mathematics education in the margins

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Theorising about Mathematics Teachers' Professional Knowledge: The Content, Form, Nature, and Course of Teachers' Knowledge

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The guiding philosophy of this theoretical work lays in the argument that mathematics teachers' professional knowledge is the integration of various knowledge facets derived from different sources including teaching experience and research. This paper goes beyond past trends identifying what the teachers' knowledge is about (content) by providing new perspectives, in particular, on how the knowledge is structured and organised (form), on what teachers' draw on their knowledge (source), and whether the knowledge is stable and coherent or contextually-sensitive and fluid (nature).

Introduction

The guiding philosophy of this work is the assumption that teachers draw on a wide range of sources as they do their work, using and transforming these in various ways for the purposes of their teaching and for the needs of their students. Thus, one of the key theoretical concerns arising in the realm of teachers' professional learning and development is the question on which sources teachers draw on their work. Researchers have reflected on resources (including knowledge), identifying them, among orientations (including beliefs) and goals, as critically important determinants in what teachers do, and why they do it (Schoenfeld, 2010). The sources of particular significance for the teaching enterprise are, from the author's perspective: (a) knowledge; (b) teaching; and (c) research (see Figure 1). These three sources are viewed as playing a complementary role in relation to each other; for instance, research can inform and enhance teachers' knowledge about particular instructional strategies, as well as equipping the teacher for the rich reflection required in professional judgement. At the same time, research itself can be enriched through greater insights into the challenges and complexities of educational practice.

The last few decades have produced a considerable body of literature that describes, theorises, and conceptualises knowledge as a source for teachers doing their work. Shulman (1987), for instance, identified three dimensions of knowledge needed for teaching; namely content knowledge (knowledge of the subject matter per se to be taught), pedagogical knowledge (knowledge of how to teach in general terms), and pedagogical content knowledge (knowledge of how to teach that is specific to what is to be taught). In this and further work, Shulman (1987) makes clear that the knowledge base necessary for teaching comprises teachers' knowledge of content in the domain being taught, knowledge of learners' common conceptions, and difficulties that learners may have when learning particular content, and knowledge of pedagogical strategies that can be used to address learners' needs in particular classroom circumstances. However, less emphasis has been put on teachers' knowledge of students' learning. To put it in other words, what is missing in Shulman's (1987) contribution on various dimensions of teachers' knowledge, as argued in this work, is teachers' knowledge of learning, in particular, teachers' knowledge of theoretical frameworks of knowing and learning. However, knowledge of approaches to, and research on, learning mathematics is taken as a crucial component of mathematics teachers' resources, and a particular focus of the theoretical work reported here.

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Another source of teachers' professionalisation is *the personal experience of being taught, or of teaching*. In analogy to *phenomenological primitive*s arising from experience with the physical world as described in detail by diSessa situated in his *knowledge in pieces* framework (e.g., diSessa, 1993), *pedagogical primitives* arise from experiences of being taught or of teaching. They provide powerful, mental resources useful for sense-making in the education instructional context, formed through a process in which individual teacher's ways of teaching are strongly shaped by their personal experience of being taught or of teaching. Researchers may refer to this as *craft knowledge* or *practical knowledge* to distinguish it from what others have referred to as *didactical knowledge'* or *mathematics education knowledge*, in particular, knowledge derived from research reported in the field. Knowledge derived from research (in short, research-based knowledge) is considered as a further source of teachers' professionalisation. In particular, research-based knowledge on: (a) students' ways of understanding and thinking; (b) ways of learning mathematics; and (c) ways of teaching particular mathematical concepts are viewed as providing a rich source for teachers' doing their work. Teachers need to engage with research, in the sense of keeping up to date with the latest developments and findings in research on students' ways of thinking, understanding, and learning, and on effective instructional techniques to inform their pedagogical content knowledge. In addition to the latest research findings, teachers should become familiar with the implications of this research for their day-to-day practice, and for education policy and practice more broadly. With this perspective, research is viewed as a key source of teachers' broader professional identity, one that reinforces other pillars of teacher quality: notably teachers' knowledge base and teaching experience.

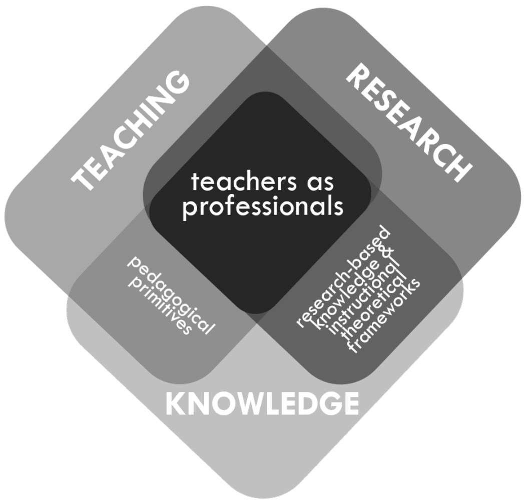


Figure 1: Sources of teachers' professionalisation

It is this conceptualisation of sources of teachers' professionalisation that enables an elaboration of knowledge resources for teaching mathematics. Consequentially, in contrast to any narrow or simplified view, the idea of teachers' professional knowledge essentially conveys the need to *integrate* knowledge from *various* sources including knowledge derived through teaching experience/practice (pedagogical primitives) and research (research-based knowledge and instructional theoretical frameworks).

Lessons from Past Approaches Conceptualising Mathematics Teachers' Knowledge

Over the past decades, a range of research work on conceptualising teachers' knowledge has been developed often taking Shulman's initial work as a point of departure, a considerable number of which has been located in mathematics education research (e.g., Adler & Davis, 2006; Ball, Thames, & Phelps, 2008; Blömeke, Kaiser, & Lehman, 2010; Even, 1990; Ma, 1999; Fennema & Franke, 1992; Kilpatrick et al., 2006; Rowland et al., 2005; Schoenfeld & Kilpatrick, 2008), and how such knowledge can be operationalised and measured (Baumert et al., 2010; Blömeke et al., 2014; Hill et al., 2007; Schilling et al., 2007; Tatto et al., 2008, 2012). Crucial lessons we have learned from these and related work on conceptualising mathematics teachers' knowledge have been identified and described elsewhere (Scheiner, 2015). In short, Shulman's (1986, 1987) conceptualisation of domains of teachers' knowledge, in particular, subject matter knowledge (SMK) and pedagogical content knowledge (PCK), has been made specific to teaching mathematics. The distinction between SMK and PCK, although being ambitious in empirical investigations, continue to be widely used, in particular since it is considered as a useful tool in describing teachers' knowledge for research purposes and in devising pre-service teachers' and professional development programs. The multidimensional nature of mathematics teachers' knowledge has been demonstrated by further refining the categories SMK and PCK and accentuating sub-dimensions that are specific for the purposes of teaching mathematics, such as describing and conceptualising a particular kind of mathematical content knowledge considered as unique for teaching mathematics.

In this work, the author wants to point to a further aspect that is about the dominating and guiding idea of most of the approaches on conceptualising mathematics teachers' knowledge developed in the past, namely the idea about teachers' *unpacking of mathematics content* in ways accessible for their students. In doing so, past approaches have centred their focus on the mathematics content; making the mathematics content a point of departure. Approaches guided by this *philosophy* often use the notion of *mathematical knowledge for teaching* in describing the teachers' knowledge base. From the author's perspective, the use of the notion of *mathematical knowledge for teaching* is insufficient since it seems not to capture other dimensions besides the subject content. Thus, this work calls for using the notion of *knowledge for teaching mathematics* including an epistemological, a cognitive, and a didactical dimension in addition to the subject content dimension. In doing so, it is intended to extend the current perspective on teachers' knowledge in the sense of going beyond a more or less purely content perspective by taking into account several other perspectives important in in this issue.

Conceptualising Mathematics Teachers' Knowledge: Past Trends and New Perspectives

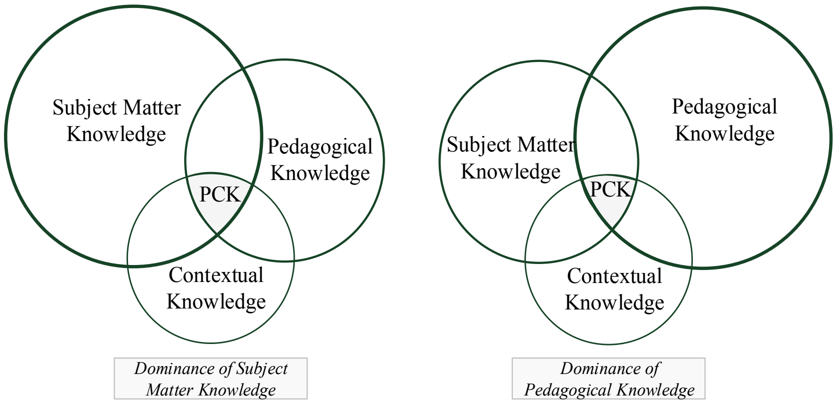
In the past, the literature concentrated its focus on what the teachers' knowledge is about. In doing so, the literature limited its focus on the *content* teachers do or should possess. Research has made progress in identifying various facets of mathematics teachers' knowledge arguing that teachers' subject matter knowledge is about substantive and syntactic structures of the discipline (Schwab, 1978); and mathematics teachers' content knowledge, in particular, seems to be about ways of understanding and ways of thinking (Harel, 2008), or school mathematical knowledge and academic content knowledge (Bromme, 1994), among others. Mathematics teachers' knowledge, as argued in the literature, is about the epistemological foundations of mathematics and mathematics learning (see, Bromme, 1994), students' cognitions (Fennema & Franke, 1992), in particular, knowledge of students' common conceptions (see Shulman & Sykes, 1986), knowledge of students' cognitive difficulties involved in concept construction (Harel, 2008), and the interpretation of students' emerging thinking (Ball et al., 2008), as well as "the most useful ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9), including teachers' illustrations and alternative ways of representing concepts (and the awareness of the relative cognitive demands of different topics) (Rowland et al., 2005) and knowledge of the design of instruction (Ball et al., 2008), among others.

However, what seems to be missing in the current landscape on various approaches of conceptualising mathematics teachers' knowledge are efforts in going beyond what the knowledge for teaching mathematics is about by taking into account: (1) how the knowledge is structured and organised; (2) on which sources teachers' draw on their knowledge; and (3) whether the knowledge is stable and coherent or contextually-sensitive and fluid. In short, the major issues that need better resolution if we are to understand teachers' acquisition of an integrated knowledge base are questions concerning: (1) the *form*; (2) the *source*; and (3) the *nature* of mathematics teachers' knowledge.

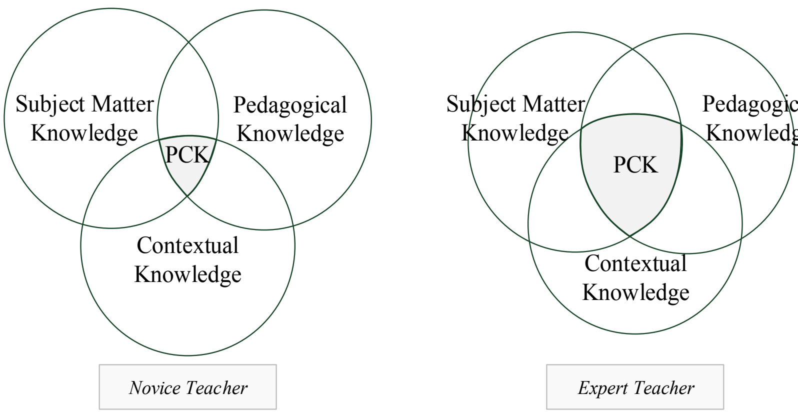
The Form of Teacher Knowledge

The initial point in this issue is the assumption that examining teacher expertise may help to advance our understanding of what makes the knowledge for teaching specialised since expert teachers are considered as focal elements in the movement towards excellence in education (Sternberg & Horvath, 1995). Findings in research on expert teachers, and, in particular, on expert teachers' knowledge show that the concept of domain-specific knowledge structures is vital. Among various differences, Sternberg and Horvath (1995) consider knowledge as "perhaps the most fundamental difference between experts and novices" (p. 10). The same authors conclude that research findings indicate that an expert in the domain of teaching differs from a novice not only in the amount of subject matter knowledge and pedagogical knowledge but also in the organisation of their domain-relevant knowledge.

Magnusson, Krajcik, and Borko (1999) illustrate one way (among several possible other ways) to think about the interaction of the domains of knowledge in the development of pedagogical content knowledge. They suppose that the knowledge bases (subject matter, pedagogical, and contextual knowledge) may unequally influence the development of pedagogical content knowledge due to differences in the amount of knowledge in each domain. However, taking the research findings on expert teachers' knowledge into account, it may be suggested that after a certain amount of subject matter knowledge, pedagogical knowledge, or contextual knowledge these knowledge bases do not have a higher relative influence on PCK. Rather, as shown in Figure 2a, it is not merely the amount of knowledge in each knowledge domain (subject matter knowledge, pedagogical knowledge, or contextual knowledge) that matters most but the degree of integration of the knowledge bases. Expert teachers, from this point of view, would show a greater overlap, symbolising increased integration of the three knowledge bases, than novice teachers (see, Figure 2b).



(a) The potential (impact) of the dominance of a particular knowledge base on PCK



(b) The (potential) impact of the degree of integration of knowledge bases on PCK

Figure 2: The (potential) impact of the dominance of a particular knowledge base and the degree of integration of knowledge bases on PCK

The Source of Teacher Knowledge

A further aspect in conceptualising the knowledge specialised for the purposes of teaching mathematics is to examine the constituent knowledge bases that influence this particular kind of knowledge. In the past, Shulman's pedagogical content knowledge was considered almost always as the only form of knowledge unique for the purposes of teaching. In Shulman (1987), pedagogical content knowledge was defined as "that special amalgam of content and pedagogy … It represents the blending of content and pedagogy " (Shulman, 1987, p. 8, italics added). However, this perspective is problematic for many reasons, including the fact that the amalgamation of content and pedagogy leads not only to a too broad category but lacks in both subject- and context-specificity. Still, the mathematics education research community has identified specific dimensions built upon Shulman's initial work on PCK. The various refinements of PCK seem to converge in three dimensions, namely: (1) knowledge of students' mathematical understandings (KSU); (2) knowledge of learning mathematics (KLM); and (3) knowledge of teaching mathematics (KTM). The former two refer to a cognitive and an epistemological perspective, while the latter refers to a didactical perspective on this issue. In this work, knowledge of students' mathematical understanding (KSU), knowledge of learning mathematics (KLM), and knowledge of teaching mathematics (KTM), together with mathematical content knowledge per se (MCK per se) and mathematical content knowledge for teaching (MCK for teaching) build the knowledge bases that constitute the particular kind of knowledge that is considered as specialised for the purposes of teaching mathematics. In doing so, past and current approaches in research on mathematics teachers' knowledge are turned on their heads in the sense of taking the identified (and refined) knowledge dimensions as building blocks for the construct of *knowledge for teaching mathematics*.

The Nature of Mathematics Teacher Knowledge

Certainly, approaches mentioned above do not converge on a clear conceptualisation of PCK. Indeed they portray differences of opinion and a lack of clarity about the nature of PCK and its development. Research approaches consider PCK as a knowledge dimension on either: (1) a cross-subject level; (2) a discipline-specific level; (3) a domain-specific level; or (4) a topic-specific level. Some researchers also hold the view that PCK can be considered as a knowledge dimension regarding several levels. In recent studies, PCK seems more often to refer to a broad and general form of knowledge, sometimes even losing its discipline-specificity. Fernández-Balboa and Stiehl (1995), for instance, analyse PCK in professors across several fields, including biology, business, and education, among others. However, in line with Hashew (2005), the author argues that PCK seems to have lost one of its most important characteristics, namely its topic-specificity. The work by Smith, diSessa, and Roschelle (1993), for instance, reminds us that knowledge is concept-specific and highly context-sensitive. For instance, the *knowledge in pieces* framework developed by diSessa calls for viewing knowledge as microstructures coming in a loose structure of quasi-independent, atomistic knowledge pieces.

Final Remarks: Future Directions

Although the various frameworks and models on the construct of mathematics teachers' knowledge have provided crucial insights on what mathematics teachers' knowledge is about, several of the discipline-specific frameworks represent conceptualisations of mathematics teachers' knowledge by a very general approach that seem ad hoc. The author, by contrast, does not believe in the existence of a general framework on teachers' knowledge but rather thinks that in investigating the form and nature of teachers' knowledge various frameworks may be discovered, which will be quite specific to particular mathematical concepts and individuals.

The author calls for paying attention to investigating what in this paper is called *knowledge for teaching mathematics* considered as a pool of personal and private constructed pieces of knowledge that have been transformed along a variety of knowledge bases identified by previous research investigating the multidimensionality of teachers' knowledge. In more detail, this work emphasises the view that teachers' professional knowledge specialised for teaching mathematics is the repertoire of *knowledge atoms* that have been transformed along: (1) knowledge of students' mathematical understanding (KSU); (2) knowledge of learning mathematics (KLM); and (3) knowledge of teaching mathematics (KTM), taking (4) mathematical content knowledge per se (MCK per se) and

(5) mathematical content knowledge for teaching (MCK for teaching) as the cornerstones (see, Figure 3). Notice that: (i) the notion of *transformation* implies that the constituent knowledge bases are inextricably combined into a new form of knowledge that is more powerful than the sum of its parts (*form*); (ii) in contrast to Shulman and his proponents' work, it is KSU, KLM, and KTM, together with MCK per se and MCK for teaching that build the knowledge dimensions that serve as the constituent knowledge bases for teaching mathematics (*source*); (iii) the notion of *knowledge atom* indicates that knowledge is of a microstructure, highly context-sensitive, and concept-specific and has to be considered as of a fine-grained size (*nature*); and (iv) The notion of *repertoire* indicates that knowledge is personal and private and that teacher education programs can only provide (as good as possible) rich resources for building up a fruitful repertoire of knowledge atoms.

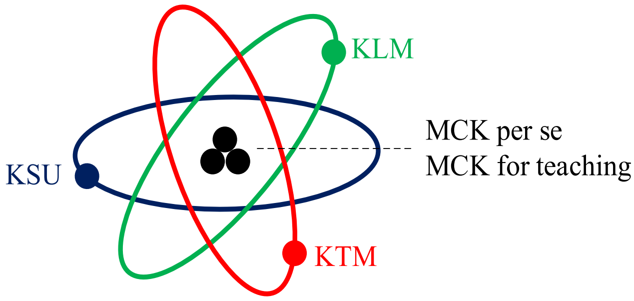


Figure 3: The *knowledge atom*

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