EDUCATIONAL DESIGNER

Issue 4 - March 2011

Welcome to Educational Designer #4

This issue of Educational Designer has been a while in the making, but we are pleased with the result and hope our readers will be as well. This issue contains a feature article by Tom Reeves on design research; a tale of designing a tool for proof comprehension by Lara Alcock and Nicola Wilkinson; and designer reflections on rich and balanced mathematics lessons by Charles Lovitt and Doug Clarke.

Along with this issue, we announce that the guidelines for contributors have been updated. In addition to invited pieces, we welcome unsolicited articles to the journal that reflect alignment with the guidelines presented <u>here</u>. Also, we would like to remind our audience that we encourage readers to respond to Educational Designer articles. While informal responses are most welcome, we are also interested in sharing extended responses, as <u>Paul Black</u> did in reaction to <u>Alan Schoenfeld's piece</u> on the cultures of educational research and design.

Susan McKenney
for the Editors

Can Educational Research Be Both Rigorous and Relevant?

Thomas Reeves
The University of Georgia

This paper addresses a complex question: Can educational research be both rigorous and relevant? The first eight years of the first decade of the 21st Century was a time when federal support for educational research in the USA emphasized rigor above most other concerns, and the last two years may mark the beginning of a shift to more emphasis on impact. The most desirable situation would be a balance between rigor and impact. Educational designers, teachers, and other practitioners would especially stand to benefit from such a balance because of the likelihood that it will enhance the impact of educational research. Educational design research is proposed as having enormous potential with respect to striking an appropriate balance between rigor and relevance in the service of the educational needs of learners, teachers, designers, and society at large.

e-Proofs: Design of a Resource to Support Proof Comprehension in Mathematics

Lara Alcock & Nicola Wilkinson
The University of Loughborough

This paper presents a theoretical basis for the design of *e-Proofs*, electronic resources to support proof comprehension in undergraduate mathematics. To begin, we frame the problems of teaching for proof comprehension, giving research background and an argument about what teacher-centred lecturing does not, and cannot, do to address these. We then describe e-Proofs, discuss the way in which they have been used in an Analysis course, and review their limitations and affordances as part of an overall educational experience. Finally, we briefly describe the development of a web-based tool for constructing e-Proofs, ways in which this tool will be used to different pedagogical ends, and associated research activity.

A Designer Speaks: Charles Lovitt and Doug Clarke

Look at any 10 classroom lessons. Two might be quite forgettable, about six might be passable, but one or two might, by general acclamation from teachers, be considered outstanding.

By analysing what we consider to be high quality lessons, we contend that it is possible to distil out a set of practical design features and qualities that are generalisable and transferable to many lessons. We would claim that in this process, working alongside teachers, there is the opportunity to empower them to create their own "rich" and "balanced" classroom lessons.

We will analyse three selected lessons chosen from classrooms. Each in our opinion passes the test of being both *rich* and *balanced.*

Guide to contributors

Rationale

ISDDE has established Educational Designer as an e-journal for the following reasons:

- To enhance communication between its members and with others
- Because rich exemplification is often essential in discussing design, a web-structure with linked examples communicates better than purely linear papers
- Because an e-journal costs less to readers and to authors' institutions

Audiences

The main audiences for Educational Designer are:

- Educational designers with a substantial commitment to design and development
- Leaders of design and development groups
- Strategists in education
- Funders and clients of systematic design and development
- Educational design researchers

Others with an interest in the quality of educational materials will find much of interest.

Criteria

All contributions are reviewed by independent referees to ensure that the following criteria are met.

Contributions should relate to the goals of ISDDE, namely to:

- Improve the design and development of educational tools and processes for other people to use
- Increase the impact of good design on educational practice
- Build a design community that will move forward toward these goals

Contributions should be original work of the author(s) and not have been published elsewhere. They must be free from copyright restrictions that affect publication here. We normally ask authors to grant Educational Designer a license to publish the article that is exclusive for as long as ED remains available; however, we will consider releasing individual articles under a Creative Commons, or similar license, where the author requests this.

Content and conceptual considerations

Educational Designer contributions address one or more of the following:

- Design and development processes – The processes that were followed, should be followed, or should not be followed. The relative merits and drawbacks of pathways to design when it comes to resource development, professional development, implementation, diffusion, and/or scaling should be discussed.

- Design premises, principles or considerations – Generalizable knowledge from one design project to another. The generalizable content can take many forms: knowledge, attitudes, skills, resources and/or collaboration that foster or hinder robust design. This could pertain to the design process, the design context, or to the design itself.

- Rich use of exemplification – Demonstrating design and development processes that can inform others through examples. Where meaningful, authors are encouraged to make full use of the on-line opportunities to link and share resources (e.g. online tools; downloadable products) or information about how designs are used (e.g. video of teachers, learners, or contexts).

- Evidence – Empirical support for the views expressed. Authors are encouraged to share empirically-grounded design insights, and to describe how the data and insights were obtained.

Possible topics

Areas on which contributions might focus include:

- What can good educational design achieve?
- From the perspectives of designers, clients and/or users, what constitutes good design and why?
- Issues in design and design research
- Development processes
- The roles of evaluation
- Building a professional design community and its influence on practice
- Research methods, including documentation of outcomes
- Theory of design
- Long term strategies

Style for contributions

- Main text – Guideline length: preferably not exceeding 10,000 words. Please supply text in Microsoft Word (.doc/.docx), Open Document (.odt) or plain text format. We will re-format the material for the web, so keep the layout simple and just use the built-in heading and list styles.

- References - We do not rigorously enforce a style but APA citation style (Wikipedia) is suitable.

- Footnotes - use these sparingly: since articles are presented online as continuous, scrolling text, all "footnotes" will be moved to the end of the article.
- Illustrative exemplification – using embedded links. The material for these links should be provided along with manuscript submission as images, PDFs or even (in consultation with the Editors) videos or interactive applets. Links to other websites will be seen as references.
  Please remember to seek permission to use any third-party materials you include.
  If you include illustrations in the main text, please also supply the original .tiff/.jpg/.png/.ai files if available.
  Software applets created in Flash, Java or other browser-based technologies can usually be integrated into your article as "pop-ups" if you supply the appropriate .jar or .swf files.
- Biography and photograph - please include a short (50-250 word) "About the Author" section in your article and, if you are willing, a portrait-style digital photograph of yourself.

Contributions

Those interested in contributing to the journal are asked to send manuscripts, abstracts or outlines to the editors: Susan.McKenney@utwente.nl and Daniel.Pead@nottingham.ac.uk

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JOURNAL OF THE INTERNATIONAL SOCIETY FOR DESIGN AND DEVELOPMENT IN EDUCATION

A Designer Speaks:
Charles Lovitt and Doug Clarke

The features of a rich and balanced mathematics lesson: Teacher as Designer

Abstract

Look at any 10 classroom lessons. Two might be quite forgettable, about six might be passable, but one or two might, by general acclamation from teachers, be considered outstanding.

By analysing what we consider to be high quality lessons, we contend that it is possible to distil out a set of practical design features and qualities that are generalisable and transferable to many lessons. We would claim that in this process, working alongside teachers, there is the opportunity to empower them to create their own "rich" and "balanced" classroom lessons.

We will analyse three selected lessons chosen from classrooms. Each in our opinion passes the test of being both rich and balanced.

What do we mean by *rich* and *balanced*?

Many authors have attempted to define the term *rich* in relation to mathematics tasks or lessons (see, e.g., Downton, Clarke, Knight, & Lewis, 2006; Flewelling, & Higginson, 2001). Although it is very rare for a single task or lesson to have all of these features, the following are often mentioned:

- It draws on a range of important mathematical content;
- It is engaging for students;
- All students are able to make a start, as it caters for a range of levels of understanding;
- It can be successfully undertaken using a range of methods or approaches;
- It provides a measure of choice or openness, leading to a sense of student ownership;
- It involves students actively in their own learning;
- It shows the way in which mathematics can help to make sense of the world;
- It makes appropriate and effective use of technology;

- It allows students to show connections they are able to make between the concepts they have learned;
- It draws the attention of students to important aspects of mathematical activity; and
- It helps teachers to decide what specific help students may require in the relevant content areas, or ways in which students might be extended.

There are many such lists in mathematics education articles, and in working with teachers, we have encouraged them to put the lessons we share with them and the ones they develop themselves under the microscope, and to ask the questions, "Is this lesson rich? What are the features which make it so?" These features therefore provide a kind of checklist, and thus form our design elements in developing lessons. In the following discussion of three particular lessons, we will summarise the features which teachers identify in relation to the lessons.

When we refer to *balanced*, we mean that the features above work "in harmony", are mutually self-supportive and not over or under weight in any aspect. Many traditional text-book type lessons are arguably overweight in the rush to skill development, while ignoring a range of other equally important teaching and learning principles. The balancing act is needed within any lesson, and by implication, across any set of lessons. We will explore the way in which the three lessons we discuss can be regarded as balanced.

The three lessons which we encourage the reader to place under the microscope in this article are titled:

- Temperature graphs
- Maths in motion
- Mathematics of Lotto

In each, we will present a description of the lesson followed by a critique designed to identify the elements of the design profile, and also compare and contrast the lesson with an equivalent stereotypical textbook version.

Our context

In the 1980s, there were major concerns with the teaching of mathematics in Australia, particularly among teachers in the middle years of schooling (Grades 5-8): We are confident that the following list generated at that time will come as no surprise to readers:

- Maths was seen as boring and irrelevant;
- Not enough thinking nor genuine understanding was required of students;
- The topic was too abstract;
- Fear of failure and poor attitudes were evident;
- Maths was seen as elitist, and designed for tertiary bound students only;
- There was too much content to address and not enough time to do so;

- Assessment approaches were too narrow;
- Catering for a wide ability range was very challenging; and
- Teachers were struggling with problematic parent and community expectations.

The authors first worked together jointly running a national program, titled the Mathematics Curriculum and Teaching Program (MCTP, Lovitt, Stephens, Clarke & Romberg, 1990). In this project, we had the opportunity to consider how best to address the above issues, in conjunction with teachers, mathematics consultants and education systems.

Our response

Our focus was, and in many ways still is, to use "the classroom lesson" as the basis of professional learning conversations. We attempted to "capture" examples of good practice from around Australia by documenting lessons, sharing these as "works in progress", and encouraging teachers in supportive professional learning environments, to consider how they could both enhance the lessons, and more particularly expand their personal comfort zone or repertoire of teaching strategies. This personal repertoire of features can be called their *design profile*—the personal collection from which a teacher draws in creating (designing) learning experiences.

The model shown in Figure 1 was used as a basis for these conversations (Owen, Johnson, Clarke, Lovitt, & Morony, 1988, p. 4.)
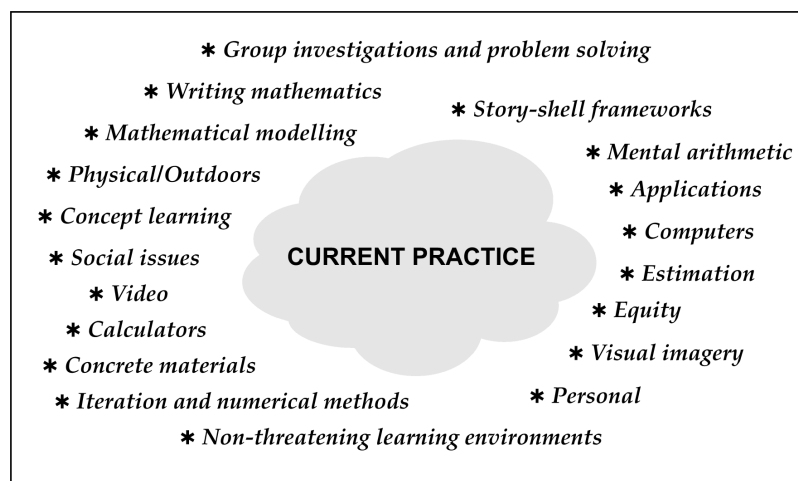

✶ Group investigations and problem solving
✶ Writing mathematics
✶ Story-shell frameworks
✶ Mathematical modelling
✶ Physical/Outdoors
✶ Mental arithmetic
✶ Concept learning
✶ Applications
✶ Social issues
CURRENT PRACTICE
✶ Computers
✶ Video
✶ Estimation
✶ Calculators
✶ Equity
✶ Concrete materials
✶ Visual imagery
✶ Iteration and numerical methods
✶ Personal
✶ Non-threatening learning environments

Figure 1: Model from Owen, Johnson, Clarke, Lovitt, & Morony, 1988, p. 4.

Each teacher comes to any learning situation with their personal current practice or 'comfort zone'. As teachers worked through lessons in the role of the student in professional learning settings, and subsequently trialled them in their own classrooms, they were encouraged to seriously scrutinise the lessons for the features they contained, particularly those new to themselves, with a view to ultimately expanding their comfort zone or teaching repertoire.

In our documentation for a given lesson, which always went through many iterations,

and, in one sense, was never finished, we attempted to tell the story of the lesson, which was always an "amalgam" of the ways in which the lesson has played out in a variety of settings.

We used photographs, sketches, samples of student work, comments from teachers, and insights from trialling to paint the picture of a lesson, in a way that the reader could hopefully visualise.

MCTP also focused significantly on the professional development (PD) settings in which these "lesson conversations'" could occur. We analysed existing PD models from around Australia, developed a set of criteria for effective PD (see Clarke, 1994) summarised the various PD offerings into eight generic models (Owen, et al., 1998), and supported consultants and curriculum leaders to use these as a basis for generating their own local PD settings.

Our work at the time was clearly informed by prevailing national policy and our understandings of the literature, but was also largely intuitive and arguably more of a grassroots approach. One memorable day, a colleague came to us with an article and said "This is what you guys are on about!" He shared with us the seminal work of Lee Shulman (1986, 1987) on *"pedagogical content knowledge", or "capturing the wisdom of practice".*

Among the statements which resonated particularly with us and the work we were undertaking at the time, were the following:

> "Teaching is characterized by a collective and individual amnesia,… the consistency with which the best creations of its practitioners are constantly being lost to both current and future peers… It is devoid of a history of practice"
> (Shulman, 1987, p. 11).
>
> "Pedagogical Content Knowledge is …an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of the most frequently taught topics and lessons … if the preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganising the understanding of learners. … the most powerful analogies, illustrations, examples, explanations, and demonstrations related to the topic one teaches. … the ways of representing and formulating the subject that make it comprehensible for others."
> (Shulman, 1986, p. 9).

We realised that in attempting to document the best lessons from around Australia, we were both highlighting the wisdom of practice, and encouraging teachers to explore those aspects of pedagogical content knowledge which could enable a teacher to take a basic idea for a lesson and make it both rich and balanced.

Our response, which started with MCTP has largely continued in several follow up

projects through to the present. One such current project using classroom lessons as the basis for professional learning conversations is Maths300. This is managed by a government agency, Education Services Australia. Its ambition is to find the 300 most 'interesting' (i.e. worthy of professional discussions) lessons from across Australia and mount these on a website (www.maths300.esa.edu.au) to be used by teachers and curriculum leaders in various PD settings. All three lessons discussed here are available to teachers via Maths300.

The three lessons

As has been mentioned, our approach has been to introduce the lessons to teachers in a workshop setting, and then to discuss the features of the lessons which are evident to them, and which may or may not be part of their current teaching repertoire.

The lessons are now discussed in some detail. The lessons have been chosen to reflect a range of design features, different mathematical content, and different grade levels. For each lesson, we describe how the lesson might play out (with teachers and with students), and then discuss the nature of the debrief which occurs with teachers in professional learning settings.

Lesson 1: Temperature Graphs

Teachers are provided with an initial worksheet (Appendix 1), as well as a one-page overview of the lesson (Appendix 2), and access to the complete Lesson Plan notes and accompanying software (Appendix 4). The lesson is recommended for students in Grades 5 to 10.

The lesson

The teacher greets the class and indicates that s/he has a challenge for them, handing out the worksheet. They then explain the following:

- There are 6 graphs on the page – one for each of the cities you can see named at the top.
- They are temperature graphs – each one has two lines – the top graph shows the average daily maximum temperature for each of the months of the year, while the lower line is the average daily minimum temperature for each of the months of the year.
- The challenge (in small groups) is to match the graphs to the cities.
- When appropriate, the teacher pauses the class, summarises the thinking so far and develops whole class agreement (appealing to the weather bureau data if necessary for validity).
- The teacher then introduces the software.
- The teacher then uses the "Choose your own cities" option to reconstruct the worksheet on screen and, once again, encourages the students to match the cities to the graphs.
- The students then access computers (in small groups), choose their own cities and answer the challenges they have set themselves on screen.
- An extension project option is also available as detailed in the full lesson plan.

The debrief

The following is a typical summary of the debrief discussions. The first aspect is often to recognise the lesson as different to the content equivalent stereotypical textbook version, which would be along the lines of: Here is the temperature graph of some (irrelevant) city. Answer the following:

i. What was the maximum in July?
ii. In which month was the minimum 17 degrees?
iii. Which is the hottest month?
iv. Etc.

Teachers recognised that the purpose of such questions target just one outcome - namely the skill of correctly reading a scale. In the alternative version, it was easily noted that this skill has not been abandoned; it is still there. But now the deliberate addition of some extra pedagogical features leads to other additional worthwhile learning outcomes. The additional features which teachers often identify during

discussions include the following:

- **Context based.** The choice of cities puts a focus on links with geography and ideas of weather in relation to location. The context is important and students can learn more about the world they live in.

- **Challenge.** Having a challenge is seen to be engaging. Teachers reported much higher levels of 'participation'. The challenge has elements of the intrigue of a puzzle.

- **Thinking, reasoning, problem solving.** The challenge requires students to think, argue and justify desirable outcomes, features missing from the text book equivalent.

- **Group work.** The higher order outcomes of reasoning and communication are seen to be developed better by setting a group challenge rather that an individual one.

- **Personal.** Teachers reported that students know something about many of the cities and were keen to share this personal knowledge. For example, student often ask, "where is Anchorage?". If students have personal knowledge to contribute, then their commitment to the learning is enhanced.

- **Technology.** The worksheet used initially is printed from the software drawn from a provided data base of cities. The capacity to produce multiple worksheets for any combination of cities, the quick feedback, the opportunity for students to choose their own questions, the opportunity for research on additional cities are all aspects provided by the structure of the software. All were seen to add value to the lesson in ways not available in the more traditional textbook version.

- **Ownership.** This was seen as a particularly valuable feature. In the text book question, the decision on what work is to be tackled is likely to be taken by some textbook author writing questions in their home office, possibly 10 years ago. It is this decision which is largely dictating the journey of both teacher and student. The software elegantly hands over ownership and control to the teacher and students. Teachers commented strongly that when students were selecting cities for their challenges they selected places of interest to themselves. They had a degree of control over their learning and were more committed to that learning.

- **Skill.** As students debate in small groups, it was clearly seen that the desirable skill of reading graph scales was still prominent and being developed and enhanced.

- **Mixed ability – multiple levels of success.** Some teachers initially commented, "my students might struggle with this; they don't know enough about these cities." But on the software there is the option to choose just two cities, say Singapore and Beijing. What do they need to know for success? Singapore is near the equator and is tropical, Beijing is up north and gets cold.

- **Open-ended.** An important aspect of being able to go further and in more depth (if the teachers wants to do so). For example, "Our school is in Mt. Isa (country Queensland, Australia) and it is not in the database on the software.

But it could be." Go to a search engine, put in "Mt. Isa Temperature" and in just a few seconds, the data will be available. Enter that into the data base as a new city (and the data will stay there). Then we could put in other regional cities and produce a worksheet for the class just for the State of Queensland.

The number of deliberate design features began to look impressive and as one teacher commented:

> "This lesson is now profoundly different, richer and more balanced that the typical textbook".

Up until now, the comments and insights from teachers are deliberately intuitive and informal. Further reinforcement for the features they have recognised can come from considering formal curriculum policy documents. One such example is the much respected International Baccalaureate program (International Baccalaureate Organisation, 2008)

In the IB Middle Years Program (MYP), the section on Teaching and Learning lists criteria series of statements (Appendix 3) about student learning. Consider just the first four of these:

Students learn best when: (p. 60)

- their prior knowledge is considered to be important
- learning is in context
- context is relevant
- they can learn collaboratively

Teachers commented:

> "This design structure of the lesson exhibits all four of these – the text book version exhibits virtually none of them."

Summary

As indicated earlier, one of the main purposes of the lessons and the discussion which surrounds them is for teachers to reflect on particular features of a given lesson, and to consider expanding their pedagogical repertoire. So, in the case of Temperature Graphs, for example, teachers might ponder whether they can give their students more ownership using a piece of software, or by other means. The whole lesson is also a transferable 'template' – the lesson structure, complete with all its features can be repeated using a different context such as rainfall, or population growth, or rates of deforestation, or indeed any measurable commodity that might be presented graphically.

Lesson 2: Maths in Motion

Teachers are provided with an overview page (Appendix 5) and access to the full lesson notes (Appendix 7). This lesson targets children in grade level K (kindergarten).

The lesson

The teacher greets the class and invites children to solve some number challenges for the class, and they get to "be" the numbers. The teacher then hands out number cards (either 0 to 10 or 1 to 10) for students to wear. Various stories about ducklings lining up behind mother duck in order, or finding pairs adding to 11, or rubbish bins in front of houses are acted out. Each scenario is a challenge requiring the group to work together. Other challenges can be generated by the students; e.g., "this group of 3 students are the numbers '4', '5' and '8'. What challenges can we create for them?

The debrief

The following is a typical summary of the kinds of features which teachers identify during debrief discussions.

- **Active / physical / personal.** All of the lesson could be done with 10 small numbers cut from cardboard, with all students working individually at their desk, but the teachers reported it was far more memorable and involving with students physically 'becoming the numbers'.
- **Group work.** Working as a group to solve the challenges increased both cooperation and communication.
- **Storyshell contexts.** The 'fantasy' stories about ducklings and other scenarios greatly increased student interest and involvement – and led to greater engagement with the mathematical content ideas.
- **Challenge, problem solving.** The challenges clearly needed the students to think and reason as well as learn number skills and facts.

- **Creativity.** This was seen as a highlight feature – given the chance to invent challenges caused many students to peer into the task differently and come up with unexpected yet profitable directions. Teachers involved in the professional development also were able to add creative twists and turns.
- **Ownership.** When tasks were created by the students and the teachers, the sense of owning the direction of learning clearly increased involvement and commitment.
- **Multiple interconnected content.** Some teachers reported starting with a narrow focus such as ordering numbers from 1 to 10, but finding themselves in other mathematical territory such as finding a strategy to find all the different numbers that can be created by students '3', '2' and '8.' This led to insights about place value and also combination counting methods.

Teachers' informal and intuitive judgments can be backed up by research on effective teaching (e.g., Askew, Brown, Rhodes, Johnson, & Wiliam, 1997;), and emphases in policy documents. This can give teachers added confidence that their observations are

valid and justifiable. For example, the emerging (currently in draft form) Australian National Mathematics Curriculum (ACARA 2010, p. 5) states in a section titled *Implications for teaching and Learning:*

> In Mathematics, challenging problems can be posed using basic content, and content acceleration may not be the best way to extend students. Choosing engaging experiences as contexts for a variety of tasks assists in making mathematics inclusive, differentiating both for students experiencing difficulty and those who complete tasks easily.

Summary

Teachers in professional learning settings readily agreed that each of the features listed above, deliberately designed into the lesson, added to the overall quality, richness and balance of the experience. Yet every single feature could have been ignored or eliminated, as often happens in some resources, where rote learning is more often the focus. Importantly, teachers agreed that the list of features by themselves would not be empowering. It is the feature backed up by the imagery of its use that gives meaning.

Lesson Three: Maths of Lotto

This lesson, currently in development, arose spontaneously because a mathematics teacher noticed newspaper reports about the social issues behind citizens over-spending on such things as Lotto Games to the detriment of their family and financial circumstances. One hypothesis is that such victims seriously do not understand their real chances of success and are lured by propaganda and mathematical half-truth advertising in a desperate attempt to improve their circumstances. Unfortunately, usually the opposite happens.

So how might a social context like this become a component of a rich and balanced mathematics program?

The teacher has a repertoire of lesson design features and considers systematically their use as a lesson structure evolves. The process is deliberate and systematic.

Recently, one of the authors used this Lotto lesson as part of a professional development session. It was received extremely well as teachers responded to its design. One participant commented that it was a creative way to tackle the issue of gambling. Our contention is that whatever creativity is exhibited, it is not some sort of artistic inspiration, but more the product of systematic and deliberate consideration of the teachers' repertoire of design features.

Teachers in a professional learning setting are provided with an overview (Appendix 6), access to the full (draft) lesson plan notes (Appendix 8) and access to the draft software.

The lesson

Initial discussion of students' current knowledge of Lotto Games leads to an invitation to explore the mathematics and the 'How and Why' of people losing their money. A simpler version (a 6:2 game) is proposed, an important 'estimation' stage to establish students' perception of success, then the game is played. This provides class data which are then analysed to develop underlying key mathematical ideas. A software simulation significantly supports the process.

The debrief

Firstly, a more typical expository approach might deliver formal instruction about the mathematics of Combination Theory leading to skill practice exercises, which may have limited engagement or relevance for large numbers of students. Redesigning such a lesson into this alternative investigative approach gives teachers the opportunity to debate the relative merits of both approaches. The lesson is very current – and has not yet gone through multiple iterations, but thus far reactions and discussions have been as follows.

- **Social Issue context.** The social context of gambling and the problems this brings combines obvious opportunities to link mathematics with moral and ethical challenges. It was seen by teachers as a very relevant context. Importantly, the major outcome of the lesson is in understanding the context and hopefully being lass exploitable. The mathematics is not the major outcome, but a pathway to social empowerment. This is a reversal of text book approaches where the context is merely the vehicle to learn more mathematics.
- **A class game.** The 6:2 class game has been a huge highlight; very engaging and personally involving for students and a powerful first step towards the investigation that follows.
- **Estimation.** An important aspect of the lesson is to find out students' current perceptions of success. The investigation is premised on the hypothesis that students will see the game as easier than it really is. Also, if students make a commitment to an initial guess, their motivation to find out if their 'guess' is accurate is greater.

- **Mathematical investigative process .** How does a mathematician go about investigating such a context? The attached poster (Figure 2) has been very effective to highlight how a mathematician works. [The mathematician in the photo is an Australian - Terry Tao – a recent winner of the Fields Medal.]
- **Strategy.** As part of the investigative process, two problem solving strategies are evident and highlighted. The real game is modeled ('make a model'), and also simplified ('solve a simpler problem'). The attached strategy poster (Figure 3) is used to highlight a toolbox of such strategies.
- **First principles approach**. When working out the chances of winning the 6:2 game, teachers reported greater understanding is developed by writing down all the pairs from a first principles approach than using algorithmic formulae from combination theory.

Figure 2: *Being a Mathematician* poster.
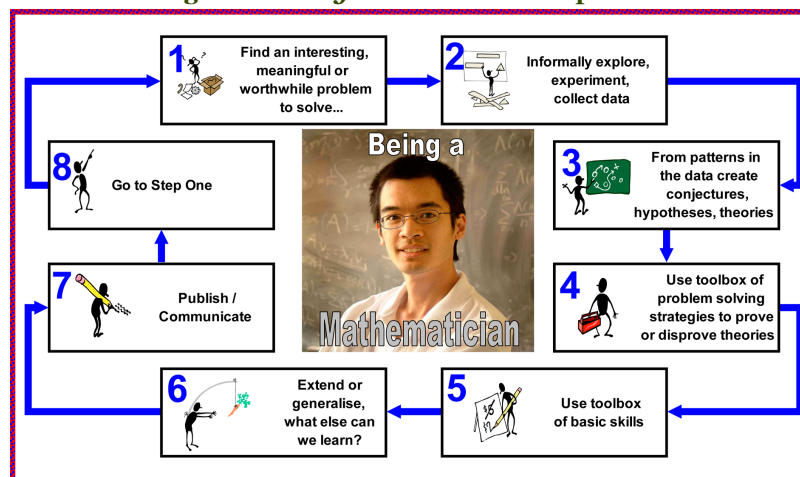
Figure 3: *Problem Solving Strategies* poster.
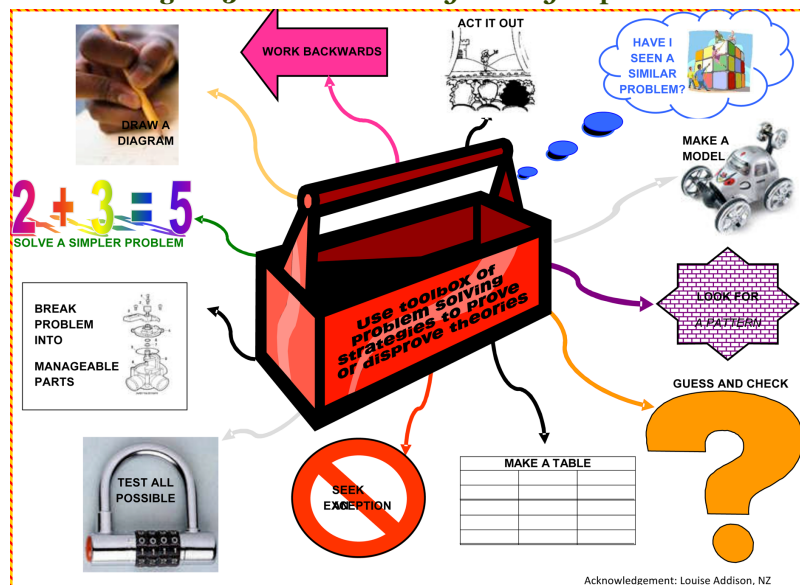
- **Technology.** The software supports the investigation in important ways, and allows the transfer from the simple 6:2 game (Figure 4) towards the more commonly used 45:6 community context Figure 5).

- **First hand data for analysis.** Playing the game produces data and hence is owned by the students. Teachers stated that this is significantly more valuable than providing second-hand data from results of other people playing the games (which is what text books so often do). It is the students' own data that delightfully exposes understandings about whether each pair is equally likely to be drawn.

- **Ownership.** The software cleverly allows students to pursue directions of interest to them. If a student decides they want to find the chances of winning a 8:3 game, the software allows this. Also, in follow up lessons students decide whether they wish to play the 6:3 or the 7:2 game.

- **Multiple content.** There are many interconnected big ideas of probability and statistics all being developed concurrently. This is in stark contrast to textbooks that partition content into separate lessons and hence fail to develop the interconnections.

- **Mixed ability.** Teachers strongly stated that all students were engaged and involved and all felt they had learned something worthwhile, even though this is different for different students.

Figure 4: Simulation of a 6:2 game

Figure 5: Simulation of "real" lottery
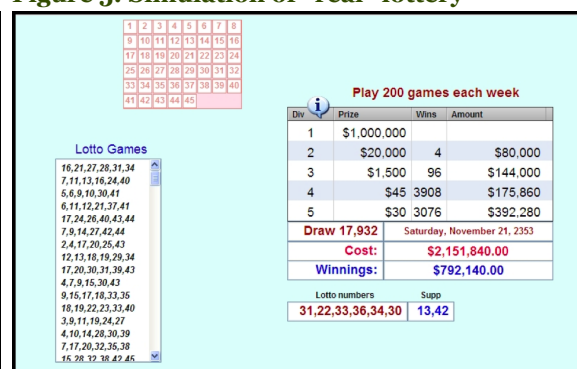
Many of the features of this lesson are advocated within various policy documents. Once again, referring to the draft Australian National Curriculum (ACARA 2010, p. 4-5):

> The curriculum lists fewer detailed topics with the intention to encourage the development of ideas in more depth and promote the interconnectedness of the mathematical concepts...
>
> The ... general capabilities of self-management, teamwork, intercultural understanding, ethical behavior and social competence are all relevant to the pedagogy used by teachers of mathematics.

Summary

This last of the three lessons we believe illustrates our general argument. If teachers

wish to create rich and balanced mathematics experiences for their students, they need command of a pool of design features. These features may be expressed intuitively, but are backed up by the imagery of their successful use in other lessons as well as supported by current research on effective teaching and prevailing educational policies.

Our design features and their relationship to Pedagogical Content Knowledge (PCK)

When we asked teachers in workshop settings to identify features of interest within the three lessons described above, and which in their opinion contributed to the quality of the learning experience, the descriptors they chose were as follows: The features chosen are expressed intuitively, informally and are born out of teacher conversations. We wanted the words used to be 'teacher words', generated by their observations and experiences and not laden with jargon. When used in discussion, the practical classroom meaning of the (design) features are illuminated by the context (i.e., lessons) in which they appear and later mediated through a background learning theory lens or a system policy lens.

The three lessons and features identified by teachers

Lesson 1	Lesson 2	Lesson 3
Temperature Graphs	**Maths in Motion**	**Maths of Lotto**
- Context based - Challenge - Thinking, reasoning, problem solving - Group work - Personal - Technology - Ownership - Skill - Mixed ability – multiple levels of success - Open-ended	- Active / Kinesthetic - Group work - Storyshells - Challenge, Problem solving - Creativity - Ownership - Multiple interconnected content	- Social Issue context (community gambling) - Mathematical Investigative process - A class game context - Strategy (solve a simpler problem) - Estimation - First principles approach - Technology - First hand data for analysis, - Ownership - Multiple content - Mixed ability

All the elements from these and other 'lighthouse' lessons aggregate into a single portfolio or personal design repertoire. A teacher's confidence in selecting from these to enrich new lessons of their own design is a function of personal experience, in seeing them used successfully within a pool or library of lighthouse classroom lessons.

The teacher is truly an educational designer.

# References

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliam, D. (1997). *Effective teachers of numeracy: Final report.* London: King's College.

Australian Curriculum, Assessment and Reporting Authority (ACARA). (2010). *Australian mathematics curriculum draft consultation.* Sydney: Author.

Clarke, D. M. (1994). Ten key principles from research for the professional development of mathematics teachers. In D. B. Aichele & A. F. Croxford (Eds.), *Professional development for teachers of mathematics* (Yearbook of the National Council of Teachers of Mathematics, pp. 37-48). Reston, VA: NCTM.

Downton, A., Knight, R., Clarke, D., & Lewis, G. (2006). *Mathematics assessment for learning: Rich tasks and work samples.* Melbourne: Mathematics Teaching and Learning Centre, Australian Catholic University.

Flewelling, G., & Higginson, W. (2001). *A handbook on rich learning tasks: Realizing a vision of tomorrow's mathematics classroom.* Canada: Centre for Mathematics, Science and Technology Education.

International Baccalaureate Organisation. (2008). *Middle years program MYP: From principles to practice.* Cardiff, Wales: Author. Retrieved from http://www.guajome.net/uploads /MYP%20From%20Principles%20into%20Practice.pdf

Mathematics Curriculum and Teaching Program (MCTP) Activity Bank – Volumes 1 & 2 (1988). Canberra, Australia: Curriculum Development Centre.

Owen, J., Johnson, N., Clarke, D. M., Lovitt, C., & Morony, W. (1988). *Guidelines for consultants and curriculum leaders.* Canberra, Australia: Curriculum Development Centre.

Lovitt, C., Stephens, W. M., Clarke, D. M., & Romberg, T. A. (1990). Mathematics teachers reconceptualizing their roles. In T. J. Cooney & C. R. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s* (Yearbook of the National Council of Teachers of Mathematics, pp. 229-236). Reston, VA: NCTM.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15,* 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational review, 57,* 1-22.

Stigler, J. W., & H. W. Stevenson (1991). How Asian teachers polish each lesson to perfection. *American Educator, 15*(12), 14-20, 43-47.

Appendix 1: Initial worksheet for *Temperature Graphs*

Temperature Graphs for Pick your own cities

1. Alice Springs   2. Anchorage   3. Hong Kong
4. Melbourne   5. Miami   6. Singapore
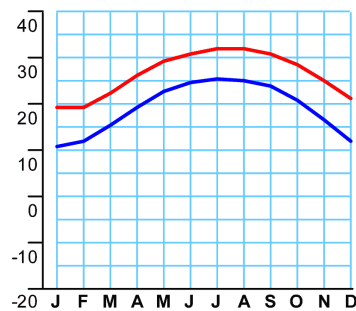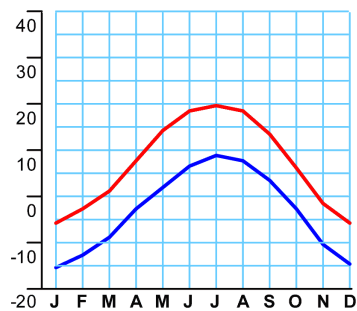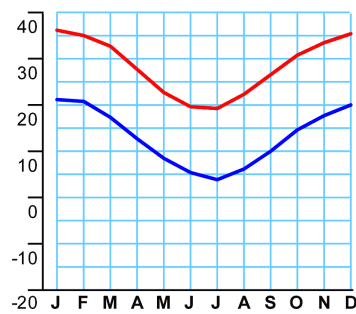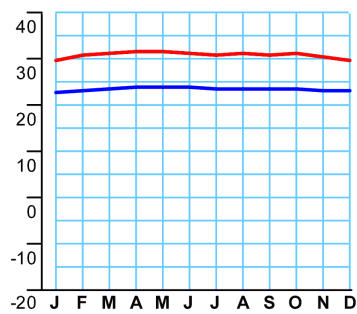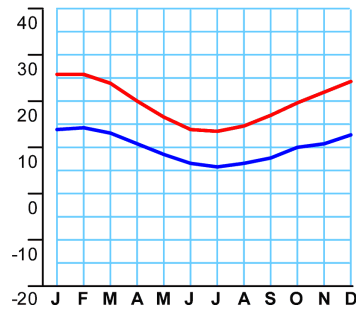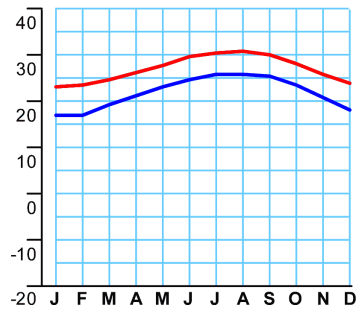
Appendix 2: Temperature Graphs - overview

Students are given several graphs of the average maximum and minimum monthly temperatures for various cities and challenged to match the graph to the cities. The resulting discussion brings out personal knowledge, links to Geography, communication skills, thinking and reasoning, as well as the skill of reading scales. The supportive computer software then allows students to challenge themselves on many other graphs as well as an extension project of researching data for regional or global cities.

Mathematics Content Outcomes\Links To Official Curriculum Documents

- Statistical inference – interpreting graphical data
- Reading Scales
- Thinking, reasoning and communication
- Interdisciplinary context

Lesson Stages

1. Small group challenge to match graphs to cities provided on a worksheet.
2. Whole class discussion to agree on results
3. Using software to generate similar challenges
4. Project to research extra data for new cities and generation of new worksheet challenges.
5. Extensions into other measures such as rainfall, population growth, rates of deforestation

Issues or Discussion Points

- How a recognisable closed text task has been altered into a group investigation
- The potential of well-chosen software to extend and enrich the learning

Straw Vote

Please rate each of the following features (out of 10) as to its contribution to the overall quality of the learning experience.

1. The context of 'real' cities [ ]
2. Interdisciplinary links eg to Geography [ ]
3. Opportunity to use personal knowledge [ ]
4. The small group challenge involving reasoning and communication [ ]
5. The role of the computer challenges [ ]
6. Mixed ability in choice of cities and difficulty level [ ]
7. Extension opportunities eg researching other cities, other contexts [ ]
8. Openness - suits many grade levels? [ ]

This lesson is documented as Lesson 83, including the software simulation, within the Maths300 project *www.maths300.esa.edu.au*

Appendix 3: International Baccalaureate Middle Years Program (MYP)

Students learn best when: (p60)

- Their prior knowledge is considered to be important
- Learning is in context
- Context is relevant
- They can learn collaboratively
- The learning environment is provocative
- They get appropriate, formative feedback that supports their learning
- Diverse learning styles are understood and accommodated
- They feel secure and their ideas are valued and respected
- Values and expectations are explicit
- There is a culture of curiosity at the school
- They understand how learning is judged, and how to provide evidence of their learning
- They become aware of and understand how they learn
- Structured inquiry, critical thinking, learning through experience and conceptual developments are central to teaching in the school
- Learning is engaging, challenging, rigorous, relevant and significant
- They are encouraged in everything they do in school to become autonomous, lifelong learners.

Link to the Middle Years Programme, MYP: From principles into practice document (2008):

http://www.guajome.net/uploads/MYP%20From%20Principles%20into%20Practice.pdf

Appendix 4: *Temperature Graphs* Lesson Plan notes and accompanying software

The lesson notes and the supportive software are part of an Australian based program titled Maths300. This is an internet based program delivering innovative mathematics lessons to teachers. It is a non-profit program run by Education Services Australia - a company jointly owned by the collective Ministries of Education across Australia. Any school can gain access to all the lessons through a subscription.

However, the Temperature Graphs Lesson and software are available to anyone free of charge. Go to the site www.maths300.esa.edu.au and to the section titled *Free Sample Tour* from where this lesson and its supportive software can be downloaded.
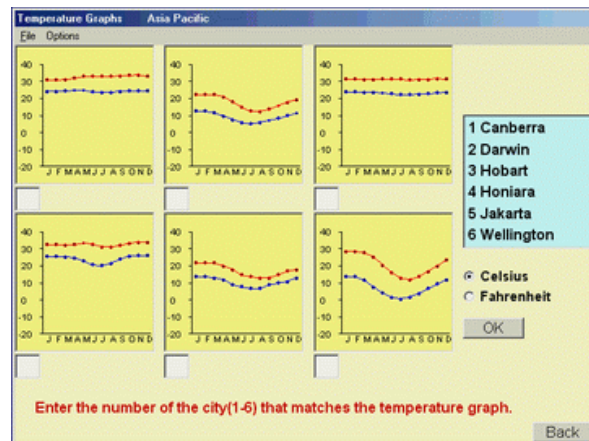
Appendix 5: *Maths in Motion* – Overview

Years: K - 3 **Time:** Many lessons **Strand:** Number

This lesson includes activities of children 'being' numbers. Groups of children wearing number cards act out events so that the whole class can talk about the mathematics involved. The activities are visited and revisited over days/ weeks/ months (threaded through the curriculum) so that children have a chance to construct and reconstruct their number concepts.

Learning outcomes and related concepts

- Number recognition (numerals)
- Sequencing
- Place value
- Number facts
- Problem solving
- Combination counting

Resources required

At least one set of cards numbered from 0 to 10 that are large enough to be worn by the children.

Additional cards as indicated by scenarios in the lesson plan.

Laminating these cards will preserve them.

Size: approx. 12 x 20 cm
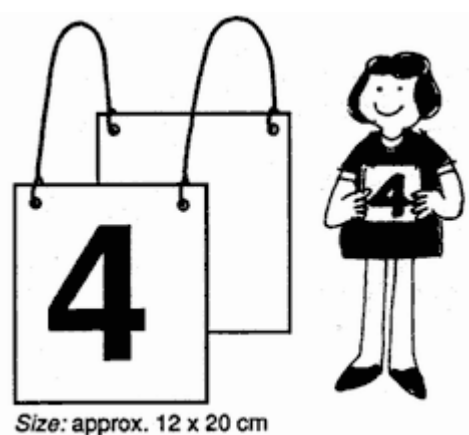
Lesson stages

1. Getting started: introducing the 'number children'
2. Mrs Number and her family
3. Discussing mathematics
4. Exploring other maths in motion activities

Issues or discussion points

1. Physical involvement
2. Small group or whole class
3. Fantasy/story telling
4. Mathematical conversation
5. Creativity
6. Easy to prepare yet yields significant learning

7.   Can be threaded through the curriculum

The children could do similar activities with small number cards on the table top. What difference does it make to the learning to present the activities in the kinaesthetic manner suggested?

Can the mathematical conversation be recorded in a way that concurrently achieves literacy objectives?

Straw vote

After teaching the lesson, please rate each of the above features (out of 10) as to its contribution to the overall quality of the learning experience. This exercise will provide a basis for staff discussion of curriculum development.

This is lesson 142 within Education Services Australia's Maths300 project www.maths300.esa.edu.au. An earlier version was published within the MCTP Activity Bank Volume 1, p189.

Appendix 6: *Maths of Lotto* – Overview

This is an anti-gambling lesson. Or at least one designed to show the mathematical realities behind Lotto games. Does the community understand the true chances? If not, then they may be exploitable. It is presented as a whole class game which becomes an investigation involving probability, combination theory, statistics and working mathematically. A computer simulation of the game allows a problem solving extension, and provides a basis for analysis of the realities of real lotto games.

> "After this lesson one of my students called them 'Looto" games

Mathematics Content Outcomes\Links To Official Curriculum Documents

- Probability and expectation
- Combination theory and simple proportion
- Statistics
- Interdisciplinary learning
- Working mathematically – mathematical modeling

Lesson Stages

1.   Discussion of real lotto games and demonstration of the simple 6:2 game
2.   Estimation of chances in the 6:2 game
3.   Play 10 games – record results on board
4.   Check results against perceptions – is there a difference? – reasons for the difference
5.   Explaining the real chances - are all pairs (or groups) equally likely?
6.   Are you superstitious? – the psychology of choices
7.   Playing and analysing other simple games eg. 7:2, 8:2, 7:3, 5:3, 10:2, ….

8. Computer simulation

9. Analysing real Lotto games and where the money goes

10. Educating the public – do they know their real chances?

Straw Vote

Please rate each of the following features (out of 10) as to its contribution to the overall quality of the learning experience.

1.	Modelling a real context as a class game	[　]
2.	Collecting initial class perceptions (Estimation)	[　]
3.	Incidental or informal learning opportunities	[　]
4.	First principles analysis of how many pairs are in the barrel	[　]
5.	Opportunity for strategy development	[　]
6.	Computer simulation	[　]
7.	The 'superstition' analysis - is each pair equally likely?	[　]
8.	Non-threatening	[　]
9.	Openness - mixed ability - suits many grade levels?	[　]
10.	Relevance outcome i.e addressing a social issue - educating the public	[　]

This lesson is fully documented (pending), including the software simulation, within the Maths300 project. www.maths300.esa.edu.au

Appendix 7: *Maths in Motion* – Lesson Notes

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Maths in motion

These activities have as a central idea, children being physically involved by acting as numbers.

Groups of children, wearing their number cards, act out the roles and discuss the various tasks with each other while the rest of the class encourages and offers advice. The tongue-in-cheek story that goes with it makes the activity live for children, letting them bring forward all sorts of creative, non threatening ideas.

Children will enjoy repeating these activities straight after the first 'performance,' the next day and/or weeks later.

Features of these activities

- Physical involvement.
- Able to be used with small groups or the whole class.
- Fantasy settings and story telling are used.
- Use of language.
- Problem solving.
- Number recognition, sequencing, place value and number facts.
- Mental imagery. Concepts and skills are recalled when mental images these activities are formed at a later date.

Preparation

- At least one set of number cards, from 0 to 10. For the *Call the number* activity you will need one number card per child. (Laminating the cards will preserve them.)
- A means of attaching these to the child, for example, shoulder straps (as shown), pins, tape or bulldog clips.
- You may choose to make additional labels — *Mrs. Number* and *Red sports car.*

Size: approx. 12 x 20 cm

Normie Nine?

HOW ABOUT TWEEDLE DUM TEN?

When you introduce the number children to the class, have them invent new names. For instance, Sunny One, Tricia Two, Francesca Five and so on.

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190 PHYSICAL / OUTDOORS

Maths in motion

The telling of the story is vital for this part of the lesson to:
- set the scene and mood,
- motivate the children for the discussion which follows,
- provide the language for the problem-solving part of the lesson.

Have children discuss ways of helping Mrs. Number organise her family. The children not participating at this stage can give advice and help. They can judge if the solution is a good one or not. Keep discussing and arranging until the Number Line sequence is gained. Try as many of the children's ideas as possible. If no one comes up with the Number Line sequence get them to remember back to the line the children were in when you were introducing the Number family. If this is still not successful you will have to pose some leading questions, for example, 'What is the next number we say when we are counting?'

Once the children have become reasonably proficient at arranging the numbers in number line sequence they can then discuss the problem... 'How quickly can you get into line? Discuss 'before' and 'after' numbers.

Mrs. Number's family (In what order should my number family be?)

The aims are to have children understand such concepts as: *next number, last number, more than , less than.*

1. Have the numbered group line up in order behind you. Tell the following story and have children act it out.

WHENEVER THE NUMBER FAMILY GO ON AN OUTING THEY ALWAYS GO IN A LINE IN THE RIGHT ORDER. ONE DAY, WHILE GOING SHOPPING, THEY HAD TO CROSS THE PEDESTRIAN CROSSING. DON'T FORGET— LOOK RIGHT, LOOK LEFT, LOOK RIGHT AGAIN. OK NOW CROSS...

2. Children jump back into an unordered bunch.

...BUT WHEN THEY WERE ONLY HALF-WAY ACROSS, A BRIGHT RED RACING CAR CAME SPEEDING AROUND THE CORNER. THE DRIVER DIDN'T SEE THE CHILDREN. SO Mrs.NUMBER SCREAMED 'RUN BACK QUICKLY!'...
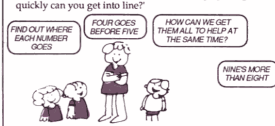
3. Children try to suggest a strategy. Normally children suggest starting at '1' and working up.

...WHEN THEY HAD GOTTEN OVER THEIR SHOCK Mrs.NUMBER TOLD HER CHILDREN TO GET INTO LINE. BUT THEY COULDN'T REMEMBER WHERE THEY BELONGED IN THE LINE. NOW HOW COULD THE CHILDREN FIND THEIR PLACES?

4. Challenge children to find more efficient strategies. Change the number cards on to other children and play the game...'How quickly can you get into line?'

FIND OUT WHERE EACH NUMBER GOES

FOUR GOES BEFORE FIVE

HOW CAN WE GET THEM ALL TO HELP AT THE SAME TIME?

NINE'S MORE THAN EIGHT

MATHS IN MOTION 191

A little help from the teacher

No — this isn't five raised to the power of one. This teacher invented yet another activity. 'Find a larger number and climb gently on its back.'

'You (two) should be on the other side of the street', cries number nine.

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192 PHYSICAL / OUTDOORS

Smallest to largest

Line up eleven children from shortest to tallest.

Give the shortest the number '1' and the tallest the number '10'.

Place the numbered cards, jumbled up, in front of the children.

Ask class to predict who will be first and last to get their right number.

Tell children to get their correct numbers.

Discuss why some people get them before others.

I MUST BE NINE 'COS I'M BEFORE TEN

I MUST BE ONE AFTER YOU

I'M TWO 'COS I'M AFTER ONE

Where do the rubbish bins belong?

Tell the numbered children that they are the rubbish bins in (your) street.

Tell the tale of how the wind blew them all down to the end of the street.

The rest of the class clap to simulate the noise of the wind clattering the bins, while the 'bins' huddle in disorder at the 'end of the street'.

Then challenge 'bins', with the help of the rest of the class to find their right places before the 'garbo' (the teacher) comes.

To reinforce the idea of odds and evens, children can tell the class what they are — e.g. 'I'm six and I'm even.'

Step forward and bow

This can be an activity for the whole class if cards are available.

Children line up and when their number is called, they take one step forward and bow.

MORE THAN TWO BUT LESS THAN FOUR

WE'RE MORE THAN TWO BUT LESS THAN FOUR

Other instructions may be:
- Numbers bigger than six.
- Three.
- Odd numbers.
- Numbers starting with 'T'.

Get behind the number

Using one group of numbered pupils, call certain groups of people to fall in behind the numbers you call until only one child is left in the class. (Ideal for a pre-recess activity)

GET BEHIND TWO IF YOU HAVE BROWN HAIR

Other instructions may be:
- ... if your birthday is in April.
- ... if you are wearing yellow.
- ... if you are wearing a digital watch.

Appendix 8: *Maths of Lotto* – Lesson Notes

The lesson is accompanied by detailed lesson notes and a software simulation. An initial draft of the notes has been made available, with permission - see the link below. The materials are being readied for publication within the Maths300 project of Education Services Australia.

Maths of Lotto – Lesson Notes

http://www.educationaldesigner.org/ed/volume1/article15/pdf/maths_lotto_draft.pdf

About the Authors

Charles Lovitt has been involved in Mathematics Professional Development for many years. He has co-directed several Australian National and State projects such as MCTP (with Doug Clarke), RIME, Initiative 5.4 for Maths and Science. He was director of Maths projects for the Australian Curriculum Corporation and generated such projects as The Maths Task Centre Project, the Chance and Data project and more recently the Maths300 project. He is now a consultant and a regular keynote and workshop presenter both in Australia and internationally.

Charles strongly believes that the wisdom of our profession is held by practicing teachers. It is tapping into this wisdom via captured images from classrooms that allows us to all professionally grow and learn from each other for mutual benefit. These captured images are essentially design pieces – weaving the pedagogical knowledge of teachers into a coherent narrative for others to consider.

Doug Clarke directs the Mathematics Teaching and Learning Research Centre at Australian Catholic University. Following ten years teaching high school mathematics, he jointly coordinated the national Mathematics Curriculum and Teaching Program (MCTP) with Charles Lovitt. While undertaking doctoral studies at the University of Wisconsin, he coordinated the development of teacher support materials for the National Science Foundation's Mathematics in Context Project. From 1999 to 2002, he directed the Early Numeracy Research Project, a joint initiative of the Victorian Department, Catholic and Independent systems, involving 70 schools, 350 students and over 11,000 students. He currently directs the Contemporary Teaching and Learning of Mathematics Project (CTLM) for the Catholic Education Office (Melbourne), which aims to enhance teacher pedagogical content knowledge and student learning.

In the past five years, Doug's major research interests have included young children's number learning, assessment with a particular focus on the power of the one-to-one, task based assessment interview, rational number learning, teacher professional learning, and pedagogical content knowledge. The most enjoyable aspect of Doug's work is the opportunity to work with teachers and students in classrooms in the ongoing challenge of making mathematics worthwhile and enjoyable.