Will this Net Work?: Development of a Diagnostic Interview

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Spatial visualisation is a subset of spatial ability and is exemplified in predicting whether or not a net will fold to form a target solid. The researchers examined video of interviews to explore the schemes of Year 5 students for determining the validity of nets for a cube and pyramid. Findings suggest the significance of imaged actions, shown through gesturing, and the importance of providing physical models during interviews as a means of validation.

Much is known about progression in students' thinking for some domains, such as number and probability. Relatively less is known about the strategies that students apply in spatial contexts. In this article we document our research process towards creating a diagnostic tool that provides insights into students' schemes for two-dimensional nets of simple three dimensional solids. A scheme, as we use it, is a triadic relationship between a genre of situations, actions and anticipated results of those actions (Steffe & Olive, 2010; Vergnaud, 2009; Von Glasersfeld, 1998). Our interest is with the mental operations employed by students to establish whether a given net will fold to form a target solid.

Spatial Visualisation

Anticipating the result of actions, personal or those of other agents, is a form of spatial visualisation that plays a critical part in people's interaction with their world. Realistic application is justification enough for the inclusion in curricula of outcomes that require students to create and manipulate mental images of spatial objects, alongside more traditional topics in Geometry, such as classification of shapes and proof. For example the Australian Curriculum for Mathematics (ACARA, 2013) includes working with two-dimensional representations of three-dimensional solids in the content descriptors from Year Two on.

Simply put spatial visualisation is the "ability to comprehend [and apply] imaginary movements in three-dimensional space or manipulate objects in imagination" (Pittalis, Mousoulides, & Christou, 2007, p. 1073) though Clements and Battista (1992) also include maintaining images in the service of other operations. Visualisation, along with orientation and relations, make up the suite of constructs thought to comprise spatial ability that associates highly with students' performance in geometry. Moreover thinking spatially goes beyond simply representing the physical world, it provides tools for structuring the organisation and interconnectedness of ideas (National Research Council, 2006). Many researchers assert the importance of spatial ability to mathematical thinking in general (Battista, 2007). For example the preference of students to attend to external perceptual characteristics of objects rather than to spatial properties in creating and structuring images may well link to their understanding of other topics such as measurement and algebra (Pitta-Pantazzi & Christou, 2010).

Working with nets provides an opportunity for students to connect two and three dimensional space and is now common practice in primary mathematics syllabi, texts and assessments. Yet few studies are available that describe the schemes that students use in

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anticipating the legitimacy of nets using spatial visualisation. Research has tended to focus on how students create nets when given the target solid, often with the support of acting on physical materials. Mariotti (1989) noted that 10 to 13 year olds consistently found the prototypical T-shaped net for a cube by unfolding physical models but often believed it to be the only possible net, frequently neglecting symmetry to predict if other nets would work.

Stylianou, Leikin and Silver (1999) reported that successful net builders relied on known composites of shapes, most notably a 'line of four squares' that wrapped to form the core of a cube and located the other two squares relatively to form the missing faces. Students worked both solid to net, usually by unfolding or rolling, and net to solid by creating speculative patterns, often non-systematically, and folding the net to confirm. The researchers' findings were similar to those of Potari and Spiliotopoulos (1992) who categorised primary students' attempts into five categories of attendance from holistic appearance to perceptual features, like a single square, to complete geometric structure when properties of the target solid were connected with features of the net. In a simpler taxonomy Pitta-Pantazzi and Christou (2010) characterised students as spatial visualisers who focused on the properties of the geometric shapes, and object visualisers who focused on physical appearance. Spatial visualisers were consistently more successful. This finding suggests that the creation of nets involves complex spatial abilities, visualisation, orientation and relations, and appropriate attendance to, and mapping of, geometric properties from net to target solid and vice versa. The significance of noticing properties to visualisation of nets was supported by Lawrie and Pegg (2000) who used both SOLO Taxonomy and the van Hiele levels of geometric understanding to classify students responses.

Work on the relative complexity of different nets also seems to support the importance of students' attending to spatial properties. Bourgeois (1986, p. 228) found that "the degree of difficulty of a net is a function of both the form of the solid and the arrangement of parts of the net." His claim omits the influence of strategic choices that students make which, in turn, impact on the required demands for co-ordination of solid to net or net to solid transformations (Mariotti, 1989; Stylianou et al., 1999). Multiple transformations place inevitable demands on students' working memories as well as on their spatial visualisation abilities. Strategies, such as assigning a base face and constructing composites of shapes, reduce the number and complexity of transformations and the corresponding load on working memory.

Method

We began with the following research question and corollary question:

1. What are the characteristics of students' schemes for establishing whether or not a given net folds to form a simple solid?
   a) Which interview tasks and questions provide insights into students' schemes?

Our intended research process is given in Figure 1. Phase One built on previous work with Grade 5 and 6 students that showed the benefit of 'hands on' experience with physical materials for the development of spatial reasoning (Knight, 2012). We also undertook a literature search of research specifically about students' engagement with nets.

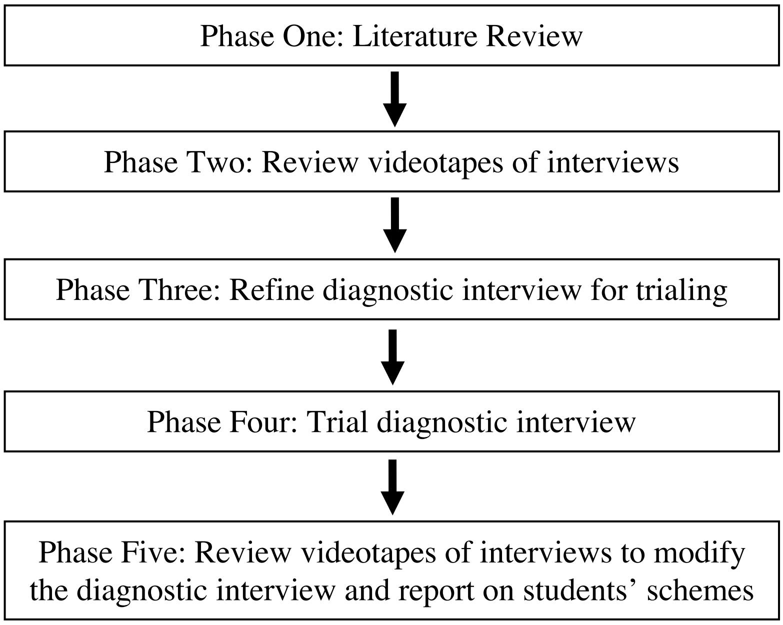

Figure 1. Research Action Plan

In Phase Two we used data collected as part of a classroom study in 2011 based in a Catholic primary school in inner Melbourne. All of the Grade Five students in one class attempted seven spatial reasoning paper tasks at the beginning of a teaching unit focused on representation of three-dimensional shapes and six tasks at the end of the unit. Nine of the students were interviewed about their responses on both occasions. One task in each interview was dedicated to students' anticipation of whether or not nets for a cube (initial) and a square-based pyramid (final) would fold to form the target solid (see Appendix 1). The interviews were videotaped and transcribed. Both researchers viewed the interviews independently at first noting the actions of students when anticipating the validity of nets and their explanations about how they determined if a given net would or would not work. Later we compared our observations and documented the findings in a research diary using Google Docs.

Using our observational notes we created a diagnostic interview with two options for the target solid, cube or square-based pyramid, and refined the interview through discussion and a small number of trials with colleagues and children. Phase Four, trialling on a larger scale, began in February, 2014.

Results

We firstly describe the data from the interviews of the nine Grade Five students. Their responses to the nets for a cube are shown in Table 1. The ticks and crosses in the first row represent whether the given net is correct or incorrect. In the body of the table ticks and crosses represent students' decision about whether or not the nets work. Consistent with previous research the prototypical T-shaped net (b) proved the simplest for students with slightly less success on the non-examples (a, d) and net e which is easily derived from the prototype. Nets (c) and (f) were the most difficult, both involved maximum lines of two or three squares and L-shaped composites of three squares. There was natural variation between students in both success rate and pattern of correct responses. The sample of nine students provided representative success rates on each net compared to that of the whole class (n=23); a(78%), b(100%), c(26%), d(96%), e(78%), f(43%).

Table 1
Results of Initial Pencil and Paper Identification of Nets for a Cube

Student/Net	a ✗	b✓	c✓	d✗	e✓	f✓	Correct
Allie	✗	✓	✗	✗	✓	✓	5
Byron	✗	✓	✓	✗	✓	✗	5
Cate	✗	✓	✗	✗	✗	✓	4
Diana	✗	✓	✗	✗	✓	✓	5
Elisa	✗	✓	✓	✗	✓	✓	6
Frank	✗	✓	✗	✗	✓	✗	4
Grant	✗	✓	✗	✗	✓	✓	5
Harry	✗	✓	✓	✓	✓	✗	4
Isabella	✓	✓	✗	✗	✓	✗	3
Total	8(89%)	9(100%)	3(33%)	8(89%)	8(89%)	5(55%)	41(76%)

The success rate for anticipating the correctness of nets for a square based pyramid was slightly higher (82%) than the rate for a cube (76%) (see Table 2). As anticipated, the prototypical star-shaped net (c) proved the easiest. However a non-example featuring a triangle composed of four triangular faces (e) was correctly rejected by all students. Another non-example that had overlapping shapes (a) and a rotationally symmetrical net (b) proved almost as easy as nets c and e to answer. Net (d) was predicted correctly by only three students. The success rates for the sample group again mirrored those of the whole class (n=22) for each net; a(95%), b(91%), c(100%), d(32%) and e(100%).

Table 2
Results of Final Pencil and Paper Identification of Nets for a Square Based Pyramid

Student/Net	a ✗	b✓	c✓	d✓	e✗	Correct
Allie	✗	✓	✓	✗	✗	4
Byron	✗	✓	✓	✓	✗	5
Cate	✗	✓	✓	✗	✗	4
Diana	✗	✓	✓	✗	✗	4
Elisa	✗	✓	✓	✗	✗	4
Frank	✗	✓	✓	✓	✗	5
Grant	✗	✓	✓	✗	✗	4
Harry	✗	✓	✓	✓	✗	5
Isabella	✓	✗	✓	✗	✗	2
Total	8(89%)	8(89%)	9(100%)	3(33%)	9(100%)	37(82%)

By viewing videos of the interviews of all students in the sample group at the beginning and end of the teaching unit we obtained data relevant to our main research question concerning the nature of their schemes for anticipating nets. Our focus was on the thought processes that students exhibited as revealed by their explanations. Firstly we discuss the data from the first interviews based on nets for a cube.

Consistent with scheme theory eight students described their thought processes as imaged actions commonly manifest by assigning one square as the base of the cube, folding squares up to form 'sides' and folding the last square to become the top face. Allie's explanation was typical:

> A: Well, I sort of imagine it in my head so I think that one's going to fold up that way, that one's going to go there, that one's going to go there, that one's going to be on that side and that one's going to be on the top (with each part that she mentions she makes folding motions with her hands).

A noticeable feature of students' explanations was their use of informal language and gesturing. For example, the word 'flip' referred to the double fold that created a side and top face commonly supported by hand movements representing the right angle of fold. The students pointed to particular shapes in the net and used pronouns like 'this' and 'that one'.

Isabella who was the least successful student with the cube nets relied on recognition of the prototype.

> I: Well, because that's whenever we make a cube it always is like that (points at the worksheet).

Her other two correct answers were derived from trust in the T-shaped prototype. She accepted net (e) as a minor variation of the prototype that would still make a cube and rejected net (a) because it needed one square on each side of the line of four squares to work.

All nine students applied criteria for impossibility of a net, in particular when they predicted overlapping faces or, often co-incidentally, missing faces. Cate's explanation of overlapping, supported by gesturing, was typical.

> C: This one because if this one...that would be one of the flaps on the side and this would be one at the top (showing the sides with her hands). And this would be one on that side but that would overlap on the top one (puts one hand on the other). So I don't think that one.

Not all tests of impossibility were correct, especially when the more difficult nets, (c) and (f), were involved. Diana who was successful with five of her six predictions considered net (c) which had a longest length of two squares.

> D: I don't think it does because that goes up (Pointing). And then sort of like a diff…weird shape. And I wouldn't…

Her explanation shows that she could not predict the direction of squares when an L shape of three squares was folded. In contrast, Byron provided a rare example of co-ordinating the L-shaped composite when considering net (f).

> B: Well that one goes up, folds out and makes a three sided shape like that one's up, standing up like that. And then that one's like that. And that one's on top (places his right hand on top of his left).

Byron saw the nets as composed of two identical composites that formed half of the cube. Inability to predict the outcome of folding composites of squares was also evident in students' assertions that particular squares would not be part of the target cube and be 'sticking out' of it.

The schemes students used for testing the nets for a square-based pyramid had similar characteristics to those exhibited in the initial interviews based on a cube. A subtle difference was that the assigning of a base was implicit in their descriptions given they naturally assumed the square in any net to be the base. Folding up and wrapping were common imaged movements supported by hand gestures. All nine students described the prototypical star-shaped net (c) as ‘easy’ since the triangles folded up to a vertex, though only Allie used that term. Seven students derived the validity of net (b) from the prototype by folding up the opposing triangles and wrapping the other two into place, noting the mapping of triangle sides to the corresponding edges in the base of the pyramid. Similar criteria for impossibility, commonly overlapping and missing triangles, were applied both appropriately and inappropriately in predicting non-examples. Net (a) was readily identified as impossible due to overlapping and net (d) was the most common incorrect non-example. Elisa rejected the net saying that the bottom triangle would be “upside down” if moved to the other side of the base.

All students’ explanations, except those of Isabella, were linked to movement. For example, Cate described net (a) as impossible since an identified triangle ‘can’t make it round’ to the other side of the base. Isabella continued her focus on her trusted prototype and accepted or rejected nets on the basis of similarity or difference from the star shaped. Hence she incorrectly accepted net (a), and rejected the other nets because there were “triangles in the wrong place.”

There was a single glimpse of use of composites of shapes in Harry’s consideration of net (d) as he described how the arrangement of four triangles made a tent-like structure that fitted onto the base.

H: That one goes there…it practically makes all the sides and you just flip it over.

There was also an example of inability to co-ordinate composites leading the student to think that a triangle would be excluded from the pyramid. In considering net (d) Grant declared that a triangle would be “stuck in the corner.”

The interviews also provided insight into ways to better elicit students thinking. In the final and trial interviews students were asked more about specific net to target solid mappings as a way to clarify their thinking or to provoke new considerations. For example, in the transcript below Allie was challenged to identify the triangles in an incorrect net that overlapped:

E: Yup. And this one with that one.

I: So which ones [triangles] would overlap in that last one?

E: This one. Or you’d fold up and then down and then...oh wait, there’d be an extra shape. Like just there.

The interviewer’s question prompted Allie to revisit the images of action in a more specific way that mapped features of the net, triangles, to features of the target solid, faces. Other mappings such as sides to edges and vertices to vertices were provoked in a similar way through specific questions.

Students rarely used mathematical language preferring to use pronouns such as ‘this’ and ‘that one’ and imprecise adverbs like ‘here’ to describe shapes, movements and locations. Lack of formal terminology did not influence whether or not they could determine which nets would form the solid. Most students used gestures as tools to think with and to illustrate the movements they imaged. In some interviews the camera shots excluded hands which made close examination of their gestures problematic. Making sense of students’ explanations and recording their strategies in real time was challenging for interviewers. Frequently students were unaware if they were correct or incorrect and had no ways to validate their answers.

Discussion

Our ultimate goal is to use the findings about students' schemes for nets to develop instructional approaches that will support improvement in those schemes. So the diagnostic interview is a means to that end. There is an established methodology for creating models of the 'mathematics of children' that inform effective teaching. Although our work is partially progressed we already see the potential benefit in predictive imaging of assigning a reference face (usually the base), folding, and wrapping actions naturally preferred by students in combination with developing their knowledge about composites of shapes. Asking reflective questions about how specific features in the net, such as shapes, points and sides, map to features of the target solid focuses students on properties rather than on holistic perceptions.

The value of the data obtained from an interview is obviously influenced by the nature of the task and the questions that are asked. We made several improvements to the draft interview through noticing students' behaviour during our trial phase. Ensuring that filming captures hand movements and that all shapes in the nets are labelled alphabetically supports the interviewer in making inferences about students' mental operations from their gestures and language. Providing manipulative models of the nets after their predictions confirms that students' explanations match their actual imaging as well as offering them an opportunity to learn. There are early indications that some nets are easier than others to predict generally and that pyramids may be marginally easier to work with than cubes. Difficulty seems a derivative of the number and complexity of transformations required which depends on the strategies students choose to use. Also of future interest is the extent to which strategies are generalizable or specific to the target solid.

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Appendix 1

Tick the nets that will make a cube/square based pyramid if they are folded up.