A Typology for Instructional Enablers of Mathematical Modelling

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Competency with mathematical modelling is increasingly important for career and informed and engaged participation in personal, civic and work life. In this paper we report on an aspect of a three-year longitudinal study that aimed to identify and describe enablers of mathematical modelling. Teacher interview data has been drawn upon to exemplify key features of a typology for instructional enablers of mathematical modelling. Findings highlight the importance of the didactical contract and socio-mathematical norms in promoting students' mathematical modelling competency, as well as teachers' anticipatory capabilities.

An inability to use mathematics limits an individual's career aspirations, social well-being, and financial security (Paulos, 2000). Competency with mathematical modelling, the use of mathematics to deal with real world problems, is increasingly important for career aspirations (e.g., STEM, economics) and informed and engaged participation in personal, civic, and work life (Geiger et al., 2018; Maass et al., 2019). Recognition of this competency is reflected in the inclusion of mathematical modelling in school mathematics curricula in a growing number of countries, including Australia (Geiger et al., 2021).

Research has provided insight into factors that influence the development of mathematical modelling competency, including: teachers' and students' mathematical and extra-mathematical knowledge (e.g., Blum, 2011); dispositions and beliefs (e.g., Jankvist & Niss, 2019); blockages between stage transitions (e.g., Galbraith & Stillman, 2006); use of digital technologies (e.g., Geiger, 2017); implemented anticipation (e.g., Niss, 2010); and the effective design and implementation of tasks (Geiger et al., 2022). Despite these and other studies, how to best develop students' modelling competency and teachers' instructional competency in relation to modelling remains an unresolved challenge in educational research and practice.

We report on an aspect of a national project that aimed to identify, apply and refine teaching approaches that support secondary students' competency development in mathematical modelling. In this paper, we address a dimension of this aim through a focus on mathematical, cognitive, social and environmental factors that *enable* students to implement the modelling process. Accordingly, we respond to the following research question:

What specific mathematical, cognitive, social and environmental aspects of instruction are conducive for assisting secondary students to implement the mathematical modelling process?

The theory/practice gap identified in this question involved collaboration with teachers and students. Our combined insights have resulted in the generation of a typology that consists of actions and conditions that foster productive activity when working on mathematical modelling tasks – both for learning and instruction. Limited space permits the presentation of findings related to instruction alone. To do so, we first present a concise synthesis of relevant research literature. Second, the methodological approach will be outlined. Third, the outcome of our analysis of data, in the form of an *instructional enablers typology,* will be described. Fourth, we will substantiate key aspects of the typology using illustrative comments from teachers. Finally, selected implications for practice and future research will be discussed.

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The Nature of Mathematical Modelling

The goal of mathematical is to understand or make predictions about real world phenomena, typically to inform decision-making or action. While there are differing perspectives on mathematical modelling in the literature, there is general consensus that the key phases consist of: (1) identifying and specifying a real-world problem; (2) developing a mathematical representation; (3) generating a mathematical solution; (4) interpreting the mathematical solution within the context of the real-world problem; and (5) evaluating validity of the solution relevant to the original context. While each of these stages, is separately important, the goal of instruction is to develop holistic competency with the total process.

Mathematical modelling often requires iterations of this process to improve the model or refine solutions. Thus, it is typically depicted as cyclic in nature. While the representation is cyclic, associated diagrams are analytic reconstructions of the process and not depictions of the routes necessarily taken by actual modellers (Niss & Blum, 2020). In the representation below (Figure 1) (Galbraith, 2013), the heavy clockwise arrows (1 to 7) depict the flow of the modelling process (stages A to G). The double headed arrows indicate that intermediate transitioning/revisiting, within and between stages, are likely as metacognitive reflection both reviews and amends progress to date and anticipates moves yet to be enacted.

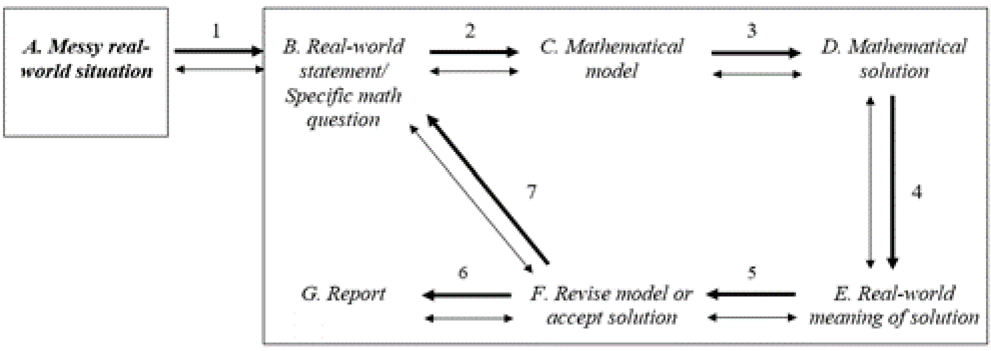

Figure 1. Representation of the modelling cycle (Galbraith, 2013).

Developing Mathematical Modelling Competency

Research into mathematics teacher competency has tended to explore cognitive aspects of performance (Blömeke et al., 2014). There is, however, increasing recognition of the situated nature of teaching, highlighting the affective dimensions of competence (Schoenfeld, 2011). Similarly, research into how teachers assist or inhibit learners' development of mathematical modelling competency has identified: a tendency to intervene and reduce cognitive challenge (de Oliveira & Barbosa 2010); disposition towards guiding students toward pre-determined solutions (Tan & Ang, 2016); ways to diagnose student difficulties as modelling competency develops (Jankvist & Niss, 2019); the importance of task authenticity (Galbraith, 2013); and advantages of openness to multiple solutions (Schukajlow et al., 2015).

Borromeo Ferri and Blum (2010) developed a model for mathematical competency by defining the cognitive demands of task creation, quality instruction and assessment of modelling activity. While providing a multi-dimensional perspective of modelling competency, this model is restricted to cognitive considerations alone. To include other factors that may influence the teachers' approaches to the development of mathematical modelling competency, we drew on Brousseau's (1984) notion of a didactical contract, which positions the actions students take to promote their own learning in the context of teacher expectations.

For example, what kind of activity will teachers expect students to complete and what is reasonable in terms of cognitive demand and time frame. Consistent with this perspective, we further considered the role of socio-mathematical norms (Yackel & Cobb, 1996), which define valued modes of reasoning and ways of working aimed at developing solutions to mathematical tasks. Socio-mathematical norms are seated within a didactical contract, which itself is shaped by these norms. Thus, if teachers are to change the established practices within a classroom, for example, introducing or placing greater emphasis on mathematical modelling, they must renegotiate the existing didactical contract as well as associated mathematical norms.

Further to these perspectives, we also drew on the notion of anticipation (e.g., Niss, 2010) in enabling modelling competency, as this requires the capacity to look forward and backwards, when evaluating progress, in order to make decisions about future action. Niss (2010) coined the term implemented anticipation and outlined three key processes (Table 1).

Table 1
Niss' (2010) Processes of Implemented Anticipation (adapted)

Structuring of an extra-mathematical situation, to prepare it for mathematization, must be focused on features that are anticipated as essential in addressing a problem situation.
Anticipation of mathematical representations that are suitable for capturing a situation must be familiar to the modeller and, ideally, the modeller would have had experience with their use in mathematizing simpler or similar situations.
Anticipation of how the mathematization and resulting model will provide a mathematical solution to the questions posed. Thus, that the outcomes of applying selected mathematical procedures and problem-solving strategies must also be anticipated after mathematization.

While these processes are vital to successful mathematical modelling, our study indicated that *anticipation* was also key for effective instruction as teachers must be capable of looking forward and backwards to evaluate the progress of students on a modelling task. This implies they must have a clear understanding of the task and how it may be solved, be capable of anticipating where students may encounter blockages, including the nature of these blockages, to their progress and have the capacity to make decisions about relevant advice *in situ*.

Research Design and Implementation

We adopted a design-based research approach (Cobb et al., 2003), a methodology suited to applied research that aims to develop contextualised theories of learning and teaching, The goal of such research is to address educational problems situated in a wide range of contexts.

Participants included six teachers and their intact Year 8–11 classes drawn from schools in Queensland (Australia). Teachers were selected purposively (Burns, 2000), volunteering because of their interest in promoting student modelling competency. Their experience with teaching modelling varied from novice to highly experienced. The curriculum context in which teachers practiced required the inclusion of modelling assessment tasks in Years 11 and 12, although relevant documents provided little advice on how to promote students' competency in this area, especially to inexperienced teachers in lower secondary.

The study utilised an iterative process of design-implement-reflect to facilitate researcher/teacher collaboration that informed the identification of instructional enablers, or dis-enablers, of mathematical modelling. This process was operationalised through three whole-day researcher/teacher meetings and two classroom observation visits per year over three years. Classroom visits took place between researcher/teacher meetings. Iterations of these activities (Figure 2) took place in each year of the project (see Figure 2). Researcher/teacher meetings focused on task development and planning for implementation as well as the identification of factors that enabled or dis-enabled mathematical modelling. This facilitated the trialling of tasks and pedagogies informed by identified enablers during classroom visits, leading to the refinement of tasks and elaboration of instructional enablers.

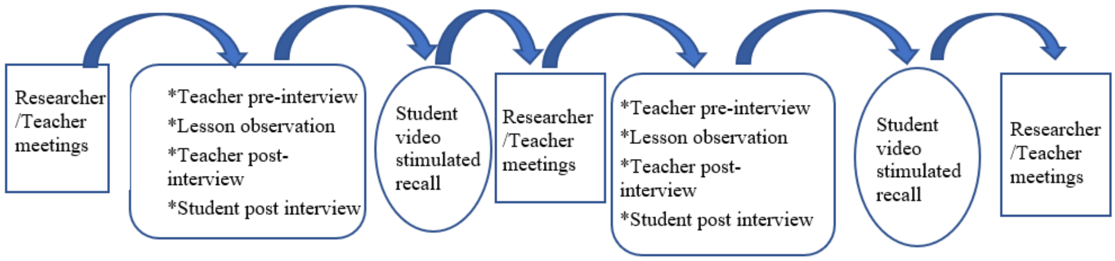

Figure 2. Yearly cycle of researcher/teacher meetings and classroom observation visits.

In this paper, we draw on data from teacher pre- and post-lesson interviews to substantiate elements of the instructional enablers of a mathematical modelling typology. Enablers identified in early stages of the project, were used as the basis for initial coding of data. This process led to the refinement of code definitions and the identification of additional enablers. Further iterations of identification and refinement were continued until definitions stabilised.

In the next section we describe a typology of instructional enablers and illustrate key features via reference to teacher commentary.

A Typology for Enablers of Mathematical Modelling Instruction

A typology for instructional enablers of mathematical modelling is presented in Table 2. Space limitations means that only selective aspects of the typology can be illustrated here.

Table 2
Typology for Enablers of Mathematical Modelling Instruction

Overarching enablers	Generic and specific enablers of mathematical modelling
Impact of learning goals – external/internal/ personal	*Generic enablers* Aligns teaching/learning with broader educational goals (e.g., promoting 21st century skills) *Specific enablers* Links task to formal curriculum/syllabus (e.g., addresses a specific content goal)
Classroom expectations/ways of working	*Generic enablers* Encourages diverse approaches and risk taking Encourages questioning Encourages student collaboration Provides opportunity for reporting findings *Specific enablers* Shapes the physical environment to support modelling Few restrictions on the use of digital technologies
Managing the learning process	*Generic enablers* Manages the learning process in an active manner *Specific enablers* Makes explicit reference to the modelling process Utilises/generates their own resources to support learning in modelling (e.g., representation of modelling process)

	Supports the development of a *mathematical question* Measured Responsiveness -Provides measured guidance in relation to aspects of the task -Provides measured guidance in relation to students' initial ideas -Provides measured in relation to students' proposed directions -Direct Responsiveness -Provides direct clarification of aspects the problem -Provides direct clarification of new terms and ideas
Teacher anticipation	*Generic enablers* Demonstrate knowledge of capabilities/needs/experiences Notice and respond to emerging issues *Specific enablers* Understands and makes explicit reference to the modelling process Identifies possible barriers to solving the problem through reference to the stages of the modelling process

The typology consists of four overarching enablers, each inclusive of both generic enablers relevant to mathematics instruction and enablers specific to mathematical modelling. The overarching enablers are: impact of learning goals–external/internal/ personal; classroom expectations/ways of working; managing the learning process; and teacher anticipation. These overarching enablers are associated with specific actions or expectations teachers employ, or classroom conditions they manage, to enable the development of students' mathematical modelling competency. We now elaborate on each of these over-arching enablers and describe actions/expectations/conditions that researchers/teachers identified as enablers of mathematical modelling activity. In addition, their absence was noted as a dis-enabler.

Findings

Impact of Learning Goals: External/Internal/Personal

Teachers indicated that external factors, for example, curriculum requirements or expectations of approaches employed for instruction in schools influenced the way they implemented modelling. Some teachers saw modelling as aligned with relevant curriculum documents or their own broader educational goals, for instance, 21st century skills.

> I like the modelling. It fits into the Scientific Method and Scientific Process and it's a big thing that we should actually be doing in 7, 8, 9, 10 … The kids are doing all the work. They're coming up with the pathway to solve it and everything … so it ticks all of the boxes for [local curriculum authority].

There were others, however, who were influenced by perceived restrictions related to curriculum requirements or school expectations about instruction, for example, the need to ensure each student acquired a high degree of fluency with set material within limited time.

> We've got a nine-week term plus all the disruptions ... kids are going off on this excursion … or whatever else, photo days. There're just constant disruptions. So, you don't have your kids there …

These are examples of conditions that can act as enablers or dis-enablers of mathematical modelling instruction which can shape the didactical contract and classroom mathematical norms (Brousseau, 1984; Yackel & Cobb, 1996).

Classroom Expectations/Ways of Working

This over-arching enabler is concerned with: what teachers expect students to do; how they work; and valued modes of reasoning and forms of student/student/teacher interaction. Thus, this enabler, consistent with the notions of the didactical contract and classroom mathematical norms (Brousseau, 1984; Yackel & Cobb, 1996), relates to the supportive classroom culture necessary for the development of modelling competency. Specific enablers that were supportive of such a culture included encouraging: diverse approaches to tasks (risk taking); questions/queries; student collaboration within and across groups; and reporting of findings to the whole class. There were instances where teachers supported modelling by shaping the physical environment to support modelling. Few restrictions were placed on the use of digital technologies. Illustrative examples of each of these actions/conditions follow.

In the following instance, a teacher noticed an approach that was different to what they had anticipated. The task had required students to determine the optimal approach to refuelling a car, given petrol stations were at different instances from their starting points and offered different prices. The teacher was comfortable with students solving the task in different ways.

> They were looking at having the most fuel in the car ... So, they were looking at a full tank is better than losing fuel when going to cheap one as it's further away. I didn't think they would come up with that but with these … problems there's a few points of view and that's a valid point to if you're looking at having the most or to have the tank full. So that's another avenue that you'd have to factor into your scheme.

In a different task related to construction, a question was welcomed by the teacher. Importantly, they linked their response to a key aspect of modelling—making assumptions.

> That was an excellent question, so you might need to decide what kind of concrete. And that's part of defining the problem … remember, once we make a decision, we need to include that in our assumptions.

A typical comment related to collaboration as an element of modelling can be seen in the following quote. The caution, however, is a salient reminder that not all collaborative work is productive, and teachers have a key role in guiding students modelling activity.

> Two-heads are better than one. Although you need to be clear what you're talking about. It's easy to get people on the same page when they're looking at the same information.

Students often reported their findings at the end of a lesson or wrote a report in order to receive feedback from the teacher. One teacher provided additional scaffolding to this end by way of a booklet in which they recorded their work in a structured fashion.

> And after we've finished the two lessons, we're going to write a little report ... So, the more you write in the booklet, the easier that report's going to be because you'll just go back to the start of the booklet and start typing what you've actually written … and the report is going to be our assessment.

Teachers also made use of the physical environment of the classroom or other resources. This was highlighted during a lesson in which desks designed to be written on were available. Students had no hesitation in writing notes, diagrams and other prompts that helped them share their ideas with members of their group. It was clear this was a regular occurrence.

> I think the set-up of the room, having particularly some shared information, is really valuable. So, I noticed the groups with the whiteboard tables are quite good because they can write, and they can see what each other are doing. And I think that enhances the collaboration straight away ....

The use of digital technologies was encouraged with few restrictions. Teachers saw such resources as supportive of students' thinking and in mediating collaboration.

> … they had graphs up and they were sharing them so that they could see them on their own computers ... So again, I think that real shared information is important, particularly when working in a group.

Managing the Learning Process

Learning was managed in an active fashion through preparatory and developmental phases. Key to preparatory phase was the identification of an extra-mathematical question that would lead to a mathematical question—a statement that encompassed the goals of students' activity expressed in a mathematical sense. The generation of this question was begun in groups but then opened-up to public scrutiny by peers. This question was the foundation for other aspects of the modelling process, such as the identification of assumptions.

> … so, I'm just going to get you to write them there when they come up in your group … feel free to chat amongst your group … What's the mathematical question we need to answer from this real-life situation?

During the developmental phase (body of a lesson), teachers employed a balance of direct and measured responsiveness to students' requests for assistance. In the case of new terms or ideas within a task, clarification was provided in a direct fashion. For example, one task was concerned with replacing the foundations of a building. An activity with which students were unfamiliar. In this case, teachers provided a direct explanation so that progress was not limited by factors unrelated to the use of mathematics to solve the problem.

By contrast, fostering students' modelling development often requires measured responsiveness (Geiger et al., 2022). This involves providing only enough information for students to make progress without directing them towards a pre-determined solution (Tan & Ang, 2016). Support in this circumstance was in the form of questions aimed at assisting students to clarify their thinking or by referring to stages of the modelling process.

> So, the question is, in a foundation, are there multiple blocks or one big block? So, what do you think? Have you seen foundations before? Is a foundation made of blocks? It's made of concrete. Is it made of concrete blocks or poured concrete? Well, you might need to decide before you do this activity.

While teachers understood the value of this approach, they also admitted that, at times, it was hard to let students lead the work.

> So, I knew once we got to that stage that they would be right. It's very hard to take that step back … I mean, I trust them, and I know how good they are and it's still hard to take that step back.

Teacher Anticipation

To provide effective instruction about mathematical modelling teachers needed to have a clear understanding of the modelling process themselves and ensured a task was worked through before implementation in the classroom. This preparation informed instructional anticipation—providing insight into where students may encounter difficulties, and why.

> … once we were looking for the prices of concrete. Not every student managed to take in all the information I gave them ... So, they didn't look on the OneNote where I provided a couple of links.

Knowledge of the modelling process also informed teachers' advice to students about possible ways forward when an unexpected difficulty or blockage occurred, taking into account students' previous experiences and individual students' current capabilities.

> The ones working with the cycle have a better grasp of where they are in terms of arriving at a reasonable solution. And so, you're seeing, "Here's an initial solution but, actually, I might need to think about these other assumptions that I haven't yet made. How is that going to affect the solution?"

Discussion and Conclusion

In this paper, we have provided insight into some of the important mathematical, cognitive, social and environmental aspects of instruction that enable students' mathematical modelling. Our analysis has identified generic and specific instructional enablers of modelling in the form of a typology. It is important to note that the absence of these enablers represent dis-enabling factors in modelling activity. The findings of this study confirm the results of previous research in relation to instructional factors that promote or constrain productive modelling activity, while providing, at the same time, insight into how teachers can manage the learning processes. In particular, the outcomes of the study indicate that attention to the didactical contract, socio-mathematical norms, and teacher anticipation are key to successful student outcomes in modelling and is thus an important area for future research in the field.

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