Leading a New Pedagogical Approach to Australian Curriculum Mathematics: Using the Dual Mathematical Modelling Cycle Framework

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The aim of this study was to investigate the use of the *dual mathematical modelling cycle framework* as one way to meet the espoused goals of the Australian Curriculum Mathematics. This study involved 23 Year 6 students from one Australian primary school who engaged in an *Oil Tank Task* that required them to develop two models in order to solve the task. Results indicate that although some students struggled to fully develop the two models there were students who engaged in both models, deepening their mathematical knowledge and its application when working in real world contexts.

The Australian Curriculum Mathematics (ACARA 2014) has real world problem solving expertise as an espoused goal for all students. In particular, the doing of mathematics is described through the proficiency strands of understanding, fluency, problem solving and reasoning with the recommendation that these be the mathematical actions teachers take in their teaching. Galbraith (2013) points out that for teachers to develop the pedagogical expertise necessary to achieve this outcome, a new approach and therefore new knowledge is required, suggesting a starting point of being able to select appropriate problems from real world contexts and to decide which mathematics could be used and how.

The issues are addressed in current research that recommends ambitious instructional practices (Grossman et al. 2013; Lampert et al. 2013) where mathematical tasks should be cognitively demanding, non-procedural in nature (Boston & Smith 2009; Stein et al. 2000) and have multiple entry points resulting in multiple solutions. While engaging in these activities with their students, teachers should identify different solutions to be compared during any whole class discussion. This gives students exposure to different applications of key mathematical ideas (Stein et al. 2008). Mathematical modelling and applications utilizes these pedagogical approaches and could therefore form the bases for the doing of mathematics for Australian students.

Literature

Supporting the mathematical modelling and applications approach to teaching is the cognitive theoretical framework developed by Blum and Leiß (2007, p. 225) that identifies seven steps in the modelling cycle: constructing, simplifying/structuring, mathematizing, working mathematically, interpreting, validating, and exposing. This cycle can be represented by the model shown as Figure 1.

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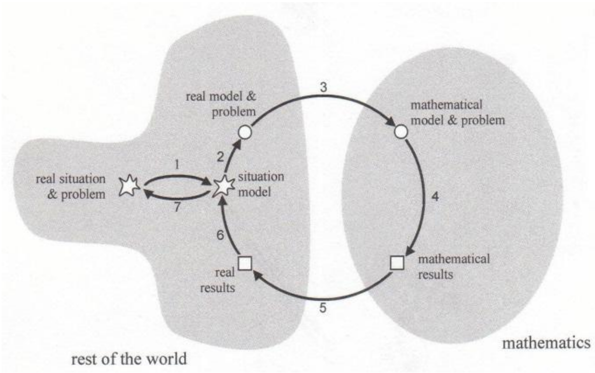
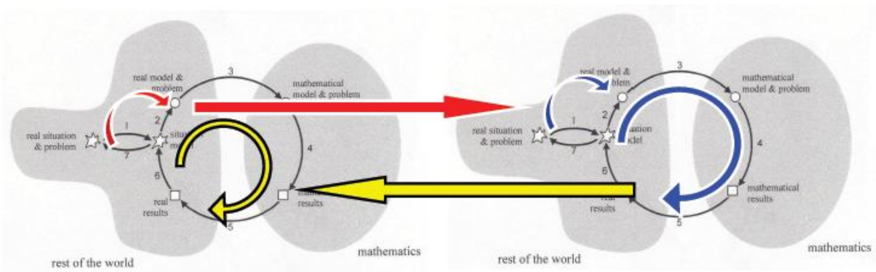


Figure 1. Mathematical Modelling cycle (Blum & Leiß, 2007, p. 225)

Blum and Borromeo Ferri (2009) outlined that for some, cognitive demands are heightened when moving from a situation model to a mathematical model. In addressing this problem Matsuzaki (2011) found that students may elect to model a similar situation if they could not model the initial task while students with more sophisticated understanding may draw on findings from their initial model as well as additional models to solve their problems. In response to the diversity of modellers' modelling processes (e.g., Borromeo Ferri 2007; Matsuzaki 2011) and variation between tasks in their individual modelling progress (e.g., Matsuzaki 2007, 2011; Stillman 1996; Stillman & Galbraith 1998) Saeki and Matsuzaki (2013) developed the dual modelling cycle framework that promotes switching between the first modelling cycle of the initial task to a second modelling task using a similar or simpler model to interpret the problem. Here the second mathematical problem is used to support the understanding of the first problem and is designed to aid its solution by broadening and deepening the conceptual understanding necessary to fully solve the first problem resulting in a far more sophisticated understanding of the problem and the mathematics at hand. This approach is based on Polya's (1988),'How to solve it' framework where in step two, he recommends, 'if you cannot solve the proposed problem try to solve first some related problem' (p. 10).

Research on this framework by Kawakami, Saeki and Matsuzaki (2012) and Matsuzaki and Saeki (2013) has found that modellers who could not solve the initial task were indeed able to advance their modelling of this task by modelling a similar but simpler second task. Students who could forecast results for the first task developed more enlightened mathematical understanding and skills by engaging with the second task. This approach allows more students to find success with mathematics more generally and with modelling tasks more specifically. See Figure 2 for Saeki and Matsuzaki's (2013) theoretical extension of Blum and Leiß's (2007) mathematical modelling cycle.



[The first modelling cycle] [The second modelling cycle]

Figure 2. Dual modelling cycle diagram (Saeki & Matsuzaki, 2013, p. 94)

How the dual modelling cycle will support Australian students' to acquire the desired mathematics outlined in the proficiency strands as *understanding, fluency, problem solving* and *reasoning* is largely unknown. In response to Galbraith's (2013) call for new approaches to teaching mathematics this current study aims to understand to what extent the dual modelling cycle supports Australian students' to develop real world problem solving expertise.

Method

This study included 23, Year 6 students from one Australian school in their second week of the school year. In this paper the successful modelling completed by one student, Millan, will be described along with typical responses from students in the class. The researchers, both Australian and Japanese, participated in the lesson delivery. The students engaged in the dual modelling tasks over two days with each lesson lasting approximately 1.5 hours.

The data collection comprised lesson video recordings, lesson artefacts and field notes. The artefacts included digital images of student work produced during this project. Field notes were kept by researchers noting any critical insights or issues as they emerge throughout the lessons. These data were analysed for evidence of student engagement with the two different modelling situations. This included each modelling situation independently and how the second modelling situation informed the first, and if this led to enhanced potential in mathematical proficiency.

Task 1: Oil Tank Task (Initial task)

Before introducing the task to the students the context for the problem was established. The students were asked about the types of *danger* signs they might see at petrol stations, where petrol was stored, where it came from and its explosive properties. The following picture displayed as Figure 3 supported the context of fireman needing to climb to the top of one of the tanks as quickly as possible to cool a tank with fire retarding foam. The problem was then presented to the students in the following way: There are several types of oil tanks in danger of exploding. Their heights are equal but their diameters are different. Is the length of the spiral stairs on these oil tanks equal or not? Conditions are the same, for example the angle of the spiral stairs around each tank, are all 40°. The firemen need to know which spiral stair will get them to the top first.

Figure 3. Oil tank image (Note: Picture reversed in class)
URL<http://blog.goo.ne.jp/kobeooi/e/b021c971381154725 fc3ee4a3d645aa8> [18 Mar 2014]

Implementing the Modelling Tasks

Initially the students were asked to work with a 3D model of the oil tanks displayed as Figure 4 and asked to produce a 2D drawing that represents this 3D model. Following this modelling the students were asked to complete Task 2: Toilet paper tube task.

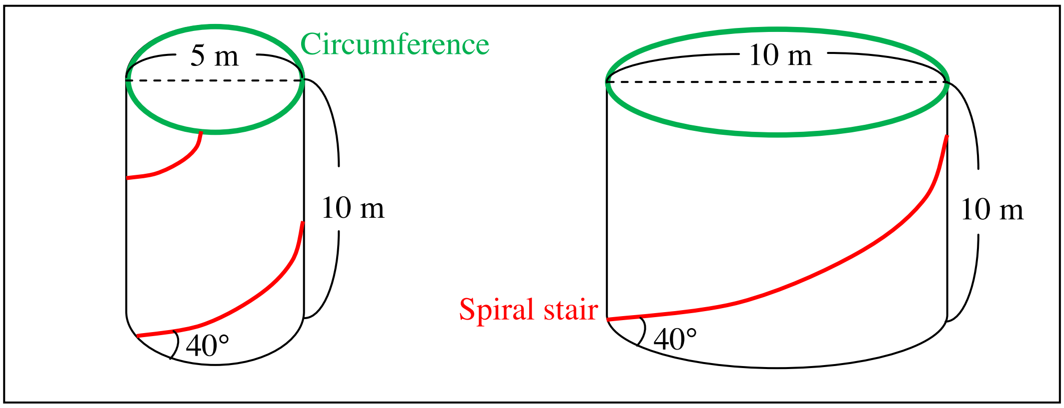
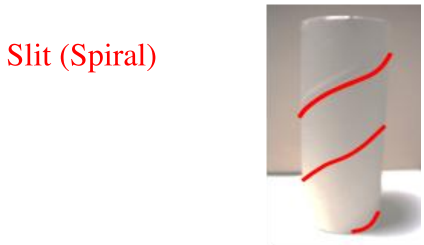


Figure 4. 3D drawing of oil tanks

Task 2: Toilet Paper Tube Task (Similar task)

It is not possible to open along the actual spiral stair of the oil tank. We can take a toilet paper tube as a similar shape to the oil tank, which is to be opened along its slit.



Results and Discussion

Day 1: Eleven of the Year 6 students in the class were able to draw the rectangular representation of the oil tank from the 3D model of the oil tanks. However, in each case the students drew the staircase as a curved line on their 2D model. The remaining students were unable to draw a 2D model tending instead to copy the 3D model provided for them. Millan’s drawing demonstrates the 2D model with a curved line to represent the spiral stair (See Figure 5). Figure 6 represents a typical drawing by students unable to draw a 2D model from a 3D model.

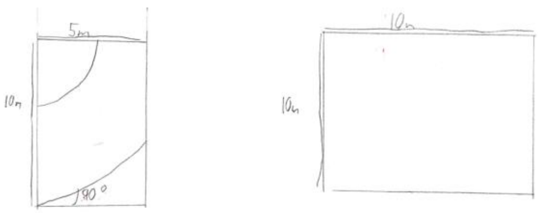


Figure 5. Millan’s 2D model

Figure 6. Typical example from class

The content descriptor for year 5 of the Australian Curriculum Mathematics ACMMG111 outlines, "Connect three-dimensional objects with their nets and other two-dimensional representations." The students in this study did not appear to have this content knowledge which highlighted two points for the researchers at this stage. Firstly, half the students could not draw a 2D model from a 3D model. Secondly, no student demonstrated an understanding that in their 2D representation, the spiral stairs would be a straight line. This is an important understanding for them to then be able to determine if the spiral stairs on each oil tank were the same or not.

Following this modelling task, the second task was introduced where the students were asked to predict what the toilet paper tube would look like when cut up the slit. This second similar task provides the students with an alternative 2D shape to use when determining the length of the spiral stairs. Six students were able to draw a shape close to a parallelogram although some evidence of the curved spiral staircase was evident for most students. See Millan's drawing as Figure 7. Most students produced diagrams similar to Figure 8.

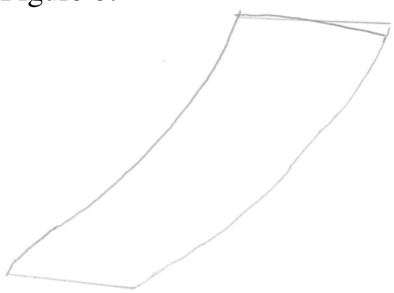


Figure 7. Millan's interpretation

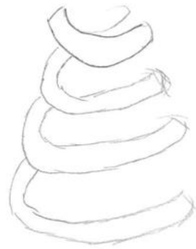


Figure 8. Typical spiral representation

The students were then asked to cut their toilet paper tube along the slit and to draw what they found. To assist in identifying the different sides of the shape the students marked the slit with a red felt pen and the top and bottom circumference with green felt pen. They were then also given a second toilet paper tube and asked to colour it in the same way. They were then asked to cut the toilet tube straight up from bottom to top allowing them to compare the two shapes, parallelogram and rectangle models of the oil tanks.

As most students in the class were unable to first visualize and then draw 2D representations of the 3D model of the oil tank or the toilet paper tube it was decided to work with the class as a whole on day two so that all students could more full engage in the dual modelling tasks.

Day 2: The lesson commenced with a whole class discussion of the problem and the two possible models that would provide information as to whether the spiral stairs were the same or different for each oil tank. The students were then asked to predict whether the spiral stairs were the same or not with the outcome of this count being 14 students predicting the stairs were equal and 9 students predicting they were different. When writing about their predictions most students simply wrote comments such as "*I think the bigger tank is faster than the spiral tank*" or "*I think they are the same because the stairs are both 10 meters* (sic)." Others wrote a little more such as, "*I believe the bigger oil tank's staircase is faster up because it goes up at an angle straight up and the smaller one has a spiral witch* (sic) *will make the staircase longer.*" Several students used models to support their explanation. See Millan's explanation of his prediction as Figure 9.

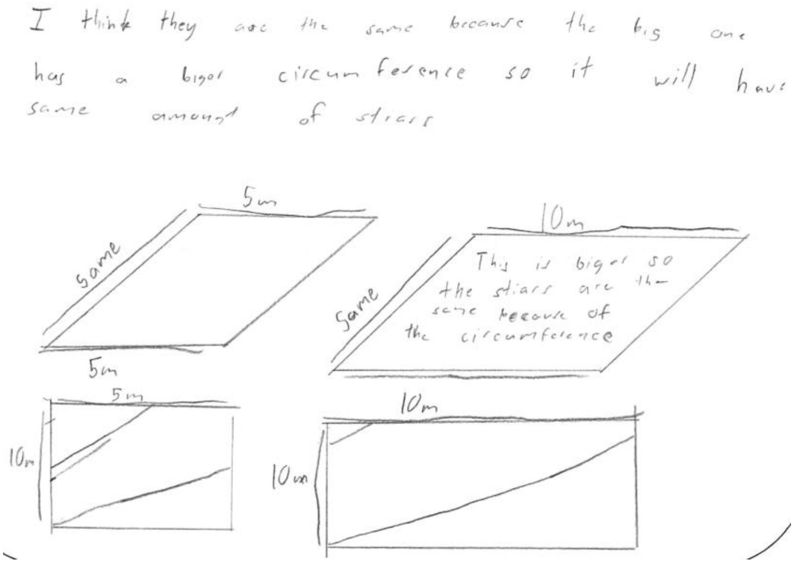


Figure 9. Millan's solution to the modelling tasks

The students were then asked to prove their prediction by creating a model to explain how they could compare the 10m and 5m diameter oil tanks. The students set about constructing models, one double the diameter of the other. The discussion held by students in small groups during this stage of the lesson was very valuable as they talked about how to construct the models and then how to cut them up to give the results they wanted. After constructing these models and cutting them up to make either the rectangle or parallelogram model the students were able to easily demonstrate and discuss that the spiral stairs were the same. These discussions led to the models being overlaid on the whiteboard. Important to note is that with the rectangle model the small section of spiral stair that came around the oil tank twice on the 5m diameter model was cut off and place on the line representing the spiral stair for the 10m diameter model. This was proof that both staircases were the same length for both the rectangle model and the parallelogram model. Figure 10 displays these processes.

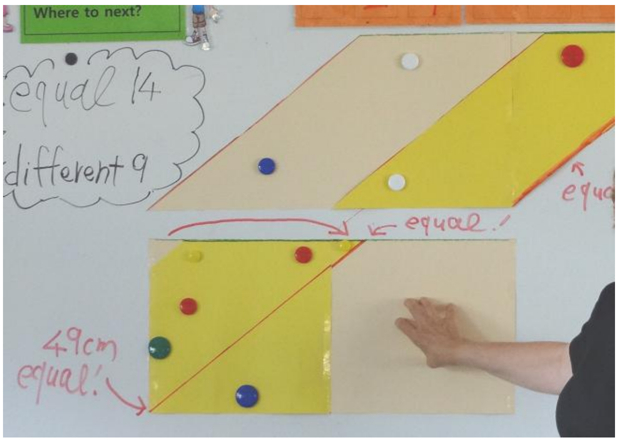


Figure 10. Overlay process for two different models

The lesson concluded with a demonstration of the red line, representing the spiral stair on the rectangle model, being translated and reflected onto the parallelogram model. This demonstration proved that both models equally represented the spiral stairs in the oil tank tasks as shown in Figure 11. The students were able to confidently conclude that the spiral stairs were the same length on both oil tanks.

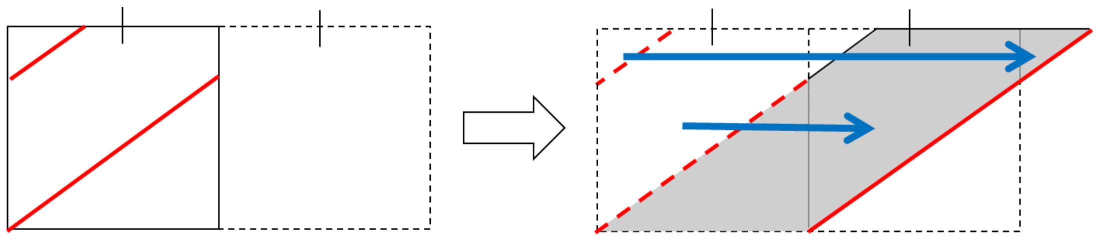


Figure 11. Relationship between the rectangle model and the parallelogram model

Conclusion

There are several important findings from this research. Firstly, the 23 Year 6 students were not able to easily move between 3D and 2D models as anticipated. This hindered their ability of most to ascertain the first model, the rectangle model. Moreover the students did not identify that the spiral stair would be represented as a straight line on a 2D model. This prevented them from constructing an appropriate model and to directly measure and compare the spiral stair on their 5m diameter to their 10m diameter model. When presented with the second task, the toilet tube task, the students were again unable to freely move between 3D and 2D models. This hindered their visualisation and ultimate prediction of their 2D model. Nonetheless, when the students saw the shape after they had cut up the toilet paper tubes they quickly engaged with the task holding valuable mathematical discussions that required precise mathematical representations to convey their thoughts and to compare and contrast the two different models. Unfortunately these representations did not appear on most students' worksheets. Rather they used simple descriptions with a tendency not to draw a model to support their description.

There were several students in the class, such as Millan, who were able to use their understanding of the first task to assist their interpretation of the second task. They worked quickly to identify the way each model could be used to prove that the spiral stairs were equal. There were students on the other hand who did not easily understand the first task and why we would cut off a section of the red line on the small rectangle and place it on the longer line on the larger rectangle. But when we came to the second task and the parallelogram model, they could easily see that the spiral stairs were equal for both oil tanks. This finding supports the work of Saeki and Matsuzaki (2013) and Kawakami, Saeki and Matsuzaki (2012) and their benefits of the dual modelling cycle framework. It also supports their fining that some students are able to engage more fully than others providing them with an opportunity to deepen their mathematical knowledge.

The findings from this research would be supportive of Galbraith's quest to promote a new approach and therefore new knowledge when working on problems with real world contexts as well as meet the intended goals of the proficiency strands in the Australian Curriculum Mathematics.

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