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EXPLORATION INTO CULTURAL ASPECTS OF MATHEMATICAL GENERALISATION: YOUNG AUSTRALIAN INDIGENOUS STUDENTS

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This paper reports on the preliminary findings from the researchers' thesis exploring how young Australian Indigenous students engage in mathematical generalisation tasks. Six students (8.5 years) were purposely selected to participate in one on one Piagetian clinical interview to explore how they engage and express generalisations with growing patterns. Initial findings of the study suggest that these young students can generalise contextual growing patterns. Representing the patterns with concrete materials and kinaesthetically engaging with tasks enhanced their ability to reach generalisations. In addition, using patterns where the relationship between the two variables was visually explicit assisted students to express this relationship in general terms. Interestingly, cultural gestures (both verbal and non-verbal) were apparent throughout the interviews. These gave insight into how Indigenous students engage in one-on-one settings in contrast to whole class interactions.
Keywords: Generalisation, Primary mathematics, Indigenous students.

INTRODUCTION

In Australian society there is a belief that Indigenous students cannot achieve strong educational outcomes. National and international reports on test results indicate that Indigenous students are achieving below benchmarks in both mathematics and literacy (MCEETYA, 2009). This may cause teachers' to have low expectations of students' abilities, thus influencing the type of mathematics being presented in the classroom. Often the type of mathematical activities, and the way in which the mathematics is presented, reflects this perspective. In Indigenous contexts, teachers have often provided lower contextual lessons based on skill and drill limiting students access to higher order mathematics (Baturo, Cooper, Michaelson, & Stevenson, 2008; Jorgensen, Grootenboer, Niesche, & Lerman, 2010). Additionally, studies have highlighted that educators have little faith in Indigenous students' mathematical ability (Matthews, Watego, Cooper & Baturo, 2005). There is a developing prospective statement that, in formal education settings with Indigenous students, little algebraic thinking is being developed. Subsequently, there is rarely opportunity for these students to engage in higher order mathematical thinking, which is needed for later school achievement.

The derivation of this is that teachers present mathematical experiences with limited opportunities for Indigenous students to explore acts of mathematical generalisation. Generalisation is a central construct in mathematics and hence forms the basis for our exploration. This exploratory study aims to extend upon the limited research available regarding how Indigenous students generalise mathematics. Also the objective is to provide insight into tasks that assist students to move through the stages of mathematical generalisation. In particular it aims to share the processes that assisted these young students to reach generalisation.

LITERATURE

Visual growing patterns are predominately the initial experience students encounter when introduced to formal algebra (Warren & Cooper, 2008). Growing patterns are characterised by the relationship between elements which increase or decrease by a constant difference. Students in the early years experience growing patterns through activities such as coping, continuing, and extending patterns. Eventually, there is a need for the student to see the relationship between the pattern and their position (stage), which can be termed a generalisation. The ability to generalise a growing pattern enables students to predict the pattern beyond terms that are provided (i.e., 10th position, 25th position, 100th position, nth position). As students engage in this function, they begin to explore the concept of co-variation as they reconsider growing patterns as functions, rather than recessive terms. This often involves students generating a visual representation (drawing) usually involving a mathematical abstraction, this is then recorded in a table, and the relationship is identified between the two data sets (Warren & Cooper, 2008). Research has shown how this often leads to recursive thinking and the relationship between the growing pattern and the term is not identified.

The ability to generalise one's learning beyond the initial experience is said to be the essence of mathematics (Cooper & Warren, 2008; Kaput, 1999; Mitchelmore & White, 2000). Consequently, it can be understood why generalisation has been described as the 'heartbeat' of mathematical thinking (Mason, 1996). Thus, generalisation is an imperative skill for success in mathematics, enhancing capability in the application of mathematical concepts across tasks and particularly for achieving higher levels of mathematics. Commonly, educational researchers have conducted studies in secondary and tertiary environments to investigate students' ability to generalise mathematics (Carpenter & Franke, 2001; English & Warren, 1995; Lee, 1996). More recently, studies have begun to explore how younger students generalise mathematic constructs in early algebraic contexts.

Current researchers have demonstrated that young students are capable of generalising mathematical structure across a range of contexts (Carraher, Schliemann, Brizuela & Ernest, 2006; Cooper & Warren, 2008). These contexts include generalising relationships between numbers and pattern rules, and generalising from particular examples in real-world situations to abstract representations (Cooper & Warren, 2008; Warren & Cooper, 2009). Researchers have identified there are different ways to generalise (Lannin, 2005; Radford, 2010). Lannin (2005) distinguishes between two types of generalisation; recursive and explicit. Recursive generalisation involves the use of a single variant, while explicit generalisation involves covariant thinking. More recently, research in the area of generalisation has focused on layers of generality to identify stages of student progress as they engage with generalisation tasks (Radford, 2010).

Factual, contextual, and symbolic generalisation are layers of generality distinguished by Radford (2010). Factual generality is an elementary level of generalisation where students engage predominately in gestures, words and perceptual activities. Within this level students would attend to the pattern presented and would not move into quasi-generalisations. Quasi-generalisation can be defined as a generalisation of a large number or position beyond the presented activity (Cooper & Warren, 2008). Contextual generalisation requires students to reduce the signs (semiotic contraction) for greater expression of meaning, moving onto quasi-generalisations. Finally, the symbolic level requires a further semiotic contraction where students replace words with symbols such as letters to express the generality of the rule. While there is agreement in the scholarly community that students progress through different stages during the generalisation process, however, how students generalise and the processes which assist learning, is still unknown.

Within the context of Australian Indigenous students, studies in mathematical generalisation are limited. A single study (Matthews, Cooper & Baturo, 2007) conducted in 2007 alluded to the subject of generalisation with regard to Australian Indigenous students and stated 'algebra is based on generalising pattern and structure, skills with which Indigenous students may have an affinity because their culture contains components that are pattern-based and which may lead to strong abilities to see patterns and structure' (p. 250). Thus, it appears essential to begin the exploration of generalisation using patterns with which Indigenous students are familiar. It is also evident that the use of concrete materials is needed for students to make the connections to generalising a pattern or predicting further pattern structures (Papic, 2007).

THEORETICAL FRAMEWORK

The theoretical frameworks underpinning this study are semiotics and Indigenous research perspectives. In adopting the theoretical perspective of semiotics, the researcher attends to the signs, gestures or symbols that assist students to generalise mathematics. In conjunction with semiotics, enacting the secondary theoretical perspective of Indigenous research perspectives, places a focus upon building relationships with students in order to facilitate the learning. It is essential to build a trusting environment for the researcher and students to share knowledge, particularly in the light of cultural variances. Acknowledging Indigenous research perspectives will frame the research with an emphasis on empowering the participants, and thus facilitating the free transfer of knowledge and reducing the likelihood of the students being inhibited by the presence of the researcher.

Semiotics

The learning of mathematics is two-fold; it involves the interpretation of signs, and the construction of mathematical meanings through communication with others (Saenz-Ludlow, 2007). These knowledges do not present immediately rather they evolve from interrelated experiences. These have been termed semiotic systems (Saenz-Ludlow, 2007) Semiotic nodes are defined as those 'pieces of the students' semiotic activity where action, gesture and words work together to achieve knowledge objectification' (Radford, 2006, p 144). Other researchers used terms, namely semiotic bundling, to describe sign as any intentional action such as speaking, writing, drawing, gesticulating, handling and use of artefacts (Arzarello, 2006). The theoretical perspective of semiotics will be utilised as a lens within this study to interpret the interactions between teacher and students, and between students and context. Semiotics is also used to assist in the selection of the types of materials used to represent growing patterns and how they are used in the interview context. The researcher will be looking at gesture, signs and symbols in isolation and then in retrospect to see if there is a relationship between these semiotic resources and the ability to generalise, and if these signs are culturally bound.

Indigenous research perspectives

A decolonized approach has been adopted for this study with a focus on valuing, reclaiming, and having a foreground for Indigenous voices (Denzin & Lincoln, 2008). By using an Indigenous research perspective, it is essential to create a space for dialogue, rather than simply closed observation. This is not to say that through observation information cannot be learnt. More so, when observing students within a particular Indigenous culture, there are practices that may not be overtly apparent to the researcher, hence, the importance of including an open dialogue with the students. For this particular study, the relationship also needs to be cultivated with Indigenous Education Officers (IEOs) to assist with knowledge that may not be explicitly recognisable by the researcher. In effect, this brings the researcher and the participants into a shared space. At the cultural interface, the researcher is conscious of building relationships so that students, IEOs and the researcher can partake in a meaningful exchange. This created, shared space is where empowerment can occur (Denzin & Lincoln, 2008). The implication of this decolonised approach dictates that the study must be viewed within the bounds of the individual community in which the research takes place and not generalised to the broader Indigenous population.

METHODOLOGY

Participants

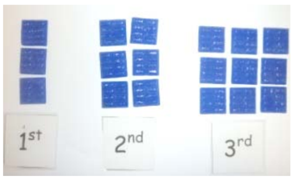
The students were from an Indigenous College situated in an urban town setting, in Queensland, Australia. Six Year 2/3 students (average age 8.5 years) participated in a Piagetian clinical interview. The researcher selected students who engaged in classroom discussions or presented interesting insight into the tasks during the week preceding the interview. Additionally, there were three categories in which the students were selected, (a) students who are experiencing difficulties in reaching generalisation, (b) those who have grown in their ability to generalise from the teaching episodes, and (c) those who can generalise. Each group had two students, one male and one female.

The interview tasks

The methodological approach taken for the study was that of design experiment (Cobb, Confrey, Lehrer, & Schauble, 2010; Steffe & Thompson, 2000). The design experiment consisted of three teaching episodes; each episode included three 45minute lessons on growing patterns. The teaching episodes took place over a six month period and they focused on opportunities for the students to draw on their own contextual knowledge to explore growing patterns for the first time. It became clear that the students responded positively when the artefacts utilised in the learning activities were related to the students’ local environment. Students responded positively to concrete artefacts used in the lessons and gave an opportunity to engage with the task kinaesthetically. Concrete materials were selected that could clearly show both variables of a growing pattern (position and pattern). After each teaching episode the six students participated in a one on one interview. The data presented in this paper focuses on the results of the first one on one interview after teaching episode one.

The interview consisted of an initial discussion about the students' understanding of a growing pattern. Two growing pattern tasks were presented to the students. First, the researcher created a growing pattern and participating students were asked to: (a) continue pattern with materials attending to the structure of the pattern, (b) predict the next position of the pattern, (c) predict the quasi-variable position, (d) identify the rule, and (e) generalise using alphanumeric notation. A second growing pattern task following the same process was then presented. The first task was a growing pattern using small plastic crocodiles. Students were asked to examine the relationship between the number of tails and the number of crocodile feet (see figure 1). In the second task the students were asked to explore the relationship between the class year level (e.g. 1st grade – represented on number cards) and the number of desks (represented by the blue tiles), see figure 1.

Task 1



Task 2

Figure 1. Pattern task 1 and 2 of initial clinical interview

Piagetian clinical interviews

Interviews were approximately 20 minutes in length. These were video recorded where both the students' gesture and the researcher's gestures were captured. The questions posed and subsequent actions were contingent on the responses given by the student. The interviews mirrored the dimensions associated with Piagetian Clinical interviews, namely, endeavouring to avoid leading the student in a particular direction, but at the same time making the most of the opportunities to formulate and test hypotheses about students' understanding.

DATA ANALYSIS

The data were analysed in a four-fold process. Firstly, the initial video footage was transcribed to capture students' verbal responses and for noting emerging themes. Secondly, the evolving data were reanalysed focusing on semiotic bundles (signs, gestures, language). Of particular note, were the students' physical gestures including the manipulation of the concrete objects and their body language. Thirdly, following this analysis the video footage was reviewed with the two Indigenous education officers, an Aboriginal woman and a Torres Strait Island woman. The Indigenous education officers watched the interview and provided feedback about the cultural signs that were displayed within the video. Their input was recorded and then transcribed to match the identified gestures and cultural signs used by the students. Fourthly, given this feedback with regard to cultural signs, the videos were reanalysed with an emphasis placed on students' physical gestures including their manipulation of the concrete objects and their body language. This process was repeated for all six student video recorded interviews.

RESULTS

Interview results from Task 1 and Task 2

For reporting purposes each student was allocated a code, namely, S1, S2, S3, S4, S5, and S6. All six students were asked to copy, extend, predict the next position, predict the quasi-variable, identify the rule, and generalise two growing pattern tasks. All students successfully copied and extended, and predicted the next position for both growing patterns. Students were then asked to predict beyond the pattern given using a quasi-variable (i.e. 25^{th} position, 100^{th} position). When asked to predict the quasi-variable most students' mathematical knowledge was limited to provide an exact answer, however they were able to provide but were able to identify how you would construct the pattern. This notion is demonstrated in an interview excerpt below.

Researcher (R): What if I had year 100? What would you have to do?

Student 3: (S3): Make 100 groups of 3. *[Student gestures the lines of three beside the example given]*

R: Do you know what 100 groups of 3 are?

S3: No

Therefore, for Task 1 (crocodile feet and tails) three students (S1, S4, S5) were able to quasi-generalise the growing pattern and for Task 2 (class year level and desks), five students could predict the quasi-variable (S1, S2, S3, S4, S6).

Students were then asked to if they could identify the rule for each growing pattern and then generalise the pattern rule. Below are examples from interviews of students demonstrating their answer the two questions. They have been categorised under Radford's layers of generality (factual, contextual, and symbolic).

Example of factual generalisation

R: What if I had two crocodile tails how many feet?

S3: 8 *[Student is nodding head and looking at crocodiles]*

R: How did you work that out?

S3: I counted in fours.

R: So if I have 12 crocodile feet how many tails would I have?

S3: You'd be having 3. *[Counts to twelve nodding head and then uses fingers to count tails]*.

Example of contextual generalisation

R: So if I have 100 tails what do I do to it?

S1: Times four

R: What if I had a million tails what would I do?

S1: Times 4 *[Student places hand over the crocodile and moves it along to demonstrate making new groups of 4]*

R: So what if I had ten times four. Do you know what ten times four is?

S1: Forty

R: It is so ten times four is forty and what is forty? What part of the crocodile is it? Tails or feet?

S1: Feet

R: So what do you think the rule is?

S1: Times four of whatever crocodile feet *[Student takes a long time to answer this and uses a lot of gesture with their explanation]*

Example of symbolic generalisation

R: What do you think my rule is for this pattern?

S2: Rows of three. *[Student gestures lines of three by moving their hand from the bottom of the pattern to the top using three fingers]*

R: What do I have to do for any grade? If I know the grade number what do I have to do?

S2: Rows of threes. *[Student repeats above gesture]*

R: What if I had 'n' grades? What would my rule be?

S2: n rows of three. *[Student repeats above gesture]*

Discussion with Indigenous Education Officers

A discussion followed at the end of the six interviews with the Indigenous Education Officers (IEO). Both the researcher and the IEOs watched the video recording of the interviews and interactions of the students. Themes that emerged from this discussion were: (a) Students could identify patterns easier when they were using contextual concrete items; (b) students gesture often when discussing the mathematics as they may not have the 'western mathematical language' to explain the concept; and (c) cultural factors contribute to communication in the interview. These included eye contact, shame and changes of manner from a classroom setting to a one-on-one setting.

DISCUSSION OF PRELIMINARY FINDINGS

From the analysis of the first set of interviews it became clear that there are four tentative findings regarding how young Australian Indigenous students appear to engage with the process of generalisation, and what artefacts or aides assist students to express mathematical generalities.

Principally, it was evident that young Indigenous students could generalise simple growing patterns. Using Radford's (2010) layers of generality the students could engage in factual, contextual and symbolic generalisation. Attending to the structure of the pattern in the early stage of the interview, assisted the students to generalise. This aligns with past research that suggests that attending to the structure or grasping the common features of the pattern (Radford, 2010) or noticing the particular in the general (Mason, 1996) proved to assist students to generalise the growing pattern to the nth position. Importantly, it is apparent that these students can engage in generalisations common to higher ordered mathematical thinking, yet this is not reflected within Indigenous students' national testing results.

Second, evidence emerged that concrete representation of the growing pattern assisted the students to generalise. The patterns presented to the students had contextual meaning and were drawn on from classroom observations and mathematical tasks. From a semiotic perspective, the patterns were set up so both variables (position and pattern) could be easily identified by the students allowing them to clearly attend to the signs within the task. In the first task the student were kinaesthetically engaging with a growing pattern using small plastic crocodiles. Students were asked if there were four crocodile tails how many crocodile feet would you have? The variables in this pattern were the tails (position) and feet (pattern). The concrete items held contextual meaning for the students and allowed them to kinaesthetically engage with the pattern when creating positions beyond what was given in the interview. Additionally, the students engaged simultaneously with the concrete items when explaining the generalisation. This use of gesture was more prevalent when the students did not have the language to verbally explain the generalisation of the growing pattern.

Third, mathematical non-verbal cues were evident while students were identifying the rule and expressing generalities. As the mathematical language became less attainable for the students they used gesture to express the generality. The drop in use of language aligns with studies suggesting that Western mathematical language creates barriers and difficulties for Indigenous students (Jorgensen, 2011). This was particularly evident with one of the students who would express his generality such as 'for the nth class it would be n of (gesture) three'. This student could not access the words 'groups of three' or 'multiplied by three' and would gesture the group of three by running his finger along the desk beside the group of three on a concrete representation. This student often mirrored the gestures that were used by the researcher during the discussions leading up to the generalisation task. In this particular instance the interplay of gesture between the student and the researcher played an observable role and impacted on the student's engagement with the generalisation tasks.

Fourth, during the analysis conducted with the Indigenous education officers it became evident that there were particular cultural verbal and non-verbal cues displayed by the students. Eye contact, shame, and change in manner of participation were three preliminary findings. Three students had limited eye contact with the researcher during the interview. Eye contact was only made when seeking confirmation that the answer given was correct. The IEOs suggested that this was the student's way of showing respect between themselves and the researcher. In contrast, three students held strong eye contact throughout the interview. Additionally, the IEOs identified moments of which the students showed shame. It is important to note that shame for Indigenous Australians can be compared to emotions such as embarrassment or shyness in westernised cultures. Shame was identified when the students might smile and turn their head or body while answering a question or in some cases when they were praised for answering questions. At other times students would become silent and require reassurance. Finally, social mannerisms changed for many of the students during the interview. A particularly interesting case was that of a girl, a high achiever in mathematics and very quiet in class. Her participation level in mathematics lessons during the teaching episodes was individualistic and she rarely answered questions. However, during the interview the IEOs were surprised how her mannerisms changed and the student was enthusiastic, loud and engaging without worry of getting the answer incorrect. At the initial stage of data collection it is difficult to determine if there are any consistencies or patterns in these cultural verbal and non-verbal views. This data collection was not anticipated initially, however has proved to be a valuable insight into the interactions students have in a one-on-one setting.

In conclusion, this is the initial stage of the researchers' thesis and further analysis of the subsequent two teaching episodes and interviews will assist in reaching more definitive conclusions. From initial analysis it is evident that the students were capable of reaching generalisation and progressed through the stages of generalisation as suggested by Radford (2010) once they understood the tasks presented to them. While this study provided insight into how young Indigenous students engaged in generalisation tasks, how they reach generalisation is still a pertinent area for analysis.

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