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ORIGINAL PAPER

The reciprocal relationship among Chinese senior secondary students' intrinsic and extrinsic motivation and cognitive engagement in learning mathematics: a three-wave longitudinal study

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Abstract
In the present longitudinal study, cross-lagged path models were applied to investigate the potential reciprocal relationships between senior secondary school students' motivation and their cognitive engagement, using data from 623 Chinese senior secondary school students across 2 years. The 623 students completed self-reported measures of motivation and engagement at three time points within 2 years. The results suggest that the participants held a mixed type of intrinsic and extrinsic motivation to learn mathematics and did not hold a deep level of cognitive engagement in mathematics learning. Compared with their extrinisic motivation, their intrinsic motivation to learn mathematics was more closely related to their cognitive engagement in mathematics learning, which points to a stronger reciprocal effect between their cognitive engagement and intrinsic motivation. The findings suggest that societal and cultural factors, such as the strong examination culture and high external expectations might be influential factors affecting the reciprocal relationships among students' motivation and cognitive engagement.

Keywords Intrinsic motivation · Extrinsic motivation · Cognitive engagement · Mathematics achievement · Senior secondary school student

1 Introduction

In the last decade, interest in the investigation of affective dimensions of mathematical learning and the connections between students' cognitive and affective learning outcomes has increased rapidly in the field of mathematics education (Hannula et al., 2019; Schukajlow et al., 2017). Affective traits in mathematics learning have been widely recognized and commonly accepted as critical factors positively influencing students' mathematics learning outcomes (Fredricks et al., 2004; Taylor et al., 2014). Among all the affective traits, students' motivation to learn mathematics has been the one that has been most widely investigated. Empirical studies conducted in various contexts have consistently suggested that students' motivation to learn mathematics can positively affect their mathematics achievement (e.g., Chiu & Xihua, 2008; Zhu & Leung, 2010).

However, relatively speaking, less is known so far about the relationship between students' motivation to learn mathematics and other affective variables for learning mathematics (Schukajlow et al., 2017). Moreover, researchers have argued that an action component such as engagement is required to mediate the effect of motivation on students' performance and achievement (Skinner et al., 2009). It is assumed that motivation will mainly underpin students' engagement in learning activities (Martin, 2012; Reeve, 2012). Therefore, the relationship between students' motivation to learn mathematics and their engagement in mathematics learning needs to be investigated specifically for a deeper understanding of its effect on students' learning.

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In recent years, the investigation of student engagement has attracted considerable attention in the field of mathematics education as it has been accepted as a "precursor" for students' mathematics achievement (Watt & Goos, 2017, p. 134). Theoretically, motivation and engagement have been regarded as two "separate, but related constructs" which will jointly influence students' learning outcomes (Reschly & Christenson, 2012, p. 14). Generally speaking, motivation has been defined theoretically as students' intention or willingness to act, and engagement on the other hand has been accepted as students' actual involvement (Gettinger & Walter, 2012). Moreover, it is believed that changes in student engagement will in turn change student motivation (Reeve & Lee, 2014). Researchers in previous empirical studies have found that students' motivation and engagement can predict one another (e.g., Reeve & Lee, 2014)—that is, the relationship between these two constructs is reciprocal.

In addition, according to the self-determination theory of Deci and Ryan (2000), there exist essentially two distinct types of motivation, namely, intrinsic and extrinsic motivation. The effects of these two types of motivation on students' learning, such as their learning behavior and achievement, are different (Deci & Ryan, 2000; Liu et al., 2019a, 2019b; Ryan & Deci, 2020). Therefore, it is reasonable to conjecture that these two types of motivation will influence students' engagement, especially their cognitive engagement in mathematics learning differently to a certain degree.

Moreover, it has been pointed out that both constructs—student-motivation and engagement—"cannot be separated or disentangled from the social context" in which they are shaped and developed (Reeve, 2012, p. 152). That is, both students' motivation to learn mathematics and students' engagement will be influenced by cultural and societal factors such as teachers' teaching behavior, parents' expectations, and examinations within a specific cultural context (Yu et al., 2018). Because of cultural differences, in previous empirical studies it was found that students' intrinsic and extrinsic motivation contribute to their mathematics achievement differently between East Asian and Western contexts (Zhu & Leung, 2010). Therefore, an investigation of the relationship between these two constructs within a specific cultural context is needed to understand the possible societal and cultural influences on this relevant relationship.

In addition, it has been pointed out that culture is not static but dynamic (Rao & Chan, 2010). Traditionally, Chinese mathematics education culture has been described in literature as being strongly teacher-directed with a transmissive teaching approach, as being more repetitive, involving rote-learning and memorization-based learning processes, and examination-driven curricula (Leung, 2001; Ni et al., 2014). However, studies have also found a complementary picture, namely that Chinese mathematics teachers not only emphasize memorization and exercises, they further emphasize the variations of problems, consolidation of newly learned knowledge by revision, frequent use of abstraction to generalize inner-mathematical relationships, and they provide timely feedback which could lead students to become deeply involved in mathematics learning (Cai & Lester, 2005; Zhang et al., 2004). Moreover, a new mathematics curriculum reform started at the beginning of this century leading to a fundamental change in Chinese mathematics education. One main goal of the present mathematics curriculum at senior secondary school level is to advocate student-centered teaching such as the use of inquiry-based mathematics learning and teaching, developing students' critical-thinking skills and the skills to apply mathematics going beyond memorization and exercises (MOE, 2003, 2018).

Since such reformed mathematics curriculum ideas have been implemented in China for almost two decades, these reforms may have gradually formed a new mathematics education culture in China, which may in turn fundamentally change students' mathematics learning experiences, such as their motivation and cognitive engagement in learning mathematics. Against this background, the present study's main aim is to investigate longitudinally the characteristics of senior secondary school students' motivation to learn mathematics, their cognitive engagement in mathematics learning and the reciprocal relationship between these two constructs within the reform-oriented Chinese mathematics educational culture.

2 Literature review

2.1 Intrinsic and extrinsic motivation to learn mathematics

Motivation has long been identified as one of the strongest correlates of students' academic achievement (Taylor et al., 2014). According to self-determination theory, two discrete motivation types may be distinguished: intrinsic and extrinsic motivation, which have been widely investigated (Deci & Ryan, 2000; Ryan & Deci, 2020). Intrinsic motivation is typically defined as motives that stem from the activity itself and emerge from within a person's inherent interest, enjoyment, satisfaction of curiosity, or personal challenge in the work (Amabile, 1996; Deci & Ryan, 2000). Similarly, intrinsic motivation to learn mathematics in the present study is conceptualized referring to students' completion of mathematics activities or learning for their own inherent interests and satisfaction with the activities themselves, and to seeking challenge from mathematics learning tasks for themselves (Herges et al., 2017; Middleton & Spanias, 1999; Ning, 2020).

By contrast, extrinsic motivation typically refers to the motives originating outside of the activity (Deci & Ryan, 2000). Within self-determination theory, four subtypes of extrinisic motivation with various degrees of internalization have been specificed, namely, external regulation, introjected, identified, and integrated regulation (Ryan & Deci, 2020). External regulation mainly refers to behaviors driven by factors external to the person such as rewards and punishments (Deci & Ryan, 2000). Introjected regulation, however, refers to those motivations that have been partially internalized such as those behaviors "regulated by the internal rewards of self-esteem for success and by avoidance of anxiety, shame, or guilt for failure" (Ryan & Deci, 2020, p. 2). Normally, these two subtypes of motivational regulations represent controlled forms of motivational styles. Similarly, in the present study, extrinsic motivation is conceptualized as having a reason to learn mathematics for attaining rewards, meeting expectations from outside or the evaluation by others, and winning the competition with others (Herges et al., 2017; Liu et al., 2017; Middleton & Spanias, 1999; Ning, 2020).

Moreover, within self-determination theory, intrinsic and extrinsic motivation are described as a continuum instead of being a dichotomy (Deci & Ryan, 2012). That means that intrinsic motivation and extrinsic motivation can be experienced simultaneously by students with different degrees (Deci & Ryan, 2000; Ryan & Deci, 2020). Furthermore, in previous research it was pointed out that some dimensions of motivation are subject-specific, which means that motivation should be understood as a domain-specific motivational construct (Green et al., 2007).

In addition, empirical studies have shown that students' motivation to learn mathematics tends to decline at every schooling stage. A longitudinal study with students from Grades 5–10 in Germany found that both students' intrinsic and extrinsic motivations to learn mathematics decreased (Murayama et al., 2013). Similarly, South Korean secondary students' intrinsic mathematical motivation declined continually throughout their entire secondary school careers (Lee & Kim, 2014). However, in other studies it was found that students' motivation to learn mathematics—particularly their intrinsic motivation—was more likely to change during the early stages of learning but became more stable during their later stages of learning (Garon-Carrier et al., 2016). Similarly, in a study with Chinese students, Liu (2015) found that senior secondary school students' intrinsic motivation to learn mathematics did not change significantly across two academic years.

Empirical studies have further confirmed that students' motivation to learn mathematics is context-dependent and is thus influenced by cultural and societal factors (Reeve, 2012). Cross-cultural studies have identified several differences between students from different contexts. For example, based on TIMSS data, Zhu and Leung (2010) found that Grade eight students from both East Asian (e.g., Japan and Korea) and Western contexts (e.g., the US, England, and Australia) exhibited significantly higher levels of extrinsic motivation, conceptualized as the pursuit of utilitarian goals. Students from Western contexts tended to exhibit even stronger extrinsic motivation, though this had a detrimental effect on their mathematical achievements. Based on PISA 2012 data, Chen and Lin (2020) reported that intrinsic and extrinsic motivation to learn mathematics of students from Taiwan were both stronger than the intrinsic and extrinsic motivation of students from the US. According to PISA data, Chinese students tend to exhibit significantly higher levels of intrinsic than extrinsic motivation to learn mathematics (e.g., Chen & Lin, 2020; Zhu, 2021).

2.2 Cognitive engagement in mathematics learning

Student engagement has attracted considerable attention in the fields of psychology and mathematics education research (Fredricks et al., 2004; Watt & Goos, 2017). Student engagement has been widely accepted as a multidimensional construct that encompasses several distinct but interrelated dimensions, such as behavioral, emotional, and cognitive engagement (Fredricks et al., 2004). Recently, a fourth dimension, social engagement, was proposed (Fredricks et al., 2016; Wang et al., 2016). Of these, cognitive engagement has been posited as the "impetus" (Hong et al., 2020, p. 3) for the other dimensions of engagement.

Cognitive engagement may be generally understood as a student's cognitive level of investment in learning, which includes "being thoughtful, strategic, and willing to exert the necessary effort for comprehension of complex ideas or mastery of difficult skills" (Fredricks et al., 2016, p. 6). Therefore, cognitive engagement relates to students' mental efforts in learning tasks, such as the comprehension of concepts, the recognition of connections among different ideas, thoughts, and solutions to problems, which emphasize inner psychological investment and profound mental involvement rather than superficial participation (Fredricks et al., 2004; Lin et al., 2018).

Based on the level of engagement, generally speaking, cognitive engagement has often been further differentiated as lying between deep and surface level learning (Li & Lajoie, 2021),or between negative and positive engagement (Wang et al., 2016). Similarly, in mathematics education, Kong et al. (2003) also differentiated surface level of mathematics cognitive engagement, which includes memorization, practicing and handling tests, and deep level of mathematics cognitive engagement, which includes understanding the question, summarizing what is learnt and connecting new knowledge with prior learning knowledge. However, researchers further argued that cognitive engagement is not a dichotomous construct between deep and surface levels; instead, essentially, it is a dynamic and consecutive process which “fluctuates over time as students immerse themselves in learning” (Li & Lajoie, 2021, p. 21).

In addition, engagement should be productively understood through a sociocultural lens (Watt & Goos, 2017) or seen as context-dependent (Li & Lajoie, 2021). Learning environment, teaching practice, the learning tasks, and other societal and cultural factors all play salient roles in shaping student engagement (Li & Lajoie, 2021; Wang et al., 2019). Cross-cultural comparative studies have found that Chinese students commonly use memorization-based learning strategies and repeated exercises to achieve proficiency or understanding (Biggs, 1998; Leung, 2001). However, researchers have also found that such strategies, used in combination with other strategies, such as variation, will largely avoid rote learning (Leung, 2001, 2017). Engagement has also been identified as relevant at a domain-specific level, for example by Fredricks et al. (2016), supported by Green et al. (2007), who examined high school students’ motivation and engagement in mathematics, English, and science. We therefore conceptualize cognitive engagement in mathematics learning in our study as referring to the cognitive level of involvement of mathematics learning, such as using deep mathematics learning strategies and necessary cognitive mathematics learning strategies for the comprehension of mathematics ideas, both from negative and from positive perspectives (Fredricks et al., 2016; Wang et al., 2016).

Another common characteristic of engagement is its malleability rather than stability across different learning situations (Li & Lajoie, 2021; Wang et al., 2019). Therefore, engagement is “not merely a static outcome” (Wang et al., p. 1097); rather, it is “a fluid set of processes that can be influenced by learners themselves and by the environment” (Greene, 2015, p. 27). Regarding the developmental trajectories of engagement, earlier studies’ findings were somewhat inconsistent. For example, Wang and Eccles (2012) observed in their longitudinal study that students’ cognitive dimensions of school engagement declined from Grade 7 to Grade 11. More recently, however, other longitudinal studies have described students’ engagement in mathematics learning to be relatively stable (e.g., Hong et al., 2020).

2.3 Relationship between motivation and cognitive engagement in mathematics learning

The discourse surrounding the relationship between student motivation and engagement encompasses different perspectives. Theoretically, there exists some overlap between motivation and engagement; especially, from the psychological perspective of student engagement, “motivation and engagement are closed intertwined” (Yin & Wang, 2016, p. 3). For example, Martin (2007) proposed an influential multidimensional motivation and engagement wheel model that integrates several types of student engagement and motivation.

However, recently, student motivation and engagement have been gradually accepted in literature as two distinct and separate but inherently interrelated constructs (Mahatmya et al., 2012; Martin et al., 2017). The distinction between the two constructs has been based on the description of motivation as an inner, private, and unobservable psychological factor, which serves as an antecedent cause of students’ observable behavior, namely, engagement (Reeve, 2012). That is, motivation has been typically accepted as an internal psychological factor having an energizing impetus, and engagement, on the other hand, has been typically accepted as a factor reflecting students’ actual involvement in the learning activity (Ainley et al., 2012; Martin et al., 2017). Essentially, student motivation has been described to be the subjectively experienced cause which leads to student engagement and is thus a precursor to engagement; that means engagement is mostly one of the outcomes of motivation (Reeve, 2012).

Moreover, researchers have also argued that motivation is merely “a necessary but not a sufficient condition for engagement” (Mahatmya et al., 2012, p. 47). Currently, such a linear or direct effect of motivation on engagement has been pointed out as “only partially valid” (Reeve, 2012, p. 152). The change in the quality of students’ engagement during learning will in turn cause subsequent changes in students’ motivation to learn (Reeve, 2012; Reeve & Lee, 2014), indicating a reciprocal relationship between motivation and engagement.

Similarly, in the field of mathematics education, researchers have also argued that student motivation to learn mathematics and engagement in mathematics learning are two “distinguishable” constructs (Durksen et al., 2017, p. 165). Student motivation to learn mathematics is also believed to strongly influence student engagement in mathematics learning (Fielding-Wells et al., 2017). Therefore, student motivation and engagement in mathematics learning have been commonly investigated together as two related factors which influence student mathematics achievement (e.g., Hsieh et al., 2021; Yu et al., 2021).

The relationship between these two constructs has been evaluated in previous empirical studies, although the findings are not consistent. For example, Lee and Koszalka (2016) reported that South Korean undergraduate students’ intrinsic and extrinsic motivations could positively predict their deep and surface-level cognitive engagement. Recently, in a blended-learning and synchronous-learning environment, Shi et al. (2021) observed that Chinese senior secondary school students’ extrinsic motivation was positively and significantly associated with their surface cognitive engagement. However, they identified no significant association between students' intrinsic motivation and their cognitive engagement. With mathematics as a specific domain, Hsieh et al. (2021) found that senior secondary school students' mathematical motivation patterns were related to different levels of mathematical behavioral engagement. Specially, those students who showed high motivation in mathematics tended to have a higher level of mathematical behavioral engagement; in contrast, those who showed low motivation in mathematics tended to have a lower level of mathematical behavioral engagement.

Moreover, previous longitudinal studies have provided evidence for the reciprocal relationship between these two constructs. For example, Reeve and Lee (2014) observed that high school students' initial engagement could predict changes in their mid-semester motivation, and such changes could further predict changes in their end-of-semester motivation. Similarly, Jang et al. (2016) found that students' perceptions of their teachers' motivating style (e.g., autonomy support) affected changes in their later engagement or disengagement only indirectly by affecting a change in students' motivational resources (e.g., need satisfaction). In contrast, students' disengagement in learning was found to have a direct effect on their perception of teachers' motivating style.

To summarize, research has demonstrated that students' engagement "bridges students' motivation to highly valued outcomes" (Reeve, 2012, p. 163). However, only a few empirical studies investigated the reciprocal relationship between these two constructs in the field of mathematics education. Specifically, it is not clear how intrinsic and extrinsic motivation longitudinally differently influence students' cognitive engagement. As Schukajlow et al. (2017) argued, it seems necessary to implement a longitudinal study design to examine causal relations among students' motivational constructs and other affective variables (e.g., engagement).

2.4 Research questions and hypotheses

Based on this discourse on the characteristics of motivation and engagement and the relationship between them, we proposed a longitudinal model to investigate the relationship between students' intrinsic and extrinsic motivation, and their cognitive engagement (see Fig. 1). Specifically, the random intercept cross-lagged panel model (RI-CLPM; Hamaker et al., 2015) was employed in the present study, which enabled us to examine both the within- and between-person variation while considering time-invariant components (e.g., Hamaker et al., 2015; Takahashi et al., 2022).

Referring to the research described above, in the present study we aimed to address the following research questions:

(1) What are the characteristics of and changes in Chinese senior secondary school students' motivation for mathematics learning and cognitive engagement in mathematics learning?
(2) How can Chinese students' original motivation and cognitive engagement predict their later motivation and cognitive engagement?

To answer these two research questions, the following research hypotheses were developed:

Hypothesis 1 Senior secondary school students' motivation and cognitive engagement will decrease significantly within the two-year period of their senior secondary schooling.

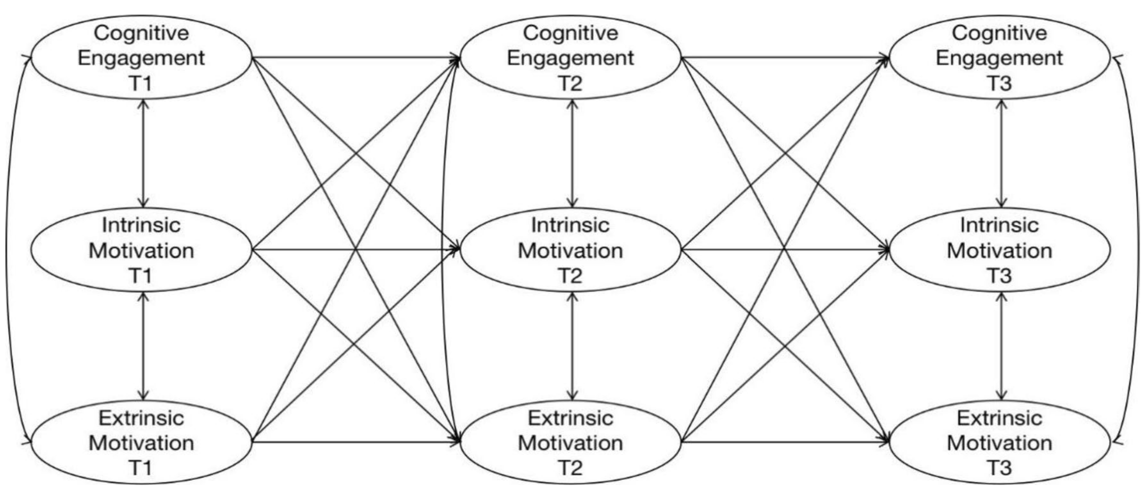

Fig. 1 Hypothesized model for the longitudinal relationship among students' intrinsic motivation, extrinsic motivation, and cognitive engagement

Hypothesis 2 Senior secondary school students' intrinsic and extrinsic motivation will predict their later cognitive engagement to different degrees.

Hypothesis 3 Senior secondary school students' cognitive engagement will predict their later intrinsic and extrinsic motivation to different degrees.

3 Methodical approach

3.1 Context of the study and participants

The present study focuses on senior secondary school students in China who will take the National University Entrance Examination by the time of their graduation (Grade 12). Senior secondary school graduates can apply to different levels of university according to their examination results. The examination is thus highly competitive and demanding. Mathematics is one of three compulsory subjects (the other two are Chinese language and English). This common examination culture and pressure may induce senior secondary school students to develop different motivations for learning mathematics and they may thus engage cognitively in mathematics learning differently.

In the present study, we surveyed senior secondary school students from two schools from two provinces. One school was in Shangdong, a province with a relatively strong economic and educational background in China; the other school was in Guizhou province, a province with a relatively poor economic and educational background in China. The data were collected in three time waves: Time 1 (T1) in November 2019 when the participants were in Grade 11; Time 2 (T2) in November 2020 when the participants were in Grade 12; and Time 3 (T3) in April 2021 when the participants were close to completing their Grade 12 examination. The senior secondary school education system in China comprises three years in total (Grades 10–12, students 16–18 years old).

We originally recruited around 623 participants at Time 1 in the two schools, but only 365 participants could be matched for the three time points. The 623 senior secondary school students (298 female, 325 male) were from 26 classes in the two schools, and 479 of them came from urban families. The students' mathematical backgrounds were varied, ensuring that the reason for non-participation was not students' differences in mathematics achievement.

3.2 Measurement instruments

Intrinsic and extrinsic motivation. The scale used in the study was based on the following two dimensions. (1) Intrinsic motivation to learn mathematics. All the seven items used in this dimension were selected from the highly accepted instrument 'Motivated Strategies for Learning Questionnaire' (MSLQ) (Pintrich & De Groot, 1990). This questionnaire was validated in a Chinese context (Hong Kong) (Rao & Sachs, 1999) and has been used in the field of mathematics education to investigate students' intrinsic motivation (e.g., Herges et al., 2017). According to the theoretical framework of intrinisic motivation as described above, the selected items were modified to investigate mainly students' interests, enjoyment, and sense of challenge in the learning of mathematics. (2) Extrinsic motivation to learn mathematics (5 items). All the five items were chosen from several previously used questionnaires to investigate students' extrinisic motivation to learn mathematics (e.g., Liu et al., 2019a, 2019b; OECD, 2003).

All selected items were translated into Chinese by the first author and were further checked by a mathematics education researcher with good knowledge of English and Chinese. Two highly experienced secondary school mathematics teachers examined the content of all items. The modified questionnaire had already been validated in an earlier study (Zhang & Yang, 2022). The final set used in the analysis of the present study included five items on the intrinsic motivation dimension (e.g., "I prefer mathematics work that is challenging", "I like what I am learning in mathematics", "I think that what we are learning in mathematics is interesting", "understanding mathematics is important to me" and "I am interested in solving mathematics problems") and four items on the extrinsic motivation dimension (e.g., "I will learn many things in mathematics that will help me get a job", "to get better mathematics grades than my classmates is very important to me", "I want to do well in mathematics because it will make my classmates regard me as a smart person", and "I want to do well in mathematics because it will make my teacher regard me as a good student").

To validate the usage of the questionnaire of the present study, exploratory factor analysis (EFA) and confirmatory factor analysis (CFA) were performed. Firstly, data collected from the non-matched 257 students at Time 1 were used for EFA. Two factors were identified that accounted for 49.78% of the variance; therefore, a few items were excluded. In addition, we conducted confirmatory factor analysis (CFA) to further determine the construct validity of the measures at the three time points with the 365 matched students' information. CFA results suggested adequate model fit for each of the three times respectively: T1: $\chi^2(26) = 86.157$, RMSEA = 0.076, CFI = 0.946, TLI = 0.926, SRMR = 0.051; T2: $\chi^2(26) = 90.568$, RMSEA = 0.069, CFI = 0.961, TLI = 0.946, SRMR = 0.043; and T3: $\chi^2(26) = 96.551$, RMSEA = 0.077, CFI = 0.942, TLI = 0.922, SRMR = 0.056. Items on the two dimensions were presented on a five-point Likert scale with 1 = "strongly disagree" and 5 = "strongly agree". The internal consistency of each dimension was estimated using Cronbach's alpha reliability coefficient. Satisfactory levels of internal consistencies for statistical consideration were identified as follows: $\alpha_{\text{intrinsic motivation at T1}} = 0.769$; $\alpha_{\text{intrinsic motivation at T2}} = 0.805$; $\alpha_{\text{intrinsic motivation at T3}} = 0.796$; $\alpha_{\text{extrinsic motivation at T1}} = 0.794$; $\alpha_{\text{extrinsic motivation at T2}} = 0.815$; $\alpha_{\text{extrinsic motivation at T3}} = 0.769$.

Cognitive engagement of mathematics learning. Items in the cognitive engagement dimension were chosen from the questionnaire developed by Fredricks et al. (2016), Wang et al. (2016) and Kong et al. (2003), translated and modified according to the current Chinese mathematics education situation by the first author and another highly experienced mathematics education professor with rich secondary school mathematics teaching experience. The chosen items were designed to investigate how students engage in mathematics learning from a negative perspective (sample item: "In learning mathematics, I prefer memorizing all the necessary formulas rather than understanding the principles behind them") and a positive perspective (sample item: "I try to understand my mistakes when I get something wrong in mathematics"). Two highly experienced secondary school mathematics teachers also examined the content of all translated items. The modified questionnaire had already been validated in our earlier study (Yang et al., 2021). The final scale consisted of six items, all of which were presented on a five-point Likert scale with 1 = "strongly disagree" and 5 = "strongly agree".

CFA was also conducted to determine the construct validity of the scale of cognitive engagement of mathematics at the three time points. CFA results suggested good model fit for the three times respectively: T1 $\chi^2(9) = 31.102$, RMSEA = 0.069, CFI = 0.947, TLI = 0.957, SRMR = 0.029; T2 $\chi^2(9) = 14.063$, RMSEA = 0.033, CFI = 0.995, TLI = 0.992, SRMR = 0.016; and T3 $\chi^2(9) = 41.096$, RMSEA = 0.075, CFI = 0.950, TLI = 0.916, SRMR = 0.036. The internal consistency was further estimated again using the Cronbach's alpha reliability coefficient. Satisfactory levels of internal consistencies for statistical consideration were identified as follows: $\alpha_{\text{cognitive engagement at T1}} = 0.811$; $\alpha_{\text{cognitive engagement at T2}} = 0.861$; $\alpha_{\text{cognitive engagement at T3}} = 0.807$.

3.3 Data analysis

First, as mentioned above, psychometric analyses were conducted at the beginning using exploratory factor analysis (EFA) and confirmatory factor analysis (CFA). The CFAs were performed for the purpose of testing for the fit of the factor structure to determine the extent of measurement invariance between assessments at the three time points. Then, descriptive statistics, t tests, and one-way ANOVA and Pearson correlation results were carried out to provide preliminary information. We then established and examined the RI-CLPM to illustrate the longitudinal relations among intrinsic motivation, extrinsic motivation, and cognitive engagement. Data analyses were conducted with SPSS 26.0 and Mplus 8.0. During the estimation of the RI-CLPM, we used the full-information maximum likelihood to estimate all model parameters; this estimator can handle missing data efficiently, and can provides robust standard errors and robust chi-square tests of model fit (Paul et al., 2019). More details on the chosen approach are provided in the results section.

4 Results of the study

4.1 Descriptive results

The item mean score and standard deviation of each dimension were computed to obtain a sample description of the students' motivation and engagement at the three times (Table 1). The item mean scores for intrinsic motivation were greater than the item mean scores for extrinsic motivation across the three time waves. The independent t test results revealed significant differences between them (T1 $t = 28.51$, $p < 0.001$; T2 $t = 30.92$, $p < 0.001$; T3 $t = 36.57$, $p < 0.001$), suggesting that the participants held stronger intrinsic motivation to learn mathematics. However, the item mean scores of extrinsic motivation for the three times were all close to 3, indicating that students' extrinsic motivation to learn mathematics was not low. Furthermore, the item mean scores of cognitive engagement in mathematics learning were all close to 3, which indicates that the participants did not exhibit strong cognitive engagement in mathematics learning.

Table 1 Descriptive statistics of cognitive engagement, intrinsic motivation, and extrinsic motivation

	Time 1 *M (SD)*	Time 2 *M (SD)*	Time 3 *M (SD)*	*F*	*df*	*P*
Cognitive engagement	3.27 (0.70)	3.21 (0.75)	3.25 (0.71)	0.48	2	0.364
Intrinsic motivation	3.99 (0.63)	3.91 (0.68)	3.90 (0.66)	1.97	2	0.079
Extrinsic motivation	3.23 (0.92)	3.08 (0.94)	3.13 (0.88)	2.39	2	0.062

In addition, a series of one-way ANOVAs were performed to examine the change of these two constructs during their high school attendance. As shown in Table 1, no significant differences were identified among the three time waves. These findings reveal that the participants' cognitive engagement and their intrinsic and extrinsic motivation neither decreased nor increased significantly from Grade 11 to Grade 12. These results thus do not support our first hypothesis.

4.2 Correlational results

The relationships among the three constructs were examined using Pearson correlation analysis (Table 2). As shown in Table 2, it was found that intrinsic motivation at the three times was strongly and significantly correlated with cognitive engagement (r values range from 0.32 to 0.69). Extrinsic motivation at the three times, however, was found to be less strongly and significantly correlated with cognitive engagement (r values ranged from 0.05 to 0.32) compared to intrinsic motivation.

4.3 Results of the longitudinal interaction model

The longitudinal measurement invariance (MI) was conducted to test whether each of the constructs of cognitive engagement, intrinsic motivation, and extrinsic motivation exhibited MI across the three times before the conduction of the RI-CLPM. Refering to Willoughby et al. (2012), a separate longitudinal CFA model was developed to fit each variable. There are four steps for the test of longitudinal MI: configural-, weak-, strong-, and strict-MI (Liu et al., 2017). Table 3 lists the results of longitudinal MI for three variables. The criteria used by the OECD (OECD, 2014) were also adopted in the current study: if ΔRMSEA is less than or equal to 0.010 and ΔCFI larger than or equal to −0.010, the strong MI is satisfied. Results show that both cognitive engagement and extrinsic motivation satisfy the weak MI; intrinsic motivation, in contrast, satisfies the strict MI. Therefore, the weak MI for cognitive engagement and extrinsic motivation and the strict MI for intrinsic motivation were used in the RI-CLPM to investigate the relationship among these three variables.

Table 2 Pearson correlations among cognitive engagement, intrinsic motivation, and extrinsic motivation in three time waves

Variable	1	2	3	4	5	6	7	8	9
1.T1 cognitive	1								
2.T2 cognitive	0.49***	1							
3.T3 cognitive	0.40***	0.60***	1						
4.T1 intrinsic	0.55***	0.37***	0.32***	1					
5.T2 intrinsic	0.37***	0.45***	0.61***	0.37***	1				
6.T3 intrinsic	0.37***	0.62***	0.69***	0.44***	0.62***	1			
7.T1 extrinsic	0.19***	0.07	0.05	0.14**	0.17**	0.09	1		
8.T2 extrinsic	0.15**	0.17**	0.18**	0.10	0.27***	0.14**	0.40***	1	
9.T3 extrinsic	0.24***	0.32***	0.34***	0.20***	0.29***	0.41***	0.33***	0.50***	1

Cognitive = cognitive engagement, Intrinsic = intrinsic motivation, Extrinsic = extrinsic motivation; **p* < 0.05; ***p* < 0.01; ****p* < 0.001

Table 3 Results from the longitudinal MI tests for cognitive engagement, intrinsic motivation, and extrinsic motivation

Scale	χ^2	*df*	CFI	TLI	RMSEA	SRMR	ΔCFI	ΔRMSEA
Cognitive engagement								
Configural MI	367.172	130	0.923	0.909	0.054	0.044		
Weak MI	395.659	142	0.918	0.911	0.054	0.060	−0.005	0.000
Strong MI	461.232	154	0.900	0.901	0.057	0.066	−0.018	0.003
Intrinsic motivation								
Configural MI	230.388	87	0.930	0.915	0.051	0.047		
Weak MI	242.796	97	0.929	0.923	0.049	0.061	−0.001	−0.002
Strong MI	272.317	107	0.919	0.921	0.050	0.073	−0.01	0.001
Strict MI	272.623	117	0.924	0.932	0.06	0.092	0.005	0.010
Extrinsic motivation								
Configural MI	173.948	48	0.932	0.906	0.065	0.047		
Weak MI	186.692	56	0.929	0.916	0.061	0.056	−0.003	−0.004
Strong MI	219.909	64	0.915	0.913	0.063	0.066	−0.014	0.002

To summarize, a cross-lagged structural equation model was developed for the longitudinal linkages among cognitive engagement, intrinsic motivation and extrinsic motivation after the longitudinal MI was tested. The results indicate that the model fit with the data is acceptable [$\chi^2(942) = 1470.373$, CFI = 0.939, TLI = 0.935, RMSEA (90% CI) = 0.030 (0.027–0.033), SRMR = 0.054] (Marsh et al., 2004). For simplicity, we present only the within-person component of the RI-CLPM (Fig. 2).

As hypothesized, students' intrinsic and extrinsic motivation were found to predict their later cognitive engagement in mathematics learning differently. As shown in Fig. 2, Chinese senior secondary school students' intrinsic motivation to learn mathematics at T2 could positively and significantly predict their later cognitive engagement in mathematics learning at T3. However, Chinese senior secondary school students' extrinsic motivation to learn mathematics could not positively and significantly predict their later cognitive engagement in mathematics learning. As shown in Fig. 2, students' extrinsic motivation both at T1 and T2 could not significantly predict their cognitive engagement at T2 and T3 respectively.

In addition, as also hypothesized, Chinese senior secondary school students' cognitive engagement in mathematics learning was also found to predict students' later intrinsic and extrinisic motivation to learn mathematics differently. As shown in Fig. 2, earlier cognitive engagement could positively and significantly predict students' later intrinsic motivation into mathematics learning. In addition, the coefficients between Time 2 and Time 3 were found to be greater than the coefficients between Time 1 and Time 2, which suggests that an even closer relationship between the two constructs could exist at a later educational stage. By contrast, as shown in Fig. 2, only students' cognitive engagement at T2 could predict their later extrinsic motivation to mathematics learning at T3, with a much smaller coefficient.

5 Summary and discussion

5.1 Characteristics and changes in motivation and cognitive engagement

A central aim of the present study was to investigate the characteristics and changes in Chinese students' motivation to learn mathematics and their cognitive engagement in mathematics learning during the two years of their senior secondary school study at the end of their schooling. However, generally speaking, the participants' intrinsic motivation to learn mathematics was significantly stronger than their extrinsic motivation, although their extrinsic motivation was still relatively strong. These findings are consistent with previous findings reported from the Chinese educational context (e.g., Chen & Lin, 2020; Zhu, 2021; Zhu & Leung, 2010). In these studies, East Asian students (including students from Shanghai, Hong Kong, and Taiwan) were all found to have relatively higher levels of intrinsic motivation than extrinsic motivation, but their extrinsic motivation to learn mathematics was also quite high. These findings suggest the mixed type of intrinsic and extrinsic motivation to learn mathematics (e.g. Deci & Ryan, 2012; Ryan & Deci, 2020), at least for Chinese senior secondary school students (e.g., Ning, 2020).

The mixed type of intrinsic and extrinsic motivation might be explained by the Chinese societal and cultural factors. First, senior secondary school students' higher level

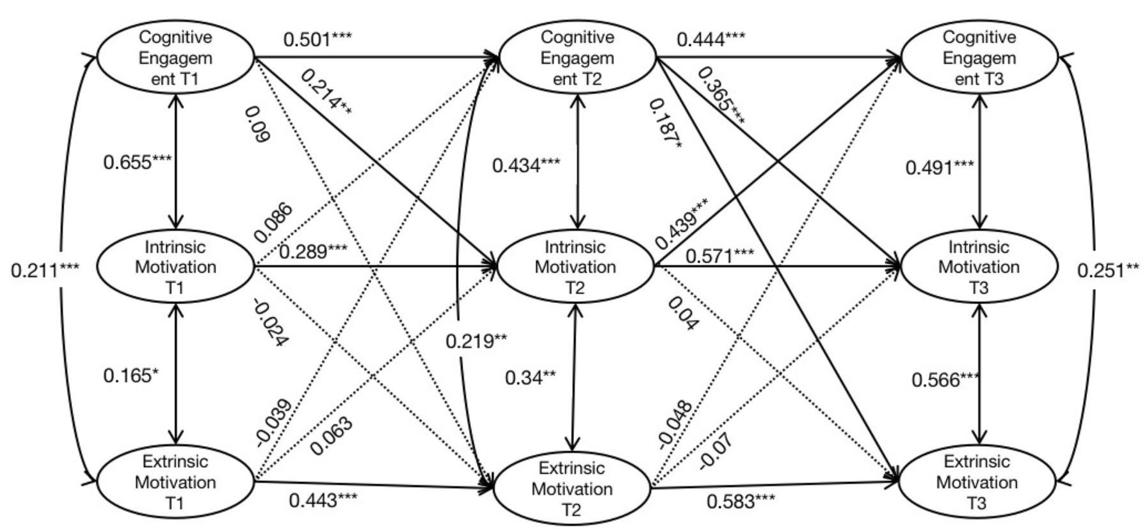

Fig. 2 Simplified model results for the longitudinal linkages among intrinsic motivation, extrinsic motivation and cognitive engagement. Dashed lines indicate statistically non-significant paths

of intrinsic motivation compared with extrinsic motivation to learn mathematics might be explained by the reformed mathematics education culture in China. Currently, student-centered teaching and learning approaches are strongly encouraged in China by official reforms (MOE,). Such a new teaching and learning culture may change students' experiences of learning mathematics, which in turn may improve students' intrinsic motivation to learn mathematics. In addition, the long tradition of valuing academic achievement or success in China acts as another main source, influencing senior secondary school students' extrinsic motivation to learn mathematics since mathematics is a compulsory subject in the National University Entrance Examination (Leung, 2001). Under the influence of such traditions, Chinese teachers and parents attach great importance to students' learning, which acts as another main influence for the development of Chinese students' motivation to learn mathematics. Such external expectations for their learning probably increase students' extrinsic motivation to learn mathematics.

In addition, we hypothesized that senior secondary school students' motivation will decrease during the two-year timeframe. But although a slight decrease was observed in participants' intrinsic motivation, one-way ANOVAs could not detect significant differences between their motivation in the three time waves. This finding is inconsistent with earlier findings from non-Chinese contexts, such as those of Germany and South Korea (Lee & Kim, 2014; Murayama et al., 2013). As mentioned above, mathematics is a very important subject in the National University Entrance Examination in China. The pressure from this examination may cause Chinese senior secondary school students to hold stable instrinsic and extrinsic motivation to learn mathematics. For example, this stable status of Chinese students' motivation is indeed consistent with earlier findings related to Chinese secondary school students' intrinsic motivation to learn mathematics (Liu, 2015).

Regarding the participants' cognitive engagement in mathematics learning, it was found that senior secondary school students in China are currently not profoundly and mentally engaged in learning mathematics, that is, in deep understanding of the content and recognition of the connections among different ideas. This situation may indicate that low cognitive-level learning strategies, such as repeated exercises, may still persist among senior Chinese students. In addition, the participants' cognitive engagement in mathematics learning was found to be relatively stable during the two year period of investigation. Similar findings were also identified in recent studies in the Chinese contexts. For example, in a recent large-scale survey in China, it was found that Chinese students still chose at least one very popular memorization strategy even though they also chose other mathematics learning strategies such as elaboration (Liu et al., 2019a, 2019b). Similarly, Guo and Wei (2019) also found that senior secondary school students in Shanghai reported a high level of rehearsal strategy usage but moderate levels of elaboration and critical thinking strategy usage in their learning of mathematics.

Similarly, the pressure associated with the upcoming National University Entrance Examination might cause senior secondary school students not to be particularly cognitively engaged but to be behaviorally engaged, perhaps completing a certain number of exercises every day to attain proficiency in mathematics (Yu et al., 2018). In addition, due to the influence of the examination culture, mathematics teachers at senior secondary school level in China were also found to "become increasingly concerned about memorization and basic practices, reducing the occurrence of group discussion or open-ended tasks" (Zhou et al., in press). Such ways of mathematics teaching and learning that are still prevalent may further lower students cognitive engagement in mathematics learning.

5.2 The relationship between motivation and cognitive engagement

The second aim of the study was to invatigate the reciprocal relationship between motivation and cognitive engagement. Firstly, it was found that students' intrinsic and extrinsic motivation was associated with their cognitive engagement, but the correlation between intrinsic motivation and cognitive engagement was stronger than the correlation between extrinsic motivation and cognitive engagement. These findings are not entirely consistent with previous similar studies conducted in the context of China. As mentioned above, Shi et al. (2021) did not identify a significant association between students' intrinsic motivation and their cognitive engagement; and their extrinsic motivation was associated positively and significantly only with surface cognitive engagement. These differences may be explained by the missing domain specificity of Shi et al.'s (2021) study, which did not focus on mathematics education as we did in the present study.

Further interpretations of these results should take into consideration China's societal and cultural background. As mentioned above, mathematics is a vital subject in the National University Entrance Examination; therefore, parents and teachers normally put quite high expectations on mathematics learning. These expectations may cause students to become more mentally involved in mathematics learning. Therefore, it is understandable that senior secondary school students' extrinsic motivation to learn mathematics is also positively associated with their cognitive engagement (Yu et al., 2018). Similarly, Lee and Koszalka (2016) found that South Korean students' intrinsic and extrinsic motivation could both positively predict their cognitive engagement.

However, in the present study, the correlation between intrinsic motivation and cognitive engagement was considerably higher than the correlation between extrinsic motivation and cognitive engagement. Moreover, it was found that students' earlier intrinsic motivation to learn mathematics could more strongly predict their later cognitive engagement than their earlier extrinisic motivation. These findings may also suggest that, like the long-term effect of intrinsic motivation on students' performance and course effort (Liu et al., 2019a, 2019b), comparatively speaking, senior secondary school students' intrinsic motivation to learn mathematics also plays a far more important role with respect to the influence on their cognitive engagement than does their extrinsic motivation.

Moreover, participants' earlier cognitive engagement could much more strongly predict their later intrinsic motivation than their later extrinsic motivation, which is consistent with previous findings (e.g., Reeve & Lee, 2014). Students with high cognitive engagement are typically characterized as having high self-efficacy beliefs (Luo et al., 2009). Subsequently, those students demonstrated high cognitive engagement and were more likely to sustain their engagement in learning activities over time (Reeve & Lee, 2014). Therefore, it is expected that students' cognitive engagement in learning activities will improve their intrinsic motivation at a later stage. Similarly, with respect to mathematics learning, it appears that if students engage more strongly at a cognitive level, they may become more intrinsically motivated to learn mathematics. Overall, Reschly and Christenson (2012, p. 14) argued that "it is possible that cognitive engagement and motivation are in fact very similar". By contrast, within the Chinese contexts, it is quite common for students to learn mathematics well to meet external factors such as expectations from their teachers and parents (Yu et al., 2018), which improves their extrinsic motivation to learn mathematics. But a deep cognitive engagement in mathematics learning will not enhance such a type of motivation.

In addition, it is worth noting that the relationship between motivation and cognitive engagement and the reciprocal effect between these two constructs from T2 to T3 were much stronger than the relationship and the reciprocal effect between them from T1 to T2. Overall, it seems that when the students were in Grade 12, the final year of senior secondary school in China, the relationship between motivation and cognitive engagement tended to be quite strong. Considering that Grade 12 is the final year of senior secondary school, and the impetus to prepare for the National University Extrance Examination, students may tend to be more motivated and more engaged cognitively in their mathematics learning. Therefore, the relationship between the evaluated constructs may have become stronger.

6 Conclusions and limitations

The study has several limitations that need to be discussed. First, all participants were chosen from one school from each province. Therefore, the findings may not be sufficiently representative of the Chinese culture. Future studies should involve students from a wider range of schools and regions. Second, we collected data only in three time waves; it is possible that these data will not fully reflect the complex interplay among these constructs. Future studies could consider collecting data at more times to yield more meaningful results. Third, we employed only a questionnaire survey without any interviews with the participants. Therefore, the factors that contribute to changes in the participants' motivation and engagement and their relationships remain unclear. Future studies could consider combining such data collection methods to ensure a more comprehensive picture.

However, as research into mathematics-related motivation is "still too scarce", especially for longitudinal studies (Schukajlow et al., 2017, p. 318), the present study is one of the few studies in mathematics education that investigated the potential reciprocal relationships between senior secondary school students' intrinsic/extrinsic motivation and cognitive engagement employing a longitudinal study design in the context of China. Therefore, the study's findings contribute to our understanding of the characteristics and changes of senior secondary school students' motivation and cognitive engagement, and the reciprocal relationships between them in China's specific societal and cultural context, which may be very different from Western contexts.

Altogether, the findings reported above suggest that Chinese senior secondary school students tend to hold mixed types of motivation for learning mathematics but with stronger intrinsic motivation. They did not manifest a deep level of cognitive engagement in mathematics learning, and their motivation and cognitive engagement levels from Grade 11 to Grade 12 were rather stable. Compared with extrinisic motivation to learn mathematics, their intrinisic motivation to learn mathematics tended to be more closely related to their cognitive engagement. In addition, the reciprocal effect between intrinisic motivation to learn mathematics and cognitive engagement in mathematics learning was much stronger than the relation between extrinisic motivation and cognitive engagement.

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