open-ended Maths Activities: Using Good Questions to Enhance Learning in Mathematics* (Sullivan & Lilburn, 1997). Different versions of all four of the problems had been presented to the students at various stages of the school year during mathematical lessons. *The Balls Task* was the first problem presented to the students for the year. When first presented to the students it was 15 beach balls. I selected this problem because I knew the students would feel comfortable with the problem and I was interested to see if the groups could identify a pattern when solving the task. I had a good idea that each group would find many solutions but I was wondering if it would be completed in a random or rational way.

In *The Tyres Task*, when first presented to the students earlier in the year, the number of tyres was 30. By the time of the study, the students had completed a unit of learning on multiplication and division. I selected this problem because although it was a combination problem it also had a multiplication tone to it. I was interested to see if the students would apply their understanding and knowledge of multiplication tables to help them solve this problem.

In *The Legs Task* the students had been presented with two types of the heads and legs problems throughout the year. The first one was similar except for the number of legs and was open-ended and allowed for multiple solutions. The second type of heads and legs problem, presented both the number of heads and the number of legs seen in the paddock. It was hoped the students would realise the problem in the study was of the first type. This is virtually the same problem as in *The Tyres Task*. In retrospect I probably should have presented this problem first, so that the groups could build on the understanding they gained from completing the problem and applying it to a larger number. Figure 3.10 illustrates a group solution to *The Legs Task*.

Figure 3.10 Group Solution to The Legs Task.

The Darts Task, was another combinations type of task for the students. The students had completed a number plate challenge earlier in the year, rearranging four letters to make as many numbers as possible. In that problem, each letter could only be used once in each combination. In *The Darts Task*, I was interested to see if the students who had a good knowledge of combinations would make the leap that a combination could be 7,7,7,7.

3.4.3 Concept Mapping

Concept maps were chosen as a tool for unearthing students' thinking about their mathematical thinking. This is a tool that has been often employed for this task in the past. Hence Roberts (1999) has noted, "A concept map is a diagram intended to illustrate the understanding of the relationships between concepts involved with a particular area of study" (p. 707). As well Afamasaga-Fuata'i argued that concept maps are a "means of assessing students' conceptual understanding, fluency with the language of mathematics and critical thinking in problem solving" (2008, p. 8). The importance of concept maps can be viewed in the light of allowing teachers to see and explore the "potential directions or avenues for students' investigations and further facilitate our anticipation of the possible avenues that student discovery will assume" (Anwar & Iqbal, 2005, p. 62).

In this study the students were required to use concept maps to complete within the small group collaboration on a variety of problems posed. The reason I decided for the concept maps to be completed in a group setting rather than individually is due to the third research question. It was my intention for the students to work as a group and build upon each other's ideas and mathematical understandings. The concept maps were an opportunity to visually represent the thinking of the students in the moment of completing the tasks during the sessions. I believed this was an important element of the study to pursue as it was documenting what the students were thinking of during the process of the tasks.

Each group was to complete a concept map during each task. As shown in Table 3.1 the concept map was to be completed at the beginning of the session. Figure 3.8 shows the *See Think Wonder* concept map the groups would complete.

An analysis of the concept map would allow me to collect data, which would facilitate a deep investigation of the mathematical thinking of the students during the sessions. This would give insight particularly for the first three research questions as follows:

With respect to research question one: When solving mathematics problems do students use the taught thinking routines?
The concept map gave the opportunity for the students to list the important elements of each problem by examining the question through the lens of the *See Think Wonder* thinking routine. When examining the data I was able to explore the ideas if the students were able to identify the important elements of each problem within the *See Think* part of the routine. Also I was able to examine the questions or *Wonders* the groups raised in the initial stages of the session.

With respect to research question two: When solving mathematics problems do students use the taught mathematical problem solving heuristics?
The concept map gave the opportunity for the students to list the variety of problem solving heuristics they were employing during that particular session. When examining the data I was able to explore the ideas if the same heuristics were been used in each session or if the groups were employing different heuristics depending on the task.

With respect to research question three: When solving mathematics problems do students use ideas developed by other students?
The concept map gave the opportunity to the students to build upon each other's ideas. As mentioned earlier, each student in each group was given a colour code for the pencil they would use during each session, so it was clear on the concept map to see students adding learning to each part of the map.

3.4.4 Concept Map – See Think Wonder

The first concept map the students created together was to use the thinking routine *See Think Wonder* (Figure 3.6). As noted in Table 3.1 the concept map was the second part of the session for the students following me reading the problem to them. The concept map was employed by the students to unpack the understanding of the problem in the following way. The use of the *See Think Wonder* uses the three questions:

- What do you **see**?
- What do you **think** is going on?
- What does it make you **wonder**? (Ritchhart et al., 2011)

Hence the use of the *See Think Wonder thinking routine* was intended to assist groups to understand the problem and decide on the way they should proceed with it. I shared with the groups that they should first complete the *See Think Wonder* concept map. The groups were told that they should only proceed onto the solution sheet (see later) once they were satisfied with their *See Think Wonder* concept map. Figure 3.11 shows a *See Think Wonder* concept map completed by one of the groups for *The Balls Task*.

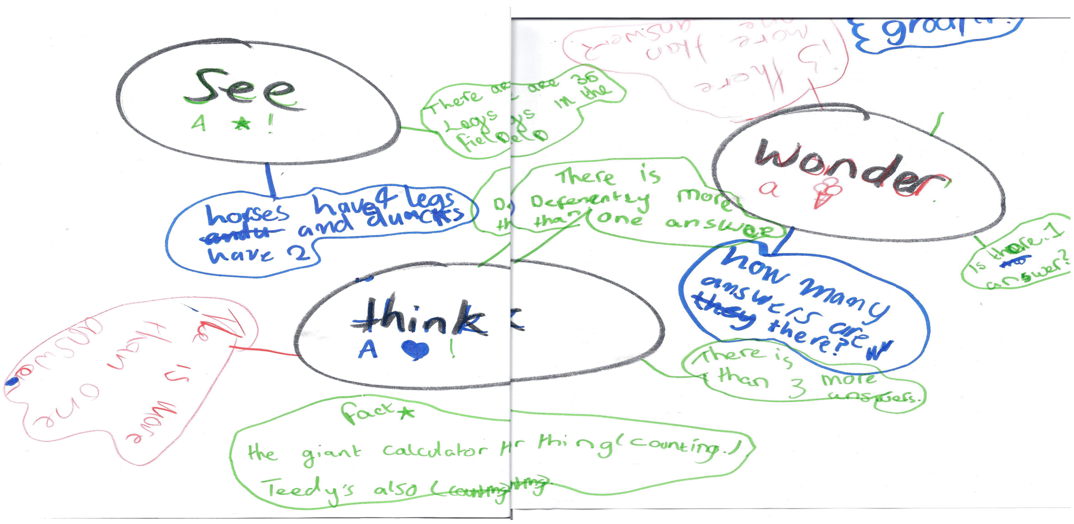

Figure 3.11 Concept Map: A completed concept map for a task.

3.4.5 Task Solutions

A solutions sheet with the task written clearly at the top of the page was provided to each group at the beginning of each of the four sessions. The solutions sheet was of A3 size. The solutions sheet had two clear functions: first to provide a reasonable amount of space for the groups to 'work out' their solutions, and secondly to provide a reasonable amount of space for the groups to write their solutions. It was decided that both the working out and solutions would be represented on the one page. The solutions sheet was important as it provided a clear 'window' into the number of solutions the groups were able to achieve. Problem solving at its most basic form is finding solutions to a task. In this study the solutions sheet is of the utmost importance as it displayed the thoughts and working out completed by the group but also the number of solutions achieved by the group.

An analysis of the solutions sheet would allow me to collect data, which would facilitate a thorough examination of the mathematical thinking of the students during the sessions.

This would give insight particularly for the second and fourth research questions as follows:

With respect to research question two: When solving mathematics problems do students use the taught mathematical problem solving heuristics?
The solutions sheet gave the opportunity for the students when solving the problem to employ any of the problem solving heuristics to assist the group during that particular session. When examining the data I was able to garner a clear idea what heuristics were been used in each session and / or if the groups were employing the same and / or different heuristics depending on the task.

With respect to research question four: How do students perform in problem solving? The solutions sheet gave the opportunity for the students to complete as many solutions as possible in the given time. When examining the data I was able to obtain a clear understanding of the number of solutions the groups had achieved. I was also able to see the methods the groups used to obtain the answers.

3.4.6 Audio Recording of Group Discussions

Due to the qualitative nature of the study, the students presented their ideas and thinking through the group concept map, the solution sheet, the *thinking routines* and discussion. The discussion is an important indicator of the groups' thinking when they recorded on the map. Hence collecting this audio data would add extra information to the concept map and solution sheet and the thinking of both the individuals and the group. Indeed some aspects of their thinking may become evident that is not recorded on the concept map or solution sheet at all. Therefore audio recording the groups' discussions while completing the tasks would capture this aspect of the mathematical thinking of the students.

Although the students had not been recorded before the commencement of the study, the students were familiar with the small group-learning environment. In the normal practice of the classroom the students were accustomed to having photographs taken of their learning and also having other teachers listening to their discussions. Hence I assumed audio recording their discussions would not impact noticeably on their behaviour, and this seemed to me to be the case. The features discovered in transcripts of recorded talk are a product of "close, repeated listenings to recordings" (Silverman, 1993, p. 117). The longevity of the recordings allowed for multiple listens for transcribing of the discussions and their thorough analysis. Furthermore, it allows other researchers access to the data.

3.4.7 One on One Interviews

I decided to interview four students at the completion of each problem solving session (Phase 5, Table 3.1). I knew from a logistical point of view, that interviewing all 17 students directly after each session would be extremely difficult. I believed by interviewing the same four purposively selected students after each problem solving session I would garner a snapshot of student's thinking. The purpose of the one on one interviews with the four students was to give an opportunity for the students to talk about their involvement and thoughts about mathematics problem solving, what concepts they were learning and employing during the problem solving sessions, the thinking routines used by the group, the group dynamics and the type of thinking employed by themselves and of the group. The data from the interviews related directly to research questions 1, 2 and 3.

The interviews were audio recorded. I initially thought the interviews would be structured (Boaler, 1996). In structured interviews, "the interviewer asks all respondents the same series of pre-established questions" (Fontana and Frey, 2005, p. 701-702). However after the first session interviews I moved into a semi-structured interview technique. This was done because at first, as the teacher-researcher I was very mindful of keeping a distance from the students and secondly, I moved into a semi-structured

interview because I discovered the students weren't elaborating their responses. Below are two excerpts of the one on one interview conducted with two of the students. The first example illustrates the more structured interview that was employed in Task 1.
Researcher - Can you say anything at all about how you solved the mathematics problem solving tasks this morning?

Student - Um, with maths strategies, like addition and subtraction and all that.

Researcher - Can you say more about what you were thinking during the mathematics problem solving tasks?

Student - About the problem?

Researcher - Yes, about anything.

Student - How to solve it.

Researcher - How did that go across your thinking?

Student - I don't know.

The second example illustrates the semi-structured interview employed in the final three tasks.
Researcher - Can you tell me what you think the main maths idea was in that activity you did today?

Student - Probably just to do with numbers.

Researcher - What do you think you had to do with the numbers?

Student - Make all the possible answers.

Researcher - Here you've got from your thinking routine, one thing I learn was to always do it in order. Why was that important to do?

Student - Because if you didn't do it in order it would take longer and you might accidentally do the same answer.

Researcher - Can you give me an example of to do it in order?

Student - First start with all the possible ones with the one at the start and then you can to do it at the start.

Each of the 17 participants was given a number from one to seventeen. The four students, who were interviewed, were done so in number order after each problem solving session. The questions used were a collection of questions I had gathered and devised myself over the last 8 years as shown in Figure 3.12.

Clarification Questions

- What do you mean by your thinking?
- Can you say a little more about how you solved the mathematics problem-solving task?
- Can you say a little more about what you were thinking during the mathematics problem-solving task?
- Can you give me some examples?

Using the 4 Ones Thinking Routine responses as a prompt

Can you add anything further to the 4 ones Thinking Routine:

- One thing I learnt was…
- One connection to mathematics was…
- One question I still have is…
- One thing I could add is…

Using the group See Think Wonder Thinking Routine as a prompt

- Can you add anything further to the See Think Wonder Thinking Routine?

Further information Questions

- What about the problem solving strategy you used?
- Does that apply to…?
- Is talking about the problem in a small group an advantage?
- Can you give me some examples?
- What did you like about working in your group?

Comprehensiveness Questions

- Have you anything more to say on the concept map?

Figure 3.12 Questions asked in one on one interview.

Students were selected for the interviews on evidence gathered throughout the year on their ability to think and to clearly convey their thinking. In selecting the students one main criterion was used. The main criterion was that the students were to be able to verbally convey their ideas and thoughts. I also wanted four students who were different in personality. The four students selected for the interviews were Steve, Eva, Cath and Terri.

Steve had been at the school since prep. Steve had shown an aptitude to verbalise his thinking and share his thoughts and ideas with the class. During whole class discussions across all areas of the curriculum Steve would take his time before he spoke. This often meant he showed a deeper understanding and appreciation of the topic. Steve enjoyed mathematics and was his favourite subject at school. Steve had worked with the thinking routines in his previous year, although the majority of these times were not in mathematics. He had a good understanding of employing the problem solving heuristics. He enjoyed working in small group learning and participated well. He shared his ideas and listened to the ideas of his peers. He was a quietly spoken student whose confidence developed as he shared his thinking.

Eva had been at the school since prep. Eva was content to share her thoughts with the class and took her time to think, showing some rich thinking with her answers across all curriculum areas. Eva enjoyed mathematics. Eva had worked with the thinking routines in her previous year, although majority of these times were not in mathematics. She had a good understanding of employing the problem solving heuristics. She enjoyed working in small groups and participated well. At times she could become unfocussed with an activity across all areas of the curriculum. She was a quiet and intelligent student.

Cath had been at the school since prep. Cath liked to share her thoughts and various 'stories' with the class across all curriculum areas. She was often slow or took her time verbalising her thoughts but had shown the ability to reason and think. Cath would often be sharing thoughts with the class when the discussion was ready to move on and had indeed moved on to the next part. This had shown that Cath was thinking deeply about her answers. Cath enjoyed mathematics. Cath had worked with the thinking routines in her previous year, although majority of these times were not in mathematics. She was still developing her understanding of employing the problem solving heuristics. She participated well in whole class and small group learning. At times she became unfocussed with an activity across all areas of the curriculum.

Terri was new to the school. Terri liked to talk and discuss ideas. Due to her thinking and verbalising her thoughts in the classroom I believed she would be good candidate for the interviews. Terri enjoyed mathematics. Terri had not worked with the thinking routines prior to her attendance at the school. She had a good understanding of employing the problem solving heuristics. She found it difficult at the beginning of the year to work collaborative in a group situation. She would often not listen and was easily distracted. As the year progressed and she got used to the routines and the classroom culture of collaborative learning, Terri became a valued member of her group. She had an outgoing personality who liked to be part of discussions across the curriculum.

3.5 Data Analysis

3.5.1 Analysis of the Number of Solutions

Analyses of the solutions focused on the type and number of solutions produced. In the analysis of the number of solutions produced by the groups for each task, I decided the solution could be labeled within three distinct categories. The solutions provided were classified as either correct, incorrect or off task. The first two categories I believe are self-explanatory. The off task solutions was selected because it gave insight into the thinking of the group. The off task solutions were any solutions that did not fit the task properly but were supplied by the groups. In The Balls Task:

a correct solution would be nine red balls, seven green balls and four blue balls;

an incorrect solution would be nine red balls, seven green balls and three blue balls;

an off task solution would be twenty-four red balls minus five green balls plus 1 blue ball.

3.5.2 Analysis of the Thinking Routine: See Think Wonder

In the analysis of the *See Think Wonder* thinking routine it was important to classify the aspects of the group thinking that was going to assist the groups to solve the task. Each of the three elements: *See, Think, Wonder* were analysed separately. The *See* element allowed me to focus on if the groups could recognise and then list the important elements of information from the written word. Each problem was displayed or written at the top of the problem solving sheet of paper, so that the students not only heard my voice read out the problem but also had the problem written and presented to them. Table 3.3, 3.4, 3.5 and 3.6 display the key elements in the *See* component for the four tasks.

Table 3.3: *Key Elements in See Component in The Ball Task*

Key Elements			
Colour (Red)	Colour (Green)	Colour (Blue)	20 Balls

Table 3.4: *Key Elements in See Component in The Tyres Task*

Key Elements		
Car	Motorcycle	86 Tyres

Table 3.5: *Key Elements in See Component in The Legs Task*

Key Elements		
Horses	Ducks	36 Legs

Table 3.6: *Key Elements in See Component in The Darts Task*

Key Elements					
Darts	Combinations	The Number 1	The Number 3	The Number 5	The Number 7

The number of key elements per task ranged from three to six as shown in Tables 3.3, 3.4, 3.5, and 3.6. Each needed to be identified to be able to successfully solve the problem.

The *Think* element allowed me to focus on if the groups could identify ways that would assist them when solving the problem. Table 3.7, 3.8, 3.9 and 3.10 display the key elements in the *Think* component for the four tasks. The number of key *Think* elements was consistent across all four tasks. All four tasks shared three identical elements; patterns, more than one answer and strategies should be used.

Table 3.7:*Key Elements in Think Component in The Ball Task*

Key Elements			
Patterns	3 colours to be used	More than one answer	Strategies to be used

Table 3.8: *Key Elements in Think Component in The Tyres Task*

Key Elements			
Patterns	Operations can be used	More than one answer	Strategies to be used

Table 3.9: *Key Elements in Think Component in The Legs Task*

Key Elements			
Patterns	Horses have 4 legs and ducks have 2 legs	More than one answer	Strategies to be used

Table 3.10: *Key Elements in Think Component in The Darts Task*

Key Elements			
Patterns	4 darts to be used	More than one answer	Strategies to be used

The *Wonder* aspect of the thinking routine allowed the groups to ponder and ask a question that might assist the group to move forward in answering the problem. The *wonders* were collated and listed for each task.

3.5.3 Analysis of the Problem Solving Heuristics

Data used in the analysis of the Problem Solving Heuristics employed by the groups included: (a) group transcripts of the tasks, (b) re-listening of group audio recording for clarification and (c) the group solution page. This allowed me to identify the various problem-solving heuristics employed by the groups when solving the tasks. Table 3.11 outlines the 10 problem solving heuristics taught to the students during the year.

Table 3.11: *Problem Solving Heuristics*

Problem Solving Heuristics	
Make a Table	Draw a Diagram
Make a List	Write a Number Sentence
Make a Model	Act it Out
Identify a Pattern	Work Backwards
Guess Check Improve	Solve a Simpler Related Problem

The data was scanned for evidence of consideration and or use of any of the ten problem solving heuristics identified in Table 3.11.

3.5.4 Analysis of the Small Group Learning

The methodology employed to analyse the data obtained from the transcribed group data and the interviews (see next sub section for interviews) for this thesis relies on coding techniques associated with grounded theory. Two analytic procedures are basic to the coding process. “The first pertains to the *making of comparisons*, the other to the *asking of questions*” (Strauss & Corbin, 1990, p. 62). The first procedure is *open coding* which fractures the data and allows one to identify some categories, their properties, and dimensional locations.

In the present study, open coding was using the approach described by Strauss and Corbin, “Data are broken down into discrete incidents, ideas, events, and acts and are then given a name that represents or stands for these” (Strauss & Corbin, 1998, p. 105).

In the present study, the observational data was taken apart line by line, or by sentence, paragraph or whole transcription, discrete incident, idea or event and given a name or code word that represented the concept underlying the observation (Merriam, 2009). Code words were selected by the researcher to elicit new insights from the data (Merriam, 2009).

The second procedure, known as *axial coding* puts the data back together in new ways by making connections between a category and its subcategories. Whereas open coding involves the process of fracturing data and exploring the data, *axial coding* put the data back together in new ways to form connections between categories and subcategories to develop several main categories or themes. (Strauss & Corbin, 1990). The second stage involves identifying categories. Categories provide the means to classify the concepts

This type of analysis permitted sense to be made of the mass of data present, and enabled me to focus on the learning of particular interest to this study. Analysis of the data led to the identification of six categories. The six categories used in the analysis for small group learning are as follows: Pulling Up (PU), Clarification (Cl), Processes Fact (PF), Confirmation (C), Reviewing (R), Suggestions (S). The definitions are presented in Table 3.12.

Table 3.12: *Categories Identified in Analysis of the Data Relevant to Small Group Learning*

Category	Code	Definition
Pulling Up	PU	Correcting a mathematical mistake
Clarification	Cl	To be clear on the task
Processes Fact	PF	A mathematical process
Confirmation	C	A corroborative statement
Reviewing	R	An examining of the task
Suggestions	S	Connections between ideas

Elaborations of each category follow the statements clarifying the meanings for each grouping and example from the study.

Pulling Up occurred when one or more students actually corrected a mistake made by another group member and this assisted the group to continue. In any form of problem solving this is a vital stage as groups can make incorrect solutions or incorrect assumptions when working towards the solutions.

Clarification occurred when the students made a statement to make the task clearer or easier to understand for themselves or other group members. This is a vital moment in the understanding of any problem as this gives the opportunity for all group members to be clear about the task and not have any misconceptions. Clarification of the task dissuades the group members going off on tangents during the task due to lack of understanding of the task.

Processes Fact occurred when students made a statement about a number facts displaying mathematical knowledge that would assist the group that could be recalled quickly and efficiently. Processes Fact included the addition, subtraction, multiplication and division of numbers. The application of Processes Facts in this instant enabled students to complete solutions during the tasks.

Confirmation occurred when students made a statement to acknowledge, corroborate or verify a statement made by another group member. This is an important part of the functioning of the group. By group members acknowledging and verifying other group member's ideas and thoughts the group is able to remain on task. Confirmation of the task allows for the group to be considered to be 'on the same page' with each other.

Reviewing occurred when students used the process of going over a task or part of the task again in order to summarise the facts. This is an important part of the group process as it allows the group to stop and take note of the progress of the task. By reviewing or summarising the facts, the group can potential fix a problem if they believe they have encountered one. Reviewing of the task allows the group to critique or evaluate their progress to date.

Suggestions occurred when students suggested something as a piece of advice. This is an interesting part of the functioning of a group because after the suggestion has been made, the group can decide to act on the suggestion or ignore it.

3.5.5 Analysis of the One on One Interviews

In the analysis of the one on one interview data I decided to use the interview data to annotate and help understand the moves made by groups at different points in their engagement with the problems.

3.6 Limitations

When considering the research detailed in this thesis the following limitations and delimitations should be taken into consideration.

1. The research is limited to a single Mornington Peninsula school.
2. The school is an accredited Primary Years Program (PYP) school, part of the International Baccalaureate Organization. Because of this difference with other primary schools some of the language used maybe unfamiliar to other teachers.
3. The study was a small sample size, it has to be acknowledged that due to the sample size it is not sufficient to make conclusive generalisations and hence the inferences had to be limited to observed trends.
4. Teaching styles around the state vary so the findings may be limited to students with teachers having a similar style.
5. This study was conducted in the researcher's own classroom

3.7 Ethical Considerations

Research and participant observation in a classroom situation involves careful planning to address potential ethical issues. As this research involved students, a formal application was made to the Human Research Ethics Committee of the Australian Catholic University (ACU), written permission was obtained from the Head of the School of the researcher, and all ethical norms were carefully adhered to. The letter of Approval for Ethics Clearance was obtained prior to the collection of data (Appendix A). First participation was completely voluntary. Students and their guardians were informed that data collected would be used for research purposes. A plain language statement was issued at the outset to all potential participants and the purpose and method of study was explained to them. A letter outlining the study was delivered to each participant. Every participant and guardian signed a consent form prior to taking part in the study.

Though these steps were taken as a precaution, there was no concern at any stage of this research contravening any ethical norms, as the instruments administered only identified mathematical problem solving strategies in general. Student identity was not revealed at any stage and all data was kept in the custody of the researcher and treated as confidential. The results of this research had no bearing on the performance of the students in their normal day-to-day classroom practice. Any information gleaned was only to improve teaching strategies for the future. In fact the nature of the research was such that most students were keen to participate and determine how small group learning, thinking routines and heuristics assist in understanding mathematical problem solving.

3.8 Conclusion

This chapter has detailed the purpose of the study investigating mathematics problem solving where students employed various problem solving heuristics, the *thinking routine*

See Think Wonder while engaged in small group learning. The situational context and the students participating have been detailed. The data collection methods and analysis tools employed and the rationale for their selection and use are explained. The results of these analyses are presented in the next chapter.

Chapter Four
Results

4.1 Overview

Having designed a study to investigate mathematical problem solving in small groups within a primary school setting and focusing on *thinking routines* and problem solving heuristics, this chapter presents the analysis of the data. The purpose of the group solution sheet was to provide an insight into the number of solutions completed and the heuristics employed when completing the task. The purpose of the concept map was to provide an insight into the students thinking as they completed each problem-solving task. The purpose of the transcripts of each group was to gain an insight into the interactions and machinations of the group as they worked through the task. The aim of the instruments was to investigate the overall ideas that were evident in the review of the literature about the positive aspects of group learning when applied to mathematical problem solving and the inherent positive belief that thinking routines and heuristics assist in mathematical problem solving.

This chapter presents the results of the research. The chapter is ordered around the four problem solving tasks. For convenience, the four tasks the students completed are presented again in Table 4.1

Table 4.1: *The Problem Tasks Presented to the Groups.*

Task Number	Task Name	Task Posed
1	Balls Task	Inside the bucket there are 20 balls. Some are red, some are blue and some are green. Their task, is to work out how many might be red, how many might be blue and how many might be green?
2	Tyres Task	My brother Jack used to sell tyres for motorbikes and cars. One day he had a sale and sold 86 tyres. How many cars and how many motorbikes may have received new tyres?
3	Legs Task	There are 36 legs in the field. There are some horses and ducks in the field. How many horses and ducks are there?
4	Darts task	List possible combinations you could get with 4 darts. The numbers on the targets are 7-5-3-1.

In describing how the students dealt with each of these problems the results are laid out in the following order:

a. *The See Think Wonder* concept map generated by each group during the initial stage of the task.

b. The transcripts of the group audio recordings were analysed and explored to garner what if any of the problem solving heuristics the groups were employed during the task.

c. The transcripts of the group audio recordings were analysed and explored to gather what if any group work ideas were being utilised by the students during the task

d. The solutions page generated by each group explored the solutions to the mathematical task.

Chapter 5 will draw these results together and focus on the research questions that were central to this study.

4.2 Problem Solving Task 1: The Balls Task

The Balls Task involved the students recognising combinations of numbers that added together to make twenty. Furthermore the groups needed to be conscious that three colours were to be represented in each solution. In this task I believed the actual mathematics of using small numbers was easily within in the reach of the groups whereas the thinking involved was more challenging. It was the idea that if patterns were recognised during the mathematical thinking in the task then a number of solutions would be forthcoming.

4.2.1 See Think Wonder

The first part of the data investigated the information gathered by each group when they employed the thinking routine *See Think Wonder*. As stated, in the literature review, problem solving requires students to have a starting point. Polya's model of *understand* the problem, make a *plan*, *carry out* the plan and *look back* (Polya, 1957) does suggest a starting point. The thinking routine *See Think Wonder* allows for students to have different starting points when they first approach the task as the *See* aspect of the routine reminds the students to focus firstly on the written word of the problem. This is an important element for successful problem solving. The model suggests that students need to first understand what the question is asking and then begin to set a plan into motion that will allow for them to begin exploring possibilities. Therefore the *thinking routine* of *See Think Wonder* was used as a scaffold for the students to focus on the problem and to make initial ideas and thoughts and importantly to write them.

In The Balls Task, there were four elements the students should identify: the three colours; (red, blue, green) and the number of balls (20). These four elements were considered the most important aspects the groups required to *see* to assist them in solving the task. For the group it was essential to know that their solutions would involve three numbers, representing the three colours. For the students it was also vital to know that the three numbers needed to add exactly to 20. Table 4.2 displays the four elements the groups needed to *see* and record at the beginning of the task.

Table 4.2: *Identification of Key Elements in See Component in The Balls Task*

	Key Elements			
Group	Colour (Red)	Colour (Green)	Colour (Blue)	20 Balls
1	✓	✓	✓	✓
2	✓	✓	✓	✓
3	✓	✓	✓	X
4	✓	✓	✓	✓
5	✓	✓	✓	✓

Note: ✓ successfully identified key element on concept map
X unsuccessful in identifying key element on concept map

The results showed that four of the five groups were able to identify or *see* the four key elements of the task. Only one group, Group 3 did not list 20 balls on the *see* part of the concept map.

In The Balls Task, there were four key elements that should be made during the *think* part of the *thinking routine*. The key elements included; that patterns could be used to assist in solving the task, that all three colours need to be used when solving the task, that there is more than one answer to the task and problem-solving strategies can be employed during the task. Table 4.3 displays student group identification or not of the four elements recorded on the concept map during the *think* part of the routine.

Table 4.3: *Identification of Key Elements in Think Component in The Balls Task*

Group	Key Elements			
	Patterns	3 colours to be used	More than one answer	Strategies to be used
1	✓	X	✓	✓
2	X	X	X	X
3	X	X	X	X
4	X	X	✓	X
5	X	X	X	X

Note: ✓ successfully identified key element on concept map

X unsuccessful in identifying key element on concept map

The results showed Group 1 was able to identify more than one key element during the *think* part of the routine. Naming patterns, more than one answer and strategies to be used but overlooked that three colours are to be used during the task. Three groups, Group 2, 3 and 5 were not able to identify any of the important elements. Group 4 were able to list one key element, more than one answer. Group 3 stated that, *I think it is about putting the balls in groups* and the selection of an inappropriate operation, *I think it is about division*.

In The Balls Task, there was not a set number of wonders or questions the students could come up with. This part of the routine allows for the students to engage in discussion and write their responses on the concept map. Again because this routine was employed at the initial stage of the task, the wonderings the groups came up with provided the impetus for further inquiry into the task. Table 4.4 shows the *wonders* the groups discussed and recorded on the concept map.

Table 4.4: *Wonders Discussed and Recorded in The Balls Task*

Group	Wonders
1	How can you get all the answers? How many answers are there?
2	How are we going to do this? I wonder how many there are each?
3	Is there more than one answer?
4	Is there more than one answer?
5	I wonder if we are right? I wonder what the answer is? I wonder if we could do more than one answer?

The results in Table 4.4 showed that each group was wondering and focussed on if there was more than one answer. Two groups, Group 1 and 2 wondered on the mechanics of how they were going to do this? And how can you get all the answers? Group 5 also wondered in a meta-cognitive way if they were right?

When Steve (Group 2) was interviewed at the conclusion of the task, one thing he wondered was how many answers there were? When asked "Do you think if you had more time do you think you could get all the answers?" His response was, "Yeah, probably". When then asked, "Do you have a feeling of how you could get all the answers?" Steve's response was, "Um…. probably do a … No I don't really think so".

When Cath (Group 2) was interviewed at the conclusion of the task she stated, “Because I didn’t think that we would have all day to do it and we wouldn’t be able to have all the answers. I was just wondering why?”

Table 4.5 outlines all the number of instances for each component of the *See Think Wonder* thinking routine employed during The Balls Task. What was of interest was that although only two groups (Group 1 and 4) listed in the *think* component that they thought there was more than one answer, all the groups in the *wonder* part of the routine wondered if there was more than one answer. The *thinking routine* employed for this task has shown the students were able to identify clearly during the *see* part the important key elements of the task but were not as clear in identifying the important elements in the *think* part of the routine.

Table 4.5: *See Think Wonder Thinking Routine Used and Discussed in The Balls Task*

	Number of Instances		
Group	See	Think	Wonder
1	4	3	2
2	4	0	2
3	3	0	1
4	4	1	1
5	4	0	3

When Terri (Group 4) was interviewed at the conclusion of the task she was asked about using the thinking routine *See, Think, Wonder* and she stated, “To help with the problem, so we knew a bit about it more”. She added, “Yeah, to break it up. To make it easier.” When asked about the breaking it up? Terri said, “I’m not sure… just like getting the easier parts first.” Terri finished by saying, “So then you can go on with it like really easy.” The above statements from Terri’s interview showcase that the thinking routine

See Think Wonder did assist the groups with their problem solving. By identifying the main parts or as Terri stated, "to break it up. To make it easier," the groups were able to focus on how to complete the problem and not become absorbed in locating the main parts of the task. At this age level the thinking routine *See Think Wonder* assists students to locate the important information of the task so they can then begin to solve the task.

4.2.2 Problem Solving Heuristics

Polya's model suggested using strategies such as using concrete materials, drawing diagrams, creating tables, making a list, trial and error processes, working backwards, looking for patterns, writing algorithms or equations, acting out the scenario and solving a simpler related problem (Polya, 1957).

In The Balls Task, each student group executed the strategies they considered through their discussion. Table 4.6 explores the problem solving heuristics the groups employed during the task. All of these are appropriate for solving this task.

Table 4.6: *Problem Solving Heuristics Employed in The Balls Task.*

Group	Problem Solving Heuristic Employed
1	Guess Check and Improve / Write a Number Sentence
2	Identify a Pattern / Write a Number Sentence
3	Draw a Diagram / Write a Number Sentence
4	Write a Number Sentence
5	Draw a Diagram

The results in Table 4.6 show that overall the groups employed four of the problem solving heuristics. Four of the groups used Write a Number Sentence. Three of the groups employed two problem solving heuristics during the task. Group 1 utilised Guess Check and Improve / Write a Number Sentence, Group 2 applied Identify a Pattern / Write a Number Sentence and Group 3 employed Draw a Diagram / Write a Number Sentence. This is quite usual as there is often two or more of the problem solving heuristics used to solve a mathematics problem. Also because the students are working in groups, they have the opportunity to be working with others whose first tendency may be to apply a different strategy. Only Group 5 did not use Write a Number Sentence as a strategy.

When Steve (Group 2) was interviewed at the conclusion of the task he stated, "For our first thinking thing we decided to just do dots for the balls and then we thought differently and then we did numbers, number patterns." When asked why they changed he said, "Because the little dots were taking up too much room." He went on to say "I learnt lots from number facts like 6 plus 6 plus… I think it was… 8 equals 20, and stuff like that. So I really had to work with them but now that I have written them down I think I can do them even quicker." Steve in his post task interview stated that his group used Make a Table and Number Patterns. Interestingly there was no evidence to suggest the group actually employed Make a Table as a problem solving strategy, although they may have mentioned it in passing without actively using it.

When Terri (Group 4) was interviewed at the conclusion of the task and asked about problem solving strategies, she stated, "The patterns and stuff." Further Terri, Cath and Eva were asked about problem solving strategies their groups used, and rather than state heuristics they stated operations such as addition, sharing and subtraction. Hence some students at least at this point of study may not have been clear as to what the heuristics actually were. On the other hand, at this age level, the formal acknowledgement of the heuristics can be hit and miss. As stated in the paragraph above, Steve was able to name two of the heuristics; Make a Table and Number Patterns, while Terri used more informal language “The patterns and stuff”. It is certainly hoped that as the students develop their mathematical vocabulary they are able to identify and name the heuristics they are employing.

4.2.3 Small Group Learning

In The Balls Task, as in all the tasks, the groups were instructed that sessions were a group or team effort. It should be recalled from chapter 3 that six themes emerged from the analyses of the group data; Pulling Up (PU), Clarification (Cl), Processes Fact (PF), Confirmation (C), Reviewing (R) and Suggestions (S) (see Table 3.12 for definitions). Table 4.7 presents the ideas the students used to assist the small group learning during the task.

Table 4.7: *Identification of Small Group Learning Ideas in The Balls Task*

Ideas Shared Within the Group	Frequency of Idea
Pulling up	2
Clarification	2
Processes fact	6

The results in Table 4.7 show that the groups employed 3 different ideas that were built upon to assist during the task. Figure 4.1 below presents the dialogue by various members of the groups to emphasise the group work shown during the task. Group 1 utilised the ideas of *Pulling Up* and *Clarification*, Group 2 employed *Clarification* and *Processes Fact*, Group 3 used *Pulling Up*, Group 4 and 5 used *Processes Fact* to assist the group to move forward. These aspects of the group work are all important facets for the groups to use so they can remain on task and move forward in solving the task. It was interesting to note that only three of the six categories were represented during this task.

There was no evidence of Confirmation, Reviewing or Suggestions by any of the groups. I expected Clarification to be represented during the task as it gives the students the opportunity to make sure all group members are clear about the task. I expected Pulling Up to be represented during the task because I assumed that mistakes would be made during the task. I expected Processes Facts to be present because of the nature of the task.

During The Balls Task, Pulling Up was employed by Group 3 to remind all the group members that the total number of balls in the task was 20.

Group 3
Val: 7 plus 7 plus 7
Andrea: Would equal 21
Eddie: 7 plus 7 plus 6 would be 20

During The Balls Task, Clarification was employed by Group 2 to clarify for all group members that three coloured beach balls were to be used in the solution.
Steve: No you see you only recorded number one and number two we haven't recorded those ones
Ariel: It doesn't matter
Steve: Yeah it does, it says how many blue, how many green and how many red
Ariel: It doesn't matter. 6R and 11 G, we will just name them by BRG
Steve: BRG? What?
Cath: BRG
Ariel: Blue Red Green BRG

During The Balls Task, Processes Facts were employed by Group 4 and 5 to show mathematical knowledge of addition.
Group 4
Grace: 11 blue and 5 red
Victor: That equals 16, so 4 green

Grace: Equals 20

Group 5
Henry: 4 green, 8 red, 8 blue
Eva: It needs to add up to 20 remember
Henry: Yeah it does 8 and 8 is 16 and then 17, 18, 19, 20

When Steve (Group 2) was interviewed at the conclusion of the task about learning and sharing in small groups, he stated, "Because in bigger groups there are lots of people thinking different stuff so then there will be like, lots of talking… and then if you say something someone else will say something different and then there will be a lot of disagreements… and everything." Steve suggested that talking sometimes slows things done but talking was still important. When Eva (Group 5) was interviewed at the conclusion of the task she stated, "It was good sharing ideas with others and coming up with ideas." When Terri (Group 4) was interviewed at the conclusion of the task she suggested that sometimes it was good to work in a group, when asked why only sometimes she stated, "Because some of the people like interrupt you a lot. Like talking while you are talking." When asked so you need to be able to work together as a group? She replied with a yes. When Cath (Group 2) was interviewed at the conclusion of the task and asked about group work, she stated, "Because then you can work it out faster."

4.2.4 Solutions

Mathematics gives the opportunity for students to become absorbed in the complex and beauty of finding solutions to problems. Thinking and understanding the mathematics involved in problem solving allows students to reflect on their own ideas and structures. The solutions were analysed on the basis of number of solutions and correctness of the solutions.

In The Balls Task, I expected, due to the open nature of the task, that the groups would discover many solutions. Table 4.8 presents the number of correct solutions produced by the groups during the task. Table 4.8 also shows incorrect solutions and off task incorrect solutions.

Table 4.8: *Number of Solutions Produced in The Balls Task.*

Group	Correct	Incorrect	Off task Incorrect
1	8	1	20
2	10	1	NA
3	2	1	NA
4	12	1	NA
5	2	NA	NA

Note: NA Not Applicable

The results in Table 4.8 show that the groups were not very successful in producing a multitude of solutions. Three groups, Group 1, Group 2 and 4 were able to produce 8 or more solutions. In contrast, two other groups struggled with the task and only produced two solutions each. Four of the groups produced one incorrect solution each and Group 1 produced 20 off task incorrect equations.

When Steve (Group 2) was interviewed at the conclusion of the task he stated, “I was thinking of … I just did random for the first two then I added those two up to see how many more would equal twenty.” I think this is an important idea Steve has suggested, the idea of randomness. When Eva (Group 5) was interviewed at the conclusion of the task she stated, “We kinda used subtraction and addition a lot.” When Terri (Group 4) was interviewed at the conclusion of the task she stated, “Um, with maths strategies, like addition and subtraction and all that.”

4.2.5 Summary of The Balls Task

The Balls Task was the first opportunity for the students to work in their groups. Four out of the five groups were able to successfully identify all four key elements in the *see* part of the *thinking routine*. Group 3 was able to identify three out of the four key elements. Overall the groups showed they were successful in this part of the routine. In the *think* component of the routine, three of the five groups failed to identify any of the four key elements, while Group 4 identified one element and Group 1 identified three key elements. All five groups were not able to identify that three colours were key elements. This is an important element to identify because without understanding this element, it would be difficult for groups to achieve correct solutions. Group 1 was the only group to identify the key element of patterns. This is a key element for groups to identify in assisting them to find as many solutions efficiently as possible. All groups produced *wonders* that focused on how many answers are there?

In The Balls Task four problem solving heuristics were employed. The most popular employed heuristic with four groups using it was Write a Number Sentence. Three groups also chose to use two heuristics during the task. All three groups chose to use Write a Number Sentence but paired it with either: Guess Check and Improve, Identify a Pattern or Drawing a Diagram. Make a table was not used as a heuristic in the task but would have been a good choice to help groups find solutions in an efficient manner.

In The Balls Task three categories were identified: Pulling Up, Clarification and Processes Fact. The most popular item was Processes Fact. This showed the groups were eager to assist each other in correcting mistakes to help in finding solutions.

The number of solutions completed by the groups was surprising low. Three groups were able to calculate eight or more solutions. Two of the groups only managed two correct solutions in the time given. This was surprising as the numbers in the task the groups had to manage were within their ability to manipulate. Group 1 did well to produce eight correct solutions but also provided twenty off-task incorrect answers.

4.3 Problem Solving Task 2: The Tyres Task

The Tyres Task involved the students recognising the mathematical ideas of multiplication and addition. It was important for the groups to identify that their knowledge of the four times and two times tables would be an important determining factor in solving this problem. This problem also highlighted the mathematical thinking of patterns.

4.3.1 See Think Wonder

In The Tyres Task, there were three elements the students should identify: the two vehicles; the car and the motorcycle, and the number of tyres (86). These three elements were considered the most important aspects the groups required to *see* to assist them in solving the task. For the groups it was vital to know that their solutions would involve either skip counting by fours and twos or the use of multiplication tables of fours and twos. For the students it was also essential to know that the number of car tyres, four on a car and the number of motorcycle tyres, two on a motorcycle must add exactly to 86. Table 4.9 displays the three elements the groups needed to *see* and record at the beginning of the task.

Table 4.9: *Identification of Key Elements in See Component in The Tyres Task*

Group	Key Elements		
	Car	Motorcycle	86 Tyres
1	✓	✓	✓
2	✓	✓	✓
3	X	X	X
4	X	X	✓
5	✓	✓	✓

Note: ✓ successfully identified key element on concept map;
X unsuccessful in identifying key element on concept map.

The results showed that three of the five groups, Group 1, 2 and 5 were able to identify or *see* the three key elements of the task. Group 4 was only able to *see* the key element of 86 tyres. Group 3 was unable to identify or *see* any of the three key elements. Group 3 listed full stops, question mark, numbers and letters in the *See* element.

In The Tyres Task, there were four key elements that should be made during the *think* part of the thinking routine. The key elements included; that patterns could be used to assist in solving the task, that operations such as multiplication and addition could be employed, that there is more than one answer to the task and problem-solving strategies can be employed during the task. Table 4.10 displays the four elements the groups could record on the concept map during the *think* part while solving the task.

Table 4.10: *Identification of Key Elements in Think Component of The Tyres Task*

Group	Key Elements			
	Patterns	Operations can be used	More than one answer	Strategies to be used
1	X	X	✓	✓
2	X	X	X	X
3	X	✓	X	✓
4	X	✓	✓	X
5	X	X	X	X

Note: ✓ successfully identified key element on concept map;
X unsuccessful in identifying key element on concept map

The results showed that three of the groups were able to identify two key elements during the *think* part of the routine. It is also interesting to note that none of the three groups identified the same two elements. As Table 4.10 shows, Group 1 identified more than one answer and strategies to be used, Group 3 identified that operations can be utilised and strategies to be used, and Group 4 identified that operations can be utilised and more than one answer. Two groups, Group 2 and 5 were not able to identify any of the important elements. None of the groups were able to identify the importance of patterns in the *think* part of the routine. Group 2 listed: I think it will be easy, it will be fun and it will take a long time. Group 5 listed: I think it is equal, tens, fives, quarters and number.

In The Tyres Task, there is no set number of wonders or questions the students could come up with. This part of the routine allows for the students to engage in discussion and write their responses on the concept map. Again because this routine was employed at the initial stage of the task, the wonderings the groups came up with provided the impetus for further inquiry into the task. Table 4.11 shows the *wonders* the groups discussed and recorded on the concept map.

Table 4.11: *Wonders Discussed and Recorded in The Tyres Task*

Group	Wonders
1	How many answers are there?
2	How many motorbikes and cars?
3	I wonder if it is about adding? I wonder about sharing?
4	How many answers are there? How many different strategies are there?
5	I wonder what the answer is? I wonder if it is using multiplication?

The results in Table 4.11 showed that two groups were *wondering* how many answers there were? While one group *wondered* what the answer was? Two groups, Group 3 and 5 *wondered* about what operations were going to assist them in solving the task. They *wondered* if multiplication, sharing or addition were the main mathematical concepts. Group 4 also *wondered* how many different strategies there were?

When Eva (Group 5) was interviewed at the conclusion of the task about the use of the thinking routine she stated, "Well, if you do with wonderings you can answer the questions and then you can find out some answers."

Table 4.12 outlines all the data of the *See Think Wonder* thinking routine employed in The Tyre Task. What was of interest, once again was that all groups were able to identify more clearly during the *see* part the important key elements of the task but were not as clear in identifying the important elements in the *think* part of the routine. Three of the groups during the *see* part of the routine were able to capture all the important elements of the task. This was in contrast to the *think* element of the routine where no groups were able to identify all the important parts of the task. During the *wonder* element of the routine the groups were able to ask, from a process point of view, I wonder if it is using multiplication? I wonder about sharing? I wonder if it is about adding? Also two groups asked, how many answers are there? Suggesting the students had an idea or notion that this wasn't a closed question.

Table 4.12: *See Think Wonder Thinking Routine Used and Discussed in The Tyres Task*

	Number of Instances		
Group	See	Think	Wonder
1	3	2	1
2	3	0	1
3	0	2	2
4	1	2	2
5	3	0	2

4.3.2 Problem Solving Heuristics

In The Tyres Task, the students as a group executed the strategy they considered through their discussion. Table 4.13 explores the problem solving heuristics the groups employed during the task.

Table 4.13: *Problem Solving Heuristics Employed in The Tyres Task.*

Group	Problem Solving Heuristic
1	Make a Model / Write a Number Sentence
2	Identify a Pattern / Draw a Diagram / Write a Number Sentence
3	Draw a Diagram
4	Make a Model / Draw a Diagram
5	Make a Table

The results in Table 4.13 show overall that the groups employed five of the problem solving heuristics. Three of the groups used Draw a Diagram as the most utilised problem solving strategy. One group, Group 2 employed three problem solving heuristics; Write a Number Sentence, Identify a Pattern and Draw a Diagram. Two groups employed two problem solving heuristics during the task. Both used Make a Model, with Group 1 using Write a Number Sentence whilst Group 4 employed Draw a Diagram. Group 3 utilized one strategy Draw a Diagram and Group 5 also only used one strategy in Make a Table. It certainly wasn't a surprise that the most used strategy was Draw a Diagram as this strategy gives the groups the opportunity to visually see their thinking and assists them when they are skip counting. What was a surprise was I expected more groups to use Make a Model to assist them with Draw a Diagram.

Five of the ten problem solving heuristics were considered and employed during task solving for The Tyre Task. This can be compared to The Balls Task where four problem solving heuristics were used. For The Balls Task the most popular heuristic employed was Write a Number Sentence whereas for The Tyre Task the most commonly used heuristic was Draw a Diagram. During The Tyre Task, Make a Model and Make a Table were employed for the first time in the study.

When Steve (Group 2) was interviewed at the conclusion of the task he stated, "Because normally I just did the number sentence but in that case when I tried doing the drawing one it actually helped me better." When asked, "Why did the drawing help you out so much do you think?" Steve responded with, "Probably because if I was stuck I could just count the wheels of it". When Eva (Group 5) was interviewed at the conclusion of the task she reponded, "make a number pattern with the number sentences." When Terri (Group 4) was interviewed at the conclusion of the task she stated that by using different resources helped in solving the task. She said they used, "The koalas and teddy bears and bugs" and they were used "To represent the cars." Terri was able to say they recorded their models on paper using drawings. When Cath (Group 2) was interviewed at the conclusion of the task she stated, "It's means like if you draw a car you count that car and then if you draw another one you count them again, so you don't just draw a couple of things and count it." Cath was able to state that their group used *Draw a Diagram* as a problem solving strategy. When asked why she was happy using *Draw a Diagram,* Cath responded with, "Because it was kind of more my type of maths thinking."

4.3.3 Small Group Learning

Table 4.14 shows the ideas the students used to assist the group learning during the task. The results in Table 4.14 show that the groups employed 4 different ideas that were built upon to assist during the task.

Table 4.14: *Identification of Small Group Learning Ideas in The Tyres Task*

Ideas Shared Within the Group	Frequency of Idea
Pulling Up	1
Clarification	2
Processes Fact	2
Confirmation	5

Confirmation was the idea identified as the most used by the groups during the task. Group 1 employed *Clarification*, *Processes Fact* and *Confirmation*, Group 2 used *Pulling Up* and *Confirmation*, Group 3 used *Processes Fact* and *Confirmation*, Group 4 used *Confirmation* and Group 5 used *Clarification* to assist the group to move forward. These aspects of the group work are all important aspects for the groups to use so they can remain on task and move forward in solving the task.

During The Tyre Task, Pulling Up was employed by Group 2 to remind all the group members that the total number of tyres equals 86.

Steve: (Skip counts by 2s and stops at 94) Look I'm on 94 already

Ariel: Take away 10 plus 2, Equals 86.

During The Tyre Task, Clarification was employed by Group 1 to clarify for all group members the total number of tyres required for both vehicles.

Lucy: We just needed 2 there to make 82
John: There is 82
Anna: But there is 86 tyres not 82 tyres

During The Tyre Task, Processes Facts were employed by Group 3 to show mathematical knowledge of addition.
Eddie: You have 84
Val: You need two more.

During The Tyre Task, Confirmation to verify a statement was used by Group 1 and Group 4. Group 1 used confirmation to show that their working out was incorrect and they had just discovered that their method would not produce a correct solution.
Anna: If you are looking for 40 plus 40 plus 6 I have done it, 96 take away ten
Lucy: (Counting in the background)
Anna: 40 plus 20 plus 20 plus 6
Lucy: But it is 2 tyres on a motorbike 4 tyres on a car
Anna: So you can't do it like that
Lucy: Yeah it doesn't work like that

Group 4 used confirmation to enhance their understanding that they had achieved a correct solution.
Grace: 4 plus, 4 plus 4plus 4 plus 4 plus 4 plus 4 plus 4 plus…
Victor: 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21
Terri: So we have some teddies and backyard bugs
Grace: (Counting quietly) 2, 4, 6, 8, 10, 12, 14…
Victor: 21 cars plus 1 motorbike
Terri: We have one working out

When Steve (Group 2) was interviewed at the conclusion of the task about group work, he stated it was good to work with the same students and work with some students who could just do it. When Eva (Group 5) was interviewed at the conclusion of the task she was able to say that talking about the task was good and then it good to put their thinking on the paper. When asked about working with the same group members, Eva stated, “Because that way you have the same thinking as last time.” I think this is an important idea to explore in the next chapter in the discussion. When Cath (Group 2) was interviewed at the conclusion of the task she stated, “Because I just like felt in the first bit I was actually quite comfortable working with those certain people so I would like to work with them again.”

4.3.4 Solutions

In The Tyres Task I expected, due to the nature of the task, that if the groups identified patterns as an essential element, that the groups would discover many solutions. Table 4.15 presents the number of correct solutions produced by the groups during the task. Table 4.15 also shows incorrect solutions and off task incorrect solutions.

Table 4.15: *Number of Solutions Produced in The Tyres Task.*

Group	Correct	Incorrect	Off task Incorrect
1	1	4	7
2	4	1	NA
3	0	1	NA
4	1	2	NA
5	0	1	NA

Note: NA Not Applicable

The results in Table 4.15 show that the groups were not very successful in producing a multitude of solutions. Group 2 were able to produce four correct and one incorrect solution. Two groups, Group 1 and 4 produced one correct solution each, while Group 3 and 5 were not able to produce any correct solutions to the task. All five groups produced at least one incorrect solution with Groups 2, 3 and 5 producing 1 and Group 4 producing 2 and Group 1 producing 4 incorrect solutions. In addition Group 1 produced 7 off task incorrect solutions. I believe one of the reasons why the number of solutions was low was due to the size of the number given in the task. On reflection 86 was too big a number for the students at this level.

When Eva (Group 5) was interviewed at the conclusion of the task she stated, "Well, you can use the times tables and keep the numbers. You can times things and you can add things and make a number pattern with the number sentences."

4.3.5 Summary of The Tyres Task

In The Tyre Task three of the five groups were successful in identifying all three key elements in the *see* component of the thinking routine. Group 4 were able to identify two elements but missed out on 86 tyres. Group 3 were not able to identify any of the key elements. This group focused on the actual structure of the writing, listing items such as words and full stops. This group did not use the *see* part of the routine to deconstruct the problem to assist them in understanding the problem. The groups continued to find it challenging to identify all the *think* key elements. Three groups were able to identify half of the key elements while two groups did not identify any key elements. Patterns were not identified by any of the groups. It is also interesting to mention that none of the three groups identified the same two elements. The wonders included how many answers are there? There were also questions about the use of operations.

During the task five problem solving heuristics were employed. The most popular heuristic used was Draw a Diagram. Make a Table and Make a Model were heuristics employed that were not used in the previous task. Again some groups decided to use two or more heuristics.

In The Tyres Task four categories were identified: Pulling Up, Clarification, Processes Fact and Confirmation. The most popular idea used by the groups to assist them during the task was Confirmation.

The number of solutions completed by the groups low but this is not surprising. On reflection the number 86 I believe was too large a number for the students to manipulate successfully. All groups successful managed to write something on the solutions sheet. Group 2 completed four correct and one incorrect solution. Two other groups produced one correct solution while two groups could only manage an incorrect solution.

4.4 Problem Solving Task 3: The Legs Task

The Legs Task, like the task before it, involved the students identifying the mathematical ideas of multiplication and addition. This task also highlighted the mathematical thinking of patterns. The students to successful complete this task needed to be thinking of combining their knowledge and understanding of multiplication and addition.

4.4.1 See Think Wonder

In The Legs Task, there were three elements the students should identify; the two animals - horses and ducks, and the number of legs (36). These three elements were considered the most important aspects the groups required to *see* to assist them in solving the task. For the group it was essential to know that their solutions would involve either skip counting by fours and twos or the use of multiplication tables of fours and twos. The students were also required to know that the number of horse legs, four on a horse and the number of duck legs, two on a duck must add exactly to 36. Table 4.16 displays the three elements the groups needed to *see* and record at the beginning of the task.

Table 4.16: *Identification of Key Elements in See Component in The Legs Task*

Group	Key Elements		
	Horses	Ducks	36Legs
1	✓	✓	X
2	✓	✓	✓
3	X	X	X
4	✓	✓	✓
5	✓	✓	✓

Note: ✓ successfully identified key element on concept map;
X unsuccessful in identifying key element on concept map

The results in Table 4.16 showed that three of the five groups, Group 2, 4 and 5 were able to identify or *see* the three key elements of the task. Group 1 successfully identified two key elements but missed out 'seeing' the 36 legs. Group 3 were not about to identify or *see* any of the three key elements. Group 3 listed: full stops, numbers, letters, question marks and words. When compared to the previous task, The Tyres Task, Group 2 and 5 were able to identify all the key elements of the task. Likewise Group 3 was not able to identify any key elements in this or the previous task. From the responses, Group 3 seemed to be focussing on seeing what was actually written on the page in the form of the task, e.g. full stops, numbers, letters, question marks and words rather than identifying the information to assist them in solving the problem.

In The Legs Task, there were four key elements that should be made during the *think* part of the thinking routine. The key elements included; that patterns could be used to assist in solving the task, identifying the importance of four legs for a horse and two legs for a duck, that there is more than one answer to the task and problem-solving strategies can be employed during the task. Table 4.17 displays the four elements the groups could record on the concept map during the *think* part while solving the task.

Table 4.17: *Identification of Key Elements in Think Component of The Legs Task*

Group	Key Elements			
	Patterns	Horses have 4 legs and ducks have 2 legs	More than one answer	Strategies to be used
1	X	X	✓	X
2	X	X	✓	X
3	X	X	X	X
4	X	X	✓	X
5	X	X	X	X

Note: ✓ successfully identified key element on concept map
X unsuccessful in identifying key element on concept map

The results showed that three groups were able to identify one key element during the *think* part of the routine. Group 1, 2 and 4 all identified the same element: there would be more than one answer to the problem. Two groups, Group 3 and 5 were not able to identify any of the important elements. Three of the four key elements were not recorded by any of the groups during the *think* part of the routine. Patterns, Horses having four legs and ducks having two legs, and strategies to be used.

In The Legs Task, again there is no set number of *wonders* or questions the students could come up with. Table 4.18 shows the *wonders* the groups discussed and recorded on the concept map.

Table 4.18: *Wonders Discussed and Recorded in Legs Task*

Group	Wonders
1	How can we get all the answers? How many answers are there?
2	How many answers are there?
3	I wonder if we share?
4	How many answers are there?
5	What are the possibilities?

The results in Table 4.18 show that the groups were mainly focussed on how many answers are there? Three groups asked this question. Group 1 also had a self-reflection *wondering* when they asked how can we get all the answers? This displays a level of thinking that goes beyond the norm of placing a number on a wondering as the previous item does. This type of wondering begins to enter into the realm of organising their thinking so they can maximise their problem solving ability. Group 3 wondered if the task was primarily a sharing operation task?

When Eva (Group 5) was interviewed at the conclusion of the task she stated, "With the wonder you write questions down and when you find the answer to one of the questions you can find another way of."

Table 4.19 outlines all the data of the *See Think Wonder* thinking routine employed in The Legs Task. Overall the results displayed in Table 4.19, show that the thinking routine employed for this task has shown the students were able to identify clearly during the *see* part the important key elements of the task but were not as clear in identifying the important elements in the *think* part of the routine. Only Group 3 was not able to *see* or identify the any of the key elements of the task. The *wonders* were mainly of the, is there more than one answer variety. Only one of the groups asked a processes question, I wonder if we share?

Table 4.19: *See Think Wonder Thinking Routine Used and Discussed in The Legs Task*

	Number of Instances		
Group	See	Think	Wonder
1	2	1	2
2	3	1	1
3	0	0	1
4	3	1	1
5	3	0	1

4.4.2 Problem Solving Heuristics

In The Legs Task, the students as a group again executed the strategy they considered through their discussion. Table 4.20 explores the problem solving heuristics the groups employed during the task.

Table 4.20: *Problem Solving Heuristics Employed in Legs Task.*

Group	Problem Solving Heuristic
1	Identify a Pattern / Write a Number Sentence / Draw a Diagram
2	Identify a Pattern / Write a Number Sentence / Draw a Diagram
3	Draw a Diagram
4	Write a Number Sentence
5	Draw a Diagram

The results in Table 4.20 show that the groups employed three of the problem solving heuristics. Four of the groups used Draw a Diagram as the most utilised problem solving strategy. Like the previous task, The Tyre Task, this task did lend itself for the groups to use the Draw a Diagram strategy. Across the first three tasks, Draw a Diagram has been a popular heuristic used by the groups. Two of the groups employed all three problem-solving strategies during the task. Group 1 and 2 employed Write a Number Sentence, Identify a Pattern and Draw a Diagram. Group 3 and 5 both used Draw a Diagram while Group 4 used Write a Number Sentence. Group 4 were also the only group not to employ Draw a Diagram as a strategy.

It was interesting to note that due to the similar nature of The Legs Task and The Tyre Task that there were no groups employing the Make a Table heuristic. If groups were able to identify a pattern then Make a Table becomes a more efficient way to find solutions compared to Draw a Diagram.

When Steve (Group 2) was interviewed at the conclusion of the task he stated, "we did number sentences and we did a pattern." When Eva (Group 5) was interviewed at the conclusion of the task she stated, "We drew a diagram, not quite sure if we numbered them though." When Terri (Group 4) was interviewed at the conclusion of the task she stated, "We had to use materials to solve it. Maybe model." When Cath (Group 2) was interviewed at the conclusion of the task she stated, "Because I drew a lot of the things." Cath added, "We were all sort of using different strategies because two of us were using make a number sentence and one of us was using draw a diagram." She believed it was good that different members employed different strategies during the task.

4.4.3 Small Group Learning

Table 4.21 shows the ideas the students used to assist the group learning during the task. The groups employed four different ideas that were built upon to assist during the task.

Table 4.21: *Identification of Small Group Learning Ideas in The Legs Task*

Ideas Shared Within the Group	Frequency of Idea
Pulling Up	1
Clarification	1
Confirmation	3
Reviewing	1

The results in Table 4.21 show that the groups employed 4 different ideas that were built upon to assist during the task. *Confirmation* was the idea identified as the most used by the groups during the task with three groups employing it. Group 2 used *Pulling Up* and *Confirmation*, Group 3 used *Clarification* and *Confirmation*, Group 4 used *Reviewing*, and Group 5 used *Confirmation* to assist the group to move forward. These aspects of the group work are all important aspects for the groups to use so they can remain on task and move forward in solving the task.

During The Leg Task, Pulling Up was employed by Group 2 to correct an addition mistake that if not rectified would have produced an incorrect solution.

Ariel: 4,8,12,16,20,24,28,32, 34, so you will need to add another duck

Steve: But that will equal 37

Anna: 36, 34 plus 2 is 36

During The Legs Task, Clarification was employed by Group 3 to remind each group member that the field needed a total of 36 legs.

Eddie: 14,16,18,20,22,24,28,33, 35, 36, aargh, no. Shivers, its easy, we just do 2 more ducks.

Val: But then we wouldn't be equal

Eddie: 33,34, 35, 36, 37 it can't be right. (Counting)14, 16, 18, 20,22,24,26,28,30, 32, 34. 32, 34, 38, we are on 38. Cross out that horse and put in a duck

Mary: There are 36 legs in the field some horse and some ducks.

During The Tyre Task, Confirmation to verify their learning was used by Groups 2 and 3.

Group 2

Ariel: Now let me see, D,D,D,D,D,D,D,D

Steve: Why you doing that

Ariel: Because it labels the ducks.

Group 3

Eddie: I had an idea they go in 4s to 30, in 2s go up to…

Mary: Thirty

Eddie: No in fours up til 28

During The Legs Task, Reviewing was employed by Group 4 to summarise the task so all group members knew where the group was up to in the solving of the task.

Terri: Grace is going to read out all our maths thinking

Grace: Adding, using maths materials, tools like number charts, teddies, and I think we are going to use skip counting

Terri: Victor will read out the See think wonders

Victor: Horses have 4 legs and ducks have 2, there are 36 legs in the field, how many answers are there, is there one answer, is there more than one answer

When Terri (Group 4) was interviewed at the conclusion of the task about group work, she stated, "Yes, wouldn't want to work with too many people". When Cath (Group 2) was interviewed at the conclusion of the task about group work she stated, "Because I felt confident with them, it's not like I have two people swapped around every single time".

4.4.4 Solutions

In The Legs Task I expected, due to the nature of the task, that identifying patterns would be of value, that the groups would discover many solutions. Table 4.22 presents the number of correct solutions produced by the groups during the task. Table 4.22 also shows incorrect solutions and off task incorrect solutions.

Table 4.22: *Number of Solutions Produced in The Legs Task.*

Group	Correct	Incorrect	Off task Incorrect
1	1	4	3
2	6	4	NA
3	1	NA	NA
4	4	5	NA
5	1	NA	NA

Note: NA Not Applicable

The results in Table 4.22 show that two of the five groups were successful in finding almost all the correct solutions. This task had seven correct solutions. Group 2 were able to produce six correct solutions and Group 4 was able to produce four correct solutions. The other three groups were all able to produce one correct solution each. Three groups, Group 1, 2 and 4 all produced incorrect solutions. Group 1 and 2 produced 4 incorrect solutions each and Group 4 produced five incorrect solutions. Group 1 was the only group to produce off the task incorrect solutions.

When Steve (Group 2) was interviewed at the conclusion of the task he stated, “Lots of answers of four and twos. Because horses have four legs and ducks have two legs.” When Eva (Group 5) was interviewed at the conclusion of the task she thought there were more than one answer to the problem, she stated, “because a lot of different numbers equal up to 36." When Cath (Group 2) was interviewed at the conclusion of the task she stated, "To make as many possible answers of 4 and 2 legs and make it to 36. Because horses have four legs and ducks have two legs."

4.4.5 Summary of The Legs Task

In The Legs Task three of the groups were able to identify all three key elements in the *see* aspect of the routine. Group 1 identified two elements but missed on identifying 36 legs while Group 3 were not able to identify any key elements. As in the previous task, Group 3 focused on the structure of the written task, listing items such as full stops and words. Again Group 3 failed to use the *see* part of the routine correctly. The groups continued to find it difficult to identify the key elements in the *think* component of the task. Three groups were able to identify one key element. That key element identified by the three groups was more than one answer. All five groups could not identify patterns, horses have 4 legs and ducks have two legs, and strategies to be used as key elements. The *wonders* ranged from how many answers are there? To what are the possibilities?

In The Legs Task three problem solving heuristics were employed. The most popular employed heuristic with four groups using it was Draw a Diagram. This task did lend itself to this heuristic so I wasn't surprised it was employed by the majority of groups. Write a Number Sentence and Identify a Pattern were the other two heuristics employed. Group 1 and 2 employed all three heuristics during the task. Again it was a surprise that Make a Table was not used.

In The Legs Task four categories were identified: Pulling Up, Clarification, Confirmation and Reviewing. This was the first task that a group used Reviewing to assist them during the session. The commonly used item was Confirmation as a verifying statement to assist the groups.

The groups were separated into two sorts: two groups found four or more correct solutions while the remaining three groups discovered one correct solution each. Three of the groups also produced four or more incorrect solutions. Group 1 also produced three off-task incorrect solutions. What was of interest was that Group 2 and 4 who produced the most correct solutions also produced the most incorrect responses.

4.5 Problem Solving Task 4: The Darts Task

The Darts Task involved the students identifying combinations of numbers. Just like trying to create a locker combination, and using the numbers 1,3,5,7, the combination has four 'blank' spaces. In this task I believed the actual mathematics of using small numbers was easily within in the reach of the groups. It was the idea that if patterns were recognised during the mathematical thinking in the task then a number of solutions would be forthcoming.

4.5.1 See Think Wonder

In The Darts Task, there were six elements the students should identify, the four numbers on the dart board; 1,3,5,7, there were four darts involved and the solutions would involve a combination of numbers. These six elements were considered the most important aspects the groups required to *see* to assist them in solving the task. For the group it was essential to know that their solutions would involve 4 numbers, representing the four darts. Table 4.23 displays the six elements the groups needed to *see* and record at the beginning of the task.

Table 4.23: *Identification of Key Elements in See Component in The Darts Task*

Group	Key Elements					
	Darts	Combinations	The Number 1	The Number 3	The Number 5	The Number 7
1	✓	✓	✓	✓	✓	✓
2	✓	X	✓	✓	✓	✓
3	X	X	X	X	X	X
4	✓	✓	✓	✓	✓	✓
5	✓	✓	✓	✓	✓	✓

Note: ✓ successfully identified key element on concept map
X unsuccessful in identifying key element on concept map

The results in Table 4.23 showed that three of the five groups, Groups 1, 4 and 5 were able to identify or *see* the six key elements of the task. Group 2 successfully identified 5 of the key elements but did not record combinations as a key element. Group 3 were not about to identify or *see* any of the six key elements. Group 3 listed: numbers, letters, full stops and multiplying. For the third task in a row, Group 3 failed to identify one key element during the *see* part of the session. Group 3 did identify numbers but did not go into enough detail to list the actual numbers represented in the task. It was pleasing that three groups were able to identify all six elements as I was then wondering if this would transfer into higher number of correct responses.

In The Darts Task, there were four key elements that should be made during the *think* part of the thinking routine. The key elements included; that patterns could be used to assist in solving the task, that four darts needed to be used each time, that there is more than one answer to the task and problem-solving strategies can be employed during the task. Table 4.24 displays the four elements the groups could record on the concept map during the *think* part while solving the task.

Table 4.24

Identification of Key Elements in Think Component of The Darts Task

Group	Key Elements			
	Patterns	4 darts to be used	More than one answer	Strategies to be used
1	✓	X	✓	X
2	✓	X	✓	X
3	✓	X	X	X
4	✓	X	✓	X
5	X	✓	X	X

Note: ✓ successfully identified key element on concept map

X unsuccessful in identifying key element on concept map

The results in Table 4.24 showed that three of the groups were able to identify two key elements during the *think* part of the routine. The three groups all identified the same two key elements on their concept map. As Table 4.24 shows, Group 1, 2 and 4 identified patterns and more than one answer. Group 3 identified patterns as a key element while Group 5 were the only group to identify the importance of using four darts in the solution. Also all the groups were not able to identify the importance of using strategies in the task.

In The Darts Task, there is no set number of wonders or questions the students could come up with. Table 4.25 shows the *wonders* the groups discussed and recorded on the concept map.

Table 4.25: *Wonders Discussed and Recorded in Darts Task*

Group	Wonders
1	How many answers?
2	How can we do it? How many answers are there?
3	Is there more answers? I wonder if you have to make patterns? I wonder if it is about counting?
4	How many answers are there?
5	Is it a game? I wonder how many possibilities there are?

The results in Table 4.25 show that each group were wondering and focused on if there were more than one answer to the task. Group 2 *wondered* how they could do it? Group 3 *wondered* if the task was about counting? They also *wondered* if you have to make patterns? This was a very insightful question to ask at the beginning of the task and would assist them as moved forward with solving the problem posed. Group 5 *wondered* if the task was game? This was an interesting *wondering* and the first time any of the groups in any of the tasks made any connections between the mathematical tasks they were confronted with.

Table 4.26 outlines all the data of the *See Think Wonder* thinking routine employed in The Darts Task. What was of interest, was that once again, the students were able to identify clearly during the *see* part the important key elements of the task but were not as clear in identifying the important elements in the *think* part of the routine. As stated above, one group asked, is it a game? The other insightful wondering came from Group 3 who asked, I wonder if you have to make patterns? Group 3 had stated in the *think* part of the routine that they thought patterns were an important element.

Table 4.26: *See Think Wonder Thinking Routine Used and Discussed in The Darts Task*

	Number of Instances		
Group	See	Think	Wonder
1	6	2	1
2	5	2	2
3	0	1	3
4	6	2	1
5	6	1	2

4.5.2 Problem Solving Heuristics

In The Darts Task, the students as a group executed the strategy they considered through their discussion. Table 4.27 explores the problem solving heuristics the groups employed during the task.

Table 4.27: *Problem Solving Heuristics Employed in The Darts Task.*

Group	Problem Solving Heuristic
1	Identify a Pattern / Make a List
2	Identify a Pattern / Make a List
3	Draw a Diagram
4	Identify a Pattern
5	Identify a Pattern / Draw a Diagram

The results in Table 4.27 show that the groups employed three of the problem solving heuristics. Four of the groups used Identify a Pattern as the most utilised problem solving strategy. This task due to the large number of solutions possible lent itself for the groups to employ the Identify a Pattern strategy. Three of the groups employed two problem solving heuristics during the task. Group 1 and 2 utilised Identify a Pattern and Make a List, while Group 5 employed Identify a Pattern and Draw a Diagram. Group 4 used Identify a Pattern, while Group 3 used Draw a Diagram as their solitary strategy. This was the first task that any groups used the heuristic Make a List. The groups that did use Make a List and Identify a Pattern, like the groups that were able to identify the key elements in the *see* part of the session, I am wondering if the this will transfer into more correct solutions.

4.5.3 Small Group Learning

Table 4.28 shows the ideas the students used to assist the group learning during the task.

Table 4.28: *Identification of Small Group Learning Ideas in The Darts Task*

Ideas Shared Within the Group	Frequency of Idea
Clarification	1
Confirmation	1
Reviewing	1
Suggestion	4

The results in Table 4.28 show that the groups employed 4 different ideas that were built upon to assist during the task. Suggestion was the idea identified as the most used by the groups during the task. Group 1 used *Suggestion* twice, Group 2 employed *Clarification* and *Suggestion*, Group 3 used *Confirmation*, Group 4 employed *Reviewing* and Group 5 used *Suggestion* to assist the group to move forward. These aspects of the group work are all important aspects for the groups to use so they can remain on task and move forward in solving the task.

During The Darts Task, Clarification was employed by Group 2 on whether the group members could use operations to assist in solving the task.

Steve: What was our maths thinking?

Cath: How do we do addition and subtraction?

Steve: Yeah we done it

Cath: But we don't do them

Steve: She does it

Cath: But we don't do it with this. So we can't do it.

During The Darts Task, Reviewing was employed by Group 4 to summarise where the group was situated in relation to solving the problem.

Grace: So you need to use combinations, on the think we have added we might need to use bit books, there has to be at least 4 answers, and we need to use a strategies

Terri and Victor: These are our numbers seven, five, three and one

During The Darts Task, Suggestion was employed by Group 1 to offer a piece of advice to group regarding the possibility of using a number multiple times

Anna: Guys do you reckon we could use numbers more than one time

Lucy: Yep okay

Anna: Numbers more than one time: 3,3,3,3, 7,7,7,7, 5,5,5,5

More than one number, using more than one number

I put down using more than one number twice

When Eva (Group 5) was interviewed at the conclusion of the task she stated, “Because you are working a lot with them and you know their strategies and when you get a different person you don’t really know their strategies to help solve the maths problems”. When Cath (Group 2) was interviewed at the conclusion of the task she stated, “I would like to stay with my same group. I’ve got to know what strategies they mostly use”.

4.5.4 Solutions

In The Darts Task I expected, due to the open nature of the task, that the groups would discover many solutions. Table 4.29 presents the number of correct solutions produced by the groups during the task. Table 4.29 also shows incorrect solutions and off task incorrect solutions.

Table 4.29: *Number of Solutions Produced in Darts Task.*

Group	Correct	Incorrect	Off task Incorrect
1	46	NA	NA
2	23	NA	NA
3	11	NA	NA
4	12	NA	NA
5	23	NA	NA

Note: NA Not Applicable

The results in Table 4.29 show that three of the groups were able to identify patterns, which enabled them to produce more solutions. Group 1 produced 46 solutions and attempted to order their solutions in a systematic manner. Groups 2 and 3 produced 23 solutions each. Groups 4 and 5 were able to produce 11 and 12 solutions each.

When Steve (Group 2) was interviewed at the conclusion of the task he stated, "Because if you didn't do it in order it would take longer and you might accidentally do the same answer." He added, "First start with all the possible ones with the one at the start and then you can to do it at the start." The researcher asked. "Is there a way you think you could have done it faster as a group?" Steve responded with "Yeah, I think so. If we thought of doing it in order quicker. We did do it in order, but at first we didn't do it in order, then we just started to do it in order." When Eva (Group 5) was interviewed at the conclusion of the task she stated, "Because you've got lots of chance and lots of possibilities for the answers." Eva also stated, "We just looked at the numbers and mixed them up and doubled them." She explained, "Well, such as seven and seven, and five and one." When Terri (Group 4) was interviewed at the conclusion of the task she stated, "To get all the numbers in order. We put them in a row from biggest to smallest." When Cath (Group 2) was interviewed at the conclusion of the task she stated, "It was finding different combinations for one, five and seven and three I think it was." She added, "I think it was just like finding different combinations for it so first you might do the ones, then you might do the threes and then you might do the five and then you might do the sevens first."

4.5.5 Summary of Darts Task

The Darts Task was the final session so I was interested to notice if the groups had built upon their previous knowledge and understandings from the previously completed tasks. In the *see* component of the routine three groups identified all six key elements and Group 2 successful identified five of the key elements. Once again Group 3 were not able to use the *see* part of the routine correctly making the same errors as they had in the previous two tasks. All the groups compared to the previous three tasks more successfully completed the *think* component of the routine. Three groups were able to identify two key elements while the other two groups were able to identify one key element. Four of the groups were able to successfully identify the key element of patterns within the task. In the *wonder* aspect of the routine the common question of how many answers are there was asked. What was of interest were two wonderings. The first; is it a game? And the second; I wonder if you have to make patterns? These are two excellent wonderings that attempt to make connections to the task.

In The Darts Task three heuristics were employed. It was pleasing to note that four of the groups used Identify a Pattern, as this would be an excellent heuristic to use to find solutions efficiently. Make a List was used for the first time during this study. Two groups used this heuristic along with Identify a Pattern. It was interesting that two groups also used Draw a Diagram as this is a more time consuming heuristic to be using for this task.

In The Darts Task four categories were identified: Clarification, Confirmation, Reviewing and Suggestion. Suggestion was employed for the first time in the study and was also the most popular item used during The Darts Task.

The number of solutions discovered by the groups was in sharp contrast to the previous three tasks. All the groups produced more solutions. This task due to its open nature had the most possible solutions available to the students. There were no incorrect or off-task incorrect solutions. The number of solutions ranged from eleven correct solutions to forty-six. Two groups were able to discover twenty-three correct solutions. Due to many groups using Identifying a Pattern as a heuristic I believe the number of solutions is also higher than in other tasks.

4.6 Conclusion

In this chapter, the problem task and the conditions of the task solving sessions were presented. The *See Think Wonder* key elements for each task were presented, the problem solving heuristics for each task were shown, the small group learning categories were itemised and examples given and finally the solutions for each task were presented. This study suggests further investigation into the use of problem solving heuristics and thinking routines in mathematics problem solving. All these findings along with the implications of these findings for teaching practice and further research will be discussed in the next chapter.

Chapter Five

Discussion

5.1 Introduction

This research investigated how one class of Year 2 students employed the *thinking routine: See Think Wonder* and mathematical problem solving heuristics when they were involved in mathematical problem solving in small groups. The study was conducted over four sessions focusing on the three aspects of thinking routines, heuristics and small group learning while completing mathematical problem solving. The preceding chapter presented the results of the research. This chapter brings these results together and the discussion compares and collates the findings of the individual parts with a view to answering the research questions that guided this study. Implications of the study and recommendations for further studies are also presented.

5.2 Answering the research questions

In this study four research questions were posed. These were:

1. When solving mathematics problems do students use the taught thinking routines?
2. When solving mathematics problems do students use the taught mathematical problem solving heuristics?
3. When solving mathematics problems do students use ideas developed by other students?
4. How do students perform in problem solving?

Each of the research questions will now be addressed.

5.2.1 Research question 1:
When solving mathematics problems do students use the taught thinking routines?

This research question essentially consists of three parts: the *see, think* and *wonder* aspect of the thinking routine. As stated in chapter 2, section 2.2.2, the groups were directed to employ the routine one aspect at a time. Therefore I will present the findings in the same order beginning with the *see* aspect before turning my focus to the *think* component and then finally present the findings of the *wonder* part of the routine. As stated in chapter 3, section 3.4.4 on concept maps, the *See Think Wonder* routine is segmented into three questions; What do you *see*? What do you *think* is going on? What does it make you *wonder*? (Ritchhart et al., 2011).

5.2.1.1 Findings of the See component of the thinking routine

When examining each of the four tasks: The Balls Task, The Tyres Task, The Legs Task, and The Darts Task, the findings show, that on the whole the groups were able to identify the *see* aspect of the task. Majority of the groups, four out of the five were able to perform at a high functioning level of performance. There was one glaring exception and that was group 3 but I will leave that discussion until later.

In the first task, The Balls Task, all the groups were very successful in identifying the four key elements, colours: red, blue, green; and twenty balls. Four of the five groups identified all four elements. Group 3 identified only three out of the four elements, as they failed to name twenty balls as a key aspect of the task. This had a bearing subsequently on the number of solutions this group achieved.

In the second task, The Tyres Task, the findings show that the majority of the groups identified the three key components, car, motorcycle and 86 tyres. Three of the groups correctly identified all three key elements, while Group 4 was only able to identify one component 86 tyres and Group 3 failed to identify any *see* key components of the task.

In the third task, The Legs Task again the findings overall suggest the groups were able to identify the three key components, horses, ducks and 36 legs. Three of the groups correctly identified all three key elements, while Group 1 only missed identifying 36 legs as an important aspect. As mentioned in the last paragraph Group 3 failed to record any key elements but again completed their concept map with full stops, numbers, letters, question marks and words.

In the fourth and final task, The Darts Task again the findings overall suggest the groups were able to identify the six key components, darts, combinations, the numbers one, three, five and seven. Three of the groups correctly identified all six elements, while Group 2 only missed identifying combinations as an important aspect. Group 3 failed to record any key elements.

When examining the findings for each group it became apparent that four of the five groups were successful in using the *see* aspect of the thinking routine at the initial stages of the task. I define the success of the *see* element as all or most *see* elements identified during the four tasks. The findings show that Group 5 working together over the four tasks were able to *see* or identify every key component, a total of sixteen. This shows Group 5 were functioning at a high level of performance during the *see* element of the task. Group 1 and 2 over the four tasks were able to *see* or identify a total of fifteen key elements. This confirms that Group 1 and 2 were also functioning at a high level of performance during the *see* element of the task. Group 4 were able to identify fourteen of the key elements. Group 4 performed at a high level for three of the tasks and only had difficulty with The Tyres Task. Group 3 were only able to identify 3 key elements, all listed in the first task. Group 3 completed the concept map with ideas such as full stops, question marks, numbers and letters. In the last three tasks, Group 3 failed to identify any *see* key components. This was an interesting development because Group 3 had completed the first task correctly. Group 3 was an eclectic group of students with four strong personalities (which I will discuss in relation to research question 3) however it was still expected that the group would be able to complete the *see* aspect of the thinking routine.

It will be interesting to note later in the discussion, if the groups that succeeded in the *see* aspect of the thinking routine were able to transfer this understanding into a high number of correct solutions. It would be an expectation from the findings of the *see* component of the sessions that Group 2 and 5 due to their high level of performance, demonstrating their unpacking of the task, they would be making substantial progress through a majority of the tasks. It would be expected that Group 1 and 4 to make some to substantial progress in the tasks. It would be expected that Group 3 to make some progress but on the findings of the *see* component it may be little or no progress in the tasks.

5.2.1.2 Findings of the Think component of the thinking routine

When examining all four tasks, the findings show, the groups were not as successful identifying the *think* key elements compared to identifying the *see* key elements. The findings show that the groups found it more difficult to identify the *think* key components. This part of the routine challenges the students to make the connections or links between the tasks and "to synthesize this information" (Ritchhart et al., 2011, p. 55).

In The Balls Task, the four key elements were; patterns, three colours to be used, more than one answer and strategies to be used. The findings for this task are clear that only Group 1 were successful in identifying three out of the four elements. It was interesting to note that the missing key element was three colours to be used, as this is a vital piece of information needed to successful produce multiple solutions. Group 4 was the only other group to identify one element and that was more than one answer. Group 3 produced some good thinking on their concept map when they wrote 'I think it is about putting the balls in groups' but more information was needed to ascertain if the group members knew that 'balls in groups' meant into three colours. Group 3 were also misguided in their thinking that the task was going to be about division.

In The Tyres Task, the four key elements were; patterns, operations can be used, more than one answer and strategies to be used. The findings show that the groups performed second best in this task in identifying key elements. Three of the groups; Group 1, 3 and 4 were all able to identify two key elements. It was interesting to note that the three groups all had different combinations of the two key elements. Group 2 and 5 were not able to identify any key elements. From the findings it can be noted that pattern was not identified by any of the groups and it will be interesting to examine later on in the discussion to see if this has any affect on the number of solutions produced by the groups.

In The Legs Task, the four key elements were; patterns, horses have four legs and ducks have two legs, more than one answer and strategies to be used. The findings show that the groups found this task the most difficult to identify the key elements in the *think* component of the routine. The only element that was identified by any group was more than one answer. Three groups recorded this on their concept map. I was quite surprised by these findings as the students had completed similar problems through out the year. Also it is important for the solving of the problem to recognise the difference in leg numbers for horses and ducks.

In the final task, The Darts Task, the four key elements were; patterns, four darts to be used, more than one answer and strategies to be used. The findings show that in this component of the *see think wonder* routine the groups performed the best in this task. Three groups, Group 1, 2 and 4 were able to identify two key elements while Group 3 and 5 identified one each. No groups identified strategies to be used as a key element.

When examining the findings for each group it became clear that the groups were more able in identifying the *see* component compared to the *think* component. This is probably due to the different level of thinking required in the *think* aspect of the routine. Students who are proficient in this part of the routine are able to make connections between the task and what they need to do to solve the problem. Group 1 were able to identify half of the key elements over the four tasks and were also only one of two groups that were able to identify at least one key element in each of the four tasks. Compared to the other groups, Group 1 was functioning at a higher level of performance in the *think* component of the tasks. The other group able to identify at least one key element in each of the four tasks was Group 4. The findings show that Group 4 was making some progress in the *think* component of the routine. Groups 2 and 3 recorded 3 key elements over two tasks and also recorded no key elements over the other two tasks. Group 5 only recorded one key element and this came in the final task. The findings show that Group 2, 3 and 5 made little or no progress in the *think* component of the routine. The findings showed that overall the groups found it difficult to make the connections necessary to assist them in solving the tasks.

5.2.1.3 Findings of the Wonder component of the thinking routine

When examining responses to the Wonder component of the thinking routine in all four tasks, the findings show, the groups were very consistent in the ideas they were wondering. Overall the wondering element of the routine is used for the students to record any questions they had at that moment that might assist them during the problem solving. By verbalising and then recording the wonder on the concept map the students are organising their thoughts, beginning a plan or strategy and remaining open-minded to the ideas of their peers.

During The Balls Task a majority of the wonders recorded were of the how many answers are there type. When you look closely at the findings you can see that four of the wonders begin with how, e.g. How can you get all the answers? Two of the wonders begin with is, e.g. Is there more than one answer? One of the wonders Group 5 asked was, I wonder what the answer is? Group 5 also asked I wonder if we are right? I'm not sure what to make of the last wonder, if it is a clarification that the group is on the right track in solving the task or if the group members want confirmation and assurance that they are on the right track. Overall the findings suggest the students were aware that this task was of an open nature due to the questions asking is there more than one answer and how can you get all the answers?

In The Tyres Task the findings show that for this task a subtle shift in questions and wonderings occurred. Although some groups still asked how many answers are there? Two groups also asked questions specific to methods needed to solve the task. For example, Group 3 asked I wonder if it is about adding? And I wonder about sharing? Group 5 asked I wonder if it is using multiplication? These three questions are refreshing to see asked and recorded as it shows the students are starting to ask more specific questions that may assist them in solving the task. All three questions are relevant to this task. The findings suggest the students were aware that this task was of an open nature due to the questions asking how many answers are there?

In The Legs Task again a subtle shift can be noticed in the type of wonderings the groups are now starting to elicit with each other as members of the group. Some groups still asked how many answers are there? But group three wondered if they share? Group 1 asked how could we get all the answers? It should be noted that Group 1 in the *think* part of the routine did not identify patterns as a key element, if they had the might not be asking the above question. The findings suggest the students were aware that this task was of an open nature due to the questions asking what are the possibilities?

In The Darts Task, I would like to highlight two wonderings from the findings. The first from Group 5 asked is it a game? This is a good connection to darts, and as Sparrow (2008) claimed students become engaged with mathematics when they can make links and connections. The second wonder was from Group 3 who asked I wonder if you have to make patterns? This is an excellent question and one that if followed to its end conclusion would provide many solutions to the group as they could record clearly the patterns that would be emerging on their solutions sheet.

5.2.1.4 Summary of See Think Wonder thinking routine

When solving mathematics problems do students use the taught thinking routines? The findings have shown that the students do use the taught *thinking routine See Think Wonder* in a successful capacity. The findings showed that the *see* aspect of the *thinking routine* was successful employed by a majority of the groups over the four tasks. The findings also showed that the students did use the *think* element of the routine but found it more difficult to identify the connections compared with the *see* aspect of the routine. The students found the *think* aspect the hardest part of the three sections of the routine. The *wonderings* showed that there was a subtle shift in the questions being asked as the tasks were being completed. The shift was towards questions where connections could be in a more specific manner. For example I wonder if you have to make patterns? Is it a game? I wonder if we share? I wonder if it is using multiplication?

At this age level (7 and 8 year olds) it was not surprising the findings showed the students were most confident and comfortable with identifying the key elements in the *see* part of the routine. In addition, they found it more difficult to replicate the same success with the *think* part of the routine. The students produced *wonder* statements that were age appropriate but it was noted that some of the *wonders* in the latter stages of the study started to display wonders that were showing more depth and connections to the specific task.

5.2.2 Research question 2:

When solving mathematics problems do students use the taught mathematical problem solving heuristics?

When selecting the tasks, I wanted the tasks to be open-ended so that multiple answers could be produced, patterns could be identified and a variety of heuristics could be applied to the same task. Becker and Shimada (1997) in their study discovered that providing open-ended problems allow students the initial stage of mathematical creativity. When examining the findings it became clear that for the students in this study certain tasks were suited towards certain heuristics and particular groups also favoured certain heuristics.

The findings show that seven of the ten problem solving heuristics were employed during the four tasks. Haylock (1985) stated that by employing learned strategies students could apply a variety of methods to solve a problem. The seven heuristics used by the groups were Guess check and Improve; Write a Number Sentence; Draw a Diagram; Identify a Pattern; Make a Model; Make a Table; and Make a List. The three heuristics not used during the study included Act it Out, Work Backwards and Solve a Simpler Related problem. This was not surprising as tasks presented in this study would not suit the first two of the heuristics not used and the students would've had to take it upon themselves to create a simpler related problem. To act it out involves going through actions, which give the problem 'concreteness'. This concreteness makes it easier to discover relationships leading to the solution. For example, three people in a group. How many handshakes would there be if each person shakes hands once. So Sally shakes with Dave (one handshake), Sally shakes with Mary (two handshakes) and Dave shakes with Mary (three handshakes). In problem solving, working backwards involves determining how the action or process ends and then working from the end position to the solution. For example, I thought of a number, added 12, then added my original number again, and the answer was 30. What was my original number? 30-12=18, 18 divided by 2=9 so 9+12+9

=30. Solving a simpler related problem involves setting aside the original problem and working through a simpler related problem. The same solution method is then applied to the original problem. One of the reasons for the small group learning I was hoping students would discuss the task and sort through any problems or issues.

In The Balls Task, the findings showed that four of the heuristics were employed during the task. Three of the groups employed two heuristics: Group 1 applied Guess check and Improve, and Write a Number Sentence; Group 2 employed Identify a Pattern, and Write a Number Sentence; and Group 3 used Draw a Diagram, and Write a Number Sentence. Group 4 employed Write a Number Sentence while Group 5 applied Draw a Diagram. The findings showed that Write a Number Sentence and then Draw a Diagram were the two most commonly used heuristics. The Balls Task did lend itself to all of the heuristics that were employed. Although the groups were successful in applying the problem solving heuristics, groups would have had a greater chance of success by producing more solutions if they had used Make a Table with Identify a Pattern. Draw a Diagram is time consuming and more often than not the groups were not finding a pattern to their solutions when applying Draw a Diagram in this task.

In The Tyres Task, the findings showed that five of the heuristics were employed during the task. Once again Draw a Diagram and Write a Number Sentence were employed. Two previously unused heuristics were employed during this task: Make a Model and Make a Table. Make a Model is applied by using concrete materials to make a representation of the task. When Make a Model was applied another heuristic must also be used to record what has been made because students were not permitted to use cameras to take photographs of their models. Therefore Group 1 used Make a Model and Write a Number Sentence, and Group 4 applied Draw a Diagram with Make a Model. The findings also showed that Group 2 applied three heuristics; Identify a Pattern, Draw a Diagram and Write a Number Sentence. It was often found in this study that students will begin with a heuristic like Draw a Diagram but change to Write a Number Sentence because it is less time consuming and patterns are more easily to be seen compared to

Draw a Diagram. Group 5 decided to use Make a Table, which if applied in an efficient manner is a good way to produce a majority of solutions, especially if it is also used with identify a pattern.

In The Legs Task, the findings showed that three of the heuristics were employed during the task. What was surprising was that this task is similar in structure to the previous task, The Tyres Task, and there were no groups that used Make a Model or Make a Table. Four groups, with two groups also using Identify a Pattern during the task, employed draw a Diagram. Write a Number Sentence was the third heuristic employed during the task.

In The Darts Task, the findings showed that three of the heuristics were employed during the task. The most common heuristic employed during the task was Identify a Pattern with four groups employing this heuristic. Two groups also used Make a List. When these two heuristics are employed together the likelihood of increasing the number of solutions is greater. Draw a Diagram was used but again it is not the most efficient heuristic to be applied for this task.

The findings showed that when examining each group, a majority of the groups employed similar heuristics over the four tasks. This is not surprising because the groups were static and although the students are open to new ideas within the group, majority of the students are more comfortable working with three or so heuristics. Group 3 used Draw a Diagram in all four tasks even though they had in their group the most capable mathematics student in the study. From the findings it can be inferred that Group 3 by consistently applying Draw a Diagram they would not achieve as many possible solutions as other groups. Group 5 also applied Draw a Diagram in three instances and also attempted to use Make a Table in the Tyres Task. Group 2 applied Identify a pattern, as the over-arching heuristic during the four tasks and paired it with Write a Number Sentence three times and Draw a Diagram two times. In the final task, Group 2 also used Make a List, which I believe is the most efficient pairings of heuristics to produce the most possible solutions. Group 4 used a mixture of heuristics during the study, employing four heuristics and on three occasions only applying one heuristic per task. Group 1 employed the greatest number of heuristics in the study. They used six heuristics and during each task applied two or more to produce as many solutions as possible.

When solving mathematics problems do students use the taught mathematical problem solving heuristics? The findings showed that the students did use the problem solving heuristics during the task to assist them providing solutions. The students employed seven of the ten heuristics. Across the five groups and four tasks, there were 11 instances of two or more heuristics being applied during the task while there were also nine instances of only one heuristic being applied to produce possible solutions. This was pleasing to note that the groups were being strategic in the applying what they believed were the best heuristics for their group to use during the task. The groups were being strategic in two senses. First the groups were clearly 'playing to their strengths' as they were selecting the heuristic they were most comfortable in using. Secondly some groups such as Group 1 and 2 in The Darts Task recognized that by using both Identify a Pattern and Make a List this would give them the greatest opportunity to produce the most solutions possible. The two groups were making a strategic plan by reflecting on their learning, as neither group had used Make a List before in the study but in The Darts Task it was a perfect choice. At this age level, it was not a surprise to find Draw a Diagram as the most common heuristic used across the study. As stated earlier van Essen and Hamaker (1990) believed the first step towards success in solving problems was to generate a diagram. Diezmann (2000) stated that being able to display written information in a diagram assists in decoding information. Weinstein and Mayer (1986) stated that during Draw a Diagram students able to reorganize information. Draw a Diagram, as the name suggests allow the majority of the students to participate in the problem solving if their own number knowledge and understanding is limited. Due to its visual nature Draw a Diagram is appealing to students because they can draw a car and know it has four tyres or a duck and know it has two legs. Karmiloff-Smith (1990) stated that throughout the process there is the potential for understanding and knowledge acquisition. The next most commonly used heuristic was Write a Number Sentence and this is also not a surprise because also at this age level the students want to show their answers using numbers rather than with drawings. It was pleasing to find that Identify a Pattern was employed on eight occasions during the study. This shows the students were attempting to make connections to their learning as the tasks selected for this study all had high levels of patterns or combinations inherent within the task.

At this age level the students had moved away from using concrete materials, so it was not a surprise to see this heuristic only implemented twice in The Tyres Task. Make a table was employed once during the study and this may be accredited to a couple of different reasons. Namely the students are unsure with the construction of a table. For example, not knowing where to place the variables. This was interesting to note because the majority of groups in the *see* component of the *thinking routine* were able to identify the variables of the task but this didn't transfer to understanding how to construct a table. Make a Table or Make a List partnered with Identify a Pattern would have been the preferred heuristics to use in The Balls Task and Darts Task. If employed correctly the possible number of solutions would be increased and the efficiency of producing those solutions would also be increased. Two groups had partnered Make a List and Identify a Pattern for the final task, The Darts Task, which was pleasing to note. For The Tyres and Legs Task, Draw a Diagram is a competent heuristic to be used at this age level and this study showed this to be the case. For more efficient solving of the task, Make a List partnered with Identify a Pattern would more often than not produce an increase in possible solutions.

5.2.3 Research question 3:
When solving mathematics problems do students use ideas developed by other students?

When examining the findings although six categories of small group learning ideas were identified by the researcher, it was clear that an even spread would not be evident in this study. The six categories included; Pulling Up, Clarification, Confirmation, Processes Facts, Reviewing and Suggestion. It was anticipated that the group members would apply these various small group-learning ideas during the tasks and these in turn would assist the groups in producing solutions to the tasks.

During the study it was discovered that in The Balls Task, three small group learning ideas were identified for a total of ten instances that assisted the students with their problem solving. What was of interest in this task was the number of Processes Fact instances that were recorded, given the students were working with combinations of three numbers that could add to twenty. It was pleasing to note that the groups used Clarification to assist the group in understanding the task so that all group members could proceed together during the session. All five groups were identified as using at least one small group learning idea during the task. Group 1 and 2 utilised two of the ideas and what was interesting was they both employed Clarification but Group 1 also utilised Pulling Up while Group 2 applied Process Fact.

During The Tyres Task, four small group-learning ideas were identified for a total of ten instances that assisted the students with their problem solving. What was of interest in this task was that the number of Confirmation instances that were recorded, were the highest small group learning idea, as in the previous task there were no recording of Confirmation as a small group learning idea. It was pleasing to note that the groups used Confirmation as a small group learning idea, as it gave the group members the opportunity to verify statements and proceed with the problem solving in a cooperative manner. Four of the five groups applied Confirmation during the task. Once again all five groups employed at least one small group learning ideas during the task. Group 1 applied three ideas; Clarification, Processes Fact and Confirmation. Group 2 and 3 employed two ideas while Group 4 and 5 used Confirmation and Clarification respectively.

During The Legs Task, four small group-learning ideas were identified for a total of six instances that assisted the students with their problem solving. Once again Confirmation was the highest recorded small group learning idea for the task. For this task the idea of Reviewing was applied by Group 4 and they would continue this in the next task as well. What was of interest also was there were no recorded instances of Process Fact errors by any of the groups. Group 2 and 3 both used Confirmation and Pulling Up and Clarification respectively. Group 5 employed Confirmation while Group 1 did not record a small group learning idea for the task. This was a surprise because in the previous task the group had employed two or more ideas.

During The Darts Task, four small group-learning ideas were identified for a total of seven instances that assisted the students with their problem solving. What was of interest in this task was that the number of Suggestion instances that were recorded, were the highest small group learning idea, as in the previous task there were no recording of Suggestion as a small group learning idea. It was interesting to note that Suggestion was not employed until the final task. I don't believe it was because of the nature of the task but I believe it was more to do with the comfort of the group members with each other. With this being the fourth and final task, the group members might have been feeling more comfortable with others to suggest ideas and to believe their ideas would be received in an open-minded manner. Three of the groups, Group 1, 2 and 5 employed Suggestion during the task. As stated in the previous paragraph, Group 4 applied the reviewing idea in this task and was the only group to do so during the study

When solving mathematics problems do students use ideas developed by other students? At this age level it is not surprising to note that the groups did not use all of the small group learning ideas identified in this study however it was pleasing that the ideas were present and used to assist in the solving of the tasks. Two of the groups, Group 1 and 2 employed five small group-learning ideas. The findings showed that Group 2 consistently employed 2 small group-learning ideas during each task which stands to reason that Group 2 was a high functioning group during the study. Group 1 also showed through the findings that they were a high functioning group. Group 3 employed four small group-learning ideas during the study with an emphasis on using Confirmation in three out of the four tasks. As mentioned earlier, Group 3 were an eclectic group that often lost focus during the study. Group 5 also employed four small group-learning ideas during the study but never displayed multiple small group learning ideas in a task. Like Group 3, they often lost focus during the study. Group 4 was an interesting group as they only recorded three small group-learning ideas during the study but were also the only group to employ Reviewing as an idea which they applied in the last two tasks. Group 4 overall were a well functioning group. The findings showed that the students did use small group learning ideas during the study to assist them in recording as many possible solutions to each task.

5.2.4 Research question 4:
How do students perform in problem solving?

Overall the findings suggested the students were situated between making some progress and making substantial progress in each task. When examining the findings, if students can identify the patterns in the task used in this study, then this could aid them in the finding of a number of solutions. All the tasks selected for this study allowed for multiple solutions for each task. Overall the findings suggested the students might not of performed as well as expected in producing high numbers of multiple solutions in each task. This could be explained by a number of factors including not selecting and using: one, the heuristic that would be the most efficient in assisting in solving the task; two, students losing focus during the tasks; and three, the time of the year the study was conducted.

Table 5.1 presents the group success of the five groups in The Ball Task. The success of each group was determined on their success: during the *see think wonder* component of the thinking routine; the group success in implementing small group learning ideas; the implementation and use of problem solving heuristics; and finally the number of solutions produced by the group in the task. As noted in Table 5.1, three groups: Group 1, 2 and 4 all had substantial success in The Balls Task. The remaining groups: Group 3 and 5 had some success in the same task.

Table 5.1: *Group success on The Balls Task*

Groups	Group success on task			
	Little or no progress	Some progress	Substantial progress	Task solved (all or most solutions found)
1			✓	
2			✓	
3		✓		
4			✓	
5		✓		

In The Balls Task, I was expecting a high number of solutions to be produced as the students throughout the year had worked with three or more addends to various numbers fewer than twenty. Overall the class had a firm grasp of numbers facts up to twenty. The findings showed that the three groups that I believe were the higher functioning and most collaborative and cooperative groups performed the best in this task. Group 4 produced the most solutions with twelve while Group 2 produced ten and Group 1 produced eight. Group 1 went 'a bit off track' as they also produced twenty off task incorrect solutions. In these off task incorrect solutions, Group 1 used a combination of operations such as multiplication and subtraction to arrive at the number twenty. I think the group became 'sidetracked' with their achievements of using other operations than addition to arrive at the number twenty. Unfortunately although the group displayed some innovative calculations for this age level, the solutions were off task incorrect. Group 3 and 5 only produced two correct solutions. This is a low number of solutions and I place it generally in the 'loss of focus' category. This unfortunately would be a trend for the remainder of the study for these two groups.

In conclusion, The Balls Task, when viewed in the light of research question 4 and the performance of the groups, overall the groups' performance was situated between some progress and substantial progress. During this task Group 1 displayed: task solved for the *see* aspect; substantial progress for the *think* and *wonder* aspect; substantial progress for the small group learning ideas implemented; and finally some progress for solutions produced using heuristics. All of the findings indicated that Group 1 was functioning at a high level for this task. Group 2 displayed: task solved for the *see* aspect; little or no progress for the *think* and *wonder* aspect; substantial progress for the small group learning ideas implemented; and finally some progress for solutions produced using heuristics. The findings suggested that Group 2 were also functioning at the higher level of performance for this task. Group 3 displayed: substantial progress for the *see* aspect; little or no progress for the *think* and *wonder* aspect; some progress for the small group learning ideas implemented; and finally little or no progress for solutions produced using heuristics. Although I have identified Group 3 with making some progress in Table 5.1, the group was on the boarder line of making little or no progress and some progress. The group displayed a loss of focus and a lack of cohesion during the task. Group 4 displayed: task solved for the *see* aspect; little or no progress for the *think* aspect; some progress for the *wonder* aspect; some progress for the small group learning ideas implemented; and finally substantial progress for solutions produced using heuristics. The findings suggested that Group 4 were also functioning at the higher level of performance for this task. Group 5 displayed: task solved for the *see* aspect; little or no progress for the *think* aspect; some progress for the *wonder* aspect; some progress for the small group learning ideas implemented; and finally little or no progress for solutions produced using heuristics. The findings suggested Group 5 were functioning at a lower level of performance like Group 3, as they too lacked focus during the task and were easily distracted.

Table 5.2 presents the group success of the five groups in The Tyres Task. Two groups: Group 1 and 2 made substantial progress in The Tyres Task while Groups 3, 4 and 5 made some progress.

Table 5.2: *Group success on The Tyres Task*

	Group success on task			
Groups	Little or no progress	Some progress	Substantial progress	Task solved (all or most solutions found)
1			✓	
2			✓	
3		✓		
4		✓		
5		✓		

In The Tyres Task, the findings show the students found this task a challenge in producing possible solutions. On reflection, if I had my time again, the number of tyres for this task would be reduced. I believe in part, one of the reasons for the small number of solutions produced was the number of tyres in the task was too big of a number for the students to manipulate. The students, although confident in skip counting by fours were not as confident applying their multiplication understanding and knowledge to the task. Therefore the students became 'lost' in solving the task. Group 2 produced the highest number of correct solutions with four. Group 1 and 4 produced one correct solution each and also four and two incorrect solutions respectively. Group 3 and 5 were not able to produce a correct solution but at least were able to produce an incorrect response each.

As in the previous task Group 1 produced seven off task incorrect solutions along a similar line of thought as in the previous task.

In conclusion, The Tyres Task, when viewed in the light of research question 4 and the performance of the groups, overall the groups' performance can be stated as some progress was made. Although the groups did not produce a high number of solutions other aspects of the groups performance can be considered as making substantial progress. During this task Group 1 displayed: task solved for the *see* aspect; some progress for the *think* and *wonder* aspect; substantial progress for the small group learning ideas implemented; and finally some progress for solutions produced using heuristics. All of the findings indicated that Group 1 was functioning a high level for this task. During this task Group 2 displayed: task solved for the *see* aspect; little or no progress made for the *think* aspect; some progress for the *wonder* element; substantial progress for the small group learning ideas implemented; and finally substantial progress for solutions produced using heuristics. All of the findings indicated that Group 1 was functioning a high level for this task. During this task Group 3 displayed: little or no progress for the *see* aspect; some progress for the *think* and *wonder* aspect; substantial progress for the small group learning ideas implemented; and finally little or no progress for solutions produced using heuristics. The findings show that Group 3 was inconsistent in their learning in this task. They were able to make progress in the *think* aspect and implementing small group learning ideas but this did not transfer to producing a satisfactory amount of solutions. During this task Group 4 displayed: little or no progress for the *see* aspect; some progress for the *think* and *wonder* aspect; some progress for the small group learning ideas implemented; and finally some progress for solutions produced using heuristics. For this task, Group 4 was performing at a good level of functioning but the findings also show that the group was performing at a lesser level compared to the first task. During this task Group 5 displayed: task solved for the *see* aspect; little or no progress for the *think* aspect; some progress for the *wonder* element; some progress for the small group learning ideas implemented; and finally little or no progress for solutions produced using heuristics. The findings show that Group 5 also performed at an inconsistent level during this task.

Table 5.3 presents the group success of the five groups in The Legs Task. This task was clearly the less well-performed task for the study. Group 2 made substantial progress; Group 4 and 5 made some progress: while Group 1 and 3 made little or no progress in the task.

Table 5.3: *Group success on The Legs Task*

Groups	Group success on task			
	Little or no progress	Some progress	Substantial progress	Task solved (all or most solutions found)
1	✓			
2			✓	
3	✓			
4		✓		
5		✓		

In The Legs Task, the findings suggested the students made a better attempt with this task compared to the previous task. With this task the Heuristic Draw a Diagram was the predominant strategies used by the groups. Due to the smaller total number in this task, thirty-six compared to eighty-six in the previous task, Draw a Diagram becomes a more manageable heuristic to use in attempting to solve the task. Group 2 used three heuristics, Draw a Diagram, Identify a Pattern and Write a Number Sentence to produce six correct solutions. They also produced four incorrect solutions so they showed themselves to be very active in this task. This was an excellent result for Group 2,as the possible number of solutions was reduced in this task compared to the other three tasks. Group 4 was able to produce four correct and five incorrect solutions suggesting they too were active in this task. As for Group 2, this was a good result for Group 4. The remaining groups all produced one correct solution. Group 1 like Group 4 also produced four incorrect solutions. What was interesting to note for Group 1 was that they did not record any small group learning ideas for this task. For the time allocated Group 3 and 5 recorded one correct solution and no incorrect solutions, which intimated that focus issues might have been a factor in this task.

In conclusion, The Legs Task, when viewed in the light of research question 4 and the performance of the groups, overall the groups' performance was some progress was made. During this task Group 1 displayed: some progress for the *see* aspect; little or no progress made for the *think* aspect; some progress made for the *wonder* element; little or no progress made for the small group learning ideas implemented; and finally little or no progress made for solutions produced using heuristics. The findings shows that Group 1 found this task the hardest out of the four tasks. This was surprising because Group 1 performed better on the previous task where the number in the task was larger and most groups found this to be more difficult. During this task Group 2 displayed: task solved for the *see* aspect; little or no progress made for the *think* aspect; some progress made for the *wonder* element; substantial progress for the small group learning ideas implemented; and finally substantial progress for solutions produced using heuristics. The findings show that Group 2 was the only group to make substantial progress in this task and to continue to perform at a high level during the study. During this task Group 3 displayed: little or no progress for the *see* aspect; little or no progress made for the *think* aspect; some progress made for the *wonder* element; some progress for the small group learning ideas implemented; and finally little or no progress made for solutions produced using heuristics. The findings show that Group 3 continued to perform at low level of functioning during the task. They found most parts of The Legs Task to be difficult and the result of this difficulty was one correct solution in the time allocated for this task. During this task Group 4 displayed: task solved for the *see* aspect; little or no progress made for the *think* aspect; some progress made for the *wonder* element; some progress for the small group learning ideas implemented; and finally some progress made for solutions produced using heuristics. The findings show overall that Group 4 were active in this task and attempted to produce as many solutions as they could. Group 4 were performing at a satisfactory level for this task. During this task Group 5 displayed: task solved for the *see* aspect; little or no progress made for the *think* aspect; some progress made for the *wonder* element; some progress for the small group learning ideas implemented; and finally little or no progress for solutions produced using heuristics. The findings showed that Group 5 continued to perform at a similar level as the other tasks they had completed during the study. Some of the good learning Group 5 produced in this task did not transfer to producing solutions, as they were only able to produce one correct solution.

Table 5.4 presents the group success of the five groups in The Darts Task. Group 1 and 2 made substantial progress; Group 3 and 4 made some progress; while Group 3 made little or no progress.

Table 5.4: *Group success on The Darts Task*

Groups	Little or no progress	Some progress	Substantial progress	Task solved (all or most solutions found)
	Group success on task			
1			✓	
2			✓	
3	✓			
4		✓		
5		✓		

In The Darts Task, the findings suggested the groups achieved their best results. Like The Balls Task this task presented a large quantity of possible solutions. It is no surprise that the four groups that employed the heuristic Identify a Pattern produced the most solutions. Group 1 employed both Identify a Pattern and Make a List and produced forty-six solutions. Likewise Group 2 and 5 (surprisingly) produced twenty-three solutions. Both groups applied Identify a Pattern but Group 2 used Make a List while Group 5 employed Draw a Diagram. Group 4 produced twelve solutions. Group 3 were the only group not to use Identify a Pattern and they produced the fewest solutions in 11.

In conclusion, The Darts Task, when viewed in the light of research question 4 and the performance of the groups, overall the groups' performance was situated between some progress and substantial progress made. During this task Group 1 displayed: task solved for the *see* aspect; some progress for the *think* and *wonder* aspect; some progress for the small group learning ideas implemented; and finally substantial progress for solutions produced using heuristics. The findings show that Group 1 performed at a high functioning level for this task. The group doubled the amount of correct solutions of any of the other groups. The group worked methodically to produce their solutions. During this task Group 2 displayed: substantial progress for the *see* aspect; some progress for the *think* and *wonder* aspect; substantial progress for the small group learning ideas implemented; and finally some progress for solutions produced using heuristics. Group 2 continued to work at a high level of performance. During this task Group 3 displayed: little or no progress for the *see* aspect; little or no progress for the *think* aspect; some progress for the *wonder* element; some progress for the small group learning ideas implemented; and finally little or no progress for solutions produced using heuristics. Group 3 continued to work at a low level of performance. Once again the group showed they were not working as a collaborative unit and therefore they found it difficult to produce a high number of solutions in the time allocated. During this task Group 4 displayed: task solved for the *see* aspect; some progress for the *think* and *wonder* aspect; some progress for the small group learning ideas implemented; and finally little of no progress for solutions produced using heuristics. The findings showed that Group 4 continued to perform at a consistent level. This level of performance in this task didn't transfer into the output of producing a high number of solutions. During this task Group 5 displayed: task solved for the *see* aspect; little or no progress for the *think* aspect; some progress for the *wonder* element; some progress for the small group learning ideas implemented; and finally some progress for solutions produced using heuristics. The findings showed that in this task Group 5 performed at a good level of functioning. Group 5 were able to focus in a clearer manner and this was confirmed in the number of solutions the group produced.

5.3 Implications of the study

From this study, it can be stated that the importance of problem solving to be continued in the primary school classroom should be seen as a priority. Dreyfus and Eisenberg (1996) stated that students who are capable of flexible thinking and exploring problems from a variety of angles display a good mathematical mind. Haylock (1985) stated that a student's understanding and knowledge of mathematics could be encouraged through applying their own creativity to solving problems. The mathematics programs for primary schools need to ensure that rigorous planning and implementation of problem solving is incorporated into all mathematics classrooms. It is therefore of the utmost importance that teachers are also led in professional development in this area. Problem solving allows students to work and learn to their ability levels and complete tasks in a variety of different areas within the mathematics domain. Davis, Maher and Noddings (1990) discussed that for learning to transfer and to occur, students' need more than drill and rote learning. Renzulli, Gentry and Reis (2004) believed problem solving creates a learning environment where students can apply relevant skills and knowledge. The implications of this study show that although students are exposed to and participate in problem solving on a consistent level, problem solving needs to be catered for students, so that all students can achieve some level of success and some students can be extended so that they can excel.

The importance of teaching the students the heuristics is another implication that has emerged from this study. Kilpatrick, Swafford and Findell (2001) emphasized the importance of giving students opportunities to access mathematical ideas through activities. The importance of teaching the heuristics can be viewed through the lens that the students were able to apply heuristics to provide multiple solutions in The Legs Task and The Darts Task. The importance can also be seen that during the small group learning more than one heuristic was being employed. Students need to be shown and given a variety of tasks that enable them to use a variety of heuristics to solve the problem. As Polya (1957) stated once you have achieved a satisfactory answer, think of another solution or another way of doing it. Students need to have the opportunity to not only use one heuristic but also have the confidence to develop a variety of heuristics and apply the 'best fit' heuristic that is going to achieve the most efficient solutions for the task.

The third implication for this study is the importance of implementing thinking routines and small group learning into the mathematics classroom curriculum and structure. The use of the *see think wonder* thinking routine to unpack and understand and begin to implement the plans for solving the task have shown to be a success from the findings. The routine used in this study is but one of many routines that could, should and are applied in schools in the mathematics classroom. More mathematics classrooms should be employing the thinking routines to assist learning and understanding. The benefits of small group learning in mathematics should not be underestimated. The students learn from each other through discussion, through listening, through modeling and through debate and reasoning.

5.4 Recommendations

In further studies it would be interesting to research the same one or two tasks completed once a term over the course of a full year. In this case, the findings would show the growth that is occurring with the completion of each task. I would also like to compare and contrast the data from this research with students in older year levels such as year 5 or 6. Would the maturity levels play a major factor in the thinking, planning and solving of the tasks?

5.5 Conclusion

This study showed that small group learning in mathematics is an important idea that should continue to be advocated in mathematics in primary schools. Advocates of collaborative learning believe working together provides more opportunities for students to discuss mathematics (Brown & Palincsar, 1989; Stevens & Slavin, 1995). In this study time and opportunity were given to the groups to explore the task through discussion and then to write the main elements of the problem, to investigate the problem through using the heuristics and to formulate possible solutions. The small group learning allowed for students to agree and disagree with each other as they sought out possible solutions. This study showed small group learning in mathematics assists students in understanding mathematical concepts. This study also showed that by using the correct thinking routine, in this case, *see think wonder,* the students are able to plan and 'unpack' the problem. The thinking routine allowed the groups to discuss the task and set a platform with which they were then able to begin to investigate the task. This study showed that by employing the thinking routine in this way it made the students discuss the problem fully. At this age level some students are not confident in reading the problem by themselves or identifying all the main elements. This study showed the use and implementation of the problem solving heuristics to assist in producing solutions as a tool that is taught and modeled to students will hopefully yield long-term benefits. The heuristics employed by the groups were age appropriate that showed the students were gaining an understanding of the mathematics they were completing.

In this study, the level of performance by the individual groups during the four tasks can be viewed as the following. Group 2 was the best-performed group of the study. They consistently achieved as a high functioning group even though there were three distinct personalities. The three students worked and learned well together and were consistent in the manner they approached the tasks and the output they produced. Group 2 were able to implement the *see think wonder* routine, the small group learning ideas and heuristics to achieve substantial success. Group 1 was also a high performing group and I expected this to be the case when I chose these three students to work and learn together. They were three strong personalities that displayed good cohesion and co-operation during the study. This group only had one poor session, The Legs Task. With this as an exception Group 1 was able to implement the *see think wonder* routine, the small group learning ideas and heuristics to achieve substantial success. Group 4 was also a well functioning group that made progress throughout the study. This group moved between making some progress to making substantial progress throughout the study. This group was the only group to employ the small group learning idea of reviewing in the last two tasks of the study. Like Group 1, Group 4 only functioned at a lower level once during the study and that was in The Tyres Task. Group 4 were able to perform well and this showed in the consistent number of solutions they were able to produce throughout the study. Group 5 was a consistent group that made some progress throughout the study. This group was inflicted with focus issues that at time distracted from some of the learning and thinking that was occurring. Group 5 was a group that looked for a leader and I believe would have benefitted from someone who could have directed this group. Group 3 was a group that consistently made little progress. They were the only group that was not successful in the *see* aspect of the routine throughout the study. They were also low performers in the *think* element of the routine as well. Group 3 consistently performed low in producing solutions. Group 3 were a fractured group that lacked cohesion and a sense of cooperation.

This study I believe showed that when students collaborate in small groups, even at seven and eight years of age, the chances of the success would increase. When students participate and act positively in a small group, discuss ideas and learn from each other, again there is a greater chance of a higher level of performance. This study I believe showed that when the problem solving heuristics are employed along with small group learning ideas and a thinking routine such as *see think wonder*, the performance of the group will be high functioning and well performed.

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Appendices

Appendix 1. Human Research Ethics Approval

ACU
AUSTRALIAN CATHOLIC UNIVERSITY

Human Research Ethics Committee
Committee Approval Form

Principal Investigator/Supervisor: Philip Clarkson Melbourne Campus

Co-Investigators: Jill Brown Melbourne Campus

Student Researcher: Gary Thomas Melbourne Campus

Ethics approval has been granted for the following project:
Thinking, Problem Solving in Primary Maths

for the period: 03/11/2011-10/12/2012

Human Research Ethics Committee (HREC) Register Number: V2011 120

Special Condition/s of Approval
Prior to commencement of your research, the following permissions are required to be submitted to the ACU HREC:
Toorak College (received)

The following standard conditions as stipulated in the *National Statement on Ethical Conduct in Research Involving Humans* (2007) apply:

(i) that Principal Investigators / Supervisors provide, on the form supplied by the Human Research Ethics Committee, annual reports on matters such as:
- security of records
- compliance with approved consent procedures and documentation
- compliance with special conditions, and

(ii) that researchers report to the HREC immediately any matter that might affect the ethical acceptability of the protocol, such as:
- proposed changes to the protocol
- unforeseen circumstances or events
- adverse effects on participants

The HREC will conduct an audit each year of all projects deemed to be of more than low risk. There will also be random audits of a sample of projects considered to be of negligible risk and low risk on all campuses each year.

Within one month of the conclusion of the project, researchers are required to complete a *Final Report Form* and submit it to the local Research Services Officer.

If the project continues for more than one year, researchers are required to complete an *Annual Progress Report Form* and submit it to the local Research Services Officer within one month of the anniversary date of the ethics approval.

Signed: .... Date: ....03/11/2011............
(Research Services Officer, Melbourne Campus)

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