



# Creativity in students' modelling competencies: conceptualisation and measurement

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## Abstract

Modelling competencies are currently included in numerous curricula worldwide and are generally accepted as a complex, process-oriented construct. Therefore, effective measurement should include multiple dimensions, like the sub-competencies required throughout the modelling process. Departing from the characteristics of modelling problems as open and often underdetermined real-world problems, we propose to enrich the current conceptualisation of mathematical modelling competencies by including creativity, which plays an important role in numerous phases of the mathematical modelling process but has scarcely been considered in modelling discourse. In the study described in this paper, a new instrument for the evaluation of this enriched construct has been developed and implemented. The modelling competencies incorporating creativity of the students were evaluated based on the adequacy of the models and the modelling processes proposed, and the appropriateness and completeness of the approaches were evaluated in detail. Adapting measurement approaches for creativity that have been developed in the problem-solving discourse, certain criteria of creativity were selected to evaluate the creativity of the students' approaches in tackling modelling problems—namely, usefulness, fluency, and originality. The empirical study was conducted among 107 Chinese students at the upper secondary school level, who attended a modelling camp and independently solved three complex modelling problems. The results reveal significant correlations between fluency and originality in students' performances across all tasks; however, the relationships between usefulness and the other two creativity aspects were not consistent. Overall, the results of the study support the importance of the inclusion of creativity in the construct of modelling competencies.

**Keywords** Mathematical modelling competencies · Measurement · Creativity · Chinese mathematics education

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## 1 Introduction

With the aim of promoting responsible citizenship, mathematical modelling and related competencies have been recognised as important in numerous national curricula, particularly in order to enhance quality-oriented teaching (Blum, 2015). For example, in the newly released national curricular standards for upper secondary school mathematics in China (Ministry of Education of China [MOE], 2018), mathematical modelling competencies are considered one of the six core competencies of mathematics education. They are, on the one hand, “relatively independent” and, on the other hand, “intertwined with one another” to promote the “right values, necessary characters and key abilities” that students should develop in mathematics learning (p. 4, translated by the first author). According to these standards, among the biggest challenges to the promotion and implementation of modelling competencies in mathematics teaching and learning are the development and implementation of valid assessments accompanied by the problems students and teachers have with mathematical modelling (Blum, 2015).

The characteristic challenges while solving real-world problems are typically described by modelling cycles (e.g., Blum & Leiß, 2005; Galbraith & Stillman, 2006; for a more recent overview see Niss & Blum, 2020), which indicate the competencies that are crucial for solving modelling problems. Overall, several descriptions of modelling cycles exist in the current discussion on mathematical modelling and in the curricula prescribed in several countries, which have the potential to influence the promotion of mathematical modelling in these countries, as curricula provide a basis for the development of textbooks and are important guidelines for teachers to implement mathematical modelling in their teaching (Borromeo Ferri, 2018; Niss & Blum, 2020). A broad body of research from the international modelling discourse has empirically investigated modelling processes and students’ barriers and identified the importance of specific sub-processes of the modelling process and the related sub-competencies to solve real-world problems by implementing a modelling process using mathematical means (Kaiser, 2017). Further, mathematical modelling problems can be distinguished by the openness and underspecification of the underlying real-world situation, thereby enabling students to approach the modelling problem in different ways with different solutions (Schukajlow et al., 2015). Openness and underspecification usually require that the students maintain open minds and deploy creativity to simplify real-world situations and make appropriate assumptions. Mathematical modelling does not entail the solution of real-world problems using standard methods; rather, the development of new methods of finding solutions or new ways of addressing real-world problems based on sound mathematical knowledge is required (Niss & Blum, 2020), which calls for the incorporation of certain aspects of creativity into the discourse on the teaching and learning of mathematical modelling.

Despite its relevance, until now, only a few studies (e.g., by Wessels, 2014) have explored the relationship between mathematical modelling and creativity but unfortunately, these do not offer a clear construct of mathematical modelling competencies that incorporate creativity. The study described here aims to propose a further development of the conceptualisation of mathematical modelling competencies with an emphasis on various dimensions of creativity, which should be included in this new construct. Further, as mathematical modelling is a complex process and a multidimensional construct, a complex evaluation instrument is proposed that aims to evaluate students’ modelling competencies incorporating creativity. This approach is of particular importance in the context of China, where mathematical modelling has been introduced recently into the mathematical curriculum and where evidence from empirical studies in support of modelling is lacking (Lu et al., 2019).

## 2 Literature survey and research questions

### 2.1 Mathematical modelling competencies

As the discourse on mathematical modelling is growing strongly, in the literature survey, we focus on mathematical modelling competencies and refer to overviews on the discussion on the teaching and learning of mathematical modelling provided by Blum (2015) and Schukajlow et al. (2018).

Departing from psychological discourse, mathematical modelling competencies encompass both the ability and willingness to tackle real-world problems using mathematical methods associated with affective issues such as motivation and volition; in addition, various sub-competencies have been identified as necessary to implement modelling processes (Kaiser, 2007; Maaß, 2006). In contrast, Niss and Højgaard (2011, 2019) emphasised mainly cognitive abilities as the core of mathematical competencies within their extensive framework, which has been recently published in an updated version.

The following four central perspectives have been identified within the discussion on the teaching and learning of mathematical modelling competencies, which have shaped the discourse in the last two decades; not all of these play a prominent role in the current discourse (for more details, see Kaiser & Brand, 2015):

- The introduction of modelling competencies in an overall comprehensive concept of competencies by the Danish KOM project focusing on cognitive abilities (Niss & Højgaard, 2011, 2019).
- The measurement of modelling skills and the development of measurement instruments by a British-Australian group, proposing quantitative measures for modelling competencies (Haines et al., 1993).
- The development of a comprehensive concept of modelling competencies based on the distinction of sub-competencies and its evaluation by the discussion on modelling in the German context (Blum, 2015; Kaiser, 2007; Maaß, 2006).
- The integration of metacognition into modelling competencies and the identification of modelling barriers by work within the modelling discussion in the Australian context (Stillman, 2011; Stillman et al., 2010).

Departing from these strands of the discourse, the current discussion on mathematical modelling differentiates global modelling competencies from sub-competencies of mathematical modelling. Global modelling competencies are the abilities that individuals require in order to successfully perform and reflect on the entire modelling process. The sub-competencies of mathematical modelling refer to the individual phases of the modelling cycle; they include the different competencies necessary to successfully complete these phases. A widely accepted version of the modelling cycle includes the following sub-competencies related to the phases of the modelling process (Kaiser, 2007; Maaß, 2006):

- Simplifying the real-world problem and making adequate assumptions.
- Mathematising the real-world problem.
- Tackling the mathematical model using adequate methods.
- Interpreting and validating the results in the original real-world situation or even before in the real-world model.

In addition to these sub-competencies, more general competencies are included within the entire modelling competencies construct: the “competency to solve at least partly a real-world problem through mathematical description (that is, model) developed by oneself” (Kaiser, 2007, p. 111) and the metacognitive competency to utilize knowledge regarding modelling processes in general to reflect on the modelling process and one’s own thinking (Maaß, 2006; Stillman, 2011; Vorhölter, 2018).

Owing to the lack of standard methods for solving real-world problems and the context-boundedness of each step of the modelling process, creativity plays a key role in all phases of the modelling cycle (Wessels, 2014). For example, when understanding a real-world situation, it is necessary to apply creativity in developing various perspectives on the problem; within the mathematical development of the results, a flexible usage of different mathematical means is important. Moreover, in the results’ interpretation and validation phases, it may be necessary to include transverse ideas to make sense of the results. Therefore, in our further development of the construct of modelling competencies, we identified the following aspects of creativity that play an important role in the various phases of the modelling process. We display these aspects in the enriched diagram of the modelling process given by Kaiser and Stender (2013, p. 279) in Fig. 1.

Modelling competencies and their development are strongly related to how they are measured (Blum, 2015; Kaiser, 2017; Niss & Blum, 2020). The extant literature has focused on the assessment of sub-competencies throughout the modelling cycle and the global competency to execute the modelling process (for an overview see Kaiser & Brand, 2015) as well as additional competencies, like metacognitive competencies (e.g., Stillman, 2011; Vorhölter, 2018). To capture the complexity of mathematical modelling, comprehensive approaches to its measurement are necessary.

To summarise, although the fostering of modelling competencies is requested in numerous curricula worldwide and there is a consensus that students should learn how to use mathematics in real life, the promotion and measurement of modelling are still not given the emphasis recommended in academic discourse.

## 2.2 Creativity and its relationship with mathematical modelling

Creativity is considered a major disposition for modern life, bringing about growing innovative changes in numerous aspects of our lives (Pellegrino & Hilton, 2012). Research interest in the field of creativity has increased in the last 30 years, and its focus varies from genius to more wider perspectives of inquiry—for example, creative behaviour in daily life (Hersh & John-Steiner, 2017; Kupers et al., 2019). Such aspects are also important for mathematics education discourse; therefore, mathematical curricula have emphasised the fostering of creativity and critical thinking (Pitta-Pantazi et al., 2018).

Thus far, there has been no consensus on whether creativity is a general cross-domain or a domain-specific concept, although current research appears to focus more strongly on the relationships between general and specific creativity (here pointing to mathematical creativity) than on the analysis of the relationship between creativity and the domain (Plucker & Zabelina, 2009). For example, Hong and Milgram (2010) proposed that general creativity is a prerequisite for the emergence of mathematical creativity, although general creativity cannot contribute to the explanation of mathematical creativity and its impact on mathematical activities. Originally, Kattou et al. (2015) claimed that mathematical creativity is not a part of general creativity due to its domain-specificity; however, more recently, the framework or evaluation

of general creativity has been transferred and adapted to mathematical creativity within empirical studies (e.g., by Pitta-Pantazi et al., 2018; Silver, 1997). Overall, it appears to be necessary to empirically analyse the role of creativity in mathematics by referring to the approaches for general creativity.

In psychological discourse, it is emphasized that the learning of mathematics can contribute to the promotion of creative thinking, not only merely in mathematics but also in general (Sternberg, 2017). Mathematical creativity is usually considered one sub-component of mathematical ability (Kattou et al., 2013) and is researched through studies on problem-solving and problem-posing, often embedded in comprehensive theory building processes (Assmus & Fritzlar, 2018).

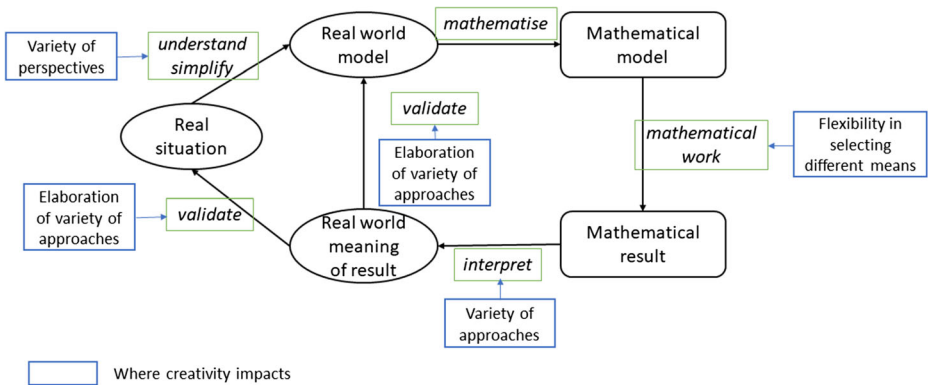
Torrance (1966), in his seminal work, defined creativity in the following manner:

“A process of becoming sensitive to problems, deficiencies, gaps in knowledge, missing elements disharmonies and so on; identifying the difficulty, searching for solutions, making guesses, or formulating hypotheses and possibly modifying and retesting them; and finally communicating the results” (p. 6).

Departing from this definition, commonalities can be identified between the two approaches of creativity and modelling: both are process-oriented and are based on cognitive, intrapersonal, and interpersonal competencies. Creativity requires originality and appropriateness in individuals' abilities to produce work (Sternberg & Lubart, 1999). A creativity perspective on mathematical modelling will contribute to a comprehensive understanding of modelling competencies in mathematics education, and the promotion of such a perspective will also facilitate insight into the promotion of creativity per se.

Before discussing the measurement of modelling incorporating a creativity perspective, we review the connections of creativity with mathematical problem-solving and problem-posing; from this, we may develop a better understanding of creativity and its potential role in mathematical modelling. For several decades, it has been indicated that problem solving includes creative processes (Guilford, 1977), particularly within the solution process, which involves divergent thinking (Haylock, 1987). Moreover, the creation of new knowledge and flexible problem-solving abilities appear to be the two major components of mathematical creativity (Kwon et al., 2006), and solving non-routine or ill-structured problems may contribute to the improvement of creativity (Chiu, 2009).

Based on the work by Torrance (1966), Silver (1997) refined the definition of the construct of creativity enhancing problem solving in mathematics education and proposed the following components: fluency as the identification of multiple solutions to a problem, flexibility as the generation of new solutions in addition to the existing one(s), and originality as the exploration of as many solutions as possible to a problem and the generation of new solutions. Leikin (2013) further developed this categorisation and proposed the use of multiple-solution tasks to develop students' mathematical creativity. She developed a scheme for the evaluation of creativity in problem solving based on solution spaces, which consists of fluency as the number of appropriate solutions, flexibility as the categories of solution, and originality as the combination of relative and absolute aspects—considering routine solutions from students' regular learning experiences and their level of insight involved in the solution process. With this fluency-flexibility-originality triad, Leikin investigated the differences between the problem-solving abilities of gifted and non-gifted students and proposed originality as the key factor in determining creativity, thereby serving as a possible means of identifying gifted students in mathematics (2013). Moreover, this model was applied to investigate prospective mathematics



**Fig. 1** Modelling cycle enriched by aspects of creativity

teachers' proof-related and creativity-related skills during problem-posing activities; the results demonstrated the relationship among creativity, mathematical skills, and the participants' knowledge bases (Leikin & Elgrably, 2020).

Problem posing is considered a form of mathematical creation as well (Bonotto & Santo, 2015) and has been used to promote and evaluate creativity (e.g., Silver & Cai, 2005). The evaluation also usually connects problem-posing skills with the three creativity categories of flexibility, fluency, and originality (e.g., Cai & Hwang, 2002; Leikin, 2013; Van Harpen & Siraman, 2013).

Like mathematical problem solving and problem posing, the mathematical modelling process is highly related to creativity. Coxbill et al. (2013) defined mathematical creativity as students' ability to create useful and original solutions in authentic problem-solving situations, in which students interpret the actual situation and understand the situation through mathematizing. With procedural tasks, students can analyse and work with mathematical models, interpret mathematical results, and, through the process, gain new insight into the situations (Tabach & Friedlander, 2018). Overall, all the modelling activities can and even must incorporate creativity (as displayed in Fig. 1), which implies that, consequently, the development of mathematical modelling is usually accompanied by the occurrence of fluency, originality, and flexibility.

### 2.3 Measuring mathematical modelling from the perspective of creativity

Mathematical modelling tasks are complex, open, and non-routine problems using various real-world contexts, which can be approached by students at different levels (Wessels, 2014). As described above, creativity can be incorporated into complete modelling processes, which provide the opportunity to measure both the global modelling competency and the sub-competencies of modelling through the perspective of originality. The three components of creativity—originality, fluency, and flexibility—have been broadly employed in the study of mathematical creativity (Pitta-Pantazi et al., 2018). However, the following open question remains: whether or not only these three components must be specifically considered in investigating modelling or whether other important criteria exist. For example, Klavir and Gorodetsky (2011) propose elaborateness, level of details provided, appropriateness, and

adequacy of the modelling approach as features of creativity, which should be considered in an integrated manner while measuring mathematical modelling incorporating creativity.

There is no consensus regarding the inclusion of usefulness into the definition of creativity. For example, in his comprehensive overview, Sriraman (2009) characterised creativity only by the features of novelty and originality as mathematical creative work may not always be applicable. This approach is in contrast to that of Sternberg and Lubart (1999), who defined creativity as original, useful, and adaptive. Wessels (2014) included usefulness—in addition to relevance and adaptability—in her framework to measure modelling while defining reusability of the modelling approaches in other real-world situations as an indicator of usefulness. Usefulness may be of specific importance in mathematical modelling, as mathematical modelling is characterised as applicable mathematics, which does not hold for mathematics in general (Pollak, 1977).

In addition to usefulness, in her theoretical framework, Wessels (2014) considered the aspects of fluency, and flexibility while evaluating pre-service teachers' work of modelling. Fluency evaluates the solution of the modelling process and is classified as low-, medium-, or high-level based on the number of different solutions represented. Flexibility entails a shift in the emphasis, direction, or approaches of problem solvers within the modelling process and is coded by the number of shifts undertaken by the study participants. This comprehensive analytical framework by Wessels (2014) requires a rich data set, particularly with regard to fluency and flexibility; this implies that the participants' entire work on the modelling process or draft ideas on the selected modelling approach and possible modelling processes are necessary. The developed theoretical framework and the evaluation instrument associated with it given by Wessels (2014) are strongly restricted to smaller samples, as an extensive database is required which records every drafted solution of the participants and every shift that occurs during the thinking procedure. In particular, the evaluation of flexibility appears to be ambitious to capture, as the various modelling cycles implemented in solving a modelling task may include several shifts in the directions of the approach, as well as smaller mini-cycles (Borromeo Ferri, 2018). Due to these evaluation difficulties and the strong relationship between flexibility and fluency, flexibility has not been included in several assessment schemes of creativity (Hébert et al., 2002).

Novelty or originality, as measured in the abovementioned problem-posing and problem-solving activities, has been recognised as the most important indicator of creativity in numerous frameworks (Leikin, 2013; Reiter-Palmon et al., 2019). This aspect should be considered in the evaluation of modelling competencies but not be limited to the originality of the mathematical means used and also include the interpretation of real-world situations. Originality is usually evaluated within a reference group to consider the reference norm, which is particularly appropriate for younger students; however, this relative originality leads to the evaluation of relative creativity (Assmus & Fritzlar, 2018). Therefore, Leikin (2013) proposed a combination of relative and absolute originality by involving more reference groups, like a group of expert solvers in addition to beginners. However, the inclusion of varied reference groups requires either large-scale or longitudinal studies, cannot be achieved easily, and has a few limitations related to mathematical modelling, as experts' solutions may employ mathematical means and heuristic strategies that go beyond school mathematics (Stender, 2017).

Based on the literature review on the various frameworks for defining and measuring creativity, considering the discussion on mathematical modelling, we include usefulness as a unique creative component in our instrument for measuring mathematical modelling competencies incorporating creativity due to the nature of modelling as applying mathematics in real-



world examples. Furthermore, we only include fluency in our instrument due to the strong orientation of mathematical modelling processes to solutions which provide an answer to the original question. Due to the difficulties described above and its close relation to fluency, we did not consider flexibility. Furthermore, we included originality as described in the literature but referred only to one reference group as our framework and measurement instrument have been newly developed without possibility to access different samples.

## 2.4 Research questions

We integrated the widely accepted components of creativity (usefulness, fluency, and originality) into the construct of modelling competencies and describe creativity as an overall characteristic of the modelling competencies important in each phase and step of the modelling cycle (see Fig. 1). Based on this conceptualisation, we conducted a study in China with upper secondary school students to measure mathematical modelling competencies incorporating creativity.

We address the following research questions:

1. Which level of modelling competencies did the students attain based on the adequacy of the modelling approaches they provided across the three implemented modelling tasks?
2. Which level of modelling competencies did the students attain based on the three creativity aspects of usefulness, fluency, and originality across the three modelling tasks?
3. Are the students' performances measured in terms of the adequacy of the modelling approaches and the correlation among the three creativity aspects and, if yes, how strongly?

## 3 Methodology and design of the study

The study adopts aspects of qualitative as well as quantitative research in the evaluation of both modelling competencies and the three aspects of creativity—usefulness, fluency, and originality—which are of importance in all phases of the modelling process.

### 3.1 Participants and data collection

The participants of the study, who were recruited in 2018, were 107 Chinese students, who were aged 16–18 years during the study. They were school students from 23 upper secondary schools in 19 cities across China and attended a summer modelling camp. The study comprised 23 girls and 84 boys.


The participants had attended 1 to 2 years of upper secondary school. They were expected to have acquired the mathematical knowledge prescribed by the centralised curricular standards (MOE, 2007; MOE, 2011); in addition, they had experiences in attending modelling competitions for secondary students, which was one criterion for them to be allowed to participate in the camp. The majority (83%) of the students had previous experiences in tackling modelling tasks in the past 1 to 2 years, and approximately 12% of them had experienced modelling for the first time while attending the modelling competition before the camp. Since modelling had not been promoted in their school education, they needed to learn about modelling in their free time; performing well in national and international




modelling competitions became the motivation for them to learn about modelling and to attend these competitions as an approach to learning. They had not experienced other creativity-oriented activities.

At the beginning of the test, the students were asked to complete a questionnaire on background information, like their previous experiences with tackling modelling tasks. Thereafter, they were required to work on three modelling tasks individually, with approximately 1 h where there was no access to the Internet or any teacher assistance. The three tasks (Fig. 2)


**I. Mathematics in pineapple**  
 The situation:  
 April is pineapple season. When we buy a pineapple, the vendor usually peels it artistically for us, leaving attractive spirals behind. Please think about this peeling process mathematically, and consider why the vendor peels the pineapple in this way. (1) Show your opinion(s); (2) Translate it/them into mathematics; (3) Provide solutions; and (4) Demonstrate your opinion(s).



The pineapple before peeling




The pineapple before removing the part inedible




The pineapple after peeling


**II. Making up a football**  
 The 2018 FIFA World Cup was just successfully held in Russia. From group stage to quarterfinals, and to the final game, it attracted many fans' attention. Actually, there are different expectations towards the football among different groups of interest. The Adidas Company supplies the footballs for FIFA. Do you know how the FIFA footballs are made by hand?  
 The following pictures shows how a manufacturer makes:




The manufacturer is sewing up the pieces of the football



Several pieces have been sewn up



A football is finally made



A football broken after several years and here we can see the thread

The manufacturer was paid related to the number of soccer balls of good quality. Please evaluate how long it takes to make a soccer ball from a mathematical perspective. Write down the process of thinking, and solve the problem.

**III. Refuelling**  
 Mr Lin lives in Shanghai. The nearest filling station in Shanghai is 20 km away from his home, and the nearest one in Soochow is 80 km away. He usually drives to Soochow to fill up his Volkswagen CC1.8T because the fuel price is 7.61 RMB/L in the Soochow filling station, but 8.04 RMB/L in the Shanghai one. Some information about Mr Lin's car CC1.8T:

FAW-Volkswagen

Length × Width × height (mm):	4799×1855×1417	Weight of the car (kg):	1535
Fuel consumption measurement (L/100km):	7.8	Capacity of the fuel tank (L):	70.00
Warranty:	2 years 60, 000 km	Capacity of the trunk (L):	532

Is it worth it for Mr Lin to go to the Soochow station to fill up on gasoline? Please provide your opinions and demonstrate your argument.

Fig. 2 The translated versions of the three modelling tasks

were task 1 *Peeling a pineapple* (a similar version can be found in Ludwig & Xu, 2010), task 2 *Making a World Cup football*, and task 3 *Refuelling* (an adaption of a task published in Blum & Leiß, 2005).

These modelling tasks were selected for the study on creativity because of their properties as open-ended and underspecified and as they have the potential to promote various modelling approaches and multiple solutions. Due to the characteristics of the tasks, they necessitate comprehensive modelling competencies and creative solution attitudes. Mathematically, the tasks involve geometrical shapes, such as cylinders and polyhedra, and algebraic contents, such as trigonometric ratios and polynomial functions. The students were expected to be familiar with the context of the first task, since scenes of pineapple peeling are commonly encountered during the pineapple sale season in China but rarely encountered in their school learning experiences. A photograph depicting how a pineapple is peeled by the salesperson was displayed in the task formulation, and the students were asked to explain why it is peeled in such a manner. For the second task, the students were familiar with the FIFA World Cup but did not know much regarding the manufacturing of footballs. Photographs of workers manufacturing a completed ball and a broken ball were shown to the students, and they were asked to calculate the time invested in manufacturing a soccer ball. The third task posed the question of where to refuel a car, given certain conditions (details see Fig. 2). This task provided more information for the students than the other two tasks and was considered the most familiar scenario for them, which was confirmed in informal conversation with the students after the evaluation. The students described this task as similar to tasks in their textbooks or daily mathematics exercises at school and considered it as a very easy task for lower secondary school students. Overall, they felt more confident when tackling this task compared to the other two; therefore, it was not surprising that they performed better on this task than the other two. The original German task contained less information and was, therefore, more complex for the students.

The students' paperwork on the three modelling tasks was collected and analysed based on the approach to qualitative content analysis by Mayring (2014), which involves employing strict quality standards through the usage of clear coding manuals that contain explicit descriptions of the evaluation of the items with clear rating scales.

### 3.2 Data coding

The students' solutions to the three modelling tasks were first analysed in terms of the adequacy of their modelling approach, and thereafter, the three aspects of creativity—usefulness, fluency, and originality—were evaluated using a three-level ordinal scale. We defined and applied these four criteria in the following manner:

Adequacy refers to the evaluation of the adequacy of modelling approaches to solve tasks, considering both the completeness of the modelling procedure—which means whether the approaches include the necessary modelling steps—and the appropriateness of the single steps of the modelling process and the overall approach. As modelling tasks usually do not have one correct answer, but are rather often characterised by multiple solutions (Achmetli et al., 2019), we evaluated the appropriateness of the modelling procedures and not their correctness and did not grade minor calculation errors. A three-level sub-category scheme was developed to grade the high-medium-low level of overall modelling competency (Table 1).

**Table 1** The sub-categories of high-medium-low levels reflecting the adequacy of modelling approaches

Examples		Task 2	Task 3
Level	Descriptions	Task 1	Task 3
High	A high level of adequacy is assigned when the modelling approaches encompassed relatively completed modelling cycles and the modeller could successfully solve the tasks.	The approaches include appropriate means to simplify the problem (e.g., representing the pineapple as a cylinder and unfolding it to the plane level) to establish the connections among the lengths of different tracks of peeling, to work mathematically on the comparison to obtain results, and to interpret the results. The approaches do not include the complete means to simplify the problem, or the clarified means to identify the important mathematical relationships.	The approaches include the consideration of necessary parameters (e.g., the number and the length of the edges, and the time required to sew up the edges) and the appropriate means to represent the parameters mathematically and work correctly and illustrate the results. The approaches only identify the mathematical relationships between the costs of refuelling at two places but did allow for obtaining the results.
Medium	A medium level is assigned when the approach had the potential for the solution of the tasks but with incomplete modelling cycles—for example, appropriate models have been created but without mathematical results or the mathematical work required to be refined to obtain correct results.	The approaches do not include the complete means to simplify the problem, or the clarified means to identify the important mathematical relationships.	The approaches only identify the mathematical relationships between the costs of refuelling at two places but did allow for obtaining the results.
Low	A low level is assigned when the approaches were not adequate to solve the tasks.	The approaches failed to adequately represent the peeling tracks.	The approaches only include the restatement or illustration of the problems, but no models/solutions are developed.

The three creativity aspects—usefulness, fluency, and originality—were then independently analysed to evaluate the students’ modelling competencies from the perspective of creativity. The analyses were based on the following definitions:

- Usefulness refers to the evaluation of the utility of all the approaches that the students employed to solve the tasks through modelling. A lower level of usefulness is assigned to an incorrect approach, where a redirection of the modelling approach is required for students to successfully solve the task, while a higher level denotes useful and sharable approaches.
- Fluency refers to the implementation of various solutions to the tasks. A lower level of fluency is assigned to a single solution and solutions within one modelling cycle and higher levels of fluency are assigned to approaches that apply various models to solve the tasks<sup>1</sup>.
- Originality is evaluated on the basis of the relative rarity of the mathematical approaches employed by the students within the group they were part of. A lower level of originality is assigned to responses that are commonly identified in the group, and a higher level of originality is assigned to those responses that apply unique mathematical approaches.

As the three tasks had been used in previous empirical studies on mathematical modelling, data on exemplary approaches to solve the tasks through modelling were available (Blum & Leiss, 2005; Ji, 2008; Ludwig & Xu, 2010; Wang, 2019). These data served as anchor exemplary approaches to the construction of the coding manual. The exemplary approaches enabled us to identify key technical strategies, the necessary mathematical knowledge, and different ways to interpret and tackle situations. The codes used within the structuring content analysis (Mayring, 2014) on the adequacy of the modelling approach as well as on the dimensions of creativity were deductively defined codes that enriched the inductively developed codes that were derived from the analysis of the students’ approaches. The descriptions of each sub-category and corresponding examples from the students’ solutions are summarised in Tables 1 and 2.

### 3.3 Data analysis

In order to test the reliability of the coding scheme, 30 randomly selected student scripts were first coded independently by the first author and a doctoral student majoring in mathematics education. The coding mainly focuses on the adequacy of the modelling approach and the three aspects of creativity embodied in students’ performance on the three modelling tasks. A weighted kappa of  $\geq 0.81$  shows a “very good” inter-rater agreement on all the dimensions, according to Altman’s criterion (Altman, 1991, p. 404). The first author completed all the remaining 77 scripts after the first 30 were coded.

For each modelling task, a descriptive analysis on adequacy and three creativity aspects was first conducted (e.g., frequency). Thereafter, a set of Friedman tests were used to compare students’ performance across the three modelling tasks on each of the four aspects (i.e., adequacy and three creativity indices). When an overall significant difference was detected, Dunn-Bonferroni post hoc tests were used to further examine pairwise differences. Further, the correlations between students’ performance on different aspects were tested by Spearman

<sup>1</sup> If more than one modelling cycle was used by the students, we would code all the modelling cycles regarding usefulness and fluency and only consider the highest level of usefulness and fluency.

**Table 2** The sub-categories of high-medium-low level of the creativity aspects

Level	Descriptions	Examples		
		Task 1	Task 2	Task 3
Usefulness	High A high level is assigned to the approaches which are not only useful to solve the tasks themselves but are also sharable to solve other similar tasks.	The approaches include the strategy of transforming a solid geometry problem to plane geometry (representing the pineapple as a cylinder and unfolding it) and provision of more potential peeling tracks.	The approaches include the relevant ways to figure out the number of edges and the lengths of the edge through the connections between pentagonal and hexagonal shapes.	The approaches apply algebraic expressions, inequalities, or functions to express the mathematical relations, with which the situation can be described.
	Medium A medium level is assigned to the approaches, with which solutions of the task are possible, but cannot be shared.	The approaches describe specific cases, e.g., the lengths of the peeling tracks take specific values or the relations between different tracks are limited to specific cases.	The approaches indicate the specific procedure to sew up a football.	The approaches calculate the cost of refuelling at two places based on arithmetic means.
	Low A low level is assigned when the approaches yield wrong solutions.	The approaches which failure to present the proper relations between different peeling tracks.	The approaches with improper consideration of the parameters.	The approaches are restricted to petrol consumption and the means to fill up.
Fluency	High A high level is assigned to the approaches which encompass various models to solve the tasks.	NA*	Two models were developed to represent the time for sewing up.	Two solutions were provided, each of which considers different groups of parameters.
	Medium A medium level is assigned to those approaches including only one modelling cycle/model.	One model provided, usually the comparison of the length of the different peeling tracks	One model provided, usually calculating the sum of sewing up every edge.	One model provided, usually comparing the cost of refuelling once at the two places.
	Low A low level indicates that no modelling cycle or an uncompleted model is included in the approaches.	No model developed, blank, or restatement of the problem	No model developed, blank, or restatement of the problem.	NA
Originality	High A high level is assigned to the approaches which include the considered parameters and	The approaches that include mathematical means (e.g., coordinate systems, sequences, and triangle inequalities) which are used by no	The approaches which calculate the time from a different perspective, such as the payment for making; or	The approaches that include the parameters considered by only a small group of students (e.g., the

Table 2 (continued)

Level	Descriptions	Examples		
		Task 1	Task 2	Task 3
Medium	<p>mathematical means used by a small number of participants.</p> <p>A medium level is assigned to the approaches employed by a relative larger number of participants.</p>	<p>more than 10% of the participating students.</p> <p>The approaches that include mathematical means (e.g., triangle rates), which are used by 10–30% of the participants.</p>	<p>include uncommon mathematical means to calculate the time to sew up</p> <p>The approaches that consider more than one parameter, like the breaks between sewing up the edges</p>	<p>time required for travel) or use ambitious mathematical means.</p> <p>The approaches that include advanced algebraic methods (such as rational expressions or inequalities with variables), or use parameters which are considered by 10–30% of the participating students (e.g., vehicle abrasion en route to filling up).</p> <p>The approaches which only consider the cost of refuelling or only use mathematical means from arithmetic.</p>
Low	<p>A low level is assigned to the approaches used by the largest number of participants.</p>	<p>The approaches which include mathematical means used by more than 30% of participating students, alternatively, restrictions to arithmetic means as key methods.</p>	<p>The approaches which only consider the parameter of sewing up edges or use arithmetic means.</p>	

NA mentions that no examples of the sub-category can be found in the students' solution

correlation analysis. Partial correlation analysis was further conducted to detect the correlations between three creativity indices with a control of adequacy.

## 4 Results

### 4.1 Students' performance based on the adequacy of the modelling approaches

As mentioned above, the adequacy of the overall modelling approaches was evaluated based on the criteria of the completeness and appropriateness of the approaches, and the criterion of appropriateness covers for both correct and potentially correct solutions. As indicated in Table 3, approximately 99% of the 107 students provided adequate approaches to solving task 3 (refuelling), while only approximately 27% presented successful approaches to task 2 (making a World Cup football), and 29% developed appropriate ways to solve task 1 (peeling a pineapple).

Based on the Friedman test, a significant difference is observed in the students' modelling competencies among the three tasks—namely,  $\chi^2(2) = 113.047$ ,  $p < 0.001$ . Dunn-Bonferroni post hoc tests indicate significant differences between tasks 1 and 3 ( $p < 0.001$ ) as well as between tasks 2 and 3 ( $p < 0.001$ ) after Bonferroni adjustments; no difference was observed between tasks 1 and 2 ( $p = 0.111$ ). Overall, these results indicate that the students performed best in modelling task 3 and that they performed poorly in tasks 1 and 2 based on the evaluation of the adequacy of their modelling approaches.

For task 1, many students recognised the importance of simplifying the pineapple's shape to a cylinder and then unfolding it to the level plane to work on it, and approximately one-third of them were able to develop successful approaches. Further, 6% of the students did not present a clear modelling procedure in their approaches, which implies that they developed only unclear ways to simplify the situation.

For task 2, although only 27% of the students provided approaches that led successfully to adequate answers, 46% of them could have provided appropriate solutions if they had known the number of edges on a football. The students were not allowed to access the Internet, which hindered their search for this information. Apparently, most students could not successfully apply mathematical means to deal with this difficulty. However, 27% of the students successfully worked out the solutions and presented their approaches by figuring out the number of edges (e.g., using Euler's formula or their chemistry knowledge of *Buckminsterfullerene* (C60)).

For task 3, 99% of the students were able to provide adequate approaches, but these were mainly restricted to the usage of arithmetic means.

**Table 3** Percentages of students at different adequacy levels across the three modelling tasks

Levels	Task 1	Task 2	Task 3
High	29%	27%	99%
Medium	6%	46%	-
Low	65%	27%	1%



**Table 4** Percentages of students at different usefulness levels across the three tasks

Levels	Task 1	Task 2	Task 3
High	32%	17%	46%
Medium	36%	15%	53%
Low	32%	68%	1%

## 4.2 Students' performance based on creativity

We analysed the students' performance on the three creativity aspects of usefulness, fluency, and originality across the three tasks.

### 4.2.1 Usefulness

Table 4 indicates that 36% of the students performed at a medium level of usefulness in completing task 1, 68% performed at a low level of usefulness on task 2, and 53% performed at a medium level of usefulness on task 3. Significant differences were identified among all three tasks:  $\chi^2(2) = 68.693$ ,  $p < 0.001$ . Further analysis indicates significant differences between tasks 1 and 2 ( $p < 0.001$ ), tasks 2 and 3 ( $p < 0.001$ ), and tasks 1 and 3 ( $p = 0.004$ ). These results indicate that the students' performances in task 3 indicated the highest level of usefulness while their performances in task 2 showed the lowest level.

### 4.2.2 Fluency

Table 5 indicates that the students did not, in general, employ multiple approaches to solve the tasks; overall, most students performed at a medium level with regard to fluency. In task 2, a few students provided two approaches—one directly calculating the time required to sew up a football and the other estimating the time based on the salary paid to the manufacturer and other parameters. Several students provided two approaches for task 3, which included different parameters. A significant difference is also found in fluency between the tasks:  $\chi^2(2) = 43.841$ ,  $p < 0.001$ . Further analysis indicates that there is a significant difference between tasks 1 and 3 ( $p = 0.001$ ), but not the other two pairs.

**Table 5** Percentages of students at different fluency levels across the three tasks

Levels	Task 1	Task 2	Task 3
High	0	3%	8%
Medium	73%	84%	92%
Low	27%	13%	0

**Table 6** Percentages of students at different originality levels across the three tasks

Levels	Task 1	Task 2	Task 3
High	13%	22%	22%
Medium	13%	33%	39%
Low	74%	45%	39%

### 4.2.3 Originality

As Table 6 illustrates, 74% of the students showed low levels of originality in performing task 1, 45% showed low levels in task 2, and 39% showed low levels in task 3. Significant differences in originality are observed between the tasks— $\chi^2(2) = 25.595$ ,  $p < 0.001$ . Further, significant differences are noted between tasks 1 and 2 ( $p = 0.004$ ) and between tasks 1 and 3 ( $p = 0.001$ ). No significant difference could be observed between tasks 2 and 3. These results indicate that the students' approaches to tasks 2 and 3 showed higher levels of originality than their approaches to task 1. As fewer parameters required consideration when solving task 1 compared to the other two tasks, originality is only reflected in the rarity of the mathematical means employed. In tasks 2 and 3, originality is apparent in students' novel ideas of including different parameters—stemming from the real-world—in the approaches.

### 4.3 Correlations between the different aspects of modelling competencies

We also analysed the relationships among the different criteria with which we measured students modelling approaches and the creativity apparent in it in order to obtain a first insight into the relational structure of this enriched construct of modelling competencies incorporating creativity. In detail, we analysed students' performance on the adequacy of modelling approaches and the three creativity aspects (i.e., usefulness, fluency, and originality) and the relationship among the three creativity aspects themselves using Spearman's correlation analysis (Table 7).

The correlations between adequacy and the creativity aspect of usefulness on tasks 1 and 2 are significant, with  $r_s(107) = 0.849$ ,  $p < 0.001$ , and  $r_s(107) = 0.710$ ,  $p < 0.001$ , respectively. The correlation on task 3 is much weaker than that on the other two tasks— $r_s(107) = 0.192$ ,

**Table 7** Correlations among students' performance on different aspects

	Adequacy	Creativity-usefulness	Creativity-fluency	Creativity-originality
Task 1	Adequacy	1	0.849**	0.120
	Creativity-usefulness		1	0.138
	Creativity-fluency			1
	Creativity-originality			
Task 2	Adequacy	1	0.710**	0.018
	Creativity-usefulness		1	0.040
	Creativity-fluency			1
	Creativity-originality			
Task 3	Adequacy	1	0.192*	0.110
	Creativity-usefulness		1	-0.005
	Creativity-fluency			1
	Creativity-originality			

\* $p < 0.05$ , \*\* $p < 0.01$

**Table 8** Partial correlations among the creativity aspects

		Usefulness	Fluency	Originality
Task 1	Usefulness	1	0.100	0.067
	Fluency		1	0.283**
	Originality			1
Task 2	Usefulness	1	0.132	0.039
	Fluency		1	0.366**
	Originality			1
Task 3	Usefulness	1	-0.011	0.547**
	Fluency		1	0.403**
	Originality			1

\*\* $p < 0.01$

$p = 0.047$ . Table 7 indicates that the only significant correlation is that between adequacy and fluency on task 1 ( $r_s(107) = 0.346$ ,  $p < 0.001$ ), and that there are no significant correlations between adequacy and originality on all three tasks. Apparently, the adequacy of modelling approaches is strongly correlated with usefulness as one of the creativity aspects, which is not unexpected as utility is one of the main characteristics of modelling processes.

With regard to the three aspects of creativity, Table 7 displays significant correlations between fluency and originality in the students' performances on all three modelling tasks, with  $r_s(107) = 0.305$ ,  $p = 0.001$ ,  $r_s(107) = 0.362$ ,  $p < 0.001$ , and  $r_s(107) = 0.404$ ,  $p < 0.001$ , respectively. It indicates significant correlations between usefulness and fluency on task 1 ( $r_s(107) = 0.343$ ,  $p < 0.001$ ) as well as between usefulness and originality on task 3 ( $r_s(107) = 0.554$ ,  $p < 0.001$ ).

In order to exclude the influence of the adequacy of modelling approaches on the measurement of creativity, we tested the correlations between the creativity aspects once more while controlling for the variable of adequacy. Table 8 indicates that significant correlations between fluency and originality still exist, which indicates that the adequacy of modelling approach does not have a significant effect on the correlation. This result also applies to the correlations between usefulness and originality but does not hold for the correlations between usefulness and fluency, which became weaker when controlling adequacy.

#### 4.4 Summary of the results

Based on an analysis of the adequacy of the modelling approaches the students provided for the three tasks, task 1 and task 2 appeared to be much more challenging for the students than task 3. Therefore, it is not a surprise that they performed much better in task 3 than in the other two.

Overall, the students did not perform well regarding the three creativity aspects. They reached the highest level of usefulness in task 3 and the lowest level in task 2, but more than half of the students only attained the medium level of usefulness, which implies that they were only able to solve the task in itself rather than go beyond a larger range of situations. The students did not perform well on task 1 based on fluency compared to tasks 2 and 3, and most of them only presented one model or modelling cycle to tackle all the tasks. With regard to originality, they performed better on tasks 2 and 3 than task 1, but only a small number of students

provided relatively novel approaches, which reflected that more parameters had been considered in the approaches.

To summarise the correlations between these variables—adequacy of modelling approaches and the three creativity aspects—the results include the following aspects:

- For adequacy and the creativity aspect of usefulness, strong positive significant correlations could be identified on the two difficult modelling tasks (tasks 1 and 2). Correlations between adequacy and fluency have been observed on the difficult tasks.
- For the creativity aspects of fluency and originality, positive significant correlations have been identified on all the three modelling tasks, unaffected by the adequacy of the modelling approaches.
- For the creativity aspects of usefulness and fluency, correlations became insignificant on the two difficult tasks when controlling for adequacy.
- For the creativity aspects of usefulness and originality, a significant correlation was noted in Task 3, which was the best-performed task.

These results indicate that the correlations between the adequacy of the modelling approaches and the creativity aspects may be influenced by the difficulty of tasks as well as the correlations between usefulness and the other two aspects of creativity. The significant correlation between usefulness and originality in task 3 may indicate that original approaches may increase the level of usefulness in the approaches if the task is not too complex and far beyond the students' current horizons.

## 5 Discussion and conclusion

Overall, our findings emphasise that it is possible to conceptualise modelling competencies incorporating creativity and measure these competencies by including aspects of creativity.

### 5.1 Examining the rationale of measuring modelling competencies incorporating creativity

Based on the students' work on the three modelling tasks, we focused on the measurement of students' modelling competencies incorporating creativity, which influences the modelling process at all phases of the modelling cycle and the related modelling sub-competencies.

Referring to more recent approaches to defining and evaluating creativity, our studies indicate that the dimension usefulness as one component of creativity should be included in our enriched construct and newly developed measurement instrument. This implies that modelling competencies including creativity need to cover the aspects of applicability and shareability, although these criteria are not included in all definitions of criteria (e.g., as indicated by Sriraman, 2009). The evaluation of usefulness supports not only the measurement of the comprehensiveness of modelling approaches (Lesh et al., 2000) but also contributes to the measurement of the sub-competencies of modelling. For example, in task 1, those approaches that successfully represented the pineapple as a cylinder and unfolded it to a level plane were categorised at a medium level of usefulness, although they failed to adequately represent the peeling tracks and were unsuitable for solving the task, as they still utilized a sharable strategy to simplify the situation. In task 2, a high level of

usefulness was assigned to approaches that calculated the number of edges of a football, which is considered necessary knowledge for this specific modelling process. The context of the task encouraged the students to employ mathematical means to understand an actual situation.

Further, the correlation analyses reveal a rather weak correlation between usefulness and the adequacy of the modelling approaches for students' performance on the less challenging task 3, which may be due to the emphasis on the shareability of the approaches. In task 3, usefulness only requires abstract mathematical means, which can be shared in similar situations, while adequacy merely examines the suitability of the approaches to the tasks themselves. This result confirms that usefulness is separate from adequacy, despite its high correlations in the tasks 1 and 2. Therefore, usefulness may be a unique indicator for creativity within mathematical modelling, especially when considering the various processes of modelling and the complexity of relation between real-world situations and mathematics

The results show that usefulness is not correlated closely with originality and fluency in contrast to other studies examining creativity (e.g., Hébert et al., 2002; Runco, 2010), which emphasize that fluency and originality are important components of creativity and strongly correlated with each other.

The aspect of fluency focuses on the completeness and variety of the modelling cycles and the models developed. Although it is widely accepted that successful modelling quite often requires multiple solutions (Achmetli et al., 2019), few approaches on modelling competencies include the evaluation of a variety of modelling cycles and models developed for one task (Kaiser & Brand, 2015).

Originality emphasises the use of special mathematical means to construct and solve the models and encouraged the students to consider more parameters from the situations of the tasks. This is different from the evaluation of originality for problem solving by Leikin (2013), which mainly considers appropriate and complete solutions. The evaluation of originality in modelling processes emphasises both a sound mathematical knowledge base and students' comprehensive understanding of real situations as part of a complete modelling process.

Overall, the results of the evaluation of students' performances on different modelling tasks incorporating creativity indicate that our enrichment of the construct modelling competencies is theoretically viable and that the enriched construct can be empirically evaluated.

## 5.2 The students' modelling performances in the study

Although the students from schools throughout China had experiences in tackling mathematical modelling compared to their peers in schools who had no experience in modelling, overall, they did not perform well within these three modelling tasks, particularly when evaluated from the perspective of creativity. Our evaluation of the adequacy of modelling approaches adopted the suggestion by Leikin (2009) of replacing the criterion of correctness of approaches by the criterion of appropriateness, which included in our evaluation as many adequate solutions as possible and allowed for more differentiated results.

Only approximately 30% of the students provided adequate approaches for tasks 1 and 2, with which students were unfamiliar since these kinds of tasks are usually not covered in their ordinary mathematics lessons. Moreover, no extensive information regarding the situations in which the two tasks were embedded were provided, except for photos of

peeling a pineapple and sewing up a football, which visually displayed the shape of the pineapple and the arrangement of the parts needed to be moved and the sewing parts of the football as hints to figure out the number of edges of the football. Task 3, on refuelling, shared more similarities with tasks that the students had already encountered in school; further, the task contained more information than the other two and, therefore, required less creativity. Thus, it was not unexpected that almost all the students provided adequate solutions to task 3.

According to the strong correlations between adequacy and usefulness and the correlations between adequacy and fluency on tasks 1 and 2, the difficulty level of modelling tasks did not allow the students to consider more approaches or shareable approaches. Task 3, the easiest task, indicates a higher level of originality than task 1, which may suggest that, on the one hand, a task must only contain a certain kind of challenge to elicit more original responses. However, as only 22% of the students showed the highest level of originality in task 3, this contrastingly indicates that students' familiarity with the tasks from their school learning may constrain their efforts to attempt different mathematical means and understand the situations differently.

Generally, the students did not perform well from the perspective of creativity in terms of covering its three aspects, particularly fluency and originality. The low levels of fluency and originality in our study indicate that the students in our study were not used to attempting multiple or diverse ways to solve the tasks and, therefore, experienced difficulties, which is in line with current research on multiple solutions in mathematical modelling (Schukajlow et al., 2015). However, these difficulties may be exacerbated by China's examination-oriented nature for mathematical education (Wong et al., 2004), where students tend to provide one approach to efficiently solve tasks regardless of the kind of approaches that have been used. However, this imprinting appears to be changing, since the new curricular reform emphasises students' comprehensive competencies to accommodate life-long learning and the development of society (Wang & Lu, 2018). From this perspective, increasing attention has been paid to the promotion of fluency and originality in the teaching and learning of mathematics in China.

### 5.3 Limitations of the study

This study has several limitations. First, the three modelling tasks were solved as individual exercises to evaluate students' individual performances in mathematical modelling; however, modelling tasks are usually implemented within group work (Vorhölter, 2019), which may stimulate higher levels of creativity, although group work increases the difficulty in measuring students' individual modelling competencies. Moreover, the modelling work did not allow the holistic identification of students' thought processes and modelling processes, which would comprise all detailed blockages and deviations associated with the modelling process. Such an analysis would require other instruments that can capture rich information, like the think-aloud methods used, for example, by Hankeln (2020).

The three high-medium-low-level categorisation of each construct—the adequacy of modelling approaches and the three creativity aspects—shows discrete and relatively approximate categorisations of the components of our construct modelling competencies incorporating creativity. We focused on the students' performances on each component of our construct across the three tasks, rather than an overall evaluation of the construct. In particular, our evaluation of originality as the relative evaluation of originality within this student group as a reference group contains important limitations, although it enables the

evaluation of the connections between the solutions and the students' previous mathematical experiences (Leikin, 2013). Therefore, Leikin (2013) proposed the inclusion of absolute measures for the evaluation as well using achievements and results from other groups—for example, solutions from more experienced modellers. However, results from previous studies in which the first two tasks were used are not available (e.g., Wang, 2019), amongst others as creativity was not focused on. The last task was changed compared to the German original (Blum & Leiß, 2005). Thus, further studies should be performed in order to overcome this weakness.

Our investigation of modelling competencies does not cover metacognition, which has been described as an important component of modelling competencies (Stillman, 2011), as this would have implied other kinds of measurement instruments that focus explicitly on metacognition.

In summary, further research is required to validate our enriched construct and the newly developed evaluation instrument concerning modelling competencies incorporating creativity. For example, the inclusion of various reference groups with different experiences and the usage of various modelling tasks with different situations may contribute to the further development of the construct. An in-depth exploration of the construct within actual modelling processes in usual classrooms may reveal a deeper understanding of the indicators of creativity and the relations among them, and capture a comprehensive understanding of modelling competencies with more components involved, such as metacognition. Overall, this study provides the first steps related to the inclusion of the construct of creativity in the discourse on mathematical modelling competencies, but further studies need to follow.

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