NAÏVE AND YET KNOWING: YOUNG LEARNERS PORTRAY BELIEFS ABOUT MATHEMATICS AND LEARNING

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This thesis contains no material published elsewhere or extracted in whole or part from a thesis by which I have qualified or been awarded another degree or diploma.

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All research procedures reported in the thesis received the approval of the relevant Ethics/Safety Committees (where required).

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Date: September 29, 2001

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Naïve and yet knowing: Young learners portray beliefs about mathematics and learning

ABSTRACT

This is a report of an investigation of children's beliefs about the nature of mathematics, the nature of learning and helping factors for learning mathematics. The study aimed to investigate whether beliefs held by eight learners of eight to nine years of age could be articulated and portrayed. It aimed also to develop procedures to facilitate this process, to portray children's beliefs from their responses to the research procedures, to provide insights into possible complexities and subtleties of young learners' beliefs, to reflect upon the significance for the mathematics classroom of the insights gained, and to reflect upon the value of the procedures developed for the study.

The research took the form of individual case studies of four girls and four boys of eight to nine years of age from two schools in suburban Melbourne. Four children were teacherperceived low achievers in mathematics and four were teacher-perceived high achievers in mathematics.

The children were each interviewed on ten occasions over a five-month period using thirty semi-structured, creative interviewing procedures that were developed or adapted for the study that included drawing, writing, discussing scenarios presented through photographs, video snippets and other children's drawings, ordering of descriptors, and responding to questionnaires presented verbally. The interview data consisted of transcripts and artefacts. Some class administered tasks, lesson observations and interviews with the mathematics teachers provided background information.

Analysis of interview responses was undertaken through a criss-cross examination in which themes were drawn from each child's data. Responses were not judged for correctness or for a match to any predetermined categories and the researcher sought to take a stance of neutrality to the phenomena under study.

The research suggests that teachers and others involved in the education of young learners of mathematics should know that

- it is possible to gain insights into children's beliefs about maths (the term used most commonly by the children), learning, and helping factors for learning maths;
- to gain insights into young children's beliefs, it is important to have dialogue with the children to avoid making assumptions about their interpretations or meanings;
- the creative interviewing procedures developed for the present research are helpful as they can stimulate reflection and prompt conversation;
- young children's beliefs can be complex, subtle, broad and deep;
- young children's beliefs are individually constructed and differ from child to child;

- children may not see mathematics concepts in the same ways as their teachers and other adults;
- beliefs are sufficiently diverse and significant to affect the way children see the mathematics learning situation;
- although the beliefs of children of eight to nine years of age may, on the surface, appear simplistic and naïve, they are not necessarily so. Young learners are able to reflect on their own and others' experiences and often construct complex beliefs. There is a lot happening in the minds of these children.

The research suggests also that it is important that educators do not to make assumptions about

- what children see as maths (or mathematics);
- what children see as learning; and
- what children see as helping factors for learning maths.

A key factor facilitating children's reflection and expression was the range of visual, verbal, and text-based creative interviewing strategies developed for the present study. The individual procedures provided suitable prompts to allow young children to articulate or represent their beliefs. The semi-structured procedures, through which ideas were explored on multiple occasions, followed by theme-based, criss-cross analysis of interview transcripts and artefacts, resulted in rich and trustworthy portrayals of beliefs, increasing the validity of the findings.

The research provides the education community with insights into young children's beliefs that are unlikely to emerge within the day to day activity of the classroom and, through the availability of the research procedures, facilitates further gaining of insights into beliefs either by classroom teachers or other researchers.

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CHAPTER ONE

MATHEMATICS, LEARNING, AND HELPING FACTORS FOR THE LEARNING OF MATHEMATICS: AN INTERWOVEN COLLAGE OF BELIEFS

The thesis is written at a time when there is great interest in education in the theme of the centrality of the individual in the construction of mathematical knowledge. It is acknowledged that children learn mathematics in varying ways and construct their own understandings (e.g., Ernest, 1991). The present research focused not on children's learning of mathematics but on their beliefs about mathematics learning. Underpinning the research was an assumption that just as children construct their understandings, they also construct or develop beliefs (e.g., Yackel & Cobb, 1996); these, in turn, may affect aspects of their learning (e.g., Hoyles, 1982).

Learning is an accepted part of human nature; it occurs throughout our lives and in many different situations. Children develop mathematical concepts and strategies from an early age (Becker & Selter, 1996; Cobb, Yackel, & Wood, 1991; Hughes, 1986; Wynn, 1998), and bring these understandings to school, the formal institution of learning for children in Western society. While there is some commonality in structure of schools, such as in the formation of learning groups with an instructor or teacher, and a general commonality in broad intention, that is, the education of individual students to their greatest potential (e.g., Cheng, 1995), specific philosophies and beliefs may vary both between and within communities of educators. Likewise for children, beliefs, such as those about mathematics and learning, may vary.

The present investigation of children's beliefs evolved in consideration of the education community's intention to educate children to their greatest potential and in response to the following assumptions that underpinned the research:

- beliefs are constructed by individuals and therefore vary within a class or grade;
- individuals' beliefs about the nature of mathematics and learning interact with their beliefs about factors that help them in their learning of mathematics;
- children may benefit from attempting to articulate beliefs about mathematics and learning through reflecting upon and possibly questioning their ideas thus building awareness of the self and potentially increasing control over their learning, and enhancing skills in critical thinking;
- with appropriate procedures it is possible for teachers to gain insights into children's beliefs;
- teacher knowledge and appreciation of children's beliefs can inform decision-making for mathematics teaching;
- it is of value for teachers, parents and researchers involved in the education process to have insights into the nature of beliefs constructed by individual children.

As each of these points contributed to the rationale for the present study, they are expanded within this thesis, some in more detail than others, and reflected upon in the concluding chapter. This introductory chapter considers other aspects of the research also; it contains the following sections:

- children can inform teacher decision-making;
- the importance of beliefs;
- benefits of articulation of learner beliefs;
- an emerging interest in children's beliefs;
- theoretical orientation of the research;
- a research model;
- defining the subject of the research portrayal;
- sketching the research portrayal.

Children can inform teacher decision making

Every day teachers make pedagogical and management decisions in their teaching of mathematics. Questions they ask themselves might include whether to break the class into groups, and if so upon what criteria, whether to use closed or open-ended tasks, whether to use an expository approach, whether to encourage pupil exploration, whether to encourage pupil communication and collaboration, and whether to have children use concrete materials. The resolution of such questions contributes to the creation of a learning environment which may or may not be favoured by, or be of self-perceived benefit to, individual learners. Traditionally teachers have made management and pedagogical decisions with pupil needs taken into account from the teacher's perspective, through consideration of matters such as the teacher survival issue of class management. Pupil perspectives as expressed by individual learners appear to have been accorded little value in mathematics classrooms and have received limited research attention.

The present study suggests that beliefs about mathematics learning also can inform mathematics teaching. As illustrated in the model presented in Figure 1 (McDonough & Gervasoni, 1997, p. 150), information can be gathered from a range of sources - by collecting products, making observations and listening to learners. This element of the model is informed by Clarke and Wilson (1994).

As the McDonough and Gervasoni (1997) model (Figure 1) illustrates, the collected information, which may relate to learning or beliefs, can provide insights or dilemmas for a teacher. In turn, teacher reflection and questioning performed individually or through consultation, such as with fellow teachers or published resources, can inform future teaching. An example is presented in Figure 2 and discussed below to illustrate this process.



Figure 1. Modelling teacher decision making.

In response to an item asking children to draw a picture of a situation in which they felt they were learning mathematics well, a Grade 5 child drew a classroom situation showing children working individually from their text books (Figure 2) and explained that what most helped her to learn mathematics was "the book because if I don't understand something it will sometimes give a demenstration [*sic*] and then I will understand".



Figure 2. "A situation in which I was learning mathematics well" - Samantha's drawing.

The teacher was surprised by the child's response: she stated that the text was "hardly ever" used in her mathematics classes. The teacher felt that she could not totally change her teaching approach; she would not necessarily use text books more than usual, but would experiment with strategies to meet the child's need for clear demonstrations. She appreciated coming to know the child's perception of what helped her to learn mathematics and thus reflected on the child's response and identified changes that could be trialed in the classroom. This example illustrates that by being prepared to collect data on children's perceptions of mathematics learning, and to reflect and respond in an informed manner, teacher decision making for mathematics teaching can usefully be informed by the articulation of children's beliefs.

With the increasing acknowledgment of children as individuals who learn in different ways and who construct their own meanings (e.g., Australian Education Council, 1991; Ernest, 1991, 1994; Mousley, 1993a), pedagogical and management decisions are made more complex for teachers. This research was based upon the premise that decision-making can be better informed and therefore more beneficial for all through the consideration of pupils' beliefs.

By gaining insights into children's beliefs about the nature of mathematics and learning, as well as children's beliefs about helping factors for the learning of mathematics, teachers can gain a multi-faceted appreciation of individual perspectives.

The importance of beliefs

The importance of beliefs is considered in terms of the stability and centrality of beliefs, as well as the impact and effect of beliefs.

Research reviews indicate that beliefs are commonly seen to be stable (McLeod, 1992; Pajares, 1992), especially as compared to other aspects of the affective domain such as emotions and attitudes (McLeod, 1992). Of importance in terms of the relevance of this study to classroom teachers is the view that "beliefs can be held with varying degrees of conviction" (Thompson, 1992, p. 129), that some beliefs are more central than others, thus more likely to resist change (Rokeach, 1968), and that some are more open to change or manipulation by outside influences such as the teacher.

Considering the general stability of beliefs, and therefore the consistent influence they may have on learners, but also the possibility of change due to the degree of non-centrality of beliefs, it is asserted that it is important that educators are aware of the beliefs of children as young learners of mathematics and thus of the perspectives they may be taking to their learning. Beliefs may impact upon children's reactions to, or interpretations of, what is stated, performed or produced in a mathematics learning situation.

Children's beliefs may affect many aspects of their learning. Beliefs may have more influence "than knowledge in determining how individuals organize and define tasks and problems and are stronger predictors of behaviour" (Pajares, 1992, p. 311). It has been suggested that perceptions of what mathematics is, and is not, may influence approaches to the solving of problems in mathematics (Frank, 1988), may influence the nature of children's participation in meaningful mathematics learning (Franke & Carey, 1997), may impact upon conceptions of specific topics in mathematics, and may affect attitudes, performance, confidence, perceived usefulness of mathematics, and choice of courses or careers (Kouba & McDonald, 1987).

Little systematic research has been undertaken on images of mathematics, with limited investigation of the views of students (Ernest, 1996). Likewise, there appears to be limited

research into young children's beliefs about factors that help in their learning of mathematics. It is appropriate and timely to research these areas.

The articulation of beliefs, as was the intention of the present study, has the potential to benefit both teachers and children as described below.

Benefits of articulation of learner beliefs

It is argued that by being inquisitive about children's beliefs, by questioning past assumptions about children's beliefs, and by developing awareness and knowledge of children's beliefs, teachers can be better informed of children's perspectives and thus may better cater for, and respond to in a more informed manner, the individuals in their mathematics classes. Concrete changes, in factors such as informed and thoughtful verbal interaction, or considered grouping for learning, stemming from insights from the present research or from use of the research procedures in a teacher's own classroom, have the potential also to assist learners. While teachers may not be able to change the curriculum, they can teach mathematics and interact with individuals from a perspective of increased awareness.

It is possible for children to benefit more directly as learners of mathematics from the experience of attempting to articulate beliefs, such as responding to procedures like those used in this study. Articulation requires thought; it may facilitate awareness, reflection and possible questioning of one's perspectives. In turn, learners may become more active in directing their learning. Student questioning can contribute to the development of critical thinking which is seen, for example from a social constructivist perspective (Ernest, 1991), as an important element of mathematical activity. Questioning of beliefs about mathematics and learning has direct relevance for individual children as it is their own learning experiences and their constructed beliefs that they are reflecting upon. This activity may contribute not only to examination and awareness of beliefs, but also to further construction of beliefs about mathematics and its importance, and about factors influencing learning.

As demonstrated in the discussion above, the present research has the potential to benefit both children and teachers. The possible benefits of the study influenced the choice of research method which included the development and use of thirty research procedures (see Chapter 3 and Appendix A). These procedures for gaining insights into young children's perspectives as learners of mathematics facilitate collection of rich and informative responses and are potentially suitable for classroom and research purposes. The procedures were developed by the researcher or adapted from the work of others and were deployed to gain insights that otherwise are not readily available. The tools of investigation and analysis, and the insights into student beliefs, are available for sharing with teachers to develop further their appreciation and understanding of the complexity and subtlety of young children's beliefs and, in turn, potentially to help them cater better for individual learners.

Thus the research had the potential to benefit those involved in the data collection reported in this thesis; it also has the potential to inform and benefit teachers, learners, parents and researchers through future uses of the procedures. The value of the procedures in generating responses used in the analysis within the present study is considered in Chapter 7.

The preparation for the data collection, including the development and use of the procedures, involved many stages but stemmed first of all from the emergence of a personal interest in children's perspectives.

An emerging interest in children's beliefs

The present research developed through a process of evolution, influenced by many experiences during my years as a primary teacher, teacher educator, and researcher.

Nine years of primary school teaching experience nurtured my already existing interest in children and learning. My interest in mathematics and particularly in individual children's perspectives of the learning of mathematics strengthened during my four years (1983 - 1986) as teacher-in-charge of the Brunswick Primary School Mathematics Task Centre visited by children from many schools in Melbourne. The centre was at the forefront of new approaches to mathematics teaching (e.g., McDonough, 1984a, 1985) and provided professional development for many visiting teachers, trainee teachers, and parents. A video of the operation of the centre was made and used for professional development in schools (McDonough, 1984b). With a focus on problem solving and the teacher as a facilitator of children's learning, the centre provided a different view of mathematics, of the organisation of classes, and of the role of children in learning mathematics from what tended to be the case in the majority of classrooms at that time. This is illustrated by the fact that it was not until 1988 that a major mathematics curriculum document was published in Victoria that included problem solving and cooperative learning as key elements of mathematics teaching and learning (Ministry of Education, 1988).

It was the philosophy of the Mathematics Task Centre that children explore mathematical problems independent of structured direction from the teacher, or *teaching by telling*; the role of the teacher was to pose mathematical problems, and question and listen to the students. During my experience in the Mathematics Task Centre I learnt to step back as a teacher, not to provide answers but to expect and allow children to be more responsible for their learning of mathematics. The most important thing I learnt was to listen to the children, paying attention not to what I expected or hoped they would say but to what they chose to say about their thinking and ideas. I found that this could lead to responses and insights I had not expected. Genuine interest and curiosity opened my eyes; the children who visited the centre provided professional development for my teaching as they taught me to listen with interest and to value individuals' contributions to their own learning of mathematics.

My interest in the learning of mathematics continued into the next stage of my career: as a mathematics education lecturer. At that time, I conducted Masters level research that involved the study of the teaching and learning of mathematics in a straight Grade 5 class and in a composite 3/4/5 class at one school through the collection of qualitative and quantitative data from both teachers and children (McDonough, 1991). One simple, but seemingly

effective approach developed for accessing the children's perspectives involved the use of drawing.

The use of drawing to gain insights into children's perspectives was developed further, and children's perspectives on mathematics learning were collected on a larger scale, through development of a procedure with the acronym PPELEM (Pupil Perceptions of Effective Learning Environments in Mathematics). The procedure involves drawing along with description through a questionnaire (McDonough, 1995, 1998a; McDonough & Wallbridge, 1994). PPELEM was used with 1816 Grade Prep to Grade 6 children from schools in Victoria, Australia, and in Alberta, Canada (McDonough & Wallbridge, 1994). The study gave interesting results but it seemed that possibly there were reasons behind the children's responses that had not been tapped fully with use of only drawings and questionnaires.

In summary, my experience in the Mathematics Task Centre stimulated my interest in individual children's perspectives about their mathematics learning. My Masters study helped crystallise my belief in the value of exploring children's perspectives in a focused manner, but it was the PPELEM research conducted in Victoria and Canada that inspired me to go further in exploring children's perspectives. After reflecting on my teaching and research experiences I re-directed my interest to focus more on individual children's perspectives, that is, on exploring subtleties and intricacies within and behind children's expressed thoughts. This doctoral study is the result of that re-direction.

Following from the intention to explore children's perspectives in a more focused manner, I identified a need for development and identification of appropriate methods to gain insights into children's beliefs. My experience as a teacher in single-class and team-teaching situations, and in teacher development, to a small degree while running the Mathematics Task Centre, and to a larger degree as a mathematics education lecturer and school in-service presenter convinced me that there was a need for development and dissemination of tasks to explore children's beliefs about mathematics learning. Literature in the field of children's beliefs suggests there has been limited research into, and information about, young children's beliefs about the nature of mathematics, about learning, and about helping factors for their learning of mathematics. Thus, in the present study, I posed myself the dual challenges of developing procedures for helping children express their beliefs, and of gaining insights into young children's beliefs through the use of these procedures. The analysis of the resultant data and subsequent reporting involved making sense of each child's responses to thirty procedures used in ten interviews, to draw out key points, and to present them in a representative and coherent manner. The search for patterns and relationships in children's responses to the procedures led to the identification and discussion of themes observed within each child's expression of beliefs, adding an analytic dimension to the research, that is, a dimension beyond simple reporting and description.

Compared to my earlier broad brush approach to research of children's beliefs through the use of PPELEM (McDonough & Wallbridge, 1994), the present study took a more focused perspective, seeking to gain depth of insight into individual young learners' beliefs. The focus on individual children's beliefs related not only to my interests but also to theoretical perspectives of children's learning.

Theoretical orientation of the research

The decision to focus on children's beliefs emerged as a result of a number of factors, as discussed above. Central to these was an underlying belief that the learner is at the centre of the individual's learning of mathematics, thus the research sought the views of mathematics learners.

The research has most in common with the ideology of Public Educators, the philosophy of which is social constructivism (Ernest, 1991). There is

a respect for each individual's rights, feelings and sense-making . . . [and] . . . children and other persons are seen as active and enquiring makers of meaning and knowledge . . . [with] . . . internal constructions resulting from social interactions and the 'negotiation of meaning'. (Ernest, 1991, p. 198)

By incorporating the articulation of beliefs about the nature of mathematics and learning, the research addressed the Public Educator call for learner "conceptions and assumptions. . . to be articulated . . . and challenged, to allow the development of critical thinking" (Ernest, 1991, p. 208), and through the articulation of beliefs about factors that help in the learning of mathematics, the research took account of the Public Educator call for learner questioning of pedagogy.

The study is not compatible with a totally child-centred ideology to school mathematics and schooling such as that of the Progressive Educator, where children are seen to require "nurturing, protection and enriching experiences to allow them to develop to their full potential . . . [and are seen as individuals] . . . whose needs and rights are paramount" (Ernest, 1991, p. 182). Underlying the present research are other factors that also are considered important to mathematics learning. These include the mathematics curriculum, an awareness of mathematics, and an engagement with mathematics. In addition, the teacher is considered to play a central role through planning learning experiences and through contributing to the construction of the learning environment; by becoming aware of children's beliefs about mathematics and learning the teacher can be better informed for teaching as discussed above.

The research is generally compatible with the social constructivist view that mathematical knowledge is individually constructed and socially determined (Ernest, 1991). As indicated in the introduction to this chapter, the research extended this view and made the assumption that beliefs as well as understandings are individually constructed (e.g., Yackel & Cobb, 1996). This assumption impacted upon the reasoning for, and structure of, this research; the focus became the essence of experience, the meaning or sense-making of individuals.

In taking a theoretical perspective, the research did not subscribe to a particular model of developmental learning such as that of Piaget (Piaget & Inhelder, 1969), or a hierarchy of

cognition such as that of van Hiele (e.g., Pegg, 1985), or a taxonomy of learning such as the SOLO Taxonomy (Biggs, 1991). The research evolved from an interest in learning from people so as to develop some understanding of how individuals see the world, and, as discussed in Chapter 3, was underpinned by elements from a range of theoretical perspectives including phenomenological, ethnographic, constructivist and interpretive. The analysis of data was not structured according to a pre-existing theory, but themes (van Manen, 1990) were allowed to emerge from the children's data. The inclusion of open-ended questions for data collection and the absence of pre-determined categories for analysis facilitated communication by the eight children, and identification by the researcher, of a range of beliefs and relationships between them. The portrayal of beliefs reflects the children's individual constructions and orientations according to insights gained, and inferences made, by the researcher.

The beliefs of individuals were approached from three perspectives, as outlined below.

A research model

Figure 3 presents a model illustrating the hypothesised relationship between the three key areas of interest in the research: children's beliefs about mathematics, about learning, and about helping factors in their own learning of mathematics. It shows the overall focus of the study as an exploration of children's perspectives. Children's beliefs about mathematics and learning are represented as underpinning their beliefs about factors in the learning environment that help in their learning of mathematics. A cyclic model is presented, with perceptions of helping factors in turn being proposed to have some impact upon personal beliefs about mathematics and learning. Thus, this model represents a reciprocal relationship between two sets of beliefs.



Figure 3. Children's perspectives - the basic model.

It was assumed that children's beliefs regarding learning and mathematics, underpin, and are intertwined with, their perceptions of themselves as learners of mathematics. Because of this relationship, individuals' beliefs about helpful factors for mathematics learning could not be examined and written about adequately without consideration of individuals' broader conceptual meanings for learning and mathematics. As Pajares (1992) notes, "subject specific beliefs, such as beliefs about . . . mathematics . . . are the key to researchers' attempting to understand the intricacies of how children learn" (p. 308).

To gain insights into children's beliefs about helping factors in their own learning of mathematics, factors perceived to hinder mathematics learning were considered also. These might have included reference to external objects and situations as well as reference to the children themselves, since their own responses, thoughts and actions could be identified as perceived factors of influence. In this research, the term *learning environment* was used to encompass these many possible factors of influence. The use of this term is discussed in Chapter 2.

Along with the Figure 3 model, which provided a basic framework for the research, three research questions were developed.

Defining the subject of the research portrayal

The study was structured around three research questions:

- 1. Do young children hold beliefs about mathematics, learning, and helping factors for mathematics learning that can be articulated and portrayed from responses to procedures developed for this research?
- 2. What beliefs do children hold about the nature of mathematics and the nature of learning?
- 3. What factors within learning environments do children believe help them to learn mathematics well?

The main purposes of the research were to

- explore whether young children's beliefs could be articulated and portrayed;
- develop procedures for use in the present study, and for potential use in classrooms and other research studies, for gaining insights into young learners' beliefs about learning, mathematics, and helping factors for mathematics learning;
- explore and discuss beliefs as portrayed by eight children of eight to nine years of age;
- gain insights into possible complexities and subtleties of young learners' beliefs;
- reflect upon the insights and their significance for the mathematics classroom; and
- reflect upon the value of the procedures developed for the study.

Eight Grade 3 children of eight to nine years of age from two suburban schools were chosen by their teachers to participate in the study, according to achievement level and gender criteria given by the researcher (discussed more fully in Chapter 3). The children completed a range of tasks including drawing, describing, discussing, sorting words and pictures, building, and some writing, conducted in a one-to-one interview situation with the researcher. The procedures and their administration, as well as a small number of supplementary forms of data collection, are discussed in more detail in the Chapter 3.

The semi-structured nature of the interview data collection and the openness of the analysis allowed for the children's perspectives and emphases to emerge from the data. The data collection procedures were developed in four domains: one to give *background*

information on the young learners so they could be introduced to the reader, one to explore beliefs about *mathematics*, one to explore beliefs about *learning*, one to explore beliefs about *helping factors*. However, some responses were found to relate to more than one domain, and therefore responses to items in one domain sometimes gave insights in relation to all of the research questions. Some overlap of beliefs had been anticipated, as represented in the reciprocity of the two sets of beliefs as portrayed in Figure 3. The overlap facilitated appreciation and understanding of the intricacies and subtleties of young children's beliefs and added validity to the research findings, as discussed in Chapter 3.

Each child's beliefs were examined from the perspectives of the three key domains within research questions 2 and 3, that is, mathematics, learning, and helping factors for the learning of mathematics. Further analysis was determined by themes that emerged from each child's responses; a detailed framework for discussion was not pre-determined prior to the analysis of data. The research was designed and conducted according to the premise that it was important to allow, and even to facilitate, the individual children's constructions, emphases and themes to frame their data and results so as to gain understanding of perspectives the children brought to their learning of mathematics. The research report provides a portrayal of young children's beliefs; verbal, and at times diagrammatic descriptions or collages of beliefs are developed by the researcher by drawing upon excerpts from interview transcripts and other interview artefacts. Theory about children's beliefs developed within the research, not prior to conducting the study.

The research facilitates appreciation of children's beliefs through the use of a range of qualitative procedures differing as a collection from those used in previous studies. Most studies on affective issues in mathematics education, including beliefs about oneself as a learner, and studies on students' views towards subject matter have involved the use of questionnaires and other quantitative methods (McLeod, 1994; Stodolsky, Salk, & Glaessner, 1991). The present use of a range of qualitative procedures provides data that add to the field by complementing previous studies. In addition, the present approach contributes to the provision of new insights and new conceptions of research on affective issues, as called for by McLeod (1994).

The research adds in a positive sense to teacher knowledge about learners in mathematics classes; it helps teachers develop insights into, and cater for, individual differences among learners, a challenge that continues for teachers (Romberg & Carpenter, 1986). It cannot be assumed that children in the one class will all learn in the same way or develop the same beliefs about learning mathematics; the idea that children are the same and learn in the same way whatever the social context, is strongly questioned (Langford, 1989). Teachers know from experience that children learn in different ways, but teachers may not be aware of the uniqueness of children's beliefs. Cobb (1985) encourages teachers to talk to children about mathematics – their mathematics, to help children come to see that mathematics involves understanding and gaining of insights. Talking with, listening to and

asking questions of students to gain insights into beliefs, feelings, understandings, experiences, and interests can assist teachers to come to know and understand individual students better, appreciate their personal perspectives in learning, and provide more effective teaching (e.g., Corbitt, 1984; Kersaint & Chappell, 2001; Lindenskov, 1993).

Teachers can be informed by the insights they gain into pupil perspectives and, in turn, children can contribute actively and constructively to classroom practice and teacher decision-making.

Within investigations of effective teaching, some major research studies (e.g., Askew, Brown, Rhodes, Johnson, & Wiliam, 1997; Clarke, 2001; Wright, 1998) have focused on gaining indepth insights into individual children's understandings. In turn these studies have impacted upon classroom practice. In the Early Numeracy Research Project (ENRP), in which the researcher is involved, one key element has been the development and use of a one-to-one interview of approximately 30-40 minutes, conducted by ENRP class teachers with each child they teach (Clarke, 1999, 2001). Although that interview focuses on gaining insights into children's understandings rather than beliefs, the experience of the researcher in focusing on individual children within the present study contributed in part to the development of the ENRP interview.

Sketching the research portrayal

The present study found that the beliefs of children as young as eight years of age can be complex, subtle and idiosyncratic. Illustrating this, detailed case studies of two children are provided in Chapters 5 and 6, along with a summary and discussion of results from all eight children in Chapter 7, substantiated further by descriptions and discussion of findings for three children in Appendix E. The detailed descriptions for five children make clear the thorough manner in which data for the children were analysed and synthesised, and illustrate the manner in which summaries for the remaining three children, as provided in Chapter 7, were constructed.

Both the simplicity and complexity of children's beliefs are reflected in the title of this thesis. Children may appear naïve on the surface but deeper investigation shows that their beliefs may be complex; thus the eight child participants in this study are portrayed as both naïve and knowing. There is some analogy with the naïve style of art in which paintings are child-like and "do not follow any particular movement or aesthetic" (Piper, 1998, p. 369). Like portrayals of landscape or everyday events, the portrayals of beliefs in this research can contain subtle detail and complexity. The analogy with art is strengthened through viewing this thesis as a *portrayal* of children's beliefs. The eight children participating in the study were the key contributors to this portrayal: their spoken, written and pictorial data gathered during research interviews provided material from which interpretations and inferences were made by the viewer, in this case, the researcher. The analogy with art is pursued in the chapter titles as well as in the thesis title.

A discussion of ideas and research findings from literature that provides background to the present study is presented in Chapter 2. A discussion of the research methodology including details of the research method, for example, the materials, techniques, and procedures contributing to the portrayal of beliefs is provided in Chapter 3. In Chapter 4 the eight children and their experiences at school are introduced briefly. The discussion of findings begins in Chapter 5 in which the responses from one research participant, Cara, are analysed and discussed in detail. Responses from another participant, Emily, are discussed in detail in Chapter 6. Through a discursive and diagrammatic approach for each child, beliefs are presented according to themes that emerged from the data, and are built up gradually with reference to interview transcripts and other interview products such as drawn and written material to validate inferences made by the researcher. Chapter 7 contains a comparative portrayal and discussion of beliefs of all eight children presented through reference to the research questions and related overall themes within the children's data. This is accompanied by reflections on the value of the procedures developed for the study. Chapter 7 includes also discussion of conclusions, recommendations and implications stemming from the study.

CHAPTER TWO

MATHEMATICS, LEARNING, AND INFLUENCES UPON THE LEARNING OF MATHEMATICS: INSIGHTS INTO THE BROADER PICTURE

One objective of the present study, as expressed in the third research question, was to investigate primary school children's beliefs about factors that help them to learn mathematics. It was anticipated that this research would provide insights into children's perceptions of their own learning process. Because the foundation of this work was mathematics learning, an important component of the study was the consideration of the conceptions of both mathematics and learning held by the children participant in the study. Consideration of factors perceived to impact negatively on mathematics learning could also provide insights regarding factors perceived to be of positive influence in learning.

Thus the research focused upon children's beliefs about

- mathematics:
 - the nature of mathematics and mathematical activity;
- learning:
 - the nature of learning;
- themselves as learners of mathematics:
 - what it is to learn mathematics;
 - what factors positively influence their learning of mathematics; and
 - what factors negatively influence their learning of mathematics.

Extending from the key areas of interest in this research, as listed above, the discussion within the present chapter of the broader picture includes consideration of

- conceptions of mathematics and mathematical activity;
- theories and conceptions of learning, particularly of the learning of mathematics; and
- previous research on children's perceptions of factors within mathematics learning environments which influence their learning of mathematics.

As each of these areas is broad, the discussion provides an overview of key ideas, drawing on major theories, on the findings of a selection of research studies, and, where appropriate, on interpretations and recommendations within curriculum documents.

Discussion of some key terms is included also. Within the second research question, "What beliefs do children hold about the nature of mathematics and the nature of learning?" the key terms *learning* and *mathematics* are introduced. The nature of mathematics and mathematical activity, and the nature of learning, have been, and continue to be, subjects of discussion within the education community. The present research contributes to the ongoing discussion by adding the voices of young learners of mathematics. The study was informed by previously published perspectives and research findings; a range of meanings and interpretations for the terms mathematics and learning are considered below. The third research question, "What factors within learning environments do children believe help them learn mathematics well?" introduces to the research the element of *learning environment* and, like the second research question, makes reference to *beliefs*. It was not the intention that children's meaning for these concepts be sought, but that the concepts provide a framework within which children's perspectives on mathematics, learning, and helping factors could be investigated. Thus the establishment of a working definition for *learning environment* and *beliefs* was important and is developed in the discussion below, along with consideration of previous research findings where appropriate.

This chapter provides a portrayal of the broader picture, that is, of the background in which the present research was situated. In response to the key ideas expressed in the research questions, the following are discussed in Chapter 2:

- beliefs;
- mathematics and mathematical activity;
- learning;
- learning environment;
- perceptions of factors of influence in mathematics learning.

Beliefs

The discussion begins with consideration of the meaning of *beliefs*, as it is beliefs that provide a base, or what might be called the canvas, for the portrayal of all other elements within the study. A range of meanings for beliefs is considered, and the meaning assumed for the present study identified.

The difficulty of defining beliefs has been documented (Hart, 1989; Pajares, 1992; Thompson, 1992) and is reflected in the range of meanings expressed in different research studies. Pajares (1992) states that

defining beliefs is at best a game of player's choice. They travel in disguise and often under alias–attitudes, values, judgements, . . . perceptions, conceptions, . . . perspectives, repertoires of understanding, . . . to name but a few that can be found in the literature. (p. 309)

He adds that while definitions differ, or beliefs are not well defined,

the chosen and perhaps artificial distinction between belief and knowledge is common to most definitions: Belief is based on evaluation and judgement; knowledge is based on objective fact. (p. 313)

Thompson (1992) considers a distinction between beliefs and knowledge. She states that beliefs "can be held with varying degrees of conviction" (p. 129) and notes that philosophers tend to associate knowledge with certainty or truth, and beliefs with disputability, which suggests that beliefs are not consensual. She suggests also that, over time, a belief can become knowledge if supported by new theories, as can the reverse occur.

Knowing whether a statement of belief expressed by a child is consensual in the community of learners in which the child operates, or even in the broader community, can be problematic. It appears that what to one person is a belief may to another be knowledge, what

at one time is a belief may become knowledge, and what at one time is knowledge may become a belief. It appears also that some overlap between beliefs and knowledge may occur.

While acknowledging the differing approaches and definitions, (see for example, Pajares, 1992), the definition of beliefs as given by Rokeach (1968) was used as the baseline for the approach taken in the present study. Rokeach defined beliefs as "any simple proposition, conscious or unconscious, inferred from what a person says or does, capable of being preceded by the phrase, 'I believe that . . ." (p. 113). As discussed in detail in Chapter 3, the present research investigated eight children's beliefs through the use of a number of procedures that produced a range of data. The variability of the procedures, which led to written, pictorial and spoken responses, facilitated the inference process and contributed to the validity of the research. The individual children's interview responses did not provide statements that began with "I believe that . . ." but provided data from which inferences could be made regarding what the children believed.

Children's beliefs about learning and mathematics and their beliefs about helping factors for learning mathematics are considered as members of the set of their perspectives. The term *perspectives* is used in this research in a broad, encompassing manner. Following the doctoral study by Janesick (1977, cited in Clark & Peterson, 1986, p. 287), a perspective is defined as "a reflective, socially derived interpretation of experience that serves as a basis for subsequent action . . . perspective combines beliefs, intentions, interpretations, and behaviour that interact continually". According to Clark and Peterson, Janesick used this term in relation to teachers; in the present research it is used mainly in relation to learners.

A similar broad approach was taken by Thompson (1992) in her study of teachers' beliefs. She used the term *conceptions* to include "conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences" (p. 132). As in the present study, Thompson did not discuss beliefs in a vacuum but described and accessed them within a broader network of interacting beliefs, preferences, views, and concepts. In the present study this network is called *perspectives*.

In the present research report, the term *perceptions* is used at times in place of *beliefs*. For the purpose of the study it was assumed that this term has the same meaning as that deployed for beliefs. Thus the terms beliefs and perceptions are used interchangeably and are encompassed within a person's perspectives.

Beliefs are related to and interact with attitudes and emotions (McLeod, 1992; Rokeach, 1968); each can be considered a member of the *affective domain* (Fennema, 1989; McLeod, 1992). This term is used in preference to *attitudes* which, although sometimes used to include beliefs about oneself as a learner and beliefs about mathematics, can also be used in a more narrow sense by referring to liking, disliking and related preferences (McLeod, 1992).

Beliefs, attitudes and emotions encompass a range of affective responses: emotions are *hot* reactions that can change rapidly (Mandler, 1989); beliefs and attitudes are generally assumed more stable in nature. Each of these three affective responses differs in the degree to

which a cognitive component is involved in its formation: "we can think of beliefs, attitudes, and emotions as representing increasing levels of affective involvement, decreasing levels of cognitive involvement, increasing levels of intensity of response, and decreasing levels of response stability" (McLeod, 1992, p. 579).

Traditionally beliefs have been associated with the affective domain (McLeod, 1992) but beliefs have the potential to draw together affective and cognitive aspects of learning by functioning as a single, unified dimension (McDonough & Clarke, 1994). Statements such as "The sum of the angles in a triangle is 180 degrees" and "Mathematics is boring" might be statements of belief: the first a cognitive belief and the second an affective belief. Importantly for this research, each statement could be justified as personal opinion or empirical fact by the person making the statement, and each could be preceded by the phrase, "I believe that", thus matching Rokeach's definition of beliefs, as discussed above. Applying Rokeach's (1968) theory, the first statement above might be classified as a descriptive or existential belief, the second an evaluative belief. This discussion of the statements shows that beliefs can be associated with both cognitive and affective domains.

From the use of the procedures designed for this study there was potential for a unified view of mathematics to arise, one that encompasses the two traditional research domains of cognitive and affective aspects of mathematics learning. It was not the intention that only *cognitive* beliefs or only *affective* beliefs be examined, but rather that through the use of procedures sufficiently non-coercive in nature, both types of beliefs could arise. Indeed, it was assumed that an interrelatedness of the *affective* and *cognitive* might emerge, depending of course on the perspectives of the children participant in the study. A response to the prompt "Maths is like", given by an eleven year old female during the research procedures trialing process, illustrates the possibility of an interrelatedness of cognitive and affective elements within the expression of beliefs. The respondent wrote,

Maths is like a fun activity but it is also a job we have to do. I like maths because it really makes you think about what you'r[e] doing. At other times maths is like a weed that grows and grows when I'm having a bad day. Maths is changing and the better you get at it the harder it gets. The things we do are quite fun. I like maths.

One conclusion to draw from this response is that the child relates to mathematics personally, that it is something about which she has developed strong feelings. A cognitive aspect, mathematical thinking, also is important to her. In contrast, a reply from another interviewee during the trialing stage related only to the cognitive side of mathematics. A twelve year old male wrote,

Maths is like adding dividing timising & takerwaying. waying things mezering distenses [*sic*].

Thus beliefs in this research were not sought solely in either cognitive or affective terms. The research responded to the increasing awareness in the mathematics education community of the importance of studying both cognitive and affective aspects when exploring our understanding of children's learning of mathematics (e.g., Clarke, 1996; Leder, 1993; Tirosh,

1993). The integrated perspective reflects the reality of the interrelatedness of affective and cognitive learning in the mathematics classroom (e.g., Clarke, 1987; McDonough & Clarke, 1994).

Beliefs expressed in response to the procedures in this research belong to the belief system of each child. The belief system is the organisation or structure in which beliefs are held, with "no beliefs in total independence of all other beliefs" (Thompson, 1992, p. 130). Belief systems may be restructured or further developed as children evaluate beliefs against experiences (Thompson, 1992), thus it is worthwhile for teachers and researchers to investigate and gain insights into children's beliefs, and, in turn, for teachers to consider maintaining or refining elements within the mathematics classroom such as style of tasks, classroom organisation, or the locus of decision-making (McDonough & Gervasoni, 1997).

The present research provides a canvas on which children's beliefs about mathematics, learning, and helping factors for learning mathematics are portrayed. To provide background for the consideration of these issues from the perspective of the eight research participants, and to inform the interpretation of the children's responses, the following discussion considers the issues from a broader perspective, beginning first with mathematics and mathematical activity.

The nature of mathematics and mathematical activity

A key element of this research is the perspectives of young children on the nature of mathematics and mathematical activity. These issues have been addressed for thousands of years and the debate continues. The beliefs of the children are not juxtaposed against the strands of the overall debate but it is useful to examine briefly the way different groups have interpreted mathematics and mathematical activity and to identify themes that can inform the analysis of the children's responses. The perspectives considered here include those of philosophers, mathematicians, students, mathematics teachers, and those related to the school curriculum particularly as conveyed in major documents. The perceptions of the broader community are considered also, followed by discussion of the concept of numeracy. The focus is mainly on perspectives of different groups of people; this focus is apparent also in the present research, but with a group of young children contributing their beliefs. The discussion begins by drawing briefly on an historic perspective as background to an examination of philosophies and conceptions of mathematics as they exist today.

Philosophical perspectives

A common theme permeating writing on the philosophical dimension of mathematics is that of competing perspectives on the nature of mathematics. At one end, mathematics is seen as fixed and either discovered or waiting to be discovered; at the other mathematics is seen as, and interpreted as, a social construction. Dossey (1992) explains that the nature of mathematics was discussed as far back as the fourth century BC, with Plato and Aristotle two major contributors. Dossey attributes the moulding of two major contrasting themes concerning the nature of mathematics to Plato and Aristotle, with Plato's view of mathematics based on a theory of an external, independent, unobservable body of knowledge, which involved abstract mental activity, and Aristotle's view "based on experienced reality, where knowledge is obtained from experimentation, observation, and abstraction" (Dossey, 1992, p. 40). Plato believed that "objects of mathematics have a real, objective existence in some ideal realm" (Ernest, 1991, p. 29), and are independent of the mind (Pateman, 1989). In contrast, Aristotle saw the empirical world, as it existed in perceptible concrete objects, as being fully real (Tarnas, 1991). According to Dossey (1992) the views of Plato and Aristotle ran as an undercurrent in later schools of thought.

Portrayal of contrasting perspectives of the nature of mathematics or of mathematical knowledge is apparent also, for example, within the works of Ernest (1991) and Fisher (1990). Ernest (1991) contrasts competing perspectives, absolutist and fallibilist, on the certainty associated with mathematics. Mathematics within an absolutist paradigm is "a body of infallable, objective knowledge" (Ernest, 1991, p. xii), and "mathematical truth is [considered] absolutely certain" (p. 3). In contrast, from the fallibilist perspective, mathematics is a "fallible, social construct . . . a process of enquiry and coming to know, a continually expanding field of human invention and creation, not a finished product" (p. xii), and "can never be regarded as being above revision and correction" (p. 18). Fisher (1990), also identifies two paradigms: firstly, mathematical knowledge as an external and objective entity "always in existence [from which] occasionally mathematicians unconceal or discover another element" (p. 82), and, secondly, mathematics as an internal and subjective entity, where knowing and doing are inseparable, and mathematics is created by social groups. Dossey (1992), Ernest (1991), and Fisher (1990) each propose a set of opposing views in which one side views mathematics as an external body of knowledge which has an existence of its own and the other side portrays mathematics as human activity where knowledge is abstracted or created. This summary reduces subtle distinctions and makes a "crude dichotomy" (Fisher, 1990, p. 82) of a more complex situation, including the existence of more than one absolutist school of thought (Dossey, 1992; Ernest, 1991). However, the discussion provides key perspectives that act as background for the consideration of further literature regarding the nature of mathematics and for the consideration of beliefs of the eight children involved in the present study.

Ernest (1991) believes all absolutist schools of thought are inadequate as philosophies as they do not account for external social and historical factors within the nature of mathematics, or the genesis and utility of mathematics. Ernest believes that the "philosophy of mathematics should include external questions as to the historical origins and social context of mathematics, in addition to the internal questions concerning knowledge, existence, and their justification" (p. 26). He finds untenable "the hypothesis that mathematical knowledge is a set of truths, in the form of a set of propositions with proofs, and that the function of the philosophy of mathematics is to establish the certainty of this knowledge" (p. 23). Drawing on, and going beyond, earlier perspectives, Ernest (1991) proposes and discusses *social constructivism* as an alternative philosophy of mathematics:

Social constructivism views mathematics as a social construction. It . . . [accepts that] human language, rules and agreement play a key role in establishing and justifying the truths of mathematics. It takes . . . [the] fallibilist epistemology . . . that mathematics knowledge and concepts develop and change. It adopts . . . [the] thesis that mathematical knowledge grows through conjectures and refutations, utilizing a logic of mathematical discovery. (Ernest, 1991, p. 42)

The genesis of mathematical knowledge is portrayed as a central focus within the social constructivist philosophy. The social group plays a key role in the development of mathematical knowledge through involvement in a cyclic process which involves both subjective and objective knowledge with each contributing to the renewal of the other:

In this cycle, the path followed by new mathematical knowledge is from subjective knowledge (the personal creation of an individual), via publication to objective knowledge (by intersubjective scrutiny, reformulation and acceptance). Objective knowledge is internalized and reconstructed by individuals, during the learning of mathematics, to become the individual's subjective knowledge, thereby completing the cycle. (Ernest, 1991, p. 43)

An individual's knowledge is subjective, but, when published through linguistic representation, typically in written form, can become objective knowledge if socially accepted following critical public scrutiny. Ernest accepts the fallibility of mathematical knowledge but also accepts an objectivity of mathematics, where mathematical knowledge and objects of mathematics can have social objectivity.

The dual acknowledgment of the socially constructed nature of mathematics, and therefore subjectivity, along with the objectivity that comes with publication or general agreement on usage, was implied by the mathematician, Hersh, who wrote,

- 1. Mathematical objects are invented or created by humans.
- 2. They are created, not arbitrarily, but arise from activity with already existing mathematical objects, and from the needs of science and daily life.
- 3. Once created, mathematical objects have properties which are well-determined, which we may have great difficulty in discovering, but which are possessed independently of our knowledge of them. (Hersh, 1986, p. 22)

Hersh's third point is consistent with the further idea of social constructivism of the existence of the world beyond the subjective realm of experience:

The humanly constructed reality is all the time being modified and interacting to fit ontological reality, although it can never give a 'true picture' of it. . . . there is a world out there supporting the appearances we have shared access to, but we have no certain knowledge of it. (Ernest, 1994, pp. 8 - 9)

However, Ernest (1991) notes that in the social constructivist view of mathematics, mathematical entities exist only while humans exist, they do not have a more permanent existence, thus the rejection of the Platonist view of mathematics as an external, independent, unobservable body of knowledge. Labinowicz (1985) writes also of the relationship between people and reality:

Rather than passively copying knowledge that exists "out there", we actively construct our knowledge of the world internally through conscious interaction with the environment. Each person's understanding is like a personal painting resulting from one's own interpretation and synthesis of reality. It is not a photograph of reality. It is a person's internal network of ideas that interacts with reality in a mutual transformation. (p. 5)

Ernest (1994, p. 9) summarises social constructivism by positing its model of the world as a "socially constructed, shared world" and its metaphor for the mind as "persons in conversation". He stresses the centrality of socially constructed knowledge which is created and modified through shared experience of an underlying physical reality.

The present research draws on the social constructivist philosophy as it is the perceptions of reality, as individually constructed by each of the eight research participants, that is of interest. The research attempts to present a portrayal of what might be called each individual's *personal painting*.

Another form of contructivism, radical constructivism, originating from Piaget and others and expressed as a theory of knowledge by von Glasersfeld (Ernest, 1994; Pateman, 1989; von Glasersfeld, 1989) continues to be considered today in relation to mathematics education.

Ernest (1991) believes that von Glasersfeld's two principles suggest a purely subjective view of knowledge. These principles are,

- a. knowledge is not passively received but actively built up by the cognizing subject; and
- b. the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality. (von Glasersfeld, 1989, p. 162)

The acceptance of both of von Glasersfeld's principles represents the view of a radical constructivist. The first principle is accepted by all varieties of constructivism (Ernest, 1994, 1996), including social and radical constructivism; both positions recognise the active, individual and personal aspects of knowing, with previously constructed knowledge providing a base. They also both accept that the world cannot be known with any certainty; the underlying epistemology for both is fallibilist. But for radical constructivism, mathematics is a subjective entity only. Ernest (1994, p. 7) comments that radical constructivism makes

no presuppositions about the existence of the world beyond the subjective realm of experience. The epistemology is wholeheartedly fallibilist, sceptical, and anti-objectivist . . . there is no ultimate true knowledge possible about . . . such realms as mathematics

... all knowledge [is] constructed by the individual on the basis of its cognitive processes in dialogue with its experiential world.

Ernest (1994) expresses the view that interpersonal concerns, and shared feelings and values, which are issues of importance in education, are in danger of not being accommodated within radical constructivism. He suggests that, in contrast, the complementarity between individual construction and social interaction is accommodated in social constructivism, as it

has both a social and individual focus; the creation of mathematical ideas is considered to involve both individual and social elements.

The discussion of philosophical perspectives has indicated that views of the nature of mathematics and mathematical activity can vary greatly. Over time there has been development in thinking about the nature of mathematics, but still today there is no consensus within philosophical views of the nature of mathematics. The discussion suggests that the extremities of beliefs can be represented by the absolutist and fallibilist perspectives, although not all writers use these terms or couch their beliefs in these philosophies. Ernest (1991) believes absolutist philosophies dominated previously, but that an increasing number of philosophers are now adopting a fallibilist position.

While making no inferences on the way the belief statements of children are to be interpreted, the general approach underlying this research is compatible with the socially constructed view of the nature of mathematics. The research is interested in the views of the children precisely because children's constructions of mathematics, learning, and helping factors for learning mathematics are likely to be a product of their experiences and their interactions in social groups during their lives of eight or nine years.

The background discussion to the consideration of the eight children's beliefs continues with consideration of the perspective of mathematicians on the nature of mathematics and mathematical activity.

Mathematicians' perceptions of the nature of mathematics

Mathematicians are a major group involved in the activity of mathematics whose views present a perspective alternate to that of philosophers to inform the analysis of the children's beliefs. The following discussion does not seek to represent the views of all mathematicians. However, it gives a flavour of the diversity of beliefs within this group.

Among mathematicians there is no commonly held view of the nature of mathematics (Dossey, 1992; Pateman, 1989; Schoenfeld, 1992). Indeed, Dossey believes that mathematicians, in general, think little about the nature of mathematics.

It is reported that the mathematician, Hardy, was considered by many to believe that mathematics exists *out there*, and that it is the task of the mathematician to find it (Pateman, 1989). Similarly, Newton considered mathematical truths objective and available for discovery for those able to do so (Pateman, 1989). However, in contrast, Einstein believed mathematics was "not discovered because it is already there, but constructed logically from relations between ideas" (Pateman, 1989, p. 14).

In recounting Hersh's description of the work of a mathematician, Dossey (1992) states that, when creating new mathematics, "the mathematician works as if the discipline describes an externally existing objective reality. But when discussing the nature of mathematics, the mathematician often rejects this notion and describes it as a meaningless game played with symbols" (Dossey, 1992, p. 42). This suggests a tendency for mathematicians to carry strong Platonic views, that is, that mathematical concepts are believed to exist outside the mind, but

when pushed, mathematicians move away from this view. Hersh believes a shift in philosophy is needed. As discussed above, mathematics as creative human activity is central to Hersh's beliefs. He states that "mathematics deals with ideas. Not pencil marks or chalk marks, not physical triangles or physical sets, but ideas (which may be represented or suggested by physical objects)" (Hersh, 1986, p. 22).

Steen (1988) portrays contrasting views of the nature of mathematics within society by speaking of the views of mathematicians, scientists and engineers. Steen believes mathematicians "see their field as a rapidly growing rain forest, nourished and shaped by forces outside mathematics while contributing to human civilization a rich and ever-changing variety of intellectual flora and fauna" (Steen, 1988, p. 611). In contrast, Steen portrays many other educated persons, such as scientists and engineers seeing mathematics "as akin to a tree of knowledge: formulas, theorems, and results hang like ripe fruits to be plucked by passing scientists to nourish their theories" (Steen, 1988, p. 611).

A study of conceptions of mathematics of different groups in society (Wallbridge & Clarke, 1989) found that mathematicians, along with Year 8 students, were most conservative in the extent to which they were willing to acknowledge mathematics in a range of everyday activities, suggesting that mathematical activity was seen by these two groups as an end in itself.

The mathematician, Polya, conceptualised mathematics as problem solving; mathematical engagement was central in his view, and mathematics, like the physical sciences, was seen to be dependent on guessing, insight, and discovery (Schoenfeld, 1992). Likewise, Halmos claimed that "what mathematics *really* consists of is problems and solutions" (Halmos, 1980, p. 519). Schoenfeld's beliefs about mathematical activity centre around pattern-seeking. He believes that mathematical scientists "engage in the science of patterns–systematic attempts, based on observation, study, and experimentation, to determine the nature of principles of regularities in systems defined axiomatically or theoretically ("pure mathematics") or models of systems abstracted from real world objects ("applied mathematics")" (Schoenfeld, 1992, p. 335). By discussing his own views and those of Polya and Halmos, Schoenfeld gives a different image of mathematics from that, for example, conveyed by Hardy and Newton, as reported by Pateman.

Hatano (1996) speaks also of mathematicians dealing with patterns, structures and relationships, attributing these ideas to Piaget. He contrasts this activity with that of the majority of other users of mathematics who he believes want to know how and when to use ready-made procedures. However, he believes that within this latter group there are those who "try to understand, adjust, or elaborate the procedures . . . like mathematicians, to *do*, and not merely use, mathematics" (p. 207). This comparison helps to clarify Hatano's view of the mathematical activity of mathematicians.

A different view of mathematics is given by the mathematicican, D'Ambrosio (1985), who describes mathematics as a cultural system. He sees culture as a broad concept, asserting

that social and age groups act within cultures of their own; the culture of each group includes behaviour, values and expectations. He goes further to say that cultural groups such as children, engineers, and farmers "develop their own pattern of behaviour, their own symbols and codes and their own way of mathematizing, in other words, their own Mathematics" (p. 42). D'Ambrosio's reference to mathematics as a cultural system suggests that he does not see mathematics as an externally existing objective reality, rather it is something that people develop or create within their own cultural group.

The discussion above indicates that mathematicians hold a broad range of views that exhibit a lack of consensus regarding the nature of mathematics. Just some of the themes regarding the nature of mathematics include mathematics as a set of truths available for discovery, mathematics as an externally existing objective reality, mathematics constructed logically from relations between ideas, mathematics as an end in itself, mathematics as problem solving, mathematics as a cultural system, and mathematics as the science of patterns. There is scope for further change in the views of mathematics held by mathematicians: with the current interest in the popularization of mathematics (e.g., Ernest, 1996; Pollak, 1992), Pollak (1992) recommends that mathematicians need to enlarge their personal view of mathematics.

This discussion of the views of mathematicians indicates that, within this group, a variety of perspectives about the nature of mathematics and mathematical activity exist. The discussion provides further perspectives informing the consideration of children's beliefs, and suggests that one should be open to the possibility of a range of beliefs within one group and should not presume the nature of beliefs underlying mathematical activity.

Philosophical perspectives and the perspectives of mathematicians are unlikely to have direct influence on the beliefs about mathematics developed by young children but may play an indirect role through possible influence on the development of curriculum documents. In the context of the present research, philosophical perspectives and the views of mathematicians have been presented as important elements of the broader picture of beliefs about mathematics and mathematical activity. Prior to considering previous research on the beliefs of teachers and students, school curriculum perspectives are examined, as the school curriculum forms part of the broader context within which teachers operate.

School curriculum perspectives

The school is the institution in which teachers and students participate formally in the teaching and learning of mathematics. The teaching of mathematics at school is informed by curriculum documents that are informed by research on the learning of mathematics. These documents are relevant therefore to the learning of young children. Mathematics in the context of the school curriculum can be viewed from many perspectives; to set a context for later discussion of the beliefs about mathematics of eight young children two paths of discussion are pursued here: the content areas of the mathematics curriculum, and the

recommended nature of mathematical activity in the school environment. Curriculum documents are a major source for information in these areas.

The content areas of the mathematics curriculum

In this section, content areas of the mathematics curriculum, and related terms, are discussed from a current and historical perspective to provide frameworks for later consideration of children's beliefs about the nature of mathematics. Because the school curriculum is one area of children's experience, it is a relevant area for consideration. As content areas are published in curriculum documents, they may influence teachers' and children's beliefs about the nature of mathematics of statements of belief.

Kilpatrick (1996) observes that in most countries of the world, computational arithmetic and basic concepts of measurement and informal geometry have remained at the core of the primary school curriculum during the twentieth century. However, while this core has remained, and given the curriculum its basic structure, it has been extended also through the addition of the key areas of chance and data (probability and statistics). In many countries, probability is no longer solely a specialised tertiary study, but is also a part of mathematics at the elementary level (Borovcnik & Peard, 1996), and data-handling is given equal recognition with other areas of the primary mathematics curriculum (Shaughnessy, Garfield, & Greer, 1996).

The *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) gives importance to all content areas of the primary mathematics curriculum: number, algebra, space, chance and data, and measurement. Similar trends are evident in documents published in the United States (National Council of Teachers of Mathematics, 1989, 2000), in the United Kingdom (Cockcroft, 1982), and, according to Shaughnessy, Garfield, & Greer (1996), in Spain.

National recommendations for curriculum content in Australia had a direct impact on curriculum in the state of Victoria, with the publication of the planning and assessment documents, the *Curriculum and Standards Framework: Mathematics* (CSF) (Board of Studies, 1995) and the revised edition, *Curriculum and Standards Framework II: Mathematics* (CSF II) (Board of Studies, 2000). The first of these publications was relevant at the time of the data collection for the present study and therefore is considered in some detail below, as well as the more recent version. These documents include content strands with focus on number (incorporating algebra at the primary level), measurement, data, space, and chance. The *Mathematical Tools and Procedures* strand within the 1995 publication, loosely based on the National Statement strands, *Mathematical Enquiry* and *Choosing and Using Mathematics*, is discussed further below. *The Mathematics Course Advice: Primary* (Directorate of School Education, 1995), which provides teachers with activity and assessment ideas, as well as a listing of quality resources, has the same content areas as the CSF. The activities are linked to CSF (1995) substrands (the breakdown of strands). The revised CSF, *Curriculum and Standards Framework: II Mathematics* (CSF II) (Board of Studies, 2000), includes the same
four content strands and a fifth strand refocused and renamed as *Reasoning and Strategies*. A recent publication called *Curriculum @ Work* (Department of Education, Employment and Training, 2000) builds on the 1995 course advice document and is based on CSFII.

Major curriculum documents and support material suggest that all content areas of the primary mathematics curriculum as listed above and as identified in the CSF and CSF II (Board of Studies, 1995, 2000), are of importance. However, this does not guarantee that emphasis is given to all these content areas within primary mathematics classrooms throughout Australia. Nor does it guarantee that teachers and children in Australian primary schools believe these content areas to be legitimate and equally important elements of mathematics. One of the key changes has been the inclusion of Chance in mathematics at the primary level. The emergence of further support material for teachers in this area, such as the Curriculum Corporation publication, *Chance and Data* (Lovitt & Lowe, 1993), which contains a wide variety of data-handling investigations for Years K-12, suggests the shift has moved beyond curriculum documents to the practical level.

Many changes have occurred in primary mathematics documents published by education authorities in Victoria. Some changes in content and terminology use are considered here to inform the later analysis and discussion of the beliefs of the eight children in the present study.

The inclusion of Chance is a relatively recent change. In addition, Visual Representation has been replaced and extended by the inclusion of the Data sub-strand. The focus on Mathematical Tools and Procedures and, more recently, Reasoning and Strategies, is another major change. Not only are there now additional areas to those recommended in previous documents, such as The Mathematics Framework P-10 (Ministry of Education, 1988), but over the years there also has been a change in organisation and terminology use for the mathematics curriculum. During the 1980s, teachers in Victoria were provided with guideline booklets, in their final form published in two volumes: Guidelines in Number (Ministry of Education, 1989) and the Mathematics Curriculum Guide Measurement (Ministry of Education, 1985). In these, the study of Number is portrayed as separate from other areas of the mathematics curriculum. The inclusion of Space and Visual Representation in the Measurement document implies that these are each part of the measurement component of the mathematics curriculum. This structure follows that used in an earlier curriculum guide in Victoria, but with a change in terminology. The 1967 document, Course of Study (Education Department, Victoria, 1967), names the two areas of the mathematics curriculum as Pure and Applied number and states that "they are not unrelated parts" (p. 3). However, the two sets of accompanying publications within the mathematics course of study clearly differentiate these areas: the first set containing the pure number course, and the second set containing the applied number course. For example, the *Curriculum Guide Pure Number Section E* (Education Department of Victoria, 1965), described as "The Study of Basic Mathematical Ideas in Terms of Numbers to 144" (p. 1), states as its aim, the "mastery of concepts such as

equality and the four operations and fractions and their inter-relation" (p. 3), demonstrating a pure number perspective only. The *Curriculum Guide Applied Number Sections D, E, F* (Education Department of Victoria, 1968) covers the concepts of Length, Volume and Capacity, Weight, Time, Money, Spatial Relations, and Statistics and Graphs. These documents from the 1960s and in use up to the late 1980s suggest a history of mathematics education in Victoria being divided into two sections according to a Pure and Applied divide, stated also as Number and Measurement. While this division of the curriculum into two main areas does not continue in current curriculum documents in Victoria, it may still impact upon use of these terms as well as upon teachers' conceptions of mathematics.

Although varying terms and meanings exist in the documents discussed, awareness of alternate meanings and the establishment of a current meaning for use in analysis of data is important. De Lange (1996) uses the terms pure and applied, linking the latter to the use of applications in the mathematics classroom. He notes that discussion of the dichotomy "pure" versus "applied" mathematics is "seemingly never-ending" (p. 50), and contains elements including whether applications should be accepted as part of mathematics, and whether pure and applied mathematics are of equal standing. De Lange sees the discussion as a false one, largely because of the importance of applied mathematics in society, for example, as related to technological requirements. He notes societal support for more applications, as mathematics has become a useful tool in a range of areas including "social sciences in general, antropology [*sic*], archeology, and so on" (p. 53).

De Lange (1996) considers also the place of applications in the school mathematics curriculum, noting that in the last eighty years there has been discussion of the desirability of including applications, with such discussions becoming more frequent by the end of the 1970s. He notes also "that there are many local interpretations of applying . . . in mathematics education" (p. 65). Since the seventies, applications have been part of the curriculum, with the intention "to show the relevance of mathematics in the concrete real world" (p. 65). Problems related to matters such as how much paint to paint a room, checking sales tax computations, building a book case, and buying a rug of the right size, have appeared in the school mathematics curriculum (de Lange, 1996).

Later in this report, when discussing the beliefs about mathematics of the eight children in the present study, some reference is made to mathematics using the *pure* and *applied* dichotomy. The real world examples listed in the above paragraph typify activities assumed for the purpose of discussing the children's responses, as *applied*, as they involve "user's mathematics" (de Lange, p. 66). De Lange suggests that a critical and reflective component should also be part of the use of applications in the mathematics curriculum. However, such a component is not assumed in the later discussion of individual children's beliefs as it is only the children's identification of such activities as mathematical activity that is considered, that is, their perceptions as voiced; carrying out of the activities did not occur in the interview situation. The above discussion regarding the content areas of the school mathematics curriculum began with consideration of the content areas as they exist in key curriculum documents today. The main content areas in documents published by authorities in the state of Victoria are number (incorporating algebra at the primary level), measurement, data, space, and chance, each incorporating a range of mathematical tools and procedures, or reasoning and strategies. It is recognised that the degree of emphasis given to each of these areas in individual teachers' *implemented* curriculum (e.g., Clarke, Clarke, & Sullivan, 1996), cannot be guaranteed. The identification of these areas provides one framework that can be considered within the discussion of children's beliefs. Likewise, the terms *pure* and *applied* mathematics provide an alternate framework. The discussion included consideration of the use of these terms, and of the terms number and measurement (used seemingly with the same meaning), in older curriculum documents, and the use of applications in the mathematics curriculum.

A focus on content was taken as this is one clear (and published) way of considering the mathematics curriculum, and one with which children may associate. Recommended mathematical activity in the primary school is a second perspective from which the portrayal of mathematics in the school curriculum is examined.

Recommended mathematical activity in the primary school

The discussion above referred to key elements of the mathematics curriculum today through the consideration of content areas. Within this discussion it became apparent also that processes have become a recognised element of the curriculum, demonstrated by the inclusion in the National Statement of the strands Mathematical Enquiry and Choosing and Using Mathematics (Australian Education Council, 1991), in the CSF document of the strand Mathematical Tools and Procedures (Board of Studies, 1995), and in the CSFII document of the strand, Reasoning and Strategies (Board of Studies, 2000). Processes as mathematical activity are considered in more detail below, providing a perspective on mathematics education which is considered important in current curriculum documents. It is suggested that this perspective reflects a shift in views of mathematics as a domain. Awareness of this shift informs the analysis of children's beliefs.

Having examined mathematics curriculum documents from the United Kingdom, The Netherlands, the United States, and Australia, Verschaffel and De Corte (1996) identify changes not only in the content, but also in the processes of mathematics learning. They conclude that the curriculum documents suggest that mathematics is no longer seen as involving mastery of a collection of concepts and skills but is viewed primarily as human sense-making and problem solving. They believe this shift in the perception of mathematics is accompanied by a shift concerning mathematical competence, that is, the acquisition of mathematical disposition has replaced the acquisition of isolated concepts and skills as the ultimate objective. Verschaffel and De Corte suggest that major shifts in the conceptualisation of mathematics as a domain have been one factor leading to these changes. Concurrent

changes are believed to have occurred in the view of learning, no longer seen as passive absorption of information and procedures but as active individual construction in a mathematical community. When considered in light of the views of Ernest (1991), as discussed above, Verschaffel and De Corte's contentions about mathematics and learning, as recommended by curriculum documents for schools today, suggest a social constructivist perspective of learning based on a fallibilist view of mathematics.

A National Statement on Mathematics for Australian Schools (Australian Education Council, 1991) gives emphasis to the observation, representation, and investigation of patterns and relationships in both social and physical phenomena, and between mathematical objects. Mathematics is seen as a creative activity; it involves, for example, invention, intuition and exploration. It is stated that the purpose of solving problems may be explicitly mathematical or may be generated from the real world. The emphases in the National Statement suggest a move away from a view of mathematics as a body of facts and procedures and the relationships between them, which are to be mastered if one is to know mathematics. It presents a contrasting view of mathematics as the "science of patterns" (Australian Education Council, 1991, p. 4). Schoenfeld (1992) believes a traditional view of mathematics as a body of facts and procedures trivialises mathematics; the science of patterns view better portrays the work undertaken by mathematicians as it involves thinking mathematically, that is, developing a mathematical point of view and competence in the use of the tools of mathematics for the purpose of sense-making. This suggests that teachers are being encouraged by the writers of the National Statement to teach children not just to accept mathematics as a passive activity of absorbing and regurgitating facts and procedures, but to consider and experience it as *active*, that is, involving higher levels of thinking. If appropriate active experiences are within the implemented curriculum, children may refine beliefs about the nature of mathematics.

The Victorian *Curriculum and Standards Framework: Mathematics* (Board of Studies, 1995), the curriculum document relevant at the time of the data collection for the present study, also moves away from a traditional view of mathematics as essentially memory of facts and procedures. This is evident particularly through the inclusion of the strand Mathematical Tools and Procedures that is in addition to the content strands. This strand emphasises and values processes of mathematics such as

- the use of a range of mathematical tools including calculators and measuring instruments;
- communication in mathematics through natural and mathematically conventional language, symbols and notations;
- identifying and solving problems through a range of processes and strategies;
- employing reasoning in mathematics; and
- recognising, appreciating and applying mathematics in a range of contexts.

Although this strand is not given a separate section in the *Mathematics Course Advice: Primary* (Directorate of School Education, 1995), the teacher support material designed for practical use by teachers, it is intended that the use of these tools and procedures occur through activities in all four content areas. The inclusion of a range of activities in the Mathematics Course Advice, including those requiring more than recall or reproduction of procedures, suggests an underlying philosophy supporting the use of a range of tools and procedures, and therefore of higher order thinking. The revised CSF, the Curriculum and Standards Framework II: Mathematics (Board of Studies, 2000) continues to include emphasis beyond content areas through the strand Reasoning and Strategies. The recently published teacher support material, Curriculum @ Work (Department of Education, Employment and Training, 2000) also includes problems that are intended to require a variety of problem solving and investigation strategies. The document indicates also that children should be required to evaluate their work. The inclusion of problem solving and evaluative activities suggests there is a move for the current curriculum materials to be like those described by Verschaffel and De Corte (1996). They state that reform documents in mathematics today include goals for mathematics education such as "the development of higher order skills like the ability to explore, to conjecture, to reason, to reflect and to communicate mathematically" (p. 101).

The move to viewing mathematics as a creative activity involving invention, intuition and exploration, and based on the science of patterns, is relatively recent. For example, major reform documents recommending such a view of mathematics have been published in the United States, Australia, Spain, and the United Kingdom during the 1980s and 1990s (Verschaffel & De Corte, 1996). The philosophy of the Australian National Statement is reflected in the practical Victorian Mathematics Course Advice document for teachers. It appears that in the past there was no such philosophy. The 1967 Course of Study (Education Department, 1967), stated that "the child's awareness of mathematics should involve more than the recall of number facts and the use of formal procedures" (p. 3). However, it included no discussion on how an alternative approach could be implemented; also there was no discussion on how a teacher could ensure that a child would "understand how mathematics is related to the world around him" (p. 3). The 1988 publication, *The Mathematics Framework P-10* (Ministry of Education, 1988), signalled a move towards a renewal of purpose in mathematics education:

We need to reconceptualise for ourselves a view of mathematics that makes sense – ordinary common sense and from there we can build our generalisations and our abstractions . . . we have drawn heavily from the discipline of mathematics as the basis for teaching mathematics. Yet the history of mathematics reflects that mathematical advances are made by people solving problems which interested them. (pp. 12-13)

In this Framework document, change in teaching methods was recommended including use of open ended investigations, making and solving problems, and groups working cooperatively, thus incorporating discussion in the learning of mathematics. An emphasis on skills was maintained but accompanied by the development also of concepts, applications and processes. Mathematical activity was seen as active, including use of processes such as estimation,

alertness to reasonableness of results, and prediction. Thus as early as 1988, recommendations were made for changes in Victorian schools based on a view of mathematics as something to be explored.

To summarise, an earlier focus on mathematical activity in schools as the essentially passive learning of facts and procedures, seemingly based on mathematical content, has been replaced partly in more recent curriculum documents by adding an emphasis on mathematics as a creative activity using a range of mathematical tools and procedures.

This discussion provides background in which to situate later consideration of the views of eight young learners, of the nature of mathematics. Within the discussion of mathematical activity recommended in a range of documents, passive and active views of mathematical activity have been contrasted. Research on student views of the nature of mathematics provides an additional perspective in the portrayal of the broader picture.

Students' perceptions of the nature of mathematics

The discussion of mathematical activity from philosophical perspectives informs the following discussion of the beliefs of students, with absolutist views of mathematics as a passive activity being suggested by the majority of students. However, literature shows that, as within the group of mathematicians, within the group of students a range of beliefs about the nature of mathematics is held.

Learners' beliefs about mathematics are important as they can shape the way in which they do mathematics as learners "perceive what they experience in the light of interpretative frameworks they have developed" (Schoenfeld, 1987, p. 195). Students' beliefs may influence the nature of participation in mathematics learning (Franke & Carey, 1997), and may be related to success in mathematics (Crawford, Gordon, Nicholas, & Prosser, 1993). Similarly, the learner's everyday knowledge of what is mathematics can influence the learner's interpretation of what is taught (Lindenskov, 1993) and consequently influence what is learned.

Findings from some studies of students' views of the nature of mathematics are discussed specifically below. Borasi's (1990) synthesis of results provides an introduction to the broader picture. Later, by reference to further studies, it is argued that these results do not represent the views of all learners of mathematics, and that there is scope for further research in this area, as has occurred in the present study. The synthesis by Borasi (1990) portrays students' views about school mathematics in relation to four aspects:

- *The scope of mathematical activity:* Providing the correct answer to given problems, which are always well defined and have exact and predetermined solutions. This applies to the activity of both mathematicians and mathematics students, though the complexity of the problems would obviously differ.
- *The nature of mathematical activity:* Appropriately recalling and applying learned procedures to solve given problems.

- *The nature of mathematical knowledge:* In mathematics, everything is either right or wrong; there are no gray areas where personal judgement, taste, or values can play a role. This applies to the facts and procedures that constitute the body of mathematics and to the results of each individual's mathematical activity.
- *The origin of mathematical knowledge:* Mathematics always existed as a finished product; at best, mathematicians at times discover and reveal some new parts of it, while each generation of students "absorb" the finished products as they are transmitted to them. (p. 176)

These findings suggest prevalence of an absolutist view of the nature of school mathematics. Similar results come from a range of other studies, as discussed below. These studies reveal also perspectives focusing on number as content, and beliefs about the existence of mathematics beyond the school environment, and about the relationship of non-school mathematics to school mathematics.

Garafolo (1989) refers to the widespread existence of students holding beliefs about mathematics as a collection of rote, mechanical procedures leading to correct answers, and of the teacher and textbook as authorities on mathematical truth. He attributes students' "narrow and constraining beliefs . . . [to the] traditional curricula, instructional practices and classroom environments" apparent within the traditional approach to teaching mathematics (Garafolo, 1989, p. 453). According to Ernest (1996), a study by Preston found that "a substantial subgroup of students [saw] mathematics as an algorithmic, mechanical and stereotyped subject" (p. 805). In other studies (Cotton, 1993; Frank, 1988; Kouba & McDonald, 1987; McDonald & Kouba, 1986; Spangler, 1992; Stodolsky et al., 1991), children were found to focus on the computational or number aspect as the essence of mathematics. Students believe that mathematical activity at school includes memorising formulas and, in a rote fashion, carrying out computations and procedures (Garafolo, 1989; Schoenfeld, 1992; Spangler, 1992) that are "divorced from real life, from discovery and from problem solving" (Schoenfeld, 1987, p. 197). The emphasis on mathematics as a rule-governed activity appears present within students both at the primary and secondary levels (Cobb, 1985). Children see mathematical activity in real-life situations as active: "In order to use or do mathematics, a person must 'do something'" (McDonald & Kouba 1986, p. 23). But they consider their role as learners at school as passive, that is, as receivers of mathematics. They see teachers as the active participants in the teaching learning process, that is, as those who transmit the mathematics (Frank, 1988) as it is the teachers who are thought to have the questions and answers (Cotton, 1993). These perspectives of student and teacher roles appear linked to the perception of the origin of mathematics: "Students tend to perceive mathematics as a static discipline where everything has already been created or discovered" (Spangler, 1992, p. 150), suggesting a more absolutist than fallibilist view of the nature of mathematics. As the authoritarian, transmission style of teaching, based on the concept of mathematics as content,

remains common among teachers (e.g., Dossey, 1992; Romberg & Carpenter, 1986), these results are not surprising.

However, not all students hold such views. Franke and Carey (1997) researched the perceptions of mathematics of 36 first-grade children from six Cognitively Guided Instruction (CGI) classrooms "where students had opportunities to consistently engage in problem solving, discuss their solution strategies and build on their own informal strategies for solving problems" (p. 10). In a structured interview of about twenty minutes duration, children were posed questions situated in the context of classroom events. The children held different perceptions of what it means to do mathematics from those traditionally held by students, with children discussing "problem solving, use of manipulatives, talking about mathematics, and solving problems in a variety of ways" (p. 14). A majority of the 36 children did not consider the teacher as sole authority in the classroom. Twenty-two children (61%) felt that they themselves could tell whether they had done a good job in solving a problem, six children (17%) gave this role to the teacher, and seven children (19%) talked of both the teacher and themselves doing this. Moreover, twenty-eight children (78%) felt they could resolve a conflict where two partners had different answers to a problem. This research suggests it is possible for a shift to occur in whom children see as playing a dominant role in the mathematics classroom. Also children can, in an environment where problems are emphasised, recognise the importance of both the process and answers in doing mathematics. It appears that the children's views reflect those inherent in the problem solving program and thus in the curriculum reform documents in the United States.

The findings from Franke and Carey (1997) contrast with those from much previous research, as discussed above. Combining the fact that Franke and Carey's study occurred in *problem solving* classrooms, with the suggestion that the transmission style of teaching remains common in mathematics classrooms, we find that the nature of a mathematics program may have an impact upon children's beliefs about mathematics. Beliefs such as those itemised in Borasi's synthesis are considered "dysfunctional [as they] could negatively affect mathematics instruction" (Borasi, 1990, p. 176); such beliefs also have been called unhealthy and undesirable (Garafolo, 1989). It appears that such beliefs, which are more likely to be associated with a traditional program, may not be readily open to change; some studies have found that conflict can arise when a teacher attempts to teach in a non-traditional way (Cooney, 1985; Doyle, 1986; Garafolo, 1989). However, as Franke and Carey's (1997) research demonstrates, it is possible for alternate beliefs to develop. Such a finding gives credence to the present study: it is important that teachers and others involved in the education of children gain insights into the perspectives of individual children as this awareness can inform and guide teachers in their decision-making for teaching mathematics.

Perceptions uncovered by Franke and Carey (1997), as discussed above, are believed by those researchers generally to be productive, as they promote learning. However, Franke and Carey alert the reader to the possibility that in a problem solving program children may

develop some beliefs that may have a negative impact upon learning. For example, many children in the study saw all solution strategies in solving a problem as equally valuable. This finding indicates the complexity of children's beliefs, and gives further support to the notion of investigating and being aware of individual children's beliefs, not making assumptions about them, and taking them into account in interactions with children. The role of the teacher and social interaction in the mathematics classroom are important considerations also; factors such as a lack of development of appropriate sociomathematical norms, for example of mathematical difference and mathematical sophistication (Yackel & Cobb, 1996), may contribute to children believing all solutions are equally valuable.

Beliefs different from those traditionally held by students were found also in a study by Wood and Smith (1993) of perceptions of mathematics of 74 students who had successfully completed mathematics at secondary school and were going on to a mathematics or engineering degree at university. As the data collection occurred in the first weeks of semester, the authors consider their survey a study of the respondents' ideas about school mathematics. It was found that "most students (71%) felt that everything in mathematics is not already known by mathematicians, that in mathematics you can be creative and discover things by yourself (86%) and that it is not true that maths problems can be done correctly in only one way (92%)" (p. 595). A fallibilist view of mathematics appears to underlie these results. The findings that over three-quarters of the respondents associated school mathematics, specifically, with facts and procedures and memorising, and that most believed that in mathematics something is either right or wrong, were thought possibly to be tempered by the students' recent completion of their Higher School Certificate examination where speed was essential. The apparent differences in some of these findings as compared to much other research may reflect a variation of beliefs within students as an overall group. For example, the beliefs of the respondents in this research may be related to the nature of the experiences of these students: overall they were successful with secondary level mathematics and had chosen to study mathematics at the tertiary level. It was noted that within this group there was variation in responses, seemingly related to the amount of mathematics studied at school and possibly therefore to success in mathematics: there was greater association of mathematics and memorising by students who had studied less mathematics at school. The fact that few other studies showing similar results were identified suggests that students holding fallibilist beliefs about mathematics are in the minority. From Wood and Smith's (1993) study it is not clear whether success or fallibilist beliefs came first.

The findings contrast in some respects with those from a group of tertiary students studied by Crawford et al. (1993) who found that among 300 first year mathematics university students, surveyed during their first week at university, the majority conceived "of mathematics as numbers, rules and formulae which can be applied to solve problems" (p. 213). These views were associated with what the authors called a "surface" approach to learning mathematics, that is, "the reproduction of knowledge and procedures" (p. 212). Also

within the research findings there were "indications that a more cohesive conception of mathematics and/or a deep approach to studying mathematics are positively and cumulatively associated with achievement at university level" (p. 213). A deep approach was evidenced by working towards a "relational understanding of the theory and the concepts" (p. 212), where ideas are interrelated and cohesive. However, the majority of respondents were found to hold beliefs about mathematical activity similar to those identified by Borasi (1990). Crawford et al. (1993) suggest an association between beliefs about mathematics and success, and a desirability of a broader view of mathematics than numbers, rules and formulae.

Within the discussion above there is some suggestion of a relationship between children's absolutist views of the nature of mathematics and an authoritarian, transmission style of teaching. However, research does not lead to a conclusive resolution of the issue of the relationship between children's beliefs and their teacher's beliefs. Although teaching practices may have some influence on student beliefs, it appears that views of a current teacher do not determine the beliefs of students in a class. For example, one study of older students revealed variation of beliefs within a group. Interviews with nine Year 13 students in England who had moved from an investigation-driven to a more formal mathematics course revealed six students with fallibilist views of mathematical truth, and three with absolutist views (Rodd, 1993). Results from trialing for the present research show variation, suggesting also that student views are not necessarily reflective of those of their current teacher. Two children from one Grade 6 class, were posed the question What is mathematics? Their word wheel responses, presented in Figure 4, show contrasting perceptions of the nature of mathematics or mathematical activity. In the first response the child focuses mainly on mathematics as content; in contrast, the second child gives little consideration to this aspect, but rather sees mathematics as a social activity that involves active individual and group processes.

These responses demonstrate that mathematics might be perceived as something external to oneself, as demonstrated through an emphasis on content of mathematics such as the operations, or might be seen as involving personal participation through active processes and therefore more internal to the person. The two children's responses are different and show that individual children's perceptions may be idiosyncratic and may not necessarily correspond to those of their current mathematics teacher. However, each response may portray or emphasise elements within the teacher's beliefs; children may be influenced by a teacher's beliefs although ultimately constructing their own perceptions of the nature of mathematical activity.



Figure 4. Contrasting perceptions of the nature of mathematics.

It is acknowledged that the data in Figure 4 come from the use of only one procedure. Nonetheless these preliminary findings, along with those of Rodd (1993), are revealing, especially as some past research and some curriculum documents suggest links between the images of mathematics of teachers and those of their pupils (e.g., Brown, 1992; Garafolo, 1989; National Council of Teachers of Mathematics, 1989). Student views of mathematics may be determined or affected by the classroom teacher to some degree, seemingly related to more concrete aspects of the classroom such as activities, and roles and responsibilities (Franke & Carey, 1997). However, it is clear that teachers should not make assumptions about the nature of the beliefs of students in their classes; students may experience the same teaching but develop idiosyncratic perspectives, perhaps influenced only to some degree by the teacher. It is through focused investigation that a teacher can become aware of the nature of individuals' beliefs. The present research illustrates the possible insights gained from such investigation and offers research tools for teacher use. The research did not aim to identify cause and effect relationships between teacher and pupil beliefs.

A further perspective from which to investigate children's perceptions of mathematics and mathematical activity is the everyday activities of people outside of the school environment. The National Council of Teachers of Mathematics (1989) states,

To some extent everyone is a mathematician and does mathematics consciously. To buy at the market, to measure a strip of wallpaper, or to decorate a ceramic pot with a regular pattern is doing mathematics. School mathematics must endow all students with a realization that doing mathematics is a common human activity. (p. 6) Many textbook writers and teachers of mathematics try to make mathematics relevant for students by embedding the school mathematics in situations that appear to relate to the children's lives. This may be through the use of *relevant* textbook or worksheet activities. However, this integration may not necessarily lead children to see outside-of-school activities as mathematical. It has been suggested that children may make "links between school mathematics and non-school mathematics in an extremely contracted way . . . and that this may be due to texts and their inherent discursive practices to which children have been exposed in their study of mathematics" (Zevenbergen & Crowe, 1992, p. 5). Reference to outside-of-school activities in mathematics classrooms may encourage children to interpret those activities at that time as belonging to school mathematics, seemingly a concept perceived by children as different from common, everyday non-school experiences. Mathematics appears to be a school-related concept for children. Kouba and McDonald (1987) found that outside activities were not seen as mathematical in their own right as the children did not recognise an underlying mathematical structure; it was against school experiences and ideas in mathematics that outside activities were judged as to whether they involved mathematics.

Children may see activities outside the classroom as mathematical, but when examples are used in the mathematics classroom, they are interpreted as *school mathematics*. It appears that to make mathematics more relevant to the everyday lives of children, it is not enough to bring examples into the classroom through text or other references; it appears that children need also to consciously experience mathematical activity in their own non-school environments.

Further evidence of differences in student and mathematics teacher perceptions were found by Wallbridge and Clarke (1989) who researched the perceptions of mathematical activity of Year 8 students, teachers, mathematicians and other community groups. Individuals were asked to express the perceived degree of mathematics in a range of activities including "using a calculator to work out the interest paid on a housing loan over 20 years", "upkeeping a domestic vegetable garden", "chopping down a large pine tree", and "painting the house". The results "suggested that year 8 students and academic mathematicians were similar in their narrow conception of mathematics" (Wallbridge & Clarke, 1989, p. 463). While the teachers in the sample were the most likely of all groups to see the everyday activities as mathematical, the students gave academic descriptions of mathematics related to "figures", "numbers", "symbols", "measurements, angles, tables", "special subject at school". The responses indicate that, as suggested by the Zevenbergen and Crowe (1992) results, teachers may believe they are relating school mathematics to a range of real life situations, but students may perceive differently the situations and the degree of mathematical activity.

In research conducted by McDonald and Kouba (1986) and Kouba and McDonald (1987), children were posed with descriptions of situations in and out of school and asked to judge whether the person in the situation was using or doing mathematics. A narrow

conception of mathematics was found among primary school students (McDonald & Kouba, 1986) and junior secondary students (Kouba & McDonald, 1987). The researchers found perceptions of mathematics as mainly a number concept. For example, in the primary sample, "overwhelmingly, students saw mathematics when they were able to recognize counting, adding, and numbers" (McDonald & Kouba, 1986, p. 24). Decisions of whether situations contain mathematical activity were influenced by developmental factors, demonstrated, for example, by younger children tending to cue in on number and counting more frequently. But for all ages the presence of explicit numbers and operations in a situation was a major factor in identifying the presence of mathematics. In the primary sample, geometry, statistics and probability generally were not accepted as being within the domain of mathematics, including reference to geometry, probability and measurement in their rationales, but the basic operations were still the major component. The change in the recognition of the content of mathematics from primary students to junior secondary students may have been due to a change in emphasis in the school curriculum.

As well as a focus on number, other beliefs about mathematics were discovered by McDonald and Kouba. In both the primary and junior secondary samples, mathematics was seen as school-related. For example, many primary school children considered mathematics occurred only at school under certain conditions, stating as necessary, factors such as a teacher, a question, a paper and pencil, and that the mathematics has to be right (McDonald & Kouba, 1986). In general, mathematics was seen also as active, that is, as something which involves doing. Mathematics was considered by many as isolated from other curriculum areas, and as a fluid or upwardly shifting domain, that is, to many children when mathematics becomes easy it is no longer mathematics. It was reported that junior high school students also saw mathematics as exact, therefore not involving estimating or approximating (Kouba & McDonald, 1987). There was no indication that primary school children considered estimating or approximating as mathematical.

To summarise the discussion of students' perceptions of mathematics and mathematical activity, we have seen that primary school children tend to have narrow conceptions of mathematics, focused mainly on number, including counting and the operations. Mathematics at school is seen by most to involve formulas and rote computations and to be exact. Generally, students appear to have more absolutist than fallibilist views of the nature of mathematics, that is, mathematics as a static discipline that has already been created by others. In contrast, primary school children experiencing problem solving programs in mathematics appear to hold different views of mathematics, and successful secondary mathematics students may see mathematics as more fallibilist than absolutist.

The discussion has shown also that for most primary school children, mathematics is closely associated with school; the teacher is the authority regarding mathematical truth, and correct answers are important. Outside activities are judged for a mathematical component in terms of how they relate to classroom mathematics experiences. It appears that children see mathematical activities in non-school situations as active as they involve the person doing something, but that the learning of mathematics is considered more as a passive task for the student, with the teacher as the active participant.

There is some evidence that children's beliefs about mathematics may correspond to those of their mathematics teacher but this is not conclusive. Children's conceptions of mathematics appear to vary, and indeed may be idiosyncratic. They may be more complex than they appear on the surface; the discussion has demonstrated that children may hold beliefs about many aspects of the nature of mathematics and mathematical activity. There is the possibility that there are further elements and complexities even than those discussed here; the present research has provided, through the use of a range of open ended procedures, the opportunity for young children to portray elements they perceive as pertinent or relevant.

As background to this research, previous findings on teachers' perceptions are considered also; these are discussed below.

Teachers' perceptions of the nature of mathematics

In traditional mathematics classrooms, mathematics is assumed as a "static, bounded discipline" (Romberg & Carpenter, 1986, p. 851). In this paradigm mathematical knowledge is viewed as external and objective, available for unconcealment or discovery by mathematicians, in turn to be transmitted by teachers and received by students (Dossey, 1992; Fisher, 1990). This suggests that mathematics is considered absolutist, "a body of infallible, objective knowledge" (Ernest, 1991, p. xii). The alternate view of mathematics as a dynamic, growing field of study (Fisher, 1990; Mathematical Sciences Education Board, 1989; National Council of Teachers of Mathematics, 1989) "posits that mathematical knowledge is internal and therefore subjective . . . [it] is not so much discovered as created by social groups . . . knowing and doing [mathematics are] inseparable" (Fisher, 1990, p. 82). In this view, mathematics also can have objectivity if socially accepted following intersubjective scrutiny (Ernest, 1991). In other words, in the traditional paradigm the focus is on mathematics as content which is external to the learner; from the alternate viewpoint mathematics is a process, and knowledge is internal to the learner. These two contrasting perspectives appear to mirror the absolutist and fallibilist paradigms discussed earlier.

Although current documents encourage the view of mathematics as an active process and some teachers do appear to have made changes in their teaching methods (Forgasz, Landvogt, & Leder, 1997), the majority of teachers appear not to have rejected an authoritarian, transmission style of teaching (Becker & Selter, 1996; Dossey, 1992; Romberg & Carpenter, 1986, Verschaffel & De Corte, 1996). This suggests that, in practice, many teachers may view mathematics from an absolutist perspective. Indeed, citing the writing of Cooney, Dossey (1992) speaks of teachers having a formal view of mathematics as an external body of knowledge ready for presentation or transmission to students. A study undertaken by Warren and Nisbet (2000) with 398 primary teachers revealed that the teachers held "a fairly limited view of what mathematics is . . . [with a focus on mathematics as] a static unified body of knowledge and a bag of tools" (p. 638). Galbraith and Chant (1993) found from a sample of 97 secondary teachers "an overwhelming view of mathematics for the majority as arithmetic" (p. 272), indicating a narrow view of mathematics. Underhill (1988) found variation in teachers' views of mathematics and suggests this may be related to teacher training. Teachers of secondary mathematics, who were trained in disciplines other than mathematics, showed "a much less divergent set of beliefs about mathematics as skills. Secondary teachers trained to teach mathematics gave more emphasis to mathematics as exploration and problem solving. These results from a range of studies suggest that teacher beliefs do vary, but that the teaching of mathematics may be influenced mainly by absolutist perspectives.

Underhill (1988) reports that a study of the beliefs of Canadian elementary teachers conducted by Dionne found a range of views. A little more than half the time mathematics was seen as a set of skills, or as logic and rigour. A little less than half the time it was seen as a process. However, Underhill cautions elsewhere in his paper that there are frequent discrepancies between teachers' espoused beliefs and their practices or actions. Therefore, it appears that although a little less than half the teachers in Dionne's study espoused beliefs of mathematics as a process, these may not be reflected in their actual teaching practice. Thompson (1992) warns also that research has found "varying degrees of consistency between teachers' professed beliefs about the nature of mathematics and the teachers' instructional practices" (p. 134). Therefore it appears that statements of belief do not necessarily reflect teaching practices in mathematics classrooms. Teachers who espouse fallibilist beliefs may teach from an absolutist perspective, and vice versa.

Some mention of the views of teachers was made earlier in reference to the findings of Wallbridge and Clarke (1989). Posed with the task of giving the perceived degree of mathematics in a range of everyday activities such as "using a calculator to work out the interest paid on a housing loan over 20 years", and "upkeeping a domestic vegetable garden", teachers in the sample were found as the most likely of all groups to see everyday activities as mathematical. This suggests that teachers may have a broad view of mathematical activity, but does not necessarily conflict with an individual holding beliefs about learning mathematics that mirror either an absolutist or a fallibilist view of mathematics. That is, mathematics may be seen as an activity undertaken in everyday life, but the learning of mathematics may be viewed, for example, as the discovery of pre-existing ideas, or as the creation of ideas and relationships.

In summary, it seems that among teachers, varying views of mathematics are held. However, those who believe mathematics to be an active process, not just the mastering of facts and procedures, may not necessarily reflect such beliefs in their teaching. Some research suggests that primary teachers tend to hold divergent views of mathematics. Secondary teachers trained to teach mathematics tend to give emphasis to mathematics as exploration and problem solving, and secondary mathematics teachers trained in other disciplines view mathematics from a traditional perspective, that is, mathematics as skills. It can be assumed that the beliefs of teachers have at least some impact on the beliefs of students.

Children are exposed also to the beliefs of adults in the broader community. Such beliefs are examined below.

Perceptions of the nature of mathematics held by other adults

As further background to the study of children's perceptions of mathematics, it is relevant to examine perceptions of the nature of mathematics held by adults in the wider community. Through children's interactions with adults other than their teachers, such perceptions may have some impact upon children's beliefs. The discussion of beliefs held by the wider adult community adds to the portrayal of a broad landscape of beliefs about mathematics and mathematical activity, and identifies further background to inform analysis of children's responses. This discussion is followed by consideration of the meaning of numeracy and of its relationship to mathematics, thus giving further insights into the concept of mathematics held within the community.

In a review of adults' beliefs about mathematics, FitzSimons, Jungwirth, Maab, and Schloeglmann (1996) state that "adults tend to identify mathematics with arithmetic. Doing mathematics is some sort of calculation" (p. 768). Galbraith and Chant (1993) found also "an overwhelming propensity among both the public and the teaching profession to view school mathematics primarily as arithmetic, both in terms of what is learned and what is useful" (p. 272). Mildren and Brandenburg (1991) found, from a survey given to a school community group of 62 parents, that although "a couple of responses referred to angles, measurement and money, only one subject thought of estimation being related to mathematics . . . [and there appeared] a narrow perception of mathematics, generally limited to numbers and operations" (p. 77). Their student-teacher survey group had a similarly narrow perception of mathematics. Civil (1990) describes the views about mathematics of four prospective teachers as focusing on the writing down of equations and other symbolic operations, rather than focusing on thinking processes. Wallbridge and Clarke (1989) indicate that the non-maths trained adult respondents in their study working in a *non-maths* job, saw less mathematics in the everyday activities than did the [mathematics trained] mathematics teachers, but saw more mathematics than did the Year 8 students. These non-maths adults showed some recognition that mathematics is not bound to the school situation.

To summarise, it appears that adults in the wider community associate mathematics mainly with computation and are unlikely to suggest higher levels of mathematical thought as mathematical activity. Some recognition is made of the use of mathematics beyond a formal learning environment such as a school. It is possible that adults' narrow views of the nature of mathematics may have some impact directly or indirectly upon views held by students. Such a relationship is not investigated in the present research, but the discussion of adult views contributes to the growing picture of the broader environment in which children learn.

Numeracy

Scholarship in regard to adults and mathematics uses two main terms: mathematics and numeracy (FitzSimons et al., 1996). Consideration of meanings given for the term numeracy provides some further insight into perceptions of mathematics.

As *literacy* is often used by the general public in regard to language, so too *numeracy* is a common term used in reference to mathematics. The term numeracy was used in 1959 to represent "the mirror of literacy" (Crowther, cited in Cockcroft, 1982). However, meanings for the term numeracy have continued to emerge, varying over time and in different contexts.

It appears that in the United Kingdom the focus of numeracy has been on number and the application of number. The Cockcroft Report speaks of the importance of an "at-homeness with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life" (Cockcroft, 1982, p. 11). In a major study carried out in the United Kingdom, Askew et al. (1997) define numeracy as "the ability to process, communicate, and interpret numerical information in a variety of contexts" (p. 4). There have been shifts in meaning for the term numeracy since 1959 (Love & Tahta, 1991), but claims also that in Britain the term was accepted generally as meaning arithmetic (Ernest, 1991; Love & Tahta, 1991) and related "to straightforward-seeming skills" (Love & Tahta, 1991, p. 256). The above definitions of numeracy appear to give emphasis to number or arithmetical aspects of mathematics and some application in everyday activities. The needs of the individual also receive some consideration. Two themes of basic skills and needs for daily living are apparent.

In the United States, the National Research Council report (1989) speaks of two types of literacy: verbal and mathematical. Mathematical literacy is numeracy. The report states that

Numeracy requires more than just familiarity with numbers. To cope confidently with the demands of today's society, one must be able to grasp the implications of many mathematical concepts – for example, chance, logic and graphs – that permeate the daily news and routine decisions. (pp. 7-8)

More recently the term *quantitative literacy* (Devlin, 2000; Wallace, 2000) has been coined. Sometimes considered ill-defined (e.g., Wallace, 2000), quantitative literacy is also called numeracy and

roughly speaking . . . comprises a reasonable sense of number, including the ability to estimate orders of magnitude within a certain range, the ability to understand numerical data, the ability to read a chart or graph, and the ability to follow an argument based on numerical or statistical information. (Devlin, 2000, p. 24)

These latter two definitions appear to relate mainly to number and its application within society. The term *mathematical literacy* was used also by the Organisation for Economic Co-operation and Development (OECD) which states that

Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual's current and

future life as a constructive, concerned and reflective citizen. (OECD, Paris, 1999, p. 41, cited in Department of Education, Training and Youth Affairs, 2000, p. 4)

This definition refers not only to number but also to mathematics, and suggests that knowledge of mathematics and individual judgement are important.

In Australia, writings in regard to numeracy give a more encompassing view of numeracy than the "mirror of literacy" concept proposed by Crowther, or the emphasis on number found in some definitions. For example, in discussing what it means to be numerate, Scott (1999) emphasises the three components of confidence, content and context. Willis (1990) emphasises the importance of attitudes, skills, fundamental mathematical concepts, and competence and confidence for developing numeracy. The Australian Association of Mathematics Teachers (AAMT) adopted the following working definition of numeracy:

To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life.

In school education, numeracy is the fundamental component of learning, discourse and critique across all areas of the curriculum. It involves the disposition to use, in context, a combination of

- underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic);
- mathematical thinking and strategies;
- general thinking skills; and
- grounded appreciation of context. (Australian Association of Mathematics Teachers, 1997, p. 15)

The identification of mathematical thinking and strategies as a component of numeracy suggests a concern with higher order skills, something not evident in all definitions above.

The Department of Education, Training and Youth Affairs (2000) state also that numeracy provides skills for students to participate in schooling and equips people for life beyond school by "providing access to study or training, to personal pursuits and to participation in the world of work and the wider community" (p. 1).

Some richness within numeracy is emphasised also in a document published by the Department of Education and the Arts, Tasmania (1995):

To be numerate is to have and be able to use appropriate mathematical knowledge, understanding, skills, intuition and experience whenever they are needed in everyday life. Numeracy is more than just being able to manipulate numbers. The content of numeracy is derived from five strands of the mathematics curriculum – space, number, measurement, chance and data, and (pattern and) algebra – as described in the National Statement and Profiles. (p. 6)

The concept of numeracy continues to be of interest in the education community as evidenced by continuing discussions on the meaning of numeracy and implications for the classroom (e.g., Askew, 2001; Girling, 2001) and by the recent publication of a range of documents as cited here including the *Numeracy Benchmarks Years 3, 5 & 7* (Curriculum Corporation, 2000). This document does not provide a definition of numeracy but gives some sense of what is assumed by the term. The numeracy benchmarks "incorporate aspects of students"

developing understanding of and competence with number and quantity (i.e., measurement), shape and location, and the handling and interpretation of interpretative and qualitative data" (introductory, unnumbered page). They thus give emphasis to the range of strands in current curriculum documents (e.g., Board of Studies, 2000). As no definition for numeracy is provided in the benchmarks document the meaning can only be assumed. It appears significant that the definition provided in the former draft document (Curriculum Corporation, 1997) and that includes reference to the effective use of mathematics to meet the demands within various aspects of life, is not included. It appears that the numeracy benchmarks refer more to what the Australian Association of Mathematics Teachers speak of as "school numeracy", that is numeracy inherent in schooling than to numeracy as a personal attribute as emphasised in the AAMT policy statement (Australian Association of Mathematics Teachers, 1998). However, it is acknowledged also that the numeracy benchmarks document does not claim to represent all aspects of numeracy.

Further statements related to the nature or meaning of numeracy include that it is context specific, that is, that an individual is numerate or not within a given context. Judgements of relative levels of numeracy need to be made within a context(s) (Australian Association of Mathematics Teachers, 1998).

The relationship of numeracy to mathematics is a further consideration in this discussion. It is noted that in German-speaking countries only the term mathematics is used in scholarship and public discussion (FitzSimons et al., 1996). However, where two terms do exist, it appears there is general agreement that they have different meanings. For example, the Australian Association of Mathematics Teachers states that numeracy is not a synonym for school mathematics, and that being numerate does not necessarily result in success in mathematics at school (Australian Association of Mathematics Teachers, 1998). Numeracy is said to be under-pinned by some mathematics but "learning mathematics at school is also about learning in the discipline of mathematics – its structure, beauty and importance in our cultures" (Australian Association of Mathematics Teachers, 1997, p. 12). It is stated also that "knowledge of, and skills with, mathematics are not sufficient to assure 'numeracy" (Australian Association of Mathematics Teachers, 1998, p. 3). Documents from the United Kingdom, as discussed above, suggest also that numeracy is not considered as being the same as mathematics. Ernest (1991) sees numeracy and mathematics as different. Some authors from Australia and the United States stress that mathematics and numeracy are different and, indeed, suggest that numeracy (or mathematical literacy) is every teacher's responsibility (e.g., Australian Association of Mathematics Teachers, 1997, 1998; Chapman, Kemp, & Kissane, 1990; Department of Education and the Arts, Tasmania, 1995; Devlin, 2000; Wallace, 2000). However, it is stated also that good teaching in mathematics contributes to the development of students' numeracy (e.g., Australian Association of Mathematics Teachers, 1998; Department of Education and the Arts, Tasmania, 1995; Department of Education, Training and Youth Affairs, 2000).

In summary, it appears that there is no consensus on the meaning of the term numeracy. Although it has generally become customary for the two terms, numeracy and mathematics to be used it is not clear where the dividing line between them should be drawn (FitzSimons et al., 1996). In some respects numeracy appears broader than mathematics. For example, there is some agreement that all teachers in schools are responsible for developing students' numeracy. There is a view also that attitudes, confidence, content and context are all relevant to numeracy. However, numeracy also appears more narrow than mathematics, for example, in that the higher order processes of mathematics discussed in recent mathematics curriculum documents do not appear inherent in many definitions of numeracy discussed above, and that understanding of all mathematical content is not needed for participation in society. Reflecting on the perspectives given, it appears that there are at least two themes within discussions of the meaning and importance of numeracy: the development of basic skills, and the need for attributes that faciliate meeting the needs of individuals within daily lives.

Looking back: Mathematics

The above discussion has indicated that there are varying conceptions of the nature of mathematics and mathematical activity, and that mathematics can be viewed from many perspectives. The discussion was structured mainly by considering the beliefs of people within different groups. It was found that beliefs about the nature of mathematics and mathematical activity can vary between and within different groups, and have varied over time. The discussion began with consideration of philosophical perspectives, providing introduction to a spectrum of beliefs, the extremes of which, absolutism and fallibilism, provided markers which, when appropriate, were referred to in discussion of perceptions held by a variety of groups. Meanings for numeracy and its relationship to mathematics were discussed also.

The nature of mathematics and mathematical activity was one of three key elements that contributed to the present research. As evident in the second and third research questions, as discussed earlier, the other two key elements were perceptions of learning, and perceptions of helping factors for learning mathematics. The discussion continues with consideration of learning in general and then the learning of mathematics in particular.

Learning

The second major element of the present research concerned children's beliefs about learning. Of prime interest were their beliefs about the learning of mathematics. However, beliefs about learning in general were sought also to give greater breadth and depth to the insights gained. The themes of the nature of learning and learning of mathematics pervade the following discussion.

The school as an entity continues to operate, as it was first developed, with two main participatory parties: pupils and teachers. Learning theories, which have been and continue to be developed, underpin but do not necessarily have immediate or direct influence upon, teaching practices (Romberg & Carpenter, 1986). Teachers develop personal philosophies of learning and teaching, and children experience the related practices, possibly developing perceptions of the learning process from these as well as from outside of school experiences that they may or may not regard as learning experiences. Just as some teachers may have implicit rather than well-articulated theories about teaching mathematics (Cooney, 1985), children may have implicit theories about learning mathematics. The research approached the investigation of children's beliefs about the nature of learning mainly by having the eight participants talk about actual mathematical and non-mathematical learning experiences to help children portray their explicit and implicit beliefs.

The investigation of the eight children's beliefs about learning was designed partly as an introduction to the third research question regarding what factors within learning environments children believe help them to learn mathematics well. Discussion of the eight children's beliefs about learning serves also to facilitate the education community's understanding and appreciation of the possible complexity of children's conceptions. As background, the following discussion includes consideration of writings on children's beliefs about learning, and researchers' and theorists' conceptions of learning, thus giving insights into the complexity of the concept of learning and providing background for the analysis of the children's responses. The discussion begins with consideration of meanings given to learning as a general concept; discussion of mathematics learning, in particular, follows.

The nature of learning

Learning is a complex concept for which there appears to be no one accepted definition. For example, Bigge (1976) defines learning as ". . . an enduring change in an individual" (p. 1). He adds, "It may be considered a change in insights, behaviour, perception, or motivation, or a combination of these" (p. 1). In contrast, the Collins Concise English Dictionary (Wilkes & Krebs, 1982, p. 640) definition has a focus on the gaining of knowledge, also with implication of change; it defines learning as

- 1. knowledge gained by study; instruction or scholarship
- 2. the act of gaining knowledge

This definition provides two perspectives on learning: outcome and process, the former of which was explicit also in the Bigge (1976) definition. The discussion below includes reference to these two aspects but in addition indicates that learning can be interpreted more broadly.

In early work in the twentieth century, learning was seen to occur in students as a response to how the teacher presented material. Theories at that time included those of behaviourists who concentrated on associations between stimuli and response (Bigge, 1976; Siann & Ugwuegbu, 1988). Learning was seen to centre around associations between stimuli and responses, with the rewarding aspect of the association forming the reinforcement, and the practice helping to stamp in the association. For behaviourists, the learning of associations

is the basis of all learning (Siann & Ugwuegbu, 1988), with learning being a change in behaviour (Bigge, 1976).

This view contrasts with the cognitive approach to learning which considers that learning involves "the active restructuring of perceptions and concepts" (Siann & Ugwuegbu, 1988, p. 158), and that "incoming information is structured and processed in memory" by the learner (Weinstein & Mayer, 1986, p. 316). In this approach, learning is considered an active process for the learner that can be influenced by the learner.

Marton (1983) suggests two ways of conceptualising learning: firstly, as a change in the way of understanding some aspect of reality, and secondly, to memorise something, to acquire some procedures and facts. He states that these two conceptions of learning "are ultimately linked with (indeed derived from) two different conceptions of knowledge; namely knowledge as ways of viewing reality on the one hand, and knowledge as a collection of right answers on the other" (p. 291).

In a study conducted by Saljo in 1979 (see Marton & Saljo, 1984, p. 52), results from interviews with a group of adults regarding what learning meant to them revealed five qualitatively different conceptions of learning:

- 1. a quantitative increase in knowledge;
- 2. memorising;
- 3. the acquisition of facts, methods, etc. which can be retained and used when necessary;
- 4. the abstraction of meaning;
- 5. an interpretative process aimed at understanding reality.

Marton and Saljo claim that within the five conceptions of learning there are two pairs of what-how relationships. The first *what* is the increase in knowledge, achieved through the *how* of memorisation. The second *what* is the understanding of reality, achieved through the *how* of abstracting meaning. Looking back at the dictionary definition of learning given earlier, it becomes apparent that, like the writers of the dictionary definition, learners can give definitions that refer to the process (how) and/or the outcome (what) of learning. The outcome might also be called the product of learning.

Alternative conceptions of learning include that inherent in the "SOLO Taxonomy" (Biggs, 1991). SOLO, an acronym for structure of the observed learning outcome, is a general taxonomy of learning that identifies five stages or levels of learning. The first three levels "are concerned with the progressive growth of knowledge or skill in a quantitative sense, the last two with qualitative changes in the structure and nature of what is learned" (Biggs, 1991, p. 12). The first levels concern accrual of knowledge, thus showing similarity to the *what* of increase in knowledge through the *how* of memorisation identified by Marton and Saljo (1984). The final levels focus more on "how knowledge may be theorised about and generalised" (Biggs, 1991, p. 13), suggesting, like Marton and Saljo (1984) that abstraction is a valued and higher order process of learning, and different from learning through memorisation.

Biggs (1991) identifies three approaches to learning called *Surface, Deep* and *Achieving*. The first two appear closely related to concepts of learning; the third, or achieving approach, relates more to motives or intentions and is evidenced in study skill strategies. A student who takes a surface approach to learning, learns by rote methods, memorising unrelated components of a task or tasks (Biggs, 1991). Such a student appears to see learning as a concrete exercise aimed at achieving short-term goals. In a deep approach, task components are integrated with each other and with other tasks, the focus is on the underlying meaning; higher order processes such as theorising and forming hypotheses are part of this approach to learning (Biggs, 1991). As mentioned earlier in relation to a study of first year university students' conceptions of and approaches to mathematical activity, Crawford et al. (1993), describe students' uses of surface and deep approaches in learning mathematics. It appears that inherent in the deep and surface approaches to learning are contrasting conceptions of learning somewhat like those identified by Marton and Saljo (1984). Learning might be a rote exercise or it might involve higher level abstraction of ideas.

The discussion shows also that there are varying perspectives and emphases within the ideas held by theorists and learners regarding the meaning of learning. While it is useful for the purposes of the present research to identify general ideas such as the focus on process or product, or learning through rote methods or by abstracting, consideration of the learning of mathematics, as follows, provides more specific background to the study of children's perceptions of factors influencing their learning of mathematics.

The learning of mathematics

This discussion of the learning of mathematics is relevant as further background to the consideration of the eight research participants' beliefs for two reasons: i) during the research interview discussions, specific examples of learning were requested mostly in reference to mathematics learning, and ii) children's beliefs about the nature of mathematics learning provide background to their beliefs about helping factors for learning mathematics.

There exist many theories, perspectives and interpretations of the learning of mathematics. For the purposes of providing background to the consideration of the children's perspectives, some key ideas are examined below. For example, constructivism is considered in terms of mathematics learning, with a range of perspectives of constructivism contributing to the discussion. It is not the intention, nor possible, that this section give a complete account of the learning of mathematics; there are many perspectives about mathematics learning still under debate (e.g., Steffe, Nesher, Cobb, Goldin, & Greer, 1996), only some of those are considered here.

To illustrate the type of teaching developed in response to conceptions of mathematics learning and to related changes recommended in curriculum documents (e.g., National Council of Teachers of Mathematics, 1989) a small number of research studies are discussed. The classrooms in these studies conform in some manner to what is sometimes referred to as the reform movement in mathematics education as they exemplify teaching and learning approaches different from those of traditional mathematics classrooms. Although the studies were developed with varying intentions and deploy differing modes of operation in the classroom, they were chosen for discussion as they concur with the perspective of learning underlying the reasoning for, and implementation of, the present research study; that is, a constructivist perspective. Each study has, as one key element of its purpose, the investigation of children's learning of mathematics; thus discussion of these studies provides some insights into the complexities of mathematics learning as researchers are seeing it today. The discussion contributes also to the development of a picture of learning of mathematics as it can be interpreted in mathematics classrooms today. As noted above, the discussion of the learning of mathematics also informs and underpins the investigation and analysis within the present research of children's beliefs about mathematics learning. One study, by Cobb and his associates, is discussed in detail to demonstrate the possible complexity of research undertaken from a constructivist perspective; the other studies are discussed more briefly. Prior to the discussion of the research studies, consideration is given to the notion of constructivism and related theories about how people learn.

Constructivism is usually interpreted (e.g., Mousley, 1993a), in terms of von Glasersfeld's first principle that "knowledge is not passively received but actively built up by the cognizing subject" (von Glasersfeld, 1989, p. 162). Pertaining to this principle, Ernest (1996) suggests the metaphor of "carpentry, architecture, or construction work" (p. 335). Understanding is not built from received pieces of knowledge, but is seen as the building of mental structures, the building blocks of which are the products of previous constructions by the learner; thus a recursive process (Ernest, 1996). It has been claimed that von Glasserfeld's first principle is acceptable within all constructivist theories (Ernest, 1991).

According to Ernest (1991), an example of a psychological theory fitting the broad sense of constructivism suggested by von Glasersfeld's first principle, is the schema theory of Skemp (1976, 1986). This theory relates specifically to the learning of mathematics; Skemp's model of understanding has been influential (Mousley, 1997) and important (Herscovics, 1996) in the mathematics education community. Skemp (1986) posits that the learning of mathematics consists of the formation of schema; that is, mental structures in which concepts, or ideas, are interrelated. His belief, that by assimilating something into an appropriate schema one comes to relationally understanding and learning (Skemp, 1976). Skemp associates schematic learning with relational understanding, where concepts are interrelated in the form of mental schema. He identifies a contrasting form of understanding, instrumental understanding, which is inextricably related to rote learning and is accomplished mainly by memorising; instrumental understanding is the product of the learning of rote rules and theorems.

Skemp (1976) does not consider instrumental understanding to be quality understanding of mathematics; he does not value learning by rote methods. He suggests that learners of

mathematics who principally are interested in knowing the *how* of mathematics can be satisfied with rote methods which can lead to competence in, and apparent understanding of, computational methods. But their understanding is instrumental in nature. This can appear to be adequate when objectives are also instrumental in nature. Skemp (1976) notes that a child who has learnt instrumentally may respond in disagreement if it is suggested that the child does not *understand* what has been learnt. As far as the child is concerned, learning has occurred and understanding is present as knowledge has been attained.

In contrast to rote learners, learners who use schematic methods to achieve relational understanding value the *why* as well as the *how* of mathematics. The schematic learner achieves a deeper level of understanding. Skemp posits that it is schematic learning that leads to true understanding as it involves the interrelationship of concepts in mental schemas. Hiebert and Carpenter (1992) suggest also a schematic conception of understanding when they state that "it is useful to think of students' knowledge of mathematics as internal networks of representations" (p. 69). They believe that understanding, a critical element of learning, "occurs as representations get connected into increasingly structured and cohesive networks" (p. 69).

The rote/schematic dichotomy of learning articulated by Skemp, and the related instrumental/relational dichotomy of understanding, are detailed here for the possibility of usefulness in examining the beliefs about the learning of mathematics of the eight children in the present research. The dichotomies alert to the possibility that when a child talks about understanding something mathematical, the understanding may be relational or instrumental in nature. The dichotomies remind also that one should not assume a person's meaning for the concept of learning, and more specifically, that in considering a child's views of factors that impact upon learning of mathematics, it is important first to gain insights into the child's concept of learning. This occurred in the present research.

Skemp's theory is encompassed within the broad sense of constructivism, as are other psychological theories such as those of Kelly and Piaget (Ernest, 1991). According to Ernest (1991), these theorists share the belief that knowledge is built up by the cognizing subject through the construction of mental structures based on experience and reflection. Relational understanding, as discussed by Skemp (1976), appears to be produced through a learner's personal involvement with mathematical problems and ideas.

The theory of contructivism has been interpreted also by classroom researchers investigating children's learning of mathematics. A small number of studies conducted mathematics classrooms, most of which were underpinned by beliefs in constructivism, are discussed here to demonstrate the application of the constructivist theory of learning to mathematical activity in the classroom. The philosophy underpinning each of these studies contrasts with that of the traditional classroom. To provide a setting for the discussion of these studies and to demonstrate the importance of such research in terms of changing views of learning and teaching mathematics, the contrast between traditional and non-traditional classrooms is reviewed.

As discussed briefly earlier, individuals' active cognitive involvement in learning is encouraged in more recent curriculum documents written for the mathematics classroom, contrasting with the activity in traditional classrooms. International mathematics curriculum documents (e.g., Australian Education Council, 1991; Mathematical Sciences Education Board, 1989; National Council of Teachers of Mathematics, 1989, 2000) are based on the view that the learning of mathematics is an active process. The National Council of Teachers of Mathematics states, "knowing' mathematics is 'doing' mathematics. A person gathers knowledge in the course of some activity having a purpose. This active process is different from mastering concepts and procedures" (National Council of Teachers of Mathematics, 1989, p. 7). Observing patterns and relationships, exploring, clarifying, refining and consolidating ideas, generating questions, justifying, generalising, and formulating and testing of conjectures are all important processes in the learning of mathematics (e.g., Australian Education Council, 1991; Board of Studies, 1995, 2000; National Council of Teachers of Mathematics, 1989, 2000). Classrooms underpinned and informed by this philosophy, or theory of learning, are "places where interesting problems are regularly explored using important mathematical ideas" (National Council of Teachers of Mathematics, 1989, p. 5).

Non-traditional classrooms, sometimes called reform or enquiry classrooms, operate according to the belief that learning is an active construction, and also that it is lifelong and collaborative in the sense of being a social phenomenon (Fisher, 1990). The student is seen to engage with the subject matter of mathematics directly; the teacher plays a role like a coach and is involved in co-inquiry with the student (Fisher, 1990). Teachers who have a *connectionist* orientation to teaching mathematics include a "strong emphasis on reasoning and justification" (Askew et al., 1997, p. 27) and a focus on efficiency and effectiveness in children's choice and use of different methods of calculation. They believe that challenge, struggle to overcome difficulties, and "purposeful interpersonal activity based on the interactions of others" (p. 31) are important within learning mathematics. Such views towards learning contrast with those characterised within traditional classrooms.

In traditional mathematics classrooms, mathematics is likely to be fragmented and divorced from reality and enquiry; mathematical ideas are "selected, separated, and reformulated into a rational order" (Romberg & Carpenter, 1986, p. 851). The mathematics is made up of a set of concepts, principles, and skills; it is the transmission of these as content that is the focus in traditional mathematics classrooms. The learner is a passive receiver of knowledge, engaging primarily with the teacher rather than with the mathematics (Fisher, 1990). Teachers with a *transmission* orientation (Askew et al., 1997) associate learning mathematics with remembering routines which are introduced one at a time to students. Such learning is an individual activity, pupil strategies are of little importance, and pencil and paper methods are emphasised. Referring back to the work of Marton and Saljo (1984), it appears

that traditional classrooms can be described as having mostly an increase in knowledge or routines as the *what* of learning, achieved through the *how* of memorisation.

A third orientation towards mathematics teaching, *discovery*, was identified by Askew et al. (1997). Teachers with such an orientation believe that learning is an "individual activity based on actions on objects [and] pupils need to be 'ready' before they can learn certain mathematical ideas" (p. 31). Pupils use their own strategies and work out or discover things for themselves.

A range of research studies in classrooms is discussed below. Most were conducted in non-traditional classrooms because of their interest in the relevance of constructivist theories of learning to the classroom situation.

The above discussion of Skemp's theory of learning introduces the concept of what might be called quality learning (leading to relational understanding) as distinct from nonquality learning (leading to instrumental understanding). Constructivists acknowledge that children construct understandings no matter what the teaching style (Cobb, Yackel, & Wood, 1992; Mousley, 1993a). However, at question is the nature and quality of children's constructions (Cobb et al., 1992). Some research studies are designed to take advantage of students' constructions, aiming to facilitate the construction of higher level understandings in mathematics; "the experience of a world structured by mathematical relationships is . . . [considered] a central aspect to meaningful mathematical activity" (Cobb et al., 1992, p. 8). These studies seemingly aim at what Marton and Saljo (1984) describe as learning with a *what* of the understanding of reality, and a *how* of abstracting meaning.

In the reform mathematics classroom, mathematics is transformed from the traditional transmission and absorption to *enquiry mathematics* (Cobb, Wood et al., 1991; Richards, 1996). Negotiation of meanings, mathematical discussions, and listening by students and teachers are important elements of mathematical activity (Richards, 1996). It is recommended that student activity includes small group work where students negotiate problems and explain ideas (Forman, 1996). Interaction and collaboration are significant because they mobilise reflection, which is the basic mechanism for abstraction at higher levels (Verschaffel & De Corte, 1996). Learning is viewed as "an active, social, problem-solving process" (Cobb, Wood et al., 1991, p. 8).

One such study, conducted in second grade mathematics classrooms by Cobb and his associates, was compatible with constructivist principles of mathematics learning (Cobb, Wood et al., 1991; Cobb, Yackel et al., 1991; Wood & Sellers, 1996; Yackel, Cobb, & Wood, 1991), with instruction "generally compatible with a socioconstructivist theory of knowledge" (Cobb, Wood et al., 1991, p. 3). The study was also "broadly compatible with the recent NCTM [National Council of Teachers of Mathematics] . . . reform recommendations on students' learning and teachers' practice" (Cobb, Wood et al., 1991, p. 4). Cobb and his associates' research occurred in the classroom situation, with negotiation and interaction recognised as central to their constructivist view of the teaching-learning process. Learning

was considered both an individual and collective activity (Cobb, Yackel et al., 1992). As stated earlier, this study is discussed in detail to illustrate the possible complexity of research undertaken from a constructivist perspective, but also to consider more closely the constructivist view of mathematics learning underpinning the study.

In the first stage of the study, individual children were interviewed regarding knowledge and abilities in arithmetic and space. The second stage of the study was informed by models of children's construction of arithmetical knowledge developed from constructivist teaching experiments (e.g., Cobb & Steffe, 1982), and by the stage one interview results (Cobb, Yackel et al., 1991; Yackel et al., 1991). This stage included the development of sample activities. In the second stage, a classroom teaching experiment in one Grade 2 classroom was conducted for one year with instruction in all aspects of mathematics provided by the regular class teacher. Teaching was compatible with constructivism, that is, was based on the psychological perspective of the learning of mathematics "as one in which students reorganise their activity in an attempt to resolve situations which they find problematic" (Cobb, Yackel et al., 1991, p. 103). Learning activities were designed to be potentially problematic and to make sense to students at a variety of conceptual levels. Instruction was thus individualised, and focused on developing the individual as an intellectually autonomous learner in mathematics (Cobb, Yackel et al., 1991; Wood & Sellers, 1996; Yackel & Cobb, 1996). Activities were designed also to facilitate concurrent conceptual and procedural developments (Cobb, Wood et al., 1991; Cobb, Yackel et al., 1991) and to challenge children to reflect so as to organise their ideas at increasingly complex levels of abstraction.

The teacher role reflected the view of children's learning. The teacher had the complex role of initiating and guiding mathematical negotiations, which included "highlighting conflicts between alternative interpretations and solutions, ... implicitly legitimizing selected aspects of contributions to a discussion in light of their potential fruitfulness for further mathematical constructions . . . and guiding the development of taken-to-be-shared interpretations when particular representational systems [were] developed" (Cobb, Wood et al., 1991, p. 7). This was facilitated by the basic instructional strategy of small group problem solving during which the teacher moved around, observed, and frequently intervened, followed by whole class discussion in which in-depth discussion of children's mathematical thinking was encouraged. Collaborative dialogue with genuine commitment to communicating was considered to provide learning opportunities for children (Yackel et al., 1991). Teacher actions included genuinely listening to each student, respecting actions or explanations as reasonable to a student, but still judging whether the student's understanding was productive to future mathematical development and, if necessary, intervening to guide reconceptualisation of the situation (Cobb, Wood et al., 1991). This contrasts with the traditional view of teacher as one who transmits knowledge and *funnels* students towards the solution the teacher has in mind (Cobb, Yackel et al., 1991). Features of the learning in the Grade 2 project class included "disequilibrium, conflict, and problem solving" (Yackel et al.,

1991, p. 396). Beliefs, knowledge and truth were not "delivered" by the teacher in this classroom, but "emerged" as a result of interaction and negotiation resulting from instructional activities (Cobb, Yackel et al., 1991). The teacher was not assumed as an authority figure but mathematics learning was controlled and monitored also by the students. The project continued in its third stage with 24 Grade 2 teachers involved as class teachers (Cobb, Yackel et al., 1991).

Children's learning, and the role of individuals and others in their learning, was of interest to the researchers. Cobb, Yackel et al. (1991) believe that in relation to the social setting of the school, where the focus is not solely on the individual, difficulties arise with what they see as the mathematics education community's general acceptance that "individuals each construct their individual mathematical worlds by reorganising their experiences in an attempt to resolve *their* problems" (p. 84). This view is addressed by Cobb and his associates within discussion of their research study. Children's constructions were not seen as natural constructions made by children on their own, but were seen as constrained by a number of factors in the classroom. These included the activities, the materials made available, the requirement to explain and sometimes to justify solutions, and the emphasis on understanding the ideas of others (Cobb, 1988; Cobb, Yackel et al., 1991). Children "engage in consensually constrained mathematical activity" (Cobb, 1988, p. 13) and construct understandings from problems encountered in the social context of classroom instruction (Yackel et al., 1991). Through interacting with others and negotiating meaning, children reorganise their experiences to be able to communicate in a consensual domain and are "acculturated to the mathematical ways of knowing of the wider community" (Cobb, Yackel et al., 1991, p. 94). In this respect, mathematics is seen by these researchers as evolving cultural knowledge rather than as mind-independent knowledge as suggested by Plato (Cobb, Yackel et al., 1991). Cobb and his associates believe children contribute to the evolution of this cultural knowledge within their educational experiences. This anthropological perspective is seen to complement the psychological perspective of the learning of mathematics as cognitive development (Cobb, Yackel et al., 1991). Mathematical learning is viewed as a process of both acculturation and individual construction (Cobb, Wood et al., 1991). Thus classroom experiences may contribute to, or constrain, the construction of children's beliefs about mathematics and learning. This suggests that, as contended by the present research, it is important for teachers to be aware of children's constructions and consider how they may relate to the environment in which children are learning mathematics.

Cobb and his fellow researchers' interpretative stance involves also a sociological perspective with the development of general classroom norms being of interest. More recently this work was extended to include the explication of sociomathematical norms, that is, "aspects of mathematical discussion that are specific to students' mathematical activity . . . [such as] normative understandings of what counts as mathematically different, mathematically sophistocated, mathematically efficient, and mathematically elegant . . . [and]

what counts as acceptable mathematical explanation and justification" (Yackel & Cobb, 1996, pp. 458, 461). The sociomathematical norms of the inquiry classroom are seen by Yackel and Cobb as related to the development of beliefs and values consistent with the reform movement in mathematics education. These norms also provide a further perspective from which to consider mathematics learning.

In assessment of the constructivist based study by Cobb and his colleagues, comparisons of ten second grade project classes with eight second grade non-project classes showed that

levels of computational performance were comparable, but there were qualitative differences in arithmetical algorithms used by students in the two groups. Project students had higher levels of conceptual understanding in mathematics; held stronger beliefs about the importance of understanding and collaborating; and attributed less importance to conforming to the solution strategies of others, competitiveness, and task-extrinsic reasons for success. (Cobb, Wood et al., 1991, p. 3)

The authors believe these findings suggest that a problem-centred approach based on constructivist principles of learning, involving teacher and student discourse in the development of meaning, and therefore compatible with recommendations in recent curriculum documents, is feasible in mathematics classrooms. Later evaluation of the approach as applied for the longer period of two years during second and third grade, and as compared to non or partial problem centred mathematics classrooms, further supports the findings of higher computational proficiency and conceptual understanding, as well as stronger beliefs about finding one's own or different ways to solve problems (Wood & Sellers, 1996). Longitudinal comparisons, including after students from problem centred classes moved to classes using textbook instruction, show benefits also of the problem centred approach (Wood & Sellers, 1997).

The research conducted by Cobb and his associates developed over time and is interpreted from many perspectives, as demonstrated above. It was an investigation of children's learning in actual classroom settings that contributed to the ongoing development of the researchers' and others' understandings about children's learning of mathematics. The study was underpinned by constructivist perspectives of the learning of mathematics, thus the teaching and learning contrasted with that of traditional classrooms.

The present study also is underpinned by constructivist principles. The learner is perceived as an individual who contributes to his or her learning of mathematics and concurrently develops individual perceptions/beliefs. Discussion of the study by Cobb and his associates provides insights into the possible complexity of a constructivist based classroom research study and shows one interpretation of the constructivist theory in relation to the classroom situation. The discussion also provides some perspectives from which to examine children's learning of mathematics when informed by constructivist principles. The present study is different in many ways but has the common perspective of seeing children as individuals who construct their own understandings, beliefs and values about mathematics no matter what the teaching/learning situation (Cobb et al., 1992; Mousley, 1993a; Yackel & Cobb, 1996), but whose constructions may be constrained by that situation.

As mentioned above, further research studies based on constructivist principles, and exemplifying classrooms informed by the reform movement in mathematics education, are also considered briefly as background to the present study. Discussion of these studies also provides insights into present understandings of mathematics learning.

In the study conducted by Cobb and his associates in a Grade 2 classroom, children were posed with problem solving activities that encouraged the combined development of conceptual understanding and procedural competency. For example, children were "encouraged to construct their own increasingly efficient nonstandard computational algorithms" (Cobb, Yackel et al., 1991, p. 104). Similarly, Kamii, Lewis, and Livingston (1993) discuss a study in which children invented their own procedures when confronted with number problems. They report that the arithmetic program in one school was based on the theory of constructivism. Work with Grade 2 children is described in which the teacher wrote one problem after another on the board and asked for a quick and easy way to solve each problem. The children worked in small groups or the class worked together with the teacher interacting with a volunteer and encouraging the rest of the class to voice agreement or disagreement and to speak up if something did not make sense. It is stated that "the exchange of points of view is very important in a constructivist program, and the teacher is careful not to reinforce right answers or correct wrong ones . . . the class will continue to think and debate until agreement is reached" (Kamii et al., 1993, p. 201). The authors believe it is better for children to invent their own procedures as then they continue to do their own thinking, strengthen their understanding of place value, and develop better number sense. This is contrasted with traditional approaches where the focus is on memorisation. Comparisons made with children in Grades 2 and 4 who were taught by teachers who elected not to follow the constructivist approach, and therefore taught algorithms, suggest that the children who invented their own procedures demonstrated more number sense and tended to think about the entire numbers and not about each column separately as did many of the other children. The authors suggest that the outcomes of their teaching approach are positive in terms of quality mathematics learning. Kamii and her fellow researchers suggest that the outcomes of teaching based on a constructivist philosophy, that is of teaching in which children invent their own procedures rather than using traditional algorithms, are positive in terms of quality mathematics learning (Kamii & Dominick, 1998; Kamii et al., 1993)

The above discussion of the work of Cobb and his associates and Kamii and her associates highlights possible features of cognitively active student learning and provides background to the consideration of ideas about learning conveyed by the children in the present study. The discussion highlights also the issue of the nature and quality of children's constructions. Assessment of the programs included consideration of factors such as children's competency and beliefs. In the present research beliefs were of interest, but these were not examined solely in terms of the objectives or parameters of a particular style of teaching. The present research left such factors open; children's beliefs were investigated in some depth, but couched in the children's experiences as they perceived them and as they chose to portray them. Individual children's beliefs about their learning in general and their mathematics learning in particular were investigated. Children's mathematics classes were not judged as to whether they were based on constructivist principles of learning.

Another major investigation of children's early number learning in reform based classrooms is examined to provide further insights into non-traditional views of mathematics learning and teaching. The Cognitively Guided Instruction (CGI) project consisted of a range of stages including a one year experimental study with first grade teachers (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989) and a four year study with teachers of first, second, and third grades (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996), with, at its core, a research-based model of children's thinking. In contrast to the two previously discussed studies, mathematics content was a key feature of the CGI study, specifically in relation to addition and subtraction in the first grade element of the study (Carpenter et al., 1989) and all four operations in the longitudinal study (Fennema et al., 1996), with a focus on the use of a range of word problems. It was envisaged that through exposure to a researchbased model about content and children's thinking, and with development of teacher knowledge of, for example, differences among problems and children's strategies for solving different problems, teachers would modify instruction to become more in line with current curriculum recommendations and children would learn mathematics with understanding (Carpenter et al., 1989; Fennema et al., 1996). Children's understandings would involve high levels of thinking and not be instrumental in nature. Development of children's thinking was an important element of the CGI approach, with each person's thinking in the mathematics classroom respected by peers and the teacher; children's informal or intuitive systems of mathematical knowledge could be used as a foundation for the development of more abstract ways of thinking about numbers and solving problems (Fennema, Carpenter, & Franke, undated).

The research-based model of children's thinking was presented to participant teachers through workshops and interpreted by teachers in relation to their own students. The model acted as a "catalyst between teachers' intuitive knowledge and principled knowledge of their own students' thinking which they developed as they taught" (Fennema et al., 1996, p. 431). Explicit guidelines for instruction were not provided; teachers had to decide, for example, "how to consider students as they selected problems, how to question children, and how to organise their classrooms" (Fennema et al., 1996, p. 432), thus unique CGI classrooms emerged (Fennema et al., undated). Teacher development of curriculum and teacher reflection were facilitated, supported, and allocated time within the project. In order for teachers to implement a CGI approach, they needed "to internalise its main components – or hold certain beliefs . . . including that children construct their own knowledge, . . . that skills should be

taught in relationship to understanding and problem solving, . . . [and] that instruction should faciliate children's construction of knowledge" (Fennema, Carpenter, & Peterson, 1989, p. 183).

Eighteen of the 21 teachers in the longitudinal study demonstrated fundamental changes in beliefs and instruction over the four years of the study (Fennema et al., 1996). Teachers came to perceive their role as active: they facilitated learning through engaging children in problem solving and reporting on solution strategies. Teachers did not perceive their role as telling children how to think. Some teachers changed more than others but the reason is not clear. Patterns in the relationship of changes in belief and instruction varied among teachers, in degree and in timing. Each year each teacher was assigned an instructional level by the researchers. The description of teachers at Level 4B, the highest level of cognitively guided instruction and fitting recommendations made in curriculum documents, indicates the style of teaching favoured by the researchers. Level 4B was described:

Provides opportunities for children to be involved in a variety of problem-solving activities. Elicits children's thinking, attends to children sharing their thinking, and adapts instruction according to what is shared. Instruction is driven by teacher's knowledge about individual children in the classroom. (Fennema et al., 1996, p. 412)

At the end of the study more teachers were at Level 3 than at any other level, that is, having children involved in problem-solving and reporting solutions, but with the teacher not making use of what was shared by children to determine future instruction. Although these teachers were judged not to be teaching at the highest level of cognitively guided instruction, the curriculum was very different from traditional curriculum as children were engaged in "problem solving, reasoning, looking for connections, and communicating about mathematics" (Fennema et al., 1996, p. 417). Higher level thinking, related to schematic learning, appears to have been valued by the researchers; although not guaranteed within the CGI project participants' classrooms as teaching methods were determined by the teachers, not the researchers; evaluation of the study suggests that such thinking occurred in the majority of classrooms involved in the study.

The research questions of the first grade CGI study (Carpenter et al., 1989) began by considering the teacher. They investigated whether teachers exposed to the research-based model of children's thinking would be different, in their instructional process, beliefs, and knowledge of children's abilities, from teachers not familiar with the model and who teach in a traditional manner. From this point, the focus turned to the students, that is, levels of achievement and confidence, and their beliefs about themselves and mathematics. But ultimately it was the quality of children's learning of mathematics that was the outcome of concern (Fennema et al., 1989). The researchers believe that learning is influenced by learners' cognitions and behaviours, these are influenced by classroom instruction, this is influenced by teacher decisions, and these are directly influenced by teachers' knowledge and beliefs; thus the investigation used teachers' knowledge and beliefs as a starting point in a chain aiming to lead to better student learning of mathematics (Fennema et al., 1989). If such

a chain is reasonable, it suggests again a reason for the present study: by gaining insights into children's beliefs, teachers' knowledge and beliefs about their students are broadened and strengthened. In turn, teacher decisions can be better informed and therefore lead to better student learning.

Outcomes for students in the CGI first grade study showed "students in experimental classes exceeded students in control classes in number fact knowledge, problem solving, reported understanding, and reported confidence in their problem solving abilities" (Carpenter et al., 1989, p. 500). In the longitudinal study (Fennema et al., 1996), changes in teaching were found to relate to changes in students' achievement. Gains were evident in student concepts and problem solving, and there was no decline in performance on computational skills. Fennema et al. (1996) suggest therefore that "developing an understanding of students' mathematical thinking can be a productive basis for helping teachers to make fundamental changes called for in current reform recommendations" (p. 403). Such changes in teaching appear to assist children in meaningful learning of mathematics.

It appears that while teacher knowledge, beliefs and instruction were central to the CGI project, these were related to a perception of learning valued by the researchers. This in turn related to the image of learning recommended for reform classrooms, which is very different from learning in traditional mathematics classrooms. The emphasis on children's construction of mathematical knowledge is apparent also within other current studies of mathematics teaching and learning. Fuson et al. (1997) write of four studies investigating problem solving approaches to the teaching and learning of multidigit concepts and operations, one of which is the CGI project. The studies share an interest in constructing overarching views of children's learning of mathematics and share a belief in the learning of mathematics "as a conceptual problem solving activity rather than . . . transmission of established rules and procedures" (Fuson et al., 1997, p. 131). In each of the four studies, the teacher poses problems, coordinates discussion of strategies, and joins the students in asking questions about strategies; "the intent is to create an environment in which teachers support students' efforts to construct their own solution strategies" (Fuson et al., 1997, p. 131). An underlying belief in children's construction of mathematics is apparent within these studies.

Each of the studies discussed above was built upon a vision related to constructivist principles of mathematics learning and conveyed a belief that children construct their own solution methods and conceptual structures. There appeared a common theme of aiming for children's mathematical thinking and learning to be in line with that recommended in reform documents. Overall, the studies gave endorsement to classes based on non-traditional principles of learning and teaching mathematics and demonstrated that constructivist theories of learning can be supported in the primary mathematics classroom. The studies suggest that teachers take an active role in children's learning of mathematics through, for example, posing appropriate problems and questioning children about their strategies and thinking. A major study undertaken undertaken in the United Kingdom (Askew et al., 1997; Brown, Askew, Rhodes, Wiliam, & Johnson, 1997), involved teachers with varying orientations towards teaching mathematics. Three main classifications were described: *connectionist, transmission,* and *discovery,* as discussed earlier in this document. The study found that the highly effective teachers focused on developing rich networks of connections between mathematical ideas, encouraged students to reason and justify to establish connections and address misconceptions, focused on the development of mental calculations and stressed the importance of making efficient choices as to method of calculation. The beliefs and practices of the least effective teachers were classified as *transmission* and *discovery*. Procedural responses, or responses less elaborated than those of connectionist teachers regarding alternative meanings and representations, were evident amongst these teachers. It appears then that those teachers described as having a *connectionist* orientation had more in common with constructivist principles of learning, as discussed above, than did the other teachers within this study.

It appears that classrooms consistent with what is sometimes called the reform movement, including those studied by researchers such as Cobb and his colleagues, Kamii and her colleagues, and within the CGI project, are based on a fallibilist perspective that knowledge is not pre-existing and is therefore created by humans. It appears that processes of mathematics, including problem solving and constructing of solution methods, are common elements of the classrooms in these studies, albeit facilitated in ways that differed in some respects. Content appears to be an important element of the CGI project but underlying the learning of this content is the belief that children learn best from meaningful construction not from transmission. Thus process appears equally important. It is possible to consider the studies discussed above in terms of a model of the learning process proposed by de Lange (1996). The emphasis is placed on "process as opposed to content or outcomes . . . knowledge is ... being continually created and re-created, not an independent entity to be acquired or transmitted" (de Lange, 1996, p. 58). The wording of the first part of the quotation perhaps would not be agreed upon by all the researchers discussed, particularly those working in the CGI project, but the focus on creation, or what might be called construction of mathematics, as opposed to transmission, is common to the studies discussed. De Lange's model of learning is cyclic, with "the student's real world adjusted continuously" (p. 58); reflection, abstraction, and "mathematizing in applications" are some key elements of his model. De Lange claims that development of mathematical concepts and ideas starts from the student's real and very dynamic world; "the picture adjusted almost on a daily basis" (p. 57). As discussed above, Ernest (1996) speaks also of a cycle or recursive process within learning, one in which student constructions build on previous constructions.

It is useful for the purposes of the present research to identify general ideas such as the focus on process and product, as above. In addition, consideration of the learning of mathematics, as discussed in the context of research studies, provides more specific

background to the study of children's perceptions of learning and helping factors for learning mathematics. The detail from the research studies facilitates insight into how a theory of learning, such as constructivism, can be taken into consideration within the teaching of mathematics. Constructivism also is a theory that also can underpin alternate research of children's learning of mathematics, such as the present study.

The discussion of the learning of mathematics included the contrasting of rote and schematic learning leading to instrumental and relational understanding. Constructivist and related perspectives were focused upon because of their relevance to the stance taken in the present study. Traditional classrooms, or classrooms where teachers have transmission orientations, were described briefly and contrasted with non-traditional classrooms, sometimes called reform or enquiry classrooms or described as classrooms where teachers have connectionist orientations. The discussion of research studies in non-traditional classrooms illustrated the philosophy and workings of such classrooms and the possible chain between teacher knowledge and beliefs, teacher decision making, student beliefs and student learning of mathematics. The latter is of particular relevance to the present study; in Chapter 1 it was suggested that teacher knowledge of learners' beliefs can inform decision making in the quest to cater as best as possible for all individual learners in mathematics classrooms.

The discussion now considers previous findings on children's perceptions of mathematics learning.

Children's perceptions of mathematics learning

The above discussion of the nature of mathematics learning essentially was of adult perspectives; a further important dimension is the perspectives of mathematics learning taken by school-age students. The following discussion illustrates that many children who have experienced mathematics teaching of a traditional nature hold a relatively narrow view of mathematics learning. It appears that in contrast, when mathematics teaching is informed more by the constructivist theory of learning, children are more likely to hold broader views of the nature of mathematics learning. The discussion begins by considering results of studies suggesting more closed views of the nature of mathematics learning.

Garafolo (1989), as discussed earlier, claims that children commonly consider mathematics as a collection of rote, mechanical procedures for the purpose of arriving at correct answers. In his observation of a lesson in a Grade 6 mathematics class, Garafolo saw no evidence of common-sense responses or everyday reasoning; students attempted to use step-by-step procedures only. Garafolo's description of the students' responses to the problem posed suggest that children held a view of the learning of mathematics as involving the mastering, remembering, and correct application of procedures. When first attempts did not produce a correct method for the particular problem, the children continued by guessing possible appropriate methods for solution. Frank (1988) found a similar view of the learning of mathematics among a group of mathematically talented middle school students enrolled in a course in mathematics problem solving with computers. Beliefs within this group included
that learning mathematics is mostly memorisation and that mathematics is a *package* of facts, rules, and procedures to be passively received by the student, from the teacher. Students in the group believed correct answers indicate understanding of material, suggesting a possible instrumental view of the understanding of mathematics. The focus by students on use of memory in learning mathematics rather than on reasoning (Frank, 1988; Garafolo, 1989; Spangler, 1992), suggests a view of mathematics learning different from that proposed in curriculum documents, in the United States and Australia at least (e.g., Australian Education Council, 1991; National Council of Teachers of Mathematics, 1989, 2000).

There is further evidence that children consider the learning of mathematics as the learning of a collection of rules (e.g., Erlwanger, 1975; Herrington, 1990). In a well-known case study conducted by Erlwanger (1975), Benny, a 12-year old judged as above-average in mathematics learning, searched for patterns from which to generate rules to solve problems and developed different rules to solve different types of problems. Although he could invent his own rules, he "believed that the rules in mathematics were all invented, and they remained unaltered although the work became more difficult" (p. 221). For Benny "the learning of mathematics became a 'wild goose chase' in which he was chasing particular answers" (p. 216). His use of patterns, while a major element of his learning of mathematics, was not consistent in nature with the pattern-searching recommended in documents (e.g., Australian Education Council, 1991; Schoenfeld, 1987) as discussed in the earlier section on the nature of mathematics.

A different perspective regarding children's beliefs about what it means to learn mathematics relates to the degree of familiarity with the mathematics being learnt. Stodolsky et al. (1991) found their 60 Grade 5 interviewees mostly considered learning mathematics as becoming proficient in new techniques. However, the authors state also, that when talking about learning mathematics, many of the children were not talking about learning something completely novel: "Implicit in the students' remarks is the view that math involves knowing how to do something. If skills already mastered are involved, they might be able to 'learn' it, but what is not known is fairly impenetrable" (p. 105). Such a view appears somewhat contradictory; alternatively, it may suggest that something in mathematics can be learnt if it is an extension of something that is known, that is, that some familiarity is necessary to have the confidence or belief that *learning* will occur. One child in the study said, "It depends on what the problems are. I don't think I could have learned dividing mixed numbers because you sort of have to know what you're doing before you can do it" (pp. 105-106), suggesting the need for familiarity with the mathematics involved. Such responses suggest also a perception of learning as an outcome rather than as a process of coming to know.

Learning is not a straightforward concept for all children and is not always associated with school. For example, Cotton (1993), found that for the children of five to nine years of age with whom he worked and talked in small mathematics groups, learning was a word with which they were generally uneasy. The youngest children made no mention of school when

discussing learning and indicated home as the most popular place to learn. For them, school was a place for doing work, not for learning. Although the other children included different activities as learning situations away from school, all ages associated school with work or with doing things such as *doing maths* or *doing spelling*. Results from McDonald and Kouba (1986) suggest a similar perception; they found that many children made comments such as "it isn't math because it isn't work" (p. 29), suggesting that, as mathematics was seen primarily as school-related, mathematical activity at school was considered as *work*. Children in McDonald and Kouba's (1986) study were not reported to have said anything to the effect that a situation was not mathematics because it was not learning. The results from Cotton (1993) and McDonald and Kouba (1986) suggest that *work* and *doing* are seen by children as different from learning when discussed in relation to mathematics.

McDonald and Kouba (1986, p. 23) report also that children see mathematical activity in real-life situations as active: "In order to use or do mathematics, a person must 'do something'". According to Frank (1988), children consider their role as learners at school as passive, that is, as receivers of mathematics, and see teachers as the active participants in the teaching learning process, that is, as those who transmit the mathematics.

However, recent studies of beliefs held by children experiencing teaching more in line with reform classroom recommendations, suggest that children can develop different views about the learning of mathematics. For example, Franke and Carey (1997) found that children in first grade problem solving classes within the Cognitively Guided Instruction project, referred to their school mathematics experiences as involving "problem solving, use of manipulatives, talking about mathematics, and solving problems in a variety of ways" (p. 14). Only two children raised more traditional notions of doing mathematics, suggesting that the majority of the CGI children held perceptions of learning mathematics different from those of children studied by Frank (1988), Garafolo (1989), and Spangler (1992). It is noted that the grade one children in Franke and Carey's study were responding to a question about what mathematics is like in first grade, not a question specifically asking about the learning of mathematics, thus some conclusions of children's beliefs about learning mathematics are made by implication only. As Cotton (1993) discovered, as discussed above, for young children, *doing* mathematics is not necessarily considered the same as *learning* mathematics. However, data from Franke and Carey's (1997) study do indicate clearly a different view of the role of the teacher in mathematics learning. Posed with a question of who would resolve a difference-in-answer situation when working with a partner, 78 percent of the children thought they themselves would do this, with only 3 of 36 children believing it was exclusively the teacher's role to say who was correct. This finding suggests a perception of less dependence on the teacher and a more active role for students when learning mathematics than apparent in studies of beliefs of children experiencing more traditional programs. It suggests also a view more in line with a constructivist perspective: that situations found personally problematic or requiring resolution of conflict are learning opportunities.

Longitudinal analyses of another problem centred mathematics program reveal that children in problem centred classes held different beliefs about doing mathematics from those in classes where textbook tasks were the focus (Wood & Sellers, 1997). Children in problem centred classes were more likely to be motivated by a task centred belief, that is, that collaboration with others can lead to understanding of mathematics, and believed that doing mathematics involves finding their own or different ways to solve problems (Wood & Sellers, 1997). The focus on resolution of problems portrays the learning of mathematics as an active process by the learner; it does not suggest the traditional association of memorising and the learning of mathematics.

The above discussion of a range of studies indicates that children may associate mathematics learning with processes such as memorisation. This appears to be associated primarily with traditional forms of teaching, and implies a passive, receiver role for learners and a more active role for teachers. Results from projects taking a more problem centred approach, as recommended in more recent curriculum documents, suggest that these students may hold different beliefs about learning mathematics, including that the learner is an active participant who solves problems and is not dependent on the teacher for resolution of conflict.

Children's beliefs also reflect complexities related to factors such as whether they are comfortable with the notion of learning mathematics as opposed to doing mathematics, and whether mathematical activity in the classroom and elsewhere are both seen as mathematics. Also, children sometimes associate learning with their beliefs of what they can achieve or do, that is, learning is associated with already known skills. Mathematics that is not known is not always seen as achievable.

Thus children's views about mathematics learning are complex and can relate to a range of elements as discussed above. In the present research, insights were gained into children's beliefs about the nature of mathematics and the nature of learning, and also into beliefs about helpful factors in the learning environment.

Learning environment

The third research question for this study, "What factors within learning environments do children believe help them to learn mathematics well?", introduced the concept of a student's learning environment, a concept interpreted in many ways in past research, as indicated below. The meaning taken in the present research is informed by previous studies, but does not replicate that in any known study. The focus of the present research on a small group of individuals and data collection by intensive and varied qualitative data collection procedures, as discussed in Chapter 3, also is generally different from known previous work in the research of learning environments. The discussion below introduces the term *mathematical learning environment* and explains that this appears a different concept from the *learning environment* concept used in the present study.

To introduce the concept of the learning environment, previous work in this field is reviewed. Early work regarding the relationship between the environment and the person included that by Lewin (1936) and Murray (1938). Lewin contributed significantly through his recognition of the importance of what he called the *whole situation*, that is, the interaction of personal characteristics and the environment in the determination of human behaviour. While his theory was not specifically developed in relation to learning in schools, it can now be seen as a major contributor to theory regarding educational life (Fraser, 1994). It was the intention of the present study that a *whole situation* approach be taken through the consideration of what might be called internal and external factors in learning environments. Briefly, these relate to factors internal to the learner such as motivation, and factors external to the learner such as grouping patterns.

Murray (1938), in referring to the work of Freud, discusses the idea that an object (a thing, person or institution) that evokes a need is said to have *cathexis*. If the object evokes a positive need (for example, achievement, recognition, affiliation), indicating that the subject likes the object, it is said to have positive cathexis, or value. But to Murray it is inappropriate only to enumerate the positively and negatively cathected objects, even though this will tell us which entities in the environment have drawing or repelling power for the individual. He believes that enumeration has limited meaning, that the objects list can only be understood by those who have had experience with those objects, and that it will be only through intuition that one can imagine why the objects repelled or appealed to the person.

For this reason the exploration of children's perspectives in this research went beyond enumeration to include more indepth exploration and discussion of children's perspectives as accessed through a range of procedures. Taking into account the ideas of Lewin (1936) and Murray (1938), the present research intended to

- identify personally perceived positively and negatively cathected items in the mathematics learning environments of the children participant in the study;
- have children describe and discuss the items in their own terms so as to gain insights into children's reasoning for identification of those items;
- meaningfully analyse the data from the children by abstracting from the concrete so as to find similarities among events (Murray, 1938); and
- explore the "whole situation" from the perspective of the child, that is, in the identification of factors children perceive to impact upon their own learning of mathematics, to allow for the expression of personal factors such as motivation, as well as more tangible external factors.

Following from these four objectives, the research used the term *learning environment* in a broad sense to encompass a range of possible positively and negatively cathected items. The factors identified by the children are the components of the learning environment as the term is used in this study. The learning environment may be made up of factors internal to the child, for example, motivations and attributions, and/or factors external to the child, for example, ecological features, and communication patterns. The openness of the interview procedures developed or adapted for the research allowed children to express their

perceptions without predetermined, researcher-imposed analytic structures underlying the data collection. It is acknowledged that the external/internal divide can be problematic, as a factor that appears external to the researcher may involve internal encoding by the child. For the purpose of simplicity the divide was retained in the initial stages but it was not intended that analysis would necessarily be according to this divide. However, the external/internal divide is useful for illustrating the breadth of the working definition of *learning environment* and suggests one perspective from which it was possible that findings be viewed.

Figure 5, an extension of Figure 3, demonstrates that the *learning environment* construct has the potential to encompass internal and external factors. It demonstrates also that in this study the learning environment construct was deployed in relation to children's perceptions of helping factors for their learning of mathematics, that is, in the investigation of the third key research question.



Figure 5. Children's perspectives - the extended model.

The term *learning environment* is perhaps one of convenience, as it was chosen to allow for the inclusion of all factors of influence upon the learning of mathematics as identified by the children. While there is some opinion that environment refers to what is outside the person (e.g., Tangiuri, 1968), the present use of the term learning environment is not totally contrary to previous studies. Within the growing body of research on the learning environment over the past two decades a range of meanings have emerged.

Nielsen and Kirk (1974) discuss the construct in a broad manner stating that "a learning environment could be everywhere and entail practically anything" (p. 57). Other researchers take differently focused, but what might also be called broad, perspectives. For example, Fraser and Walberg (1991, p. x) state that "educational environments can be considered as the

social-psychological contexts or determinants of learning". Insight into the meaning intended by these authors is found by referring to the instruments discussed in the introduction to the Fraser and Walberg (1991) publication, and discussed by Fraser (1991), and by noting the kind of environmental factors upon which responses are sought. These instruments, which aim to measure the perception of and/or preference for the learning environment, include items grouped according to scales such as satisfaction, teacher support, affiliation, participation, material environment, independence, speed, competitiveness, and difficulty.

The *material environment* scale, as listed among those above, is the only scale of those discussed by Fraser (1991) that explicitly refers to external, material type factors. This scale is found only in the Learning Environment Inventory and in the Science Laboratory Environment Inventory, instruments designed for use in secondary school classes (Fraser, 1991). Learning Environment Inventory items (Fraser, Anderson, & Walberg, 1982) such as "The room is bright and comfortable", "The classroom is too crowded" and "The books or equipment students need or want are available to them in the classroom" indicate clearly that Fraser and Walberg's definition of educational environments does not exclude the physical and organisational aspects of the classroom. However, material environment is one of many scales and is not included in the My Class Inventory (Fraser, Anderson, & Walberg, 1982), the only instrument Fraser (1991, 1994) discusses that is designed for use in primary classes.

Results from trialing of procedures for the present study, and from previous studies by the author using a drawing and written description data collection approach at the primary level, indicate that factors which children perceive to help their learning of mathematics may be internal to the child such as persistence and wanting to learn, or may be external to the child, such as availability of materials or presence of a *teacher* (teacher in this sense refers to any person in a teaching role; this person could be a parent or older sibling, for example) (McDonough, 1993; McDonough & Wallbridge 1994).

It is proposed that an alternate conceptualisation of the learning environment encapsulate physical and architectural factors (e.g., MacAulay, 1990; Moos, 1973; Weinstein, 1979; Tangiuri, 1968), structural and organisational factors (e.g., MacAulay, 1990; Moos, 1973; Tangiuri, 1968), *teacher* characteristics (e.g., MacAulay, 1990; Wittrock, 1986), and learner characteristics (e.g., MacAulay, 1990; Tobias, 1994; Wittrock, 1986), as shown in Figure 6, an adaptation of conceptual frameworks developed by Moos (1979) and MacAulay (1990).

Categories and possible elements are

- physical and architectural factors: space, privacy, noise, tiredness, light, equipment/materials, technology, seating arrangements, location;
- structure and organisation: (mathematics) task type, time, grouping, teacher direction, discussion, communication patterns, rules and procedures, competition, cooperativeness;
- teacher characteristics: teaching style, feedback, expectations, warmth, friendliness, communicativeness;

 learner characteristics: desires, attributions, interests, motivations, expectations, attitudes, beliefs, emotions, self-efficacy, gender, cognitive processes (for example, attentional processes, perceived intelligence, memory, problem solving ability).

Figure 6 presents a conceptual model of the learning environment that is based on the hypothesis that social, physical, organisational, and psychological factors may all be components within a child's perception of the learning environment. The focus in this research was on perceptions of helping factors within learning environments for the learning of mathematics. The model proposed in Figure 6 includes possible reference to mathematical tasks, content, and processes through the *structure and organisation* and *learner characteristics* categories.



Figure 6. Determinants and outcomes of classroom environment.

The interpretation of the term learning environment in relation to the learning of mathematics appears to vary across previous studies. Saxe and Bermudez (1996) report that some researchers use features of the environment such as social class and economic organisation, properties of individuals such as race and gender, social conflicts, and cultural aspects. Basing their own work on children's engagement with mathematical problems, Saxe and Bermudez (1996) speak specifically of *mathematical learning environments*, and argue that a core construct is children's emergent goals during an activity. They believe that learning environments are not presented to individuals and are not directly observable. The present research takes an approach different from those discussed by Saxe and Bermudez (1996): it assumes that both children and teachers can contribute to the creation of learning environments for mathematics and does not differentiate broader features such as social class and race. It is not a study of learning environments as they are known to exist, if such *knowing* can be common across individuals, but a study of children's perceptions of helpful learning environments and therefore of elements they may identify as important. It is possible that elements or factors may appear to be controllable, to some degree, by the teacher.

Clement (1991) states that in a classroom based on constructivist views of learning mathematics, teachers are "constructors of learning environments through their efforts to modify or construct (rather than transmit) the curriculum" (p. 423). Also suggesting that the teacher can play a role in contributing to the creation of learning environments for mathematics, Franke and Carey (1997) speak of the potential influence of a teacher-created classroom focus on children's thinking. Garafolo (1989) appears to suggest learning environments are at least partly external to the child; he speaks of the nature of the classroom environment strongly influencing how students view the subject of mathematics and the way they believe mathematics should be done. Beliefs regarding the nature of mathematics and how mathematics should be done may be related to students' views of what they perceive helps them to learn mathematics.

It is suggested that the teacher plays a role in the creation of a classroom environment for the learning of mathematics, and therefore that at least some elements of this environment may be observable and open to change. The present research is of value in leading to greater awareness by the mathematics education community of a possible range of factors that children believe help in their learning of mathematics. Increased awareness can lead to greater appreciation of young learners' perspectives, to increased reflection, and to consideration and implementation of possible responses by the teacher (McDonough & Gervasoni, 1997).

In the present research, some factors taken as possible elements of learning environments may be directly observable, such as those that are external to the learner. However, others, such as factors internal to the learner, are not observable. Indeed, observability is not crucial as it is the child's perception of factors experienced that is of interest. This is not dependent on identification by observation by another person or on children's direct interaction with a mathematical task. The learning environment studied by Saxe and Bermudez (1996) is a *mathematical environment* that appears to focus on mathematics learning in progress, and is different from a more general *learning environment* assumed for this research. The model for the present research (see Figure 6) was developed from a selection of literature regarding learning environments and is applicable to many areas of learning including mathematics. Its openness and breadth makes it suitable for informing analysis of semi-structured interview self-report data from young children.

It was not intended that the categories in Figure 6, or their possible elements, as discussed above, pre-determine the analysis of children's responses. The identification of categories situates the present study within a framework of ideas from previous studies and indicates the possible breadth of areas in which children may identify factors perceived to impact upon their learning of mathematics. The internal/external divide could be *laid over* the four interacting domains to provide a more complete, although not necessarily exhaustive, model of learning environments. However, it is the children's perspectives that are considered in the final analysis. It was recognised that, in relating or describing situations, children may not choose to pay attention to the breadth of ideas presented in Figure 6; it is their

interpretations and perspectives that are of importance in this study, as it is knowledge about the way in which individuals perceive and interpret situations which provides the basis for understanding individuals' behaviour (Nystedt, 1983). It was the intention to examine the learning environment from the "subject's own interpretation of the phenomena he [or she] perceives" (Murray, 1938, p. 122).

In summary, for children as young learners of mathematics it is possible that a broad range of learning environment features, differently focused for example from the psychosocial perspective of Fraser (1991) and the emergent goals perspective of Saxe and Bermudez (1996), may be considered to be important in their learning of mathematics. While it is recognised that, as Murray (1938) has written, "the physical environment and behavioural (psychological) environment are two different things" (p. 117), it is considered that they both contribute to the environment in which young learners operate. Indeed, as suggested in Figure 6, social, physical, organisational, and psychological factors may be components within a child's perception of a learning environment.

It should be noted that the term instructional environment, as used by McLeod (1989) for example, is avoided in this research because of its perceived strong implication of a situation of instruction, one in which an instructor is present. McLeod (1989) lists a variety of factors within instructional environments such as material use, teacher direction, and grouping structure. The present research recognises that any of these may be indicated by children as being of influence in their learning of mathematics. However, the operational definition of *learning environment* within this study does not restrict the learning environment to a situation in which the presence of an instructor is assumed. It is also important to reiterate that in the present research the learning environment is not a concept restricted to a formal learning situation such as a school provides. Children were free to nominate any situation in which they perceived they were learning mathematics, or were given the opportunity to learn mathematics.

In summary, the term *learning environment* is used in the present research to group the perceived factors of influence as expressed by the children, no matter whether these factors, as perceived by the researcher, are external or internal to the children. The use of the term *learning environment* is based on use in some previous research, but in this study the term is extended or varied in meaning compared with much previous work. Children and teachers are seen as possible contributors to the creation of learning environments for mathematics.

Factors impacting upon learning

The present research was designed with a focus on student perspectives, as children are considered active participants who have an impact upon their learning outcomes (Koehler & Grouws, 1992; Wittrock, 1986). As the teaching and learning situation is complex and interactive (Koehler & Grouws, 1992; Romberg & Carpenter, 1986), with many possible factors impacting upon students' learning, there was opportunity for consideration of beliefs

covering a vast range of factors of influence, some suggested in the model developed for this study (see Figure 6).

Koehler and Grouws (1992) demonstrate that research has come to recognise, and to respond to, the immense and increasing complexity of the teaching process. Of the four research models Koehler and Grouws present, their Level 4 model (1992, p. 118) most shows this complexity. Inherent in this model is recognition of the active contribution of pupils in the learning process: pupil outcomes are based on their own actions and behaviours; outcomes are influenced also by pupil characteristics and attitudes, and by teacher behaviours. The twoway connection between teacher and pupil behaviour gives further reason for investigation of children's perspectives on the nature of, and reason for, their behaviours and perceptions of learning situations, particularly in relation to their learning of mathematics. As further background to the third research question, previous findings regarding children's perceptions of factors of influence upon their learning of mathematics are considered. It is suggested that although there has been research in the area of student perceptions of learning environments (e.g., Fraser, 1994), research of young learners' perceptions is limited, including in relation to the learning of mathematics. The use of indepth, semi-structured, qualitative data collection procedures also appears to have been an infrequently used approach for the investigation of the perspectives of learners of mathematics.

Children's perceptions of factors influencing mathematics learning

One key element of the present research was an investigation of children's beliefs about factors positively and negatively influencing their learning of mathematics, with a focus on factors perceived to assist in the learning of mathematics. It is acknowledged that investigation of how students believe they learn mathematics best is not necessarily the same as finding out how they actually do learn best (Rodd, 1993). However, such investigation does inform regarding students' individual perspectives towards learning mathematics. The beliefs held by students, although not openly visible, are a reality in the classroom learning situation.

Previous research suggests that students are aware of their learning of mathematics and do hold beliefs about helping and hindering factors; children's beliefs may not be available to teachers through day to day interactions in the classroom but indepth investigation may offer insights which add to teacher knowledge of learner perspectives and therefore can inform teaching. Children's perceptions about the nature of a learning environment for mathematics may differ from those of the teacher in that environment, just as they have been found to differ in relation to environment for the learning of science (Fraser, 1994). Perceptions of the mathematics learning environment may be linked to beliefs about the nature of mathematics and learning which also may differ from those of the teacher. Some previous studies which include consideration of students' perceptions of factors impacting upon the learning of mathematics are discussed below.

Herrington's (1990) semi-structured interviews of upper primary school children reveal that teacher help was considered by children as one important factor in helping to learn

mathematics. The following response, given by one of Herrington's interviewees, suggests that allowing children to express their perception of helping factors can assist teachers to become more aware of a child's reaction or perspective, and help teachers to reflect meaningfully on their own actions:

Well she could sort of explain it easier. Like there's a kid that might not be able to learn it that much, and instead of yelling at them and saying listen harder, when they're really listening, just go through it again until they sort of get the idea of it. (p. 8)

Better explanations and being willing to help individuals were identified by a number of Herrington's respondents as beneficial teacher actions. Teacher assistance was identified as being helpful particularly when learning difficulties are encountered. Teacher explanation was identified also by older students in Year 13 as helpful for mathematics learning (Rodd, 1993).

In addition to the role of the teacher, Herrington's interviewees saw potential to help themselves in learning mathematics. For example, a question regarding how they would learn for a mathematics test lead to suggestions of "checking previous work, using others to ask questions, asking oneself questions, repeated practice and using mnemonics" (p. 15). Herrington believes these strategies relate generally to a rote understanding of algorithms; strategies to extend understanding were not present.

A questionnaire, also given by Herrington (1990) to upper primary pupils, revealed that many students saw "practising the same question over and over again or copying notes from the blackboard, as the best way to learn mathematics" (p. 15). Again these responses suggest a perception of the learning of mathematics as a rote activity, seemingly leading to instrumental understanding.

Responses from first-grade children in the CGI classrooms studied by Franke and Carey (1997) indicate that concrete materials were considered helpful in learning mathematics, suggesting a perception of helpful factors different from that held by children studied by Herrington (1990). The difference in age, and therefore possibly of recent learning experiences, may have contributed to the differing responses, but the type of teaching style might have contributed also. The children in the problem solving classrooms of the CGI project suggested that it was helpful if "not all children use the same materials and that the children themselves could choose the materials they wanted to use" (Franke & Carey, 1997, p. 20). Such a teaching approach appears to foster student reflection and student choice in the learning of mathematics, particularly in terms of helpful student actions.

During interviews with 60 Fifth Grade pupils, Stodolsky et al. (1991) asked students whether they felt they could learn mathematics on their own or whether they would need school. Responses suggested a dependence on the teacher for the learning of mathematics: only seven interviewees felt that they could learn mathematics on their own. One interviewee explained "Well (the teacher) tells me if I'm wrong, gives me the right answer" (p. 105), suggesting that teacher feedback is important for them for learning mathematics. Teacher guidance was suggested also as helpful and nesessary: "you sort of have to know what you're doing before you can do it" (p. 106). For many students in the Stodolsky et al. (1991) study,

mathematics was not amenable to independent learning, thus the school learning environment where a teacher is present was valued.

Interviews by Stodolsky et al. (1991) with Year 2 pupils revealed also a dependence on the teacher for learning mathematics, followed by reliance on textbooks and parents. It appears that a knowledgeable authority was considered helpful, if not necessary, for giving guidance and explanations for the learning of mathematics; generally students believed they could not learn mathematics on their own. As indicated above, Franke and Carey's (1997) research in First Grade classrooms gave different results, with students electing to focus more on use of materials and student choice. Differing results may have related to the manner in which the questions were posed, but alternately may have related to differing experiences in the learning of mathematics at least partially resulting from differing teacher beliefs about teaching, learning, and mathematics.

Studies of the perceptions of middle school and upper secondary students reveal factors some older students associate with helping in the learning of mathematics. Forman (1996) studied two middle school mathematics classes taught by the one teacher, Ms Hanes, who allowed students to participate in a manner, considered by Forman, to be consistent with reform classrooms. The classes were characterised by "whole-class recitation led by the teacher, whole-class presentations led by one or more students with the teacher's intermittent assistance, individual seat work, and unofficial peer group activities" (p. 121). Interviews with small samples of students revealed that, as was expressed by the respondents in the study by Herrington (1990), learners in Ms Hanes' classes saw clear explanations as important. However, in Ms Hanes' classes the focus was on clear explanations from members of student groups; small groups were seen as providing a supportive community, for example, people would listen, and help could be offered and received; students would be exposed to a variety of ideas also. These findings suggest that students found activity in small groups helpful for their learning of mathematics. It appears that, for most interviewees, the teacher or other authority was not perceived as the most helpful element for the learning of mathematics.

However, there were some students who appeared not to consider such a classroom environment favourable or helpful for their learning of mathematics. Forman (1996) discusses the change in authority source in the classroom, pointing out that some students did not like this. Some students appeared also to feel frustrated by the reduced teacher direction, and some missed the direct teacher interaction, and teacher initiation and evaluation of work. In contrast, one student who resisted authority in the classroom now participated in a taskfocused manner when in small group problem solving activities. This indicates some differing preferences and needs for learning of mathematics among the individual learners in the two middle school classes, but with the majority finding that a structure different from a traditional mathematics classroom could meet their need for clear explanations and support from others. The above discussion, and further research as discussed below, suggest that traditional and non-traditional or reform classrooms provide different experiences and different challenges for learners. From each there emerge themes in children's perceptions of factors of influence in the learning of mathematics that can be examined, for example, by reference to the ideas summarised in Figure 6.

Herrington (1990), Rodd (1993), and Stodolsky et al. (1991) identified perceptions of helpful factors such as the teacher giving guidance through explanation and the teacher listening. Referring to the categories and elements identified for Figure 6, these can be classified as Teacher direction within the category of Structure and Organisation, and Teacher actions and responses to students within the category of Teacher Characteristics.

Herrington (1990) found reference to student actions; these may be related to beliefs about the nature of the learning of mathematics and therefore to Learner Characteristics. Franke and Carey (1997) found articulation of factors related to the material or physical environment, and also to learners' responsibility and choice in learning. These appear to relate to the following categories:

- physical and architectural factors because of the use of materials;
- structure and organisation through procedures and teacher direction which in this case appear different from those in a traditional classroom;
- teacher characteristics through expectations; and possibly to
- learner characteristics through interest, beliefs and motivations.

Learners' perceptions of factors of influence in learning of mathematics appear somewhat diverse, with elements categorised in each of the four determinents of the classroom environment identified in Figure 6. Discussion below illustrates additional and seemingly interacting themes in student perceptions: feedback, supportive environments, confidence, and stress on learners which may be related to those already discussed.

A belief that feedback is helpful for learning may be held by school learners: a positive perception of teachers has been related to the provision of specific feedback to the class (Heroman, 1990). However, Forman's (1996) study suggests that when students experience a non-traditional teaching/learning situation for mathematics, traditional teacher actions such as feedback can be considered by the majority of learners to be met best in non-traditional ways, that is, by peers. It appears, as discussed below, that needs for feedback and support may be related to student confidence and the degree of stress in a learning environment.

Perceptions of an environment being helpful for learning may be related in part to the degree of stress that the situation imposes on learners. The work of Hoyles (1982) with 14-year-old pupils, suggests that stress can be related to bad learning experiences in mathematics in part because it can lead to negative effects on student confidence and to feelings of inadequacy as a learner. Hoyles found that students seemed to want teachers of mathematics "to 'make it easy' or 'tell them the way" (p. 368). They also "were appreciative of a secure, encouraging environment . . . and liked teachers to provide a logical structured progression in

their work, with plenty of patient explanation, encouragement and friendliness" (p. 368). Forman's findings suggest such needs can be met by others; Hoyles' results suggest that the degree of pressure put on learners by the teacher may have an impact on whether the situation is judged as being positive or negative for the learning of mathematics. It appears from Forman's work that the traditional role of teacher, assumed by a trained adult, can be replaced, and even reformulated, to the satisfaction and self-perceived benefit of the majority of learners in a non-traditional mathematics learning situation, when there is support from other group members. It appears that such a situation may not be as highly stressful for learners of mathematics as might a more traditional situation. Confidence is an element which is identified by some learners as of influence in the learning of mathematics (Hoyles, 1982; Rodd, 1993); students appear to hold greater confidence when a situation poses less stress (Hoyles, 1982). The themes of feedback, supportive environments, confidence, and stress on learners suggest again a breadth in student perceptions of factors of influence, drawing on elements of the Figure 6 categories of structure and organisation, teacher characteristics, and learner characteristics.

The studies discussed above, of student perceptions of factors of influence, appear to relate primarily to the learning of mathematics in the school environment and mostly investigate views of learners in upper primary or secondary school. There appears scope for further study of perceptions of factors of influence in the learning of mathematics, through more investigation of the views of younger learners, and studies that consider the learning of mathematics in situations other than school. The present study attempted to address this need, with a particular focus on beliefs about helping factors for learning mathematics.

CHAPTER THREE METHODOLOGY: THEORETICAL UNDERPINNINGS, TOOLS, AND TECHNIQUES OF THE RESEARCH

As indicated in earlier chapters, this study involved the investigation of eight Grade 3 children's beliefs about the nature of mathematics, learning, and helping factors for the learning of mathematics. Data collection occurred over a five month period and involved

- collection of verbal, drawn and written interview data from individual children;
- observations during the interviews;
- collection of whole class data in written and drawn form;
- observations of class lessons; and
- interviews with the two class teachers.

Each of these approaches is elaborated within this chapter.

These methods of data collection can be called qualitative in nature as they include the "three kinds of [qualitative] data collection: in-depth, open-ended interviews; direct observation; and written documents", as described by Patton (1990, p. 10).

However, it is not only data collection techniques that make a study qualitative. This chapter expands upon the idea of the research as being qualitative in nature, detailing the research processes and, importantly, focusing also on the purpose of the research as it relates to its theoretical underpinnings.

Theoretical underpinnings of the research

The discussion of research methodology begins with an outline of the theoretical underpinnings of the study and concurrently presents assumptions of the study for the reader. This is approached through discussion of interpretations and relevance of the following terms: qualitative, constructivist, phenomenological, ethnographic, and interpretive. While it is acknowledged that such terms can have many interpretations and are used variously by researchers, the discussion below explores the meanings of the terms mainly as they relate to the purpose of the present study, as it is the intent of the users that gives such terms their particular meanings (Schwandt, 1994).

Qualitative and constructivist underpinnings of the study

The term *qualitative* implies for many the use of non-quantitative approaches such as those listed above, but it has also a broader interpretation related to the intent or purpose of a study, that is, "interest in human meaning in social life and its elucidation and exposition by the researcher" (Erickson, 1986, p. 119). Erickson (1986) classes qualitative research under an *interpretive* umbrella, also because this broader term allows the possibility of inclusion of some quantification. Erickson stresses that for research to be interpretive or qualitative is not a matter of technique, but of the "substantive focus and intent" (p. 120). The present research was qualitative not only because of its methods but also, for example, because of its interest in

making sense of phenomena through the meanings people bring to them (Denzin & Lincoln, 1994), and its view of the form and nature of reality (Guba & Lincoln, 1994).

Qualitative research has traditions from anthropology, psychology, and sociology, but coming to understand the meaning of an experience is the key objective. Coming to understand involves finding out what the world looks like from the perspective of the research participants (Merriam, 1988). The analysis endeavours to achieve depth of understanding, openness and detail (Patton, 1990). According to Merriam (1988), a qualitative research view of the world underpins a study where such approaches are used, although this does not negate the possibility of deployment of quantitative methods. Qualitative research, in contrast to the "'traditional' or 'scientific' paradigm, . . . assumes there are multiple realities–that the world is not an objective thing out there but a function of personal interaction and perception" (Merriam, 1988, p. 17).

For Guba and Lincoln (1994), questions of paradigm, that is, of "the basic belief system or worldview that guides the researcher" (p. 105), are paramount. Under the qualitative umbrella they identify the paradigms of Positivism, Postpositivism, Critical Theory and related ideological positions, and Constructivism. It is the last of these that relates to the present study. In the constructivist paradigm no one "real" world is assumed, but,

realities are apprehendable in the form of multiple, intangible mental constructions, socially and experientially based, local and specific in nature (although elements are often shared among many individuals and even across cultures), and dependent for their form and content on the individual persons or groups holding the constructions . . . Constructions are alterable, as are their associated "realities". (pp. 110-111)

The present research assumed multiple realities; it was the realities of individuals, over the period of the data collection, that the research sought to investigate and interpret. The study did not search for one truth that might be expressed as a single right answer but assumed "multiple perspectives and multiple 'truths' depending on different points of view" (Patton, 1987, p. 166). The credibility of the research was increased by its neutral stance; the research did "not set out to prove a particular perspective or manipulate the data to arrive at predisposed truths" (Patton, 1990, p. 55).

The assumption of multiple realities and multiple truths underpinned the expectation of beliefs differing from child to child and took into account children's varying experiences. It was anticipated that beliefs would be idiosyncratic to each child. Just as, according to the social constructivist theory of learning, children learn mathematics in different ways and through negotiation of meaning construct their own understandings (e.g., Ernest, 1991), they also construct or develop their own beliefs about the learning of mathematics (e.g., Yackel & Cobb, 1996). Thus links to constructivism in the present study related to individuals' construction of beliefs about the nature of mathematics, of learning, and of helping factors for mathematics learning.

Although the study drew on qualitative research methods and a constructivist perspective, as demonstrated above, the study was eclectic in its methodology as it had mixed

theoretical underpinnings, and did not fit neatly into any one paradigm. For example, the interest in individually constructed realities links the research also with phenomenological and ethnographic theoretical orientations as discussed below.

Links with phenomenological research perspectives

Phenomenology is one research perspective with which the current study can be linked. Points of intersection as well as differences are explored below.

According to Patton (1990, p. 69), "phenomenological enquiry focuses on the question 'What is the structure and essence of experience of this phenomenon for these people?'". A key element of phenomenology is the idea of shared experiences of phenomena. Descriptions entail a shared perspective; they are developed from consideration of "experiences of different people [that] are bracketed, analysed, and compared to identify the essences of the phenomenon" (Patton, 1990, p. 70). Textual interpretations are then constructed using a particular writing approach based on semiotics (van Manen, 1990). In the present study the essence of experience for individuals was of interest. Individual children's beliefs were bracketed, with presentation and interpretation of data structured according to themes that appeared to emerge from the data. Some comparison of individual children's beliefs was made but not to the degree of some phenomenologists (e.g., van Manen, 1990).

A summary of elements of the theoretical underpinnings and assumptions of the present research in common with phenomenology as described by Patton (1990) and van Manen (1990), are listed below. These relate, firstly, to the purpose of the research in the immediate sense of gaining insights into children's beliefs, secondly, to the purpose of the research in terms of its capacity to inform teaching, and, thirdly, to underlying elements of a general nature.

Common intentions of phenomenology and the present research are

- study of the meaning or essence of experience or phenomena (in the present study these phenomena were mathematics, learning, and helping factors for learning mathematics);
- gaining of insights from people's descriptions of experiences.

Common assumptions of phenomenology and this study related to informing teaching are

- teaching requires a sensitivity to children's lived experiences and realities;
- teaching requires the ability to make sense of children's interpretation of phenomena and situations so as to see the pedagogic significance of those situations;
- the gaining of insights can help us appreciate better what mathematical learning experience is like for children.

Other underlying common elements are

- the conjecture that beliefs are one form of objectification of "mind, thoughts, consciousness, values, feelings, emotions, actions and purposes" (van Manen, 1990, p. 3);
- the belief that it is appropriate for research to want to know the world and to question the way we experience the world;

• the belief that the gaining of plausible insights brings us in more direct contact with the world, or more fully part of the world.

This research facilitates the gaining of insights into the world as seen by a small number of individual eight to nine year olds, and the portrayals of beliefs allow adult readers to review their own assumptions, as well as their beliefs and experiences, in relation to the perspectives communicated by the children. The research facilitates awareness of, and sensitivity to, the experiences of other individuals. It may encourage adults to continue to explore children's beliefs; thus the reflection may become an ongoing or cyclic process that can inform pedagogy.

Although commonalities exist between the present research and a phenomenological research perspective, the present research is not strictly labelled as phenomenological as differences are apparent also. For example, while phenomenology involves writing from a shared perspective, as discussed above, the present study focused on reporting and interpreting individual perspectives.

Another difference is apparent in the nature of the descriptions. In phenomenological enquiry as described by van Manen (1990), shared experience descriptions are said to be compelling, insightful, and eloquent. The writing in the present report portrays individual children's beliefs and reflects more of the analytic and writing process than does phenomenological writing. The present writing does not neglect the difficulties of the research/writing process and therefore, according to van Manen's criteria would not be called eloquent. However, it does provide insights and is potentially compelling. Presentation of beliefs consists of transcript excerpts and other forms of interview data (for example, drawings), followed by analysis and reflection within the text of the thesis.

To summarise, the present study is not described as phenomenological, but is underpinned partly by this theoretical orientation as it does have some common features including the study of the meaning or essence of experience or phenomena, and the gaining of insights from people's descriptions of experiences. The present study has less focus than phenomenological research on the idea of shared experience of phenomena although the method of analysis, as detailed later, allowed for such to arise if identified by the research participants.

Another theoretical position partly underpinning this study is that of ethnography. Even though not described as ethnographic, the study shares some common features.

Links with ethnographic research perspectives

The present study is similar to ethnographic research in terms of the purpose of research, that is, a common interest in how people define the world. Ethnographic research provides the opportunity to learn from people (Spradley, 1980), it facilitates finding out how people "define the world" (Spradley, 1979, p. 11), and it identifies meanings held by research participants (Eisenhart, 1988). The present study had these features also.

A number of other features the present research had in common with ethnographic data collection methods and analysis, as discussed by Eisenhart (1988) and Spradley (1979), are explored here.

For example, stemming from the common interest in how people define the world, there is common use of open-ended questions followed by more focused questions in response to answers. There is also the collection of artefacts. In common also is observation; in the present study observation occurred during the ten interviews with each child, and class observation occurred on a small number of occasions.

Stepping back and reflection by the researcher on the research activities, data and context are to some degree common activities. In the present study, the use of semi-structured data collection procedures entailed some stepping back and reflection by the researcher. During interviews, decisions were made for follow-up questions to the planned key introductory open-ended questions or tasks depending on the response of an individual child. Stepping back and reflection occurred also within the latter stages of the research when interpreting and reflecting on all data from each of the individual children so as to build theory within the project. Overall, while common to the present research and ethnographic research, stepping back and reflection influenced the overall course of data collection in this study to a lesser degree than appears to be the case in ethnographic research.

Another common feature is that like some ethnographic research, data from the present study were organised into themes. In the present study these themes were identified primarily after all data were collected rather than throughout the data collection as occurs in ethnographic research (Spradley, 1979).

A further common feature is that, like ethnographic research, the present study was unlike much research in mathematics education that aims "to identify psychological, psychosocial, or instructional factors and processes that affect mathematics education and then to design and implement treatments to achieve better results" (Eisenhart, 1988, p. 100). The goals of the present study were like those of ethnographic research in that they might be better described as descriptive and theoretical rather than descriptive and prescriptive (Eisenhart, 1988). Emergent theory, which may include elements related to culture and social relations, was developed from the data as in ethnographic research (Eisenhart, 1988). The results from the present study can inform mathematics teaching by providing illumination and possible direction for the reader, but the study did not aim to prescribe actions for teaching.

While there were common features between the present research and ethnography, there were also differences; the most significant difference related to the unit of analysis.

At the heart of ethnographic research is the study of communities: the result is a description or case study of the life or the culture of a group; social structures as well as the behaviour of individuals as members of the group are described and interpreted (Patton, 1990; Taft, 1988). In the present study, the open nature of the research procedures, as detailed later in this chapter, gave the potential for the social context to emerge as a factor within individual

children's beliefs regarding mathematics, learning, and helping factors for mathematics learning, but a focus on social groups was not determined by the researcher. The research aimed to describe key elements as identified by individual children.

A difference is found also in the organisation for data collection. In the present research, data were collected from children in one-to-one interview situations for which children were withdrawn from class. In ethnographic research the investigator becomes closely involved with the group and thus experiences partial acculturation (Taft, 1988). In the present research acculturation occurred in a lesser form through interaction during interviews, through observation of classes, and through collection of data from the eight children when within their class groups. The researcher's ten years of experience in teaching primary level classes also contributed to an understanding and appreciation of such learning situations. However, this acculturation was different from that of an ethnographic study.

In summary, an ethnographic perspective partly underpinned the present study. Similarities are found in the overall purpose or intent of study, that is, to learn from people and to come understand how they see the world, as well as in some aspects of data collection and analysis. Differences are apparent in aspects such as the unit of analysis and the organisation for data collection. While there was common use of a case study approach, in ethnographic research the cultural or social group is the unit of analysis, whereas in the present study the individual was the unit of analysis. Rather than describing a social group, the study focused on meanings constructed by individuals, although there was the potential to provide insights into a larger group, that is, learners of mathematics at the Grade 3 level.

As discussed above, the present study is partially underpinned by ethnographic and phenomenological research. Erickson (1986) places each of these, along with qualitative approaches, under the umbrella term of interpretive research; the theoretical underpinnings of the present research are now drawn together under this broader perspective.

Drawing together the theoretical underpinnings: The present study as a form of interpretive research

The present research is perhaps summarised best as a form of interpretive research as this broader umbrella term allows for commonality with a range of interpretive or qualitative theoretical underpinnings, for example, as demonstrated above for phenomenological and ethnographic traditions. The theory and method of "ethnographic, qualitative, participant observation, case study, symbolic interactionist, phenomenological, constructivist, or interpretive [approaches to research] . . . are slightly different, but each bears strong family resemblance to the others" (Erickson, 1986, p. 119). Erickson goes on to use interpretive to refer to what he calls this family of approaches. Similarly, the present research uses the term interpretive as an encompassing term.

Commonalities with the broader interpretive perspective include concern for "What is going on here? . . . [and] . . . meanings [that] underlie these 'goings on'" (Eisenhart, 1988, p. 104). This has appeared as a common concern throughout the above discussion. In the

interpretive tradition underlying meanings are intersubjective; in the present research, insights were sought into the beliefs of individual children in their learning of mathematics. Openended questions provided opportunity for the children to acknowledge shared experiences and related constructions if they so chose.

As in interpretive research, the concern in the present study was for particularisability rather than generalisability. Erickson (1986, p. 130) states that "one discovers universals as manifested concretely and specifically, not in abstraction and generalisability . . . [but] the paradox is that to achieve valid discovery of universals one must stay very close to concrete cases". The present research aimed to gain insights into the possible nature and complexity of beliefs held by young children and to explore whether these could be articulated and portrayed. This was done through studying in detail the beliefs of a small number of children to provide illumination and possible direction for the reader.

As the research aimed to make statements about how the eight research participants understood their worlds, interpretive methods were required (Eisenhart, 1988). Qualitative approaches, for example, as defined by Patton (1990), and as outlined at the beginning of this chapter, were used. But, as illustrated above, full ethnographic approaches to data collection and researcher reflection (see e.g., Eisenhart, 1988; Spradley, 1979; Taft, 1988) were not deployed; nor could the research be described as essentially phenomenological in nature.

To summarise, the research was underpinned by a range of theoretical perspectives, fitting broadly under an interpretive umbrella, and deploying interpretive or qualitative methods of data collection. As qualitative research may have "no theory, or paradigm, that is distinctly its own" (Denzin & Lincoln, 1994, p. 3), it is appropriate that the present research drew on theories of constructivism, phenomenology, and ethnography to underpin its interpretive or qualitative approach. The study is described as eclectic as it drew on quality ideas and methods from a variety of sources.

Related to the theoretical underpinnings of the research are the elements of research method, that is, the actual processes and actions that occurred. These are discussed below.

Research method

The research method encompassed many elements that followed from the identification of the purpose of the research and the development of research questions. These included deciding a unit of analysis, identifying research participants, developing and using data gathering procedures, and analysing responses to those procedures. The discussion of the research method is approached firstly through consideration of the unit of analysis in the present study.

The research as a set of case studies

A case study approach is selected when it is the most effective strategy for obtaining information to answer the research questions (Denzin & Lincoln, 1994). Such was the situation in the present research.

The research took the form of case studies as it was defined by interest in individual cases, that is, eight children at the Grade 3 level of schooling. This was not a methodological

choice, but a choice of unit or group to be studied (Stake, 1994). The study met the key elements of case study research as stated by Stake (1994): "the object of study [was] a specific, unique, bounded system" (p. 237).

Merriam (1988), in a review of case study literature, states that while writers have identified case studies by criteria expressed in a variety of ways, it can be concluded that case studies that are qualitative in nature have four essential properties: the studies are particularistic, descriptive, heuristic, and inductive.

The present study was *particularistic* as it focused on a particular phenomenon, beliefs about the learning of mathematics, expressed as beliefs of eight to nine years olds about mathematics, learning, and helping factors for learning mathematics. The study focused on a particular group and their view of a phenomenon, researched through focus on a small number of individuals.

Secondly, the study was *descriptive* as it provided a rich description drawing on detail from a range of sources including quotes from interviews and artefacts produced during interviews and class activities. Through the use of data extracts, the research provided literal descriptions from the research participants, and within discussion of these data the research provided interpretation of meaning. The openness within the research procedures allowed many variables to emerge from the data and be portrayed in this report.

The study was *heuristic* as it provided illumination for the reader of the phenomena studied (Kemmis, 1982; Merriam, 1988). The illumination from the research can emancipate: it provides action possibilities for the reader that "are grounded in the situation itself [and] not imposed from outside it" (Kemmis, 1982, p. 109). The depth of detail and the "criss-crossed" reflection (Stake, 1994) provide insights into the complexity of development of children's beliefs. With little previous indepth research of young children's beliefs regarding mathematics, learning, and helping factors, the report can extend the reader's experience. For those who have developed some appreciation of children's beliefs, the report may confirm as well as elaborate what is known. It also may activate or motivate teachers to build up awareness of children's beliefs and take them into account in decision making for teaching mathematics.

Fourthly, the study was *inductive* as it relied on inductive reasoning; concepts and generalisations about individual children's beliefs emerged from the data. The study was characterised not by verification of pre-determined hypotheses, but by the "discovery of new relationships, concepts, and understanding" (Merriam, 1988, p. 13) within and about young children's beliefs. Beginning with specific observations, the researcher was then able to build toward seeing general patterns (Patton, 1987). Such an approach works well "when the terrain is unfamiliar and/or excessively complex, a single case is involved, and the intent is exploratory or descriptive" (Huberman & Miles, 1994, p. 431). The complexity and subtlety of Cara's beliefs (see Chapter 5), for example, demonstrate the appropriateness of an

inductive approach where concepts and generalisations, expressed within emergent patterns or themes, enable the reader to gain insights into a child's beliefs.

As demonstrated in the above discussion, the present study met the four criteria for a case study that is qualitative in nature, as identified by Merriam (1988). In addition, as stated in the chapter introduction, the study used the three kinds of qualitative data collection as outlined by Patton (1990).

As well as qualitative being an appropriate description for the present study as demonstrated above, the research can be described as case studies that were instrumental and *intrinsic* in their purposes. Here the word instrumental has a different meaning from that discussed in Chapter 2 in relation to understanding. Instrumental case studies provide insight into an issue (Stake, 1994), in this instance, whether children as young as eight to nine years hold beliefs about mathematics, learning and helping factors that can be articulated and portrayed. Intrinsic case studies are conducted for the purpose of coming to understand better a particular case (Stake, 1994), in this instance, some individual young children's beliefs about mathematics, learning and helping factors for learning mathematics. Stake posited an additional category of purpose for studying cases, that is, *collective*, where the purpose of choosing and focusing on individual cases within a group is to generalise to a still larger collection of cases. It was not the intention of the present research to generalise to other individuals although it may inform as to the possible nature of beliefs of other eight to nine year olds and may activate investigation of a larger number of cases, at both the day to day classroom and broader research levels. The categories of intrinsic, instrumental and collective indicate the variation in concern and methodological orientation that can be possible within the case study domain, but according to Stake (1994), "authors and reports seldom fit neatly into such categories" (p. 238). It is reasonable therefore that the purpose of the present study relates to two of these categories.

To summarise, the above discussion outlined that the present research took the form of case studies that were qualitative in nature. The research was particularistic, descriptive, heuristic, and inductive and used qualitative methods of data collection; it was intrinsic and instrumental in its purposes. The discussion now moves on to other elements of the method of the study, or the journey of the research.

Preparing for the research portrayals

In response to the key elements within the three research questions, as presented in Chapter 1, the discussion in Chapter 2 centred around the constructs of mathematics, learning, and perceptions of helping factors for mathematics learning, providing for the reader an illustration of the broader context in which the present study was situated. Within Chapter 1 and the introductory section of this chapter, there was discussion of the purposes of this study, the evolution of the study, and its theoretical underpinnings.

The remainder of the present chapter details the methods of the study and includes discussion of

- reliability and validity issues;
- the research participants;
- development of data collection procedures;
- collection of data; and
- analysis and presentation of data.

Reliability and validity issues

Reliability and validity issues are important within this qualitative or interpretive study. Just as quantitative research strives for findings that can be believed and trusted, so too does research that uses qualitative methods. In a qualitative study measures are taken to ensure plausibility and meaningfulness of results; the study should be as trustworthy as possible (Merriam, 1988), and the findings should be credible and authentic (LeCompte & Goetz, 1982). Traditional evaluation criteria for research include reliability and validity but rethinking of these terms has occurred in relation to qualitative studies (Denzin & Lincoln, 1994). As a range of alternative terms, including authenticity, credibility, truth value, consistency, transferability, dependability, and confirmability have been deployed in discussion of qualitative research (Denzin & Lincoln, 1994; LeCompte & Goetz, 1982; Merriam, 1988), the present discussion is structured around the traditional and more common terms, reliability and validity, but from the perspective of qualitative research.

Discussion later in the chapter regarding the interview procedures, particularly that concerning the creative and varied nature of the interview tasks, provides additional insights into ways in which this qualitative study addressed reliability and validity issues.

Reliability

Reliability is concerned with "replicability and consistency of the methods, conditions, and results" (Wiersma, 1995, p. 9). In traditional research there are two aspects to reliability. One is external reliability, that is, "whether independent researchers would discover the same phenomena or generate the same constructs in the same or similar settings" (LeCompte & Goetz, 1982, p. 32). The other, internal reliability, "refers to the degree to which other researchers, given the same set of previously generated constructs, would match them with the data in the same way as did the original researcher" (LeCompte & Goetz, 1982, p. 32).

Theoretically a researcher who uses the same methods and design as another researcher can obtain the same results. However, it is argued that no study can be replicated exactly as human behaviour is never static (LeCompte & Goetz, 1982; Merriam, 1988), and that, because ethnographic or qualitative research occurs in natural settings and the process is personalistic, there are some constraints on reliability (LeCompte & Goetz, 1982; Merriam, 1988). Nonetheless, measures are taken by qualitative researchers, including in this study, to enhance external and internal reliability.

External reliability

LeCompte & Goetz (1982), believe that qualitative research "may approach rather than attain external reliability" (p. 37), but state that external reliability can be enhanced in a qualitative study if the following five issues are recognised and handled:

- researcher status position;
- informant choices;
- social situations and conditions;
- analytic constructs and premises;
- methods of data collection and analysis.

The measures taken in each of these respects to reduce obstacles to external reliability of the present study are discussed below and limitations are acknowledged.

• Researcher status position: This factor concerns the positions held by the researchers and the extent to which researchers are members of the group studied.

To address the issue of researcher status position, the researcher's role and status are discussed briefly below and illuminated in other parts of this report. The history of the researcher is described in Chapters 1 and 3 and insights into the relationship that developed with each of the children through the interview experiences are discussed in Chapter 4. It is acknowledged that, even with this information, other studies may not be comparable in this respect; the important factor is that the information is supplied so that readers can judge for comparability.

One of the goals of the researcher was to be accepted by the participants, and for them to feel comfortable in the interviews. To conduct the interviews in the present study, the researcher entered each of the two school settings on ten occasions over a five month period. During each visit, individual interviews were conducted for approximately 30 minutes per child. Although immersion or participation in the situation did not occur to the degree recommended for ethnographic research (e.g., Taft, 1988), some degree of social relationship did emerge between the researcher and each of the eight participants in the present study because of the number of interviews with each child. As in other qualitative studies, this relationship with the participants was a feature that cannot necessarily be replicated by others as it had a personal and social dimension. It is important that this is recognised as a feature of the study and a possible threat to reliability. However, it is a possible strengthening factor for internal validity as discussed later.

The social role of the interviewer also plays a part in determining whether external replicability of research is possible (LeCompte & Goetz, 1982). In the present study, the background of the interviewer influenced the way in which questions were asked. If a child's class teacher asked the same questions he or she would not conduct a replicative study as their social role and relationship to the child would be different; theirs would be a supplemental study (LeCompte & Goetz, 1982). Although results of the current study may be narrowly applicable, the results are legitimate. The present study provided what might be called *slices*

of data (LeCompte & Goetz, 1982), giving insights into individual children's beliefs. Different researchers working with different children would gain insights into different children's realities, that is, different perceptions of the world. Indeed, replication with the same children also may not provide the same results, but this does not discredit the results of the present study as "several interpretations of the same data can be made, and all stand until contradicted by new evidence" (Merriam, 1988, p. 172). The study did not seek to expose one truth. Nor is it claimed to be objective. It is possible that neither qualitative nor quantitative methods are synonymous with objectivity; just as observation notes and the asking of questions may be open to researcher bias so too may be the construction of tests and questionnaires (Patton, 1987). Responses from the eight children interviewed in the present study contribute to the picture of beliefs of these Grade 3 children, as portrayed by the researcher.

• Informant choices: As "knowledge gathered is a function of who gives it" (LeCompte & Goetz, 1982, p. 38), it is important that the types of people participant in the research, and the decision process in their choice, are delineated. As later discussion elaborates, participants were selected to provide a cross section according to key characteristics. In addition, researcher impressions of personal factors for each child are provided in Chapter 4.

• Social situations and conditions: In a qualitative study, it is necessary that social settings and physical and interpersonal contexts in which data are collected are delineated (LeCompte & Goetz, 1982). Unlike many qualitative studies, the present research was not a study of communities. Social situations were not necessarily an element of the data as they are in ethnographic research, nor did they contribute to the process of collection of data. Individual interviews were conducted, thus social interaction took place only between two people at a time. In relation to this social situation, relevant details of the relationship between the interviewer and each interviewee are discussed in Chapter 4.

• Analytic constructs and premises: According to LeCompte and Goetz (1982), outlining assumptions of a study aids replication; likewise, replication is enhanced by outlining "theoretical premises and defining constructs that inform and shape the research" (p. 39). The thesis to this point has included discussion of theoretical underpinnings of the study and has identified and provided meanings for terms relevant to the purpose of the study. Assumptions, such as that there is no one truth or reality, are identified also.

Analytic constructs must be addressed to enhance reliability. The development of data categories is a low level construct that has the potential to create problems for external and internal reliability (LeCompte & Goetz, 1982). Units of analysis need to be clearly identified and an analytic framework adds to the possibility of replication.

In the present study the individual children were the units of analysis, as stated above. Beliefs about mathematics, learning, and factors that help in the learning of mathematics are identified within this report as major categories. However, further breakdown of these categories was made by the researcher in response to each individual child's responses. This approach did not standardise the data, nor did it trivialise the rich and complex findings. The provision of interview excerpts and other data sources such as drawings, allows the reader to see the basis for the categorisation. Categories or themes emerged from the data rather than being predetermined.

• Methods of data collection and analysis: Description of the scope, development and use of research procedures enhances the external reliability of a qualitative study, as does delineation of strategies for analysing data (LeCompte & Goetz, 1982).

The development of interview data collection procedures suitable for eight to nine year old children was a major feature of the present study. In response to the purposes of the research and the age of the children, certain criteria, outlined later in this chapter, were identified and met when developing appropriate procedures. The resultant procedures are detailed in this chapter and in Appendix A. The appendix can act as an operating manual: tasks for the children and key interview questions for each interview provide an overall structure for other researchers. Set questions are not provided for use throughout each interview as the direction of interviews depended upon children's responses to the tasks. Strategies for data collection, and purposes for the interviews, are detailed within this chapter.

The detailing of recording of responses also enhances reliability. In the present study, recordings of responses were made using audio equipment, with some interview artefacts, such as drawings, collected from the children. Field notes were completed when appropriate during and immediately following the interviews, on the prepared interview record sheet (Appendix B).

The methods of data analysis are detailed within this chapter, thus also enhancing reliability of the present study. Beyond the three main areas of interest, that is, beliefs about mathematics, learning, and helping factors for learning mathematics, analysis of data was not predetermined. Themes from the data allow outsiders, such as readers of this thesis, to make some sense of the eight children's views of the world. The research was a serious attempt to understand the children's thoughts and meanings (Spradley, 1979), therefore the analysis was based on the children's concepts.

In summary, the above discussion illustrates that the five potential problems for external reliability identified by LeCompte & Goetz (1982), that is, for whether the same phenomena would be identified by other researchers and the same constructs generated, were addressed in the present study. The next section describes how issues related to internal reliability were addressed.

Internal reliability

LeCompte and Goetz (1982) identify five strategies to reduce threats to internal reliability:

- low-inference descriptors;
- multiple researchers;
- participant researchers;
- peer examination; and

mechanically recorded data.

In the current research, measures were taken to implement these strategies where possible, as detailed below.

• Low-inference descriptors: Data in the present study were gathered in recorded interview conversations. Interview excerpts were drawn upon for discussion of themes that emerged from each child's data. The interview excerpts, or narratives, were low inference but higher inference interpretive comments were added by the researcher. The inclusion of multiple examples of interview excerpts substantiates inferred categories of analysis for the reader and increases the credibility of the research.

• Multiple researchers: In the discussion of multiple observers, LeCompte and Goetz (1982) refer to the need to reach agreement about the meaning of observations. In the present study, data were collected primarily through interview conversations and related tasks. The data collection did not involve multiple researchers who could discuss, corroborate and confirm the meaning of such observations but discussion of the meaning of such observations did occur through peer review as discussed above and below. Internal reliability of the research was enhanced also by low-inference descriptors and mechanically recorded data.

• Participant researchers: Participation by the eight children as arbiters of the research was desirable in theory but was not possible in practice. Partnerships in the keeping of notes or providing reactions to working analyses were not appropriate with children of eight to nine years of age. However, confirmation or otherwise of analyses did occur through the use of multiple methods or instances of data collection over the five month period. This form of triangulation strengthens reliability as well as being an important contributor to validity (Merriam, 1988).

• Peer examination: Reliability in the present doctoral study was facilitated by three forms of peer support. Firstly, there was involvement of a supervisor who questioned conclusions looking always for truthfulness and accuracy. Peer support was provided also by a colleague who examined data and commented on the researcher interpretation. The publication of results (e.g., McDonough, 1996, 1998b), and the associated conference presentations also constituted the offering of material for peer review.

• Mechanically recorded data: As stated previously, audio tapes were used to record and preserve interview data collected within the present study, thus enhancing reliability.

The above discussion outlined measures taken to increase internal and external reliability of the present study and acknowledged limitations to reliability.

In traditional experimental research, phenomena are considered to be constant and the focus is on discovering causal relationships between variables; it is assumed that repeated research of a single entity will give the same results (Merriam, 1988). The present research contrasts with traditional experimental research in purpose, theoretical underpinnings, and method as it sought to describe and explain aspects of the world from the perspectives of eight

children. Thus the present study can only be replicated to an extent. Nonetheless, specific steps, as outlined above, were taken to enhance internal reliability.

Validity

Just as it is imperative that a study be reliable, so too it should be valid. Validity involves two concepts: internal validity, "the extent to which results can be interpreted accurately and with confidence" (Wiersma, 1995, p. 6), and external validity, "the extent to which results can be generalised to populations, situations, and conditions" (Wiersma, 1995, p. 5). In research of all forms, validity is a matter of degree; it is virtually impossible to attain "perfect" validity (Wiersma, 1995).

Measures taken to ensure traditional studies are scientific or rigorous or trustworthy are based on a different view of reality from that of qualitative studies thus appropriate standards for assessing the validity of qualitative research should be identified (Merriam, 1988). Measures taken in the present research are examined in terms of each type of validity.

Internal validity

Internal validity relates to truth value, or how one's findings match reality (Merriam, 1988). The primary rationale of the present study was to gain insights into how the world of mathematics learning looked to young children as learners of mathematics. Expression of beliefs was open to change over the period of the research and presentation of beliefs involved the researcher as an interpreter or translator thus the report does not present one true reality for the reader (Merriam, 1988). Nonetheless, it was important within the study that measures were undertaken to ensure trustworthiness of findings. These measures are examined firstly in terms of five threats to internal validity (LeCompte & Goetz, 1982):

- history and maturation;
- observer effects;
- selection and regression;
- mortality;
- spurious conclusions.

The present research took each of these into account in the following ways.

• History and maturation: In ethnographic research, history relates to changes in the social scene (LeCompte & Goetz, 1982). The present study was qualitative in nature but, as discussed earlier, is not claimed to be an ethnographic study as its main focus was not the culture of a social group. Changes may have occurred in the children's experiences over the period of the data collection, but these are not seen as contaminants of the data; they are acknowledged where children's responses indicated they occurred as they are part of the data. However, as the research did not seek to identify cause and effect, such occurrences have less significance than they might in another study. The collection of data over a five month period gave some insights into the range of experiences Grade 3 children choose to refer to in their discussion of mathematics and learning.

Maturation refers to development within individuals. In experimental research, maturation concerns "those biological or psychological processes which systematically vary with the passage of time, independent of external specific events" (Campbell & Stanley, 1963, pp. 7-8). However, in the ethnographic view, maturation varies according to cultural norms and researchers control for its effects "by identifying explicitly what behaviours and norms are expected in different sociocultural contexts" (LeCompte & Goetz, 1982, p. 45). In the present research, maturation of the individual was of relevance as the individual was the unit of analysis. Data were collected over a period of five months, and therefore maturation changes were likely to have occurred. These were acceptable as a descriptive account was built up over the data collection period; this account portrayed children's beliefs as they could be accessed by the procedures developed for the research. Where maturation changes were apparent, they are reported within the discussion.

• Observer effects: This threat to internal validity is seen as parallel to threats of testing and instrumentation in experimental studies (LeCompte & Goetz, 1982). There are a number of aspects relevant to the present study as discussed below.

One factor related to observer effects concerns reactivity. Campbell and Stanley (1963) state that "In general, the more novel and motivating the test device, the more reactive one can expect it to be." (p. 9). Approaches such as drawing, construction, and children talking about their own personal experiences, reduced the threat of reactivity to the internal validity of the research as these were familiar activities for eight to nine year olds and as they related closely to the children themselves. The activities were less of a novelty for children than would be, for example, placing a video camera in a classroom. However, the lesson observations by the researcher may have had some novelty value for the children. To minimise this factor, observations were conducted towards the end of the data collection period, once the children had become comfortable with the researcher. The intention was for the researcher to develop a perception of the nature of the mathematics classes in which the eight research participants were members as background data for the study. Class processes and procedures that involved the teacher and all the children were observed. The interviewees were not given special focus.

A further element of reactivity is the effect of the relationship between the interviewer and the informants (LeCompte & Goetz, 1982). The equal status given to each of the eight participants in the present study through equal time and the use of the same sets of tasks, minimised the possibility of special relationships that would distort data or affect the researcher role. Nonetheless, relationships between the researcher and the participants were not completely neutral; some rapport was built up through the regular interactions. This may have lead to the interviewees developing a feeling of trust in the interviewer (Burns, 1997). It potentially strengthened internal validity also as it minimised the possibility of inference by the participants of indifference or hostility on the part of the researcher and thus minimised consequent "paranoiac reactions" that would affect the quality of the data (LeCompte & Goetz, 1982, p. 46). Just as it is not possible to do a good ethnographic study without rapport with informants (Spradley, 1979), this study benefited from a positive relationship between the researcher and the participants. Following the advice of Patton (1990), efforts were made to build rapport with, and hold respect for, the children as people, but to be neutral towards the content of what they were saying as it was "their knowledge, experiences, attitudes and feelings" (Patton, 1990, p. 317) that were important. Ginsburg (1997) also stresses the importance in the interview situation of showing respect for children, for example, by conveying deep interest in their thinking and acknowledging as genuine their attempts to create meaning and make sense of the world.

Another aspect of reactivity is that "the process of measuring may change that which is being measured" (Campbell & Stanley, 1963, p. 9). If children reflected upon responses to tasks used early in the period of the data collection period, it is possible that responses to further tasks may have been influenced. For this reason the order of the ten sets of procedures was given by random arrangement to each child (see Appendix C).

The developmental nature of the research was unavoidable but acceptable. The children were developing over the period of the data collection and they may have been influenced from exposure to the research procedures. If the research experience caused the children to increase their awareness of their own learning of mathematics, as may have happened, this is viewed as an unavoidable and positive outcome of the research. Awareness of one's learning is desirable (Fennema, 1989; National Council of Teachers of Mathematics, 1989; Spangler, 1992); student awareness of their own beliefs toward mathematics may be as important as teachers' awareness of students' mathematical beliefs (Spangler, 1992). Any influence from exposure to the research procedures that became apparent to the researcher is reported.

The credibility of responses in interviews is another potential problem for internal validity of research related to observer effects (LeCompte & Goetz, 1982). The possibility of artificial responses from children was reduced in the present study by the extended period of data collection and the use of multiple procedures addressing beliefs about a particular factor. The cross analysis of data from all interviews in coding data and identifying emergent themes addressed this potential problem. Because children were interviewed on a number of occasions, they also may have anticipated to some degree what was to come, or at least the subject of questions for future interviews. As indepth investigation of their beliefs was intended, this was unavoidable. However, as question and task types were varied as much as possible, children were not able to prepare responses.

It is important that the research represent the perspectives of the participants, not the researcher's own ethnocentrisms and biases (LeCompte & Goetz, 1982). This threat to internal validity was addressed through the inclusion of interview excerpts throughout the discussion of the children's beliefs ensuring that "the categories are meaningful to the participants, reflect the way participants experience reality, and are actually supported by the data" (LeCompte & Goetz, 1982, p. 47).

The research addressed this observer effect threat to internal validity also by not imposing predetermined categories upon children's responses. The following discussion of the use of questionnaires in previous research on children's perceptions of learning environments demonstrates that questionnaire analysis utilising predetermined categories may have introduced more of a researcher perspective than appropriate for a qualitative study with the theoretical underpinnings of the present study.

In the early stages of this study, investigation of possible strategies for gaining insights into children beliefs about helping factors for learning mathematics lead to the examination of previous research on children's beliefs about learning environments through the use of questionnaires (e.g., Fraser, 1994). Those questionnaires that were examined, give children the opportunity to respond about factors perceived to influence their learning, but appear largely to limit responses to factors predetermined by the questionnaire developers. Also, in many such questionnaires, items are worded to focus on a "student's perceptions of the class as a whole, as distinct from that student's perceptions of his or her own role within the classroom" (Fraser, 1994, p. 528). An instrument designed specifically for use with primary students, the My Class Inventory (Fraser, Anderson, & Walberg, 1982), includes items of this type such as: "The children enjoy their schoolwork in my class", "Children often race to see who can finish first", and "In our class the work is hard to do" (Fraser, Malone, & Neale, 1982, p. 195). The analysis and comparison is carried out in terms of class means. While responses to the instrument can be targeted specifically at mathematics classes as, for example, in the study by Fraser, Malone, and Neale (1982), the items themselves do not necessarily address issues that are perceived as related directly to the learning of mathematics for the actual respondents in any one study. The My Class Inventory can be useful for teachers wishing to study the overall climate or environment in their classes but the factors it studies, for example satisfaction, competitiveness, and difficulty, are determined by the instrument and are different from those of interest in the present study.

The procedures deployed in the present study were chosen because they have features that were suitable for the particular purposes of the study. It is posited that

- the research procedures encouraged the children to focus largely on their own experiences, thus facilitating reflection and increasing researcher insights into the world of individual learners of mathematics;
- the openness within the procedures allowed factors and issues to emerge that were perceived by the individual children to relate specifically to mathematics and learning;
- the study focused specifically on beliefs about mathematics and learning so that indepth portrayals of the individuals' beliefs are presented; and
- the presentation and discussion of data within categories that emerged from the children's responses allowed the individual children's beliefs to be portrayed with confidence.

In terms of the above discussion of the potential threat to internal validity of the *researcher's ethnocentrisims and biases*, the latter point is relevant. By allowing individual children's categories or themes to emerge from their data, such observer effects were lessened.

The above discussion identified other potential threats to internal validity also that relate specifically to *observer effects*. As discussed, such factors include reactivity, credibility of responses in interviews, and influence of researcher bias on analysis of data. A further threat to internal validity for qualitative research is selection and regression (LeCompte & Goetz, 1982).

• Selection and regression: In experimental research, this factor relates to the comparison of outcomes for groups, with the assumption that, upon recruitment, the groups do not differ (Campbell & Stanley, 1963). In the present research, similarity was not assumed nor was it important or necessary. Comparison of results across the eight children also was not a key aspect in the sense that it might be for quantitative research. However, to avoid distortions in data and conclusions, a diversity of participants was sought (LeCompte & Goetz, 1982). Firstly, principals from two schools in different locations and with different socio-economic populations were approached. Upon agreement, the selection of children from one class in each school was in accordance with categories identified by the researcher, as discussed later in this chapter. The eight children were chosen by their class teachers according to those categories.

• Mortality: Mortality was not a threat to the internal validity of the present study as all eight children selected for the study participated for the five month data collection period.

• Spurious conclusions: Of importance in qualitative studies is that data are examined so that all possible causes or explanations are delineated (LeCompte & Goetz, 1982). This requires "effective and efficient retrieval systems . . . and the scrupulous use of corroboratory and alternative sources of data" (LeCompte & Goetz, 1982, p. 50). Negative instances and disconfirming evidence for emerging constructs should be sought (LeCompte & Goetz, 1982).

The portrayal of children's beliefs in the present study was built up by reference to data gathered in multiple interviews using a large number of interview tasks and procedures that ensured revisiting of ideas from different perspectives. As discussed within this chapter, tasks incorporated a range of visual, verbal, and text-based components for both the interviewer and participants thus producing different task types addressing the same ideas, and data that varied in nature. Results enabled portrayals of individual children's beliefs to be built up.

In many instances data from a range of interviews confirmed or extended emerging themes, but there were instances where disconfirming data emerged. This is acknowledged and drawn upon; it gives insights into the possible complexity of young children's beliefs. For example, in Cara's beliefs about the nature of mathematics there are suggestions of contradiction or uncertainty in relation to whether informal measurement is, or is not, mathematical activity. This is not ignored but is portrayed within the discussion and within the figures that provide an emerging schematic reference of the findings from this research. To conclude, a range of threats to internal validity were addressed in the present study. Referring to the strategies outlined by Merriam (1988), the internal validity of the study was strengthened by triangulation through multiple methods and sources of data collection within the interviews, long-term gathering of data, peer examination or comment upon findings, and clarification of the researcher's assumptions and theoretical orientation.

External validity

As stated above, external validity refers to the generalisability of results. It relates to qualitative research differently from experimental research. For example, the intention in the present study was not to investigate a representative random sample from a population but rather to gain insights into meanings or beliefs of individuals. Therefore, statistical sampling, as used in experimental studies, was not relevant or appropriate.

The insights gained facilitate depth of understanding (Patton, 1990) of the nature of beliefs held by children of eight to nine years. The study allows for translatability, that is, theoretical constructs and research procedures are described in a way so that other researchers can understand the results, and allows also for comparability, that is, the characteristics of the research are described adequately so that other researchers can use the results to extend their knowledge (Wiersma, 1995).

As "comparability and translatability of findings, rather than outright transference to groups not investigated" (LeCompte & Goetz, 1982, p. 34) are relevant to qualitative research, measures were taken in the present study to reduce obstacles to these. The research report includes "a careful description of settings and people, the conditions of study, and the constructs used [to] give other researchers the information necessary to assess the typicality of a situation and thus the appropriate comparison groups and translation issues" (Eisenhart, 1988, p. 109). For example, research methods are detailed in this chapter and characteristics of the research participants are considered within this chapter and Chapter 4. The development of analytic categories is discussed in this chapter with further justification provided, through the incorporation of interview excerpts and reference to interview artefacts such as drawings, within the presentation and discussion of data for each child.

Generalisability, in the traditional sense as applied in experimental research, was not relevant in this study. Indeed, if the primary aim of a researcher is to assess the generalisability of a finding, qualitative or interpretive research, which depends to a large degree on the active and personal involvement of the researcher, is not an appropriate choice (Eisenhart, 1988; Erickson, 1986).

To conclude the discussion of reliability and validity issues, it is reiterated that the present qualitative study had different theoretical underpinnings, different purposes and different methods from quantitative research. As discussed above, validity and reliability issues, while also relevant to qualitative research, are addressed sometimes in different ways from quantitative research, but also with the intention of ensuring credibility and authenticity of the study so that the research is of value (LeCompte & Goetz, 1982; Patton, 1990). The

preceding discussion has established the steps taken to ensure the validity and reliability of the data collection and reporting.

A further key issue is that clearly the report is open to interpretation by the reader. The hope is to maximise the match between the writer's intended portrayal and the reader's interpretation of the children's beliefs. Steps taken to make the communication clear and to minimise disparities between writer and reader interpretations include intending to

- be comprehensive;
- collect similar data from multiple sources;
- collect similar data by using a variety of question types;
- draw upon different forms of data (visual, verbal, and text-based);
- report the process for data collection fully, describing the development and use of procedures;
- use the data themselves as the organising mechanism;
- report honestly, fully and faithfully;
- establish links between data and inferences; and
- write frequent syntheses and summaries.

The introduction to the discussion of reliability and validity issues stated that qualitative research strives to be believable, plausible, meaningful and trustworthy. As discussed in the preceding sections of this chapter, many measures were taken to ensure these outcomes within the present study. Included among these were measures, as listed above, to increase the likelihood of common researcher and reader interpretations of the data.

In the practical component of the study, one of the first occurrences was the selection of the research participants.

The research participants

In the present study, eight children of eight to nine years of age were interviewed during their Grade 3 year of schooling. The children's mathematics teachers and peers also contributed in a small way to the research. The discussion below details background concerning the eight children including the criteria and procedures for their selection, the participation of their teachers and peers in the study, and the mathematics teaching at their schools.

Selection of the research participants

During the early stages of the study, two primary school principals were contacted and, following their approval, a Grade 3 teacher from each of the schools was approached. These teachers agreed to participate in the study. The schools featured differing socio-economic levels and differing locations, one in inner suburban Melbourne and the other a middle suburban school. Although such differences existed, the schools were not chosen for particular features. Generalisability was not a concern of the research, therefore representativeness was not sought, nor could it be with such a small sample. As stated above, unlike many quantitative studies, the present study did not aim for direct transference to groups not studied. It was intended that the research aim for comparability and translatability of findings, factors crucial to the application of qualitative studies (LeCompte & Goetz, 1982). The delineation of characteristics of the group studied (LeCompte & Goetz, 1982), in this section of the report and in Chapter 4, lays the foundation for translation and comparison by the reader. Description of the research participants and criteria for their selection in part informs the reader for possible replication of the methods used in the present study (LeCompte & Goetz, 1982).

The participants, or informants, who provided data in the study were not selected as the study progressed, but were determined prior to data collection. They were not selected as typical or atypical of Grade 3 children, but, through reference to school location, gender, and achievement, there was some variation within the group. This approach to selection illustrates one difference from much qualitative inquiry in which participant selection is not finalised prior to the study but unfolds as part of the ongoing exploration of the setting (Patton, 1990). The difference emanates from the purpose of the study, that is, to investigate individual perspectives regarding elements of the phenomenon of learning mathematics rather than to study a setting or group.

After the two Grade 3 teachers agreed for children in their classes to participate in the study, the teachers were each requested to select four children for the research, two males and two females with one of each a high achiever in mathematics and one a low achiever in mathematics (according to teacher perception). Like the differences between the schools, this gender and achievement mix gave some variation among the participants but was not deployed with the intention of selecting a representative sample. For this reason it was considered acceptable that teacher perception of achievement act as a guide for participant selection; for the purpose of the research it was not necessary to use more formal assessment or selection procedures.

Following teacher selection of the students, their parents were contacted by mail to ensure they were fully aware of the intention of the processes of the research and able to make informed decisions on their children's participation. In response to the letter sent to parents, giving information on the purpose and methods of the study, parents signed a form giving their child permission to participate.

Table 1 illustrates the distribution of school, gender, and teacher-perceived achievement level for the eight key participants in the research.

| Child | Anna | Ben | Cara | David | Emily | Filip | Gina | Harry |
|-----------------|--------|------|--------|-------|--------|-------|--------|-------|
| School | S | S | S | S | Ι | Ι | Ι | Ι |
| Gender | female | male | female | male | female | male | female | male |
| Achievement | high | high | low | low | high | high | low | low |
| (teacher rated) | | | | | | | | |

Summary of details of child participants in the study

Table 1
Names used in the table (and throughout the report) are fictitious and developed for easy reference in the study. Names beginning with the letters A to D represent children from the middle suburban school, School S, and names beginning with the letters E to H represent children from the inner suburban school, School I.

More detailed introductions to each of the children are provided in Chapter 4, drawing on information from the children themselves, their teachers, and researcher impressions. Included is discussion of the children's disposition during, and in response to, the interview situation, as well as their perception of their own achievement in mathematics. Stemming from the participation of the eight children was minor participation in the study by their classmates and mathematics teachers.

Contribution of the research participants' teachers and peers

The two class teachers became contributors to the research, although to a much lesser degree than the eight children, once they agreed to have children in their class take part in the research. The teachers participated by responding to the researcher in one interview concerning their beliefs about mathematics and learning, and their mathematics teaching practices. They also allowed observations of three mathematics lessons and a small amount of data collection in the whole class situation. Although the teachers' time commitment and direct contribution were minimal, their assistance was vital to the study, especially as they each allowed four children to leave the class, each for approximately a half hour on 10 occasions over a five month period, and thus incurred disruption to their teaching.

As some data were collected from the whole classes, the members of those classes became participants in the research also. However, their contribution was minimal; lesson observations provided some contextual data for the research, and drawings and writing completed by the eight children within the whole class setting provided an alternative data source for those interviewees.

Thus the class teachers and all the children in the two classes contributed to the research, but the key contributors to the study remained the eight children who each participated in ten one-to-one interviews with the researcher.

As an overriding purpose of the study was to gain insights into the beliefs of those eight children, it was necessary to use data collection procedures that would facilitate the expression of young children's beliefs. The development of such research procedures for use in the interviews became one major element of the study. The development of procedures appropriate for use with young children, the implementation of these, and the interpretation of data were challenging tasks. The detailed discussion of the one-to-one interview procedures is followed by a brief discussion of the procedures used for the whole class collection of data, lesson observations, and teacher interviews.

Development of interview data collection procedures

The research began with an interest, an idea, that evolved over a period of time through my experiences as a primary teacher, university lecturer, and researcher, as discussed in Chapter 1. My initial interest was in children's perceptions of factors in learning environments that they believe help them to learn mathematics well. I first asked this question during my Masters level study (McDonough, 1991), and found that the use of a drawing and description procedure gave children the opportunity to formulate their thoughts and represent them on paper in a familiar format before being asked to write or speak about the situation and the helping factors. As discussed below, this approach was used also in the present study. The Masters level study suggested that children in Grade 3, thus of eight to nine years of age, could communicate beliefs through responses to procedures involving drawing. The present study took into account these findings regarding the age of the children and the mode of response but the development of data collection procedures went beyond the use of drawings.

Little previous research was identified in the area of young children's beliefs and thus the development and/or identification of a range of suitable procedures was of importance for answering the questions of whether children of eight to nine years of age hold beliefs about mathematics, learning, and helping factors for learning mathematics that can be articulated and portrayed, and what those beliefs are. The research aimed to take an indepth approach through the use of multiple interviews with a small number of children. It was believed that interviews could provide richer insights into children's beliefs than those gained through pencil and paper measures, just as data resulting from a think aloud interview of a student's attempt at a problem solving task are very different from those from a standardised test of mathematics achievement (Hart, 1989). Thus the development and use of interview procedures were key elements of the research. Development of whole class data collection and class observation procedures, and teacher interview questions were elements of the study also. The responses to these supplementary forms of data collection contributed to the discussion of background or context related to the main focus, the beliefs of the eight children.

The discussion of procedures begins by considering the key data collection opportunity, that is, the interviews with the eight main research participants. It is broken into the following sections:

- objectives for the development of the interview procedures;
- purposes and summaries of the interview procedures;
- creative and varied interview procedures.

Objectives for the development of the interview procedures

The development of interview procedures was guided by a broad set of objectives. Taking into account the research questions and purposes of the present study, as detailed at the beginning of Chapter 2, the research sought to

i. develop a range of procedures focusing on beliefs about the nature of mathematics;

- ii. develop a range of procedures focusing on beliefs about the nature of learning;
- iii. develop a range of procedures focusing on beliefs about helping factors for the learning of mathematics;
- iv. develop procedures that would allow children to express themselves in a variety of ways and that therefore would provide rich and illustrative data for each child;
- v. develop procedures each with an open-ended component that would a) accommodate and facilitate the gaining of insights into the perspectives of each individual respondent, and b) allow for the direction taken by each child to be pursued within the interviews;
- vi. incorporate activities and ideas of interest to eight to nine year old children;
- vii. accommodate attention spans of the children;
- viii. identify wording, appropriate for use with young children, for key and possible followup questions;
- ix. incorporate and build on the work and procedures of previous researchers where possible and appropriate; and
- x. retain the objective of gaining insights into how the children see the world of mathematics and learning.

In the following section the resultant thirty research procedures are summarised and purposes are clarified.

Purposes and summaries of the interview procedures

Twenty-seven research procedures were grouped in ten sets with the intention that one set be used in each interview with a child. An eleventh set of three procedures, developed during the course of the data collection, was broken up and used when interview time allowed. Procedure Sets 1 to 5 incorporated tasks targeted at children's beliefs about the nature of learning and the nature of mathematics. Procedure Sets 6 to 10 targeted the gaining of insights into children's beliefs about helping factors for learning mathematics. The full reference sheets used for the interviews, that contain further detail of the key questions and tasks posed, are provided in Appendix A. Purposes for and summaries of all procedures are provided below. Numbers such as 7.2 indicate the more specific task numbers, referred to in discussion of findings in later chapters.

Prior to summarising the procedures, it is necessary to explain that the term *maths*, the common abbreviation of *mathematics* (Wilkes & Krebs, 1982), is used both when detailing the procedures and considering children's responses as it was the term used in the interviews. Although I had planned to use the terms mathematics and maths interchangeably during the data collection, I became aware that I should not assume that children see the terms as having the same meaning. The possibility of different meanings became evident to me from interview incidents in the beginning weeks of data collection, which I detail later in this report. Thus, in subsequent conversations with the children I used the term *maths activity* and *mathematical activity* were used occasionally. Likewise, in later chapters, I refer to *maths*, rather than

mathematics, when describing procedures and discussing responses and do this intentionally, to mirror the terminology use in the interviews. Included in later analysis and discussion of data is discussion of each child's meanings for *maths*, a term familiar to all, and for *mathematics*. For some of the children the terms were interchangeable; for others, such as Cara, they were not. The formal term, *mathematics*, and the common term, *maths*, are used below as appropriate.

Set 1. Word association quiz; Password; Word wheels; Subject most like maths; Subject least like maths

Purpose:

- To gain insights into what children most readily associate with maths and learning as a basis for a more detailed and developing picture of the children's beliefs.
- To facilitate briefly written but multiple responses.
- To have a written record that provides the basis for further discussion.
- To provide opportunities for children to articulate or describe their beliefs in a variety of ways, such as through comparison, so as to gain insights from varying perspectives.

Procedures:

- *1.1 Word association quiz.* A list of single words (including Maths and Learning) is read out one by one. Children are asked to say the first word they think of.
- *1.2 Password.* The scenario is set that a word game is being played with a friend. The first player begins with a word for the other player to guess. But only a <u>one</u> word clue can be given, which must not contain any part of the word being guessed (Spangler, 1992). Everyday words (for example, film, cat) are given as well as Maths and Learning.
- *1.3 Word wheels* (Maths; Another subject); and related discussion (Lewis & Davies, 1988). Two prepared sheets are available for children to add words or phrases. In the middle of one sheet is the question 'What is Maths?' (Task 1.3.1). The second sheet refers to another curriculum area with which the children are familiar (Task 1.3.2). Children are asked to write phrases or words, on the stems or in the spaces, which describe what each mean to them.
- *1.4 Subject most like maths; Subject least like maths? Why?* This discussion builds on from the previous activity in which maths and another subject have been compared.

Set 2. Personal dictionary; Learning situations; Maths situations

Purpose:

- To provide a familiar context, the structure and use of a dictionary, so as to stimulate children to think about their meaning for *maths* and *learning* and key words they associate with each of these concepts.
- To provide opportunities, through the children's descriptions, for informed inference by the researcher about the children's beliefs about learning and maths.

To provide an opportunity for children to possibly give insights into their beliefs about • factors that help in their learning of maths, thus adding to the data from tasks developed specifically for this purpose.

Procedures:

- 2.1 Personal dictionary Learning. The task requires children to pretend they are writing • their own personal dictionary. Words are provided, including Learning for which the children describe the definition they would put in their own dictionary. The exercise begins with other, more easy to define words, such as House, Eat, and Pet.
- 2.2 Descriptions of learning situations. Children are asked to tell about situations in which they have learnt something recently. From the children's descriptions words are selected by the interviewer for the child to consider whether they would be added to the personal dictionary definition for learning.
- 2.3 Personal dictionary Maths. Children pretend they are writing their own personal dictionary definition for the word Maths. Maths situations are discussed in terms of the personal definition.

Set 3. Mathematical activity – Draw; Show; Questionnaires

Purpose:

- To elicit children's beliefs about the nature of mathematical activity by asking them to • represent two mathematical situations of their choice, and so contribute to building up a picture of children's beliefs about the nature of maths.
- To facilitate children's articulation/description of their beliefs about mathematical activity through the use of a drawing and another mode of communication of their choice.
- To gain insights into whether mathematical activity is seen as occurring only, or even mostly, in a formal learning situation.
- To provide opportunities, through the children's descriptions, for informed inference by the researcher about the children's beliefs about maths, and the relationship of these to everyday activities.

To gain insights into whether children's beliefs about the nature of mathematical activity • are constant no matter whether a person is in a school situation or elsewhere.

Procedures:

- 3.1 Draw a mathematical activity. Children are asked to draw a picture of someone doing • some sort of mathematical activity. They then are asked to write about what the person is doing and why they think it is a mathematical activity. Discussion follows.
- 3.2 Show another mathematical activity. Children are asked to show another mathematical activity by either acting it out, drawing, or writing. Discussion follows.
- 3.3.1 and 3.3.2 Questionnaires. Using questionnaires adapted from the work of McDonald and Kouba (1986), and Wallbridge and Clarke (1989), children are asked to classify activities as to whether they believe them to be mathematical or not mathematical. They explain reasoning behind each answer.

Set 4 Maths is like; Drawings; Planning for an integrated unit of work Purpose:

- To identify whether, when given a suitable open response item, children choose to discuss • affective or cognitive aspects of maths, or a combination of the two.
- To identify the situation (school or elsewhere) that children most readily associate with doing maths, that is, to gain insights into whether or not mathematical activity is seen as occurring only, or even mostly, in a formal learning situation.
- To ascertain whether children's beliefs about the nature of mathematical activity are constant no matter whether a person is in a school situation or elsewhere.
- To investigate whether children's conception of mathematical activity identifies doing maths and learning maths as identical activities, or as different activities.
- To gain insights into the breadth of possible classroom activities that children consider as mathematical

Procedures:

- 4.1 Personal writing: "Maths is like"; and discussion. Children are asked to add to • the above prompt in writing. Children are encouraged to talk about what they have written and to describe examples of situations to which their response would apply.
- 4.2 Drawings people doing maths.

4.2.1 Children are asked to draw a picture of someone using or doing maths. Children describe picture.

4.2.2 Children are asked to draw another picture of someone using or doing maths - either at school or not at school. (For each child, the situation is chosen by the researcher to contrast with that depicted in Task 4.2.1.) Children describe picture. Discussion for each picture focuses on what the person is doing, and in what way the activity is mathematical. Children are asked whether the person is learning maths as well as doing or using maths in those situations.

• 4.3 Planning for an integrated unit of work. Children are asked whether they have ever been taught maths at school where everything has been related to one topic such as The Olympics or Houses. Children choose from the topics Holidays, Birthday Parties, or The Environment, or one of their choice, that they would like to learn about and learn maths at the same time, and then brainstorm activities that might be done at school.

Set 5 Alien; Photographs

Purpose:

- To gain insights into the children's beliefs about the nature of mathematical activity, that is, the type of activities they consider as mathematical.
- To discuss activities, at school and elsewhere, so as to ascertain the common features of those activities considered by the children as mathematical.
- To ascertain whether children's beliefs about the nature of mathematical activity are constant no matter whether a person is in a school situation or elsewhere.

• To gain insights into whether or not mathematical activity is seen as occurring only, or even mostly, in a formal learning situation.

Procedures:

- *5.1 Describing maths to an alien* (Stodolsky et al., 1991). Children are asked to pretend that an alien has arrived in their suburb and does not know what is going on. The child's job is to tell the alien what maths is.
- 5.2 Photographs-mathematical activity? A series of photographs is shown. Children are asked to describe what they believe is happening in each photograph and to say whether they believe there is any maths in what the person(s) is doing. Photographs belong to the following categories (according to the perception of the researcher):
- i. a school mathematics context,
- ii. a non-school mathematics context, and
- iii. a non-mathematical or ambiguous context (for example, sitting on a school bench). (Zevenbergen & Crowe, 1992)

Set 6 Learning maths well; Hindered from learning maths well Purpose:

- For each child to re-create his or her image of selected situations through visualising and representing using the familiar medium of drawing, prior to describing in words, so that the child is comfortable with and can complete the initial portrayal before further analysis.
- For each child's drawings to provide somewhat complete portrayals of the chosen situations on which to build discussion and reflection of helping and hindering factors.
- To have children identify and rank order, helping and hindering factors for learning maths, so that some depth of insight built on each child's own perspectives can be gained.

Procedures:

- *6.1 PPELEM: Situation in which learning maths well.* Children draw and describe their selected situation. Helping factors for learning maths in the situation are identified from the verbal descriptions, written on cards, and checked with the respondents. Helping factors are rank ordered and discussed.
- *6.2 PPELEM: Situation in which hindered from learning maths well.* Children draw and describe their selected situation. Hindering factors for learning maths in the situation are identified from the verbal descriptions, written on cards, and checked with the respondents. Hindering factors are rank ordered and discussed.

Set 7 Scenario; Video clips

Purpose:

- To present a situation regarding a hypothetical child, so as to facilitate, from a different perspective, children's consideration of difficulties in learning maths and perceptions of helping factors.
- To present a number of hypothetical situations so as to provide a broad range of situations for consideration by the children and as the basis for discussion.

- To present features of maths classrooms such as using calculators, responding to open questions and discussing with other children in the easily the recognised format of video footage, to facilitate interpretation of the situations as such, and therefore to facilitate reflection upon such factors.
- To present situations in which one and two way interactions occur, so as to identify whether interaction is significant in children's perception of factors that impact upon their learning of maths.

Procedures:

- *7.1 Scena*rio. Children are posed with the situation that a child they know is having difficulty learning maths. They are asked what they would suggest to help this child.
- 7.2 Video clips. Children are shown a range of classroom situations and describe each in their own words telling what they perceive is happening. They are encouraged to describe whether they would be learning maths well or would you be having some difficulties in those situations, and to discuss reasons.

Set 8 Easier and harder experiences; Photographs of home and school; Discussion of feelings Purpose:

- To broaden the image held by the researcher of situations in which children consider they learn maths, through the opportunity for children to describe situations possibly different from those given in response to other procedures.
- To ensure that home and classroom situations are given consideration as to whether they are situations in which each child believes he or she would learn maths.
- To provide an opportunity for affective factors related to learning maths to emerge, so as to give further insights into each child's perception of him or her self as a learner of maths.
- To portray situations containing a varied range of factors so insights into beliefs about what makes a situation mathematical might also be gained.

Procedures:

- 8.1 Easier and harder experiences in the learning of maths. Instances are discussed in which children find maths easy to learn and hard to learn. Differences between these instances, particularly in terms of perceived effectiveness of learning, and factors influencing the learning, are discussed.
- *8.2 Describe and sort photographs (home situations and school situations).* Children describe and sort photographs, according to whether they would learn maths well in those situations. Grouping: Yes, No, Not sure.
- *8.3 Discussion of feelings*. Children are asked to express their feelings about the learning of maths. Feelings are discussed in relation to the situations mentioned and the quality of the learning in those situations.

Set 9 I could do better in maths if; Children's drawings

Purpose:

- To facilitate reflection by children on their own participation in maths learning experiences and what helps them to learn by providing a sentence for completion.
- For children to consider the learning of maths within a range of scenarios so as to extend the situations and factors under consideration.
- To facilitate communication of perceived helping factors for learning maths, through free choice and interpretation of situations significant to other children.

Procedures:

- 9.1 I could do better in maths if Children write and talk about their learning of maths, and suggest factors that they perceive help them to achieve in their learning.
- 9.2 Visual vignettes Children's drawings of classroom, home, and other out of school situations. Would this help you do better in maths? Children view a selection of drawings. Children describe the drawings and comment on whether the factors portrayed in the drawings would help them to learn maths well.

Set 10 Written descriptors; Duplo; What is learning?

Purpose:

- To facilitate communication of perceived processes and helping factors for children's own learning of maths, by presenting to the children in brief written form, a range of factors to be considered.
- To use a different material, that is, building blocks, for children to portray learning situations, to once again foster reflection upon, and communication about, learning maths.
- To gain insights into children's perceptions of situations in which they do and do not feel comfortable about their learning (and therefore possibly learn maths well or are hindered from learning maths well), through the perspectives of feeling good and not feeling good.
- To address the issue of learning in a focused manner through the use of four terms, so as to assist in clarification of children's meaning for learning.

Procedures:

- *10.1 Ranking task* (adapted from Stodolsky et al., 1991). Children are presented with phrases or words describing activities or processes. The children sort into which they use in learning maths (Yes, No, Not sure), and rank the factors from the most used to the least used. There is some discussion of potential of the chosen factors for helping in learning maths.
- *10.2.1 Duplo* create a situation in which you feel good about learning maths. Children build the situation using Duplo, then describe and discuss.
- *10.2.2 Duplo* create a situation in which you did not feel good about learning maths. Children build the situation using Duplo, then describe and discuss.

• *10.3 Learning - What does this mean to you?* Children select, from four key words presented on cards, the terms that they associate with learning. Discuss in terms of the children's own learning.

Set 11 How good at maths; I think mathematics is; A good maths teacher Purpose:

- To gain insights into children's perceptions of their own abilities or achievements in maths by having children rate, on a scale of one to ten, how good they believe they are at maths.
- To make direct reference to the concept of *mathematics*, to ascertain whether this concept is seen as different from *maths*, by having children complete a sentence.
- To discuss possible perceived teacher behaviours for helping children to learn maths, so to gain further insights into helping factors for learning maths, and possibly into beliefs about the nature of mathematical activity.

Procedures:

- *11.1 How good at maths 1 10.* Children are asked where they would place themselves on a continuum, with 1 being "for a person who is not very good at all at maths", and 10 being "for a person who is really, really good at maths".
- *11.2 I think mathematics is* Children write and talk about their perception of the nature of mathematics.
- *11.3 A good maths teacher* Children write and talk about their perception of the characteristics of a good maths teacher.

The majority of procedures were developed by the researcher for the present study but some were identified from other sources and adapted where necessary. Table 2 aligns the code number assigned to each procedure, the title, the features of the tasks, that is, verbal, textbased or visual, the anticipated main focus, that is, information about self (S), beliefs about maths (M), beliefs about learning (L), or beliefs about helping factors for learning maths (HF), and the author and date where use or adaptation of previously developed procedures occurred.

The development of the interview procedures was a major element of the present study. Piloting occurred initially with Grade 5 and 6 children, and after some refinement of tasks, further piloting was conducted with Grade 3 children. Responses indicated that the tasks made sense to, and could be responded to, by children as young as eight years of age (Grade 3), and therefore were suitable for use in the present study.

In the remainder of this chapter following Table 2, the word *maths* is used when talking specifically of instructions given to the children; otherwise the more formal term, *mathematics*, is used, with the understanding that the same concept is being discussed, unless otherwise stated.

Table 2Summary of interview tasks

| Code no. | Verbal | Text- based | Visual | Focus | Task title |
|-------------|--------|----------------|----------|-------|--|
| 1.1 | ~ | | | M, L | Word association quiz |
| 1.2 | ~ | | | M, L | Password: Word game (Spangler, 1992) |
| 1.3 | ~ | ~ | | Μ | Word wheels (Lewis & Davies, 1988): Words or phrases to describe "maths" and another suitable subject, e.g., "music" 1.3.1 Maths 1.3.2 "Other" subject |
| 1.4 | ~ | | | М | Subject most like maths; subject least like maths? Why? |
| 2.1 | ~ | | | L | Personal dictionary: Learn. Children give own definition and discuss in context of personal experiences |
| 2.2 | ~ | | | L | Descriptions of learning situations |
| 2.3 | ~ | | | М | Personal dictionary: Maths. Children give own definition and discuss in context of personal experiences |
| 3.1 | ~ | | ~ | Μ | Draw a mathematical activity |
| 3.2 | ~ | ~ | ~ | М | Show another mathematical activity (act out, write, draw etc.) |
| 3.3 | ~ | | | М | Questionnaires - Maths in everyday activities (presented orally) 3.3.1 (Adapted from McDonald & Kouba, 1986) 3.3.2 (Adapted from Wallbridge & Clarke, 1989) |
| 4.1 | ~ | ~ | | М | Personal writing: "Maths is like", and discussion |
| 4.2 | ~ | | ~ | М | Drawings: People using or doing maths inside and outside school 4.2.1 Drawing one 4.2.2 "Other" drawing |
| 4.3 | ~ | ~ | | M, L | Planning for an integrated unit of work |
| 5.1 | ~ | | | М | Describing maths to an alien (Stodolsky et al., 1991) |
| 5.2 | ~ | | ~ | М | Photographs - mathematical activity? (Zevenbergen & Crowe, 1992) |
| 6.1 | ~ | ~ | ~ | HF | PPELEM: Situation in which learning maths well: Draw, describe, and identify most helpful factors (ranking) |
| 6.2 | ~ | ~ | v | HF | PPELEM: Situation in which hindered from learning maths well: Draw, describe, and identify most hindering factors (ranking) |

| 7.1 | ~ | | | HF | Scenario: Child having difficulty, how would you help? |
|------|---|---|---|-------|--|
| 7.2 | • | | ~ | HF | Video clips - Describe: Would you learn maths well in depicted situations? |
| 8.1 | ~ | | | HF | Easier and harder experiences in learning maths: Perceived effectiveness of learning and factors influencing the learning are discussed 8.1.1 Easier 8.1.2 Harder |
| 8.2 | • | | ~ | HF | Describe and sort photographs (home situations and school situations): Learn maths well? |
| 8.3 | ~ | | | S | Discussion - how feel about learning maths |
| 9.1 | ~ | ✓ | | HF | I could do better in maths if |
| 9.2 | V | | ~ | HF | Visual vignettes - Children's drawings of classroom, home, and other out of school situations. Select situations in which you would: learn maths well; be hindered from learning maths well. Discuss |
| 10.1 | ~ | ~ | | HF | Written descriptors. Phrases/words provided, sorted by child and ranked (Stodolsky et al., 1991). Discussion of use of these in learning maths |
| 10.2 | ~ | | ~ | HF | 10.2.1 Duplo: Construct/build situation in which you felt good when learning maths |
| | | | | | 10.2.2 Duplo: Construct/build situation in which you did not feel good when learning maths |
| 10.3 | ~ | | | L | Learn: What does this mean to you? Discussion |
| 11.1 | ~ | | | S | How good at maths: 1 - 10 |
| 11.2 | ~ | ✓ | | Μ | I think mathematics is |
| 11.3 | ~ | ~ | | L, HF | A good maths teacher |

Creative and varied interview procedures

Ten objectives for development of procedures are listed above. Following recognition of the need to incorporate activities and ideas of interest to children of eight to nine years of age, to accommodate attention spans of the children, and to gain insights into how the children see the world, it became clear that a range of procedure types would be required as stimuli for discussion. From experience with children of eight to nine years of age, it was concluded that it would be inappropriate to expect the research participants simply to make statements of belief. Likewise, merely talking about experiences would be challenging and perhaps of little interest to young children. One tool used by previous researchers to maintain children's interest was to alternate questions about beliefs and mathematical problems (e.g, Kloosterman & Coogan, 1994). This approach was not deployed in the present study as it would have brought in elements that did not address the research questions.

The decision was made to use creative interviewing (Patton, 1990, p. 340), rather than just asking questions, as the strategy of investigation. A variety of media were incorporated into the presentation of questions and the form of student response making the tasks "especially effective for interviewing children" (Patton, 1990, p. 341). For example, children reacted to stimuli that included photographs, drawings, video footage, and sets of written probes, such as the words and phrases used in Task 10.1. A small amount of writing was included within the interviews and children responded at times by drawing or constructing with materials. Contexts for response or comment were a feature of some tasks, provided through reference to the children's own experiences or through the posing of scenarios of the experiences of others. In the present research the range of tasks and response types created interest for the young participants and facilitated the investigation of ideas from a number of different perspectives thus increasing the depth and validity of the research findings. The qualitative methods deployed in this study allowed the researcher to delve into students' thinking about the nature of mathematics and how learning occurs (Stodolsky et al., 1991).

The variety within the tasks and the reference to previous research for task development are discussed below. As the development of creative interviewing strategies was a major feature of the research, the reader is introduced to the tasks through a discussion broken into sections according to the main feature of the task, that is, visual, verbal or text-based. The manner in which the reasoning for, and development of, data collection procedures was informed by previous research is discussed also.

Visual tasks

Visual refers to any task that had a visual element as its main component for either researcher use or student response. Visual includes children making drawings, children responding to drawings, to photographs, to video clips, and children expressing their perceptions through construction using Duplo materials. Obviously, these interview tasks contained a verbal component also, as discussion was an element, but visual means were the primary way by which tasks were introduced or responded to.

As indicated above, in the discussion of objectives for consideration in developing the interview data collection procedures, the age of the children was one important factor to take into account. It appeared, from previous studies, that use of drawings would be appropriate for children of eight to nine years of age. For example, the work of Muscella (1986), indicated that even younger children could respond to drawings. Muscella (1986) developed a perception and preference classroom environment instrument that she used with kindergarten pupils who were asked to select favourite learning situations from representations given in adult-drawn sketches of the actual class situations. Muscella's instrument, administered in an interview situation, facilitated expression of ideas by children younger than those who are able to respond to many written questionnaires, a common form of instrument in learning environment research (e.g., Fraser, 1994).

The approach of using already-drawn representations was adapted for the present study: drawings previously completed by other children were used as the stimulus for discussion in Task 9.2. Children were asked for their interpretation of what the child who drew the picture was showing that helped him or her to learn maths well, and then were asked whether they thought that factor would help themselves to learn maths. Whereas with older children written vignettes are a way to illustrate a range of scenarios or experiences about which to seek comment, in this case drawings were used.

Drawing and description as a mode of expression by the research participants was another approach used in this study. Previously this approach was used successfully with six to twelve year olds in a family counselling context (Habenicht, Shaw, & Brantley, 1990) and with children from Preparatory level (age 5) to Grade 6 (ages 11-12) in an education context (McDonough & Wallbridge, 1994). The decision to use the drawing and description method for data collection in an interview situation in the present study was influenced particularly by this previous research by the author, but with adaptations made.

As mentioned in Chapter 1, McDonough and Wallbridge (1994) deployed the drawing and written questionnaire response version of PPELEM. The instrument was administered by class teachers to 1816 Grade Prep to Grade 6 children from two countries. The researchers analysed the end products using quantitative procedures, and identified and discussed results for groups based on gender, grade level, class type and country. The approach was of value for an overall perspective and for group perspectives. However, when considering the implications it became apparent that, because of the large numbers involved and the form of analysis, the study did not inform the researcher of individual children's perspectives or perceived needs. The study provided detail neither of any possible network of beliefs held by individuals, nor of personal reasoning for development of beliefs. It is possible also that the limited utterances that could be given, due to the use of only one procedure, may have provided distorted or incomplete data (Gellert, 2001). A further limitation was that the study was based on assumptions of children's perceptions of mathematics and learning, assumptions that may or may not have been correct.

It became apparent that for further research to make sense of children's beliefs about helping factors for learning mathematics, investigation of children's beliefs about the nature of mathematics and learning would be necessary. In addition, more extensive research of individuals' perceptions would provide greater insight into possible complexities and relationships within individuals' networks of beliefs.

As one element of the investigation of individuals' beliefs about helping factors for learning mathematics, the present research extended previous PPELEM research through the development and use of an interview version of PPELEM (Tasks 6.1 and 6.2). Drawings allowed the children to formulate responses through a familiar form of expression, before being asked to detail verbally the elements of the chosen situation. The one-to-one interview allowed for the drawing to act as a stimulus for discussion and therefore facilitated more

detailed responses than probable from young children when written expression only, such as with most questionnaires, occurs. It gave the opportunity for clarification and follow-up questions that were stimulated by an individual child's response. The child also had the opportunity to spontaneously extend ideas.

Drawings were used also, with similar reasoning as discussed above, in Tasks 3.1, 4.2.1, 4.2.2 and as an optional form of communication in Task 3.2 where children could draw, act out or write their response as preferred.

A further form of visual representation, iconographic, occurred in Tasks 10.2.1 and 10.2.2 in which Duplo was provided for the children to construct situations in which they felt good and did not feel good when learning maths. These tasks were approached through the construct of feelings, as a way of possibly moving towards gaining further insights into perceived helping factors for learning mathematics.

Photographs also provided a form of visual stimuli within data collection tasks. For example, in Task 5.2 children were shown a series of 20 photographs and asked to describe each photograph and decide whether there was any mathematics in the activity depicted. This followed the work of Zevenbergen and Crowe (1992) in which the perceptions of 51 upper primary school children were investigated. Although some of the original photographs were replaced, the collection for the present study encompassed the three options identified by Zevenbergen and Crowe:

- i. a school mathematical context (e.g., teacher at the front addressing a class, an activity using MAB blocks)
- ii. a non-school mathematical context (e.g., shopping, weighing fruit)
- iii. a non-mathematical, or ambiguous context (e.g., an outdoor scene showing two people looking at flowers)

The photographs provided a focus and stimulus for discussion, but, like all approaches used in the present study, were discussed in the context of the children's interpretations of what was happening; the researcher did not assume that the children and the researcher saw the same things in each photograph. This task, along with many others designed to gain insights into beliefs about mathematical activity, allowed for the possibility of non-school as well as school experiences to be identified as mathematical.

In Task 8.1 the children were shown a series of photographs of potential mathematics learning situations in home and school settings. They were asked to tell what was happening in each photograph and then decide whether they thought they would learn maths well in that situation. The children sorted the photographs into *Yes*, *No*, and *Not Sure* piles. Photographs in the *Yes* pile were discussed with the children telling what in that situation would most help them to learn maths well. As comprehension or interpretation was not dependent on reading skills, the Task 8.1 photographs were a simple and effective way of presenting a range of scenarios to the research participants.

Similarly, Task 7.2 presented school situations with the potential for mathematics learning shown in video format. Use of the video clips was based on the same reasoning as Task 8.1 but, because of the active nature of the image, gave more likelihood that group interaction would arise for consideration as a possible helping factor for learning mathematics.

As demonstrated above, tasks based on a visual component included the use of drawings, photographs, video clips, and manipulatives in the form of Duplo. These gave the opportunity for children to portray beliefs about mathematics, learning, and helping factors for learning mathematics on many occasions and though a range of familiar media. Where the interviewer presented photographs, drawings or video clips, the children were asked for their interpretation of the situation prior to further discussion. As the research was an investigation of individual children's beliefs, the possibility of differing interpretations was not of concern, as is generally the case in research using standardised procedures (Ginsburg, 1997). Indeed, idiosyncrasies added richness to the study. The use of tasks with a visual component contributes to the confidence one can hold in the research results because of the use of the multiple data sources, the opportunity for clarification of children's meanings and beliefs, and the triangulation of data.

Verbal tasks

Verbal exchange occurred in all thirty tasks used in the interviews, but in some it was the key data collection feature. Those tasks are discussed within this section of the thesis.

In some tasks verbal responses replaced the usual written responses. For example, this occurred where questionnaires, based on the work of other researchers, were deployed in Tasks 3.3.1 and 3.3.2, with all items presented verbally by the researcher. Children's responses were verbal, with any written record made by the researcher backed up by an audio-taped record of the interview.

Task 3.3.1 was informed by McDonald and Kouba's (1986) use of a questionnaire as outlined here. In McDonald and Kouba's study, primary aged children were asked whether they believed a person was doing or using mathematics in a range of situations. To enable ease of administration to a large number of children and to avoid reading difficulties, the questionnaire was read item by item by each teacher to their class and children responded by circling *Yes* or *No* on a response sheet. After completion of the questionnaire a discussion was held in which children's rationales were explored. Although kindergarten and first grade children in McDonald & Kouba's study appeared often to not know what was going on and, thus, to randomly choose Yes or No responses, the research was believed to be successful with older children as steadily increasing patterns in responses were found.

The form of administration taken by McDonald and Kouba did not facilitate clear identification of beliefs of individual children but this was possible in the one-to-one situation of the present research. The administration of Task 3.3.1 was an adaptation of the approach of

McDonald and Kouba: in the present research items were read to the individual children, *Yes* or *No* responses were made verbally, and discussion of items followed.

The Task 3.3.2 items of the present research drew on the work of Wallbridge and Clarke (1989), who gave a written questionnaire of perceptions of mathematical activity to a range of groups including Year 8 students. The respondents were posed with everyday situations and asked to rate how mathematical they believed each activity to be. A similar approach was used in the present research but items were read out loud by the researcher. The children were asked whether they believed each activity had *Lots of maths, Some maths,* or *No maths* in it and discussion followed to elicit reasons behind the responses. The personal meaning of each item was discussed prior to the children explaining their perception of mathematical content. The items concerned the same or related activities as those used by Wallbridge and Clarke, for example, the item "Playing a musical instrument" was taken directly but "Using a calculator to work out the interest paid on a housing loan over 20 years" was changed to "Using a calculator to work out the money to pay the bank" and "Driving a car" was changed to "Travelling to school". Changes were made to wording so that items would be understood by, and have meaning for, the younger children.

Another form of written document is the dictionary. Tasks 2.1 and 2.3, in which children were asked to give personal dictionary definitions *verbally*, drew on experiences of the use of dictionaries, enabled children to reflect on their own personal meanings for words, but avoided any writing by the children. The idea of giving a definition was pursued firstly for common everyday words before the terms Maths and Learning were introduced. Task 2.2, which asked for a verbal description of a learning situation, enabled children to expand upon the ideas given when defining learning in Task 2.1.

A fantasy scenario provided the stimulus for discussion of the meaning of maths in Task 5.1. Following from the work of Stodolsky et al. (1991), the children were posed with the situation that an alien, who knew nothing about earth, had landed in their suburb and that the child's role was to tell the alien what maths is.

Three tasks which sought to gain insights into factors perceived to help in the learning of mathematics and which used verbal interaction only were Tasks 7.1, 8.1 and 8.3. Through discussion of easier and harder learning experiences, of how the child felt about learning maths, and of what help he or she would give to another child experiencing difficulty in learning maths, the conversation moved towards and gave insights into beliefs about helping factors for learning mathematics.

Task 11.1, where children were asked to rate how good they believed they were at maths, on a scale of one to ten, was another task that used verbal interaction only. This provided background information; it provided further insights into beliefs about mathematics and the child's context in learning mathematics.

Although all tasks developed or identified for the present study involved some verbal component, in some tasks this was the key feature as discussed above. In some cases verbal

responses replaced what would be more common written responses. This change enabled young children to respond more easily to tasks that, in a written format may have caused problems. The research therefore had more likelihood of gaining insights into and portraying the children's beliefs.

Text-based tasks

Text-based tasks are those where text was created by the children through writing and those where the children were asked to read, discuss and sometimes sort words or phrases, either transcribed from their verbal descriptions or provided by the researcher.

Children wrote responses in some tasks. In Tasks 1.3.1 and 1.3.2, word wheels (Lewis & Davies, 1988) were used as a strategy to facilitate briefly written but multiple responses. In Tasks 4.1, 9.1, 11.2, and 11.3 sentence starters stimulated thinking and encouraged elaboration. The written responses were not seen as end products but provided concretely expressed ideas that could be explored and elaborated further through verbal interaction. In Task 4.3, Planning for an integrated unit of work, children reflected on school learning situations and their potential for integrating mathematics with other curriculum areas. They recorded their key ideas on paper. Responses had the potential to give insights into children's beliefs about the relevance and meaning of mathematics.

Text was a key feature, along with drawing, in Tasks 6.1 and 6.2 which used the interview version of PPELEM. Words or phrases from within children's verbal descriptions of their pictures that related to helping or hindering factors for learning maths, were recorded on cards by the researcher and then sorted by the children according to perceived degree of impact on learning maths. In Task 10.1 phrases and words provided by the researcher were grouped and ranked by the children, building on the work of Stodolsky et al. (1991). Elaboration and justification were possible elements of the accompanying discussion. Words written on cards provided a stimulus for discussion of children's meanings for learning in Task 10.3.

To summarise, the above discussion provided an overview of the tasks used in the research interviews and illustrated how the development and use of the tasks were influenced by other research. It demonstrated also that the objectives listed for the development of data collection procedures were addressed. For example, tasks explored meanings for mathematics and learning and facilitated the gaining of insights into beliefs about factors that help in the learning of mathematics. The range of task presentation and response types enabled beliefs about each construct to be explored from a number of perspectives and on multiple occasions, thus giving greater credibility to the research findings.

The discussion turns now to the process of collection of interview data.

Collection of interview data

A range of data was collected in this research including verbal, drawn and written interview data from the individual children, and observations during the interviews. The collection of data in the whole class situation in written and drawn forms, the observations of class lessons,

and the interviews with the two class teachers are discussed later in this chapter as they were supplementary for the research. The discussion of the collection of interview data is broken up into the following sections:

- organisation for interviews;
- use of the interview procedures;
- guiding principles for interview questioning.

Organisation for interviews

The children were each interviewed on ten occasions over a five month period. Each school was visited approximately once every two weeks, with visits alternating between schools. An interview lasted approximately half an hour; during a morning's visit to a school four interviews were conducted, usually with each using a different set of procedures. The order of use of procedure sets varied from child to child.

In preparation for the administration of procedures, questions were noted (Appendix A) and materials were collected through reference to a prepared checklist (Appendix D). Interviews were conducted in a room separate from the classroom, usually with only the child and researcher present.

Use of the interview procedures

The administration of the interview tasks involved in-depth, intensive interviewing, "the major way in which qualitative researchers seek to understand the perceptions, feelings, and knowledge of people" (Patton, 1990, p. 25). The study can be described as qualitative in nature, not only because of its interest in people's beliefs and meanings, but also because the full details of questioning were not completely specified in advance of the fieldwork (Patton, 1990). The key questions and tasks for the data collection procedures were developed and decided upon before data collection began but the openness of these questions and tasks led to the possibility of differing follow-up questions for each child, depending upon each individual's initial responses. Only minor changes were made to the overall structural framework as the study progressed. The study design provided some opportunity for unfolding during the data collection but less than appears to be the case in much qualitative research (e.g., Patton, 1990). One set of procedures, Set 11, was added after data collection commenced.

The interviews were semi-structured in nature (Burns, 1997), that is, the study did not prescribe fixed wording throughout the interviews and the order of procedures varied from child to child. The interviews used creative data collection tasks (Patton, 1990) that provided a framework for the children's responses; they directed responses to the idea about which we were talking but they allowed the children to portray their own views about the world with accuracy and thoroughness (Patton, 1990). Follow-up questions were developed or improvised (Ginsburg, 1997) keeping in mind the issue or idea under consideration but wording was not predetermined; questioning and discussion followed the children's paths of thought in relation to the issue or idea. Responsiveness to the interviewees was a key element

of the research. The semi-structured interviews provided greater flexibility than closed tasks and permitted "a more valid response from the informant's perception of reality" (Burns, 1997, p. 330); the open-ended questions provided a "forum for elaborations, explanation, meanings, and new ideas" (Patton, 1987, p. 11).

Guiding principles for interview questioning

In developing the interviews a number of guiding principles for action were taken into account. The discussion of the principles includes

- giving explanations to children;
- posing clear and understandable questions;
- taking care with *why* questions;
- using a variety of question types;
- accepting and valuing children's responses; and
- seeking elaboration, clarification or further information.

Giving explanations to children

During the course of the interviews a variety of explanations were given to the children as it was important that informants had some idea of the purpose and direction of conversations (Spradley, 1979). These explanations included a project explanation, a recording explanation, interview explanations, and question or task explanations (Spradley, 1979).

At the first interview the project was explained. I told each child that I was a teacher from the university and that I was wanting to learn from children so I could be a better teacher and I could help other teachers. I said that I was interested in finding out about what they each thought, that I was interested in their own ideas.

A recording explanation, was given also during the first interview. I explained that I would be using a tape recorder and perhaps writing a few notes, so I would not forget what they said and could go over these later. At the beginning of each interview we tested the tape recorder so the children could hear their voices and so I could be sure the equipment was operating correctly.

Interview explanations included statements such as "We will be doing three different things in today's interview", to help the children know what to expect.

Question or task explanations helped the children to know what was happening in a task. For example, David was told at the beginning of Task 4.3 that "This one is different" because the task used a structure completely different from any used up to the time. A further example is found in Task 6.1: "Emily, I asked you to draw a picture where you were learning maths well. What I'm going to do is get you to describe that to me and while you're describing it to me I'm going to write down some words. Okay, so tell me about your picture".

Each of these explanation types contributed to the children being informed and assisted them to respond.

Posing clear and understandable questions

One principle, that questions are understandable, that is, that the interviewee is clear about what is being asked (Patton, 1990), was particularly relevant in this research with young children. Strategies to address this principle included providing the opportunity for children to become familiar with the structure of a question or task before introducing elements related directly to the research questions. This is demonstrated in Tasks 1.1 and 1.2, two different word association tasks, where a range of words, potentially familiar to the children and simple to respond to, were introduced before interviewees were asked about *maths* and *learning*.

This principle influenced the decision to use the more common term *maths* with the children, rather than *mathematics*. Justification for this decision emerged in Cara's second interview when she was asked to draw a mathematical activity. She questioned my instruction by asking "mathematics?". Assuming Cara equated maths and mathematics, I responded in the affirmative. Cara then drew a situation that appeared to involve gymnastics rather than mathematics. This event is explained in more detail in the discussion of Cara's data, but at this point illustrates clearly the importance of the interviewee being clear about what is asked. This event reinforced this principle for the interviewer and gave the message that one should never assume children's meanings.

Taking care with "why" questions

A further principle, suggested by Patton (1990), relates to taking care about asking *why* questions.

Questions such as *why* and *what do you mean* were used minimally in the present study because they may suggest that the informant is not clear (Spradley, 1979) or that the response is inappropriate (Patton, 1990). Patton states also that these questions "move beyond what has happened, what one has experienced, how one feels, what one opines, and what one knows to the making of analytical and deductive inferences" (1990, p. 313).

The study was informed by this principle that applies to ethnographic research; *why* and *what do you mean* questions are absent from ethnographic interviewing (Spradley, 1979). Where possible in the present research, the asking of such questions was replaced by a broader range of question types.

Using a variety of question types

The research deployed a variety of question types that included asking for use questions, funnelling questions, descriptive questions, role play and simulation style questions, structural questions, and contrast questions.

As an alternative to asking *why* questions, Spradley (1979) recommends *asking for use* questions. This approach, as utilised in the present research, is illustrated in the following examples:

• Task 2.2 "Tell me something you have learnt recently, say, in the last few weeks". In seeking insights into children's meaning for the term learning, they were first asked for a

definition of learning that they would put in their personal dictionary, but were then asked to tell about a learning experience. Learning was considered in both an abstract and concrete way. This approach used a *funnelling* technique, that is, it commenced with a broad, general question and progressively focused onto the topic with more specific questions (Burns, 1997).

- Task 6.1 "Tell me about a time when you were learning maths well". Children were asked to describe through drawing and verbal communication one instance of maths learning.
- Task 8.1.1 "Tell me about a time when maths was easy for you to learn". Once again, a particular experience, of the child's choice, was to be described.

The above are categorised also as *descriptive* questions, that is, they ask the children to talk about an event or experience. This approach was favoured in the study as it "is like offering informants a frame and canvas and asking them to paint a word-picture of their experience" (Spradley, 1979, p. 85). As evident in the discussion of data for each of the children, the word-pictures contribute in a major way to the portrayal of beliefs as was the intention of this research.

Role play and *simulation* style questions (Patton, 1990), were helpful in providing a context for the children to respond to questions. For example, the following tasks asked the children to respond as if they were someone else or to respond from someone else's point of view:

- Task 4.2 "Draw a picture of someone using or doing maths". The children drew their picture and then described what the person was doing.
- Task 3.3.2 "Who is someone else who lives in your house? Would also say this has (some/no/lots of) maths in it?" The children were given an opportunity to demonstrate their awareness of an alternative perspective or viewpoint.
- Task 4.1 "Maths is like". "What about someone else, would they write something different?" This was a spontaneous question asked of Cara following her initial response to the task.

Structural questions (Spradley, 1979), were used in the collection of data. For example, the word wheels in Tasks 1.3 provided an opportunity for children to portray information about a particular construct, that is, to show how they had organised their ideas.

Contrast questions (Spradley, 1979), were a feature of the research also. For example, although one intention was to gain insights into children's beliefs about factors that helped them to learn mathematics, Task 6.2 asked for a situation in which something was stopping them or making it hard for them to learn maths well. The research looked for difference to help in the identification of the dimensions of the children's meaning for helping factors.

The use of a range of questions contributed to the research opportunities to gain insights into young children's beliefs. A further important element of the research was to accept children's responses to the questions and tasks posed.

Accepting and valuing children's responses

As stated earlier, efforts were made by the researcher to be neutral to the statements made by the children; it was important that the children felt their responses were not being judged. Likewise, it was important that the children felt their responses were accepted and valued. Strategies used to convey these messages included listening with interest to the children, using eye contact during the interviews, and using minimal encouragers such as "Mm", "Yes", "Right" and non-verbal forms of communication such as a nod of the head (Burns, 1997; Patton, 1990; Spradley, 1979).

Seeking elaboration, clarification or further information

As a major aspect of the research was gaining insights into children's perspectives, strategies were employed to encourage children to articulate these. For example, the following prompts were among those used to invite children to elaborate upon their responses:

"Tell me about that"

"Please give me an example of"

"Please tell me a little bit more"

Repeating the final few words spoken by a child was a further strategy employed to encourage elaboration. This parroting or mirroring technique can be an effective way of keeping an informant conversing (Burns, 1997).

Probing within follow up to children's responses included questions beginning with *who, where, what, when,* and *how* as, for example, listed in Appendix A for Task 4.2.1. The question "How was that maths when?" also demonstrates this probing approach taken when seeking clarification of meaning.

During the interviews, opportunities were taken to summarise and check out any inconsistencies (Burns, 1997). For example, after writing words and phrases on cards in Task 6.1, the researcher asked Anna "Was there anything else I had not written down or that you had forgotten?".

As described earlier in this chapter, the tasks were designed to interest young children and to enable them to respond in a meaningful way. The questioning was a key element within the initial tasks as posed but also within follow-up discussion that sought further elaboration and clarification. The principles discussed here assisted in the gaining of insights into the eight children's beliefs and therefore increased the likelihood that children's perceptions, rather than the researcher's perceptions, would be portrayed in the research.

Analysis and presentation of interview data

As described earlier, the thirty tasks developed for the research resulted in verbal data from the children as well as drawn and written products, stimulated by a range of approaches including photographs, learning scenarios (presented through other children's drawings and video clips), sentence starters, phrases and words, game simulations, and questionnaires posed verbally. The creative style of interviewing differed from traditional interviews, as discussed above.

The analysis and presentation of the interview data took into account the nature of the thirty tasks used in the study and the nature of the resulting data. Thorough analytic and presentation procedures were followed. The discussion of the analysis and presentation of data is organised in the following sections:

- portrayals of children's beliefs;
- thematic approach to data analysis;
- criss-cross analytic approach;
- incorporation of interview excerpts;
- a focus on two children's beliefs.

Portrayals of children's beliefs

The research report provides individual portrayals of children's realities or perspectives about mathematics and learning; each portrayal was developed from drawn, written and verbal data collected primarily from responses to thirty procedures used during ten interviews. The children played the initial, and the key role, in the production of the portrayals; thus the title of this report attributes the creation of the portrayals to the children. The researcher carried out the subsequent stages of the composition of the portrayals by working with the material provided by the children to compose the portrayals into cohesive and communicable reports that incorporated and reflected the children's responses. As stated by Stake,

Even though committed to empathy and multiple realities, it is the researcher who decides what is the case's own story, or at least what of the case's story he or she will report. . . . It may be the case's own story, but it is the researcher's dressing of the case's own story [with] the aim of finding the story that best represents the case. (Stake (1994, p. 240)

Thus the present report presents the researcher's dressing of a story provided by the children. The children were the key contributors to the portrayals.

A portrayal may vary at different times. In a study such as the present one, a portrayal is dependent on the information provided by the research participants and is to some degree open to the interpretation of the researcher/composer. The word portrayal is used deliberately to describe the interpretation and presentation of data for two reasons. First, the children chose their personal responses to the procedures on the occasions of the interviews; on another occasion they may have provided differing responses. Therefore the portrayals provide insights rather than one true reality. Second, in the following stage of the portrayal construction, the researcher worked through inference, thus the portrayals are an interpretation of the data collected. As discussed earlier, measures were taken to maximise the match of researcher and reader interpretations of data. Thus, hopefully the portrayals will be interpreted as intended.

As an intention of the research was to create portrayals of young children's beliefs about mathematics and learning, it was important that analysis of data was based on the children's

experiences and perceptions rather than those of the researcher (Spradley, 1979). Along with other measures discussed earlier to increase the validity of the research, a theme based analysis of data increases the reader's confidence in the accuracy of the portrayals from the data collected.

Thematic approach to data analysis

Due to the size, complexity and subtlety of the data collected, the research could not report all responses but sought to identify from the data key aspects of each child's beliefs. To do this, themes were drawn from the data (van Manen, 1990) to provide relevant and significant insights.

Within each case, data were sorted into groups to enable themes to be identified. As described below, the same beginning stage of data analysis, that is, the creation of categories, was used for each child's data. The more detailed data handling occurred with the assistance of a computer program for Cara, the first child whose data were analysed, but manual handling followed for the other children. This was not perceived to impact negatively upon the management of the data. Although differing tools were used for part of the data sorting, a category and theme-based analysis was undertaken in each case and a common philosophy and general approach underpinned the handling of each child's data.

Three common overall categories or domains, that is, mathematics, learning, and helping factors for learning mathematics, were created for each child's data as these addressed the research questions. However, identical or common sub-groupings of data for each of the eight children were considered but rejected. Such an approach could have imposed inappropriate and irrelevant categories on children's data and could have given precedence to issues that may not have warranted focus for some children. Such misrepresentation of individual children's perspectives would have been in contradiction to the underlying principles and intentions of the research. As it is important that a coding system relates to the theoretical framework or research questions (Burns, 1997), subcategories of data within the overall groupings of mathematics, learning, and helping factors for learning mathematics were created for each child in response to their individual data.

Cara's data were chosen as the first to be examined because of the researcher perception that hers were the most complex data of the eight participants. Beginning by reading and rereading Cara's transcripts, the process commenced of grouping together ideas identified within her responses (Burns, 1997). Thus, the first stage recommended for use of the qualitative data handling computer program, NUD•IST (Non-numerical Unstructured Data Indexing Searching and Theorizing) (Richards & Richards, 1990), was applied and Cara's data were organised into manageable categories. In accordance with the design of NUD•IST, an index tree structure made up of categories and subcategories was created. This was recorded on paper.

Diagrammatically an index tree resembles the roots of a tree, with main roots stemming out to smaller ones. Index trees are made up of key elements or categories called nodes, and sub-elements or sub-categories also called nodes. As each root branches out to smaller roots the classification of data becomes more refined. The numbering system of nodes reflects this. For example, node 5.1 is broken into 5.1.1, 5.1.2, 5.1.3 etcetera. In turn, node 5.1.1 is broken into 5.1.1.1, 5.1.2, and 5.1.1.3. Likewise, node 5.1.3 is broken into 5.1.3.1, 5.1.3.2, 5.1.3.3, 5.1.3.4 etcetera. Branching continues until data are classified into the finest categories that emerge from those data. Each of the most refined nodes, or categories of data, can be traced back its key node or category.

Four index trees were created for Cara, one for each of the concepts of mathematics, learning, and helping factors for learning mathematics as these were the key ideas within the research questions, and one tree on background or contextual factors, focusing on factors such as perceived ability in mathematics. Each of the other children's data were classified initially according to the same four categories, but then broken down into themes which differed for each child in response to the trends identified within their data. Variation across the children's sub-nodes or sub-groups of categorisation occurred where appropriate, enabling the research analysis to reflect and build on their individual experiences and perspectives. Insights into Cara's beliefs evolved from the creation of NUD•IST index trees. For the other children, categories were created manually but these also were in response to ideas identified within each child's responses and therefore reflected and built upon their experiences and perspectives.

Cara's word-processed transcripts were entered into the NUD•IST program resulting in each interview transcript being broken up into numbered text passages. In the present study, a number was assigned each time the interviewer or interviewee commenced talking, that is, each time the speaker changed. Through reading and re-reading, the text passages were then sorted into common ideas or themes as listed on the index trees, and assigned a number or number from the nodes of the index trees. This system allowed one text passage to contribute to the development of more than one theme.

For Cara's data, lists of relevant text passage numbers were entered into the computer and print outs were produced of the selected text passages according to each theme. However, it was found that additional brief handwritten summaries of the interview excerpts according to themes, available for immediate reference, improved the researcher's ability to work through the complex data and develop a coherent written report of the issues and themes that emerged from the responses.

As the same form of summaries could be manually created for the other children, whose data were not as complex, and reference to full print outs of interview transcripts could be made, the decision was made not to use NUD•IST. The summaries were made using Task numbers (for example, Task 3.3.1) for quick reference rather than numbered text passages. The numbering allowed easy cross-reference between the interview transcripts and the brief handwritten summaries of the interview excerpts that related to each child's themes. They allowed also for data to contribute easily to more than one theme for each child. Reading and

re-reading of interview transcripts and the use of handwritten summaries was common to the analysis of data for each child. Reflection on these summaries allowed categorisation to occur and themes to emerge for each child. The themes were different in all cases.

In summary, through the development of themes identified through the sorting and categorisation of data, meanings and concepts were developed from the children's verbal, drawn and written interview responses. Themes were identified under the central ideas of interest in the study but then varied from child to child according to the emerging ideas within the interview responses. Themes were not predetermined by the researcher. Work by the researcher began with a "naive ignorance", not preconceived ideas, and the informants, or research participants, defined what was important for the researcher to find out (Spradley, 1979, p. 29). "Depth, openness, and detail" were facilitated by the absence of predetermined categories of analysis (Patton, 1990, p. 13).

The theme-based organisation of data described here resulted in data that required careful and accurate reporting. The identification and reporting of themes within the present research was facilitated by a criss-cross analytic approach, discussed below.

Criss-cross analytic approach

The creation of index trees and summary notes facilitated the criss-cross analysis of data. Responses to any one task could contribute potentially to themes related to the domains of learning, mathematics, or helping factors for learning mathematics, even if the interview procedure was not developed with that element of the research as the focus. During the analytic process children's responses were not assumed to contribute necessarily only to one aspect of the research. Criss-cross analysis was planned to some degree, as demonstrated in the listing of focus concepts in Table 2, but eventuated as more extensive than anticipated, as discussed in Chapter 7 where it is pointed out that many tasks contributed to insights for multiple domains.

The possible contribution from various interview transcripts to any one theme within a child's beliefs is demonstrated, for example, in the discussion of Cara's beliefs (see Chapter 5), in which data are presented through the incorporation of excerpts from a range of procedures. The overlap in the expression of perspectives that occurred across tasks and across interviews added to the likelihood of the report presenting a comprehensive view of an individual child's beliefs and increased the credibility of the findings in this qualitative study.

Incorporation of interview excerpts

This inclusion of interview excerpts is a major feature of the presentation of data in this report. Excerpts were chosen and included within discussion of beliefs to

- exemplify themes identified from a child's responses;
- show the reader the bases upon which inferences were made by the researcher;
- show examples, and counter-examples where apparent, to build up a portrayal of a child's beliefs;
- demonstrate the detail of the data; and

• demonstrate the complexity and subtlety of beliefs held by the young research participants.

The inclusion of interview excerpts assists also in the discussion of the issue, as articulated in the first research question, of whether young children hold beliefs that can be articulated and portrayed through the data collection and analysis approaches used in this research.

Because of the number of themes within each child's portrayal, the complexity of beliefs, and the quantity of data required to illustrate those beliefs, the thesis reports in full detail on the beliefs of two of the children interviewed, and in briefer forms on the beliefs of the other six children.

A focus on two children's beliefs

The original plan, for full reporting of data from all eight participants in the research was not pursued as this became inappropriate in terms of size, space, complexity and significance. As is the nature of qualitative studies (Patton, 1990), the present study evolved during its progression, particularly at the stage of data analysis and interpretation where the detail available for each child made indepth reporting for all eight research participants an unrealistic outcome.

The study focused on indepth analysis and reporting of two children's beliefs, chosen not for representativeness in the traditional sense, but for the "opportunity to learn" (Stake, 1994, p. 243). Detailed findings from Cara and Emily are reported not for the purpose of direct comparison, but to illustrate in a more general sense the possible subtleties and differences in young children's beliefs. The briefer accounts of the other six children's beliefs add breadth to the study but not for the purpose of comparison leading to generalisations. Sufficient detail is provided for Cara and Emily for the reader to make comparisons beyond the case at hand (Stake, 1994).

The beliefs of three of the other children are presented in briefer accounts in Appendix E and summarised portrayals of the beliefs of all children, including the remaining three research participants, are presented in Chapter 7. This approach illustrates clearly

- the data upon which the research portrayals were based;
- the manner in which children's responses were drawn upon to make inferences and identify themes;
- the comprehensiveness of the approach taken in analysing and reporting data;
- the nature and complexity of themes that emerged from interview data;
- the idiosyncratic nature of children's beliefs; and
- the insights gained into the beliefs of the eight children.

It was considered unnecessary to present the reader with full accounts of the insights gained into all eight children's beliefs. The summary tables (see Chapter 7, Tables 4 to 11), present the breadth of the insights gained, the reports for Cara and Emily illustrate the complexity of the process of gaining and portraying insights, and the reports for Gina, Ben,

and David provide additional insights and evidence of the thoroughness of the analytic approach.

In summary of the discussion of the analysis and presentation of interview data, the research provides portrayals of children's beliefs, data were sorted to identify themes that emerged from each child's responses, criss-cross analysis enabled data for each theme to be gained potentially from responses to any procedure, themes were explored, exemplified, and justified for the reader through the inclusion of interview transcript excerpts, and the discussion of findings focused on two children's beliefs while also providing an overview of beliefs for the other six children.

The above discussion detailed the development and use of the interview procedures, and the analytic and presentation approach. It is followed now by a brief discussion of the supplementary data collection procedures that added background context to the reporting of interview data, as mentioned earlier.

Development, use, and analysis of supplementary data collection procedures The interview tasks and data, as described above, were supplemented by data collected from the eight research participants when in the whole class situation, by observations of class lessons, and by interview data from the teachers. These data were not intended as the main sources of insights but provided a contextual framework and alternative view points from which to build a picture of the children's beliefs. Insights gained from these supplementary data are discussed mainly Chapter 4, where the research participants are introduced to the reader, and in Chapter 7 where there is some focused discussion of the eight children's responses to the pencil and paper tasks completed in the whole class situation.

The discussion below elaborates in turn each of these forms of supplementary data collection.

Whole class data collection

The whole class data collection consisted of two types: i) whole class tasks involving the collection of data on paper from each child in each of the two classes from which the eight interviewees were members, and ii) the observation of three lessons in each of the two classes.

Whole class tasks

Two tasks used in the interviews were adapted for whole class use so as to collect additional data from each of the eight children. Responses might have confirmed or disconfirmed emerging themes from interview data and might have provided new insights. These tasks, a questionnaire adaptation of the interview drawing and description tool, PPELEM (Task 6.1), and an adaptation of Task 5.1 (telling an alien about maths), were administered towards the end of the data collection period.

For the questionnaire version of the PPELEM task, each child in the class was given a plain piece of paper on which they were asked to draw a picture of a time when they were learning maths well. Upon completion of the drawings, questionnaire sheets were distributed.

The questions were read through and children filled in their own responses. Details of the procedure are provided in Appendix F.

The whole class *alien* task involved the same fantasy situation presented in interview task 5.1. The children were told that an alien had come to their suburb and, after brief discussion of what the alien might look like, the children were asked to pretend that the alien had arrived on earth and did not know what was going on. Each child was asked to tell the alien what maths is, by writing a letter to the alien.

Analysis of responses to these tasks was determined by the purpose of their administration, that is, to provide further insights into the perspectives of the eight research participants. It was their data, provided in a setting different from the interviews, created in response to slightly different prompts, and given on a different occasion from the interviews, that were of interest to the researcher. The *alien* task was designed to provide insights into beliefs about the nature of mathematics, and the PPELEM task to provide insights into perceived helping factors for learning mathematics. Responses are drawn upon when relevant in the discussion of each child's themes.

Lesson observations

Three mathematics lessons were observed in each class, one where the teacher taught a lesson developed from an idea provided by the researcher and two where the teacher taught her normal mathematics lessons as planned at that time.

The observed lesson taught by the class teacher for which the task was chosen by the researcher followed the work of Mousley (1993b). The teachers were asked to conduct a lesson in which children were given the task "From a given piece of cardboard, make a regular shape which holds one cup of birdseed. Make a similar shape which is twice as big" (Mousley, 1993b, p. 127). Teachers were not given further directions for the lesson. As the lesson could be conducted in a number of different ways, with different degrees of child or teacher direction, the lesson gave the opportunity to observe teacher and student decision-making and processes. Pasta was provided in lieu of birdseed. In this document this lesson is referred to as the *Pasta lesson*.

When observing lessons, field notes were made by the researcher on aspects of the lessons such as the tasks used, lesson procedures, teacher role in discourse, student role in discourse, organisational structures, use of materials, and reflection by students. The teachers were interviewed prior to each lesson to discuss their purpose and planned procedure and, after the lesson, were asked to reflect on the actual happenings and the degree to which they were consistent with their usual style of mathematics class. These lesson observations and discussions with the teachers provided insights into the eight children's experiences of mathematics learning at school and contributed mainly to the discussion of background in Chapter 4.

Teacher interviews

The two teachers each spoke to the researcher for approximately one hour, describing their mathematics lessons and providing insights into the beliefs that underpinned their teaching. The interviews focused on beliefs about the nature of mathematics, beliefs about the nature of learning, and the teachers' perceptions of the *big ideas* in mathematics for Grade 3 (see Appendix G). The teachers talked also about organisational factors related to their mathematics teaching. The results contributed to discussion of background in Chapter 4, and informed discussion of children's beliefs in Chapters 5, 6, and 7 and Appendix E.

In summary, supplementary data collection procedures were developed and used to provide further background for the eight participants in the study. Data were collected from the children in whole class situations, observations were made of three mathematics lessons in each class, and a background interview was conducted with each of the class teachers.

Methodology: Drawing together the key elements

A model of the proposed relationship between the major elements of the research as presented in Figure 3 (see Chapter 1), and in more detail in Figure 4 (see Chapter 2), provided an overall structure to the research, but allowed for further development from the children's perspectives. Such development was in keeping with the theoretical underpinnings of the research. Key interest lay in gaining insights into children's perspectives or into how children see the world. Mathematics, learning, and helping factors for learning mathematics were identified as key structural elements for the research but no further structure was applied to the data analysis: categories were not pre-determined but themes emerged from each child's responses.

The data collection procedures discussed in this chapter were designed in accordance with a list of objectives, identified earlier, that took into account the purposes of the research, the theoretical underpinnings, and the needs and interests of the research participants. Openended questions within semi-structured interviews allowed for a broad range of possible beliefs to emerge. Further insights were gained through the use of a small number of supplementary data collection procedures.

The variety and open-ended nature of tasks and questions in the data collection facilitated expression by the children. Leading from this, the criss-cross nature of the analytic approach resulted in theme-based portrayals of individuals' constructions and orientations. It has been argued that measures undertaken within the research resulted in a credible and authentic report of the children's beliefs from which there is opportunity for the reader to learn.

The following chapter, Chapter 4, introduces the reader to the eight key research participants and explores the school situations in which they learned mathematics. The discussion contributes to an appreciation of some contextual factors that may have impacted upon the findings within the study.

CHAPTER FOUR SKETCHING THE BACKGROUND

Introduction

The child is a vital element in the learning process; indeed it seems that the child is the central element, with his or her involvement critical to the learning process. Children as mathematics learners may hold beliefs that may sit consciously or unconsciously in the background of their learning experiences. However, encouragement or opportunity rarely exists for young children to express such beliefs. Having young children voice their beliefs, by responding to procedures developed to aid their expression, allows the education community to listen to children and to build up awareness of children's perspectives of their own learning. The portrayal of children's beliefs provides an avenue for adults to see, know, and appreciate better some aspects of mathematical concepts and experiences as seen through the eyes of children.

The present research focused on accessing, identifying, analysing, reflecting upon and making available to interested members of the education community those views of eight young learners of mathematics that could be accessed. One interest in this research was what young children's views might be and whether they vary or are basically the same across the eight Grade 3 children. Of interest also were data that provide insights that go beyond the findings of previous research.

As stated earlier, it was not the intention to report all data gathered, but rather to identify from the data key elements within the children's beliefs. To do this, themes were drawn from the data to provide relevant and significant insights. Prior to discussing such themes, the reader is provided with some contextual background to the portrayals.

Introducing the children and some broader factors concerning their learning of maths Eight Grade 3 children from two Melbourne schools were involved in the present study. The main purpose of this chapter is to introduce those children to the reader. This enables the reader to develop some familiarity with and a mental picture of each child, and develop an appreciation of their individuality. Further insights become apparent also within later chapters. This chapter provides insights into factors such as the researcher's impressions of each child, the children's ages during the data collection, the children's perceptions of their own achievement in mathematics, the children's disposition during the interviews, and their attitudes towards mathematics. Information provided by the children in their interviews, by their teachers, and through researcher impressions is drawn upon. As further background to the presentation of data, a discussion of the perspectives of the children's mathematics teachers and of some features of the class mathematics program, as ascertained from the teachers, the children, and from a small number of lesson observations, is included also. As the lesson observations are discussed briefly only and contributed to the thesis in a minor way, further data and discussion are not included elsewhere in the thesis.

The schools

Children participating in this study came from two schools, referred to for the purposes of this report as School S and School I. School S is a middle suburban Melbourne Catholic primary school. Of the two Grade 3/4 composite classes, one class of 28 students was involved in the study. School I is an inner suburban Melbourne Catholic primary school of which the Grade 3 class of 24 children participated in this study.

The individual children

The main participants in the study were eight Grade 3 children, four from each of the two schools. The participants in the study attending School S are referred to by the pseudonyms of Anna, Ben, Cara, and David. Children referred to as Emily, Filip, Gina, and Harry were from School I. For key summary details of each child, please see Table 1.

Further information on each of the eight children is provided below. The descriptions in this chapter vary in length according to the data that were available and the insights gained by the researcher. Cara and Emily, the two children whose beliefs are discussed in detail in the main body of this report, are introduced in greatest detail below. The descriptions for Cara and Emily each conclude with a summary, but these are unnecessary for the other children as the discussions are briefer.

Where children's responses from interviews are quoted in the discussion of findings in this and the following chapters, the task or set used, or the number of the interview for that child, is made clear within the discussion. Task, set and interview numbers can be cross-checked by referring to Chapter 3 (Table 2) and Appendix A. The discussion begins by introducing the eight children. As explained earlier, the word *maths* is used when speaking of the procedures and responses as this term was used in the data collection. Otherwise the word *mathematics* is used with implication of the same meaning unless otherwise stated, for example, as when children's individual meanings for the two words are considered.

Anna

Anna was bright and friendly, but a quietly spoken child who communicated her ideas well in the interview situation. On each interview morning Anna was usually the last of the four children at School S to talk to me. She appeared settled and to enjoy the interviews. She patiently took the time she needed to think about her answers.

Anna was chosen by her teacher as a higher achiever in mathematics, one of the better achieving females at the Grade 3 level in a composite Grade 3/4 class. However, Anna believed she was "probably a five" on a scale of one to ten "because I'm not really good, 'cos my mum gives me the hard ones and I don't really remember and then the next day on the real test I'm not really good at it" (Task 11.1, Interview 6). At times she showed lack of confidence in her ability (e.g., Tasks 7.2, 11.1) although in the same interview but prior to responding to Task 11.1, when talking also of a test, said she "might get them all right" suggesting some confidence in her ability (Task 8.2). She reported also that the teacher called on herself or one or two others when she wanted a child to explain to the class (Task 7.2),

suggesting that she was called upon for her ability to complete the work correctly. Within Task 4.1 Anna stated that she found maths "sometimes very hard and sometimes very easy" but enjoyed it. On another occasion she stated that there were no times when she found maths difficult or hard to learn (Task 6.2). Anna indicated a desire to learn more difficult maths such as when she stated she wished to do more "pluses" rather than "times tables" because she believed she already knew her times tables but did not know all pluses, and a desire to do "the hardest times tables" not just "easy" ones (Task 6.2) but on another occasion reported that when allowed to do any maths she wanted she would chose pluses because "they're the easiest" (Task 10.2.1). Anna turned nine after the second of the ten interviews.

Ben

Ben, a Grade 3 male from School I, was chosen by his teacher as a high achiever in mathematics. In relation to his perception of his achievement in maths, Ben believed he was a seven or eight on a scale of one to ten "because most of my sums are correct" (Task 11.1). Ben was a pleasant and quietly spoken child with what could be described as a twinkle in his eye. Ben stated that he liked learning maths (Task 8.3) and appeared happy to talk about his beliefs. It was usual for us to meet after morning recess. While he sometimes came in feeling hot from playing sport with his friends, he always settled down and applied himself well to whatever task we did. He seemed to enjoy the one-to-one interaction in the interviews. Ben turned nine about three months before the interviews commenced.

Cara

Cara was chosen by her class teacher following my request for a low achieving Grade 3 female. She was bright in character and able to communicate well in the interview situation.

Cara began the interviews as an eight year old, and turned nine half way through the data collection period. Cara seemed an outgoing, happy child who enjoyed talking. She was friendly, first evidenced on my initial visit to the school by her coming up and welcoming me before any official introduction had been made. She indicated that she was happy to be interviewed by me on regular occasions, a willingness that appeared to continue throughout the study, sometimes tempered slightly by tiredness. She joined me happily on each visit, but occasionally showed slight frustration and perhaps some boredom at talking predominately about maths. When I visited the classroom I observed that among her peers she seemed to convey a liking for the idea of being one of the four children involved in my research study.

During the interviews Cara displayed a sense of humour evidenced for example in the following two interview excerpts. The first was from my third interview with Cara in which we were doing Task 1.4. I spoke to Cara about other things she did at school and whether she felt they were like maths. We discussed similarities and differences between maths and language and maths and science. I then asked a rather open question in response to which Cara showed her sense of humour:

Interviewer: Can you think of anything that is nothing like maths? Cara: Yep I: What?

C: Me!

The last statement was offered with a big chuckle! In the fifth interview, using Task 5.2, we looked at photographs and discussed whether there was any maths in what the people were doing. Rather than a simple answer of "No" for one photograph, Cara responded "No way Jose!".

With Cara's outgoing nature and propensity to talk, our interviews sometimes went a little over the planned time of twenty to thirty minutes. However, she contributed well in the interviews, in many cases explaining and elaborating without extensive further questioning. When probing did occur, some confusion resulted at times, as becomes apparent in the discussion of results.

Overall Cara was neither strongly negative nor strongly positive about her ability in maths, but tended to consider herself not to be very good at maths. She was asked during her sixth and tenth interviews how good she felt she was at maths, that is, for her perception of her achievement in maths (Task 11.1). She was told that someone who was not very good at all in maths would be a "1", and someone who was really, really good at maths would be a "10". On the first occasion she responded, "the (person) at number 1 would be a Prep and (the person) at number 10 would be a Grade 6". Therefore she rated herself, as a Grade 3 child, at number five. However, she did appear to understand when asked to imagine the scale represented Grade 3 children only, and in this situation saw herself "in the middle 'cos some times tables I know and some times table(s) I don't". On the second occasion, which occurred six weeks later, she saw herself as a five because "I'm not that very good at maths … I'm really bad at maths". We revisited this conversation later in the interview and she stated she was a five "because I'm not that good at maths … 'Cos sometimes I get mixed up and I don't know what the answer is and I forget, I know the answer but I've forgotten the answer and I put, like, if the answer's fifty-three and I put thirty-five".

Cara also appeared neither strongly negative nor strongly positive in her feelings about learning maths. When asked about her feelings about learning maths, she was able to identify a situation in which she felt good about her maths learning and another in which she did not feel good (Tasks 10.2.1, 10.2.2). Her responses indicated varied experiences in maths learning, including both positive and negative encounters. When asked an open question about how she felt about learning maths, Cara gave a more negative response: "funny . . . if I forget what the answer is . . . sort of upset" (Task 8.3).

Cara's perceptions of maths learning experiences and achievement did not appear to impact significantly upon her willingness to talk with me about maths and its meaning to her; she appeared at most times to have a positive attitude toward participation in our ten interviews.

To summarise, Cara was a personable child who talked willingly with me during the interviews, but whose perception of her own achievements in maths, and whose feelings about

learning maths, did not always match her generally very positive attitude to discussing maths in the one-to-one interview situation.

David

David was chosen by his School S teacher as a Grade 3 male who was a low-achiever in mathematics. David appeared as a child who thought about things that happened around him, developed ideas, and liked to discuss these. He gave the impression that he enjoyed the one-to-one interaction and appreciated the opportunity to talk about his thoughts, often seeming to enjoy telling his story for whatever it was that we were discussing. Some of his responses suggested that he was exploring language; he expressed his ideas using many words with which he seemed to be in the process of developing familiarity.

At times he was fidgety during interviews, distracted by objects around him. However, he talked willingly with me at each interview and participated well. David turned nine years of age about half way through the data collection.

David gave the impression that he took the learning of mathematics seriously; he reported the needed to get a good education (Tasks 7.2, 9.1). Although chosen as a low-achiever in mathematics, David assessed his achievement as "a seven or eight" on a scale of one to ten "because I'm getting better and better at maths" (Task 11.1, following Set 9). When given Task 11.1 a second time he once again judged himself as a seven or eight but "because I like it" (Task 11.1, following Set 10). He did recognise that he found maths hard at times (Task 8.1.2) and experienced some difficulty: "maths is hard things, sometimes I even have trouble doing maths" (Task 1.1b). He reported that sometimes he was bored with maths but that "most of the times [he] enjoy[ed] doing maths" (Task 9.2).

Emily

Emily, a Grade 3 student at School I, was eight years of age throughout the data collection period. She was selected by her teacher, Ms I, as a high-achieving female.

Emily appeared a happy and welcoming child, always entering the interview room with a big "Good morning Andrea!" and continuing in this cheerful manner throughout the interview. Her eyes lit up often; she appeared fascinated by many of the materials and procedures used in the interviews. For example, she showed delight when posed with the idea of a personal dictionary (Task 2.1), and when viewing many of the drawings by other children (Task 9.2). Emily seemed very willing to participate in the research and appeared comfortable and at ease; she spoke quietly, and frequently took time to think about her responses, making an effort even when having difficulty giving detailed responses.

There were two occasions when Emily seemed unsettled and could not concentrate as well as usual. An unforeseen room change led to the occasional presence in the room of other people undertaking activities such as photocopying. Emily had difficulty concentrating when there were disturbances around her, both in the interview situation and, according to Emily, in the maths classroom, as discussed more fully in Chapter 6. Concentration was important for Emily, but in two interviews this was difficult, thus perhaps affecting some responses.
Emily perceived herself as a high achiever in maths. On a scale of 1 to 10, (Task 11.1) she rated herself as "8 or 9", adding that she was "the second most smartest [*sic*] in the grade because there's a person . . . he is the smartest . . . everyone knows that . . . when the teacher asks him a question he will always put his hand up". After further questioning, Emily added that although she could not always answer the questions that the "smartest" child could answer, sometimes she could answer questions he could not, coming to the conclusion "so we're about even in that". Emily's perception of smartness in maths appeared related to the ability to answer the teacher's questions.

Emily suggested that she liked to do well in maths, when in response to a Task 9.2 item, in which she was asked how she would feel if she did not get things correct on a maths test, she said "I feel disappointed". Emily indicated a desire for, and expectation of, more difficult maths in her future experiences at school; when asked what changes she would like in the way she does maths at school, for example when she is in Grade 4, Emily stated, "make things more harder [*sic*] . . . because I'm in Grade 4" (Interview 10).

Further insights into Emily's perception of herself as a learner of mathematics, including perceptions regarding attribution and confidence to call on others, are available from reports regarding the few occasions when she felt she did not achieve in maths. When asked for a time when she did not feel good when learning maths (Task 10.2.1), Emily replied, "when we always do the fractions this year, maybe I didn't listen how to do it or something like that". In response to a question of how she felt at this time, she replied, "stupid", but added that this happened "not much" when she was learning maths. Given that Emily saw herself as a high achiever in maths, it is not surprising that when she had difficulty, she felt "stupid" and that her strategy to overcome the situation was to "peek of [*sic*] other people's work", and not to ask the teacher as "she was teaching". In response to a question, Emily stated that she asked one person for help, but seemingly in defence of this, Emily stressed that "they didn't show [me] how to do it". It appears that Emily encountered difficulty in maths infrequently and may have felt some unease in this situation.

Emily saw herself as a high achiever in maths and could feel "disappointed" or "stupid", and perhaps uneasy, in the few situations in which she perceived she did not know her maths. Within the discussion of the fraction situation in which Emily had difficulties, the statement "maybe I didn't listen how to do it or something like that", suggests that Emily attributed the difficulties she encountered in learning maths that year to not listening rather than to a lack of ability. It appears that this is a factor over which she could exercise some control.

Emily's demeanour and responses suggest that she felt some liking for maths, perhaps related to her perception that generally she achieved in maths. Her cheerful interview discussions of her learning of maths made clear that she felt comfortable in discussing maths and that maths was not an irksome topic for her. Some liking of maths was conveyed in her interview responses also, as demonstrated in the following Task 8.3 (Interview 3) excerpt:

Interviewer: How did you feel when you were learning maths [this week]?

Emily: I feel happy because I sometimes like maths

- I: Do you?
- E: And yesterday I done some homework was for homework [sic]
- I: What was it?
- E: Um you had to write some numbers in words and if you complete the three, six, nine and then there's the line and then you have to keep on writing
- ••
- I: Do you enjoy doing things like doing patterns or do you enjoy things like writing the number words? Which ones did you enjoy better?
- E: Writing the words, because you have to spell it and then I feel like it's fun

When talking about her liking of maths, Emily described homework in which she had to write numbers in words. It is possible that the reader may not consider this as mathematical activity, but rather as language activity involving naming and spelling. For the primary school child the writing of number words may be perceived to contribute to the development of knowledge in the domain of mathematics. Emily recognised the spelling element of the activity but seemed to consider it as a part of her school-related mathematics, a part that she enjoyed. This suggests that an activity with a language element or focus, linked in some way to maths, may be considered primarily as mathematical activity by a Grade 3 child. This homework situation is referred to once again in Chapter 6.

The above homework scenario interview excerpt shows that Emily felt positive towards the activity of writing number words, an activity she offered as mathematical. The language aspect seemed to account for the liking of this activity. Emily may have felt positive about other maths activities also but did not choose to speak of them at that time.

When asked during her final interview to describe a situation in which she felt good when learning maths (Task 10.2.1), Emily was unable to do so. After the task was posed, Emily paused for a long time, during which there was much background activity and noise. When she was asked whether she had a time, Emily answered: "Yes, but I forgot it". Her brief response may have been due to the environmental conditions, as this was one of the two occasions when we were in another room. Emily may have had difficulty concentrating on the task, and therefore thinking of and describing the details of a situation in which she felt good when learning maths. The response should not be interpreted as evidence that Emily disliked maths and the learning of maths.

It is concluded that Emily considered herself a high achiever in maths, that at times she felt uneasy when she incurred difficulty with maths, that she attributed some of her difficulty in learning maths to an external factor, not listening, that she was looking forward to harder maths in Grade 4, that she was motivated to achieve, that she found distractions unsettling, that she felt some liking for maths, and that, in the interviews, she communicated a positive attitude towards maths. From this combination of findings it can be inferred that Emily felt more positive than negative towards what she perceived as maths and her learning of maths.

Filip, chosen as a high-achieving male from School I, appeared happy to participate in the interviews but was not outgoing in his approach and seemed more reserved than some of the other children. Interviews with Filip tended to be shorter than some others as he mostly expressed his opinions succinctly. However, Filip always willingly elaborated upon responses when requested. On a couple of occasions he appeared slightly restless or less focused than usual. Filip was eight years of age throughout the interview period.

On a scale of one to ten, Filip assessed himself as a "ten . . . everybody knows I'm the best in the grade . . . every time I get all of the things right" (Task 11.1). Filip expressed a liking for maths, stated that it was fun (Task 11.1) and his favourite subject (Task 4.1). He indicated an interest in learning new or harder maths (Task 11.3), such as when he asked his older brother to teach him square root and indices (Task 2.3). He liked to do well in maths (Tasks 9.2, 10.2.2) and to learn or complete his maths quickly (Tasks 5.1, 8.2, 9.2). He appeared motivated, at least partly, by his parents' desire for him to do well and become a doctor (Task 11.3).

Gina

Gina, a Grade 3 female from School I chosen by her teacher as a low achiever in mathematics, assessed her achievement in maths as a five on a scale of one to ten. Gina was nine years of age throughout the interview period. Gina was a very pleasant child and seemed to enjoy the one-to-one interaction. In her first interview Gina appeared a little quiet and shy, but soon settled in and appeared at ease in the interviews, indicated on one occasion by her becoming a little distracted within her response and talking about her football team and the colour of the team clothes!

Gina communicated some belief in herself as being able to learn mathematics but believed that sometimes her "brain [did] not work because [she heard] some noise". In this response she appeared to attribute difficulties more to the external factor of noise disturbance rather than to lack of ability. When asked to think of a time when she felt good about her maths learning (Task 10.2.1), Gina was unable to do so. However, she could think of a time when she felt bad about her maths learning (Task 10.2.2), an occasion when she had homework that she found hard and she worried about trying to finish it late at night. It appears that while Gina believed she could learn maths, she did not see herself as always achieving with ease.

Harry

Harry was a Grade 3 male from School I chosen by his teacher as a low achiever in mathematics. He was eight years of age throughout the interview period. Harry stated that he would sometimes feel embarrassed when given praise for good work (Task 9.2), or would sometimes "get shy and embarrassed" if he came out the front of the class and "got it wrong" (Task 9.2), suggesting that he was a shy child. In the first interviews Harry was relatively quiet and reserved, rarely expanding upon his usually brief responses. I did not feel him unsuited to, or extremely uncomfortable in, the situation but thought that perhaps he was a

little less mature than the others and therefore less able to participate in any real depth. However, it was in Harry that I saw the greatest change. By the end of the interview period he was outgoing and talkative and appeared very comfortable. What this was due to I cannot say with certainty; it may have been that he needed to become familiar with my expectations and approach, or that for some other reason he developed confidence over this period.

On a scale of one to ten, Harry assessed himself as a "ten because I learn them already and I can say it really fast and my teacher and everyone tests me, so I'm really good" (Task 11.1). Harry considered learning to be "very fun" (Task 2.3) and maths to be "fun" or "very fun" (Tasks 2.2, 2.3, 4.1). He found some maths easy, for example, "plus sums and takeaway" (Task 2.3), and some hard, for example, "the dot and the line and the dot" (division) (Task 8.3). Harry was motivated to learn hard maths so that when he grew up he would "know better" (Task 4.1). He felt good in maths when it was not hard and he could "work really fast" (Task 8.3). He appeared to have little appreciation of the possible use or application of maths in his future life but was motivated to do harder work in maths so that "when I grow up I know them better".

As can be seen from the discussions above, there was much variation among the eight children in factors such as self-assessment of achievement in maths and feelings about maths. Most responded positively to the interviews although some were shy and some less verbose than others.

The eight children were taught at two schools, and by different teachers who had differing beliefs and practices. Some of these factors are explored below.

The teaching of mathematics at School S and School I

As further background to the children and their experiences of learning mathematics at school, discussion of organisational features and philosophical underpinnings of the mathematics teaching is provided below. These factors were not investigated to attribute cause and effect but as background contributing to development by the reader of a picture of the children and their experiences. The discussion below also gives context to references to classroom organisation made by some of the children in the interviews.

School S

The following information comes mainly from interviews held with Ms A, the Applied Maths teacher, on November 16 and with Ms S on December 11. Some references are made also to lesson observations.

Anna, Ben, Cara and David were members of a Grade 3/4 class at School S taught maths by two teachers. Their class teacher, Ms S, taught mainly number lessons in the classroom, and problem solving lessons in the Maths Task Centre. Ms S reported that she also taught some measurement tied in with number work, and some maths concepts, such as graphing, as part of other lessons. The children were taught measurement, space, and chance and data by Ms A, the Applied Maths teacher, in the Maths Task Centre for one hour per week as part of a rotation program. According to the classroom teacher, Ms S, problems with the timetabling of the rotation system caused the children not to receive as much teaching in measurement, space, and chance and data during the year of the study as she would have liked or as she would have done without the rotation system. Ms S was new to the school and therefore had not contributed to the decision regarding structure of the mathematics program, including the rotation system. Lessons with Ms A had ceased for the year by mid November when I enquired regarding the possibility of observing such a lesson.

Ms S's teaching in the classroom, with lessons normally conducted for "the best part of forty five minutes every day . . . [would typically concern] number work, tables, counting, some sort of blackboard work, [one group working with the teacher] and then swapping over . . . and correcting it together". Her responsibility was the teaching of number; she focused particularly on the four operations, although she indicated she would have taught number through measurement contexts if given more control of the teaching of mathematics to her class. To help children learn she liked to "make it simple, finding ways to link things together like decimals and money, decimals and measurement". She had taught some "money, time and length, because [she could] not do sums without them, like you need something to latch the sums on to". Indeed, as indicated below, the concepts of time, area and perimeter were included in the lessons I observed within the ongoing program. Ms S made little use of computers or calculators in mathematics lessons during the year in which the data collection occurred, the latter reportedly because she did not have calculators in the room for every child to use as she would prefer.

Ms S believed that mathematical activity was solving problems, anything from simple sums to more complex problems such as those in the Maths Task Centre. Ms S reported that "in life" the children had done a lot of problem solving, but in mathematics, "an average amount . . . it's difficult to say". In response to items presented in the interview, Ms S said that she felt in the class there had been a lot of *applying maths to real life problems*. There had also been a lot of *discussing maths* and *children justifying maths*. Ms S wanted also for the children to appreciate that "maths is all around us . . . and there are so many things that involve mathematical skills".

When Ms S, a teacher with fifteen or sixteen years teaching experience, was asked for her perception of the big or key ideas in mathematics for Grade 3, she replied,

Understanding number, And then again, like some sort of logical thing. I mean to teach logic, number and place value . . . and to try to link it together into things that are around you . . . relationships.

When talking of how she would define mathematics, Ms S stressed the importance of knowing "about numbers [and how] to use them to your advantage and in real life experiences . . . patterns . . . problem solving, and visualness [*sic*]".

It is not surprising, that although Ms S felt the children had not received enough time in the areas of space, chance and data, and measurement, and although she "absolutely love[d]

geometry", she focused on number in her teaching. She was working according to the school program and focusing on the area of mathematics that she saw as most important for Grade 3.

When asked to define learning Ms S replied,

To be able to know how to, to have enough skills to know how to find the answer even if you can't, if it's not at your fingertips. You can't teach them everything that they need to know, learning is all about how to get that answer even if, I have a process that I can go through to find out, so it's simple to work this out, I need a dictionary or I need a ruler or I need a library or I need someone who has more knowledge than me. So the learning part may be accepting that, realising that.

She believed that she herself would know she had learned something when she could explain it to somebody else, when she had internalised and did not care to know any more.

Ms S was concerned that even by Grade 3, some children considered mathematics to be hard. She did not want children to have a fear of mathematics but rather to think "it's not hard, you just have to think it through". She believed the role of the mathematics teacher was "to teach, explain, show, and then to kind of re-explain back or re-do, re-show the way the sum is worked out". She thought the children would think a good mathematics teacher would "explain". Ms S believed children could help themselves to learn mathematics by "sort of paying attention, having a go, like coming back to keep trialling something, practise what's been done, but not hundreds of sums but revisiting it often".

I observed three mathematics lessons taught by Ms S, two as part of the ongoing mathematics program in the classroom, and the third the *Pasta less* on requested by me but with the instructional and management details decided upon by Ms S. I did not observe lessons by Ms A as by that time of the year her mathematics lessons with these Grade 3/4 children had ceased.

Each of first two lessons I observed took place in the classroom and began with the 28 Grade 3/4 children seated in pairs at tables arranged in curved rows facing the front. Children from the different grade levels were mixed. Once a brief introduction was completed the children were broken into two groups, one working Ms S on the floor at the front of the room, the other children doing application work at the tables. Half way through each lesson the groups changed position and activity type. The teacher reported that the groups were formed according to the needs of the children, but usually roughly according to grade levels. As well as whole class time at the beginning of each lesson, in the second lesson there was group counting at the end. In these respects the lessons reflected the *typical lesson* as described by Ms S and as referred to above. The content of the first lesson included area (in the context of *house safety*, a topic discussed in other curriculum areas also), decimals and money. The second lesson content was area, perimeter, the calendar, decimals and counting. During these lessons the focus appeared to be on the acquisition of facts, the development of skills, and the understanding of concepts. The teacher highlighted some relationships between mathematics and real life, and both the teacher and children used concrete aids.

In a third lesson observed, the *Pasta lesson*, the children were posed the challenge of making from a piece of paper a container to hold one cup of pasta, and then making a container twice as big. The lesson was held in the Art Room due to the hands-on construction and the materials used. Ms S introduced the activity, discussed the need for planning, passed out a container holding one cup of pasta to the pairs as the lesson progressed, gave some encouragement to some, disciplined the children, made management and timing decisions (e.g, with whom the children were to work), and questioned children in the discussion at the end. Some children were observed to plan their two-cup container, including comparison to the properties of the original container, showing evidence of planning and visualising in thinking mathematically. Some children worked more through trial and error, making a container with the piece of paper, then cutting it down to hold two cups.

In an organisational sense, this lesson was not representative of Ms S's *typical* mathematics lessons as it was taken in the Art Room; it was a spatial activity, and these were usually taken by Ms A in the Maths Task Centre; and both grade levels were taken together for the one activity. It appeared also to leave more decision making to the children than apparent in the lessons in the classroom.

Although lessons taken by Ms A were not observed, they were described for me using the example of a measurement lesson. Ms A stated that lessons began with a discussion of informal measurement and the need for formal measurement. Half of each lesson was *hands on* and half was a worksheet from a published source. To summarise, lessons were described as Discussion, Activity (equipment), Worksheet. Ms A gave an example of an activity: If you built a cubic metre, how big would it be, how many balloons would fit in? She spoke of this as a visual activity, as it calls on children to imagine. Ms A tried to make the mathematics relevant to the real world, conducting discussions of when the children had to use the mathematics. She stated that she tried to draw on the real world in all lessons. She felt that among the children there was some lack of motivation and interest, that it was only the ones who were driven to do well who made the connections. Ms A stated also that in each session she asked the children one thing they learnt in that session. She reported that she tended to get responses such as "We learnt to measure" which she thought were "totally off the track", she believed the children found this question "hard as they don't think about it".

To conclude, the four research participants from School S were taught mathematics by two teachers, Ms S and Ms A. The majority of the children's mathematics lessons were taught by Ms S with a focus mainly on number concepts. Both teachers were concerned about children's perceptions of mathematics and learning and reflected on these with the children.

School I

The following is informed mainly by an interview held with Ms I on November 13. References are made also to lesson observations.

Ms I was the class teacher and only teacher of mathematics to the Grade 3 class in which Emily, Filip, Gina, and Harry were members.

When asked how much emphasis Ms I had given different aspects of the mathematics program, choosing from *a lot, some, not much*, or *not at all*, Ms I reported that she had given *a lot* of emphasis to problem solving, through word problems and puzzles, to estimation, such as through measuring and adding numbers, to use of the computer, the four operations, discussing, children justifying, games, multiplication tables, and solid shapes. She said she had only given *some* emphasis to spatial work, measurement, and data. At that stage of the year she had not taught chance and had used calculators only once, in the lesson I observed four days prior to our interview. At another time in the interview she said that she had not given much emphasis in that year to mass and volume and had "really concentrated that they all [knew] how to do their addition". When asked earlier in the interview to give her perception of the *big ideas* in mathematics for Grade 3, Ms I included number, such as the four operations, real life situations, computers and calculators, different kinds of measuring, and problem solving.

Ms I reported that she had spent most of the time in terms one and two on the four operations and multiplication tables, and continued these through the year but varying the program to "try to make each day different". She did not like to teach content in blocks as she felt that children forget after a period of time; she liked to come back to things. She also had a revision day each week when she would cover "a whole lot of things" with all children "get[ting] the same thing". She liked to "make [mathematics] interesting, not to have the same kind of lessons for the whole year [and for children to be] exposed to all different kinds of maths and to know that maths isn't just from the blackboard and is just not five plus five". But she stated also that she did do "quite a lot of the traditional kind of maths from the blackboard".

When asked to define mathematics, Ms I, a teacher of two and a half years classroom teaching experience, replied, "you can find mathematics in all things . . . numbers . . . computer . . . real life situations . . . concrete material" but felt that she was not really defining mathematics. This task seemed to cause her some difficulty. Her description of mathematical activity appeared to focus on school activity: "using concrete material, it could be doing worksheets, work from boards, mathematical activity is really everything".

In summary, Ms I appeared to associate mathematical activity mostly with schooling, thought there should be a link between school mathematics and real life situations, and, when defining mathematics and when teaching mathematics, focused more on number than other content areas.

When asked to define learning, Ms I talked of "being exposed to something and having enough time and maybe experience and hands on activities . . . that's the only way they will grasp it, just by exposure to the concept that you're teaching". Ms I felt that children's learning was evident when they "grasped [concepts] . . . if they've explained it properly and if they've got the activity correct and done it right". Evidence of learning for herself was described as having the final product correct, meeting criteria within your objective, "fully understand[ing] something so you can explain it to someone else", and "being able to apply what you learnt in different situations".

Ms I considered it important that children understand that making mistakes and talking to other children are part of learning:

I think having the experience of actually doing it and also making mistakes and then having those mistakes pointed out and re-directed. . . . I think it's important for the children to be able to discuss, and talking amongst themselves. . . . Sometimes children explain better than yourself.

Ms believed understanding was important and did not want children to be afraid when they did not understand. She felt that children could be helped in their learning by

making mistakes, being exposed to different kinds of activities where they can actually work out for themselves because that's the only way you learn, with hands on experience, and talking. I think that's very important, to talk to each other and to the teacher. Because if they just sit there and not talk they won't learn.

In reflecting on her role as teacher in a mathematics lesson, Ms I stated that she felt the teacher should act as a facilitator and role model, and should "springboard ideas" to help children learn together. It is clear that Ms I did not see learning as a purely individual endeavour.

To conclude, when asked about aspects of learning, Ms I appeared to immediately relate learning to classroom situations and children's experiences. Making mistakes, hands on experience, talking to other children and a variety of activities were believed to foster learning.

I observed four lessons in this class, two of which were on line symmetry, and part of the same rotation and therefore basically the same, one on the use of calculators and the *Pasta lesson* that I had requested.

I had asked that for my visits Ms I teach whatever mathematics lesson she would normally teach. The symmetry lessons involved group work. This approach had been introduced in Term 3, and was not used for Number work. The children were accustomed to the arrangement in which a set of four activities on one topic would rotate between the four groups, each of approximately six children, for four lessons. I did not see any class review presumably because the rotation had not completed. This structure was not representative of the majority of mathematics lessons; it had been used three times during the year (Interview, November 13). However, it appears that with less of an emphasis on number in the second half of the year, Ms I was covering a range of other topics also and using more than one organisational approach.

The first symmetry lesson began with a teacher-led discussion with the whole class. The teacher asked the children to give an explanation of symmetry and to tell her "different things that have symmetry". Discussion of the meaning of symmetry ensued and a listing was made. The introduction was concluded with a teacher summary preceded by a definition of

symmetry. Throughout this work children participated with interest, willingly offering suggestions and taking part and listening to discussions.

In the remainder of that lesson and in the second lesson observed, activities included making Blob pictures (using paints), exploring shapes for the property of line symmetry (on prepared sheets enlarged from teacher resource books), use of mirrors, and completing prepared half pictures by folding, cutting and drawing. Ms I moved around the room encouraging discussion of ideas.

The third observation on calculators focused mainly on the features of the calculator such as the different keys. Ms I stated that she had taught all topics in her program for the year except "calculators". This lesson was not representative in structure of the lessons on number concepts. The latter would involve one group of "kids who didn't grasp it . . . say addition or subtraction" working with her on the floor. In the calculator lesson, the children worked individually at tables and were each given a calculator and worksheets. Children were enthusiastic participators and listeners during the initial discussion, listening to Ms I and the other children. When children gave responses Ms I asked them to explain, giving time for them to do so. Activities at tables focused on use of the CE and other buttons, the layout of the calculator, making words on the calculator, such as ShELL, and a calculator keys, and on increasing knowledge of the functions of a calculator. There was ongoing interaction between the children and teacher as she moved around the class. Ms I was comfortable and honest with the children, such as when she learned something new about the calculator.

The *Pasta lesson* appeared more open than Ms I would normally have taught, and gave the children an opportunity to show that, generally, they could work on such a task. Their responses were, in some respects, to the surprise of the teacher. In this lesson, as in the others I observed, the children appeared interested, motivated and to enjoy the activity.

To conclude, the lesson observations did not represent the majority of mathematics lessons during the year. However, they did provide some background perspectives on the experiences of the children in this Grade 3 class and added to the insights gained through other means.

Summary

The present research investigated the beliefs about mathematics and learning of eight Grade 3 children. Those beliefs were sought through a range of data sources and procedures, all of which were administered individually. Further, some background data were collected from the eight children, their mathematics teachers, and from lesson observations.

Prior to presenting and discussing the findings from the research, this chapter introduced the eight children who were the key participants in the research. The chapter illustrated not only the individuality of the eight children, but also demonstrated that, even at the young age of eight or nine years, children may react differently in similar circumstances and may develop personal beliefs and attitudes about themselves learning mathematics that they are able to express.

This chapter provided some insights into the background context of the school mathematics learning situation at each of the two schools, although it is acknowledged that this provides a limited view of the child's learning experiences. Home and other learning situation data gathered informally during interviews are included in the discussion of findings later in this report when relevant to research questions but an extensive study of learning situations from the perspective of the researcher was not undertaken. The main interest in this research was the perspectives of the eight learners. The purpose of the introduction to the eight children, their teachers and school learning situations was to provide some insights into the individuality of each child and into their experiences.

CHAPTER FIVE A PORTRAYAL OF CARA'S BELIEFS

The portrayal of Cara's beliefs is broken into three sections:

- beliefs about the nature of maths and maths activity;
- beliefs about learning;
- beliefs about helping factors for learning maths.

These sections are broken into subsections, structured and titled according to themes that arose during the data collection period or as a result of post-collection examination of the data. The discussion within the first section focuses on Cara's beliefs about *maths*; as an aside a subsection elaborates Cara's beliefs about, and difficulties with, the term *mathematics*.

Maths and maths activity

The issue of different meanings for some children for the terms *mathematics* and *maths*, and the familiarity of all the participants with *maths*, has led in this report to use of the latter term when discussing meanings for all children, as explained earlier. Nevertheless, it is considered appropriate to discuss the participants' meanings for *maths* in light of previous research of children's meanings for *mathematics*, as in those studies no such terminology issue was reported.

Cara's interview responses indicated that maths was seen to involve an array of concepts, concepts that did not correlate with those she associated with *mathematics*. Cara's meaning for *maths*, the term with which she was more familiar, was found to be more complex than children's meanings identified in previous research. For example, it had been found that children commonly associate number or computation with mathematics (Frank, 1988; Garafolo, 1989; Spangler, 1992; Stodolsky et al., 1991). Cotton (1993) reported from his study with 5 to 9 year old children that "the older children saw mathematics as nothing more than number" (p. 15).

Cara's portrayal of beliefs during the interviews unveiled a complexity and multidimensionality that is perhaps one of the more revealing aspects of this research, as it contrasts markedly with findings from previous research as discussed above. A thematic approach was taken to facilitate identification and discussion of major elements of Cara's beliefs. The resultant themes are

- measurement;
- estimating and guessing;
- maths as answers;
- maths for an everyday or non-school purpose;
- maths for a combined school/non-school purpose;
- maths for a school-related purpose; and
- segmentation of maths: maths as content or action; number and measurement.

Sub-themes are identified and discussed within some of these themes.

The discussion within each theme draws data mostly from a number of interviews, each piece of data contributing to the growing portrayal, and at times appearing to complicate that portrayal. Cara demonstrated that her beliefs about maths and mathematics formed what appeared, at times, a logical schema, but at other times an intricate and even entangled web. Throughout the writing up of results, schematic representations are presented to clarify for the reader major elements of Cara's beliefs.

The discussion of Cara's beliefs about maths and maths activity begins with consideration of measurement within maths.

Measurement

Measurement as an aspect of maths, was one of the main themes that emerged from Cara's data. The key elements of this theme are displayed in schematic representations, built up as the ideas emerge. The discussion of Cara's beliefs about measurement as maths activity is introduced through consideration of the meaning as generally held to provide some context for consideration of Cara's beliefs.

Introduction

The Concise Collins Dictionary (Wilkes & Krebs, 1982) defines the noun measure as

the extent, quantity, amount, or degree of something; a device for measuring distance, volume, etc. such as a graduated scale or container; a system of measurement; *metric* measure; a standard used in a system of measurements; a basis or standard for comparison. (p. 700)

The definition of the verb form of *measure* includes "to determine the size, amount etc., of by measurement; to make a measurement; to estimate or determine" (p. 700).

The term *measurement* has been described variously, for example, as "a system of measures based on a particular standard" (Wilkes & Krebs, 1982, p. 700), and as "a practical activity which enables us to associate a number and a unit with a specific property of an object" (Ministry of Education, 1985, p. 3). A recent education community publication which discusses measurement in more detail includes the statement, "Essential ingredients for measurement are *units* and *numbers*" (Board of Studies, 2000, p. 18).

These quotations suggest that measurement is a practical activity that utilises standards or systems that operate with the use of units and numbers. Marked devices for measuring in formal units include rulers, scales, and graduated containers. In the early levels of the primary school children "become familiar with the concepts of length, mass, capacity, time and money ... [they compare] objects and quantities with respect to particular attributes ... [and come to] see the need for unit measures" (Board of Studies, 2000, p. 19). The curriculum document, current at the time the data were collected, recommended that children's schooling experiences with measurement begin with the use of informal units and that children are introduced to formal units or standards by Level 3, that is, by the end of the Grade 3/4 learning experience (Board of Studies, 1995). The more recent version of this publication also recommends introduction to formal units by Level 3 (Board of Studies, 2000).

It was possible that each of the eight Grade 3 participants in the research had had some experience with formal units, at home, at school or elsewhere, and may have been in the process of developing understandings about the meaning of these. It was assumed that having operated in the society for at least eight years and having experienced approximately three and a half years of schooling each child would have experienced some measurement concepts, if only through an informal approach.

For all of the eight research participants except Harry, measurement appeared as a element within their concept of maths, as discussed later in this report. The views expressed by Cara, a child who gave emphasis to measurement when identifying mathematical situations, are considered below, with discussion of her reference to measurement in general, and to specific concepts within measurement, both in informal and formal terms.

Cara's beliefs about measurement and its relationship to maths

As gleaned from Cara's interview data, her beliefs about maths contrast in part with findings from previous research in that she saw maths as more than number. Cara did refer to situations involving number but had a broader view of mathematical activity. This view included measurement, a concept frequently referred to by Cara. For example, when given the word *maths* in a word association activity (Task 1.1) Cara gave the response "measuring". In the password activity (Task 1.2), when asked for a synonym for *maths* Cara gave the word "estimating", and when asked whether she could think of any other words to do with maths she added "measuring". These responses suggest that measuring and estimating may have been the aspects of maths with which Cara was most familiar, or of which she was most conscious. As discussed below, many times in interviews Cara referred also to measuring and estimating in relation to activities at home, school, work, and in other contexts. In exploring Cara's understandings of the concept of *measurement*, we build an appreciation and understanding of intricacies and complexities within the beliefs Cara, as an individual, had formed.

Cara demonstrated that she considered a range of measurement concepts, including mass, length, and capacity, to be mathematical. For example, in her third interview Cara spoke of herself in the school situation measuring mass in Science, and of herself "playing with things [as] . . . part of measuring . . . have to measure them on them weight things". When discussing a photograph of a child measuring another child with string (Task 8.2), Cara referred also to measurement of length in the school situation, but indicated that she used a tape measure.

Cara made a number of references to the use of measurement outside of school, in which she indicated that she saw measurement as something to be used in real situations and as having a purpose. For example, in her third interview she described a real life use of maths when she spoke of her father using measuring jugs when baking cakes. In response to Task 3.2 in which Cara was asked to show a maths activity, she referred to herself measuring with her father when building a cubby house. In the discussion she demonstrated that she saw a purpose to measurement when building: "Yeah 'cos if we didn't measure it, it would be too big and too small and too wide and too tall". When shown, in her fifth interview (Task 5.2), a photograph of a woman weighing fruit at a supermarket Cara described it as, "she's using maths, she's measuring, to see how heavy the something that she bought [*sic*]". Cake-baking was mentioned in Cara's eighth interview (Task 4.3), again with evidence of an understanding of a purpose behind the measurement activity: "Measuring the cake, how big it can be . . . If you do it too big and there's not enough people to eat it, it's not fair if you get, someone gets two pieces and the others only get one". The above interview excerpts indicate that for Cara, measurement activities involving length, mass, and capacity were not confined to the school situation but were relevant to the home, workplace, and elsewhere, with the maths being used to meet a need of the person involved.

Formal units of measure for mathematical activity were mentioned or implied at times such as when speaking of her father making cakes. In her second interview (Task 3.2) Cara stated, "He gets a jug that goes up to 150 and he puts the cream in", and "He has to measure how big they want it like on the bottom, he goes, oh how big is the circle, and then he gets a ruler and he measures it everywhere and then he cuts around the circle and sticks it on whatever and it's a [wedding] cake". Formal units were referred to specifically when Cara described two photographs from Task 8.2 that she interpreted as a girl reading a packet and baking a cake: ". . . she's seeing like, she's seeing on the back here to see if it's 1 mil (mL) of water or 2 mils (mL), and there she's putting the water in".

Cara was involved in, or could identify, a number and a variety of activities that she considered to be measurement situations. Cara showed that she understood the existence of, and a purpose in, the use of formal units in measuring length, mass, and capacity. Cara had had personal involvement with hands-on measurement activities, especially through her father's work and leisure activities, and in her descriptions of these demonstrated that she saw a purpose to the use of maths in these situations. The excerpts suggest that Cara saw a link between formal units, measurement, and maths. These links are summarised in Figure 7.



Figure 7. Cara's beliefs about measurement and maths - Image one.

This figure is developed further in the following discussion as more insights are revealed, providing an emerging schematic reference of the findings from this research of Cara's beliefs about measurement and its relationship to maths.

Cara's perception of a link between maths, measurement, and formal units was evident also in the discussion of two photographs during Cara's fifth interview (Task 5.2). The links

were perhaps a little tenuous in the discussion of the first photograph which Cara interpreted as a painter pouring paint into a tin. At first she was unsure whether there was any maths in what he was doing, however, she tended to think that yes, it was maths. She spoke about him "estimating to see how much he needs in that bucket". The estimation element seemed instrumental in Cara coming to say that this was a maths activity. However, when questioned further, Cara stated that she thought he was not doing maths because "if he was doing maths ... he would have a ruler". She did not mention *measuring* when describing this picture, but implied the use of formal measurement through reference to the ruler.

It is possible to infer from this excerpt that for Cara the use of a formal tool of measure was necessary for a situation to be considered *maths*. This message is different from, and could be described as more extreme than, that gained from much of Cara's other interview data in which she spoke of activities and concepts such as estimating, guessing, times tables, "plusing", and halving as maths. These are not dependent on the use of formal tools of measure. In the present case Cara stated the need for use of a formal measurement tool, the ruler, for a situation to be called *maths*. From her other interview data it seems that in fact she may have felt that formal tools were necessary for a situation to be considered as measurement, but not for a situation to be considered maths. Thus the statement "if he was doing maths . . . he would have a ruler" appears extreme and non-representative of her beliefs about maths as expressed in other responses. Nevertheless, the fact that the statement was made could indicate that Cara had not fully established her beliefs and may have been in the process of developing her views about the criteria for a situation to be considered a maths activity. Alternately she may have chosen not to verbalise the specific term measurement in that particular situation; she had been asked about maths and she responded by using that term. Although not expressing it, she may have been thinking of the term *measurement*, generally considered to be a more specific term than mathematics or maths.

When shown the next photograph, which Cara described as a man measuring to see how heavy a big fish was, she said that it was a maths situation. When queried, she said that he was measuring even though he was not using a ruler "because he [had] a scale". The face of the spring scale with the graduated measurements was evident in the picture. Once again Cara associated maths and formal measurement, but did not say the latter was a necessary criterion for the former. She appeared to link the use of formal units to the measurement concept.

The latter excerpt and those from other interviews cited above suggest that Cara believed that when formal units are deployed a situation can be considered as *measuring*, and in turn as *maths*. To summarise the findings discussed above, the Figure 7 schematic diagram is repeated, but with the addition of mathematical activities, each of which was identified by Cara, included for reader reference (see Figure 8). These are sample or representative activities only and included for illustrative purposes to remind the reader of what has come before. Thus each category does not necessarily include reference to all relevant activities

mentioned by Cara. For example, *fruit at supermarket* was chosen as a sample mass activity; *the man weighing the fish* could equally have been chosen but was not.

As the informal measurement/formal measurement dichotomy appears as an issue within Cara's beliefs, as explored below, each sample activity is labelled either F for Formal or IF for Informal. The discussion above does not point to any recognition of the use of informal units as measurement, thus the inclusion of only formal measurement activities in Figure 8.



Figure 8. Cara's beliefs about measurement and maths - Image two.

The interview data suggest overall that Cara was aware of measurement as a concept within maths, that she identified and proffered a range of situations involving measurement as maths but that mostly these included formal units or measuring tools. There was evidence in one interview that Cara was not clear on the different purposes of two measuring tools and on the measurement concepts to which they related. When concluding the discussion regarding the photograph of the man measuring the big fish with the spring scale she said he used the scale instead of the ruler because "the ruler wouldn't fit that big". This suggests that while Cara was able to talk about a number of situations in which measurement was occurring, there were some limitations in Cara's understanding of the concept of measurement and the purpose of tools of measure.

The discussion above shows that Cara associated formal measurement situations with both measurement and maths. In contrast, it appears that informal measurement situations were considered as maths but not as measurement, as becomes apparent in the discussion below, in which Cara's beliefs about informal measurement are explored.

In her fourth interview (Task 8.2), Cara was shown a photograph of a girl holding a string against the height of another child. Cara described the situation as someone using a "rope", and said that they were measuring. When discussing whether using a rope like that for measuring would help her to learn maths, she added, "Well they are measuring it's just that there's no numbers to tell them what to do." This statement seemed to be a qualifier to Cara's earlier statement that she considered the children to be measuring. In an attempt to check Cara's assessment of the situation she then was asked, "Did you say they are or they're not measuring?". Cara answered: "I think they're not measuring, 'cos they do have a rope but how can they tell what the numbers are? How can they tell if they're spaced or not?". This response indicates that the question may have caused Cara to reconsider her earlier response.

It is possible also that now she was paying attention to her most recent experiences in the school situation in measuring length, as demonstrated as the conversation continued:

Interviewer: So have you ever measured anyone with a string or rope like that before? Cara: No, we use long rulers, like not wood ones, like skinny ones around our waist and then up and down

- I: What about when you were in grade one or grade two, did you use rope like that?
- C: (Non-verbal: No)
- I: No. Okay

Initially Cara identified the informal situation in the photograph as involving measurement, but during the interview seemed to change her view. She came to indicate that for her, measurement of length involved the use of formal units as would be found on a tape measure. The questioning in the interview may have caused Cara to reassess her view, perhaps to question her own beliefs. It is possible that she did develop an understanding of measurement from an informal perspective in earlier years at school, but in the interview did not recall such as a measurement experience. Cara's account of the use of tape measures indicates that her recent school experience of measurement of length involved the use of formal units. Cara's responses in this interview suggest that her beliefs about measurement were in transition and were not established fully. It may be that she had not thought a great deal about informal measurement as a concept and thus was partially formulating her views within the interview. Cara's indecision, or evidence of partially formed beliefs, in the above excerpt, indicate some conflict within her beliefs. This conflict, which contrasts markedly with her seemingly established belief that a situation utilising formal units constitutes measurement, is portrayed in schematic form in Figure 9. The zigzag line portrays the conflict with Cara's beliefs about formal measurement discussed previously, the broken line portrays her uncertainty of the relationship between measurement and the informal measurement of length situation.



Figure 9. Cara's beliefs about measurement and maths - Image three.

In her second interview, Cara was presented with a range of similar situations that involved both formal and informal measurement. Cara's responses give further insight into her beliefs, particularly into those regarding informal measurement and its relationship to measuring and to maths.

Using Task 3.3.1, a number of situations were read to Cara. In one of these, a measurement tool and formal units were included: "Bradley measured his book and his pencil with a ruler. His book measured 25 centimetres. His pencil measured 20 centimetres. He said his book was longer". When asked whether there was any maths in what Bradley was doing, Cara immediately identified this as a measurement situation. She stated, "Yes, he was doing a little measure". Her response indicated that she saw measurement as maths. She had linked formal units, measurement, and maths, endorsing the findings discussed above.

A following situation involving informal measurement was responded to differently by Cara. She identified the activity as *maths*, she spoke of measurement concepts, but she did not label the situation as *measurement* as she had in the previous example:

Interviewer: Bradley laid his pencil next to his book and he said his book was longer. Did he use or do maths?

Cara: Yes

I: How come?

C: Because, if he weighed a book against a ruler, the book would still be bigger

The absence of direct reference by Cara to measuring may have been because formal units and the word *measured* were not included in the second item as they were in the first.

The latter item had been selected to gain some insight into children's understanding of informal measurement, in particular regarding the attribute of length. It is unclear whether Cara related this situation to length or to mass. She used the term "bigger", not necessarily associated with length or mass, but also used the verb "weighed", suggesting that she thought of measuring mass. If this was the case, it is not clear whether she was thinking of weighing in formal units using a scale, or weighing informally, for example, by hefting.

Cara then was shown in the interview situation the comparison of two objects, a book and a magazine, and asked whether the comparison involved maths. She replied, "Yes it would be guessing", suggesting that she believed guessing to be a mathematical activity. However, she did not choose to indicate that she saw this situation, chosen specifically because of its use of informal measurement through comparison, as a *measurement* situation.

In her fifth interview (Task 5.2), Cara was presented with another situation selected as one involving informal measurement through matching or comparison. The interview excerpt gives further insights into Cara's beliefs about informal measurement. When shown a picture of a woman checking the length of a blouse sleeve against her arm, Cara stated, "I think she's using maths because she's trying to see what size she is". The conversation continued:

Cara: That's all I know

Interviewer: So that would be maths, no ruler or anything like that but it's okay is it, it's still maths?

C: Yeah I think she's trying to see what size she is

I: Okay and what, what is the maths she is doing there, what sort of, why is seeing what size you are maths?

C: You're trying to estimate if you don't know what size you are in pyjamas

- I: Yeah
- C: Well it looks like she's estimating what size she is

Cara believed the situation did involve *maths*, suggesting again that her statement regarding the painter requiring a ruler to be doing maths was an aberration. She did not make specific mention of *measurement*. However, reference to "trying to see what size she is" makes an implication that the measurement concept of length was being considered, although not labelled as such. Whether Cara associated the measurement concept of length with "what size you are in pyjamas" is not known. We cannot conclude with certainty that in this situation Cara associated an informal measurement activity with *measuring* but it appears this link may have been made, although she did not choose to consciously voice such an association.

The conversation regarding the *pyjamas* does indicate that Cara saw a relationship between maths and estimation. "Estimating what size she is" suggests also a recognition of a relationship between measurement and estimation, although not specifically stated as such. A measurement/estimation relationship certainly is not unreasonable, with a similar view stated in the Wilkes and Krebs (1982) definition of *measure* given above, and with estimating included as a major element of measurement activity within recent curriculum documents in Victoria (Board of Studies, 1995, 2000). It may be that Cara saw the estimating in this situation as being similar to the guessing in the situation where the book and magazine were compared, as discussed above, and that this cognitive activity made these maths situations for Cara. Although Cara appeared to see these informal measurement situations as maths situations, she did not call either of these situations *measuring*. It is clear that Cara's views regarding estimating and guessing as mathematical activities are worthy of further consideration. They are discussed in more detail in the following section, and included in a schematic diagram at that stage.

In summary, the interview data presented above indicate the possibility that Cara saw *informal* measurement situations as *maths*, but did not consciously see them as *measurement*, and that she saw *formal* measurement situations as both *maths* and *measurement*.

From the excerpts cited of Cara's reference to situations involving measurement, there was one instance where she appeared to confuse two measurement tools, the ruler and spring scale, and their relationship to mathematical concepts. Other than this, Cara's discussion of measurement cited to this point signifies that her conceptual understandings of length, mass and capacity are appropriate in terms of the definitions provided earlier.

As discussed below, there were some situations described by Cara that appear not to fit the definitions of *measurement* and *measure*, and that show some uncertainty in her views. For example, in her second interview (Task 3.3.1), Cara questioned whether a particular sporting situation constitutes measuring. Having been read a statement in which a person was playing soccer and asked whether there was any maths in what he was doing, Cara replied that she did not know. To determine any possible links Cara saw between maths and sport she was

asked whether she played some other game and whether there was any maths in her doing so.

The interview excerpt shows some uncertainty as to the meaning and application of

measurement:

Cara: I play netball

Interviewer: Netball, and when you play netball do you use or do any maths when you play netball?

C: Well you have to, you're not allowed to move and I don't know if that's measuring

When asked about *maths* in this situation Cara appeared immediately to look for the use of measurement; she did not show evidence of considering use of other mathematical concepts such as number or space. Cara drew on the concept of mass when talking of measurement in the netball situation:

- Interviewer: Right so you're not sure if that's measuring. Is it like the measuring you do at school and at home?
- Cara: Oh well measuring we have to like measure scissors and a ruler and a thing on a, I don't know what it's called. Like you have two plates on one side and then you have things hanging and you have to make it balance or make it up
- I: Alright so on a balance, yeah so you think
- C: A scale
- I: On a scale, right, so you're wondering if it's a bit like that are you?
- C: Yes
- I: Because, why would you think it might be a bit like that, are you sort of balancing too are you?
- C Yeah 'cos you have to balance the ball to get it in the net
- I: Right okay
- C: You balance, you go to hold the long, you jump up so, yes

Although Cara talked about using a scale to create a balance situation when measuring, she seemed to have difficulty in identifying the exact relationship between this and netball. The term balance was drawn upon for further questioning, perhaps giving some unintended direction to what Cara was saying.

Cara went on to state that playing football and soccer were not maths, but playing netball was. However, in her fifth interview (Task 5.2), she stated that when playing netball or some game like that there would not be any maths in what she was doing.

These data suggest that at one time Cara was uncertain whether to identify netball activity as measuring. Once again we see lack of clarity and definition in Cara's views, suggesting that those views were in the process of formation, perhaps being stimulated by the experience of being interviewed. Cara's reference to measuring as finding balance is limited but acceptable as it relates to the concept of mass; by considering an association between netball and maths through the concept of balance, she was contemplating a linkage more tenuous than might generally be made. The possible association between maths and netball evident in her second interview did not extend to her fifth interview.

In Cara's third interview (Task 1.3.2), she described an activity that occurred at her school. Her identification of this activity as measuring suggests that some of her beliefs about

the meaning of measurement are not in accordance with beliefs generally held, as becomes apparent when considered in light of the definitions cited earlier.

Cara: And we have to measure how many times we can go on the grass area Interviewer: What do you measure, how many times you go on the grass? I don't understand

- C: 'Cos you have to take turns to go on the grass like fives, sixes and you've got to look on the board to see
- I: What, how much time you have?
- C: Yeah, to see when it's rest week, to see if it's playing week
- I: Oh, is that what you have at this school is it, so you share the grass?
- C: Yeah
- I: What do you mean you said you have to measure how many times you go on the grass?
- C: How many goes you get, when it's your week and when it isn't
- I: Like counting them or something?
- C: Yeah

As the interviewer I interpreted this situation as possibly a quantifying or basic counting activity, as it did not seem to use formal units of measure. Alternately, Cara may have meant something more like reading a timetable; I was unable to get further clarification from her. Cara's use of the term *measure* in speaking of this situation is interesting as it indicates a possible application of the word beyond situations that might generally be accepted. The situation seems more about keeping track of the number of times a grade goes on the grass area and when it is their turn.

Earlier in the same interview, when asked whether her responses in the maths word wheel (Task 1.3.1) applied to anything out of school, Cara stated, "I made a swing at home and I had to measure how many times I could swing on it, estimating . . . And I estimated three but I done a hundred ha ha". Here, once again, is an indication that for Cara measuring may refer to counting, but without units of measure, as well as a parallel being drawn between measuring and estimating.

Figure 10 extends the image of Cara's beliefs about measurement and maths from that presented in Figure 9. Cara's reference to counting as measurement is shown in the Figure 10 schematic diagram using the example of playing on the swing at home, with a solid line linking this to the word measuring, indicating her belief that this was a measurement situation. As the researcher believes that a "counting - playing on the swing at home" situation generally would not be considered to involve the use of either formal or informal measurement concepts, it is not tagged either "F" or "IF".

Cara appeared to see the possibility of balance as a mass situation in the netball scenario, but was unsure whether this situation constituted measuring. Her uncertainty is represented by a broken line. The situation is labelled "IF", for informal measurement, as a mass activity involving balance in this way does not involve the use of formal units, but rather takes an informal approach to the concept.



Figure 10. Cara's beliefs about measurement and maths - Image four.

In drawing together the beliefs about measurement expressed by Cara, it appears from the interview excerpts discussed above, that, for Cara, measurement was a real and relevant aspect of mathematical activity; measurement was mentioned frequently by Cara. She associated measurement with maths; she saw measurement as involving the concepts of mass, length, and capacity in formal measurement situations, as represented in Figure 10, and showed a belief that measurement involves both physical activity through hands-on use of equipment, and mental or cognitive activity, for example, through estimation. Cara demonstrated some uncertainty as to what constitutes measurement, particularly when contemplating informal measurement situations, as represented in Figure 10 by the netball and rope situations. In addition, there is some questioning on my part as to whether what she considered as a measurement activity always matches with socially agreed meanings as expressed in the documents quoted earlier. To demonstrate this perspective upon her beliefs I have included in Figure 10 the playing on the swing activity which appeared to involve counting, not measurement, but which Cara referred to as measurement.

Cara's transcript excerpts communicate a strong sense of measurement as applicable in the school situation and in other locations such as the home and workplace, most particularly for her parents and herself. There is evidence that Cara attempted to relate school measurement with outside situations such as when she associated using a balance at school with playing netball. Evidence of some appreciation of the application of measurement is found in discussions of matters such as making cakes, shopping, and building. Cara appeared to link measurement and estimation, particularly when presented with situations involving informal measurement, as examined in more detail below.

Estimating and guessing

Prior to discussing Cara'a beliefs regarding estimating and guessing as mathematical activity, the concept of estimating is introduced.

Introduction

A definition of the verb *estimate* includes, "to form an approximate idea of (size, cost, etc.); calculate roughly; an approximate calculation; judgement, appraisal" (Wilkes & Krebs, 1982, p. 379). In forming an approximate idea of size, estimation can be related to many forms of measurement. A relationship between measurement and estimation is emphasised in recent curriculum documents in Victoria, as stated above (Board of Studies, 1995, 2000). The Wilkes and Krebs (1982) definition in referring to approximate calculations suggests also a possible relationship between estimation and number. In the school context, estimation is viewed also as an aspect of the number content area, paired sometimes with mental computation (e.g., Board of Studies, 1995, 2000).

Of interest in this research was any degree of importance accredited by the individual children to estimation, that is, whether it was identified by the children as related to mathematics, with estimation as a concept or process, and if so, whether children provided evidence of believing estimation is related to number and measurement.

Cara's beliefs about estimating and guessing and the relationship to maths As discussed previously, Cara associated the word estimating with maths when responding to the password activity (Task 1.2). Similarly, she mentioned estimation immediately in relation to maths when asked what she would tell an alien maths is (Task 5.1). She stated, "I would tell them what maths, estimating was, I would give them a lesson and then he will get up to times tables and he would learn what dividing by and equals and half millions". This statement demonstrates that Cara did consider estimating and various number concepts to be part of maths. It suggests also a notion of equivalence between maths and estimating, a notion not suggested by Cara in her other interviews.

Cara appeared in the main to see estimating as an element of maths activity. In describing situations in which estimating occurs, Cara included estimating when she has to measure mass in Science (Task 1.1), estimating how much water will fit in a container, in a computer game (Task 5.2), estimating how many times she will swing on her swing (Task 1.3.1), and estimating the number of steps to take in another computer game (Task 10.1). These examples involve the number concept of counting, and measurement concepts, although not necessarily stated as such, nor necessarily involving formal units. A further example, described as estimating when pouring paint in a bucket (Interview 5), was linked to maths initially because of the perceived estimation component, but then questioned as mathematical activity because a *ruler* was not used. This scenario revealed a possible conflict for Cara, as discussed earlier, between the role of estimation and formal measurement in determining mathematical activity; Cara demonstrated some uncertainty within her perception of the relationship between maths, measurement and estimation. The conflict portayed within her beliefs is represented schematically in Figure 11 by the zigzag line.

In contrast, Cara's statement made in response to the alien question suggests that she believed maths and estimation to be equivalent. The unbroken connecting lines represent this equivalence. Cara's *aliens* response was not presented as an example of an estimation or maths activity as such, thus arrows are not used.

Figure 11 makes reference to situations discussed in relation to estimation; only where there is an overlap with situations discussed and presented previously in relation to measurement (Figure 10) are these are included in Figure 11. At the end of the section regarding Cara's beliefs about maths a full schematic diagram illustrating all aspects of the discussion will be presented.



Figure 11. Cara's beliefs about estimation/guessing and maths - Imasge one.

The above analysis of Cara's data illustrates some inconsistencies and uncertainties and suggests that Cara's beliefs were being constructed and were not fully formed at that time of the interviews, illustrated for example by one counting activity being identified by Cara as both measurement and estimation, and another identified only as estimation. Importantly, the interview responses reveal in addition that Cara, a child considered as a Grade 3 lower achiever in maths, was able and willing to think about, talk about, and question her own beliefs.

In an Interview 2 scenario, chosen for discussion with Cara as a number situation because of its place value component, Cara referred only to estimation as a possible mathematical element. Initially Cara indicated that she did not see any mathematical activity but qualified her assessment of the situation by referring to estimating:

Interviewer: Melanie had to tell her teacher which number was bigger 50 or 30. Did she use or do maths?

Cara: She um, no

- I: How come?
- C: Because estimating she could have estimated and wrote it down
- I: And if she estimated was she doing maths then?
- C: Yes
- I: So I'll read it to you again. Melanie had to tell her teacher which number was bigger 50 or 30. Did Melanie use or do maths?

C: No

Cara made clear that if Melanie had estimated she would have been doing maths, indicating a belief that an estimation component would render this situation mathematical. The inclusion of numbers in the described situation did not cause Cara to see a maths component as might have been expected in light of results from previous research (e.g., Cotton, 1993; McDonald & Kouba, 1986).

Cara indicated that it was not only in what had been chosen as a number situation that she considered estimation could render a situation mathematical, but that this could apply also in relation to informal measurement. As discussed earlier, Cara appeared to identify a photograph of a woman comparing the length of a blouse sleeve to her arm, which she interpreted as "she's trying to see what size she is . . . in pyjamas . . . estimating what size she is", as depicting a maths activity because of what she saw as an estimation component (Task 5.2). When an informal measurement by comparison situation had been presented in the second interview Cara did not refer to measurement or estimating; in this case she chose to use the verb guessing. When she was asked whether there would be any maths in the following situation, "Bradley laid his pencil next to his book and he said his book was longer", Cara stated "Yes it would be guessing". For Cara the mathematical status was due to what she expressed as guessing. It is clear that, in these two situations chosen for their different informal measurement by comparison contexts, Cara saw estimation and guessing as mathematical activity. It appears that when discussing these situations Cara may have used the words estimating and guessing with a similar meaning. Cara's use of the words estimating and guessing is explored further below.

Cara did describe one situation as estimating that appears to have a different focus from those presented in documents as discussed above. When shown a Task 5.2 photograph that she described as a person "planting plants" she said at first that there was no maths in this activity. When asked about the possibility of maths when you have to think about where to put the plants, Cara spoke of "estimating where a plant should go in a garden . . . and if you put it somewhere wrong it will grow in a different way and if you put it in the right spot it will be okay". Her meaning in this discussion was unclear to the interviewer, even after attempts at further probing. Cara's comments suggest that she had constructed her own meanings for the concept of estimation, as she had with other mathematical concepts, although perhaps in this case she responded as she did to accommodate the further probing. It is possible that Cara used the word guessing. This view is taken, firstly, as it would seem reasonable for a person to guess an appropriate position for a plant, perhaps taking into consideration its future growth, as Cara said, "to see where it should go or what it equals", and secondly, as on other occasions Cara did relate directly the words estimating and guessing.

The data cited above indicate that Cara did recognise estimating as a mathematical concept or process. It appears that guessing may have been seen similarly. Indeed, Cara's

descriptions and explanations of situations where estimating was occurring reveal a relationship between estimating and guessing that appears to be of significance to her.

Cara made a reference to estimating and guessing in her second interview when she spoke of a game she plays with her father in which he counts and she guesses. She stated, "It's like estimating ... You've gotta guess what does it equal". This example related to number. When I asked Cara to show what she was talking about she drew four circles, each with five lines in it. I asked her to guess how many lines there were. She replied twenty, explaining her answer, "I done circles, I thought, I thought five and five is ten and another five and five is ten so ten and ten is twenty". Her explanation suggests that the method she described as being a guess could be described fairly as exact mental calculation. It does not seem to involve an uncertain estimate or conclusion as does guessing (Wilkes & Krebs, 1982, p. 496). Of course, it may be that only this particular case in the interview situation did not involve guessing as Cara herself drew the circles and lines. Alternately, perhaps Cara thought of this game as a guessing activity to add a fun element. This possibility was reinforced in her fifth interview when Cara spoke of a number guessing game, a fun activity, that she played with her mother to help Cara get ready for school quickly! From these examples we see that Cara held some perception of a relationship between maths and guessing, and estimating. They demonstrate also that Cara believed that the playing of some games could be considered as mathematical activity.

When talking in her ninth interview about playing a maths game on a computer Cara stated, "you have to try and guess how many steps it has to take, so it's estimating" (Task 10.1). In this statement Cara interchanged the words estimating and guessing, suggesting a sameness or similarity between these concepts. Also in describing the activity, Cara reinforced the notion of playing a game as mathematical activity. In her third interview Cara stated that "maths . . . and science are like guessing" in both "you learn also how to do things ... measuring and estimating" (Task 1.3.2). In this case Cara suggested a likeness between estimating and guessing. However, perhaps she was suggesting that estimating was the mathematical activity. Earlier in her third interview Cara wrote estimating and guessing separately on a word wheel that asked "What is maths?", indicating that both activities were considered mathematical and that perhaps they were considered distinct (Task 1.3.1). When discussing her response she indicated that the latter would be an incorrect interpretation; Cara asked the interviewer whether estimating was guessing or whether it was like guessing. After being encouraged to answer her own question she stated "A little bit". Through this response Cara indicated that she considered both estimating and guessing to be mathematical activity, and that she saw a relationship between the concepts of estimating and guessing, but did not necessarily see them as the same. Her interchange of these words when responding to some interview tasks was perhaps unconscious; it was when she stopped and thought about the relationship that she questioned it. It appears that Cara had not made a definitive judgement

on the relationship between estimating and guessing in the mathematical context, represented in Figure 12 by a broken line connecting these two concepts.

It is generally accepted, as indicated in the documents referred to above, that estimating relates to both number and measurement. Likewise, it appears that Cara held a perception of this relationship although she did not always label those situations considered by the researcher as involving informal measurement, as *measurement*. Links between measurement and estimation were apparent in her data but not always acknowledged by Cara.

The evolving relationships within Cara's beliefs are represented schematically in Figure 12 through a collection of examples. This is of necessity a complicated diagram, to communicate the complexity of Cara's beliefs about maths and estimating, and the apparent links to guessing, to the number concept counting, and to a range of socially agreed measurement concepts, but in these situations not always directly voiced as such by Cara. As stated above, the broken line indicates that Cara had not made a definitive judgement on the relationship between estimation and guessing in the mathematical context.



Figure 12. Cara's beliefs about estimation/guessing and maths - Image two.

Whereas the word *estimating* was at times interchanged with *maths* or with *guessing*, as demonstrated in some of the above examples, and in Figure 12 through the inclusion of the *aliens* and *capacity - water* examples, at one time in her third interview Cara implied a similarity or sameness of the words estimating and measuring: ". . . I had to measure how many times I could swing on it, estimating". As discussed previously, Cara made an association between maths and measuring, the data now indicate a link between maths and estimating, and measuring and estimating. Cara associated also estimating and guessing in the general context of mathematical activity. It appears that a range of Cara's responses illustrate a circle of associations and interchange of terms related to maths, as illustrated in Figure 13.



Figure 13. Cara - an apparent circle of word associations involving estimating/guessing and maths.

The associations are demonstrated further in Cara's description of a photograph within Task 5.2: "She is using maths. She's playing the computer and she's using maths to see how much water will fit in then she has to try and guess . . . it's estimating". From this quote, that includes the words maths, guess and estimating, and that implies the use of measurement, and from the above discussion, it can be concluded that estimating, guessing, maths, and measuring were for Cara related concepts. However, Cara appeared not to have established firm or consistent beliefs about relationships between these concepts. The circle of associations involving estimating and maths (see Figure 13) became apparent from some responses but Cara was not always consistent in making these associations.

Cara expressed also a perceived link between guessing, estimating and chance situations. For example, when shown a Task 7.2 video snippet, in which a class of children was refining estimates in a chance situation involving coloured objects hidden in a bag, Cara stated, "They were estimating and it's maths. Trying to guess what was in the bag . . . estimating is guessing for maths". Cara demonstrated again that she saw estimating and guessing as components of maths. She appeared also to suggest that there are different types of guessing, and that within maths estimating fills that role. Thus Cara expressed a belief of a direct relationship between estimating and guessing, with mathematical estimation possibly being seen as a subset of guessing. Other data as referred to above indicate that this was perhaps a partially or temporarily formed view. However, it may explain the instances in which Cara interchanged the words estimating and guessing and therefore is a plausible possibility within her set of beliefs about maths.

On some occasions Cara spoke of guessing within maths without mentioning the words estimating or measuring. This occurred, for example, when Cara was read two separate chance scenarios in which two girls were reaching into their lunch bags that in the first case held "three yellow lollies and five red lollies" and in the second case held "more red lollies than yellow lollies" (Task 3.3.1). Cara was told that each girl thought she had a better chance of getting a red lolly than a yellow lolly. Cara judged the girl in the first scenario as using or doing maths ". . . because she was trying to guess she was going to get a red" but the second person was not doing or using maths because ". . . she really wanted to get a red one. First person didn't care". For these scenarios, Cara's decision to classify an activity as

mathematical seemed based on her interpretation of whether the situation involved guessing. The guessing element may have been identified in the first case and not in the second because the first girl knew the number of each colour lolly in her bag. Cara seemed also to interpret the first scenario as having a cognitive element and the second an affective element. The perception that the second girl really wanted a particular colour, rather than focusing on guessing what colour she might get, seems to have caused Cara to consider only the first of these situations as mathematical. Perhaps Cara was implying that she believed one cannot influence the outcome in such a situation and therefore that for the person's actions or thoughts to be considered mathematical it is appropriate for the person to guess, but not appropriate for the person to take a personal interest perspective through caring about the outcome. As there was no discussion of affective factors in relation to Cara's judgement of whether mathematical activity was occurring in other situations, no further insights were gained into this aspect of her beliefs.

An emphasis on guessing in maths, with no mention of the words estimating or measuring, occurred also on other occasions. In the following instances, which involve number contexts, Cara's emphasis seemed to be on guessing the right response or right answer, implying that for Cara this may have been an important aspect of maths. For example, as discussed above, in a game Cara played with her mother she had to guess the number her mother had thought of (Task 5.2). Within Cara's explanation of how she would tell an alien what maths is (Task 5.1), she stated she would give a lesson about estimating and times tables, she would give him a sheet, he would try to guess a number, and "if it's right he had a lucky guess". In this statement Cara implied that getting the right answer through guessing is an element of maths, and one that can depend on luck. Cara's perception of getting a right answer as a result of luck related both to the alien (Task 5.1) and to herself (Task 10.2.1). Her focus on getting the right answer in number contexts through guessing was demonstrated also in other interviews. For example, in Interview 3 Cara stated that times tables are like maths, she explained, "you have to guess what the answer is" (Task 1.3.2).

As illustrated above, it seems, for Cara, that guessing or estimating can render an activity mathematical. In Figure 12, the second image of Cara's beliefs regarding estimation/guessing and maths, it was shown that, for Cara, estimation can relate to situations involving number and measurement concepts. Image three (Figure 14) adds the chance concept to this portrayal of Cara's beliefs. The discussion of the lolly situation revealed a belief of an association between chance and guessing and maths.

While at times Cara interchanged the words guessing and estimating it appears that she did not necessarily consciously consider these to have exactly the same meaning but that perhaps she saw estimating in maths as a subset of guessing. She did appear to believe there is a relationship between estimating, guessing, and maths. Cara's data revealed to the researcher beliefs linking estimation and measurement but these connections were not always

acknowledged by Cara, particularly in the case of informal measurement situations which Cara did not tend to classify as measurement.



Figure 14. Cara's beliefs about estimation/guessing and maths - Image three.

The discussion of interview excerpts has highlighted language usage by Cara that suggests a schematic linkage between a number of terms. Figure 15 expands the structure presented in Figure 13, with a difference being the direct linking of guessing and maths.



Figure 15. Cara - an expanded circle of word associations involving estimating/guessing and maths.

In this discussion of *estimating* and *guessing* in maths, it has become apparent that while Cara saw these two concepts as related, she did not believe that they were necessarily the same. She did see them at times, however, as mathematical activity.

Maths as answers

In examining Cara's use of the word guessing, the phenomenon of maths as getting answers became apparent within her beliefs. In fact, it seems that it was in attempting to get an answer that guessing was deployed mainly, particularly when problems related to number. A range of further interview responses indicate that Cara gave emphasis in some number activities to the getting of an answer, but without the guessing element. Due to the absence of reference to guessing, the discussion of these responses is separated from that above.

When Cara was asked in Task 3.3.2 whether and to what degree she would consider there to be maths in the activity of using a calculator to work out money to pay the bank, she said there would be lots of maths "cos if you calculate something up it tells you the answer". The item of "planning a two week holiday for a family" was considered to have some maths in it "because it's telling you how much money you should spend . . . and how much you shouldn't so you're timesing it together and see how much it equals to go". Using a calculator to add nineteen and four was considered mathematical "because she was equalling". In each of these varied examples of everyday activities, Cara saw maths activity. She gave focus to the final result or answer in explaining the reason for each of these to be considered a maths activity, with statement or implication of number calculation also.

When asked whether she believed the person was using or doing maths when "Terry went to McDonald's and she paid \$3.20 for a hamburger and coke" (Task 3.3.1), Cara replied, "No, or, let me think she bought . . . No . . . Maybe because if, if you had \$3.00 only left and she asked for \$3.20 I wouldn't be measuring I wouldn't be doing things, you wouldn't be like, maths is like measuring and equal and take away and times but she wasn't equalling". In this statement Cara once again gave evidence that the breadth of concepts she considered to be maths included measurement and number. The inclusion of numbers in the situation did not render it mathematical, but Cara indicated a belief of an importance of equalling. Perhaps she was referring to getting answers such as through use of one of the four operations.

Cara's tendency to consider the answer as an element of a mathematical situation involving number was evident also in her interpretation of two photographs. The first was of a classroom situation (Task 5.2), the second of a home situation (Task 8.2).

In response to the first photograph Cara stated, "They're writing times tables so they're doing maths", showing that she considered a multiplication tables activity mathematical. In an attempt to make clear Cara's belief about the mathematical activity she was asked whether it was the teacher or the children who were doing maths. She included the teacher "Cos [he's] writing the answer down too". It is possible that Cara considered that if the teacher in the photograph had been writing the number problems only, and not calculating the answers, he would not have been undertaking mathematical activity. If this was her interpretation, it could be seen as a sophisticated view for a young child; a view that maths is a cognitive or thinking activity, not just a recording activity. We cannot be certain that this was her interpretation.

To clarify whether Cara considered the getting of an answer in such a number activity to be essential for the activity to be considered mathematical she was asked, "So if you didn't write the answer would you be doing maths, if you just wrote the sums without the answer?". She answered, "Um. No. I don't think so anyway". This response gives some support to the proposition that Cara did not consider mere recording of mathematical symbols or algorithms as constituting mathematical activity. It indicates also that Cara saw the calculating of

answers as mathematical activity. However, it does not discount the possibility that other activity can be considered mathematical, thus it does not contradict other findings from the range of procedures completed with Cara.

In response to a Task 8.2 photograph of a girl using a pen, paper and calculator at her desk at home, Cara stated, "She's using maths. She's definitely definitely definitely using maths. She's writing down ques, answers there [*sic*]". In this case Cara pointed very strongly to a belief that the getting of answers in a number activity is an example of using maths. Her correction in the final sentence gives emphasis to her belief that answers are an important element of maths.

Gaining answers sometimes through calculation, sometimes through guessing, or possibly through luck, was considered mathematical activity by Cara. Her responses indicate further her belief that cognitive activity is an element of mathematical activity. The discussion of responses regarding getting answers adds to the data which indicate that number was a significant aspect of maths for Cara. Figure 16 provides a schematic overview of the *number* concepts considered by Cara as part of maths, with sample activities included where available from interview data. This figure will be linked to the previous figures highlighting estimating, guessing, and measuring at the end of this section. At present the focus is on Cara's beliefs regarding number concepts as maths.



Figure 16. Cara's beliefs about number and maths.

Cara identified a range of number concepts when discussing maths, as demonstrated in Figure 16, some of which were in response to interview items that described a number-specific situation. The few situations that she proffered from her own experience in which number was the main mathematical concept appeared to stem mainly from a school-related purpose, contrasting with her responses regarding measurement, that stemmed more from a non-school purpose than from a school-purpose, as considered further below.

The purpose or intent in using or doing maths

From Cara's descriptions of mathematical situations in school and non-school environments, three categories of intent become apparent for using or doing maths: for an everyday or non-school purpose, for a combined school/non-school purpose, and for a school-related purpose. An examination of Cara's perceptions of mathematical activity in different environments and

for different purposes involves further consideration of her perceptions of concepts that are mathematical.

Maths for an everyday or non-school purpose

Previous discussion illustrates that Cara identified a range of situations, of her own experience and of other people's experiences, as mathematical. Cara was not hesitant in considering the possibility of mathematical activity in everyday or non-school contexts. For example, she talked of her parents and others measuring when cooking, and of her father and her measuring when building a cubby (Interviews 2, 3, 4, & 8). When shown photographs in Task 5.2, she saw maths activity in weighing fruit, weighing a fish, using a "ruler" (tape-measure) to measure piping, laying tiles for a footpath, and estimating one's size in pyjamas. Other than for the final of these scenarios, in which Cara identified the maths as estimating, the maths in these non-school contexts was described in terms of measuring. Cara also identified maths activity in number contexts such as counting when putting stickers on pages (Task 3.3.1) and counting the number of people when planning a birthday party (Task 4.3). The maths activity in each of these scenarios appeared purposeful for those involved.

Cara showed also that she was able to select situations she considered not to contain maths activity. She was a discriminate judge, in terms of her own beliefs, when considering whether a situation contained maths activity. For example, the following, presented through photographic representation (Task 5.2) and described by Cara, were seen as having no maths in what the people were doing: playing chess, tightening a pipe (scaffolding), looking at flowers, blowing out candles on a birthday cake, shopping, playing drums, and hanging paintings. It was anticipated that the birthday activity may have been considered mathematical because of the link with a person's age. However, Cara paid attention to the children's activity in the manner that she possibly had experienced it. She related the photograph to the purpose in blowing out birthday candles, "blowing out a candle is a birthday wish", not to the purpose of mathematical activity.

Seemingly in contrast to Cara's judgement of the Task 5.2 photograph involving shopping, Cara did at times see working with money as mathematical activity, as implied within the discussion above of the theme Maths as Answers. Money situations identified as mathematical included working out the money to pay the bank (Task 3.3.1), planning a two week holiday (Task 3.3.1), and buying clothes at a sale (Task 3.3.2). These were seen not only to involve use of money but were believed by Cara to involve calculation. She talked about "timesing" and "getting the answer", processes not mentioned in the discussion of the shopping scenario in Task 5.2. Cara described the Task 5.2 scenario as, "the lady's shopping ... paying that lady money" but added "she's going to collect her shopping and bring it home". It appears that Cara may have been focusing on the management of the articles purchased rather than on any monetary transaction. It seems it was only when Cara perceived calculating to occur that she considered an activity involving money to be mathematical. In other words, the presence of cognitive mathematical activity, not just of mathematical tools or

symbols in the form of notes or coins, rendered a money situation mathematical. This distinction by Cara, a young learner of mathematics, demonstrates her ability to reflect upon situations and make judgements based on beliefs she had constructed. Cara's response to these situations, for example, suggests also that she believed number to be a relevant and useful maths concept in everyday, non-school activities. In addition, Cara's response to these situations suggests that the schooling process did not dominate her perception of what constitutes maths activity.

The photographs of tile-laying for the footpath and placement of pictures on a wall (Task 5.2) had been chosen for their possible spatial attribute. Cara did not identify these as spatial activities. In the tile-laying scenario, a non-school context, she did interpret the activity as tile-laying but identified "measuring how big or small" as the maths. Once again she showed her belief that maths, in the form of measurement, can be used to assist in performing a task unrelated to the schooling process. When shown a video clip (Task 7.2) of children creating and discussing shapes on geoboards, Cara recognised that what the children were doing was "about shapes" but said that it was not maths because "they weren't measuring or doing things, except doing a shape". It seems she did not believe school activity related to shape to be maths. Perhaps Cara did not consider shape activity to be purposeful in the same way that measurement, estimation, or number calculation could be, and therefore was less likely to consider it as maths activity. The presence of a shape component in an activity, whether in a school or a non-school situation, did not seem to be instrumental for Cara when considering its possible identification as mathematical activity.

Two scenarios presented verbally and without an identified setting (Task 3.3.1) also challenged Cara's perception of whether activity with shapes was mathematical. When given a scenario involving identification of shapes, that is, of a child finding squares in the pictures on a page, Cara said there was no maths in what the child was doing. In response to another scenario, of a child cutting out squares and circles and making a design, Cara stated that he was doing things with shapes. She considered this maths, but clarified her statement by adding, "like how long he wants the straight lines to be, like one little line and one big line". It appears that while she stated that the shape activity was maths, she actually gave attention to the attribute of length. This concurs with her reaction to the tile-laying and geoboard situations discussed above where measuring was referred to as maths activity.

It appears therefore that while Cara could recognise a spatial activity, generally she did not label the spatial element as maths activity. This was the case both for non-school and school situations. In some scenarios provided for their potential spatial component, Cara displayed a tendency to find a measuring component, and labelled this as maths. This response is not surprising considering her inclination often to associate measurement with maths, as demonstrated, for example, in her replies to the automatic response type tasks presented in Set 1. Cara's interpretation of, and responses to, the scenarios chosen for their potential spatial component, inform us not only about Cara's beliefs but indicate the importance in research such as this of first gaining from participants their interpretation of what is happening in a situation, whether the situation is presented in verbal, pictorial or written form. The participant's belief as to whether the situation, as it is interpreted, involves maths, should follow the personal interpretation.

The above discussion has demonstrated that Cara saw maths activity contributing to needs in real life situations; it is clear that maths for an everyday purpose is an appropriate summary of one aspect of Cara's beliefs. Within the theme of the intent or purpose in using or doing maths I have identified maths for an everyday or non-school purpose as one category of intent. The other two categories of intent are discussed below.

Maths for a combined school/non-school purpose.

When asked to clarify her beliefs by giving examples of actual situations, Cara appeared to refer less often to maths in the school environment than in a non-school environment. When she did talk about school located maths activities she appeared to show a school-related purpose with little, if any, meaningful link to outside activities or purposes. Some maths experiences described in her home environment showed also an element of school-related purpose, that is, the activities were related to Cara's schooling. The following discussion considers the maths situations she offered in which it appears that school and non-school purposes are combined, and is followed by discussion of maths situations that appear to have a school-related purpose only.

The second category identified within Cara's responses, of an apparent combination of maths for an everyday purpose and maths related to school, is potentially problematic as aspects of the examples might be considered a little contrived or fantastical. The discussion of the data is included as it is not the place of this research to be judgemental or selective in the sense of discarding data due to the possibility of it not representing real life situations. The purpose of the research is to investigate and attempt to present the child's perspectives as an insight into the child's beliefs. Therefore it is important to present the child's reality as it was expressed in the interviews and to make meaning from this as best as possible.

When asked in her second interview to show a second maths situation (Task 3.2) Cara wrote about her chosen situation. She read from her paper: "Home. I was playing when my dad called me. 'Can you measure this?' 'Yes'". Her statement suggests purely a measurement activity, but further description of the situation indicates that it not only also contained a computation element but that Cara perceived this to relate to learning maths for school:

I was measuring a wall 'cos my daddy was trying to build a cubby house for us and he asked me what's one times one and he asked me all these sums and then he told me to measure like one times one equals one. So I had to put like, I had to measure one cardboard and then put it up and then I had to write sums on the cardboard . . . like homework
The building of a cubby house utilised maths for a direct and relevant purpose in Cara's non-school life. Cara indicated that she saw a purpose to the maths; she stated she had to measure the cardboard "so [the cubby] would fit the tree". However, in this situation, everyday maths and school maths appear to have become intertwined; her mathematical activity appeared to have a dual purpose. Cara indicated that not only did she measure, but also that she computed in a school-like manner. It seems that a part of her description of the computation was illustrative only, as Cara may have found difficulty in measuring "one times one equals one". The choice and description of activity suggests that Cara did not always see a distinction between maths performed at home for a practical, everyday purpose, and maths performed at home for a school purpose.

A further example of the phenomenon was apparent later in the same interview. Cara had introduced to the conversation the idea of her father making birthday cakes. I asked whether he used any maths at that time. Cara replied,

He gets a jug that goes up to 150 and he puts the cream in and then he measures it and he asks me to do a times table when he pours it into this special thing that makes it come out all curly . . . And when he does that I go one times one and he goes it's one and then he draws a one in curly writing on my birthday cake

In responding, Cara began by talking about measuring cream with a jug, a seemingly realistic and purposeful mathematical activity. However, Cara spoke also of herself and her father doing "a times table". From Cara's description, it seems this activity may have been seen by her father as a time-filler, as related to the number to be written on the cake, or as specific practise of number facts. The fact that the example was "one times one", as given at other times in interviews, for instance as in the cubby house anecdote cited above, suggests once again that this was illustrative and that the number problem did not necessarily relate always to the number to be written on the cake. A later statement supports this interpretation: "When he does fake [birthday] cakes that he wants to throw away he writes on them sums and goes one plus six hundred or something like that". Cara did not indicate any reason for the choice of this particular problem; perhaps once again it was illustrative. It is clear that the answer to the problem would not be a person's age, thus suggesting that the purpose of the activity was for time-filling or for practise of automatic response of number facts. My impression is that it was more likely the latter. Importantly, the telling of this situation where maths for measurement in cake-making and maths for sums were combined suggests the merging in Cara's mind of real-life maths and school-related maths activity.

Cara's description of cake decorating combined with a times table activity, and her description of cubby building combined with a times table activity, suggest that she was not always able to differentiate between maths for a non-school purpose and maths for a schoolrelated purpose. The descriptions suggest that Cara's father encouraged her to practise schoolrelated maths at home. It is possible that Cara described these activities in a fanciful way, but, as stated above, it is approriate to take Cara's descriptions at face value rather than judging them for their degree of realism. Thus, from these scenarios, it is reasonable to conclude that maths activity was undertaken for a useful purpose in everyday tasks, but that the activity was at times combined with the practise of school-related maths. As a result, Cara had a tendency sometimes to combine the maths for the two purposes.

Maths for a school-related purpose

Although much of the discussion so far, particularly regarding Cara's descriptions of measurement activities, may lead to the impression that schooling was not a dominating factor in Cara's beliefs about the nature or purpose of maths, some interview responses, as discussed below, indicate that Cara did associate schooling only with some mathematical activity.

During her second interview (Task 3.2), Cara spoke of a home situation, one in which it appears that the maths had essentially a school-related purpose. Cara spoke of her father making up sums, what she called [......] (the country of her family origin) sums. Cara stated,

he writes all these [......] words down and I have to write them in my book and then he writes [.....] sums down and then he writes English and then I learn them into my homework book and then in class we have to learn them and I read them out

Although the activity occurred at home, and contained an element that was related more closely to the family than to school, that is the labelling of the sums firstly as [......] sums, the activity essentially was a schooling activity. It appears that it was for the purpose of knowing sums at school that her father engaged Cara in this activity. Although this activity is reported to have occurred at home, it appears to fit best into what has been identified as Cara's third category of intent in doing or using maths, that is, maths for a school-related purpose. Cara's reference in her third interview to learning times tables as homework fits also into this category.

A range of activities was spoken of or identified as maths at school, both as part of the maths and science curriculums. For example, Cara talked about using scales for measurement activities, measuring height with tape measures, rolling dice and doing "times, . . . plus, . . . and divided by", playing a computer game and estimating water to fit, writing times tables, "putting two MAB (Multi-base Attribute Blocks) pieces together and seeing what they equal", "breaking bread . . . dividing it together and seeing how much there is" (Sets 1, 8, and 5, Interviews 3, 4, and 5). These responses make reference to the study of number and measurement concepts in the school environment, indicating some breadth to Cara's concept of mathematical activity at school.

In her eighth interview Cara was asked to complete a sentence: "Maths is like" (Task 4.1). She read from her written response: "times tables, pluses, that's all". This response, which may have related mostly to school maths, is somewhat limited in terms of many of Cara's other interview responses, suggesting the possibility of drawing wrong inferences if a child's response to one task is taken at face value. When probed further, Cara added to her response only "divide by", a seemingly school-related term. However, when asked whether she thought other people would write something different, she replied in the affirmative, suggesting an acknowledgment of differing experiences and perceptions and a possible broader idea of maths. In the activity that followed (Task 4.2.1), Cara was asked to draw a person using or doing maths. She remarked that it was hard, but then drew a situation in which the child who sits next to her in the classroom was answering a question asked by the teacher (see Figure 17).



Figure 17. Cara's portrayal of a school maths situation.

Cara made few references to her own experiences of the teaching and learning of number concepts at school. It is notable that in responding to Task 4.2.1 Cara chose a school situation when not specifically asked for one, and when the choice obviously caused some difficulty. Cara indicated in other interviews that she associated maths with a range of non-school and school activities, especially in the area of measurement, but did not choose to recount one of those on this occasion when she was asked to draw someone using or doing maths.

Cara's difficulty in her eighth interview in identifying a maths situation continued in Task 4.2.2 in which she was asked to draw a picture of someone using or doing maths, but not at school. She replied, "I don't know anything" and when encouraged further said "I just can't think of anything". When asked about places to which she went, she suggested restaurants, dancing, and another person's house, but did not think that the children or the adults in those situations used or did any maths. These responses suggest that Cara associated maths with school activity and not with activity in non-school locations. This finding was somewhat unexpected considering the number of non-school maths situations she spoke of at other times. It is noted that during this conversation in the eighth interview neither Cara nor the interviewer made reference to her home situation, a situation mentioned a number of times in other interviews. It is possible that had Cara or the interviewer done so, the outcomes from that interview may have been different, and would have provided insights to clarify her beliefs. We do not know whether she considered home as a possible place for maths activity when addressing this task.

The findings indicate overall that Cara did associate maths with school, but that while this was a dominant view during her eighth interview, it did not appear the dominant view through the whole data collection period. Cara did not make clear a reason for the Interview 8 responses but it is possible that they were associated with recent experiences or with developing views. It is noted that by the time of Cara's eighth interview, Applied Maths classes, taken by Ms A within a rotation program, had ceased for the year, and all maths lessons were taught by Ms S, the classroom teacher. Of the two in-class maths lessons taken by Ms S and observed at around the time of Cara's sixth and seventh interviews, the first included counting, and work on decimals and area, and in the second, counting, decimals and the calendar were studied. These two lessons were representative of the lessons taken at that time, they were taught as part of the planned program and were not structured especially for my benefit as an observer. The content of these lessons supports the possibility that number work was given more precedence than other concepts at that time. With school mathematics experiences no longer giving specific emphasis to measurement, or Applied Maths, through a part of the program labelled as such, and with a seeming emphasis on number, it is not surprising that Cara was seeking to relate number experiences when asked about maths at that time. In contrast to her propensity in earlier interviews to talk about measurement situations when talking about out-of-school experiences, it seems that a conflict or difficulty arose when attempting, in her eighth interview, to select maths experiences of the non-school type. With the decreased coverage of measurement in the school curriculum, Cara may have been less likely or even have felt less confident to refer to activities in this area, whether they were school or non-school located activities.

Further possible insight into Cara's attention in Interview 8 to number in maths, is provided from Cara's responses in Interview 10, in which she was asked to give a personal definition for maths (Task 2.3). When asked what meaning she would put for the word maths, she replied,

Cara: Times tables

Interviewer: Mhm

C: Pluses, dividing by, that's all

- I: That's all? Nothing else? That would be enough, that would give the meaning of maths?
- C: Yes, they'd go and look for times tables and then they will know, oh that's maths.
- I: What about when you do maths? What about in . .
- C: Measuring
- I: Yeah, what about in your head when you do maths?
- C: Memorising, thinking

Cara included reference to number concepts and measurement in her definition, with the latter added only after probing. It appears that Cara considered times tables would act as a key indicator in determining a mathematical situation. However, her points added after further questioning indicate that her initial response did not eliminate the possibility of other concepts and processes being considered mathematical. It appears that when asked for a meaning for maths, she interpreted the instruction as asking for a key or defining element of maths

activity. Her immediate reference to a number concept, which appears associated with the learning of maths in the school environment, and her reference to measurement, memorising, and thinking when she was asked to think of her own activities, suggests a tendency to view maths from different perspectives, depending upon the question asked and perhaps upon the setting envisaged for the mathematical activity.

The development and breaking up of the theme of maths for different purposes into subthemes focused around school and non-school purposes, occurred during data analysis; this theme had not been anticipated. The discussion shows that Cara recognised that she undertook mathematical activity in a range of environments, on some occasions associating maths with school activities, but that at other times referring more readily to non-school maths activities. She discussed also some activities in which school and non-school purposes seemed intertwined. This variety of responses by Cara, related to the purpose she associated with maths activity, reiterates that she had a broad concept of mathematical activity. It suggests also a possible segmentation of maths, an idea explored in more depth and in relation to other perspectives regarding maths, below.

Segmentation of maths

Further segmentation of maths within Cara's beliefs becomes apparent through considering content and action perspectives and number and measurement perspectives. Cara's responses are discussed also for the degree to which they suggest she held a personal affinity with maths.

Maths as content or action

A demonstration of the action and content perspectives from Cara is provided in the personal dictionary (Task 2.3) quotation from Interview 10 cited in the preceeding section. A content emphasis seemingly was given preference in the initial act of defining maths as "Times tables Pluses". In stating that times tables can help one know a maths situation Cara indicated that this was predominantly an identification or naming exercise through which she signified an abstract or out-there view of maths in which maths is separate from real-life experience. The second perspective, of maths as action, also is apparent in the quotation. This entails personal involvement, that is, activity in a physical or cognitive sense, stated in this case through the use of the verbs measuring, memorising, and thinking. The two perspectives are discussed further below.

Cara at times gave emphasis to the action perspective. For example, as discussed earlier, automatic response type questions regarding maths elicited from Cara the replies of "estimating" and "measuring" (Tasks 1.1 and 1.2). Likewise in her Task 1.3.1 maths word wheel response (Figure 18), Cara gave focus to activity:



Figure 18. Maths word wheel - Cara.

Cara read her word wheel response: "Measuring, estimating, playing with things, guessing things, using things, doing things, games, learning things like maths". Cara portrayed maths as activity-based, involving both physical and cognitive action.

Measuring and estimating, or guessing, were activities Cara spoke about freely when talking of her own mathematical experiences. These appeared to be a part of her personal life in a non-school environment, and at times in a school environment, and seemed to be alive and of personal relevance to her through her own activities and those of her parents. It is hypothesised that Cara held a degree of personal affinity with maths, influenced largely by her use and familiarity with measurement in her own and her parent's everyday lives, but perhaps influenced also by her Applied Maths lessons in which the teacher, Ms A, reported that she tried to draw on the relevance and use of maths in the real world in each of her lessons (conversation, November 16). However, such activity-based maths with personal relevance was not always portrayed by Cara.

At times Cara appeared to see maths in a more abstract sense, as a content-based entity, indicating that her apparent affinity with some aspects of maths such as measurement and estimation, did not relate to all aspects of what she considered as maths. For example, when giving a definition type statement, Cara chose sometimes to list concepts or topics in maths. The personal dictionary Interview 10 response cited above of "times tables, pluses, dividing by" is one example of this. Cara gave the impression that from this perspective, maths was something to be done, something separate from her, something with which she did not always hold a personal association. Likewise, when talking about her own involvement when learning maths at school, she stated that the children "work, do times tables, and then we learn them, and homework" (Task 1.2). The concept of work appears very different from many of the maths activities Cara undertook with, or observed of, her parents; she gave the impression

that maths as work at school entailed less personal involvement, relevance or fun. Children's focus on maths at school as work, as opposed to learning, was found in previous research by Cotton (1993) and therefore some indication of this was not unexpected. Cara appeared to believe that work in maths was followed by learning, most particularly the learning of number facts.

The examination of Cara's definition-type responses under a content/action lens has revealed, that in defining maths, Cara at times gave focus to maths as action, and at other times gave emphasis to maths as content. In addition, the discussion of Cara's data so far reveals that she appeared to link action mainly with measurement ideas, and content with number concepts and processes. Through measurement activities, for example, *maths as action* seemed to be related to Cara personally in a way that was not apparent for *maths as content*. The latter seemed more divorced from her personal experiences of the use of maths in real-life situations, and related largely to number concepts, mostly for school-related purposes. The above discussion reveals a possible segmentation of maths, built around content and action perspectives, and seemingly linked mainly to number and measurement aspects of maths. In defining maths, as discussed below, Cara tended to make reference to measurement and/or number concepts and not others.

Number and measurement

Although tending to refer, when defining maths, to number or measurement concepts, Cara showed some inconsistency in choosing one or both of these aspects. The number and measurement perspectives were visible also within many of the maths situations Cara proffered, or identified, as mathematical in interviews. Cara's apparent number/measurement segmentation of maths was perhaps related to her school and non-school experiences in maths.

As demonstrated in Figure 18, Cara made reference, in her Interview 3 (Task 1.3.1) word wheel response, for which she had been asked to define maths, to measurement only of the commonly recommended main content components of the primary school mathematics curriculum: number, chance, measurement, data and space (e.g., Board of Studies, 1995, 2000). Some focus on measurement is not surprising in light of the number of times Cara referred to measurement in discussing her own maths experiences. The absence of specific reference to other maths concepts, particularly number, that were referred to, or identified, at other times in interviews, is notable.

Other tasks that similarly asked for, or prompted, a definition type statement did not always elicit the same measurement-focused response. For example, when talking of a situation of a person buying food at a McDonald's store, Cara gave a definition type statement for maths in which she chose to refer not only to measuring but also to number concepts and equalling (Interview 2, Task 3.3.1). In addition, in her discussion of informing an alien of what maths is (Interview 5, Task 5.1), Cara made no reference to measuring, she talked mainly of number concepts: "estimating . . . times tables . . . dividing by and equals and half

millions, . . . and applied maths" which she thought was something to do with halving. When asked whether this would be enough for the alien to know what maths is she added "pluses and taking away, and he'll know enough by then".

It is clear that measurement and number were the concepts within maths that Cara referred to most when defining maths. Her tendency to give different emphases in different interviews may have been related to Cara's possible affinity with certain maths activities, as hypothesised above, and to her school and home experiences, that she perceived as mathematical, or that were given emphasis at around the time of each interview.

The structure of the mathematics program at Cara's school may have had influence upon her responses. Firstly, it appeared that number and measurement were given some emphasis in the mathematics program. Measurement, space, and chance and data were taught in the Maths Task Centre by Ms A, the Applied Maths teacher, as part of a weekly rotation program that operated for most of the year but with some interruptions (Conversation with Ms A, November 16; & Interview with Ms S, December 11). Ms S also incorporated measurement into some classroom mathematics lessons, as observed in the lessons discussed in Chapter 4. However, Ms S gave focus to number concepts in the classroom and saw these as important. For example, when asked what she considered as the big ideas for Grade 3, Ms S referred to "understanding number" and added "logic, number, place value ... patterns ... tables ... relationships in numbers" (Interview, December 11). It appears that in the maths lessons Cara experienced, number and measurement were given emphasis with some coverage of other areas such as space, chance and data, and problem solving.

Cara saw both Ms A and Ms S as her maths teachers but appeared to see the maths taught by each as distinct. In Interview 4 (Task 8.1.2), Cara referred to the two teachers: Interviewer: Who teaches you for maths?

Cara: Ms A

I: All the time?

C: Yes. But sometimes Ms S teaches us

Cara's response suggests that she saw Ms A, the Applied Maths teacher, as her main teacher of maths.

It is possible that this response linked not so much to the teachers themselves but to the maths they were teaching. As demonstrated in the discussion of Cara's meaning for maths, Cara indicated that she perceived she learnt about a range of maths concepts at school. This was reiterated also in Interview 4. Cara spoke of her learning of times tables and pluses and, later in that interview referred also to measurement activity at school, as in the continuation of the conversation quoted above:

- I: And do you do the same sort of maths in the Task Centre or different sort of maths?
- C: Different. It's estimating and measuring. And in class we do like times tables, take away, equals, and all these things

This quotation suggests again a segmentation of maths by Cara, a segmentation in which number and measurement concepts dominated, a segmentation that may have been

influenced by the school curriculum. Noted is the absence of reference to spatial activities, a mathematical area seemingly absent from Cara's perception of maths.

Later in that interview Cara gave emphasis again to measurement as maths; she stated that a situation was not maths because "she's building something, not measuring something". When asked the question of what it would need to be, to be maths, she responded: "measuring, estimating". It appears that in this interview Cara was indicating that while she saw multiple aspects within the discipline of maths, when trying to identify whether a situation contained maths activity she would look first for evidence of activity she would classify as measurement. This contrasts with her Interview 10 response in which she seemed to see times tables as a key factor for determining whether a situation was mathematical.

Cara's response regarding Ms A as her maths teacher indicated the possibility of a tendency more readily to associate measurement than number with maths. In view of Cara's responses to a range of tasks, such as those in Set 1 and Set 8, it is not surprising that she nominated Ms A, her teacher of measurement (Applied Maths), as her main maths teacher. Many other interview responses indicated also the saliency of measurement for Cara. For example, when talking of maths at home and in her father's workplace, and when identifying maths situations, many of Cara's examples related to measurement. Some of her definition type statements called also on these aspects of maths activity. When asked about the possible presence of maths in scenarios presented in Task 3.3.1, Cara at times chose to refer specifically to measurement rather than talking of the broader concept of maths. Earlier in the same interview confusion resulted from my asking Cara to draw someone doing a mathematical activity (Task 3.1). As discussed in more detail later, she associated the task instruction with gymnastics, but from my responses came to see that I had intended her to think about maths. In changing her account of the situation, seemingly to meet my expectations as the interviewer, she chose to speak of a measurement situation, the measurement of height with "a masking, er measuring tape". She might have chosen any maths situation.

Like Cara's apparent tendency sometimes to call automatically on measurement as maths, so too she may have called automatically on Ms A, her teacher of measurement concepts, as her main maths teacher. While measurement was not the only concept Cara saw as mathematical, it clearly was important and relevant in Cara's life. As discussed earlier, it appears that it was in the later interviews that Cara gave more emphasis to number concepts as maths, particularly in the school context, seemingly linked to the absence of lessons, taken by Ms A and labelled as measurement, at that time of the year.

In the school context, Cara appeared to conceive number and measurement concepts separately, perhaps mirroring the structure of the mathematics program and teaching at her school. Cara's final statement in the Interview 4 quotation above, in which she recounted what happened in the two school locations, the classroom and the Maths Task Centre, could be seen also as a statement differentiating the activities not only according to the

mathematical concepts, but also according to their relationship to the learner. Once again, in talking of measurement she appeared to indicate action by the learner, whereas the words Cara chose when talking of number were more content oriented, suggesting that she saw this facet of maths as an object separate from herself.

Segmentation of maths - a summary

Within the theme Segmentation of maths, two sub-themes evolved. From the discussion of the first of these, Maths as content or action, and from earlier discussion of Cara's responses, a number of conclusions regarding Cara's beliefs about maths are suggested. Firstly, it appears that Cara's beliefs involved multiple perspectives. For example, she demonstrated that she saw maths as physical activity through responses such as "Measuring", "Playing with things", and "Using things", as cognitive activity through responses such as "Estimating", and "Guessing", and as content as expressed in responses such as "Times tables", "Pluses", and "Equals". The discussion revealed that in the main Cara appeared to link number with content and measurement with action. Cara's beliefs contrast in part with those of children studied in previous research. In addition, earlier discussion showed that Cara perceived a range of purposes in doing or using maths in both school and non-school situations. In many situations at school, or related to school, she portrayed maths as something she learns to do, or to know, such as pluses or times tables. In some other situations maths was portrayed as applicable to real life as, for example, when measurement was used to achieve some end such as baking a cake. Cara did not associate maths only with schooling or as a learning activity, but readily identified maths activity in other settings and with other purposes; her school experiences did not appear to dominate when she talked about mathematical situations.

Transcript excerpts discussed under the sub-theme of *Number and measurement* suggest that Cara referred mainly to two maths concepts when defining maths, that is, measurement and number. Cara's segmentation of maths into two main concept areas, number and measurement, may have related to the segmentation of maths within the school's teaching structure. It also may have related to what appeared as a segmentation of maths in her non-school environment. The maths Cara reported that was used by her parents and herself to perform tasks was clearly relevant to their lives, perhaps mirroring the relevance Ms A tried to bring into the Applied Maths (measurement) lessons. Cara may have made links between the purposes in her measurement activities at home and those at school, thus contributing to the salience and meaningfulness of measurement for her, but this is not known with certainty. Likewise, it is not known whether Cara's understanding of the home activities as *maths* was enhanced by her school experiences. Cara's use of number in non-school situations, such as learning times tables or number facts, appeared largely content-based and related to schooling, but measurement appeared more related to problems to be solved in the Maths Task Centre, home or work environment.

Cara's responses indicated that she did not seem to identify spatial activity as maths. Although the reasoning for this is not apparent, this finding does warrant consideration. As discussed in Chapter 4, the interviews with Cara's maths teachers indicated that Cara had experienced spatial activities at school, that they were conducted mainly in the Maths Task Centre, but that they were given less emphasis than the number and measurement aspects of the school maths curriculum. It may be of value for the school to re-examine this arrangement if the teachers consider the perception of spatial activities as mathematical desirable for the children they teach. The research findings discussed so far suggest that Cara constructed her own beliefs about maths; it appears that for Cara to come to consider spatial concepts as maths, a similar process of construction would be necessary. For Cara, construction might be facilitated by exposure to a greater range of spatial situations and activities with the opportunity to reflect, in a group learning situation, on the attributes, meaning and application of these. Because of the purposeful nature of many of the activities identified by Cara as maths, it may be opportune for adults around her to examine their own beliefs about the relevance of spatial concepts and their relationship to everyday needs and the place of this in the maths curriculum.

The discussion of segmentation of maths within Cara's beliefs has highlighted two main perspectives: firstly, content and action, secondly, number and measurement. In addition the previous section on the purpose or intent in using or doing maths identified three subthemes: maths for an everyday or non-school purpose, maths for a combined school/non-school purpose, and maths for a school-related purpose. The discussion of all of these elements demonstrates that Cara held a broader view of maths than many other children, but also that at times her beliefs appeared variable.

Cara's beliefs about maths - A summary

Cara's beliefs about maths were complex and subtle, thus not to be ascertained from responses to just one procedure. The discussion revealed nuances and complexities within her beliefs not expected from a child of eight to nine years. Cara appeared to hold some degree of personal affinity with maths, particularly with the ideas of measurement. This affinity seemed related to the construct maths as action, to her beliefs about different topics or concepts within maths, to her perceptions of the structure of the mathematics curriculum as she experienced it at school, and to her experiences and observations of the application and purposes of maths in non-school environments such as the home and workplace, and in the school environment. Measuring and estimating in maths appeared to be salient and of personal relevance to Cara, and the latter was related at times to guessing. The discussion revealed that number appeared significant to Cara also, indeed towards the end of the interview period number appeared more salient than measurement.

Figure 19 was developed as a schematic synopsis of Cara's beliefs about maths, particularly related to the mathematical activities she described. Each aspect of Figure 19 was covered in more detail in earlier discussion and schematic diagrams, but this diagram serves as a summary, and indicates the complexity of Cara's beliefs, and the interrelatedness she portrayed. It indicates also what appeared as two uncertainties for Cara, firstly, whether or not to classify as measurement, some activities included by the researcher as possible examples of informal measurement, and, secondly, whether estimation and guessing are the same.



Figure 19. Cara's beliefs about maths - a schematic summary.

To portray the complexity and multi-dimensionality of Cara's beliefs, a further diagram is included, Figure 20, which was developed from viewing her beliefs from an alternate perspective. As outlined in the introduction to the discussion of Cara's beliefs about maths, in this approach number and measurement are portrayed as the two key content areas. Within each of these content dimensions three continuums are portrayed. On each of these a point is placed representing the researcher's view of the appropriate location of Cara's beliefs, taking into account the discussion above. Content areas of little salience to Cara, such as Space and Chance, are not portrayed in the diagram as there is insufficient information to judge for the dimensions provided.



Figure 20. An alternate schematic portrayal of Cara's beliefs about maths.

The alternate portrayal of Cara's beliefs about maths, presented in Figure 20, makes clear in particular the contrast in Cara's beliefs about number and measurement. In regard to number she seemed to be mostly product orientated, with the product obtained through the use of calculation, and she portrayed situations in a school environment, or in an environment where the activity had essentially a schooling purpose. She did identify also some non-school number situations as mathematical.

In contrast, the measurement situations discussed included both school and non-school environments, but with an emphasis on the use of formal measurement in non-school environments for those situations she proffered. While the product appeared an important element of the situations she described at home, Cara gave at least equal, if not more emphasis to the process, which often involved physical activity through measuring and sometimes cognitive activity through estimating. The situations in which informal measurement was a feature were at times not considered as *measurement* by Cara, but were considered to be *maths*, often because of their estimation component. These included both school and non-school activities.

Figure 20 brings attention to a segmentation of maths within Cara's beliefs, as discussed earlier, which particularly is evident when contrasting her beliefs about number and measurement. The diagram shows also that her beliefs about maths appear to represent greater breadth and complexity than has been found of children studied in previous research suggesting Cara had constructed beliefs different from those of other children. The results from the eight children interviewed for this study indicate this is highly probable as the children revealed eight different sets of beliefs. It may be that many children, if given the opportunity to express their beliefs in the depth that has been possible in this research through the use of a range of procedures and multiple interviews, also would reveal idiosyncratic and complex views. It was only through the use of a variety of tasks over a long period, and through the reappearance of themes within the data, that the nature and complexity of Cara's beliefs became obvious to the eye of the adult researcher.

Further insights into young children's beliefs about maths are presented through the examination of the beliefs of the other seven research participants. But first, in response to the research questions, Cara's beliefs about learning and helping factors for learning maths are examined, preceded by a discussion of Cara's beliefs about *mathematics*, as distinct from *maths*, an aside from the main focus of the research, but a discussion that reveals further complexities and nuances within a young child's beliefs.

An aside: Cara's meaning given to "mathematics"

Cara's beliefs about maths were different from those she held about mathematics. Such an outcome was unexpected and warrants further examination as it adds to the insights Cara's data provides about young children's beliefs about mathematics and mathematics learning.

Introduction

As stated earlier, children's interview responses in the present study indicted that for some there was a difference in meaning for the terms *mathematics* and *maths*. The research began with the intention to use the words mathematics and maths interchangeably. As the researcher and interviewer, I soon learnt this to be naivety on my part, a naivety not precluded by results from trialing of research procedures, nor by reports I had read of previous research. The literature did not indicate any issue related to the use and understanding of the formal term as distinct from the informal term.

It is common for the formal expression, *mathematics*, to be used in major curriculum materials in this discipline (e.g., Australian Education Council, 1991; Board of Studies, 1995, 2000; Lovitt & Clarke, 1989), whereas in everyday speech the abbreviation *maths* is commonly substituted. The familiarity of the latter term and the acceptability of the exchange of the two terms, are illustrated, for example, in a newspaper article (Hunting, 1995) in which the title and the summary statement, designed to catch the attention of readers and inform as to the main message of the article, use the term *maths*, whereas the text that talks about children's learning deploys the term *mathematics*.

Maths is a recognised abbreviation of *mathematics* (Wilkes & Krebs, 1982), with an assumption of the same meaning. However, the present research shows that young children may not always understand the relationship between the two terms. This finding is of significance to teachers and others in the education community as it reminds us that children, even at the young age of eight or nine years, are individuals who construct their own meanings which should be acknowledged. The research illustrates that one should not assume the meanings children hold, but should take a stance of *listening* to individual students. This listening may occur through verbal, written and pictorial communication modes as well as by watching.

Each of the eight children was questioned on the meaning of *mathematics*, and it was only for some that there was some uncertainty. This was expressed in varying ways and to varying degrees as discussed in this document. It was from the children's responses that the theme concerning the meaning of the term *mathematics*, as distinct from *maths* arose. This theme had not been planned as an issue for analysis. The findings discussed below were not only unexpected, but illustrated early in the data collection the value of intensive interviewing in seeking the young child's perspectives.

Cara's beliefs about "mathematics"

Cara was comfortable with the term *maths*, and would willingly and without hesitation consider whether she believed a situation to contain maths activity. However, she did not appear to feel the same level of comfort or familiarity with the term *mathematics*, perhaps partly due to the fact that she did not appear to consider these terms to have the same meaning.

In my first interview with Cara I asked her to draw a situation in which she felt she was learning mathematics well (Task 6.1). In response she said, "I don't know anything about mathematics". However, she said she did know what maths was and went ahead to complete the task.

From this point, which was the beginning of the first interview I conducted, I was aware of the possibility of children having differing meanings for the terms *mathematics* and *maths*. The second interview with Cara provided an opportunity for validation of her first interview response.

When reflecting on Cara's response in her second interview to the task of drawing a person doing some sort of mathematical activity (Task 3.1) I became truly alerted to the possibility and complexity of differing understandings.

Interviewer: Cara, the first thing I'd like you to think about is, or what I'd like you to do for me, is I want you to draw a person doing some sort of mathematical activity

- Cara: Mathematics?
- I: Yes
- C: On sports we done mathematics we done, you had to have a partner that was just a tiny bit taller than you and you have to go on their back and go around the wicket and back around and back
- I: Mhm
- C: And you had to, you know the way you do a pyramid
- I: Mhm
- C: You had to do that just with two people like one person down the bottom and one person on top
- I: Right, so what sort of mathematics were you doing there?
- C: Er prac, getting warm up for gymnastics
- I: For gymnastics. And you said you were doing maths there at that time?
- C: No it wasn't maths, it was seeing if we could do things without falling

At this stage of the interview I was not alert to the significance of Cara's first reference to *mathematics*, that is, of her taking on the meaning for the word *gymnastics* when I said *mathematics*. I did not recognise her subtle shift. As the interview progressed a difference in her meaning for the terms *mathematics* and *maths* began to become more apparent:

- I: Right, so I said can you think of, can you draw someone doing some sort of maths activity
- C: Maths
- I: Maths activity. What did you think I said?
- C: Mathematics
- I: What's the difference between mathematics and maths?
- C: Mathematics can do, er you, I don't know
- I: What is mathematics?
- C: I don't know

My lack of full awareness of the possibility of miscommunication resulted in unconscious continued interchange of the terms *mathematics* and *maths*. Cara's response caused me some confusion which, it appears, was conveyed to Cara. My confusion, combined with a sensed lack of acknowledgment of her view, seemed to cause Cara to change what she was saying. Cara's following statement suggests that she received from my responses the message that maths and mathematics are the same and that she responded by adjusting her story to show an understanding of this:

- I: Well why did you choose that activity when you were doing pyramids and things when I said to think of someone doing some mathematics activity?
- C: W.e.l.l er I thought you meant um, measuring

At the end of the conversation regarding the gymnastics activity I asked Cara, "So was that maths?". She answered, "Er, well it was really sports". So Cara was fairly sure that she hadn't been doing maths in that situation but she was not certain as to whether she had been doing mathematics, as she was not sure of the meaning of the term. From the vividness of this discussion regarding gymnastics I, as the interviewer, took on an understanding that Cara had associated the words mathematics and gymnastics rather than mathematics and maths. In addition, I was alerted to the possibility of differing understandings and perspectives of the terms mathematics and maths among the eight children.

In response to this incident I added to my planned procedures a simple task through which I could directly address the issue of children's meanings for *mathematics*. In her response to the sentence starter "I think mathematics is" (Task 11.2), given in her eighth interview, Cara showed that she had begun to form a meaning for the term mathematics. Cara responded to this task by writing her first response but then the exchange became verbal:

Cara: I think mathematics is great

Interviewer: Why did you say that?

- C: 'Cos sometimes you can get it right, sometimes you can get it wrong (giggle)
- I: Alright. And what do you mean, sometimes you can get it right?
- C: Like when, what is mathematics? Like if you do something wrong you don't get a tick and if you do something right you get a tick

Cara's second statement indicates that she may been seeing mathematics and maths as the same thing. However, by asking herself the question "What is mathematics?" Cara suggests that she remained unsure of the meaning of mathematics, that she was questioning her own understanding of this concept.

- I: Did you just say what is mathematics? Is that the question?
- C: Yeah
- I: Well what would your answer to that question be? What do you think it is?
- C: Err, learning things, er doing things,
- I: Mm
- C: About maths

Once again Cara associated mathematics and maths, suggesting not only that they are closely related, but that they are an active process, made up of learning and doing. Cara continued by giving examples that reinforce this notion of action:

- I: What sort of things?
- C: Like, um, um, umm, like, like you don't know what dividing by is and you learn how to do it. And um it's like learning how to swim, learning how, how width the pool is and how long it is
- I: Mhm. Is that mathematics is it?

- C: Yeah that's what I think
- I: So how's that different from maths?
- C: Width and length is mathematics I think
- I: Is it also maths?
- C: Well, yeah
- I: Is maths the same as mathematics or is maths different from mathematics?
- C: I think maths, maths is different, maths is different from mathematics
- I: Can you explain that difference to me?
- C: Um,
- I: I'm pretty interested in this, so I sort of want to know what you think
- C: I think that maths is different from mathematics because mathematics is doing something else than maths. Maths is like times tables, and well sometimes you can learn things from maths but I think mathematics might be good to learn

In this segment of the conversation Cara began by giving examples of activities that suggested that she saw mathematics and maths as the same. However, when asked directly whether they were the same or different she stated that she thought they were different. This change of view suggests some possible confusion or uncertainty. The last statement by the interviewer, suggests also that Cara was tiring of, or beginning to have some difficulty with, the conversation, perhaps aware of her uncertainty and therefore finding the probing frustrating. This is endorsed by Cara's following statement in which she says she doesn't know what mathematics is:

- I: Can you still tell me a bit more about what mathematics is? It's good to learn
- C: I don't know what mathematics is, I think it's something that you learn and then you can do and then you can like learn it when you grow up, more
- I: So is it more when you're grown up that you learn mathematics is it, or and children at school learn maths?
- C: No, children at school learn mathematics and when you grow up to be in that school I think you learn more mathematics
- I: The school across the road? (Reference made to the secondary college)
- C: Yeah
- I: So you're still, you are learning mathematics now are you?
- C: Yes, I think
- I: You think so?
- C: No we're learning maths
- I: Oh you're learning maths
- C: We've only done mathematics once
- I: Alright, when did you do that? Tell me about the once you did it
- C: Ages ago. Term three
- I: Tell me about it
- C: We were in with Ms A and we were doing a worksheet and then she was telling us a bit about mathematics and I forget what she said
- I: Alright. So it was just that once with Ms A was it that you did mathematics?
- C: (Non-verbal: Yes)
- I: Right. So you don't usually do mathematics at this school?
- C: No I don't do it anywhere

Cara stated again that she did not know what mathematics was, but thought it was something you learn and then do and then learn more. She was unsure as to the timing of this, suggesting that mathematics is learnt mostly in the secondary school. She believed that at primary school she learned maths but because of one experience which the teacher may have called mathematics, she thought she had learnt mathematics at one time. The conversation moved on to consider the use of mathematics by adults and continued to explore Cara's understanding of the relationship between mathematics and maths:

- I: Do you think adults use mathematics at all?
- C: Yeah, when they're measuring
- I: Like your dad?
- C: No, when they go shopping and they're measuring how heavy is a ton of bananas or about three or four bananas and they're doing that, I think that's mathematics
- I: Is that also maths?
- C: Yes
- I: And that's mathematics too?
- C: Mhm
- I: In your classroom if you did some weighing would you be doing mathematics or maths?
- C: Maths
- I: Not mathematics?
- C: Yep

In the above excerpt, Cara's tendency to associate mathematics with people older than herself, is apparent again. She indicated during the interview that she saw mathematics either as something you learn as a student in the upper primary grades or at the secondary level, or as an activity done by adults when measuring as part of shopping. For these people and activities she equated the terms mathematics and maths but generally did not do so in relation to her own experiences in the classroom.

The interview continued with an attempt to have Cara draw together what she had said through this whole interview segment:

- I: Now is there anything else that you can write on here for me as well as I think mathematics is great?
- C: Uh uh (No)
- I: What else could you write after we've had this discussion? What would you write there? I think mathematics is
- C: (No response)
- I: Don't know?
- C: Mm mm (No)

This limited response from Cara suggests that the beliefs she had been expressing in the interview were at the formation stage. Cara's beliefs may have been in the process of construction and thus were open to fluctuation, uncertainty, or even contradiction, as illustrated in the quotations. In this eighth interview Cara attempted to communicate her ideas on what mathematics is, but lacked definition and certainty. Some of her statements appear to hold contradiction; for example, we see that she swayed between mathematics being maths and it not. However, we see also her attempts to make sense of what she observed, for example by suggesting that as mathematics is learned mostly in secondary school, it is used or done by adults rather than by children.

In this eighth interview Cara gave a sense that for her *mathematics* was a verb, rather than a noun, that when she thought of mathematics she thought mainly of action rather than of content. This was endorsed by Cara in a brief conversation at the end of her tenth and final interview:

Interviewer: What did you say you thought mathematics was? Cara: Learning maths I: You thought it was the same did you?

C: Yes. Just learning maths

During the interview I indicated that I thought Cara meant that mathematics and maths were the same, but in retrospect I believe that she may have been seeing mathematics as the *learning* and *doing*, that is, the activity within the domain of *maths*. This reinforces two statements she made in the eighth interview when she said, "[Mathematics is] err, learning things, er doing things" and that adults use mathematics when they are "measuring things".

In conclusion, it appears that Cara's meaning for mathematics was a personal one, with her thoughts influenced by a range of experiences and thus with her belief statements showing some inconsistency. It is clear that although Cara at one time said she knew nothing about mathematics, she did have some idea of the meaning of the term. In response to probing Cara was able and willing to express her meaning, although this meaning for mathematics appeared not to have been formed in a definitive sense at the time of the interviews. Cara's meaning for mathematics seemed still in the developmental stage; it is possible that her beliefs continued to develop or evolve during the two-term interview period. The interviews themselves may have formed part of a process of Cara coming to construct her views.

Cara demonstrated attempts at sense-making during her interviews. For example, when in her second interview Cara was asked to draw someone doing some sort of mathematical activity, it is of interest that although Cara questioned the interviewer to check that a mathematics situation was required, she then drew a gymnastics situation. While the word mathematics had been stated clearly, it appeared that Cara did not *hear* this term, she appeared in fact to hear what she could make sense of. It seems that Cara did not use the term mathematics in her everyday communications and therefore may not have been expecting to discuss this term in the interview situation. Cara appeared to relate the request to something she knew; she showed that as an interviewee of only eight years of age her inclination was to attempt to make sense of the situation. She continued to do this throughout that interview as demonstrated in her change of direction to talk more about maths and mathematics, possibly in response to seeing the reaction of the interviewer.

Cara was comfortable with the term and concept of maths, and did at times relate mathematics to this. Some of her responses suggested that she saw mathematics as an activity that follows after the learning of maths; that mathematics is studied by older students at upper primary and secondary school and used by adults when measuring as part of shopping. In relation to older people Cara at times interchanged the terms maths and mathematics but did not do this in relation to herself as she perceived she had had only one mathematics learning experience but many maths learning experiences. Her beliefs regarding possible links between mathematics and maths did not appear fully established but it appeared that she may have seen mathematics as the activity element of maths. She appeared to see maths as a more encompassing term, perhaps because of her familiarity and comfort with the concept.

Cara: "Maths" and "Mathematics" - A reflection

Cara was a friendly participant in the research and was generally happy to talk about maths and mathematics, revealing complex beliefs. Data from Cara demonstrate that her beliefs about the nature of maths were not restricted to number concepts but encompassed a spectrum of concepts and processes. Mathematical action or process appeared important to Cara, and at least equally important as mathematical content. The former appeared largely related to the use of measurement, especially in non-school situations. Analysis of Cara's interview data suggests she related number concepts more to schooling, and to a content-based approach to mathematical activity, and less to the needs of herself and others in real-life situations. Cara's beliefs about maths reveal a segmentation that analysis suggests is related to process, that is, estimating or calculating; to content; to the needs met by maths in non-school situations; and to schooling experiences. Through the data collection there was some inconsistency in Cara's responses, influenced by a range of possible factors, but perhaps due largely to a continuing construction of beliefs over the research period. This does not de-value the research findings but suggests that beliefs, like knowledge, are constructed by individuals by building on their experiences.

Cara held different meanings for the terms *mathematics* as and *maths*; meanings for the former were not always easy to access. When Cara responded to the tasks investigating her meanings of the two terms, one noticeable difference was her level of confidence in responding. She appeared much more familiar, comfortable, and confident with the term *maths* than with the term *mathematics*. Cara perceived that she had had little experience with mathematics, but her responses suggested that she may have seen it as an action element within maths. Some inconsistency in responses regarding mathematics was apparent, seemingly due to the continuing development of Cara's beliefs.

Learning

Another dimension within the study of Cara's beliefs related to learning and learning processes. This emanated from the second key research question: "What beliefs do children hold about the nature of mathematics and the nature of learning?".

When Cara was posed with a task that asked for one word she would associate with the word *learning* (Task 1.1), she responded, "I don't know anything about that". It appears that Cara was expressing some reluctance to discuss her meaning for learning, indicating a lack of experience in discussing or reflecting on learning as a concept or process. Cara had given the same response when posed with the term *mathematics*, but revealed in further discussion that she did have some beliefs about mathematics; it was not a totally foreign term to her. Likewise, further discussion of *learning* revealed beliefs, that, like those Cara held about *mathematics* and *maths*, were complex.

As discussed earlier in this report, some procedures in the study facilitated accessing of children's beliefs about learning in general. In addition, responses to some other tasks provided insights into children's beliefs about learning maths in particular. Thus, the discussion below draws on any relevant data for Cara, which may be from any of her ten interviews. The discussion is informed, and to some degree contextualised for the reader, by the previous discussion of Cara's beliefs about maths and mathematics.

Cara's beliefs about the learning of maths and learning in general encompassed a range of perspectives. However, there are less data available on which to base discussion of the meaning of learning for Cara than was the case when considering her beliefs about the nature of maths and maths activity. This is due perhaps to Cara having some difficulty in formulating and articulating beliefs about an abstract concept such as learning. A second factor may have played a part in this result also; because of the open nature of many of the interview tasks it is possible that while some may have been developed with the intention of accessing beliefs about learning, they may not have acted as stimulants for Cara in particular to reflect on and talk about her meaning for learning. Thus the findings below are expressed in more brevity than those above, but nonetheless reveal much about those beliefs that Cara had formulated and was willing and able to express.

On balance, as discussed further below, Cara appeared to emphasise learning as knowing and remembering, saw the role of the learner as thinking, remembering, listening, and working, and portrayed the teacher as an organiser, facilitator, encourager, motivator, and affirmer. When talking about her meaning for learning, at times Cara communicated an uncertainty about the difference between learning and teaching to the extent that in some interviews she interchanged the terms.

Cara's beliefs about learning are discussed in four sections. These are

- knowing and remembering;
- the role of the child in learning;
- the role of the *teacher* in learning; and

• learning and teaching.

These section titles each signify a theme related to learning that arose in the analysis of Cara's data.

Knowing and remembering

A theme that emerged strongly from Cara's discussion regarding learning was that of learning as knowing and remembering.

In her ninth interview (Task 10.3), Cara was asked to consider single words that were based on different conceptions of learning held by adults (Marton & Saljo, 1984). The words *know, remember, do*, and *understand*, were presented on cards for Cara to choose those that she associated with learning. She was given the option of adding words if desired. Cara responded by choosing the words know and remember. When asked whether her response would be same for the learning of maths she answered in the affirmative, and clarified by adding "Know what to do and remembering what the answer is". It is noted that of the four words presented in Task 10.3, *remember* and *know* were selected by Cara, *do* was used in her explanation, but the word *understand* was not called upon. This response suggests that Cara associated remembering, knowing, and doing with learning maths, but did not necessarily consciously link understanding with learning maths. It appears that Cara may have seen maths learning as procedural, rather than leading to an understanding of mathematical relationships.

Further data pertinent to this discussion was provided in Interview 10 (Task 2.1) when Cara defined how she would know that she had learned:

Interviewer: How do you know when you've learned then?

Cara: When [the teacher] tells you all the answers and you've learned them and memorised them, then you know you've learned

This reply gives emphasis to memorising as fundamental to learning; it implies a procedural or instrumental view of learning or understanding (Skemp, 1976), a view that focuses on rote learning. A contrasting form of understanding, sometimes referred to as relational understanding, is deeper, more readily extended to other situations and based on linking concepts in a schema (Skemp, 1976, 1986) or a network of representation (Hiebert & Carpenter, 1992). Relational understanding involves knowing why as well as how (Skemp, 1976).

The timing of Cara's selection of the words knowing and remembering in her ninth interview suggests that this response related primarily to the learning of number concepts. In interviews at around this time she tended to emphasise number when giving definition-type responses regarding maths, whereas in earlier interviews the emphasis within such responses seemed to be on measurement, for example as seen in her maths word association type responses of "measuring" and "estimating" to Tasks 1.1 and 1.2, and in her maths word wheel response (Task 1.3.1) (see Figure 18). The focus on number in later interviews was shown, for example, in an early comment in Cara's ninth interview: "You have to memorise something for a times table test", and in Interview 10, when, in giving her personal dictionary definition

for maths (Task 2.3), Cara began, "Times tables . . . Pluses, dividing by". Cara appeared to show more tendency, when not prompted specifically, to speak of number situations in later interviews than in earlier interviews.

Cara gave the impression that she associated the learning of number mainly with a purpose of getting an answer, in contrast to understanding a method, once again suggesting an instrumental view of learning or understanding rather than a relational one. For example, in her fourth interview (Task 8.2), Cara stated that the use of a calculator helps her to learn maths because "You get answers from it", indicating the perceived importance of getting or knowing answers when learning maths. She did not talk of the thinking that might be involved in the process of deciding which calculator buttons to push and in what order. It is noted also that the examples covered in the earlier discussion, under the theme of Maths as Answers, indicated that for Cara knowing or remembering the answer appeared to relate most strongly to number concepts.

Cara referred to two other maths situations that further indicated an instrumental view of learning. In her third interview she spoke of learning times tables by writing them out and writing answers, as homework, and doing them again the next day at school. Here the focus seemed to be on right answers and on automatic response, not on using strategies for manipulating numbers with meaning, such as using doubles or commutativity (e.g., Board of Studies, 1995, 2000; Sullivan, 1985). In her second interview (Task 3.3.1), Cara was asked whether a person would use or do maths if he said one hundred cents is the same as one dollar. Her response suggested a rote approach to doing maths: "Because you've got one cent and one hundred cents, well if you take away the zeros it will be one". Cara gave the impression of an instrumental understanding of the concept when she explained why this would be:

If you took away the one there would be, there wouldn't be one cent at all. It would just be one hundred again so you took away the zeros and it would be one there and that would be one cent

Cara seemed confused as to whether the one that is left after the zeros are taken away represents one dollar or one cent. Her elimination of the zeros appeared to be without any real understanding of the process or result; rather it is a procedure to follow. This example provides a situation in which Cara expressed the manipulation of number in maths in a rote fashion suggesting that, for Cara, knowing what to do and getting the answer may involve instrumental understanding with little, if any, links to the mathematical meanings and relationships inherent in the task.

The above discussion of Cara's choice of the words knowing and remembering in relation to learning, and of her accounts of some mathematical situations, suggest that Cara perceived learning as procedural with a focus on getting right answers. It suggests also that this view related mainly to learning situations in mathematics involving number facts and procedures.

However, in consideration of the broad view of mathematical activity held by Cara, as discussed earlier, there is the possibility that a conclusion that she believed learning maths to be procedural, could be narrow and inaccurate. Cara indicated that she considered number and measurement, for example, as mathematical activity, the latter of which is less likely to be considered procedural although it may involve some learning of procedures in using a range of tools such as scales and tape measures. For Cara measurement related mainly to real-life uses or the solving of problems in school and non-school environments. Procedural tasks in mathematics seemed more often to be associated with number tasks isolated from real-life problems, for example, as in the use of algorithms for the four operations.

Cara's responses in a range of interviews suggest that she saw and valued varied types of cognitive activity as mathematical activity, that is, in addition to rote learning. For example, as demonstrated earlier, estimation was seen by Cara as a significant element of mathematical activity. Figuring out or guessing for oneself before being told answers was valued also as Cara demonstrated when she talked about the help a learner having difficulty might need (Task 7.1), and when talking about what she believed to be the features of a good teacher (Task 11.3). Cara liked to figure out first before her dad would tell her the answer (Task 10.1), but expressed a preference to figure out with someone else, not alone.

Cara's interview data stemming from tasks specifically designed to gather insights into beliefs about the nature of learning suggest that she considered the learning of maths as a rote exercise leading to instrumental understanding. But data resulting from other interview tasks, as discussed in the previous paragraph, suggest that this may be a limited view of Cara's beliefs. In the responses cited above, there is the implication that figuring out will lead to learning. Figuring out for Cara may have related to solving a mathematical problem such as when she was constructing a container in the *Pasta lesson* (refer to Chapters 3 & 4), or when using measurement in everyday life. It is acknowledged that in other mathematical situations, figuring out might have related to knowing what to do, and to the instrumental recall of facts and procedures. Thus learning maths for Cara might have been mostly of an instrumental nature requiring knowing and remembering, but perhaps at times also involved more complex cognitive activity.

Further insight into Cara's beliefs about learning in maths is found from her Task 11.1 response, in answer to how good she thought she was in maths. In explaining her implied belief of a knowledge or ability correspondence with age, Cara stated,

a grade three just learns about times tables and when you're in grade four you learn more about pluses and all this stuff, but when you're in grade five six you learn, what's it called um, a, I forgot what it was, but it's something like learning how, where to put um, er, pluses and times tables

In this conversation Cara implied a perception of greater complexity in maths learning as a child progresses through the primary school. She talked also about *learning how*, a concept most commonly associated with instrumental understanding (Skemp, 1976), and suggested that learning maths at school relates to number concepts. Cara appeared to see

herself as a learner of mathematical facts and procedures related to number, and she implied an instrumental approach to learning and, by inference, to understanding.

It appears from the quote and other data discussed above, that Cara associated learning in maths mainly with number concepts. Measurement concepts were at times associated with learning, for example as in Task 6.1 when Cara chose a weighing activity as one in which she was learning maths well, but in general measurement seemed to be associated more with doing or using maths, and number with learning maths. It is noted that of six responses given in Sets 6, 8, and 10 where Cara described learning activities, only this Task 6.1 response related to measurement and not to pure number. In describing a range of measurement situations, Cara communicated an understanding of a purpose to the maths, other than just learning how, and did the activity with meaning. The purpose in number situations at school was largely getting the correct answer.

An important element within Cara's beliefs seemed to be whether she perceived she was learning maths, or doing or using maths. Cara may have seen maths activity at home with a non-school purpose, and school-related maths activity as different forms of maths with different intentions. It seems that the former did not relate to learning to any great degree, in contrast to the latter. It is noted that many of the maths activities discussed earlier which Cara seemed to do with meaning and purpose, especially those performed at home, involved measurement concepts and real-life tasks with problems to solve. In contrast, school maths was expressed as something you "get on with" learning (Task 7.2), and seemed to relate mainly to number.

From Cara's interview data discussed under the theme knowing and remembering, it appears appropriate to conclude tentatively that Cara's instrumental approach involving knowing and remembering was primarily in relation to *learning* maths, as distinct from doing or using maths, to *number* concepts, as distinct from measurement concepts, and to activities undertaken for a *school-related* purpose, as distinct from a non-school related purpose.

The role of the child in learning

In identifying Cara's beliefs about learning, a further key theme that emerged was the role of the child in learning.

From Cara's response to Task 10.3, where she portrayed learning as knowing and remembering, it could be inferred that Cara believed herself to have little active role in creating her own understandings as opposed to mimicking the procedures and understandings of others. However, Cara indicated this was not the case in all instances of her learning. She suggested for example, that in one non-maths learning situation she was able to learn without a person such as an adult playing a dominant *teacher* role. She told of learning during a weekend spent with her aunt by the sea:

I learned that the water, it rised, up to all the sand, see [my aunt] lives next to the beach. And every night the water, it goes up, it goes higher, and it goes on the sand. And I learned that any time it does that, that if you walk in it you could like go on the wrong spot and drown It appears that in this situation Cara learned mainly through her own observations, thus taking an active cognitive role in her learning about a new concept.

In at least two interviews Cara spoke also of the role of thinking in her learning. In Task 1.1, a word association type exercise, Cara had some difficulty in associating a word with learning. She replied that she knew nothing about learning but after some encouragement, that is, after being asked "Do you know the word learning? It doesn't make you think of another word?", she replied, "Learning er thinking". Later in the interview she explained that her thinking would be in response to a teacher request for her to think about what she was doing, but added that "you have to talk and think and learn", indicating a perception of an important role of thinking in learning. In her seventh interview, when prompted to respond to a child's drawing portraying thinking as helpful in learning maths (Task 9.2), Cara appeared to attribute great importance to this cognitive activity: "It doesn't matter if you get the answer right or wrong, it's just that you've got to think about it". This statement gives a new and different perspective from that gained from Cara's comments regarding getting answers as part of learning maths. It appears from her other comments that she saw getting the right answer as a prime objective in learning maths, but does not seem so concerned with that factor here. Her overall responses suggest though that the present comment may be an exception. Alternately, the contrasting statements may be due to partially formed or variable beliefs.

As discussed above, Cara believed that remembering was a key element in learning. She felt that there were undesirable consequences if unsuccessful. When asked in her fourth interview how she felt about learning maths (Task 8.3), Cara replied "funny". She recounted a situation involving being asked by her teacher for answers to problems such as "one times one" and "a hundred times a hundred" so as to explain that "sometimes it makes me feel funny if I forget what the answer is". After my querying the word *funny*, she elaborated: "I mean like sort of upset . . . like the whole world's on you if you get something wrong . . . some people laugh at you". Cara was conscious of those around her when she was learning maths, and seemed to relate this to the school setting. In this maths learning situation, which was reported to involve number processes, Cara perceived that it was important for her to remember answers. She appeared to believe that by remembering she would learn maths, and avoid distress in the process.

Cara communicated an assumption of a largely passive role in learning when in her tenth and final interview (Task 2.1), in response to the question "And when you're learning what are you doing?", she said "Listening". A different role, but potentially passive, was communicated in response to a question asking what she did when learning maths. Cara talked of the role of children in learning maths, explaining that they "work" and "then learn" (Interview 3, Task 1.2), implying that, for herself, work is a major and potentially passive element in learning maths, with the implication of "a duty, task or undertaking" (Wilkes & Krebs, 1982, p. 1359).

The above discussion indicates that Cara may have assumed a partly passive role, as a worker and listener when learning maths, but also saw remembering and thinking, both active elements of learning, within her own efforts to learn. Cara indicated that she wanted to learn maths "just to know what they're about and to learn what they do, what they are, what it means, and what it is" (Task 11.3). In this statement she appeared to portray maths as isolated from herself and from her everyday experiences but nonetheless expressed a desire to learn maths. As discussed above, learning for Cara, as "know what to do and remembering what the answer is", seemed mainly to relate to number activities with a school-related purpose. Measurement, although sometimes talked about in terms of learning, appeared a more active and personally relevant pursuit that involved maths but not necessarily the learning of maths.

The role of the "teacher" in learning

In discussing Cara's beliefs abut the role of the *teacher* in her learning, the word teacher is interpreted in a broad sense, to encompass others, such as parents, in a teacher role.

Cara's interview responses portrayed the teacher in a variety of roles including organiser, facilitator, encourager, motivator, and affirmer. For example, Cara considered the teacher important for telling her what should be done, that is, for organising and giving direction for appropriate activities:

If we didn't write anything down, the teacher didn't tell us what to measure, she just goes, go and estimate this, and she wouldn't tell us what to measure, then you wouldn't know what to measure, or just like get anything and the teacher goes, no, not that (Interview 1, Task 6.1)

Cara felt that without the teacher presence the children would not have direction. She suggested that the teacher should be, or is, the judge and final decision-maker on what to measure; Cara appeared not to see it as the child's role to make such decisions. This attitude may reflect the teaching approaches she had experienced.

As well as being an organiser or instructor for learning the teacher was seen to facilitate learning in other ways, as expressed, for example, in response to Task 1.1: "she tells you to build something and I go I can't build that and she goes think about how to build it". The teacher would encourage Cara to think about what she was doing and, as stated by Cara, would ideally "help me . . . to learn things . . . let me measure things, learn how to use scales . . . learn dividing by, mathematics" (Task 11.3). The concept of the teacher helping was a general one, but one that Cara appeared at times to have some difficulty specifying further. However, Cara elaborated upon part of this statement by saying "I think the teacher should let us see what the answer is for ourselves, on the scales". In this case Cara gave emphasis to finding an answer in a situation involving measurement, contrasting with her tendency to indicate an association of getting answers with number activities. She suggested also that the teacher and the child each had an important role in relation to the finding of the answer in this situation; Cara expressed preference for the teacher to allow the children to discover for themselves, as she believed "if she tells us that's giving it away" (Task 11.3), thus suggesting

that the teacher should act as a facilitator. The response has built into it the assumption that the mathematical knowledge the children are seeking is already held by the teacher, and that the children are trying to learn what the teacher already knows.

The attitude of the teacher and the perceived support from the teacher gave solace to Cara when she felt less capable in maths than she would have liked, as expressed in an excerpt from Cara's first interview:

It's okay to get some things wrong. You don't always have to get them right . . . Because you don't, it's not against the law to get them wrong . . . It's okay to get them wrong once in a while . . . I think that and my teacher thinks that

It appears that Cara felt pressure from other children when playing maths games like times tables games and, according to Cara, getting one wrong and having everyone laugh at her. At times like this her strength of character and belief in the teacher as authority came to the fore:

I just sit down and I think for a little while and I say they can't treat me like that I, I won't listen to them, I can do whatever I want to do, I'm allowed to get them wrong. I don't always have to get them right because the teacher says (Interview 1, Task 6.2)

This excerpt appears to contradict findings from other statements by Cara where she gave emphasis to getting the right answer in maths. However, this statement may have been made in an attempt to maintain her self esteem in situations in which she perceived herself as unsuccessful. It may have been related also to the role of authority and support-person that she gave to the teacher in this situation. The responses indicate that, for Cara, getting answers in maths has an affective component that can be made easier by the teacher playing a role where consciously or unconsciously she gives support to the children in their efforts to learn maths.

When Cara was prompted to repeat what she had said, in her first interview as quoted above, she added, "You don't have to get them right, if the teacher goes please get these right I'll just try". This statement suggests that for Cara the teacher acted not only as an encourager for thinking, but also as a motivator for trying to get right answers, and therefore, it appears, as a motivator to learn maths.

Cara saw the teacher also as affirmer, perhaps related to Cara's perception of the child coming to know and remember as a learner. In a range of interview responses Cara portrayed the teacher as the one to ratify a child's efforts as correct. For example, in Interview 9 (Task 10.1), Cara stated, "Like a block, and you're writing something down and then the teacher corrects it and says it's right, you've really learned something about a cube or a block". In an earlier interview (Interview 3, Task 1.2), Cara had conveyed a similar view:

Cara: We have a work sheet that we have to bring home and stick in our homework book and um then it's got times tables and we have to write them out and then write the answers and then in school we get a big times table sheet and we have to do them and if we get them correct we have learned them

Interviewer: Oh right, I see, so you've learned, is that when you know that you've learned them if you get them correct is it?

- C: Er yeah
- I: Is there any other way that you know you've learnt something?
- C: I think, yeah, when um when let me see, when the teacher corrects it and says excellent or something or she puts an A plus that means we know we've learned it

Getting her maths correct in this Interview 3 situation possibly was associated with remembering, as the learning of times or multiplication tables is traditionally often undertaken through quick answer, rote learning activities. In each of these situations, Cara interpreted knowing that work is correct as an indication of learning. The teacher signified correctness through verbal communication, a grade, or some other external communication, such as a tick (Task 11.2). In the continuation of the above Interview 3 conversation, Cara gave emphasis not only to the purpose of this role for the teacher, but to the belief that in the school situation it is the teacher who is the person to perform the role of affirmer:

- I: What about when the teacher hasn't come up and had a look at it yet, might you already know that you've learned it?
- C: W e e ell she always has to tell you

Cara acknowledged a similar role for her parents when, for example, in Task 8.2 she spoke of her mother telling her the answers when helping her learn times tables at home. The teacher-figure as affirmer was important and could be in the form of a teacher or parent. Cara did not voice the possibility of herself or other children playing a role in the affirmation of her learning.

It seems also that tools in the maths learning situation could affirm Cara's answers as correct. A range of tools such as a calculator, blocks, fingers, a tables chart at home, a ruler, a weight scale, a "trungle wheel" [*sic*], and an exercise book with tables on the back could all give Cara answers (Interviews 1, 4, 6, 7, 9). Because they gave answers, it appears that Cara saw these tools also as affirming her growing knowledge of maths.

The teacher was perceived by Cara to play a somewhat dominant role in a child's learning through being an instructor or organiser, an encourager, a motivator, and an affirmer, but also was perceived to be a facilitator of children's thinking and children's efforts to discover answers for themselves. The role of affirmation in learning could come from sources in addition to the teacher, including parents and tools. It appears that neither Cara nor other learners were seen as appropriate affirmers of Cara's learning.

Learning and teaching

In responding to the word *learning* in the Task 1.2 password activity in her third interview, Cara gave the word *teaching*, indicating that she associated this with learning. When asked if teaching and learning were the same or a bit different, she replied, "The same". She explained further:

The teacher teaches, the teacher teaches you and she learns you things and then she learns you, she tells you and it's teaching

This rather convoluted sentence indicates that Cara saw the teacher in a teaching role, but perceived that at the same time the teacher had an active role in *learning* the children.

Clarification of this statement did not appear in the same interview as we went on to talk about the child's role which she stated as "we work", but further insights were gained when the concepts of teaching and learning were revisited in Cara's final interview in which she was asked for personal dictionary definitions (Task 2.1). The conversation began:

Interviewer:What does *learn* mean? What would you put in your dictionary?Cara:Someone who tells you what to do

- I: Tell me a bit more. I'm not quite sure why that's learn
- C: Someone that tells you something like one times one. She is telling me the answer and learning me what it is and I'm learning

Once again we see evidence that Cara appeared to give credence to knowing an answer as evidence of learning. Some form of evidence seemed to be important for Cara in identifying a situation as a learning situation, not only in relation to maths, but also in other areas as demonstrated in a response in Task 4.3: "like go somewhere, to a playground and see if there's rubbish, and if there's rubbish . . . and if you pick it up that's learning, to take care with the environment". In telling this story she appeared to emphasise the need for physical evidence as proof that learning was happening, as an affirmation of learning. In this case the evidence was displayed in a particular behaviour; a change in behaviour has been identified as one facet of learning (Bigge, 1976).

It is possible that Cara saw the getting of correct answers in maths in a similar way. She judged herself as not being very good at maths as sometimes she did not know answers (Interview 10), therefore may have perceived the getting of correct answers as a change in behaviour, indeed a desirable change in behaviour, and thus evidence of learning. The getting of correct answers might have been seen as evidence also of a change in knowledge held. For Cara, proof of learning in maths related to the role of the teacher in affirming children's work; the teacher indicated not only that the maths was right, but by consequence that learning was occurring.

In Cara's Task 2.1 dictionary definition for learn, quoted above, she referred to another person playing a role with a number of elements. In an effort to clarify Cara's meaning for learning and the relationship with teaching, I asked Cara for a personal dictionary definition for the word *teach* (Task 2.1). She began, "It's a teacher that can teach you what things are", giving the teacher an important role in her learning, as she had when asked for a definition of *learn*. Responding to this statement, and to her previous statement that the teacher would be "learning" her, I asked, "So does the teacher teach you and does the teacher learn you?" She replied, "Yes", and added that the teacher teaching her and the teacher learning her were the same thing. Her explanation, however, focused on the teacher as a learner: "She's learning me and she's learning something at the same time . . . she asks us a question and we tell her the question that she's trying to figure out what the answer really is . . . 'Cos she doesn't know the answer". This statement appears to contradict the meaning conveyed in many other responses in which Cara appeared to perceive the teacher as the holder of knowledge, the one who knew the answers and could correct the children's work. It may be that Cara believed

both situations to be possible and to have occurred. However, her emphasis in the majority of responses related to this point appear to portray the teacher in the latter role. Following on from this conversation Cara added to her explanation:

If she teaches you she's . . . teaching and telling you what to do and learning is something like telling you the answers. It's a bit different.

This statement implies that the teacher has two roles running side by side: one as the instructor or organiser and one as the affirmer. It appears that Cara may have believed the teacher was acting as affirmer in *learning* the children because they were learning, that is they were coming to know the right answers. This supports the conclusions discussed in regard to maths, under the previous theme concerning the role of the teacher.

In contrast to Cara's perception of the role of the teacher in teaching maths, she gave evidence of seeing teachers in other curriculum areas in a variety of ways. When asked of the physical education teacher, "Does he teach you or does he learn you?", she replied "He just plays". The teacher in the art room was perceived to have a different role, that is, to teach: "Teach you what to do. Telling you what to do", overlapping with one of the roles of the maths teacher. Cara stated that the language teacher would be learning, and added, "She's, we're learning the spelling words, 'cos she's writing them up on the board and we're learning them and then we have to spell them without looking at them. And then she corrects them and tells you what, how to spell them". In actuality it seems that the children were portrayed as learners in this description, with memorising being a key aspect of their learning. The teacher was *learning* the children. It appears that Cara may not have seen children as learners in those situations in terms that, for her, corresponded to the children's activity.

Cara gave emphasis to the role of the children as the learners when later in the interview she stated, "Well [the teacher] teaches you, tells you what to do and then we learn, learn what she's telling us, learn and then we do". When I attempted to question her further to ascertain whether there was consistency with earlier statements, as discussed above, she stated "You're mixing me up", and "I don't quite know". Her voice expressed a little frustration, perhaps because of my continued efforts in probing her beliefs regarding teaching and learning, perhaps because of a tiredness in discussing and clarifying her beliefs of the relationship between these two concepts, or perhaps because of some uncertainty in her own views. She did make the statements as discussed above, but also varied of her earlier response regarding the language teacher, when at the end of this portion of the interview she stated that the language teacher would teach her as well as learn her. It appears that Cara did hold these views of the teacher as *teaching* the children and *learning* the children but that perhaps these views were not fully established. Once again, the questioning process in the interviews may have been instrumental in the formation or at least the expression of her views as it is possible that Cara undertook little, if any, similar indepth discussion of such views in other contexts. She may have become more conscious of her beliefs as she expressed them in the interview and therefore it is not surprising that for an eight to nine year old child there was some apparent lack of certainty or only partial formation within her beliefs.

Cara: Learning - Summary

Although the interview data were not always consistent, it does appear that a tentative summary of Cara's beliefs about learning can be made. It seems, firstly, that the teacher was judged to be *learning* the children in the situations where the children were perceived to be learning, that is, where the children were memorising and coming to know something new, with success indicated by teacher correction of the children's work. The children's learning appeared to consist mainly of knowing what to do, as informed by the teacher when she was *teaching*, and, importantly, of getting the right answers, most often by memorising. Learning seemed to be perceived by Cara to have some passive aspects, such as listening, and some cognitively active aspects, such as thinking and remembering.

The above conclusions appear to apply to Cara's beliefs about learning in general and to her beliefs regarding the learning and teaching of maths in particular. In contrast, Cara appeared to see differently her activities in some of the other curriculum areas at school, and the related teacher role. Of importance in Cara's assessment of a situation as a *learning situation* seemed to be whether it involved learning for the child, that is, the acquisition of knowledge or a change in behaviour through remembering and possibly through thinking, and whether the teacher acted not only as an organiser but also as an affirmer of children's learning.

Cara portrayed complex beliefs about learning that at times appeared to hold some contradiction but from which key themes evolved. These centred around learning as knowing and remembering, the child's role in learning, the teacher's role, and a close and at times seemingly entwined relationship between learning and teaching.

Helping factors for learning maths

The third main focus of the research was children's beliefs about helping factors for learning maths. To possibly gain further insights into beliefs about helping factors, hindering factors were considered also.

Cara displayed a perception of a range of factors of influence, as demonstrated immediately below through reference to responses made during Task 6 activities, and as then discussed in more detail through reference to these and further interview responses.

In her first interview, Cara was asked to think of a situation in which she was learning maths well (Task 6.1). She chose a situation of learning the measurement concept of mass in a lesson taken by Ms A, the Applied Maths (measurement) teacher, seemingly reflecting the

attention Cara gave to measurement, particularly in her early interviews. Cara referred to three factors as most helping her to learn maths well:

- i. calculators, blocks, pencil case, scissors (that is, the materials that were weighed)
- ii. measure
- iii. write it down

Cara's drawing of this situation showing the concrete materials to be weighed, the balance scales and the writing down is depicted in Figure 21.



Figure 21. Cara's drawing of a situation in which she was learning maths well.

As discussed in more detail later, in response to the following task (Task 6.2), Cara was unable to think of a situation in which something was stopping her or making it hard for her to learn maths well. Instead she described a second situation in which she was learning maths well. For this second situation, a number situation, Cara ranked different helping factors as her first three:

i. think;

ii. maths book with times tables;

iii. count with my fingers.

In each case the teacher was identified as the fourth most helpful factor.

Within these responses Cara made reference to the use of materials or tools, physical mathematical activity, and cognitive mathematical activity, implying personal involvement by the child as the learner. The teacher was valued also to assist in the process of learning maths.

These responses suggest a breadth within Cara's concept of helping factors, a breadth that involves people, tools, and processes. Thus the fuller discussion that follows, regarding Cara's beliefs about factors that assist in her learning of maths, explores her references to each of these factors in greater depth. Subsets of these categories and additional factors of influence also become apparent. Where possible and appropriate, the analysis and discussion draws links to the previous discussions regarding Cara's concepts of maths and learning. Taking a theme-based approach, the discussion begins with an examination of Cara's responses regarding her own activities and processes in helping herself learn maths. Following that is a discussion of Cara's perception of the input of others, of the use of mathematical tools, and of luck in assisting her learning of maths. Possible hindering factors are discussed also. The key themes to evolve are

- Cara helping herself to learn maths;
- other learners helping Cara to learn maths;
- the role of adults in helping Cara to learn maths;
- tools helping Cara to learn maths;
- luck as a helping factor for learning maths; and
- factors hindering Cara from learning maths.

The discussion concludes with a brief summary and comment.

Cara helping herself to learn maths

Cara perceived that she played a role in her own learning of maths, and was able to take certain measures, both cognitive and physical, to assist in this learning. Cara's responses included a number of factors that she saw as helping herself to learn maths, including,

- thinking;
- listening;
- practising;
- concentrating;
- estimating/guessing;
- seeing a pattern; and
- writing.

These are considered in the discussion below, with subheadings providing direction for the reader.

It became apparent during the data collection that Cara held multi-dimensional perspectives on a number of these factors. Cara expressed some factors as positively contributing to her learning, but the discussion of the contribution of some others was ambiguous and even contradictory. As in earlier sections, this appears due to the complexity of the constructs for Cara and the seeming emergence or re-development of some of her beliefs during the interviews. The direction or degree of influence is discussed below and summarised in Table 3.

Thinking

One of the helping factors mentioned by Cara in the scenario described for Task 6.1 was *think*. However, later in this first interview, Cara indicated that thinking is sometimes not very helpful, such as when "there's a times table that's a hard one . . . like twelve times twelve . . . I try to think and then I put a wrong answer . . . it's not very helpful for me". In this excerpt Cara conveyed some concern about hard problems and not getting correct answers. She did not appear to perceive that her efforts in thinking were of benefit to her learning in this

situation. For Cara, it is possible that thinking for a problem such as twelve times twelve mainly may involve remembering, although there is the potential in solving this problem for more creative manipulation of number.

When responding to a Task 7.2 video snippet of children mentally calculating number problems and comparing this approach to her own where she has "to think what the answer is and then write it down", Cara commented that she preferred "thinking . . . writing down and thinking, both I think", conveying in this conversation a positive perspective to thinking for the purpose of gaining answers. In response to the Task 10.1 written descriptor Think about it, Cara stated, "Well I have to think about what I'm doing before I do it, just like discussing with someone . . . talk or discuss". This comment can be interpreted in two ways. That is, the action of talking and discussing in Cara's latter quote may have related firstly to organising herself for learning, or knowing what to do, a theme identified within Cara's responses and discussed previously. Alternately, it may have related to thinking about the maths element within the task at hand. Such a situation was witnessed in the *Pasta lesson* in which Ms S asked the children to plan the container they would construct before beginning the task. A brainstorm with the children as to what the planning could include resulted in suggestions such as talk to your partner, share ideas, discuss an idea, design, draw a sketch, draw something or write something down. The children obviously were familiar with this approach to beginning a new task which involved them thinking about the mathematics they would choose to employ. Such thinking would involve higher level thought than does remembering.

It appears that Cara did perceive that she undertook *thinking* in her learning of maths, and believed that it could be of assistance as, for example, when she talked or discussed with a partner before tackling a problem, or that it could lead to unsatisfactory results as, for example, when thinking by herself about an answer to a difficult times table problem. Thinking had the potential to be of assistance in her learning of maths, but this was not necessarily the case, influenced it seems by the purpose or intention of the thinking, by the difficulty of the problem, and by whether she was doing the thinking alone or with others.

Listening

Listening was another factor identified by Cara through which she could help herself learn maths; Cara linked listening with doing better in maths.

Cara was presented with a task designed to access beliefs about helpful factors in learning maths through a less direct approach than that employed in Task 6.1. She was posed with the sentence starter, "I could do better in maths if" (Task 9.1, Interview 7). She added, "If I listen, learn my times tables better and my pluses even better". This response suggests the holding of a belief that she could participate actively in behaviour aimed at achieving better in maths through taking control of and directing her own attention, with implication that this attention would be to a teacher figure.

Practising

A further factor discussed in terms of helping to learn maths was practise, but the discussions of this factor lead firstly to ambiguous responses from Cara.

Following the Interview 7 conversation quoted above, further questioning took place regarding how Cara was going to learn her times tables better. Her reply of "Keep on doing them over and over again", suggests that Cara believed repetition or practise would help in her learning of maths. The response suggests also a belief that learning requires remembering and low level or instrumental understanding; repetition is unlikely to result in relational or deep understanding.

It had been planned that the concept of practise be examined with the children: *Practise it* was presented as a written descriptor within Task 10. Cara was asked whether *practise it* is something she does when learning maths. It was intended then to ask whether she believed this to help with her learning of maths. However, Cara's response to the question was "No", indicating that she did not believe that she used practise in her learning of maths. As this Interview 9 response seemed to contradict the belief conveyed in Interview 7, as discussed above, I asked Cara whether she knew what *practise it* meant. She replied, "Yeah practising, practising this and that. How to do". This explanation is not detailed and seems not necessarily to match a common definition of practise as, for example, "to do repeatedly in order to gain a skill" (Wilkes & Krebs, 1982, p. 891). Cara's explanation did not include the aspect of doing repeatedly, something, that according to her Interview 7 response, she did consider to help her learn maths better.

It seems that repetition, commonly referred to as practise, was perceived by Cara to play a role in helping her learn maths. This insight became evident particularly from the Interview 7 response which Cara offered and which was phrased in her own words. The seeming contradiction in Cara's responses to the concept and the term *practise it* appear due to her not consciously linking the term with the activity of repetition. Cara's responses regarding repetition or practise as a helping factor support once again the inclusion in this research of multiple modes and instances of data collection for investigating a child's beliefs about a concept, and warns that a researcher should not assume a child's meaning for a term even if it is considered likely to be familiar to the child.

Concentrating

Another factor identified by Cara as helpful in her learning of maths was concentration. In Interview 7 Cara made two references to concentration. The first was in response to a drawing shown to Cara in which a child had portrayed quiet areas as those best to learn in. When asked whether this would be the case for her, Cara stated, "Loud noise makes me lose my concentration" and added that she preferred quiet areas. Open-ended tasks, such as Tasks 6.1 and 6.2, did not stimulate Cara to give such a response, but her response to the situation chosen and depicted by another child does suggest that a quiet environment was considered by Cara to be of assistance in learning maths. The child's drawing presented the possibility of
a helpful factor in the way a questionnaire does, but through the use of a pictorial rather than a written item.

In the subsequent drawing shown to Cara, a child had portrayed a maths test as the situation in which maths was learnt well. Cara was asked whether a maths test would help her to learn maths well. She replied, "No it's really trying, nup, you can really get it wrong, it's like oh I done this, and you're really not concentrating so maths tests is really hard for me". Cara spoke of losing her concentration in a test situation, one in which she believed a possibility of getting something wrong. While not stated explicitly, it appears that Cara may have felt some anxiety in such a situation. This possibility is apparent also in her comment that followed:

Oh, it's like I um, I'm having a maths test like, I'm working alright, then the teacher gives me a maths test and I don't know what to do and then I get it all wrong then like, I like, I don't want to get in trouble . . . It's like I'm scared

The anxiety Cara felt in a test situation did not appear to be a stimulant for her learning. When asked, "So it doesn't make you try harder to learn?", she replied: "Well it's hard for me". Cara believed she had to memorise for a maths test (Task 10.1), a process in which she was not always successful. She also felt the need to concentrate in a test, but perceived herself as unable to do this adequately, seemingly because of the anxiety associated with the possibility of getting things wrong. When asked whether she had had a maths test Cara stated: "Yeah we had it once and I got some answers wrong". Through her responses, Cara showed that she was familiar with the concept of tests in maths.

The responses to these two Task 9.2 drawings indicate that Cara perceived concentration to be one factor that helped her to learn maths, fostered in most situations by a quiet environment. In a test situation, anxiety would negate the possibility of concentration being a helping factor.

Estimating/Guessing

Cara appeared to show some inclination to perceive estimating and guessing, a theme that evolved from her discussion of maths activity, as influential in her learning of maths. For example, when presented in her seventh interview with a drawing that illustrated the belief of a child that maths games help in the learning of maths, Cara described the activity of the children portrayed in the drawing as guessing and indicated that she felt maths games would help her do better in maths "because you're estimating and you're seeing if the guess is right or wrong". This excerpt suggests that Cara considered guessing and estimating to help in her learning of maths.

In describing her situation chosen for Task 6.1, Cara first nominated estimating as something that was helping her to learn maths well. When, during the interview, we looked at ordering the helpful factors, thus approaching this task in a more structured way, initially she placed estimating as the second most helpful factor, but then withdrew it as she decided that estimating had been a "bit hard . . . like if we guess we might get it wrong". She considered

that guessing could lead to an incorrect response. While correctness "wasn't that important" in that situation, the association between guessing, getting something wrong, and the difficulty of the estimation in that maths task seemed to result in the elimination of estimating from the helpful factor list. Cara concluded that estimation had "not really" helped her to learn maths that day.

It appears that estimating or guessing could potentially be considered of help in maths learning, but that this depends on factors such as the perceived difficulty of the task and whether it is likely that estimating or guessing will result in incorrect answers for that task. Cara liked to get right answers as she stressed when discussing the use of the ruler in her fourth interview. She spoke of the teacher wanting the children to guess and then measure, but that they liked to measure first "cos you get it right". Her telling of this story communicated an understanding that measuring first was not favoured by the teacher but that it was more fun for the children! Perhaps Cara liked to begin by measuring as it might have caused her to feel good as she was more likely to get the answer right the first time. Her view may reflect also her assessment of her ability in maths.

Seeing a pattern

A further mathematical activity identified by Cara as helping her learn maths was seeing a pattern. In her seventh interview (Task 9.2), she recounted a situation in which a friend had been helping her but then stopped so Cara went on by herself and got the right answer because, as she described, "I saw a pattern and I was going ten, twelve, fourteen, sixteen, eighteen, and she didn't know it was nineteen and I knew it was nineteen 'cos I was following a pattern". Cara implied that getting a correct answer is learning; this concurs with her belief, as discussed earlier, that evidence demonstrated through a change, in this case the getting of a right answer which is possibly seen as a change in knowledge held, is proof of learning. Cara indicated also that she believed that her chosen mathematical activity, following a pattern, was instrumental in helping her to learn maths better. In the two maths lessons that I observed in the classroom, number pattern work was included. The class teacher, Ms S, indicated to me in discussion that she considered work with number patterns to be essential in learning maths: "... knowing enough about the patterns and I think if you know enough about the numbers up to one hundred then you've got it made" (Interview, December 11). She ensured that she devoted time regularly to counting and number pattern work. The class teacher's emphasis on pattern work in maths may have had some influence on Cara's attempted use and identification of a pattern as a helping factor in her learning of maths.

Writing

In Cara's response to Task 6.1 in which she ordered the three most helpful factors in a situation in which she thought she was learning maths well, as indicated earlier, she chose *write it down* as the third most helpful factor. She explained that "we knew what to measure" indicating that writing was for the purpose of giving direction to the children in their work. Cara believed it helped with learning because the children needed to be told by the teacher,

and to record what to do, so that they would know what to do and could learn from the activity. This links with Cara's perceived need, identified and discussed earlier, for organisation and direction in learning situations. Perhaps as a low achiever this gave her some success, at least in feeling that she could begin a task.

At other times Cara implied also that writing contributed to her learning as when speaking in Task 1.2 of learning times tables by doing a worksheet at home: "We have to write them out and write the answers", and as when, in response to a Task 7.2 video snippet, she commented that both writing and thinking are "better for you to learn". However, it seems that Cara had not fully established a belief of value in writing for learning maths as demonstrated in a conversation during Task 8.2:

- Interviewer: What about writing, do you usually write things or do you sometimes just do them in your head?
- Cara: Write them and then we guess them in our head
- I: And does writing help you to learn maths?
- C: No, you have to write down the answer but writing doesn't help us
- I: So writing the answer does that help you to remember the answer?
- C: Yeah if you look back on the answer

In this quote Cara seems to isolate the practice of writing the answer from learning the answer. Writing is not seen to assist learning, although Cara does indicate that it might help her to remember the answer if she looks back at that writing. In this case it appears that the act of looking back at a written answer is believed to assist remembering, which might be seen as learning, but the act of writing the answer is not. This comment is not necessarily in contradiction to the previous comments regarding writing, but may be an adjunct, a clarifier or a qualifier that applies to those as well.

Cara helping herself to learn maths: Summary and reflection

The discussion within the theme regarding Cara helping herself to learn maths revealed that Cara saw herself as playing a number of roles in assisting her own learning. As stated earlier she sometimes expressed what appeared as contradictory or ambiguous views. To summarise the above discussion of Cara's beliefs of how she could help herself learn maths the factors discussed are listed in Table 3, with a listing of the status of the factors as indicated by Cara in our interview conversations, and an explanatory comment for summary and/or clarification.

Cara's descriptions portray a mix of factors through which she could help herself learn maths. As discussed following Table 3, the factors vary in being of an internal or external nature, and in the degree of control that Cara could direct consciously or unconsciously over their implementation.

| Factor | Status for helping herself learn maths | Explanatory comment |
|-------------------------|---|--|
| listening | helping | - a part of taking control of and directing her own learning |
| seeing a pattern | helping | - pattern leads to correct answer, that is, evidence of learning |
| practising | ambiguous/ contradictory | concept of repetition (practise) as helpful term practise it not defined as repetition |
| concentrating | ambiguous/ contradictory | helpful in quiet environments not helpful in test situation as is negated by anxiety |
| thinking | ambiguous/ contradictory | helpful for gaining answers helpful to think about what doing knowing what to do thinking, brainstorming, planning stages in solving a problem not helpful for a hard times table problem (here, thinking may equal remembering) |
| estimating/ guessing | ambiguous/ contradictory | helpful for learning when playing a game not helpful if leads to an incorrect response |
| writing | ambiguous/ contradictory | helpful for knowing what to do and for writing answers written answer, or product, is helpful for remembering; the process of writing, is not |

Table 3Cara helping herself to learn maths

Thinking, estimating, and guessing are cognitive or internal processes. Thinking for Cara could also involve an external component, talking and discussing, undertaken with someone else. Repetition is an activity that may be apparent to an observer, that is, if undertaken through speaking, acting, or writing, or may be an internal process undertaken without any outward sign. Concentration, indicated by Cara as potentially helpful in her learning of maths, is like repetition; it may have some visible signs or it may be a private and individual activity. For Cara, concentration could be improved by an external element, a quiet environment.

By indicating that she considered listening, concentrating and thinking as helping in her learning of maths, Cara implied preference for a degree of personal decision-making or control for helping her leaning of maths. In contrast, factors such as writing and estimating may be more teacher-directed in the school learning situation.

The factors discussed above relate to Cara helping herself learn maths, thus her own internal or external actions were the focus of the discussion. Cara's interview responses indicate that others in a learning situation, such as the teacher and other learners, also could

influence the way in which Cara could help herself learn maths, and could give her direct assistance, as explored below.

Other learners helping Cara to learn maths

In some cases Cara felt negatively about the potential of other children to help with learning maths, and at other times saw positive outcomes in working with other children.

Where other learners were present, the role of the teacher, and other factors, seemed to influence some of Cara's statements regarding the potential for those learners to help in her learning of maths. When shown a photograph of three girls working in a group (Task 8.2) and asked whether she thought working in groups in maths would help her to learn, Cara replied,

It helps, that helps, it helps really much. Working in a group you ask the answer and the group comes together and then you say, okay now um, does anyone know this answer, and then they all divided it up on a calculator, and they go blah blah and then the answer comes out and you say thanks, and then the teacher gives you more and then the group helps but if you're not working in groups it's really boring, you can't help yourself

It appears that Cara saw the purpose of the maths activity in this scenario as getting answers and identified the activity with the learning of maths. Use of the words "you say thanks" implies that the group was seen possibly as providing the direction for each person to know how to calculate the answer on their calculator, or possibly as just providing the answer. It seems Cara believed there was a greater probability of getting the correct answer, thus of achieving the goal or purpose in doing the task, and thus of producing evidence of learning in maths, when working in a group than when working alone. The final statement in the quotation above implies that Cara perceived that when working by herself she may need help, but that she does not have the strategies or knowledge to provide help for herself. This may cause her to lose interest in the task. The group was seen as helpful in learning maths. The teacher in this situation provided the maths tasks or activities, thus playing the role discussed earlier as organiser for learning; the teacher was not portrayed in a more active role such as discussing with, or explaining to, the children. The group appeared to play the active role of working together supported by the calculator, a tool that provided the answer. In Cara's description of this situation, the group was portrayed as the helping factor in what was seen as her learning of maths.

Three Task 10.1 written descriptors also prompted Cara to talk of the benefits of working with other learners in maths. In response to the first, *work with other students*, which addressed this concept directly, Cara talked of her Maths Task Centre partner, her *fire* buddy (in accordance with the theme, *Fire*, being studied by the class), and her maths partner with whom she would solve problems. The conversation continued:

Interviewer: Does that help you though, working with the other student? Cara: Yep

I: In what way?

- C: By solving problems
- I: Could you solve them by yourself though?
- C: (Laughter) I couldn't

Cara's laughter indicates that some problems could be so difficult for her that she did not have confidence in being able to solve them. A partner could give support with mathematical activity which she perceived as difficult.

When posed with the next Task 10.1 written descriptor, *figure it out myself* Cara took the opportunity to add to her previous statements. She responded,

- C: Well sometimes I know things and sometimes I don't
- I: And does that help you to learn it better if you figured it out yourself?
- C: No
- I: Is it better if you do it with someone else?
- C: Yep

Cara suggests in this statement that she learns better if she works with someone else. In view of her previous statements this may mean that she believed she is more likely to get a correct answer when working with someone else. Because of her perception of herself as "not that good at maths", she might not assume that *figuring something out* herself means that she will come to a correct answer by herself. It appears that she felt there was a greater possibility of success when working with another child in learning maths.

When asked whether *do problems*, as presented on the third of these written descriptors, would help her to learn maths, Cara replied, "Yes. By working with a partner we solve problems and then we figure them out the answer [*sic*]". Apparent once again is the implication that getting an answer is evidence of learning, coupled with a belief that her own learning is assisted through her and her partner figuring out or solving problems together.

As discussed above, Cara's response to the descriptor *Work with other students* indicated that she appreciated support from other students when working on a difficult maths task. Cara made further reference to this type of help from other learners in two statements in Task 9.2. In the first of the situations she was shown a child's drawing depicting two children sitting with their teacher while eating play-lunch and being asked verbally the question " $4 \times 10 = ?$ ". When asked whether she would find Ms S questioning her in that way helpful for learning maths, Cara stated, "I would like a friend with me . . . I might get some wrong and then the teacher was going, I thought you wanted this, like I'd have a friend". Once again Cara's tendency to relate her maths activity to the possibility of getting wrong answers is apparent; in this case it appears to result in some worry about the teacher's response. Cara implies that it is the teacher's expectation that she know correct answers; the presence of a friend seems preferred either for moral support or for giving those answers. Cara was asked about learning but did not use this word in her response. It appears once again she was linking correct answers with learning.

The next Task 9.2 drawing shown to Cara in the interview depicted two children doing maths problems together, with one saying "Thankyou Robert". I explained that the child found his friend, Robert, his biggest help in maths, because he helps him to learn and understand. Cara related to this situation, speaking of the help a friend had given her and commenting, "If you get it wrong you need a friend. And if you get it right it's like thank you

for reminding me". The moral support from a friend and the help in giving answers seemed to be perceived by Cara as helpful for her learning of maths.

The interview excerpts discussed above suggest that Cara believed that when she experienced difficulty with a maths task the presence of other children contributed to her own success in completing the task. As Cara appeared to see successful completion as evidence of learning it is not surprising that she identified the presence of other children in such situations as helping her learn maths. Cara appeared to perceive that having another student with whom to do the task would at times give her moral support, especially when she felt some pressure from teacher expectations. It appears that having other learners present gave Cara the confidence to give answers, and perhaps therefore to learn maths by engaging herself in the activity without fear of failure or criticism.

However, Cara did give some indication that help from other learners was not essential for the learning of maths. For example, when told of a hypothetical situation of a person in her class finding maths hard to learn (Task 7.1) she suggested that a private lesson would be appropriate because "if the children were all together the children would tell him the answer, he wouldn't learn for himself". She said it is the teacher who is most helpful "because children don't know much, that's why they go to school to learn". A view that the teacher holds the necessary knowledge of maths for sharing with their children is inherent in this statement. It appears important to Cara that to help a child having difficulty the child must be given the opportunity to learn for him or herself; that the other children and teacher must give a chance for the child to have a go. The process would involve the child "guess[ing] the answer and then if it's wrong the teacher will tell him the right answer". The subject of the role of the teacher will be discussed in more detail in the following theme; the emphasis at this point is on the potential of other learners to help in learning maths.

Cara's assessment of how good she was at maths and her concern about getting incorrect answers suggest that she might at times have been in the situation of the child in the hypothetical situation (Task 7.1). However, her perception of appropriate involvement of other students in the hypothetical child's learning does not appear to concur with the above responses regarding involvement of other students in her own learning. The comparison of her responses suggests that Cara did not associate personally with the hypothetical child, just as she could not associate her experiences with the scenario of something stopping her or making it hard for her to learn maths well; she was unable to respond to Task 6.2 as it was posed. It is possible therefore that what she saw as helpful for herself she did not necessarily see as helpful for others, or that her beliefs differed either because of the particulars of the situation portrayed or perhaps because of her own experiences around the time of any one interview.

It is noted that in one response in her first interview Cara chose *partner* as the least helpful factor for her learning, explaining that in the measurement learning situation portrayed for Task 6.1 she would not like to have "a partner that would tell you all". When compared to

her many comments extolling the merits of working with a partner, it seems that there may have been some uncertainty or even contradiction in her beliefs. A possible explanation for Cara's apparent contradictory view is that she may have found other learners helpful in a number situation, but not needed in a measurement situation as in the latter she may have perceived less emphasis on knowing an answer and therefore have felt less pressure. Alternately, Cara may have perceived a difference between the cognitive activity in working on a problem and solving it together, and being told the answer by another child without any cognitive activity of her own. Indeed if she was making this distinction, and gave preference to the former view as more beneficial for learning maths, there would not seem to be contradiction or uncertainty in her beliefs regarding the helpfulness of other learners for her learning of maths.

Later in the same interview Cara recounted a time when she and her partner were fighting because of an disagreement about whether there was "one in a half". She said that as the teacher had said that Cara was right, her partner had not helped her at all. This assessment of the partner's lack of assistance is logical in terms of Cara's apparent association of correct answers with learning. However, this does not discount the possibility that in cases where Cara had difficulty with a maths problem she preferred to have the help of fellow learners, particularly when she perceived this would lead to a correct answer.

As discussed above, Cara stated in Interview 7 that she preferred to work in a quiet place when learning maths. When posed with another task focused on this construct (Task 10.1, Interview 9) she indicated that she sometimes preferred to sit in a quiet place "so [other children] will [not] go 'what's the answer, what's the answer?' when I know it". This may have related to a resultant lack of ability to concentrate, or a dislike of being interrupted. It is apparent that Cara perceived herself as knowing the answer in this situation.

It seems that when Cara felt confident with maths, that is, when she felt she would achieve correct answers, she did not perceive that the assistance of fellow learners would help her with learning maths. As stated above, it seems that it was when Cara personally had difficulty with a maths task that she saw the presence of a partner or friend as beneficial.

The role of adults in helping Cara to learn maths

A further theme related to helping factors in Cara's learning of maths is the role of adults, including both parents and teachers. Generally adults were not seen to hinder Cara's learning of maths, although they may have caused some anxiety because of expectations, but were seen to be a factor of help and of motivation for maths learning.

Cara referred a number of times to mathematical situations undertaken with her parents. In at least two of these she indicated that her parents assisted her learning of maths by playing the role of affirmers. For example, in her second interview she spoke of her father giving her "sums" that helped her to know plus and take away. She explained that the activity was like a game, that sometimes it was hard for her, but that if she got any wrong her father would tickle her. Cara did not convey concern about this situation as was sometimes apparent in situations spoken of at school that she found difficult or for which she expected to get wrong answers. When shown a photograph of a mother working with her daughter at the kitchen table (Task 8.2), Cara answered that if that was her mum she would be "telling you the answers and helping you". Cara perceived therefore that affirmation by her parents helped in her learning of maths.

The teacher was seen in various guises or roles as a helper for learning maths. In the discussion of the hypothetical child scenario in Task 7.1, Cara believed that it was the role of the teacher to tell the child the answer to help him learn maths. Likewise in her first interview, she explained that the teacher gives the answer, but also that the teacher provides the maths tasks or questions:

well if the teacher goes one times one and you write it down and then she tells, then write down, we write the answer next to it and then she goes one times one equals one and then you correct it with the red pen . . . the teacher tells you the answer after you finish

The above was Cara's explanation of why she had put *listen to the teacher* in her list of helpful factors, and how the teacher was helpful for her learning of maths. It seems that the teacher was perceived to be helpful not only in her role as affirmer but also in her role as organiser for learning or provider of tasks. However, affirmation could also be provided by other people or by tools such as calculators, as discussed earlier. The teacher acting in the roles of facilitator, encourager or motivator could also be seen to potentially help Cara to learn maths. For example, trying to learn maths to please the teacher may have caused Cara to behave appropriately such as by listening or concentrating better. However, it seems that it was as the affirmer and provider of tasks that Cara saw the teacher as most helping her learning of maths.

Tools helping Cara to learn maths

An earlier discussion of affirmation in learning maths included mathematical tools such as the calculator and blocks as contributing to this task for Cara. Some tools were seen also to help in Cara's learning. For example, Cara stated that blocks can be "helpful [for learning maths] if you're counting with them" (Task 9.2). Times table sheets on the wall or on the back of a book were believed helpful for learning by providing answers (Tasks 9.2, 10.1). The calculator was considered to help children "that don't know" as it provides answers, but Cara felt in the context of that particular conversation that working with others would be of more help to her (Task 7.2). In another conversation Cara indicated that a calculator does help her to learn maths: "By typing one times one and then you know what it equals. Or really hard one". Learning was perceived as finding the answer, thus the calculator could help the user achieve this step, and according to Cara, learn maths.

A further tool seen to be of possible help in learning maths was a balance scale. In discussing a Task 7.2 video snippet Cara stated that the children working in groups could

learn without the teacher present because "the scale can tell you too . . . the scale can help you too". The learning in this situation was perhaps finding an answer or result.

However, Cara did indicate that tools can cause confusion if one does not know how to use them properly, as demonstrated in the following Task 8.1.2 conversation:

Cara: Times tables are too hard for me now

- Interviewer: Do you think someone can do something that will make them easier for you, or you can do something, or someone else?
- C: Well, someone told me to use my hands like nine times two you would take away the two I: Mm
- C: And then I don't know how to use them and you go one, two, three, four, five, six, seven, eight, nine, nineteen
- I: Right, so you're not sure what to do for the rest of it?

C: Yeah

Cara did appear to believe that tools played some role in assisting in her learning of maths, but overall it appears that the assistance of people, particularly adults, was considered of greater value. Luck was another factor present in Cara's responses.

Luck as a helping factor for learning maths

On two occasions Cara attributed success in maths to luck. When speaking of the alien (Task 5.1) she stated that "he tries to guess a number . . . If it's right he had a lucky guess". As discussed earlier, this assessment of the reason for the alien's correct response may have related to Cara having been told that the alien knew nothing about maths. Luck might be a reasonable explanation for success in this situation where the task was totally foreign. However, on another occasion Cara spoke of getting some maths problems correct "a long time ago we done a times table sheet and I got them all right and I was very lucky" (Task 10.2.1). This was evidence that Cara saw luck contributing to some degree to her success in learning maths. However, while present in her beliefs, luck seems a factor of lesser significance in helping maths learning than others discussed. The research included also consideration of factors hindering maths learning.

Factors hindering Cara from learning maths

In the pursuit of insights into helpful factors for learning maths, it was planned that factors perceived to hinder the learning of maths would be discussed with participants. It was believed that factors hindering learning could help the researcher, and possibly the child, to identify and clarify helping factors. However, Cara seemed unable to portray concretely her own difficulty in learning maths.

Two scenarios offered by Cara, as discussed in the introduction, resulted from Set 6 procedures that were designed to target beliefs about helping factors for learning maths. When asked in Task 6.1 to think of a situation in which she perceived she was learning maths well, Cara responded positively and without hesitation. However, for Task 6.2, in which she was asked to think of a situation in which she felt something was stopping her or making it hard for her to learn maths well, that is, to identify factors perceived to hinder maths learning, she

responded, "There isn't any time . . . it could be for someone else but not for me". This response was unexpected from the child who had been chosen as a lower achiever among the Grade 3 females in her class, and who considered herself as "not that very good at maths", as was revealed in her sixth and tenth interviews. Although she was aware of her own difficulties in learning maths, she was unable in this first interview to isolate a particular situation in which something was stopping her from learning maths well. It was for that reason that when Task 6.2 was posed she related another situation in which she was learning maths well, although this was not originally intended. Flexibility in the interview procedure accommodated the child's perspectives and allowed further insight into one aspect of her beliefs.

In three interviews Cara did mention factors that she appeared to perceive to affect negatively her learning of maths. During Interview 7 Cara stated that "Loud noise makes me lose my concentration" and in Interview 9 indicated that she did not like other children to ask "what's the answer, what's the answer?". It is clear that at times she did not like noise or interruption and wanted to be able to concentrate. However, in a test situation, anxiety would negate the possibility of concentration being a helping factor. The Task 8.1.2 (Interview 4) discussion of another child unsuccessfully showing Cara how to use her fingers in maths, as discussed above, suggests also the possibility that a new method could be a hindrance if not understood properly. The method of using fingers was not understood by Cara and therefore had a detrimental effect by causing confusion.

Little information was gained from Cara regarding factors perceived to impact negatively upon her learning of maths, except those mentioned above and those assumed by implication, that is, where appropriate, the opposites of those she identified as positively impacting upon her learning of maths. However, the insight gained from this element of the research was Cara's inability to reflect deeply on this aspect of her learning.

Cara had been chosen by her teacher as a low achiever in maths. Therefore, had Cara been able to identify factors hindering her learning, the findings would have given her teachers and others information to reflect upon, and perhaps some direction for re-structuring the learning environment to advantage Cara. Cara's inability to reflect deeply on hindering factors may be related to her level of cognitive thought. Seeing progress in learning, in terms of one's concept of learning, and seeing related helping factors, may be more real or concrete than factors hindering learning which seemed mostly abstract for Cara. Affirmation by the teacher or other was demonstration of progress of learning for Cara. Failure in learning may have been communicated also by the teacher, but Cara appeared to find the reasons for the underlying difficulty less easy to identify.

Helping factors for learning maths - Summary

Cara attributed many factors to helping her learn maths including other learners, the teacher, her parents, a variety of mathematical tools and luck. In Interview 6 (Task 7.1) the teacher was portrayed as playing an important role: "The teacher is the most helpful person because

children don't know much. That's why they go to school, to learn". Importantly, in addition, by taking responses from a number of interviews and thematising the data, another person emerged as helping Cara learn maths; that person was Cara. Cara did not make strong overriding statements about her own role in learning maths, like the statement above of the teacher, but she did, within the interviews conducted for this research, portray herself as a key factor in her own learning of maths.

Cara saw school as the main centre of learning; just as the school is generally recognised by Western society. For Cara, doing or using maths related either to solving a problem or to learning for school. Home-related maths appeared to be seen as having little role in assisting learning. Indeed, when Cara was asked whether she would learn well at home, like two girls depicted in a photograph, she stated "Well you really need school so I'm not sure" (Interview 4, Task 8.2). It is significant that while Cara recognised that she performed much maths at home, for example, with her parents, the home-purpose maths was not associated with learning.

Learning maths was associated mainly with school, and could be helped by a range of factors, although at times, Cara's beliefs about the effect of some factors seemed ambiguous or contradictory, as discussed earlier.

Cara: A Brief Reflection

Cara was chosen for participation in the research as a lower achiever in maths. Her data indicate that

- Cara held complex views about the nature of maths and mathematics.
- Cara was able to talk confidently and extensively about maths and seemed to hold some personal affinity with this concept.
- Cara's data about the concept of learning were less detailed, perhaps reflecting less previous thought or discussion about this concept.
- Cara was able to articulate views portraying helping factors for her learning of maths but had difficulty identifying situations and factors that hindered her learning of maths.

Cara's beliefs were idiosyncratic; the portrayals of beliefs of the other seven research participants also show individual perspectives. The beliefs of one other research participant, Emily, chosen as a high achiever in maths, are discussed in full in Chapter 6. Following that, Chapter 7 includes a comparative discussion of findings from the eight research participants. Reference is made to Appendix E which provides portrayals of the beliefs of Gina, Ben and David.

CHAPTER SIX A PORTRAYAL OF EMILY'S BELIEFS

The three-part structure stemming from the key research questions, and developed for the discussion of Cara's beliefs is deployed also for the discussion regarding the child for whom the pseudonym Emily is used. The portrayal of Emily's beliefs is broken into the sections:

- beliefs about the nature of maths and mathematical activity;
- beliefs about learning;
- beliefs about helping factors for learning maths.

In the bulk of the discussion, the more common term, *maths*, is used in preference to *mathematics* as this word was used mostly during the interviews with Emily. However, as discussed later, the understanding and use of the terms maths and mathematics did not pose an issue for Emily as she considered them basically synonymous (Tasks 1.2, 1.3.1, 11.2).

The discussion within each of the three sections listed above is broken into subsections, structured and titled according to themes that were seen by the researcher to emerge from Emily's data. The compilation of responses from a range of interview tasks provides a rich portrayal of Emily's beliefs, as the theme-based analysis allows for an interweaving of data, that is, for data from a number of tasks to contribute to insights gained about any one particular aspect of Emily's beliefs. As a metaphor, when preparing for a visual portrayal of an area of countryside, not only are main routes or highways traversed, but views are taken also from many criss-crossing backroads giving a richer appreciation of the variations and complexities that lie within. The same theme-based approach was taken in analysing Cara's data, resulting, as has been seen, in a complex, and at times challenging, analysis and portrayal of beliefs.

Overall, Emily is portrayed as a child who was confident at maths, who in general focused on maths as related to number, counting and operations, but whose views were not as straightforward as they at first appeared. Her beliefs about the nature of maths and maths activity contained subtlety, depth and some degree of complexity. Emily appeared unused to talking about the nature of learning but gave some insights into her beliefs, which appeared related to her beliefs about helping factors for learning maths.

Beliefs about maths and maths activity

Emily's beliefs about the nature of maths and mathematical activity are explored through a theme-based analysis, followed by a summary. Themes that emerged from Emily's data are introduced below.

Introduction to Emily's beliefs about maths and maths activity

Emily's beliefs about maths, as obtained from the interview responses, reveal less breadth than those of Cara; indeed, on the surface, Emily appeared to have beliefs like those of the children in Cotton's (1993) research who "saw mathematics as nothing more than number" (p. 15). Such a statement may imply that a child's beliefs about maths are narrowly focused.

However, the present study indicates that even within the domain of number, a child's beliefs can have a range of perspectives, some of them unexpected. The discussion of Emily's interview data illustrates that while her beliefs might not appear as complex as those held by Cara, they were not always straightforward and were subtle, deep, and to some degree complex. The data are analysed and discussed according to the following themes that were seen by the researcher to emerge from Emily's interview responses:

- maths as numbers and operations;
- numbers as necessary for maths but not sufficient;
- maths as addition and counting;
- measurement as maths when numbers, counting or operations are present;
- comparisons related to number and formal measurement;
- maths primarily as a schooling experience.

The characterisation of Emily's data as having an emphasis on number-related aspects of maths, as portrayed throughout the discussion of her beliefs, is present in her Task 1.3.1, maths word wheel, response (Figure 22). However, as the thematic-structured discussion develops, a greater depth within Emily's beliefs becomes apparent, accessed by the cross-analysis of responses to a range of interview tasks.



Figure 22. Maths word wheel - Emily.

The maths word wheel response illustrates an emphasis on number, and provides also some insights into Emily's seeming equivalence of meaning for the terms *maths* and *mathematics*. Emily's views provide an interesting contrast with Cara's differentiation of the terms. When probed following this response, Emily stated that "mathematics is nearly the same as maths", but could not elaborate. In the password task, undertaken earlier in the same interview (Task 1.2), Emily had offered mathematics as a same-meaning substitute for maths, and explained her choice: "because it is like maths, with addition and all those things". In response to a question asking when she had heard the word mathematics used, Emily answered "when I do it", as the teacher "uses both of them". Emily believed she had always

thought the two terms to have the same meaning. She thought her home language, unlike the English language, only had one term. This may have had some influence, along with the teacher's language use, on the development of Emily's belief that the terms maths and mathematics have the same or virtually the same meaning. An attempt was made later in the interview to understand further the relationship Emily perceived between the two terms. She was asked what changes she would make to her word wheel if the question was changed from "What is maths?" to "What is mathematics?". She replied that on the outside of the word wheel she would write *maths* instead of *mathematics* and keep the other outside words; the simple interchange of terms suggests an equivalence of meaning.

Unlike Cara, Emily did not seem to hold different meanings for the terms *maths* and *mathematics*. Therefore, as terminology use did not pose an issue for Emily, this will not be discussed further. The term, maths, as seemingly most commonly used by the research participants, is deployed in the discussion of Emily's beliefs.

Maths as numbers and operations

Emily gave two definition-type statements regarding maths that referred to maths as numbers and operations and thus confirmed and elaborated her response given in the maths word wheel (Figure 22).

When asked in her second interview to give a personal dictionary definition of maths (Task 2.3), Emily replied, "Maths is when you learn numbers and plus sums, times tables, take aways, and divided by". When given the responsibility of helping an alien to understand what maths is (Task 5.1), Emily suggested she would

show the alien all the numbers, from one to ten, and show them the plus sums and minus sums and all different kinds of sums, and divide, and all those and, I'll show the alien like, if I wrote five plus two I'll show the alien five fingers and then you add two more.

The message was similar in each statement, showing an emphasis on number, but a view that expanded upon one aspect of the word wheel response; in the Task 2.3 and Task 5.1 quotes Emily made specific mention of some types of *sums*, that is, times tables, take away/minus, and divide/divided by. The responses suggest a maturity of language use for a Grade 3 child: Emily used the term "minus" as well as "take-away", and referred to "divided by" and "divide" as well as "how many". This sophisticated use of language at the Grade 3 level appears consistent with Emily's high level of achievement in the maths classroom, as assessed by the teacher and by Emily.

When the response to the Task 5.1 alien task is considered in terms of other descriptions of number situations, it is clear that the range of numbers given by Emily was more limited than that with which she worked at that time. Although Emily suggested that knowing the numbers one to ten would be useful for the alien, she had talked in the previous interview of 153–12=141 as being easy for herself to learn. Perhaps in the alien response Emily was portraying what she saw as the essential elements of maths, as these would be suitable for a

creature who had had no previous experience with maths as Emily knew it. She may also have chosen these numbers as an exemplar or signifier of maths on earth for the alien.

As an ongoing summary of the insights gained into Emily's beliefs about maths, schematic diagrams are built up through the discussion, beginning with Figure 23. These diagrams are similar to those developed within the portrayal of Cara's beliefs, with key elements present, and sample activities, as proffered or identified by Emily, added for reader reference. Emily's emphasis on number and operations are portrayed in Figure 23, the first stage of Emily's schematic portrayal. Her reference to writing number words as a maths homework task, as discussed in Chapter 4, is included also.



Figure 23. Emily's beliefs about maths - Image 1.

Responses such as those quoted above portray a view of maths based on number. Some of Emily's responses, as discussed in the section immediately below, suggest a narrow view, that is, that numbers can render a situation mathematical, but closer examination of a range of responses shows that Emily's beliefs contained greater depth and subtlety.

Numbers as necessary but not sufficient

Emily appeared to believe that numbers are necessary to make a situation mathematical, but that their presence within a context would not guarantee that she would consider that context or situation mathematical.

Emily stated the need for numbers in maths in response to a range of interview procedures. Having difficulty identifying whether a child in a photograph was doing maths, because of an inability to see the writing on the page, Emily stated, ". . . when I see numbers I think it's going, something that's going to do with maths" (Task 8.2, Interview 3). Similarly, in the word association quiz task (Task 1.1, Interview 9) Emily responded with the word *numbers* when given the word *maths*. Later in the same interview she stated, "You need numbers to do maths". When shown a photograph of a man constructing scaffolding, Emily again suggested that numbers are considered necessary for maths: "I think he's screwing something on . . . it's nothing to do with maths and there's no numbers" (Task 5.2, Interview 4). She thought a teacher depicted in a photograph was teaching maths because he was writing numbers on the blackboard (Task 5.2, Interview 4). In fact, the blackboard had many numbers on it, presented in number santences and vertical algorithmic form, thus the numbers were to be used in the context of pure number tasks. Emily's focus on the importance of numbers for maths was consistent, for example, as illustrated here ranging from Interviews 3 to 9.

Discussion of a task included by the researcher as potentially involving space concepts, reinforced the importance Emily attributed to numbers within mathematical activity. The discussion also gave further insights into the perceived relationship between maths and language, and gave insights into Emily's beliefs regarding the place of space, and more specifically shape, within maths. A video snippet of children making shapes on a geoboard (Task 7.2) elicited the response that Emily did not think they were learning maths because "you have to have something to do with numbers". She appeared to perceive her own learning about shapes to be more language centred: "[we learnt] what the shapes are called, like octagon and all those stuff". In contrast with the earlier discussed homework situation where the writing of number words was classified as mathematical activity, Emily appeared to perceive her learning of what shapes are called to relate to language and not to maths. When asked whether it would be maths if the activity was done in maths time she replied laughingly, "No!". Emily's responses to these two situations suggest that where numbers are involved, a language-focused activity may be classified as maths but an activity integrating language and shape, but without numbers, may not be classified as maths. Emily's belief that numbers are necessary for mathematical activity appeared established; she was not tempted to change her view even if activity without numbers was labelled as maths through timetable allocation.

Spatial activities did not appear to be considered as mathematical activity by Emily. Even after she had experienced two lessons on symmetry between her seventh and eighth interviews, Emily did not offer space or related concepts as an element of maths in any of her last three interviews. Emily might, for example, have made reference to this in her Interview 8 "Maths is like" response, in her Interview 8 drawings of mathematical activity, or in her Interview 9 maths word wheel response. It appears that even the presence of the researcher as an observer of the two symmetry lessons did not cause Emily to re-examine her views regarding the relationship of spatial activities to maths. Emily appears to have associated her experiences with shape not with maths, but with language. Perhaps if she had consciously linked number to her learning of the names of shapes such as octagons, she might have considered this not only as a language activity but also as a maths activity, just as she seemed to do with the homework writing of number words activity.

Although Emily thought numbers were necessary for maths, she did not seem to consider maths and numbers as synonymous as for Emily the presence of numbers did not necessarily make a situation mathematical. This was demonstrated, for example, in Emily's responses to some items in Task 3.3.1 in which numbers were intentionally included as potential distracters. The people in the items "George cleaned up room number 7 which was really messy" and "Susie ran over to Anna's house to see her first dog" were each judged by Emily not to be using or doing any maths. Also, a situation which Emily described by using some number words was not considered to be mathematical: when shown a child's drawing of children and a teacher working with an abacus or concrete graph drawn on a blackboard (Task 9.2), Emily stated that "it's just like putting four things down and five and I don't think that's

got to do with maths". The presence of numbers in a money transaction also did not ensure that Emily would judge this as mathematical activity. For example, she believed there was no doing or using of maths in the situation "Terry went to McDonald's. She paid the salesperson \$3.20 for a hamburger and coke" (Task 3.3.1). In considering this situation, Emily may have seen the numbers only in relation to money, the use of which she did not necessarily consider to involve maths, as illustrated below in discussion of other interview responses. The absence of any operation with the numbers, such as addition or counting, may have been instrumental in Emily not identifying these Task 9.2 and Task 3.3.1 situations as involving mathematical activity. Her responses to these situations indicate also that the presence of numbers would not necessarily render a situation mathematical for Emily.

Clearly Emily did associate maths and numbers, and suggested that for a situation to be mathematical, numbers were required. Yet the use of numbers in describing a situation did not lead Emily automatically to categorise that situation as mathematical. When Emily made the statement "when I see numbers I think . . . maths", she had begun by talking in the context of operations with numbers: "groups of things, two minus one" (Task 8.2). It appears that, when deciding whether a situation was mathematical, Emily would look for numbers as a first criterion, but then seek evidence of use of those numbers in a context such as a number operation. As illustrated in Emily's personal dictionary definition of maths (Task 2.3) and in her response regarding the alien (Task 5.1), maths was associated with the application of numbers through the operations of addition, subtraction, multiplication and division. Adding and counting situations were given some emphasis by Emily and are discussed in more detail below.

Maths as addition and counting

The processes of addition and counting appeared salient for Emily within her domain of mathematical activity; situations containing either addition or counting were believed by Emily to contain maths.

Addition

Emily mentioned addition frequently when defining maths or talking of mathematical situations. For example, when explaining why she considered mathematics to be like maths, she added "with addition and all those things" (Task 1.2). When asked in Task 3.1 to draw a person doing some sort of maths activity she drew "a girl, she's not sure what the sum is, like she's not sure that, like two plus two equals four". The girl was described as using her pencil and her brain, she was writing down a sum, and was in a school situation. Emily might have chosen any school or non-school mathematical activity when responding to this task. Emily had not depicted herself in the drawing but had "made up" a person. When asked in the next task for someone doing a maths task different from adding, Emily chose to talk of "times tables". In each situation Emily portrayed a pure number activity; as the second involved multiplication, it might have been associated with addition also.

In Task 4.2.2 Emily was asked to draw someone using or doing maths in any situation except at home (as she had drawn a home situation for Task 4.2.1). Her drawing depicted "a person and a person's classmate . . . trying to finish their sum up and this girl got the answer and she got it right". The drawing is included as Figure 24 as a sample response illustrating the salience of addition for Emily.



Figure 24. Emily's portrayal of someone using or doing maths - not at home.

Emily identified addition as mathematical activity in discussion of a Task 5.2 photograph that she described as "two boys figuring out what the sum is" and identified as "something to do with maths because there's sums and they're like four plus one and all those things". Addition as a possible criterion for defining a mathematical situation was suggested when two children in another photograph, described as "playing a dice game", were considered to be doing maths because Emily believed they were adding numbers.

When asked whether there is any maths in playing sport (Task 3.3.2, Interview 2), Emily said that when she played basketball and cricket there was "some [maths], because when you win points, you add up the score". In this situation, addition was used as a tool to calculate progress. However, when speaking during Interview 4 (Task 5.2) of her playing of cricket, football, and basketball, Emily gave a different view by stating that there was no maths in playing these sports. It seems, therefore, that maths was not associated readily with sport, unless a process such as addition appeared salient. Perhaps the differing responses related to Emily's experiences at around the time of each interview, particularly whether addition had been used to calculate scores.

When asked about using a calculator to work out money to pay the bank (Task 3.3.2), Emily interperpreted this in relation to payment of bills. She stated there would be "lots [of maths] because um like you're using a lot of hot water or something like that and um then you have to use a calculator and then you have to keep adding 25 plus 25 dollars plus 29". When asked whether this was maths because of the calculator or money she stated "both", but in light of comments at other times, it appears that the question may have led Emily to give a response that did not fully represent her views. From other responses it appears more likely that the presence of addition, which she identified within the hot water bill-paying situation, caused Emily to say that there was lots of maths in this situation. Emily associated the calculator with maths, but pressing numbers was not seen as sufficient for the activity to be mathematical, as shown, for example, in response to a Task 8.2 photograph of three children using a calculator: "I think there's no maths because they're just pressing the numbers and they're not doing anything and they're not writing the numbers or whatever . . . they're not using their brain to know what the sum is". The sum (addition), writing numbers, and using one's brain were suggested as elements of mathematical activity. Money also may not have been intuitively associated by Emily with mathematical activity. Her assessment of paying for a hamburger at McDonald's as not being maths (Task 3.3.1) suggests that the presence of money in the hot water bill-paying situation may not have been a critical factor for her in identifying mathematical activity. As the presence of the calculator also may not have been instrumental in her judgement of this as mathematical activity, it seems the presence of numbers that were added may have been the pertinent factor. Having been asked only about money and the calculator as the link to maths, Emily may have been steered away from discussing addition. The potential for the interviewer to direct discussion is highlighted here; the importance of listening carefully and responding to children's responses is made clear. A more open question might have led to more insights.

As demonstrated in the above discussion, and as shown in Figure 25, the presence of addition appeared to meet Emily's criteria for identifying a mathematical situation.



Figure 25. Emily's beliefs about maths - Image 2.

Written algorithms, for addition and possibly for other operations, were considered mathematical activity, as portrayed in the Figure 24 drawing, and as present in the Task 5.2 photograph, as discussed above, of a teacher writing numbers (within algorithms) on a blackboard. Emily's seemingly contradictory views regarding maths in sport are indicated in Figure 25 by a broken line. Through the discussion of addition as mathematical activity, situations using number in an applied sense have been introduced within the portrayal of Emily's beliefs about maths. To signify this aspect of the activities to the reader, a coding system is introduced: activities judged as pure number are labelled (P); applied activities are

labelled (A). To clarify for the reader the growth within this image of Emily's beliefs, and within subsequent images, the elements added on each occasion are presented in italics. Addition clearly was a key element of maths for Emily. Counting was seen also as mathematical activity, at times closely associated with addition.

Counting

Like addition, counting was identified as maths by Emily in situations that can be interpreted as involving both pure and applied maths. As stated in the discussion above, the handling of money in the hot water bill-paying situation possibly was identified as mathematical activity by Emily because it involved addition. Discussion below shows that counting of money was considered also as mathematical activity. Counting was seen as occurring in school and nonschool environments.

Emily's identification of the counting of money as mathematical activity gives further insights into the subtlety of her beliefs. When shown a photograph of a person paying for an item at a supermarket checkout (Task 5.2), Emily thought that there was maths in the purchaser's activity because "you count, because when it's fifteen dollars and sixty, you can give a ten dollar note and a five dollar note and a one dollar coin and a fifty cent coin and a ten cents coin, so you have to count". The response to this photograph suggests that it was not money that rendered the situation mathematical, but the activity of counting, which appears to have been operating as addition to ensure the right amount of money was paid.

As described above, Emily identified the McDonald's hamburger situation (Task 3.3.1) as not being mathematical, and the Hot water bill-paying situation (Task 3.3.2) and the Checkout situation (Task 5.2) as mathematical, thus appearing to hold some contradiction within her beliefs regarding situations in which money was present. However, closer examination suggests a more subtle view; for Emily it was not the presence of money that made a situation mathematical, but rather the action that accompanied the use of that money. Where counting or addition operated in the handling of money, the situation was believed mathematical, without either of these it was not. The fact that the handling of money does not necessarily involve maths, that is, mathematical thinking or mathematical activity, is perhaps a sophisticated insight for a Grade 3 child.

Responses to two further interview items confirm that counting could be seen as mathematical activity in situations involving money. In response to the Interview 1 (Task 3.3.1) item which followed the McDonald's hamburger scenario, that is, "Tom said that 100 cents is the same as one dollar. Did Tom use or do maths?", Emily responded that he did because "maybe he had to count 100 cents, that makes a dollar". When posed with the Interview 2 (Task 3.3.2) item regarding the buying of clothes at a sale, and given the choice of lots of maths, some maths, or no maths in the situation, Emily stated that she thought there would be some maths "because when you buy clothes, just say they were like 19 dollars, and then you have to count 19 dollars and counting is maths". Counting or addition of money

were applied situations identified by Emily as mathematical, and involved maths in nonschool settings.

Within Emily's responses regarding counting at school as mathematical activity, both pure and applied number contexts are apparent. Emily made one reference (Task 6.1) to her own counting at school: "I was, um learning minus and take away sums and it was very easy because I had to use my fingers and count . . . I was in Grade 1". She had recalled an experience from two years earlier where she had used fingers as concrete aids. A school situation in which counting was associated with addition as mathematical activity was in a Task 5.2 photograph: Emily believed four children were "playing a game . . . adding a sum up . . . they're Ten Blocks (that is, longs in a base 10, Multi-base Attribute Block set) . . . I think they had to count ten to make one of these ones . . . counting is . . . something to do with maths". Emily described a later Task 5.2 photograph as "This boy is peeling the bananas in half, and maybe the teacher told the boy to fill in three pieces of it or four, so he has to count out how many pieces there are and I think that's part of doing maths". Counting of items was seen as mathematical activity. The counting in these three situations was a pure number activity.

Counting was identified also in applied number, or measurement, contexts. However, it was not the measurement that Emily saw as mathematical, it was counting. At times, Emily associated counting with the measurement concept of capacity, although not identifying the latter as a mathematical concept. In Interview 2 (Task 3.3 2), Emily said that she thought there was no maths in using a recipe for cooking. Later in the interview, when asked whether she thought using a recipe would have maths in it for her mother, Emily answered "Maybe, like she has to cook like two teaspoons of salt and then you have to count two teaspoons 'cos you have to count two times of the teaspoon". In Interview 8 (Task 4.1), Emily said that when her mother cooks she uses maths when "she has to measure like how many cups of rice or something like that". Because of Emily's earlier Interview 8 (Task 4.1) response that "maths is like something you use with numbers and sums", she was asked whether her mother would be using numbers and sums when cooking. Emily responded "Yes, numbers and sums because at home we've got something like a little cup and she has to count how many cups to put in the rice cooker". Emily spoke of the concept of measurement, she had used the word measure within the description of her mother's activity which involved the use of informal but uniform units of a teaspoon and a cup, yet the maths in cooking was believed to be counting, which seemed to be associated with sums, perhaps in this case addition. At no time when asked for a definition type statement did Emily use any form of the word measure, but counting was included in her maths word wheel response (Task 1.3.1, see Figure 22). Emily's discussion of her mother's cooking is significant not only because measurement was mentioned, but also because it was one of only a small number of non-school situations that Emily proffered as having mathematical activity. The use of two different units of measure of capacity was not identified as maths; it was counting, perhaps closely related to addition, that

Emily saw as mathematical activity. In these situations, counting was operating as an applied number concept.

A further situation that Emily accepted as mathematical, when the situation was presented in a video snippet by the researcher, was the making of a graph (Task 7.2). It is possible that Emily saw the purpose of the activity as related to counting as she stated that the previous day when her class had made a graph the teacher had been "showing us how many people want to be when they grow up, wanted to be a scientist, a doctor, builder, bus driver and all those things". The discussion demonstrated that Emily believed the presentation of data through graphical representation to be a mathematical activity, perhaps related to counting because of its purpose of finding out *how many*.

To summarise, counting was a process with which Emily was familiar, identifying it in many situations. It appears that counting was an important aspect of maths for Emily. Counting for the purpose of adding the number of blocks, for learning subtraction when in Grade 1, for calculating amounts of money, and for informally measuring capacity were included as mathematical activities. The situations discussed in this section are portrayed in Figure 26 by selected representative examples, with the rice measuring situation labelled (IF) indicating informal measurement, but classified as maths because of the counting that occurred. As the discussion to date does not indicate that measurement was considered a mathematical activity in its own right, a separate measurement section is not included in Emily's schematic portrayal. The selected examples show that counting was seen to occur both in school and non-school environments.



Figure 26. Emily's beliefs about maths - Image 3.

Although Emily did not mention measurement when asked for a definition type statement of maths, she did use the term measure in reference to the cooking of rice, identified in this document as the use of informal units of measure. At times she also referred to or implied the use of formal units of measure, as discussed in the following section.

Measurement as maths when numbers, counting or operations are present Emily at times associated measurement with maths, but mainly when numbers, counting or adding were involved, which in turn related to informal and formal units of measure. Emily's use of the term *measure* in describing a situation did not ensure that she saw the activity as mathematical; she did not make an automatic association between measurement and maths.

Responses within Task 8.2, given in Emily's third interview, provide some insights into her beliefs about measurement and its relationship to maths, although the task was designed to access beliefs about factors perceived to help in her learning of maths. Task 8.2 involved the describing of photographs followed by the sorting of them according to whether Emily perceived she would learn maths well in the portrayed situations. Emily described the first photograph as "she's measuring things" and explained that the girl was measuring with a ruler. Emily said that she would not learn maths well in this situation. At that stage she was not asked why. The second photograph, which showed one child informally measuring the height of another with a piece of string, was described by Emily as "measuring how tall she is". Emily went on to explain that she would not learn maths well in the second situation "because the string is not a ruler . . . so the boy might not know how tall she is because it's just a string and it doesn't say how many centimetres she is". Emily had stated that she would not learn maths well in the two situations. It is worth noting that we do not know for sure whether Emily believed each situation portrayed mathematical activity, as the questioning did not pursue this directly.

When asked then to explain her response to the first photograph, in which a ruler with centimetres was used, Emily could only say "I don't know", but later changed her mind for photograph one, deciding that she would learn maths well in the first situation "because they're measuring how long it is". In this response Emily implied that the situation was considered mathematical. The second situation, measuring with string, a comparison situation, was seen as portraying measurement but not necessarily maths; certainly Emily did not perceive that she would learn maths in that situation.

These responses suggest that Emily identified both the formal and informal measurement of length situations presented in the photographs as *measurement*, but the second situation possibly not as *maths*. The conclusion that she may not have thought of this informal, measuring with string, situation as maths is given further support in Emily's response to the fourth Task 8.2 photograph: "She is trying to measure how long is the bench and maybe that's not maths because, mm, if, no numbers or counting or whatever". Emily did not recognise comparison of lengths in this photograph as mathematical activity, although such activity can legitimately be labelled as informal measurement of length within the primary mathematics curriculum (Board of Studies, 1995, 2000). It is significant that Emily spoke of the situation in the context of measurement, but looked for numbers or counting to decide whether the activity was mathematical.

Responses to the second photograph (string) and fourth photograph (bench) suggest that when Emily used the word *measure* to describe a situation, she did not necessarily consider that situation as *mathematical*. Numbers and counting seem to have been instrumental in determining whether Emily saw posed situations as mathematical. Emily's response to the third Task 8.2 photograph, in which she described why the making of shapes would not be a situation in which she would learn maths well, gives further insight into the subtlety of her beliefs. From her explanation that "it's nothing to do with maths because maths is like measuring and counting", it seems that use of units of measure may have been considered maths when their use involved counting. Emily's explanation suggests that she saw a direct link between maths and formal measurement, with counting as the necessary connector. This view was articulated by Emily only on this occasion but implied in some other responses discussed above.

However, it seems that counting was not the only possible connector; other number concepts and processes could perform this role for Emily. In further references by Emily to measuring or units of measure, there appears a link to a range of number concepts or processes, the latter of which were possibly the reason for the situation being seen as mathematical. For example, when asked whether there is any maths in her walking to school, Emily replied, "Some, because you walk four miles or something like that and add another one". It appears that while Emily talked of a measurement situation, on this occasion involving the use of formal units, she might equally have been considering the addition as the mathematical element.

Emily made further references to the use of formal units when discussing maths, and again seemed to do this in a number context. For example, when asked during Task 4.1 whether her father used maths when driving a car to work she replied, "Maybe, because there might be how many miles or kilometres or something like that". She might have related this to counting. In Task 4.3, in which Emily gave suggestions for maths activities for children studying the topic of The Olympics, she referred to a quantity but also implied the use of units when she explained her inclusion of the study of speed: "The good guy could run about twenty or something like that".

Emily demonstrated indecision in her beliefs regarding the possibility of measurement of capacity as maths. In Interview 3 she was uncertain whether measuring when making jelly was mathematical: "I'm not really sure because you have to measure like you have to count two teaspoons of sugar [but] it's like no adding or something like that, but there's still numbers" (Task 8.2). Emily seemed unsure whether numbers and measuring through counting were enough for the jelly-making situation to be mathematical; she was looking for application of the numbers through addition or something similar, suggesting that counting for its own sake was not always considered mathematical, but that counting related to addition or one other of the four operations would be mathematical. Although Emily saw measuring and counting in the jelly-making, but she was unsure whether this was maths, mainly because of the absence of operations with number. However, in Interviews 2 and 5, counting cups of rice and pieces of banana were seen as mathematical activity. Emily's response in Interview 3 shows some uncertainty in her beliefs.

Two mass situations presented in Task 5.2 (Interview 4), and involving the use of formal units, were not considered by Emily as mathematical activity. A photograph of a woman weighing fruit at a supermarket using a graduated circular scale was described as "The lady is weighing something and seeing how many kilos or grams it is". When asked whether there was any maths in what she was doing Emily seemed a little undecided but tended towards the negative: "um, um, mmm, mmm no", explaining that it was not maths because "she's just weighing something". Similarly a photograph shown two later was described as a "man caught a big fish, holding it up and weighing how many kilos". Emily thought that this also was not maths. Emily's statements suggest that she may not have considered weighing to constitute mathematical activity. It is possible also that she might have been looking for the presence of counting or an operation with numbers. The numbers on the supermarket scale were visible in the photograph, their purpose was understood by Emily, but it appears Emily did not perceive their use on this occasion as mathematical.

It seems that for Emily, the use of the word *measuring* in length situations, and the use of the word *weighing* in mass situations, cannot be taken to imply mathematical activity. It appears that Emily identified measurement situations as mathematical activity only if she perceived counting or operations with number to be present. She was consistent on most occasions in accepting counting and/or operations with number as mathematical activity, but her Interview 3 response regarding the making of jelly, discussed above, suggests that counting might not always be sufficient for a situation to be considered mathematical.

As stated above, at no time when asked for a definition type statement regarding maths did Emily offer *measuring*, however, when posed in Interview 3 with photographs of situations involving informal and formal measurement of length the latter were identified as maths, but only when they involved counting. The presence of addition could also cause a situation, which was perceived to involve measuring, to be identified as mathematical. Thus, a situation described by Emily as measuring, may or may not have been considered as mathematical.

It seems from a range of interview data that Emily saw measurement as maths only when it involved the use of numbers through counting or addition. This finding suggests that in Emily's responses to the maths word wheel and other definition type tasks, the terms *counting* and *adding* may have encompassed part of what she considered as *measuring*, that is, the measuring that involved formal or informal units which she perceived would be counted or added. This did not include informal measurement of length through comparions or formal measurement of mass, seemingly as counting was not perceived to be involved. This suggests that when a child speaks of *measuring* or of *weighing* it should not be assumed that the child is seeing the activity as mathematical. Emily's paucity of reference to her own experiences, both at school and elsewhere, suggests also that she saw little relationship between measurement and her own life. In this respect, among others, she was different from Cara.

Further interview responses by Emily suggest that the presence of comparison of numbers or matching of numbers also could be instrumental in her identifying an activity as mathematical. This occurred in response to the measurement situations discussed below, and two non-measurement contexts.

Comparisons related to number and formal measurement

Some responses regarding measurement suggest that Emily saw a link to maths not necessarily through number, counting, or adding, but also through matching or comparing. For example, in response to a Task 5.2 photograph of a person measuring spouting with a measuring tape, Emily stated,

this man is measuring something for the house and I think there's something to do with maths because when you measure something you have to use the right piece to fit in that place and if you measure like it was um, 30 centimetres, and then you do it, you do it in another place, and that brick wall, that brick wall is 30 centimetres . . . And I think that's something to do with maths.

Emily's response suggests that the person was measuring formally for the purpose of comparison, that is, to carry the measurement to another location. Similarly, when speaking, in response to another Task 5.2 photograph, of a person having to fit blocks into a path, Emily stated that it was maths as "he has to measure how long or wide it is, and then when you measure you remember it" and then "measure on the big block and cut with a saw". This procedure would enable the person to fill the space with the correct sized block, as the two would match.

In response to a situation posed in Task 3.3.1, involving comparison through measurement using formal units of length, Emily seemed to focus on the comparison of number values rather than the use of measurement as the mathematical element. Emily was asked whether Bradley was doing maths or using maths in the item "Bradley measured his pencil and his book with his ruler. His book measured 25 centimetres. His pencil measured 20 centimetres. He said his book was longer". Emily replied, "Yes", but at first could not explain why. When the item was re-read she added "Because 25 centimetres is longer that 20 centimetres". Emily explained that she worked this out by "saying which number is greater". The following item described an informal measurement situation: "Bradley put his pencil next to his book and said his book was longer". This elicited a different response. Bradley was considered not to be doing or using maths because he "just swapped the pencil round with the book". Emily's responses suggest that in these situations, chosen for their potential measurement component, Emily identified comparison where numbers were present as mathematical but not comparison of objects alone. Likewise, as discussed above, she did not consider measurement of a bench length and a person's height to be mathematical activity when these involved comparison of lengths without numbers (Task 8.2).

Comparison of numbers as mathematical activity was indicated also in two other Task 3.3.1 items. Firstly, Emily identified the situation in which "Melanie had to tell her teacher which number was bigger, 50 or 30" as maths. Secondly, Kirsten was judged to be doing or using maths when she knew she had a better chance of getting a red lolly than a yellow lolly from her lunch bag which contained five red and three yellow lollies, because "five is greater than three". The significance of the numbers making this a mathematical situation was highlighted when a similar situation, "Kirsten's lunch had more red lollies than yellow lollies ... she knew she had a better chance of getting a red one", was judged as not mathematical "because she was just guessing". There was no evidence that Emily saw guessing as mathematical activity. This contrasts with Cara's belief that estimating and guessing can be mathematical activities.

Emily closely associated numbers with maths and included comparison where numbers were involved as mathematical activity, as represented in Figure 27.



Figure 27. Emily's beliefs about maths - Image 4.

The examples provided in Figure 27 suggest that Emily recognised the use of maths within a variety of situations. Closer analysis of data, shows, however, that while such recognition was made, there were few examples offered by Emily that included maths for a non-schooling purpose.

Maths primarily as a schooling experience

Emily saw maths as being relevant to herself, her parents and other people. She associated her own use and learning of maths mainly with school and a schooling purpose, but the few situations she suggested where maths was used by others linked both with school and nonschool experiences. Emily perceived that she did not use maths at home in any way other than in homework (Task 4.2.1); when probed about her use of maths at home she spoke only of her mother calling her to learn her times tables.

Further association of maths with school was implied in Emily's Task 4.1 response: "Maths is like something you use with numbers, and sums". She explained that the use of numbers and sums is "to make a question, a maths question". In seeking insights into whether Emily perceived a purpose for maths in non-school situations she was asked for a time when she would use maths. Her response of "when Ms I was teaching me, teaching the whole grade, times-tables" did not suggest an appreciation of possible real-life uses of maths. Further questions revealed little more:

Interviewer: Do you know somebody else who uses maths?

Emily: Maths person . . . like maybe people who write those boards with numbers on it

(referred to multiplication tables sheet on display)

- I: Can you think of any other adult who uses maths?
- E: My parents
- I: Let's choose one of those people
- E: Um
- I: Say, dad
- E: My auntie
- I: How might your auntie have used maths in the last few days?
- E: She goes to school and sometimes she uses maths

Emily did proceed to speak of her mother measuring rice as maths activity, but felt that her father, a "computerist [*sic*] . . . not often does use maths, not much" (Task 4.1), or only "sometimes, maybe" uses maths (Task 2.3). Although Emily had been prompted to give an expanded view of the application of maths in Task 4.1 it was only her mother's use of counting in cooking that was clearly a non-school use of maths.

To provide an opportunity to relate maths at school with non-school activities and purposes, Emily was asked to suggest maths activities for children studying a topic of her choice from the list provided (Task 4.3). She attempted to relate the Olympic Games to the learning of maths at school, but had difficulty giving clear ideas. It was only in her own playing of sport that she said, on one occasion but not on another, that she used maths in a potentially non-school situation. Thus there was very little evidence of intuitive linking in her mind of maths with non-school purposes.

As discussed above, Emily proffered very few instances for herself or others of the use of maths in non-school situations. However, she did identify situations that included mathematical activity from among those posed by the researcher, as discussed earlier. These included working out the money to pay a hot water bill and purchasing an item in a supermarket, seemingly because the counting and/or addition in these situations was considered mathematical activity. Measuring the length of something in the construction of a house or in laying a path seemed to be considered mathematical because of the purpose of comparing numbers or lengths. Measurement in school and non-school situations appeared to be seen as maths only when it involved comparing, adding, or counting. Real life informal measurement of length and use of formal units of mass were not perceived to involve maths.

Emily's overall portrayal of the purpose of maths seemed linked mainly to schooling. She recognised some other situations as mathematical, but rarely proffered such situations of her own accord. Emily's perception of the purpose of maths contrasts with that of Cara who included many references to real life uses of maths by herself and her parents.

Emily's beliefs about maths - A summary

As stated earlier, the overall impression from Emily's data is an emphasis on maths as number and related operations, suggesting a simplicity of belief. However, her beliefs hold subtlety, depth and some complexity that firstly became apparent from the analysis structured according to themes, as presented above, and that became more clear when viewing the data from a different perspective as portrayed in Figure 28. This representation portrays vividly the interweaving of concepts within Emily's beliefs. It makes clear also her belief that measurement is not always maths, and that numbers are necessary for a situation to be mathematical although they are not always sufficient.



Figure 28. Emily's beliefs about maths portrayed from a different perspective.

The broken line for the *Informal capacity: Rice* entry represents the indecision voiced on one occasion regarding whether this situation is mathematical. However, on another occasion the same situation was volunteered by Emily as mathematical, thus the arrow head is placed in the maths section of the diagram. Two italicised entries at the top of the diagram represent number examples discussed but identified as not being mathematical activity. The italicised

entries regarding informal length and formal mass are combined with a broken line to represent some indecision but placed outside the area representing maths, as Emily's responses suggested seemingly more a belief that the situations were not mathematical.

Emily in some ways held more narrow views of maths than Cara, but perhaps held more established views, with less uncertainty or contradiction in her responses. Emily seemed to relate maths more closely to addition and counting, as well as to comparisons, multiplication, subtraction, division, and graphing, whereas estimating and measuring seemed to be the most salient aspects of maths for Cara. The paucity of reference to space, chance, and data as maths concepts by Emily and Cara is an important finding of the research and one of concern.

Emily saw maths mainly as a schooling activity with some application for others in nonschool environments. In contrast, Cara volunteered many mathematical activities from school and non-school environments both for herself and others, and for school and non-school purposes.

The second main perspective of interest in examining Emily's interview responses was her beliefs about learning in general and about maths learning as an element of that.

Beliefs about learning

Emily did not appear accustomed to talking about the concept of learning. She answered questions posed by the researcher but tended not give detailed responses. She gave the impression that she held a clear concept of her own learning, particularly of maths, and that she had established an understanding of the purpose of learning maths at school. These aspects of her beliefs are explored below. The length and depth of the discussion reflects the amount and type of data gained from the interview procedures. Following the method used to date in this report, themes were drawn from Emily's data by the researcher. These were

- learning as knowing and remembering;
- thinking as an element of maths learning;
- the child's role in learning;
- the role of the *teacher* in learning of maths; and
- the purpose of learning maths.

Emily gave some indication that knowing and remembering were what she considered as the key elements of the learning process. However, these concepts were not raised by Emily in all interview procedures regarding learning; at times Emily did not exhibit any real thought about her meaning for learning, especially when the tasks probed her understanding through an abstract, non-subject related approach.

When posed, early in the Task 1.1 word association quiz activity (Interview 9), with the word *learning*, Emily responded simply with the word *learn*. When the prompt was repeated towards the end of the set of words it elicited once again the response of *learn*. When the word *learn* was posed in the following password task (Task 1.2), Emily paused, seemingly unable to think of another word which would convey the same meaning. Her responses to other words posed indicated that she had understood the purpose and structure of the

password game, thus it seems that it was the concept of learning, or the abstract manner in which it was discussed, that posed the challenge. Emily was able to discuss specific learning experiences and, through her description of these, provided insights into her beliefs about learning which suggest that knowing and remembering were recognised processes.

Learning as knowing and remembering

It appears that for Emily, knowing is the outcome of learning, with remembering an important part of the process of achieving this outcome. In giving her personal dictionary definition (Task 2.1) for the word *learn*, Emily explained that "when you learn you know things". When asked how she knows when she has learned things, she replied, "by remembering them". In Task 10.1 Emily stated also that "you learn and then you know it". It is not clear whether Emily was referring to, for example, the learning of facts or procedures. This point is discussed further below.

In talking of her maths learning (Task 8.1.1), Emily spoke of doing some maths problems in her head: "the easy ones, ten take away four or something like that . . . because I always remember them. I always know them". It appears that this remembering involved the recall of number facts. Emily identified also the use of remembering when doing written problems, for example the three-digit take two-digit problem she volunteered as an example of maths she does in Grade 3, that for her incorporates simple number facts (see Figure 29).



Figure 29. A mathematical problem encountered by Emily in Grade 3.

It appears that the remembering of basic number facts in this problem was not a conscious experience for Emily as the facts were well known, it was the method of working through the problem, that is, the procedure, that had to be remembered: "You have to remember how to do it. Sometimes you have to carry the numbers . . . if you've forgotten . . . you have to use your hands and that is very hard, it takes long [*sic*]". Emily witnessed a change in the remembering required in the doing of maths, from only basic number facts at the Grade 1 level to more emphasis on procedures at the Grade 3 level. The discussion of Emily's example as given in Figure 29 suggested that she was very conscious of having to remember procedures. As an eight year old she also was conscious of changes in her maths learning.

Emily suggested that for subtraction, and indeed for addition also, basic number facts were recalled easily and therefore could be applied in more difficult written problems.

However, her responses indicated that she was aware still of her need to remember in the learning of multiplication facts; she referred to remembering in her learning of *times tables* on a number of occasions (e.g., Tasks 2.2, 3.2, 5.2). Emily's discussion of remembering in maths involved pure number examples.

When posed with the task of choosing words associated with learning, from the words *know, remember, do*, and *understand* (Task 10.3, Interview 10), Emily chose the word understand. She explained "understand how to do it", suggesting an emphasis on method or procedure. This appeared consistent with her emphasis on method or procedure as discussed in relation to subtraction. When asked whether remember has anything to do with learning she responded "Sometimes". She had difficulty expanding upon this response, perhaps because the discussion was not contextualised. In earlier interviews, when talking of the learning of maths, Emily had given insights into her use of remembering for learning maths, as illustrated above.

It appears that in Emily's learning of maths, remembering was of two types: *remembering things*, such as number facts, and *remember[ing] how to do it*, that is, with a focus on procedures (Task 6.1). The essence of this summary was given also in response to Task 8.1.1. Emily linked her remembering in maths also with thinking.

Thinking as an element of maths learning

Thinking, as a process within learning, was discussed with Emily on more than one occasion.

After Emily paused when posed with the word *learn* in the password activity (Task 1.2), she was asked "What happens inside your head when you are learning?". She replied, "Thinking". It is acknowledged that the question lead her to focus on the mental processes in her learning, nonetheless her response gave some hint of her awareness of thinking as a process she undertakes when learning. Responses to other procedures indicated that Emily believed thinking to occur within the learning of maths. For example, Emily's response to a Task 8.2 photograph suggested she believed that once sums are present a situation is mathematical, and thinking and remembering occur. She thought there was no maths in the Task 8.2 photograph "because maybe she doesn't have to remember or think because it's the sum and in the picture there's no sums or anything".

Discussion of a Task 5.2 photograph led also to the expression of a link between maths, sums, and thinking and remembering. The photograph was seen to contain mathematical activity because two boys were figuring out a sum. Emily had difficulty responding to the question of whether the maths was happening in their heads or on the paper, but when asked whether doing maths happens in her own head she replied, "Mmm, no it's happening on paper". When asked specifically what happens in her head she said she had to think and remember. Emily seemed to associate the mathematical activity with the purpose of figuring out the sum, or perhaps in other words, figuring out the answer. Thinking and remembering would be used within this process.

Emily's beliefs regarding remembering, knowing, and thinking as discussed above, appear to determine the role she saw for herself in her learning of maths.

The child's role in learning

It is clear from the above discussion that Emily perceived her role in learning in general, and in the learning of maths, to include remembering. Her remembering might have been of number facts or of procedures in maths. She may have sensed that she had some personal control over her learning as indicated in her statement that when learning times tables she would learn by "looking at the numbers and remembering it" with help from her "brain" (Task 3.2). Likewise, when learning to spell words, Emily had used "remembering" as well as "sounding it out". Emily perceived that she had not been taught to sound out, but that she had "worked it out" herself, again suggesting that she felt she played an active role in her own learning (Task 2.2).

Emily's role in learning maths appeared also to include the use of tools where appropriate. She reported the use of writing tools for recording and in earlier grades had used her fingers for working out (Task 6.1). She recognised the use of blocks as being a tool used by others in learning maths (Task 7.2).

Overall, Emily gave few insights into her perception of her own role in her learning of maths, but did suggest that as well as using tools, thinking, and remembering facts and procedures, concentrating was important. She spoke of her difficulty in learning maths when she was disturbed by others (e.g., Task 6.1).

The role of the *teacher* in a child's learning of maths, as portrayed by Emily, complemented what she portrayed as the essence of learning and as the role of the child.

The role of the "teacher" in learning of maths

Emily believed that two important elements within learning maths were knowing maths, that is, recalling number facts, and knowing how to do maths, that is, correctly following a procedure. Seemingly with consideration of the latter element, that is, method or procedure, Emily described a "good maths teacher" (Task 11.3) as one who would "show you how to do it". When asked, during Task 10.1, who teaches her *how to do it* Emily stated, "It could be anyone - like my mum, my dad, or my teacher", interpreting the concept of teacher in a broad sense, and seemingly including both school and non-school contexts. Emily stressed the importance of the "teacher . . . showing her how to do it" but not giving the answer as she perceived in the latter case she would not be learning.

In discussing a situation in the maths classroom, Emily spoke also of the teacher giving an explanation. She added, when asked, that the teacher would write on the board also (Task 10.2.2).

The teacher also seemed to play some part in directing or motivating Emily for learning maths. For example, after Emily had encountered difficulty with the learning of fractions (Task 10.3) she went on to learn more about fractions "because our teacher wanted us to do it". Emily may have been doing simply what the teacher asked, that is, responding to the

teacher as organiser, or may have been learning in an effort to please the teacher, that is, responding to the teacher as motivator for learning. In her Task 4.2.2 drawing, Emily portrayed the teacher as responsible also for one form of extrinsic motivation: "whoever gets it first [the sum correct] gets a lolly".

Emily perceived the teacher to play a multi-faceted role in a child's learning of maths, although with less breadth in the role than was portrayed by Cara. Emily portrayed the teacher role as to explain, to demonstrate, and to direct or motivate. She included members of her family as her *teachers* of maths, but appeared to portray the majority of her learning of maths as occurring at school for a school-related purpose.

The purpose of learning maths

Emily's perception of the purpose of learning maths appeared to be limited to school recognition and achievement, and for progressing to higher grade levels.

Her portrayal of the situation in which a teacher gave a lolly to the first child to get a sum correct (Task 4.2.2) implied a valuing of a correct result, a very short-term purpose for learning maths. Likewise, a valuing of completion of work, as a short-term achievement, was suggested in her statement in Task 6. 1 that she "was the first to finish". Emily did not appear to have a concept of the learning of maths at school for more long-term use or benefit.

The importance of learning for others and for achievement at school was conveyed in a Task 1.3.1 conversation:

Interviewer:Why do you think you come to school and they teach you maths at school?Emily:Because they want you to learn

- I: Mmm, and why is that?
- E: (pause) To be smart
- I: And why do you want to be smart at maths? What is good about being smart?
- E: Um, that you can go to another grade
- I: Mmm. Is that the only reason so you can go to another grade?
- E: (sigh) Um, maybe
- I: Can you think of any other reason?
- E: (pause) No

Emily appeared to have no broader view of the purpose of learning maths than school recognition and advancement, and achievement related to the wishes of teachers. She appeared to see little, if any, relationship between maths learning at school and possible application in the outside world, her world of the future.

Emily: Learning - Summary

Emily gave little evidence of in-depth thought about the concept of learning. She appeared to associate it mainly with school. The few examples proffered of learning in the home environment appeared to have a school-related purpose.

The learning of maths at school was seen to involve remembering of facts and procedures, leading to knowing, and ultimately to academic advancement. Little, if any, appreciation of long-term uses or benefits of learning maths was demonstrated.

Emily gave some indication that she considered herself to play an active role in her own learning of maths through thinking, remembering, and concentrating. She also used tools in her learning. The *teacher* was seen to help the learner through giving explanations, demonstrating, directing, and motivating.

Helping factors for learning maths

The third focus within collection, analysis and presentation of Emily's data is her beliefs about helping factors for learning maths. The themes within the discussion are

- Emily's perception of learning maths well;
- feedback helping Emily to learn maths;
- tools helping Emily to learn maths;
- Emily helping herself to learn maths;
- help from a *teacher;*
- help from other children; and
- importance of a quiet working environment.

Emily's perception of learning maths well

Emily's concept of learning maths well appeared intertwined with her concept of learning, which, as discussed above, appeared mainly to involve remembering facts and procedures. If something was remembered, it was known and therefore had been learned. If Emily did not remember something, she perceived that she was having difficulty and that she still had to learn. Emily's interview responses suggest that when she talked of learning maths well she was referring to the fact that she had already learnt. For Emily there appeared to be two states: knowing, evidenced by remembering and finding work easy (therefore having learnt), and not knowing, evidenced by not remembering (therefore not having learnt). Learning appeared not to be seen as a process of coming to know or coming to understand, but as evidence that one had come to know. A range of interview responses suggest the presence of these ideas in regard to helping factors for learning maths.

The interview task that most directly broached the subject of helping factors for learning maths was Task 6.1 in which Emily was asked to draw and describe a situation in which she was learning maths well. She chose a school situation from Grade 1 in which she "was learning minus and take away sums . . . I was learning maths well by using my fingers because I went five plus three and stuff like that and that was very easy". The conversation continued:

Interviewer: So what does learning maths well mean?

Emily: It means that you have to remember things, and you have to remember how to do it

- I: Why was that [using fingers and counting] the most helpful thing?
- E: Because the sums were below ten and when you use your fingers, you have ten fingers and perhaps like five, five plus three and then you put five fingers up and then you put up another three and then you go one, two, three, four, five, and then you go six, seven, eight, and so that's why it's very easy.
It appears that it was the ease of getting the answer, and therefore knowing the answer, that caused Emily to choose this situation as one in which she was learning maths well. Her fingers were a ready and appropriate tool, but it was the smallness of the numbers and their match to the tool that made the situation easy, and therefore in Emily's view, a situation in which she was learning maths well.

In commenting on a situation depicted in a Task 8.2 photograph, Emily stated that if she were in that situation she would be learning maths well. In explaining her affirmative response she included comment about what she saw the girl doing: "because you have to do the maths, be doing it right now and she knows it, she knows it so she's writing it down". Emily's comment suggests that it was knowing the maths that made the situation one in which she would learn maths well. A similar response was given to a drawing presented in Task 9.2. A child had portrayed herself as doing maths by herself in her bedroom. Emily did not see the need for a teacher in this situation; she explained "[as] I know how to do it", showing that learning maths well appeared to be associated with already knowing.

To summarise, two key elements of learning maths well appeared pertinent for Emily: knowing the work or the answer, and ease in getting an answer.

The discussion now considers factors that Emily's responses suggested helped her in learning maths.

Feedback helping Emily to learn maths

The role of feedback in helping to learn maths seemed to relate mainly to situations where Emily gave wrong answers. For example, she identified having a maths test as helping her to learn maths "a little bit because if you write the wrong sum you have to remember not to write that sum again" (Task 9.2). Similarly, she suggested that playing a number maze game on the computer might help her learn maths. In Interview 4 (Task 5.2) she stated that

sometimes it helps me to learn what it is, when you press the right number then something happens and when you press the wrong number then something happens. So the next time you have to remember not to press that number again.

Emily volunteered the same example of the number maze game in Interview 3 (Task 8.2) saying that it could help her "a little bit" to learn maths because she would come to know not to press the wrong number. Emily's responses suggest an instrumental understanding of mathematics focused on producing answers which are judged by outside sources.

It was suggested in Task 9.2 that Emily pretend she was the child shown in a drawing working out " $4 \times 10 =$?" mentally for the teacher in an informal lunchtime setting. When asked whether having the teacher ask her the question at that time would help her to learn maths Emily stated, "Yes, because when you said the answer maybe . . . I said it wrong so the teacher will say I said it wrong so next time you know not to say it again". However, she said the best way to find out the answer would be to "write four groups of ten in each . . . then you count all of the things together". Making the four groups with "a pencil and paper . . . and

some counters" was preferred for finding the answer. Perhaps Emily believed also that the latter method would better help her to learn.

As discussed below, tools were seen to contribute to some degree to Emily's learning of maths.

Tools helping Emily to learn maths

Emily portrayed the use of tools as helpful for learning maths in both classroom and nonclassroom contexts.

The Task 6.1 drawing and description of using fingers and counting for learning minus and take away sums in Grade 1, as discussed above, was one example of learning in a classroom context. Similarly, in Task 9.1, Emily wrote of hands and counters helping her to do better in maths. In Task 9.2 Emily spoke of fingers helping her learn maths in Grade 1 but added that she did not like to use fingers any more as her hands got tired!

Counters in a maths game were seen also to help in the learning of maths because you "use the counters to make numbers . . . and you count all the counters . . . and you get the right answer" (Task 9.2). Both use of counters and writing on paper were believed to help her to learn maths but Emily preferred to write on paper as she "couldn't be bothered putting the counters down". Emily's preference for writing is not surprising considering the type of the problems she associated with school maths situations (e.g., see Figure 29). She may have found writing the most efficient way to solve such a problem, and perhaps was encouraged in Grade 3 to record and complete the problem column by column, as she did in the interview. It appears she was able to operate without physical aids, but nonetheless believed they helped in her learning of maths.

However, in speaking of the use of calculators, Emily had a different view: she believed calculators would be useful in giving answers but would not be a tool she could learn from (Task 10.1).

In a non-school situation, shopping was seen by Emily to help her learn maths "a little bit because you have to count your money". Seemingly the presence of the physical objects was perceived to help in learning maths.

Emily's responses suggest she believed tools provided one opportunity to help herself in learning maths. Emily believed also that there were other ways in which she could help herself to learn maths.

Emily helping herself to learn maths

Emily gave some indication that she could undertake actions that would assist in her own learning of maths.

For example, in Task 10.1 Emily was posed with a range of words and phrases for which she was asked firstly whether she used them in learning maths, and secondly whether they helped her to learn maths. Responses to three items are discussed below, each of which Emily indicated was used and helped in some way in learning maths. In response to *figure it out myself*, Emily explained, "Sometimes I don't remember how to do it and then I try to

remember so I figure it out, and sometimes I have to keep remembering". In response to the word *memorise* she said, "It [my mind] sometimes helps me remember", and in response to *talk or discuss* she replied, "Yes [it helps]". When asked who she talked or discussed with, Emily replied, "With myself . . . I said to myself, how to do it, and sometimes I ask myself, um, sometimes remember". From the responses to each of these items Emily showed she could help herself when learning maths. She recognised also that such actions were not required when she knew her maths, as she showed when replying to *think about it*. Emily's response of "think about how to do it when I don't know the sums" suggested that she would not need to think for learning in a situation where the sums did not pose any difficulties. She would not need to *think about it* in a situation where she had already learned the maths.

As discussed earlier, Emily did resort to helping herself by having a "peek of other people's work" at a time when she did not feel good about learning fractions. She also asked one person for help. Indeed, help from the teacher, and sometimes from classmates, was considered to be of assistance in learning maths, as discussed below.

Help from a "teacher"

The teacher was considered important in helping children to learn maths. In response to the sentence starter (Task 11.3), "A good maths teacher", Emily added, "is smart [and] teaches you how to do maths". Emily appeared to look up to the teacher as a person who could help children to learn, that is, "to be smart [so they could] go to another grade" (Task 11.3). The teacher appeared to be considered an authority on maths; when Emily was having difficulty understanding she did not like other children to help as "other people won't say it properly than Ms I because Ms I is the teacher" (Task 6.2).

On multiple occasions Emily referred to the teacher telling her "what to do" and "how to do it". For example, in Task 10.1 teacher explanations were described as helpful for learning maths as the teacher "explains" and "teaches you what to do". The importance of teacher explanations for learning and telling "how to do it" were mentioned elsewhere also (Tasks 7.2, 8.2, 9.2).

It appeared that Emily's parents could act in a role of a *teacher* also. Emily's mother and father were perceived to help her learn maths when they told her *how to do it*. Emily did not want them to give her answers as she believed she would not learn (Task 10.1).

Emily perceived that children, as well as adults, could help in her learning of maths.

Help from other children

The assistance preferred from children was similar to that from adults, that is, help in how to do maths.

The smartest person in the grade was identified by Emily as a possible helper for her to learn maths if he showed her "bit by bit . . . how to do it" (Task 7.1). A further reference to help from other children was made in Task 9.2 when Emily was shown a drawing of a child thanking his friend, Robert, for helping him to learn maths. Emily thought that if Robert was explaining the maths she would learn: "I'm learning by him and you should know how to do it

because he taught you how to do it". However, she believed she would not be learning if he told the answer.

However, it appears that to achieve in maths, Emily felt a need to work alone and in a quiet environment.

Importance of a quiet working environment

Emily stated that working in quiet areas (Task 9.2) "so no-one disturbs me" (Task 10.1) and "thinking and calling people to be quiet" in her class (Task 3.1) helped her to learn maths. Emily felt that people talking hindered her from learning maths (Task 6.1); she reported that she felt annoyed when there was loud noise. In response to the sentence starter "I could do better in maths if", Emily included the statement "Nobody disturbs me" (Task 9.1). She liked at times to work by herself "because sometimes people disturb me . . . because I can't concentrate and I always make mistakes" (Task 8.1.2). Similarly, as described earlier, Emily found difficulty concentrating during one of the interviews because of occasional activity and noise when others used a part of the room in which the interview was conducted.

The interview excerpts referred to here indicate a preference for a quiet working environment and a belief that this helped her to learn maths. It appears that being able to concentrate was favoured by Emily. Stemming from this need for concentration was a preference to work by herself rather than with others (Task 6.1), although there was acknowledgment on one occasion that, when the maths was difficult for her to learn, she would like to work with someone else as friends might know and tell her the answer and help her to remember the "special way of doing these things" (Task 6.2).

Emily: Helping factors for learning maths - Summary

In summary, it appears that Emily regarded an adult *teacher* as the main source of tuition for learning maths but that children could assist for learning maths also if they showed *how to do it*. She mostly preferred not to be told answers. Emily perceived herself as playing a role in helping her learning of maths. Tools such as fingers or concrete materials could be of assistance, a quiet environment allowed her to concentrate better and therefore to make less mistakes, and feedback could inform Emily of what not to do on another occasion.

Emily: A Brief Reflection

Overall, Emily's responses to the interview questions revealed the following:

- Emily could describe her beliefs about the nature of maths although at times her responses suggested some uncertainty in her criteria for deciding whether a situation was mathematical (such as whether measuring involving counting and no operations was mathematical).
- Emily's appeared to define maths in relation to number, but upon further analysis, her beliefs were shown to be more subtle, as demonstrated by Emily including formal measurement situations within her meaning of numbers in maths.

- Emily could provide insights into her views about learning, from responses to tasks set in context or asking for examples.
- Emily encountered some difficulty describing her views about learning when responding to more abstract tasks, suggesting she had not previously thought consciously about the meaning of learning.
- Emily's examples of learning were limited mainly to experiences in school or homework settings and for the purposes of schooling.
- Emily was able identify and articulate helping factors for learning maths.

As was the case for Cara, Emily's beliefs can be described as idiosyncratic. Chapter 7 demonstrates the variation within beliefs of the eight research participants and draws out and reflects upon key issues that emerge from the research. The discussion is informed by data collected in interview and whole class settings from each of the eight children, and as summarised in Tables 4 to 12. The summaries draw upon the careful and thorough analysis taken for each child's data, as described in Chapter 3 and demonstrated in Chapters 5 and 6, and in Appendix E.

CHAPTER SEVEN

DRAWING TOGETHER, DRAWING OUT AND LOOKING FORWARD

Drawing together: Summary of research purposes and methodology

The present qualitative study investigated the beliefs about mathematics, learning, and helping factors for learning mathematics, of eight children of eight to nine years of age. The definition of beliefs stated by Rokeach (1968, p. 113), as "any simple proposition, conscious or unconscious, inferred from what a person says or does, capable of being preceded by the phrase, 'I believe that'", formed' the baseline for the study.

The research study was developed around three research questions:

- 1. Do young children hold beliefs about mathematics, learning, and helping factors for mathematics learning that can be articulated and portrayed from responses to procedures developed for this research?
- 2. What beliefs do children hold about the nature of mathematics and the nature of learning?
- 3. What factors within learning environments do children believe help them to learn mathematics well?

The study aimed to investigate whether beliefs held by the research participants of eight to nine years of age could be articulated and portrayed, to develop procedures to facilitate this process, to portray children's beliefs from their responses to the research procedures, to provide insights into possible complexities and subtleties of young learners' beliefs, to reflect upon the significance for the mathematics classroom of the insights gained, and to reflect upon the value of the procedures developed for the study.

The eight children who participated in the study came from two classes in two schools in Melbourne, Australia. Data for analysis resulted mainly from ten face-to-face interviews with each of the children, conducted approximately once a fortnight over a five-month period. Discussion was stimulated by thirty procedures involving activities such as drawing, writing, sorting words and phrases, and responding to photographs, drawings, verbal descriptions and video clips. Each procedure began with a planned question or task that allowed the children to respond in a manner that reflected their perspectives. The semi-structured interviews allowed the direction taken by the children to be pursued. Background data on beliefs about personal achievement in mathematics were collected also in the interviews.

Additional data were collected from the eight research participants from two tasks administered in the whole class situation. The data provided substantiation, or lead to questioning, of findings from interviews. Mathematics lessons were observed in each class and interviews were conducted with the children's teachers leading to insights into the context of the children's learning of mathematics at school.

The analysis, interpretation, and reflection upon data occur in many parts of this thesis. Introductions to the children are presented in Chapter 4, in which reference is made to their perceptions of themselves and to researcher perceptions, and brief descriptions are given of the children's settings for mathematics learning at school. In Chapters 5 and 6, two children's beliefs are portrayed. The length of these chapters, particularly of Chapter 5 that concerns Cara's beliefs, is due to the complexity and depth of beliefs portrayed, and the attempt to present and analyse these as completely as possible. Beliefs of three other interviewees are presented in Appendix E. The present chapter includes synthesis of findings from the interviews and class-administered tasks for all eight interviewees, and reviews the research procedures.

In the case study discussions of children's beliefs, interview data are presented and discussed within the individual cases; through this choice of unit of analysis the researcher attempts to portray individuals' beliefs "in depth, in detail, in context, and holistically" (Patton, 1987, p. 19). Where seeming contradictions or ambiguities arose within data from a child, these are reported; the intention was to represent each child's reality as responses to the research tasks suggested he or she perceived it. The richness in each child's responses facilitated the development of an appreciation of subtlety and complexity within young children's beliefs. Reference to the specific and concrete, that is to the expression of beliefs by the individual children, provides insights into universals for the reader (Erickson, 1986).

The choice of method and unit of analysis within this research reflected the underlying assumption that realities and truths are influenced by historical and cultural factors and are socially dependent. The children's responses were not judged for correctness or for a match to any predetermined categories; the researcher attempted to take a stance of neutrality to the phenomenon under study, adding to the credibility of the research (Patton, 1987).

The discussion, below, of research findings and of the value of the data collection procedures is a result of the researcher stepping back and taking an overall or reflective view of the research. Themes that emerged from the entirety of the data are explored and each research question is addressed. Reflection upon the meeting of the research purposes occurs at the end of this chapter. As in the previous chapters, the word *maths* is used in preference to *mathematics* when speaking of the data and the children's responses.

Drawing out and looking forward: Research conclusions and future directions The ten interviews with each of the eight children provided rich and complex data of which a synthesis and overview for each child is presented in Tables 4 to 11. While these tables cannot represent the fullness of detail, complexity and subtlety of findings for the children, they do facilitate appreciation of, and insights into, each of the eight children's beliefs.

As stated earlier, additional data were collected from each of the eight children in the whole class situation through two pencil and paper tasks, *Alien* and *PPELEM*. Although the resultant data, summarised in Table 12, were not intended to contribute to the research in a major way, they are an additional source of insights into children's beliefs. They are drawn upon in this chapter in the discussion of the research questions and general themes, and in the reflection upon the value of the procedures

Table 4Summary of Anna's responses to interview tasks

| Overall comments | Maths/Mathematics | Learning | Helping factors |
|--|---|---|--|
| Anna | • Main elements of maths related to number, e.g., "sums", "adding", "counting", "take away", "times tables" | • Learning associated with keeping something in her mind, being able to do something and remembering. | • Materials considered helpful included blocks and hands/fingers (although Anna did not always need to use these as could do things "in her head"). |
| High-achiever | "equals".Use of calculator seen as maths when "adding up". | Learning associated with trying to find an answer.Knows she has learned when she has | • MAB blocks helpful for counting when you "don't have enough fingers" and for more difficult sums such as take aways. |
| Female School S | • Some shape activities, from prompts, identified as maths but due to counting (e.g., of number of sides). | correct answers - the teacher gives a tick or Anna uses calculator to check answer and gives herself a tick. | • Calculator helpful: - when no blocks and cannot use fingers because of large numbers |
| | • Formal measurement of length and mass considered maths. | • Doing something that she already knows is not considered as learning. | - because it "tells answers and then you can know [the answer]". |
| • Judged own achievement in maths as "5" on a scale of 1 to 10. | • Informal comparison of heights using string seen as measuring and as maths. | • Teacher role is mainly to tell children, so they will learn, and to | • Working in a noisy area hinders: "I lose the number in my head". |
| "5" on a scale of 1 to 10. Sometimes responses suggested lack of confidence in her ability to learn maths. Enjoyed learning maths. Patiently took the time she needed to think about her interview responses. | Other informal comparison of length situations considered guessing, not maths. Chance situations, seen to involve guessing, not considered maths. | correct children's work. Seemed to see the purpose in learning maths as getting or knowing the answer. Correct answers on tests would then lead to going to a higher | Group helpful: teacher gives blocks (thus materials helpful, not the group) can tell answer if you don't know (seemed happy to be told answer). |
| | • When asked to volunteer mathematical situations, only school-related maths situations given (i.e., school or homework). | Motivated to learn harder "sums" to be able to do them successfully on a test. | • Ambivalence - Helpful to work out a problem yourself and then be told the answer. Also helpful to be told answer by classmate, by calculator, or by seeing answer in the book thus "cheating for a test". |
| | When given prompts, some non-school situations identified as maths because of counting or adding. Mathematics considered different from maths. Mathematics was for those who "know maths really well", such as | | Mother helps by giving practise tests at home. Games helpful motivation to learn listen to others so know. |
| | maths. Mathematics was for those who "know maths really well", such as "Grade 4 girls". | | |

Table 5Summary of Ben's responses to interview tasks

| Overall comments | Maths/Mathematics | Learning | Helping factors | | | |
|--|--|--|---|---|---|---|
| D | • Maths considered to include number (numbers, counting, four operations). | • Learning considered to occur by helping others or listening to others. | • Believed teacher helps and plays an important role e.g., by showing how to do sums, asking | | | |
| Ben | • Reading numbers seen as mathematical activity (e.g., numbers as markers on a | • Believed that learning is about getting better and better at new things. | questions, and giving guidance and encouragement. | | | |
| High-achiever | volume control). | • Indicated that learning is built upon prior | • But Ben appeared not to be dependant on the teacher. | | | |
| Male School S | • Referred to absence of measuring of numbers when explaining why an activity was not mathematical. | knowledge and need. Saw the following processes as elements of learning: thinking remembering | • Believed peers help: - at times can be more helpful than the teacher | | | |
| School 5 | • Identified activities involving informal measurement through comparison as maths. | | Ben can learn from being shown by another child appreciated a hint from a friend Ben felt negitive short working in groups some | | | |
| T 1 1 1' /' | Formal measurement situations identified and proffered as measurement and maths (often referred to family experiences of building). Space activities identified sometimes as maths due to counting, measurement, angle or straightness. Competing considered as mathematical | practising making mistakes | - Ben feit positive about working in groups some of the time. | | | |
| Judged own achievement in maths as "7 or 8" on a scale of 1 to 10. Appeared to think about and reflect on his interview responses. Responses suggested he was a reflective student, with potential for controlling his own learning; aware of need for expert guidance at times. | | trying different ways to solve a problem. Liked to be shown first part of hard sum only, and then try himself. | Believed he helps himself to learn maths by waiting, watching and thinking as teacher explains to another child listening having a go at different or hard sums using known facts and strategies to solve | | | |
| | | | | activity (perhaps due to comparing or ordering of results). | - learning from his mistakes - trying to remember, or looking back at | learning from his mistakes trying to remember, or looking back at, earlier |
| | | | | • Mathematics seen as a school and non- school activity. | | work - talking to himself. |
| | | | • Did not know what the term mathematics meant; thought it was not the same as maths but might have similar meaning. | | Equipment that helps: calculator counters fingers backs: to look up now ways to do things | |
| | | | | • Parents seen as potentially helpful. | | |

Table 6Summary of Cara's responses to interview tasks

| Overall comments | Maths/Mathematics | Learning | Helping factors |
|--|--|--|--|
| | • Beliefs were complex, subtle and multi- dimensional. | • Learning seen as knowing and remembering. | • Believed teachers and parents help by |
| Cara | Measurement considered an element of maths: formal measurement seen as maths and as measurement | • Appeared to relate knowing and remembering mainly to school situations (rather than non-school | providing affirmation giving the answers providing tasks facilitating associate an analysis |
| Low-achiever | - understood existence of and purpose for formal units of length, mass, and capacity | situations), to learning maths (rather than doing maths), and | motivating. |
| Female | - informal measurement seen as maths but not consciously seen as measurement. | mostly to number (but some recognition of opportunities for | • Cara believed she helps herself to learn maths by listening and seeing a |
| School S | • Conveyed a strong sense of measurement in non- | learning when doing measurement activities). | pattern. |
| • Judged own achievement in maths as "5" on a scale of 1 to 10. | Estimation and guessing were seen as important mathematical activities | • Saw a focus of learning as getting correct answers. | • Held ambiguous or contradictory beliefs about help from practising, thinking, concentrating, writing, |
| • Able and willing to think about, talk about and question her own beliefs. | • Appeared to hold a personal affiliation with maths, | • Saw role of the learner as | estimating/guessing. |
| • Evidence that ideas were being constructed and | especially through measurement and estimation. | and working. | • Other learners can help Cara learn maths (especially when maths difficult |
| appeared not fully formed at the time of the interviews. | • Number seen as an element of maths, e.g., equalling/getting answers. | •Believed children work and then learn | for Cara) by providing - direction and help in getting answers |
| • There was a seeming emergence or re- development of some of her beliefs during the | • Maths seen as purposeful through estimation, measurement and number calculation. | • Saw role of the teacher* as | moral support (leading to higher likelihood of success). |
| interviews: - some change in emphasis in defining maths over | • Maths considered to include content (more number) and action (more measurement) | motivator, and affirmer. | • Other learners could hinder learning by telling answers |
| the period of the interviews. | • Maths seen as school and non-school activity. | • Believed that the teacher, not children, knows mathematics. | • Tools considered helpful for Cara to |
| helping herself to learn maths. | • Conveyed an increasing association of maths with school over the 5 month interview period | • Sometimes uncertain about the | learn maths (by giving answers): blocks, times tables sheets, |
| • Cara was a discriminating judge of mathematical activity in accordance with her beliefs. | • Uncertain of relationship between terms maths and | between learning and teaching. | calculators, balance scale. |
| • Experienced difficulty in reflecting on factors hindering her learning of maths. | mathematics. More familiar and comfortable with term maths. | | • Believed luck could help in learning maths. |

Table 7Summary of David's responses to interview tasks

| Overall comments | Maths/Mathematics | Learning | Helping factors |
|---|--|---|---|
| | • Focused more on behaviours than content or cognition as maths. | •Learning seen to involve - listening and obeying the | • Believed could help himself by - listening to the teacher |
| David | • Hard work, concentrating, listening to and obeying the teacher required for maths and for some other | teacher - working hard | - concentrating - thinking |
| Low-achiever | activities subsequently associated with maths, e.g., art. | - receiving information from the teacher | - trying - learning from his mistakes. |
| Male | • Making mistakes and not understanding seen as a | - remembering - thinking | • Directions or instructions for learning seen as important. |
| School S | associated with maths (e.g., sign language). | completing work. Loorning score to load to gotting | • Ambivalence towards working with |
| | • Believed that mathematical activity does not include talking (e.g., suggested counting aloud is not | correct answers. | other children: - Considered peers can be helpful e.g., |
| • Judged own achievement in maths as "7 or 8" on | maths). | • Teacher considered main source of information and direction for | to give directions, to work out answers |
| a scale of 1 to 10. | • Maths is designing things and solving big | learning. | - Believed learning should be |
| • A key concern: To get a "good educate [sic] in maths". | • Formal measurement seen as an element of maths | | legitimate, e.g., not occur by hearing or watching other children. |
| •Keenly participated in interviews, often | (e.g., with rulers). | | • Calculators can be helpful (to give |
| expanding upon responses but sometimes communicating ideas that appeared a little | • Number concepts and processes seen as an element of maths (e.g., putting in groups, counting, adding). | | answers you do not know) and for learning. |
| confused. | • Own mathematical experiences associated with school | | • Blocks and fingers are helpful for |
| • Interview responses gave evidence of exploration of language and ideas. | • Appeared to be in the process of making meaning | | then finds out answer straight away. |
| | of the term mathematics, with no clear concept of its meaning or relationship to maths. | | • Ambivalence towards the importance of a quiet working environment for learning maths. |

Table 8Summary of Emily's responses to interview tasks

| Overall comments | Maths/Mathematics | Learning | Helping factors |
|--|---|---|---|
| | • Associated maths mainly with numbers and operations. | •Learning portrayed as knowing and remembering. | • Emily's perception of learning maths well: knowing the work or the answer, |
| Emily | •Numbers seen as necessary for maths but not sufficient. | • Perceived two states in the learning process: - knowing, evidenced by remembering and finding | Believed feedback helps her to learn |
| High-achiever | • Maths seen to include addition and counting (may have assumed some formal measuring within). | work easy (therefore having learnt) - not knowing, evidenced by not remembering (therefore not having learned). | maths - after she gives incorrect answers.Tools help Emily to learn maths: |
| Female | • Mixed beliefs regarding measurement: | • Saw the child's role in learning as remembering, | - fingers |
| School I | measuring not always seen as maths measuring seen as maths when numbers or operations were present | • Believed outcomes of learning maths were to recall number facts and know how to do maths. | - counters - writing on paper - (calculators give answers but not help to learn). |
| • Judged own achievement in maths as "8 or 9" on a scale of 1 to 10. | some inconsistency in benefits whether measuring by counting is mathematical saw little relationship between | Believed majority of her learning of maths occurred at school for a school-related purpose. Saw the role of the teacher* in learning of maths | •Emily helps herself to learn maths by - figuring it out |
| • Frequently took time to think about her responses. | measurement and her non-school experiences. | • Saw the fole of the teacher in fearing of maths as to explain, demonstrate, direct, motivate. (Teacher considered an authority on | memorising talking to herself. |
| • Subtlety in her views, e.g., it was not the presence of money that made a | • Considered comparisons that involve number values or formal measures as | mathematics.) | • Teacher helps by telling what to do and how to do it (not to tell answers). |
| situation mathematical, but rather the action that accompanied the use of that money. | mathematical.Saw maths primarily as a schooling experience. | Saw multiple purposes for learning maths. to be smart for school recognition and achievement for progressing to higher grade levels. | • Emily could be helped to learn maths by being shown by a child "who knows". |
| • Gave little evidence of in-depth thought about the concept of learning. | • Terms maths and mathematics considered basically synonymous. | • Difficulty in learning maths attributed to not listening. | • Quiet working environment important as need to concentrate. Preferred mostly |
| • Responses suggested valuing of an instrumental understanding of mathematics focused on producing answers that are judged by outside sources. | | - | to work alone. |

| Table 9 |
|---|
| Summary of Filip's responses to interview tasks |

| Overall comments | Maths/Mathematics | Learning | Helping factors |
|---|---|--|--|
| Filip | • Maths associated mainly with number concepts including counting, operations, fractions and percentages (Counting seen to include addition and subtraction). Also associated operation | Defined learning as "to know how to do it". Learning explained as remembering (what to do), for the purpose of understanding (what to do / how to do it). | Learn best by being shown examples of "how to do it": by others (his mother, class teacher, brother) from a book do the examples over and over until remember / understand. |
| High-achiever | signs.Mathematical activity also seen to | • Distinguished between doing maths (already "knows how to do it"), and learning maths. | • Believed he learns more quickly through private tuition e.g., by his mother. |
| Male | include - estimating numbers - graphing | • Learning associated with tuition by a teacher*: - know you have learnt because taught by a | • Teacher more helpful than children: knows when you need help. |
| School I | • Contradictory statements whether shape activities are part of maths. | teacher - teaching done by a teacher, not by children (e.g., Filip does not teach other children). | Inconsistent regarding help from working in groups: Yes: discuss a new problem, tell each other what is known; "can teach you how to do it" |
| • Judged own achievement in maths as "10" on a scale of 1 to 10. | • Considered formal measurement as an element of maths in school and real-life situations. | •On one occasion defined both learning and teaching as "to show somebody how to do it". | No: "you need the teacher" Unsure: maybe everybody in the group "doesn't know how to do it". |
| • Tended to communicate succinctly but elaborated when asked. | • Identified a small number, but proffered few real life situations for application of maths. | Motivation to learn: to know more things so can be smart because mother's ambition for him to be a | Unsure about working with other individuals: happy to work with a partner to "tell each other what we know", but |
| •Occasionally a little impatient in interview when | • Home mathematical activity involved being taught by a parent or sibling. | doctor - to achieve in tests and win games. | - has not experienced friends explaining things really well to him (because he already knows). |
| tasks involved many parts. • Little uncertainty or | • Saw maths primarily as a "schooling" experience. | • Competitive in learning: - stated proudly, without provocation, that he | Actions to help self in learning: not cheat: by figuring out for himself, working alone, and not using a calculator. |
| contradiction in responses regarding the nature of mathematics. | Saw maths as work.Associated use of maths with school. | and Emily were the best at maths in the class - wanted to learn more to achieve in tests and win games. | "concentrate, listen, don't talk" write down so know for next time ignore any distractions |
| • Concerned about possible cheating in maths such as through use of a calculator. | • Terms maths and mathematics considered synonymous. | | •Materials help Filip to learn maths e.g., ruler, tape measure, blocks, fingers (in Grade 1). |

| Table 10 |
|--|
| Summary of Gina's responses to interview tasks |

| Overall comments | Maths/Mathematics | Learning | Helping factors |
|---|---|--|--|
| Gina | • Number seen as an important element of maths, with a focus on counting and some operations. | • Associated learning with schooling, that is, learns when the teacher teaches her. | Working alone seen as helpful: to be able to concentrate not to be told answers. |
| | • Number seen as not necessary or sufficient. | •Learning maths seen as a school- related activity. Believed did not learn | • Believed should not listen to or observe others, as she would be copying. |
| Low-achiever | • Measurement considered an element of maths: | maths elsewhere, except through homework. | • Ambivalence regarding learning mathematics in a group situation: |
| Female | included informal measurement (comparison) | • Remembering considered an element of learning. | - helpful when not know what to do - not helpful if told answers. |
| School I | identified use of formal units as maths some confusion in matching attributes and units. | • Learning maths referred to as "how to do it" or "what to do", not as just getting answers. | • Teacher* portrayed as helping Gina to learn maths by - telling what to do or how to do the maths |
| • Judged own achievement in maths as | • Space, that is, shapes and corners, seen | • Learning seen to involve thinking. | - giving clues but not answers. |
| "5" on a scale of 1 to 10.Communicated some belief of being able to learn mathematics but also | as an element of maths.Maths as a school-based activity most pertinent for Gina. | • Could not say how she would know when she had learned, other than because she had been taught by the | Had confidence in the teacher as believed teacher knows more than the children. Gina believed helps herself to learn maths by |
| experienced difficulty. | • Maths as a non-school activity | teacher. | concentrating, getting enough sleep so she can |
| • A broader view of mathematics than some of the other children: included | identified only in situations put forward by the researcher. | • Portrayed a multiplicity of reasons to learn: | listen to the teacher, listening to the teacher, putting more effort into her work, using her |
| number, informal measurement, formal | • Did not generally differentiate between | - to go up to another grade | brain, thinking, working quickly and counting. |
| • A seemingly instrumental approach to | comfortable to use either. | - to become a teacher to teach others - to not be tricked when shopping. | • Believed tools help her to learn maths: fingers, geoboards, blocks. |
| learning maths, with the immediate purpose of getting correct answers. | | | •Believed calculators help her to learn maths, but also that their use is cheating. |
| • Doing maths quickly was valued. | | | C C |
| • Portrayed a strong image of working alone in mathematical activity. | | | |

Table 11Summary of Harry's results from interview tasks

| Overall comments | Maths/Mathematics | Learning | Helping factors |
|---|---|--|--|
| Harry | In defining maths or describing mathematical activity referred mainly to number concepts. Referred also to "learning and concentrating" as maths, what might be seen by Harry as maths in action in the school context. | Learning maths appeared associated mainly with knowing number facts and procedures. Believed "learning and thinking are the same". | • Testing and feedback (by class teacher, parents, sister, computer program) help by indicating what needs to be learned. Helpful to play a maths game at home with a friend and test each other. |
| Low-achiever Male | • Suggested view of mathematical activity influenced by the class teacher, e.g., "graphs is maths and so is shapes because Ms I said so", although not consistent in seeing shapes as maths. | • Stated "[I know I have learned when] the maths goes into my head, the brain can remember it, and I say the sum really quick". | Teacher writing on blackboard helps as "everyone can see the sum", and Harry "writes answers and teacher says wrong or right". Helps himself by |
| School I Judged own achievement in maths as "10" on a scale of 1 to 10. Seemed shy in the beginning interviews but became more confident and outgoing over the five month period. | Gave contradictory responses regarding whether work with shapes is maths. Spoke of, but did not identify as mathematical, informal and formal measurement concepts and tools such as "how heavy", "big or small", "see how long it is", "how many kilos", "measuring each other with string", and "ruler". Mathematical activity undertaken by Harry at school and at home for tuition purposes only. | Stated "[I know I have learned when] I know it, I understand it, and I can remember it, and I can do it. [Understand means] I know already. All these things are the same meaning". Role of teacher*: to teach "how to do it" / "how to do maths". Seemed to perceive teaching and learning process in maths as: "Teacher" | thinking and concentrating counting using a pencil and book writing the sum practising. Quiet working environment helps, so can concentrate. Encouragement from friend helps. Friend says "just have a guess if I don't know". |
| Considered maths and learning to be fun. Found some maths easy and some hard. Valued speed in working out answers to "sums". Seemed to consider testing and feedback to play key roles in teaching and learning. | Demonstrated difficulty in proffering real life uses of maths. Process of working with and coming to know maths: testing, feedback and response. Believed terms maths and mathematics have the "same meaning" but associated the latter with "grown ups high school harder maths". | gives a question, child writes the answer, teacher ticks whether right or wrong. Child would then "learn" those incorrect and be tested again. Class teacher, father, mother, sister all seen as teachers as "they know it already". Stated adults learn maths "because it is fun and you get smarter, you learn better". | Working with other individuals or in groups helps when maths is "hard": if you're stuck "they write the answers I don't know" if do hard homework together feel good (Preferred to work alone when easy maths, "because it is quicker"). Blocks, fingers, and counters help (especially when younger). Calculator helps "if I don't know the answer". |

Table 12Summary of the eight children's responses to whole class tasks

| Child | Mathematics | Learning | Helping factors |
|-------|--|---|---|
| | (Insights provided by class administered Alien task) | (Insights provided by either class administered task) | (Insights provided by class administered PPELEM task) |
| Anna | [Anna was not present to complete this task with the class.] | _ | "The teacher is the most helpful thing because she might tell you the answer." |
| Ben | "Maths is about numbers and measuring. [The alien] needed to count all his fingers." | | "I think I learn maths well at home. [My dad] was showing me what to do [and] helping me. The maths I am learning is how to measure using a tape measure and hammer we are measuring how [much] wall to push down." |
| Cara | "Sums" mentioned only: "1 x 1 = 1; 2 x 1 = 2; 1 x 3 = 3". | _ | Remembering the answer (Doing times tables at school) |
| David | "Maths helps us if we have maths problems like times division Maths Task Centre problems." | "We learn maths because it is important to us we learn from maths books." | "The maths book because it explained what to do." (At school doing answers to questions). |
| Emily | "Numbers also call it mathematics which is nearly the same meaning as maths" "Add up numbers make a sum write the answer." "maths has a special group of things to use to add up a sum and some of them are fractions, add, take-away and division." Mum uses maths at home when cooking, e.g., counting. | Learn: "remember how to count and do it by your mum teaching you, your dad and your teacher". | "Dad teaching me times tables with cards, by timing what it says on the card at home." |
| Filip | Numbers and signs. Might use at school, work or home - "if need to plus or subtract something." Need to learn maths so it is not hard is fun teacher teaches us. | _ | "Why I learned maths well because in was fun [<i>sic</i>]. The teacher teaches us until we know what to do." |
| Gina | "Maths is learning lots of sums like, plus, take-away and we have to thing [<i>sic</i>] and using fingers. We do maths at home or school and lots more places." | "Learning is important Our teacher teaches us math and we learn." | "when the teacher explained it well I am working alone I was feeling glad that I knew." |
| Harry | "you use maths like thinking and learning to learn how to do the sum really fast" "do maths by useing [<i>sic</i>] a pencil or test-book or a test sheet at home or at school take time to do it." | "If you learn and think you should know it you should know it off-by-heart and it is important for you." | "maths test because when you get it is roung [<i>sic</i>] you will have a cross or right and it helps me." |

The following discussion addresses each of the research questions in order and focuses mainly on issues that appear interesting and enlightening for the reader. These issues draw together the data and provide for the reader alternate or additional categories to the themes identified in Chapters 5 and 6 and Appendix E, facilitating the development of multiple-perspective insights into young children's beliefs. The children's data are analysed to abstract from the concrete to identify common themes and issues (Murray, 1938; van Manen, 1990).

Children's articulation and portrayal of beliefs

The first research question asked whether young children hold beliefs about mathematics, learning, and helping factors for learning mathematics and whether these could be articulated and portrayed in response to procedures developed for this research.

The discussion of findings from five of the research participants (Chapters 5 and 6 and Appendix E) and the summary of interview and class task results from all eight research participants (Tables 4 to 12), illustrate that these children of eight to nine years of age did hold beliefs about mathematics, learning, and helping factors for learning mathematics. Responses demonstrate that the children were able to reflect upon their experiences, and articulate beliefs when prompted.

The research gave children opportunities for expression of beliefs. The children appeared comfortable in the interview situation and generally were able to respond to the procedures. The range of specific affective responses included the display of increasing confidence by Harry and Gina, delight by Emily, occasional impatience by Filip, and appreciation of the opportunity to express perspectives and explore ideas by Cara and David.

Each child was able to respond to each procedure in some manner that gave insights into beliefs. The detail and depth within responses for any one procedure may have varied from child to child. However, the number of procedures addressing each of the areas of interest and the range of media in task prompts and for communication of responses provided many opportunities for exploration and expression of beliefs.

Some children's thinking was challenged on occasions, for example, when asked to talk of learning in an abstract sense in the personal dictionary definition for learning (Task 2.1). This was not unexpected and for this reason few interview tasks of an abstract nature were included. For those children who could give reflective or considered responses to such tasks, insights were gained into the extent of their thinking and ideas about the particular concepts. In addition, alternative tasks of a more concrete nature, designed to give insights into beliefs about the same concepts, provided the opportunity for all the children to express their beliefs.

During the interviews it became apparent that some of the children's beliefs were in the process of formation or development. For example, Cara's beliefs regarding the nature of maths changed in emphasis during the five-month period. David also showed evidence of the exploration of ideas within the interviews. The exposure to tasks may have caused children to think about ideas that they had not considered consciously before, as appeared the case when Cara posed a question to the interviewer when discussing the relationship between *maths* and

mathematics. As stated in Chapter 3, the developmental nature of the research was unavoidable but acceptable. Any incidental increase by individual children in awareness of their own learning of mathematics was considered a positive outcome of the research. Where changes were apparent, they were reported.

The children were able to respond to the tasks presented to the whole class, although sometimes giving less detailed responses than in the interviews, as discussed further in the reflection upon the value of the tasks, later in this chapter.

In summary, the research participants, of eight to nine years of age, held beliefs about mathematics, learning and helping factors and could articulate beliefs when prompted by the procedures developed for this study. Children generally responded positively to the interviews and appeared comfortable using the variety of media available. Some beliefs appeared in the process of formation or development. As indicated in the detailed discussions of beliefs held by five individuals, and as explored for all children within this chapter, the study revealed depth, breadth, complexity and subtlety within individual children's beliefs. These features are reflected upon specifically later in this chapter.

The second research question inquired as to what beliefs children hold about the nature of mathematics and the nature of learning. The discussion below considers themes in relation to each of these in order.

Issues related to the nature of mathematics

In considering the results from the eight children, a range of issues related to beliefs about the nature of mathematics arise. These include

- confirmation of the importance of number within young children's beliefs;
- challenge to and refinement of the notion that children perceive maths as mainly number;
- children's perceptions of measurement as maths;
- experiences of, beliefs about, and affinity with measurement as mathematical activity; and
- a paucity of reference to other examples of maths.

Confirmation of the importance of number within young children's beliefs

As discussed in Chapter 2, some previous research found that children believe maths to be mainly number. Results from the interviews and class administered tasks in the present study, as illustrated in Tables 4 to 12, indicate that number concepts were important elements of maths for each of the eight research participants. The importance of number was confirmed with counting, addition, subtraction and times tables referred to often when children gave definition type statements for maths.

The importance of number for children was confirmed also through the finding that some children's references to number appeared to include other areas of the mathematics curriculum as well as pure number. For example, Anna and Emily appeared to see number as the key idea within activities that to others may be seen to encompass other mathematics concepts. Ben also gave some emphasis to number at times. Anna's interview responses revealed that number can be the key concept seen in activities with a space component. For example, counting the number of sides of shapes was given as the reasoning for classifying some space activities as maths. It is possible that when Anna gave the response of "maths is also a thing that you count" in her maths word wheel response, she may have been thinking of space activities as an example of this counting. This finding suggests that statements within brief, one off expressions of belief may not communicate the subtlety of young children's beliefs.

Emily's interview responses revealed that not only did she include counting and the four operations within her concept of maths as number, but she also perceived comparisons and some measurement as mathematical activity because of the relationship to number. Through the presence of numbers, counting or addition (which deploy numbers), some measurement activities were considered mathematical. Thus when Emily wrote in her maths word wheel that maths is "mathematics, numbers, sums, counting, adding" these terms may have encompassed formal measurement activities for Emily, but essentially because of the number element.

Ben referred to many measurement situations and concepts including weight, area, height, width and length when defining maths but described some situations as mathematical because of the number element. For example, when describing weighing vegetables as mathematical activity he referred to the scales, the weight, and the use of numbers. He referred at another time to counting centimetres on a ruler. These responses suggest an importance of number but indicate also that reference to number can include concepts such as formal measurement. For young children, it seems that moving to the use of formal measures may cause focus upon reading of numbers and/or counting of units. It is understandable therefore that some children may focus on the number element of these activities.

The two examples from Anna and Emily suggest that some children may see maths through what might be called a *number lens*. Their perceptions of maths as number can include other concepts but possibly without conscious awareness of the relationship or differences between concepts. This research finding suggests that if awareness by children of a range of mathematics concepts, as distinct from number, is valued, the nuances of those concepts should be highlighted by teachers. This could be done through conversation with children related to the normal varied range of tasks with a mathematical component that occur in the mathematics classroom and at other times in the school day. Experiences might be followed by reflection, conducted verbally or through written means, on the nature of what children have experienced in those activities. Such reference can assist in building awareness of the variety of activities and concepts that might be called mathematical (e.g., Frid, 2000). Focused investigation of the effectiveness of such strategies for building up children's awareness is a possible avenue for further research. As illustrated above, subtleties in children's beliefs became apparent within this research, Indeed, insights into further subtleties, as discussed below, suggest that the notion of children believing mathematics is mainly number should be challenged and refined.

Challenge to and refinement of the notion that children perceive maths as mainly number All children referred to number as an important element of maths. However, the notion of children believing mathematics to be mainly number is challenged further by four findings as listed here and as discussed below:

- there was evidence that children can hold beliefs about maths being more than number;
- responses indicated that the presence of numbers in a situation does not guarantee that young children will consider that situation to involve maths;
- children may identify a situation without a number element as mathematical suggesting a belief that maths does not require numbers;
- children may choose to identify number as maths as an exemplar or signifier of maths although this may not fully represent their beliefs about the nature or extent of mathematical activity.

Cara, Ben, Filip, Gina and David articulated concepts of maths that appeared to encompass more than number. This was particularly the case for Cara. Although number was an important element of maths for Cara, measuring and estimating appeared frequently as salient aspects of mathematical activity. For example, she referred to measuring with her father, and to her father measuring in his work as a pastry cook. Cara's focus on measurement emerged also in relation to other mathematical concepts at school. For example, one activity, presented for its potential spatial content, was identified as maths by Cara because the children were seen to be focusing on length. Some situations, chosen by the researcher for their inclusion of informal measurement by comparison, were considered by Cara as maths indicating that, for her, numbers were not necessary for a situation to be considered mathematical.

Number was an important element of maths for Ben but he also identified formal measurement, and informal measurement through comparison activities, as mathematical. In addition, Ben identified as maths a range of activities including throwing, running and blowing out candles that appeared to relate to ordering without the use of numbers, or to competing to achieve higher and higher levels of outcome. His responses indicate that he believed that maths was not only about numbers and indeed did not require numbers.

Filip associated maths mainly with number but also included formal measurement, graphing, estimating and two examples of making shapes as mathematical. Likewise, Gina associated maths mainly with number, but also believed it to encompass informal measurement, formal measurement and some elements of space. For Anna, the main elements of maths appeared to relate to number concepts but she did include an informal measurement through comparison activity as mathematical even though numbers were not involved.

David at times associated maths with concentrating and working hard, and, indeed, appeared to use these as criteria for deciding whether an activity was mathematical.

These findings suggest that although some children may appear to see maths through a *number lens* this is not the case for all children of eight to nine years of age. Views about maths can be subtle and complex, and maths can be seen to encompass a range of concepts and behaviours, not just number.

The responses suggest also a belief held by some children that although numbers may be considered important for maths, they are not sufficient. For example, Emily did not identify *cleaning room number* 7 as mathematical and did not believe a person was using or doing maths when *paying \$3.20 for a hamburger*, although numbers were mentioned in each case. Responses from Gina suggested also that, although important, numbers were not sufficient for a situation to be identified as mathematical. Once again these findings emphasise the danger in making general statements about children's beliefs about associations between number and maths.

Another challenge to the notion of stating that children think of maths as mainly number is posed by considering the possibility that references to number in some cases may be made as signifiers or exemplars of maths. It was found that although children may refer to number when giving a definition type statement of maths, or giving an example of mathematical activity, this may not represent their beliefs fully. While a child's beliefs may on one occasion appear uni-dimensional and simplistic, they may be more complex. This issue is illustrated firstly by comparing Cara's interview responses and her response to the class administered alien task. In the latter (see Table 12), Cara referred only to multiplication sums. Although this response was limited to number, and number was clearly an important concept for Cara, in the interview tasks she referred to maths as encompassing a greater range of concepts. It is clear from the total of her interview responses as discussed above, that a statement such as that she believed maths to be mainly number would not represent the entirety, complexity or subtlety of her beliefs.

A further instance of a child possibly choosing a signifier to communicate the essence of maths is noted in Emily's interview response to Task 5.1. Emily suggested she would show the alien a range of operations and the numbers one to ten, numbers that were much simpler than those with which she was familiar. It is possible that she chose those numbers as they were simple for the alien or that she chose them as signifiers or exemplars of maths.

The discussion of children making references to number possibly as signifiers of maths, but as not necessarily representing their beliefs fully, brings to the fore two points. The first is the subtlety of young children's beliefs, and the second is the fact that some procedures, when used alone, give partial insights only into potentially subtle and complex beliefs. Such procedures include those that allow single word answers or, for some young children, those that require a written response. Such procedures appear valuable when deployed in combination with others. Limitations of pencil and paper, one-off, and single word response tasks are recognised, as are potential benefits.

In summary, the findings

- confirm previous results of the importance of number for most children in defining mathematics and in identifying situations as mathematical;
- challenge the notion that children see maths as mainly number;
- suggest that at the young age of eight or nine years, children have developed beliefs about the nature of maths that are idiosyncratic and subtle.

As a result of these findings, it is suggested that teachers might consider regular reflection and communication of perceptions of mathematical experiences in the mathematics classroom and elsewhere. With such activity there is potential to further enlighten both children and teachers and make them more critically aware of the others' perspectives and of outcomes, purposes, and the nature of, mathematical experiences. Ways to foster such communication, especially for children in the earlier grades, is an avenue for possible future research.

Children's perceptions of measurement as maths

Measurement is a key strand of the mathematics curriculum (e.g., Board of Studies, 2000) and, as such, children's beliefs about this area of maths are of interest. Themes that arise from the children's responses include the following:

- children vary in their beliefs as to whether formal measurement and/or informal measurement situations are considered as maths;
- when speaking of measuring, children are not necessarily speaking of maths;
- children may hold idiosyncratic meanings for commonly used terms related to measurement;
- the question of whether it is important for children to see a relationship between formal measurement and informal measurement.

It is likely that the Grade 3 children in the present study had been introduced by their teachers to some formal units of measure, which is likely to have been preceded by informal measurement activities, as recommended in the then current curriculum document (Board of Studies, 1995).

Data indicate that for Anna, Cara, David, Emily and Filip, measuring was linked mostly to the use of formal units. Cara at times referred to measurement when giving definition type statements and included formal measurement of length, mass, and capacity situations as mathematical. Ben used the word measuring in some of his definition type statements and referred to a range of measurement concepts as mathematical but sometimes referred to the use of number such as through counting as the mathematical element. Ben appeared to look for the presence of numbers or measurements when deciding whether a situation involved maths. Anna, David and Filip all considered formal measurement situations as mathematical showed an appreciation of the measurement of length using formal units as mathematical activity, but seemingly mainly because of the application of number through counting or addition.

Links between informal measurement and maths were more complex. It is noted that the informal measurement situations presented to the children included direct comparison and indirect comparison through use of a third object. Measurement using informal units was not included. In retrospect, inclusion of such situations may have given further insights. However, children did have the opportunity to refer to such tasks in the open questions requesting examples of mathematical activity but none chose to do so. The measurement by comparison tasks do give interesting results.

Situations that were classified by the researcher as involving informal measurement were considered by two children, Gina and Ben, as measurement and as maths. As indicated above, these children also saw formal measurement as maths.

Anna held mixed views about the informal measurement activities posed to her: using string as a tool for comparing lengths was considered maths, but informal measurement through direct comparison was seen as guessing and not maths.

Cara referred to informal measurement by comparison situations as *maths* but not as *measurement*. It appears that *measurement*, as Cara saw it, was a subset of maths that included formal measures and thus numbers. Informal measurement by comparison situations presented by the researcher appeared to belong, for Cara, to a different part of *maths*, that did not include measurement, suggesting Cara believed measurement required numbers but maths did not.

In contrast, Emily seemed to believe that measuring could exist as a mathematical or non-mathematical activity, that is, she tended to think of formal measurement situations as maths but called informal measurement situations *measuring* but said they were not *maths*. Emily may have called informal measurement situations *measuring* because of the language use of those around her, but it appears that she did not call them maths due to her need to identify number in a mathematical situation. Emily described situations as using "kilograms" and "grams" and she called the activity "weighing", she did not call these situations maths.

Harry also used the word measuring along with other measurement terms, including names of formal units such as "kilos", and measurement tools such as rulers, but did not indicate that he associated these with maths.

Cara appeared to use the word measuring with an alternate meaning also, that is, she explained it as counting in the situation of swinging on her swing.

These data indicate that the understanding of what might be considered familiar language of mathematics is not common to all, although the same words, such as measuring or weighing, may be uttered. The results suggest that modeling and discussion of mathematical language is an important role for teachers of young children, and a role of which teachers can be conscious. There are also questions about children's understandings of the relationship between informal and formal measurement. All of the children except Harry identified formal measurement situations as maths but six of the children either did not recognise informal measurement by comparison situations as maths or had mixed beliefs about the different situations posed by the researcher. As stated above, none proffered a situation with informal units as mathematical activity. This suggests that links recognised by adults should not be assumed for young children.

A further question concerns the importance of informal measurement and the emphasis it should be given in the mathematics curriculum, a question of debate in the mathematics education community (e.g., Ainley, 1990). Although this issue is not related directly to the research questions of the current study, responses from the children suggest investigation of the use of informal units is of value. As an aside, some perspectives to date are examined briefly below.

Recommendations in curriculum documents about the teaching of length have been questioned by Boulton-Lewis and fellow researchers (Boulton-Lewis, 1999). Lessons involving informal measurement may help children to operate in real life situations requiring such skills but may increase cognitive load and not lead to understandings by young children of principles of formal measurement in the way that is assumed in many curriculum documents. Boulton-Lewis and fellow researchers (Boulton-Lewis, 1999) suggest that young children prefer to use a standard measuring device and that with explicit instruction can learn to avoid errors of usage. Ainley (1990) also wonders "what sense children make of measuring things in foot lengths when they have perfectly good rulers in their bags" (p. 74). She questions lessons involving the use of non-standard units and suggests that there is scope for research on the value of such experiences at school.

In light of findings such as those in the present study and in studies by Ainley and by Boulton-Lewis and her associates, it is recommended that further research occur that examines the value of, and the meaning children make of, lessons involving non-standard or informal units of measure. It is possible that following such research, the teaching of measurement might be treated differently in the future.

Measurement is a key mathematical concept for some children as demonstrated in responses from Cara and Ben. It appears that the beliefs of these two children were influenced partly by experiences in non-school environments as discussed below.

Experiences of, beliefs about, and affinity with measurement as mathematical application The apparent belief held by Cara and Ben that measurement was a significant element of maths appeared to relate to the children's personal experiences, the influences of significant others, and the development of an affinity with maths.

Ben gave emphasis to formal measurement as an element of maths, especially in the non-school situation of his father, grandfather or uncles building houses. Ben was involved sometimes in the building activity. Cara spoke of measuring with her father, and of her father measuring in his work as a pastry-cook. Through the occupations of family members, Cara and Ben were exposed to the application of mathematics in purposeful, real life situations and saw these activities as mathematical in their own right. David appeared also to see some association between maths and non-school situations illustrated in his discussion of maths as solving problems.

Some of the other children in the present study were more like those in a previous study (Kouba & McDonald, 1987) who appeared to judge outside activities against school experiences, in deciding whether the outside experiences were mathematical. For example, Anna, Emily, Filip, Gina and Harry appeared to associate mathematical activity mainly with schooling, as outside of school experiences cited were generally homework or tuition by a parent or sibling.

It appears that Cara and Ben developed an affinity with maths, and understanding of the application of maths in real life situations, largely because of purposeful experiences of, and with, family members who were important to them. This finding confirms the key role that families, particularly parents, can play in the development of children's beliefs. The research supports the message that it is desirable for parents to feel comfortable with their own use of mathematics and involve children in real, rather than contrived, mathematical activities in their daily lives. Schools might consider working with parents to examine appropriate mathematical activities to undertake with children in non-school environments in a relaxed and non-contrived manner.

The outcomes of the research did not suggest whether experiences of other mathematical concepts in purposeful, non-school situations would have the same impact upon young children's beliefs as measurement experiences did for Cara and Ben, but this is possible. Indeed, it seems that for the children interviewed there was limited recognition of other concepts as mathematical activity as discussed below.

A paucity of reference to other examples of maths

In confirmation with previous research, when defining or giving examples of mathematical activity, children made little reference to other elements of the mathematics curriculum such as space concepts, chance, data, and pattern ideas. The discussion below focuses mainly on space.

Only Ben volunteered a situation with any space component, in this case straightness and angle in positioning a house, as an example of mathematical activity. He also described a shape activity as maths because of attention to straightness. However, he appeared to feel that naming, drawing and making shapes were not mathematical activity. When posed with such situations, Ben indicated that he looked for the presence of measuring or counting to decide whether the activity was mathematical. Anna identified one shape situation as maths, but because she believed she would be counting in that situation. Cara generally did not believe working with shapes constituted maths but, as stated above, did identify one shape activity as maths because she believed the children were measuring. She only spoke of ideas of space when discussing one situation, the planting of a garden. Filip gave inconsistent views of whether working with shapes was mathematical activity, more often saying that this was not the case. There were elements of the space curriculum for children of his age, including recognition of shapes (Board of Studies, 1995, 2000) that he did not consider as maths. Gina described "doing shapes" and "corners and shapes" as mathematical activity. Harry gave conflicting views on whether working with shapes was maths. He explained on one occasion that shapes are maths because his teacher said so, on another occasion he said he could make shape patterns when exploring patterns in maths, but in an earlier interview stated that shapes are not part of maths. Emily did not identify any maths when presented with situations with a spatial element.

The findings suggest that Space, encompassing elements such as Location, Shape and Transformation (Board of Studies, 1995, 2000), was not readily identified by these young children as mathematical activity. This may have related partly to the degree of emphasis given to space concepts in their mathematics classes. The teachers reported that space was included in the teaching of mathematics in the year of the data collection but seemingly its inclusion in the mathematics programs the children experienced was limited. Ms S indicated that while the children had been taught space concepts, she would like to have included space in the maths program "every week, but that hasn't happened" (Interview December 11). When Ms I was asked for the degree of emphasis she had placed on space in mathematics she was given the choices of "A lot, Some, or Not much/None at all". She replied "Some" but added later that she had "done solid shapes . . . a lot", and also listed geoboards among the concrete materials used (Interview November 13). The children's previous spatial experiences within the school curriculum are not known. However, Emily's response regarding Space reminds us that attributing beliefs to schooling experiences only may be inadequate and inaccurate. Although she had experienced spatial concepts in recent mathematics lessons taught by Ms I, and the researcher had observed her completing activities on symmetry in mathematics time, she portrayed the belief in one interview that even if space activities were studied in maths time they would not be classified automatically as maths.

To summarise, the children suggested and identified limited instances of spatial activities in their school environment and made few references to space as a maths concept in non-school environments. If children's perspectives on their learning of mathematics are valued and considered to impact upon their learning, as suggested within this thesis, there appears scope for development of an awareness of space as a maths concept in the children's school and non-school environments.

The eight or nine year old children in the study also made few references to other concepts, such as chance, data, and pattern ideas as maths. It may be worthwhile for educators, including teachers and parents, to identify a variety of situations as involving maths. The importance of talking to children about their developing understandings of the

meaning and use of maths in school and non-school situations is suggested also by this research.

In considering elements of mathematics, it is appropriate to consider not only content, as has been done above, but also processes. There is scope for developing a concept of mathematics in which processes are seen as key elements. As discussed in Chapter 2, curriculum writers and mathematicians include in their definitions and descriptions of mathematical activity, terms such as reasoning and strategies, invention, exploration, the science of patterns, human sense-making, problem solving, conjecture, reflection and communication (e.g., Australian Education Council, 1991; Board of Studies, 1995, 2000; Hersh, 1986; National Council of Teachers of Mathematics, 1989, 2000; Pateman, 1989; Schoenfeld, 1992; Verschaffel & De Corte, 1996). However, the children in this study gave a generally limited sense of recognition of such elements of mathematical activity. David did communicate some sense of solving problems and Ben reported that he used strategies and liked to solve problems in different ways, but, for the majority of the children, maths appeared more straightforward and more defined in terms of content ideas. There is scope for children in the first four years of schooling to have greater experiences of such processes in mathematical contexts, and to reflect upon them as mathematical activity. Children can be invited to explore maths by teachers "providing opportunities for children to explain their mathematical understandings, encouraging various ways of solving mathematics problems, and encouraging children to reflect on answers and strategies" (Montgomery & Cheeseman, 2000, p. 5). A range of strategies has been used successfully with children in some or all of Grades Prep, 1, 2, and 3. These include questions that are open-ended or have more than one answer (e.g., Clarke, 2000; Frid, 2000; Groves, 1999; Sullivan & Lilburn, 1997), investigations (e.g., Becker, 2000; Diezmann, Watters, & English, 2001; Lovitt, 2000; Stephens, Montgomery, & Waters, 1997), children solving non-routine problems (e.g., Lowrie, 1999a), children posing problems (e.g., Diezmann, Watters, & English, 2001; English, 1998; Leung & Wu, 2000; Lowrie, 1999b, 2000; Stephens, Montgomery, & Waters, 1997), and children inventing and evaluating their own procedures or algorithms for solving number problems, (e.g., Cobb, Wood et al., 1991; Kamii & Dominick, 1998; Kamii et al., 1993). Such activities might involve children exploring a problem, and sharing, explaining and justifying their strategies and solutions. Individual written reflection, possibly most suitable for children in Grade 2 and beyond, can focus on various elements. These include observations, patterns, theories, strategies and skill use (e.g., Lovitt, 2000), what children have learned about being a mathematician (e.g., Frid, 2000), and problem posing and solving undertaken and recorded at school and/or at home (e.g., Leung & Wu, 2000; Stephens, Montgomery, & Waters, 1997). Verbal reflection, perhaps recorded by a teacher during a lesson review with the whole class, is suitable for younger children. Daily reflection may be targeted at highlighting key mathematical ideas and thinking strategies. Children might be posed questions such as "What did you learn today?", "What is the smartest thing you did in

maths today?", and "Tell me something you did today that surprised you." (e.g., Gervasoni, 2001, p. 68).

In a different study in which the researcher in involved, the Early Numeracy Research Project (ENRP), that focuses on improving mathematics learning of children in Grades Prep to 2 (approximately 5 to 7 years of age), teachers are finding that exploring problems and encouraging children to reflect on answers and strategies is appropriate and effective (e.g., Clarke, 2001). This is demonstrated by a teacher within her third year of the project:

I'm just sold on open-ended questions. I'm just sold on summing up during the lesson or at the end, which ever is applicable. Getting that teaching point and ramming it home. I really like just the fun aspect of maths . . . you link those activities to make a really enjoyable and worthwhile maths session. And you can only do that by pulling out what you want out of it at the end or during the session. [The children] are confident and . . . they're really willing to talk about how they solved things and how they've done things [for example, they say] doubles helped me with that. I've done this or I've done that and they talk about it all the time. It's strategy, strategy.

Solving problems requiring "hard thinking" has been found also to have positive outcomes in a mathematics recovery program for young learners. Problem solving with challenging and difficult tasks within a child's zone of proximal resulted in satisfaction and positive motivation (Wright, Martland, & Stafford, 2000).

In summary, findings from the present study suggest there is scope for children's beliefs about the nature of mathematics to be challenged through experience and reflection upon a range of mathematical content and activity that is close to that of mathematicians. Thinking, reasoning, exploring and problem solving can be integral components of children's mathematical activity and, facilitated by reflection, can come to be seen by children as a part of the mathematical activity they undertake.

A related consideration is children's beliefs about where mathematical activity occurs and of what it consists. Discussion within Chapter 2 examined not only differing perceptions of the nature of mathematics within and between different groups in the community, but also the current emphasis on numeracy. Responses from children in the present research suggest the majority saw mathematics relating more to the development of basic skills than to the needs of people within daily lives. Although there was appreciation by some children of the usefulness of mathematics in everyday situations, there appears scope for teachers to help children develop further such perspectives. For primary level children, linking mathematics with other areas of the curriculum might be useful.

A further factor that may impact upon what happens in classrooms is the beliefs held by teachers about the nature of mathematics and mathematical activity. Discussion in Chapter 2 highlighted variation within primary teachers' beliefs and identified the view that a person's beliefs about one thing are not in total independence of that person's other beliefs. Indeed, nor are beliefs and practices independent (e.g., Thompson, 1992). It appears then that teachers' decisions related to the mathematics curriculum may be linked to their beliefs about the nature

of mathematics and mathematical activity. For systems and those educating teachers and conducting professional development, this is a consideration.

The discussion, above, of key themes emerging from the data related to children's beliefs about the nature of mathematics, focused to some degree on beliefs about the content of mathematics and nature of mathematical activity. Subtleties within beliefs were identified and considerations for teachers highlighted.

The research investigated also children's beliefs about the nature of learning, including the learning of mathematics. There is an inherent link between the concepts of the nature of mathematics and the nature of learning, although they were separated to some degree in the present study for ease of investigation and discussion. The discussion focuses now on beliefs about learning but continues to give insights into beliefs about the nature, scope and purpose of maths and mathematical activity because of the links between the key areas of interest in this study.

Issues emerging from beliefs about learning

Of the three main concepts under study, *learning* appeared the one that the children had least thought about previously. However, some themes did emerge, including

- learning of maths portrayed to include remembering;
- knowing or having learned maths indicated for most by correct answers;
- children's beliefs about the teacher's role in their learning of maths;
- purposes for learning maths;
- mathematical activity as work or learning.

The discussion focuses on beliefs about the learning of maths, informed by responses to all procedures related to learning in general and the learning of maths in particular as appropriate.

Learning of maths portrayed to include remembering

Each of the interviewees appeared to make some association between the learning of maths and remembering. For example, Anna spoke of keeping something in her mind, being able to do something and remembering. Gina spoke of learning "how to do it" and "what to do", suggesting that these involved remembering. Harry spoke of maths going into his mind, remembering it and being able to give answers quickly. He stated also that knowing, understanding and remembering all have the same meaning.

Emily suggested she believed there were two states in learning: not knowing or not having learned, evidenced by not remembering; and, knowing, evidenced by remembering and finding work easy. For Emily, as for all the other children, remembering was an important element within the learning of maths. Concentrating, as mentioned by David and Emily, may have facilitated remembering.

Although Ben included remembering as an element of learning maths, he portrayed a different view of learning. For example, Ben felt that he learned also by listening to and helping others. He saw making mistakes and trying different ways to solve problems as

elements of learning. He also showed evidence of the use of strategies that did not depend on remembering, such as applying a pattern, to work out an answer.

Further perspectives on the learning of maths were given by Filip and Harry who used the word *understand*, and by Ben, Cara, David, Emily, Gina and Harry who believed learning involved *thinking*. However, it is not clear that understanding and thinking always indicate a high level of cognitive activity. As Skemp (1976) points out, when a child states he or she understands maths, that understanding may be instrumental in nature.

In summary, it appears that, for the children interviewed, remembering was an element of learning maths. It is possible that this was the prevalent view as, in the first years of schooling, some children may be too young to monitor, or describe in different ways, the range of their mathematical activities. Activities that might be seen by teachers as encompassing higher level processes, may be described by children as remembering. For example, while materials such as Tens Frames build up understandings of part/part/whole relationships (e.g., Gervasoni & McDonough, 2000), in the long run, they also develop automatic number facts. Children may focus on the latter as the outcome although relational understanding is being developed along the way. Even knowledge such as addends to ten may be seen to be remembered even though they may be developed through reasoning and patterning. Thus, although children may describe their maths learning as remembering, it is possible that the children participate in activities that facilitate and use a range of processes.

In addition, learning was described by some children as encompassing thinking and understanding. These terms suggest higher level cognitive activity, but as discussed above, it should not be assumed that this is necessarily the case. Ben stood out among the group of eight children as the one who articulated different perspectives on learning, for example, by seeing applying a pattern as part of his learning process.

The results from this study suggest that, while children of eight to nine years of age may associate learning mathematics mainly with remembering, it is possible for learning to be seen as more than remembering. The discussion above suggests also it is possible that, for children, the word *remembering* may encompass a broader approach to mathematics than adults may associate with this word.

Knowing or having learned maths indicated for most by correct answers

A number of the children, including Cara and David, considered that gaining correct answers was an important outcome in learning maths. For Harry a tick from the teacher was a clear indication that he had learned his maths. Indeed, if he were not successful, he would expect to re-learn and be re-tested. He appeared to see this cycle of learning and testing as important within mathematical activity. His job was to learn; the teacher's job mainly was to test. Anna conveyed her perception of the important role of teacher correction, or correction by herself after checking her work with a calculator.

Correct answers were related not only to cognitive aspects of learning but also to affective aspects as expressed by Filip who indicated that he felt good when he had correct answers on a maths test and bad when did not do well on a maths test.

In contrast, Ben's responses suggest that it is possible for a young child to focus on aspects other than correct answers as the outcomes of mathematical activity. Ben included getting better at new things as an outcome of learning and appeared to see learning more as a process than an endpoint. This appeared to relate to Ben's apparent view of learning as a higher order process. There was little evidence of other children valuing learning processes in the way that Ben did.

In summary, responses suggest that some children focused mostly on low level cognitive activity, with learning of maths demonstrated by correct answers, but that it is possible for other beliefs to be held by young learners. Valuing correct answers is clearly important, but it also may be helpful also for children to value processes for learning mathematics. Mathematical activity within classrooms might be examined, with a view to identifying whether processes such as reasoning, exploring, problem solving, conjecturing, and reflecting are present and whether their contribution is emphasised as a part of learning. It is important that children experience a range of mathematical cognitive activity including activity that promotes high level thinking and the understanding of relationships (e.g., Skemp, 1976). As discussed above, tasks such as investigations that focus on process as well as answers, or tasks that focus on more than one answer can be posed to young learners. Teachers also might consider assessing such processes, as what teachers choose to assess communicates to children what is valued (e.g., Becker & Selter, 1996; Clarke, 1988).

As indicated here, correct answers were important for the children in the study. Linked to this, the teacher role of correcting answers was considered important by some children. As discussed below, the teacher was perceived to contribute to children's learning of mathematics in other ways also.

Children's beliefs about the teacher's role in their learning of maths

The responses from the children portrayed the teacher as playing a variety of roles in supporting the children's learning of maths including explaining, demonstrating, showing, organising or directing, correcting, facilitating, motivating, and affirming. Sometimes family members such as a father, mother or sibling were portrayed as playing a teacher role but in the home environment.

Most of the children suggested they favoured a transmission style of teaching. For example, Emily liked the teacher to explain and demonstrate, and David believed it was important for the children to listen, obey and receive information from the teacher. Anna, Gina and Harry liked the teacher to tell them how to do the maths and to correct answers. Filip felt it was the teacher's role to show how to do the maths through the use of examples.

Ben, in contrast, appeared to value more of an enquiry style of learning maths. Although he was aware of the need for expert guidance at times, he did not appear to be as dependent on the teacher as were some of the other children. Ben liked to be shown partly how to do a sum; he was happy to be challenged to work out problems himself.

As demonstrated here, the children did not all see learning, or the role of the teacher, in the same way. However, the insights gained into children's beliefs about learning maths suggest most of the children could be described as receivers of knowledge who interacted more with the teacher than with the mathematics. The children mostly portrayed the focus within learning maths as the transmission of mathematical content and skills; that is, the *what* of learning maths was seen mostly as an increase in knowledge of what to do or how to do it, and was achieved through the *how* of memorisation.

Compared to the other children, Ben seemed to focus more on interacting with the maths as well as with the teacher. The earlier discussion of Cara's affiliation with maths suggests she, also, interacted with the maths. However, this appeared mainly in the home situation in which she experienced measurement and estimation, particularly with her father. The situations she identified with *learning* maths related mainly to schooling. Although measurement was identified on one occasion, learning maths was more frequently associated with number and correct answers were sought.

In summary, responses showed that those children who interacted more with the teacher than the maths liked the teacher to tell them how to do the maths. They appeared to see the teacher as an authority on maths who passed on knowledge and skills. In contrast, Ben appeared to interact with the maths as well as the person teaching him maths in both school and non-school situations and thus was not as dependent on the teacher. One possible inference from the research is that it is appropriate for students to have experiences of exploring mathematics, taking risks, and attempting problems without step by step directions from the teacher. However, research suggests there are students who may resist that orientation (Doyle, 1986). Catering for a range of learners of age eight to nine years, including those who respond positively to and those who resist such an orientation as described above, is suggested as an area of further fruitful research.

The second research question asked what beliefs children hold about the nature of mathematics and learning. A related factor of interest that was not consciously planned for in the data collection but that emerged from the data was beliefs about purposes for learning maths. Views communicated by the children help the reader to appreciate more fully the children and their perspectives to learning.

Purposes for learning maths

Four of the eight children gave responses that provided specific insights into their beliefs about the purposes for learning maths. Emily's purposes appeared to relate mainly to schooling in that she wanted to be smart, seemingly in comparison to other learners, and she learned maths for school recognition and achievement. Filip learned maths to be smart and to know more things, but also seemed influenced by a long-term ambition, fostered by his mother's views, related to a possible future career as a doctor. Gina gave reasons for learning maths as to progress to another grade, to become a teacher to teach maths to others, and not to be tricked when shopping. Harry believed adults learn maths because it is fun, and to become smarter.

Learning mathematics for the purposes of fun or of becoming smart suggest a belief of learning mathematics for the sake of more learning. This view appears intrinsic (e.g., Brophy, 1986) and essentially immediate in nature. Such a view may motivate children to learn mathematics for achievement and progress. It contrasts with views that portray the importance of knowing mathematics for more extrinsic purposes such as to use mathematical knowledge to complete tasks and to solve problems in real life situations.

Some extrinsic motivations also seemed apparent in responses within the study such as learning maths to progress to another grade, to become a doctor, and to have control over money when shopping. In making suggestions of helping the alien to come to know about maths, Ben portrayed a belief that learning is built upon prior knowledge and occurs for meeting needs within the everyday environment. Gina's reference to money and Ben's discussion of helping the alien suggest a perception of extrinsic purposes for learning maths emanating from personal needs.

Although insights were gained only into four children's beliefs about purposes for learning mathematics, the results show a range of views and suggest that, for children of eight to nine years of age, purposes for learning mathematics can relate to intrinsic and extrinsic motivations. The children did not consider mathematics to be learned only to meet immediate school or teacher requirements. These data suggest that people live their lives with a mix of motivations.

Being aware of the importance of motivation in mathematics learning, and knowing individual students' motivations can be beneficial to teachers and, in turn, to students' learning of mathematics. Assisting students to develop intrinsic motivation to learning mathematics is generally superior to providing extrinsic rewards (Middleton & Spanias, 1999), and is particularly important for students encountering difficulty (McComb & Pope, 1994). Although it may be a reality of life that people do some things to obtain a reward or avoid censure, in the mathematics classroom such motivations may not always be productive. Middleton and Spanias (1999) conclude that,

when individuals engage in tasks in which they are motivated intrinsically, they tend to exhibit a number of pedagogically desirable behaviours including increased time on task, persistence in the face of failure, more elaborative processing and monitoring of comprehension, selection of more difficult tasks, greater creativity and risk taking, selection of deeper and more efficient performance and learning strategies, and choice of an activity in the absence of an extrinsic reward. (pp. 66-67)

Middleton (1995) recommends that teachers focus on "learning for its own sake as a primary goal" (p. 254) in the mathematics classroom. Also, a teacher might be supportive and authoritative, and act as a model and a friend who "gives children feelings of confidence and self-worth necessary to be comfortable in mathematics" (Middleton & Spanias, 1999, p.82).

It may be useful for teachers to gain insights into their students' motivations to learn mathematics, to reflect upon their own beliefs about motivation to learn mathematics, and to examine their instruction in terms of how it effects students' motivations to learn mathematics.

An associated issue of mathematical activity as work or learning relates to beliefs about both the nature of maths and the nature of learning.

Mathematical activity as work or learning

There was suggestion in previous research, as discussed in Chapter 2, that some children see mathematical activity at school as work rather than as learning. However, the children in the present study seemed to associate maths at school with learning as well as with work.

In the present study David used the word *work* frequently, mainly in the context of working hard as mathematical activity. Indeed, as stated above, at times he appeared to have difficulty differentiating between hard work, concentrating and maths. For example, chess was cited as having some maths in it because it required concentration; art was identified as maths, or like maths, because it required thinking, concentration and hard work. David appeared to believe also that hard work would lead to success in maths. Cara included the word *work* when describing her mathematical activity at school, suggesting that learning would follow. She appeared to see this activity differently from much of the non-school mathematical activity she undertook with, or observed of, her parents. Emily used the term *work* when she described her "peek[ing] at other people's work", suggesting that work was the written product or the production of such. Filip described a photograph of children copying algorithmic problems from a blackboard as "they're doing work", adding, when asked for more information, that it was "maths".

These responses suggest these research participants may have associated the activity at school with work but with the belief that work or working hard would lead to learning or was a part of the learning process. Thus both work and learning seemed to be associated with mathematical activity at school for at least some of the students. It appears that, in general the children were committed to their learning of mathematics. They saw the subject as something important that required work to achieve learning.

A question for teachers is whether, with children showing such commitment, they are given appropriate support, for example, whether children are helped in knowing what learning entails to be able to have some control over their learning. Working hard is perhaps commendable but does not necessarily alone foster understanding of mathematical operations and relationships. At present our knowledge of children's learning does not allow us to prescribe one best way of teaching. However, it seems that activities that foster thinking and reasoning followed by reflection will help children to learn at a higher cognitive level. If children are taught in a manner that fosters understandings of relationships in mathematics, their mathematics learning should be easier to remember and more adaptable to new tasks than mathematics learned instrumentally (Skemp, 1976). Relational mathematical knowledge

can also be "effective as a goal in itself" (Skemp, 1976, p. 24) thus reducing the need for external motivations to learn, and "relational schemas are organic in quality [in that the learner is likely to] seek out new material and explore new areas" (p. 24).

The findings from the present study suggest, therefore, a concurrence with current curriculum documents (e.g., Board of Studies, 2000; National Council of Teachers of Mathematics, 2000) that thinking and reasoning should be significant elements of a mathematics program, fostering for some children an expanded or different view of the nature and role of work within their learning of mathematics. Strategies discussed earlier to broaden children's experiences of mathematical processes can promote such reflection and understanding of relationships.

It is difficult to summarise insights into children's beliefs about learning. There is the temptation to conclude that the children associated the learning of maths mainly with number, thought maths was learned mainly for schooling purposes, portrayed remembering as the key process of learning, and focused primarily on attaining correct answers. These inferences give an overview of the main trends but do not represent fairly the exceptions nor do they portray the variation and the complex and idiosyncratic nature of the children's beliefs. Even at the age of eight or nine years, children do hold beliefs about mathematics learning that appear in some way influenced by their school and non-school mathematics learning experiences.

Trends within the research findings identified and discussed above include that,

- learning maths was portrayed by all the children to include *remembering*, but that this may have encompassed broader activity than that which may be assumed by adults;
- references by children to *thinking* and *understanding* may not necessarily have included higher level mathematical activity;
- levels of dependence on the teacher, as a person who would tell or show, varied but held some relationship to the degree to which the children interacted with the teacher rather than the maths;
- purposes for learning maths were both intrinsic and extrinsic;
- the children appeared to associate mathematical activity at school with learning as well as with work; and
- the children were committed to their learning.

The research suggests that there is scope for actions that help children to

- experience mathematical learning as an active process involving thinking and reasoning;
- interact with mathematics as well as with the teacher; and
- reflect upon and know what learning mathematics entails to be able to have some control over their learning.

It is recommended that teachers well might consider

- their own beliefs;
- their classroom practices; and
- the messages conveyed explicitly or implicitly to young children.

The discussion continues by responding to the third research question. Insights into children's beliefs about helping factors for learning mathematics, as presented in this chapter, earlier chapters, and Appendix E, are examined to identify broad themes.

Themes related to helping factors for learning mathematics

The investigation into children's beliefs about helping factors for learning mathematics resulted in the expression of beliefs about a variety of factors, as illustrated in Tables 4 to 11. As occurred for beliefs about the nature of mathematics and the nature of learning, data are drawn together for discussion and reflection. The discussion below identifies and reflects upon seven themes that emerged from the children's data:

- the calculator as a helping factor for learning maths;
- other materials helping with learning maths;
- the teacher assisting children to learn maths;
- other children as helpers for learning maths;
- children helping themselves to learn maths;
- obtaining answers in maths from sources other than oneself;
- a quiet environment assisting the learning of maths.

These themes are discussed in turn. Data discussed include those that posit the factor as a possible helper and those that give a different view. Further consideration of possible significance of the children's beliefs for the mathematics classroom follows.

The calculator as a helping factor for learning maths

The calculator was included among materials discussed as potentially helping with learning maths. Some of the eight children believed calculators helped with learning maths but, for others, calculators were described as providing answers, thus might or might not have been considered a helping factor. The calculator also held negative connotations for some as its use was considered cheating.

Emily believed that a calculator would give answers but would not help her to learn maths. She appeared to make a distinction between attaining correct answers and learning maths. Gina believed use of a calculator helped her to learn maths, but, at the same time, would constitute cheating. Filip also did not want to cheat in maths. To this end he thought he should figure out for himself, work alone and not use a calculator.

In contrast, David felt a calculator could help by giving unknown answers and would help with learning maths. Anna perceived the calculator could assist her learning also by giving answers, and could be used for correcting her work and thus affirming her learning. Cara also believed that when given answers by a calculator she would be helped in her learning. Harry felt that a calculator could help him to learn maths if he did not know an answer. Ben also felt a calculator helped him to learn maths, it could make a problem easier, could show "you what maths you're doing", or help with difficult maths. However, Ben also liked to work out problems without a calculator, suggesting that he did not necessarily see his goal in maths as gaining correct answers quickly.
In summary, there was indication that some children perceived the calculator to help with learning maths, sometimes by providing correct answers. For some others the calculator had more negative connotations in terms of learning maths, such as being associated with cheating.

It appears that when given a task involving the use of calculators, children of eight to nine years of age may hold beliefs that negatively impact upon their comfort with and perceived value of the task. This may be related to the type of task that is given and the role of the calculator in working on that task.

One the one hand, there are many resources that suggest activities where the calculator is a tool for exploring maths and for empowering students' investigations. On the other hand it is possible that some teachers use calculators as a tool for checking answers. Whether the focus by some children on the role of the calculator in giving answers is an outcome of their school experience, or of an orientation to explaining the role of answers in their learning, it seems appropriate for teachers to arrange at least some experiences using calculators in broader ways, and to discuss with the children the nature of the role of the calculator in such experiences. Calculators may "liberate both teachers and children to focus on the teaching of number sense rather than slavishly following a text or set of procedures" (Swan & Sparrow, 1998, p. 458). Calculators can foster exploration, discussion and explanation by children, sometimes through the recycling of good tasks for which parameters are changed or slightly different questions are asked (e.g., Swan & Sparrow, 1998). Through such experiences children may develop flexible and efficient strategies for dealing with number problems, may become less dependent on outside sources such as the calculator for gaining or correcting answers, and may come to see their learning of mathematics and their own role, and that of the calculator, from new perspectives. Acknowledgement is made of such possible outcomes of calculator use. However, there appears to be value in teachers being aware of the possible tensions, as outlined above, which may arise for children through the use of calculators.

Other materials helping with learning maths

All eight children portrayed other materials as helping with learning maths, although with some variation in beliefs.

Ben, Cara, Emily and Gina identified a range of materials, including blocks, balance scales, rulers, tape measures, fingers, and counters, that were perceived to help in the learning of maths. There was a suggestion by Anna, David, Filip and Harry that materials such as counters and blocks were helpful when they were younger but were less suitable once they were in Grade 3. Filip's goal also was to complete maths quickly without the aid of materials indicating that he knew it. However, although Anna felt that she could now do some maths in her head, she liked to use blocks for harder problems or for when numbers were too big and she did not have enough fingers, suggesting she believed she had not fully grown out of materials.

Written materials were mentioned also. Cara felt that times tables sheets could assist in her learning, Ben identified books as helpful as he can look up new ways to do things, and Emily, Filip and Harry were helped to learn maths by writing on paper.

Cara felt that, like the calculator, other materials would help her to learn maths as they would give answers. However, although David felt calculators could help by giving answers and would help for learning maths, he indicated that other materials that gave him answers immediately would not help him to learn maths.

These responses suggest that children may approach a situation in which materials are present with varying beliefs as to the value of those materials for their learning of mathematics. They may hold a negative view towards the potential for learning mathematics in a situation where they are to use materials.

That some of the research participants perceived materials as less suitable for Grade 3 children than for younger children raises questions of the suitability of materials for Grade 3 children. There is a long history of debate about whether concrete materials help or hinder the learning of mathematics (Szendrei, 1996). However, materials are believed by many teachers to help the learning of mathematics in various ways including to "facilitate retrieval of information from memory, mediate transfer between tasks and situations, [and] indirectly facilitate transition to higher levels of abstraction" (Boulton-Lewis, 1999, p. 5). The proposition that materials aid in the development of abstract ideas in mathematics suggests that materials may be useful at times for all learners in the primary school. What is concrete and what is abstract is relative, for example, "to one child joining two blocks and four blocks is concrete, but 2 + 4 is not, another child may view 2 + 4 as concrete and x + y as abstract" (Reys, Lindquist, Lambdin, Smith, & Suydam, 2001, p. 12).

However, as indicated above, the use of materials is questioned by some researchers (Szendrei, 1996). The need for children to abstract mathematical ideas is an important consideration when choosing tasks in the mathematics classroom (e.g., Mitchelmore & White, 2000; Sullivan, Clarke, Cheeseman, & Mulligan, 2001). While materials may play a role in helping to form conceptual understandings, it is important that children do not become dependent on physical materials or models, as the abstraction of ideas may become a barrier in mathematics learning. In consideration of this, when children can solve problems by modeling with materials, teachers might consider the use of materials for posing problems rather than for providing answers and thus trivialising calculations (e.g., Sullivan et al., 2001). Indeed, a major study, involving 90 teachers and 2000 children in the United Kingdom, found that "teachers who gave priority to the use of practical equipment rather than developing effective methods, and delayed the introduction of more abstract ideas until they felt a child was ready for them, produced lower numeracy gains" (Askew et al., 1997, p. 3).

It is posited also that materials can increase the cognitive load in solving a mathematical problem (Boulton-Lewis, 1999) and can impede learning of mathematics as they do not "speak by themselves" but bring to the situation additional subject matter for the child to learn

(Becker & Selter, 1996, p. 519). There is also the danger that teachers can misuse materials (Szendrei, 1996). Boulton-Lewis suggests that teachers should find out what it is that a child knows at any point in time and then teach the child mathematics "in the most meaningful, straightforward, interesting way possible" (1999, p. 3). This might or might not include the use of concrete materials. She states that teachers need to understand the materials well, use them frequently and use them appropriately. In addition, the classroom climate should be examined as materials are not useful where children see them used for remedial purposes with those encountering difficulty, as children will not want to use them.

It appears that materials have the potential to extend or to limit children's mathematical thinking depending on factors such as when, what, how and by whom materials are used. In addition, when deciding whether to incorporate use of materials in mathematics lessons, teachers might take into account children's beliefs about the value of materials for helping in the leaning of mathematics as these may impact upon learning.

The teacher assisting children to learn maths

Earlier discussion of the teacher's contribution to learning gave some insights into perceptions of possible helping roles performed by the teacher. For example, as stated earlier, Harry and Anna saw the teacher playing a key role in their learning by correcting their work. It appears Harry perceived that teacher correction provided him with direction to follow in future work. Anna believed her mother also helped her to learn maths by providing practice tests at home. Filip believed that having a *teacher*, that is, his class teacher, a sibling or a parent, show him how to do the maths by using examples over and over was most helpful. He seemed especially to value private tuition by his mother. Cara's responses suggest she believed that by affirming, encouraging, providing tasks and giving answers, her teacher could help her to learn maths.

David favoured the teacher telling him what to do. Gina and Emily also found it helpful, when they did not know, for someone in a teacher role to tell them what to do, but they stated that they did not want to be told answers. Ben suggested that the teacher did not only help his learning by telling him what to do, but also helped by asking questions and giving guidance and encouragement. He felt also that parents could be helpful.

In summary, the class teacher, a parent or sibling were valued in helping children with their learning of maths. Helping roles ranged from providing tasks, giving direction and correcting work to providing encouragement. In previous research, as discussed earlier, clear explanations were cited as a helping factor for learning mathematics; these might be provided by the teacher or peers. In the present study, children did not refer specifically to clear explanations but some did identify the teacher telling them what to do or how to do it as a helping factor. Likewise, teacher feedback was identified by some of the children in the present study as helpful. It is noted that, as discussed elsewhere in this chapter, feedback from peers or other sources such as calculators were also considered by some to be helpful.

Other children as helpers for learning maths

Children gave a variety of views about receiving help for the learning of maths from classmates.

Anna believed that a group could help her to learn maths by giving answers when she did not know. She did not appear concerned about receiving answers from a classmate, a calculator, or from a book. However, Anna was concerned that a person in her group might not be a friend and might give her the wrong answer. When the teacher gave blocks to the group, Anna was helped to learn maths, but seemingly due to the materials rather than the group situation.

Ben felt he could learn from a friend and appreciated being given a hint. In contrast to the other research participants, Ben felt at times that other children could be more helpful than the teacher for his learning of maths, demonstrating once again that he did not feel dependent on the teacher. He felt positive about working in a group some of the time. His belief that being given hints rather than answers is helpful, also differs from some others.

Emily felt that another child who knew the maths could help her but she preferred mostly to work alone. Harry appreciated encouragement from a friend and was happy to work with others when the maths was difficult. He felt good when he did homework with another child. He liked them to test each other, indicating once again that Harry believed testing assisted his learning of maths. Otherwise Harry preferred to work alone and give answers quickly.

Responses from Cara, David, Filip and Gina suggested ambivalence towards help from other learners. Cara found other children giving moral support and direction to work towards an answer helpful but believed that others could hinder her learning of maths by telling answers. Gina also showed ambivalence towards working with others. She found group situations helpful when she did not know what to do but did not like to be told answers by others in her group. Cara and Gina's ambivalence to working in groups appeared to result from concern about being given answers. David felt that other children or the teacher could help by giving directions for what to do, but that his learning of maths should be legitimate, that is, it should not occur by hearing or watching other children. Gina also believed that listening to or observing a conversation between the teacher and other children would be inappropriate, as she would be copying. Filip had not experienced another child explaining things to him, but felt happy to share knowledge with a partner or to discuss a problem with a partner or a group. If he did not know something he would prefer to ask the teacher, but if s/he was not available, children in the group could help each other. However, Filip did express some fear of cheating when not working alone. He felt also that he needed a person in a teacher role to be able to learn mathematics as that person would know when he needed help.

The present study illustrates varying views about the value of working with others in the maths classroom, with some positive attitudes and some reluctance towards working with others. A meta-analysis of research studies investigating the impact of group work found that

children learning in small groups can achieve "significantly more than students not in small groups" (Lou et al., 1996, p. 439). However, reluctance was displayed by some children in the present study. This may stem from beliefs about the nature of learning and the children's role in that learning. The nature of the task may also be a factor as small group work has been found in some research as less effective than whole class teaching for routine algorithms, but more effective for higher cognitive skill work (Brown, 1999).

For some children in the present study, the teacher was the expert and the person in whom they had confidence to provide guidance. In view of these perspectives, it may be helpful for teachers to have explicit discussions with students on the nature of the mathematics and learning, and the value of working with others. The nature of the tasks and mathematical activity provided by the teacher might also be considered.

Children helping themselves to learn maths

The research participants appeared to feel some responsibility for their learning of maths and gave various suggestions as to how they could contribute to this learning.

Ben, Cara, David, Filip, and Gina identified listening as helping them to learn maths. Gina liked also to have sufficient sleep, as she believed it helped her to listen better to the teacher. Anna, Cara, David, Filip, Gina and Harry stated that concentrating was a helpful action. Filip and Gina found that distractions from other children made learning difficult.

When discussing doing maths at home, Anna indicated that it is helpful to try to work out a problem herself first and then be told an answer. Thus Anna appeared to believe she could help herself to learn maths, as well as learning with assistance from other sources that provided answers, such as the teacher or a calculator, as discussed above. Ben included having a go at difficult sums and solving problems among his self-help strategies for learning maths. Ben indicated that he used known facts and strategies to solve problems and would look back at earlier work or try to remember to assist in his maths learning. Ben and Emily reported that they talked to themselves to help with learning maths and Emily also would figure out and memorise.

David believed he needed to think, to try, and to learn from his mistakes. Ben also believed he could learn from his mistakes. Gina believed she should use her brain, think and put more effort into her work to help her learning of maths. Cara believed that she could help herself also by seeing a pattern. Emily, Filip and Harry identified writing down on paper as a helping factor.

Working quickly also was identified as a helping factor. Gina felt that working quickly would help her to learn maths perhaps suggesting that by focusing and concentrating on her work she would achieve in maths. Harry's preference for working quickly resulted in him wanting to work alone when he found the maths easy.

The data show that these children of eight to nine years of age had developed beliefs about how they could help themselves to learn maths. The children appeared highly aware that learning is an active process that requires attention, self-application, concentration and focus on their part. This is perhaps a surprising finding for children of eight to nine years of age. It suggests that learning is not a process of which young children are unaware but rather that learning operates through complicity by the children, and related factors such as effort and goals.

As such beliefs are formed by this early age, it is recommended that beliefs about helping factors and the child's role in learning are expressed and discussed in the mathematics classroom. Such expression might be stimulated by use of procedures from this research such as the PPELEM tasks for helping and hindering factors (Tasks 6.1 and 6.2). Reflecting in this way may contribute for children to an increase in their control of their learning of mathematics, an important element of cognition (Schoenfeld, 1992).

Obtaining answers in maths from sources other than oneself

A further theme related to helping factors for learning mathematics concerns obtaining answers from sources other than oneself. This theme overlaps to some degree with themes discussed above but because of its possible impact on attitudes and participation in the maths classroom, is identified as a separate theme.

Some children appeared comfortable with being told or given answers, particularly when they were encountering difficulty, but being given answers, by whatever means, was not always considered helpful.

Some children indicated they should be responsible for their own learning and should not be helped by materials, other children, the teacher or other sources that would provide answers. These children appeared concerned about their learning of maths and seemed to have developed a moral stance against gaining the correct answer by what they saw as an illegitimate means such as copying or being told. Some work with or help from others was not necessarily out of the question, as long as answers were not provided when the intention was to learn.

However, for others this was not of as much concern. For example, Anna and Ben did not appear fearful of answers, or worry about possible cheating, as did some of the other children, but seemingly for different reasons. Anna appeared happy to be told answers as she associated learning with getting correct answers. In contrast, Ben preferred to be given a hint or shown part of a problem as he considered important the processes within learning maths such as thinking, learning from mistakes and solving problems in different ways.

Beliefs specifically related to help from other learners varied in some respects. For some children there was a distinction between help in knowing what to do, which was believed to assist learning, and help in giving answers, which was believed to hinder learning. But children's expression of beliefs sometimes suggested inconsistency, possibly because the children's beliefs were in a state of development. For example, inconsistency is evident in Cara's belief that materials help learning by giving answers, but that other children hinder by giving answers. Perhaps there is also a subtlety in her beliefs that was not identified in the

study. David's beliefs that calculators help learning by giving answers and other materials hinder by giving answers also suggest inconsistency.

The research findings suggest that children may hold beliefs regarding legitimacy of how answers are achieved and, in turn, that this may have an impact on the learning of mathematics. If a child feels that he or she is cheating or not working responsibly when working with a calculator, other materials, or other children, this potentially could have a negative impact on the child's affective and cognitive outcomes in that situation. It appears that children not wanting to be told answers by other children, a teacher or other person in such a role, the calculator or other materials, may believe this inappropriate for either of two reasons. They may not wish to have their thinking closed off, and/or, they may feel personally responsible for their own learning of maths. The first seems to relate to wanting to interact with the maths and the second to a moral perspective on learning maths, it appears that he was the child most likely to have related to the first reason. Filip also gave some indication that he liked to think through problems. The second reason may have applied, for example, to Gina and David who seemed to believe in a personal responsibility for learning maths and who seemed to focus more on lower level thinking, such as on recalling answers.

The findings regarding cheating and the fear of illegitimately gaining answers suggest that even at the young age of eight or nine years, children can be conscious of doing the right thing, that is, of wanting to learn maths in what they perceive as the right way. The question arises as to whether such beliefs are influenced by classroom procedures and policy, and, if so, whether such outcomes are desirable. A second question is whether such views impact negatively upon how children work in the classroom. When a child is asked to work in a group or to use a calculator he or she may feel uncomfortable if there is an opportunity to obtain answers from others and if it is felt that this is not the right thing to do. If children cannot learn in a way they believe morally correct, stress may result, possibly leading to negative effects on confidence and participation. The existence of such beliefs, the dilemma of the situation into which the child is placed, and the child's potential reaction, may not be realised and appreciated by the child's maths teacher. The findings from the present research highlight this possible dilemma. They also highlight, once again, the importance of talking with children about the nature of learning, of different ways in which people can learn, and the outcomes of learning experiences.

A quiet environment assisting the learning of maths

Some children believed that the noise level in a learning situation had an effect upon their learning of maths. For example, Anna believed that working in noisy areas hindered her learning of maths as she would "lose the number in [her] head". Emily felt that a quiet working environment was important as it helped her to concentrate. Cara, Gina and Harry also favoured a quiet working environment to facilitate concentration when learning maths. Gina believed that working alone also would facilitate concentration as she would not be disturbed

by others talking. Ben felt that a medium noise level would not have a negative impact upon his learning. David was ambivalent about the help a quiet environment could give for his learning.

The research showed that some children held preferences about the level of noise as an element of the physical environment. Such preferences could be taken into account by the teacher and the children within a learning community. The type of mathematics for which quiet conditions are preferred could be considered also. It is possible that children's beliefs about this aspect of the physical environment relate to what they perceive as maths and appropriate activity when learning maths. This relates back to student and teacher goals and the manner in which they are communicated and enacted in the classroom.

Further reflection upon children's beliefs about helping factors for learning maths The above discussion identifies insights gained into young children's beliefs about helping factors for learning maths. These include that

- children at the Grade 3 level of schooling vary in their beliefs about the value of concrete materials and the value of other learners as helping factors for learning maths;
- children portray teachers in a range of roles that are believed to help in learning maths;
- at eight to nine years of age, some children are aware that learning is an active process that requires attention, self-application, concentration and focus on their part. The research posits that the following teacher actions would be valuable:
- arrange some experiences for children to use calculators in broad ways, and discuss with the children the nature of the role of the calculator in such experiences;
- discuss with children the value of help from teachers and other learners;
- discuss with children processes for helping themselves learn mathematics, assisting them to utilise their commitment to learning in productive ways;
- discuss with children the nature of mathematics and learning, and purposes and outcomes of learning mathematics, to underpin and inform discussions of the range of helping factors for learning mathematics.

In conclusion of the discussion of insights into beliefs about helping factors for learning mathematics, brief reflection is now made upon possible elements of the learning environment as listed in Chapter 3. The learning environment was conceptualised to include the following categories and elements:

- physical and architectural factors: space, privacy, noise, tiredness, light, equipment/materials, technology, seating arrangements, location;
- structure and organisation: (mathematics) task type, time, grouping, teacher direction, discussion, communication patterns, rules and procedures, competition, competitiveness;
- teacher characteristics: teaching style, feedback, expectations, warmth, friendliness, communicativeness;

• learner characteristics: desires, attributions, interests, motivations, expectations, attitudes, beliefs, emotions, self-efficacy, gender, cognitive processes (for example, attentional processes, perceived intelligence, memory, problem solving ability).

In the present research, the analysis and discussion of children's beliefs regarding helping factors within learning environments were not structured necessarily to consider these factors. In contrast, the intention was to portray the children's beliefs using the themes that emerged from their responses as demonstrated in Chapters 5, 6, and 7 and Appendix E. The above list provided some possible elements that might have been observed by the researcher. However, it was not treated as a definitive list because of the interest in and intention to portray the children's perspectives.

Within the discussions of the eight children's beliefs there was some reference to each of the four categories listed and to a selection of elements within each of the four categories. For example, the physical factor of noise level was discussed and tiredness was significant to one child. However, few other references to physical and architectural factors emerged within the children's responses, suggesting that for these mathematics learners of eight to nine years of age such factors were not perceived to have impact upon their learning of maths. Materials were referred to, but in the context of concrete materials in the mathematics classroom. These materials might also be thought of as an element of the mathematics task. Grouping for learning and discussion arose also within the expression of beliefs. As demonstrated above, the teacher featured also within consideration of helping factors with a variety of teacher roles referred to. Of interest also, was the finding that children as young as eight or nine years of age thought about their own role in their learning of maths and how they might help themselves.

Having gained these insights, it is worthwhile to consider what a teacher might do in response. Some specific suggestions were made above and considerations identified, but in some situations further reflection is possible. As suggested in discussion of the model presented in Figure 1, and in the discussion of the teacher's response to Samantha's drawing (Figure 2), in the situation where a teacher learns that a child believes a particular factor hinders or assists learning, the teacher can reflect upon the child's beliefs. In response, the teacher may or may not make changes. For example, if a child feels uncomfortable because of group work or the use of calculators in maths, the teacher may not necessarily eliminate these particular elements from the maths program. The teacher will be aware of the attitudes of the child and will take these into account in selection of activities and in discussions with students. For example, where a child is uncomfortable with group work the teacher may choose a group that will encourage thinking and reasoning and will demonstrate that answers are not always given immediately. The teacher may also re-examine the types of tasks that are presented as mathematics – more focus may be given to the process rather than to the answer, through open-ended, problem solving or investigation tasks.

Just as it appears that for adults change in beliefs follows change in behaviour (e.g., Guskey, 1985), it is posited that children's beliefs are most likely to change following a change in practice. The teacher may focus on the positive aspects of the program, making these elements more obvious, such as through choice of tasks, and discussing their purpose in maths learning experiences and their contribution to individuals' thinking.

Further investigation of beliefs might occur also. For example, where individual interviews as occurred in the present study are not possible, the class-administered version of PPELEM (Appendix F) is one procedure to gain some insights into and provide a basis for response to children's beliefs. The teacher is informed by the students, can address their concerns in some way such as through elements such as the choice of task and interactions with students, and is aware of the benefits of listening to students' perspectives. In this way, teacher decision-making, as discussed in Chapter 1, can be informed by the students.

As children's beliefs about helping factors appear to relate at least in part to beliefs about the nature of learning and mathematics, it is suggested once again that there is potential value in having discussions with children about the nature of mathematics and learning. Not only can the teacher become more aware of individual children's perspectives but the children also can become more conscious of their own beliefs, of the beliefs of others, of reasoning for beliefs and of the way beliefs may impact upon learning.

The above discussion centred around the three research questions related to whether the young children in the study could articulate and portray beliefs, what beliefs they held about the nature of mathematics and learning, and what beliefs they held about helping factors for learning mathematics. Each research question was responded to, and conclusions drawn.

Taking a broader view of the study and the data, further insights can be gained, particularly into the idiosyncratic nature of young children's beliefs.

The idiosyncratic nature of young children's beliefs

The following discussion draws out features judged to address and go beyond the research questions for this study. The data from the study, gathered over a period of five months from responses to thirty interview procedures and two class administered tasks, provide the opportunity for further understanding of the nature of beliefs held by young learners of mathematics.

The summaries of data collected in the interview and whole class situations, provided in Tables 4 to 12, indicate that differences did exist between the beliefs of the eight children, as was hypothesised in this study. These differences varied in degree: some were small and others more obvious. There were some similarities also. Frequently within the discussion of results for the individual children, mention was made of the subtlety and complexity of the children's beliefs. These features are reviewed here, along with depth and breadth of beliefs.

Complexity of young children's beliefs

For the purpose of this discussion, complexity is taken to mean connectedness, that is, it is identified where there are interconnections between elements of a young child's beliefs.

Connectedness may exist within beliefs related to one domain only. The schematic portrayals of beliefs about the nature of maths for Ben, Cara, David, Emily and Gina show connectedness as it was inferred by the researcher from the children's responses to a range of research procedures. For example, for Cara, counting emerged as a theme relating to number and to measurement through estimating. As a further example, guessing was suggested as a common feature in relation to length, chance and times tables. The schematic portrayals were developed firstly within the presentation of Cara's beliefs because of the complexity of her beliefs and as an attempt to manage the complexity. They illustrate for the reader, in an efficient format, the complexity possible within young children's beliefs and some key ideas within those beliefs in ways that may not have been available previously to educators of these children.

A further element of complexity within the children's beliefs relates to the connectedness or linkages between beliefs related to each of the three domains of the nature of mathematics, the nature of learning and helping factors for learning mathematics. These were hypothesised by the researcher as illustrated in Figure 3. However, the children's responses suggest that the degree of these linkages vary. For example, David's responses suggest a close relationship, such as through the recurring theme of working hard, and at times a seeming inability to separate or clarify ideas within each of the three domains. Filip's focus on "how to do it" was evident particularly within his considerations of learning and helping factors for learning maths. The phenomenon of *answers* permeated Anna's beliefs within each of the three domains: answers were to be calculated, to be remembered, and could be found with help from a number of sources.

These are samples of the instances of connectedness evident within the data and illustrate the complexity within young children's beliefs both within one domain and across the three domains that were the main focus of this study.

Subtlety of young children's beliefs

There were instances in the research where children gave responses that might superficially have been taken to mean one thing but that with further investigation and through comparison to a range of responses concerned with the same topic or issue, can be seen possibly to mean more. This is what is meant by subtlety of beliefs.

An example of this phenomenon can be seen by referring back to the discussion within this chapter of children seeing maths mainly as number. The discussion illustrated that while this would not be an incorrect statement, it is a statement that masks the true nature of the children's beliefs. Individual children saw the relationship between maths and number in different ways. For example, Emily seemed to define maths in terms of number, such as counting and the four operations, but further discussion revealed that she also perceived comparisons and some formal measurement activities as mathematical activity because of the relationship to number. The argument was made also that sometimes children may refer to number concepts as a signifier for, or sample of, maths. Thus a single response, such as one that suggests maths is number, gives some insights but may not represent the fullness and subtlety of a child's beliefs. Insights into detail of young children's beliefs may require ongoing investigation as occurred in the present study. It appears that caution should be taken in drawing conclusions from responses to single data collection procedures as such may not reveal subtlety within beliefs.

Breadth of young children's beliefs

For this study, breadth is defined as the ability to give a range of answers to questions about one topic.

The breadth of the research participants' beliefs is illustrated in the five individual write-ups of beliefs and for all eight participants in the summaries of results in Tables 4 to 12. For each child a number of perspectives, resulting from the children's responses to many tasks, and in relation to each of the domains of mathematics, learning and helping factors are portrayed.

Insights into breadth of beliefs were facilitated by the thematic analysis undertaken within the study. By using criss-cross analysis, many insights were gained into beliefs about individual issues or topics; sometimes such insights did not come from tasks designed specifically for that purpose. Two themes taken at random illustrate the breadth that emerged as evidenced by the use of a range of procedures for insights. Firstly, the discussion of Cara's theme of *Maths as Answers* drew on responses to Tasks 3.3.1, 3.3.2, 5.2 and 8.2. Secondly, in discussion of factors Ben identified as helping for learning maths, reference was made to responses to 13 different tasks.

The study illuminated breadth within beliefs, facilitated by the form of data collection and analysis, and either confirms or adds to the reader's appreciation of young children's beliefs.

Depth of young children's beliefs

In the context of this study, depth of beliefs is evidenced when a child can make a statement about a phenomenon or topic and add another statement, either in the same or another interview, that adds to the original idea.

This phenomenon is illustrated by children's responses to tasks that asked for single word or brief responses and to tasks on the same topic that encouraged deeper reflection. For example, for beliefs about the nature of mathematics, children gave one word answers for Task 1.1 (Word association quiz) and Task 1.2 (Password). However, there was the opportunity to give a more detailed response when discussing their word wheels (Tasks 1.3.1, 1.3.2), and in Task 5.1 (Alien) and Task 5.2 (Photographs - mathematical activity?). Children showed the ability to respond in a deeper or more detailed manner to the latter tasks.

Cara's data on beliefs about the nature of maths provide a specific example of depth. In response to Tasks 1.1 and 1.2 she gave the words "measuring" and "estimating". Later in the same interview she elaborated her understanding of measuring when speaking of her father using measuring jugs. She elaborated her beliefs about measuring as maths also, for example, when discussing scenarios presented in photographs in Task 8.2. Cara was able to express extended beliefs about the nature of maths also in Task 5.1 when she spoke of maths for an alien.

In addition, children's responses to tasks identified for focus on a particular domain, such as helping factors for learning mathematics, sometimes gave insights into beliefs about other domains, such as the nature of mathematics. The reference to insights into Cara's beliefs about the nature of maths from Task 8.2 is an example of this. Task 8.2 was developed by the researcher primarily for the purpose of focusing on beliefs about helping factors for learning mathematics. However, it provided children with the potential opportunity to add to what had been said elsewhere. Where this occurred it was due to the children's depth of beliefs.

The above discussion illustrates that the young children's beliefs were not only subtle and complex but also can be described as having breadth and depth. Indeed the research suggests also that young children's beliefs are idiosyncratic. This is a major finding of the present study as it provides the reader with a perspective that, in relation to individual children of only eight or nine years, appears to have received limited acknowledgment in the mathematics education research community to date. For teachers the research suggests that assumptions should not be made of simplicity of beliefs because of the young age of learners. Within a group of children of eight to nine years of age, beliefs may be held that are complex and that are particular to the individuals although members of the group may appear to have very similar mathematics learning experiences at school. Children bring their own perspectives to the learning situation and these may impact upon their learning of mathematics. The impact of such beliefs upon learning outcomes for young learners is another avenue of possible further research.

A further element of the study was the development of research procedures. Reflection on these procedures highlights another element of the outcomes of this study.

Evaluation of research procedures, and possible future research directions Participation by students is paramount in the quest to come to know, and understand better, the personal perspectives of young learners of mathematics. The research procedures used in the present study were designed to facilitate students to reflect actively and constructively upon, and express beliefs about, the nature of mathematics, learning, and helping factors for learning mathematics.

Data from the eight child participants in this research came from a total of 80 interviews conducted over a period of five months and two class tasks administered to each of the classes.

The research procedures were developed to access beliefs, that is, to come to see mathematics and learning through the experiences and perspectives of the young children. The development of procedures appropriate for use with such children and the interpretation of data were challenging tasks.

Reflection and evaluation of the tasks begins with consideration of the interview procedures, followed by discussion of the class tasks.

Interview procedures

Thirty interview procedures or tasks were developed or identified for use in the interviews. The procedures were divided into ten sets, used as one set per interview, with one extra set of three tasks (Set 11) broken up and used with each child when convenient.

As discussed in Chapter 3, the development of procedures was guided by ten objectives. Reflecting on the use of, and results from, the thirty procedures it is apparent that,

- the procedures allowed the gaining of insights into beliefs about the nature of mathematics, learning and helping factors;
- the research procedures facilitated the provision of rich and illustrative data showing complexity and subtlety within children's beliefs;
- the open-endedness of the procedures allowed children's individual perspectives to be pursued in the interviews and the depth, breadth and idiosyncratic nature of their beliefs to emerge;
- the procedures were suitable for eight to nine year olds, with all children responding, although there was evidence at times of some children being challenged to think in ways, or about concepts, with which they were unfamiliar (This point is discussed further below);
- the procedures successfully used a range of strategies to stimulate discussion and incorporated ideas for children to respond to through a variety of media;
- the procedures were successful generally in maintaining the children's interest and accommodating their attention spans, although on a small number of occasions a little impatience was apparent (This is explored further below);
- follow-up wording to the interview tasks was suitable for children to respond further in the interview situation;
- overall, suitable tasks were developed by the researcher or adapted from the work of previous researchers.

It is possible also to consider the value of the procedures by examining reference to children's responses from the different procedures within portrayals of beliefs.

For this purpose a tally of use within the discussion of themes from the interview data was calculated. Appendix H contains a full account for each child; a summary version is provided in Appendix I. It is pointed out that these data concern the responses to tasks that were drawn upon and referenced by the researcher in the write-up of the children's themes. Children may have responded to more tasks than those indicated, but with their responses not

called upon in the presentation of themes, for reasons such as duplication of data. The data in Appendices H and I are organised according to the three main domains of interest, that is, beliefs about mathematics, learning, and helping factors for learning mathematics but also to the fourth, background domain of self. In turn, the discussion below considers the procedures in terms of providing data and insights into these four domains.

The following conclusions, based on data in Appendices H and I, can be made about the value of the tasks:

• Although each procedure was designed to focus primarily on beliefs about self, mathematics, learning, and helping factors, they generally gave insights into beliefs about more than one of these domains.

This finding is a positive outcome of the research and suggests value in future use of these procedures for gaining insights into different aspects of children's beliefs. Where a task did not contribute to insights into beliefs about more than one domain this may be due to task specificity. For example, Task 10.3, a focused task that asked children to discuss four words chosen for their potential to relate specifically to learning, gave insights only into beliefs about the nature of learning. Also, as mentioned above, not all responses were referred to in the write-up of results and thus not all responses to individual tasks that were made by children in the interviews are included in Appendices H and I.

• All tasks gave insights within the discussion of themes for the domain for which they were intended, at least for some of the children.

This finding also suggests the value of the tasks developed for this study for future research, particularly where just one domain is the focus. It suggests also that the value of the task for gaining insights into a domain is not guaranteed from any one task for all children. However, once again, the fact that not all responses to tasks are recorded in these appendices, should be taken into account when considering this outcome.

• More tasks gave insights into children's beliefs about mathematics than any other of the domains within the study.

In retrospect this is not surprising as for most of the tasks, mathematics was the common underlying element. Although the emphasis, for example, may have been on helping factors for learning mathematics, children may have made responses that gave insights into their beliefs about the nature of mathematics. It is perhaps partly for this reason of the permeating nature of the theme of mathematics, that the write up of each child's beliefs tends to give most emphasis or consideration to this domain.

Tasks 4.1, 5.1, 7.2, 8.2, and 9.2 gave insights into each of the four domains of self, mathematics, learning, and helping factors, for at least one child.
 This indicates that although Tasks 4.1 and 5.1 were developed mainly to give insights into beliefs about mathematics, they can provide insights into multiple domains within young children's beliefs. These tasks are simple to administer and therefore possibly suitable for

further research exploring beliefs across the domains and exploring the use of procedures with class groups.

Tasks 7.2, 8.2 and 9.2 were developed mainly to give insights into beliefs about helping factors for learning mathematics but the data in Appendices H and I indicate these tasks potentially can provide insights into a broader range of beliefs. It is noted that each of these tasks was made up of multiple elements, for example, a range of photographs, drawings, or video clips, giving many opportunities for discussion and providing concrete examples to which children responded, and for the interviewer to use as the beginning point for discussion. The research findings suggest that such creative interviewing tasks that incorporate visual portrayals of scenarios or other people's beliefs are suitable and powerful prompts for stimulating discussion of young children's beliefs when used in the interview situation.

• Fifteen, or half, of the tasks provided insights into beliefs within three domains for at least one child.

This finding emphasises also the potential multiplicity of insights to be gained into children's beliefs from responses to many of the tasks developed for this study. Further tasks also provided insights into two domains.

- No tasks gave insights into each of the four domains for all of the eight children. This finding indicates that the potential for multiple insights cannot be guaranteed for any one task. The responses to tasks and subsequent insights depend on many factors. These include the children's interest in the task format, their feelings and willingness to contribute in detail on the particular day a task is used, and the ability of the interviewer to use the children's initial responses in a meaningful way to explore beliefs further.
- All tasks provided responses that were used at some time within the discussion of children's beliefs within this thesis.

This finding indicates all of the tasks hold potential for gaining insights into young children's beliefs, although in this study some tasks gave more insights than did others.

• All tasks except one provided data that were used within discussion for at least five of the eight children.

This finding suggests that tasks are suitable generally for use with children of eight to nine years of age. As stated above, although some tasks were not used in the discussion of individual children's themes this does not necessarily indicate that the children could not respond to the tasks. Responses that did not clearly add value to the discussion were not included in the discussion. An example is Ben's response to Task 11.3. This included much information that was presented more succinctly at other times and therefore was referred to from those responses.

• Task 1.4, concerning subjects most and least like maths, provided the least insights into children's beliefs and was referred to only in the discussion of Anna's and Cara's beliefs.

This task appeared challenging for some children who had difficulty identifying and/or talking about other subject areas in the curriculum as easily as they could maths. This suggests maths is more recognisable than other curriculum areas for some children of eight to nine years of age.

Further reflection on the value of the procedures also reveals other aspects to be considered:

• Some impatience was evident within interviews.

Some impatience was displayed on a small number of occasions with tasks that included multiple elements. However, this was an infrequent occurrence, suggesting that the tasks would engage most children of eight to nine years of age. It seems undesirable to break up or reduce the number of items in such tasks as multiple elements give increased potential for gaining insights into beliefs. Where impatience occurs, the task items could be broken into subsets.

• Some tasks provided challenges for young children.

As stated above, occasionally children found a task challenging. For example, Cara was unable to identify a situation in which something was stopping her or making it hard for her to learn maths (Task 6.2). Following Cara's difficulty with Task 6.2, she continued the conversation by giving a second example of a situation in which she was learning maths well (Task 6.1). Cara's response to Task 6.2 was somewhat unexpected as she had been identified by her teacher as a low achiever in maths and it was thought that she would have found maths difficult to learn at some point. Cara reported on another occasion that she sometimes found she would get mixed up or forget answers in maths (Task 11.1).

Tasks asking for definitions or asking for reflection upon learning also posed some challenges at times. The personal dictionary task (Tasks 2.1, 2.3) and learning situation task (Task 2.2) called for children to reflect upon learning, a concept that seemed the least thought about by the children. It is noted that some children had difficulty reflecting upon or recalling recent learning experiences. For example, Emily could identify only learning the times tables even when she was encouraged to think of whether she had learned other things out of school.

• Where children did experience difficulty with tasks, it was possible that valuable insights would be gained.

This finding is illustrated by once again considering Cara's reaction to Task 6.2. Cara's difficulty gave insights that added to the researcher's appreciation of her view of the world and of herself as a learner. Her non-response suggested she had difficulty recognising, reflecting on, and/or articulating her difficulties in learning maths. Importantly, knowledge of such could lead Cara's teacher to implement strategies to help Cara become more aware of specific difficulties, of factors potentially affecting these, and

of potential outcomes, so she might have greater control over her learning of mathematics (e.g., McComb & Pope, 1994).

- Tasks appear to have the potential to build up young children's self-awareness as learners. Although young children are learning all the time some appear to have little to recall when asked to reflect on their learning. It is possible that if they took part in discussion or reflection more often, they would become more aware of their learning and their accomplishments, perhaps leading to an increase in confidence or an improved selfconcept. It seems desirable for children consciously to see themselves as learners who are making progress. The present research may have fostered such beliefs for the research participants. The research suggests that teachers might incorporate such reflection in their mathematics programs to foster children's awareness of themselves as learners of mathematics.
- Tasks with multiple elements, such as Tasks 3.3.2, 5.2, 7.2, 8.2, and 9.2 tended to provide much potential for discussion and thus for insights into beliefs.
 A possible challenge for future research is to adapt these tasks and their administration to be suitable for use with groups of children.
- Tasks that are simple to administer hold potential for use with class groups. Simple to administer tasks include the word wheels (Tasks 1.3.1, 1.3.2), Drawings (e.g., Task 4.2.2) and the PPELEM tasks (Tasks 6.1, 6.2). Following their success in the interview situation, it can be inferred that these tasks hold potential for use with class groups. Some PPELEM research has occurred with class groups (e.g., McDonough & Wallbridge, 1994) but limitations became apparent, as discussed in Chapter 3. It may be possible to improve the use of PPELEM and the word wheels in classroom situations. One possible strategy is to follow the completion of the drawings and written descriptions with brief discussion. This could apply also to the administration of the Task 3.3 questionnaires. It is noted that research by McDonald and Kouba (1986) incorporated group discussion following the administration of their questionnaire from which Task 3.3.1 was adapted. Future research could investigate for other tasks also, the possibility and value of using the approach of McDonald and Kouba.

As indicated here, the research procedures or tasks were successful generally in meeting the objectives set for the development and use of procedures and in gaining insights into young children's beliefs. It is suggested that there is scope for further research regarding the adaptation of tasks for use with groups of children within school classes.

A further strength of the research was the use of more than one procedure in multiple interviews to gain insights into each domain of beliefs, allowing themes to emerge and insights to be validated, as discussed below.

The value of one-to-one, semi-structured, multiple interviews over a period of time The research demonstrates the power of the semi-structured one-to-one interview for allowing views and issues to arise, to become illuminated slowly and possibly to develop further within an interview. The incorporation within each interview procedure of conversations between the research participant and the researcher following the posing of the initial task was a strength of the research. Such conversations allowed the researcher to explore children's beliefs further, and for children to expand upon, clarify, and add responses. It is recommended that future research with young children incorporate such conversation, stimulated by suitable varied prompts, where possible.

The use of multiple procedures over a period of time facilitated the collection of *slices of data* (LeCompte & Goetz, 1982) and allowed the compilation of growing portrayals of beliefs or what might be called personal paintings (Labinowicz, 1985). By having multiple tasks related to the same research question, data were gathered by more than one means and were credible. Such an approach provided greater potential also for differences between individual children's beliefs to become apparent than might be the case from the use of a small number of procedures. Earlier discussion regarding the possibility that children see maths as encompassing more concepts than number, but with a number focus, indicates that possible subtleties of meaning in children's beliefs may not emerge from one-off or brief statements of belief but require further investigation as occurred in the present study.

The combination of investigation of beliefs through multiple procedures, including those where beliefs were explored within contexts or situations, and the ongoing discussion within interviews demonstrated that the children's beliefs were not one-dimensional but complex. Insights were not based on single, isolated utterances; the use of multiple procedures over a period of time allowed the researcher to make inferences with confidence.

In summary, the results from the present study suggest that insights into children's beliefs ideally should be gained from more than one task, with the range of tasks administered over a period of time, and where possible with verbal communication so that initial responses can be explored further. Such an approach enables children to provide insights that require less inference by the researcher or teacher, and allow children to reveal complex linkages, subtlety, breadth and depth within beliefs. The responses to any one procedure should be interpreted as giving insights into what children choose to represent on that day. Use of multiple procedures over a period of time appears to give a more balanced representation of a child's beliefs.

Class tasks

It is important also to evaluate the two class-administered tasks, not because they contributed a great amount of data to the study, but because they can easily be used by class teachers in future.

The eight research participants (and their classmates) were able to respond to the class tasks administered by the researcher. As the children in the present study were of the Grade 3 level, it is anticipated that children of Grade 3 and upwards would encounter few difficulties.

A second perspective is the quality of the data received. Compared to the data from the interviews (see Tables 4 to 11 for summaries), data from the whole class tasks (see Table 12) were generally limited in breadth and depth although offering some insights. On the surface the data appear simplistic and, for most children, do not represent the complexity of beliefs revealed within the interviews. It is acknowledged that such complexity may be difficult for children to portray within pencil and paper tasks and therefore if only one or two such tasks are used this is a potential limitation. Some children also may not feel inclined to provide detail when asked to respond in writing as this may be less desirable and/or more demanding than speaking in an interview situation.

A further perspective is that class tasks may have some value in prompting children to give what they consider to be the key ideas or those most clearly representing the concept at hand and thus provide some insights. However, this can also be seen as a limitation as insights are not necessarily gained into the potential complexity or subtlety of a child's beliefs. This appears the case for some children in the present research. For example, although Cara revealed in her interviews that she believed measurement, estimation, and number to be important elements of maths, she wrote only of number in the class administered alien task. She may have thought this was the most salient or most useful aspect of maths of which to inform the alien. A limitation of the pencil and paper task is that no follow up conversation occurred and therefore more indepth insights were not available. Cara's intentions and further beliefs could not be known from such a limited response to the pencil and paper alien task. This suggests that for those children for whom a teacher is particularly interested in the meaning of a response, a follow-up conversation may be worthwhile to discuss the response and what was not, but might have been included. Some other tasks from the present study could be used within child/teacher interactions.

Even for Emily, who wrote a lot more than Cara in response to the class-administered alien task, the complexity of beliefs that was apparent from the interview data was not communicated, for example, in relation to beliefs about measurement. Gina also gave a more limited view of her concept of maths in the written alien task, for example, not including any reference to space or measurement. In contrast, it seems that Harry's written response to the alien task does summarise much of his beliefs about the nature of maths. However, Harry's pencil and paper PPELEM response focuses only on one of the important helping factors identified from his interview responses. To conclude, it appears that the class administered written tasks do facilitate the gaining of some insights into children's beliefs but that with one-off usage do not necessarily communicate the depth, breadth, complexity and subtlety of those beliefs. It is recommended that the alien and PPELEM pencil and paper tasks be used by classroom teachers but with recognition of the potential limitations, and thus combined with the use of other procedures to gain insights into children's beliefs, possibly administered over a period of time. The administration of pencil and paper tasks might be followed by whole class discussion also. Follow-up discussions with children of particular interest are recommended.

Limitations of the class-administered tasks are recognised. However, it may be possible to overcome these limitations, as discussed above and as further research could judge. The possibility of the use of such procedures by classroom teachers is not dismissed; the tasks have potential to provide teachers with avenues to gain insights into the beliefs of the children in their classes and thus to come to know and cater better for those children. Easily administered single response tasks, such as Task 1.1 (Word association quiz) and Task 1.2 (Password), or easily administered multiple response tasks such as Task 1.3.1 (Word wheel – maths) also have potential for use with class groups. However, as in the interview situation, it may be best that they are followed by discussion. Future research could explore the value of such discussion with a class group for stimulating reflection and conversation by children.

The discussion of the interview tasks and class-administered tasks developed for and used in the present research suggests that the tasks did facilitate students to actively and constructively reflect upon and express beliefs about the nature of mathematics, learning, and helping factors for learning mathematics. Strengths and limitations of the tasks have been acknowledged and recommendations made for possible further research and use of tasks in the class situation.

Following specific reflection of research findings of children's beliefs and the value of the procedures deployed in the study, a more overall view of the study is now taken, with consideration of achievements and recommendations.

Achievement of the research purposes

As stated in Chapter 1, the main purposes of the research were to

- explore whether young children's beliefs could be articulated and portrayed;
- develop procedures for use in the present study, and for potential use in classrooms and other research studies, for gaining insights into young learners' beliefs about learning, mathematics, and helping factors for mathematics learning;
- explore and discuss beliefs as portrayed by eight children of eight to nine years of age;
- gain insights into possible complexities and subtleties of young learners' beliefs;
- reflect upon the insights and their significance for the mathematics classroom; and
- reflect upon the value of the procedures developed for the study.

Each of these objectives was achieved. Exploration was a key feature of the present study, so the choice of semi-structured interviews and thematic analysis of data was

appropriate. The study produced portrayals of beliefs that are honest and, as demonstrated by the inclusion of interview excerpts, faithful to the children's communications. The research demonstrated that children of eight to nine years of age are able to articulate beliefs when prompted. The findings from the study also lead to possible considerations for teachers in the mathematics classroom as articulated in this chapter.

As well as the portrayal of beliefs, a key element of the study was the development or identification of procedures suitable for use with young children. The research procedures generally were successful when used in the interview situation. To develop recommendations for optimal use of the research interview procedures in whole class situations, further research is required regarding adaptation of procedures and the identification of appropriate analytic techniques. Such research would lead to more specific recommendations for teachers wishing to investigate the beliefs of pupils within their classes, and to reflect upon and respond in an informed manner to the insights gained.

The use of the thirty interview tasks and two whole class tasks developed or identified for the present study, and the implementation of criss-cross analytic techniques led to detailed and rich portrayals of young children's beliefs. Insights were gained into the complexity, subtlety, depth, breadth, and the idiosyncratic nature of beliefs held by the eight research participants. The insights gained add to those from previous research, as demonstrated in the discussion within the present chapter, and lead to possible further research. They have the potential also for informing teachers of the possible nature of beliefs held by young children.

Conclusion

To conclude this report, the reader is taken back to a range of assumptions that underpinned the study as presented in Chapter 1. Those assumptions are reflected upon in turn in light of the research results, followed by a statement of the key messages from the present research.

• Beliefs are constructed by individuals and therefore vary within a class or grade. As discussed in Chapter 1, the research focused on the meaning or sense-making of individual children, and was generally compatible with the social constructivist view that mathematical knowledge is individually constructed and socially influenced. The results of the study suggest that beliefs as well as understandings are individually constructed, and differ from child to child.

Although only a small number of children were interviewed, the research confirmed that beliefs are individual and suggested that beliefs vary within a class. The depth of the investigation and detail of the write-up for Cara and Emily revealed idiosyncratic and at times complex and subtle beliefs. For all the other children's beliefs, whether reported in an appendix or in a summary table, the observation and conclusion of idiosyncrasy can be made. Subtlety and complexity also often were apparent. The nature of beliefs as individual constructions is conveyed strongly by the present research.

- Individuals' beliefs about the nature of mathematics and learning interact with their beliefs about factors that help them in their learning of mathematics.
 - Responses to the research procedures suggest interaction of beliefs, as was hypothesised in Chapter 1 (see Figure 3). Each interview procedure was developed to gain insights into beliefs about one of three main domains, that is, the nature of mathematics, the nature of learning, or helping factors for learning mathematics. In addition, a small amount of data was collected for the domain of self, giving background information. However, within the presentation and discussion of data and the discussion of the research procedures, there were many instances of data for one domain being provided by responses to tasks developed specifically to explore beliefs about another domain. The deployment of a criss-cross analytic approach, as described in Chapter 3, allowed beliefs about any of the three key domains to emerge potentially from any of the research procedures. However, there was no guarantee that this would occur; it was dependant upon an interaction of beliefs within a child's perspectives. This interaction is demonstrated for the reader in a clear and succinct manner in the summary tables of beliefs (Tables 4 to 12 and in Appendix I). For many of the children there is evidence of an interrelationship of ideas between the main research elements. For some, such as David, the interrelationship is obvious immediately as it is evidenced by a repetition of words and phrases.
- Children may benefit from attempting to articulate beliefs about mathematics and learning through reflecting upon and possibly questioning their ideas thus building awareness of the self and potentially increasing control over their learning, and enhancing skills in critical thinking.

At times the children's responses suggested that they were reflecting upon ideas and questions that they had not consciously thought about previously. Although changes may have occurred in awareness of self, control over learning, and critical thinking skills, these were not evaluated systematically as part of the research. This is one possible area of further research extending from the present study.

• With appropriate procedures it is possible for teachers to gain insights into children's beliefs.

Although the present study did not include the use of the data collection procedures by individual teachers, the effectiveness of their use in the interview and whole class situations suggests that the procedures are potentially useful for classroom teachers. As the use of one procedure may give limited and possibly unrepresentative insights, the use of a greater number of procedures is proposed. Such procedures would best be used over a period of time so as to build up a slowly emerging picture of children's perceptions, or with a small number of children for whom the gaining of insights by the classroom teacher would be most valuable. Some procedures such as the Task 9.2 drawings and the Task 6.1 and Task 6.2 PPELEM procedures lend themselves to adaptation for use with large

groups. Further research is desirable to learn how such procedures could best be used and responses analysed by class teachers.

• Teacher knowledge and appreciation of children's beliefs can inform decision-making for mathematics teaching.

This research has the potential to add to the insights gained through day to day contact with learners of mathematics, and therefore to assist teachers in better knowing about and catering for the mathematics learning perspectives of individuals. It is appropriate for findings from this study to be shared with classroom teachers, for them then to determine relevance and application.

It is of value for teachers, parents and researchers involved in the education process to
have insights into the nature of beliefs constructed by individual children.
It was posited that the research would give insights that would inform teachers and parents
about the depth and complexity of the perspectives of their children, and therefore assist
them in coming to know and understand those children and assist in making decisions for
teaching.

In general terms, the research informs the educational community of the idiosyncratic, complex and sometimes subtle nature of young children's beliefs and suggests that other children, including those of eight to nine years of age, potentially have idiosyncratic and complex beliefs. The research results assist researchers, educators and parents, in a general sense, to further their appreciation of children as individuals. In a specific sense, it has the potential to assist parents to interact with and support their children in a more informed and focused way, and assists teachers by indicating the value of building up processes for coming to know further the individuals in their classes. It is acknowledged that teachers do not have the time available to interview children to the extent deployed in this study. However, analysis above suggests those interview procedures that were most informative and could be recommended to teachers. As stated above, the research suggests also that a single procedure may give limited insights but through use of a selection of potentially informative procedures, with the whole class or a selection of pupils, useful insights may be gained. As a result of the present research, teachers and parents also may be more alert within day to day interactions to explore individual children's beliefs in an ongoing or regular manner, through listening, watching and collecting products, and therefore coming to know and, in turn, to support learners better. As discussed in Chapter 2, and illustrated in the discussion of results, beliefs may relate to affective and cognitive elements of the learning of mathematics thus insights can be gained into both.

To conclude this discussion three quotations from Corbitt (1984) are drawn upon for application to the present study. Following from her investigation focused on eighth grade

students' liking for mathematics and how important they perceived the subject to be, Corbitt (1984) states the following:

You might be amazed to discover how much a discussion about students' feelings about mathematics can do to increase communication about the subject on other levels. (p. 16)

Although benefits are derived from talking to students about their feelings about mathematics, teachers seldom do so. (p. 19)

Talking with students about mathematics should be an enlightening experience for any mathematics teacher. (p. 20)

Considering Corbitt's statements in relation to the present study the following can be said:

- By making an effort to gain insights into individual students' perceptions, such as through the use of procedures developed in this study, it is possible to expand or open up communication channels between students and teachers.
- As teachers seldom talk to students about their feelings (and possibly their beliefs), teachers may be assisted by the availability of procedures such as those developed in the present research. The research adds to a teacher's repertoire of strategies for communication with students, particularly with children in the primary school.
- The present research is potentially enlightening (Corbitt, 1984) or illuminating (Kemmis, 1982; Merriam, 1988) for the reader as the findings indicate the possible complexity and subtlety of young children's beliefs. The availability of the procedures for use by classroom teachers and their students provides action possibilities (Kemmis, 1982), that is, they foster the possibility of further enlightenment or illumination and, in turn, informed response in the classroom.

Looking Forward: Future research

Within the discussion of research procedures, above, some comments were made of possible directions for future research. It is suggested the following are issues that could form the basis of future research:

- the ways in which children's beliefs affect their learning of mathematics;
- the adaptation of the procedures from this study for use with class groups;
- the use of such procedures by class teachers and insights that might be gained;
- the pedagogical implications of the knowledge resulting from use of such procedures.

The research has demonstrated that it is possible to work with children of eight to nine years of age in an ongoing and indepth manner. For others undertaking such research, the following are recommended:

- interview children over a period of time and on a number of occasions;
- use more than one task to gain insights into each aspect of interest;
- include the use of tasks with multiple elements;
- build up rapport with the children prior to beginning each interview;
- to facilitate reflection and expression, use a range of different tasks of interest to young children that incorporate a variety of media within the discussion prompts;

- provide opportunities for children to give their interpretation of a prompt such as a photograph, drawing or video clip before discussing further, that is, do not assume children see the same meaning in interview prompts as does the interviewer;
- use a semi-structured questioning style and deploy some flexibility when questioning;
- provide opportunities for children to explain and clarify beliefs in a variety of ways and in response to a variety of on-going prompts and questions;
- be patient when working with young children, for example, provide wait time after presenting tasks and after children's responses;
- listen carefully to allow the children's perspectives to be heard and followed.
- have the children record beliefs through a variety of media (e.g., drawings);
- collect artefacts and discuss with the children what they represent or say;
- make a record of what is said (e.g., audio recording followed by transcription).

The research suggests the following also:

- the opportunity for children to express depth and breadth of beliefs is important;
- it is valuable to use a variety of trialed, appropriate procedures to gather different insights on each research question;
- creative interviewing procedures offer the opportunity for working with young learners.
- it is valuable to use criss-cross analysis of interviews as this allows multiple perspectives to emerge;
- representation of complexity is important.

Key messages from the research

Finally, the research suggests that teachers and others involved in the education of young learners of mathematics should know the following:

- it is possible to gain insights into children's beliefs about maths, learning and helping factors for learning maths;
- to gain insights into young children's beliefs, it is important to have dialogue with the children to avoid making assumptions about their interpretations or meanings;
- the *creative interviewing* procedures developed for the present research are helpful as they can stimulate reflection and prompt conversation;
- young children's beliefs can be complex, subtle, broad and deep;
- young children's beliefs are individually constructed and differ from child to child;
- children may not see mathematics concepts in commonly accepted ways, for example, measurement is not always seen as a part of maths, and the terms *maths* and *mathematics* may be considered to hold different meanings;
- beliefs are sufficiently diverse and significant to affect the way children see the mathematics learning situation, for example, regarding the purpose of learning maths or the value of working with other children in maths classes;
- although the beliefs of children of eight to nine years of age may, on the surface, appear simplistic and naïve, they are not necessarily so. Young learners are able to reflect on their

own and others' experiences and often construct complex beliefs. There is a lot happening in the minds of these children.

The research suggests also that it is important that educators do not to make assumptions about

- what children see as maths (or mathematics);
- what children see as learning; and
- what children see as helping factors for learning maths.

APPENDIX A

Reference sheets for semi-structured interviews

Each set of procedures is designed as material for an interview of approximately 30 minutes duration. The tasks begin usually in a semi-structured way but progress to more conversational interaction that builds on each child's individual responses while keeping in mind the purpose of that interview as detailed within Chapter 3. Some follow-up questions were prepared and used when appropriate.

Set 1 Word association quiz, Password, Word wheels, Subject most/least like maths

1.1 Word association quiz

We are going to do an activity, sort of like a quiz. I will say some words. For each I want you to say the first word that comes into your head, the first word you think of. For example, if I said bed, you might think sleep. Or if I said cat, you might think dog.

Words posed: Chair; Games; Maths; Tree; Science; Music; Problem; Bicycle; Learning; Reading; Builder; Measuring jug; Language; Farmer; School; Heavy; Shapes; Hard work; Artist; Thinking.

1.2 Password

There is a game I have been told about called *Password*. Do you know it? I'll describe it for you:

You play with a friend. You have a word that you want your friend to guess. You can give a one word clue, which must not contain any part of the word being guessed. For example, if I was playing and I wanted to pass the word 'cat' to you I couldn't say it but I could say something else that means the same. I might say 'pussy' as a clue. If you wanted a friend to guess 'film' what clue might you give? What clue might you give for your friend to guess:

Words posed: Mother; Father; Language; Learning; Maths.

1.3 Word wheels

1.3.1 Maths word wheel: I have a piece of paper for you to write about what you think maths is. You can write words or sentences anywhere on the paper.

Read to me what you have written. Tell me about what you have written.

1.3.2 "Other" word wheel: What is another thing that you do at school, another subject? Which would you like to write about?

Read to me what you have written. Tell me about what you have written.

In what ways are maths and (the other subject) alike?

In what ways are they different?

1.4 Subject most like maths / least like maths? Why?

What else do you do at school that is most like maths? In what ways are they alike? What else do you do at school that is least like maths? In what ways are they different?

Set 2 Personal dictionary, Learning situations, Maths situations

2.1 Personal dictionary: Learn

Do you know what a dictionary is? [Do you have a dictionary?]

What does it have in it?

Now, I want to you pretend that you are writing your own personal dictionary with your own meanings of words in it.

What would you write, what meaning would you write for the word?

Words posed: House; Pet; Eat; Jump; Learn.

2.2 Learning situations

Tell me something that you have learned recently, say in the last few weeks.

How did you know you'd learned something?

(If appropriate ask:)You used the word when talking about learning. Would you put that word in your dictionary for your meaning of learning?

2.3 Personal dictionary: Maths

What would you write, what meaning would you write, for the word maths?

Tell me about one time when you were doing or using maths in the last week or the last few days.

(If appropriate ask:)You used the word when talking about maths. Would you put that word in your dictionary for your meaning of maths?

Set 3 Mathematical activity – Draw; Show; Questionnaires

3.1 Draw a mathematical activity

I would like you to draw a person doing some sort of maths activity. Tell me about what the person is doing (the maths)

Possible question starters: Who; What; When; Where.

3.2 Show another mathematical activity

Can you please show me one maths activity now? (For example, act out, write, draw). Tell me about the maths.

3.3 Questionnaires

3.3.1 On this paper I have written some things people are doing. I want you to tell me whether you think the person is doing or using maths (yes or no).

Discuss in what way it is or is not maths.

Questionnaire items included:

- Melanie had to tell her teacher which number was bigger, 50 or 30. Did Melanie use or do maths?

- Terry went to McDonalds. She paid the salesperson \$3.20 for a hamburger and Coke. Did Terry use or do maths?

- Mark visited the museum to see the dinosaurs. Did Mark use or do maths?

3.3.2 Now for the next questions, please tell me whether you believe the activity has lots of maths, some maths, or no maths in it.

Go through each of these and ask for reasons, for example: Do you play sport? Do you use maths then? What maths do you do or use? Any other?

Who is someone else who lives in your house? Would also say this has maths in it?

Questionnaire items included: Playing a sport; Planning a two week holiday for a family. Painting the house.

Set 4 Maths is like; Drawings; Planning for an integrated unit of work

4.1 Maths is like.....

I would like you to add to what I have written on this page. I would like you to write as much as you can.

Sentence starter written on sheet: Maths is like

Read to me what you have written. *Discuss, for example:* Tell me about it. What were you thinking of when you wrote this? Tell me an example of this.

4.2 Drawings - people doing maths

4.2.1 I would like you to draw me a picture of someone using or doing maths. Describe the picture to me. What is the person doing? What is the person thinking? *Possible question starters:* Who, what, when, where.

4.2.2 I would like you to draw me a picture of someone using or doing maths at school/not at school (whichever was not portrayed in the previous picture).

Describe the picture to me. What is the person doing? What is the person thinking?

Possible question starters: Who; What; When; Where.

Is the person (in each picture) learning maths or doing maths? Is learning maths the same as or different from doing maths?

4.3 Planning for an integrated unit of work

When you've been at school this year or another year your teacher might have taught maths for a while where you were learning maths but also learning about a topic such as The Olympics or Houses or something like that. Has this happened to you? Please you tell me about it.

I want you to pretend you are helping to plan the maths work for a grade that learns maths like this. I am going to tell you three topics. You can choose one of these or one of your own. I want you to think of all the maths things a class could do, or could learn about, for that topic. The topic might be Holidays, Birthday Parties, The Environment, or one of your own. Which one do you think you would like to learn about and learn maths at the same time?

I will write the topic on this paper. I want you to tell me some maths things the children could do or learn. You can write them or I will. What would you prefer? Tell me about the activities you have written. *Discuss*. Any more?

Set 5 Alien; Photographs

5.1 Describing maths to an alien (Stodolsky, Salk, & Glaessner, 1991).

Do you know what an alien is? Suppose someone said to you that an alien had come to [your suburb]! What do you think that alien might look like?

I would like you to pretend that the alien has arrived and that he or she doesn't know what is going on. They don't know about things on earth. Your job is to tell them what maths is. How and what would you tell or show the alien about what maths is? Tell me everything you would do or say.

5.2 Photographs (Zevengergen & Crowe, 1992)

I have some photographs here that I would like you to look at.

For each photograph, in turn: I would like you to tell me what the person or people are doing. I want you to decide (and tell me) whether there is any maths in what the person is doing.

Now that you have looked at the photos is there anything else that you can think of that you think you should tell the alien so that you have said what maths is?

Photographs included: A woman weighing fruit at a supermarket; Two children playing chess; A man laying paving stones.

Set 6 Learning maths well, Hindered from learning maths well

6.1 PPELEM: Situation in which learning maths well

Close your eyes. I would like you to think about different times when you have been learning maths, maybe this year, maybe last year or even earlier, maybe at school, maybe at home, or somewhere else.

I want you to take your time and choose one time when you felt you were learning maths well. Make a picture in your mind of that time. When you're got your picture open your eyes and draw on this piece of paper.

Discuss the situation portrayed by the child. As child describes the situation, write key words or phrases on strips of card.

Tell me about that time when you were learning maths well. (Describe the picture to me). Tell me about what was helping you to learn maths well. In what way was helping you? Was there anything important in that situation that was helping you that you couldn't draw? What maths were you doing? What maths were you learning?

Possible question starters: Who; When; Where; In what way.

Rank ordering and discussion *(Using key words/phrases recorded by interviewer):* I have written some words because I want to find out what was helping you to learn maths. I will read them to you.

Are there any other things I should have written that were helping you?

Are there any things that it wasn't possible to draw?

Which was most helping you to learn maths well?

Which was next most helping you to learn maths well?

Which was the least helpful? etc.

6.2 PPELEM: Situation in which hindered from learning maths well (something stopping you or making it hard for you to learn maths well)

Close your eyes. I would like you again to think about some different times when you have been learning maths, maybe this year, maybe last year or even earlier, maybe at school, maybe at home, or somewhere else.

I want you to take your time and choose one time when you were trying to learn maths but feel that something was stopping you or making it hard for you. Make a picture in your mind of that time. When you have your picture, open your eyes and draw that time.

Discuss the situation portrayed by the child. As child describes the situation, write key words or phrases on strips of card.

Tell me about that time when something was stopping you or making it hard for you to learn maths. (Describe the picture to me).

Tell me about what was stopping you or making it hard for you to learn maths. In what way was stopping you or making it hard for you to learn maths?

Was there anything in that situation that was stopping you or making it hard for you to learn maths well that you couldn't draw?

What maths were you doing? What maths were you trying to learn (or supposed to learn)? *Possible question starters*: Who; When; Where; In what way.

Rank ordering and discussion (Using key words/phrases recorded by interviewer):

I have written some words because I want to find out what was stopping you or making it hard for you to learn maths. I will read them to you.

Are there any other things I should have written that were stopping you or making it hard for you to learn maths?

Are there any things that it wasn't possible to draw?

Which was most stopping you or making it hard for you to learn maths?

Which was next most stopping you or making it hard for you to learn maths? etc.

Set 7 Scenario; Video clips

7.1 Scenario

Pretend that there is someone you know who is finding maths hard to learn. What would you suggest to help that child?

For each factor suggested by the child, ask: In what way would help them? What maths were you thinking of that they might be finding hard to learn?

7.2 Video clips

I am going to show you videos of some children doing some maths at school. *For each piece of video ask:*

- i. Tell me what is happening.
- ii. Tell me what maths the children are doing.
- iii. If you were one of the children in that situation do you think you would be learning maths well or would you be having some difficulties? (Would something be making it hard for you or stopping you from learning maths well?).

Video clips included: Making shapes on geoboards; Predicting and listing colour of tiles from a bag; Mentally making number sentences for 20; Constructing a graph.

Set 8 Easier and harder experiences; Photographs of home and school; Discussion of feelings

8.1 Easier and harder experiences

8.1.1 Tell me about a time when maths was easy for you to learn.

What made it easy for you to learn maths? In what way did this (these) help you to learn maths?

Possible question starters: Who; When; Where.

8.1.2 Tell me about a time when you had some difficulty learning maths.

What made it difficult for you to learn maths? In what way did this (these) stop you from learning maths? What would you have liked changed?

Possible question starters: Who; When; Where.

8.2 Photographs of home and school

I have some photographs. I would like you to tell me what is happening in each photograph and then decide whether you think you would learn maths well in that situation. Put each photo in one pile: *Yes* (I would learn maths well); *No*; or *Not sure*.

Yes: In what way does this help you? What is it in that situation that would most help you to learn maths well?

No: In what way does this stop you or make it hard for you to learn maths?

Photographs of school situations included: A child measuring a bench in the playground with a tape measure; Three children using a calculator; A teacher explaining maths to the whole class.

Photographs of home situations included: A child making jelly in the kitchen; A child doing homework assisted by her mother; One child measuring another's height with a piece of string.

8.3 Discussion - feelings

Tell me how you feel about learning maths.

How did you feel today/yesterday in your maths lesson?

What makes you feel good about learning maths?

What makes you feel not good about learning maths?

What about when you learn maths but not at school? (Where? What makes you feel good/not good?)

Set 9 I could do better in maths if; Children's drawings

9.1 I could do better in maths if

I would like you to add to what I have written on this page. I would like you to write as much as you can.

Read to me what you have written.

Tell me about it:

- i. What does "do better in maths" mean to you? In what way would you do better?
- ii. How do you know when you are doing better? Tell me a little more about this.
- iii. What maths were you thinking of when you wrote this?

9.2 Visual vignettes - Children's drawings: Would this help you do better in maths?

I have some drawings from other children that I would like you to look at. Each shows what that child thought helped them to do learn maths well (or to learn maths better). For each drawing please tell me:

- i. What you think the child has shown that helps him or her to learn maths well (or to do better in maths)?
- ii. Would that help you to learn maths well (or to do better in maths)? In what way do you think it would help you?

A selection of twenty drawings collected from primary school children within a previous project (McDonough & Wallbridge, 1994), portrayed situations including: Playing a maths game; Thinking; Using a calculator; Shopping.

9.1 (Revisited)

So now you have looked at the drawings, is there anything you could add to what you have written for "I could do better in maths if"?

Set 10 Written descriptors; Duplo; What is learning?

10.1 Written descriptors

Think about different times when you are learning maths - at home, at school or other places. You could be learning any maths.

I am going to show you some words or phrases that I want you to sort into piles. The piles are:

Yes: I use this when learning maths

No: I don't use this when learning maths

Not sure: I'm not sure if I use this when learning maths.

Can you add anything to the Yes pile that I have missed?

Go to the Yes pile. Take out the one that you use the most when learning maths, then the one you use the second most and the third most.

Discuss with the child the most used etc., in turn. Sample questions:

Tell me about that one that you said you use the most when learning maths.

Can you give me an example of when you use it? In what way do you use it?

Does it help you to learn maths? In what way?

Are there some here that you don't use when learning maths, but think you would like to use? How would these help you to learn maths?

Would you like to change anything about the maths you do at school?

Would you like to change anything about the way you learn maths?

(Further questioning could consider instruction as well as content).

Words and phrases included: Practise it; Use a calculator; Sit in a quiet place; Ask someone. (Stodolsky, Salk, & Glaessner, 1991)
10.2 Duplo

10.2.1 I want you once again to think about learning maths. I want you to think about some different times when you have been learning maths, maybe at home, at school or somewhere else. You might have been learning any maths.

I would like you to think of a time when you felt good about your maths learning. Please build with the Duplo to show me that situation.

Tell me about that time when you felt good about your maths learning.

What were you learning?

Was something helping you to learn the maths?

Possible question starters: Who; What; When; Where.

What was happening that you couldn't show with the Duplo? (e.g., What was happening in your mind? What were you trying to do in your mind?)

10.2.2 I want you once again to think about learning maths. I want you to think about some different times when you have been learning maths, maybe at home, at school or somewhere else. You might have been learning any maths.

I would like you to think of a time when you did not feel good about your maths learning. Please build with the Duplo to show me that situation.

Tell me about that time when you did not feel good about your maths learning.

What were you trying to learn?

Was something stopping you or making it hard for you to learn maths?

Possible question starters: Who; What; When; Where.

What was happening that you couldn't show with the Duplo? (e.g., What was happening in your mind? What were you trying to do in your mind?)

What would you have liked changed so that you could have learned maths better? What maths was it that you were trying to learn?

10.3 Learning

I would like to get some better idea of what you mean by learn.

I have written some words on cards. How do you know when you have learned something? Is it because you now: know, remember, do, understand, or is it something else? Tell me.

Words on cards: Know, remember, do, understand.

Set 11 How good at maths; I think mathematics is, A good maths teacher

11.1 How good at maths 1 - 10

I would like you to think about how good you believe you are at maths. I want you to give yourself a number from 1 to 10. Ten is for someone who is really, really good at maths; one is for someone who is not very good at all at maths. What number would you give yourself?

11.2 I think mathematics is

I would like you to add to what I have written on this page. I would like you to write as much as you can. Read to me what you have written. Tell me about it.

11.3 A good maths teacher

I would like you to add to what I have written on this page. I would like you to write as much as you can. Read to me what you have written. Tell me about it.

APPENDIX B

Interview Record Sheet

For each interview conducted, this record sheet was completed.

| Child's Name: | | Age: | Date: |
|-----------------------------------|-----------|------|----------------|
| School: | Birthday: | | Interview No.: |
| Location of interview: | | | |
| Time of interview: | | | |
| Interview procedure/s intended: | | | |
| Materials required: | | | |
| | | | |
| Interview procedure/s eventuated: | | | |
| Opening remarks/feelings etc.: | | | |
| | | | |
| Child's disposition: | | | |

Comments/Field Notes:

APPENDIX C

| CI II | e | • • | • |
|--------------|-----|-------|-------|
| Schedule | OT. | inter | views |
| Schedule | ••• | | |

| | Anna | Ben | Cara | David | | Emily | Filip | Gina | Harry |
|----------|--------|---------|--|----------|-----------|------------|-------|------|----------|
| | HA | HA | LA | LA | | HA | ΗÂ | LA | LA |
| 18/7 | 5 | 2 | 6 | 3 | 26/7 | 3 (not | 5 | 8 | 6 |
| | | | | | | completed) | | | |
| 1/8 | 6 | 5 | 3 | 8 | 9/8 | 2 | 1 | 3 | 5 |
| | | | (not | | | (Set 3 | | | |
| | | | completed) | | | completed) | | | |
| 15/8 | 2 | 8 | 1 | 6 | 23/8 | 8 | 10 | 6 | 1 |
| | | | | | 6.12 | | | | |
| 29/8 | 4 | 6 | 8 | 1(a) | 6/9 | 5 | 3 | 4 | 8 |
| | | | | (Set 2 | | | | | |
| | | | | planned) | /- | - | | _ | |
| 19/9 | 10 | 1 | absent | 4 | 20/9 | 6 | 8 | 5 | 10 |
| 10/10 | 0 | 10 | | ~ | 10/10 | 0 | | 10 | 1 . |
| 10/10 | 8 | 10 | (19/10) | 5 | 18/10 | 9 | 4 | 10 | absent |
| | | | 5 | | | | | | (Set 3 |
| | | | | | | | | | planned) |
| 24/10 | 7 | 7 | 7 | 7 | 1/11 | 4 | 7 | 1 | 9 |
| 0/11 | 2 | 4 | 0 | 1 / | 1 7 / 1 1 | | | | 4 |
| 2/11 | 3 | 4 | 9 | absent | 15/11 | 1 | 2 | 1 | 4 |
| 1 (/1 1 | 1 4 | 0 | 4 | 1(1) | 20/11 | 1 | (| | 7 |
| 10/11 | absent | 9 | 4 | 1(0) | 29/11 | 1 | 6 | 2 | / |
| 22/11 | 0 | abcont | 10 | 0 | 12/12 | 10 | 0 | 0 | 2 |
| 23/11 | 7 | auseilt | 10 | フ | 13/12 | 10 | 7 | 7 | 2 |
| 5/12 | 1 | 3 | 2 | 10 | | | | | |
| | - | - | (Set 3 | - • | | | | | |
| | | | completed) | | | | | | |
| | | | ····r································· | | | | | | |

Tasks 11.1, 11.2, and 11.3 were scattered throughout the interviews, as indicated below:

| | 11.1 | 11.2 | 11.3 |
|-------|-------------|------------------------------|------------------------------|
| Anna | 10/10 | 23/11 (Planned but not done) | 23/11 (Planned but not done) |
| Ben | 18/7 | 16/11 | 16/11 |
| Cara | 24/10, 5/12 | 16/11 | 16/11 |
| David | 23/11, 5/12 | 23/11 | 23/11 |
| Emily | 13/12 | 29/11 | 29/11 |
| Filip | 13/12 | 29/11 | 13/12 |
| Gina | 29/11 | 29/11 (Planned but not done) | 29/11 (Planned but not done) |
| Harry | 13/12 | 13/12 | 13/12 |

APPENDIX D

Interview Checklist

| Date | School | Day |
|--------------------------|--------|------------------------|
| Tape recorder/Dictaphone | | Pens |
| Microphone | | Pencils |
| Spare batteries | | Lined paper |
| Tapes (2) | | Highlight pen |
| | | Rubber bands |
| | | |
| | | |
| Set 1 | | Set 7 |
| Procedure/Questions | | Procedure/Questions |
| Word wheel sheets (2) | | Video tape/s |
| | | TV/Video recorder |
| Set 2 | | |
| Procedure/Questions | | Set 8 |
| Nil | | Procedure/Questions |
| | | Home/school photos |
| Set 3 | | Yes/No/Not sure cards |
| Procedure/Questions | | |
| Paper | | Set 9 |
| Textas | | Procedure/Questions |
| 3.3 Questionnaires | | I could do better |
| | | Children's drawings |
| Set 4 | | |
| Procedure/Questions | | Set 10 |
| Maths is like | | Procedure/Questions |
| Paper (3) | | Written descriptors |
| Textas | | Duplo |
| | | Learn cards |
| Set 5 | | Yes/No/Not sure |
| Procedure/Questions | | Blank cards |
| 5.1 Photographs | | |
| | | Set 11 |
| Set 6 | | Procedure/Questions |
| Procedure/Questions | | I think mathematics is |
| Paper | | A good maths teacher |
| Textas | | |
| Blank cards | | |

APPENDIX E

Brief portrayals of three children's beliefs

Detailed portrayals of the beliefs of Cara and Emily are provided in Chapters 5 and 6. More condensed portrayals of Gina, Ben and David's beliefs are provided below, and as an overall account of the research findings, summary data for all eight children's beliefs are provided in Chapter 7. As stated in Chapter 3, data from each of the eight children were sorted and categorised. Themes identified for Gina, Ben and David are discussed below.

Following the procedure explained in Chapter 4, the discussion of children's beliefs uses the more commonly used term *maths*, as deployed in the interviews with the children, in preference to the term *mathematics*, except when a child's meaning for the latter is discussed specifically.

Gina

Summary

Gina was chosen as a low achieving, Grade 3, female in maths, from School I. Gina's data indicate that she held beliefs related to the nature of maths, the nature of learning, and helping factors for learning maths. She referred to number, measurement and space concepts as mathematical, and saw maths as both a school and non-school activity. Gina liked to work alone but saw some merits in working in a group. *Teachers* were considered helpful for her learning of maths. Gina liked to be told "what to do" or "how to do", or to be given clues to help her to learn maths but considered that being told answers would not help her to learn. Gina believed she could help herself in a number of ways and some tools were considered helpful for learning maths. Gina's beliefs are discussed more fully below.

Beliefs about the nature of maths

Gina's responses about the nature of maths suggested three content-based themes: number, measurement and space. The discussion begins by considering Gina's references to number concepts.

When asked to define maths or describe mathematical situations Gina referred mostly to *sums* in reference to which she frequently included "plus, take away, times tables" (e.g., Tasks 1.3.1, 2.2, 3.1, 4.1, 5.1). On some occasions division was mentioned also, such as in relation to adult mathematical activity, but was not always readily associated with the concept of maths. Along with sums, counting was proffered frequently, usually in relation to the use of fingers or the use of blocks, mostly for the school-related purpose of finding an answer, although Gina did speak in Task 1.3.1 of her mother counting using her fingers and her father counting using his mind as mathematical activity. Counting was seen also as an element of addition. For example, Gina suggested that to work out five plus ten "you just count by fives" (Task 1.3.1).

Many non-school situations were presented to Gina through the use of photographs and verbal questionnaires. From responses to such procedures there emerged once again a significance for Gina of counting and number operations as mathematical activity. Gina identified three Task 5.2 photographs as portraying mathematical activity that she described as counting. Gina said the people in the photographs were counting candles on a birthday cake, counting bricks to lay a garden path, and counting money to buy something or to check the right money. In Task 3.3.1, mathematical activity was identified in seeing 100 cents as the same value as one dollar, transactions, and counting of money. A further reference to counting arose in Task 3.3.2 where Gina identified playing a musical instrument as containing some mathematical activity because "you have to put the people in order like tambourines, a group of tambourines over there and a group of xylophones over there . . . and you have to count". Deciding which number was bigger (Task 3.3.1) and "the teacher putting children in order like 1, 2, 1, 2 like that" for sport (Task 3.3.2) also were seen as mathematical activity.

To summarise, the most pertinent elements of number as mathematical activity for Gina appeared to be counting and the operations of addition, multiplication (specifically "times tables") and subtraction.

The second theme about the nature of maths that emerged from Gina's responses was measurement. Gina did not refer spontaneously to measurement situations when talking about maths. However, when prompted she did identify measurement situations as mathematical.

References that Gina made to measurement situations as mathematical included concepts of mass, length, and capacity. An outside length activity that Gina described as "measuring the chair" (Task 8.2), was seen as maths. She believed there was "lots of maths" in using a recipe because "we have to write the measure, the how many measure, how many flour we have, centimetre" (Task 3.3.2). In relation to mass, Gina spoke of using scales for measuring "how heavy" for vegetables, fish and herself (Task 5.2). When speaking of the vegetables, Gina described the photograph as

the woman's buying food and she is measuring in the um, what's that called [scales] what centimetre it is . . . because you're looking at the, what kilometre, um, what metre was it, how heavy

The responses suggest that Gina was aware of the use of formal units of measure but that she had some confusion as to the appropriate units of measure for particular attributes.

Gina spoke also of units of measure when talking about measuring metres to know how much paint to buy to paint a room (Task 3.3.2) and described "how many miles" as mathematical activity when she travelled by car to school (Task 3.3.2). When shown a photograph in Task 5.2, Gina correctly linked "measuring how tall" to centimetres.

Similarly, informal measurement of length, that is, a child putting a pencil next to a book and saying which was longer (Task 3.3.1), and a woman comparing a sleeve length to her arm (Task 5.2), were identified as mathematical activity. Gina explained the latter as "we buy clothes . . . have to measure what it, need a body and measure how long is the sleeves".

In addition to number and measurement as content areas of maths, there was some indication that Gina saw space concepts as mathematical. In Task 8.3 Gina said that the children had "talked about shapes" in the previous maths lesson. She believed that a video clip showing children working with geoboards and rubber bands (Task 7.1) showed mathematical activity. She described the lesson as "doing shapes" and the maths as "corners and shapes".

From the above discussion of the references to the content of mathematics a range of themes emerged. These include

- number as an important element of maths, with a focus on counting and some operations;
- formal measurement and informal measurement by comparison as elements of maths; and
- space as an element of maths.

Other themes that are inherent in the above and that are explained further below are

- number as an important element but as not necessary or sufficient;
- maths as a school-based activity; and

• maths as a non-school activity.

For Gina, although number appeared as a key element of maths, it was not a necessary element as evidenced by the identification as mathematical activity of comparison of length in informal measurement situations including comparing a pencil and a book to find out which was longer and comparing the length of a sleeve to an arm.

The presence of number in a situation also did not ensure that the situation would be identified as mathematical by Gina as demonstrated in responses to three Task 3.3.1 items. Gina did not identify "visiting Anna's first dog", "Cleaning room number 7" or "having a better chance of getting a red lolly from a bag that contains five red and three yellow lollies" as mathematical activity although numbers were mentioned.

As stated above, Gina tended to suggest number activities as mathematical. Discussion of different types of *sums* suggested association with school-based activity, as did her response to the Task 4.1 sentence starter "Maths is like". Gina wrote "a little bit hard and a bit easy sometimes and sometimes you get confused". When asked for something else maths is like she added "homework and like, work". Later in the same interview (Task 4.2.1), Gina described her drawing of someone using or doing maths as "a girl and the thing that you do maths with [blocks], and a sheet of paper, you need to answer all the sums . . . She's thinking how to do it and listening to the teacher, how to do it". When asked for another situation of someone using or doing maths but not at school (Task 4.2.2), Gina drew a picture of a homework situation. These examples suggest that Gina readily associated maths with both number and school-related tasks.

The identification also of mathematical activity in situations presented to Gina such as weighing fruit in a supermarket or counting bricks to lay a path, indicates that maths was seen as a non-school related activity as well as a school activity.

The above discussion of Gina's beliefs about the nature of maths illustrated the emergence of eight key themes. From the discussion it becomes apparent also that other possible elements of the content of maths such as Chance and Data were neither identified as mathematical within presented situations nor proffered as mathematical. The process of estimation, which featured prominently for Cara, also did not emerge as a feature of mathematical activity for Gina. Guessing was identified within one situation (Task 7.2) but the maths of that situation was described as "they were counting".

As a summary of Gina's beliefs about the nature of maths, a schematic portrayal is provided below demonstrating that number, measurement and space concepts were all considered mathematical.



A further element that was considered for Cara and Emily was the side issue of the use and understanding of the terms *maths* and *mathematics*. Like Emily, who was a member of the same class at School I, Gina did not generally differentiate between the two terms. For example, in Task 2.2 she spoke of learning mathematics, and when questioned as to what mathematics is, she responded "You do sums - you do maths". Gina appeared comfortable to use either term although the common abbreviation, *maths*, was used more often in the interviews.

Beliefs about the nature of learning

The second key focus of the study's research questions was *learning*. Discussed below are Gina's beliefs about the nature of learning in general, beliefs about the nature of learning maths in particular, and her beliefs about reasons for learning maths.

Gina's ideas appeared to be under development, sometimes with inconsistency or seeming contradiction appearing. Key themes that emerged from the interviews are

- an association of learning with schooling;
- learning maths as a school-related activity;
- remembering as an element of learning;
- learning "how to do it" or "what to do";
- learning as thinking; and
- a multiplicity of reasons to learn.

It appeared that Gina associated learning in general, and the learning of maths in particular, with schooling. For example, when asked in Task 1.2 (Interview 7), "what does learning mean?", Gina replied, "School - where you learn something". Gina was asked how she knew when she had learned. In Task 1.2 she replied, "I don't know", but when asked the same question in Task 2.1, she stated, "Because you go to school and you learn . . . and the teacher teaches you". This response suggests that Gina perceived that the teacher played a key role in her learning. Likewise, in Task 2.2, Gina said she knew she had learned because the "teacher had taught us". When asked for a recent learning experience (Task 2.2), Gina

described learning sums and maths such as "take-away, plus and divided by". When then asked for something else learned recently she talked of learning about fractions at school (Task 2.2). This was not the only reference to learning maths as a school-related activity. Gina believed that everyday she did maths at school she learnt something, but when asked whether she learned maths elsewhere, answered that she did not (Task 8.3). In response to further probing, she talked about her brother helping her to learn maths at home. It appeared that this was associated with homework, thus with schooling requirements.

The third theme listed above is "remembering as an element of learning". The Task 2.2 description of learning about fractions, referred to above, suggested an instrumental approach to learning:

Um, I don't know what um that thing's called - um - um that there's um that thing and there's numbers on the bottom and plus and there's like that number on the bottom as well and you have to plus the top and you can't plus the bottom.

Gina's recording of the task on paper showed addition of two tenths and six tenths. Thus she was following a correct method but stated that she was not sure why she had to leave the bottom as it was. When asked whether she understood this task she replied, "Yes, because the teacher has taught us lots of times and I remembered it". Perhaps these two responses by Gina suggest an instrumental rather than relational form of understanding of the addition of fractions (Skemp, 1976). Gina's other references to remembering as an element of learning, such as in Task 7.2, also support the conclusion that she took an instrumental approach to learning. She felt that not learning something the teacher had taught was because "I forget some things" (Task 2.1). However, when asked in Task 10.3 whether remembering is important for learning, Gina responded "No", but understanding was considered important. Gina clarified her response by explaining her meaning for remember as "remember all the stuff you learned" and for understand as "when a teacher tells you what to do and you have to understand it". There appears some contradiction over the use of the term remember and its application to learning. Many responses suggested remembering was considered part of the learning process; perhaps it was not in contradiction to say that it was not important, or perhaps the word important was understood differently by Gina from the understanding of the researcher but her meaning for the word was not revealed in the interview.

As cited above, in Gina's description of her Task 4.2.1 drawing, she included the words "thinking how to do it". Knowing maths was considered not merely as getting answers (Tasks 1.3.1, 7.1). For example, the calculator could give answers, it would "help me but you won't learn . . . because when you use [the] calculator, you're cheating" (Task 7.1); knowing "what to do" or "how to do it" were valued for learning (Tasks 7.1, 10.3).

Gina gave the word "thinking" as the first word she thought of when posed with the word *learning* (Task 1.1). However, later in the same interview, when asked for a definition of learning (Task 1.2), she could only respond by equating learning with schooling. In a later interview, when asked to tell what learning is, Gina stated, "you use your mind and you think

and you write it down" (Task 2.1). It appears from other responses, such as cited in the previous paragraph, that the thinking may have been about what to do or how to do it.

A further theme related to the reasons for learning. Knowing how to do the work and getting the right answer were considered important so a child could "go up" to another grade (Task 6.2). When asked why she followed a particular procedure when dividing, Gina stated, "so when we grow up we become a teacher so we can teach other kids so they can learn" (Task 2.2). These purposes again link learning to schooling. When asked about people who don't become teachers Gina responded they need to know mathematics "because then they can be clever, like you go shopping, like people trick you, so you know" suggesting that knowledge of mathematics gives power and control.

As illustrated in the discussion, the six themes listed above that related to beliefs about the nature of learning, of learning maths, and the reasons for learning maths, emerged from Gina's responses to the interview tasks. Further themes emerged regarding helping factors for learning mathematics.

Helping factors for learning maths

Gina gave a number of responses about factors that she perceived help her to learn maths. At times non-helping or hindering factors were discussed to assist in clarifying the helping factors. Key themes identified within Gina's responses are

- working alone;
- learning maths in a group situation;
- the "teacher" helping Gina to learn maths;
- Gina helping herself to learn maths; and
- tools helping Gina learn maths.

Gina indicated that she believed working alone was a helping factor for learning maths. For example, when asked in Task 3.1 what helps her to learn, Gina stated, "doing it by yourself, you learn more". In the previous interview Gina stated that the children in her class sometimes worked together but that they worked more often by themselves because they learned more (Task 8.2). This opinion may have been influenced by the views of significant others but, nonetheless, was held by Gina. In her drawing for Task 6.1 Gina portrayed herself doing homework. When sorting the statements of helping factors, "by myself" was ranked second most helpful after "concentrate". Indeed, Gina believed that working alone in a quiet environment, such as away from the television when at home, facilitated concentration, and in turn, facilitated the learning of mathematics (Task 6.1). A quiet environment would allow "you to think of things you want to write" (Task 9.2). In Task 4.1 she described problems arising from being disturbed:

Like when someone's talking, when you're thinking or something, and you're gone back, gone back to write it, and when someone's talking to you, you are confused when you lost that question or something.

Noise from people talking also was perceived to cause her brain sometimes not to work, because she could not concentrate (Task 9.1). A quiet environment that facilitated concentration and working alone appeared important helping factors for Gina's learning of maths.

Gina may have preferred to work alone also as she feared that another child could tell her wrong answers (Task 6.2) or could hear the wrong thing and tell her the wrong thing (Task 8.2). She was concerned also that if "people helps us work, does it for us then we won't be able to know first off" (Task 6.1). On many occasions Gina stressed the importance of not being told answers as she believed that she would not learn. She believed that if other children called out answers in class she could not learn (Task 8.2). She believed also that if she observed and listened to a conversation between a teacher and another child she would be copying and therefore that this would not help her to learn maths (Task 8.2).

Overall, working alone appeared as a strong theme for Gina related to helping factors for learning maths. However, Gina did see some benefits in group work.

A second theme that emerged in relation to helping factors for learning maths was learning in a group situation. There appeared some inconsistency in Gina's responses about working in groups. On two occasions Gina indicated that group work could help for learning maths. When asked about a specific lesson on symmetry where the children worked in groups, Gina stated that if one child had found it hard, others in the group may have been able to help (Task 9.2). Working in a group could be considered "good . . . so you know what to do" (Task 7.2).

However, other responses, discussed above in relation to working alone, suggest that Gina did not always identify discussion with other children as helpful for learning maths. She did not take the opportunity to select group work as a helping factor when posed with scenarios in photographic or video form. Perhaps she feared that such a situation would provide her with answers.

It appears that Gina's ambivalence to working in groups may have related to two possible forms of help from a group; help in knowing what to do was favoured but giving of answers was not considered helpful.

Gina's responses suggest that she believed the "teacher" played an important role in helping her to learn maths. The role of the teacher is the third theme identified from Gina's responses.

One element of the teacher's role was telling what to do or how to do the maths. Gina talked of her school teacher helping her in this way (Tasks 6.1, 7.1, 8.2), which might involve the teacher writing on the blackboard (Task 7.1). Her brother, interpreted here as another *teacher* figure, helped by "telling us to put our hands up, like fingers up and count" (Task 8.3), and her father might help by telling her to count or write on a piece of scrap paper (Task 6.2). Gina believed it was helpful also when a teacher chose a child to go out the front as "you had to listen to the [person] so you know what is happening" (Task 7.2).

Gina did not want her teacher to give answers when helping her at school (Task 8.2), nor family members, such as her brother or father, to give answers when helping her at home (Tasks 6.2, 8.2).

A form of assistance preferred in contrast to being given answers was being given clues. When Gina had difficulty and asked the teacher for help, she liked the teacher to give clues "so I can know" (Task 8.2). Although Gina stated that when she was confused in maths she would "ask the teacher or [her] friend next to [her]" (Task 4.1), she appeared to value help from the teacher more than from another child as she felt her teacher knew "all that stuff ... the teacher knows more stuff than the children do" (Task 7.1).

A further theme that emerged for Gina from the discussion of helping factors concerned Gina helping herself to learn maths. As well as concentrating (Tasks 6.1, 9.2), she felt she should get enough sleep so she could listen to the teacher (Task 6.1), listen to the teacher regarding "how to do it" or "what to do" (Tasks 4.2.1, 6.1), put effort into her work by "work[ing] harder at school" (Task 9.1), use her brain (Task 9.1), think (Tasks 4.2.1, 9.2), and count (Task 9.1, 9.2).

Seemingly in contradiction to what is cited above, Gina did not believe that the phrase "think about it" (Task 10.1) suggested a helpful action for learning. She explained that "when the teacher tells you to do something quickly you can't wait because you waste our time". Gina's father also suggested doing "it quickly, then we know it very fast" (10.1). It appeared that although concentration was considered helpful for learning maths, spending time on thinking was not always valued for learning maths.

Gina could help herself to learn maths when playing games such as tables races: "when you're going to say the answer, you remember the last time you said the answer was wrong" (Task 9.2). The sad feeling remembered from the time of a previously incorrect response may have been a factor of influence also.

Tools as helping factors emerged as a further theme for Gina. Fingers were considered helpful (Tasks 5.1, 9.1), especially for times when "you forgot, don't have your concentrate [*sic*] in your head" (Task 9.2). Other tools that were considered helpful for learning maths included geoboards for hands-on work of making shapes (Task 8.2) and blocks for counting (Tasks 4.2.2, 5.1, 7.2). Gina felt that the calculator was helpful, as "you learn to remember something you learnt . . . when you keep doing it" (Task 7.2) but that its use was cheating (Task 7.1), suggesting ambivalence towards the use of calculators to assist learning of maths. Similarly, she felt the computer was helpful "but sometimes gives us the answer" and therefore she preferred to write in her maths book.

Five themes related to helping factors for learning maths were identified from within Gina's interview responses, as discussed above. Overall Gina portrayed a strong image of working alone in maths and appeared ambivalent towards a group situation for learning maths. Mostly she liked to work quietly and not be interrupted. She believed the teacher was the best person to help but did not want to be given answers. She could help herself in various

ways and considered a number of tools as helpful. Although chosen as a low achiever in mathematics, Gina communicated a self-awareness of being able to learn mathematics.

To conclude it is apparent that

- Gina did hold beliefs about the nature of maths and mathematical activity that she was able to articulate in response to the research procedures.
- Her beliefs about the nature of maths were broader than those of some of the other children, as her beliefs included concepts related to number, space, informal measurement and formal measurement.
- Gina held some beliefs about learning that she was able to communicate in the interviews.
- The concept of learning tended to revolve around schooling with the teacher a key element in the child coming to remember or coming to know how, or what, to do.
- Gina was able to identify a range of helping factors that included reference to herself, other children, the teacher and tools.
- Gina showed some ambivalence towards working with others and the use of the calculator to help learning.
- Gina seemed to take her learning of maths seriously and wanted to learn in a responsible manner, for example, by not getting answers from other sources.

Ben

Summary

Ben was chosen as a Grade 3 male from School S, who was a high achiever in mathematics. His definitions of maths referred mainly to number and measurement concepts. As described in more detail below, Ben also included *competing* as mathematical activity. This appeared to relate to ordering without the use of numbers. Situations that used numbers as a marker or for ordering also were suggested as mathematical activity. Ben recognised maths as both a school and non-school activity, the latter mainly in relation to building, an activity with which he appeared familiar through the occupations of family members. Other than in relation to straightness and angle, space concepts were not proffered as mathematical activity.

For Ben, learning was associated primarily with getting better and better at something, and thinking, remembering and practising were perceived to play a role in the process.

Ben's responses suggest that he believed that other people helped him in learning maths, that he could help himself in learning maths, and that use of materials assisted in his learning of maths.

Ben's beliefs about the nature of maths, of learning, and helping factors for learning maths are now discussed more fully in that order.

Beliefs about the nature of maths

Six themes about the nature of maths emerged from Ben's interview transcripts:

- number as an important element of maths, with a focus on counting and adding;
- using numbers as markers or for ordering as mathematical activity;
- competing as mathematical activity;
- measurement as an important element of maths;
- maths as a school-based activity;
- maths as a non-school activity.

In his first interview (Task 2.3), Ben gave his dictionary definition of maths as "All about numbers and adding up, divided bys and going to another number, equal". When asked in the following interview what he would tell an alien about maths (Task 5.1), Ben stated, "It's a subject to do with numbers and people add them up and they divide and take away and subtract them, and people use them with adding up like money and they might use them for bills". In his fifth interview (Task 1.1), Ben gave "numbers" in response to maths in the word association quiz and read from his maths word wheel (Task 1.3.1): "sums, weight, height, equals, plus, numbers, division, adding up, minus, groups, times tables, plus, area of something, width". In his eighth interview he wrote "Maths is like all different numbers, weight, height, length, width, minus, adding up, counting, times tables, division, units, tens, hundreds, thousands and millions" (Task 4.1). He stated that he believed others could have a different meaning from his for maths, that is, that "everyone would probably have a different mind about maths" (Task 2.3). When he spoke of his Mum's meaning he referred to "try and

add up all your sums and what they equal and you put words in some of them" (Task 2.3), seemingly suggesting a partially similar meaning to his own.

In summary of the above, Ben's responses suggest a view of maths as involving numbers, the four operations and measurement concepts, as a school activity, and as used by people outside of school for purposes such as paying bills. Each of these elements was mentioned many times in different forms throughout the interviews. Some are expanded upon in the discussion below.

The presence of numbers and measurements were used to justify maths in some situations. For example, weighing vegetables in a supermarket was explained as mathematical because "the scales and the weight 'cos it's using numbers" (Task 5.2). Ben highlighted the importance of numbers when he gave handwriting as a non-example of maths: "Like you don't use numbers in handwriting, and you don't, you just write" (Task 1.3.2). Likewise, a situation in Task 3.3.2 was described as not mathematical because "he's not using any measurements or any numbers or something".

Many situations were judged as mathematical because Ben believed they involved counting. These included, from Task 3.3.1, telling which number was bigger - fifty or thirty, putting six stickers onto three pages, saying that one hundred cents is the same as one dollar, paying a salesperson \$3.20 for a hamburger and coke, saying that half a chocolate bar is better than a third of a chocolate bar ("she needed to count how much, the exact amount"), finding all the squares in the pictures on the page, measuring with a ruler ("counting how many centimetres"), and, from Task 3.3.2, looking after a vegetable garden ("counting how much vegies are there and if the rabbits took some").

Adding was mentioned frequently also. Using a calculator to work out the money to pay the bank was judged to involve *lots* of maths "because you're adding the bill, putting the amount of money you want in the bank" (Task 3.3.2). The following situations, also posed in Task 3.3.2, were judged to involve *some* maths because of adding: playing a sport ("you'd have to score the points and add them all up at the end"), planning a two week holiday for a family ("adding up how much days you'll be away"), and playing a musical instrument ("because you had to add up the tunes . . . like, you have to know which tune to play"). In response to a Task 3.3.1 scenario where a child paid for a hamburger and coke but no mention was made of the amount of money, Ben stated the maths would be adding. The similarity of this situation to the \$3.20 hamburger and coke situation mentioned above suggests a possible similarity in meaning for *counting* and *adding up*. Certainly these were two activities that Ben identified frequently as criteria for determining whether a situation was mathematical.

Ben made reference to a range of situations as mathematical where he appeared to perceive the use of numbers as markers: windspeed, temperature ("the number of the heat"), and loudness of a microphone or tapes ("like you might have um a loud of three, er, they might go a bit louder like three, volume three, volume twenty [*sic*]" Task 1.3.2). A further situation, "[running] to Anna's house to see her first dog" (Task 3.3.1) was considered

mathematical activity because the person going to the house would "need to know the number of the house". In this case the number was portrayed as a marker for visitors to read. It appears that knowing and reading numbers was seen as mathematical activity. In the situations of windspeed, temperature and loudness, the numbers also may have related to ordering, that is, a higher number was used as the attribute increased.

Although number appeared as a key element of maths for Ben, he did identify situations that he perceived as mathematical that did not involve the use of number. These included informal measurement situations such as "measuring how big she needs the clothes" (holding a piece of clothing against one's body to check for size) (Task 5.2), comparing a pencil to a book (Task 3.3.1), and a girl "measuring her sister" with a piece of string (Task 8.2). Ben referred also to working out smallest to biggest and cutting thick and thin pieces of banana as mathematical activity (Task 5.2).

In addition, Ben talked about *competing* against others or oneself as mathematical activity. He explained through examples including band members "trying to get better and better at [playing instruments] and you know they're probably getting higher levels of it", running across the playground and trying to get faster and faster each time, and a person "trying to blow as hard as he can go to get the candles [on the birthday cake] out". The latter was identified as maths "cos he's trying to do it harder and harder 'cos he's trying to get every candle out" (Task 5.2). Maths was explained as "strength", for example, like "trying to get the longest throw or the shortest throw". Ben stated that "you can compete against fastness or heaviness [or] strength" as mathematical activity (Task 5.2). It appears that the competing may have related to ordering or higher and higher levels of outcome and therefore was seen as mathematical. Alternately these situations may have been seen as informal measurement situations, where a comparison is made but numbers are not used. Compared to the other children within this research study, the identification of competing against others or oneself as mathematical activity was the most idiosyncratic aspect of Ben's beliefs.

Measurement in non-school situations appeared familiar to Ben in a similar way to that for Cara, that is, through experiences with family members (Ben's "grandfather and two or three uncles" were builders). Ben drew the building of a house in Task 4.2.2 and chose Houses as an integrated topic in Task 4.3. Ben spoke of measuring wood for building (Task 1.3.2) and measuring when "cutting wood and mucking around" before he started school (Task 6.1) as mathematical activity. Other real-life measurement situations were cited also, such as mass and capacity activities when making a cake or jelly (Tasks 1.3.2, 3.3.2, 8.2) and his grandmother measuring when making a quilt (Task 6.2). Although Ben referred often to measuring, even sometimes looking for evidence of measuring as a criterion for maths (Task 8.2), he referred to estimation only when describing two situations portrayed in photographs in Task 5.2. In the first he thought the person was estimating the depth of a hole and that this was mathematical, but stated for the second photograph that, although he thought the people were "estimating how heavy", the mathematics was "weight or length". When defining or

listing elements of maths Ben did not refer to estimation on any occasion. In relation to estimation, Ben's associations with, or meanings for, maths, differed from those of Cara despite both children appearing to have had practical family experiences with measurement and both being in the same class at school. Perhaps the measurement tasks of a builder were seen to call for much more precision than those of a pastry cook (Cara's father's occupation).

Ben appeared to make little association between maths and space concepts. He did state that if studying an integrated topic of Houses, consideration could be made of the "angle of the house, what angle it's going to be on, like diagonal, straight or back to front" (Task 4.3). However, he did not relate to space concepts when identifying "Chris cut out shapes of squares and circles and made a design" as mathematical activity. Instead he spoke of the maths as "he has to make the same amount of paper to cut out, if he made it too big it wouldn't look as good, [he has to] rule off and measure it where you have to cut it off" (Task 3.3.1). Two further situations that potentially involved spatial concepts were identified by Ben as mathematical because of counting (Tasks 3.3.2, 5.2). In one, when asked about a child who found "all the squares in the pictures on the page", Ben said it was maths "because he's counting all the squares". He was then posed the question "What about if you're saying 'that one's a square, that's a triangle and that's a diamond', would you be using maths then?" Ben responded "I haven't had that one. I don't know". After further discussion in which he drew triangles and rectangles, Ben was asked "so when you tell me about triangles - different triangles, are you using maths?" He answered "Yes, because you have to make, see if it's straight or not". When viewing a photograph of a child making shapes on a geoboard, Ben described the activity as "making little shapes", but said the child was "not doing maths, like it's not measuring" (Task 8.1). Naming, drawing and making shapes seemed not to be considered mathematical by Ben. Working with straightness, perhaps related to building experiences, was identified as mathematical activity.

In addition, Ben did not spontaneously refer to any chance or data activities as mathematical and did not identify such within any of the tasks posed. It appears that number and measurement concepts were perceived by Ben to be more significant elements of school and non-school activities than were spatial, chance or data concepts. This may have related to his perceived greater use of number and measurement in school and non-school situations. When asked "Do you ever do things to do with shapes and things like that?", Ben stated, "Not much now 'cos like I'm in the Senior School. I don't have to much more". The teacher stated that the children had done some spatial and some chance and data work during the year with the Applied Maths teacher but, due to school organisational structures for the year, not as much as she would usually have done (Interview with Ms S, December 11).

Ben appeared to see mathematical activities both in school situations and non-school situations. His drawing in response to Task 4.2.1 showed a school situation, of a girl using a "calculator and MAB minis . . . trying to do a sum . . . using the equipment to help her". Likewise, his Task 3.1 drawing was of learning maths in a school situation, suggesting that

Ben's immediate association of maths may have been with learning at school. However, he also related maths to non-school activities, as demonstrated in his Task 3.2 drawing of people building a house, his Task 3.3.2 reference to measuring the "wall length and how much paint" as maths when painting a room, and in other references to house building and cooking as referred to in the above discussion.

As a summary of Ben's beliefs about the nature of maths, a schematic portrayal is provided below displaying the interrelationships within those beliefs.



The use of the term *maths* in the interviews in preference to *mathematics* is a side issue that has been discussed in relation to the other children. This choice of terminology was appropriate for Ben who explained "I don't know what mathematics means but I don't think it's the same as maths as it's a longer word than maths" (Task 11.2). He thought the two may be similar but believed that no one had really talked to him about *mathematics* (Task 11.2).

Beliefs about the nature of learning

The interviews with Ben revealed beliefs about learning related to

- learning as a process of working with others;
- getting better and better at something;
- learning built upon prior knowledge and need;
- remembering;
- thinking;
- practising;
- making mistakes; and
- trying different approaches.

In his first interview, when asked to give a personal dictionary definition of learning, Ben responded "People learn by listening to others and by helping by other people [*sic*]" (Task 2.1). He went on to say that you know you have learned "when you get better and better at it". In the automatic response quiz Ben gave the response "finding out" for learning (Task 1.1) and for the password game he suggested the word "discovering" for learning (Task 1.2). He explained on another occasion how he discovered how to learn to spell a word - by asking his father who spelt it for him and then by sounding it out himself (Task 2.1). He had learned to play chess by people helping him (Task 2.1). Ben's references to others helping suggest he saw a role for a variety of *experts* in assisting learning.

Ben's response to the question of how he would tell an alien what maths is (Task 5.1), suggested a thoughtful approach to learning, that is, that he had come to believe that learning is built upon prior knowledge and need. Ben said he would ask the alien if he knew anything and ask what the alien wanted to know. Ben suggested also showing the alien "a bit of maths that people are doing and why, like to show him how to do it". Taking the alien to "shopping centres, banks and homes" suggests also that Ben felt one can make more sense of something new if it is observed or experienced in action.

When asked specifically in relation to maths, Ben stated that "[you have learned when] you get better at [new things] and you get to do them off by heart, and you don't need to use any equipment or materials, and you don't need anyone helping you" (Task 10.1).

Remembering, although considered as part of the learning process, was not given as much prominence by Ben as by some of the other children. It was mentioned in relation to remembering how to do a sum or what the answer was (Task 10.1) and was mentioned in relation to the playing of games (Tasks 8.2, 9.2). Ben felt that games helped him "to remember some of the sums that I haven't done for a long time" and therefore to learn the maths (Task 8.2).

It appears that Ben believed thinking was deployed when learning and possibly to a lesser degree when doing known maths: "Sometimes you might think about how to do the maths or sometimes you just think it out and just write it because you might know it already" (Task 10.1). Thinking was mentioned in relation to doing a hard sum (Task 6.1).

Ben suggested that practice was an element of learning (Tasks 1.3.2, 2.1, 10.1), for example, "you keep playing [games] and you get better at the maths, you might learn something that you didn't know" (Task 10.1). When posed with *practise it*, Ben said "you might practise part of the maths games or you might practise the sum" (Task 10.1). In some respects Ben took this concept further: he talked about having another go and "keep[ing] trying to get really good at [the sum]" indicating that this was not just practising but was making mistakes and "re-doing" or "think[ing] of another way to do it" (Task 10.1).

In summary, Ben associated learning mainly with getting better and better at something, perhaps by listening to or helping other people. He suggested that he believed learning is built upon prior knowledge and need, and that remembering, thinking, practice, making mistakes, and trying different approaches are all used to some degree within the learning process.

Helping factors for learning maths

Within the interviews Ben identified a number of factors that he believed could help him in learning maths. These are discussed below within the following themes:

- learning by listening to, and being helped by, other people;
- Ben helping himself to learn maths;
- the use of equipment to help in learning maths.

Ben felt that others could help him to learn maths. These others might be the teacher, his parents, or other children.

Ben's response to the PPELEM task, developed specifically to investigate perceptions of helping factors for learning maths, revealed his belief in the teacher as a helper in his learning of maths (Task 6.1). Ben described his drawing: "One of my teachers were showing me how to do the sums, how to work the sum out". The teacher "might know a bit more" than the children so would be the best person to explain (Task 6.1). However, Ben reported that when in a group, he would probably ask another child for help before asking the teacher (Task 7.2). Ben felt that the teacher helped him learn maths by asking questions when playing a times tables race game as this helped him to remember (Task 9.2), by writing things on the board (Task 8.2), or, if Ben was "not [doing] so well" with maths, the teacher "might say you could try it with little blocks" (Task 8.2).

Although Ben felt that the teacher played an important role in his learning of maths, he appeared not to be dependant on the teacher but could help himself in learning maths. For example, he stated, "I'd prefer the teacher to give me some instructions like what to do first and like if I can't do that sum properly on my own I could like skip it and wait and at the end I might do it" (Task 6.1). In response to Task 6.1 the most helpful factor for learning maths was identified as Ben "waiting and watching" as the teacher was explaining. In this way he was helping himself to learn maths. During this time he would be thinking about whether he would use counters or fingers. This was ranked as the second most helpful factor. Ben reported that the teacher would "tell [him] how to work it out and then [he] would work it out in [his] head and understand it and then do it" (Task 6.1). Watching and listening were cited as helpful also in Task 8.2.

Ben's response to Task 6.1 suggests that he believed the teacher helped him to learn maths but also that he had a perception of having an active and reflective role in his own learning. His involvement in learning maths also included self help such as sometimes looking back in his maths book to remind him of "what some sums were about", or "if [he] didn't do it much, trying to remember the last time [he] did it in [his] maths book" (Task 9.2). He believed he could help himself also by learning from his mistakes after the teacher had corrected his work (Tasks 8.1.1, 9.2, 10.1). Ben spoke also of talking to himself while doing maths, telling himself "the number and counting with [his] fingers" (Task 9.2). This would be facilitated by working at home in a quiet environment. He believed that doing things a couple of times and writing them down (Task 9.2) or trying to work out a hard problem by having a go on a scrap of paper (Tasks 8.1.2, 9.2) could help in learning maths. He felt also that if he was called up to the blackboard to explain some maths this would be helpful for his learning of maths as he would "be doing it himself without anyone else" (Task 9.2). Ben liked other people to tell him how to do the first part of a "hard sum", and then he would try and work out the second part; he liked also to have "a go at different sums, like the harder ones" (Task 8.1.1). Ben appeared to be willing and able to use strategies to help himself with maths. For example, when he encountered nine times seven, a problem that he considered "hard", he solved the problem by "going up by the sums of the seven times tables" (Task 8.1.1). Although Ben did not have an automatic response to the problem, he had an understanding of the relationship between addition and multiplication that allowed him to work though the problem.

Ben believed that parent help was potentially a helping factor for learning maths (Task 8.2). On one occasion he suggested that he may have received such help (Task 7.1), but when shown a drawing representing a child working on her maths at home with her mother, and asked whether he sits down like that with someone at home he responded "I don't do that, no" (Task 9.2).

Ben appeared to have a strong perception of the value of other children in his learning of maths. He stated that "if you didn't know a sum . . . you could learn off other people" (Task 2.1). He felt that another child going up to the front of the class would be as helpful as the teacher (Task 9.2). Ben believed that the alien could be helped by other people to learn maths (Task 5.1), just as he himself had been helped by other people to add dice (Task 5.2). He preferred to be shown how to do something in maths rather than trying to learn by watching someone else doing the maths. He appreciated a friend helping and "giving you a hint" (Task 9.2) and would do the same for someone who was finding maths hard to learn after he had asked them what their problem was (Task 7.1). Ben was positive towards working in a group as other people could help (Tasks 7.2, 9.2) and did not seem unduly disturbed by noise from others; he stated that he didn't need to work in a quiet area, it could be "medium-ish, not too low, not too loud" (Task 9.2). He felt that working by himself "wouldn't be as easy as working in a group" but liked to work with others some of the time and by himself some of the time (Tasks 9.2, 10.2.2).

Ben felt also that the use of equipment could help him to learn maths (Tasks 4.2.1, 6.1, 7.2, 8.2, 9.2, 10.1). In describing his Task 4.2.1 drawing of a girl using a calculator and MAB minis, he stated that "she's trying to do a sum and she might be having trouble, so she's using the equipment to help her and make it easier for her". In response to Task 8.2, he stated that he thought a calculator would help him learn maths. However, on another occasion he stated that he would probably prefer to work something out without a calculator, that is, to work it out on his own (Task 7.2). The use of equipment to make it easier "to work it out" was stressed in Task 9.2. Ben could use his fingers if he didn't have counters, and the calculator

could "sometimes" help him to learn maths as it "shows you what maths you're doing" (Task 9.2) or could help "if you want to know some of the hard, difficult maths" (Task 10.1).

Ben believed that copying something from a book could help him to learn maths (Task 8.2) or that looking up a book "might tell you what type of maths materials they are or what type of maths you're doing" so that you are finding out new things you did not know (Task 10.1).

In conclusion, Ben appeared to have a degree of confidence in his learning of maths, that is, he believed he could help himself and did not appear fearful in any way of help from other children or the use of equipment such as calculators or counters. He did not express any concern about cheating or being told the answer as did some of the other children. He felt it was appropriate to be shown how to do something as he would learn from the experience. Indeed, Ben appeared a highly reflective student who was aware of his own learning and of the need for expert guidance at times. However, he did not appear to have the same dependence on the teacher that some of the other children displayed although he did think the teacher played an important role in his learning of maths.

Overall, Ben's responses to the interview questions revealed that

- He could describe his views about maths.
- His views about maths were broad to some degree but did not appear to include all main content aspects of the curriculum.
- He could describe his views about learning.
- His views about learning were broad.
- Ben was highly reflective when considering helping factors for learning maths and was able to identify the possibility of help from a range of sources.

David

Summary

David, a Grade 3 boy from School S, was chosen for participation in the research as a low achiever in mathematics.

The content areas that David appeared to associate with maths were number and measurement. But equally, if not more important for David seemed to be his association between maths and the processes of working hard and concentrating. David talked also about maths as solving big problems and designing things.

David believed the teacher played an important role in his learning and thus that it was important for him to listen to the teacher as well as to work hard to achieve satisfactory completion of his work. Remembering and thinking also were associated with learning.

Concentration was identified as the most helpful factor for learning maths. Directions or instructions from the teacher or other children could help also.

Beliefs about the nature of maths

David's beliefs about the nature of maths were complex and seemingly under development. David included some content as an element of maths but appeared not to give this as much emphasis as did some of the other children within the study. David seemed to focus just as much, if not more, on processes such as working hard. Indeed, when he identified in other activities processes that he associated with maths, he often called those activities maths.

David's beliefs about the nature of maths appeared to revolve around the following themes:

- maths as something that requires hard work, listening, and concentrating;
- solving big problems and designing things;
- formal measurement as mathematical activity;
- number concepts and processes.

The first theme came through strongly in David's responses. For example, he stated that "in maths you have to do work" (Task 4.1) and later in the same interview he stated, "if you're doing maths you're working and you're thinking and you're concentrating". Another response was "in maths you work hard, you listen and you obey the teacher" (Task 1.3.1b). This focus on maths as something that requires hard work, listening, and concentrating appeared to lead David to associate closely, and sometimes classify, other activities with these features as mathematical. For example, when asked to describe "another mathematical activity" (Task 3.2), David spoke of learning art at school: "In art you get to do a lot of things like draw pictures and sometimes you have to do activities with hard ones and you have to concentrate on it and listen to the teacher and instructions because if you don't listen, you might get the thing wrong". The following discussion revealed that David saw no further mathematical content within the art activity, that is, he did not, for example, identify any number or space concepts as justification. David appeared to associate maths and art because, for himself, they were both schooling experiences that required thinking, concentrating and

hard work. In Task 4.1, drawing was stated to be *like* maths "because when you're drawing you're thinking and when you're doing maths you're thinking and you're concentrating on work. That's what we do in drawing and maths". In each of these tasks David referred to art although one task asked for a mathematical activity and the other asked him to think of something *like* maths. The subtle change of language may not have been obvious to David or perhaps was not judged as important. The hard work, listening, and concentrating seemed to be the common features that caused David to make a link between maths and art.

A range of activities from Task 5.2 were identified as mathematical because they involved concentrating. For example, David said that playing chess has "a little bit [of maths in it] because you have to concentrate". Building a house and putting up pictures for a classroom display were also considered mathematical because of the need to concentrate. In Task 3.3.2 painting a house was considered to involve maths because "you have to concentrate and do a lot of hard work". Once again, hard work and concentration appeared to be the criteria applied in deciding whether a situation was mathematical.

As demonstrated in the Task 3.2 reference to art, as discussed above, David related lack of concentration to the possibility of "getting things wrong". He stated also that "some people are deaf and they have to use sign language and other people have to understand it, that's a part of maths because if you don't understand you might get things wrong" (Task 1.3.2a). He believed that in maths "you have to learn how to do things so you don't end up having mistakes" (Task 1.3.1b). David's association of maths with hard work and the possibility of making mistakes may have related to his own experiences and a perceived need for himself to concentrate and work hard so he would achieve. As indicated above, David had some difficulty in learning maths. However, he also saw himself as improving in his maths.

Difficulty was used also as a criterion for judging whether an activity was mathematical. When posed in Task 3.3.1 with the scenario "Chris cut up shapes of squares and circles and made a design", David stated that there was no maths because "maths has got harder things, harder and things that you have to concentrate on". In response to a later scenario of "Louis found all his squares in the pictures on the page", David stated, "That's an activity". When asked whether there was any maths in what Louis was doing, he said "not quite". He explained: "Groups have, you know, hard things sometimes they have activities but not like finding squares". These responses indicate David's focus on maths as *hard*.

David's association of working hard with maths extended also to science. Science and maths were considered "quite the same but different . . . they both do experimenting, learning, working hard, doing dangerous things" (Task 1.3.1a). David made reference to experiments such as "how to get people to stay alive . . . when in a coma". Maths was seen to contribute to the scientific activity: "If you are doing science and you haven't learned maths yet you won't know how [to] cope with dangerous stuff" (Task 1.3.1a). David talked also of solving big problems such as "how to buy food without wasting money because you need the money for your fare" (Task 4.1) and of designing things in maths (Task 1.3.1b). Maths at school was

considered to include the solving of problems when in the Mathematics Task Centre but not in the classroom where "we just do times tables" (Task 10.1). These responses suggest David encountered applications that involved use of maths that for him was not straightforward. However, potential applications of mathematics in real-life situations were not always recognised by David, as illustrated below in the discussion of his references to measurement.

David included measure and measurement within his maths word wheel response (Task 1.3.1a) and stated that "in maths you add up things and measure them" (Task 5.2). A photograph of a man measuring spouting with a measuring tape was identified as maths because of the "the man measuring . . . the length of the house" (Task 5.2). Measuring objects with a ruler was identified as mathematical activity (Tasks 3.3.1, 6.1, 8.2). When viewing a picture of someone using a computer David said there was no maths "because you use rulers and other things in your own pencil case [for maths]" (Task 5.2). Two situations posed that might be considered to involve informal measurement were not judged as mathematical, one justified as such as "you usually use a ruler" for maths (Task 3.3.1). It appears that rulers and tape measures were tools that for David indicated mathematical activity. David stated also that "length is in maths and width and that" (Task 3.3.1) indicating the concepts also were seen as mathematical.

David classified the use of scales to weigh vegetables in a supermarket as maths because "the weight tells you how much you have to pay and that's like a calculator that people use in maths" (Task 5.2). His reference to paying suggests an understanding of one purpose of weighing vegetables. The scales appear to have been seen as a tool for calculation. Reference to weighing as mathematical activity was repeated two photographs later when the use of scales to weigh a fish was considered mathematical as "he's scaling up again . . . measuring how fat or how big he is" (Task 5.2).

Measuring using standard unit tools such as rulers, trundle wheels and scales was considered mathematical by David. However, the activity shown in Task 6.1, of measuring objects with a ruler in the home environment but for a school purpose, was the only measurement activity volunteered by David; all other instances were identified from prompts introduced by the interviewer. Also, some activities that were identified by some respondents as involving measurement were identified by David as mathematical because of "hard work" and "concentrating" as discussed above. David did not make reference to specific measurement activities carried out by himself or others. Unlike Cara and Ben, David did not appear to have non-school, family based measurement experiences and did not demonstrate an affinity with maths through such measurement activities. For one photograph of a person laying a garden path, David was "not quite sure" whether there was maths because "he's a grown man and he would have had maths a long long time ago, it's just a man working on a house" (Task 5.2), suggesting a posssible association between maths and schooling. In summary, there was limited recognition of adult measurement activities as mathematics because of the measurement features. Nor did David consider two activities chosen as involving informal measurement to be either measurement or maths. In addition, although David identified many non-school activities presented to him as mathematical, he tended to associate maths more with school than non-school situations. On one occasion he stated that "I think maths is a lot of hard stuff at school" (Task 8.2), but other responses as discussed above indicate that non-school "hard" tasks could also be identified as mathematical. When proffering mathematical activities, David focused on school or school-based situations.

Number concepts and processes emerged also from the interviews as elements of mathematics for David. Division and times tables were mentioned (e.g., Tasks 4.1, 8.1.1), and using calculators was seen as mathematical activity (Tasks 3.3.1, 5.2, 7.2). Within Task 3.3.1 David referred several times to groups, counting in groups, or making groups. For example, he stated that "maths has numbers and you try to put them together in groups and figure them out, like times", and said that adding nineteen and four on a calculator was maths as "she counted them and put them in divided groups". For the scenario where a person said that one hundred cents is the same as one dollar, David identified maths as "he counted in groups". His response was the "same, in groups" for a person paying \$3.20 for a hamburger and coke. In Task 3.3.2, buying clothes at a sale was described as having *some* maths as "you're buying things and you, um, are putting them in the groups for children to wear". Using a calculator to work out the money to pay the bank was judged as having *lots* of maths, because "in different groups because, you have to get money from the bank and they pay the house if you're renting it".

In Task 5.2 the processes of "adding up" at school and "adding up their money" when paying for shopping were described as maths. In addition, "writing down" of algorithms by the students and the teacher were described as mathematical activity. David explained that counting was only maths if a person was "counting in [their] mind so other people don't hear". Although counting was referred to as mathematical activity (Tasks 3.3.1, 7.2), counting with fingers was not considered to be maths because "other kids would learn and you would have spoke up and kids would have knew what the answer was, other kids would have knew what the answer was" (Task 5.2). David appeared concerned about children hearing answers from other children, indicating that such would not be considered mathematical activity. During the previous interview David had stated that "counting is like doing maths but not, but doing it in a different way. In maths you don't talk and in counting you do talk" (Task 4.1), again suggesting that when thinking about mathematical activity he was focusing on behaviours and not on cognitive or content aspects.

The presence of numbers did not guarantee that David would identify a situation as mathematical. For example, items in Task 3.3.1 in which numbers were distracting elements, such as "George cleaned up room number 7 which was really messy" and "Susie ran over to Anna's house to see her first dog" were not considered as mathematical activity. "Samira said that half a chocolate bar is better than a third of a chocolate bar" also was not judged as mathe

although David did see another Task 3.3.1 situation as maths that he described as "thinking of trying to divide them in half".

Two items in Task 3.3.1, "Chris cut up shapes of squares and circles and made a design", and "Louis found all his squares in the pictures on the page", made reference to shapes but were not identified by David as mathematical. Similarly, David responded that cutting a banana into "different shapes" was not maths (Task 5.2). These responses suggest that David did not readily associate working with shapes as mathematical activity. In Task 7.1, David did not comment upon whether the use of geoboards to make shapes was or was not mathematical activity, but in Task 8.2 stated that it was "just making pictures". It appears that David did not think readily of activity with shapes as mathematical. Likewise, he did not make any reference to chance and data concepts when discussing maths. A chance situation where children were predicting colours of objects to be taken from a bag was described as "Guessing game, it wouldn't be maths. They'd be playing" (Task 7.2). David went on to talk about the need to listen and concentrate and work hard for maths, a theme discussed in detail above.

In summary of David's beliefs about the nature of mathematics, a schematic portrayal below portrays for the reader key themes including those related to content areas, and the interrelationships between themes, such as between Concentrating, Listening and Working hard.



Finally, as for the other children, it is of interest to consider David's beliefs about the terms *maths* and *mathematics*. In Task 4.1, when asked for an example of a maths project, a topic that he had initiated, he replied,

mathmatic, mathsmatic [*sic*], it's the same as, it's the same as maths but in different ways. Because in mathsmatic [*sic*] you, um, figure out problems and in maths you do, um, you figure out things, you have to try and solve them. Because in maths you use rulers and that so have to solve problems. But in mathmatic [*sic*] you don't use any rulers.

He went on to talk about something he had seen on television, about an experiment involving a car. The discussion appeared confused and confusing suggesting that mathematics was a word to which David had been exposed, but of which he was still trying to make meaning. On another occasion (Task 11.2), when asked about mathematics, David stated that he was not sure what mathematics was but thought it was different from maths. Once again he talked about a television program that he thought involved mathematics, where people made things. David's beliefs about the nature of mathematics and its relationship to maths appeared still in the process of development.

Beliefs about the nature of learning

David appeared to associate learning mainly with listening to the teacher, working hard and completing work, remembering, and thinking.

For the password game, David suggested that in lieu of the word *learning* he would pass the word *listening* because "when you're learning you're listening". When then asked what else he does when he is learning, he replied, "I don't know, I don't have a clue". (Task 1.1a). The conversation continued:

Interviewer: Well how do you know when you've learned something? David: You listen, then you learn, things

I: Always, if you listen do you always learn?

D: Not always

I: Mhm, but even if you do, how do you know, say a week later, that you've learnt it?

D: Um

I: How do you know that you can say to yourself 'Oh I learnt that'?

D: Because you've done it, you've tried really hard to do it but then you done it.

Although David had the opportunity on this occasion to talk of other things demonstrating that he had learned, such as remembering or understanding, he seemingly related learning more to listening, hard work and completion.

It appears that David believed that listening would result in correctness, an important element of learning in mathematics for David: "You learn things, how to do, you learn how to design things in maths, so you don't end up having mistakes" (Task 1.3.1b). As an example of his learning in maths, David talked of learning his multiplication "tables", and then spoke of "learning things about maths and listening to what the teacher has to say, and working hard" (Task 1.3.1b). Later he referred to listening to and obeying the teacher suggesting the teacher has a role as director or instructor and the student has a more passive role. When asked

whether he listens to anyone else in his learning, David responded in the negative (Task 1.1a). The teacher appeared to be considered also as an imparter of knowledge: "I need to listen to my teacher . . . because there might be important things that I have to get". (Task 6. 2). It appears that for David, learning begins with receiving instructions, directions or information from the teacher. Seemingly related to this, David talked of telling the teacher "the answers she wants you to know" (Task 7.2) suggesting that he may have perceived a purpose for learning maths was to complete work to the satisfaction of the teacher.

Remembering was also seen by David as an element of learning. When asked to draw a picture of a person doing some sort of maths activity, David produced a drawing of a boy in a classroom "trying to learn his one times tables and twos" (Task 3.1). David described the boy's learning: "He's using his mind to try not to forget any of it or the questions" implying that remembering was considered a part of learning mathematics. When posed with the word *memorise*, David stated that he used this in his learning of maths (Task 10.1), and when given a range of words to choose those he associated with learning, David selected *remember* "because you remember what you done the other day so you can remember what you done today. You need to remember things so when you grow up you might do the same thing and you might know the answer" (Task 10.3).

It appears that David believed that as well as listening and remembering, thinking is a part of learning. In one interview he was not sure whether this was the case when learning maths (Task 1.1a) but thought on another occasion that "in maths you think a little bit" (Task 1.3.1b). David contrasted this with art: "When you're thinking in art you have to design. And you might get it wrong all the time, so you do things and you keep on thinking until you get it right" (Task 1.3.1b). It appears that in this interview David did not associate thinking in learning or doing maths with such an active process he saw in art. David stated that in maths "you just think about what you're going to do and then you just write it down" (Task 10.1), suggesting thinking may have related to remembering. Thinking was mentioned as a factor that could help David do better in maths (Task 9.1) and that could help him to learn maths (Task 9.2), suggesting that it was considered part of the learning process.

When posed with the term, *practise it*, David indicated that believed he did not use this when learning maths. He stated that "you don't practise in maths, we just go in and do maths" (Task 10.1).

In summary, David appeared to associate learning mostly with listening to the teacher and getting correct answers as well as with remembering and thinking. David believed that learning is evidenced by hard work and completion of a task. He appeared to see the purpose of learning maths was to design things and not make mistakes. It appears that David believed that learning should be legitimate, that is, achieved through hard work and without cheating, as discussed in more detail below.

Helping factors for learning maths

As discussed below, David identified factors that he believed both helped and hindered his learning of maths. The main themes that emerged in regard to helping factors for learning maths were

- helping oneself by listening, concentrating, thinking, and learning from mistakes;
- the teacher as the main source of information and direction for learning;
- the importance of directions or instructions for learning;
- ambivalence towards working with others;
- the role of materials in helping for learning maths; and
- ambivalence regarding whether a quiet environment helped in David's learning of maths.

David's description of a time when maths was easy for him to learn (Task 8.1.1) gives insights into the elements he saw within the activity and into his perception of helping factors for learning maths. David described that he was writing in his book, using blocks, counting, doing hard work, listening to what the teacher was saying, and concentrating. He identified concentrating as the most helpful factor for his learning in that situation. He identified this as a helping factor also in response to Task 7.1. David believed that circumstances that hindered concentration, such as measuring outside, would not help in his learning of maths (Task 8.2).

The teacher was perceived to play an important role in helping David to learn maths. She would help by explaining, telling, writing on the board what David had to do (Tasks 7.1, 7.2, 8.2, 9.2, 10.1), or by doing examples (Task 8.2). In recounting a harder experience of learning maths, David referred to a time when his teacher did not give instructions (Task 8.1.2). David felt also that the teacher could help by giving clues for maths problems (Tasks 9.1, 11.3). David believed that, in turn, he could help himself by listening to the teacher (Tasks 7.1, 8.1.1, 9.1). When talking of a child in his class who had difficulty in maths, David attributed her problems to not listening to the people who gave her instructions (Task 10.1). When asked what he would suggest to help her, he stated, "just listen!".

David believed also that he could help himself to learn maths by thinking and trying (Task 9.1) and by learning from his mistakes "because sometimes people make mistakes in maths and then they get better at them" (Task 10.1).

The need for instructions extended also to the situation of working with others. David believed that working in a group situation would be "pretty good" with the children explaining to each other what they had to do (Task 7.2). David believed having children tell others in the class what they were thinking was a "little bit good [as they could] have a good educate [*sic*]" and that the situation would help them to learn a little. However, when he elaborated he said that his partner (in the Mathematics Task Centre) "told [him] a few things that [they] had to do and that helped [him]" (Task 7.2). David's explanation suggests that other children would help in his learning through giving information or instructions about what had to be done, just as the teacher would help. A further response confirmed David's perception that a group could help because of the assistance it could give with instructions:

"Because if you're not in a group and . . . you're not [learning] and you have hearing problems, and there's no one in your group to help you, um you might not know what to do" (Task 8.2). The focus on others helping by explaining what to do and not giving answers was evident also in Task 9.2.

David believed a partner could help him to learn maths also as "you might have worked out to get the answers together" (Task 8.2). He thought that a partner "might know more than you and . . . you can learn their things off them" (Task 9.2). David was not sure whether he talked and discussed with other students when learning maths, but felt that the best student at maths could help David "get better, get good at maths" if they worked together (Task 10.1). Other children giving answers in a times tables race game was seen as an opportunity for learning: after you get a problem wrong "you keep listening so you can learn more" (Task 9.2).

However, David displayed some ambivalence towards working with others. He held a seeming concern for legitimate learning, that is, learning through one's own work and not by hearing or seeing the work of others. David expressed concern about people learning counting by hearing others or by seeing others' use of fingers (Task 5.2), as discussed earlier. He did not want other children to give him answers as "you need to learn and you need to think" (Task 9.2). David preferred to do maths in his head rather than on paper "because people look at your work when you're doing it on paper" (Task 7.2). These responses suggest David believed a person should learn for him or herself and that seeing or hearing answers from others was cheating rather than learning, although he did not use that term.

David was comfortable with working by himself, saying that both working alone and working with others could help him to learn maths (Tasks 9.2, 10.1), but, as discussed above, also showed some ambivalence towards working with others.

In relation to help from parents, David replied that, yes, a person could be helped to learn maths by working "at home with their mother asking them questions", if it was maths, as they "might get a good educate [*sic*] in maths" (Task 9.2). He was no more specific in regards to potential help from parents.

In the discussions with David, materials including calculators, computers, published maths books, blocks and fingers were considered for their potential in helping him to learn maths. When shown situations of children using calculators, David stated that the calculator would help him to learn maths (Tasks 7.2, 8.2, 9.2) when doing a "hard sum [such as] division" (Task 8.2) and as it can give "the answer so you can learn the times table that you don't know" (Task 9.2). However, he had not had experience of using a computer in learning maths and therefore was not sure whether it would help him (Task 8.2). Looking in published maths books also was considered potentially helpful (Tasks, 8.2, 9.1, 9.2) as they would give information (Task 8.2) or problems to work on (Task 9.2) but David stated that he did not read books or look up things in maths (Task 10.1). Although David stated that he used blocks when learning maths (Task 10.1), he did not identify the use of blocks as helping him to learn

maths as he felt that he would "find out the answer straight away" (Task 9.2). He believed blocks could be helpful for younger children but not for himself as he would "think in [his] brain" (Task 9.2). Likewise, he felt that counting on fingers could "help little kids" to learn maths as they "don't know how to count" but he would use his "brain to think" (Task 9.2). These responses suggest for David a differentiation between doing maths (with blocks) and learning maths (by thinking). There is also the suggestion that finding an answer straight away with assistance is not legitimate when one is *learning* maths as distinct from *doing* maths.

David gave some seemingly ambivalent views regarding the impact of the physical environment on his learning of maths. When asked whether he needed to learn in quiet areas (Task 9.2), he stated, "I can learn anywhere". However, as mentioned above, he believed that measuring outside would not help in his learning of maths (Task 8.2) as he would not be able to concentrate. When talking of the home environment, he stated that although sometimes he can learn at home, he cannot when his parents and brother or sister are at home as "they're around the house making noise [and] you can't think, you can't hear yourself think" (Task 9.2). A quiet environment to enable thinking was cited as a helping factor for learning maths also in Task 10.1.

To summarise, David considered the teacher to play a major role in his learning of maths, by giving information on the maths he had to do. He believed that he could help himself in learning maths by concentrating, listening, thinking, trying, and learning from his mistakes. He appeared comfortable to work by himself. He showed some ambivalence towards working with others: he found a group helpful, particularly when others could give information on what had to be done, but appeared to hold some concern about the legitimacy of getting answers from others in a group when learning maths. David felt that a parent could potentially help with learning maths. From the range of materials considered, calculators and published maths books were identified as helpful. David felt that younger children could also be helped by the use of blocks and fingers but because he knew how to count he would use his brain for thinking. He appeared ambivalent also about whether a quiet environment assisted in his learning of maths.

Overall, it can be said that David was able to respond to the tasks posed, although at times his views, such as about the nature of mathematics, appeared a little confused. His views about the nature of maths were idiosyncratic: it appeared that elements related to working hard and concentrating featured for David. Elements of number such as adding up and counting featured in his discussions of the content of maths, although formal measurement was recognised also. However, even within some of these, his views were at times related to behaviours as much as the content, for example, as demonstrated in his beliefs about counting only being maths if it was done silently. He did see a purpose to maths, as demonstrated in his discussion of designing things and solving big problems.

David's views about the nature of maths, learning maths, and helping factors appeared closely related. Thus, although at times his views did appear a little confused, he portrayed a

consistent picture. As stated earlier, he appeared to believe that learning of maths began with instructions from a teacher, followed by hard work by the child. Having received information from the teacher, a child could think and remember for learning, but also would need to work hard, concentrate and listen. By focusing on the latter, David would help himself to learn maths.

In conclusion, it can be stated that

- David held views about the nature of mathematical activity that he was able to express in response to the research procedures.
- David's image of his own mathematical activity focused mainly on behaviours to prepare himself for learning from the teacher rather than on cognitive or content aspects.
- David had some initial difficulty formulating and expressing ideas about the nature of learning but was able to provide some insights.
- David had a somewhat narrow view of learning maths illustrated by the salience of behaviours such as listening and working hard, compared to less emphasis given to cognitive activities such as thinking and remembering.
- David was able to reflect upon and express beliefs about helping factors for learning maths but showed ambivalence towards some factors.
- The range of factors identified was broad but for some factors the type of help suggested was narrow. For example, the teacher was identified as helping David mainly by giving directions for what he had to do.

APPENDIX F

PPELEM: Pupil Perceptions of Effective Learning Environments in Mathematics

A class administered task

| NAME | | GRADE | | |
|------|--------|--------|--|--|
| | | | | |
| DATE | . MALE | FEMALE | | |

- 1. What in the situation you have chosen is <u>most</u> helping you to learn maths well, and why?
- 2. What things could you not draw which were helping you to learn maths? How were they helping you?
- 3. <u>Where</u> does the picture show you to be?
- 4. <u>Who</u> is in your picture?
- 5. What maths are you <u>doing</u>?
- 6. What maths are you <u>learning</u>?
- 7. What things are you <u>using</u> in your picture?
- 8. <u>When have you experienced what you have drawn?</u>
- 9. Add anything else you can to tell about the situation you have chosen.
APPENDIX G

Teacher Interview

Name:

School:

Date:

Time:

- 1. I'd like to start by asking you to define mathematics.
- 2. Some people talk about mathematical activity. What do you see as mathematical activity?
- 3. What do you see as being the big or key ideas of mathematics for grade 3?
- 4. Now I'd like you to define learning.
- 5. How long have you been teaching?
- 6. Describe your teaching experience.
- 7. What part of maths do you most enjoy teaching? Why?
- 8. What part of maths do you think you have given most emphasis to this year? Why?
- 9. Is there any part of maths you haven't given much emphasis to this year? Why not?
- 10. Describe a typical maths lesson in your class.
- 11. What do you see as your role in a maths lesson?
- 12. What actions do <u>you</u> take that you think are <u>most</u> important in helping the children in your class to learn mathematics?
- 13. What do you believe are important actions for the <u>children</u> to help them help <u>themselves</u> learn mathematics? How do you build these into your lessons?
- 14. What do you believe are important actions for the children to help them help <u>each other</u> learn mathematics? How do you build these into your lessons?
- 15. You have mentioned that the children in your class are given homework. How often would they be given maths homework? What would they typically be asked to do?
- 16. In what way and to what degree do you think the parents of the children you teach are involved in their children's learning of mathematics?

17. For the following, please tell me how much emphasis you think you have given each of these in your maths program this year.

| | A lot | Some | Not much, or |
|------------------------------|-------|------|--------------|
| | | | none at all |
| Problem solving | | | |
| Use of calculators | | | |
| Use of estimation | | | |
| Measuring | | | |
| Use of computers | | | |
| Applying mathematics to real | | | |
| life problems | | | |
| Writing mathematics | | | |
| The 4 operations | | | |
| Counting | | | |
| Concrete materials | | | |
| Spatial work | | | |
| Chance and data | | | |
| Discussing in mathematics | | | |
| Children justifying in | | | |
| mathematics | | | |
| Games | | | |
| Multiplication tables | | | |
| | | | |
| | | | |

Possibly probe further such as: Tell me about this.

- 18. For yourself, how do you know when you have learned something?
- 19. Is maths taken at a set time, i.e. the same time each day?
- 20. What do you hope for the children to leave your class with in terms of how or what they think about maths?
- 21. Tell me about the teaching of space.
- 22. Tell me about the teaching of measurement.
- 23. Tell me about the teaching of chance and data.
- 24. Tell me about the teaching of number.

APPENDIX H

Reference to interview procedures in reporting of results

Detailed version

*Each task referred to in the write-up of results is identified with a tick, irrespective of the number of references made.

Anna

High-achiever Female School S

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|--|-------|------|--------------|----------------------|----------------------|
| 1.1 | Word association quiz | M, L | | ~ | | |
| 1.2 | Password: Word game | M, L | | ~ | | |
| 1.3 | Word wheels: Words or phrases to | | | | | |
| | describe "maths" and another suitable | | | | | |
| | subject, eg."music" | | | | | |
| | 1.3.1 Maths | Μ | | ~ | | |
| | 1.3.2 "Other" subject | Μ | | | | ~ |
| 1.4 | Subject most like maths; subject least | М | | ~ | | |
| | like maths? Why? | | | | | |
| 2.1 | Personal dictionary: Learn | L | | | ~ | |
| | Children give own definition and discuss | | | | | |
| | in context of personal experiences | | | | | |
| 2.2 | Descriptions of learning situations | L | | | ~ | |
| 2.3 | Personal dictionary: Maths | М | | ~ | | |
| | Children give own definition and discuss | | | | | |
| | in context of personal experiences | | | | | |
| 3.1 | Draw a mathematical activity | М | | ~ | | |
| 3.2 | Show another mathematical activity (act | М | | ~ | | v |
| | out, write, draw etc.) | | | | | |
| 3.3 | Questionnaires - Mathematics in | М | | | | |
| | everyday activities | | | | | |
| | 3.3.1 (Adapted from McDonald & | | | ~ | | |
| | Kouba, 1986) | | | | | |
| | 3.3.2 (Adapted from Wallbridge & | | | \checkmark | | |
| | Clarke, 1989) | | | | | |
| 4.1 | Personal writing: "Maths is like", | М | ~ | | | |
| | and discussion | | | | | |
| 4.2 | Drawings: People doing maths inside | Μ | | | | |
| | and outside school | | | | | |
| | 4.2.1 Drawing one | | | v | | ~ |
| | 4.2.2 "Other" drawing | | | ~ | | ~ |
| 4.3 | Planning for an integrated unit of work | M, L | | ~ | | |
| 5.1 | Describing maths to an alien (Stodolsky, | Μ | | ~ | | |
| | Salk, & Glaessner, 1991) | | | | | |
| 5.2 | Photographs - mathematical activity? | Μ | | ~ | | |
| | (Zevenbergen & Crowe, 1992) | | | | | |
| | | | | | | |

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|---|----------|------|-------|----------|---|
| 6.1 | Situation in which learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | | ~ | ~ |
| 6.2 | Situation in which hindered from learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | ~ | | | ~ |
| 7.1 | Scenario: Child having difficulty, how would you help? | HF | | ~ | | |
| 7.2 | Video clips - Describe: Would you learn maths well in depicted situations? | HF | • | | ~ | ~ |
| 8.1 | Easier and harder experiences in the learning maths: Perceived effectiveness of learning and factors influencing the learning are discussed 8.1.1 Easier 8.1.2 Harder | HF | | | | ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ |
| 8.2 | Describe and sort photographs (home situations and school situations): Learn maths well? | HF | ~ | ~ | ~ | ~ |
| 8.3 | Discussion - how feel about learning maths | S | | | | |
| 9.1 | I could do better in maths if | HF | | | | |
| 9.2 | Vignettes - Children's drawings of classroom, home, and other out of school situations. Select situations in which you would: learn maths well; be hindered from learning maths well. Discuss | HF | | | ~ | ~ |
| 10.1 | Written descriptors. Phrases/words provided, grouped by child and ranked (Stodolsky, Salk, & Glaessner, 1991). Discussion of use of these in learning maths | HF | | | V | |
| 10.2 | 10.2.1 Duplo: Construct/build situation in which you felt good when learning maths 10.2.2 Duplo: Construct/build situation in which you did not feel good when learning maths | HF | ~ | ~ | V | |
| 10.3 | Learn: What does this mean to you? Discussion | L | | | ~ | |
| 11.1 | How good at maths: 1 - 10 | S | ~ | | | |
| 11.2 | I think mathematics is | М | | | | |
| 11.3 | A good maths teacher | L, HF | | | | |

High-achiever Male School S

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|--|-------|------|-------|---|---|
| 1.1 | Word association guiz | L, M | | ~ | v | |
| 1.2 | Password: Word game | L, M | | | v | |
| 1.3 | Word wheels: Words or phrases to | 2 | | | | |
| | describe "maths" and another suitable | | | | | |
| | subject, e.g., "music" | | | | | |
| | 1.3.1 Maths | М | | ~ | ✓ | |
| | 1.3.2 "Other" subject | Μ | | ~ | | |
| 1.4 | Subject most like maths; subject least | М | | | | |
| | like maths? Why? | | | | | |
| 2.1 | Personal dictionary: Learn | L | | ~ | v | ~ |
| | Children give own definition and discuss | | | | | |
| | in context of personal experiences | | | | | |
| 2.2 | Descriptions of learning situations | L | | | | |
| 2.3 | Personal dictionary: Maths | М | | | | |
| | Children give own definition and discuss | | | | | |
| | in context of personal experiences | | | | | |
| 3.1 | Draw a mathematical activity | М | | ~ | | |
| 3.2 | Show another mathematical activity (act | М | | ~ | | |
| | out, write, draw etc.) | | | | | |
| 3.3 | Questionnaires - Mathematics in | М | | | | |
| | everyday activities | | | | | |
| | 3.3.1 (Adapted from McDonald & | | | ~ | | |
| | Kouba, 1986) | | | | | |
| | 3.3.2 (Adapted from Wallbridge & | | | ~ | | |
| | Clarke, 1989) | | | | | |
| 4.1 | Personal writing: "Maths is like", | М | | ~ | | |
| | and discussion | | | | | |
| 4.2 | Drawings: People doing maths inside | М | | | | |
| | and outside school | | | | | |
| | 4.2.1 Drawing one | | | ~ | | ~ |
| _ | 4.2.2 "Other" drawing | | | ~ | | |
| 4.3 | Planning for an integrated unit of work | M, L | | ~ | | |
| 5.1 | Describing maths to an alien (Stodolsky, | Μ | | ~ | Image: A start of the start of | Image: A start of the start of |
| | Salk, & Glaessner, 1991) | | | | | |
| 5.2 | Photographs - mathematical activity? | Μ | | ~ | | Image: A start of the start of |
| | (Zevenbergen & Crowe, 1992) | | | | | |

Ben

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|---|----------|------|-------|----------|---------|
| 6.1 | Situation in which learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | ~ | ~ | ~ |
| 6.2 | Situation in which hindered from learning maths well: Draw, describe, and identify most influential factors | HF | | ~ | | |
| | (ranking) | | | | | |
| 7.1 | Scenario: Child having difficulty, how would you help? | HF | | | | ~ |
| 7.2 | Video clips - Describe: Would you learn maths well in depicted situations? | HF | | | | ~ |
| 8.1 | Easier and harder experiences in the learning maths: Perceived effectiveness of learning and factors influencing the learning are discussed 8.1.1 Easier 8.1.2 Harder | HF | | | | v |
| 8.2 | Describe and sort photographs (home situations and school situations): Learn maths well? | HF | | ~ | ~ | ~ |
| 8.3 | Discussion - how feel about learning maths | S | • | | | |
| 9.1 | I could do better in maths if | HF | | | | |
| 9.2 | Vignettes - Children's drawings of classroom, home, and other out of school situations. Select situations in which you would: learn maths well; be hindered from learning maths well. Discuss | HF | | | V | V |
| 10.1 | Written descriptors. Phrases/words provided, grouped by child and ranked (Stodolsky, Salk, & Glaessner, 1991). Discussion of use of these in learning maths | HF | | | ~ | V |
| 10.2 | 10.2.1 Duplo: Construct/build situationin which you felt good when learningmaths10.2.2 Duplo: Construct/build situationin which you did not feel good whenlearning maths | HF | | | | v |
| 10.3 | Learn: What does this mean to you? Discussion | L | | | | |
| 11.1 | How good at maths: 1 - 10 | S | ~ | | | |
| 11.2 | I think mathematics is | M | | ~ | | |
| 11.3 | A good maths teacher | L, HF | | | | |

Low-achiever Female

School S

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|---|--------|------|---|--------------|----------|
| 1.1 | Word association quiz | L, M | | ~ | ✓ | |
| 1.2 | Password: Word game | L, M | | ~ | ✓ | v |
| 1.3 | Word wheels: Words or phrases to describe "maths" and another suitable subject e g "music" | | | | | |
| | 1.3.1 Maths 1.3.2 "Other" subject | M M | | ~ ~ | ~ | |
| 1.4 | Subject most like maths; subject least like maths? Why? | М | • | | | |
| 2.1 | Personal dictionary: Learn Children give own definition and discuss in context of personal experiences | L | | | V | |
| 2.2 | Descriptions of learning situations | L | | | | |
| 2.3 | Personal dictionary: Maths Children give own definition and discuss in context of personal experiences | М | | ~ | V | |
| 3.1 | Draw a mathematical activity | М | | ~ | | |
| 3.2 | Show another mathematical activity (act out, write, draw etc.) | М | | ~ | | |
| 3.3 | Questionnaires - Mathematics in everyday activities | М | | | | |
| | 3.3.1 (Adapted from McDonald & Kouba, 1986) | | | ~ | \checkmark | |
| | 3.3.2 (Adapted from Wallbridge & Clarke, 1989) | | | ~ | | |
| 4.1 | Personal writing: "Maths is like", and discussion | М | | ~ | | |
| 4.2 | Drawings: People doing maths inside and outside school 4.2.1 Drawing one 4.2.2 "Othor" drawing | М | | ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ | | |
| 12 | Planning for an integrated unit of work | МТ | | ~ | ~ | |
| 5.1 | Describing maths to an alien (Stodolsky, Salk, & Glaessner 1991) | M | | · · | • | ~ |
| 5.2 | Photographs - mathematical activity? (Zevenbergen & Crowe, 1992) | М | • | ~ | | |

Cara

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|--|----------|------|-------|--------------|-----------------------|
| 6.1 | Situation in which learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | ~ | ~ | ~ |
| 6.2 | Situation in which hindered from learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | | ~ | v |
| 7.1 | Scenario: Child having difficulty, how would you help? | HF | | | • | ~ |
| 7.2 | Video clips - Describe: Would you learn maths well in depicted situations? | HF | | ~ | ~ | ~ |
| 8.1 | Easier and harder experiences in the learning maths: Perceived effectiveness of learning and factors influencing the learning are discussed 8.1.1 Easier 8.1.2 Harder | HF | | | | v |
| 8.2 | Describe and sort photographs (home situations and school situations): Learn maths well? | HF | | ~ | ~ | ~ |
| 8.3 | Discussion - how feel about learning maths | S | ~ | | ~ | |
| 9.1 | I could do better in maths if | HF | | | | ✓ |
| 9.2 | Vignettes - Children's drawings of classroom, home, and other out of school situations. Select situations in which you would: learn maths well; be hindered from learning maths well. Discuss | HF | | | V | V |
| 10.1 | Written descriptors. Phrases/words provided, grouped by child and ranked (Stodolsky, Salk, & Glaessner, 1991). Discussion of use of these in learning maths | HF | | ~ | ~ | V |
| 10.2 | 10.2.1 Duplo: Construct/build situation in which you felt good when learning maths 10.2.2 Duplo: Construct/build situation in which you did not feel good when learning maths | HF | ~ | V | | |
| 10.3 | Learn: What does this mean to you? Discussion | L | | | ~ | |
| 11.1 | How good at maths: 1 - 10 | S | ~ | | ~ | |
| 11.2 | I think mathematics is | Μ | | ~ | ~ | |
| 11.3 | A good maths teacher | L, HF | | | \checkmark | |

David

Low-achiever Male School S

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|---|-------------|------|--------------|----------|----------|
| 1.1 | Word association quiz | L. M | | ~ | | v |
| 1.2 | Password: Word game | L. M | | | | |
| 1.3 | Word wheels: Words or phrases to |) | | | | |
| | describe "maths" and another suitable | | | | | |
| | subject, e.g., "music" | | | | | |
| | 1.3.1 Maths | Μ | | ~ | ✓ | |
| | 1.3.2 "Other" subject | Μ | | ~ | | |
| 1.4 | Subject most like maths; subject least | М | | | | |
| | like maths? Why? | | | | | |
| 2.1 | Personal dictionary: Learn | L | | | | |
| | Children give own definition and | | | | | |
| | discuss in context of personal | | | | | |
| | experiences | | | | | |
| 2.2 | Descriptions of learning situations | L | | | | |
| 2.3 | Personal dictionary: Maths | Μ | | | | |
| | Children give own definition and | | | | | |
| | discuss in context of personal | | | | | |
| | experiences | | | | | |
| 3.1 | Draw a mathematical activity | Μ | | | ~ | |
| 3.2 | Show another mathematical activity | Μ | | ~ | | |
| | (act out, write, draw etc.) | | | | | |
| 3.3 | Questionnaire s- Mathematics in | М | | | | |
| | everyday activities | | | | | |
| | 3.3.1 (Adapted from McDonald & | | | ~ | | |
| | Kouba, 1986) | | | | | |
| | 3.3.2 (Adapted from Wallbridge & | | | V | | |
| | Clarke, 1989) | | | | | |
| 4.1 | Personal writing: "Maths is like", | Μ | | \checkmark | | |
| | and discussion | | | | | |
| 4.2 | Drawings: People doing maths inside | Μ | | | | |
| | and outside school | | | | | |
| | 4.2.1 Drawing one | | | | | |
| | 4.2.2 "Other" drawing | | | | | |
| 4.3 | Planning for an integrated unit of work | <u>M, L</u> | | | | |
| 5.1 | Describing maths to an alien | Μ | | | | |
| | (Stodolsky, Salk, & Glaessner, 1991) | | | . 4 | . 4 | |
| 5.2 | Photographs - mathematical activity? | Μ | | V | V | |
| | (Zevenbergen & Crowe, 1992) | | | | | |

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|--|-------|------|-------|----------|----------|
| 6.1 | Situation in which learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | ~ | | |
| 6.2 | Situation in which hindered from learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | | ~ | |
| 7.1 | Scenario: Child having difficulty, how would you help? | HF | | ~ | | ~ |
| 7.2 | Video clips - Describe: Would you learn maths well in depicted situations? | HF | ~ | ~ | ~ | ~ |
| 8.1 | Easier and harder experiences in the learning maths: Perceived effectiveness of learning and factors influencing the learning are discussed 8.1.1 Easier 8.1.2 Harder | HF | ~ | ~ | | |
| 8.2 | Describe and sort photographs (home situations and school situations): Learn maths well? | HF | • | ~ | | v |
| 8.3 | Discussion - how feel about learning maths | S | | | | |
| 9.1 | I could do better in maths if | HF | | | ✓ | ~ |
| 9.2 | Vignettes - Children's drawings of classroom, home, and other out of school situations. Select situations in which you would: learn maths well; be hindered from learning maths well. Discuss | HF | ~ | | V | V |
| 10.1 | Written descriptors. Phrases/words provided, grouped by child and ranked (Stodolsky, Salk, & Glaessner, 1991). Discussion of use of these in learning maths | HF | | ~ | ~ | V |
| 10.2 | 10.2.1 Duplo: Construct/build situation in which you felt good when learning maths 10.2.2 Duplo: Construct/build situation in which you did not feel good when learning maths | HF | | | | |
| 10.3 | Learn: What does this mean to you? Discussion | L | | | ~ | |
| 11.1 | How good at maths: 1 - 10 | S | ~ | | | |
| 11.2 | I think mathematics is | М | | | | |
| 11.3 | A good maths teacher | L, | | | | ~ |
| | | HF | | | | |

Emily

High-achiever Female School I

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|---|-------|------|--------------|--------------|-----------------------|
| 1.1 | Word association guiz | L, M | | ~ | v | |
| 1.2 | Password: Word game | L, M | | ~ | ~ | |
| 1.3 | Word wheels: Words or phrases to | , | | | | |
| | describe "maths" and another suitable | | | | | |
| | subject, e.g., "music" | | | | | |
| | 1.3.1 Maths | Μ | | ~ | ~ | |
| | 1.3.2 "Other" subject | М | | | | |
| 1.4 | Subject most like maths; subject least | Μ | | | | |
| | like maths? Why? | | | | | |
| 2.1 | Personal dictionary: Learn | L | ~ | | ~ | |
| | Children give own definition and | | | | | |
| | discuss in context of personal | | | | | |
| | experiences | | | | | |
| 2.2 | Descriptions of learning situations | L | | ~ | ~ | |
| 2.3 | Personal dictionary: Maths | Μ | | ~ | | |
| | Children give own definition and | | | | | |
| | discuss in context of personal | | | | | |
| | experiences | | | | | |
| 3.1 | Draw a mathematical activity | М | | ~ | | ✓ |
| 3.2 | Show another mathematical activity | Μ | | | \checkmark | |
| | (act out, write, draw etc.) | | | | | |
| 3.3 | Questionnaires - Mathematics in | Μ | | | | |
| | everyday activities | | | | | |
| | 3.3.1 (Adapted from McDonald & | | | \checkmark | | |
| | Kouba, 1986) | | | | | |
| | 3.3.2 (Adapted from Wallbridge & | | | ~ | | |
| | Clarke, 1989) | | | | | |
| 4.1 | Personal writing: "Maths is like", | Μ | | \checkmark | | |
| | and discussion | | | | | |
| 4.2 | Drawings: People doing maths inside | Μ | | | | |
| | and outside school | | | | | |
| | 4.2.1 Drawing one | | | V | | |
| | 4.2.2 "Other" drawing | | | v | | |
| 4.3 | Planning for an integrated unit of work | M, L | | V | | |
| 5.1 | Describing maths to an alien | Μ | | ~ | | |
| | (Stodolsky, Salk, & Glaessner, 1991) | | | | <u> </u> | |
| 5.2 | Photographs - mathematical activity? | Μ | | ~ | \checkmark | |
| | (Zevenbergen & Crowe, 1992) | | | | | |

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|--|----------|------|-------|----------|----------|
| 6.1 | Situation in which learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | ~ | ~ | ✓ |
| 6.2 | Situation in which hindered from learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | | | v |
| 7.1 | Scenario: Child having difficulty, how would you help? | HF | | | | ~ |
| 7.2 | Video clips - Describe: Would you learn maths well in depicted situations? | HF | | • | ~ | v |
| 8.1 | Easier and harder experiences in the learning maths: Perceived effectiveness of learning and factors influencing the learning are discussed 8.1.1 Easier | HF | | | V | |
| 8.2 | Describe and sort photographs (home situations and school situations): Learn maths well? | HF | | ~ | ~ | <u> </u> |
| 8.3 | Discussion - how feel about learning maths | S | ~ | | | |
| 9.1 | I could do better in maths if | HF | | | | v |
| 9.2 | Vignettes - Children's drawings of classroom, home, and other out of school situations. Select situations in which you would: learn maths well; be hindered from learning maths well. Discuss | HF | • | ~ | | V |
| 10.1 | Written descriptors. Phrases/words provided, grouped by child and ranked (Stodolsky, Salk, & Glaessner, 1991). Discussion of use of these in learning maths | HF | • | | ~ | ~ |
| 10.2 | 10.2.1 Duplo: Construct/build situation in which you felt good when learning maths 10.2.2 Duplo: Construct/build situation in which you did not feel good when | HF | | ~ | V | |
| 10.3 | learning maths Learn: What does this mean to you? | L | | | ~ | |
| 11 1 | | C | | | | |
| 11.1 | How good at maths: 1 - 10 | <u>S</u> | V | | | |
| 11.2 | I think mathematics is | M | | V | | |
| 11.3 | A good maths teacher | L, HF | | | V | V |

High-achiever Male School I

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|----------|---|-------|------|-------|--------------|--------------|
| 1.1 | Word association quiz | L, M | | ~ | ~ | |
| 1.2 | Password: Word game | L, M | | ~ | | |
| 1.3 | Word wheels: Words or phrases to | | | | | |
| | describe "maths" and another suitable | | | | | |
| | subject, e.g., "music" | | | | | |
| | 1.3.1 Maths | M | | ~ | | |
| <u> </u> | 1.3.2 "Other" subject | M | | V | | |
| 1.4 | Subject most like maths; subject least like maths? Why? | М | | | | |
| 2.1 | Personal dictionary: Learn | L | | | | |
| | Children give own definition and | | | | | |
| | discuss in context of personal | | | | | |
| | experiences | | | | | |
| 2.2 | Descriptions of learning situations | L | | | v | |
| 2.3 | Personal dictionary: Maths | М | ~ | ~ | v | |
| | Children give own definition and | | | | | |
| | discuss in context of personal | | | | | |
| | experiences | | | | | |
| 3.1 | Draw a mathematical activity | М | | ~ | | |
| 3.2 | Show another mathematical activity | М | | ~ | | |
| | (act out, write, draw etc.) | | | | | |
| 3.3 | Questionnaires - Mathematics in | М | | | | |
| | everyday activities | | | | | |
| | 3.3.1 (Adapted from McDonald & | | | V | | |
| | Kouba, 1986) | | | | | |
| | 3.3.2 (Adapted from Wallbridge & | | | V | | |
| 4.1 | Clarke, 1989) | | | | | |
| 4.1 | Personal writing: "Maths is like", | М | V | V | | |
| 12 | Drawings: People doing maths inside | M | | | | |
| 4.2 | and outside school | IVI | | | | |
| | 4.2.1 Drawing one | | | ~ | ~ | |
| | 4.2.2 "Other" drawing | | | ~ | ~ | |
| 4.3 | Planning for an integrated unit of | M, L | | ~ | | |
| | work | | | | | |
| 5.1 | Describing maths to an alien | Μ | ~ | ~ | \checkmark | \checkmark |
| | (Stodolsky, Salk, & Glaessner, 1991) | | | | | |
| 5.2 | Photographs - mathematical activity? | Μ | | ~ | | |
| | (Zevenbergen & Crowe, 1992) | | | | | |

Filip

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|--|----------|------|----------|----------|----------|
| 6.1 | Situation in which learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | | | ✓ |
| 6.2 | Situation in which hindered from learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | | | v |
| 7.1 | Scenario: Child having difficulty, how would you help? | HF | | | | ~ |
| 7.2 | Video clips - Describe: Would you learn maths well in depicted situations? | HF | | ~ | | ~ |
| 8.1 | Easier and harder experiences in the learning maths: Perceived effectiveness of learning and factors influencing the learning are discussed 8.1.1 Easier 8.1.2 Harder | HF | | | | V V |
| 8.2 | Describe and sort photographs (home situations and school situations): Learn maths well? | HF | • | ~ | ~ | ~ |
| 8.3 | Discussion - how feel about learning maths | S | | | | |
| 9.1 | I could do better in maths if | HF | | | | ✓ |
| 9.2 | Vignettes - Children's drawings of classroom, home, and other out of school situations. Select situations in which you would: learn maths well; be hindered from learning maths well. Discuss | HF | ~ | | V | ~ |
| 10.1 | Written descriptors. Phrases/words provided, grouped by child and ranked (Stodolsky, Salk, & Glaessner, 1991). Discussion of use of these in learning maths | HF | | | ~ | ~ |
| 10.2 | 10.2.1 Duplo: Construct/build situation in which you felt good when learning maths | HF | | | | |
| 10.2 | 10.2.2 Duplo: Construct/build situation in which you did not feel good when learning maths | T | | ~ | v | |
| 10.3 | Learn: What does this mean to you? Discussion | L | | | | |
| 11.1 | How good at maths: 1 - 10 | S | ~ | | | |
| 11.2 | I think mathematics is | Μ | | ~ | | |
| 11.3 | A good maths teacher | L, HF | ~ | | ~ | ~ |

Low-achiever Female

School I

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|---|-------|------|----------|--------------|---------|
| 1.1 | Word association quiz | L, M | | | v | |
| 1.2 | Password: Word game | L, M | | | ~ | |
| 1.3 | Word wheels: Words or phrases to describe "maths" and another suitable subject, e.g., "music" | | | | | |
| | 1.3.1 Maths | Μ | | ~ | | |
| | 1.3.2 "Other" subject | М | | | | |
| 1.4 | Subject most like maths; subject least like maths? Why? | М | | | | |
| 2.1 | Personal dictionary: Learn | L | | | \checkmark | |
| | Children give own definition and | | | | | |
| | discuss in context of personal | | | | | |
| | experiences | | | | | |
| 2.2 | Descriptions of learning situations | L | | v | v | |
| 2.3 | Personal dictionary: Maths | Μ | | | | |
| | Children give own definition and | | | | | |
| | discuss in context of personal | | | | | |
| 2.1 | experiences | | | | | |
| 3.1 | Draw a mathematical activity | M | | V | | V |
| 3.2 | Show another mathematical activity | Μ | | | | |
| | (act out, write, draw etc.) | | | | | |
| 3.3 | Questionnaires - Mathematics in | Μ | | | | |
| | 3.3.1 (Adapted from McDonald & | | | ~ | | |
| | Kouba, 1986) $2.2.2(A + A + A + A + A + A + A + A + A + A +$ | | | 1 | | |
| | 3.3.2 (Adapted from Wallbridge & | | | • | | |
| 4.1 | Clarke, 1989) | М | | ~ | | ~ |
| 4.1 | and discussion | IVI | | | | • |
| 4.2 | Drawings: People doing maths inside | М | | | | |
| | and outside school | | | | | |
| | 4.2.1 Drawing one | | | ~ | v | |
| | 4.2.2 "Other" drawing | | | V | V | |
| 4.3 | Planning for an integrated unit of | M, L | | | | |
| 5 1 | Describing maths to an alien | М | | ~ | | ~ |
| 5.1 | (Stodolsky Salk & Glaessner 1001) | TAT | | - | | • |
| 52 | Photographs - mathematical activity? | М | | ~ | | |
| 5.4 | (Zevenbergen & Crowe, 1992) | 141 | | - | | |

Gina

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|--|----------|--------|-------|----------|----------|
| 6.1 | Situation in which learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | | | ~ |
| 6.2 | Situation in which hindered from learning maths well: Draw, describe, and identify most influential factors (ranking) | HF | | | ~ | ~ |
| 7.1 | Scenario: Child having difficulty, how would you help? | HF | | ~ | ~ | ~ |
| 7.2 | Video clips - Describe: Would you learn maths well in depicted situations? | HF | | ~ | | |
| 8.1 | Easier and harder experiences in the learning maths: Perceived effectiveness of learning and factors influencing the learning are discussed 8.1.1 Easier 8.1.2 Harder | HF | | | | |
| 8.2 | Describe and sort photographs (home situations and school situations): Learn maths well? | HF | | • | ~ | ~ |
| 8.3 | Discussion - how feel about learning maths | S | | • | ~ | ~ |
| 9.1 | I could do better in maths if | HF | | | | v |
| 9.2 | Vignettes - Children's drawings of classroom, home, and other out of school situations. Select situations in which you would: learn maths well; be hindered from learning maths well. Discuss | HF | | | | V |
| 10.1 | Written descriptors. Phrases/words provided, grouped by child and ranked (Stodolsky, Salk, & Glaessner, 1991). Discussion of use of these in learning maths | HF | | | | V |
| 10.2 | 10.2.1 Duplo: Construct/build situation in which you felt good when learning maths 10.2.2 Duplo: Construct/build | HF | ~ ~ | | | |
| | situation in which you did not feel good when learning maths | | | | | |
| 10.3 | Learn: What does this mean to you? Discussion | L | | | ~ | |
| 11.1 | How good at maths: 1 - 10 | S | ~ | | | |
| 11.2 | I think mathematics is | Μ | | | | |
| 11.3 | A good maths teacher | L, HF | | | | |

Harry

Low-achiever Male School I

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|------|---|-------|------|----------|--------------|--------------|
| 1.1 | Word association quiz | M, L | | | ~ | |
| 1.2 | Password: Word game | M, L | | ~ | v | |
| 1.3 | Word wheels: Words or phrases to | | | | | |
| | describe "maths" and another | | | | | |
| | suitable subject, e.g., "music" | | | | | |
| | 1.3.1 Maths | Μ | | V | | |
| | 1.3.2 "Other" subject | Μ | | ~ | | |
| 1.4 | Subject most like maths; subject least like maths? Why? | М | | | | |
| 2.1 | Personal dictionary: Learn | L | | ~ | ~ | ~ |
| | Children give own definition and | | | | | |
| | discuss in context of personal | | | | | |
| | experiences | | | | | |
| 2.2 | Descriptions of learning situations | L | ~ | | ~ | ~ |
| 2.3 | Personal dictionary: Maths | Μ | ~ | ~ | ~ | |
| | Children give own definition and | | | | | |
| | discuss in context of personal | | | | | |
| | experiences | | | | | |
| 3.1 | Draw a mathematical activity | М | | | | |
| 3.2 | Show another mathematical activity | Μ | | | | |
| | (act out, write, draw etc.) | | | | | |
| 3.3 | Questionnaires - Mathematics in | Μ | | | | |
| | everyday activities | | | | | |
| | 3.3.1 (Adapted from McDonald & | | | | | |
| | Kouba, 1986) | | | | | |
| | 3.3.2 (Adapted from Wallbridge & | | | | | |
| 4.1 | Clarke, 1989) | | | | | |
| 4.1 | ", and discussion | Μ | V | V | V | V |
| 4.2 | Drawings: People doing maths inside | Μ | | | | |
| | and outside school | | | | | |
| | 4.2.1 Drawing one | | | V | \checkmark | \checkmark |
| | 4.2.2 "Other" drawing | | | <u> </u> | | |
| 4.3 | Planning for an integrated unit of | M, L | | V | | |
| | work | | | | | |
| 5.1 | Describing maths to an alien | Μ | | V | | |
| | (Stodolsky, Salk, & Glaessner, 1991) | | | | | 4 |
| 5.2 | Photographs - mathematical activity? | М | | V | | V |
| | (Zevenbergen & Crowe, 1992) | | | | | |

| Code | Description of procedure | Focus | Self | Maths | Learning | Helping |
|----------|---|-------|------|--------------|----------|-----------------------|
| 6.1 | Situation in which learning maths well: Draw, describe, and identify | HF | | | | ~ |
| <u> </u> | most influential factors (ranking) | | | | | |
| 6.2 | Situation in which hindered from | HF | | | | V |
| | learning maths well: Draw, describe, | | | | | |
| | and identify most influential factors | | | | | |
| | (ranking) | | | | | |
| 7.1 | Scenario: Child having difficulty, | HF | | V | | V |
| | how would you help? | | | | | |
| 7.2 | Video clips - Describe: Would you | HF | | \checkmark | | \checkmark |
| | learn maths well in depicted | | | | | |
| | situations? | | | | | |
| 8.1 | Easier and harder experiences in the | HF | | | | |
| | learning maths: Perceived | | | | | |
| | effectiveness of learning and factors | | | | | |
| | influencing the learning are discussed | | | | | |
| | 8.1.1 Easier | | | | | |
| | 8.1.2 Harder | | | | | ✓ |
| 8.2 | Describe and sort photographs (home | HF | | \checkmark | | \checkmark |
| | situations and school situations): | | | | | |
| | Learn maths well? | | | | | |
| 8.3 | Discussion - how feel about learning | S | ~ | ~ | | |
| | maths | | | | | |
| 9.1 | I could do better in maths if | HF | | | ~ | ~ |
| 9.2 | Vignettes - Children's drawings of | HF | ~ | ~ | ~ | ~ |
| | classroom, home, and other out of | | | | | |
| | school situations. Select situations in | | | | | |
| | which you would: learn maths well; | | | | | |
| | be hindered from learning maths well. | | | | | |
| | Discuss | | | | | |
| 10.1 | Written descriptors. Phrases/words | HF | | | ~ | ~ |
| | provided, grouped by child and | | | | | |
| | ranked (Stodolsky, Salk, & Glaessner, | | | | | |
| | 1991). Discussion of use of these in | | | | | |
| | learning maths | | | | | |
| 10.2 | 10.2.1 Duplo: Construct/build | HF | | | | ~ |
| | situation in which you felt good when | | | | | |
| | learning maths | | | | | |
| | 10.2.2 Duplo: Construct/build | | | | | \checkmark |
| | situation in which you did not feel | | | | | |
| | good when learning maths | | | | | |
| 10.3 | Learn: What does this mean to you? | L | | | ~ | |
| | Discussion | | | | | |
| 11.1 | How good at maths: 1 - 10 | S | ~ | | | v |
| 11.2 | I think mathematics is | Μ | | | | |
| 11.3 | A good maths teacher | L, | | | ~ | ~ |
| | | HF | | | | |

APPENDIX I

Reference to interview procedures in reporting of results

Summary version

The data in the table below are organised in two ways:

Firstly, letters are used to indicate the names of the children for whom reference is found to the particular procedures in the discussion of themes (that is, A = Anna, B = Ben, C = Cara, D = David, E = Emily, F = Filip, G = Gina, H = Harry).

Secondly, the table is divided into columns. The numbers within the titles of these columns refer to the number of domains within which children's beliefs are discussed with a possibility of up to four domains: Self, Maths, Learning, and Helping factors for learning maths. Entries in the column titled 4 indicate that reference was made to a response to the particular task for the nominated child or children (according to letter) for all of the four domains. Likewise the column titled 3 indicates reference was made for those children in three domains, and so on. The final column indicates that tasks did not provide responses that were referred to specifically in the discussion of themes. Where the child's letter is in brackets no data are available as the task was not presented to that child either because the child was absent on that day or because the data were not recorded by the audio tape. These instances are unfortunate but are minimal in number. Although they lessen the data available from the children involved, they do not negatively affect the meaningfulness of the insights gained as the focus of the research was the construction of individual portrayals rather than comparative studies. It is noted that even where the same tasks are used with two children, different directions or emphases can be taken and different insights gained. Full data from which this summary was constructed are presented in Appendix H.

| Code | Description of procedure | | Reference in number of domains | | | | |
|-------------|---|-----------|--------------------------------|-------|---------|------------|--|
| | | 4 | 3 | 2 | 1 | Not used | |
| 1.1 | Word association quiz | | | BCDEF | AGH | | |
| 1.2 | Password: Word game | | С | HE | ABFG | D | |
| 1.3 | Word wheels: Words or phrases to | | | | | | |
| | describe "maths" and another | | | | | | |
| | suitable subject, e.g., "music" | | | | | | |
| | 1.3.1 Maths | | CDE | | ABFGH | | |
| | 1.3.2 "Other" subject | | В | С | ADFH | GE | |
| 1.4 | Subject most like maths; subject least like maths? Why? | | | | AC | BDEFG H | |
| 2.1 | Personal dictionary: Learn | | BH | Е | CAG | F(D) | |
| | Children give own definition and | | | | | | |
| | discuss in context of personal | | | | | | |
| | experiences | | | | | | |
| 2.2 | Descriptions of learning situations | | Н | GE | AF | BC(D) | |
| 2.3 | Personal dictionary: Maths | | FH | С | AE | BG(D) | |
| | Children give own definition and | | | | | | |
| | discuss in context of personal | | | | | | |
| | experiences | | | | | | |
| 3.1 | Draw a mathematical activity | | | GE | ABCDF | (H) | |
| 3.2 | Show another mathematical | | | А | BCDE | FG(H) | |
| | activity (act out, write, draw etc.) | | | | | | |
| 3.3 | Questionnaires - Mathematics in | | | | | | |
| | everyday activities | | | | | | |
| | 3.3.1 (Adapted from McDonald & | | С | | ABDEFG | (H) | |
| | Kouba, 1986) | | | | | | |
| | 3.3.2 (Adapted from Wallbridge & | | | | ABCDEFG | (H) | |
| 4.1 | Clarke, 1989) | TT | | 50 | ADODE | | |
| 4.1 | Personal writing: "Maths is like | Н | | FG | ABCDE | | |
| 4.2 | , and discussion | | | | | | |
| 4.2 | brawings: People doing maths | | | | | | |
| | 4.2.1 Drawing one | | п | ADEC | CE | D | |
| | 4.2.1 Drawing one 4.2.2 "Other" drawing | | AFEG | ADFU | BCH | D D | |
| 13 | Planning for an integrated unit of | | C | | ABEEH | GD | |
| H .J | work | | C | | ADLITI | UD | |
| 5.1 | Describing maths to an alien | F | BA | GC | EH | D | |
| 5.1 | (Stodolsky Salk & Glaessner | 1 | DIT | 00 | | D | |
| | (Stodolský, Salk, & Glacssher, 1991) | | | | | | |
| 5.2 | Photographs - mathematical | | BCDE | | AFG | | |
| | activity? (Zevenbergen & Crowe | | Н | | | | |
| | 1992) | | | | | | |
| 6.1 | Situation in which learning maths | | BCE | А | DFGH | | |
| | well: Draw, describe, and identify | | | | | | |
| | most influential factors (ranking) | | | | | | |

| Code | Description of procedure | Reference in number of domains | | | | |
|------|---|--------------------------------|--------|----------|---------|----------|
| | | 4 | 3 | 2 | 1 | Not used |
| 6.2 | Situation in which hindered from | | | AG | BCDEFH | |
| | learning maths well: Draw, | | | | | |
| | describe, and identify most | | | | | |
| | influential factors (ranking) | | | | | _ |
| 7.1 | Scenario: Child having difficulty, | | G | CDH | ABEF | |
| | how would you help? | | | | | |
| 7.2 | Video clips - Describe: Would you | D | ACE | FH | BG | |
| | learn maths well in depicted | | | | | |
| 0.1 | situations? | | | | | |
| 8.1 | Easier and harder experiences in | | | | | |
| | the learning maths: Perceived | | | | | |
| | effectiveness of learning and | | | | | |
| | factors influencing the learning are | | | | | |
| | alscussed 9.1.1 Engine | | D | | ADEE | ССН |
| | 0.1.1 Edstel 9.1.2 Harder | | D D | | | G |
| 82 | 0.1.2 Haluel Describe and sort photographs | ٨F | BCEG | лμ | ADCEFII | U |
| 0.2 | (home situations and school | AI | DCLU | DII | | |
| | situations): Learn maths well? | | | | | |
| 83 | Discussion - how feel about | | G | СН | BE | ADF |
| 0.5 | learning maths | | 0 | en | DL | |
| 9.1 | I could do better in maths if | | | HD | CEFG | AB |
| 9.2 | Vignettes - Children's drawings of | Н | DEF | ABC | G | |
| | classroom, home, and other out of | | | | | |
| | school situations. Select situations | | | | | |
| | in which you would: learn maths | | | | | |
| | well; be hindered from learning | | | | | |
| | maths well. Discuss | | | | | |
| 10.1 | Written descriptors. Phrases/words | | CDE | BFH | GA | |
| | provided, grouped by child and | | | | | |
| | ranked (Stodolsky, Salk, & | | | | | |
| | Glaessner, 1991). Discussion of | | | | | |
| | use of these in learning maths | | | ~ | | |
| 10.2 | 10.2.1 Duplo: Construct/build | | А | С | EGH | BDF |
| | situation in which you felt good | | | | | |
| | when learning maths | | | Б | DOFILO | |
| | 10.2.2 Duplo: Construct/build | | | F | BCEHG | AD |
| | situation in which you did not leef | | | | | |
| 10.2 | Learn: What does this mean to | | | | BCDECH | BE |
| 10.3 | You? Discussion | | | | рсицип | DI |
| 11 1 | How good at maths: 1 - 10 | | | СН | ABDEEG | |
| 11.2 | I think mathematics is | | | <u>C</u> | BDEFH | (A)(G) |
| 11.3 | A good maths teacher | | F | EH | CD | B(A)(G) |

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