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AN INTERDISCIPLINARY APPROACH TO MATHEMATICAL MODELLING IN SECONDARY TEACHER EDUCATION

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The purpose of this paper is to describe and analyse the nature of an interdisciplinary approach to the development of an online learning module designed for secondary mathematics Initial Teacher Education Students (ITES). In developing a module on mathematical modelling, team members crossed both disciplinary and institutional boundaries. Semi-structured interviews were used to gain insight into the perspectives of team members on the collaboration and analysed through the frame of boundary crossing. The analysis revealed the process of collaboration was advantageous in a number of ways but brought with it complexities that required accommodation.

INTRODUCTION AND BACKGROUND

The performance of Australian students in international comparative assessment regimes such as the Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMMS) is a source of increasing concern government, educational jurisdictions and the public at large. For example, across 2003-2015 PISA results, Australia was ranked 20th for mathematical literacy in 2015, down from 19th in 2012, 13th in 2009 and 8th in 2006. Further, PISA results show that 22% of Australian 15 year olds did not meet the international proficiency Level 2 for mathematical literacy – indicative of the level of competence necessary to use mathematics effectively in real-life situations. These results are paralleled by falling participation in mathematics, science and technology in Australia, raising serious questions about Australia's capacity to sustain a knowledge-based economy and society.

In a response to a report aimed at providing a blueprint for turning around such trends (Office of the Chief Scientist, 2012), the Australian government providing funding for a number of initiatives including the Enhancing the Training of Mathematics and Science Teachers (ETMST) scheme (2013-2017). A fundamental principle for the funding of projects under this scheme was that mathematicians, scientists, mathematics educators and science educators be brought together to develop programs aimed at strengthening Initial Teacher Education Students' (ITES) discipline knowledge. This principle was a challenging demand within the Australian context as there was little by way of existing culture related to this type of collaborative activity.

Under the umbrella of the ETMST scheme, *Opening Real Science: Authentic Mathematics and Science Education for Australia (ORS)* was developed and implemented over a period of 4 years through the support of seven universities and research institutions. The aim of the ORS project was to engage pre- and in-service

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teachers with “real” science and its authentic practice — dynamic inquiry and subsequent action related to real world phenomena. In support of pre-service teachers' learning, the ORS teacher education program developed 25 on-line learning modules across mathematics and science, eight of which focused on mathematics, that utilised authentic contexts and enquiry-based pedagogical approaches.

The project's approach focused on student-centred learning, employing problems in which students were genuinely interested, utilising investigative approaches, coupled with scaffolded applications of digital technologies.

The purpose of this paper is to describe and analyse the nature of the interdisciplinary collaboration that was integral to the design and development of the learning module on mathematical modelling – *Modelling the present: Predicting the future*. In attending to this issue we address the following research questions:

- Did the collaboration produce a quality outcome??
- What were the opportunities when collaborating across disciplines?
- Were there any limitations associated with interdisciplinary collaboration?

CONCEPTUAL FRAMEWORK

In his analysis of groups involved in shared practices within and across trades and professions, Wenger (1998) developed the notion of *communities of practice*. Within communities of practice, group members come together for the purpose of a mutual endeavour within which they contribute to each others' learning by engagement in a common activity. Wenger proposed three dimensions of collaborative pursuit within such communities: mutual engagement, joint enterprise and shared repertoire. He also described different ways of participating within communities of practice:

- Engagement: doing things together, talking, and producing artefacts.
- Imagination: constructing an image of ourselves, of our communities, and of the world, in order to orient ourselves, to reflect on our situation, and to explore possibilities.
- Alignment: a mutual process of coordinating perspectives, interpretations, and actions so they realise higher goals.

(Wenger, 1988)

Communities, by their existence, are defined by boundaries that separate groups of participants and non-participants. Such boundaries can both divide and connect communities (Akkerman & Baker, 2011) but where it is advantageous, members of different communities will seek out opportunity for boundary encounters (e.g., Sztajn, Wilson, Edgington & Myers, 2013). Such encounters represent points at which coordinated and coherent shared action and interaction can be established.

According to Akkerman and Baker (2011), the concepts of *boundary crossing* and *boundary objects* are central to describing the ways in which different communities can engage with learning sharing, coordinated action and gainful interaction.

Boundary crossing refers to the transitions of individuals across communities and their interactions with new and different ideas and cultural norms. *Boundary objects* are those artefacts that act as bridging mechanisms by which a *crossing* is affected. The concepts of boundary crossing and boundary objects are of interest within educational contexts because of the potential for learning at intersections between communities who create and value different types of knowledge.

Suchman (1994) has argued that the term boundary crossing denotes the transition of an expert into an arena in which they are far less qualified. Such transitions have the potential for new learning and the development of new knowledge as those crossing boundaries must bring together their expertise with the unfamiliar knowledge and new ways of knowing and reasoning that exist within the community to which they have transitioned.

Within mathematics and science education, the ideas of boundary crossing and boundary objects have been utilised to analyse one-way transitions of different types including: school to work (e.g. Wake, 2014) and teachers who are required to work "out of field" (e.g., Hobbs, 2013). Additionally, these concepts have also been used to explore bilateral exchanges including: collaborations between educational researchers and teachers in-service (e.g., Goos, 2013); mathematics teacher educators and teachers involved in teacher professional development (Sztajn, Wilson, Edgington & Myers, 2013); and mathematicians and mathematics educators collaborating to strengthening initial teacher educations students discipline knowledge (Goos, 2015). Few, if any studies, however, have investigated how more diverse groups have collaborated on joint endeavors, such as in the case discussed in the sections which follow that involve mathematicians, scientists, mathematics educators and instructional designers.

CROSSING BOUNDARIES TO DEVELOP THE MODULE

Module development was carried out by a team of eight academics with backgrounds including biological evolution, financial mathematics, astrophysics and environmental science as well as mathematics educators with experience in the teaching and learning of mathematical modelling and instructional design. Members of the team either self-identified by responding to an expression of interest distributed to relevant staff (mathematicians, scientists, and mathematics and science educators) of participating universities or were invited on the basis of their expertise.

The process of module development began with introducing team members to the framework used to guide the development of the every module in ORS the Biological Sciences Curriculum Study (BSCS) 5Es Instructional model approach (Bybee, 2009). The 5Es enquiry-based approach to science education consists of five phases: engagement, exploration, explanation, elaboration and evaluation. Each phase has a role in developing students' understanding of scientific and technological knowledge, attributes and skills (Bybee, 2009). There were then four additional phases consisting of: selection of content, identifying structure, and planning for subsequent phases; initial case study development; draft case study review; and finalisation of the module by linking of case studies. Case studies were based on authentic uses of mathematical modelling.

In order to identify potential case studies the module leader asked members of the Module Development Team (MDT) to talk about their personal research interests and how these were connected to mathematical modelling – to provide ideas about content and to provide opportunity for team members to share aspects of the communities in which they typically worked. Presented topics were diverse and included: evolution and transmission of disease-causing agents (epidemiology), effect of market forces on the stock exchange in relation to investment and risk (financial mathematics), nature of eclipsing binary stars (astrophysics) and impacts of pollution in waterways (environmental chemistry). After a discussion of these topics in relation to the module, the group came to the conclusion that each could be authentically represented as a case study from which students could gain an understanding of the use of mathematical modelling. This decision led to a subsequent discussion of how to organise the case studies within the module in a manner consistent with the 5Es model and within the constraint of 36-40 hours of study over 4-5 weeks allocated for a module. The outcome of this deliberation was agreement that the module would consist of: an introduction; a case study mandatory for all students; a second case study chosen from three options; and a final reflection tied to a capstone assessment. Consultation with, and review by educational designers took into account the views of the larger project team and selected teacher education student representatives in a cycle of review and development.

After the initial meeting, members of the Module Development Team (MDT) worked on developing draft versions of their case studies, some in teams and some as individuals, in collaboration with the instructional designer and the MDT leader. Draft case studies were presented at a second face-to-face meeting so that members of the MDT could provide critique and feedback. Comments and suggestions were accommodated into the existing drafts and then finalised.

EVALUATION OF THE COLLABORATIVE PROCESS

After the design of the module and trial with ITES, semi-structured interviews were conducted with each member of the MDT. The instructional designer, a member of the MDT, conducted six interviews within one month of the completion of the module. Interviews were digitally recorded and later transcribed by an independent researcher for the purpose of analysis.

Interview Protocol

Interviews were based on a protocol developed by the larger project team consisting of three core open-ended questions. Relevant to this report is Question 3 that included response eliciting prompts as set out below:

Describe, from your perspective, the experience of working in a cross-disciplinary team to develop the module as a whole. For example:

- What do you believe was the value in including contributions from different disciplines? Describe advantages/disadvantages.
- Are you satisfied/happy/impressed with the module as an outcome of the collaboration?
- Outline the opportunities/advantages for educators/mathematicians/scientists working together in promoting STEM education.
- Describe any limitation/constraints/barriers for educators/mathematicians/ scientists working together in promoting STEM education.

The interviewer also made use of additional prompts when she saw it necessary to clarify a response or probes when seeking greater depth in a response. Interview duration was between 35 and 55 minutes.

PERSPECTIVES ON THE EXPERIENCE OF INTERDISCIPLINARY COLLABORATION AND DISCUSSION

Participants' transcribed responses were coded through a process of constant comparison (Strauss & Corbin, 1990).) against the research questions and a frame informed by Wenger's ways of participating in a community of practice and the concept of boundary crossing. While not all comments could be categorised against the elements of the model, all noteworthy episodes were documented.

Did the collaboration produce a quality outcome?

Participants were unanimous in their views that the outcome of the collaboration was of high quality:

Leonard: Yes I'm happy with it, I've spent most of my time looking at the binary stars and looking at the epidemiology. I'm quite happy with them, part of me, the mathematician in me would like to take them both a little bit further mathematically but at the level they're aimed at that would not be appropriate, I think we stopped at the right level.

Martin: I thought that the end product was fantastic…Whether you naturally attracted to maths or not, and the big problems on this planet, I don't think we can solve outside of, without modelling...We have to model to foresee the future and we are all resource limited.

While most participants indicated they were pleased with the finalised module, they also viewed the product of the collaboration from the perspective of their own discipline – as in the case of Leonard, a mathematician, in the excerpt above, who had to hold back from arguing for the inclusion of more sophisticated mathematics. An exception was Martin who could see the value of bringing aspects of another discipline (mathematics) to her teaching of first year biology; via a collaboration with a mathematician that would complement his expertise as a scientist.

Martin: You know I think if I do first year Biology, I will also need to bring in the mathematical expertise into it and it's not with me, it will be with someone

> that comes and helps me develop the maths behind it. But I know what the context is in which the maths is needed.

Participants' comments indicate the module had acted as a boundary object that allowed team members from different disciplines to cross disciplinary boundaries, there was an understandable tendency to view the product of their collaboration from the perspective of the discipline in which they were expert. Thus, while boundaries were crossed during the process of module development most developers crossed the bridge back to their own discipline when viewing the final product.

What were the opportunities when collaborating across disciplines?

All six interviewees spoke about the advantages of the problem-based approach that embedded mathematical modelling and situated their disciplinary knowledge and practices in real contexts, satisfying a broader goal of solving real life problems.

Many raised the challenge of knowing enough about other disciplinary knowledge but in some ways saw this as an opportunity rather than a disadvantage.

> John: The advantages…being able to use contexts that are really authentic and that they address real problems…[teacher] educators may not be quite okay with some of these current edge scientific problems such as the spread of disease if they haven't got an expert that can really help them inform how they should, or what datasets they should use and how they should be interpreting data.

This comment makes it clear that teacher educators, at least, can be advantaged through the input of discipline experts. Others commented on the usefulness of having a teacher educator's perspective on the implementation of teaching ideas within science or mathematics a discipline.

> James: So, I think we can do with a lot more learning support in academia [refereeing to science and mathematics disciplines] in general. I particularly liked that this module was collaboration, in the full sense, between scientists and educators.

Another interviewee looked at the issue more broadly.

> Leonard: There are certainly advantages for people to work together to promote STEM [Science technology Engineering and Mathematics]…I think we should take every opportunity to promote it. If people can work together, then perhaps we can create things that have more depth and breadth.

These comments indicate that there was advantage to both teacher educators and mathematics and science experts by crossing discipline boundaries – both in a reciprocal sense but also for the broader STEM agenda.

Were there any limitations associated with interdisciplinary collaboration?

Participants commented that the pressure to meet discipline-based content outcomes in their teaching limited the way that scientific disciplines worked together let alone looking for synergies with education.

> Martin: I think the limitations are if we think too specific and too small and if we go "we don't have room in our curriculum to link across because I need all my time to stuff it full of Biology knowledge".

> John: Could this collaboration happen easily and effectively where there is no science or mathematics faculty attached to a university with a teacher education program?

These comments indicate that there are institutional constraints that make collaboration between different communities of practice more difficult. Such restrictions need to be acknowledged and accommodated for it collaborative boundary crossing is a desired outcome.

One respondent expressed concern about how students' would receive the explicit embedding of mathematics in her discipline of environmental science.

> Irene: When I first heard about this, I thought, mathematics? Environmental chemistry? Ah, from my experience with dealing with classes both at university, high school and primary school, my experience is generally that the idea of doing the maths would turn students off straight away.

Thus, not only was there risk associated with interdisciplinary collaboration in terms of mapping out new relationships and approaches to teaching but also in how the product of their collaboration was received by end users – their students. This is a reminder that boundary crossing between two communities of practice is not a simple matter as the outcome may influence and have impact on other communities.

CONCLUSION

The means by which the MDT interacted in the development of the online learning module within the ORS was consistent with Wenger's ways of participating within communities of practice. There was *engagement* as members of the MDT worked together to produce an artefact in the form of a module on mathematical modelling. The way in which MDT members represented their own disciplines while exploring the potential benefits (and risks) of interdisciplinary collaboration was consistent with the *imagination* mode of working within a community of practice. The outcome of the collaboration, the modelling module, required *alignment* of perspectives and actions to realise the goal of producing a quality outcome. Thus, representatives of different communities across mathematics science and education came together work as a community of practice for the purpose of a tangible outcome.

While MDT members were unanimous in their view of the high quality of the product of their work together and acknowledged the advantages of interdisciplinary collaboration, they also identified a number of constraints. These included disciplinary demands within their teaching roles that required attention to a large body of content, leaving little opportunity to include aspects of knowledge and practice from other disciplines. Such challenges are reminders of the complexities that must be accommodated when crossing boundaries in search of interdisciplinary collaboration. Thus, if interdisciplinary collaboration is seen as a priority in mathematics and science education, further research is needed into how to best enable the necessary boundary crossings in realising this goal.

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