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ORIGINAL PAPER

Investigating teacher noticing and learning in Australia, China, and Germany: a tale of three teachers

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Abstract
Teacher noticing can be an important element in improving teaching and students' mathematical success. While the focus of the international project *Learning from Lesson*s was on teacher learning, in this paper we report what mathematics teachers noticed and claimed to learn through the process of planning, teaching, and reflecting on their lessons. The study involved teachers and research teams from three countries (Australia, China and Germany) with different cultures, contexts and pedagogies. The explicit goal of the current study was to identify commonalities and differences with respect to those aspects that teachers noticed during their teaching. A multiple case study with three teachers, one from each of the three participating countries, was conducted using prompting questions to facilitate teacher reflection. The process of defining and refining categories for teacher noticing was implemented in the methodology. The findings suggest that there were many commonalities across the cases despite the different cultural and individual backgrounds of the teachers. The specific topic and individual lessons as well as the teachers' expectations based on their lesson planning seemed to influence what the teachers noticed in their teaching process. The study highlights the importance of attending to the situational aspects of teacher noticing and learning.

Keywords Teacher noticing · Teacher reflection · Teacher learning · Cross-cultural comparison · Multiple case study · Lesson planning

1 Introduction

Both teacher knowledge and teaching quality are the focus of current international research (e.g., MET, 2013; OECD, 2014; Peng et al., 2014). However, teacher knowledge construction in situ is less well researched. The international research project *Learning from Lessons*[1] *(LfL)* particularly focused on studying the construction of teacher knowledge in situ through their day-to-day teaching practice. The major premise of this project was that teachers learn from the act of planning and teaching a lesson. Rather than asking what it is that a teacher must already know in order to teach effectively, the LfL project sought to investigate what a teacher might learn through their teaching activities in the classroom. Our study deliberately involved three countries with different cultures, contexts and pedagogies (Australia, China and Germany) since its primary goal was to identify differences as well as commonalities with respect to how teacher learning takes place in different countries.

The research design of the project was piloted in an earlier study (D. M. Clarke[2] et al., 2015) in which teacher learning was operationalised in terms of teachers' declarative "claim to know" (*epistemic claim*) and a recounted intended or enacted change in the individual practice (*adaptive practice*) based on teachers' reflections of their lesson planning and teaching. The teachers' reflections provide an indication of teacher learning (e.g., Hiebert et al., 2003; Mason, 2010). But as Mason (2010) states, "before […] this retrospective

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[2] D. J. Clarke refers to David Clarke, D. M. Clarke to Doug Clarke—David's brother.

work can be done, there has to be some data. Something has to be discerned and noticed" (p. 33).

Therefore, this paper particularly focuses on teacher noticing as a part of teachers' in situ learning. As teacher noticing emphasises classroom interactions as a central object in teaching practice, it is important to study teacher noticing in the act of teaching (Jacobs & Spangler, 2017). Teacher noticing has been described as "socially and culturally constructed" (Louie, 2018, p. 61), which is more thoroughly understood within a particular socio-cultural context (Yang et al., 2021). Our study aims to examine what three middle school teachers from contrasting contexts (Australia, China and Germany) notice as part of their classroom teaching.

2 Teacher Noticing

Every act of teaching depends on noticing: noticing what children are doing, how they respond, evaluating what is being said or done against expectation and criteria, and considering what might be said or done next (Mason, 2002, p. 7).

Teacher noticing, in the context of the mathematics classroom, has been the focus of many research projects in the last two decades (e.g., Mason, 2002; Santagata et al., 2021; Sherin et al., 2011). An early conceptualisation of noticing involved seeking to understand what teachers "look and listen for while they teach" (Erickson et al., 1986, p. ii). Since then, various conceptualisations have developed, starting with what teachers attend to within an instructional setting, to later including interpreting and responding to particular events (V. R. Jacobs et al, 2010). Each of these are situated in classroom interactions which are seen as central to teacher noticing (van Es & Sherin, 2002). König et al., (2020, under review, cited from Santagata et al., 2021) identified four categories in which theoretical frameworks of teacher noticing can be allocated: a cognitive psychological perspective, a socio-cultural perspective, a discipline-specific perspective and an expertise-related perspective. The cognitive psychological perspective puts a focus on the mental processes of teachers' noticing. It is based on van Es and Sherin (2002), who defined teacher noticing as having the following three components: "(a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom interactions" (p. 573). In 2021, van Es and Sherin introduced a new facet to the construct of noticing that they termed *shaping*. This refers to teachers' active construction of interactions, in the midst of noticing, to access additional information about student thinking. A recent conceptualisation of teacher noticing that Santagata et al. (2021) also allocated to this perspective is the perception, interpretation, decision-making (PID) model (Kaiser et al., 2015). The authors explicitly distinguished between three facets: "(a) perceiving particular events in an instructional setting; (b) interpreting the perceived activities in the classroom; and (c) decision-making, either as anticipating a response to students' activities or as proposing alternative instructional strategies" (p. 374). In contrast to research focussing solely on classroom interactions and in particular student thinking (e.g., Colestock & Sherin, 2015; Dreher et al., 2021; Huang & Li, 2012), the PID model broadens the focus to a variety of teaching aspects (Yang et al., 2020). The socio-cultural perspective comprises theoretical frameworks of noticing to a "socially organized and situated" (Santagata et al., 2021, p. 121) concept. This perspective is based on the work of Goodwin (1994). Within this perspective the focus is "on communities of practice, use of artifacts, as well as social and cultural norms that frame and inform teacher noticing" (Santagata et al, 2021, p. 121). For example, in the work of Dreher et al. (2021), the discussion on teacher noticing is situated in a cross-cultural context between Asian and Western countries. They argue that most studies implicitly or explicitly use a normative frame of reference of what teachers *should* notice in order to demonstrate teaching expertise (e.g., Choy et al., 2017; Stockero & Rupnow, 2017). This study emphasises the influences of cultural norms on teacher noticing. The third perspective, a discipline-specific one, draws from the work of Mason (2002) and emphasises the "importance of raising teachers' presence, awareness, and sensitivity of students and their understanding of the subject matter" (Santagata et al., 2021, p. 121). Although Santagata et al. (2021) acknowledge that this approach shows similarities to the cognitive psychological perspective, the focus is on the teachers' sensitized awareness according to Mason (2002). Research allocated to the last perspective identified by Santagata et al. (2021), the expertise-related perspective, compares experts and novices noticing skills. As an example, Yang et al. (2021) conducted a comparative study between experienced and less experienced teachers in China, revealing how teacher noticing is influenced by teaching experience.

Our theoretical framework best fits into the cognitive psychological perspective, as "studies that are grounded in this perspective also contribute to our understanding of the nuances of noticing and provide frameworks and tools that other teacher educators can use to design their own teacher learning activities" (Santagata et al., 2021, p. 121),

Table 1 Teacher demographics and work-related characteristics

Teacher pseudonym	Country	Gender	Age	Years of teaching experience	Number of students	School type (grades)	MCK & PCK scores
Sally	Australia	female	28	4	17	Primary school (1–6)	24/29 & 17/18
Ruo	China	female	33	10	40	Primary school (1–5)	24/29 & 15/18
Anna	Germany	female	48	7	22	Inclusive comprehensive school (5–13)	23/29 & 15/18

which we seek for in this paper. Like Kaiser et al. (2015), the LfL team expands on the idea of teacher noticing by focusing not only on student thinking but also on instruction, mathematical content, and teacher's knowledge. These categories emerged from the project's pilot study (D. M. Clarke et al., 2015) and were intended to cover a broader focus of teaching and teacher knowledge (e.g., Johnson et al., 2017). Also, the context for noticing in the current study went beyond the classroom to include the teachers' planning or preactive activities, as described by Dindyal et al. (2021). In the case of LfL, preactive activities included the teachers being observed as they engaged with the mathematical tasks, as a student might in the lesson to come. Dietiker et al. (2018) proposed the notion of *curricular noticing* to describe how teachers recognize affordances and constraints of curriculum materials. The current research design seeks to provide evidence of what teachers identify as important or noteworthy in classroom and planning situations in a cross-cultural study. This study aimed to investigate teacher learning as situated practice, in curricular, organisational and cultural terms, so we also consider the socio-cultural perspective. In line with Ball (2011), we claim that noticing is a "culturally shaped perception" (p. 21), and therefore the cultural and individual backgrounds of the case study teachers are carefully considered in the discussion of the findings.

Jacobs and Spangler (2017) described three major approaches for studying teacher noticing. These were using researcher-selected artefacts of practice such as video and student work samples; retrospective reflections on teaching through interviews; and inferring teacher noticing from observation of teachers. In this paper, the approach of retrospective reflections on teaching a lesson has been extended by including reflections on the lesson planning through interviews conducted prior to the lesson. While video data of each lesson in the case studies were collected, these were not used to study teacher noticing. Rather, the video was employed to provide stimulus for teachers during the post-lesson interviews (see Chan et al., 2018). Through investigating teacher noticing in a broader way, the LfL research team sought to contribute to the body of research on teacher noticing during the teachers' own practice and their retrospective reflections.

3 Methods

The LfL study employed multiple case studies in three countries, Australia, China, and Germany. The combination of the three countries was constructed in this project due to their differences in student achievement (OECD, 2014), in teacher professionalism and expertise (Tatto et al., 2012), and in the cultural diversity of the school communities (Thomson, De Bortoli & Buckley, 2013). Although three teachers from each country participated in the case studies, owing to space restrictions, only one teacher from each country is discussed in this paper.

In this paper, we examined three cases, one from each country, to address two research questions:

- What do the mathematics teachers from each country notice and claim to learn through the process of planning and teaching lessons when given researcher-developed lesson plans?
- What is similar (and different) about what they notice?

To gather background information on teachers' knowledge, each case study teacher completed a set of assessment questions, drawing from the TEDS-M study[3] (Tatto et al., 2012) that included two kinds of items:

- Mathematical Content Knowledge items (MCK; e.g., teachers are shown a picture of two items wrapped with ribbon, one a cube and another a cylinder, and are required to determine which required the most ribbon)
- Pedagogical Content Knowledge items (PCK; e.g., given four addition and subtraction story problems, teachers are asked to nominate the two problems which Grade 1 children would typically find most difficult to solve)

The results shown in Table 1 indicate that the teachers demonstrated very similar overall assessment results, which provides a strong case for comparability. A (multiple) case study methodology was chosen, as it allows for

[3] The official permission from the TEDS-M study authors is gratefully acknowledged.

the investigation of the interaction between individual teachers and their lessons in sufficient detail to identify teacher noticing, while accounting for classroom, school, and teacher characteristics. The research teams in the three participating countries agreed on the three topics for comparison–fractions, area of polygons, and transformations of figures. These topics were chosen because they have a prominent place in the curriculum of all three countries. In Australia and Germany, these topics are usually taught in Grade 6; while in China they are taught in Grade 5.

3.1 Participants

The national case studies were conducted over a period of several weeks between March 2018 and March 2020. Parental consent was obtained for all students in the observation classes taught by the teachers. Table 1 provides an overview of the participating teachers' demographics, professional characteristics, and performance on TEDS-M items relating to MCK and PCK.

The three case study teachers demonstrated very similar knowledge scores, but they varied in age as well as their teaching experience.

3.1.1 Sally—The Australian (AU) case study teacher

Sally was teaching at a Catholic primary school with around 300 students in a semi-rural town. Sally taught within the Grade 5/6 learning space. Only students in Grade 6 constituted the class that Sally taught for the three lesson pairs for the project. The Australian school education system consists of preschool, primary schools (Grades K–6), and secondary schools (Grades 7–12). Sally had completed a Bachelor of Education and during her participation in the study was undertaking a Masters of Education in Mathematics Leadership while teaching full time. She also held a leadership role as the Grade 5/6 Coordinator. Owing to her additional studies and interests in mathematics education and her role in supporting other staff with mathematics planning and curriculum, she considered herself a mathematics specialist.

3.1.2 Ruo—The Chinese (CN) case study teacher

Ruo was working in a public primary school (Grades 1–5) located in a small city in south-western China, with 24 classes and more than 1000 students. The Chinese school education system has mainly three levels. Primary school (usually Grades 1–5, some schools include Grade 6) is followed by lower secondary (Grades 6–9) and higher secondary (Grades 10–12). During the LfL data collection in 2018, Ruo was teaching Grade 5. She had spent four years completing her bachelor diploma, including one year of mathematics teacher training. Prior to 2018, she had been working as a teacher for ten years, including three years in the current school. Other than mathematics, she was also teaching Morality and Law (a civic education course). In her school, Ruo led the Mathematics Teaching Research Group (TRG) and acted as a liaison person between the school and the researchers during the school's participation in classroom research.

3.1.3 Anna—the German (DE) case study teacher

Anna taught at an inclusive comprehensive school (Grade 5–13) with more than 1000 students. *Inclusive* in this context means that students with and without special needs attend the same classes. In Anna's class, 5 out of 22 students were identified as students with special needs (predominantly learning difficulties). An additional teacher for those 5 children was present in the majority of the lessons. In Germany, teacher pre-service education is mainly threefold, depending on the school type and grade level teachers later want to teach: primary (usually Grades 1–4), lower secondary (Grades 5–10), or higher secondary (up to Grade 13). In this general context, Anna's teacher training qualification is not typical. The highest year level she could teach in mathematics is Grade 7, as she took additional courses in this subject as part of her primary teacher training. Even though Anna has already had seven years of teaching experience, she taught mathematics in Grade 6 for the first time during the data collection in early 2020.

3.2 Data collection and preparation

Considering previous research findings that suggest that it can be difficult for teachers to articulate how they make sense of a classroom event or how they decide to respond to classroom events (V. R. Jacobs, 2017; Sherin et al., 2011), a research design was developed which involved the provision of researcher-designed lesson plans as stimuli. In Australia and Germany, the same lesson plans were used. Due to the tighter curriculum requirements in China, it was not possible to use the same lesson plans in Chinese Grade 6 classrooms. Therefore, the lesson plans on the three chosen topics used for the Australian and German data collection were re-developed to meet the needs for Chinese Grade 5 classrooms (see Chan et al., 2021 for descriptions of all lesson plans). For all three topics, the case study teachers were invited to adapt the provided lesson plan to meet the needs of their students and then teach the lesson to their class. After teaching the adapted lesson, the teachers were asked to design a follow-up lesson based on a provided template (see IDM, 2021) and teach this lesson to the same class in the following lesson. The reason for the provision of researcher-designed lesson plans in the first place as well as a lesson

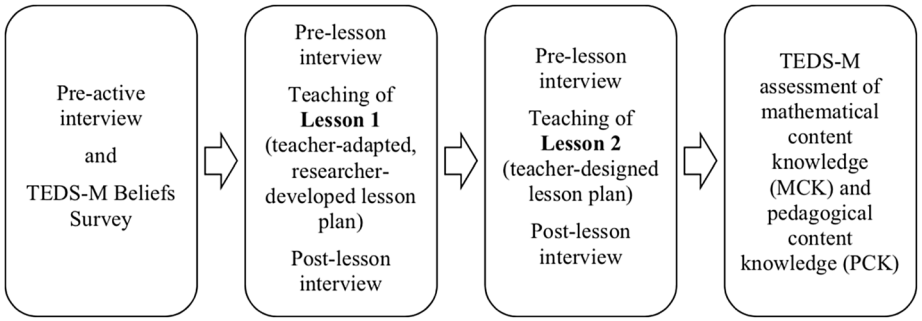

Fig. 1 The overall research design (Lesson 1 refers to the adapted lesson and Lesson 2 to the follow-up lesson) (Chan et al., 2018)

plan template was to allow for the teachers to explicitly decide and articulate what they chose to change with respect to the lesson plan provided and what they planned to teach in the follow-up lesson. This process resulted in the teaching of three adapted lessons and three follow-up lessons (three lesson pairs) by each participating teacher. Pre- and post-lesson interviews were conducted with all teachers for every lesson (see Fig. 1).

In order to identify what teachers noticed and claimed to learn through the planning and teaching of the LfL lessons, the study was designed to generate data on the teachers' adaptation of the provided lesson plan, the teachers' reflective thoughts about the lesson and their actions during the lesson, and the respective consequences for the planning and teaching of the follow-up lesson. In order to establish how teachers develop professionally through their classroom teaching, the data analysis specifically focused on categories identified in the responses in terms of their selective attention as in "noticing something significant" (Mason, 2002, p. 75). Table 2 lists the seven questions that were used in the interviews. The teachers' responses to these learning-related questions highlight what they identified as noteworthy.

3.3 Data analysis

Prior to the data analysis, the LfL research team developed a codebook to establish coding guidelines and document examples and non-examples for each category. The coding process was conducted by each national research team for their respective three lesson pairs. Within each country, the research team members were coding separately and compared their results afterwards to ensure coding consistency. The process of the data analysis mainly followed the thematic analysis steps of Braun and Clarke (2006). In order to allow for the comparison of the case study teachers in terms of the focus of the teachers' response, standardised units for coding statements—so called idea units—were generated. These idea units describe "a distinct shift in focus or change in topic" (J. K. Jacobs et al., 1997, p. 13). The partitioning of the teachers' responses into idea units was based on different ideas and topics rather than the number of words or sentences a teacher used to express an idea. Therefore, a teachers' statement could generate several idea units even if it was a response to only one question, for example:

> Question: *Was there anything that happened during the lesson that was really unexpected by you?*
> Response: *Look, the time management of the lesson was a bit unexpected. I was worried at first thinking, oh, I don't want to finish it too quickly and rush them. And then I guess at the end it was like, oh, we're really running out of time here* [idea unit 1]. *At first, I was a bit surprised that not many kids were able to sort of write down their reasoning […]* [idea unit 2].

Only the idea units related to the interview questions were selected for further coding. The idea units were then allocated to two learning categories: *New Realisations* (NR) and *Consolidations* (C). If the case study teachers made explicit

Table 2 List of relevant interview questions

Interview session	Interview questions
Pre-adapted lesson	Please describe anything you have learned as a result of participating in the task activity, and in reading and planning the lesson. Explain your response.
Post-adapted lesson	Was there anything that happened during the lesson that was really unexpected by you? Which moments in the lesson do you think provided learning opportunities for you? What did you learn?
Pre-follow-up lesson	Please describe anything you have learned as a result of planning/preparing this lesson. Explain your response.
Post-follow-up lesson	Was there anything that happened during the lesson that was really unexpected by you? Which moments in the lesson do you think provided learning opportunities for you? What did you learn? Is there anything else you have learned over the course of the two lessons and your participation in this project?

suggestions for changes (planned or enacted) in future lessons in terms of instruction, the idea unit was additionally coded as *Adaptive Practice* (AP). This paper reports only *what* teachers noticed and claimed to learn, and so we do not distinguish between Consolidations or New Realisations and will not elaborate on the learning categories here (see Chan et al., 2020). To gain a better understanding of the aspects the teachers noticed based on their responses, and in line with Kaiser et al. (2015), the idea units were then additionally coded in terms of attentional foci. Four broad categories were identified:

- Mathematical content [M]: What reference does the teacher make to mathematical content?
- Students [S]: To what aspects of students' knowledge, behaviour or needs do teachers refer?
- Instruction [I]: To what instructional actions or considerations do teachers refer?
- Teachers [T]: To what aspects of themselves do teachers make reference?

These categories were used separately and exclusively for each idea unit if an idea unit referred to only a single category. For example, if an idea unit focused on a teacher's instructional practice without mentioning students, mathematics, or themselves, it would be coded as I. The categories were used in combination if the idea unit covered more than one category, such as noticing a students' mathematical misconception, which was coded Mathematics and Students [MS], for example (for more examples see I-M-S-T code in Table 3):

> *He* (the student, therefore S) *didn't understand that dividing into two equal parts means the denominator is two* (about mathematics, therefore M).

The 1st level coding corresponds to the I-M-S-T categories, except that we merged M and T together into *mathematical and teaching knowledge*, because we did not find clear evidence of statements only about T. We then analysed the data of the three cases to identify more refined 2nd level categories. Since there were still categories that included statements about very different classroom aspects, subcategories on a 3rd level emerged. For example, the following two statements are both about student knowledge, but different aspects:

> Statement 1: *I was surprised...The exercise should be done very quickly, because it is very easy.*
> Statement 2: *So generally most of them changed their thinking so I could take that little bit of a next step next time, rather than just doing pretty much exactly the same thing.*

In statement 1, the teacher claimed to have overestimated her students, while in statement 2, the teacher talked more general about her students' knowledge. Table 3 provides an overview of the categories that emerged as well as exemplifying quotes. More categories on 2nd level and 3rd level might arise, if more cases are examined in further studies in this project.

The category *preparation and use of equipment* includes statements about the preparation and use of learning materials as well as other equipment. Teacher responses which contained aspects of the lesson that had worked well or less well were collated to the category *effectiveness of lesson elements*. Statements in which the teachers described how specific moments of the lesson affected their teaching practice, such as insights and conclusions referring to the progress of the current or future lessons with respect to the students' way of thinking or difficulties, were added to the category *teaching process*. The category *lesson content* includes statements about the mathematical content of the particular lesson, such as considerations about its meaning or connections to other mathematical topics. If teachers commented on students' general behaviour or particular challenges the students were facing without reference to the mathematical content, the statements were coded as *student characteristics and challenges beyond mathematics*.

4 Results

In this section the results on what the three case study teachers noticed through planning, teaching, and reflecting of their lessons are summarised. For a better understanding and comparability, the three lesson pairs are numbered: LP1 (lesson pair 1) was about fractions; LP2 about area of polygons (in China more specifically about parallelograms); and LP3 about transformations of figures (see IDM, 2021 for descriptions of all lesson plans used in the study).

4.1 Overview

In the interviews following the teaching of the three lesson pairs, altogether the Australian case study teacher Sally made 42 statements, the Chinese teacher Ruo 51 statements, and the German teacher Anna 30 statements (see Table 4). Some statements (4 in the Australian data, 2 in the Chinese data) were assigned to two coding categories. For example, Sally (AU) commented that using the model of a fraction wall as a game presented an entirely new idea to her. In this case the statement was coded *task design* and *pedagogical content knowledge*.

With respect to all three lesson pairs Sally (AU) mainly commented on the *effectiveness of lesson elements*,

Table 3 Overview of the refined coding categories of teacher noticing

Consolidations (C) / New Realisations (NR) (+Adaptive Practice (AP))	Refinement of Categories based on I-M-S-T Coding			Exemplifying Quote [I-M-S-T Code]
	1st level	2nd level	3rd level	
	Instruction	Time Management		Ruo (CH): "I originally wanted to invite two more groups to report. Due to time constraints, I only invited that one group." [IS]
		Preparation and use of equipment	Preparation and deployment of equipment	Sally (AU): "Maybe I would've had the dice out on the table" [I]
			Students' use of equipment	Anna (DE): "That fact that they hardly cut anything out was rather surprising for me" [IMS]
		Effectiveness of lesson elements	Task introduction	Sally (AU): "I didn"t expect that my explanation probably wasn't clear enough with that" [IMS]
			Task design	Anna (DE): "For them it was perfectly clear: I need to identify the identical parts – done. However, they did not realise what they were supposed to do [highlight edges and angles]. I should have pointed out what I expected them to do or rather choose a different task." [IS]
			Task explanations	Sally (AU): "I probably should have gone into it in more detail like I did the first time" [IS]
			Lesson design	Ruo (CN): "For the children in my class, I think it's necessary to divide the lesson content (into two lessons). Today, I taught these two parts of content together, and I found that the children's performance is not as good as usual. Many children are at a loss."[IS]
		Teaching process		Ruo (CN): "Because it was not mentioned, the child did not understand. In fact, this kind of conflict is quite good." [IMS]
	Mathematical & teaching knowledge	Lesson content		Anna (DE): "And then I thought: What do you need this for? But this way of calculating the area makes you think: When I cannot do it from the inside, it might help to look what is around and then start backtracking." [IM]
		Teacher knowledge	Content knowledge	Sally (AU): "One student asked me what an 11-sided shape was. I didn't know it off the top of my head so that was one" [MT]
			Pedagogical knowledge	no data found
			Pedagogical content knowledge	Sally (AU): "And that idea of using the fraction wall for a game, is not, you know, something that I've done a lot with" [IM]
	Students	Student knowledge	Underestimating students	Anna (DE): "She provided the explanation with the halves – well I didn't anticipate this explanation" [MS]
			Overestimating students	Ruo (CN): "I was surprised...The exercise should be done very quickly, because it is very easy." [MS]
			Realisation of student misconceptions	Ruo (CN): "He (the student) didn't understand that dividing into two equal parts means the denominator is two. He thought that two shares are six, so the answer should be one-sixth. That is wrong, he confused quantity with ratio." [MS]
			More general statements about student knowledge	Sally (AU): "So generally most of them changed their thinking so I could take that little bit of a next step next time, rather than just doing pretty much exactly the same thing" [IS]
		Students' characteristics and challenges beyond mathematics		Anna (DE): "That was unexpected, that they remained calm for so long." [IS]

Table 4 Number of statements for each lesson pair and case study teacher

Case study teacher	Number of statements			
	Total	LP1	LP2	LP3
Sally	42	14	13	15
Ruo	51	16	18	17
Anna	30	8	10	12

especially the task design. Furthermore, she took notice of what her students could or could not do and this appeared to influence her decisions about the nature of the task in the next lesson (e.g., how difficult she would make it) or the structure of her follow-up lesson (e.g., how long she might spend on aspects of the lesson). Sally was also able to reflect on her own knowledge in terms of what was challenging for her mathematically and what she understood about her students' proficiency.

Ruo (CN) reflected more on *instruction* and *students*, rather than *mathematical and teaching knowledge*. Ruo's reflection can be summarised as results-oriented or students-oriented. Her students' performance was an important indicator in evaluating her current lesson and adjusting the following one. She was still getting to know her students, which would help her to plan future lessons in a more flexible way.

Anna (DE) mainly commented on *instruction*, specifically on the *effectiveness of particular lesson elements*. These statements were often related to student knowledge and participation in the lesson. On numerous occasions, the mathematical challenges Anna experienced herself seemed to limit her judgement of student abilities and her lesson planning and adjustment.

4.2 Contrasting the results of the three case study teachers

While we are aware of the differences in the teachers' responses, there are also several commonalities that are noteworthy. Figure 2 provides an overview of the distribution of the coding categories for the 2nd level for each case study teacher.

Across all three teachers and all three lesson pairs, most of the teachers' statements were about the *effectiveness of lesson elements*: Sally (AU) 15/46 (32.6%); Ruo (CN) 24/53 (45.3%); Anna (DE) 11/30 (36.7%). These statements often included references with respect to adaptive practice: Sally (AU) 5/15 (33.3%); Ruo (CN) 20/24 (83.3%); Anna (DE) 7/11 (63.6%). All three teachers, for example, explained the way in which they would change a task in the future to make it more accessible for their students. However, the foci in these statements clearly varied across the teachers. Table 5 provides an overview of the 3rd level coding for the *effectiveness of lesson element*s, including examples of statements. While the Chinese and German case study teachers focussed on the overall *lesson design,* Ruo (CN) 19/24 (79.2%); Anna (DE) 6/11 (54.5%), the Australian case study teacher showed an equal number of idea units about *lesson design* and *task design* (6/15, which is 40.0%).

Student knowledge was the category with the second largest number of statements for all three teachers: Sally (AU) 13/46 (28.3%); Ruo (CN) 16/53 (30.2%); Anna (DE) 7/30 (23.3%). Most of the statements about *student knowledge* occurred after a lesson was finished, indicating that the teachers were taking note of what their students did or did not know during the lesson. Unlike the Chinese and German teachers, Sally's statements about *student knowledge* often included references with respect to adaptive practice (Sally

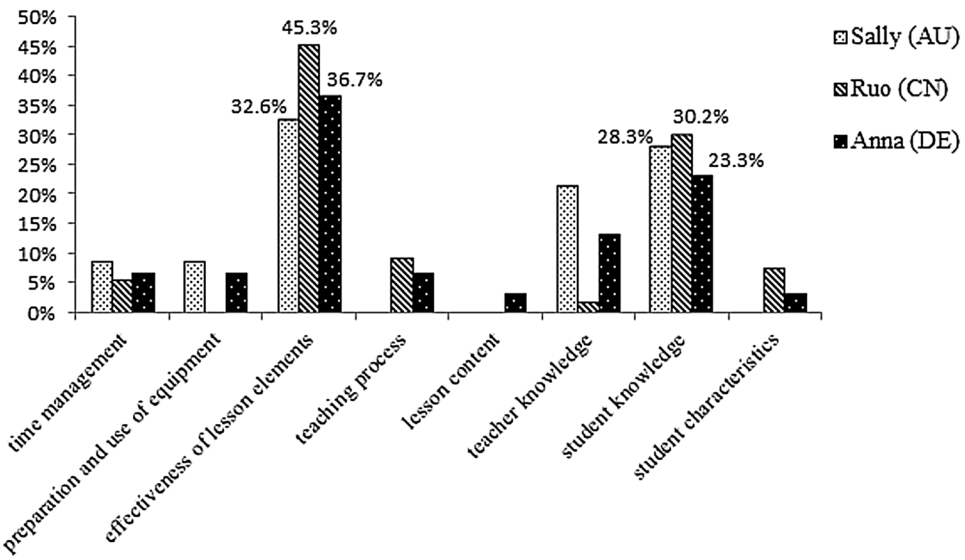

Fig. 2 Comparing the distribution of coding categories among the three case study teachers

Table 5 Comparing overview and examples of coded statements on 3rd level for the *effectiveness of lesson elements*

Category & no. of statements	Sally (AU)…	Ruo (CN)…	Anna (DE)…
Task introduction Sally (AU) 1/15 Ruo (CN) 1/24 Anna (DE) 0/11	…reflected that the way in which she introduced the game Colour in Fractions to her class was not clear enough as she noticed some students were not following the rules of the game appropriately (LP1)	…expressed that she should have stated the requirements of a group task more explicitly and directly (LP2)	no data available
Task design Sally (AU) 6/15 Ruo (CN) 0/24 Anna (DE) 3/11	…explained that tasks she had designed in addition to the given lesson plan or for the follow-up lesson were too complex for some students, or that her task could have been adjusted to make the mathematics easier (LP1)	no data available	…expressed surprise that some students did not understand which part of a shape they should have coloured, and she commented that in future lessons on this topic she would choose another task or give more detailed instructions (LP3)
Task explanation Sally (AU) 2/15 Ruo (CN) 4/24 Anna (DE) 2/11	…reflected that it was important in the lesson to explain things in different ways, and to make connections to previous lessons to enhance students' understanding (LP3)	…mentioned that she should use a description of rotation suggested in textbooks for her teaching instead of a description summarised by herself, because using terms from textbooks seem to be better regarding mathematical expression (LP3)	…reflected on the statement of a student that "you can build one whole out of two half pieces." In reaction to this, she thought about the best way to visualise and explain this process of merging/assembling and stated that she might structure it in another way in the future (LP2)
Lesson design Sally (AU) 6/15 Ruo (CN) 19/24 Anna (DE) 6/11	…learned the importance of revisiting content (transformations) for student learning and suggested she would do more of this (LP3)	…explained that even though some students were willing to express their thoughts, they were struggling with it, and they did not know what they were learning. Therefore, Ruo indicated that next time she would design the two tasks of this lesson step-by-step (LP2)	…realised that in some situations it might be better to reduce teacher explanation in order to give students more time and opportunity to work on their own. She explained that these phases of individual work provided possibilities for her to support students individually. Her statement also involved plans to take up this approach in future lessons (LP3)

6/13; Ruo 4/16; Anna 1/7). Table 6 provides an overview of the results of the 3rd level coding for *student knowledge*.

For the Australian case study teacher, most of these statements were *general comments about student knowledge*, i.e., 9/13 (69.2%). For example, in LP1, Sally either specifically identified mathematics content that the students appeared to know or not know (e.g., defining transformations), or she explained how her questioning or assessments contributed to her knowledge of students' understanding without naming the content. The Chinese teacher made many general statements about student knowledge as well, i.e., 6/16 (37.5%), but she talked even more about having *overestimated* her students, i.e.,7/16 (43.75%). In contrast, the German case study teacher predominantly claimed to have *underestimated* her students, i.e., 5/7 (71.4%).

Both the Chinese and the German teachers provided at least one statement about how specific *teaching processes* provided learning opportunities for them. For example, Ruo stated that the teaching conflict that occurred during one lesson was conducive in helping students to understand the concept of mixed fractions (see quote in Table 3). Anna perceived personal learning opportunities on occasions when the students were experiencing difficulties understanding the content and she found it difficult to relate to their difficulties. As a consequence, she intended to plan her lessons more carefully in this respect, considering smaller steps in the future.

All three case study teachers reflected on their *time management* (Sally 4; Ruo 3; Anna 2). Respective statements mostly addressed tasks that took more time than expected or the intention to give students more time to work on their own in future lessons. For example, with respect to her reflections on her follow-up lesson (LP2) Sally indicated that she would teach her planned task over two lessons in future.

Ruo did not mention the *preparation or use of equipment* in the interviews, but the Australian and German teachers did (Sally 4; Anna 2). For example, they noticed their students' use of provided material, but expressed that in different ways. Sally was pleased about the fact that she had thought to supply some pencils for the task rather than pens as these made it easier for students to erase and redo the task. In contrast, Anna expressed her surprise that students did not use the provided material for solving a task.

At least one statement about *teacher knowledge* was found in all three of the teachers' data (Sally 10; Ruo 1; Anna 4). Across all three teachers, most of these statements were about their own *content knowledge*. Ruo stated only once explicitly that her own knowledge was lacking. She claimed to have learned about the origin of and necessity for fractions during her lesson preparation. Sally and Anna both mentioned that it was challenging for them to notice and define transformations of figures. Only Sally made statements which were assigned to *pedagogical content knowledge* (see exemplifying quote in Table 3).

5 Interpretation and discussion

The previous section provided detailed descriptions of what the three case study teachers noticed and claimed to have learned during their lesson planning and teaching. In the following part, the three most revealing observations regarding our research questions are discussed.

Observation 1: The three case study teachers commented frequently on *the effectiveness of lesson elements.*

This appears to be a strong indication that teachers particularly noticed these elements during the process of planning and teaching of a lesson. However, this may also be influenced by our research design that employed researcher-designed lesson plans. Moreover, in the statements regarding the *effectiveness of lesson elements*, the three teachers elaborated frequently on the implications for their future lessons, which indicates that these moments in particular provided learning opportunities for them. Many of these statements were related to their students' knowledge and thinking. This resonates with the literature on noticing and suggests the three teachers were making connections between the specifics of classroom interactions and the teaching and learning occurring in their classrooms (van Es & Sherin, 2002).

However, not all statements about the effectiveness of lesson elements addressed student thinking. For example, many focused on their pedagogical actions and choices in the act of teaching and planning the lesson. This indicates that teachers also identified other aspects as noteworthy and reflected about them. Therefore, in line with the PID-model, which comprises a broad understanding of noticing and does not limit noticing to special incidents or features such as error detection or students' thinking processes (Kaiser et al., 2015, p. 374), we would argue for a broad focus in teacher noticing research.

The international research team defined the effectiveness of lesson elements as noticing which aspects of the lesson had worked well or not well, *in the teacher's view*. This is something all three teachers found noteworthy, therefore their lesson planning and the resulting expectations influenced what they noticed, which aligns with similar observations and statements by other researchers (e.g., Erickson, 2011; Mason, 2002).

Table 6 Comparing overview and examples of coded statements on 3rd level for *student knowledge*

Category & no. of statements	Sally (AU)…	Ruo (CN)…	Anna (DE)…
Underestimating students Sally (AU) 2/13 Ruo (CN) 1/16 Anna (DE) 5/7	…reflected that she should not underestimate her students' abilities (regarding the lesson on Pick's Rule; LP2)	… said that a student suggested to divide the pie into more pieces when presenting his ideas, which Ruo thought was great and beyond her expectation (LP1)	…was surprised that a student, who usually doesn't say much in class, could explain how to identify congruency (LP3)
Overestimating students Sally (AU) 2/13 Ruo (CN) 7/16 Anna (DE) 1/7	…admitted to being surprised that some students had not done as well as she had anticipated (LP1)	…was surprised that many students were confused about the meaning of midpoint. She explained that students had already learned about midpoints in Grade 3, which is why she believed that it should not be difficult for them (LP2)	…expressed her surprise that some students without special needs experienced great difficulties with marking fractions (LP1)
Realisation of student misconceptions Sally (AU) 0/13 Ruo (CN) 2/16 Anna (DE) 0/7	no data available	…explained for example that after asking the students to divide 12 into two parts, some students falsely thought the solution was 1/6. Ruo realised that students might confuse quantity and ratio. She commented that this might be a serious problem for students (LP1)	no data available
More general statements about student knowledge Sally (AU) 9/13 Ruo (CN) 6/16 Anna (DE) 1/7	…learned about her students' thinking about the notion of congruence (LP3)	…realised that the students in her class are not very clear about the concept of 30°, so she changed 30° into a blank space according to the lesson plan designed by the researcher (LP3)	…was confirmed in her belief that students would have difficulties with the concept of angles (LP3)

Observation 2: In contrast to Sally (AU), Ruo (CN) and Anna (DE) repeatedly reported that they had overestimated or underestimated their students' capabilities.

Anna (DE) taught Grade 6 mathematics for the first time during the study and the transcripts of the videorecorded lessons and interviews indicate that there were occasions when she found the content personally challenging. As a result, she expected her students to have similar difficulties with some of the activities. It seems that this might be the main reason why she repeatedly stated that she had underestimated her students, as the students generally solved the tasks without major difficulties.

Ruo (CN), in contrast, claimed on several occasions that she had overestimated her students. She appeared to have high expectations of her students: "I used to be strict with them in my daily teaching […] It is still very hard, they are not doing well." At the time of her participation in the LfL project, Ruo had had 10 years of teaching experience. Because she was the head teacher of the class, it could be expected that she knew her students and their abilities well. Ruo explained that she hoped that having higher expectations would help improve her students' mathematical skills. This is in line with the old Chinese saying "strict teachers produce outstanding students (严师出高徒)" (Wang, 2013) and China having a dominant exam culture as well as an emphasis on basic knowledge and skills in mathematics (Zhang et al., 2005). Thus, Ruo's high expectations might be due to those culture specific educational factors.

Sally's responses appear to reflect the wider Australian context where primary school teachers are unlikely to stream their classes by ability and therefore need to cater for a wide range of abilities in one class (e.g., see Thomson & Hillman, 2019). To do so, many Australian teachers are familiar with the use of *enabling and extending prompts* (Sullivan, 2011) to support students who may struggle to begin the task and to extend students who complete the task quickly, respectively. Sally was familiar with differentiating tasks using these prompts. While she did comment on four occasions across the three lesson pairs that some students had not done as well (or did better) than she had anticipated, her expectations overall were that her students would have a range of levels of understanding, and it was her role to anticipate and cater for these.

Observation 3: For the three teachers, the number of statements in each category varied with respect to different lesson pairs.

For example, Anna's mathematical content knowledge was challenged on several occasions, but specifically regarding one lesson pair (LP3 about transformations). This might have influenced the quantity of her statements in general (for LP3 it was the largest number over all three lesson pairs), and her statements regarding her own *content knowledge,* in particular. It may account for why she commented on *student knowledge* more often than other teachers, particularly with respect to *underestimating* her students. Not surprisingly, as Dindyal et al. (2021) noted, the specific mathematical topic in the lesson may have an impact on what teachers notice about students' thinking. Our findings indicate that the specific topic and individual lesson do indeed seem to play a role in influencing teacher noticing and learning. This highlights the need to consider the lesson topic and the teachers' knowledge of and familiarity with it when studying teacher noticing.

6 Limitations of the study

Analysing and comparing data from individual teachers with varying cultural and educational backgrounds working in different school systems provided detailed insights in what it is that teachers notice and claim to learn in their classroom practice. However, the authors are aware that this paper is based on case study data and hence the results are not globally generalisable. Owing to tighter curriculum requirements, the Chinese case study teacher was provided with different lesson plans, so the comparison has potential limitations for finding more specific commonalities. Furthermore, she taught the three lesson pairs in Grade 5, while the Australian and German teachers taught their lessons in Grade 6, which may have influenced the findings in some way.

A more general challenge in conducting international research of this kind is the difference in the backgrounds of the researchers. In this project, three different languages and cultural backgrounds had to be considered. D. J. Clarke (2013) identified the two dilemmas of (1) validity and (2) comparability that can compromise cross-cultural studies in mathematics education. Dilemma 1, which is the cultural specificity of a cross-cultural code, should be considered within this context. The international research team deliberately tried to weaken the "use of culturally-specific categories for cross-cultural coding" (D. J. Clarke, 2013, p. 1857) by discussing elements of categories within the international team on many occasions. However, international cross-coding for example can be challenging as other teams could not code the original statements as they were in different languages. Another challenge was interpreting teacher statements and defining and identifying coding categories. This was in part due to the different cultural backgrounds of the researchers, as well as the teachers' narrative. For example, the idea units may be interspersed with comments in anticipation of later questions, or reference to earlier comments. It was therefore particularly important that the national teams were consulted on the local interpretation of the teacher statements informing

the analysis and cross-cultural comparison. In this process of negotiating the categorisation of teacher noticing by the international research team, Dilemma 2 highlights the difficulties of using "inclusive categories to maximise applicability across cultures, thereby sacrificing distinctive (and potentially explanatory) detail" (D. J. Clarke, p. 1858). The international LfL research team experienced this dilemma in defining and refining categories but was still able to find commonalities in the teacher statements regarding more fine-grained categories for teacher noticing at the level being described in the findings.

7 Conclusions and implications

The LfL study provided detailed insights into what mathematics teachers from Australia, China and Germany noticed and claimed to learn through the planning and teaching of their lessons when provided with a researcher-developed lesson plan. The study considered their cultural and individual backgrounds, as well as the commonalities and differences in what they noticed.

The research design of this project enabled the three teachers to reflect on their lessons in as much detail as possible with regard to their prior lesson planning (see in particular *observation 1*). The provision of researcher-developed lesson plans as well as carefully chosen prompting questions seemed to help teachers not only to notice and reflect on their students' thinking and knowledge but on their teaching practice (e.g., the effectiveness of lesson elements) in general. Further research is needed to investigate whether the chosen approach (lesson plans and prompting questions) and a broadened focus of teacher noticing will lead to an improvement of teaching quality. If so, this approach can possibly be used in professional development courses to support teachers' noticing abilities more systematically. (See, e.g., Chan et al., 2021 for a discussion of different contexts in which this approach could be embedded.)

Overall and not surprisingly, the more fine-grained the categorisation was, the more distinct were the particular aspects about planning and teaching of lessons that the three teachers noticed. The reasons for these differences can be cultural or individual (see *observation 2*) but appeared to be also influenced by the particular lesson and its content (see *observation 3*). In particular, what teachers noticed seemed to be influenced by their expectations of their students.

The findings also suggest that the influence of the provision of (researcher-developed) lesson plans on what teachers notice should also be further investigated. It seems that both the teachers' backgrounds and their (reflections of) lesson planning influenced what they expected and therefore what they noticed.

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References

Ball, D. L. (2011). Foreword. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: seeing through teachers' eyes* (pp. 21–25). Routledge.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Chan, M. C. E., Cao, Y., Barton, D., Damrau, M., Wang, C., & Clarke, D. J. (2020). Learning from lessons: a framework for categorizing different forms of mathematics teacher in-class learning in Australia, China, and Germany. In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.), *Interim Proceedings of the 44th Annual Meeting of the International Group for the Psychology of Mathematics Education* (pp. 90–98). Khon Kean, Thailand: PME.

Chan, M. C. E., Clarke, D. J., Clarke, D. M., Roche, A., Cao, Y., & Peter-Koop, A. (2018). Learning from lessons: Studying the structure and construction of mathematics teacher knowledge in Australia, China and Germany. *Mathematics Education Research Journal, 30*(1), 89–102. https://doi.org/10.1007/s13394-017-0214-6

Chan, M. C. E., Roche, A., Clarke, D. M., & Clarke, D. J. (2021). *Learning from Lessons: Tools to support mathematics teachers' in-class professional learning*. Classic Press.

Choy, B. H., Thomas, M. O. J., & Yoon, C. (2017). The FOCUS framework: Characterising productive noticing during lesson planning, delivery and review. In E. O. Schack, M. H. Fisher, & J. A. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 445–466). Springer.

Clarke, D. J. (2013). The validity-comparability compromise in cross-cultural studies in mathematics education. In B. Ubuz, C. Haser, & M. A. Mariotti (Eds.), *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education* (pp. 1855–1864). Manavgat-Side, Antalya: CERME.

Clarke, D. M., Clarke, D. J., Roche, A., & Chan, M. C. E. (2015). Learning from lessons: Studying the construction of teacher knowledge catalysed by purposefully-designed experimental mathematics lessons. In M. Marshman, V. Geiger, & A. Bennison (Eds.), *Mathematics education in the margins: Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia* (pp. 165–172). Sunshine Coast, Australia: MERGA.

Colestock, A., & Sherin, M. G. (2015). What teachers notice when they notice student thinking: Teacher identified purposes for attending to student thinking. In A Robertson, R., Scherr, & D. Hammer (Eds.), *Responsive science teaching* (pp. 126–144). New York, NY: Routledge.

Dietiker, L., Males, L. M., Amador, J. M., & Earnest, D. (2018). Curricular noticing: A framework to describe teachers' interactions with curriculum materials. *Journal for Research in Mathematics Education, 49*(5), 521–532.

Dindyal, J., Schack, E. O., Choy, B. H., & Sherin, M. G. (2021). Exploring the terrains of mathematics teacher noticing. *ZDM Mathematics Education 53* (pp. 1–16). https://doi.org/10.1007/s11858-021-01249-y

Dreher, A., Lindmeier, A., Feltes, P., Wang, T.-Y. & Hsieh, F.-J. (2021) Do cultural norms influence how teacher noticing is studied in different cultural contexts? A focus on expert norms of responding to students' mathematical thinking. *ZDM Mathematics Education 53*, (pp. 165–179). https://doi.org/10.1007/s11858-020-01197-z

Erickson, F. (2011). On noticing teacher noticing. In M. G. Sherin, R. Phillip, & V. R. Jacobs (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 17–34). Routledge.

Erickson, F., Boersema, D., Kirschner, B., Lazarus, B., Pelisser, C., & Thomas, D. (1986). *Teachers' practical ways of seeing and making sense: A final report* (Contract No. 400–81–0014) Institute for Research on Teaching.

Goodwin, C. (1994). Professional vision. *American Anthropologist, 96*(3), 606–633.

Hiebert, J., Morris, A. K., & Glass, B. (2003). Learning to learn to teach: An "experimental" model for teaching and teacher preparation in mathematics. *Journal of Mathematics Teacher Education, 6*, 201–222. https://doi.org/10.1023/A:1025162108648

Huang, R., & Li, Y. (2012). What matters most: A comparison of expert and novice teachers' noticing of mathematics classroom events. *School Science and Mathematics., 112*(7), 420–432. https://doi.org/10.1111/j.1949-8594.2012.00161.x

IDM. (2021). *Learning from lessons. Lesson plan description and lesson template*. https://uni-bielefeld.de/fakultaeten/mathematik/fakultaet/idm/arbeitsgruppe/ag-4/lfl/lfl-material/. Last accessed: September 20, 2021.

Jacobs, J. K., Yoshida, M., Stigler, J. W., & Fernandez, C. (1997). Japanese and American teachers' evaluations of mathematics lessons: A new technique for exploring beliefs. *The Journal of Mathematical Behavior, 16*(1), 7–24. https://doi.org/10.1016/S0732-3123(97)90004-3

Jacobs, V. R. (2017). Complexities in measuring teacher noticing: Commentary. In E. O. Schack, M. H. Fisher, & J. A. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 273–279). Springer International Publishing.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202. https://doi.org/10.5951/jresematheduc.41.2.0169

Jacobs, V. R., & Spangler, D. A. (2017). Research on core practices in K-12 mathematics teaching. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp.776–792). Reston, VA: The National Council of Teachers of Mathematics.

Johnson, H. L., Coles, A., & Clarke, D. J. (2017). Mathematical tasks and the student: Navigating "tensions of intentions" between designers, teachers, and students. *ZDM Mathematics Education, 49*(6), 813–822. https://doi.org/10.1007/s11858-017-0894-0

Kaiser, G., Busse, A., Hoth, J., König, J., & Blömeke, S. (2015). About the complexities of video-based assessments: Theoretical and methodological approaches to overcoming shortcomings of research on teachers' competence. *International Journal of Science and Mathematics Education, 13*, 369–387.

König, J., Santagata, R., Scheiner, T., Adleff, A.-K., Yang, X., & Kaiser, G. (2020). Teacher noticing and teacher professional vision: A systematic literature review on conceptualizations and research designs (under review, manuscript submitted).

Louie, N. L. (2018). Culture and ideology in mathematics teacher noticing. *Educational Studies in Mathematics, 97*(1), 55–69.

Mason, J. (2002). *Researching your own practice. The discipline of noticing*. Routledge.

Mason, J. (2010). Attention and intention in learning about teaching through teaching. In R. Leikin & R. Zazkis (Eds.), *Learning Through Teaching Mathematics: Development of teachers' knowledge and expertise in practice* (pp. 23–47). Springer.

MET. (2013). *Ensuring fair and reliable measures of effective teaching*, Bill and Melinda Gates Foundation. https://eric.ed.gov/?id=ED540958. Last accessed: January 21, 2022

OECD [Organisation for Economic Co-operation and Development]. (2014). *TALIS 2013 results: An international perspective on teaching and learning*. OECD.

Peng, W. J., McNess, E., Thomas, S., Wu, X. R., Zhang, C., Li, J. Z., et al. (2014). Emerging perceptions of teacher quality and teacher development in China. *International Journal of Educational Development, 34*, 77–89. https://doi.org/10.1016/j.ijedudev.2013.04.005

Santagata, R., König, J., Scheiner, T., Nguyen, H., Adleff, A.-K., Yang, X., et al. (2021). Mathematics teacher learning to notice: A systematic review of studies of video-based programs. *ZDM Mathematics Education*. https://doi.org/10.1007/s11858-020-01216-z

Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (2011). Situating the study of teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 3–13). Routledge.

Stockero, S. L., & Rupnow, R. L. (2017). Measuring noticing within complex mathematics classroom interactions. In E. O. Schack, M. H. Fisher, & J. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 281–301). Springer.

Sullivan, P. (2011). Teaching mathematics: Using research-informed strategies. *Australian Education Review, 59.* Australian Council for Educational Research. Available from https://research.acer.edu.au/aer/13/

Tatto, M. T., Schwille, J., Senk, S. L., Ingvarson, L., Rowley, G., Peck, R., et al. (2012). *Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries findings from the IEA teacher education and development study in mathematics (TEDS-M)*. International Association for the Evaluation of Educational Achievement.

Thomson, S., De Bortoli, L., & Buckley, S. (2013). *PISA 2012: How Australia measures up.* Camberwell, Australia: Australian Council for Educational Research.

Thomson, S., & Hillman, K. (2019). *TALIS 2018 Australian report: Teachers and school leaders as continuous learners* (Vol. 1). Camberwell, Australia: Australian Council for Educational Research.

van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education, 10*(4), 571–596.

van Es, E. A., & Sherin, M. G. (2021). Expanding on prior conceptualizations of teacher noticing. *ZDM Mathematics Education, 53*, 17–27. https://doi.org/10.1007/s11858-020-01211-4

Wang, J. (2013). *Chinese mathematics education: Tradition and reality*. Jiangsu Educational Press.

Yang, X., Kaiser, G., König, J., & Blömeke, S. (2020). Relationship between pre-service mathematics teachers' knowledge, beliefs and instructional practices in China. *ZDM-Mathematics Education, 52*(3), 281–294.

Yang, X., König, J., & Kaiser, G. (2021). Growth of professional noticing of mathematics teachers: A comparative study of Chinese teachers noticing with different teaching experiences. *ZDM Mathematics Education, 53*, 29–42. https://doi.org/10.1007/s11858-020-01217-y

Zhang, D., Li, S., & Tang, R. (2005). The "Two basics:" Mathematics teaching and learning in Mainland China. In L. Fan, N. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics (in Chinese)* (pp. 189–207). Jiangsu Educational Press.