Conditional heresies

Cariani, Fabrizio and Goldstein, Simon

CONDITIONAL HERESIES *

Fabrizio Cariani and Simon Goldstein

Abstract

The principles of Conditional Excluded Middle (CEM) and Simplification of Disjunctive Antecedents (SDA) have received substantial attention in isolation. Both principles are plausible generalizations about natural language conditionals. There is however little discussion of their interaction. This paper aims to remedy this gap and explore the significance of having both principles constrain the logic of the conditional. Our negative finding is that, together with elementary logical assumptions, CEM and SDA yield a variety of implausible consequences. Despite these incompatibility results, we open up a narrow space to satisfy both. We show that, by simultaneously appealing to the alternative-introducing analysis of disjunction and to the theory of homogeneity presuppositions, we can satisfy both. Furthermore, the theory that validates both principles resembles a recent semantics that is defended by Santorio on independent grounds. The cost of this approach is that it must give up the transitivity of entailment: we suggest that this is a feature, not a bug, and connect it with recent developments of intransitive notions of entailment.

1 Introduction

David Lewis's logic for the counterfactual conditional (Lewis 1973) famously invalidates two plausible-sounding principles: simplification of disjunctive antecedents (SDA),[1] and conditional excluded middle (CEM).[2] Simplification states that conditionals with disjunctive antecedents entail conditionals whose antecedents are the disjuncts taken in isolation. For instance, given SDA, (1) entails (2).

*Special thanks to Shawn Standefer for extensive written comments on a previous version of this paper and to anonymous reviewers for *Philosophy and Phenomenological Research* and the Amsterdam Colloquium. Also thanks to Andrew Bacon, Ivano Ciardelli, Simon Charlow, Kai von Fintel, Branden Fitelson, Jeremy Goodman, Nathan Howard, Daniel Rothschild, Jeff Russell, Paolo Santorio, Mark Schroeder, Alexis Wellwood, Lee Walters, Dan Waxman, audiences at LENLS 14, the 2017 Amsterdam Colloquium, and the USC Mind and Language Group.

[1] See Fine 1975; Nute 1975; Lewis 1977; and Nute 1980*a*.

[2] See Stalnaker 1981; von Fintel 1997; DeRose 1999; Williams 2010; and Klinedinst 2011.

(1) If Hiro or Ezra had come, we would have solved the puzzle.

(2) If Hiro had come, we would have solved the puzzle and if Ezra had come, we would have solved the puzzle.

Conditional Excluded Middle is the claim that the (boolean) disjunction of $A > C$ and $A > \neg C$ must be a logical truth. Against the background of classical logic, a distinctive consequence of CEM is that the negation of $A > C$ entails $A > \neg C$. For instance, (3) entails (4).

(3) It is not the case that if Hiro had come we would have solved the puzzle.

(4) If Hiro had come, we would not have solved the puzzle.

Much attention has been devoted to these heretical principles in isolation, but virtually no work has considered their interaction. Since there are strong reasons to accept both principles, it is urgent to investigate how they might be made to fit.

The pessimistic finding at the center of this paper is that the heresies do not mix easily. We present a battery of impossibility results showing that no traditional theory of conditionals or disjunction can allow them to coexist. The general shape of these results is that given a variety of minimal assumptions about conditionals, disjunction, and logical consequence, the combination of our two heresies requires that the conditional either be the material conditional or share some undesirable property with it.

Despite these negative findings, we argue that the project of combining CEM and SDA is not hopeless. To validate both principles, we synthesize two different tools that can independently be used to validate each principle individually. Speaking abstractly, these tools are operations on possible semantic values of the conditional: given a possible semantic value m and some inferential pattern P, the operation produces a new semantic value m' which is related to m in some way and validates P (whether or not m did).

In the case of SDA, we turn to the idea that disjunctions involve alternatives (for example, Alonso-Ovalle 2006), introducing a general mechanism that turns any candidate semantic value for the conditional into a derived one that is guaranteed to validate SDA. Unfortunately, this tool alone does not yield CEM in full generality—even if we start out with a conditional connective that validates CEM. In particular, CEM won't be guaranteed for conditionals with disjunctive antecedents. In the case of CEM, we turn to the growing semantic tradition that invokes homogeneity inferences (von Fintel 1997; Križ 2015*b*). Building on the idea of homogeneity, we identify a tool that forces the validity of CEM, no matter what conditional we begin with. The drawback of this tool is that some appealing generalizations of SDA fail. In particular, simplification fails for *might* conditionals like "if it rains or snows, you might need boots".

We show that these problems can be solved by using *both* tools in sequence. This, then, is the shape of our final proposal: start with any conditional semantic value; inject alternative sensitivity, thus securing SDA; finally, inject homogeneity presuppositions, thus securing CEM. It turns out that the resulting conditional need not be trivial or equivalent to the material conditional. in fact, this procedure can yield a conditional connective that was recently defended, on different grounds, by Santorio 2017.

The resulting theory leads, however, to one last heresy: the entailment relation must fail to be transitive. In particular, while CEM is valid in the resulting theory, other principles are invalid that are logical consequences of CEM. We conclude by discussing this feature of our view and connecting it with other work on intransitive entailment. We also note some options for approximating the explanatory insight behind our proposal within more classical frameworks in which the transitivity of entailment is secured, but the intransitive behavior is emulated by some other means.

Our negative results and our positive discussion should be of particular significance to those theorists who are committed to frameworks that are founded on endorsing one of the heresies. For example, validating SDA is a founding assumption of truth-maker semantics (Fine, 2012). Given our results, friends of truth-maker semantics who also want to endorse CEM need broader revisions than one might otherwise anticipate. The same is true of those frameworks on which conditionals denote selection functions (Stalnaker, 1968, 1973, 1981), which in turn are founded on the idea that CEM is valid.

2 The Case for the Heresies

In this section, we restate and defend our two principles. Letting '$\vdash$' denote the consequence relation, simplification of disjunctive antecedents can be formulated as the following entailment:[3]

SDA. $(\mathsf{A} \text{ or } \mathsf{B}) > \mathsf{C} \vdash (\mathsf{A} > \mathsf{C}) \wedge (\mathsf{B} > \mathsf{C})$

The main argument for SDA seems to consist entirely in the observation that instances like the one from (1) to (2) sound extremely compelling (see Fine 1975, p.453-454).

Of course, a conclusive case for the validity of SDA does require some kind of defense beyond "it sounds pretty plausible". For instance, it is well known (Fine 1975; Ellis *et al.* 1977) that SDA has troublesome downstream consequences: in the presence of a principle of substitution for logical equivalents, SDA entails antecedent strengthening ($\mathsf{A} > \mathsf{C} \vdash \mathsf{A}^+ > \mathsf{C}$, where $\mathsf{A}^+ \vdash \mathsf{A}$). We will investigate these consequences and their significance below, but the point for

[3] In formulating, SDA, we use *or*, as opposed to '$\vee$' because much of the evidence for SDA relies on the intuitive plausibility of its natural language instances and we do not assume at the outset that natural language disjunction is boolean.

now is that the preliminary motivation for SDA tends to be clear and strong judgments about the validity of its instances.

Conditional excluded middle can be formulated abstractly as a validity claim:

CEM. $\vdash (A > B) \vee (A > \neg B)$

Unlike SDA, CEM is not typically justified by direct intuitions of validity about its instances. Instead, defenders of CEM propose that various linguistic phenomena fall into their proper place once we posit the validity of this schema. For instance, one might think that (3), which is of the form $\neg(hiro > puzzle)$, intuitively entails (4), which is of the form $hiro > \neg puzzle$. Given CEM, this inference turns into an application of disjunctive syllogism.

More generally, conditionals involving *will* and *would* consequents fail to enter into scope relations that would be expected if CEM failed. (The playbook for this sort of argument is laid out in the seminal discussion of Stalnaker 1981, p.137-139). A recent version of this argument relies on data involving attitude verbs that lexicalize negation (see Cariani and Santorio, 2018, for a version of this argument involving *will*). In the case of *would*, the argument centers around the observation that (5) and (6) sound equivalent:

(5) I doubt that if you had slept in, you would have passed.

(6) I believe that if you had slept in, you would have failed.

The equivalence is easily explained if CEM is valid (and assuming that failing equals not passing). The speaker doubts *sleep* > *pass*; if there was a way for this conditional to be false other than by *sleep* > *fail* being true, it should be possible to accept (5) without accepting (6). By contrast, it is hard, if not impossible, to explain without it.

This argument streamlines an older argument for CEM involving the interaction between conditionals and quantifiers.[4] Consider:

(7) No student will succeed if he goofs off.

(8) Every student will fail if he goofs off.

(7) and (8) are intuitively equivalent. They appear to involve quantifiers taking scope over conditionals. Given CEM and this scope assumption, they are predicted equivalent. Take an arbitrary student, and suppose it is false of him that he will succeed if he goofs off. By CEM it follows that he will fail if he goofs off. This batch of data involving *will*-conditionals looks equally compelling when considering counterfactual conditionals (Klinedinst, 2011).

(9) No student would have succeeded if he had goofed off.

[4] See Higginbotham 1986; von Fintel and Iatridou 2002; Leslie 2009; and Klinedinst 2011 for discussion.

(10) Every student would have failed if he had goofed off.

On reflection, we take the interaction of conditionals and quantifiers to also favor CEM, for both indicative and counterfactual conditionals.

Yet another argument for CEM is based on the interaction between *if* and *only*.[5] CEM can help explain why *only if* conditionals imply their *if...then* counterparts. Consider the following conditionals:

(11) The flag flies only if the Queen is home.

(12) If the flag flies, then the Queen is home.

(13) The flag flies if the Queen isn't home.

(11) entails (12). In von Fintel 1997 this entailment is derived compositionally, on the assumption that *only* in (11) takes wide scope with respect to the conditional. *Only* then negates the alternatives to the conditional *the flag flies if the Queen is home*, which are assumed to include (13). Given some background assumptions, CEM and the negation of (13) imply (12).[6]

This short catalogue does not exhaust the motivation for CEM,[7] but it provides sufficient motivation to explore the relationship between CEM and other plausible principles like SDA.

Before moving on, it is worth highlighting that these arguments for CEM require that the relevant conditional connective not be the material conditional. The material conditional, of course, does validate CEM, but it does so in a way that is incompatible with the explanatory benefits we just noted. For instance, the quantifiers argument relied essentially on the 'Weak Boethius' Thesis'(WBT)—the claim that $A > C$ and $A > \neg C$ cannot both be true (when A is consistent).[8]

WBT. $A > C; A > \neg C \models \bot$

Without this assumption, it could be that some student will fail if he goofs off, but will also succeed if he goofs off. But the material conditional invalidates this principle, allowing both conditionals to be true when A is false. In this case, the negative conditional (10) would be stronger than the positive conditional (9), since the positive conditional can be true simply because the relevant individuals did not actually goof off. Similar points can be established for our other arguments.[9]

[5] See Barker 1993 and von Fintel 1997 for discussion.

[6] For some experimental evidence that support the inferential judgments involved in this argument, see Cariani and Rips ms..

[7] For other arguments and further discussion, see Cross 2009, Goodman ms., Williams 2010.

[8] For discussion, see Pizzi and Williamson 2005.

[9] Both arguments are incompatible with the material conditional because the negation of a material conditional tends to be quite strong, entailing the antecedent and the negation of the consequent. But the attitudes argument requires negating the conditional "if you had slept in,

This last point is relevant to the interpretation of our negative results. For example, we will shortly show that SDA and CEM jointly imply collapse to the material conditional. One might reason as follows: CEM gives you something good (it explains the data in this section) and something bad (together with SDA it yields implausible consequences); the way to have both CEM and SDA is to go in for the material conditional; so if CEM's goodies outweigh the bad consequences, we have an argument for the material conditional. However, this interpretation would be incorrect, because the Weak Boethius Thesis (which the material conditional violates) is essentially involved in delivering the goodies. Without WBT all of the positive arguments in favor of CEM are undercut.

3 Incompatibility Results

Having introduced our favorite conditional heresies, we explain why it is difficult to jointly accept them. In §3.1, we show that, given modest logical assumptions, CEM and SDA collapse to the material conditional. The next result (in §3.2) dispenses with most of the logical assumptions and yields the conclusion that CEM and SDA collectively entail that the conditional is, in a certain respect, trivial. While we derive these results syntactically, we note in §3.3 that there are related collapse results that exploit semantic reasoning only.

In keeping with a distinction we have drawn in the previous section, we appeal to two notions of disjunction: (i) natural language *or* (which we used in stating SDA) and (ii) boolean disjunction, symbolized '$\vee$' (which we used in stating CEM). It will strengthen our argument to refrain from assuming that these connectives have the same semantic value. Our results require classical assumptions about the logic of '$\vee$' but very few assumptions about the logic of *or*.

3.1 Collapse

CEM and SDA together imply collapse to the material conditional, given relatively modest assumptions about the logic. We assume standard sequent rules for classical connectives as well as the standard structural rules governing classical logic.[10] Among the structural rules, the transitivity of entailment–which is itself a consequence of the Cut rule–will play a very important role in our discussion.

Transitivity. if $A \vdash B$ and $B \vdash C$, then $A \vdash C$

you would have passed" and the argument involving *only if* requires negating (13). One specific problematic consequence of this would be that, according to the material conditional analysis, *only if* conditionals would imply the truth of their consequent and the falsity of the antecedent. (11) clearly does not have this implication.

[10] For contemporary sources on the sort system we presuppose, see Buss 1998, Troelstra and Schwichtenberg 2000; Restall 2000; Bimbó 2014; Negri and von Plato 2001.

Cut. if $X \vdash \mathsf{A}$ and $Y, \mathsf{A} \vdash \mathsf{B}$, then $X, Y \vdash \mathsf{B}$

(notation: X and Y denote sets of sentences and that X, A denotes $X \cup \{A\}$.) Several of our proofs rely on disjunction rules, so it is worth stating them explicitly.

Cases. if $X, \mathsf{A} \vdash \mathsf{C}$ and $Y, \mathsf{B} \vdash \mathsf{C}$, then $X, Y, (\mathsf{A} \vee \mathsf{B}) \vdash \mathsf{C}$

$\vee$-Intro. if $X, \mathsf{A} \vdash \mathsf{B}$, then $X, \mathsf{A} \vdash \mathsf{B} \vee \mathsf{C}$

To these, add specific assumptions about conditionals (three dedicated sequent axioms and one new rule):

Modus Ponens. $\mathsf{A}, \mathsf{A} > \mathsf{C} \vdash \mathsf{C}$

Reflexivity. $\vdash \mathsf{A} > \mathsf{A}$

Agglomeration. $\mathsf{A} > \mathsf{B}, \mathsf{A} > \mathsf{C} \vdash \mathsf{A} > (\mathsf{B} \wedge \mathsf{C})$

Upper Monotonicity. if $\mathsf{B} \vdash \mathsf{C}$, then $X, \mathsf{A} > \mathsf{B} \vdash \mathsf{A} > \mathsf{C}$

While these assumptions are not entirely uncontroversial, they are generally accepted in the literature.[11]

For ease of reference, we call this combination of assumptions *the classical package*. We can now state our result more precisely.

Fact 1 *Given the classical package,* CEM *and* SDA *imply that* $\mathsf{A} > \mathsf{C} \dashv\vdash \neg \mathsf{A} \vee C$.

> We prove this equivalence in stages. First stage: SDA and CEM jointly imply the True Consequent paradox of material implication.
>
> True Consequent. $\mathsf{C} \vdash \mathsf{A} > \mathsf{C}$
>
> Next stage: proving this is enough to reach full collapse, in combination with the classical package.
>
> Comments on notation: Individual lines in the proofs below abbreviate multiple reasoning steps in the full sequent proof. At step 5 we note an implicit application of classical reasoning by citing something (LEM) that is not a rule in the actual proof system. Each line is annotated with a list of all the rules on which the suppressed piece of reasoning depends.
>
> First, we prove that True Consequent follows from SDA and CEM.

[11] For instance, many of those who dispute the validity of modus ponens accept the validity of some special forms of the inference. In particular, conditionals that do not themselves contain further modals or conditionals are generally believed to validate modus ponens even by those who doubt the general validity of the inference (e.g. McGee 1985, Kolodny and MacFarlane 2010). These restricted versions of modus ponens are sufficient to yield our results.

1. $\vdash ((\mathsf{A} \text{ or } \neg\mathsf{A}) > \mathsf{C}) \vee ((\mathsf{A} \text{ or } \neg\mathsf{A}) > \neg\mathsf{C})$ CEM
2. $\vdash (\mathsf{A} > \mathsf{C} \wedge \neg\mathsf{A} > \mathsf{C}) \vee (\mathsf{A} > \neg\mathsf{C} \wedge \neg\mathsf{A} > \neg\mathsf{C})$ 1, cases, SDA, cut
3. $\mathsf{C}, \neg(\mathsf{A} > \mathsf{C}) \vdash (\mathsf{A} > \neg\mathsf{C} \wedge \neg\mathsf{A} > \neg\mathsf{C})$ 2, cases, cut, weakening
4. $(\mathsf{A} \vee \neg\mathsf{A}), (\mathsf{A} > \neg\mathsf{C} \wedge \neg\mathsf{A} > \neg\mathsf{C}) \vdash \neg C$ cases, ponens
5. $\mathsf{C}, \neg(\mathsf{A} > \mathsf{C}) \vdash \neg\mathsf{C}$ 3, 4, cut, LEM
6. $\mathsf{C} \vdash \mathsf{A} > \mathsf{C}$ 5, negation rules

We can then show that another paradox of material implication, False Antecedent, follows from True Consequent plus the classical package.

False Antecedent. $\neg\mathsf{A} \vdash \mathsf{A} > \mathsf{C}$

1. $\neg\mathsf{A} \vdash \mathsf{A} > \neg\mathsf{A}$ TC
2. $\neg\mathsf{A} \vdash \mathsf{A} > \neg\mathsf{A} \wedge \mathsf{A} > \mathsf{A}$ 1, reflexivity, conjunction rule
3. $\neg\mathsf{A} \vdash \mathsf{A} > (\neg\mathsf{A} \wedge \mathsf{A})$ 2, agglomeration, cut
4. $\neg\mathsf{A} \vdash \mathsf{A} > \mathsf{C}$ 3, upper monotonicity, cut

Finally, we can use the classical package to show that TC and FA imply collapse.

1. $\neg\mathsf{A} \vee \mathsf{C} \vdash \mathsf{A} > \mathsf{C}$ cases, FA, TC
2. $\mathsf{A} > \mathsf{C}, \mathsf{A} \vdash \mathsf{C}$ ponens
3. $\mathsf{A} > \mathsf{C} \vdash \neg\mathsf{A} \vee \mathsf{C}$ 2, material conditional intro

Previous work on SDA has shown that it sits in major tension with the substitution of logical equivalents (Fine 1975; Ellis *et al.* 1977). Interestingly, our own result makes no use of this principle. More generally, we make no assumptions about the semantic or logical properties of *or*, except that it supports SDA.

3.2 The Interconnectedness of All Things

Fact 1 requires a number of assumptions about the classicality of the conditional, disjunction, and logical consequence. Furthermore, the argument appeals to rather artificial instances of CEM—ones with antecedents of the form A or ¬A. If we are satisfied with a slightly weaker conclusion, a closely related result can be derived with far fewer assumptions and without appealing to conditionals with tautological antecedents.

The result is that combining CEM and SDA forces the conditional to validate an undesirable schema, which we call IAT for "the Interconnectedness of All Things".

IAT. $(A > C \wedge B > C) \vee (A > \neg C \wedge B > \neg C)$

The reason why validating IAT is undesirable is that it requires an extreme level of dependence among arbitrary distinct sentences. Consider an instance of the above in which A="Abe flies", B="Bea runs" and C="Cleo swims". Then it must be that either both *Abe flies* > *Cleo Swims* and *Bea runs* > *Cleo swims* are true or both *Abe flies* > *Cleo does not swim* and *Bea runs* > *Cleo does not swim* are. Among other things, this appears to entail that it is incoherent to reject both of the following:

(14) If Abe flies, then Cleo swims.

(15) If Bea runs, then Cleo does not swim.

It would be incorrect to say that nothing that is recognizably a conditional validates IAT. For one thing, the material conditional does.[12] Nonetheless, we comfortably assert that only unsatisfactory conditional connectives satisfy IAT.

With an eye to our later discussion, we provide an explicit statement and proof of the result.

Fact 2 *Given disjunction rules, cut,* CEM*, and* SDA, IAT *must be a logical truth.*

1.	$\vdash [(A \text{ or } B) > C] \vee [(A \text{ or } B) > \neg C]$	CEM
2.	$(A \text{ or } B) > C \vdash (A > C \wedge B > C)$	SDA
3.	$(A > C \wedge B > C) \vdash$ IAT	$\vee$-Intro
4.	$(A \text{ or } B) > C \vdash$ IAT	2, 3, cut
5.	$(A \text{ or } B) > \neg C \vdash (A > \neg C \wedge B > \neg C)$	SDA
6.	$(A > \neg C \wedge B > \neg C) \vdash$ IAT	$\vee$-Intro
7.	$(A \text{ or } B) > \neg C \vdash$ IAT	5, 6, cut
8.	$\vdash$ IAT	1, 4, 7, cases, cut

In addition to the intuitive reasons we gave against the validity of IAT, there is an important theoretical reason which helps put Fact 2 in perspective. Suppose we toss the Weak Boethius thesis into our cauldron of assumptions; then, IAT yields the further absurd consequence that $A > C \wedge B > \neg C$ is inconsistent

[12]This is to be expected given that it validates CEM and SDA. Note that the material conditional is widely rejected as an analysis of the indicative conditional. One important class of references here is the seminal work in Edgington 1995 (but see also the survey Edgington, 2014). But rejection of the material analysis of the conditional is a tenet in the relevance logic tradition stemming from Anderson and Belnap 1975. What is even more important for our purposes is that we take our assumptions to be equally valid for counterfactuals as well, and virtually nobody believes that counterfactual conditionals are material conditionals.

(whenever A and B are consistent).[13] We submit that this would be an absurd consequence.[14]

3.3 Strict conditionals

As is to be expected, these proof-theoretic results correspond to related results at the semantic level. We approach the semantic landscape by considering how CEM interacts with semantic theses that are related to SDA. Let's begin with the idea that > is a strict conditional. That is, > universally quantifies over a fixed domain of worlds. More precisely, letting R be an accessibility relation over worlds, and letting R^w be the set of R-accessible worlds from w:

(Δ1) $[\![\mathsf{A}> \mathsf{C}]\!] = \{w \mid R^w \cap [\![\mathsf{A}]\!] \subseteq [\![\mathsf{C}]\!]\}$

Alonso-Ovalle 2006 notes that strict conditionals validate SDA.[15]

Unfortunately, this framework cannot accommodate both CEM and modus ponens. While this result follows already from the syntactic result of the previous sections, it can be further illuminated by thinking in purely semantic terms. In the strict framework, CEM and modus ponens correspond to constraints on the accessibility relation–respectively, uniqueness and reflexivity.

Uniqueness. $\forall w, v, v' : wRv \wedge wRv' \Longrightarrow v = v'$

Reflexivity. $\forall w : wRw$

The nature of the correspondence is the usual one from modal logic. That is to say: Uniqueness is the weakest constraint that guarantees the validity of CEM. Similarly, Reflexivity is the weakest constraint that guarantee the validity of modus ponens.

The combination of these two constraints trivializes R, by entailing that every world only accesses itself.

[13] Here is a sketch of the proof: this conjunction is not compatible with IAT under WBT because it entails that whenever A > C, we also must have ¬(A > ¬C)—ruling out the second disjunct of IAT— and whenever B > ¬C we must also have ¬(B > C)—ruling out the first disjunct of IAT.

[14] One response to our incompatibility results would be to bite the bullet, and allow that even counterfactuals are logically equivalent to material conditionals. We think the best way to pursue this strategy might be with some form of the dynamic conditional in Gillies 2007 and Gillies 2009. Indeed, Gillies 2009 proposes that the indicative conditional is logically equivalent to the material conditional, in order to validate Import-Export. But while the two conditionals are logically equivalent, they are not semantically equivalent; their semantic value are more fine grained than logical equivalence, in a way that predicts different behavior of the two conditionals under higher operators, like negation. While we think this is an interesting strategy to pursue, especially for indicative conditionals, we do not think that it is plausible set in full generality. Recall that our baseline target is the subjunctive conditional, and it is much less plausible to bite the bullet on subjunctives going in for the paradoxes of material implication.

[15] When > is a strict conditional, it is downward monotone—that is, A ⊢ B guarantees B > C ⊢ A > C. In this setting, SDA follows from the validity of disjunction introduction.

Isolation. $\forall w, v : wRv \Longrightarrow v = w$

Once again, $>$ collapses onto the material conditional—its domain of quantification being limited to the singleton set of the world of evaluation.

Recent defenders of strict conditionals (von Fintel 2001; Gillies 2007) might shrug at this result, since they implement the view in the more complex framework of dynamic semantics. In the context of dynamic semantics, the correspondence results between axioms and constraints on models play out differently. However, reflection on the consequence relation $\vdash$ once again yields a collapse result even for such sophisticated views. No matter how sophisticated, strict analyses must accept, for some choice of $\Box$, the validity of:

Strictness. $\mathsf{A} > \mathsf{C} \dashv\vdash \Box(\neg\mathsf{A} \vee \mathsf{C})$

This too collapses in the presence of basic assumptions. To see this, note that, given strictness, CEM corresponds to the validity of $\Box(\neg\mathsf{A} \vee \mathsf{C}) \vee \Box(\neg\mathsf{A} \vee \neg\mathsf{C})$. Consider the instance $\Box(\neg\top \vee \mathsf{C}) \vee \Box(\neg\top \vee \neg\mathsf{C})$. Given the identity rule for disjunction, $\mathsf{C} \dashv\vdash (\neg\top \vee \mathsf{C})$. and because $\Box$ is closed under substitution of logical equivalents, $\vdash \Box\mathsf{C} \vee \Box\neg\mathsf{C}$. Suppose now that $>$ validates modus ponens. In light of strictness, this corresponds to the principle that $\Box$ is strong, so that $\Box\mathsf{C} \vdash \mathsf{C}$. This last assumption allows us to infer the triviality of $\Box$. That is, it allows us to deduce $\Box\mathsf{C} \dashv\vdash \mathsf{C}$. In particular, $\Box(\neg\mathsf{A} \vee \mathsf{C}) \dashv\vdash (\neg\mathsf{A} \vee \mathsf{C})$, which chained with strictness yields collapse.

We've now seen that CEM is quite difficult to reconcile with a strict conditional analysis, on both model theoretic and more abstract grounds. This is a surprising result. For von Fintel 1997's analysis of *only if* conditionals, one of the major arguments for the validity of CEM, requires that contraposition be valid. This in turn implies the validity of Antecedent Strengthening, and more generally implies that the conditional is strict. This suggests there is a serious tension within that analysis.

4 Heresies in isolation

The results in the previous section suggest that the space to validate both SDA and CEM is at best narrow. Before exploring it, we will constrain it further. Faced with these results, defenders of SDA might be tempted to simply reject CEM. After all, despite the evidence that supports it, the principle remains controversial, and Lewis 1973 presented several potential counterexamples to it.

In this section, we want to highlight a few results to the effect that SDA *alone* requires us to tread carefully. We have already noted a classic result (Fine, 1975; Ellis *et al.*, 1977; Santorio, 2017) to the effect that SDA together with substitution of logical equivalents yields antecedent strengthening (which pattern is invalidated by many theories). We present two more results in a similar vein. The first establishes that a fully general statement of SDA—specifically one that involves modal consequents—is not compatible with the semantic assumption that conditionals are strict conditionals. The second shows that we can get collapse just by adding SDA to a more modest relative of CEM—the principle that $\mathsf{A} \wedge \mathsf{C}$ entails $\mathsf{A} > \mathsf{C}$ (this result was first presented in Nute 1980*b*).

4.1 *Might* counterfactuals

Alonso-Ovalle 2006 observes that simplification of disjunctive antecedents also occurs with *might* counterfactuals, as in the inference from (16) to (17).

(16) If Hiro or Ezra had come, we might have solved the puzzle.

(17) If Hiro had come, we might have solved the puzzle.

Additionally, he shows that strict accounts of counterfactuals cannot validate this form of simplification, given a boolean semantics for disjunction.

It will be convenient to take *If A, might B* as idiomatic. Formally, we write this as $\mathsf{A} >_{\Diamond} \mathsf{B}$. With this in hand we can state:

$\Diamond$-SDA. $(\mathsf{A} \text{ or } \mathsf{B}) >_{\Diamond} \mathsf{C} \vdash (A >_{\Diamond} C) \wedge (B >_{\Diamond} C)$

To explore whether $\Diamond$-SDA is valid, we must provide $>_{\Diamond}$ with a semantics. Towards this goal, suppose that *might*-counterfactuals existentially quantify over the very same domain that *would*-counterfactuals universally quantify over.

(Δ2) $[\![\mathsf{A} >_{\Diamond} \mathsf{C}]\!] = \{w \mid R^{w} \cap [\![\mathsf{A}]\!] \cap [\![\mathsf{C}]\!] \neq \emptyset\}$

Suppose finally that the accessibility relation R is reflexive–which as we noted corresponds to the validity of modus ponens.[16]

Note that, because we do not derive $>_{\Diamond}$ compositionally, $\Diamond$-SDA is not simply a special case of SDA. Nonetheless, $\Diamond$-SDA is very much in the spirit of SDA itself, and plausibly supported by many of the same intuitive considerations that support SDA.

We can prove a surprising result that constrains the range of acceptable semantic values for disjunction. Under the assumptions we made about $>_{\Diamond}$, the semantic value of disjunction cannot be a proposition, even if the disjuncts are propositional. (By proposition, we mean a set of worlds.)

Fact 3 *Assume (Δ1), (Δ2), the reflexivity of R and the validity of both* SDA *and* $\Diamond$-SDA. *Then disjunction is not propositional.*

[16] The result of this subsection can be proven on the basis of a weaker assumption than reflexivity: that for every world w, there is a world v such that vRw. However, the main path to a justification of this constraint is via reflexivity.

Proof: Consider arbitrary propositions $[\![\mathsf{A}]\!]$ (abbreviated: $\mathbf{A}$) and $[\![\mathsf{B}]\!]$ (abbreviated: $\mathbf{B}$). We prove that there can be no function $\mathbf{or}$, such that $\mathbf{or}(\mathbf{A}, \mathbf{B})$ (which we write in infix notation as $\mathbf{A}$ $\mathbf{or}$ $\mathbf{B}$) such that $\mathbf{A}$ $\mathbf{or}$ $\mathbf{B} = [\![\mathsf{A} \text{ or } \mathsf{B}]\!]$. We establish that (i) $\Diamond$-SDA requires $\mathbf{A}$ $\mathbf{or}$ $\mathbf{B} \subseteq \mathbf{A} \cap \mathbf{B}$; and (ii) SDA requires $\mathbf{A} \cup \mathbf{B} \subseteq \mathbf{A}$ $\mathbf{or}$ $\mathbf{B}$. These two requirements are inconsistent whenever $\mathbf{A} \neq \mathbf{B}$.

Ad (i): let $\mathbf{C}$ be an arbitrary proposition. The validity of $\Diamond$-SDA requires that if $R^w \cap \mathbf{C}$ overlaps $\mathbf{A}$ $\mathbf{or}$ $\mathbf{B}$, then it overlaps $\mathbf{A} \cap \mathbf{B}$. But now suppose $\mathbf{A}$ $\mathbf{or}$ $\mathbf{B} \not\subseteq \mathbf{A} \cap \mathbf{B}$. Then there is a v in the former set, but not the latter. By reflexivity $v \in R^v$. Now, consider the special case $\mathbf{C} = \{v\}$. Then we must have that $\mathbf{C}$ overlaps both $\mathbf{A}$ and $\mathbf{B}$, but because it's a singleton, it cannot do that without overlapping both, so $v \in \mathbf{A} \cap \mathbf{B}$ after all, which is contradictory.

Ad (ii): let $\mathbf{C}$ be an arbitrary proposition and w an arbitrary world. The validity of SDA requires that if $(R^w \cap (\mathbf{A}$ $\mathbf{or}$ $\mathbf{B})) \subseteq \mathbf{C}$, $(R^w \cap \mathbf{A}) \subseteq \mathbf{C}$ and $(R^w \cap \mathbf{B}) \subseteq \mathbf{C}$. Now, suppose for *reductio* that $\mathbf{A} \cup \mathbf{B} \not\subseteq \mathbf{A}$ $\mathbf{or}$ $\mathbf{B}$. Then there is a u that belongs to the former set but not the latter. By reflexivity $u \in R^u$. Now let $\mathbf{C} = (\mathbf{A}$ $\mathbf{or}$ $\mathbf{B})$. We claim that $u \in \mathbf{C}$ which would be contradictory, since u was chosen so that $u \notin (\mathbf{A}$ $\mathbf{or}$ $\mathbf{B})$. This must follow because $u \in \mathbf{A} \cup \mathbf{B}$, and hence either $u \in \mathbf{A}$ or $u \in \mathbf{B}$. In the first case $u \in (R^u \cap \mathbf{A}) \subseteq \mathbf{C}$; in the second, $u \in (R^u \cap \mathbf{B}) \subseteq \mathbf{C}$. Either way $u \in \mathbf{C}$.

In fact, this proof establishes something slightly stronger. There can be no sentences A and B with distinct semantic values where SDA and $\Diamond$-SDA hold for them, paired with any consequent.

4.2 Strong Centering

Our main result so far has been that CEM and SDA lead to Collapse. In one respect, this is unsurprising. Nute 1980*b* (p. 40), Butcher 1983, and Walters 2009 have developed incompatibility results showing that SDA leads to trouble in the presence of Strong Centering.

Strong Centering. $\mathsf{A} \wedge \mathsf{C} \models \mathsf{A} > \mathsf{C}$

For example, Butcher 1983 shows that SDA and Strong Centering imply that the conditional is strict. Walters 2009 observes that these principles together quickly lead to the validity of True Consequent, discussed above. But CEM and Modus Ponens imply Strong Centering, and so in one respect these results rely on weaker premises than our own.

On the other hand, these results all rely on more assumptions about disjunction than our own. Butcher 1983 relies on the absorption rule—the rule encoding the equivalence of p and $(p \vee q) \wedge p$. Walters 2009 relies on the validity of Disjunction Introduction. These assumptions are absent from our results. For comparison and analysis, it will be useful to consider a simplified version of the result in Walters 2009. Assume that *or* is minimally classical, in the sense that it obeys the following instance of Disjunction Introduction.

Identity. $\top \vdash \top \text{ or } \mathsf{A}$

Given identity, SDA and Strong Centering imply collapse to the material conditional.

Fact 4 (Walters) *Given the classical package*, SDA, *Strong Centering, and Identity:*

$$\mathsf{A} > \mathsf{C} \dashv\vdash \neg\mathsf{A} \vee \mathsf{C}$$

Proof. The crucial step in this proof is to show that our assumptions validate the True Consequent inference, that $\mathsf{C} \vdash \mathsf{A} > \mathsf{C}$. From there, the proof proceeds like the proof of Fact 1.

1.	$\mathsf{C} \vdash (\top \text{ or } \mathsf{A}) \wedge \mathsf{C}$	identity, classical package
2.	$(\top \text{ or } \mathsf{A}) \wedge \mathsf{C} \vdash (\top \text{ or } \mathsf{A}) > \mathsf{C}$	Strong Centering
3.	$(\top \text{ or } \mathsf{A}) > \mathsf{C} \vdash \mathsf{A} > \mathsf{C}$	SDA
4.	$\mathsf{C} \vdash \mathsf{A} > \mathsf{C}$	1-3, transitivity

As anticipated, we note that Fact 4 is in one sense stronger than Fact 1, because it relies on a principle weaker than CEM. This does not make Fact 1 redundant since, unlike that earlier result, it appeals to a specific assumption about natural language *or*. Even more significantly, it does not make Fact 2 redundant, since that result did not depend on assuming modus ponens for $>$. These considerations are important because the fewer the assumptions, the narrower the space for the heresies to co-exist. Furthermore, as we move to a more constructive part of the paper, it is heuristically valuable to focus on results like Fact 2—results which do not depend on entertaining artificial disjunctions such as $\top \text{ or } \mathsf{A}$.

5 Alternatives

We have developed a variety of incompatibility results, showing that SDA and CEM are in considerable tension with one another. Given these results, the prospects for reconciling these principles might appear bleak. We now turn to developing strategies for dealing with this tension.

5.1 The alternatives package

Our first attempt builds on earlier attempts to validate SDA using alternative semantics in the style of Alonso-Ovalle 2006. While that framework holds fixed the variably strict analysis in Stalnaker 1968 and Lewis 1973, we show that the crucial ideas from alternative semantics are independent of the underlying theory of conditionals. We provide a general mechanism for taking any truth conditional semantics for the conditional and producing an alternative semantics from it that validates SDA. The resulting theory restricts the validity of CEM to non-disjunctive antecedents, which is how it blocks the proofs of the incompatibility results.

In alternative semantics, sentences do not denote propositions, but instead sets of propositions (or 'alternatives'). In many cases the operations that lift semantic values to this higher type are trivial. An atomic sentence p denotes the singleton set of worlds at which p is true (i.e. $\{\{w \mid w(p) = 1\}\}$). Negation denotes the set of worlds that are not in the union of its argument; conjunction is pointwise intersection.

(Δ3) $[\![\neg \mathsf{A}]\!] = \{W - \bigcup [\![A]\!]\}$

(Δ4) $[\![\mathsf{A} \wedge \mathsf{B}]\!] = \{\mathbf{A} \cap \mathbf{B} \mid \mathbf{A} \in [\![\mathsf{A}]\!], \mathbf{B} \in [\![\mathsf{B}]\!]\}$

These operations are classical in the sense that when the semantic values are singleton sets of propositions they collapse on the standard Boolean connectives (more precisely: they collapse on the intensional analogues of complementation and intersection).

The main element of non-classicality at the ground-floor of alternative semantics is the treatment of disjunctions. The idea is that a disjunction A or C presents both of A and C as alternatives. In the system we adopt here, this means that the semantic value of a disjunction is the union of the semantic values of the disjuncts.

(Δ5) $[\![\mathsf{A} \text{ or } \mathsf{B}]\!] = [\![\mathsf{A}]\!] \cup [\![\mathsf{B}]\!]\}$

This has the non-classical effect that a disjunction can denote a non-singleton set even when its disjuncts both denote singletons.

This behavior of disjunction is essential to the treatment of simplification. To validate SDA, we let the conditional operate on each alternative in this set.[17]

The main idea is to derive the meaning of the conditional from an underlying propositional conditional operator >—the 'proto-conditional'—which

[17] We can think of several other approaches to SDA as offering different proposals about what exactly the alternatives for a disjunction are. For example, the state-based semantics from Fine 2012 and Briggs 2012 can be interpreted so that the meaning of a sentence is a set of propositions, in which case roughly it claims that the alternatives for a disjunction are the proposition expressed by each disjunct, as well as their intersection. These alternatives can then be processed further through different notions of truthmaking. Similarly, Santorio 2017 suggests a syntactic procedure for determining the alternatives for a sentence.

maps a pair of propositions to a new proposition. The conditional $\gg$ then applies to the proto-conditional operator to every alternative in the antecedent.

$$(\Delta 6) \qquad [\![\mathsf{A}{\gg}\mathsf{C}]\!] = \left\{ \bigcap\{\mathbf{A} > \mathbf{C} \mid \mathbf{A} \in [\![\mathsf{A}]\!]\} \mid \mathbf{C} \in [\![\mathsf{C}]\!] \right\}$$

To simplify a bit more, suppose the set of propositions in $[\![\mathsf{A}]\!]$ is $\{\mathbf{B}_1,...,\mathbf{B}_j\}$ denoted by the sentences $\mathsf{B}_1,...,\mathsf{B}_j$. Then $\mathsf{A} \gg \mathsf{C}$ is true just in case each of the conditionals $(\mathsf{B}_1 > \mathsf{C}),...,(\mathsf{B}_j > \mathsf{C})$ is true. In general, the alternative sensitive conditional is a generalized conjunction of a series of protoconditionals, distributed over the antecedent alternatives.[18] To recycle one of our early examples, the truth-conditions of *Hiro or Ezra* $\gg$ *puzzle* demand the truth of both: *Hiro* > *puzzle* and *Ezra* > *puzzle*.

To define entailment, we flatten the semantic values of the relevant sentences. Arguments are valid just in case the union of the conclusion is true whenever the union of all the premises are true.

$$(\Delta 7) \qquad \mathsf{A}_1,\ldots,\mathsf{A}_n \models \mathsf{C} \text{ iff } \bigcap_{i\in[1,n]}(\bigcup[\![\mathsf{A}_i]\!]) \subseteq \bigcup[\![\mathsf{C}]\!]$$

This proposal guarantees that disjunction behaves as classically as possible. Since entailment is only sensitive to the closed form of a sentence, the alternative sensitive disjunction *or* must satisfy both disjunction introduction and proof by cases. In addition, the proposal yields the DeMorgan equivalence that A or B is equivalent to $\neg(\neg\mathsf{A} \wedge \neg\mathsf{B})$.

A further consequence is that logical equivalence is less fine grained than equivalence of meaning. While A or B and $\neg(\neg\mathsf{A} \wedge \neg\mathsf{B})$ are co-entailing, they do not have the same meaning. The disjunction, but not the negated conjunction, denotes a set of alternatives. For this reason, our logic for conditionals is hyperintensional in the sense that substituting logical equivalents in conditional antecedents does not guarantee equivalence of the resulting conditionals. Conditionals with disjunctive antecedents simplify, while conditionals with negated conjunctions in the antecedent do not.[19]

It is time to turn to our collapse results. In this framework, regardless of what > means,

$$[\![(\mathsf{A} \text{ or } \mathsf{B}) \gg \mathsf{C}]\!] = [\![\mathsf{A}{>}\ \mathsf{C}]\!] \cap [\![\mathsf{B}{>}\ \mathsf{C}]\!]$$

This evidently guarantees that SDA is valid.

Whether CEM is valid depends in part on the choice of proto-conditional >. Suppose for instance that, following Stalnaker 1968, we interpret > in terms of a selection function f that, given a world w and proposition $\mathbf{A}$, returns the unique closest world to w where $\mathbf{A}$ holds.

$$[\![\mathsf{A}{>}\ \mathsf{C}]\!] = \{w \mid f(w, [\![\mathsf{A}]\!]) \in [\![\mathsf{C}]\!]\}$$

[18]For an implementation of the same idea in inquisitive semantics, with a similar purpose to the one we have here, see Ciardelli 2016 and Ciardelli *et al.* 2017.

[19]This is why this proposal avoids the classic collapse result in Ellis *et al.* 1977, connecting simplification with antecedent strengthening.

Then CEM is valid for $\gg$ when the antecedent is not disjunctive.[20]

More generally, the validity of CEM for non-disjunctive antecedents corresponds to the following condition restricted to non-disjunctive A:

$$W = [\![\mathsf{A}{>}\ \mathsf{C}]\!] \cup [\![\mathsf{A} > \neg\mathsf{C}]\!]$$

Because of this correspondence, even this restricted validation of CEM will fail if the proto-conditional does not itself validate CEM.

It is a simple corollary of our negative result that there is no non-trivial choice of proto-conditional that validates CEM for disjunctive antecedents. In a case where some alternatives guarantee C and some guarantee ¬C, CEM fails.

We summarize the two signature properties of the semantics above in a single statement.

Fact 5

1. *For any operator* $>$, $(A \text{ or } B) \gg C \models (A \gg C) \wedge (B \gg C)$
2. *For any operator* $>$, *if* $>$ *validates* CEM, *then* $\gg$ *validates* CEM *for any* A *not containing or .*

One last remark: the alternatives approach does not require that conditionals with disjunctive antecedents always go in for simplification. To avoid this result, we can introduce a closure operation which flattens alternatives.

(Δ8) $[\![!\mathsf{A}]\!] = [\![\neg\neg\mathsf{A}]\!] = \{\bigcup[\![\mathsf{A}]\!]\}$

We can then allow ! to occur in the antecedent of conditionals, generating the form $!(\mathsf{A} \vee \mathsf{B}) \gg \mathsf{C}$. This would account for some localized failures of simplification, such as the classic example *If Spain had fought on the side of the Allies or on the side of the Nazis, it would have fought for the Allies.*[21]

5.2 Evaluation

The alternative semantics approach dodges our first two theorems because those results rely on applying CEM to a disjunctive antecedent, and then applying simplification. By blocking CEM for disjunctive antecedents, both proofs are blocked. The approach reflects a conservative response to our incompatibility results: it quarantines conditionals with disjunctive antecedents and allows CEM only for conditionals that cannot be manipulated *via* SDA.

[20] Our flattened definition of entailment plays an important role in proving this. We have:

$$\models (\mathsf{A} \gg \mathsf{C}) \vee (\mathsf{A} \gg \neg\mathsf{C}) \text{ iff } W \subseteq \bigcup[\![(\mathsf{A} \gg \mathsf{C}) \vee (\mathsf{A} \gg \neg\mathsf{C})]\!]$$

But $[\![(\mathsf{A} \gg \mathsf{C}) \vee (\mathsf{A} \gg \neg\mathsf{C})]\!]$ is the set containing $[\![\mathsf{A}{>}\ \mathsf{C}]\!]$ and $[\![\mathsf{A} > \neg\mathsf{C}]\!]$, so its union is the set of worlds where one of these conditionals holds. Since either C or ¬C is guaranteed to hold at $f(w, [\![\mathsf{A}]\!])$, this last is guaranteed.

[21] See McKay and Inwagen 1977 and Alonso-Ovalle 2006 for discussion.

This immediately raises the question whether the motivation for CEM requires its validity for disjunctive antecedents. If it does not, then we can rest content with the proposal of this section as a solution to our motivating concerns. Unfortunately, however, the arguments for CEM do not appear to discriminate against disjunctive antecedents. Let us run through those arguments again with the specific case of disjunctive antecedents in mind.

I. Scope relations. (18) and (19) sound equivalent in just the same way that (5) and (6) do.

(18) I doubt that if you had slept in or goofed off, you would have passed.

(19) I believe that if you had slept in or goofed off, you would have failed.

Similarly, we observe a duality effect with disjunctive antecedents under *no* and *every*. As before, (20) and (21) appear equivalent.

(20) No student would have succeeded if he had goofed off in class or partied the night before the exam.

(21) Every student would have failed if he had goofed off in class or partied the night before the exam.

By restricting CEM to non-disjunctive antecedents, the analysis renounces these predictions. Suppose that goofing off does imply failure, but that partying does not. In this case, the analysis predicts that the scope of (20) is false for any student, and so (20) is true. By contrast, (21) is false, since partying does not guarantee failure.

This gives rise to several questions. First, do (20) and (21) have any reading on which they are not equivalent? Such a reading is predicted to exist by the account above, by removing the closure operator. If no such reading is available, the account above would need supplementation with a theory of the distribution of existential closure operators. Perhaps such operators are for some reason obligatory when disjunction occurs under the scope of a quantifier. This allows disjunction to behave classically, leading to the equivalence of (20) and (21). Moreover, this is a tool that is independently needed.

Another natural question here is whether there is a reading of the quantified conditionals (20) and (21) on which they are consistent, and yet both go in for simplification. Indeed, Santorio 2017 suggests that at least negative quantified conditionals display exactly this effect:

(22) None of my friends would have fun at the party if Alice or Bob went.

(23) None of my friends would have fun at the party if Alice went.

(24) None of my friends would have fun at the party if Bob went.[22]

[22]See Santorio 2017 10.

Such a combination of effects would be quite paradoxical, since it seems to require the validity of SDA and CEM even for disjunctive antecedents, and yet also require the validity of the Weak Boethius Thesis (WBT). Yet these principles seem jointly inconsistent, since the former two principles imply collapse to the material conditional, which is inconsistent with WBT. Concluding, we do think that the interaction between quantifiers and conditionals with disjunctive antecedents presents a problem for the current analysis.

II. 'Only if' conditionals. we saw that CEM helps derive the meaning of *only if* conditionals compositionally—i.e. on the basis of the interaction of *only* and conditionals. But, as above, it is implausible to restrict this phenomenon to conditionals with non-disjunctive antecedents.

(25) The flag flies only if the King or Queen is home.

(26) If the flag flies, then the King or Queen is home.

(27) The flag flies if the King or Queen isn't home.

Here, it is clear that (25) does imply (26), just as we saw earlier that (11) implied (12). This is a problem for the analysis above, which denies CEM for conditionals with disjunctive antecedents. For, again, a natural way to predict this entailment is through the idea that *only* negates alternatives, and that (27) is an alternative to the conditional in (25). But if CEM fails for disjunctive antecedents, then the negation of (27) will not imply the contraposition of (26), which is essential in von Fintel 1997's account.

6 Homogeneity

In the previous section, we developed a tool for taking any theory of conditionals and enriching it with alternatives. While this theory provides an elegant treatment of SDA, it faces problems with CEM. The theory invalidates CEM for disjunctive antecedents.

In this section, we develop an approach with complementary features. Building on von Fintel 1997, we now develop a tool for taking any theory of conditionals and enforcing CEM. This new tool will have an advantage: any pattern that is valid relative to the underlying conditional remains valid when the conditional is enriched with presuppositions. So if we start with an SDA validating conditional, we can force it to validate CEM. To avoid our collapse results, the theory gives up the transitivity of entailment.

6.1 Homogeneity presuppositions

The theoretical device that will yield this result is the idea of homogeneity. Homogeneity presuppositions have been invoked to explain certain otherwise problematic variants of excluded middle for plural definites.[23] It is worth indulging on plural definites because there is a parallel problem to one of our impossibility results providing us with a template for how we might go about addressing it.

Here is the problem: observe first that predications involving plural definites, like (28), plausibly license inferences to universal claims like (29).

(28) The cherries in my yard are ripe.

(29) All the cherries in my yard are ripe.

If some but not all cherries are ripe, one would not be in a position to assert (28). Furthermore, plural definites plausibly exclude the middle. That is, the following sounds like a logical truth:

(30) Either the cherries in my yard are ripe or they (=the cherries in my yard) are not ripe.

If someone were to utter (30), they would sound just about as informative as if they had made a tautological statement (although you might learn from it that they have cherries in their yard). The problem is that, starting with (30) and exploiting entailments like the one from (28) to (29) as well as standard validities for disjunction, we can reason our way to

(31) Either all the cherries in my yard are ripe or all the cherries in my yard are not ripe.

That seems puzzling: did we just prove, from logical truths and valid inference patterns, that my yard cannot have some ripe cherries and some non-ripe ones? Of course, something must have gone wrong. The homogeneity view of plural definites explains what that is: first, plural definites carry a presupposition of homogeneity: *the F's are G's* presupposes that the *F*'s are either homogeneously *G*'s or homogeneously not *G*'s. If this presupposition is satisfied, their content is that all *F*'s are *G*'s. The sense in which (30) sounds tautological is that it cannot be false if its homogeneity presupposition is satisfied. Similarly, the sense in which (28) entails (29) is that if the presupposition of (28) is satisfied and (28) is true, (29) cannot fail to be true. But even if we can exploit these to deduce (31) we do not have license to claim that (31) is valid: our justification for (30) and for the (28)-(29) entailment did not discharge the homogeneity presupposition.

6.2 Homogeneity without presuppositions

More recently, some authors have sought to disentangle the idea of homogeneity from the presuppositional implementation of von Fintel 1997. Križ (2015*a*,

[23] See for example von Fintel 1997 and Križ 2015*a*.

ch. 1) argues that the presuppositional implementation is dubious for definite plurals because homogeneity effects do not project in same way as presuppositions. Križ views homogeneity as a requirement of truth in the context of a trivalent intensional semantics. Another possible motivation for this trivalent stance is that the theory of homogeneity presuppositions seems to force unintuitive predictions about the subjective probabilities that it is natural to assign to the contents of sentences that do carry homogeneity requirements (Cariani and Santorio 2018; for an experimental study of judgments of probability in the context of homogeneity, see Cremers *et al.* 2017).[24]

While we speak of 'homogeneity presuppositions', we want to be ultimately neutral about which treatment of this phenomenon is to be implemented. Our framework can take on board whatever analysis of homogeneity turns out to be best (for example, it will be clear in the next section that our formalism for managing presupposition could be interpreted much more abstractly).

Before moving on, however, it is worth mentioning another way in which the homogeneity of definite plurals and the homogeneity of conditionals might come apart. Definite plurals can sometimes carry exceptions. The truth of *the team members look happy in this picture* seems compatible with there being an ordinarily grumpy team member who also is not happy in the picture. As a more extreme example of this phenomenon, consider *from some desert locations, you can see the stars* which is perfectly compatible with lots of stars not being visible. Linguists refer to this phenomenon as 'non-maximality'.[25] The relevant point is that the non-maximality of plural definites needn't correspond to non-maximality phenomena for conditionals. Conditionals too may carry exceptions in the sense that not all antecedent-worlds need be consequent-worlds. However, we do not assume at the outset that the explanation for these non-maximality phenomena must be identical.

6.3 Forcing CEM via homogeneity

A treatment of CEM using homogeneity presuppositions is found in von Fintel 1997. It allows that there may be more than one relevant world where the antecedent of a conditional is true. The key idea is that $A > C$ presupposes that either all of the relevant worlds where A is true are worlds where C is true, or they are all worlds where C is false. The A-worlds must be "homogeneous" with respect to the consequent.

For simplicity, we think of presuppositions as definedness conditions, and we model definedness conditions by introducing a third truth value # into

[24]That said, the trivalent intensional semantics does require developing a non-classical probability theory to make sense of the relevant probability judgments—so, it doesn't come cost-free. Still, there is work on non-classical probability over trivalent logics with direct applications to conditionals, such as Rothschild 2014.

[25]For some relevant recent work, see Malamud 2012; Križ 2015*b*; Križ and Spector 2017.

our semantics. Now the semantic value of a sentence is a total function from worlds into $\{0, \#, 1\}$. The advantage of introducing an explicit third truth value is that we can then model presupposition projection by specifying the way in which # interacts with the ordinary boolean operations of function complement, union, and intersection. For simplicity, we'll work with a weak Kleene theory of projection, on which any definedness failures percolate upwards through boolean operations.[26]

(Δ9) Where p and q are total functions from worlds to $\{0, \#, 1\}$:

1. $p - q = \lambda w : p(w) \neq \# \neq q(w) \,.\, p(w) = 1$ and $q(w) \neq 1$
2. $p \cap q = \lambda w : p(w) \neq \# \neq q(w) \,.\, p(w) = 1$ and $q(w) = 1$
3. $p \cup q = \lambda w : p(w) \neq \# \neq q(w) \,.\, p(w) = 1$ or $q(w) = 1$

We can now define semantic values for our language in terms of the usual set-theoretic operations, and know that projection will work just as we intended it to. An additional benefit is that we could, if we wanted, change our theory of projection to another simply by modifying the definition above, rather than intervening on the semantic clauses.

It is also possible to generalize von Fintel 1997 by reformulating the theory without any appeal to quantification over worlds. Instead, we provide a general recipe for taking any conditional operator >, and enriching it with homogeneity presuppositions to create a new conditional, ⤳>.

(Δ10) $[\![\mathsf{A} \rightsquigarrow> \mathsf{C}]\!](w) = \lambda w : ([\![\mathsf{A} > \mathsf{C}]\!] \cup [\![\mathsf{A} > \neg\mathsf{C}]\!])(w) = 1.\ [\![\mathsf{A} > \mathsf{C}]\!](w)$

To talk about SDA and CEM, we also need appropriate assumptions about $\neg$ and $\vee$. These connectives must allow homogeneity presuppositions to project in the right way.

Our earlier work on trivalent set-theoretic operations pays off here, since we can just use the usual Boolean definitions of $\neg$ and $\vee$. Where W is $\lambda w.\ 1$:

(Δ11) $[\![\neg\mathsf{A}]\!] = W - [\![\mathsf{A}]\!]$

(Δ12) $[\![\mathsf{A} \vee \mathsf{B}]\!] = [\![\mathsf{A}]\!] \cup [\![\mathsf{B}]\!]$

The result of combining these clauses with (Δ9) is a weak Kleene projection pattern. Notably, for instance, (Δ12) entails that a disjunctions is defined only if each of its disjuncts is defined.[27]

[26] Throughout, we let '$\lambda x : f(x).\ g(x)$' denote the smallest function mapping any x which satisfies f to 1 or 0, depending on whether x is g, and which maps any other x to #.

[27] This assumption is slightly stronger than what we need in order to predict the validity of CEM. For example, we could also allow a more complex pattern of presupposition projection for disjunction, where the second disjunct treats the first disjunct as part of its local context, as in Heim 1992. But this won't be relevant in what follows, so we stick to the current formulation for simplicity.

Finally, to get predictions about our collapse results, we need a definition of consequence. The leading candidate for languages involving presuppositions is Strawson-validity (Strawson, 1952; von Fintel, 1997, 1999, 2001). According to this notion, an argument is valid just in case the conclusion is true whenever the conclusion is defined and the premises are true.

(Δ13) $\quad \mathsf{A}_1;\ldots;\mathsf{A}_n \models \mathsf{C}$ iff $[\![\mathsf{C}]\!](w) = 1$ whenever:

- $[\![\mathsf{A}_1]\!](w) = 1$ and ... and $[\![\mathsf{A}_n]\!](w) = 1$.
- $[\![\mathsf{C}]\!](w) \neq \#$.

Now let's turn to our collapse results. The first important result is that CEM is valid regardless of the choice of proto-conditional. That is, for any operator $>$, $\dashrightarrow$ satisfies CEM. The key idea that supports this is that $\dashrightarrow$ builds in a homogeneity presupposition—that either $\mathsf{A} > \mathsf{C}$ or $\mathsf{A} > \neg\mathsf{C}$ is true. Further, the validity of CEM requires that $(\mathsf{A} > \mathsf{C}) \vee (\mathsf{A} > \neg\mathsf{C})$ is true whenever defined. Since disjunctions inherit the presuppositions of their first disjunct, we know that this whole disjunction is therefore defined only if it is true.

Our conditional $\dashrightarrow$ is guaranteed to validate CEM. To deal with our collapse results, let's now consider SDA. Here, the key result is that any proto-conditional $>$ that validates SDA induces a new conditional $\dashrightarrow$ that also validates SDA. Indeed, this is not unique to simplification. To see why, note first that any conditional $>$ differs from its homogeneous counterpart $\dashrightarrow$ only in their presuppositions. But this guarantees that any inference containing $>$ remains valid when $>$ is replaced with $\dashrightarrow$. After all, the resulting formula differs from the original only by containing extra presuppositions. But given our definition of validity, this only makes it easier for the relevant inference to be valid. So in particular if we know that $(\mathsf{A} \text{ or } \mathsf{B}) > \mathsf{C}$ implies $(\mathsf{A} > \mathsf{C}) \wedge (\mathsf{B} > \mathsf{C})$, we can infer that $(\mathsf{A} \text{ or } \mathsf{B}) \dashrightarrow \mathsf{C}$ implies $(\mathsf{A} \dashrightarrow \mathsf{C}) \wedge (\mathsf{B} \dashrightarrow \mathsf{C})$. This follows from the monotonicity of classical entailment. After all, an argument is Strawson valid just in case the result of strengthening the argument's premises with the presuppositions of the conclusion is classically valid.

Fact 6

1. *For any operator $>$, $\models (A \dashrightarrow C) \vee (A \dashrightarrow \neg C)$*

2. *For any operator $>$, if $>$ validates* SDA, *then $\dashrightarrow$ validates* SDA.

We now have a completely general recipe for validating both SDA and CEM. But is it a recipe for triviality? That is, do we have that for any operator $>$ that validates SDA, $\dashrightarrow$ collapses to the material conditional? The answer to both questions is "no".

There are many choices of protoconditional for which $\dashrightarrow$ is not trivial. A first example is if we let $>$ be a generic strict conditional. To see how this theory avoids triviality, let us look at the semantic correlates of some of the entailments we used in the proof of our first collapse result. For simplicity, let's simply see how a generic strict conditional > deals with the problem. The first step of the proof corresponds to this semantic fact: (32) is a logical truth.

(32) $[(\mathsf{A} \vee \neg\mathsf{A}) \dashrightarrow \mathsf{C}] \vee [(\mathsf{A} \vee \neg\mathsf{A}) \dashrightarrow \neg\mathsf{C}]$

Although (32) is true whenever defined, it is quite difficult for it to be defined. Given our account of $\vee$, the definedness of $(\mathsf{A} \vee \neg\mathsf{A}) > \mathsf{C}$ is equivalent to the requirement that either $R^w \subseteq [\![\mathsf{C}]\!]$ or $R^w \subseteq [\![\neg\mathsf{C}]\!]$. One of C and ¬C must be necessary at w for (32) to be defined.

Now, the reasoning connecting the first two steps of our proof also has a matching semantic fact: (33) entails (34).

(33) $[(\mathsf{A} \vee \neg\mathsf{A}) \dashrightarrow \mathsf{C}] \vee [(\mathsf{A} \vee \neg\mathsf{A}) \dashrightarrow \neg\mathsf{C}]$

(34) $[(\mathsf{A} \dashrightarrow \mathsf{C} \wedge \neg\mathsf{A} \dashrightarrow \mathsf{C}) \vee [(\mathsf{A} \dashrightarrow \neg\mathsf{C} \wedge \neg\mathsf{A} \dashrightarrow \neg\mathsf{C})]$

This holds because if (32) is defined, then the domain R^w uniformly consists of C-worlds or it uniformly consists of ¬C-worlds. Either way, (34) must be true.

Despite the validity of (32) and the entailment from (32) to (34), (34) is not itself valid. The definedness conditions of (34) are laxer than those of (32): for this reason (34) has a much better shot of being false. For instance (34) is false in a model that contains two worlds w and v with w verifying A and C and v verifying ¬A and ¬C.[28] But such a model does not impugn the validity of (32) under Strawson entailment, because its disjuncts are undefined.

In broad strokes, an instance of transitivity—in particular, one of the form $\models \mathsf{A}$, $\mathsf{A} \models \mathsf{B}$, therefore $\models \mathsf{B}$—fails for Strawson entailment (Smiley, 1967). This is possible because $\models \mathsf{A}$ only requires that A be true if defined; meanwhile, $\mathsf{A} \models \mathsf{B}$ also holds because the presuppositions of A are essentially involved in guaranteeing the truth of B. But $\models \mathsf{B}$ fails because here we are not allowed to assume that the presuppositions of A are satisfied. The same diagnosis applies to our second impossibility result. The first step of the proof claims the validity of $[(\mathsf{A} \text{ or } \mathsf{B}) > C] \vee [(\mathsf{A} \text{ or } \mathsf{B}) > \neg\mathsf{C}]$. The argument establishes that this claim entails IAT.[29] However, the validity of IAT does not follow.

Finally, it's worth considering how our theory handles incompatibility results connecting SDA and Strong Centering, like those in Butcher 1983 and Walters 2009. For illustration, we'll focus on Fact 4. Suppose we have a proto-conditional that satisfies modus ponens (so that CEM entails Strong Centering) and SDA. Then, we accept

(i) $(\top \text{ or } \mathsf{A}) \wedge \mathsf{C} \vdash (\top \text{ or } \mathsf{A}) > \mathsf{C}$.

[28] Here we assume $\wedge$ is definable in terms of $\neg$ and $\vee$, with the analogous definedness conditions.

[29] Despite involving two applications of transitivity, the argument up to step (7) can be replicated for Strawson entailment.

Second, we validate SDA, so that

(ii) $(\top \text{ or } \mathsf{A}) > \mathsf{C} \models \mathsf{A} > \mathsf{C}$.

But we disallow the chaining together of these two entailment claims to yield (iii).

(iii) $(\top \text{ or } \mathsf{A}) \wedge \mathsf{C} \models \mathsf{A} > \mathsf{C}$

The crucial distinction is that (ii) presupposes that the worlds that satisfy $(\top$ or A) are homogeneously C or $\neg C$. Under this extra assumption, the conclusion that $\mathsf{A} > \mathsf{C}$ follows. But this modal information is not contributed by the premises of (iii) which instead must fail.

6.4 Synthesis

Let us take stock of what we have argued: a generic strict conditional $>$ can validate both SDA and CEM, when enriched with homogeneity presuppositions. Here, however, we must take care. The resulting theory validates SDA, but invalidates $\Diamond$-SDA. That is, the analogue of simplification of disjunctive antecedents for *if ... might ...* fails. This is a problem because $\Diamond$-SDA sounds no less plausible than SDA itself.

We faced a symmetrical problem with $\gg$. There, simplification was unrestrictedly valid. But CEM was valid only for non-disjunctive antecedents. To fully validate simplification, we propose a synthesis of our two tools. In particular, we suggest that the English conditional recruits *both* alternatives and homogeneity presupposition. To signal this, we introduce the new connective $\cdot\!\cdot\!\gg$. Start with any conditional meaning $>$. Then apply the alternative sensitive enrichment from (Δ6), to get $\gg$. In light of Fact 1, the resulting semantics validates both SDA and $\Diamond$-SDA, but invalidates CEM for disjunctive antecedents. To force the unrestricted validity of CEM, enrich this conditional with homogeneity presuppositions according to the recipe in (Δ8), so as to get $\cdot\!\cdot\!\gg$.

More precisely, given an arbitrary proto-conditional $>$, we characterize $\cdot\!\cdot\!\gg$ by the clause:

(Δ14) $$[\![\mathsf{A} \cdot\!\cdot\!\gg \mathsf{B}]\!] = \{\lambda w : \exists v \in \{0,1\}\ \forall \mathbf{A} \in [\![\mathsf{A}]\!]\ (\mathbf{A} > \mathbf{B})(w) = v\ .\ \forall \mathbf{A} \in [\![\mathsf{A}]\!]\ (\mathbf{A} > \mathbf{B})(w) = 1 \mid \mathbf{B} \in [\![\mathsf{B}]\!]\}$$

Not every result of applying this recipe to a proto-conditional is guaranteed to yield a non-collapsing conditional. For example, if we choose the material conditional as a proto-conditional, we get back a conditional that agrees with the material conditional whenever defined. On the other hand, no choice of proto-conditional can generate the identical definedness conditions and truth conditions of the material conditional.

Importantly, however, there are choices of proto-conditional for which the recipe does not yield a collapsing conditional. In particular, a natural option for the proto-conditional is the Lewisian variably strict conditional. The underlying Lewisian operator allows that there may be multiple worlds where the antecedent is true that are relevant to the evaluation of the consequent. Then the conditional that results from applying the procedure above is doubly homogeneous. First, the conditional presupposes that the antecedent alternatives either all guarantee the consequent, or all guarantee the consequent's negation. Second, for each antecedent alternative, the conditional presupposes that either all of the relevant worlds where that alternative holds are worlds where the consequent is true, or they are all worlds where the consequent is false. Perhaps surprisingly, this theory more or less has already been developed and endorsed, for somewhat different reasons, in Santorio 2017. Another option would be to start with a strict proto-conditional, and apply both of our procedures. Furthermore, subjunctive and indicative conditionals might differ in exactly this respect: in which proto-conditional operator they are generated from.[30]

7 Intransitive Entailment?

It may seem that rejecting the transitivity of entailment is too high a price to pay. If entailment is understood as necessary truth-preservation (which of course is not how we characterized it), transitivity should be a basic property. How much do we give up by moving to an intransitive notion of entailment? And is it worth it?

The first thing to notice is that the intransitivity of Strawson entailment is not an artifact of our development. For example, if Strawson Entailment is

[30]Our proposal results from applying the two tools in a specific order: the *alternatives-first* strategy we follow, denoted by $\cdot\!\cdot\!\gg$, takes a protoconditional and transforms it into an alternative sensitive conditional. Then it adds on a layer of homogeneity presuppositions, requiring that either every antecedent alternative make the protoconditional true, or every such alternative make it false. . But, of course, there is another option: the *homogeneity-first* strategy, represented by $> \cdot\!\cdot\!>$, takes a protoconditional and first adds homogeneity presuppositions. It then adds in a layer of alternative sensitivity, where $A > \cdot\!\cdot\!> C$ says that for every antecedent alternative, either every relevant scenario where that alternative holds makes the consequent true, or every such scenario makes the consequent false.

It turns out that this homogeneity first strategy makes different predictions than the alternatives first strategy. In particular, it leads to a conditional with strictly weaker presuppositions than our own, but with the same truth conditions whenever defined. For consider some arbitrary disjunction $A \text{ or } B$ (with A and B non-disjunctive) and suppose we have $A > C$ and $B > \neg C$ for some proto-conditional satisfying the Weak Boethius Thesis. Then our preferred conditional above makes $(A \text{ or } B) \cdot\!\cdot\!\gg C$ undefined. By contrast, in the homogeneity first approach the relevant disjunctive conditional $((A \text{ or } B) > \cdot\!\cdot\!> C)$ is defined. After all, our layer of alternative sensitivity simply says that every alternative in the antecedent leads to true when plugged into a protoconditional. If each input to this operation is defined, the whole thing should be as well. For this reason, the homogeneity first approach gives up CEM on the disjunctive fragment of the language. It thus offers no advance with respect to our incompatibility results, compared with an ordinary alternative sensitive treatment of SDA.

adopted to characterize entailment in a language with presuppositions, transitivity can fail in garden variety inferences involving definite descriptions — Strawson's original application of the framework. Consider:

(35) The King of France is either bald or not bald

This is Strawson valid (that is: assuming its presupposition is satisfied it must be true). (35) also Strawson-entails this quantified claim:

(36) There is a unique x that is King of France and x is either bald or not bald.

Whenever the presuppositions of (35) and (36) are satisfied, and (35) is true, then so is (36). However, (36) is not itself Strawson valid.

Furthermore, the violations of transitivity in our framework are highly localized. Recall our formulation of Transitivity and Cut:

Transitivity. if $\mathsf{A} \models \mathsf{B}$ and $\mathsf{B} \models \mathsf{C}$, then $\mathsf{A} \models \mathsf{C}$

Cut. if $X \models \mathsf{B}$ and $Y, \mathsf{B} \models \mathsf{C}$, then $X, Y \models \mathsf{C}$

Generally speaking: Transitivity (and Cut) can only fail, in the context of Strawson entailment, if B plays an essential role in satisfying the presuppositions of C.

As a consequence, various restricted forms of Transitivity and Cut are unaffected. Specifically, there are no violations of Transitivity and Cut when C is presupposition-free. A bit more generally, there are no violations of Cut when Y alone guarantees that all the presuppositions of C are satisfied (the case in which C is presupposition-free is a limit case of this condition).

Depending on what we think motivates Transitivity (and Cut), these restricted forms might be all we need.[31] If we thought that generalizations about entailment are supported by the intuitive plausibility of their instances, we should not be troubled by the retreat to the restricted forms of the rules. The usual norms for theory building apply here: at each theoretical choicepoint it is imperative to map out the available options, and we think that *among these options* one ought to consider letting entailment be intransitive. The cases in which transitivity fails are not only relatively localized, but they are also not obviously cases in which transitivity is supported.

[31] For other discussion of restrictions of Cut or Transitivity, see Smiley 1958, Tennant 1992, Tennant 1994, Ripley 2013, Ripley 2015, Ripley forthcoming, Cobreros *et al.* 2012, Cobreros *et al.* 2015. These last two papers provide a particularly interesting point of connection, since they draw on a notion of entailment ('strict-to-tolerant') that can also be understood as Strawson entailment in a three valued semantics. Since one of the main selling points of this definition of entailment is the ability to retain a constellation of plausible principles and theses in the face of paradoxes, the synergies between these approaches reinforces the individual arguments for each.

But perhaps the justification of these structural rules is not empirical or intuitive, but purely conceptual. For example, Dummett 1975 (p. 306) famously objects that giving up transitivity "seems to undermine the whole notion of proof and, indeed, to violate the concept of argument itself". Such theorists would likely find our rejection of transitivity to border on the contradictory (or, worse, to just be contradictory).

While we are not convinced that intransitivity is contradictory, it is worth exploring the question of what could be done to reconcile our results with the more conservative, "say no to intransitivity", stance. We consider three such options.

7.1 Strawson entailment as pragmatic overlay.

Instead of claiming that *entailment* is intransitive, we might adopt a two-level explanatory scheme on which the apparent validity of certain inferences is accounted for in terms of a secondary, pragmatic relation. The playbook for this reconciliation is set by Stalnaker's discussion of *reasonable inference*. Stalnaker famously characterizes reasonable inference as follows:

> an inference from a sequence of assertions or suppositions (the premises) to an assertion or hypothetical assertion (the conclusion) is reasonable just in case, in every context in which the premises could appropriately be asserted or supposed, it is impossible for anyone to accept the premisses without committing himself to the conclusion. (Stalnaker, 1975)

There is a tight connection between this pragmatic notion and Strawson entailment. Without necessarily trying to reduce them to a common notion, we might notice some important similarities.

First, reasonable inference is also intransitive. For example, a signature application of reasonable inference is to the direct argument, from A or C to the indicative conditional $\neg A \to C$. In Stalnaker 1975, this argument is invalid, but counts as a reasonable inference. Likewise, because any classically valid inference is a reasonable inference, the inference from A to A or C is also reasonable. Nonetheless, these two inferences cannot be chained together: the inference from A to $\neg A \to C$ is unreasonable, because the intermediate step A or C can be unassertable in some scenarios in which A is true and assertable.

More generally, Stalnaker's notion of 'reasonable inference' aims to tracks preservation of assertibility. Strawson entailment can also be viewed as tracking preservation of assertibility. The difference is that the two are after somewhat different senses of "appropriate assertibility". In Stalnaker's characterization this concept is entirely general, while Strawson's is slightly more specialized and focuses on failures of assertibility due to presupposition failure.[32]

[32]One point of difference is that reasonable inference doesn't involve the supposition that

7.2 Three values and two-levels of entailment

The second option consists in replacing Strawson entailment with two transitive notions. The account of presupposition we have operated with is in essence an intensional three-valued system with *true*, *false* and # as truth-values.[33] In such a system, we have (at least) two transitive options for defining entailment. We could think of entailment as preservation of truth, or we could think of it as preservation of non-falsehood. These are often taken to be in competition, but there is room for a kind of pluralism between them: perhaps preservation of truth and preservation of non-falsehood can serve distinct but complementary explanatory roles. In particular, judgments of consequence are to be accounted for in terms of preservation of truth, while judgments of logical truth have to be systematized in terms of preservation of non-falsehood.[34]

A pluralist who accepts this idea can diagnose our leading puzzles as equivocations. In particular, the pluralist can claim that CEM is undeniable (in the sense that it can never be false), but it is not guaranteed to be true. In particular, instances of the form $(\mathsf{A} \text{ or } \mathsf{B}) > \mathsf{C} \vee (\mathsf{A} \text{ or } \mathsf{B}) > \neg\mathsf{C}$ cannot be false. By contrast, SDA is truth-preserving but does not preserve non-falsehood. So, if $(\mathsf{A} \text{ or } \mathsf{B}) > \mathsf{C}$ is true, $(\mathsf{A} > \mathsf{C}) \wedge (\mathsf{B} > \mathsf{C})$ must be. But if $(\mathsf{A} \text{ or } \mathsf{B}) > \mathsf{C}$ is undefined, $(\mathsf{A} > \mathsf{C}) \wedge (\mathsf{B} > \mathsf{C})$ might be false. Similar moves can be made for the parallel cases of intransitivity of Strawson entailment: *the cherries are ripe or the cherries are not ripe* is undeniable; whenever *the cherries are ripe* is true, *all the cherries are ripe* must be (at least under standard semantic assumptions); but when *the cherries are ripe* is undefined because of homogeneity failures, *all the cherries are ripe* must be false.

The task for this kind of pluralist is to spell out more carefully the intended division of labor between the various notions of entailment in such a way that the approach doesn't end up being a notational variant on Strawson entailment. (In other words, why do we need two notions, when Strawson entailment might do the job of both?) We leave it up to those who wants to defend this view to spell out how this account is to be developed.

7.3 Indexed Entailment in Type Theory

For our last option, we build on a suggestion that was given to us in conversation by Jeff Russell. Borrowing an idea from theoretical computer science, we might characterize a whole family of indexed notions of entailment. For instance, it is standard in type theory to index the entailment relation to an

the conclusion is assertible. But at some crucial junctures Stalnaker 1975 does appeal to a variant of reasonable inference that involves that further supposition.

[33] The 'intensional' bit is important: we don't want the conditional to be a truth-function.

[34] Compare Schroeder's structurally parallel discussion in Schroeder (2010, fn. 12). Note that Schroeder is engaged with a very different theoretical context. His goal is to study the prospects for an expressivist view about truth (see also Schroeder forthcoming).

assignment of variables (see Lambek and Scott, 1986, for a canonical presentation, and in particular pp. 130-131). This means effectively that there is not *one* notion of entailment, but a (typically infinite) set of them.

The key modification in our case would be to index entailment to a set of presuppositions. A colorful way of thinking about this is that this proposal relativizes entailment to a context (where in the background we have the idea that contexts fix the relevant presuppositions). The work that is required to make this approach work consists of explaining how the cluster of problems we have identified should be addressed. One possible path towards this goal is to think that instances of CEM only hold on those entailment notions that are indexed to a homogeneity claim. In other words, we only get CEM in those contexts in which the A-worlds are presupposed to be homogeneously C-worlds or homogeneously ¬C-worlds.

With this background, suppose that SDA was valid according to every notion of entailment in the target class. This is how an analysis of our central result might play out on such an account. Recall that the problematic instances of CEM have the form

$$(\mathsf{A} \text{ or } \mathsf{B}) > \mathsf{C} \vee (\mathsf{A} \text{ or } \mathsf{B}) > \neg\mathsf{C}$$

Such claims would only be valid in those contexts in which the set of presuppositions includes or entails the corresponding instance of homogeneity. But then, the corresponding instance of IAT would only be a logical truth for those notions of entailment whose indexing presuppositions establish the corresponding instance of homogeneity (and hence of CEM). It may fail when the indexing presuppositions do not establish the relevant CEM instances. This, needn't be a problematic result, and it would mirror our claim that IAT does follow from CEM, but it doesn't follow from the empty set of premises. But, again, it needs to be shown that this is, in any substantive sense, different from accepting one intransitive notion of entailment.

7.4 Taking stock

Our task in this positive part of the paper was to find room to reconcile CEM and SDA, by blocking some of the problematic derivation. What is key to this effort is the idea that, on the aggregate, the collection of notions that are involved in explaining the acceptability of inferences must be capable of providing explanations for intransitive-seeming behavior. This could be accomplished in a variety of ways, some of which are consistent with the idea that entailment is transitive and some that are not. It is beyond the scope of this essay to argue conclusively that a boldly intransitive explanation in terms of Strawson entailment, understood as the primary notion of entailment, is superior to the milder kind of pluralism we identified in §7.2. What we did accomplish, however, was advancing the idea that any plausible picture on which CEM and SDA might co-exist requires a pattern of explanation that can mirror the apparent intransitivity of the underlying phenomena. More substantively, we developed the framework of *Homogeneous Alternative Semantics*, which shows that a semantic framework for conditionals can be built on an intransitive notion of entailment without obvious drawbacks and even without drastic revisions to the non-conditional fragment of the semantics.

Appendix: Homogeneous Alternative Semantics

Definition 1 *Where p and q are total functions from worlds to $\{0, \#, 1\}$:*

1. $p - q = \lambda w : p(w) \neq \# \neq q(w)\ .\ p(w) = 1$ *and* $q(w) \neq 1$
2. $p \cap q = \lambda w : p(w) \neq \# \neq q(w)\ .\ p(w) = 1$ *and* $q(w) = 1$
3. $p \cup q = \lambda w : p(w) \neq \# \neq q(w)\ .\ p(w) = 1$ *or* $q(w) = 1$

Definition 2 *Let $[\![\cdot]\!]$ map sentences to sets of total functions from worlds to $\{0, \#, 1\}$:*

1. $[\![p]\!] = \{\lambda w\ .\ w(p) = 1\}$
2. $[\![\neg A]\!] = \{W - \bigcup [\![A]\!]\}$
3. $[\![A \wedge B]\!] = \{\boldsymbol{A} \cap \boldsymbol{B} \mid \boldsymbol{A} \in [\![A]\!], \boldsymbol{B} \in [\![B]\!]\}$
4. $[\![A \vee B]\!] = [\![A]\!] \cup [\![B]\!]$
5. $[\![A \cdots\!\!\gg B]\!] = \{\lambda w : \exists v \in \{0,1\}\ \forall \boldsymbol{A} \in [\![A]\!]\ (\boldsymbol{A} > \boldsymbol{B})(w) = v\ .\ \forall \boldsymbol{A} \in [\![A]\!]\ (\boldsymbol{A} > \boldsymbol{B})(w) = 1 \mid \boldsymbol{B} \in [\![B]\!]\}$

Definition 3 $A_1, \ldots, A_n \models C$ *iff* $\bigcup [\![C]\!](w) = 1$ *if:*

1. $\forall i \in [1, n]\ \bigcup [\![A_i]\!](w) = 1$
2. $\bigcup [\![C]\!](w) \in \{0, 1\}$

Observation 1

1. $(A \vee B) \cdots\!\!\gg C \models (A \cdots\!\!\gg C) \wedge (B \cdots\!\!\gg C)$
2. $\models (A \cdots\!\!\gg C) \vee (A \cdots\!\!\gg \neg C)$
3. $A \wedge C \models (A \cdots\!\!\gg C)$

Observation 2

1. $A \models A \vee B$
2. $\models A \vee \neg A$
3. *If* $X, A \models C$ *and* $Y, B \models C$*, then* $X, Y, (A \vee B) \models C$

Observation 3

1. $A; A \cdots\!\!\gg C \models C$
2. $\models A \cdots\!\!\gg A$
3. $A \cdots\!\!\gg B, B \cdots\!\!\gg C \models A \cdots\!\!\gg (B \wedge C)$

4. *If* $X, B \models C$*, then* $X, A \mathrel{\cdot\!\!\cdot\!\!\gg} B \models A \mathrel{\cdot\!\!\cdot\!\!\gg} C$

Observation 4

1. $\neg A \vee B \not\models A > B$
2. $\not\models (A \mathrel{\cdot\!\!\cdot\!\!\gg} C \wedge B \mathrel{\cdot\!\!\cdot\!\!\gg} C) \vee (A \mathrel{\cdot\!\!\cdot\!\!\gg} \neg C \wedge B \mathrel{\cdot\!\!\cdot\!\!\gg} \neg C)$
3. *Not: if* $X \models A$ *and* $Y, A \models B$*, then* $X, Y \models B$*.*

Bibliography

Alonso-Ovalle, Luis (2006), *Disjunction in Alternative Semantics*, Ph.D. thesis, University of Massachusetts Amherst.

Anderson, Alan R. and Belnap, Nuel D. (1975), *Entailment: The Logic of Relevance and Neccessity, Vol. I* (Princeton University Press).

Barker, Stephen (1993), 'Conditional Excluded Middle, Conditional Assertion and *Only if*', *Analysis*, 53, 254–261.

Bimbó, Katalin (2014), *Proof theory: Sequent calculi and related formalisms* (CRC Press).

Briggs, R.A. (2012), 'Interventionist Counterfactuals', *Philosophical Studies*, 160, 139–166.

Buss, Samuel R. (1998), 'An introduction to proof theory', in Samuel R. Buss (ed.), *Handbook of proof theory*, 1–78 (Elsevier).

Butcher, David (1983), 'An incompatible pair of subjunctive conditional modal axioms', *Philosophical Studies*, 44(1), 71–110.

Cariani, Fabrizio and Rips, Lance (ms.), 'Experimenting with (Conditional) Perfection', Manuscript, Northwestern University.

Cariani, Fabrizio and Santorio, Paolo (2018), '*Will* Done Better: Selection Semantics, Future Credence, and Indeterminacy', *Mind*, 505, 129–165.

Ciardelli, Ivano (2016), 'Lifting conditionals to inquisitive semantics', in *Proceedings of SALT*, volume 26, 732–752.

Ciardelli, Ivano, Zhang, Linmin, and Champollion, Lucas (2017), 'Two switches in the theory of counterfactuals: A study of truth conditionality and minimal change', .

Cobreros, Pablo, Egré, Paul, Ripley, David, and van Rooij, Robert (2012), 'Tolerant, classical, strict', *Journal of Philosophical Logic*, 41(2), 347–385.

——— (2015), 'Vagueness, truth and permissive consequence', in *Unifying the philosophy of truth*, 409–430 (Springer).

Cremers, Alexandre, Križ, Manuel, and Chemla, Emmanuel (2017), 'Probability Judgments of Gappy Sentences', in Salvatore Pistoia-Reda and Filippo Domaneschi (eds.), *Linguistic and Psycholinguistic Approaches on Implicatures and Presuppositions* (Palgrave).

Cross, Charles B. (2009), 'Conditional Excluded Middle', *Erkenntnis*, 70(2), 173–188.

DeRose, Keith (1999), 'Can it be that it would have been even though it might not have been?', *Philosophical Perspectives*, 13, 387–413.

Dummett, Michael (1975), 'Wang's paradox', *Synthese*, 30(3), 301–324.

Edgington, Dorothy (1995), 'On Conditionals', *Mind*, 104(414), 235–329.

——— (2014), 'Indicative Conditionals', in Edward N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy* (Metaphysics Research Lab, Stanford University), winter 2014 edition.

Ellis, Brian, Jackson, Frank, and Pargetter, Robert (1977), 'An Objection to Possible-World Semantics for Counterfactual Logics', *Journal of Philosophical Logic*, 6, 355–357.

Fine, Kit (1975), 'Critical Notice of *Counterfactuals*', *Mind*, 84(335), 451–458.

——— (2012), 'Counterfactuals without possible worlds', *Journal of Philosophy*, 109(3), 221–246.

von Fintel, Kai (1997), 'Bare Plurals, Bare Conditionals, and *Only*', *Journal of Semantics*, 14, 1–56.

——— (1999), 'NPI-Licensing, Strawson-Entailment, and Context-Dependency', *Journal of Semantics*, 16(1).

——— (2001), 'Counterfactuals in a Dynamic Context', in Michael Kenstowicz (ed.), *Ken Hale: a Life in Language* (The MIT Press).

von Fintel, Kai and Iatridou, Sabine (2002), 'If and when 'if'-clauses can restrict quantifiers', manuscript, Massachusetts Institute of Technology.

Gillies, Anthony S. (2007), 'Counterfactual Scorekeeping', *Linguistics and Philosophy*, 30, 329–360.

——— (2009), 'On truth-conditions for *if* (but not quite only *if*)', *Philosophical Review*, 118(3), 325–349.

Goodman, Jeremy (ms.), 'Consequences of Conditional Excluded Middle', Manuscript, University of Southern California.

Heim, Irene (1992), 'Presupposition Projection and the Semantics of Attitude Verbs', *Journal of Semantics*, 9(3), 183–221.

Higginbotham, Jim (1986), 'Linguistic theory and Davidson's program in semantics', in Ernie Lepore (ed.), *Truth and Interpretation: Perspectives on the Philosophy of Donald Davidson*, 29–48 (Basil Blackwell).

Klinedinst, Nathan (2011), 'Quantified Conditionals and Conditional Excluded Middle', *Journal of Semantics*, 28(1), 149–170.

Kolodny, Niko and MacFarlane, John (2010), 'Ifs and Oughts', *Journal of Philosophy*, 107(3), 115–143.

Križ, Manuel (2015*a*), *Aspects of homogeneity in the semantics of natural language*, Ph.D. thesis, University of Vienna.

——— (2015*b*), 'Homogeneity, non-maximality, and all', *Journal of Semantics*, 33(3), 493–539.

Križ, Manuel and Spector, Benjamin (2017), 'Interpreting Plural Predication: Homogeneity and Non-Maximality', .

Lambek, Joachim and Scott, Phil J. (1986), *Introduction to Higher-Order Categorical Logic* (Cambridge University Press).

Leslie, Sarah Jane (2009), '*If*, *unless* and quantification', in R.J. Stainton and C. Viger (eds.), *Compositionality, Context and Semantic Values: Essays in Honour of Ernie Lepore*, volume 85 of *Studies in Linguistics and Philosophy*, 3–30 (Springer Netherlands).

Lewis, David (1973), *Counterfactuals* (Blackwell).

——— (1977), 'Possible-world semantics for counterfactual logics: a rejoinder', *Journal of Philosophical Logic*, 6, 359–363.

Malamud, Sophia (2012), 'The meaning of plural definites: A decision-theoretic approach', *Semantics and Pragmatics*, 5, 1–58.

McGee, Vann (1985), 'A Counterexample to Modus Ponens', *The Journal of Philosophy*, 82(9), 462–471.

McKay, Thomas and Inwagen, Peter Van (1977), 'Counterfactuals with disjunctive antecedents', *Philosophical Studies*, 31(5), 353–356.

Negri, Sara and von Plato, Jan (2001), *Structural Proof Theory* (Cambridge).

Nute, Donald (1975), 'Counterfactuals and similarity of words', *The Journal of Philosophy*, 72(21), 773–778.

——— (1980*a*), 'Conversational scorekeeping and conditionals', *Journal of Philosophical Logic*, 9(2), 153–166.

——— (1980*b*), *Topics in Conditional Logics* (Reidel).

Pizzi, Claudio and Williamson, Timothy (2005), 'Conditional Excluded Middle in Systems of Consequential Implication', *Journal of Philosophical Logic*, 34, 333–62.

Restall, Greg (2000), *An Introduction to Substructural Logics* (Routledge).

Ripley, David (2013), 'Paradoxes and failures of cut', *Australasian Journal of Philosophy*, 91(1), 139–164.

——— (2015), 'Naive set theory and nontransitive logic', *Review of Symbolic Logic*, 8(3), 553–571.

——— (forthcoming), 'On the 'transitivity' of consequence relations', *Journal of Logic and Computation*.

Rothschild, Daniel (2014), 'Capturing the relationship between conditionals and conditional probability with a trivalent semantics', *Journal of Applied Non-Classical Logics*, 24(1-2), 144–152.

Santorio, Paolo (2017), 'Alternatives and truthmakers in conditional semantics', *Journal of Philosophy*.

Schroeder, Mark (2010), 'How to Be an Expressivist About Truth', in Cory D. Wright and Nikolaj J. L. L. Pedersen (eds.), *New Waves in Truth*, 282–298 (Palgrave-Macmillan).

——— (forthcoming), 'The Moral Truth', in Michael Glanzberg (ed.), *The Oxford Handbook of Truth* (Oxford University Press).

Smiley, T. J. (1958), 'Entailment and Deducibility', *Proceedings of the Aristotelian Society*, 59(n/a), 233–254.

Smiley, Timothy (1967), 'Mr. Strawson on the Traditional Logic', *Mind*, 76(301), 347–385.

Stalnaker, Robert (1968), 'A Theory of Conditionals', *American Philosophical Quarterly*, 98–112.

——— (1973), 'Presuppositions', *Journal of Philosophical Logic*, 2, 447–457.

——— (1975), 'Indicative Conditionals', *Philosophia*, 5, 269–86.

——— (1981), 'A Defense of Conditional Excluded Middle', in William L. Harper, Robert Stalnaker, and Glenn Pearce (eds.), *IFS: Conditionals, Belief, Decision, Chance and Time*, volume 15 of *The University of Western Ontario Series in Philosophy of Science*, 87–104 (Springer Netherlands).

Strawson, P. F. (1952), *Introduction to Logical Theory* (London: Methuen).

Tennant, Neil (1992), *Autologic* (Edinburgh University Press).

——— (1994), 'The Transmission of Truth and the Transitivity of Deduction', in Dov Gabbay (ed.), *What is a Logical System? Volume 4 of Studies in Logic and Computation*, 161–177 (Oxford University Press).

Troelstra, A. S. and Schwichtenberg, Helmut (2000), *Basic Proof Theory* (Cambridge University Press).

Walters, Lee (2009), 'Morgenbesser's coin and counterfactuals with true components', in *Proceedings of the Aristotelian Society*, volume 109, 365–379 (Oxford University Press Oxford, UK).

Williams, J. Robert G. (2010), 'Defending Conditional Excluded Middle', *Nous*, 44(4), 650–668.