Affordances: Ten Years On

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Ten years ago the construct, affordance, was rising in prominence in scholarly literature. A proliferation of different uses and meanings was evident. Beginning with its origin in the work of Gibson, we traced its development and use in various scholarly fields. This paper revisits our original question with respect to its utility in mathematics education research. We explore accepted meaning(s), the clarity of operationalising these meanings within research, and how the construct is being used to move the field forward.

In 2004, we wrote a paper (Brown, Stillman, & Herbert, 2004) addressing the question: Can the notion of affordances be of use in the design of a technology enriched mathematics curriculum? In this we noted that the construct, affordance, was beginning to be increasingly used in the scholarly mathematics education literature and in technology within mathematics education in general. In that paper we explored various ideas as to what affordances were and specifically how they might be of use in mathematics education. The term was first brought to our attention when Professor Kaye Stacey proposed that affordances be one of three themes for the RITEMaths Project (an Australian Research Council funded linkage project—LP0453701). The 2004 paper arose from a project meeting on the meaning of the construct and its possible potential within a research project capitalising on what technology had to offer in enhancing mathematics learning. At the time, independently, Johnston-Wilder and Mason (2004) identified affordances as one of the "key constructs in mathematics education … that have proved fruitful in research and which have informed choices made by teachers" (p. i).

Now ten years on we look back to see how the ideas discussed in that paper have evolved and to ask if the construct is now accepted and well understood within mathematics education and as such what it might be contributing rather than being one of the many blooming flowers in mathematics education that Schoenfeld (2002) suggested might need pruning. In cognitive science (Caiani, 2013), general education (e.g., Dawson, 2010; Day & Lloyd, 2007) and ICT literature (Conole, 2013) affordances have certainly gained in prominence. Conole (2013), for example, undertook a similar quest to the one we wish to carry out within ICT and concluded "that the use of affordances as a means of describing the relationship between technologies and users and in particular resultant actions is useful" (p. 88) but is this the case in mathematics education?

We begin by posing the following questions:

- What is the accepted meaning(s) of affordances in mathematics education? Is there clarity in operationalising these meanings within mathematics education research?
- How is the construct being used in mathematics education? Is it moving the field forward?

Accepted Meanings of Affordances

At least three distinct definitions of the construct affordance can be found in the mathematics education literature. The Springer Reference Encyclopaedia of Mathematics

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Education (Lerman, 2014) refers the reader to Harry Heft's entry on affordances in the Encyclopaedia of Sciences and Religion for clarification, namely,

> The perceived functional significance of environmental features taken relative to an individual. For example, a surface at approximately knee high to an individual is perceived as affording sitting on. To perceive an object's affordances is to perceive its meaning relative to the actions of the self/another individual. Affordances point to a domain of values/properties that is neither "in" the environment, considered apart from an individual, nor "in" the mind, considered apart from the environment, but rather are relational properties of an environment-organism system. Formulated by James Gibson in his ecological approach to perception – action. (Heft, 2013, p. 32)

The first conception of affordance prevalent in mathematics education is in line with Gibson (1977). If an affordance exists, in order to avail oneself of the opportunity for interactivity, the actor must act (on or with the object). The precursor to acting is perceiving—without which the actor cannot act when the affordance is for teaching or learning. Whilst a user might chance upon an affordance bearer, for example by noticing the graphing calculator being used to enact function identify-ability via the linear regression capability also has quadratic regression capabilities, there are limited opportunities for this to occur when it comes to the affordance itself. This is due in part to the complex nature of mathematics and in part to none of the affordances of interest being trivial. Affordances for learning are seen as positive affordances. Whilst Gibson has often been cited (e.g., Vera & Simon, 1993, p. 11) as suggesting that affordances are always directly perceived, this is not the case. "It is true, some … specify their affordances directly … others … do not" (Gibson, 1977, p. 80). This misunderstanding may have resulted from Gibson's major focus on affordances related to locomotion, surfaces, substances and objects. Greeno (1994), for one, does not see "direct perception [of affordances] as a defining characteristic" (p. 341), but contrary to our previous paper, we now realise this is not a defining characteristic in Gibson's final work either.

Gibson used a particular linguistic form to express affordances, namely, verb-{preposition}-able where the curled brackets indicate the preposition is optional. In Gibson (1977) a surface is described as being stand-on-able or walk-on-able. Given particular features, the surface may be sit-on-able thus affording sitting-on or sit-ability. In the context of mathematics education, Brown (2013), for example, uses this linguistic form (e.g., function view-ability) when identifying affordances present in a technology rich teaching and learning environment when students are learning about functions. Others such as Hollebrands (2007) stop short of using the term herself when referring to drag-ability affordances of dynamic software environments but point out how Hoyles and Noss (1994) used the term "messable" to refer to what she calls "dragging test" as a form of dragging (i.e., dragging to see the properties of a geometrical construction will remain invariant).

Johnston-Wilder and Mason (2004) attribute the introduction of the construct in mathematics education to Greeno (1994). In contrast, we argued in Brown, Stillman, and Herbert (2004) that it was Pea (1993) in his chapter focusing on education design to facilitate the practices of distributed cognition that was widely read and cited that brought the idea of affordances into prominence within the mathematics education community. Pea argued at the time that research focussed on affordances was critical for better understanding how "one can get a learner to attend to pertinent properties of the environment … such that the learner can join in to contribute to distributed intelligence in activity" (pp. 51-52). In the same decade, Gaver (1991) and Norman's (1990) appropriation of the term in human-computer interaction (HCI) literature occurred (see Brown, Stillman, & Herbert, 2004). This literature, and that based on it, was read by those in mathematics education with a focus on technology use. Pea himself used ideas from Norman. Thus, a second conception sees an affordance as a property of an object.

Much of the subsequent view of an affordance as a property of the object, rather than as an opportunity for interactivity between object and actor, can be attributed to this. This use is understandable in Human-Computer Interaction (HCI) where the focus is often on the design of a technological artefact, although redefining a construct is unhelpful to the broader community, resulting in ineffective communication within and across fields. However, in mathematics education, if the focus is on technology use (e.g., linking geometry and algebra using Geogebra, Hohenwarter & Jones, 2007) or not, this reducing of an affordance to something inherent in an object (i.e., a property or capability of the object) is not particularly useful. This difference is apparent when researchers use the term 'affordances of the technology' where technology can be replaced by something more specific such as spreadsheet; as opposed to the work of Brown (2013) for example who refers to the affordances of the environment (in her case, a technology rich teaching and learning environment—TRTLE) where the relationship between the object and actor or technological artefact and learner is explicit, that is both are part of the environment and must interact for an affordance to be enacted. Sedig and Sumner (2006) follow the typical HCI interpretation of affordance as "the way an onscreen object … advertises its usage cues – i.e., what sorts of operations can be performed on it" (p. 6) when they present a categorisation and characterisation of computer-based interactions by which learners can explore and investigate visual mathematical representations such as graphs and diagrams. Similarly, when authors refer to the affordances of a mathematical task per se whether these be mathematical or pedagogical (e.g., Liljedahl, Chernoff, & Zazkis, 2007), Gibson's notion of interactivity is lost and the affordance apparently lies in the task alone, irrespective of the learner(s) interacting with the task or the contextual factors of the environment where that learning is meant to take place.

The third conception of affordances arises from Vera and Simon (1993), ardent information theorists, who like Norman redefined Gibson's construct for their own purposes. In doing so, they rejected Gibson's idea that affordances are relationships or opportunities and see affordances as mental symbols, in line with their view that all cognition is symbolic. Affordances reside "in the head" of a person as "simple mappings between our functional models of what is out there" in the external world "and our functional actions" (p. 21).

In 1994, Greeno acknowledging the work of Gibson and also Vera and Simon (1993), writes of affordances in the context of situation theory as a means of extending the utility of Gibson's construct. Greeno agrees that agent-system interaction involve conditions arising from properties of both the agent and the system. To Greeno, "affordance refers to whatever it is about the environment that contributes to the kind of interaction that occurs" (p. 338) but he then introduces "abilities" to refer to "whatever it is about the agent that contributes to the kind of interaction that occurs" (p. 338). Greeno then adds the notion of constraints from situation theory (i.e., regularities involving classes of situations with objects having a specified relational property (Greeno, 1994)) to provide a broader explanation of activity in terms of agent-system interactions. In doing so, he points out that in situation theory "attunement to constraints" is a way of analysing and thinking about expertise; that is, knowing how to do something in a skilled manner.

Several other so-called extensions of the affordance construct (e.g., potential affordances) have appeared in the mathematics education literature we have surveyed but these often result from re-inventions of ideas already put forward and explored much earlier in other literature than their authors claim. Others are recent extensions (e.g., cultural and ergonomic affordances, see Monohan and Mason, 2012) that, although generating some discussion in mathematics education research, have not had the time to be fully evaluated. Similarly, extensions arising in other fields (e.g., see Caiani, 2013) have not had the time to transfer let alone become established, if indeed they ever do so.

Images for Theory Use

Simon (2009) noted there are an increasing number of theories of learning in mathematics education and whilst this brings challenges it should be seen in a positive light. Importantly, "each theory brings with it basic assumptions about the nature of learning, the phenomena of interest, constructs that can be used in the conception and implementation of research, and the types of explanations that can be generated" (p. 477). He also notes that contrary to the often assumed view that a "new theory supersedes its antecedent" and, in fact, "the pre-existing theories continue to do important work" (p. 479). Each theory affords its own set of possibilities and mathematics education research is richer as a result. In addition to arguing for the benefits of multiple theories, Simon (2009) proposes two images for the functions of theories: tools and lens.

Theories as Tools

According to Simon (2009), the notion of theory functioning as a tool relates to the idea that both a theory and a tool offer advantages in undertaking certain work but neither are universal. "Every tool [and every theory] offers the possibility of doing certain kinds of work well (when used optimally) and being less helpful for other kinds of work" (p. 482). Either can have uses beyond those originally intended but neither will always be useful to do all the kinds of work needed. Bill (2012), for example, used "affordances and constraints" (p. 67) as a theoretical tool to introduce and effectively use Fathom statistical software (Finzer, 2006) in high school attributing the idea in part to Brown et al. (2004) but his interpretation seems to miss the point made in that paper that "constraints" are "the structure for action" in keeping with Kennewell (2000), not some form of opposite to affordances as he writes of affordances as "potential for action" and constraints as "obstacles [that] inhibit the potential for action" (p. 67).

When considering theories as functioning tools we can ask, are affordances the tool of choice in designing a technology enriched mathematics curriculum? A critical aspect of having a good tool is to know when to use it; that is, to know its affordances and limitations! Hollebrands (2007), for example, mused that "understanding how students make use of different affordances of interactive geometry software programs … provides insights into what students do with different features … [but] it does not provide a complete picture of how students use and interpret dynamic sketches" (p. 169) nor should it.

Theories as Lenses

In contrast, a theory can be viewed as functioning as a lens. When looking "at a situation through a particular theoretical lens, some phenomena are prominent, whereas others are not" (Simon, 2009, p. 482). The lens directs our attention to particular aspects and equally ignores other features or phenomena. These are not denied or of less importance, they are simply not what is being attended to. Different lenses show different realities. The choice of lens necessarily focuses our questions and "the types of phenomena that are researched" (p. 482).

Björklund Boistrup (2012), for example, by considering affordances "as a quality of an object, or an environment, that allows an individual to perform an action," uses "affordances for students' active agency and learning in the mathematics classroom" (p. 2) as a lens on the assessment discourses in five Swedish Year 4 mathematics classrooms. The focussing nature of this lens is evident in all four research questions posed for the study, for example, "What are the focuses of the assessment acts in the mathematics classroom and what affordances can be connected to students' learning?" (p. 6).

Use of Affordances in Mathematics Education Research

From conducting a recent review of research and scholarly literature in mathematics education, it would appear that use of the construct, affordance, is far more frequently an incidental use than a theoretical one with many of the papers (e.g., Kong, 2008) that resulted from a Google Scholar search using "affordance + mathematics education", containing one or two instances of forms of the verb "afford" or the noun "affordance(s)". Still other papers (e.g., Beatty & Moss, 2006) mention affordances in the title and possibly the abstract but nowhere else! As our interest is primarily in the utility of the construct in advancing the field of mathematics education, we now use Simon's images of the functions of theories to examine exemplar studies to gain an insight into how the construct with its various meanings is being used in, and contributing to, mathematics education research.

Several writers (e.g., Chick & Pierce, 2008; Liljedahl, Chernoff, & Zazkis, 2007; Watson, 2007) use affordances as an analytical tool in researching task design and implementation in lessons to identify "the opportunities that are inherent in a task, lesson, or example" (Chick & Pierce, 2008, p. 2). We will examine a study by Watson (2007) as an example. Watson argues that "focusing on affordances can be a powerful method for analysing how some teaching might be differently effective than some other teaching, and how learning can be understood by examining ways in which learners might participate in what is available in the learning environment" (p. 111). From this statement, it appears that she is following the Gibsonian notion of affordances being "relational properties of an environment-organism system" as pointed out by Heft (2013, p. 32). She continues by pointing out "learning is [not] predictable and determined by teaching" but rather "it offers insight into the possibilities for action in a situation" and these actions can be mediated by the teacher or "spontaneous and unexpected" (Watson, 2007, p. 112).

Watson illustrates her interpretation of affordances by describing a task where a rectangle is modified by cutting and pasting to create new rectangles and the area and perimeter of these compared to that of the original. She describes the choices a learner has in creating new rectangles as being constrained by the cutting and parting relationship. Following this approach, Watson sets out to identify "the mathematical affordances of lessons" (2007. p. 118). Her analysis of one lesson resulted in a description of:

> the sequence of activities afforded [as] association of ideas, use of prior knowledge, exemplification, comparison, identifying relationships, new definitions, defining terms, copying, doing numerical examples, informal induction, formalising, creating objects with one feature, being offered objects with multiple features, classifying, explication, applying to other contexts. (p. 122)

Watson explains that the order in which elements occur in a lesson "influence what is afforded" (2007, p. 123). For example, a lesson that begins with a discussion of mathematical terminology offers something quite different to learners from a lesson that concludes with this activity. Clearly, Watson is specifying affordances as being particular mathematical activity; that is, somehow stimulated by the use of a particular teaching element, including the task. Her focus is very much on what is offered, not on what offerings are taken up, although she argues, “unless we can legitimately expect most students in a lesson to respond in hoped-for ways there is little point in teaching” (p. 123). This seems more in keeping with the Greeno (1994) interpretation of affordances.

Watson has her focus on learning mathematics and how this is influenced by task or lesson design. The direction is very much from the teacher perspective despite her caveat, mentioned above, that the teacher presents the opportunities. There is no doubt that she sees affordances as useful in achieving her research purpose and the ultimate production of an instrument that allows the identification of how mathematical activity that is able to be controlled by the teacher is afforded, that is, what Watson (2007) calls “the mathematical affordances of lessons” (p. 18). We concur with Monaghan and Mason (2012) that this is a “legitimate application of the construct” (p. 132) but see that the influence of Greeno and “attunements” has brought a deliberate narrowing of the light passing through her potential theoretical lens to highlight the activity of one particular actor in the environment.

Considering the influences of Pea and the followers of Norman, it is not surprising that the affordances construct to date has received most attention in mathematics education research that has a technology focus. Brown (2013), for example, used the Gibsonian theory of affordances as a lens in her work. The teachers and students in her study were in a technology-rich teaching and learning environment (TRTLE). Her focus was on how the actors, the teacher and students, interacted with objects in the environment, in particular technological devices, to enact, or not, opportunities for interaction in their study of mathematical functions. This is exactly what Gibson was interested in, albeit in a different environment. Many opportunities exist in a given environment, by focusing through the lens of affordances, and Brown focussed her attention on what was helping or hindering learners and teachers to maximise learning opportunities.

The research questions of her study all focus on affordances of TRTLE’s ranging from what affordances of the TRTLE’s were perceived by the teachers as useful in developing students’ understanding of functions, how the teachers acted to allow students to perceive these, and what features of the TRTLEs allowed students to perceive particular affordances for learning and enact these to develop their understanding of functions. Brown (2013) described in detail what affordances exist, how these may support learning if perceived and enacted during task solving (in general class situations or independent work) or how they hinder learning either by not being perceived or unsuccessful enactment.

Brown identified sixteen affordances across three focus TRTLE’s (two including Year 11 students and one with Year 9 students) as being perceived as useful in developing students understanding of functions. Most affordances were manifest in a variety of ways for different purposes within the broad purpose of the particular affordance. For example, function view-ability could be manifest via global function viewing or local function viewing. Brown found that the emphasis and extent of the particular manifestations varied across the TRTLE’s. Furthermore she identified seven teaching roles, each with different intentions, taken on by the teachers at various times. These roles tended to be deliberately planned and included those diagnosing student misunderstandings to those intended to enable enactment of a particular affordance. In addition, in-the-moment teachers used seven teacher management tactics in managing student enactment of affordances. Subsequent development of a grounded theory of student management of enactment of affordances identified strategic decision-making occurring through enactment management tactics by students.

Discussion and Conclusion

Affordances can be used theoretically as both a tool and a lens (Simon, 2009) by researchers to gain various insights into interactivity between agents with other agents and physical systems. This is clearly useful in classroom research whether the focus be on assessment discourse, lesson implementation, or technology use, to name but a few examples. Within the context of mathematics education research into digital technology use, for example, we concur with the sentiments of Hoyles and Lagrange (2009) that "how far [these] studies have taken on board the challenges of the use of digital technologies and their potential for the improvement of mathematics teaching, learning and the curriculum, remains a matter of debate" (p. 2). When the theory of affordances is used in this research we propose that this is in part due to the limited uptake of the conception of affordances as opportunities for interactivity and its use as a theoretical lens to capture *all* the interactivity. Use of the conception based on Gibson, which requires careful reading of Gibson (and more than one of his many papers) on the construct, appears to allow researchers to undertake a more detailed examination of what is occurring. The subsequent micro-analysis positions researchers to make specific recommendations regarding teaching and / or learning in technology rich environments. In turn this has potential for teachers to make changes to teaching practices thus, in turn, increasing opportunities for improving learning of mathematics.

Although the construct, affordance, was one of "a thousand flowers [let] bloom" by the end of the 20th century in mathematics education (Schoenfeld, 2002, p. 443), when used in a deliberate rather than an incidental manner, it can contribute to our understanding of the various complexities of mathematics teaching and learning in classrooms by its use both as a tool to achieve some clearly defined purpose or as a theoretical lens to focus our attention on particular phenomena and particular aspects of those phenomena. This is evident in the sample of studies examined in this paper. This is not to say that its use in some scholarly work does not suffer from looseness of definition or slippage between meanings when operationalised. However, as researchers in one field are made aware of the evolution of the meanings of such constructs and problematics posed by their indifferent use, the field has the opportunity to be reminded of how well documented our research trail needs to be so as not to add to confusion, but rather consolidation, of a construct appropriated from another field as in this instance, namely the field of perceptual psychology.

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