

# Trade, Migration, and Inequality in a World without Factor Price Equalization

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## Abstract

The behavior of trading economies in the absence of factor price equalization is not well understood, although empirical evidence against factor price equalization is overwhelming. We map regions of diversification and specialization for competitive world economies with different factor endowment partitions. Goods and factor price responses as economies move within and across different regions of specialization are explored using a series of novel diagrams. The usefulness of endogenizing patterns of specialization is illustrated by considering the impact on inequality of migration flows (such as US–Mexico), the substitutability of trade and migration, and the impact of the entry of a large unskilled labor-intensive economy (such as China) on factor prices and factor flows.

## 1. Introduction

There have been many advances in the theory of international trade in recent years (surveyed, for instance, in Grossman and Rogoff, 1995), but most trade modeling and policy analysis still operates with fully diversified economies where factor price equalization holds. This emphasis is problematic as empirical studies such as Davis and Weinstein (2001) and Schott (2003) suggest incomplete diversification and failure of factor price equalization is the norm.

We know surprisingly little about the behavior of trading economies in the absence of factor price equalization, even for the simplest competitive models. Krugman (1995, p. 1247), in his survey, comments that determining what happens outside the factor price equalization region is a “fairly nasty business,” and Dixit and Norman (1980, p. 113) observe that it is “very complicated.” Standard graduate texts such as Dixit and Norman (1980) and Bhagwati et al. (1998) err in their discussions of nonfactor price equalization cases. The recent text of Feenstra (2004, pp. 22–25) offers brief comments on the complications involved.

Some work, such as Wood (1994), Leamer (1995), and Oslington (2002), considers specialized economies but imposes a particular pattern of specialization rather than linking it to underlying endowment, technology, and taste parameters. An important paper which takes up the challenge of linking patterns of specialization to underlying parameters is Leamer (1987), who considers a three-factor  $n$ -good model, showing how the range of products produced in different countries depends on their endowment ratios. While an extremely rich paper, its usefulness for the problem considered here is limited by a fixed production coefficients technology, ruling out the changes in factor

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intensity that flow from the factor price changes which occur outside the factor price equalization region. Another strand of the literature that endogenizes patterns of production and trade is inframarginal economics—for instance, Cheng et al. (2000) or Tombazos et al. (2005)—where interactions between technology, economies of scale, and transaction costs generate different patterns.

The first aim of the paper is to map the regions of specialization as for the standard competitive trade model, as no satisfactory account exists in the literature. To make the problem tractable we use Cobb–Douglas tastes and technology, and explore numerically the shapes of the regions of specialization. For each region of specialization we will then explore relationships between endowments, factor prices, and goods prices for different trading worlds. The second aim is to clarify relationships between trade, migration, and inequality outside the factor price region, for we interpret the factors of production as skilled and unskilled labor and consider migration due to factor price differentials. The third aim is to illustrate the usefulness of a world economy model with endogenous patterns of specialization for debates about the relationship between inequality and migration flows (e.g. US–Mexico), the substitutability of trade and migration, and the impact of the entry of a large unskilled labor-intensive economy (e.g. China) on factor prices and migration flows.

The paper is structured as follows. The first aim occupies sections 2 and 3, which are a series of novel diagrams showing regions of specialization and factor prices in different regions. Sections 4 and 5 introduce the definitions of inequality and migration pressure in a nonfactor price equalization world. Sections 6, 7, 8, and 9 illustrate the model, meeting the third aim.

## 2. Integrated Equilibrium Analysis

Our mapping of regions of specialization builds on the technique of integrated equilibrium analysis developed by Dixit and Norman (1980, pp. 100–25), who took up Samuelson's (1949, pp. 194–95) parable of an angel splitting the world factor endowment between countries in different ways.<sup>1</sup> Integrated equilibrium analysis allowed Dixit and Norman to cut through the previous debate on factor price equalization by reframing it as a question of what joint restrictions on technology, preferences, and factor endowments supported factor price equalization.<sup>2</sup> It has been fruitful in other ways: Deardorff (1994) further clarified the conditions for factor price equalization; Helpman and Krugman (1985) and Kreickemeier and Nelson (2006) have extended it to consider trading worlds with imperfect competition; Davis (1998) called it a truly global approach when deriving some startling results about the consequences for different countries' factor markets of factor accumulation in different parts of the world.

A simple model with two countries, two factors and two goods, will be used, along with standard assumptions of perfect competition, concave constant-returns-to-scale technology that is the same across the world, and identical homothetic preferences. It will be assumed that equilibrium factor proportions are unique, and degenerate combinations of technology, endowments, and tastes, which mean a good is produced nowhere in the world will be ruled out.

An integrated equilibrium is shown in Figure 1.<sup>3</sup> The dimensions of the box are the world endowment of the factors, unskilled labor  $L$  and skilled labor  $K$ .<sup>4</sup> The two products are  $X$  and  $Y$ , with  $X$  relatively unskilled labor. The world endowment of  $L$  and  $K$  is split between countries  $A$  and  $B$  in the proportions represented by  $V$ . Since  $V$  is

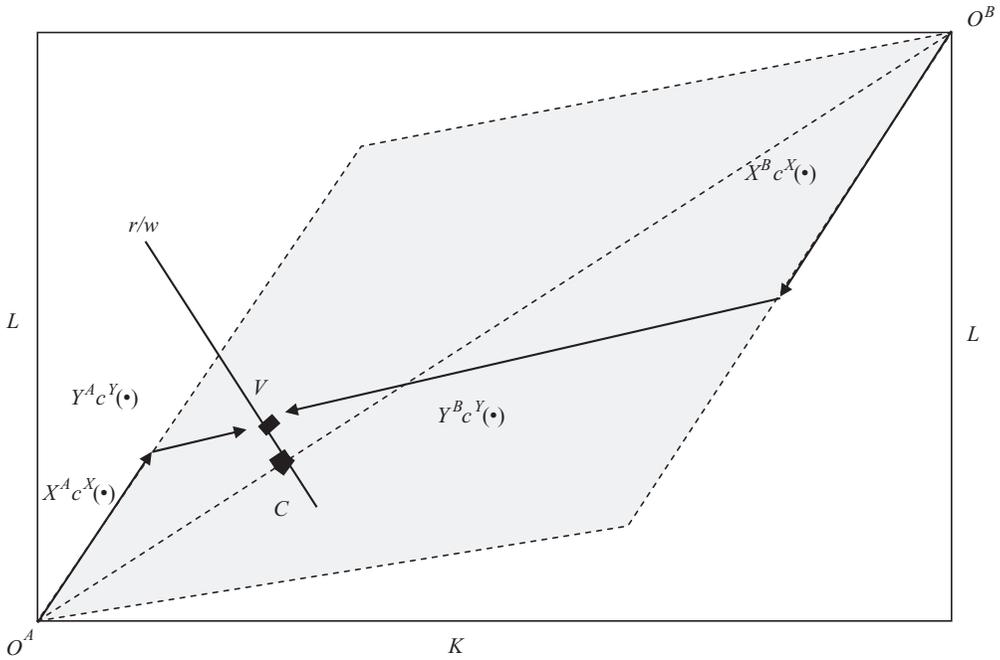


Figure 1. Integrated Equilibrium

within the shaded parallelogram, which is the area enclosed by the equilibrium factor usage vectors as the world with free movement of goods and factors, both countries produce both goods using the same factor proportions as the world with free movement of goods and factors. Factor prices and goods prices will be identical to the undivided world. Since preferences are identical and homothetic, individuals will consume the products in the same proportions as the undivided world, so the factor content of consumption in the two countries will be a point on the diagonal of the box such as *C*. The factor content of trade will thus be the vector *VC*.

For splits of the endowment outside the shaded parallelogram in Figure 1, such replication of the equilibrium is not possible and factor price equalization breaks down. This has been widely noted in the literature, but there is considerable uncertainty about what exactly happens. Dixit and Norman comment, “In order to be able to say what happens outside the factor price equalization region, we need more information concerning technology and demand functions” (p. 113) and that this can “make matters very complicated” (p. 113).

None of the discussions in the literature of what happens outside the factor price equalization region are completely accurate. Dixit and Norman’s textbook, an excellent and widely used reference, errs in suggesting that there are four regions of specialization outside the factor price equalization region<sup>5</sup> (see Dixit and Norman, 1980, pp. 113–14, and especially Figure 4.4). As will be shown below, there are in fact six regions—Dixit and Norman miss the possibility that both countries specialize completely in different goods. Bhagwati et al. (1998, pp. 87–90) repeat the error that there are four regions and miss the regions where both countries specialize. There seems to be no satisfactory account in the literature of what happens outside the factor price equalization region.

### 3. What Happens Outside the Factor Price Equalization Region?

As suggested by Dixit and Norman (1980, p. 113) the analysis outside the factor price equalization region is “very complicated,” and we will follow their approach of numerical simulation with a particular production technology to map the regions. The case illustrated has Cobb–Douglas production and utility functions, production share of  $K$  in  $X$  ( $\alpha = 0.45$ ), share of  $K$  in  $Y$  ( $\beta = 0.55$ ), and consumption share  $\sigma^Y = 0.5$ , but we have experimented with a range of parameter values.<sup>6</sup>

The six regions of specialization and diversification are shown in Figure 2. The regions are best explained by tracing how a trading world switches between equilibria as endowments change. Begin with an endowment split in the diversification region marked with a plus sign (+).

Give country B more skill and country A correspondingly less, so that we move through the region from + in the direction of the arrow in Figure 2. In country B, factor and goods prices do not change and the output of the labor-intensive good  $X$  will fall, and  $Y$  rise following the Rybczynski theorem. Eventually the output of  $X$  in country B will fall to zero at the boundary of the diversification and specialization regions. Further increases in the endowment of skill in country B will make it impossible for B to fully employ its endowment of both factors producing both products at the integrated equilibrium factor proportions. There is not enough labor to absorb all country B’s skill, and to maintain full employment in B production of the labor-intensive good  $X$  must cease and  $Y$  alone be produced in B. The reverse effects will occur in country A, and production of  $Y$  in A ceases.

Now the world economy is in the specialization region in Figure 2. Continue taking skill from country A and giving it to country B. Responses are now more complex because factor prices and proportions change outside the factor price equalization

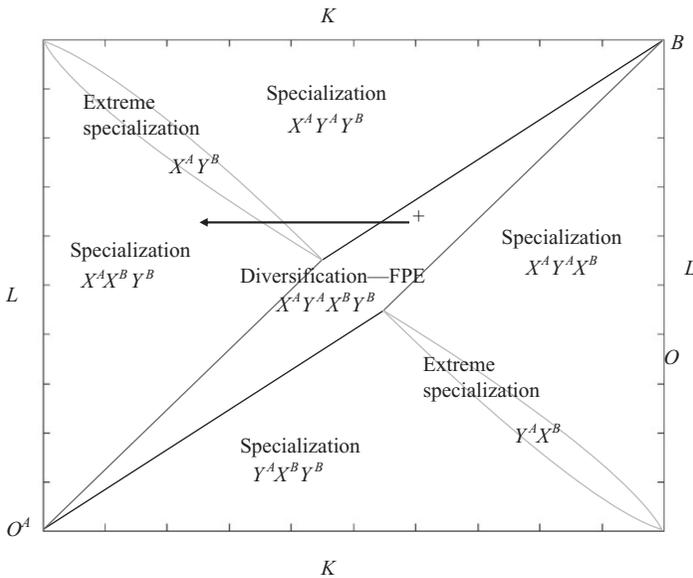


Figure 2. Regions of Specialization and Diversification

Note: Goods produced in each region are listed in that region; for instance,  $X^A$  means good  $X$  is produced in country A.

region. There will be a Rybczynski-like response at constant factor prices in country A, reducing output of the skill-intensive good  $Y$  and increasing output of  $X$  in country A. However, factor prices are not constant, and in country A the return to skill will rise and skill intensity of both goods fall. In country B the output response is straightforward as the additional skill will increase output of the only good produced  $Y$ , the return to skill will fall and production become more skill intensive. In A the return to skill rises. The relative supply of good  $Y$  falls and the relative world price of good  $Y$  rises. This reduces relative demand for good  $Y$ , tending to push it out of production. Eventually the combined effects will close down the  $Y$  industry in country A.

The world economy is now in the extreme specialization region where country A produces only  $X$  and country B only  $Y$ . Continuing to take skill from country A and giving it to B increases output of  $X$  in A and increases  $Y$  in B. The return to skill rises in A and falls in B. These changes drive down the world price of good  $Y$  until eventually it becomes so low that it is profitable to recommence production of  $X$  in country B, taking the economy to a specialization region analogous to the one previously considered.

The extreme specialization region (the region missed in some previous discussions) region has the convex lens shape shown in Figure 2.<sup>7</sup> If the production technology was fixed coefficient then the boundaries of the extreme specialization region would be straight-line extensions of the factor usage vectors which enclose the diversification region, as illustrated by Deardorff (1994, p. 169). However, in our more general case factor price changes induce factor intensity changes which delay the switch to extreme specialization described above.

We can map goods prices  $p^Y$  and factor prices  $w^A, w^B, r^A, r^B$  for all possible endowment splits, as for  $P^Y$  (Figure 3). These maps fully characterize the responses to endowment changes for the standard model, encompassing the local comparative static responses within regions and responses as we switch between regions.<sup>8</sup>

Some responses reverse well-known properties of factor price equalization economies; for instance, the correspondence between relative goods prices and factor price. As we have seen in the specialization region, reallocating capital from country A to B causes the relative world price of the capital-intensive good  $Y$  to rise and the relative return capital to rise in A and fall in B. In the extreme specialization region, the same reallocation of capital causes the price of good  $Y$  to fall, but the relative return capital still rises in A and falls in B. Stolper–Samuelson “reversals” like this were previously noted by Cheng et al. (2000), but it is perhaps unfair to describe these as failures of the Stolper–Samuelson theorem, because country B produces a single good so the usual resource reallocation mechanisms are not operating.

#### 4. Inequality

We now move to the second aim of clarifying relationships between trade, migration, and inequality outside the factor price region. To do this we will interpret the factors as unskilled and skilled labor, and the ratio of skilled to unskilled wages  $r/w$  will be our measure of inequality.<sup>9</sup> In a society of two individuals with given endowments where one owns all the unskilled labor and the other all the skilled labor, this measure would correspond to the ratio of the incomes of the two individuals, and would also be proportional to the Gini coefficient. This simple measure of inequality ties into recent debates over trade and wage inequality.

Using the factor price solutions from the previous simulations we can find values of inequality for the two countries for different partitions of the world factor endowment. These values are shown in Figure 4. The inequality surface for country A labeled  $r^A/w^A$

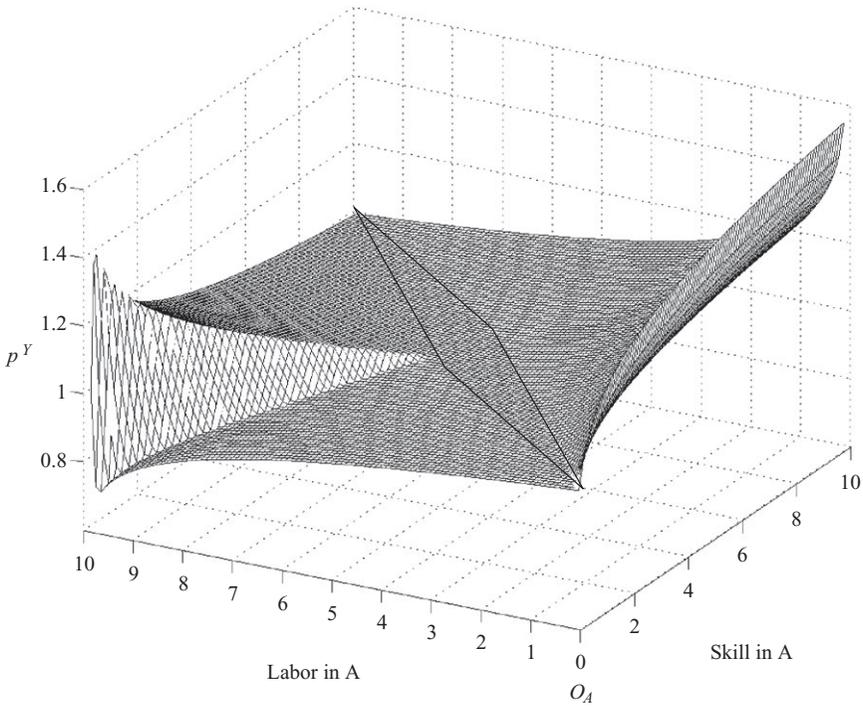


Figure 3. Price of Product Y

Notes: This figure shows the  $p^Y$  surface from a perspective that gives the clearest view of the transitions from the specialization to extreme specialization regions in the left portion, but at the cost of obscuring the symmetric right portion. All the surfaces in Figures 3–7 are symmetric.

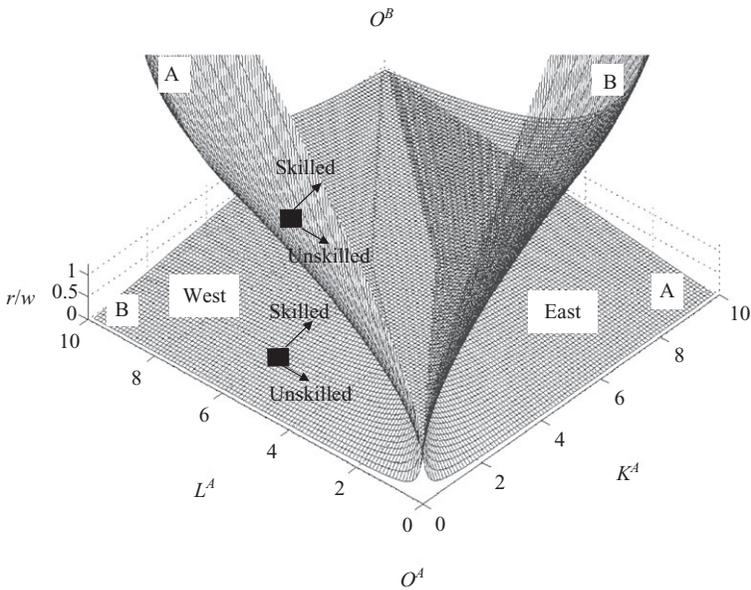


Figure 4. Inequality Surface

runs from the top left (or west) of Figure 4 to the bottom right, and for B from the top right to the bottom left. The flat central region that is common to both surfaces indicates the level of inequality in the countries when factor returns are equalized across countries, which is the level of inequality that would prevail in a borderless world.<sup>10</sup> In the west part of Figure 4, where country A is relatively well endowed with unskilled labor country A has greater inequality than country B. In the east, where country A is relatively well endowed with skilled labor, it has less inequality. These differences in inequality come entirely from differences in factor abundance between countries.

## 5. Migration Pressure

We need to specify what drives migration flows, and in the illustrations that follow, migration pressure will come entirely from factor price differentials which reflect differences in factor endowments between countries. When individuals migrate, the endowment partition changes and income is assumed to be spent in the destination country. It is recognized that migration decisions in reality are more complex; other influences include technological differences between countries,<sup>11</sup> networks created by previous migrants, ease of remittances, risk, and locational preferences, as discussed by Massey (1990) and the specialist literature on migration.

For our simplified world the sign and magnitude of migration pressure between different countries can be read off the inequality surfaces in Figure 4 for different endowment splits. In the factor price equalization region there is no pressure for labor to move between countries. Outside the factor price equalization region, in the west of Figure 4, where  $r^A/w^A > r^B/w^B$ , unskilled labor will flow from A to B and skilled labor will flow from B to A, and in the east of Figure 4 the reverse flows occur.

## 6. Migration and Inequality

The third aim of the paper is to illustrate the usefulness of a world economy model with endogenous patterns of specialization for some contemporary debates. The relationship between migration and inequality has been an important and contentious policy issue for a long time, as discussed by Hatton and Williamson (2005) and many others.

Consider an endowment point in the west part of Figure 4 where country A is relatively well endowed with unskilled labor. As explained in the previous section, opening up migration of unskilled labor induces migration flows from country A to B, pushing the endowment point for country B up its inequality surface so inequality rises in B, and pulling the endowment point for A down its inequality surface so inequality falls in A. Opening up migration of skilled labor induces the opposite flows, skilled labor moves from B to A, but inequality moves in the same direction, rising in B and falling in A. These movements are indicated by arrows in Figure 4.

Migration and the induced changes to factor prices will continue until the edge of the FPE plane is reached. If we know the starting endowment partition for the world economy and only one factor is mobile we can predict exactly where on the edge of the FPE plane the world will end up, but with two mobile factors a range of possible points on the edge of the FPE plane are possible.

### *Result 1*

Opening up migration of either factor pushes countries towards the factor price equalization plane, i.e. the level of inequality that would prevail in an integrated world economy.

*Result 2*

Opening up migration of either factor reduces inequality in the most unequal country (the labor-abundant country) and increases inequality in the other country (the skill-abundant country).

These results mean that if a country like the US is relatively well endowed with skilled labor, relaxing barriers to migration will reduce inequality in Mexico, and increase inequality in the US. They are consistent with previously derived comparative static effects of exogenous endowment changes (e.g. Woodland, 1982) for a single country, but more general.

**7. Trade and Inequality**

Relationships between trade and inequality are also contentious, especially the impact of imports of unskilled labor-intensive products on US inequality over the last 20 years; see, for example, Wood (1994) or Bhagwati (2004). In our model, comparing the free-trade inequality surfaces with autarky inequality surfaces gives the effect of opening up trade on inequality in each country.

Autarky inequality surfaces for the two countries are similar to the free-trade surfaces, but they are steeper and lack the flat central region.<sup>12</sup> When trade is opened up we move from autarky inequality surfaces to the corresponding free-trade surfaces, giving the following result.

*Result 3*

Opening up goods trade reduces inequality in the most unequal country (the labor-abundant country) and increases inequality in the other country (the skill-abundant country).

**8. Trade and Migration and Substitutes**

It is well known that for endowment partitions that support factor price equalization, trade and migration are substitutes in the sense that opening up either trade or migration will equalize factor prices (Mundell, 1957, p. 321). For world economies with endowment partitions outside the factor price equalization region free trade is insufficient, but migration will equalize factor prices.

There is another sense in which trade and migration are substitutes, which applies beyond the factor price equalization region. Comparing Results 2 and 3, opening up either trade or migration of either factor has the same effects on inequality, reducing inequality in the most unequal country and increasing inequality in the other country.

*Result 4*

Trade and migration are substitutes in the sense that opening up either reduces inequality in the most unequal country (the labor-abundant country) and increases inequality in the other country (the skill-abundant country). They are “inequality substitutes” for all endowment partitions.

**9. Unskilled Labor Growth**

The model can also be used to consider the effects on inequality and migration flows of adding a large pool of unskilled labor to the world economy. An example would be the

growth of China's unskilled labor endowment, either from demographic forces or from unskilled workers moving into the market economy.

In Figure 5, an increase in country A's unskilled labor endowment can be represented by stretching the world economy box away from the country A origin from  $O^A$  to  $O^{A'}$ . This stretching of the box alters the boundaries of the regions, perhaps leaving the endowment point in a different region. For example,  $V$  in Figure 5 previously supported a diversified factor price equalization equilibrium but is now specialized, and  $U$  was previously a specialized equilibrium now diversified.

The inequality surfaces are raised and stretched by the additional unskilled labor, and the new inequality surface is shown in Figure 6 lighter hatched, over the old darker surface. In the western part of Figure 6, the new lighter country A and B inequality surfaces lie wholly above the darker old surfaces. In the eastern part, the new A surface is wholly above the old, but the new B surface cuts the old along a line, which is shown in Figure 7. This line comes from our numerical simulations. Unskilled labor growth in A thus increases inequality in all countries, except in the case where A is skill abundant, when inequality may fall in B. The endowment partitions for which inequality falls in B are shaded in Figure 7.

So, if country A is skill-scarce China then growth of its unskilled workforce increases inequality everywhere, whereas if country A is skill-abundant Europe then growth of its unskilled workforce increases inequality in Europe but can reduce inequality in the rest of the world.

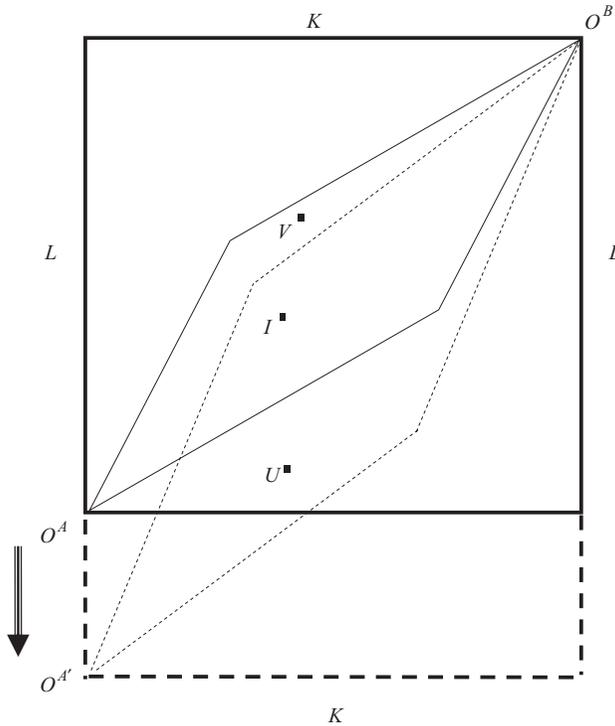


Figure 5. Labor Endowment Expansion

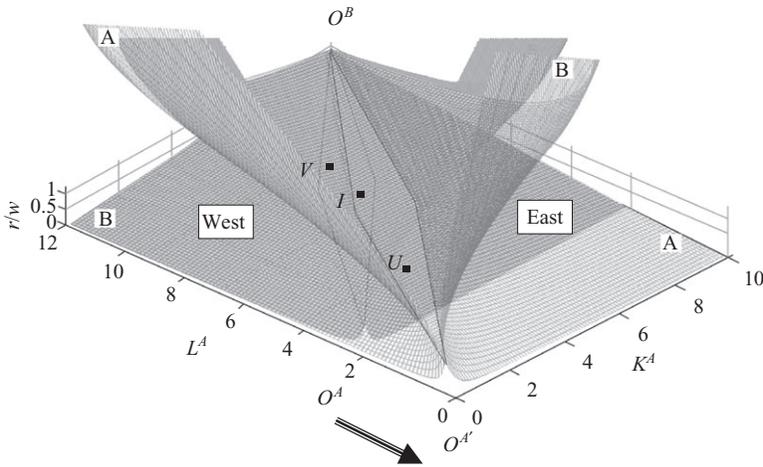


Figure 6. Effect of Labor Endowment Expansion on Inequality

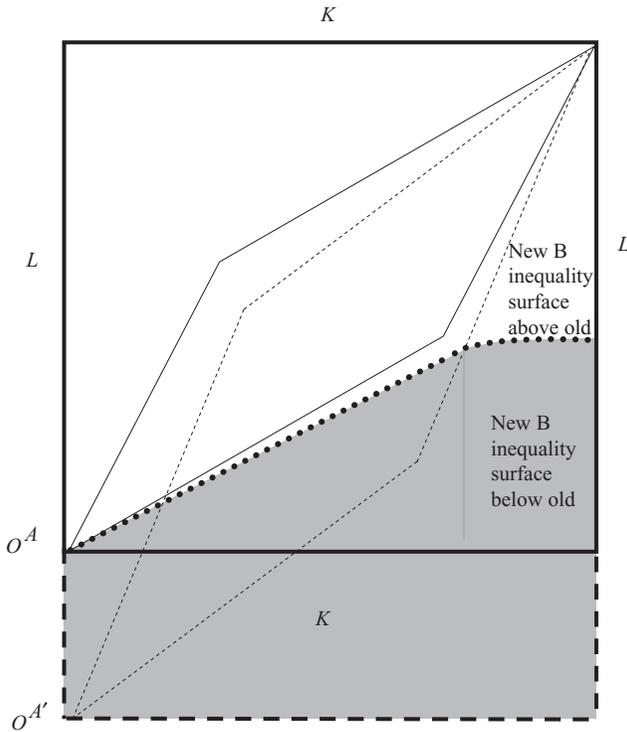


Figure 7. Where do the Surfaces Cross?

*Result 5*

With free trade but no migration, inequality rises in a country which brings additional unskilled labor to the world economy, but inequality may rise or fall in the rest of the world.

An alternative experiment is the effect of unskilled labor growth with completely open borders. With completely free migration countries always end up on the factor price equalization plane, inequality is the same everywhere in the world, and depends on world endowments. We know that increasing the world endowment of unskilled labor increases world inequality, so we get Result 6.

### *Result 6*

With free migration, growth of the unskilled labor endowment in any country increases inequality in all countries.

With completely open borders, increasing the world endowment of unskilled labor generates migration flows, but to identify these flows it is necessary to isolate migration induced by the additional unskilled labor from migration that would otherwise have occurred at the original endowment partition. Let us say we begin at  $V$  in Figure 6, where there was no migration before growth. After growth the endowment point is pushed up the country A west slope, inducing unskilled labor flows from A to B, and skill flows from B to A.

### *Result 7*

Growth of a country's unskilled labor endowment creates migration pressure for the country to shed unskilled labor and attract skilled labor.

## **10. Conclusions**

The paper has provided a full mapping from endowment partitions to patterns of production, goods, and factor prices for the simplest competitive trade model. This fills an important gap in the literature, not least because the few existing discussions err.

This mapping of the regions of diversification and specialization opens the way to consider some important issues in the context of nonfactor price equalization economies. Our two moves of interpreting the factors as skilled and unskilled labor, and assuming migration to be driven by factor price differentials allowed us to generalize existing results about connections between trade, migration, and inequality beyond the much analyzed factor price equalization case. Some sharp results were derived—especially the result that opening up either trade or migration reduces inequality in the most unequal country and increases inequality the other country. The mapping also allowed us to consider the inequality and migration impacts of adding unskilled labor to a competitive world economy.

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## Notes

1. Some of the following draws on a paper on teaching integrated equilibrium analysis (Oslington and Towers, 2009).
2. A common approach in the literature is to construct cones of diversification and argue that economies with endowments inside the cone will be diversified, while those outside the cone specialized. This is sometimes useful, but cones are drawn for particular goods prices, which are endogenous in a world economy model.

3. Equilibrium conditions are given in a longer version of the paper, available from the authors.
4. Capital can be thought of an intersectorally and internationally mobile third factor.
5. In correspondence on this issue Avinash Dixit mentioned that his colleague Gene Grossman independently realized the error in the Dixit and Norman text—see Grossman (1990) and Grossman and Helpman (1991, p. 190)—as well as a related error in the earlier Helpman and Krugman book. My letter to Avinash Dixit contained an error about the shape of one of the regions and I thank him and Gene Grossman for pointing this out. Deardorff (1994, p. 169) includes a diagram that divides the area outside the factor price equalization region into six regions, but draws linear boundaries for the special case of fixed production coefficients.
6. The figures have been generated using *Matlab*, after some initial experimentation with *Mathematica*.
7. An explicit expression for the boundary of the extreme specialization region has been derived for the general Cobb–Douglas case, but is extremely complex and given in an appendix available from the corresponding author.
8. The other maps are available in the longer version of this paper.
9. This interpretation suggests  $w \leq r$ , and appropriate choice of units for labor and skill can ensure this. An alternative, not explored in this paper, would be a production technology that allowed skilled workers to be substituted for unskilled, but not vice versa, so the skilled wage can never fall below the unskilled.
10. In the simulations  $r/w = 1$  for the flat region since  $K = L$  and the technologies are Cobb–Douglas.
11. Technology and endowments are alternative (and sometimes equivalent) analytical boxes. It can be argued (e.g. Woodland, 1982) that technology differences are omitted factors. If we allow technology differences between countries this stretches the logic of integrated equilibrium analysis, and makes the analysis of migration messier because both technological and endowment differences contribute to the factor price differentials that drive migration flows. However, we could still consider the effects of migration and trade on inequality in a world with technological differences.
12. The autarky surface and its derivation are given in the longer version of this paper.