Young Indigenous Students' Experiences in Mathematics: An Exploration in Pattern Generalisation

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Keywords

Abstract

There is limited research that focuses on young Australian Indigenous students learning specific mathematical concepts (Meaney, McMurchy-Pilkington, & Trinick, 2012). To date, there has been no study conducted within an Australian context that considers how young Australian Indigenous students engage in mathematical generalisation of growing patterns. Mathematical growing patterns are a sequence of shapes or numbers characterised by the relationship between elements, which can increase or decrease by a constant difference (linear growing pattern). Additionally, growing patterns can also exhibit quadratic and exponential growth. The purpose of this study is to explore how young Australian Indigenous students generalise growing patterns. Patterns are a common route for young students to engage with in early algebraic thinking.

Algebra has been labelled as a mathematical gatekeeper for all students, having the potential to provide both economic opportunity and equal citizenship (Satz, 2007). It has been proposed that algebra is one link in reducing the exacerbated inequalities between ethnicity and socioeconomic groups (Greenes, 2008). Concerns about students' poor understanding of algebra in secondary school have contributed to early algebra becoming a focal point for mathematics education. Early algebra is its own unique subject, and is not to be confused with the teaching of algebra early. Rather, the concept of early algebra is integrated with other early mathematical concepts as students engage in the gradual introduction to formal notation (Carraher, Schliemann, & Schwartz, 2008). In addition, early algebraic thinking leads to a deeper understanding of mathematical structures (Blanton & Kaput, 2011; Carraher, Schliemann, Brizuela & Ernest, 2006; Cooper & Warren, 2011). Recent studies indicate that young students are capable of engaging with early algebraic concepts (e.g., Blanton & Kaput, 2011; Cooper & Warren, 2011; Cooper & Warren, 2008; Radford, 2010a; Rivera, 2006)

A review of the literature generated three research questions that, in turn, informed the research design. These were: (1) How do young Indigenous students engage in growing pattern generalisation? (2) What teacher actions assist in enhancing young Indigenous students to generalise growing patterns? (3) How does

culture influence the way in which young Indigenous students engage in growing pattern generalisation?

Given that this study focused on exploring how young Australian Indigenous students construct unique personal knowledge and communicate their individual understandings of generalising growing patterns, a constructionist epistemology was adopted. As young Indigenous students construct new mathematical knowledge through interactions with others and working with, hands-on materials, the theoretical perspectives of semiotics and Indigenous research paradigms were used as lenses to analyse the data. Semiotics provides a lens to interpret the new signs being constructed by students. A sign stands for something other than itself, it is a means through which meaning is communicated (Peirce, 1958). Semiotic signs include speaking, writing, gesticulating, and using hands-on materials as students engage in learning. Students also bring their own cultural signs to the learning process. The second theoretical perspective, Indigenous research paradigms, allows for the analysis of new knowledge with respect to culture and empowerment of young Indigenous students. The methodology for the study included teaching experiments. Data collection methods incorporated: observations, pre-assessments, lessons from two teaching experiments (Students N=18) (including six 45-minute mathematics lessons), and Piagetian clinical interviews with a smaller sample of students (n=3) at the conclusion of both teaching experiments. Indigenous Education Officers provided a cultural perspective on this data after watching the video recordings of the lessons and Piagetian clinical interviews.

Findings from this study provide a positive story in relation to young Indigenous students engaging with, and learning mathematics. Major findings of the study were, first, that these young Australian Indigenous students were capable of engaging in early algebraic thinking and generating generalisations from growing patterns, including multiplicative patterns with a constant. In order to explore these structures, young Indigenous students engaged in a series of teaching and learning actions.

Second, particular teacher actions assisted these students to see the structure of the growing pattern. These actions included the use of semiotic bundles (e.g., gesture, language, questioning, hands-on materials). Additionally, the selection of

materials and how the patterns were displayed visually, also contributed to young Indigenous students' understanding of the pattern structure.

Finally, teachers need to work closely with the Indigenous Education Officers during class to better understand cultural aspects of the lessons. As these Indigenous students relied on one another to enrich their understanding of the tasks, it was imperative to provide the opportunity for them to communicate freely in class to enable sharing of ideas. While this way of learning as a classroom community assisted many students, it was also evident that providing students with a one-on-one setting, that is, just the student working with the researcher away from other students in the class, gave different insights into how they obtained mathematical understanding.

This study contributes to the understanding of how young Indigenous students engage in early algebraic thinking, in particular, growing pattern generalisation. Theoretical contributions to new knowledge include the cultural interactions that occur between the non-Indigenous teacher, the Indigenous Education Officer and Indigenous students in the mathematics classroom. Implications for future classroom practice include a hypothesised learning-teaching trajectory, which considers the semiotic interactions that occur as these Indigenous students move towards identifying generalities for growing patterns. This trajectory highlights the specific actions that teachers and Indigenous Education Officers can provide to assist young Indigenous students to engage with, and deepen their understanding of, pattern generalisation.

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List of Abbreviations

ABS   Australian Bureau of Statistics
ACARA   Australian Curriculum, Assessment, Reporting Authority
AAMT   Australian Association of Mathematics Teachers
COAG   Council of Australian Governments
CYI   Cape York Institute
DEST   Department of Education, Science, and Training
DI   Direct Instruction
IEO   Indigenous Education Officer
MCEECDYA   Ministerial Council for Education, Early Childhood,
                Development and Youth Affairs
MCEETYA   Ministerial Council on Education, Employment, Training and
                Youth Affairs
NAPLAN   National Assessment Program – Literacy and Numeracy
NCTM   National Council of Teachers of Mathematics
OECD   Organisation for Economic Co-operation and Development
PCI   Piagetian Clinical Interview
PISA   Programme for International Student Assessment
QSA   Queensland Studies Authority
ROLEM   Representations, Oral Language, and Engagement in
                Mathematics
TIMSS   Trends in International Mathematics and Science Studies

Statement of Original Authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made. All research procedures reported in the thesis received the approval of the relevant Ethics/Safety Committees.

Signature: J. Miller

Date: 4th July 2014

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To my beautiful daughter, Breeanna, though you may not realise this now, I did this for you. You will forever inspire me.

Glossary

Aboriginal English	This term is given to the various dialects of English spoken by Aboriginal people. Aboriginal English is the first language, or home language, of many Aboriginal children. In subtle ways this language, a distinctively Aboriginal kind of English, is a powerful vehicle for the expression of Aboriginal identity.
Aboriginal and Torres Strait Island people	An Aboriginal person or Torres Strait Islander is defined as someone who is of Aboriginal or Torres Strait Islander descent, identifies as an Aboriginal person or Torres Strait Islander and is accepted as such in the community where he or she lives or comes from.
Aboriginal	This term is related to the First Nation people of a country. An Aboriginal person is of Aboriginal descent, identifies as Aboriginal and is accepted by the community in which he/she lives. Throughout the thesis, there are sections that refer exclusively to Aboriginal and Torres Strait Islander women and students.
Closing The Gap	*Closing The Gap* is part of the Indigenous reform agenda. *Closing The Gap* is a commitment by all Australian governments to improve the lives of Indigenous Australians, and in particular provide a better future for Indigenous children.
Covariational thinking	Covariational thinking is representational thinking that focuses on the dynamic relationship between two varying quantities. In this thesis it is the relationship between the pattern quantity and pattern term.
Embedded variable patterns	An embedded variable pattern is where both the pattern quantity and pattern term are represented within the one pattern structure.
Embodiment	Embodiment is where thought is connected with the word and embodied in it. Within mathematics, it is where meaning in concrete enactments or material experiences appears to enhance the mathematical representation for students. The concept of a material carrier implies that the gesture, the actual motion of the gesture itself, is a dimension of thinking.
Environmental growing patterns	Environmental growing patterns are patterns that draw on elements of the natural environment to depict growth. For example: small tree,

	medium tree, large tree.
Far Generalisation	Far generalisations are generalisations for large pattern terms. Students identify the pattern quantity for large pattern terms (e.g., 157th position).
Functional thinking	Functional thinking is strongly connected with the concept of function. It is a process of building, describing, and reasoning with and about functions.
Generalisation	Generalisation refers to mathematical generalisation. This involves a claim that some property or technique holds for a large set of mathematical objects or conditions.
Geometric growing patterns	Geometric growing patterns are patterns that are constructed with mathematical geometric shapes, such as, squares, circles and triangles.
Growing Pattern	Growing patterns are characterised by the relationship between elements, which increase or decrease by a constant difference (linear growing pattern). Growing patterns can also exhibit quadratic and exponential growth. These patterns often have associated algebraic expressions that allow students to express how this change occurs. When students generalise growing patterns they are considering the functional relationship between co-varying quantities.
Hands-on Experience	Hands-on experiences are mathematical activities that use concrete materials (e.g., counters, tiles, pattern number cards) when students are exploring the mathematical concept.
Indigenous people	When the term Indigenous is used in this thesis, it encompasses both Australian Aboriginal and Torres Strait Islander people.
Indigenous Education Officers	Indigenous education officers are Aboriginal or Torres Strait Islander people who work in schools where there are significant numbers of Indigenous students. They work closely with teachers to develop culturally-appropriate resources and programs. Additionally, they encourage students and support parents.
Indigenous Knowledge	Indigenous knowledge is the knowledge that people in a given community have developed over time, and continue to develop. It is based on experience, dynamic and changing and often tested over centuries of use, adapted to local culture and environment.

Indigenous	When used in the Australian context, 'Indigenous' refers to a person of Aboriginal or Torres Strait Islander descent, who identifies as an Australian Aboriginal or Torres Strait Islander and is accepted as such by the community in which s/he lives
Kinship	The kinship system is a feature of Aboriginal social organisation and family relationships across Australia. It is a complex system that determines how people relate to each other and their roles, responsibilities and obligations in relation to one another, ceremonial business and land. The kinship system determines who marries who, ceremonial relationships, funeral roles and behaviour patterns with other kin.
Kriol	Kriol is generally classified as an English-based Creole; however, it also borrows much from the phonology, lexicon and syntax of traditional languages. Kriol emerged in northern Australia as a means of communicating across cultures that is, between Aboriginal and Torres Strait Island people and English-speakers. There are two major creoles in Australia: one spoken in Queensland, the Northern Territory and the West Australian cattle-station belt (Kriol); and one spoken in the Torres Strait and Cape York (Torres Strait Creole).
Learning-Teaching Trajectory	In contrast to the learning trajectory (Clements & Sarama, 2004), the learning-teaching trajectory has three interwoven meanings, each of equal importance. These are; (a) a learning trajectory that gives an overview of the learning process of students; (b) a teaching trajectory that describes how teaching can most effectively link up with and stimulate the learning process; and finally, (c) a subject matter outline, indicating which core elements of the mathematical curriculum should be taught (Van den Heuvel-Panhuizen, 2001). It provides a 'mental education map', which can assist teachers to make didactical decisions as they interact with students' learning and instructional tasks. It allows for a degree of flexibility in the learning sequence, and acknowledges that quality teaching is a key dimension of effective learning.
Near Generalisation	Near generalisations are generalisations for small pattern terms. Students identify the pattern quantity for small pattern terms (e.g., 12th position).

Non-Indigenous	Non-Indigenous refers to those who do not identify as either Australian Aboriginal or Torres Strait Islander people.
Objectification	Objectification can bring about knowledge formation with the use of particular mathematics activities led by semiotic systems.
Pattern Quantity	Pattern quantity refers to the variable that represents the structure of the pattern (e.g., 15 counters are needed for pattern term 5). This is the dependent variable.
Pattern Term	Pattern term refers to the variable that represents the pattern position in the growing pattern (e.g., position 5 or pattern term 5). This is the independent variable.
Piagetian clinical interviews	This is a diagnostic tool used to study the naturalistic form of knowledge structures and reasoning processes. Essential to the Piagetian clinical interview is the use of hands-on materials. It is conducted in a one-on-one setting between the researcher and the student.
Quasi-Generalisation	A Quasi-generalisation is where students are able to express the generalisation in terms of specific numbers. For example, generalising the pattern for position 4587.
Quasi-Variable:	Quasi-variables are used as a bridge between arithmetic and algebraic notation. It means that a number sentence or group of number sentences indicate an underlying mathematical relationship that remains true whatever the numbers used.
Repeating patterns	Repeating patterns have an identifiable unit of repetition and can range in levels of complexity. For example, ABABABAB or ABCCABCC.
Semiotic bundling	The semiotic bundle consists of sign systems produced by one or more interacting subjects. It is a term used to describe signs used in the interactions between students and teachers (e.g., speaking, writing, drawing, gesticulating, using artefacts).
Semiotic Contraction	Semiotic Contraction refers to the reduction and refinement of signs by students.
Semiotic Nodes	Semiotic nodes are the parts of students' semiotic systems where action, gesture and word work in conjunction.

Semiotics	Semiotics is the study of signs.
Shame	This Aboriginal English term differs from the non-Indigenous definition of shame. Shame in these contexts is expressed as shyness, embarrassment, or the breaking of a protocol.
Split variable pattern	A split variable pattern is where both the pattern quantity and pattern term are represented in two different structures such as, pattern quantity (counters) and pattern term (pattern term card). See Tables 5.4 and 5.6 for examples of split variable patterns.
Terra Nullius	Terra Nullius is a term derived from Latin meaning 'a land belonging to no one'. Captain Cook claimed Australia under the title of Terra Nullius when arriving on Australian shores, despite the presence of Indigenous people.
Visually explicit pattern	Visually explicit patterns have visual cues (sign vehicles) that attend to both variables (term and pattern).

Chapter 1: Introduction

1.1   INTRODUCTION

The fundamental objective of this thesis was threefold: first, exploring how young Indigenous students generalised mathematical growing patterns; second, identifying what teaching actions assisted students to generalise; and third, investigating the role of culture in the teaching and learning process. Both semiotics and Indigenous research perspectives provided the analytical lens for data collected during this study. Specifically, the aim of this study was to explore how Year 2/3[1] Indigenous students (7-8 year old students in a composite class) engage in the learning process that enables them to construct generalities from mathematical growing patterns. This study was undertaken in response to the lack of current research in the area of early algebra with young Indigenous students.

This chapter defines, illuminates, and justifies the research problem underpinning this thesis. First, it presents the positionality of the researcher (Section 1.2). It then describes the background and context of the research (Section 1.3), provides the impetus of the study (Section 1.4), poses the research questions and aims (Section 1.5), and establishes how the research will be conducted (Section 1.6). The significance and scope of this research are considered (Section 1.7). Finally, an outline of the remaining chapters of the thesis is provided (Section 1.8). Figure 1.1 displays a diagrammatic overview of the chapter. Figures such as this appear at the beginning of each chapter of the thesis to assist the reader.

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[1] Australia students' age in Year 2 (7-8 years old), are equivalent to 2nd and 3rd Grade students' age in the US.
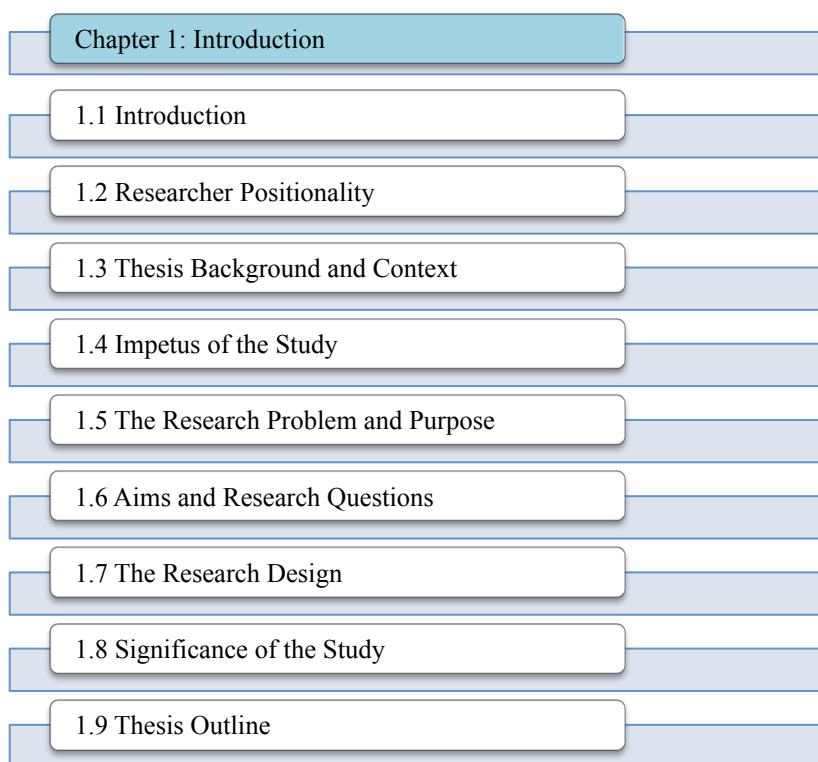
Figure 1.1. Overview of Chapter 1.

1.2 RESEARCHER POSITIONALITY

I acknowledge the past historical and social traumas that both Australian Aboriginal and Torres Strait Islander people have experienced. I recognised that these sufferings are intergenerational and still impact on students today.

As a non-Indigenous education researcher, I understand that, although my won culture deeply influences the perceptions of the world around me, those views are not a defining assessment. In the context of this study, it is pivotal that we recognise that Aboriginal and Torres Strait Peoples bring unique life experiences to the classroom. By appreciating the nuances of cultural differences, we enhance educational experiences for students. By addressing the diverse mindsets of all those involved in classroom interaction, we acknowledge and celebrate students as knowledge-makers. Through this perspective of rich, personal experience, all participants in the study contribute knowledge, which adds to the collective intellectual capacity of this research. This thesis demonstrates how selected students achieve and engage in algebraic concepts, thus providing a voice for these young Indigenous students.

1.3 THESIS BACKGROUND AND CONTEXT

Algebra has been labelled as a mathematical gatekeeper for all students, having the potential to provide both economic opportunity and equitable citizenship (Satz, 2007). This gatekeeping status is vitally important marginalised communities (Gonzalez, 2009). It has been proposed that algebra is one link in reducing the exacerbated inequalities between ethnic and socioeconomic groups (Greenes, 2008). By studying algebra, students are presented with educational building blocks which may lead to a range of opportunities enabling access to university and potential career prospects, with socioeconomic equality the ultimate goal (Moses, 1994; Satz, 2007) and this can potentially contribute to aspects of socioeconomic equality. In the year 2000, the National Council for Teaching Mathematics (NCTM) asserted, in the *Principles and Standards for School Mathematics*, the importance of all students having opportunities to access algebra within their school curriculum. Consequently, the notion of 'algebra for all' was generated (National Council of Teachers of Mathematics, 2000).

Concerns about students' poor understanding of algebra in secondary school have contributed to early algebra becoming a focal point for mathematics education. Prior to 2000, the notion of young students engaging in early algebra appeared to be unconsidered and impracticable (Carraher, Schliemann, & Brizuela, 2000). Research until this time was engrossed with difficulties secondary students have with understanding the concept of algebra (e.g., Kieran, 1989; Mason, Pimm, Graham, & Gower, 1985; Stacey & MacGregor, 1994), and concerns whether young students were in fact capable of engaging with algebraic concepts (e.g., Filloy & Rojano, 1989; Herscovics & Linchevski, 1994; Kuchemann, 1981; Linchevski & Herscovics, 1996). Recent research findings, however, indicate young students do have this capability to engage with algebraic concepts and generalise mathematical structures (Blanton & Kaput, 2011, 2005; Carpenter et al., 2003; Carraher et al., 2006; Cooper & Warren, 2011; Cooper & Warren, 2008). Despite these new insights within the field, little is known about the teaching actions that assist young students to pose mathematical generalisations. Furthermore, there has been no study conducted, despite an extensive search of the literature, that considers how young Australian Indigenous students engage in mathematical generalisation, the role of culture in this process, and the teaching actions that assist these students to generalise.

Australian Indigenous students continue to be the most disadvantaged demographic in education (Gonski et al., 2011; Matthews, Howard, & Perry, 2003). Importantly, being Aboriginal or Torres Strait Islander does not make you disadvantaged, rather, factors including poverty, racism, unemployment, and state of health contribute to Indigenous students experiencing disadvantage. Though these factors exist and impact on Indigenous students' learning, research indicates that current national and international testing results (e.g., TIMMS, PISA, NAPLAN) are inconsistent with students innate abilities in mathematics (Warren & De Vries, 2009; Warren & Miller, 2013).

Current national reporting indicate Australian Indigenous students are two years behind national and international benchmarks derived from standardised testing (Commonwealth of Australia, 2007). However, there have been very few critiques in the Australian context about particular aspects (e.g., language and contexts) of these tests and how they impact students' results. In particular, there are a number of complex issues that impinge upon these types of assessments, such as the language that the tests are delivered in (Edmonds-Wathen, 2011), the content assessed, and the equality of the test in terms of privileging one culture group (Klenowski, 2009). It has been contended that "the continual focus of Indigenous students' poor achievement in these tests is likely to produce [in] teachers, policy makers, the general public, and Indigenous students themselves a belief that Indigenous students cannot learn mathematics in their everyday lives" (Meaney et al., 2012, p. 69). From the perspective of this thesis, having poor numeracy skills does not preclude Indigenous people from accessing university or employment. Rather, poor numeracy may contribute to an inability to access these opportunities. Importantly, this can potentially be conquered. In contrast, other factors such as racism, remote living, and health are more deeply engrained and are challenging to transform.

As algebra provides a contribution towards equality and opportunity, it is important to consider the significance of this mathematical concept for young Australian Indigenous students. In reflecting upon the premise that young Indigenous students are capable learners in mathematics, consideration needs to be paid to how they construct their knowledge. In doing so, it must be acknowledged that Indigenous students have their own cultural values and these are reflected in the way they learn and how they display that learning.

1.3.1 Early Algebra Thinking

Early algebra is its own unique subject, and is not to be confused with the teaching of algebra early. Rather, the concept of early algebra is intertwined with other early mathematical concepts as students engage in the gradual introduction to formal notation (Carraher, Schliemann, & Schwartz, 2008). The central ideas promoted in the national algebra standard for young students are (1) patterns, (2) mathematical situations and structures, (3) models of quantitative relationships, and (4) change (NCTM, 2000). To view algebra only as 'generalised arithmetic' is to misrepresent and oversimplify the essential role of generalisation in algebraic thinking (Driscoll, 1999).

Algebraic thinking does not appear spontaneously in knowledge systems. Rather, a process of conceptualization and reconceptualisation is needed in the development of algebraic thinking (Radford, 2012). Algebra has stemmed from diverse cultural backgrounds. Hatfield, Edwards, & Bitter (1997) documented that "Africans invented rectangular coordinates by 2650 B.C. and used them to make scale drawings, and star clocks …the word 'algebra' is Arabic in origin" (p.71). Additionally, interpretations by the Greeks of Babylonian mathematics, which was reconceptualised by the Arabs (9th century), and then reconsidered by the Renaissance mathematicians (16th century) (Radford, 2001), contributed to the beginnings of algebraic thinking.

While it is acknowledged that algebraic thinking is fundamental to the understanding of mathematical logic later in schooling, there remains a persistent belief that young students are not capable of engaging in this type of mathematical thinking (Carraher et al., 2006). It has been argued that introducing algebra to young students is ineffective, as they are not developmentally ready due to limitations in students' cognitive development (Filloy & Rojano, 1989). Research has proposed that students have difficulty transitioning from arithmetic to algebra due to a cognitive gap between these two concepts, and with this, difficulties arise when working with unknowns (Herscovics & Linchevski, 1994). It has been suggested that algebraic thinking develops through a series of stages (Filloy & Rojano, 1989; Sfard & Linchevski, 1994). Hence, teaching algebra has been delayed until adolescence.

Fundamental to the development of algebraic thinking and concepts is the ability to generalise patterns (Cooper & Warren, 2008; Mulligan & Mitchelmore,

2009; Papic, 2007). Teachers commonly use a patterning approach when introducing early algebra concepts. Research has highlighted that young students can generalise the mathematical structure of the patterns from a range of pattern contexts. For example, students can identify the structure of repeating patterns as multiplicative, and the structure of growing patterns as functions (Blanton & Kaput, 2004; Cooper & Warren, 2011). From identifying this structure, young students have demonstrated aspects of early algebraic thinking, including the ability to generalise pattern structures (Becker & Rivera, 2008; Moss & Beatty, 2006; Radford, 2010a; Warren & Cooper, 2008a).

1.3.2 Generalisation

The ability to generalise mathematical structures beyond the initial learning experience has been highlighted as one important of the important components of mathematics (Cooper & Warren, 2008; Kaput, 1999; Mitchelmore & White, 2000). Consequently, it can be understood why generalisation has been described as the heart or the heartbeat of mathematical thinking (Mason, 1996). It can be implied that this is a skill that is intrinsic to success in mathematics, because it enhances our capability to apply mathematical concepts across mathematical tasks (Mason, 1996). Commonly, literature pertaining to students' ability to generalise in mathematics has been conducted in secondary and tertiary learning environments (Carpenter & Franke, 2001; English & Warren, 1995; Lee, 1996). Recently, there has been a growing body of literature exploring mathematical generalisation with younger students. Results of this research have shown that young students are capable of generalising mathematical structure across a range of contexts (Carraher et al., 2006; Cooper & Warren, 2008). These contexts include generalising relationships between numbers and pattern rules, and generalising from particular examples in real-life situations to abstract representations (Blanton & Kaput, 2011; Carraher, Martinez, & Schliemann, 2008; Carraher, Schliemann, & Brizuela, 2001; Cooper & Warren, 2011; Cooper & Warren, 2008; Leung, Krauthausen, & Rivera, 2012).

1.3.3 Growing Patterns

Growing patterns are characterised by the relationship between elements, which increase or decrease by a constant difference. These patterns often have associated algebraic expressions that allow the student to express how this change

occurs (Warren, 2005). Students are asked to form the functional relationship between growing patterns and their position. That is, they are asked to reconsider growing patterns as functions (covariational thinking – relationship between the pattern and its position), rather than as a variation of one data set (recursive thinking – relationship between successive terms within the pattern itself) (Warren, 2005). Growing patterns are often represented as geometric visual patterns. At times, both variables (i.e., pattern and pattern position) are clearly identified in the representation, and at other times only the pattern is represented (see Figure 1.2). These types of patterns are commonly used in the initial exploration of algebra to generate algebraic expressions (Bennett, 1988). An example of this: the algebraic expression $2n – 1$ used to generalise the relationship of the first growing pattern illustrated in Figure 1.2.

$2n$-1 $2n$+1
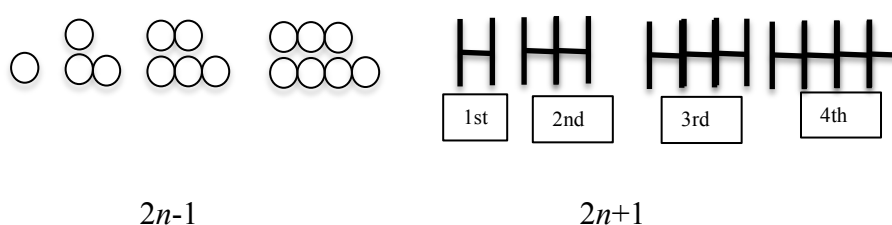
Figure 1.2. Examples of geometric visual growing patterns and algebraic expressions.

Additionally, growing patterns support the transition for students to engage in functional thinking. The concept of a function is fundamental to virtually every aspect of mathematics and every branch of quantitative science (Warren et al., 2011). Students are believed to be capable of thinking functionally at an early age (Blanton & Kaput, 2004). This form of thinking involves understanding the dynamic relationship between two variables, termed covariance – covariational thinking (Slavit, 1997).

1.3.4 Patterns and Algebra: Research with Young Indigenous Students

There is limited research considering Australian Indigenous students and the learning of a specific mathematical concepts (Meaney et al., 2012). Within Australasia, much research has focused on pedagogical practice that supports Indigenous students' learning (Harris, 1984; Hurst & Sparrow, 2010; Jorgensen, 2009; Warren, Baturo, & Cooper, 2010) and studies concerning the language of

instruction (Edmonds-Wathen, 2011; Meaney, Trinick, & Fairhall, 2012; Niesche, 2009; Owens, 2010). The minimal studies that have been conducted focus on a specific mathematical concept and are predominately in the area of number (i.e., Butterworth & Reeves, 2008; Warren & deVries, 2009), with few studies conducted specifically on algebra or algebraic thinking (Matthews, Cooper, & Baturo, 2007; Miller & Warren, 2012).

Within the area of algebra, to date, one study has mentioned the importance of the concept of mathematical patterning in relation to algebraic thinking for Australian Indigenous students. Matthews, Cooper, and Baturo (2007) conjectured that Indigenous students have an affinity with the notion of pattern, as an understanding of patterning underpins aspects of Aboriginal culture. For this, is the construction of their kinship system, which indicates that "their culture contains components that are pattern-based and which may lead to strong abilities to see pattern and structure" (Matthews et al., 2007, p. 250). As there is minimal research pertaining to how Indigenous students engage with growing patterns, and whether their perceived affinity with pattern assists them in this engagement to generalise, an opportunity arises to explore this phenomenon within this thesis.

1.4   IMPETUS OF THE STUDY

Many policies have been implemented both nationally and internationally to improve the educational outcomes for all students (e.g., Closing The Gap; Melbourne Declaration on Educational Goals for Young Australians; No Child Left Behind Act). Additionally, there have been a number of specific policies in Australia to improve the educational outcomes for Indigenous students (e.g., Closing The Gap). The implementation of such policies has initiated changes in curriculum. Within the algebraic domain, these changes include the incorporation of mathematical concepts that encapsulate the groundings of algebra. With this there is a strong push for young students to be 'algebra ready' (NCTM, 2000).

This thesis aims to reflect on these changes and give structure to the current practice of how to best engage young Indigenous students in algebraic thinking. Through working in Indigenous schools, as part of a larger early numeracy research project, I witnessed how young Indigenous students engage in aspects of mathematics. Additionally, I was working on an early algebraic research project.

Thus, the impetus of this study arose out of the enquiry to understand how young Indigenous students engage in algebraic thinking.

1.5   THE RESEARCH PROBLEM AND PURPOSE

Young Australian Indigenous students have been identified as one of the most disadvantaged groups in education (Gonski et al., 2011). National and international measures in mathematics and numeracy indicate young Indigenous students are underperforming compared to non-Indigenous students (Commonwealth of Australia, 2008; Queensland Studies Authority, 2003; Thomson, De Bortoli, & Buckly, 2013). Though this is the case, it has been identified that understanding and having success in mathematics empowers and assists Indigenous students' life decisions that concern social and economic disadvantage (Council for the Australian Federation, 2007). Thus, young Indigenous students need to be positioned for mathematical success. Being successful in algebra has been linked to students' post-school and employment opportunities. The ability to generalise mathematically is fundamental to achieving at higher levels of education. It underpins algebraic thinking. Yet, it is conjectured that, in Indigenous contexts, teachers present mathematical experiences with limited opportunities for Indigenous students to explore acts of generalisation. Research suggests that, Indigenous students are more susceptible to teachers' low expectations, and this influences the types of learning experiences presented in their classrooms (Good & Nichols, 2001). Commonly, teachers provide lessons based on skill and drill-learning experiences (Baturo, Cooper, Michaelson, & Stevenson, 2008; Jorgensen, Grootenboer, Niesche, & Lerman, 2010), and this aligns with the little faith educators have in Indigenous students' mathematical ability (Matthews, Watego, Cooper, & Baturo, 2005). Therefore, it can be conjectured that at times, young Indigenous students are perceived as not being able to achieve in mathematics, thus influencing mathematical standards and teaching practices.

The purpose of this study is to explore how young Indigenous students engage in early algebraic thinking. In particular, it focuses on how these students generalise growing patterns. By studying these phenomena, it is conjectured that the study will formulate an approach to the teaching and learning process, which will impact positively on how young Indigenous students engage in the generalisation process.

1.6 AIMS AND RESEARCH QUESTIONS

Aims

The first aim is to explore how young Indigenous students generalise growing patterns.

Second is to identify the teaching actions that assist young Indigenous students to generalise growing patterns. These include the consideration of semiotic interactions (signs, gestures, language) that are involved in the generalisation process, and how this impacts students' learning and communication.

Third is to identify the role of culture in relation to how young Indigenous students engage in growing pattern generalisations. In order to attain these aims three research questions are posed.

Research Questions

The overarching research question for this study is:

How do young Indigenous students generalise mathematical growing patterns?

After considering the literature (Chapter 3) three research questions were generated that, in turn, inform the design of the research and guide data collection and analysis:

1. How do young Indigenous students engage in growing pattern generalisation?

2. What teacher actions assist in enhancing young Indigenous students to generalise growing patterns?

3. How does culture influence the way in which young Indigenous students engage in growing pattern generalisation?

1.7 THE RESEARCH DESIGN

1.7.1 Epistemology

As this study explores the ways in which young Indigenous students construct new knowledge while engaging in pattern generalisation tasks, constructionism is the appropriate epistemological lens adopted for the research. Constructionism suggests 'meaning' or 'knowledge' is a product of social interaction and experiences (Stahl, 2003), rather than individual perceptivity. From the stance of constructionism,

meaning is socially constructed, thus, individuals may construct knowledge to the same phenomena in different ways (Crotty, 1998). Constructionism acknowledges that individuals build meaning through language, symbolism, culture, and interaction. This epistemology lends itself to the exploration of mathematical generalisation as students construct their knowledge from a known context and extrapolate this core content to the general through social interactions.

1.7.2 Theoretical Perspective

As individuals build knowledge through language, symbolism, culture and social encounters, the theory of semiotics provides a lens to interpret these interactions. Semiotics is the study of cultural sign processes, analogy, communication, and symbols (Peirce, 1958). Furthermore, mathematics as a discipline is considered to be abstract and heavily based on perceivable signs. Mathematics has been described as an intrinsic symbolic activity that is accomplished through communicating orally, bodily, written texts or utilising other signs (Radford, 2006). As this study is considering the teaching interactions that assist young Indigenous students to generalise mathematical growing patterns, semiotics provides the lens to interpret the signs within and between all social interactions. Thus, semiotics informs exploration of the teaching and learning activities in mathematics,

In researching these cognitive interactions in young Indigenous students, it is important to acknowledge the potential for unique cultural variations with regard to how the outward displays of thought processes may be expressed. To appropriately account for these cultural sensitivities, this research acknowledges Indigenous research perspectives as a theoretical perspective.

1.7.3 Research Methodology

The research methodology for this study was draw from conjecture-driven teaching experiments. This methodology provided in a natural setting a platform to investigate the interactions that support the development of students' ability to generalise, in a naturalistic setting. Teaching experiments were used in this study for the primary purpose of directly experiencing students' mathematical learning and reasoning in relation to their construction of mathematical knowledge (Cobb, 2000; Steffe & Thompson, 2000). The aims of this study were concerned with ascertaining

the type of hands-on materials, teacher actions, and classroom discussions that promoted students' engagement with the generalisation process. Additionally, it provides a platform to explore cultural aspects within an Indigenous context when young students engage in mathematical tasks.

1.7.4 Participants

The research was conducted in one Year 2/3 classroom (7-9 year olds) of an urban Indigenous school in North Queensland. Pattern School (pseudonym) is a co-educational school. In total, 18 students participated in the study; however, this number fluctuated in the pretest and mathematical lessons due to absenteeism. Additionally two women, an Aboriginal Indigenous Education Officer (IEO1) and a Torres Strait Islander Indigenous Education Officer (IEO2) were consulted during the study for cultural information in relation to students' learning. Finally, I was a participant of the study as my role was 'researcher as teacher' during the data collection.

1.7.5 Data-Gathering Strategies

To explore how students engaged in mathematical generalisations, in a naturalistic classroom setting, data-gathering strategies used in this study were:

1. Initial Classroom Observations.

2. Pretest 1 and Pretest 2 conducted at the beginning of Teaching Experiment 1 and 2.

3. Teaching Experiments with the whole class (N=18), comprising six 45-minute mathematics lessons in total (three in Teaching Experiment 1 and three in Teaching Experiment 2).

4. One-on-One Piagetian clinical interviews with students (n=3) after each teaching experiment.

1.8   SIGNIFICANCE OF THIS STUDY

This study aims to make a significant contribution to mathematics education research within the domain of early algebra. In particular, it provides a hypothesised learning-teaching trajectory for the teaching of generalising growing patterns.

It is important in determining how Indigenous students can access algebra.

Importantly, this research offers an opportunity to contribute to our understanding of how Indigenous students conceptualise mathematical pattern structure and how they generalise these structures. It also adds to prior research that has indicated that young students are capable of engaging in complex mathematics, particularly their ability to generalise (Cooper & Warren, 2008; Warren & Cooper, 2002; Warren, 2006). Despite recent research exploring this area of mathematics, little is known about how young students, and especially young Indigenous students, engage in mathematical generalisation tasks.

Furthermore, this study adds to the types of research that have been conducted with young Indigenous students in an Australian context. Few studies have conducted teaching experiments and follow up clinical interviews with Indigenous students to further deconstruct the learning that is occurring.

Finally, this study challenges the negative image often associated with Indigenous students' educational outcomes.

1.9   THESIS OUTLINE

Chapter One: Introduction This chapter has presented the significance of the research problem that underpins this thesis. Three research questions were identified. These provide direction for both of the data collection strategies and the method of analysis of this data.

Chapter Two: Context of the Research Chapter 2 positions the research problem and purpose within international and national contexts. It provides the reader with an understanding of the complex issues that surround education for young Australian Indigenous students.

Chapter Three: Literature Review Chapter 3 reviews the literature relating to the teaching and learning of early algebra, and describes the processes involved in young students generalising mathematical patterns. Additionally, the examination of the pedagogical approaches to teaching young Indigenous students mathematics is considered.

Chapter Four: Design of the Research Chapter 4 describes and justifies the research design and methodological approach adopted for this study. This includes

the range of data-gathering strategies employed to inform the research questions. Additionally, the method of analysis is outlined.

Chapter Five: Findings Teaching Experiment Chapter 5 presents results from Teaching Experiment 1 and 2. The results comprise two pretests that were conducted at the commencement of each teaching experiments and six 45-minute mathematics lessons (three lessons from each teaching experiment). The ongoing analysis of data was considered in light of mathematical learning, semiotics, and culture.

Chapter Six: Findings Piagetian Clinical Interview Chapter 6 presents the findings from three case students who participated in one-on-one Piagetian clinical interviews at the conclusion of each teaching experiment. The ongoing analysis of the data was considered in light of mathematical learning, semiotics, and culture.

Chapter Seven: Discussion of the Findings Chapter 7 provides a synthesis of results and insights from Chapter 5 and Chapter 6.

Chapter Eight: Conclusion and Recommendations Chapter 8 addresses the research questions; identifies the contribution made to existing research, theory, and knowledge; presents the limitations of the study; and, makes recommendations for further research.

Chapter 2: Context of the Research

2.1 CHAPTER OVERVIEW

This chapter documents the contexts within which the research problem is situated, that is, how young Indigenous students generalise mathematical growing patterns. Chapter 2 provides the reader with a background of the complex educational issues facing young Australian Indigenous students. The chapter begins by presenting a brief history of Indigenous Australia. This is followed by an outline of the pertinent issues concerning Indigenous students' experiences in learning contexts, and international and national perspectives for improving Indigenous education. Within this chapter, the importance of mathematics will be explored across two themes: Australian students' mathematical achievement, and mathematics in relation to Australian Indigenous people. Finally, this chapter will justify the research problem and define the research purpose. Figure 2.1 presents an overview of Chapter 2.

Chapter 2: The Context of the Research
2.1   Chapter Overview
2.2   Indigenous Australia
2.3   An International Perspective of Indigenous Education
2.4   An Australian Perspective of Indigenous Education
2.5   The Importance of Mathematics
2.6   Australian Students and Mathematics
2.7   The Research Problem
2.8   The Research Purpose
2.9   Chapter Review
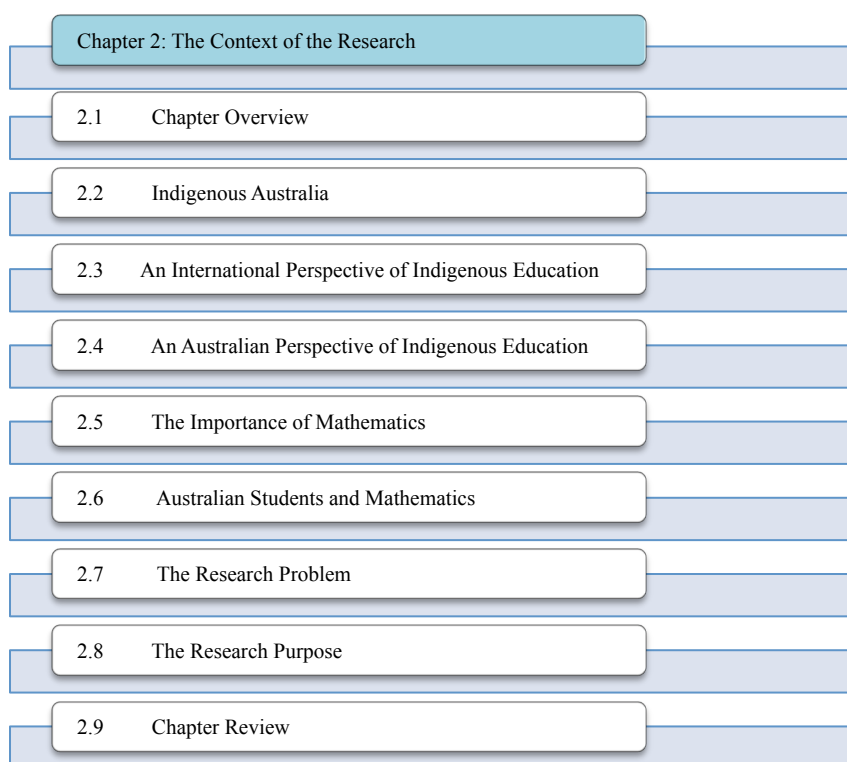
Figure 2.1. Overview of Chapter 2.

2.2 INDIGENOUS AUSTRALIA

> If you want to put it visually, if you take the clock face of 60 minutes and give each one of those minutes a thousand years, then you have the recorded time that our people [Aboriginal people] have been on this land. That means Plato was here a minute and a half ago (Rose, 2009)

Australian Aboriginal and Torres Strait Islander people have been the traditional custodians of this land and surrounding islands of Australia for approximately 70,000 years. Both groups have their own distinctive cultures and societies. Their diverse relationships with, connections to, and understanding of the Australian land and surrounding waters have been passed down from generation to generation (Dudgeon, Wright, Paradies, Garvey & Walker, 2010). Both Aboriginal and Torres Strait Islander people have experienced disadvantage and dispossession as a result of colonisation (Australian Institute of Health and Welfare, 2013; Dudgeon, et al., 2010).

Europeans first arrived on Australian shores in the 1600's. First contact was a hostile engagement between Aboriginal people and Dutch Europeans in 1606 on the western coast of Australia. One hundred and fifty years later, the east coast of Australia was 'discovered'. Then ownership claimed in the name of England by Captain James Cook and his fleet in 1788. Cook claimed the land under the title of *Terra Nullius*, a land belonging to no one. The doctrine of *Terra Nullius* remained until 1992, when the Mabo decision recognised Aboriginal and Torres Strait Island people had native title under Australian law (Commonwealth of Australia, 2003). The decision acknowledged that *Terra Nullius* should have never been applied to Australia; it was a major turning point in recognising the timelessness of Aboriginal and Torres Strait Island peoples' history.

The concept of *Terra Nullius* ran deeper than the notion of a land belonging to no one; it essentially marginalised Indigenous people from white society (Ross, 2006). Aboriginal and Torres Strait Island people were denied basic human rights and their existence was denied (Ross, 2006). The process of colonisation was rapid and disempowering for many Indigenous people. Stanner (1969) in the 1968 Boyer Lecture, describes colonisation as a process that had "decimated the Aboriginal peoples' deep wells of cultural, scientific and spiritual knowledge, had [has] disempowered their complex social networks, and had [has] marginalised Aboriginal

peoples and their issues" (cited in Askell-Williams et al., 2004, p. 58) from the broader community. The devastating result of this continues today. In 1967 a Referendum was held in Australia to change the Australian constitution. As a result, the 1967 Referendum was the first time within Australian history that Aboriginal and Torres Strait Islander people were to be included in the national census of the population, and not merely counted as a part of the flora and fauna.

Aboriginal people experienced colonisation first hand through missions. Missions were set up as a means to 'save' or 'assist' the souls of the Aboriginal and Torres Strait Islander people (Nakata, 2008). The forced relocation of Indigenous people to missions is not an uncommon experience in history. It also occurred in other colonised countries, such as Canada and the United States of America. Missions were founded by religious organisations and forcibly removed Indigenous people from their traditional lands to one central location (State Library of Queensland, 2012). On missions, Indigenous people were used as cheap labour and attempts were made to evangelise Aboriginal people to Christianity (Short, 2008). During this period (late 19th and early 20th century), there was a push to assimilate Indigenous peoples to Western society both culturally and through biological absorption (Ellinghaus, 2003). Biological absorption is defined as "the imagined process by which Indigenous identity would disappear through interracial sexual liaisons" (Ellinghaus, 2009, p. 59). It was apparent at the end of the late 19th century that there was a decline in full descent Indigenous people and an increase in Indigenous people of mixed descent (Australian Human Rights Commission, 1997). Additionally, "most colonists saw them [Indigenous peoples] as being in a state of racial and cultural limbo" (Haebich, 1988, p. 48).

As a means to further 'absorb' or 'assimilate' Indigenous peoples, children were forcibly removed from their families. These children are known as the stolen generations. Missionaries, and both state and federal governments under their respective government acts, forcibly removed children. As Indigenous children were forcibly removed from their families and sent to work for non-Indigenous people, many young Indigenous women fell pregnant to non-Indigenous men. Government officials 'theorised that this mixed decent population would, over time, 'merge' with the non-Indigenous population' (Australian Human Rights Commission, 1997, p. 24). For the most part, children who were forcibly removed experienced lasting

effects ranging from psychological harm, sexual abuse, and loss of cultural identity (Australian Human Rights Commission, 1997). Despite the 1967 Referendum to extend rights to Indigenous people, these disadvantages are intergenerational and still impact Australian Indigenous people today. It is acknowledged that the present situation for many Indigenous people includes dislocation, poor health, lack of employment, poor educational outcomes, and poverty stricken living conditions as a direct result from this past (Australian Human Rights Commission, 1997).

2.3 AN INTERNATIONAL PERSPECTIVE FOR INDIGENOUS EDUCATION

Research and literature in education have often provided a deficit perspective of Indigenous students' achievements, focusing on their failures in education and the ways Indigenous communities have contributed to this (Deyhle & Swisher, 1997). A number of education policies since the 1967 Referendum have been implemented to improve education for Australian Indigenous students (Schwab, 1998), and thus far few have improved the numeracy outcomes for Indigenous students (MCEETYA, 1999). Recently, there has been a shift in research focus and greater emphasis has been placed on the failures associated with presenting a Western curriculum model to Indigenous students. Subsequently, there has also been research conducted about the lack of acknowledgment of Indigenous practices in the curriculum and lack of understanding about the ways in which Indigenous students learn (Deyhle & Swisher, 1997; Klug & Whitfield, 2003). Parallels can be drawn with the research and literature conducted within other colonised Indigenous communities, particularly that of Canadian First Peoples. Therefore, the literature for the following section draws from both Australian and other Indigenous communities to enhance and focus our understandings of appropriate education required for Indigenous people.

To move away from this disempowerment or deficit model of education is to find the motivational and appurtenant steps forward for empowering Indigenous students in education. A report from the Expert Mechanism on the Rights of Indigenous Peoples (EMRIP) to the United Nations Human Rights Council (2009) highlights some key factors for improving the educational experience for Indigenous students and provides a shift to empowerment. Key factors of this report include:

- Deficiency of access to quality education is a major factor that continues to contribute to social marginalisation, poverty and dispossession of Indigenous peoples.

- Designing programs for specific Indigenous communities so that the needs of the community are met.

- Indigenous people cannot be forced into mainstream education and mainstream education should integrate Indigenous culture.

- Mother-tongue based bilingual and multilingual education must be integrated in teaching programs.

- Indigenous peoples, if they so choose, have the "right to educational autonomy" including "the right to decide their own educational priorities […] as well as the right to establish and control their own educational systems and institutions.

- The report also recommends that human rights education be included in schools to encourage cooperation between the different cultures; and

- States must provide funding for appropriate teaching materials and the recruitment of Indigenous teachers.

These key factors highlighted by the EMRIP provide an international model for Indigenous education. Unquestionably, this model is about providing a mainstream education standard that is enhanced by Indigenous knowledges and ways of learning to provide a pathway for an empowered culture. Indigenous knowledges are "understood to be the traditional knowledge of Indigenous peoples" (Nakata, et al., 2005, p.7). Importantly, Nakata et al. (2005) highlights that 'in Australia, a common misunderstanding is that this equates Indigenous knowledge to 'past' knowledge, when in fact Indigenous people view their knowledge as continuing' (p.7).

2.4 AN AUSTRALIAN PERSPECTIVE FOR INDIGENOUS EDUCATION

Indigenous students are enrolled in schools situated in rural, regional, urban and remote communities across Australia (MCEETYA, 2006). A study conducted by the Australian Bureau of Statistics (2006) identified that approximately 33% of all Indigenous Australians lived in major cities of Australia', 21% in Inner Regional Australia, 22% in Outer Regional, 9% in Remote Australia, and 15% in Very Remote Australia (ABS, 2006). Evidently, Indigenous students are from a diverse group of communities and should not be viewed as a homogenous group. Rather, Indigenous

students reflect the cultural, social and economic diversity of the communities in which they live (MCEEYTA, 2006). Indigenous Australians comprise of 2.5% of the Australian population and have a multitude of dialects including their own Indigenous languages, Kriol, or Aboriginal English as their first language.

Australian Indigenous students have been identified as one of the most educationally and socially disadvantaged groups in Australia (Frigo, 1999; Gonski, et al., 2011; Howard, 1997, 1998; Matthews et al., 2003). Several educational studies have articulated the issues Indigenous students face and the disadvantages that effect their schooling (e.g., Bourke, Rigby, & Burden, 2000; Gonski, et al., 2011; Sarra, 2003). These issues include complex home life, poor health, disengagement from school, language barriers, rural or remote living, low self-esteem, absenteeism, and a lack of aspirations for higher education (Bourke et al., 2000; Sarra, 2003; Thomas, 2006). Additionally, on a social level, national statistics conclude that Indigenous children have higher risks of infant mortality, a reduced life span, increased health problems, disengagement from education, criminal justice issues, and employment difficulties (Frigo et al., 2003). Generally, Australian Indigenous students perform lower than the state and national standard in all subjects. When considering numeracy, Indigenous students are found to be at least two years behind non-Indigenous students within the first four years of formal schooling (Foundation - Year 3) (Commonwealth of Australia, 2007; Frigo et al., 2003; Queensland Studies Authority, 2003; Storry, 2007). This is often referred to as the educational '*gap*'.

Government Policies to Addressing Indigenous Education

In order to address the complexities that Indigenous students face, a number of government education policies and intervention programs have been implemented. The report, Australian Directions in Indigenous Education 2005-2008 (MCEETYA, 2006), refers to Indigenous specific intervention programs delivered by education systems. Although these intervention programs have provided some assistance for Indigenous students, only a small portion of the population has been able to access them. The positive impact thus far has therefore been minimal. More recently, MCEECDYA released a report – The Aboriginal and Torres Strait Islander Education Action Plan 2011-2014 (Ministerial Council for Education Early Childhood Development and Youth Affairs, 2010). In the field of literacy and numeracy the document stipulates that it will achieve outcomes by:

- implementing a culturally inclusive and relevant national curriculum;
- supporting teachers to improve their teaching of literacy and numeracy, through implementing whole-of-school approaches to teaching literacy and numeracy and the better use of data and diagnostic instruments through the Literacy and Numeracy National Partnership;
- piloting new approaches to teaching literacy and numeracy to Indigenous students and sharing the evidence from these pilots; and
- lifting transparency of outcomes at the school level (MCEECDYA, 2010, p.13).

The report further indicates that within the Queensland context, the *Closing the Gap* education strategies need to:

- support highly mobile Indigenous students to stay in one school for longer periods to improve achievement in literacy and numeracy;
- provide targeted support to students whose first language is not Standard Australian English;
- support numeracy intervention by assisting schools to recognise Indigenous student learning needs, and raise numeracy education outcomes for Indigenous students; and
- implement a whole-school approach for improving literacy and numeracy programs involving partnerships with Aboriginal and Torres Strait Islander students and their families (MCEECDYA, 2010, p.23).

Notably absent is a strategy for specifically targeting the beliefs and attitudes of teachers within Indigenous communities. In particular, little reference is made to combating the issue of Indigenous students being taught in environments where teachers have low expectations (Commonwealth of Australia, 2008; Department of Education Training and Youth Affairs, 2000).

Currently, the National Indigenous Reform Agreement provides a strategic direction for schools to close the gap in Indigenous education disadvantage (COAG, 2009). This strategy from the reform agenda put in place in 2007 by the Council of Australian Governments (COAG). COAG consists of Indigenous leaders and government officials who are committed to overall change in Indigenous education, early childhood, health, life expectancy and employment. COAG specifically states in the agreement that they are working to "halving the gap for literacy and numeracy

by 2018" (COAG, 2012, p. 37) as part of their commitment to assisting Indigenous people.

2.5 THE IMPORTANCE OF MATHEMATICS

Within contemporary society, it is recognised that being numerate is imperative (Commonwealth of Australia, 2008). It influences the choices made regarding issues that occur within one's lifetime. Individuals are often presented with mathematical concepts that they must interpret, analyse and respond to in real-world contexts. Being deficient in the understanding of mathematics, or being mathematically illiterate, places one at a profound disadvantage (Paulos, 1988). Without this proficiency, the individual can be seen as lacking in the essential skills required to be an informed citizen and attain economic prosperity (Steen, 2001). The Organisation for Economic Co-operation and Development (OCED) define mathematics literacy as:

> an individual's capacity to identify, and understand, the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen. (OECD, 2009, p. 19)

In this light, it is reasonable to conclude that it is fundamental for individuals to have an extensive understanding of mathematical concepts.

Mathematical literacy is commonly referred to as numeracy in an Australian context. The Australian Association of Mathematics Teachers (AAMT) describes the relationship of numeracy and school mathematics thus; "Numeracy is not a synonym for mathematics, but the two are clearly interrelated. All numeracy is underpinned by some mathematics; hence school mathematics has an important role in the development of young people's numeracy" (AAMT, 1997, pp. 11–12). Being numerate provides the bridge connecting the mathematics taught in schools to application of mathematics and generally problem-solving in out-of-school contexts (Geiger, Goos, Dole, Forgasz, & Bennison, 2013).

From an educational perspective, it is evident that being numerate is more than being competent at performing mathematical operations within a classroom context. Mathematics impacts on other subject areas, home, workplace and community (Frigo

& Simpson, 2001). Additionally, mathematics is deeply embedded within political science and assists in building a democratic society (D'Ambrosio, 1999, 2001). Within education, there is a need for three important strands in mathematics. These three strands have been identified as literacy, matheracy, and technoracy (D'Ambrosio, 1999, 2001). There is a need to integrate these strands into the curriculum in order for students to develop the skills that are intrinsic to being effective citizens (D'Ambrosio, 1999, 2001). Literacy involves processing and communicating through spoken and written mediums and may also employ sign, gesture, numbers and codes. It is, therefore, viewed as a communicative instrument, which also encompasses aspects of numeracy. Matheracy involves inferring, hypothesising and concluding from data and is seen as the 'first step towards an intellectual posture' (D'Ambrosio, 2001, p. 237) The final perspective is technoracy, which is the critical familiarity of technology. Mathematics is strongly linked to all three of these perspectives (D'Ambrosio, 1999; 2001), and impregnates all dimensions of life, and must be taken into serious consideration when educating students.

In essence, being numerate allows for people to make informed judgements, ask questions that change perceptions and deepen their comprehension of societal issues (Frankenstein & Powell, 1994). As an extension to this concept, Frankenstein (1989) coined the term for this ability as being 'critically numerate'. It is crucial that all people are numerate, as it assists them to understand politics, social class, welfare, economics, preservation of resources, cultural and natural institutional structures, and government funding (D'Ambrosio, 2001; Frankestein & Powell, 1994). This in turn, implies that these citizens will participate in society and competently make informed decisions with regard to the processes and structures society uses to manage its citizens and distribute its resources.

For the proactive citizen, an understanding of mathematics is crucial to enabling better employment opportunities. It has been suggested that there are three ways mathematics enhances 'employability'. Firstly, it results in having sufficient skills that one can utilise when applying mathematics to economic, work and social matters. Significantly, this ability has been labeled as *functional numeracy* (Ernest, 2010). Secondly, *practical work-related knowledge* is a higher level of mathematical application that informs one's ability to solve problems using mathematics

specifically related to one's work environment (Ernest, 2010). Finally, there is *advance specialist knowledge*, used in areas such as engineering, medicine and science, where a sophisticated application of mathematics to tasks is required (Ernest, 2010). This high-level of mathematics, which involves the ability to generalise, is commonly a prerequisite for remunerative employment opportunities. Early engagement with mathematics influences the types of choices students make in regard to their further study and career (Jolly, Goos, & Smith, 2005). Thus, it is necessary in the early years of schooling for students be given opportunities to develop high mathematical thinking. Hence, the aspiration of the schooling system is to provide an egalitarian system in which all students are provided an equal platform for later life opportunities.

Higher levels of mathematics provide a gateway to further life opportunities. From the early years of school, students need frequent opportunities to experience higher-level mathematics (Commonwealth of Australia, 2008). Students need to be given the chance to explore more complex problems within a relevant context (Commonwealth of Australia, 2008). One aspect of mathematics that assists young students to engage in higher-level thinking is algebra, and in particular, engaging in generalising mathematical structures (Blanton & Kaput, 2011; Cooper & Warren, 2011). Providing young students with an opportunity to engage in such thinking is providing a pathway for a potential range of prospects post-school.

2.6 AUSTRALIAN INDIGENOUS STUDENTS AND MATHEMATICS

Australian students participate in the *Programme for International Student Assessment* (PISA) every three years. This assessment reports on students' understanding and skills in reading, mathematics and science. The sample age is approximately 15 years old as they are assessing students who are in their final compulsory schooling year. Within this assessment, Indigenous students are performing below non-Indigenous students and are below the OECD average (Thomson, DeBortoli, & Buckley, 2013) Table 2.1 presents the mean and standard deviations for Indigenous and non-Indigenous students for achievement in mathematics, compared to the mean and standard deviations for the OECD cohort.

Table 2.1

Means and Standard Deviations for Students on the overall Mathematical Scale for PISA

Student group	PISA 2000		PISA 2003		PISA 2006		PISA 2009		PISA 2012	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Indigenous	499	7.5	440	5.4	442	7.3	441.2	5.3	417	4.8
Non-Indigenous	535	3.4	526	2.1	522	2.3	516.8	2.5	507	1.6
Australia	533	3.5	524	2.1	520	2.2	514.3	2.5	504	1.6
OECD average	500	0.7	500	0.6	498	0.5	495.7	0.5	494	0.5

Note: SD= Standard Deviation

It is evident that there has been a downward trend in Australia's mean score on the PISA assessment since 2000 for both Indigenous students and non-Indigenous students. More importantly, Indigenous students are significantly below the OECD average mean score, and constantly lower than non-Indigenous students over the four PISA assessments. In fact, the gap between the two groups has increased, posing a distinct political and social concern. In 2012, 1991 Indigenous students participated in the PISA test. There was a mean score difference of 90 points between the non-Indigenous students and the Indigenous students in 2012 (Thomas et al., 2013). This equates to more than two-and-a-half years of schooling (Thomas et al., 2013). These trends are also reflected in the 2011 TIMMS study. Year 4 Indigenous students' mathematics score (Mean = 458, SD = 7.8) was 64 points lower than non-Indigenous students (Thomson et al, 2012). Furthermore, 55% of Indigenous students did not reach the international intermediate benchmark in mathematics. The results suggest there are complex issues stemming from the scope of the mathematics curriculum and pedagogical practices in both Indigenous and non-Indigenous classrooms, issues that are particularly prevalent within the Queensland context. These trends are replicated in data from recent Australian assessment.

Australia students are assessed at a national level four times during their time at school, and the results are presently being used to make judgements about schools' (and their students') achievements in mathematics. The *National Assessment Program for Literacy and Numeracy* (NAPLAN) commenced in Australian schools in 2008. The annual national tests replaced the tests administered by the separate Australian states and territories, thus enabling all students to be compared on the

same measure. The tests enable governments, education authorities, schools and communities to determine whether young Australian students are attaining significant educational outcomes (ACARA, 2011). This assessment program was designed to test all students in Years 3, 5, 7 and 9 in their reading, writing, language conventions and numeracy skills. This evaluation enables schools to monitor a cohort's progress and make comparisons about their students' achievements against those from other states and territories. As the participants for this study are young Indigenous students in Years 2 and 3, the Year 3 NAPLAN numeracy results will be considered as a context for the research. Table 2.2 displays the mean and standard deviation for Year 3 Indigenous and non-Indigenous students for numeracy for 2008-2013.

Table 2.2.

Mean and Standard Deviation for Indigenous and Non-Indigenous Students Between 2008-2013 for Numeracy (NAPLAN)

Year	Indigenous		Non-Indigenous	
	Mean	SD	Mean	SD
2008	327.6	-	400.5	-
2009	320.5	76.0	397.7	70.6
2010	325.3	71.2	399.0	69.8
2011	334.4	65.0	401.7	69.1
2012	320.1	75.0	399.5	70.2
2013	332.3	65.5	400.6	63.9

Note: SD = Standard Deviation, - no SD reported

The results exhibit a significant difference in scores between Indigenous and non-Indigenous students. These results clearly show that Indigenous students are performing at a lower level of mathematics than non-Indigenous students within the national context. A dissection of the results into geo-location illustrates more vividly the discrepancy between Indigenous and non-Indigenous students. For the purpose of this study, a comparison will be made between the national context and the state of Queensland in Table 2.3.

Table 2.3

Achievement of Year 3 Students in Numeracy by Geo-location 2013

Geolocation	Australia		Queensland	
	Indigenous (n= 13229)	Non-Indigenous (n=250773)	Indigenous (n= 4102)	Non-Indigenous (n=53023)
Metropolitan	348.1	405.2	339.4	394.1
Provincial	341.7	387.7	342.4	381.4
Remote	304.8	377.7	307.4	374.4
Very Remote	272.7	376.7	302.0	373.0

Recurring trends in these results show that as the school distances from metropolitan areas increase, Indigenous and non-Indigenous students' results on the NAPLAN test decrease. It is also clear that non-Indigenous students perform better on NAPLAN numeracy assessments than Indigenous students irrespective of geo-location. When comparing Indigenous students to non-Indigenous students in a very remote location, the gap is large between the two cohorts and it appears that non-Indigenous students, while still falling below the national average (M = 396.9), are achieving higher scores. In stark contrast is the comparison between the Australian mean score for 2013 (M =396.9) and that of very remote Indigenous students (M = 272.7), a gap of 123.7 points. This trend also exists within the Queensland context.

Due to the widespread media exposure of NAPLAN, publication of these results has social ramifications, and potentially negatively positions Indigenous students in the public arena. Without contextual examination, the results may be viewed in isolation, and portray Indigenous students as being incompetent in mathematics. It appears that while geo-location provides some explanation for lower scores, be it through lack of resources or the level of teaching staff experience dedication and continuity, it does not definitively explain why non-Indigenous students are performing at much higher levels. One explication for this is that there is minimal reference to socio-cultural contexts for Indigenous students, in other words NAPLAN reflects 'mainstream' school students contexts (Howard, Cooke, Lowe, & Perry, 2011; Klenowski, 2009).

As a means of overcoming social and economic disadvantage, understanding and having success in mathematics empowers Indigenous students in both post-school options and life decisions (Council for the Australian Federation, 2007). The Department of Education, Science and Training (2009) iterates that, mathematical empowerment is particularly important for Indigenous students from disadvantaged

backgrounds. It provides Indigenous people with the ability to make their own informed choices and judgements on social issues, and gain a deeper understanding of the societal issues. When one can identify, interpret, evaluate and critique the mathematics embedded in society one is socially empowered (Ernest, 2010). Additionally, social empowerment provides Indigenous people with the opportunity to make informed conclusions as to whether government policies relating to their situation are fair and are implemented using practices that best support the needs of their people.

Educational trends from national and international numeracy and mathematical measures paint a negative image for Indigenous students. These tests are presented from a Western mathematical perspective and at times do not capture students' true capabilities. Additionally, teachers present low-level mathematics to Indigenous students, focusing on drills and rote learning (Baturo, Cooper, Michaelson, & Stevenson, 2008; Jorgensen, Grootenboer, Niesche, & Lerman, 2010) rather than building conceptual understanding of mathematical structures. Hence it can be conjectured that young Indigenous students rarely engage in mathematical tasks that support algebraic thinking

Within the Australian context, the mathematics presented to young Indigenous students is heavily situated in a Western perspective, a worldview largely about economics (Commonwealth of Australia, 2008). In light of the current body of research, Indigenous students need to be set up for success in mathematics and have the same opportunities in career and tertiary education that non-Indigenous students have. In a mathematical context, for this to be achieved, the experiences within the early years of schooling must be rich in complex mathematics. As the function of generalisation is important in relation to mathematical success in the highest sense, providing students with early experiences in early algebra will potentially enhance their mathematical knowledge and aptitude in later school years. In essence, providing students with a diverse mathematical experience incorporating concrete and abstract principles is providing foundation for future opportunities.

2.7 RESTATEMENT OF THE RESEARCH PROBLEM

Young Australian Indigenous students have been identified as one of the most disadvantaged groups in education (Gonski et al., 2011). National and international

measures in mathematics and numeracy indicate young non-Indigenous students outperform Indigenous students (Commonwealth of Australia, 2008; Queensland Studies Authority, 2003; Thomson et al., 2012; Thomson et al., 2013). Though this is the case, it has been identified that as a means of overcoming social and economic disadvantage, understanding and having success in mathematics empowers Indigenous students in both post-school options and life decisions (Council for the Australian Federation, 2007). Thus, young Indigenous students need to be set up for mathematical success. Being successful in algebra has been linked to students' opportunities in post-school and employment opportunities. Comparatively, the ability to generalise is fundamental to achieving at higher levels of education. It underpins algebraic thinking. Yet, in Indigenous contexts, teachers present mathematical experiences with limited opportunities for Indigenous students to explore acts of generalisation. Research suggests that Indigenous students are more susceptible to teachers' low expectations and this influences the types of learning experiences presented in their classrooms (Good & Nichols, 2001). Commonly, teachers provide lessons based on skill and drill learning experiences (Baturo, Cooper, Michaelson, & Stevenson, 2008; Jorgensen, Grootenboer, Niesche, & Lerman, 2010). This aligns with the little faith educators have in Indigenous students' mathematical ability (Matthews, Watego, Cooper, & Baturo, 2005). Therefore, it can be conjectured that at times, young Indigenous students are perceived as not being able to achieve in mathematics, thus influencing mathematical standards and teaching practices.

2.8 RESTATEMENT OF THE RESEARCH PURPOSE

The purpose of this study is to explore how young Australian Indigenous students generalise growing patterns. By studying these phenomena, it is conjectured that the study will formulate an approach to the teaching and learning process, which will impact positively on how young Indigenous students engage in the generalisation process.

2.9 CHAPTER REVIEW

In specific contexts, this chapter has provided perspectives of how the education of past and present Indigenous students has impacted this study. These are essential factors to take into consideration when exploring how Indigenous students

engage with mathematics. Finally, the research problem and purpose were restated and will be considered in light of the literature in Chapter 3.

Chapter 3: Literature Review

3.1   CHAPTER OVERVIEW

The purpose of this chapter is to review literature pertaining to how young Australian Indigenous students generalise mathematical growing patterns. The literature that elucidates the research purpose is presented through three major themes: mathematics, semiotics, and Indigenous education. Each theme overarched emergent sub-themes. The themes mathematics is concerned with are generalisation, transfer and analogical reasoning, and early algebraic thinking. Semiotics encompasses Peircean semiotic theory, and semiotics in mathematics. Finally Indigenous education will cover literature including the current issues for Indigenous students in education, Indigenous ways of learning, and Indigenous learners and mathematics. Research questions will be presented at the conclusion of each theme. Figure 3.1 displays and overview of Chapter 3.

Chapter 3: Review of Literature

3.1   Chapter Overview

3.2   Mathematical Thinking: Generalisation, Transfer and Anological Reasoning

3.3   Mathematics: Early Algebraic Thinking

3.4   Semiotics

3.5   Indigenous Learners and Mathematics

3.6   Chapter Review
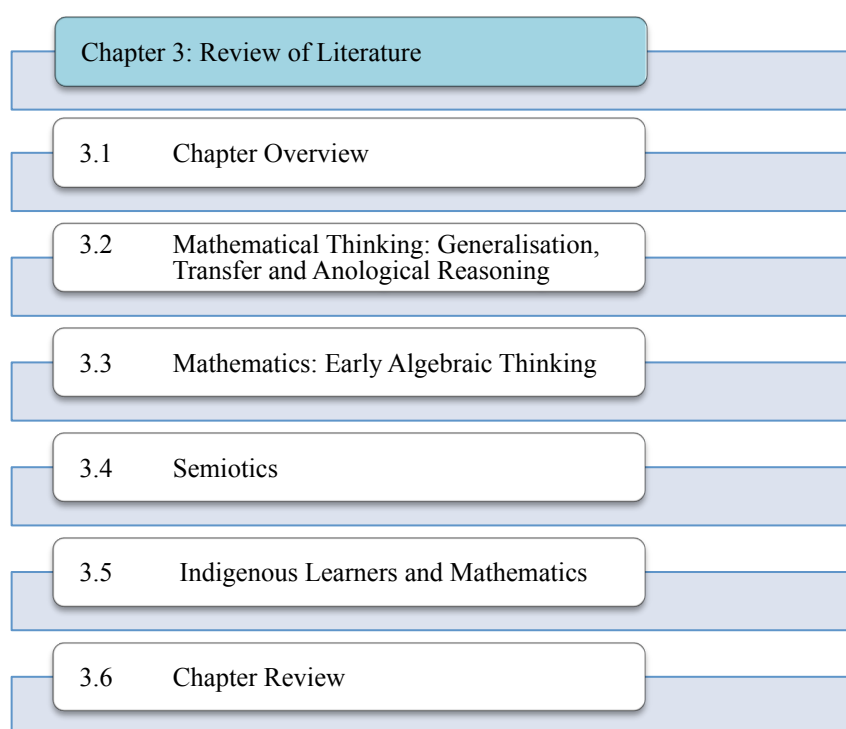
Figure 3.1. Overview of Chapter 3.

The literature is drawn from a variety of contexts, and this review will distil the central conclusions as they apply to students in the early years of formal education.

In addition, within the theme of Indigenous education, the literature will cover the sub themes: ways of learning; culture; and mathematics.

3.2 MATHEMATICAL THINKING: GENERALISATION, TRANSFER AND ANALOGICAL REASONING

The ability to transform base level mathematical experiences into abstract concepts requires one to engage in mathematical thinking and reasoning. Mathematical reasoning is of particular importance in this process, as it draws on a number of devices that assist with the transformation from the concrete to the abstract. An indication of one's capacity for mathematical reasoning is the ability to reach generalisations between and across contexts, or to transfer knowledge within or across situations. Additional importance lies in the ability of the learner to justify how the generalisation occurs. Literature in the areas of generalisation, transfer learning and analogical reasoning provides insight into how one thinks and reasons mathematically. Literature with regard to mathematical generalisation presents itself in two distinct fields of research: first, research relating to generalisation and second, research relating to transfer. Both of these fields interconnect with the notions of analogical reasoning (English & Halford, 1995) and structural mapping (Halford, 1993). Research relating to generalisation is situated in the mathematics education field, while transfer research is explored across fields such as education, psychology, and cognitive science (Ellis, 2007a).

3.2.1 Mathematical Generalisation

According to results of research, generalisation is the key conceptual premise underpinning mathematics. The ability to generalise one's learning beyond the initial experience is said to be the essence of mathematics (Cooper & Warren, 2008; Kaput, 1999; Mitchelmore & White, 2000). Consequently, it can be understood why generalisation is the heartbeat of mathematical thinking (Mason, 1996). It can be implied that this is a skill intrinsic to success in mathematics, enhancing our capability in the application of mathematical concepts across tasks. By extension of this theme, the ability to generalise is pertinent for success in high levels of mathematics. Commonly, past literature about generalisation in mathematics education has been limited to investigating students' ability to generalise mathematics, and for the most part has been conducted in secondary and tertiary

learning environments (Carpenter & Franke, 2001; English & Warren, 1995; Lee, 1996). However, more recently literature about generalisation in mathematics has extended to include young students (Blanton & Kaput, 2004, 2005, 2011; Cooper & Warren, 2008, 2011; Radford, 2010a, 2011; Rivera & Becker 2009, 2011; Warren, 2005; Warren & Cooper, 2008a, 2009)

While there is agreement in the academic community that the ability to generalise is important in mathematics, how one generalises remains unclear. Different ways to generalise have been identified in a number of research studies. Lannin (2005), for example, distinguishes between two types of generalisation: recursive and explicit. Recursive generalisation involves the use of a single variant, whilst explicit generalisation involves a covariant. This distinction was identified in a study conducted with year six students during a series of design experiments, and was concerned with their ability to develop and justify generalisations within pattering tasks whilst using computer spreadsheets as an instructional tool. Following whole class discussion, students were by and large able to provide appropriate generalisations and justify their generalisations using generic examples. When students participated in a small group discussion however, they rarely justified their generalisations, and tended to focus on the particular values (explicit generalisation) rather than on the general relations that existed between and across the values (Lannin, 2005).

From an alternative perspective, Harel and Tall (1991) theorised that there are three types of generalisation: expansive; reconstructive; and disjunctive. Expansive generalisation focuses on existing schemas being expanded in a broader context. Reconstructive generalisation is where the current schema is reconstructed to fit into the broader context. Disjunctive generalisation is where a new schema is constructed when moving to the broader context. These types of generalisation have been linked to Piaget's process of abstraction. Abstraction is the ability of students to generalise by expanding the range of reasoning beyond the case considered (Dubinsky, 1991; Harel & Tall, 1991). It should be highlighted that disjunctive generalisation is often misleading for the observer, as it can seem as though the student was successful in the broader task, although, there has been no cognitive reconstruction of prior schemas.

Generalising mathematical concepts must go beyond just the act of noticing (Radford, 2006). Students must also develop the capacity to address and express concepts algebraically for all elements of the sequence (Radford, 2006). Underpinning this assertion is Radford's (2010c) 'layers of generality': factual, contextual and symbolic generalisations. Factual generality is an elementary level of generalisation where students engage heavily in gestures, words and perceptual activities. On this level, students attend to the pattern presented and do not move into quasi-generalisations: that is, the generalisation of a large number or position beyond the presented activity (Cooper & Warren, 2008, adapted from Fujii & Stephens, 2001, notion of quasi-variable). Contextual generalisation requires students to reduce the signs (semiotic contraction) for greater expression of meaning, moving onto quasi-generalisations. Finally, the symbolic level requires a further semiotic contraction, where students replace words with symbols such as letters to express the generality of the rule.

While there is agreement that students move through different stages during the generalisation process, how one generalises, and the processes which assist students to move through these stages, remains a largely unexplored realm. There have been few research studies that have focused on the act of grasping a generalisation. Grasping a generality is to notice a commonality that holds across all terms (Cooper & Warren, 2011). Radford (2006) conjectures that the act of grasping a generalisation rests on perception and interpretation. This is an active process, and is dependent on the use of signs (gesture, speech, concrete objects) that indicate where the perceived object is located. This study focused on better understanding the role of signs in students' perceptive processes underpinning generalisation of number and geometric patterns. By contrast, Mason (1996) brings further itemisation into algebraic thinking as an activity. He sees the roots of algebraic thinking and generalising in detecting sameness and difference, in making distinctions, in classifying and labelling, or simply in 'algorithm seeking'. The very formation of this algorithm in the mind of the student, in whatever form it is envisioned, is algebraic thinking. Algebraic symbolism, according to Mason, is the language that gives voice to this thinking, the language that expresses the generality.

Radford presents (2003, 2010c) a three-level model of generalisation: (a) factual, (b) contextual, and (c) symbolic. Although this model stemmed from a study

conducted with junior high school students, it is more than feasible that the same applies to younger students, as the first stage begins with the concrete and moves through to the symbolic stage. Interestingly, this study also stressed the importance that patterning has in enhancing the understanding of generalisation (Radford, 2006). The three-level model gives a strong grounding for teachers in structuring tasks so that students can develop the ability to move beyond the act of noticing.

Another model that departs somewhat from Radford's, focuses on the actions that the student must engage in to assist with attempts to generalise. Ellis (2007a; 2007b) developed a taxonomy based in the premise that there are three major generalisation actions that middle school students participate in: (a) relating, in which one forms an association between two or more problems or objects; (b) searching, where one repeats an action to locate an element of similarity; and (c) extending, in which one expands a pattern or relation into a more general structure. Further studies by Ellis (2007b) have contributed to this taxonomy with the addition of a reflective component that refers to the ability of the student to identify and use a generalisation. Whilst these taxonomies and theories have been developed and tested on adolescent students in pre-algebraic stages of their schooling, there is minimal evidence suggesting that this is true for young students or whether the approach would be suitable for Indigenous students, thus posing the question: How do young Indigenous students engage in mathematical generalisation?

Research is currently failing to provide sufficient knowledge of how young students generalise mathematics. Internationally, research in mathematical generalisation in early years settings has focused on large scale professional development projects and teaching experiences (Carpenter, Franke, & Levi, 2003; Carpenter & Levi, 2000; Carraher & Schliemann, 2007; Dougherty & Zilliox, 2003). Furthermore, research has focused on the initial steps required to form early algebraic concepts (MacGregor & Stacey, 1995; Warren, 1996) and experience (Warren & Cooper, 2009). Hence, few studies focus on students' ability to generalise mathematical concepts in the early years setting. Those who have researched this concept have suggested that young students (age 8 to 11) can generalise across a variety and diversity of contexts (Cooper & Warren, 2008; Mulligan & Mitchelmore, 2009). These studies have been conducted in the mathematical realms of structure and early algebra, and while adding to the current literature, they do not delineate

how young students engage in the act of generalising or express their generality and what, if anything, enhances or detracts from the capacity to do so.

3.2.2 Transfer Learning

Intertwined with the notion of generalisation is the concept of transfer, as it is concerned with the identifying commonalties. Transfer learning has predominately focused on the ability to transfer one knowledge, principle or process from one task to the next or one context to the next. The capacity to solve one problem may assist in the capacity to solve the next, depending on the connections between the two tasks. Transfer has often been presented in a problem-solving context, where researchers try to derive a theoretical assumption. To date however, no general theory has been discovered. Studies have often attempted to explain how the adaption of prior knowledge has affected students' ability to solve new problems (Marton, 2006). The boundaries that define how one transfers information are taken from the perspective of the observer, and an attempt is made to identify the underlying mechanisms that contribute to students' ability to transfer. Consequently, as this type of research is taken from the observers' perspective, a student's correct result or response is often subject to a degree of predetermination on the researcher's behalf. This has led to many critiques of transfer research, and it has been identified that there is minimal agreement between scholars about the nature and mechanics of transfer and to what extent it occurs.

The classical transfer approach refers to the commonalities of elements within the transferring of prior knowledge to the new situation. According to Thorndike's (1906/1913) interpretation, transfer occurs when the learning that has taken place in the first situation influences the capability of performing the second situation. This occurs and is dependent on the shared identical elements between the two situations. 'Identical elements' refers to specific attributes within the task that the learners can recognise, such as the attribute of colour (Marton, 2006). The focus of research in this classical transfer perspective is on the function of similarities between situations and only the researcher determines this. A similar thematic approach in regards to the ability to learn concepts and understand the basis of the similarities has been the focus of studies by psychologists such as Dienes (1961), Piaget (1971/1973) and Skemp (1976).

Later studies, conducted by Gagné (1977), identified divergent elements within classical transfer. Gagné discussed the differences between lateral transfer and vertical transfer. Lateral transfer is defined as the generalisation of what is learnt in one situation and then transferred to a new situation that is presented with a similar level of complexity (Anderson, Corbett, Koedinger, & Pelletier, 1995; Bassok & Holyoak, 1993; Ellis, 2007a). This is comparable to studies that have focused on Peirce's (1878) notion of induction within a mathematical context, addressing the identification of commonalities (Dreyfus, 1991; Kaput, 1999). By contrast, vertical transfer is the learning of lower skills that correlate with and can be utilised in more complex situations (Ellis, 2007a). This behaviourist perspective based on Thorndike's studies has attended to the relationships between stimuli and responses and has been challenged by other paradigms, such as researchers concerned with deeper structures within the tasks presented.

Early cognitivists such as Judd (1908) go further than Thorndike's (1906/1913) perspective on transfer, to suggest that transfer is about how aware the leaner is of the underlying principles or deep structures of the task (Lobato, 2006). Judd was interested in the structural commonalities of the task rather than the surface commonalities of identical elements as suggested by Thorndike. Studies performed by Judd focused on the relationships of the task, not through the surface features of each situation, but rather through the processes used to solve the problems. Judd presented two groups of students with a situation (A). One group had an opportunity to learn the principle that relates to the problem, whilst the other had little knowledge given. Students were then given situation (B) where the principle and processes from situation (A) could be applied. Students who were provided with the information in situation (A) performed better, as they could transfer the learned processes across the tasks. A well-known example of this is where students are throwing darts at underwater targets. Students who were provided with the principle of refraction performed better in the task than those who were in the group given no prior information. This highly replicated experiment supports Judd's notion of transfer. While Thorndike's behaviourist perspective regards learning as the foundation of bonds between stimuli and response, Judd's cognitivists' paradigm, in contrast, stipulates that learning involves the foundation of more and more powerful representations of a general principle in the immediate world (Marton, 2006). A

powerful logic in this concept means the majority of recent studies in transfer have been seated in this perspective.

From the many attempts to explain transfer learning, common elements have emerged from the literature (Beach, 1999; Bransford & Schwartz, 1999; Carraher, Nemirovsky, & Schliemann, 1995; Lobato, 2003; Mestre, 2005; Tuomi-Gröhn & Engeström, 2003). These common elements ask four significant questions: (a) What is learned? (b) Who defines the relations between situations? (c) How many situations are involved? (d) Where does transfer happen? (Marton, 2006). Nevertheless, many different views have been defined regarding these questions in research. Thus far, the commonality of transfer learning consensus is that learners are capable of doing comparable things in diverse situations because of similarities between the circumstances being posed. These are similar views fielded by scholars in the area of generalisation.

3.2.3 Analogical Reasoning

A reasoning process commonly associated with one's ability to transfer is one's ability to reason analogically. Analogical reasoning in the broadest sense refers to one's ability to compare and identify the similarities between new and understood concepts, and to use this to gain understanding of the new concept. Thus, analogical reasoning links closely to the concepts of transfer learning and generalisation (Halford, 1993). Researchers have concluded that analogical reasoning is a skill that is developed later in life; hence most studies have focused on adult cohorts (Inhelder & Piaget, 1958, Sternberg 1997). However, other studies have concluded that children as young as 1 and 2 years of age can reason analogically (Goswami, 2001). In these instances, children have used their understanding of familiar situations to assist them in constructing new knowledge. Like generalisation and transfer learning, studies in analogical reasoning have determined ways to identify how one transfers or 'maps' one's learning. It has been concluded that there are three potential ways for a learner to map their learning: from problem to problem, structure to structure and procedure to procedure. Figure 3.2 below demonstrates this relationship (English, 2004, p. 6).
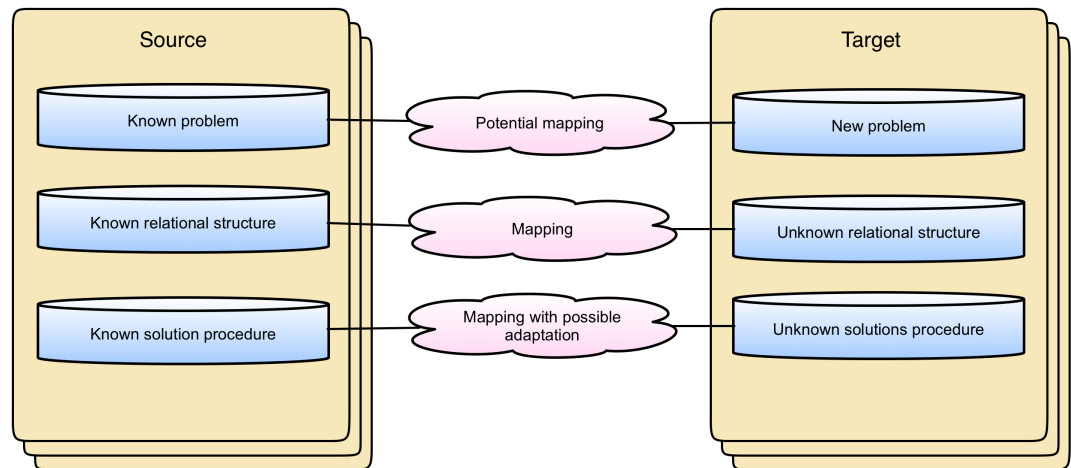
Figure 3.2. Three types of analogical reasoning.

While the learning is being mapped, there are tools that one may engage to assist this process. The incorporation of analogy, metaphor, metonymy and imagery assist with one's ability to map one's learning and think analogically (Davis & Maher, 1997). Analogy is embedded in decision making and problem solving in our everyday lives. It is defined as the transfer of structural information from one system, known as the base, to another system, known as the target (Gholson, Smither, Buhrman, Duncan, & Pierce, 1997; Holyoak & Thagard, 1995). By identifying the relationship between the two systems, the learning is transferred by a process of mapping or matching (English, 1997). An example of this in mathematics is the mapping between the relationships expressed in concrete materials and the pictorial diagrams of these relationships. The mapping should be unambiguous so that the student can see the relational structure between the target and the source (English & Halford, 1995). Once students can identify the underlying structure of the two systems, it allows them to make the shift into abstraction and to see the generality (Mason, 1989).

3.3 MATHEMATICS: EARLY ALGEBRAIC THINKING

In the early years of schooling, mathematics has predominately focused on basic concepts with little opportunity for high levels of thinking. At present, there are still many misconceptions about the perceived ability of young students' readiness for high levels of mathematics. Subsequently, teachers do not present complex mathematical tasks as they feel young students are not ready to learn this type of mathematics, and they focus on simple numbers and shapes within lessons (Sun Lee

& Ginsburg, 2009). It can be argued that teachers who have engaged in these types of lower contextual teaching actions have limited the accessible knowledge of students by not providing a challenging learning environment to extend their mathematical thinking. It has been suggested that the presentation of low level mathematics is due to teachers' poor interpretation of Piaget's theory (Sun Lee & Ginsburg, 2009). Piaget's theory implies that young children's thinking is immature and therefore they are not capable of engaging with abstract mathematical concepts in the early years context.

However, over the last decade, advice has emerged from literature suggesting that early years mathematics classroom must embrace students' engagement with a variety of areas of mathematics, including number, geometry, early algebra, measurement, and also should promote problem solving (Balfanz, Ginsburg, & Greenes, 2003; NCTM, 2000, 2006; Sarama & Clements, 2009). These areas of mathematics give students an advantage in mathematical thinking, and provide fundamental skills that will be developed in later schooling. The recommendation made by NCTM in 2000 in the *Principles and Standards for School Mathematics,* to include algebra in the early years curriculum, means that this research area is still growing and there is much to discover about students' abilities and learning styles within this mathematics context.

Algebra is seen as an interwoven aspect of mathematics that can be found in strands such as number, geometry and data analysis. It can be viewed as a system in which symbols are apportioned to indeterminate objects to allow analysis of their relationship to one another (Radford, 2006). Kaput (2008) defines algebra as having two aspects:

1. Algebra as systematically symbolising generalisations of regularities and constraints; and

2. Algebra as syntactically guided reasoning and actions on generalisations expressed in conventional symbol systems. (p. 11)

These core two aspects are embodied within three strands:

1. Algebra as the study of structures and systems abstracted from computations and relations, including those arising in arithmetic and in quantitative reasoning.

2. Algebra as the study of functions, relations and joint variation.

3. Algebra as the application of a cluster of modelling languages both inside and outside mathematics. (Kaput, 2008, p. 11).

Algebraic thinking in the early years of schooling is different from both arithmetic and traditional secondary algebra. Like arithmetic thinking, it focuses on number and operations. However, it has a different purpose to arithmetic. Early algebra focuses on arithmetic or number sense relationships (Fujii & Stephens, 2001; Steffe, 2001) such as those related to the field properties (Warren & Cooper, 2003; Warren 2004), while, in contrast, arithmetic focuses on computation. This distinction between early algebra and arithmetic has been characterised by Malara and Navarra (2003), as the difference between process (e.g., the properties of adding) and product (e.g., the result of adding). The first is generic to different forms of number while the latter is specific to given numbers.

In addition, early algebraic thinking leads to a deeper understanding of mathematical structures (Blanton & Kaput, 2011; Carraher et al., 2006; Cooper & Warren, 2011). As such, it is the basis of later algebraic understanding and the powerful and transportable forms of mathematics that underlie modern technology, problem solving and planning. In contrast, arithmetic thinking at primary school (elementary school) tends to focus on calculating and little focus is on the representation of relations of operations (Kieran, 2004; Kilpatrick, Swafford, & Findell, 2001). This in turn leads to many difficulties faced by students as they move from arithmetic reasoning to algebraic reasoning (Bednarz, Kieran, & Lee, 1996).

Due to the fact that early algebra has only recently been introduced into the curriculum most prior research has been conducted with adolescents. Studies have addressed the issues that students have when applying a variable to express a generalisation in algebraic terminology (MacGregor & Stacey, 1997; Rico, Castro & Romero, 1996; Sasman, Olivier, & Linchevski, 1999). Students are unsure what the letters represent in these equations and as an adjunct; students are often exposed to teaching practices that incorrectly influence their early knowledge and understanding of variable. These studies have often been conducted with students beginning in high school, and it is noted that exploration of the use of variable as a form of expressing a generalisation should not be the initial medium of exploring this concept in the early years. Similarly, studies conducted by Warren (2000) allude to the difficulties

experienced by adolescents, including a deficit in language when students reason their generalisations, and the ability to visualise spatially or complete patterns. This further highlights the inadequacy of literature pertaining to the understanding of what younger students know, and how they engage with, generalisation in an algebraic context. Consequently, it has been suggested that to overcome this impediment there is a need for opportunities for early years learners to experience algebraic thinking within a patterning context (Warren, 2005).

In the early years pattern activities are often initially experienced as sorting and classifying tasks. These experiences then progress to activities involving repeating patterns experiences usually involving physical movement, space, hands-on manipulatives, pictures and numbers. The use of patterns in the classroom context assists students to apply rules, reason and move to abstract notations in mathematics. Young students generally explore three types of patterns: repeating patterns, recursively defined patterns and linear growing patterns. Repeating patterns have an identifiable unit of repetition and can range in levels of complexity (Zazkis & Lijedahl 2002). An example of a repeating pattern is ABABABAB, with the 'AB' being the unit of repeat. Within this type of pattern structure it is essential to see the particular of the pattern, that is, seeing the unit of repetition. Importantly, research has evidenced that the exploration of repeating patterns with 4-year-old students positively impacts on their understanding of mathematical concepts two years later (Papic, 2007). Linear growing patterns are characterised by the relationship between elements, which increase or decrease by a constant difference. These patterns often have associated algebraic expressions that allow the student to express how this change occurs; an example of this would be 2x+b. Recursively defined patterns are patterns were successive terms are related, and this is where the basis of the pattern exists (Driscoll, 1999). An example of this is Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, when each new number is the result of adding the previous two numbers together. Consequently, patterns assist students' engagement in, and understanding of, generalisation.

The ability to pattern in the early years of schooling has been shown to positively impact on students' later mathematical achievement (Papic, 2007). Research has highlighted that young students are capable of recognising the mathematical structure of a range of patterning contexts including repeating patterns,

growing patterns and functions (Blanton & Kaput, 2004; Cooper & Warren, 2011). In addition, Papic (2007) found that one year after students had engaged in an intervention focusing on creating and interpreting a range of mathematical patterns, these young students where achieving at higher levels in mathematics when compared to students who had not engaged with these experiences. It is conjectured that the early patterning experiences helped these students to engage in 'seeing the structure of mathematics', and that students experiencing difficulty in learning mathematics do not always recognise pattern and structure (Mulligan, Mitchelmore, & Prescott, 2005).

The ability to discern patterns is the precursor to generalising mathematics (Threlfall, 1999), and generalising a pattern is the key to algebra and mathematics (Lee, 1996). Subsequently, generalisation is often researched in the algebraic context and focuses on the abstraction of pattern and structure supported with mathematical reasoning. There is a strong link between a child's ability to think and reason mathematically and their ability to perform high levels of mathematics later in life. In the early years of schooling students should be familiarising themselves with abstraction so that in latter years they can focus on complex formalised abstractions (Davydov, 1975). For instance, this thinking would advocate successful generalisation of prior schemas to new complex contexts. Studies have been based on Davydov's (1975) work, and have indicated that very young children can generalise the equivalence class principle from number and numberless contexts (Doughterty & Zilliox, 2003; Warren, 2005; Warren & Cooper, 2007). It has been concluded that numberless situations provide a substantial understanding of mathematical structures in older students, and this enhances the likelihood of achievement in mathematics at secondary and tertiary levels (Morris, 1999). Additionally, numberless situations provide opportunity for students to engage in powerful schemes of thinking (Carpenter, Franke, & Levi, 2003). However, before students can reach a generalisation they must see the structure. It has been highlighted that, students who attend to seeing the underlying structures and engage with structural thinking have deeper experiences in mathematics (Mason, Stephens & Watson, 2009). Thus, early algebraic thinking leads to a deepening understanding of mathematical structures (Blanton & Kaput, 2011; Carraher, et.al, 2006; Cooper & Warren, 2011).

There has been extensive research into how older students generalise growing patterns (e.g., Lannin, 2005; Radford, 2006). These studies required students to coordinate two variables, where one is explicitly represented (e.g., the visual representation of the growing pattern) and the other variable is more abstract (e.g., the position of each term). In addition, studies have often incorporated recording the variables in tables of values and encouraging students to search for relationship within this representation. The progress made in these studies has been mainly with regard to expressing generalisations. For example, Cooper and Warren (2008) found that most students express generalisations of pattern rules initially for small numbers (numercic), then large numbers (quasi-generalisation), and then with language, and finally with symbols (generic level). There have also been some findings with regard to visual (unnumbered) vs tables (numbered) situations and the exploration of growing patterns. Warren and Cooper (2008a) have found that un-numbered situations were more difficult for students than numbered, but resulted in richer answers (students showed greater ability to justify their pattern and could see more than one form of generalisation). Studies in patterning have indicated that this is a useful avenue when developing young students' ability to generalise.

Younger students have been shown to generalise numerically and visually with the assistance of questioning techniques that assist with the development of pattern structure (Beckner & Rivera, 2009). In a study where students looked at the relationship between puppy eyes and tails, the conclusion was that they were capable of generalising patterns to functional relationships (Blanton & Kaput, 2004). Lannin (2005) also found that patterning activities assisted adolescent students in understanding generalisations. Radford (2006) conducted a study which focused on algebraic thinking within the context of pattern generalisations. He found that teachers needed to identify the difference between naïve inductions or guessing strategies, with that of true algebraic generalisations, which should include reasoning. Within this study Radford also considered the role of gesture and identified that these signs, or semiotic nodes, are those 'pieces of the students' semiotic activity where action, gesture and word work together to achieve knowledge objectification' which assist young students to generalise (Radford, 2006, p 144).

3.3.1 Concluding Comments

Studies in early years mathematics, both nationally and internationally, have predominately focused on counting, number concepts and numeration with little exploration into generalisation. Whilst these other mathematical concepts are significant, it is important for students to develop a deep understanding of algebraic concepts and it is imperative that students need to be able to generalise mathematical concepts, therefore research into generalisation is necessary.

While there is minimal research that addresses how young students generalise mathematics, it is clear that exploring patterns would be beneficial to begin generalisation concepts. A study conducted in 2007 lightly touches on the subject of generalisation in regards to Indigenous students, states that 'algebra is based on generalising pattern and structure, skills with which Indigenous students may have an affinity because their culture contains components that are pattern-based and which may lead to strong abilities to see patterns and structure' (Matthews et al., 2007, p. 250). In the light of this knowledge, it appears that the exploration of generalisation using patterns with Indigenous students is essential. It is also evident that the use of concrete materials would be beneficial in aiding students to make connections in generalising a pattern or predicting further pattern structures (Papic, 2007). This type of exploration would substantially add to the research already present in the national and international early years context, and contribute to the limited research known about how Indigenous students generalise mathematics.

The growing focus on generalisation reflects the belief, on the part of researchers, that this is an imperative skill students need to adopt and learn to understand algebraic concepts. Above all, having this skill may give Indigenous students the opportunity to engage in high status employment after school. With research limited in this area, it is difficult to address the strategies and types of mathematics that need to be presented in the classroom to enhance these skills. Currently, as there is a deficit of knowledge in regard to Indigenous students' ability to generalise mathematical concepts, the premise is built for the first research question:

How do young Indigenous students engage in growing pattern generalisation?

3.4 SEMIOTICS

3.4.1 Semiotic Theory

Semiotics is the study of cultural sign processes, analogy, communication, and symbols. The idea of semiotics was developed during the 19th century, principally by Peirce (1839–1914) and Saussure (1857–1913). For the purpose of this study, Peirce's stance was used. Peirce (1958) defines sign as:

> Anything which is so determined by something else, called its Object, and so determines an effect upon a person, which effect I call its interpretant, that the latter is thereby mediately determined by the former. (Peirce, 1958, p. 478)

Peirce highlights three main components: a sign (signifier), an object (signified), and an interpretant (understanding or making sense of the sign/object relation). While this triadic relationship seems simple, it is actually very complex in nature. Peirce states that:

> A sign... [in the form of a representamen] is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the interpretant of the first sign. The sign stands for something, its object. It stands for that object, not in all respects, but in reference to a sort of idea, which I have sometimes called the ground of the representamen. (Peirce, 1958, p. 228)

An example of Peirce's model of the sign, in a mathematical context, would be the equals sign (=) consisting of two short parallel lines (the sign/representamen); the equation (the object) and the idea that the equation must balance (the interpretant). Recently, Saenz-Ludlow and Zellweger (2012) have adapted the Peircean theory by providing a more direct use of vocabulary to overcome the unclear terms used by Peirce particularly in regards to the use of the word 'sign'. Figure 3.3 displays the triadic concept of sign with the classifications of sign object, sign vehicle and sign interpretant used by Saenz-Ludlow and Zellweger (2012).
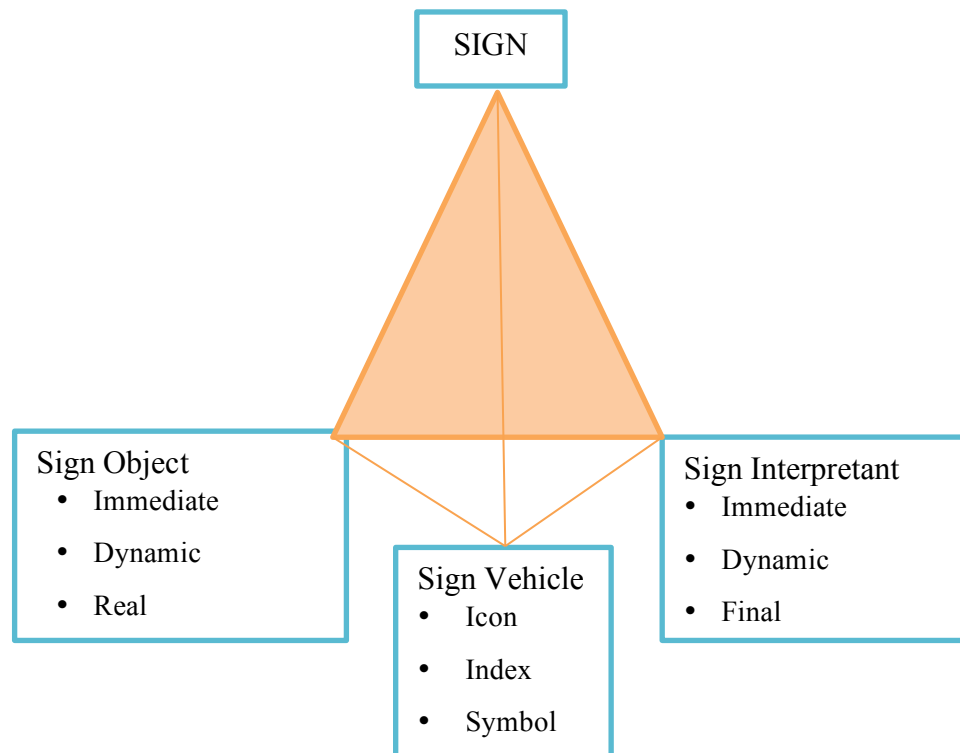
Figure 3.3. The tridactic concept of sign with terminology adapted by Saenz-Ludlow and Zellweger, 2012.

In the simplest terms a SIGN stands for something other than itself, that something is the Object. The Object can be represented by different sign vehicles that capture particular aspects of the Object. Multiple sign vehicles are required as one sign vehicle cannot encapsulate the entire Object. These sign vehicles are mediators between the Object and interpretant (sign-interpretant/student), and are then deduced to attempt to build understanding of the overall concept. To come to a complete understanding of the Object, students must be exposed to interrelated sign vehicles. These sign vehicles can be classified as iconic, indexical or symbolic as a student shifts from the immediate object, the immediate aspects of the real object, to the real object, the overall concept being taught (Saenz-Ludlow & Zellweger, 2012).

There are three kinds of sign vehicles, which are indispensable in all reasoning: icon, index and symbol (Peirce, 1958). The iconic sign exhibits a similarity or analogy to the subject of discourse (Object); the indexical sign, like the pronoun in language, forces the attention to the particular object without describing it; and the symbolic sign or description signifies its object by means of an association of ideas or habitual connection between name and the character signified. These sign vehicles

can be both static and dynamic. An example of dynamic sign is gesture (Radford, 2006; Saenz-Ludlow, 2007).

3.4.2 Semiotics and Mathematics

As the teaching of mathematics draws on a variety of representations and resources to assist students to engage with mathematical processes, semiotics provides the tools to understand these processes of thought, symbolisation, and communication. Mathematics is described as an intrinsic symbolic activity, where the outward manifestations of the processes are communicated using oral, bodily, written and other signs (Radford, 2006). Peirce (1958) defines a sign as anything that is so determined by an object that brings meaning to the interpreter who is making sense of the sign and object relationship. In the discipline of mathematics it is essential that signs can be both static and dynamic (Saenz-Ludlow, 2007; Radford, 2006). An example of a dynamic sign is gesture or kinaesthetic movement. Within the context of functions, the object can be considered as the relationship that exists between the two variables where the variables themselves are the signs that give the function meaning. Gestures, forms of dynamic signs, are defined as all of those movements [hands, arms, eyes] that subjects perform during their mathematical activities (McNeill, 1992; Sabena, 2008). From this perspective, our cognitive relation to reality is mediated by signs which can be objectified. The relationship between the language and gesture has been described as 'unsplittable' (McNeill, 1992). When language is not apparent, or is mismatched with home language, students will use gesture to assist conversation (Goldin-Meadow, 2002). Additionally, gesture may be the first place students display a new thought (Goldin-Meadow, 2002).

Semiotic signs assist students in developing mathematical understanding. At times, this mathematical understanding may have remained unseen until the use of semiotic signs (Sabena, Radford, & Bardini, 2005). Radford suggests that signs (such as bodily movement, oral language, concrete objects) play the role of making the mathematics apparent, a semiotic means of objectification (Radford, 2003). This semiotic means of objectification can engender knowledge formation with the use of particular mathematics activities led by semiotic systems, often referred to as semiotic nodes (Radford, Demers, Guzmán, & Cerulli, 2003). In relation to mathematical generalisation, gesture has played a crucial role in assisting older

students to focus on particular structural aspects of the pattern, which in turn assists in the expression of generalities (Radford, Bardini, & Sabena, 2005). When considering algebraic language, research has highlighted that it is more than the use of alphanumeric symbolism. Algebraic language is a combination of semiotic nodes/instruments (language, gesture, written) (Radford, Bardini & Sabena, 2007). Studies by Radford (2006) have found that these nodes become more refined as students move through the learning experiences. It is also essential to consider all semiotic systems as students generalise, as mathematical thinking will not be captured from written formula (Radford, Bardini & Sabena, 2007).

While the theory of semiotics has been long established, it is only recently that studies in the area of pattern generalisation have considered how semiotics impacts the learning process. Current research considers the aspects of the various semiotic resources used within the classroom when working on mathematical problems related to functions (Arzarello et al 2009; Radford 2009; Radford & Roth 2011; Warren & Cooper, 2009; Warren, Miller, & Cooper, 2012). Additionally, studies have displayed the benefits of young non-Indigenous students using hands-on materials when engaging in generalising patterns (Blanton & Kaput, 2005; Cooper & Warren, 2011). Studies have used semiotics as an analytical tool to understanding how students come to generalise constructs in mathematics (Radford, Bardini, & Sabena, 2005; Radford, 2006; Warren & Cooper, 2009). Furthermore, research has considered semiotics in relation to cognitive models for pattern generalisation tasks. Rivera (2010) has created this model, and draws on theories of Gestalt in relation to older students engaging in generalisation tasks.

Finally, semiotics is considered a means of creating a series of chaining processes to shift from culturally-embedded mathematics to Western mathematics (Presmeg, 1998). In a study based on Presmeg's (1998) model, she demonstrated a series of activities that assisted secondary students to shift from a game of dominoes embedding in students' culture (culturally-embedded mathematics) to a general formula (Western mathematics). Students used the rules and process of dominoes to assist them in conceptualising mathematical abstractions. Figure 3.4 displays the semiotic chaining process used in Presmeg's (1998) study.

Recursive and general formula

Systematization of number of pieces n for all double
x sets in a table of x and n values

Determination of number of pieces in various sets

Set of dominoes
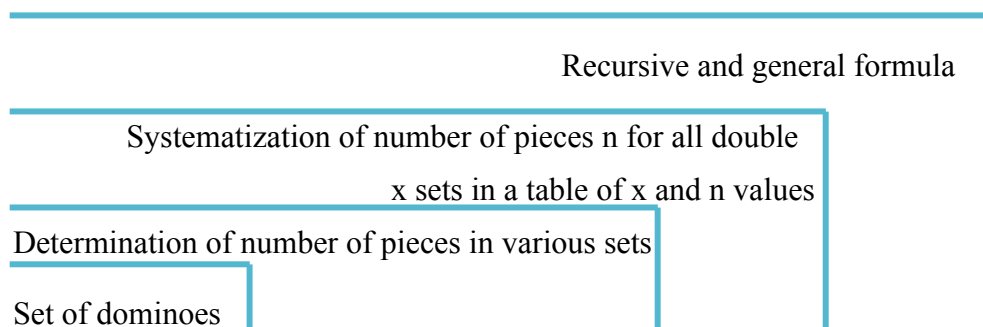
Figure 3.4. Semiotic Chaining (Presmeg, 1998).

3.4.3 Concluding Comment

While the studies outlined in this section have focused on the semiotic interactions that occur when students' are engaging in mathematical learning, no studies to date consider what teaching actions specifically assist young Australian Aboriginal and Torres Strait Island students to generalise. Additionally, Indigenous students potentially bring their own semiotic signs that assist them to engage with generalising mathematical growing patterns. Whilst there is literature describing how Indigenous students go about learning mathematics, little is known about the interaction between Indigenous students and teachers, and what particular teaching methods assist with Indigenous students' ability to generalise. Therefore it is necessary to consider the following research question:

What teacher actions assist in enhancing young Indigenous students to generalise growing patterns?

3.5   INDIGENOUS EDUCATION

Research results emphasise that there is a disparity between educators, education systems and Indigenous students (Agbo, 2001; Deyhle & Swisher, 1997; Howard, 2006; Klug & Whitfield, 2003; Pewewardy, 2005; Reyhner & Jacobs, 2002). In addition, it has also highlighted other complex factors that have contributed to the academic underachievement of Indigenous students, and has identified the potential educational areas that need to be addressed. The struggle to remove

disparity still continues while a dominant Western education model is presented in Australian schools.

Further contributing factors in this struggle are: the low expectations that teachers hold for Indigenous students (Sarra, 2003); their lack of prior teaching experience on entry to many Indigenous communities (Gibson, 1994); non-Indigenous people teaching Indigenous students (Barnhardt & Kawagley, 2005); and a lack of cultural understanding between Indigenous and non-Indigenous groups working within these school environments (Warren, Cooper, & Baturo, 2004). Evidently, there are still barriers at the cultural interface between the teacher and the student.

The curriculum presented in Australian schools is often viewed as problematic for Indigenous students. It has been suggested that lack of attention to cultural diversity is at the root of the problems within the past curricula in Australia (Frigo, 1999; Matthews, Howard, & Perry, 2003). Further, there are limited links to specific content related to Indigenous people and their culture within the mathematics curriculum (Cronin, Sarra & Yelland, 2002; NSW Board of Studies, 2000; Sarra, 2003). Research has argued that 'Western' mathematics curriculum for many Indigenous students is challenging and difficult to negotiate, particularly when it fails to be inclusive of Indigenous culture (Aikenhead, 2001; Howard, 1997; Howard & Perry, 2005). The exclusion of Indigenous culture from the teaching of mathematics presents a view that potentially devalues Indigenous culture (Matthews, et. al, 2005). It is in this paradigm that the intangible effects of students' feelings of being disenfranchised may arise. Arguably, it is understandable why Indigenous students have difficulty seeing the relevance of mathematics in their everyday lives. The relevance of studying mathematics, together with teacher perceptions of Indigenous students' mathematical ability, are fundamental issues within education and for Indigenous students' achievement (Matthews, et. al, 2005).

Teacher perceptions of Indigenous students' abilities to achieve at school are instrumental in shaping students' outcomes. If low expectations are held by teachers, which much of the current data on Indigenous student achievement suggests, a sense of futility may arise, with a limitation of understanding to the subject matter offered to Indigenous students. Sarra (2003) stated that teachers from his school would associate underachievement as an Aboriginal thing. This inherently devalues

Indigenous students. It suggests that it is an expectation of these teachers that students will underachieve, and that this underachievement is something that is culturally embedded for these students, a social norm of sorts. It is conjectured that, if teachers have such low expectations of Indigenous students, it can only be assumed that the mathematics taught within the classroom is, accordingly, basic.

Indigenous students' experience a range of factors that impact their schooling with regards to teachers. Some of these factors include: low expectations that teachers hold for Indigenous students (Sarra, 2003); lack of prior teaching experience in many Indigenous communities (Gibson, 1994); non-Indigenous people teaching Indigenous students (Barnhardt & Kawagley, 2005); and a lack of cultural understanding between Indigenous and non-Indigenous groups working within these school environments (Warren, Cooper, & Baturo, 2004). Whether this is because non-Indigenous people do not have the right to know these cultural practices or information is not shared with them, it still forms a barrier at the cultural interface between the teacher and the student.

While much emphasis is placed upon these socio-demographic factors contributing to Indigenous educational issues, it has been noted that minimal focus falls on the education system itself (Sarra, 2003). In reflective practice, educators must understand that it is not just the external influences Indigenous students' experience that contribute to low educational achievement, rather it is a combination of the education system, curriculum, as well as these external factors. Beginning to understand the problem also requires a more innate understanding of who Indigenous Australians are in broader social context.

3.5.1 Indigenous Ways of Learning: Experiences in Western Education

The need to provide students with the opportunity to embrace both Indigenous knowledges and pedagogies in conjunction with Western knowledge is paramount. 'Two ways' or 'both ways' education ensures a learning experience that connects both Western and Indigenous knowledge within the school experience. In the past the term 'two way schooling' considered Indigenous and Western knowledges as two separate paradigms within education (Harris, 1990). 'Both way' education draws on an interconnectedness of both knowledge systems to provide Indigenous students with an empowered education. This 'new' education system provides opportunity for

strengthening and retaining Indigenous identity while negotiating within both cultures.

Being 'two/both way smart' is more than just adopting the language (mother tongue) of Indigenous students. It is about providing a learning environment that combines the Western education model and Indigenous knowledges. The result of such a model is that it leads to student empowerment (Nakata, 2007, 2008). This is similar to 'both ways' education (Harris, 1990) where students engage in Indigenous culture within a mainstream Western education. Providing Indigenous students with the opportunity to be 'two-way' smart assists then to be successful in the acquisition of skills and knowledge that are highly regarded by both cultures.

Recently, two-way education has come to the forefront in Australia. Noel Pearson and Chris Sarra, two prominent Indigenous figures, have established models so as to best enhance students' access to two-way education. These models are based on the two key features, Indigenous culture and Western education. The following section will elaborate on each model.

Cape York Institute

Australian Indigenous leader Noel Pearson is providing such an education model through the Cape York Institute. The ultimate goal of the institute is to:

> Close the gap on Indigenous Australian education outcomes, so that children leave school literate, numerate and equipped with the skills and confidence to make informed and empowered choices about their lives. Children should have the necessary tools to go on to further education, gain productive and stimulating employment, participate in the real economy and contribute to Australia (Cape York Institute (CYI), n.d., para 1).

This model provides an educational experience so students can 'orbit' between both the Western and Indigenous perspectives. That is, Pearson advocates that Indigenous students experience an education model where students maintain their cultural identity while learning to become functional participants in Australia's economic and civic life. Pearson (2009) further elaborates on this notion, and suggests that Indigenous people move towards 'bi-cultural capacity' (p. 56). That is, moving between the two cultures by preservation of Indigenous culture with a parallel focus on engagement in the wider community.

Within Pearson's model, school students participate in an education program comprising of: Class, Club, Culture, and Community. Class is about teaching mainstream education. Lessons for literacy and numeracy are implemented using Direct Instruction (DI), a model developed in America to provides teachers with scripted material to deliver to students (e.g., Slavin, et al., 1992). It focuses on the need for students to succeed in one concept before moving on to the next. Club scaffolds students and socially prepares them to 'orbit' between home and school life. The program encourages students' participation in extracurricular activities such as sport, music and art. Culture supports students' learning by enhancing their own cultural knowledge. Community plays an active role in education and considers other factors that influence students' lives such as health, school readiness and attendance.

The CYI further states that 'English literacy is of utmost importance, and cultural education and Indigenous language education programs should, where appropriate, be provided' (CYI, n.d., para 1). While Pearson's education model is providing *two-way* education for Indigenous students, Western literacy and numeracy is discrete from the Indigenous cultural program. This model represents early notions of keeping the Indigenous and non-Indigenous domains separated (Harris, 1990).

Finally, the CYI model places the assumption that the mainstream Western education model is of benefit for the participation of Indigenous students beyond the school experience, though one would question if integrated links between Indigenous knowledges and Western literacy and mathematics are being drawn upon. This interconnectedness of the West and Indigenous knowledges provides students with the links to be cognisant of the relatedness between their community context and education, thus limiting the rejection of education often seen to be unrelated to their everyday experience (Guerin, 2008).

Stronger Smarter

Australian Indigenous educator Chris Sarra advocates that Indigenous students must develop and maintain a strong cultural identify while engaging in a Western civic life (Sarra, 2006). Sarra was the executive chairperson of The Stronger Smarter Institute. The vision of the Institute is for 'Stronger Smarter communities, enabling all people to honour and affirm positive identities and cultures, whilst thriving in contemporary societies' (Sarra, 2006). The term 'stronger smarter' arose out of

Sarra's time as a school principal at Cherbourg primary school. The *strong* symbolised Aboriginal and Torres Strait Island people having a positive and strong sense of cultural identity (Sarra, 2005). The *smart* equated to academic outcomes for students. The two-way approach for Sarra is built on the strong foundations of a sense of self and academic achievement so as to create positive realities for Indigenous students. Within the education context a whole school approach is needed where teachers need to embrace students' cultural identity while building students' confidence in order for success.

Indigenous learning styles

It has been argued that there is not one learning style that encompasses the diversities of Indigenous students (e.g., Warren, Cooper & Baturo, 2004). This reflects the fact that: (a) Indigenous students are widespread within the Australian geographic landscape (ABS, 2004); (b) some Indigenous students have had very little engagement in Western society (remote communities), and some have had little engagement with their own culture (urban communities); and (c) there is limited homogeneity among Indigenous groups within these communities, especially within the Queensland context. At the same time, it has been suggested that there is an over emphasis in research on defining learning styles within Indigenous cultures which may lead to inaccurate labelling and stereotype (Alfred, 1995; Williams, 2000). These stereotypes have the potential to be wrongly used by non-Indigenous people as they work in these communities. More importantly they have ramifications for Indigenous students in wrongly stereotyping themselves.

Work has been done on identifying common attributes of Indigenous learning styles. Purportedly, Indigenous students prefer to global, creative and reflective styles of learning (Pewewardy, 2002). Indigenous students prefer holistic learning styles, where they actively engage in the overview of the subject and build conscious relationships of ideas (Christie, 1994; Grant, 1998). It has been also claimed that Indigenous students have different assumptions to those of Western students. Research suggests that Indigenous students prefer opportunities for reflection, critical thinking, observation and autonomy (Four Arrows, 2003). Findings from research conducted in remote communities indicate that these characteristics have stemmed from traditional teaching practices within the culture.

A recent study has confirmed that there are several distinctive attributes of traditional Indigenous ways of learning (Williams & Tanaka, 2007). These include: mentorship and apprenticeship learning, learning by doing; learning by deeply observing; learning through listening; telling stories and singing songs; learning in a community, and learning by sharing and providing service to the community (Williams & Tanaka, 2007). This multifaceted style of learning is often conducted through initial observation of an elder, so the responsibility for the engagement is placed upon the student. This student centred approach facilitates respect and independence within the broader interdependence in the group (Goulet, 2001). Additional studies support the claim that Indigenous students engage in learning that is based on repetitive observation by an expert, with the student practising the skills in private until confidence is gained (Bergeson, Griffin, & Hurtado, 2000). The transferability of these findings to other Indigenous communities in other geographical locations needs further investigation. Past research reporting on the improvement of mathematics achievement for Indigenous students has occurred generally in remote contexts (Frigo, 1999; Harris, 1984; Harris, 1991). The application of these findings to an urban Indigenous student cohort from a different context (the focus of this study) could prove advantageous.

Development of Aboriginal pedagogy, acknowledging the diversity of Indigenous culture, provides a broad framework to consider as one engages Indigenous students in learning activities. Literature suggests that frameworks for Indigenous learning should consider adopting a holistic approach, and recognise that Indigenous students are: (a) imaginative learners, (b) contextual learners, (c) kinaesthetic learners, (d) cooperative learners, and (e) person-orientated learners (Hughes, More, & Williams, 2004; Nichol, 2008; Nichol & Robinson, 2000). Nicol (2008) highlights these elements of Indigenous pedagogies and perspectives:

- Holistic learning – tasks that are cooperative and integrated, non compartmentalised, with the need for creative learning approaches.
- Imaginative, creative and flexible learners – allow students to see the whole rather than piecemeal approach. Allow for own thought development and experiences.
- Auditory learners – this stems from oral traditions; however, there is a need for concrete examples before abstract understandings can develop.

- Imaginative and referential learners - tasks should provide visual images, symbols, diagrams, maps and pathways.

- Hands on participation – the need for tactile manipulations, kinaesthetic participation in tasks with observation before participation.

- Communal, cooperative, shared learning – encourage peer learning, cooperation rather than competition.

- Situational, contextual and experiential learners – place context in pedagogy; and

- Participatory, relational and person-orientated learners – positive learning environments; make personal connections as students will be more willing to take risks.

Similarities to Nicol's suggested Indigenous pedagogy are found in Yunkaporta's (2009) eight ways of learning for Indigenous students. The research is based on past research, and reports on 50 teachers' experiences while implementing the eight ways of learning framework (Bindarriy & Mingalpa, 1991; Cameron, 2003; Christie 1986; Craven, 1999; Harris 1984; Hughes, 1992; Hughes & More, 1997; Robinson & Nicol, 1998; Stairs, 1994; Wheaton, 2000). The findings suggest that, teachers observed there was a sense of pride and confidence in students' intellectual capacity when they engaged in this framework. Furthermore, the framework provided a platform for students to understand mainstream content, and assisted teachers to have an environment conducive to Indigenous learning. The framework consists of the following approaches:

- Deconstruct/ reconstruct - provide holistic, global, scaffolded and independent learning orientations.

- Learning maps – make learning explicit in a visual way. Images serve as an anchor or reference point for the learner.

- Community links – localised and connected to real life purposes and contexts.

- Symbols and Images – visual spatial learners, symbolic learners as a strategy rather than an orientation. In traditional Aboriginal ways teachers would use all the senses to build a symbolic meaning in support of the learning: that is, concrete and abstract imagery symbols at the micro level of content, rather than the macro of process (learning maps).

- Non-Verbal – kinaesthetic, role of body language and the use of silence. Test knowledge through experience.

- Story Sharing – make use of personal narrative in knowledge transmission and transformation.

- Non-Linear – multiple cycles that occur continuously with an indirect rather than direct orientation to learning. Western pedagogy is presented in a linear fashion, marginalising Aboriginal people and preventing from constructing own identities. It is not about presenting learning in cyclic and indirect ways. It is also about avoiding dichotomies by finding common ground and creative potential between diverse viewpoints; and

- Land links – relating to land and place and engaging in personal relationships to the land. Use place-based education with links between Western places and narrative pedagogies.

While these approaches are gaining local momentum in some communities, there has also been a large move in the use of Direct Instruction (DI) models for Indigenous learners to improve reading and mathematics (e.g., Abbott, 2009; Pearson, 2009). There has been fervent debate as to whether DI is an appropriate method of classroom practice. This didactic style of teaching is highly structured and knowledge is transmitted to the class with minimal or no discussion. It also poses the issue of the teacher as the authoritative possessor of knowledge, and students the passive recipients of selected aspects of that knowledge (Ewing, 2011). Common to this approach is the use of drill or rote learning for memorisation, with testing being central. Knowledge acquisition, usually through pen and paper tasks, can disengage the learner from the learning process (Ewing 2011). This approach can also be seen as a top-down approach in addressing Indigenous education, a focus found to be ineffective (Ewing, 2011; Sarra, 2011). This diverges from innovative delivery to increase participation as suggested by Dobson, Sarra, and Calma at the Indigenous Strategy 2008 for the Education Action Plan 2009. It has been evidenced to the point of almost being irrefutable, that empowering students and providing ownership of the learning, increases students' participation and learning outcomes (Sarra, 2005).

While these are overarching approaches for engaging Indigenous students in learning, it is important to note that complexities arise when making comparisons about learning styles between Indigenous student groups. However, some

generalisations from prior research findings can be made. One facet of this dilemma is the minimal experience some Indigenous students have with the language of Western mathematics. For some Indigenous students, involvement with mathematical learning requires them to engage with a second language in which mathematical terms, relationships and meanings can be explored (Graham, 1988). There is alternatively the need to adapt the mother tongue of these students so that meaning can be obtained (Graham, 1988). This could be seen as a colonised approach to dealing with this issue.

3.5.2 Indigenous Learners and Mathematics

It is acknowledged that young Australian Indigenous students enter school with their own cultural understanding of mathematics. However, there is a view that these students need to engage with Western mathematics. In recent times, the call to effectively engage in both Indigenous and non-Indigenous knowledge has become increasingly stronger (Nakata, 2007; Pearson, 2009; Sarra, 2011; Yunkaporta & McGinty, 2009). The view that the knowledge of Western mathematics is culture free has long been challenged (Bishop, 1994). Other mathematics exists in society. For example, "time" in many Aboriginal cultures is represented as circular, with the structure being strongly related to the gathering of food. Thus, these calendars can vary from location to location (e.g., seasons in Arnhem Land).

It is important for Indigenous students to participate in Western mathematics. It has been described as an empowering process acting as a tool in identifying differences between socioeconomic classes (Gustein, 2003). Other ideology suggests that being innumerate can be profoundly disabling in every sphere of life including home, work and professional pursuits (Orrill, 2001). It is also important to recognise that Australian Indigenous students enter school with intuitive knowledge about mathematics, and this knowledge may be different from the knowledge of non-Indigenous students.

The following teaching strategies for engaging Indigenous students in mathematics have been identified in research (Frigo & Simpson, 2001; Warren, deVries & Cole, 2009):

- Learning pathways the provision of a gradual progression along a learning path, with the teacher first modelling what is required, followed by small group work, and finally individual participation.

- Integrated experiences: listening, reading, writing, recording, and speaking about concepts to enhance transference of skills.

- Focused teaching: direct or explicit teaching in conjunction with modelling, giving clear explanations of experiences and setting high expectations.

- Building language: encouraging students to move between home language, mathematical language, and SAE as they communicate their mathematical learning; and

- Making connections: integrating mathematics with the home and the community.

Within the Australian context, Garma mathematics presents an example of Western mathematics working in harmony with Indigenous culture. This model has been referred to as 'both ways' education where Western mathematics is taught in conjunction with Yolngu mathematics (Jones, Kershaw, & Sparrow, 1996). This style of mathematical learning also fits with beliefs that parents of Indigenous students want their children to be bicultural by living and learning in both realities (Partington, 1998). It also reflects studies in Canada where students agreed that there was an increased need for First Nations culture to be incorporated in the curriculum (Alfred, 1995). This type of educational model provides empowerment for Indigenous students and presents a relevance to mathematical concepts (Matthews et al., 2005), though for this model to occur, the necessity lies in building relationships between the teacher and the student.

It is understood that the basis for a culturally responsive pedagogy, such as Garma, begins with the relationship of the teacher and the student (Moje & Hinchman, 2004). Consequently, it is essential for non-Indigenous teachers to view themselves as the learner within this context. Attending to forming culturally relevant connections within the classroom, the non-Indigenous teacher must learn about the culture and prior experiences that students bring to the classroom, before they can respect and celebrate it mathematically (Cummins, Chow, & Schecter, 2006; Sharp & Stevens, 2007). This stems from studies suggesting the need for a

blended education that respects and identifies the similarities in knowledge of both cultures (Basttiste, 2002). One can postulate that once connections have been made within the cultural context and the mathematics, there will be a point where the two become intertwined. Differing from a 'both ways' approach utilised by Garma, an intertwined approach to learning would begin within the Indigenous context and then Western mathematical knowledge would be taught within this context, thus providing a 'thoroughness' of mathematics (Ma, 1999).

By providing opportunities within the classroom setting for students to freely engage in pattern generalisations, it is possible that students may engage in ways of thinking that are not similar to that of non-Indigenous students. They may draw on a range of their own cultural signs, gestures and symbols to assist in the generalisation process. Most algebra-based textbooks provide students with an array of word problems within Western contexts, thus limiting the accessible personal knowledge of cultural contexts to draw upon (Sharp & Stevens, 2007). An example of how this has been overcome emanates from the Yolngu students involved with Garma mathematics. They started with the basis of the Yolngu kinship system and made connections between the cyclical recursive patterns evident in this complex system. Such patterns are also found within number contexts in other areas of mathematics (Divola & Wells, 1991; Jones et al, 1996). Hence, this may be a positive foundation for this research.

Mathematics taught in Australian schools largely reflects the Western perspective (Perso, 2003). Research has argued that engaging Indigenous students in decontextualized Western knowledges invites failure for both learners and teachers (Jones et al., 1996). Thus, teachers need to bridge the divide between the two cultures "in the learning of the mathematics taught in schools and numeracy acquisition, but [also] in scaffolding students' home language to Standard Australian English" (Commonwealth of Australia, 2008, p.52).

A number of studies have explored the reasons as to why policies and initiatives that focus on supporting and improving mathematical achievement for Indigenous students have failed. These findings include: (a) students not responding to written survey items (Dawe & Mulligan, 1997); (b) mathematics curriculum impacting student learning (Howard, 1997); (c) students having difficultly negotiating between cultures and seeing the any value of what is being taught

(Aikenhead, 2001; Cooper, Baturo & Warren, 2005); and (d) teaching fails to be inclusive (Howard & Perry, 2005).

Mathematical Language

The ability to communicate mathematically is seen as central to learning mathematics (Setati, 2008). In the last twenty years, the emphasis on this ability has become even more pronounced, with the discourse of argumentation, and situating mathematics in 'real world' contexts emphasised at all levels of mathematics (Moschkovich, 2002). The language of mathematics has particular nuances that differ from everyday English. For example, in mathematics the use of *table* has a specific meaning referring to a table of values; and the use of the word *between* is subtly nuanced. Students are required to learn the mathematics vocabulary, construct meaning and participate in discourse (Moschkovich, 2002). While for all students the language of mathematics can be difficult to understand, for some Indigenous students, it is far removed from their everyday speech.

Mathematical language plays a central role in students' development of numeracy, however, it can be a 'foreign language' for many students (Commonwealth of Australia, 2008). For many Indigenous students mathematics is delivered in their second or third language. The difficulties students may have with mathematical language occurs when their home language is at odds with the language of delivery (Goldin-Meadow, 2002). Research has highlighted that if the discourses of the home are at odds with school, this impacts Indigenous students' long-term achievement in numeracy (DEST, 2009; Dickinson, McCabe & Essex, 2006). Teachers need to provide the link between home language and mathematical contexts presented in their classrooms. The National Report on Numeracy (Commonwealth of Australia, 2008) provides an example where home language is at odds with classroom mathematical language:

> In some traditional/non-urban or rural Indigenous contexts for example, numbers may be familiar to students in a nominal sense through everyday contexts such as numbers on vehicle number plates or football jumpers, but not in a cardinal sense, such as for comparing quantities (e.g. I have 20 pens and he has 25). (Commonwealth of Australia, 2008, p. 58)

Additionally, the report highlights these aspects with respect to language and engaging in mathematical discussion:

- The need for students to be taught explicit mathematics language (e.g., positional language used in the home replicated with gestural language).

- The need for teachers to provide opportunities for students to 'code switch' between home and school languages. Though research has indicated that this too can be problematic, as at times students' home language (Aboriginal English) does easily match some Western mathematical concepts (Niesche 2009).

- The need for teachers to encourage and specifically teach students to discuss their mathematical thinking.

- Teachers should not direct questions that probe incorrect thinking or publicly draw attention to errors in recognition, that there is potential for Indigenous students to be shamed by their peers. (Commonwealth of Australia, 2008).

Recent research has found that a focus on an oral language approach to teaching Western mathematics proved to be an important dimension of young Indigenous students' success in mathematics (Warren & Miller, 2013). The study pre and post-tested 230 young Indigenous students (average age 5.76 years) on purposely-developed language and mathematics tests. Results of a multiple regression analysis determined that Mathematical language proved to be the greatest predictor of Indigenous students' success in mathematics when identifying pattern and structure (Warren & Miller 2013), the language developed in conjunction with modeling mathematical concepts using a variety of representations.

3.5.3 Concluding Comment

When considering the themes of generalisation and Indigenous ways of learning, there are evident gaps. If the researcher is exploring how young Indigenous students engage in generalisations, these disparities should be considered. First, while it is widely acknowledged that knowledge is influenced by culture, little is understood of how culture impacts on students' ability to generalise, particularly within the cultures of Indigenous Australians. Second, little is known about what cultural aspects students could bring to the mathematics context. Third, there is

minimal understanding as to what mathematical language will need to be explicitly taught to students in order for them to generalise growing patterns. These three gaps have provided an additional question for consideration:

How does culture influence the way in which young Indigenous students engage in growing pattern generalisation?

3.6 CONCEPTUAL FRAMEWORK IN THE CONTEXT OF THIS STUDY

The framework represents a conceptual view of the literature in relation to the context and purpose of the study. That is, it considers mathematics, semiotics and culture in relation to how young Indigenous students generalise mathematics. Within the broader context of the study, the framework encapsulates the contextual factors that influence how young Indigenous students participate in mathematics, including personal empowerment and post-school opportunities. As a result, this framework will guide the data analysis in relation to mathematics, semiotics, and Indigenous culture. Figure 3.5 illustrates the conceptual framework of the literature.
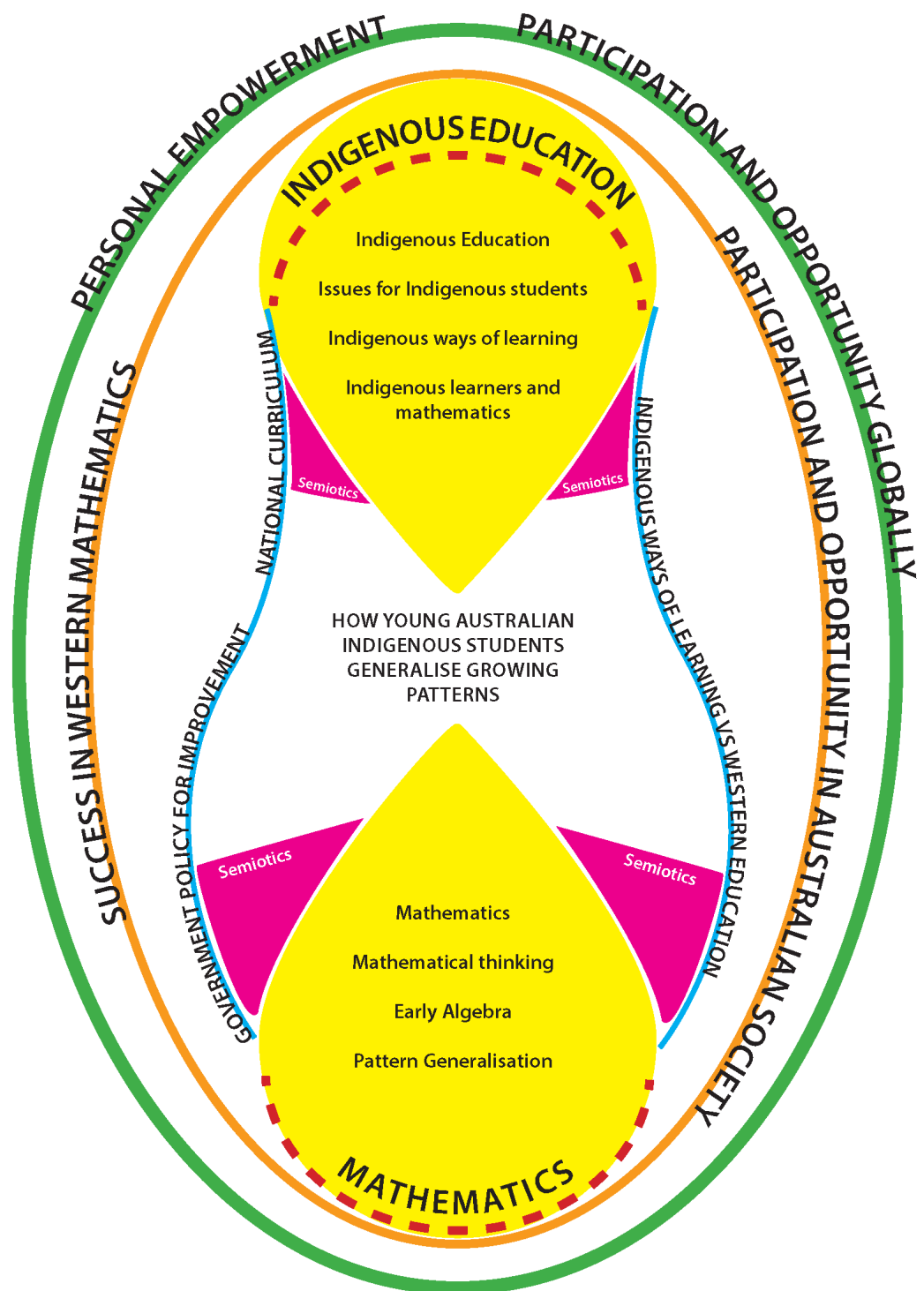
Figure 3.5. Conceptual framework of the literature.

3.7 CHAPTER REVIEW

This chapter has summarised and highlighted the important elements in the literature across three main areas: mathematics, semiotics, and Indigenous education. The first section of the literature addressed mathematical thinking, including generalisation and early algebra. This is an important area of the literature, as this study considers how young Indigenous students generalise growing patterns. The review highlights the need for further understanding of how young students generalise mathematical structures, particularly as there is a gap in the understanding of how young Indigenous students engage in this early algebraic concept. The second section considered semiotics and the important role of sign in mathematics teaching and learning. Finally, the third section considers Indigenous education, an important area given the participants of the study. Three research questions have been derived from the literature in order to explore how young Indigenous students generalise growing patterns. The next chapter will outline the research design required to address the research questions.

Chapter 4: Design of the Research

4.1 CHAPTER OVERVIEW

The research design for this study has been determined by the nature of the research problem. That is, because the study is situated in a classroom setting and seeks to explore how young Indigenous students engage with mathematical generalisations, an interpretive research paradigm is utilised. As students will be constructing their knowledge while engaging with hands on materials, constructionism is selected as the epistemological stance for the study. Building on the review of literature presented in Chapter 3, this chapter rationalises and describes the design of the research adopted. In addition, these research questions frame the theoretical perspectives and inform the data collection strategies in this investigation.

The research questions that inform the research design are:

1. How do young Indigenous students engage in growing pattern generalisation?

2. What teacher actions assist in enhancing young Indigenous students to generalise growing patterns?

3. How does culture influence the way in which young Indigenous students engage in growing pattern generalisation?

In this chapter, overviews of the participants, a description and justification of the teaching experiments and Piagetian clinical interviews, and methods of data analysis are provided. The chapter also includes a description of the results reported in Chapters 5 and 6. There is consideration for the trustworthiness and limitations of the study. Figure 4.1 displays the overview for Chapter 4.

Figure 4.1. Overview of Chapter 4.

4.2 THEORETICAL FRAMEWORK

This study seeks to explore and understand how young Indigenous students construct their own knowledge as they engage in pattern generalisation in a naturalistic classroom setting. Thus, an interpretive research paradigm was adopted. By adopting an interpretive approach, this study provides a deep insight into lived experiences from the Indigenous students' point of view (Schwandt, 1994). This study is embedded in an interpretive research paradigm, exploring what is the reality; what is the researcher/inquirer relationship with the knowledge being explored; and, how the researcher gains knowledge of the phenomenon (Denzin & Lincoln, 2005). This paradigm is based on the principle that humans build constructs to understand their world (Candy, 1989), a process unique to each individual.

Interpretive research accepts that reality is socially constructed. It is based on the perspective that humans use constructs such as culture, social context and language to influence the construction of knowledge (Gibbons & Sanderson, 2002). Through observation of social interactions of individuals within their natural context, the researcher arrives at perceptions and elucidation of how individuals generate and preserve their social environment (Neuman, 2006). This research paradigm is crucial to the study, as the researcher observed the constructs that students use to negotiate an understanding of pattern generalisations in mathematics.

In order to distil this broader philosophy, and make meaningful reference to the ways in which young Indigenous students form their own knowledge, the epistemology of constructionism was adopted. Due to the social constructs involved in this process, the research best lends itself to the epistemological approach of constructionism (Candy, 1989). Extending from the principle in which the researcher acts as inquirer, the yielded knowledge and observations regarding the construction of this knowledge is examined through the perspectives of semiotics and Indigenous methodology.

The design drew on the theoretical perspectives of Indigenous research perspectives and semiotics, as it explores what assists young Indigenous students to engage in mathematical generalisations. The role of semiotics is two-fold within the study; first it provides a lens through which to interpret the social interactions in the learning process (e.g., signs, gestures, language). Second, it informs the selection of materials and the structure of the growing patterns. An Indigenous research perspective facilitates the building of relationships with students. For the researcher and students to share knowledge, it is essential to build a trusting environment, particularly in the light of cultural variances. Referencing Indigenous research perspectives frames the research with an emphasis on empowering the participants, and thus, facilitating the free transfer of knowledge and reducing the likelihood of students being inhibited by the presence of the researcher. In conjunction with Indigenous research perspectives, enacting the secondary theoretical perspective is semiotic theory

The purpose of the research design is to provide the framework to explore and interpret young Indigenous students' actions while participating in mathematical generalisation tasks. Additionally, this framework relates the philosophical stance of

the research and makes links to practical components of the exploratory study. Table 4.1 displays the theoretical framework for the study.

Table 4.1

Theoretical Framework of the Study

EPISTEMOLOGY	Constructionism
THEORETICAL PERSPECTIVE	Semiotics Indigenous Research perspectives
METHODOLOGY	Teaching experiments
DATA-GATHERING STRATEGIES	Observations of students working with teacher Pretesting of students Video Recorded Teaching episodes Video Recorded Piagetian Clinical Interviews with students Audio Recorded Interviews with Indigenous Education Officers

4.3 EPISTEMOLOGY

In this study, young Indigenous students' construct and reconstruct their knowledge while engaging in experiences of pattern generalisation within a social setting. Thus, constructionism is the epistemology that is employed for the study. Constructionism as a theoretical approach suggests 'meaning' or 'knowledge' is a product of social interaction and experiences (Stahl, 2003), neither as a result of the individual nor from a single instance. Meaning is therefore socially constructed and advocates the influence of culture in determining the interpretation of the phenomena for the individual (Crotty, 1998). This lens provides a view that "all knowledge, and hence all meaningful reality, is contingent upon human practices being constructed in and out of the interactions between human beings and their world, and developed and transmitted within an essentially social context" (Crotty, 1998, p.42). Constructionism acknowledges that individuals build meaning through language, symbolism, culture, and interaction.

Constructionism can be summarised:

   a) There are no absolute truths;
   b) Knowledge does not come through the senses alone;
   c) Research focuses on the construction of meanings;

d) Meanings are not fixed but change according to people's interaction with the world;

e) Meanings do not exist before a mind engages them; and

f) The world is constructed by the people who live in it

<div align="right">(Sarantakos, 1998, p.37).</div>

Constructionism is not waiting for perceived reality of 'truth' to be found but rather, as the name suggests, constructing and reconstructing knowledge through relationships and negotiations between community members (Crotty, 1998; Lincoln & Guba, 2000; Schwandt, 2000). This flexibility when exploring the 'ways of knowing' (Guba & Lincoln, 1994, p.100) allows the emergence of relative 'truths' Consequently, 'truth' is determined by the best-informed view of a phenomenon upon which consensus is met (Guba & Lincoln, 1994; White, 2004). In the pursuit of a deeper understanding of how meanings are constructed by the participants, acknowledgement is given that each individual student 'makes sense' of their world, an explication seen as valid and worthy (Crotty, 1998; Schwandt, 2000).

Constructionism lends itself to the exploration of mathematical generalisation as students construct their knowledge from a known context and extrapolate this core content to the general. During this process students use a set of signs, gestures and symbols that influence and assist with creating understanding. Within the research frame of constructionism, semiotics is one of the theoretical perspectives that was employed. The second theoretical perspective that drives this study is that of Indigenous methodologies, as this study seeks to analyse Indigenous students' understanding of pattern generalisation.

4.4   THEORETICAL PERSPECTIVE

4.4.1 Semiotics

Learning mathematics involves a two-fold process; it involves the interpretation of signs, and the construction of mathematical meanings through communication with others (Saenz-Ludlow, 2006). These knowledges do not present themselves immediately; rather they evolve from interrelated experiences. The experiences have been termed *semiotic systems* (Saenz-Ludlow, 2006). *Semiotic nodes* are defined as those 'pieces of students' semiotic activity where action, gesture and words work together to achieve knowledge objectification' (Radford, 2006, p.

144). Other researchers used terms, namely *semiotic bundling,* to describe sign as any intentional action such as speaking, writing, drawing, gesticulating, and handling artefacts (Arzarello, 2006).

Mathematics is an intrinsic symbolic activity that is accomplished through communicating orally, bodily, by writing or utilising other signs (Radford, 2006) and thus, lends itself to semiotics. Semiotics informs exploration of the teaching and learning activities in mathematics, as this discipline is considered abstract and heavily based on perceivable sign. Semiotics assists in the understanding of mathematical processes of thought, symbolisation and communication as the teaching and learning of mathematics draws on a variety of resources. In a mathematics research context, the prime impediment to understanding the student's competence is that the researcher is external to the mind of the subject. The derivation of this concept is that only the signs, symbols and gestures, the outward manifestations of thought, are measurable (Seeger, 2008). There are two important tenets to semiotic theory: firstly, that of body and the way in which it interacts with artefacts, and secondly, the use of sign in representation (Sabena, 2008). This theory is strongly related to cognition being perceivable through bodily actions interfacing with social and cultural experiences (Lakoff & Nunez, 2000).

Within this study semiotics is used as a lens through which to interpret the interactions that occur in the learning process. That is, the data collected will be analysed to determine the semiotic signs that assisted young students to generalise growing patterns. In researching these cognitive interactions, it is important to acknowledge the potential for unique cultural variations as to how the outward displays of thought processes may be expressed. Additionally, to appropriately account for these cultural sensitivities, this research acknowledges Indigenous research perspectives as a theoretical perspective.

4.4.2 Indigenous Research Perspectives

In recent years when studying Indigenous people, there has been a shift in research approaches from the once colonised stance, to a decolonized approach that is embedded in Indigenous methodologies (Wilson, 2004). Indigenous methodologies are predominantly about two notions: that of relationships, and that of empowerment. Though the researcher is non-Indigenous, and understands that there

are implications for applying an Indigenous research perspective, she acknowledges that this study aligned with, and was respectful of, Indigenous ways of knowing (Martin, 2003). In essence, every attempt was made to ensure that the findings of this study best reflect how Indigenous students construct knowledge and engage in the learning process.

For the past decade debate has surrounded how non-Indigenous researchers conduct research about Indigenous people (Martin, 2003; Porsanger, 2004; Smith, 2000). Researchers have argued that research is more accurate if there are shared experiences between the researcher and the researched (Woodson, 1933/2000 cited in Dunbar, 2008), thus enabling greater understanding of the subject. On the other hand, literature has justified that non-Indigenous people can also conduct meaningful studies with this population, providing the study presents a decolonised and, therefore culturally-sensitive approach to research (Battiste, 2008; Grande, 2004; Smith, 2006). Within the Australian context, the quandary facing non-Indigenous researchers studying Indigenous people is a derivative of past colonisation practices. During that period there was a push for assimilation of the Indigenous people in an attempt to convert them to Western traditions and culture (Sikes, 2006). Much of this is suggested to have stemmed from the doctrine of Christianity, or as some scholars have stated, 'Christian guilt' (Sikes, 2006). Consequently, the Indigenous people were seen as the 'other' and became 'objects under investigation' (Porsanger, 2004). As research continued with Indigenous people, Western beliefs were heavily applied to Indigenous culture, and bias was evident in the style of research being conducted (Sikes, 2006).

During the decade 1994-2004, Indigenous people addressed the problems associated with Western epistemologies and methodologies, and Indigenous scholars requested that academia decolonise its research practices (Battiste, 2008; Grande, 2004; Smith, 2006). Decolonisation as it applies to research is concerned with the valuing, reclaiming, and foregrounding of Indigenous voices (Denzin & Lincoln, 2008). The emphasis of such studies is in opening narrative spaces and dialogue between the researcher and the subject, with the aspiration of building a cross-cultural partnership between the two. Additionally, the studies undertaken should acknowledge methodological approaches that privilege Indigenous knowledge, voices and experiences (Smith, 2006). It is essential then, that at the cultural

interface, the researcher is conscious of building relationships so that both student and researcher can participate in a meaningful exchange.

Indigenous methodology is a means of creating dialogue, rather than simply closed observation. This is not to say that through observation information cannot be learnt. Moreover, when observing an Indigenous culture, there are practices that may not be overtly apparent to the researcher; hence, the importance of including an open dialogue with students. For this particular study, the relationship also needs to be cultivated with Indigenous Education Officers (IEO) to assist with knowledge that may not be explicitly recognisable to the researcher. It is thus imperative to create space for critical collaborative dialogue within the study; hence the choice of Piagetian clinical interviews to collect data. In effect, this brings the researcher and the participants into a shared space. This created shared space is where empowerment can occur (Denzin & Lincoln, 2008). The implication of this decolonised approach dictates that the study must be viewed within the bounds of the individual community in which the research takes place, and not generalised to the broader Indigenous population.

Both theoretical perspectives adopted for this study complement each other. One is about creating dialogue, and the other is about interpreting the communication. That is, interpreting the language, gestures and signs that Indigenous students bring to the classroom, which is an essential aspect of respecting Indigenous culture and acknowledging the unique contributions it makes to learning.

4.5   RESEARCH METHODOLOGY: TEACHING EXPERIMENTS

A teaching experiment is a methodology that seeks to answer teaching-research questions (Czarnocha & Maj, 2002). It presents a living methodology where students' mathematical learning and transformations of knowledge can be explored within their own classroom environment (Cobb, 2000; Steffe & Thompson, 2000). This methodological approach provides opportunity to address development of instructional sequences by testing conjectures related to students' learning of a mathematical concept (Confrey & Lachance, 2000). It builds on the constructivist theory that believes students are able to construct mathematics (Czarnocha & Maj, 2000). With this, it is essential to include arguments for how mathematics is constructed from diverse constituents. This study utilised a teaching experiment for

the primary purpose of directly interpreting young Indigenous students' mathematical learning and reasoning in relation to their construction of mathematical knowledge (Cobb, 2000; Steffe & Thompson, 2000). This study was concerned with ascertaining the type of teacher actions that promoted student engagement with the generalisation process. Therefore, this methodology is suitable to investigate the interactions that support the development of young Indigenous students' ability to generalise.

Confrey and Lachance's (2000) model for conjecture-driven teaching experiments arose from the need to provide more equitable instructional approaches for students in mathematics (Confrey & Lanchane, 2000), and this methodology aligns with this thesis. Furthermore, in line with Indigenous research perspectives, the goal is to produce a convergence of cultural aspects of the Indigenous context with mathematics as students engage in the mathematical experiences of the study. Thus, conjecture-driven teaching experiments were conducted.

A crucial aspect of the conjecture-driven teaching experiment is the conjecture itself. It needs to be aimed at both theoretical analysis and instructional innovations (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003). Conjectures are based on inferences, and within mathematics education, these inferences may pertain to how mathematics is organised, conceptualised, or taught in order to reconceptualise the content and pedagogy (Confrey & Lachance, 2000). Confrey and Lachance (2000) argue for a guiding conjecture driven by: (a) what should be taught, and (b), the pedagogical dimensions linked to this content. Thus, the guiding conjectures for this research related to the need to engage young Indigenous students in early algebra, and to determine what teaching actions assisted young Indigenous students accessing this type of algebraic thinking. Important to this process is the need to revise and elaborate the conjecture while the research is in progress (Confrey and Lachance, 2000). Analysis is ongoing throughout the teaching experiment, and provides information about students' learning and teaching actions that assist students to engage in the mathematics. Essential to this process in this particular study, was the input provided by the Indigenous Education Officers at the end of each lesson and clinical interview (see Section 4.7.5). Figure 4.2 displays the ongoing development of conjectures across the teaching experiment.
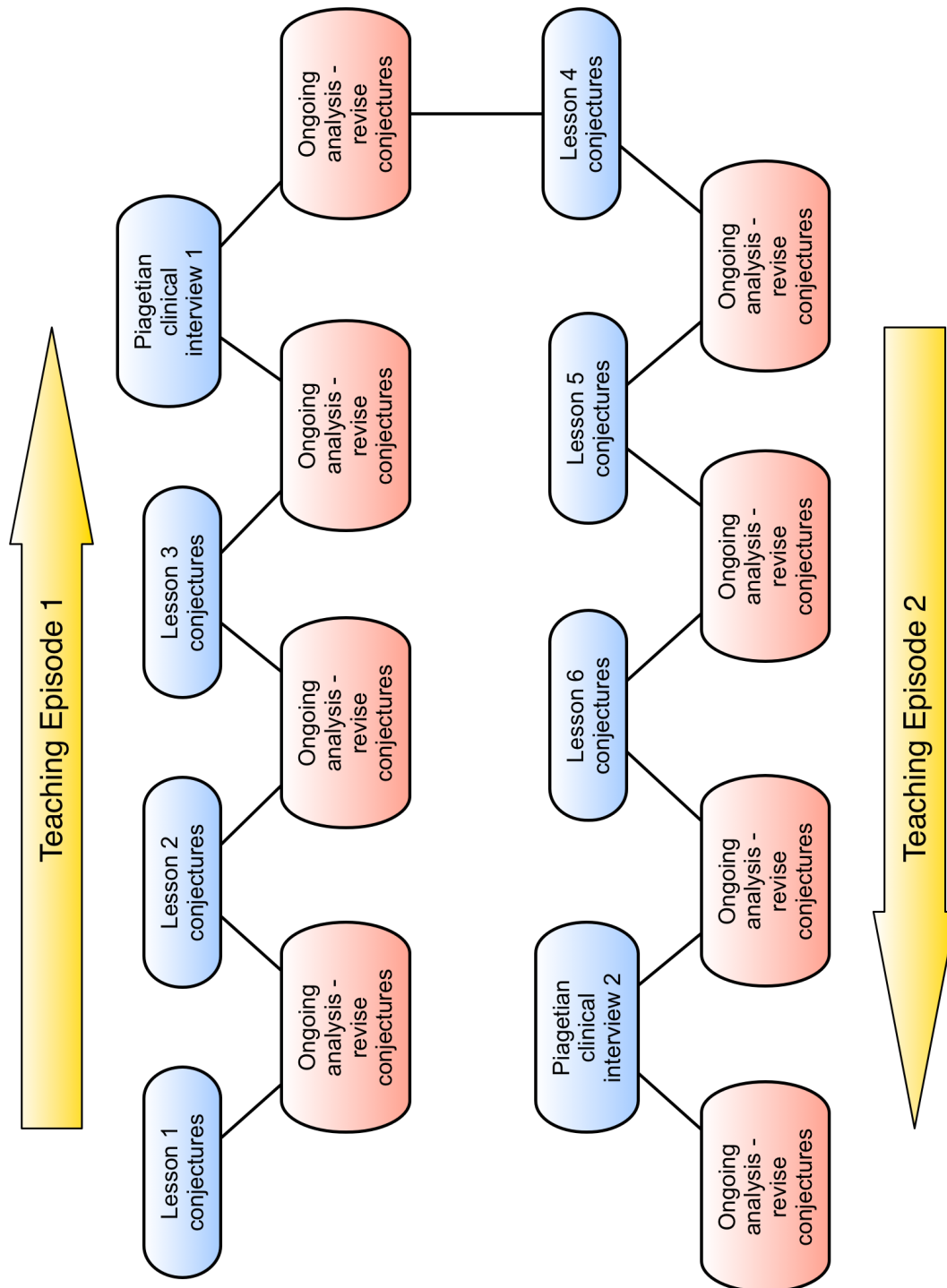
Figure 4.2. Ongoing development of conjectures in the teaching experiment.

There are four elements to a conjectured-teaching experiment: *curriculum, classroom interactions, teaching,* and *assessment*. The *curriculum* refers to the content area explored. In this study, curriculum explored was early algebra with a focus on growing pattern generalisation. Growing pattern generalisation does not form part of the curriculum content addressed in Year 2 and 3 for the selected research site. Therefore, when conducting the research there was a need to adopt a 'responsive and emergent' approach that was 'evolving and elaborating' (Confrey & Lachance, 2000). That is, responding to, and adapting learning experiences, as students provide insights into their learning. Conjectures are refined and modified throughout the evolving data collection. To accomplish and consider all aspects of how young Indigenous students' engaged in pattern generalisation, a model was developed for approaching all aspects of the research that arose from the literature (See Figure 4.3).
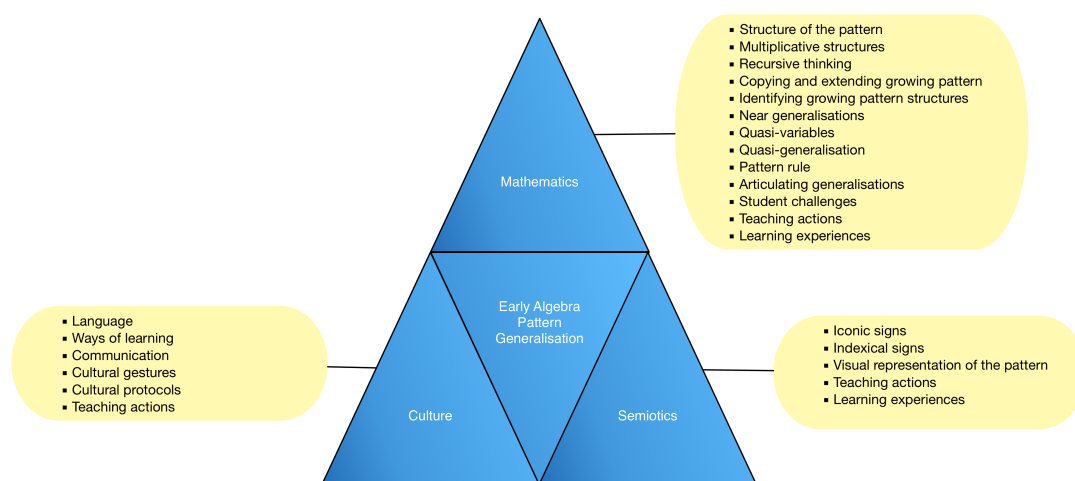
Figure 4.3. A model for approaching aspects of the literature when building conjectures: how young Indigenous students engaged in pattern generalisation.

Central to the study was building an understanding of how young Indigenous students engaged in pattern generalisation. There were three overaching themes from the literature that impacted how conjectures were developed for the study, namely, mathematics, semiotics, and culture. Key aspects of these themes were used to assist building and refining conjectures throughout data collection. Each lesson had its own conjectures that fed into the two overarching conjectures being explored. These lesson conjectures were refined at the end of each lesson. Table 4.2 displays the

conjectures explored throughout the data collection phase in relation to the three themes.

Table 4.2.

Themed Conjectures Explored Across the Teaching Experiment

	Themes		
	Mathematics	Semiotics	Culture
Lesson 1	Manipulating hands-on materials that represent the variables of a growing pattern allows students to better attend to the pattern structure.	Making both variables of growing patterns visually explicit assists students to identify the co-variational relationship	Exploring growing patterns from environmental contexts assists Indigenous students to relate growing patterns to their prior experiences.
Lesson 2	Exploring growing patterns where the structure is multiplicative (e.g., double) assists students to generate the pattern rule.	Providing growing patterns where the variables are embedded in the pattern ensures that students attend to both variables.	Embodying the mathematical structure of growing patterns assists students to explain the pattern structure.
Lesson 3	Transferring mathematical knowledge between patterns with the same multiplicative structure is difficult.	Providing growing patterns where the variables are embedded and cannot be physically separated from each other assists students to attend to both variables simultaneously.	Providing students with the mathematical language used to describe multiplicative structures assists students to generalise the pattern.
PCI1	The conjectures presented in Lesson 1- 3 were explored deeper in the one-on-one Piagetian Clinical Interviews.		
Lesson 4	Explicitly modelling the relationship between the variables in the growing pattern assists students to use the alphanumeric notation to describe the generalisation.	Using semiotic bundling, (i.e., using gesture, language and manipulation simultaneously) assists students to identify the structure of the pattern.	Exploring growing patterns from environmental contexts assists Indigenous students relate growing patterns to their prior experiences.
Lesson 5	Exploring pattern structure by attending to both variables will assist students to generalise	Providing growing patterns where the variables are embedded in the pattern and visually explicit ensures that students attend to both variables.	Manipulating hands-on materials that represent the variables of a growing pattern allows students to better attend to the pattern structure.
Lesson 6	Exploring patterns with a constant value are more difficult than exploring patterns where the structure is multiplicative	Using an iconic symbol (e.g., colour) to represent the constant in a growing pattern assists students identify the constant.	Creating a 'story' about how the two variables are related assists students see the co-variational relationship.
PCI2	The conjectures presented in Lesson 4 - 6 were explored deeper in the one-on-one Piagetian Clinical Interviews.		

Note: PCI1 – Piagetian Clinical Interview 1; PCI2 – Piagetian Clinical Interview 2

Classroom interactions occurred in whole-class lessons where students interacted with one another as they constructed new knowledge. All students used

hands-on materials during the lessons. It was essential to provide opportunity for students to be observed as a whole class to better understand the cultural aspects that arise when teaching young Indigenous students mathematics. It also provided for rich data to be collected with regards to the positioning of the Indigenous students' voice in mathematical learning experiences and in the reporting of the study's findings.

The researcher within the study carried out *the role of the teacher*. This is similar to studies conducted by Carraher, Schliemann, and Brizuela (2001), where the teacher/researcher entered the classroom at pertinent times of the year to conduct small interactive teaching interventions focusing on new knowledge content. It was decided that as the mathematical concept did not form part of students' curriculum, it was necessary for the researcher to deliver the lessons. Additionally, to maintain trustworthiness of the study, the researcher assumed the role of teacher, because of the complex and specialised nature of semiotic theory in relation to students engaging with mathematics. It was acknowledged that Indigenous students will bring their own unique contribution to the lessons and therefore the researcher remained respectful and interested in these alternative approaches.
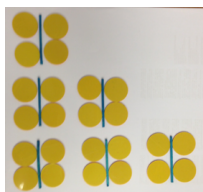
Lessons were conducted drawing on a social-constructivist approach to learning (Vygotsky, 1978). Central to this approach, is the cultural and social contexts in which learning takes place. Students engage in a series of learning experiences where they require support. Vygotsky (1978) describes this as the zone of proximal development. It is the "distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined by problem solving under adult guidance or in collaboration with more capable peers" (p.86). Thus, students collaborate with more experienced or knowledgeable members of the learning community to develop their understanding. The teacher and Indigenous Education Officers have an active role in the learning process, and provide scaffolding to students.

With regard to *assessment*, students participated in one-on-one Piagetian clinical interviews that examined their ability to generalise growing patterns. This form of assessment was selected as it provided young Indigenous students the opportunity to use the hands-on materials as they explained their mathematical thinking.
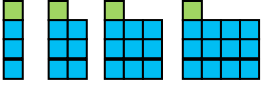
The teaching episodes took place over a two-week period, one week in Term 3 (July) and one week in Term 4 (September). Each week consisted of three 45-minute lessons exploring and developing understandings of pattern generalisation (6 lessons in total). All lessons were designed with Indigenous learning styles and semiotic structures in mind as discussed in the literature review in Chapter 3. Additionally, these experiences provided opportunities to further develop relationships with students so that, when in a one-on-one setting during the interviews, they would feel comfortable in engaging in conversation. Table 4.3 provides an overview of the lessons, tasks and materials used in the teaching episodes. Concepts of the lessons are further elaborated in Chapter 5 and Chapter 6.

Table 4.3

Overview Lessons, Tasks Presented and Materials used in Teaching Experiment 1 and 2

Teaching experiment	Lesson		Tasks Presented	Materials
Teaching episode 1 Contextual growing patterns	Lesson 1	Exploration	Explore the concept of what is a growing pattern. What does it mean to grow? What is a pattern? Focusing on structural awareness.	Butterflies
		Concept Application	Apply this knowledge to copy and continue a growing pattern.	
	Lesson 2	Exploration	Explore the difference between a repeating pattern and a growing pattern.	Number track and feet
		Concept Application	Apply this knowledge to copy, continue, predict and create elements of both a repeating pattern and a growing pattern. Transfer this knowledge between two different tasks presented.	
	Lesson 3	Exploration	Explore the concept growing pattern through patterns that have embedded signs that can not be physically separated (i.e., the relationship between kangaroo tails and kangaroo ears).	Concrete materials (Australian animals)
		Concept Application	Apply this knowledge to create a growing	

Teaching experiment	Lesson		Tasks Presented	Materials
			pattern and tell a story about how it is growing. See if students identify the relationship between the number track task and the kangaroo task (same growing pattern rule). Provide a scenario where students need to generalise their own pattern rule.	
Teaching episode 2 Mathematical abstraction	Lesson 4	Exploration	Explore a growing pattern through representations that require mathematical abstraction.	Caterpillar pattern – double-sided counters Day 1 Day 2 Day 3
		Concept Application	Apply this knowledge by students engaging in copying, continuing, predicting and creating a growing pattern. Contextualise this by the use of storytelling.	
	Lesson 5	Exploration	Explore a growing pattern using mathematical abstractions without a contextual story.	Flower pattern – double-sided counters
		Concept Application	Apply this knowledge by presenting a pattern that has a constant. Have students identify what is the same and what is different.	
	Lesson 6	Exploration	Explore a growing pattern that has a constant.	Classroom pattern – coloured tiles and number cards

Teaching experiment	Lesson	Tasks Presented	Materials
	Concept Application	Apply this knowledge by having a student create a growing pattern using mathematical abstractions, and have students identify what stays the same and what changes in their pattern. Ask students to identify the rule and justify. Provide a scenario where students need to generalise their own rule.	

4.6   PARTICIPANTS

The research was conducted in one Year 2/3 classroom of an urban Indigenous school in North Queensland. Pattern School (pseudonym) is a co-educational school that is currently implementing the Australian National Curriculum. The purpose for selecting 'Pattern School' is that there has already been a relationship established with students and teachers in this community. The Year 2/3 students selected for this study are part of a larger longitudinal mathematical study which the researcher has been part of, and thus forged relationships with the school community. There were three pertinent participants in the study: (a) Indigenous students, (b) Indigenous Education Officers, and (c) the researcher.

4.6.1 Students

All students in this study identified themselves as Aboriginal or Torres Strait Islander people, and they spoke a mixture of *Aboriginal English* and *Standard Australian English*. Initially, the class consisted of 16 students ranging from 7 years to 9 years (10 girls, 6 boys). Of those students 7 were in Year 2 and 9 students were in Year 3. Low attendance is one of the complexities within the classroom context. While there were 16 students in the class, these were very few occasions when all students were present. Additionally, two students left (S13 and S15) the school

before Teaching Experiment 2, and new students had joined the class (S17 and S18). Therefore, in total, 18 students participated in some aspect of the study. In addition, there were a core group of students who had a high attendance at school (n=10).

In 2011, the Year 3 students from this study participated in the National Assessment Program for Literacy and Numeracy (NAPLAN). Students' results, results from similar schools, and the overall Australian average for reading, persuasive writing, spelling, grammar and punctuation, and numeracy are presented in Table 4.4

Table 4.4
NAPLAN Results for Year 3 Cohort in 2011

	Reading	Persuasive Writing	Spelling	Grammar and Punctuation	Numeracy
Participating school	207	259	285	178	297
Australian Average	416	416	406	421	389

As the results indicate, the Year 3 students selected for this study were performing below the Australian national average across all areas tested in NAPLAN.

There were two phases of purposive selection of students.

First, as the study considers young Indigenous students, it was necessary to select a class that met this demographic. Additionally, students of this age (7-9 year olds) were selected as they had begun to learn the fundamentals of patterning in mathematics. Also, it was believed that students in Year 2/3 were better able to articulate their understandings and justifications, than students in lower year levels.

Second, in order to gain greater insight into student activity in the observation phase, three students were selected from the cohort in order to conduct Piagetian clinical interviews. These are referred to as case students. There were three categories from which the researcher selected these students: (a) students who are experiencing difficulties in reaching generalisation (S6 – Student 6); (b) those who have grown in their ability to generalise from the teaching episodes (S2 – Student 2); and (c) those who can generalise (S1 – Student 1). These students were purposively

sampled from the observations of the teaching experiments. Students represented a range of mathematical levels. Data from a pretest (RoleM Warren & deVries, 2011) taken from a larger study was used to determine the mathematical level of the student, as well as information from the consultations with the teacher and Indigenous Education Officers.

4.6.2 The Researcher

The role of researcher as teacher was a key part of the teaching experiment. The researcher observed students as they participated in learning experiences presented in the lessons. For this particular study, the researcher was working from two paradigms, that of the qualitative research perspective and also the Indigenous research perspectives. The researcher aims to preserve the unity and integrity of the data through adopting an emic perspective of the participants (students) and the etic perspective of the researcher (Merriam, 1998; Stake, 2005), thus providing a clear context and perspective of the study. An important dimension of Indigenous research is the relationships that form between the non-Indigenous researcher and the Indigenous students (Smith, 1999), and the sense of trust that ensues. This is seen as crucial to gaining reliable insights into the teaching actions that assist students to engage in mathematics. As the researcher was non-Indigenous, it was essential to build relationships with the Indigenous Education Officers and include them in the ongoing data collection and analysis. To ensure that the findings from the study were true to the Indigenous students the researcher involved both Indigenous Education Officers to ensure sensitivity and cultural nuances were identified. Finally, the data collected contributed to the advancement of young Indigenous students education.

4.6.3   Indigenous Education Officers

As the student participants were both Aboriginal and Torres Strait Islander it was essential to have Indigenous Education Officers from each of the Indigenous groups. This was essential as Aboriginal and Torres Strait Islander people have differing cultural practices. Both Officers were women and worked in the classroom with students.

Research pertaining to Indigenous students observed within the classroom experience can pose certain problems. There are cultural cues that could be missed or misinterpreted due the cultural differences between the researcher and students. It is

for this reason that two Indigenous Education Officers provided guidance during all phases of the research. Within this study, the researcher worked in collaboration with the Indigenous Education Officers to:

- Identify differing cultural interpretations of gesture and actions within the class as well as in interview interactions.
- Ensue a mathematical context relevant to students was used in the teaching episodes.
- Assist students with communicating their ideas, particularly with regard to language.
- Provided on-going cultural information for each individual video recording of the teaching experiments and for students who participated in the one-on-one interviews.

4.7 DATA-GATHERING STRATEGIES

Guided by the research design, multiple data-gathering strategies were adopted during the study. This method of data gathering is consistent with the teaching experiment methodology. Gathering data using multiple methods allows for a deeper understanding of how these students engage in mathematics. Further, multiple methods allows for methodological triangulation of findings, which ensures trustworthiness of the data (Hitchcock & Hughes, 1995).

The data-gathering strategies of this study are:

- co-researcher and witness observations
- daily discussions and reflections with principal supervisor and Indigenous Education Officers at the conclusion of lessons,
- pretests at the beginning of each Teaching Episode,
- whole class video-recording of lessons,
- video-recorded one-on-one Piagetian clinical interviews (case students and researcher),
- audio-recordings of Indigenous Education Officers feedback in relation to one-on-one Interviews,
- photographs of students' work.

4.7.1 Observations

Initial Observations

In line with Indigenous research perspectives prior to Phase 1 and Phase 2 of the data collection (see *Figure 4.4*), initial observations were conducted to gauge the general dynamic of the classroom, and to begin to establish relationships with the participating students. It also served to construct a deeper understanding of students' interactions with the teacher, Indigenous Education Officer, and other students. Annotated notes were collected during regular classroom lessons (1 full day of teaching) conducted by the classroom teacher prior to the first teaching episode. Additionally, the researcher discussed with the teacher and IEO1 (on separate occasions) their experiences with the class. These notes were collated and analysed to better understand the classroom dynamic. This shaped the approach to the way in which the researcher would teach the class during the teaching episodes.

Co-researcher and witness observations

During the lessons taught by the researcher in the teaching experiment, the principal supervisor acted as a co-researcher and the Indigenous Education Officer (IEO1) acted as a witness. Both were not restricted to only an observing role. Rather, the principal supervisor took photographs of students' work and recorded field notes about the lessons, particularly the teacher and student interactions. And both principal supervisor and IEO1 worked with students, encouraging and assisting them during the lessons. All observations and field notes were discussed at the end of each lesson by the researcher, principal supervisor and IEO1. The researcher took notes of the discussions. These discussions were pivotal and used in the ongoing analysis process of the research design (see Section 4.8). That is, they informed the proceeding lesson tasks and conjectures.

4.7.2 Pretests

The purpose of the pretest was to gauge students' understandings of growing patterns, that in turn would lead to the development of the teaching episode. The pretests were based on existing instruments and tasks presented to young students (Blanton & Kaput, 2005; Moss & Beatty, 2006; Papic & Mulligan, 2007; Warren & Cooper, 2009). There were two pretests (Appendix D and Appendix E). Each test was conducted at the commencement of the teaching episodes for the week (Pretest 1

and Pretest 2). All students participated in the pretests. To ensure validity of the test, two classes from a middle socio-economic school in Brisbane were selected to participate in a trial of the instrument. The sample consisted of students from Year 2 (n=23) and Year 3 (n=25) from mixed cultural backgrounds. The tests were trialled to ascertain any issues that arose concerning the language, visual representations, and mathematic content of the test. The researcher was present when the pretest was conducted to note issues that arose from the participating trial students. Data from the trial pretest were analysed to ensure that the test questions were reliable. As a result of the trial and analysis no test items were changed.

Administration of pretests occurred at the beginning of each teaching experiment in the students' classroom. The test was administered under normal test conditions for that classroom. That is, students sitting at individual desks working independently. The questions were read to students and time given to respond. The pretests were of 15 minutes duration. Field notes were taken by the researcher during this time to note any interesting ways students approached answering questions from the test.

Pretest 1 focused on students' initial understandings of patterning and what they believed to be a growing pattern. Each question was presented in a contextual manner in an attempt to ensure that students could access the mathematical ideas (10 questions in total). Pretest 2 focused on common growing patterns presented to students in the format of mathematical abstractions (12 questions in total). For the purpose of this study, mathematical abstractions are the symbols used in the growing pattern to represent the numerical value (i.e., triangles, circles, squares). These symbolic representations need to be abstracted by the student to understand the pattern in numerical terms. The pretests informed the teaching episodes and Piagetian clinical interviews.

4.7.2 Video-recordings of Lessons from Teaching Experiments

In order to capture the student-teacher, student-student, and student-IEO interactions, all lessons were video-recorded. Three video cameras were utilised within the classroom. Two were set up on tripods, one focusing on students and the other on the researcher. The third camera was a roaming video camera where the researcher, IEO or principal supervisor could capture students' individual work in lessons. The aim of using the video-recordings was to support triangulation within

the study, and provided a broader scope for data collection. More specifically, it documented teacher actions, how students used the hands-on materials, and classroom discussions that promoted student engagement with the generalisation process. All video-recordings were downloaded at the conclusion of the lesson, and later transcribed for analysis.

4.7.5   Video-recorded Piagetian Clinical Interviews

To further probe how individuals engage in the act of generalisation, students were selected to participate in a Piagetian clinical interview stage. These clinical interviews were used as data-gathering strategy to continue the testing of conjectures presented in the teaching episodes. Pioneered by Piaget (1975), the clinical interview is a diagnostic tool used to study the naturalistic form of knowledge structures and reasoning processes (Clement, 2000). This form of interviewing is predominately concerned with the testing of conjectures, where the observations made by the researcher can infer students' competency in the presented task (See Table 4.5). Inferences can be drawn in the following manner. Firstly, students are presented with an experiment in order to obtain cognitive understandings of a particular mathematical task or concept. This experiment first needs to be presented in a concrete setting, and then to a verbal problem related to the situation (Opper, 1977). From previous interactions observed during the teaching experiment tasks, the researcher was guided in conjecturing what types of thinking students engaged in. Secondly, there are a number of tasks presented to the student to expose his or her reasoning or shed light on the hypothesis (Opper, 1977). During these tasks, it is essential that the student and the interviewer participate in physical and spatial manipulation (Opper, 1977). Finally, the interviewer asks a series of questions assessing the child's ability to predict, observe and explain the results from the manipulation of the concrete materials. These predictions, observations and explanations provide a useful insight into students' view of reality and thought process (Opper, 1977). At this point in the interview process, the researcher can confirm whether the hypothesis is legitimate or invalid. If the conjecture proves to be valid, the researcher may provide further questioning to consolidate the hypothesis or move on to the next task. However, if the conjecture cannot be confirmed, the researcher needs to take the student's response into account and re-conceptualise the original conjecture. The sources of data within this method stem from the verbal

explanations of the underlying mathematical process the student is engaging in. The interviewer needs to further probe the student response to elaborate on the student's statements made within particular tasks. Data also come from the actions and manipulations of the presented tasks.

Piagetian clinical interviews have some defining features. The researcher attended to these features in the following ways:

- ensuring there is an initial introduction before the commencement of the interview to prepare the student and make them feel more at ease.
- adjusting the language and tempo of the interview so that students can understand and have the best opportunities to reason with the tasks presented.
- encouraging students to elaborate on their statements by providing counterarguments.
- refraining from bias towards desired answers
- critically observing responses for authenticity; and,
- taking students' point of view and affirming their perspective throughout conducting the interview.

The one-on-one interviews were conducted with the three selected students. The first clinical interview was conducted at the end of Teaching Episode 1 and the second interview was conducted at the completion of Teaching Episode 2. A written form of consent was sent home to parents to agree for their child's participation in the on-on-one interviews. Then, at the time of the interview, students were also asked if they agreed to continue their participation in the study. The interviews were conducted in a small room adjoining the classroom. The approximated length of each interview was 20 minutes.

All interviews were video-recorded. One video camera was used to capture the teacher-student interactions. These video-recordings were downloaded at the conclusion of the interviews, and were later transcribed for analysis. Tasks and conjectures presented to students will be discussed further in Chapter 6. Table 4.5 presents an overview of questions asked and content covered in the three interviews.

Table 4.5

Overview of Piagetian Clinical Interview Tasks

Piagetian Clinical Interview	Content	Questions
Interview 1	Contextual pattern	What is a growing pattern?
	Exploring the relationship between crocodile legs and crocodile tails (4n)	Continue the growing pattern
		Predict the next term of the growing pattern
	Exploring the relationship between students in class and year level (3n)	Predict the 10th term, 25th term, 100th term, n^{th} term.
		What is the rule?
		Create your own growing pattern
Interview 2	Mathematical abstraction	What is a growing pattern?
	Exploring a growing pattern with a constant (3n+1). Using double sided counters	Continue the growing pattern
		Predict the next term of the growing pattern
	Exploring a growing pattern with a constant (4n+1). Using square tiles.	Predict the 10th term, 25th term, 100th term, n^{th} term.
		What is the rule?
		Create your own growing pattern

4.7.3 Audio-recordings of Indigenous Education Officers

At the end of each teaching episode, separate interviews were conducted with Indigenous Education Officer 1 and 2. The women, from whom written consent was obtained, watched the individual video footage of each student and provided feedback with respect to cultural interactions. These discussions were audio-recorded and later transcribed for analysis.

4.7.4 Photographic Evidence of Students' Work

Digital photographs were taken during lessons as a means of capturing students' individual work. This provided further evidence of how students were engaging with the pattern structure and the hands-on materials. Names and faces were removed from each photograph to assure anonymity.

Overview of the Data-Gathering Strategies

Table 4.6 summarises the multiple data-gathering strategies. In addition, the table presents the chronology of the study, participants, and data-gathering strategies.

Table 4.6

Chronology, Participants, and Data-Gathering Strategies

Chronology	Participants	Data-Gathering Strategy
February and April 2011	Whole class	Initial observations Interview with classroom teacher Interview with IEO1 and IEO2
July 2011	Whole class (n=18)	Pretest 1
July 2011	Whole class (n=18)	Teaching Episode 1 Three video-record lessons Observations by Principal Supervisor and IEO Photographs of students work
July 2011	Three case students	Video-recorded one-on-one Piagetian Clinical Interviews Audio-recorded discussions with IEO1 & IEO2 about interviews
September 2011	Whole class (n=18)	Pretest 2
September 2011	Whole class (n=18)	Teaching Episode 2 Three video-record lessons Observations by Principal Supervisor and IEO Photographs of students work
September 2011	Three students	Video-recorded one-on-one Piagetian Clinical Interviews Audio-recorded discussions with IEO1 & IEO2 about interviews

4.8   ANALYSIS OF DATA

The teaching experiments and clinical interviews required two phases of data analysis. First, ongoing analysis occurred at the conclusion of each lesson of the teaching experiments, and this then informed the next stage of data collection (see Figure 4.2 and Figure 4.4). Second, a more in-depth analysis was conducted based on an iterative approach. This second analysis was designed by the researcher and is discussed in detail below. It was used in the retrospective analysis phase that considered all data that had been collected.

Ongoing and Preliminary Analysis

In qualitative research there is a need for the data gathering and analysis to be simultaneous activities (Creswell, 2008). In this study, data analysis was ongoing and formative during the data-collection phase. It occurred after lessons, and this analysis informed the next stage in the data-collection process and assisted in refining conjectures (Confrey & Lachance, 2000). That is, data gathering and theory building occur simultaneously or, at least, one leads to the other in the cycle. The initial teaching experiment was informed by the analysis of the pretest data and initial classroom observations.

Data were collected from the Pretest 1 and Pretest 2 to discern students' understanding of growing patterns before each Teaching Experiment. Data were coded and analysed using a statistical data program, SPSS, to generate descriptive data. The data were analysed to provide measures of centre and variability on the test, as reported in Chapter 5. The analysis of the quantitative data was used to feed into qualitative data, the teaching episodes and clinical interviews.

Ongoing Analysis of Lessons and Clinical Interviews

At the conclusion of each lesson, the video data were observed and this information, combined with field notes, identified emergent themes. Peer debriefing occurred at this point between the researcher, Indigenous Education Officer and supervisor to determine consistency with noticed themes. This analysis refined and informed the conjectures explored in the proceeding lesson and the accompanying tasks (Confrey & Lachance, 2000). The following lesson informed the next and so on, until the conclusion of Teaching Experiment 1. This process continued for Teaching Experiment 2. The themes that emerge from the teaching experiments also

form the basis for the conjectures explored in the Piagetian clinical interviews. The cycle continued: each stage of the data-gathering process was contingent on the previous. Figure 4.4 displays an overview of the data-collection stages and highlights where data analysis occurred.
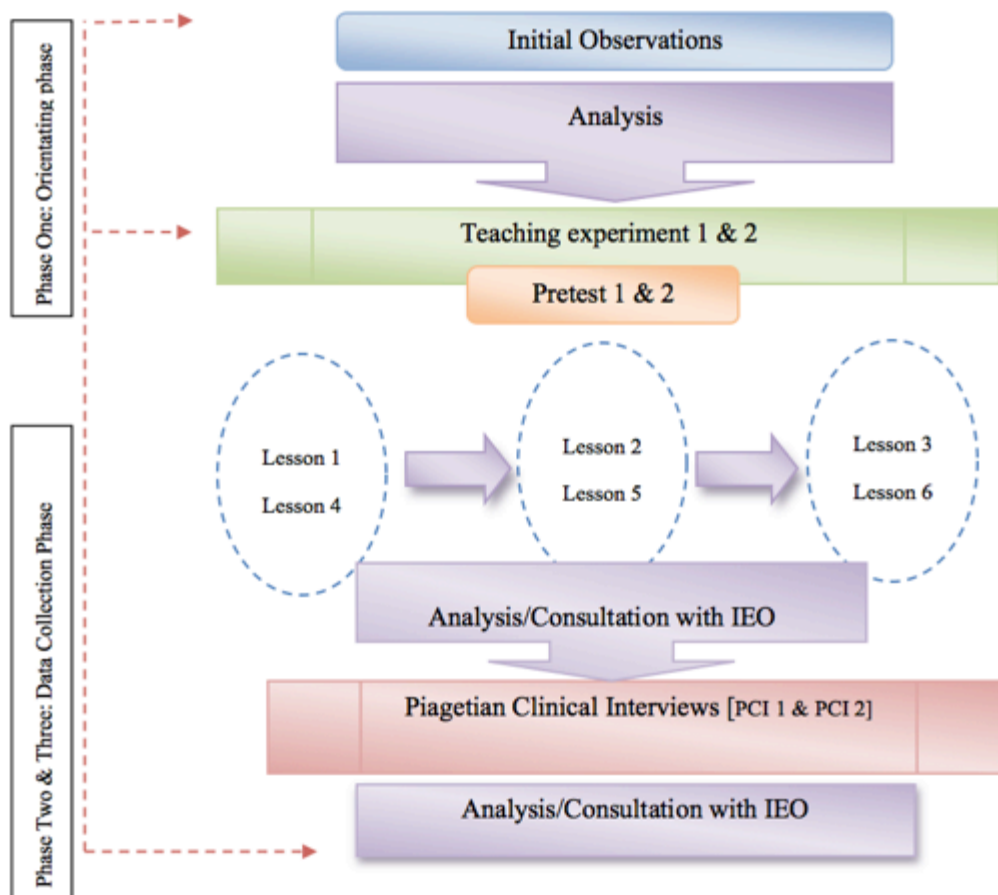
Figure 4.4. Data collection and continual analysis.

In-depth Analysis

At the conclusion of the data-collection phase, all data were reanalysed. This analysis was based on an iterative approach. This approach is a deeply reflexive process of continuous meaning-making and progressive focusing (Srivastava & Hopwood, 2009). An iterative approach is described as:

> a loop-like pattern of multiple rounds of revisiting the data as additional questions emerge, new connections are unearthed, and more complex formulations develop along with a deepening understanding of the material. (Berkowitz, 1997, p. 42)

The researcher, due to the unique application of mathematics, semiotics and culture, constructed the data-analysis model (See Appendix F for an example of in-depth data-analysis). Within the model there are four key features to analysing the data.

First, the initial video-footage was transcribed to capture students' verbal responses. These transcriptions were then analysed to consider emerging key mathematical themes from the lessons and interviews.

Second, semiotics was utilised as a lens through which to reanalyse the data. The evolving data were reanalysed, focusing on semiotic bundles (signs, gestures, language) of both the student and researcher in the lessons and interviews. This analysis provided an interpretation of the learning interactions between the researcher and students. Of particular importance were the students' and researcher's physical gestures, including the manipulation of hands-on materials and bodily language. These iconic and indexical signs were coded. Following is an example of the semiotic interactions recorded within the transcript:

108 S2:   Two times three equals six [*R separates the pattern into two rows of three*]

109 R1:   [*R points to the number card with 3rd written on it*]

110 S2:   Three times six equals nine [*R separates the pattern into three rows of three and begins to identify the multiplicative structure*]

Third, the data were reanalysed inline with the cultural perspective provided from the Indigenous Education Officers. Their input was audio-recorded and then transcribed. These transcriptions were to match the students' actions from the above analysis (See Appendix F). This process was repeated for all lessons and student interview data.

Finally, a cross analysis of cases was conducted to identify key teaching actions that assisted the students to generalise (See Appendix G). This cross-case analysis informed (a) how students engaged in generalisation, (b) what teaching actions assisted students to generalise, and (c) how culture influenced Indigenous students engagement in mathematical generalisation. Table 4.7 presents an overview of the research questions, data-gathering strategies and data analysis.

Table 4.7

Overview of the Research Questions, Data-Gathering Strategies and Data Analysis

Research Question	Data-Gathering Strategies	Steps for Data collection	Data analysis
	Classroom Observations *Whole class*	Field notes and reflections	Analysed to determine how the Indigenous students interact within their own classroom setting. Including their interactions with their classroom teacher and Indigenous Education Officers.
Research Q1 How do young Indigenous students engage in growing pattern generalisation?	Pretest 1 and 2 *Whole class*	Data gathering and reflection	Analyse responses for trends and patterns in relation to mathematical knowledge. Determine what students currently understand about patterning.
Research Question 1, 2 and 3 (1) How do young Indigenous students engage in growing pattern generalisation? (2) What teacher actions assist in enhancing young Indigenous students to generalise growing patterns? (3) How does culture influence the way in which young Indigenous students engage in growing pattern generalisation?	Teaching Episode 1 and 2 *Whole class*	Video camera, field notes, photographs Select participants from Teaching Episode 1 for Piagetian Clinical Interview 1 (repeat at the end of Teaching Episode 2)	Analyse responses to activities for trends and patterns. Discuss students' interactions with Indigenous Education Officers. Reanalyse data considering the mathematics, semiotic, and cultural perspectives.

Research Question	Data-Gathering Strategies	Steps for Data collection	Data analysis
Research Question 1, 2 and 3 (1) How do young Indigenous students engage in growing pattern generalisation? (2) What teacher actions assist in enhancing young Indigenous students to generalise growing patterns? (3) How does culture influence the way in which young Indigenous students engage in growing pattern generalisation?	Piagetian Clinical Interview 1 and 2 *(n=3)*	Interview selected participants. Video camera, field notes, photographs, hand written artefacts.	Have Indigenous Education Officer 1 and Indigenous Education Officer 2 analyse student interviews for/cultural perspective. Analyse data gathered from interview considering the mathematics, semiotic, and cultural perspectives.

4.9   TRUSTWORTHINESS

Validation and reliability within interpretivist research is determined by the trustworthiness of the data quality. Underpinned by a constructionist epistemology, meant that within this study multiple data sources were used to better understand the knowledge constructed by students and their social interactions. Trustworthiness can be established through attention to criteria established according to Lincoln & Guba (1985). The criteria that ensue the trustworthiness of this study are credibility, dependability, confirmability, and transferability (Lincoln & Guba, 1985). By adopting these four criteria, the researcher can claim that the data is trustworthy (Trochim, 2006).

Credibility

This research employs a number of techniques to ensure the credibility of the data. It has been suggested that credibility within qualitative research deals with the

question, 'how congruent are the findings with reality?' (Merriam, 1998). This was the premise for determining trustworthiness in this research (Lincoln & Guba, 1985). For the purpose of this study the techniques selected to ensure credibility were:

- *Persistent engagement*: This was achieved by conducting a number of lessons where students were observed frequently during the data collection phase. In order to obtain an understanding of what teacher actions assist in Indigenous students engaging in generalisation, it is necessary to dedicate time to collecting data and cross-checking for misinterpretations. As it is also an essential component of Indigenous methodologies to build a relationship with students, this was achieved by prolonged engagement within the classroom context for this study. Consequently, this enhanced the potential to gather rich data from the teaching experiments and clinical interviews (Lincoln & Guba, 1985; Merriam, 1998).

- *Persistent observation*: Persistent observation improved the scope and provided a depth to the study (Lincoln & Guba, 1985). During the teaching experiments and clinical interviews, observations gave a deeper understanding and informed the researcher what semiotic processes Indigenous students used when engaging with pattern generalisations. Additionally, by video-recording each lesson and interview it was possible to review the data collected on numerous occasions.

- *Peer debriefing*: Peer debriefing occurred at the conclusion of each lesson and clinical interview with both Indigenous Education Officers and the principal supervisor of the study. Additionally, peer reviews (by supervisors and other research colleagues) were conducted at the data-collection stage and with respect to the data analysis. Within the data-collection stage, peers had the opportunity to critique and discuss the interpretation of the collected data for the teaching experiments and clinical interviews. Once the final data were coded and themes were identified, peers again reviewed and crosschecked the analysis. By engaging in this process, the peer group critically review the data so as to discern bias in the research and to enhance credibility (Cohen et. al., 2007, Yin, 2003).

- *Member checking:* Member checks occurred during the ongoing analysis of the data. Member checks increased the accuracy of the findings by providing an opportunity for respondents to review the findings (Lincoln & Guba, 1985). These member checks were conducted during the lessons and interviews with students to ensure that the researcher had interpreted the intended response of the student (Shenton, 2004). Parents of the participants, participants themselves, Indigenous Education Officers and the classroom teacher had the opportunity to review the data within the form of a report following the teaching experiments and clinical interviews.

Dependability

Dependability of the data is another criterion for assurance of the trustworthiness of the study. In this research there were two ways in which this was achieved. Firstly, by conducting an independent audit of the data by an external reviewer, namely, two research supervisors and specifically selected workplace peers, at particular points in the study (Cohen et al., 2007). Secondly, dependability was also addressed by the employment of overlapping data-gathering strategies within this study (Shenton, 2004). That is, an overall picture was developed of each student through the use of pretesting, observations and analysis of classroom lessons and interactions, and finally, observations and analysis of one-on-one interviews (See Table 4.3).

Confirmability

The concept of confirmability is the qualitative investigator's comparable concern for objectivity (Shenton, 2004). This study adopted the use of an audit trail to display the systematic collection of the data. Subsequently, an independent audit occurred at the data-gathering and data-analysis stages by two research supervisors. By incorporating this process further confidence is gained in relation to the trustworthiness of the study (Lincoln & Guba, 1985).

A criticism that is often met by the qualitative researcher is that of bias. Bias or subjectivity of the researcher is where the propensity to verify preconceived notions is evident in the study (Flyvbjerg, 2007). Triangulation was used to reduce the investigator bias within the study. This was obtained by collecting both qualitative and quantitative data. This aimed to provide a rich description of students'

experiences in regard to mathematical generalisation, and provide a deeper understanding of the mathematics from their perspective (Merrian, 1998; Stake, 2005). Triangulation and peer reviews were utilised by the researcher as a way to address validity with findings of the study, as it is acknowledged that there can be multiple interpretations for a particular instance (Stake, 2005).

Transferability

It has been stated that, the ability to transfer findings is the essence of qualitative inquiry and is the responsibility of the researcher (Lincoln & Guba, 1985; Yin, 2003). It is noted that one of the limitations of this study is that it is bound to both context and time, and to overcome this limitation it has been suggested that conducting the same study in 'multiple environments' may provide transferability (Gross, 1998). Whilst the researcher agrees, this study remained bound within the one context to allow completion of her thesis within time constraints. Alternatively, literature suggests that by providing the reader with rich descriptions, vicarious generalisations can be made from the findings (Stake, 2005). Unlike positivist research, the ability to transfer the research is determined by the reader. Thus, the potential for transferability will be based upon the reader obtaining knowledge from the background data to establish the context of the study, and, in conjunction, detailed descriptions of the phenomenon in question may allow comparisons to be made (Shenton, 2004). Ultimately, the results of this qualitative research must be clear within the context of the study. By doing this, similar projects employing the same methods but conducted in different environments, could well be of significance.

4.10 ETHICAL ISSUES

Pivotal to research within Indigenous methodologies is the concern for groundings of self-determination and cultural autonomy for justice and equity for all participants (Denzin &Lincoln, 2008). It is noted that the researcher has an important obligation to respect the participants in this study in regard to their rights, needs, values and desires (Creswell, 2008). Use of Indigenous knowledge is at the discretion of the Indigenous people. The information gathered was always the property of the student and their parents. Indigenous protocols avoid cultural harm and it contributes to Indigenous people (Smith, 1999). Ethical procedures were

employed to obtain the data used in this study (See Appendix A). A letter of support for conducting the research was obtained from the Indigenous Higher Education Unit at the Australian Catholic University (See Appendix B).

While the researcher purposefully selected the class from the larger project, all students and IEO's participation in this study was voluntary and parental consent was obtained. Participants were invited to participate within the study without coercion or pressure, and permitted withdrawal from the study at any point. A letter of informed consent was obtained in writing from the school principal as well as student consent forms sent home to parents. The letters of consent clearly outlined the objectives of the study, and how the data would be collected and used within the given timeframe. Data collection did not commence until the consent was obtained. Table 4.8 outlines the ethical considerations for each of the data-gathering strategies.

Table 4.8

Data-Gathering Strategies and Ethical Considerations

Data-Gathering Strategies	Ethical Considerations
Teaching Experiments	• Letter of Invitation to Principal, Classroom teacher, Indigenous Education officers, Parents/Caregivers of students (See Appendix C). • Signed letters of consent • Codes assigned for each participant (e.g., S1 – Student 1; IEO1 – Indigenous Education Officer 1).
Piagetian Clinical Interviews	• Letter of Invitation to Principal, Classroom teacher, Indigenous Education officers, Parents/Caregivers of students (See Appendix C). • Signed letter of consent. • In-person explanation of the interview process to each student. • Signed student consent after verbal explanation for each interview (PCI1 and PCI2)

Cultural protocols were adhered to by the use of a gatekeeper within the community. The gatekeepers within this classroom community were considered to be the Indigenous Education Officers. The research conducted was consistent with the ethics employed by Australian Catholic University and was approved by the Human Research Ethics Committee at Australian Catholic University (Q201115 See Appendix A). At any time during the project the participants were free to withdraw their consent and discontinue participation without giving any reason. Confidentiality was protected during the conduct of the research by using coding and pseudonyms. That is, once data collection commenced, identification of individuals was concealed so as to provide anonymity to the participants. Permission was granted from all employing authorities to conduct the research in their school. All data was stored in accordance to Australian Catholic University guidelines and protocols, and access was restricted to those people authorised by the researcher. Copies of the interview transcripts were available to participants on request. In regards to publication of the study's findings and conclusions, participants will be consulted.

4.11 CHAPTER REVIEW

The purpose of this chapter was to justify and describe the research design adopted pursuant to exploring how young Indigenous students engage in generalisation tasks. The chapter outlines the theoretical framework and design adopted for this study. Constructionism was appropriate as the epistemology for the study, as the study explores how students construct their own knowledge within the classroom context. Semiotics and Indigenous research perspectives were employed as the two theoretical lenses for the study, to assist with exploring how young Indigenous students engage in pattern generalisations. Data-gathering strategies include teaching experiments and one-on-one Piagetian Clinical interviews. The following chapter will outline and report the data collected during Teaching Experiment 1 and 2.

Chapter 5:   Findings Teaching Experiments

5.1   CHAPTER OVERVIEW

This chapter of the thesis presents the findings of the study in relation to the pretest and lessons conducted in Teaching Experiment 1 and 2. The findings from the Piagetian clinical interviews are reported in Chapter 6.

The chapter begins with establishing the orientating phase of data collection. This section provides the reader with a cultural lens for the classroom context of, and interactions between, students and teachers. The remainder of the chapter is divided into two sections: Teaching Episode 1 and Teaching Episode 2. The two teaching episodes present findings of the pretest and describe the student learning across mathematics lessons. Conjectures and summaries are presented at the end of each teaching episode with regards to three themes: (a) mathematics, (b) semiotics, and (c) culture. These conjectures are further investigated in the clinical interviews in Chapter 6. The chapter concludes with a summary of findings across the teaching episodes. Figure 5.1 illustrates an overview of Chapter 5

Chapter 5: Findings of the study
5.1   Chapter Overview
5.2   Background to the Data Collection
5.3   Orientating Phase of Data Collection
5.4   Teaching Episode 1
5.5   Teaching Episode 2
5.6   Chapter Review
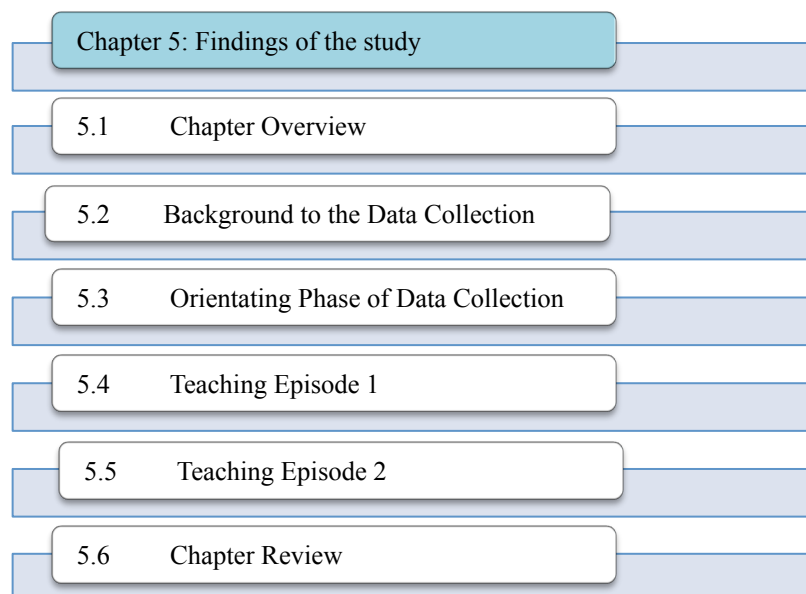
Figure 5.1. Overview of Chapter 5.

5.2   BACKGROUND TO THE DATA COLLECTION

The purpose of this study was to explore how young Australian Indigenous students generalise growing patterns, and in particular, generalise the structures that underpin growing patterns. The data collection began with an *orientating phase* (Phase 1). This consisted of classroom observations followed by a discussion of these observations with the Indigenous education officers and the classroom teacher. The data collection consisted of two distinct subsequent phases (Phase 2 and Phase 3). Each phase comprised two stages, namely *teaching episodes* (pretest and mathematics lessons) and one-on-one *Piagetian clinical interviews* (see Figure 4.4).

Each teaching episode began with a pretest followed by three mathematics lessons. In all, up to 18 students participated in the teaching experiments. Each student was allocated a code (e.g., S1, S2, S3... S18). These codes were maintained across each phase of the data collection. At the completion of each teaching episode, separate Piagetian one-on-one clinical interviews were conducted with a smaller cohort of purposely selected students (n=6) so as to explore more deeply students' ability to generalise growing patterns presented in differing contexts. This smaller cohort was consistent for both Piagetian clinical interviews (Phase 2 and Phase 3). Additionally, the voices of the Indigenous education officers were captured during each phase of the data collection. They examined all the videos recorded during teaching episodes and clinical interviews. These conversations assisted in exploring the cultural aspects of the research. The Indigenous Education Officers were also allocated anonymous codes (IEO1 – Aboriginal Indigenous Education Officer; IEO2 – Torres Strait Indigenous Education Officer).

The analysis of each stage (pretests, lessons, and interviews) in the data collection process resulted in a number of conjectures that informed the type of activity that occurred in subsequent stages. For example, the lessons taught in Teaching Episode 1 resulted in conjectures that informed the activities and questions asked in the following Piagetian clinical interviews. Hence, each stage of the data collection process was contingent on the previous stage.

Two theoretical perspectives informed the study: Indigenous research perspectives and semiotics. From an Indigenous research perspective, it was essential to create a space for dialogue, rather than simply closed observation. When observing students within a particular Indigenous culture setting, there are cultural

nuances that may not be overtly apparent to the researcher; hence, the importance of including an open dialogue with students and Indigenous Education Officers. For this particular study, the relationship also needed to be cultivated with the two Indigenous Education Officers to assist with gathering knowledge that may not be explicitly recognisable by the researcher. In effect, this brought the researcher and the participants into a shared space.

Additionally, the theoretical stance of semiotics was utilised to interpret the interactions between teacher (researcher) and students, and between students and learning context. Semiotics was also used to assist in the selection of the types of materials used to represent growing patterns. The researcher, together with the assistance of the Indigenous Education Officers, explored the relationship between the semiotic vehicles (signs, gesture, language), used by both students and researcher, and students' ability to generalise. These conversations between the researcher and Indigenous Education Officers also assisted in determining if these semiotic vehicles were culturally bound.

5.3 ORIENTATING PHASE OF DATA COLLECTION

In line with Indigenous research perspectives prior to Phase 1 and Phase 2 of the data collection (see Figure 4.4), observations were conducted to gauge the general dynamic of the classroom and to begin to establish relationships with the participating students. It also served to construct a deeper understanding of students' interactions with the teacher, Indigenous education officer, and other students. Annotated notes were collected during regular classroom lessons (1 full day of teaching) conducted by the classroom teacher prior to the first teaching episode. Additionally, the researcher discussed with the teacher and IEO1 (on separate occasions) their experiences with the class. These notes were collated and analysed to better understand the classroom dynamic. To provide the reader with an understanding of the research setting, the next section presents a summary of the data collected during these initial observations.

5.3.1 Summary of Initial Observations

The classroom dynamic is complex and differs from that of non-Indigenous settings. Class numbers are constantly changing and students can be absent for long periods of time. These changes impacted on both the overall class dynamic and

student learning. From IEO1's perspective, students often move between communities due to cultural/family commitments. At times, cultural/family commitments can be extensive, with students moving to other communities to be with relatives. Within this particular class one student had moved across the state and thus changed schools for Term 2 and then returned for Term 3. At other times students may have shorter cultural/family commitments and therefore be absent from school for a smaller amount of time (up to 3 weeks). In addition, some students have been absent for lengthy periods due to health issues. It is also common for new students to enrol mid-term, further impacting on the classroom dynamic. The teacher felt that all these changes influenced the behaviour management of the class.

IEO1 had a palpable influence on the dynamic of the classroom. As stated by the classroom teacher, "This may be either because she is Indigenous, relational [has a strong relationship with students], or has been with the same class cohort for two years" (TI1_22[2]). The teacher felt that, "There is a respect that these Indigenous students have for other Indigenous people" (TI1_24). This was also observed in the initial observations. The classroom dynamics shifted and positive individual and group behaviours were more evident when the Indigenous education officers were present.

The teacher stated that she generally created a learning environment where students were working in small groups. She elaborated that this approach was taken to assist students to feel comfortable when answering questions, limit teasing between students, and to make it easier to address other behaviour management issues. The formation of the groups was based on students' mathematical ability determined by the teacher. There were three mathematics groups: low, average, and high achievers. During mathematics lessons students worked within their ability group. When one group was working with the teacher, they participated in mathematical tasks that focused on symbolic representations, which heavily involved visual representations. Simultaneously, another student group worked with the Indigenous Education Officer, engaging with hands-on experiences in mathematics. Finally, the third group worked together playing mathematical games that focused on the same topic. Many of the activities presented to students were *hands-on*

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[2] TI1_22 – Teacher interview 1_Line 22

experiences. The teacher felt that, "Students worked best [when they were] using hands-on experiences in mathematics." (TI1_25)

The classroom observations provided further insights into how students and their teacher communicated. Students rarely participated in whole class discussions. Generally, when the class was addressed as a whole, the teacher directed the learning that occurred and students rarely posed questions. The teacher endeavoured to encourage students to 'have a go' answering questions in class, and it was apparent that some students were comfortable doing this, while others were not. Language barriers existed, particularly in the mathematics lessons. Students exhibited difficulty understanding the mathematical terminology. They were self-effacing when they had a misunderstanding and would seek help from other students or the Indigenous Education Officer when required. Rarely did they approach the classroom teacher.

Additionally, observations provided evidence that the relationship between students and IEO1 had a positive impact on students' learning. Students felt comfortable in taking risks with IEO1 during classroom interactions. This relationship appeared to encourage students to enter conversations about their learning or to share challenges they were experiencing through class time. As a result, it seemed that this 'feeling of safety' supported these students to be reflective learners and engage in deeper conversations about mathematical concepts.

Observations of students' interactions with their classroom peers provided further insight regarding inter-communication. Many students in the class are related (e.g., cousins) and this family connection also played an important role in the classroom. Students were generally playful with one another in class, often joking with each other. At times this playfulness led to moments where students teased one another if an incorrect answer was given. As a result, some students experienced a feeling of '*shame*' during class interactions and ceased to participate in class discussions. Shame within these contexts can be experienced through shyness, embarrassment, or the breaking of cultural protocol. IEO1 would normally address these matters in class. However, it was obvious that this was an ongoing issue for all class participants. During class time students generally worked together and conversed with one another. They looked to people who they perceived as answering questions correctly to find further information. It appeared that students supported each other's learning through this interaction. These interactions were either verbal

or nonverbal cues. It was as if the entire student body became one student. This occurred irrespective of whether the task was a group or an individual activity. Overall, students had a close working relationship with one another in class.

5.4   TEACHING EPISODE 1

5.4.1 Student Responses to Questions Presented in Pretest 1 Teaching Episode 1

Prior to the commencement of the first teaching episode, 16 students participated in a short test comprising 10 questions (see Appendix D). The aim of the test was to ascertain students' initial understanding of growing patterns to inform the development of Lesson 1. The patterns used in Pretest 1 were taken from environmental contexts. Following discussions with the Indigenous education officers, it was decided to draw on environmental contexts (e.g., houses, fish, possums) so that students could easily engage with the pattern tasks. The reason for this was to give students the opportunity to draw on a context that may be familiar to them, rather than constraining their exploration to the mathematical patterns commonly used when teaching growing pattern concepts (i.e., geometric growing patterns). All questions were read out loud to students. Students were asked to create, copy, continue, predict, and identify quasi-generalisations for a variety of growing patterns. In addition, for three questions, students were required to provide an explanation for how the pattern was growing (Question 4, 8 and 10b).

Students' Prior Understanding of Patterning

The test began with an item that required students to create a pattern. Of the 16 students who completed Question 1, 11 drew repeating patterns. The repeating patterns students drew were either ABABAB patterns (n=5) or ABCABCABC patterns (n=6). Of the remaining students, five did not create a repeating pattern; three created repeating patterns with errors and two students drew pictures. Figure 5.2 illustrates patterns drawn by students in Question 1.

ABABABAB repeating pattern   ABCABCABC repeating pattern
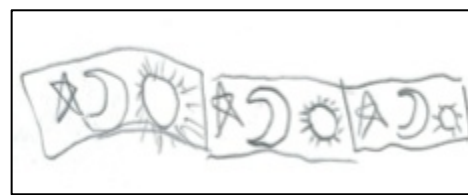
Figure 5.2. Patterns drawn by students in Question 1.

Students' prior classroom mathematics experiences had focused on repeating pattern tasks. Hence, students drew on this prior knowledge to complete Question 1.

Question 2 also required students to create a pattern. This time, students were provided with red and yellow stickers. Stickers were provided for Question 2 to eliminate difficulties some students may experience when drawing a pattern and thus provide an opportunity for these students to display their ability to create patterns. Fourteen students created a pattern using the stickers. Eleven students created an ABAB repeating pattern, and three students created an ABB repeating pattern. The remaining two students placed two stickers on the sheet (S8) or created two circles out of the stickers with no identifiable pattern (S16). Figure 5.3 illustrates the repeating patterns created by students in Question 2.
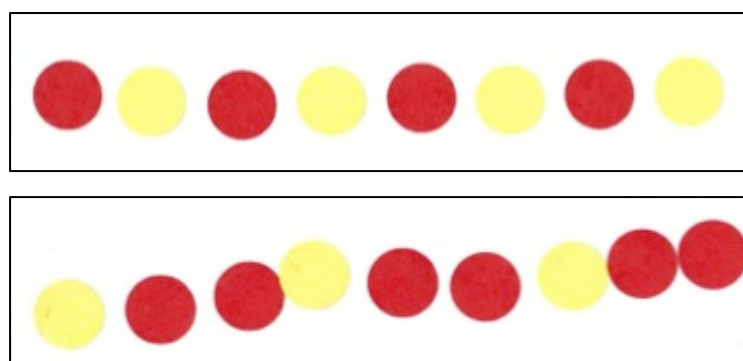
Figure 5.3. Repeating patterns created by students in Question 2.

It appeared that the use of stickers prompted students to create repeating patterns as only two different coloured stickers (yellow, red) were provided. As mentioned above, students' tendency to create repeating patterns reflected their prior experiences in this area of mathematics.

Copying Growing Patterns

Question 3 required students to copy a growing pattern (House pattern - See Figure 5.4). The pattern consisted of a geometric component (houses made of triangles and squares) and a numerical component (the number of houses in each term). Fourteen students copied the growing pattern. Students responded to this task in three different ways: (a) students copied both the geometric component and the numerical component (complete copy); (b) students only copied the geometric component of the pattern (partial copy); and (c) students did not copy either component of the pattern (incomplete copy). Figure 5.4 illustrates the three categories in which students copied their pattern and the number of students for each category.

Complete copy (n=5)   Partial copy (n=9)   Incomplete copy (n=2)
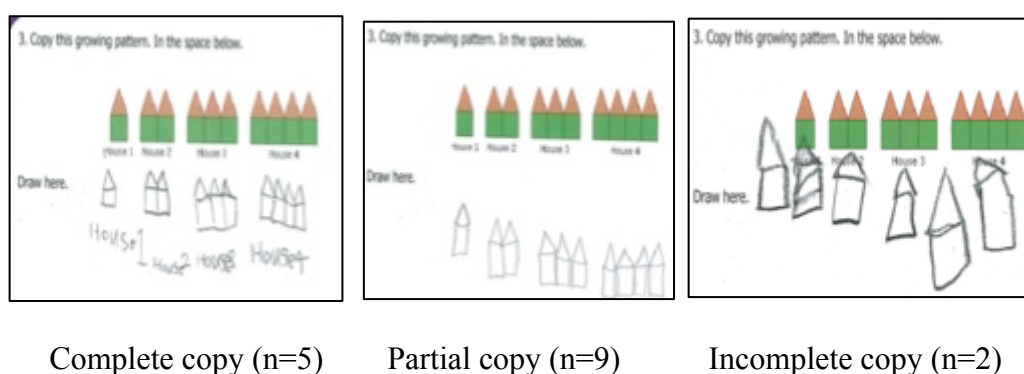
Figure 5.4. Categories of copying a pattern displayed by students in the Pretest 1.

Students were then asked to provide an explanation as to how they believed the pattern was growing (Question 4). These responses fell into four categories: (a) no response – students did not provide an explanation for the task; (b) spatial/visual response – this explanation reflected the actual visual structure of the pattern; (c) numerical response – focused on the numerical component to the pattern; (d) environmental response – drew on the context of the pattern in relation to the natural environment. Table 5.1 displays categories of explanation and the frequency of students for each category.

Table 5.1

Categories, Examples, and Frequency of Students' Responses for each Category for Question 3 of the Pre-test

Category	Example	Frequency
No response		5
Spatial/visual response	It's getting bigger and bigger It's getting longer and longer	6
Numerical response	It's growing with numbers Numbers 1, 2, 3, 4, 5	2
Environmental response	It is growing with water They built it	3

A majority of students, that is 6 out of 11 responses, provided a spatial/visual explanation where the language that was used reflected the actual structure of the pattern.

Continuing Growing Patterns

A new growing pattern was provided and students were asked to continue the pattern (Question 5 – fish pattern). Students could choose to continue the pattern in both directions, that is, to the left (decreasing) and to the right (increasing), or just one direction. Six students continued the growing pattern; three of these students continued the pattern in both directions and three continued the pattern only to the right. Some students copied the pattern rather than continuing the pattern. Students exhibited more accuracy in copying growing patterns (n=14) (Question 3) than in continuing growing patterns (n=6) (Question 5). Figure 5.5 illustrates an example of students' responses to Question 5 continuing a growing pattern.
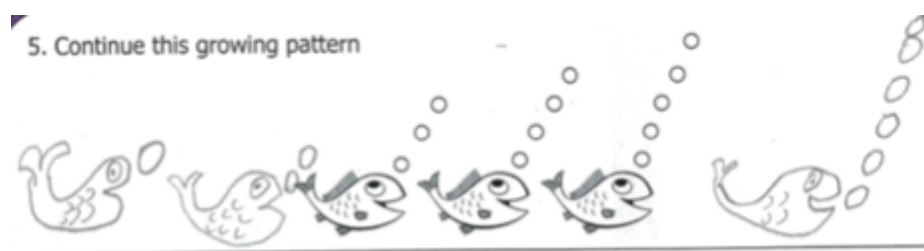
Figure 5.5. Example of students' responses to continuing a growing pattern.

Predicting Quasi-generalisations (near and far)

Students were asked to predict terms (4th and 10th) beyond the pattern presented for Question 6 (possum pattern). Table 5.2 displays frequency of students with the correct response to each prediction.

Table 5.2

Frequency of Students with Correct Predictions for Question 6

Prediction	Frequency
4th term (8 eyes)	7
10th term (20 eyes)	6

It is clear that most students did not understand how to predict terms beyond the pattern provided in the question. There was little difference between the number of students correctly predicting the 4th and 10th term.

Students were asked to provide a way to determine how many possum's eyes there would be if there were 376 possum tails (Quasi-generalisation - Question 9). Two of the 16 students provided an explanation. One student wrote that they would 'count in twos' and the other student wrote that they would 'draw the possums and count the eyes off'. It is conjectured that the student who responded 'count in twos' was beginning to demonstrate recursive thinking in relation to this growing pattern. No students wrote that they would 'double the number of possum tails', the quasi-generalisation.

Creating Growing Patterns

Finally, students were asked to create their own growing pattern and provide an explanation of how it was growing. Students provided patterns that fell into three categories, repeating patterns (n=9), environmental growing patterns (n=5), and geometric growing patterns (n=2). Repeating patterns are classified as patterns that have a discernable unit of repetition (i.e., ABABAB). Environmental growing patterns are patterns that draw on elements of the natural environment and display growth. Geometric growing patterns are a type where the growth is displayed using geometric shapes. Figure 5.6 illustrates the three different types of patterns presented by students for Question 10a.

Repeating pattern (n=9)   Environmental Pattern (n=5)   Geometric growing pattern (n=2)
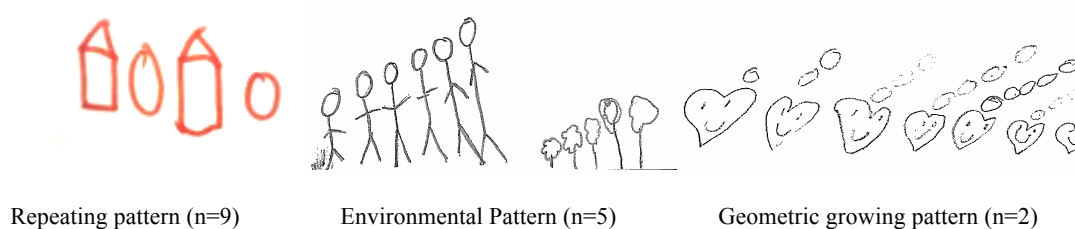
Figure 5.6. Examples of patterns presented by students for Question 10a.

Four students provided an explanation as to how their pattern was growing. The remaining students did not respond to Question 10b. Table 5.3 displays the explanation category, pattern and written response provided by the four students for Question 10b.

Table 5.3

Explanation Category, Pattern and Student Response Provided for Question 10b

Response category	Pattern drawn	Written Response
Environmental response		My pattern is growing by eating. The plants are growing by sitting in the sun.
Environmental response		My pattern is growing by shade, sun and water.
Numerical response		It is growing by one block, two blocks, three blocks, four blocks and five blocks.
Numerical and Spatial/visual response		It is growing by 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. It is getting bigger and bigger

Further opportunity was given, upon the completion of Pretest 1, for students to verbalise their process for developing their own growing pattern. This gave students the opportunity to articulate their understandings without having to write. Additional responses given by the other students were:

- Environmental response: They grow up. You have to grow up [Student gestures R hand to indicate growing up]. It is eating healthy things like fruit and vegetables. It's on more bigger, bigger, and bigger [Student gestures low first with both hands and then to chest height and then above head].

- Environmental response: It's like gardens, flowers and plants. You need water and food then you go to work when you grow up.

- Spatial/visual response: In maths its patterns like triangles and rectangles, it gets more bigger.

- Numerical response: Putting more and more stuff into it [Student gestures as if putting things into and imaginary box]. It is about putting more numbers into it. It just keeps growing and growing.

- Spatial/visual response: It gets higher and higher.

- Numerical and Environmental response: It grows one, two, three, four – like plants grow big from water.

The verbal responses were accompanied with gestures as students articulated their understandings. It was apparent that students' responses were enhanced when given the opportunity to talk through their thinking rather than providing a written response. Students also could make links between their own environmental contexts and link this with the mathematics, as demonstrated in the last response.

Summary of Pretest 1

Pretest 1 provided information regarding students' prior understanding of patterning; in particular, students' understanding of growing patterns preceding the formal introduction of the concept. As a result of this analysis, key themes were considered and served as a platform for developing ideas for lesson 1, 2, and 3 of Teaching Episode 1. The following links the key themes from the initial data collection analysis with mathematics, semiotic and cultural perspectives.

First, from a mathematical perspective, the results of Pretest 1 demonstrated that most students were familiar with repeating patterns. However, students also proved that they were capable of engaging with growing patterns. It was evident that students could copy growing patterns (n=14); yet many students were not copying both variables of the pattern (house and number). The majority of students were only copying the single variable, the house. Students had difficulty continuing growing patterns and predicting further terms of patterns in the test. Nevertheless, some students were beginning to demonstrate that they could work with patterns where they needed to coordinate the relationship between the two variables (term and pattern). Finally, some students were able to create growing patterns. As a result of these observations, the lessons in Teaching Episode 1 continued to explore students'

understanding of developing the relationship between two variables. It was conjectured that this would support students' development of co-variational thinking, and serve to provide them with an enhanced understanding of the structure of growing patterns.

Second, from a semiotic perspective, students displayed an aptitude in Pretest 1 for working with patterns where both variables were explicit in the pattern (e.g., making the variables explicit in the pattern appeared to assist students to attend to the co-variational nature of the pattern). While some of these signs were explicit (e.g., house and the number), other patterns embedded both signs into the one object (e.g., possum eyes and tails). This second pattern meant students had to attend to the pattern as a singular structure rather than two separate abstractions. While Pretest 1 provided some insight about how students engaged with the semiotic structure of these patterns, further investigation was needed. Thus for Teaching Episode 1, growing patterns were provided where both the variables were visually explicit and represented as two separate concrete items. The intention was to allow students to manipulate both variables of the pattern. At other times, patterns were provided for Teaching Episode 1 where the variables were embedded in a singular hands-on item (e.g., Kangaroo pattern).

Third, from a cultural perspective, discussions with the Indigenous education officers provided clear directions with regard to the types of patterns deemed appropriate to begin the study. They confirmed that there were repeating patterns within both Aboriginal and Torres Strait Island culture. These patterns could be seen in environmental contexts, such as art and dance. It was evident from Pretest 1 that some students used environmental contexts to display patterns and provide explanations as to how the pattern was growing. IEO1 and IEO2 also shared information that while there were some growing patterns in their culture, for example kinship groups, there were none they were aware of that would be suitable for these young students. Thus, the results of Pretest 1, and consultations with the Indigenous Education Officers identified that the exploration of growing patterns should be initially situated in an environmental context for Teaching Episode 1, as students appeared to naturally engage with patterns from an environmental context.

Finally, the conjectures proposed from the analysis of Pretest 1 and discussions with the Indigenous education officers were:

- Lesson 1 Conjecture 1: Exploring growing patterns from environmental contexts assists Indigenous students to relate growing patterns to their prior experiences.

- Lesson 1 Conjecture 2: Making both variables of growing patterns visually explicit assists students to identify the co-variational relationship

- Lesson 1 Conjecture 3: Manipulating hands-on materials that represent the variables of a growing pattern allows students to better attend to the pattern structure.

These conjectures were explored in lesson 1 of Teaching Episode 1.

5.4.2 Student Learning Across Lesson 1, 2, and 3 of Teaching Episode 1
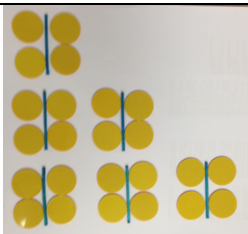
Analysis of the data indicates students' learning interactions, and demonstrated learning challenges that students experienced across the three lessons. Particular attention was paid to what teaching actions/strategies, resources, or interactions assisted students in overcoming learning barriers. While the initial data-collection phase provided a benchmark for the lessons, ongoing analysis occurred at the conclusion of each individual lesson and contributed to the subsequent lessons. This was achieved through continued discussion with IEO1, development of semiotic theories, and ongoing examination of students' interactions. IEO1 provided further insights into students' learning and cultural signs. The two-fold approach of semiotics being used in this study intended that (a) the structure of the pattern was determined on semiotic theories, and (b) semiotic signs were specifically selected to assist students to attend to the pattern. At the conclusion of each lesson, this analysis was conducted to determine the new conjectures focusing on student learning and teacher actions for the following day.

During Teaching Episode 1, all lessons focused on how students attended to the structure of a growing pattern. The growing patterns presented to students were multiplicative patterns, which are patterns where the two variables directly relate to each other (e.g., the number of petals for the flowers was four times the number of flower centres). Additionally, hands-on materials were used to create all the growing patterns. The inclusion of hands-on materials was deliberate, as it provided the opportunity for students to physically manipulate the pattern. For example, students copied and extended simple growing patterns (e.g., butterfly pattern) using hands-on

materials (e.g., match sticks and double sided counters). Furthermore, the structures of the patterns were visually explicit, meaning that students were provided with a visual cue (sign vehicle) that attended to both variables (term and pattern). The growing pattern tasks used during lessons 1, 2, and 3 are referred to frequently throughout the following section. Table 5.4 displays the lesson, pattern name, pattern, and semiotics of the pattern students engaged with during Teaching Episode 1.

Table 5.4

Lesson, Pattern name, and Patterns Students Engaged with During Teaching Episode 1

Lesson	Pattern name	Pattern	Semiotics of pattern
Lesson 1	Butterfly $4n$		Visually explicit 2 sign vehicles – counters (pattern – dependent variable) and sticks (term – independent variable) The signs embedded in the pattern have the potential to be physically separated.
Lesson 2	Feet and body on a number track $2n$		Visually explicit 2 sign vehicles – feet (pattern – dependent variable) and body (term – independent variable) The signs are embedded in the pattern, however, they cannot be physically separated from each other.
Lesson 3	Kangaroo $2n$		Visually explicit 2 sign vehicles – ears (pattern – dependent variable) and tail (term – independent variable) The signs are embedded in the pattern, however, they cannot be physically separated from each other.

It appears that sign vehicles play the role of a cognitive tool in assisting students to attend to the structure of the pattern. For the three lessons both iconic and indexical sign vehicles represented the immediate sign object, the concept of attending to the structure of a growing pattern. The sign vehicles served as a mediator between the sign object (structure of a pattern) and the sign interpretant (student) (Saenz-Ludlow & Zellweger, 2012). The sign vehicles used in the butterfly

pattern were both iconic (visual and hands-on materials) and indexical (gesture from the researcher during teaching actions). The iconic sign vehicles for the butterfly pattern can be seen in the representation and hands-on materials used to create the pattern. These iconic signs were also separable as students could physically manipulate the two variables. The indexical signs included language, such as indexical words (that, this, here, and there), and gestures by the researcher during the lessons, including pointing to the iconic signs.

The two variables were visually *explicit*, and *embedded* and were easily individually manipulated (separated). The pattern was visually explicit as the two variables were easy to identify in the pattern. This was achieved by the use of colour (blue and yellow) and materials (matchsticks and counters). Additionally, to construct the pattern students needed to attend to the two variables that were embedded in the butterfly pattern. The matchsticks (term) and the counters (pattern) were both integral elements in the construction process. Each sign however, could be individually manipulated into the two variables. This pattern was purposefully structured to frame students to attend to both variables through the use of these iconic sign vehicles.

During lesson 1, students copied and continued the butterfly pattern using hands-on materials provided. As students were working with the materials, it was evident that they attended to the iconic sign vehicles. By attending to the signs, students copied and continued the structure of the pattern in different ways. Three categories describe the nature of how students attended to the structure of the pattern when copying and continuing: high structural awareness, partial structural awareness, and low structural awareness. Figure 5.7 illustrates three structural awareness categories demonstrated by students.

Low structural awareness   Partial structural awareness   High structural awareness

S4, S7, S8, S9   S5, S10, S11, S14, S16   S1, S3, S6, S13

Note: Results from S2, S12, and S15 are missing, as students were absent from school
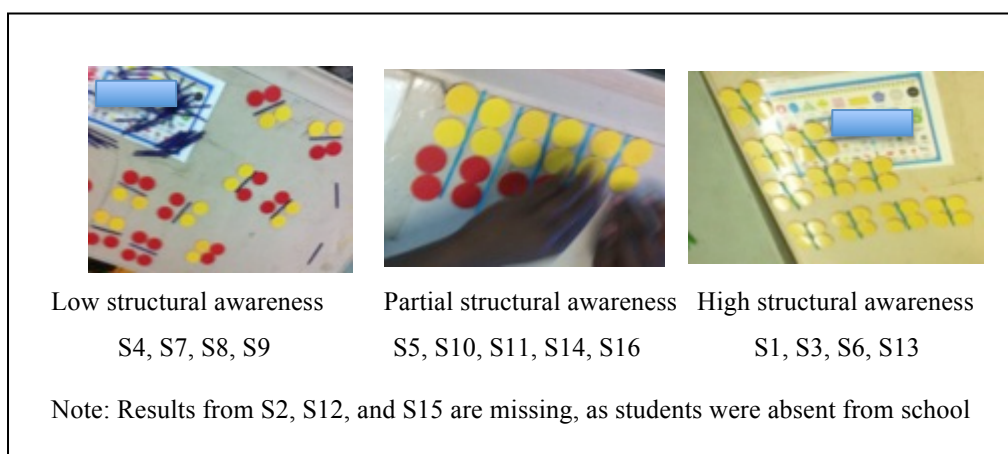
Figure 5.7. Structural awareness demonstrated by students copying the butterfly growing pattern.

Students that demonstrated high structural awareness could copy and continue the butterfly pattern displayed in the lesson. Also, those students who had high structural awareness also performed better on Pretest 1 than those students who had low structural awareness. Partial structural awareness included students who had copied the structure of the butterfly, but had formatted their pattern in a straight-line. Students that displayed low structural awareness randomly constructed butterflies on the desk and could not copy the example provided by the researcher. It was evident that some students experienced challenges while copying the pattern, which impacted on their ability to see the structure. The common problem students exhibited while copying the pattern was the ability to accurately attend to the visual representation created by the researcher on the whiteboard. Students with low structural awareness placed butterflies all over the desk. These students required explicit direction from either IEO1 or the researcher. This included a gestural cue (indexical sign) to the visual display at the front of the room and then to their pattern. Each pattern term, that is butterfly wings and body, was clearly identified for students to copy the pattern successfully.

From a semiotic viewpoint, while students were copying their pattern, it became evident that they were attending to the hands-on material in different ways. The majority of students initially placed a singular butterfly body (matchstick) on their desk and then immediately added the four butterfly wings (counters). These students were considered to be attending to the term and pattern simultaneously. By contrast other students were observed to split (S6 and S10) sign vehicles. These

students first placed an array of matchsticks on their desk to represent the butterfly bodies and then added the wings retrospectively. Students who copied the pattern in this manner were considered to be separating the two iconic signs. This provided an insight into how students were engaging with, and attending to, the structure of the pattern. Figure 5.8 provides an example of how students attended to the sign vehicles in the growing pattern.
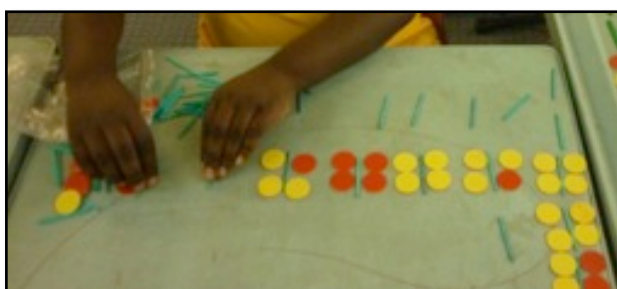
Figure 5.8. Student 10 separating the two signs while copying the growing pattern.

During the same lesson students were beginning to make conjectures about the butterfly pattern, that is, the relationship between the number of bodies and the number of wings. Students began to describe the relationship between the two variables (the number of body parts and the number of wings), and some students began to make spontaneous predictions and provide justifications for their thinking. At this point, it was clear that some students had begun to shift their understanding and focus to the relationship between the number of butterfly bodies and the number of wings. An example of this was a discussion S10 had with the researcher.

S10: 25 Miss.

R: Twenty-five for what?

S10: Altogether for butterflies

R: Can you make it for me? *(S10 makes five butterflies with four wings for each butterfly)*

R: Let's count the butterfly bodies

S10 & R: one, two, three, four, five *(R pointing to each body)*

R: Let's count the wings

R & S10: four, eight, twelve, sixteen, twenty *(R gestures to the wings)*

R: So if I have twenty wings how many butterflies?

S10: oh it's five Miss.

This student had separated the two sign vehicles while copying the pattern. S10 had all the matchsticks arrayed on his desk and then placed the counters on the matchsticks to form the butterflies. It is uncertain if this assisted the student to see the relationship between the two variables. What was clear was that the student had difficulty explaining the multiplicative structure or the 'fourness' of the pattern. Possibly the mathematical language to describe this multiplicative structure was not accessible to the student.

At the conclusion of Lesson 1 new conjectures were posed for Lesson 2. These conjectures were:

- Lesson 2 Conjecture 1: Exploring growing patterns where the structure is multiplicative (e.g., double) assists students to generate the pattern rule.
- Lesson 2 Conjecture 2: Providing growing patterns where the variables are embedded in the pattern ensures that students attend to both variables.
- Lesson 2 Conjecture 3: Embodying the mathematical structure of growing patterns assists students to explain the pattern structure.

Aiming to expand on the insights from Lesson 1, Lesson 2 focused on students continuing and predicting patterns where the variables could not be separated. The feet and body pattern had two iconic sign vehicles representing the variables; both were embedded in the pattern; however, the signs could not be physically separated. This pattern required students to physically manipulate the pattern: that is, physically engage with the pattern, as they themselves were the sign vehicles (e.g., student feet and body). From Lesson 1, it was apparent that students had difficulty describing the 'fourness' of the butterfly pattern, therefore the pattern presented in Lesson 2 focused on the multiplicative structure of 'twoness' or 'doubling', a concept that was well known to students.

It was apparent in Lesson 2 that there was a shift in student thinking as they made predictions about the pattern. Students began to see the 'twoness' of the pattern in relation to the number of students on the number track and the number of feet. Students were gesturing as they were counting the number of feet on the track (see Section 5.6.2 and 5.6.4). This was a student-created indexical sign. Evidently,

students were beginning to attend to the structure of the pattern and explore co-variational thinking. While students were beginning to see the 'twoness' of the pattern, some had difficulties expressing the multiplicative relationship between the two variables. It is possible that the shift in sign vehicles for this pattern (both iconic sign vehicles that could not be physically separated) also caused a shift in students' ability to see the structure of the pattern.

At the conclusion on Lesson 2 new conjectures were posed for Lesson 3. These conjectures were:

- Lesson 3 Conjecture 1: Transferring mathematical knowledge between patterns with the same multiplicative structure is difficult.
- Lesson 3 Conjecture 2: Providing students with the mathematical language used to describe multiplicative structures assists students to generalise the pattern.
- Lesson 3 Conjecture 3: Providing growing patterns where the variables are embedded and cannot by physically separated from each other assists students to attend to both variables simultaneously.

The notion of a pattern where both variables were unable to be physically separated in the pattern structure was explored in Lesson 3. Many students displayed a marked shift in attending to the structure of the growing pattern during this lesson. Students began to demonstrate co-variational thinking to explain the relationship between the two variables. The pattern used in Lesson 3 was the kangaroo pattern. This pattern was visually explicit, embedded, and the variables (term and pattern) were unable to be separated because the ears and tail (sign vehicles) were combined in the one structure.

Students also began to make generalisations about the pattern in Lesson 3. This was the first time students began to generalise in a lesson. Students could state that the rule for the pattern was, "Doubling it (the tails)." During the lesson students were asked if they could predict further terms ("If I had 10 kangaroo tails how many ears would I have?") and identify the relationship between the kangaroo tails and the number of ears. Below is an example of Student 3 predicting beyond the pattern presented and generalising the pattern structure.

R: If I had 10 kangaroo tails, how many ears would there be?

S3: Twenty (called out)

R1: How do you know that?

S3: I just added the same number.

R1: What do you mean?

S3: There were 10 tails so I just added another 10 to get the answer.

Additionally, some students were beginning to identify the pattern rule. Below is an excerpt from the lesson.

R: So if I know how many tails I have, how do I work out how many ears there are?

S1: You're doubling it.

R: You're doubling what?

S1: The tails

5.4.3 Summary of Teaching Episode 1

From a mathematics perspective, students indicated aptitude in copying, continuing and creating growing patterns prior to formal teaching. After the three lessons, students were beginning to identify rules for simple growing patterns. For all students the multiplicative language for seeing the 'fourness' was challenging to explain, and thus Lesson 3 multiplicative structure focused on 'doubling'. The setting up of the activities allowed students to see the structure of the pattern. It appeared beneficial for the signs of both variables (pattern and position) to be embedded in the visual pattern. The use of hands-on materials represented the growing pattern in such a way that students could attend to both signs and assisted some students to shift towards co-variational thinking. Using the kangaroo tail and ears assisted students to begin to see the relationship between the two variables, as both position (tail) and pattern (ears) were explicit.

From a semiotic perspective, there was a tendency toward gesture during the three lessons on growing pattern. The gesture was two-fold: gesture as embodiment of the task, and gesture working with language for communication of mathematical concepts. Gesture as the embodiment of the task requires students to interact with the hands-on materials. Both researcher and IEO1 agreed that the interaction with the

hands-on items assisted students to think about the growing pattern. These physical processes assisted students to objectify the task. Secondarily, the use of gesture as an adjunct to language for communicating ideas about growing patterns was observed. Many students used gesture to supplement the language used in communicating their ideas about growing patterns. This theme was consistent with literature, suggesting that gesture and language play significant roles in the learning of new mathematical concepts.

From a cultural perspective, it became apparent that students responded positively when the hands-on materials utilised in the learning activities were related to students' local environment. The use of contextualised patterns provided opportunity to discuss the pattern in terms of language which was already accessible for students, such as in the case of kangaroo ears and tails. This too was evident through the discussion with the Indigenous Education officers. The opportunity for students to engage with the hands-on materials assisted students to discuss the structure of the pattern. Students manipulated the hands-on items and used them during their explanations. Engaging with hands-on materials acted as a medium between the mathematical structure and students' thinking as they began exploring abstraction.

Additionally, students worked well together, supporting each other through the learning process. The first lesson provided cultural insights about how students communicated about the mathematics. It was evident that students would ask assistance from each other while completing the task. There were many discussions about whether students' patterns were correct. Often students would check their work with the high performing students in class.

It appeared that storytelling assisted students to develop their ideas about the patterns presented. The researcher began Lesson 2 by telling students a story about how there were butterflies in her garden. During the story, the researcher highlighted to students the relationship between the number of butterflies and the number of wings. For example, 'On the first day there was one butterfly with four wings and on the second day there were two butterflies with eight wings altogether'. When students had continued their butterfly pattern, they too used storytelling to describe how the pattern was growing.

The following section presents findings from Teaching Episode 2. Though the first clinical interview was conducted after TE1, these findings will be presented later in the chapter in Section 5.6.

5.5   TEACHING EPISODE 2

5.5.1 Student Responses to Questions Presented in Pretest 2 Teaching Episode 2

Prior to the commencement of the second teaching episode, 14 students participated in a short test comprising 12 questions (see Appendix E). The number of students had changed between Teaching Episode 1 and Teaching Episode 2, three students had left the school (S12, S13, S15) and two new students had arrived at the school (S17 and S18).

The aim of the test was to ascertain students' initial understanding of geometric growing patterns, similar to those presented in mathematics textbooks and prior research. This data also informed the development of lessons for Teaching Episode 2 of the data collection. The geometric growing patterns presented to students contained geometric shapes such as, triangles, circles, squares, and irregular dodecagon (shape of a cross) (See Figures 5.9, 5.10, and 5.11). All questions were read out loud to students. Students were asked to continue, create, make predictions, and identify the pattern rule for a variety of growing patterns.

Continuing Geometric Growing Patterns

Students were asked to continue a geometric growing pattern constructed from triangles (Question 1). The first four terms of the pattern were presented to students and they were asked to continue the growing pattern (See Appendix D). Two out of 14 students correctly continued the growing pattern. The remaining students either copied the pattern (n=3), or incorrectly continued the pattern (n=9). Students who incorrectly continued the pattern either (a), did not draw enough triangles (n=5), or (b), drew the correct number of triangles but did not draw them according to the structure of the given pattern (n=4). Figure 5.9 illustrates examples of students continuing the pattern with incorrect structure or orientation of the triangles.

Figure 5.9. Student examples of continuing geometric growing patterns with incorrect structure.

Creating Geometric Growing Patterns

Students were given the opportunity to create their own geometric pattern using a combination of triangles and circles (Question 2). Results indicated that eight students created repeating patterns and six students created geometric growing patterns. Figure 5.10 illustrates examples of students' geometric growing patterns.
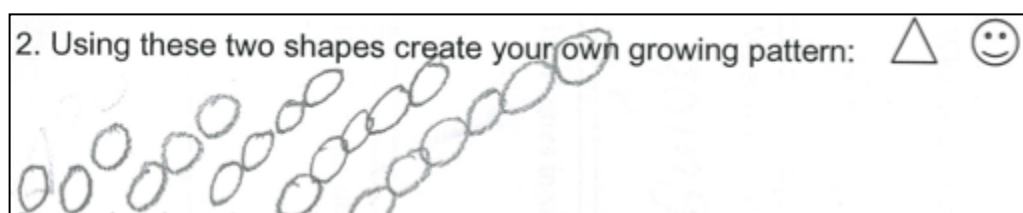
Figure 5.10. Student examples of geometric growing patterns created in Pretest 2.

Predicting Geometric Growing Patterns

A major concept explored in Pretest 2 was predicting further terms in a geometric growing pattern. Question 3 explored students' ability to identify the structure of a growing pattern and to use this understanding to predict the 5th, 6th, 10th, and 14th terms in the pattern. The pattern was structured so that the two variables were explicit and embedded. From a semiotic perspective, there were three iconic sign vehicles used in this pattern: (a) black squares, (b) white squares, and (c) numbers positioned under each structure to identify the term of the pattern. For example, the black squares and the numbers under the pattern highlighted the pattern position for the students. All three signs were considered to be explicit. Figure 5.11 illustrates the sign vehicles in relation to the pattern used in Question 3.
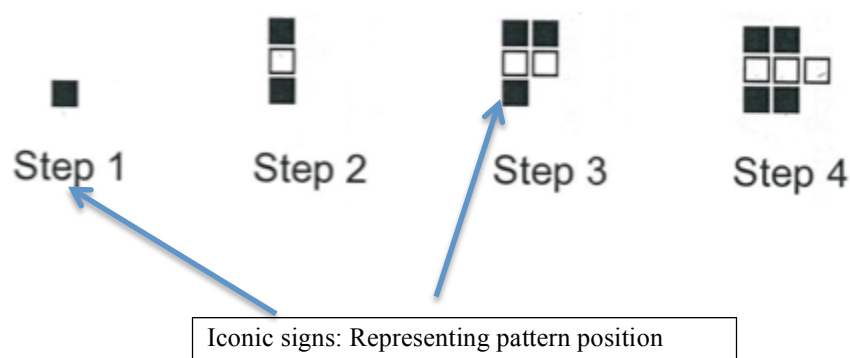
Figure 5.11. Sign vehicles displayed in the geometric pattern used in Question 3 of Pretest 2.

Part A of Question 3 asked students to attend to only the white coloured squares in the pattern. Students were asked to predict how many white squares there would be in Term 5, Term 6, and Term 10. Part B asked students to predict how many black squares there would be in Term 5, Term 6, and Term 10. Table 5.5 displays the frequency of correct responses for part A and B of Question 3.

Table 5.5

Percentage of Correct Responses for Part A and Part B of Question 3 in Pretest 2 (n=14)

	Frequency		
	Term 5	Term 6	Term 10
Part A - White squares	86%	79%	43%
Part B - Black squares	71%	71%	64%

Most students were able to identify the number of white squares needed for Term 5 (n=12) and Term 6 (n=11). However, when students were required to identify the white squares for Term 10, less than half of the cohort could identify the required number of white squares (nine white squares) needed for the growing pattern. It appears that students were more competent in determining the number of black squares required in Part B of the question.

Part C of Question 3 required students to determine which term had 13 white squares and 14 black squares present in the pattern (predicting the 14th term). Two students could identify correctly that this pattern would be present at Term 14. It is evident that many students did not make the link between the number of black squares and the term number of the pattern.

Question 4 (cross pattern) also required students to predict terms beyond those presented in the test. Students needed to draw the pattern for the 5th (10 crosses) and 10th term (20 crosses). Eleven students correctly drew 10 crosses at the 5th term. When asked to predict the 10th term, six students correctly drew 20 crosses. It is evident that students found it easier to predict near terms (5th term) than far terms (10th term).

Identifying a Rule for Geometric Growing Patterns

Students were required to provide a rule for the pattern presented in Question 4. Four students provided the following responses:

S1: You can double it, *n* equals double (Written response).

S2: Double it. (Verbal response)

S6: *n* and then count another *n*. (Verbal response)

S4: You can double it. N d 10. (Written response).

Summary of Pretest 2

Pretest 2 provided information regarding students' understanding of patterning when using geometric growing patterns. Once again, as a result of this analysis, key themes were considered and served as a platform for developing ideas for Lesson 1, 2, and 3 of Teaching Episode 2. The following links the key themes from this analysis with mathematics, semiotic and cultural perspectives.

First, from a mathematical perspective, the results of the Pretest 2 demonstrated that most students had difficulties when engaging with geometric growing patterns. It was evident that students were challenged when attempting to continue the geometric growing pattern presented in Question 1. Two students could continue the triangle pattern presented in this test, however, when considering Pretest 1, six students continued the growing pattern (fish pattern). There are two potential reasons for this. First, the context was accessible to students, and second, the fish pattern was much easier for students as it was only increasing by one each time. Pretest 2 predominately focused on predicting elements beyond the pattern provided. Students appeared to have more success when predicting terms that followed sequentially to the pattern, rather than terms that were further from the original pattern presented. Finally, some students could generalise the rule for the geometric

pattern in Question 4. Students used alphanumeric notation when generalising a rule for the growing pattern. Potentially, the structure of the pattern presented in two rows may have assisted students to see the doubling nature of the pattern when generalising.

Second, from a semiotic perspective, the geometric pattern presented in Question 3 provided interesting results for consideration. The pattern was structured so the black tiles clearly linked to the term position of the pattern. This iconic sign was also visually related to the number presented under each pattern structure. It was anticipated that students would see the link with the black tiles and the number, and thus the pattern would be easier to predict than the white tiles. However, this proved not to be the case. Students were more successful predicting the number of white tiles needed in further terms of the pattern. The white tiles were placed in a single row through the centre of the each structure, unlike the black tiles that were separated above and below the white row (see Figure 5.11). The sequence of white tiles was easier to predict as the visual representation framed students to see the number of white tiles growing by one each time. Therefore, when predicting the 5th and 6th term students just continued counting on. However, when predicting the 10th term less than half the cohort answered correctly. Evidently, further exploration of iconic signs in geometric growing patterns was needed in Teaching Episode 2.

Third, from a cultural perspective, discussions with the Indigenous education officers yielded a belief that students were more successful when working with growing patterns that presented a context familiar to students. Thus, this was considered in the selection of patterns presented to students in Teaching Episode 2.

Finally, the conjectures proposed from this analysis for lesson 4 of Teaching Episode 2 were:

- Lesson 4 Conjecture 1: Exploring growing patterns from environmental contexts assists Indigenous students in relating growing patterns to their prior experiences.
- Lesson 4 Conjecture 2: Explicitly modelling the relationship between the variables in the growing pattern assists students to use the alphanumeric notation to describe the generalisation.

- Lesson 4 Conjecture 3: Using semiotic bundling, (i.e., using gesture, language and manipulation simultaneously) assists students to identify the structure of the pattern.

5.5.2 Student Learning Across Lessons 4, 5, and 6 of Teaching Episode 2
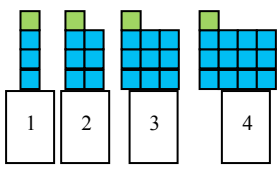
Data from the lessons shows students' learning, interactions, and demonstrated learning challenges that were experienced across the three lessons in Teaching Episode 2. Similar to the data presented in Section 5.5.1, particular attention was paid to what teaching actions/strategies, resources, or interactions assisted students in overcoming learning blocks. While Pretest 2 and data analysed from Teaching Episode 1 provided a benchmark for the lessons, ongoing analysis occurred at the conclusion of each lesson and contributed to the subsequent lessons. This was again achieved through continued discussion with IEO1 at the completion of each lesson, development of semiotic theories, and ongoing examination of students' interactions. The two-fold approach of using semiotics in this study meant that the considerations of the structure of the pattern, and the signs developed to assist students to see the structure, were also considered. At the conclusion of each lesson, this analysis was conducted to determine the content of the lesson and teacher actions for the following day.

During Teaching Episode 2, all lessons focused on how students attended to the relationship between variables (co-variation) presented in geometric growing patterns. These patterns were given an environmental context. For example, geometric patterns commonly explored in mathematics texts were refined to represent a context accessible to students, such as flowers. These flowers were created using double-sided counters so that they still contained the geometric component of the growing pattern, rather than pictures of real flowers. The growing patterns presented to students were multiplicative patterns, patterns where the two variables were directly related to each other (e.g., the number of petals for the flowers was four times the number of flower centres). Students were also given the opportunity to explore geometric patterns with a constant. The constant is unchanging in each of the pattern positions. In a growing pattern $2x+5$, '+5' is the constant in the pattern. It was believed that adding a constant increased the complexity of the pattern.

Again, hands-on materials were used to create all the geometric growing patterns. Additionally, the patterns were visually explicit, meaning that students were provided with a visual cue (sign vehicle) that attended to both variables (term and pattern). This was of particular importance in the introduction of the constant term to students. The growing pattern tasks used during lesson 4, 5, and 6 of Teaching Episode 2 are referred to frequently throughout the following section. Table 5.6 displays the lesson, pattern name, patterns, and semiotics of the pattern students engaged with during Teaching Episode 2.

Table 5.6

Lesson, Pattern Name, Semiotics of the Patterns Students Engaged with During Teaching Episode 2

Lesson	Pattern name	Pattern	Semiotics of pattern
Lesson 4	Caterpillar 2n	Day 1 Day 2 Day 3	Visually explicit 2 sign vehicles – counters (pattern – dependent variable) and numerical identification of days (term – independent variable) The signs are not embedded in the pattern and have the potential to be physically separated.
Lesson 5	Flower 5n		Visually explicit 2 sign vehicles – petals (pattern – dependent variable) and centres (term – independent variable) The signs are embedded in the pattern and have the potential to be physically separated.
Lesson 6	Classroom pattern 3n+1	1 2 3 4	Visually explicit 3 sign vehicles – blue tiles (pattern – dependent variable) and numerical identfication of the term (term – independent variable), constant term highlighted by a green tile. The signs are not embedded in the pattern however, and can be physically separated from each other.

Continuing on from Teaching Episode 1, the immediate sign object focused on the concept of attending to two variables within a growing pattern across the three

lessons in Teaching Episode two. The sign vehicles used in the caterpillar pattern were both iconic (visual and hands-on materials) and indexical (gesture from the researcher during teaching actions). The iconic sign vehicles for the caterpillar pattern can be seen in the representation and hands-on objects used to create the pattern. These iconic signs could be physically separated, as students could physically manipulate the pattern. The indexical signs included language, such as indexical words (that, this, here, and there), and gestures by the researcher during the lessons, including pointing to the iconic signs. The flower pattern was visually explicit; this was achieved by the use of colour, and the iconic signs were separable as students could physically manipulate the term (red centres) and the pattern (yellow petals). The construction of this pattern ensured that students attended to the two variables as they were embedded within the flower pattern. The red centre (term) and the yellow petals (pattern) were both integral elements in the construction process.

Further semiotic structures were considered for the classroom pattern used in Lesson 6. This pattern of indexical signs was similar to that of the previous two lessons; however, the iconic signs changed. Iconic signs were used to frame students to see the structure of the pattern. The term was not represented by colour or a number card, but was displayed in the structure of the pattern. The term of the pattern was structured into rows, and students needed to be framed to attend to this structure. For example, for Grade 1 there was one row of three tiles, for Grade 2 there were two rows of three tiles (see Table 5.6) Framing students to attend to the structure involved heavy use of indexical signs within the lesson. The indexical signs used were gesturing to the pattern structure and 'chanting/singing' the structure of the pattern with students. This also assisted students to attend to the variables within the classroom pattern. Another iconic sign used in the classroom pattern was the green tile to display the constant within each term. The green tile represented the teacher in each Grade level (term). By using the green tile, students could clearly identify what was the same in each of the patterns and thus clearly identify the constant term. This teaching process and semiotic structure of the pattern acted as a cognitive scaffold to assist students to attend to all variables in the pattern.

The patterns used in Teaching Episode 2 were initially constructed to represent models students would see in mathematic textbooks. These types of patterns often require students to make a mathematical abstraction of the representation. It was

decided to provide students with a story, as story telling appeared to be an integral part of students' explanations in Teaching Episode 1. Traditional Aboriginal and Torres Strait Island cultures used storytelling as one means of sharing knowledge from one generation to the next (Archibald, 2008). By providing students with a story for the pattern, they were able to explore the representation that would otherwise require mathematical abstraction if just displayed on its own.

During Lesson 4 of Teaching Episode 2, students copied, continued, predicted and created growing patterns using the materials provided. As mentioned above, the use of storytelling in Teaching Episode 1 engaged students; therefore the same teaching approach was used in this lesson with the caterpillar pattern. It was explained to students that there was a caterpillar in a garden and each day his body grew. Initially, students were asked to copy and continue the pattern, and many students could do this. It was anticipated that students who physically manipulated the hands-on materials to create the structure of the pattern made links between the structure and the pattern rule or generalisation.

Students were asked to form a generalisation about how the caterpillar pattern was growing. Some students could explain that the pattern was doubling, while others displayed recursive thinking when considering how the pattern was growing. For example:

- "It is growing by two each day" - Recursive thinking
- "It is adding two more each day" - Recursive thinking
- "How many reds you need and four flower centres around that red" - Generalising

At this point it was clear that students could see the arithmetic relationship (recursive thinking) between each term (number of days) but not the relationship between the two variables. It was decided to frame students to see the structure of the pattern (length of caterpillar) and term (number of days) by splitting the pattern. As it can be seen in Table 5.6, the caterpillar pattern clearly represents the added two on the end of each day, due to the comparison of length from the caterpillar above. It was decided to assist students to perceive that the structure the pattern needed to be separated into groups of two. Figure 5.12 illustrates the new structure of the caterpillar pattern presented to students.
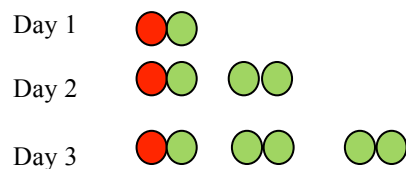
Figure 5.12. The caterpillar pattern with new structure.

Students were presented with the new caterpillar pattern where the structure was split into groups of two. The splitting of this pattern into groups of two provided a display for students to visually attend to the 'twoness' or 'groups of two' within the pattern. This then needed to be linked to the day number. The focus of the lesson at this point shifted, and students were being made to attend to the day number and the twoness of the pattern. This was achieved by gesturing to the day number (indexical sign) and then gesturing to the groups of two. It was apparent that this eliminated the recursive thinking that students were using to explain the generalisation of the pattern.

By clearly attending to the two variables, students could then predict the pattern beyond the terms provided in the lesson, and begin to justify their answers. Students were asked to predict the length of the caterpillar for day 30 (far quasi-generalisation); S2 stated "Sixty... it's like three plus three...thirty plus thirty". This student had earlier identified that the relationship between the days and the length of the caterpillar was doubling (multiplying by 2). This is further discussed in case study 2 (see Section 5.6.3).

Students were required to predict the number of days for a given caterpillar length (day 80 and day 100). Students could identify that for a caterpillar 80 long it would be day 40 (S16) and for a caterpillar 100 long it would be day 50 (S18). Most students found it difficult to explain how they worked out the answer. For example for length 100, 'It is day 50 because 50 plus 50 is 100' (S18). However, they could not see the relationships in terms of dividing 100 by 2.

Students then generalised the pattern for '*n*' days. Below are examples of students' generalisations:

R: What if I wanted to know the length of the caterpillar for day *n*?

S6: *n+n= m*

R: What if I wanted to know the length of the caterpillar on day s?

S6: *s+s=8*

R: Ok what about for day '*g*'?

S18: *g+g=gg* because it is doubling *g*

Students did not use the mathematical language of multiplying '*n*', '*s*', or '*g*' by two. This appeared to be a difficult stratagem for them.

Students were then given the opportunity to create their own growing patterns and the teachers (researcher, IEO1, and classroom teacher) moved around the class discussing near and far generalisations, and asked students to define their rule. Many students continued to use story telling as a medium to convey their knowledge and generalities.

Drawing on some of students' own work, the caterpillar pattern was revisited. However, this time instead of the pattern being times two it was times three. The first three days were displayed on the board. Students could relay that pattern was growing in threes. To break the cycle of students beginning to explain the pattern in terms of recursive thinking, the days were then randomly selected so that they would need to explain the relationship between the two variables. Students were asked, "How would you construct the caterpillar for day 21?". S2 explained that, "It would have seven groups of three in it." We revisited day one, two and three to determine if they also had groups of three. Students would chant/sing, "One group of three, two groups of three, three groups of three." Then students were asked, "What if it was day 100?" Students responded as a whole class, "100 groups of 3." This use of chanting/singing the structure assisted students to make connections between near and far generalisations, and also that the rule remained the same for each term in the pattern.

At the conclusion of lesson 4, new conjectures were posed for lesson 5. These conjectures were:

- Lesson 5 Conjecture 1: Manipulating hands-on materials that represent the variables of a growing pattern allows students to better attend to the pattern structure.

- Lesson 5 Conjecture 2: Exploring pattern structure by attending to both variables will assist students to generalise.

- Lesson 5 Conjecture 3: Providing growing patterns where the variables are embedded in the pattern and visually explicit, ensures that students attend to both variables.

Lesson 5 did not use a number structure to display the term position (e.g., number card). The centre of the red flower represented the pattern term with the pattern being the four petals surrounding the red centre. Students continued the pattern using their own hands-on materials. Most students continued to term five of the pattern. Students were then asked explain the structure of the pattern to determine the rule. Rather than using the red centre as the pattern term, students included it in their expression of the rule by stating that they see five petals in total. Students explained the structure of the pattern as one red centre five petals (one red and four yellow), two red centres 10 petals, and three red centres 15 petals. Though this was not the initial rule intended for the pattern, it was decided at this point to continue with what students had identified. S3 predicted the 4th pattern term and stated, "If there are four red centres then there are 20 counters."

S3 was then asked to explain how she predicted that 20 counters were needed for four red centres. Many students began calling out to explain how she solved this problem. This was a common occurrence in class. Some remarks from students included, "She counted with numbers", "She thinks with her brain". It was common for students to answer for others during classroom tasks or attempt to describe what they felt others were thinking. Rarely was the student who answered the question given the opportunity by their peers to explain their thinking. It was a challenge to break this mould in the classroom. When S3 finally was given the opportunity to explain her thinking she stated that she was thinking about, "Four pot plants with four red centres and each flower had five counters."

As the lesson continued, students displayed an aptitude in predicting beyond the terms presented to them. Remarkably, S2 explained that when determining how many counters he would need if there were 12 red centres (60 counters), he used the

classroom clock to help him count the groups of five. S2 stated that, "If I had 12 red centres I would need 60 counters because 12 groups of five are 60." However, when S2 was asked how many red centres would there be if I had 70 counters altogether, he had difficulty solving the problem. This is possibly because he could not use the structure of the clock anymore, and that 70 divided by five was beyond his current arithmetic knowledge. This is explored further in case study 2 (see Section 5.6.3)

It was found that with this task there was a faster shift for students between the additive generalisation (adding on 5 each time) and the actual generalised rule for the flower pattern. Students could identify that you were making groups of five each time. Asking students to solve this for any number, assisted them to quickly shift their thinking from an additive rule to that of a multiplicative relationship. When asked what I would have to do for any number, S17 stated that, "You are timesing the number of red centres by 5 to give you the number of petals."

At the conclusion of lesson 5 new conjectures were posed for lesson 6. These conjectures were:

- Lesson 6 Conjecture 1: Creating a 'story' about how the two variables are related assists students see the co-variational relationship.
- Lesson 6 Conjecture 2: Exploring multiplicative growing patterns with a constant are more difficult than exploring multiplicative growing patterns without a constant.
- Lesson 6 Conjecture 3: Using an iconic symbol (e.g., colour) to represent the constant in a growing pattern assists students identify the constant

Semiotics was used in the pattern presented in Lesson 6 (classroom pattern) to visually frame students to attend to the variables, and in particular to identify the constant. This was the students' first experience with a geometric pattern with a constant. Using semiotics to structure the pattern assisted with students to clearly identify the commonality and differences in the pattern. Students needed to determine what elements were the same in the pattern, and what was different. A green square tile was used in the pattern to display the constant. Students could easily identify this and explain that there was always one green square in each pattern. This square represented the teacher in our pattern story. Additionally, the pattern was structured into rows of three, and the number of rows signified the pattern term. Figure 5.23 illustrates the semiotic components in the pattern.
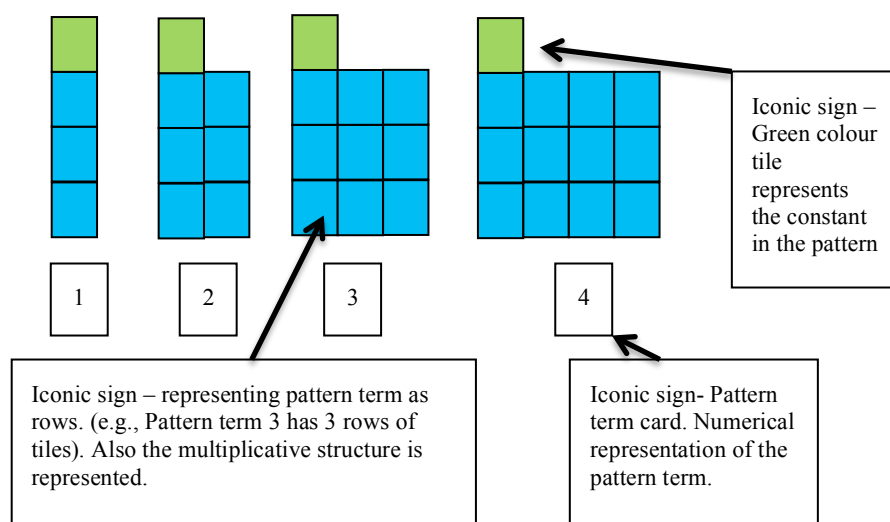
Figure 5.13. Semiotic components of pattern used in lesson 3.

Once students determined the constant in the pattern, the lesson focused on the structure of the rows to make links to the pattern term. The pattern story focused on the number of blue tiles, which became the desks in a classroom, and the number of rows that represented the grade level for that particular class. Therefore, five rows of three blue tiles represented 15 desks in Grade five. The researcher asked students, "How many rows of desks (blue tiles) in Grade 1?"; "How many rows of desks in Grade 2?"; and so on till Grade 4. The semiotic set up of the pattern framed students to attend to the structure of the rows. Additionally, the gestures (indexical signs) were used by the researcher to point to the pattern term (number under the pattern) and then point to the pattern rows. This was intentional, as the researcher wanted students to visually attend to the two components of the pattern and make connections between them. Initially, it was challenging for students because they wanted to provide an answer for the total number of tiles needed, rather than focusing on the number of rows in the pattern.

Students were asked to then explain what Year 10 would look like. S9 responded "Ten rows of three, which is twenty. No it would be thirty." Another student then commented that S3 had left off the teacher (constant - green tile). S3

then changed her answer to 31. Students were then asked to explain to IEO1 how she could construct Year 6. Initially, the student stated that she would need 12 blue tiles and 1 green tile. At this point, the researcher purposefully decided not to correct students, as there really needed to be 18 blue tiles to complete the pattern. The researcher then asked, "Is that just 12 blue tiles anywhere?" and the researcher randomly placed the 12 blue tiles on the desk. S2 then explained, "You would need to put the 12 blue counters into rows of three". This was then structured on the board for all students to see. Quickly, students identified that you actually required 18 blue tiles, as 12 tiles only gives you four rows of three and not six rows of three. S2 stated, "You need six rows of three plus one teacher". This was the first instance of a student using the mathematical language of 'plus' when discussing the constant.

Students were then asked to generalise the classroom pattern for *n* and then attempt to answer a quasi-generalisation for the pattern. Below are three types of responses provided by students:

- *n* rows of *m* plus *n*
- *n*3 in a row
- *n* rows of 3 +1

S6 responded "*n* rows of 3 +1". To further probe the student's understanding the researcher asked, "And what will *n* rows of 3+1 tell us?" S6 could not provide an answer. He could not express that it would give you the total number of people in the classroom or the total number of tiles you need to create the pattern.

5.5.3 Summary of Teaching Episode 2

From a mathematical perspective, students displayed the ability to copy, continue, create, and identify simple geometric growing patterns. Students also identified quasi-generalisations, pattern rules and generalised using alphanumeric expressions. Again, hands-on materials assisted students to attend to the variables in the pattern. Students found it challenging to work with growing patterns with a constant.

From a semiotic perspective, the selection of materials and the construction of the pattern appeared to be important aspects for assisting students to generalise. Gesture and chanting/singing the pattern provided a cognitive scaffold for students. Furthermore, consideration of how the geometric pattern was structured (e.g., iconic

signs) to highlight the variables, appeared to assist students to attend to the structure. This too assisted students beginning to relate the two variables as they engaged in covariational thinking.

From a cultural perspective, students appeared to be taking more risks (e.g., asking questions, answering questions) in class as they engaged in discussions. When discussing their mathematical thinking, students were continuing to use gesture to support their mathematical language. Furthermore, storytelling remained an important aspect, as students discussed their growing patterns with both the researcher and the Indigenous Education Officer.

5.6   CHAPTER REVIEW

In conclusion, this chapter presented data from Pretest 1 and Pretest 2 together with Lessons from the Teaching Episodes. Students engaged in the mathematical tasks and demonstrated they could copy, extend and create growing patterns. Furthermore, as students began to identify growing pattern structures, this assisted some students to then generalise these patterns. The data demonstrated that there are clear mathematical, semiotic and cultural processes that occur in the classroom context. To further understand these interactions, and how young Indigenous students generalise, Piagetian clinical interviews were conducted. The findings from the interviews are presented across three case studies in Chapter 6.

Chapter 6: Findings Piagetian Clinical Interviews

6.1 CHAPTER OVERVIEW

In the preceding chapter data from the teaching episodes were reported. This chapter builds upon these, and presents the findings of the Piagetian Clinical interviews. The interviews were conducted at the conclusion of Teaching Episode 1 and 2. A small sample of students (n=3) were selected and participated in one-on-one 20-minute interviews. Conjectures and summaries are presented at the end of Teaching Episode 1 and 2 are further investigated in the clinical interviews. The Piagetian Clinical interviews are presented as three case studies. Each case presents *a learning journey* of one student across two one-on-one interviews. Additionally, the case studies provide a deeper analysis of the data collected during the teaching episodes specific to each student (S1, S2, S6). The chapter concludes with a summary of findings across the clinical interviews. Figure 6.1 illustrates an overview of Chapter 6.

> Chapter 6: Findings of the Study
>
> 6.1   Chapter Overview
>
> 6.2   Overall Analysis of Piagetian Clinical Interviews 1 and 2
>
> 6.3   Case Study 1: Student 1
>
> 6.4   Case Study 2: Student 2
>
> 6.5   Case Study 3: Student 6
>
> 6.6   Chapter Review
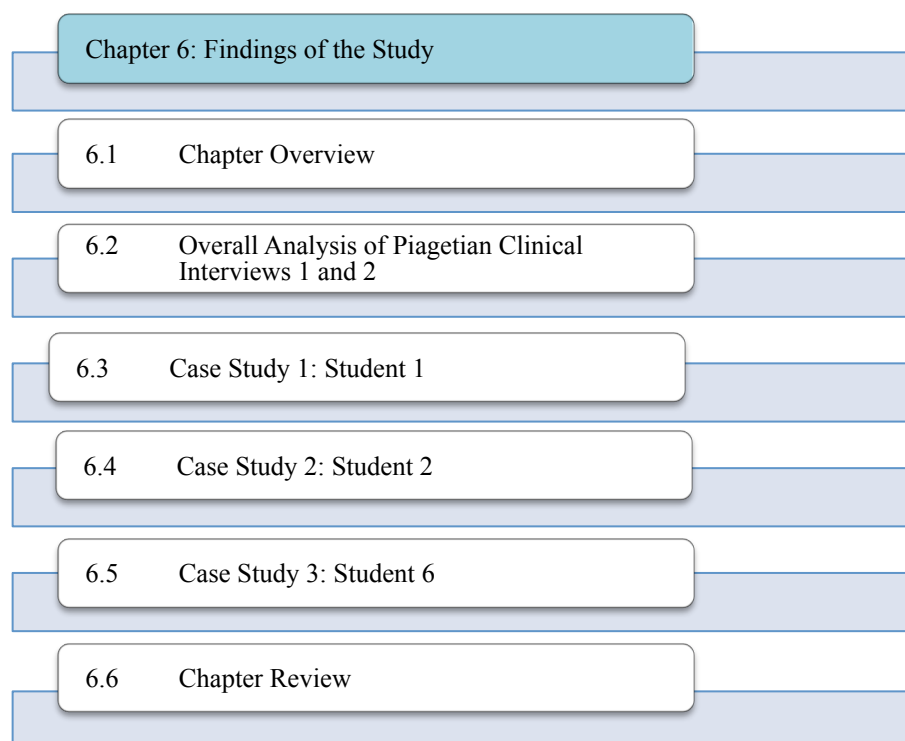
Figure 6.1. Overview of Chapter 6.

Piagetian Clinical interviews (PCI) were conducted with six purposefully selected students. For the intention of this study three students have been considered

for deeper analysis and have formed three separate case studies (see sections 6.3, 5.6.3, and 5.6.4). These three students were selected in consultation with the teacher and Indigenous education officers. Students attended school regularly, were good communicators, and identified as Aboriginal (S1 – female, S6 – male) and Torres Strait Island (S2-male). Each student participated in a test at the beginning of the school year (RoleM test). The RoleM test was used as an indicator for student mathematical achievement. Three students were then selected to provide a range of mathematical achievements based on these scores (S1 high achiever, S2 average achiever, S3 low achiever).

The clinical interviews were conducted at the conclusion of each teaching episode by the researcher, to probe students further about their understandings of growing patterns. These interviews also provided opportunities to trial new ideas and discuss with students their mathematical thinking in terms of the activities being presented to them. It also provided an environment for students to explain their mathematical knowledge without being interrupted by other students. Each interview was videotaped and all videotapes were viewed by IEO1 and IEO2 to provide cultural perspectives. These are reported in each case study. Based on themes stemming from the literature, theoretical perspectives, and the research questions the three case studies will present an in-depth analysis of: (a) mathematics in terms of generalisation, (b) semiotics and the use of sign and gesture, and (c) cultural aspects of learning.

Interviews were approximately 20 minutes in length. The interviews were video recorded so that both students' gestures and the researcher's gestures were captured. All questions were posed to students in a flexible manner. The questions posed and subsequent actions were contingent on the responses given by the student. The interviews mirrored the dimensions associated with Piagetian Clinical interviews, namely, endeavouring to avoid leading the student in a particular direction, but at the same time making the most of the opportunities to formulate and test hypothesis about students' understanding.

6.2   OVERALL ANALYSIS OF PIAGETIAN CLINICAL INTERVIEW 1 AND 2

The first PCI (PCI 1) interview began with an initial discussion about students' understanding of a growing pattern. Then two tasks followed, each consisting of five

parts. After initial creation of pattern by the researcher, the student is asked to (a) continue the pattern with materials attending to the structure of the pattern, (b) predict the next position of the pattern, (c) predict the quasi-variable position, (d) identify the pattern rule, and (e) generalise using alphanumeric notation. The first task presented in PCI 1 was a growing pattern using small plastic crocodiles (Pattern rule: number of feet = 4 x number of tails). Students were asked to examine the relationship between the number of tails and the number of crocodile feet (see Figure 6.2). The second pattern presented had a part A and a part B. Part A students were asked to explore the relationship between the class year level (e.g., 1st grade – represented on number cards) and the number of desks (represented by the blue tiles) (pattern rule number of desks = 3 x class number). Part B of the task introduced a constant to the classroom pattern; this constant was described as the teacher's desk to students. Figure 6.2 illustrates the patterns used in Piagetian clinical interview 1.

Task 1   Task 2a   Task 2b
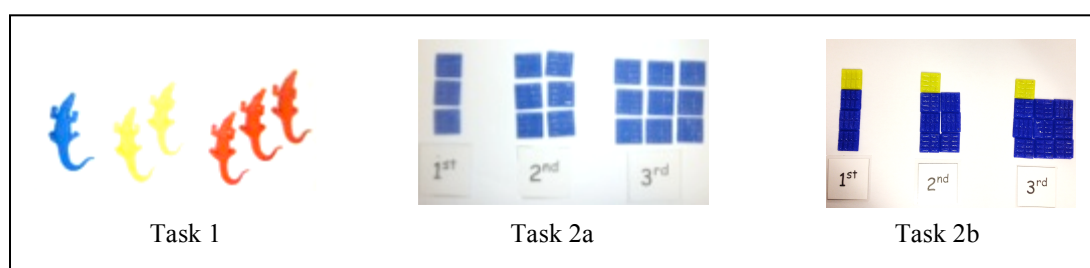
Figure 6.2. Pattern task one and two of initial Piagetian clinical interviews (PCI1).

Overall, all students successfully copied and extended, and predicted the next term for both growing patterns. When students were asked to predict beyond the pattern given using a quasi-variable (i.e., 25th term, 100th term), most students' mathematical knowledge limited their ability to provide an exact answer. However, they were able to identify how one would construct the pattern. This notion is demonstrated in the interview excerpt below.

> Researcher (R): What if I had year 100? What would you have to do?
>
> Student 3: (S3): Make 100 groups of three. [Student gestures to the lines of three beside the example given]
>
> R: Do you know what 100 groups of three are?
>
> S3: No

Thus, for Task 1 (crocodile feet and tails) three students (S1, S4, S5) were able to *quasi-generalise* the growing pattern, and for Task 2 (class year level and desks) five students could predict for the *quasi-variable* (S1, S2, S3, S4, S6).

Students were then asked if they could identify the rule for each growing pattern and then generalise the pattern rule. Students were able to provide generalisations for each pattern. Below are answers to two questions from student interviews. They have been categorised under Radford's (2010c) layers of generality (see Section 3.2.1) that is, factual, contextual, and symbolic to demonstrate the varying levels of sophistication.

Example of Factual Generalisation

R: What if I had two crocodile tails how many feet?

S6: 8 [Student is nodding head and looking at crocodiles]

R: How did you work that out?

S6: I counted in fours.

R: So if I have 12 crocodile feet how many tails would I have?

S6: You'd be having 3. [Student counts to twelve nodding head. Student then uses fingers to count tails].

Example of Contextual Generalisation

R: So if I have 100 tails what do I do to it?

S1: Times four

R: What if I had a million tails what would I do?

S1: Times 4 [Student places hand over the crocodile and moves it along to demonstrate making new groups of 4]

R: So what if I had ten times four. Do you know what ten times four is?

S1: Forty

R: So if ten times four is forty and what is forty? What part of the crocodile is it? Tails or feet?

S1: Feet

R: So what do you think the rule is?

S1: Times four of whatever crocodile feet [Student takes a long time to answer this and uses a lot of gesture with her explanation]

Example of Symbolic Generalisation

R: What do you think my rule is for this pattern?

S6: Rows of three. [Student gestures lines of three by moving their hand from the bottom of the pattern to the top using three fingers]

R: What do I have to do for any grade? If I know the grade number what do I have to do?

S6: Rows of threes. [Student repeats above gesture]

R: What if I had 'n' grades? What would my rule be?

S6: n rows of three. [Student repeats above gesture]

The second Piagetian Clinical interview (PCI 2) revisited students' understanding of a growing pattern. Three tasks followed: the first task students were required to create their own growing pattern, explain how their pattern was growing, and provide the pattern rule. In the second and third tasks, students were asked to (a) continue pattern with materials attending to the structure of the pattern, (b) predict the next position of the pattern, (c) predict the quasi-variable position, (d) identify the rule, and (e) generalise using alphanumeric notation.

The second task presented in PCI 2 was a growing pattern using double-sided counters to create a flower pattern. Students were asked to examine the relationship between the number of red centres and the number of yellow petals (see Figure 6.3). The third task presented a pattern using blue and green tiles to create a robot. Students needed to identify the relationship between the pattern term (number card) and the number of tiles. Figure 6.3 illustrates the patterns presented for both task two and three of PCI 2.
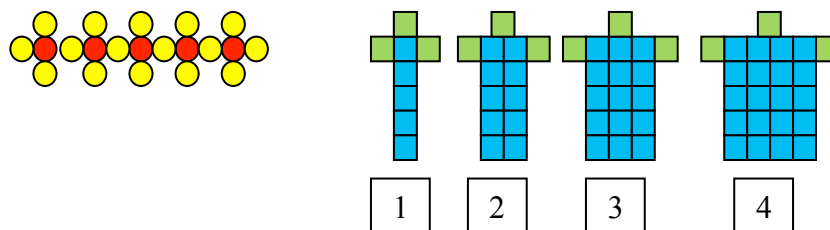
Task two: Flower pattern   Task three: Robot pattern

Figure 6.3. Patterns presented to students in PCI 2 for task two and three.

A discussion followed at the end of the interviews with the Indigenous Education Officers. Both the researcher and the Indigenous Education Officers watched the video recording of interviews and interactions of students. Themes that emerged from these discussions were: (a) students could identify pattern structures when they were using contextual hands-on items, as in the 'Flower Pattern' task; (b) students often gestured when discussing their mathematics as they may not have possessed the 'Western mathematical language' to explain the concept (both S8 and S6 displayed this in their interviews); and (c) cultural factors contributed to communication in the interview. For example, S2 revealed changes in eye contact and displayed *shame*. This observation was supported by IEO2. Meanwhile, S3s' manner changed from the classroom setting to the one-on-one setting, she became louder, more confident, and participated more in the clinical interview. Cultural elements of the study will be explored further in the case studies.

6.3   CASE STUDY 1: STUDENT 1

During Teaching Episode 1 and 2, Student 1 (S1) demonstrated that she was a high achiever in mathematics; this was also identified by the classroom teacher and Indigenous education officers. S1's RoleM math score was 21/30. S2 required encouragement to participate in class discussions. Often students would seek answers to mathematics questions from S1. Additionally, S1 participated in both pretests and all lessons presented in the teaching episodes. S1 attended school regularly. It was these aspects that influenced her selection for further analysis of S1 in the Piagetian Clinical Interviews of the study.

The data will be presented in terms of a narrative. In this section of the chapter, the emphasis is on the chronological events of the study that assist in telling the

story/journey of the student's learning across the Teaching Episodes and Piagetian Clinical Interviews.

6.3.1 General Observations from Lessons

On initial observations during lessons, it appeared that S1 was a diligent student but displayed a low level of participation, and interacted with other students in a limited way in whole class discussions. S1 rarely volunteered to answer questions posed during lessons. She was very quietly spoken, timid and made little eye contact during class with the researcher. Quite often the researcher needed to direct questions to S1 encourage her participation. The classroom teacher and IEO1 concurred that these reserved behaviours were also displayed when they were conducting lessons.

On further analysis of the video recordings from the three lessons it was evident that students looked to S1 for answers to questions. At times, other students in the class made gestures to communicate with S1, and then she would either give them the answer, or indicate accuracy through gestures such as a nod or shake of her head. Additionally, students would copy her work and present it as their own. The researcher was only aware of this in retrospect, when conducting further analysis with IEO1, watching students' interactions during lessons on the video recordings. Prior to that, these subtle gestures had been missed during teaching time. It could be said that S1's peers saw her as the smartest student in class, and this was further evidenced by comments such as, "Ask S1, she will know the answer." IEO1 felt that this dynamic placed great pressure on S1, particularly if she answered incorrectly during class discussions.

6.3.2 Student 1 Learning in Relation to Conjectures Presented in Teaching Episode 1

The following section presents the conjectures delineated at the conclusion of Pretest 1 and after each lesson of the Teaching Episode1 in relation to Student 1. Data was drawn from both Pretest 1 results and her participation in lesson one. The butterfly pattern presented in lesson 1 was purposefully created as it encompassed the three conjectures. It was patterns with an environmental context that both Indigenous Education Officers believed would be accessible to students, both

variables were explicit and embedded in the pattern (wings and body), and hands-on materials were provided to students.

Lesson 1 Conjecture 1: Exploring growing patterns from environmental contexts assists Indigenous students to relate growing patterns to their prior experiences.

Student 1 proved that she was able to work with growing patterns presented in environmental contexts in Pretest 1. She demonstrated that she successfully copied (partial copy), continued and extended environmental growing patterns. It was also evident that S1 made links to growing patterns within her own environmental context. During a discussion that followed the test, S1 stated, "It's growing like animals, plants, and humans growing in different ways from medium to large."

During lesson 1, S1 continued the butterfly pattern and explained how the pattern was growing. In discussion with the researcher, S1 was able to express that on day five there were five butterflies in her garden with 20 butterfly wings. Though she needed to count the butterfly wings, it was evident that she was able to think of both variables by presenting her own story. She drew on the environmental context of the butterfly in the garden to explain how her pattern was growing. However, she had difficulty describing the multiplicative structure ('fourness') of the pattern.

Lesson 1 Conjecture 2: Making both variables of growing patterns visually explicit assists students to identify the co-variational relationship

While S1 demonstrated that she copied the house pattern in Pretest 1, she did not attend to both variables in the pattern (the drawing of the house and the corresponding number card). S1 only copied the houses in the pattern. However, S1 started to work with both variables in the possum pattern presented in Pretest 1. S1 drew the possum tails and eyes to predict the 10th term, and provided a written explanation for her working. Figure 6.4 illustrates Student 1's results for Question 6 and 7 of Pretest 1.
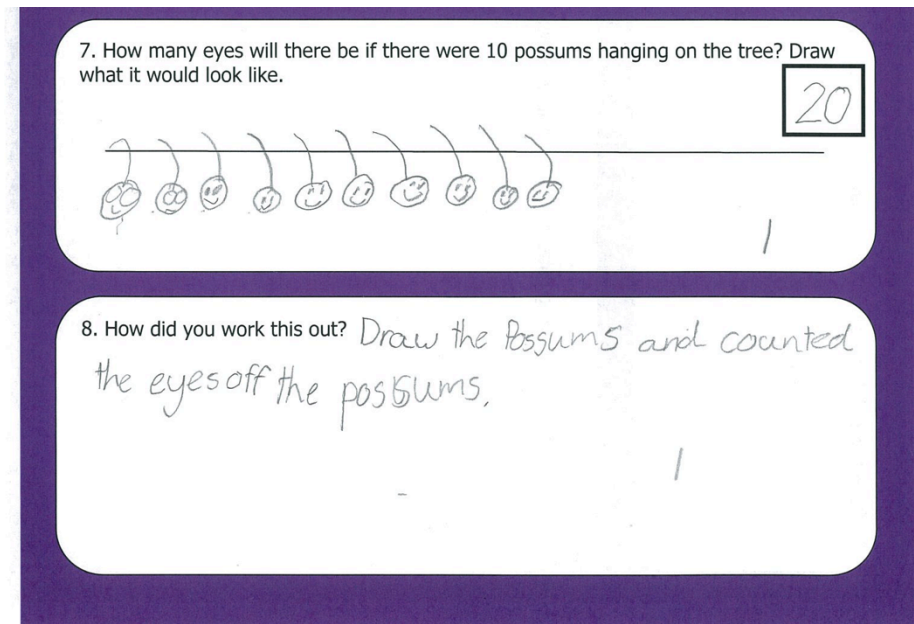
Figure 6.4. Student 1 working for Question six and seven of Pretest 1.

It appears that this particular pattern (possum pattern) made S1 attend to both variables (tail and eyes) as they were embedded in the single pattern structure. It is believed that this type of pattern structure frames students to attend to both variables, and this leads them to 'seeing' the co-variational relationship of the pattern. Additionally, the pattern also provided a structure enabling her to generalise (e.g., 376 possums how many eyes?): S1's written response was 'I can count in twos'.

During lesson 1, while both variables were visually explicit and embedded in the butterfly pattern, students attended to the sign vehicles (matchsticks and counters) separately. When considering a butterfly in the natural environment the body and wings cannot be separated. However, when using the hands-on materials to model butterflies the matchstick represented the body and the counters the wings. Thus the components of the pattern were easily separated. S1 did not split the two signs; S1 placed one matchstick on her desk and then immediately placed the four counters around that matchstick before constructing the next butterfly. S1 was able to copy and continue the structure identical to that represented on the board. It was for these reasons that S1 was considered to have high structural awareness of the butterfly pattern

Lesson 1 Conjecture 3: Manipulating hands-on materials that represent the variables of a growing pattern allows students to better attend to the pattern structure.

As S1 did not attend to both variables when copying the house pattern, it was conjectured that physically manipulating the two variables assisted them in seeing the structure. It was thought that this was particularly useful in patterns where both variables were separated, such as the house pattern (pattern and term). Furthermore, as students did not work with hands-on materials during Pretest 1, it is uncertain how S1 would have performed on such a task.

During lesson one, S1 worked with both variables using the hands-on materials provided. It is suggested that the hands-on materials prompted her to consider both variables in the task. This was evidenced through her explanation in conjunction with gestures to the hands-on materials, of the relationship between the number of butterfly bodies and the number of butterfly wings when continuing the growing pattern. As mentioned above, S1 had high structural awareness of the butterfly pattern, and this was demonstrated when she copied and continued the pattern during the lesson.

At the conclusion of lesson 1 new conjectures were considered and then trialled in lesson 2. The following section presents results from S1 collated from lesson 2 in response to the conjectures.

Lesson 2 Conjecture 1: Exploring growing patterns where the structure is multiplicative (e.g., double) assists students to generate the pattern rule.

Though it was thought that students would see the structure of doubling in this pattern, they did not. More was needed than just simplifying the multiplicative structure of the pattern. S1 demonstrated that she saw the pattern was growing by two each time. From the video analysis, S2 was attending to the additive structure (counting in two's – recursive thinking). However, her gestures suggested otherwise, and are discussed in lesson 2 conjecture 2. On her verbal explanation of the pattern, it appears S1 did not make the links between the two variables. She did not move beyond recursive thinking. S1 was not able to generalise as a result of only simplifying the structure: other teacher actions were required.

Lesson 2 Conjecture 2: Providing growing patterns where the variables are embedded in the pattern ensures that students attend to both variables.

S1 was able to predict how many people there were if 20 feet were on the ladder (10 people). When asked to explain how she arrived at her answer, S1 responded, "Counted in two's till I got to ten people." (TE1_L2_70[3]) S1 was then asked, "What would you do for 60 feet?" (TE1_L2_73), S1 responded, "Count forwards two, four, six, eight, ten.... 22." (TE1_L2_74). This response by S1 suggests that she was attending to only one variable (feet), and therefore only the additive structure of the pattern (plus two each time). However, her gestures were suggesting otherwise. S1 gestured using her second and third finger and moved them along an imaginary ladder. As she was moving her fingers along each ladder she was also looking at the ladder spaces. This suggests she was coordinating the two variables. Her hands were acting as the feet in the pattern, and her eyes were focusing on the ladder spaces where the bodies were standing. This indexical sign was created by S1. It appears that she was beginning to attend to the structure of this pattern and gesturing was assisting her to work with both variables. Interestingly, S3, who was sitting beside S1, started to mimic her gestures as she worked through the problem. Figure 6.5 illustrates the gestures used by S1 and S3 while counting the number of feet in the pattern.

Student 1   Student 3

Figure 6.5. S1and S3 gesture use while counting the number of feet in the pattern.

Lesson 2 Conjecture 3: Embodying the mathematical structure of growing patterns assists students to explain the pattern structure.

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[3] TE1_L2__70 – Teaching Experiment 1 (TE1) _Lesson 2 (L2)_ Line 70 (70).

S1 participated in the embodiment process of lesson 2. This provided S1 with a means to test her predictions and justify her response to other students (e.g., 20 feet would be 10 people). Additionally, S1 began to gesture after she saw students on the ladder. It is difficult to determine whether this was as a result of students standing on the ladder and thus providing visual stimulus for S1 to gesture, or whether her gesture were spontaneous (regardless of students standing on the ladder).

At the conclusion of lesson 2, new conjectures were considered and trialled in lesson three. The following section presents results for S1 collated from lesson 3 in response to the conjectures.

Lesson 3 Conjecture 1: Transferring mathematical knowledge between patterns with the same multiplicative structure is difficult.

S1 did transfer the additive relationship that she saw in lesson 2 (feet and ladder pattern) to the pattern she created in lesson 3. Figure 6.6 illustrates the growing pattern created by S1 in Lesson 3.
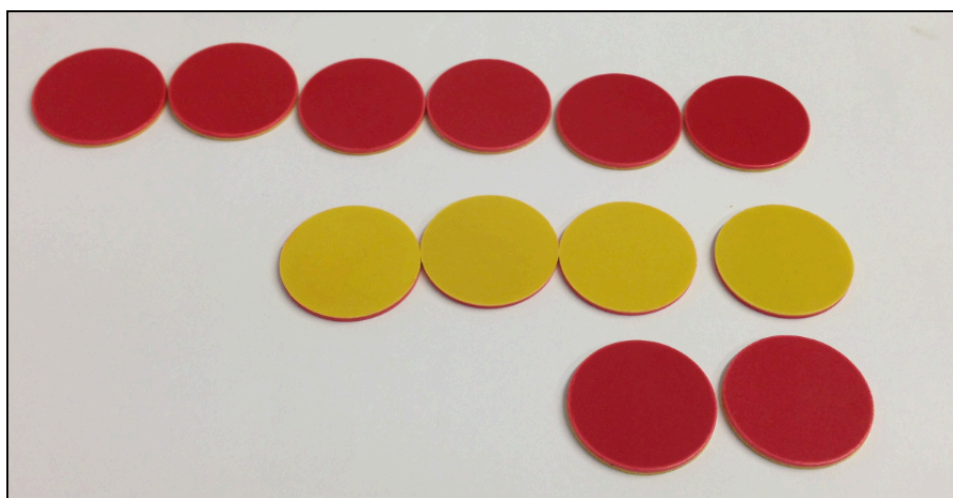
Figure 6.6. S1 growing pattern created in Lesson 3.

She was able to describe that this pattern was growing by two each time. It was evident that explicit teaching of the structure and language was needed to assist S1 to attend to the multiplicative structure of the pattern.

The explicit teaching and exploration of mathematical language (doubling, times two, multiplied by two – as discussed in lesson 2 conjecture 2 below) was needed to explain the structure of S1's created pattern. Additionally, providing a position term (second variable) and then gesturing (researcher gesturing – pointing to

each variable) between the position term and the pattern assisted S1 to identify the multiplicative structure of the pattern. Figure 6.7 illustrates the gesture used by the researcher to assist S1 to 'see' the pattern structure.
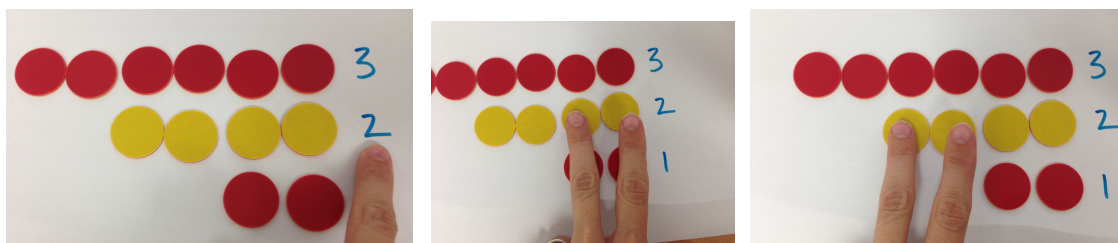
Language: In position 2 (gesture to number) we have two groups of two (gesture 2 and 3). Another way of saying groups of 2 is 'multiply by 2'. So, two (gesture to number) multiplied by two (gesture to groups of 2) is four.

Figure 6.7. Explicit teaching of language with gesture used by the researcher to assist students to attend to the multiplicative structure.

It appears that once the explicit teaching of the mathematical language combined with the gesture was experienced by S1, she was able to easily transfer this to the kangaroo pattern presented later in this lesson. S1 was able to explain that the pattern rule for the kangaroo was doubling the number of tails.

Lesson 3 Conjecture 2: Providing students with the mathematical language used to describe multiplicative structures assists students to generalise the pattern.

Prior to presenting the kangaroo pattern, students created their own growing pattern and explained how it was growing. S1 had created a pattern that was doubling each term (2n). Her pattern was used to explore growing patterns with the class. During the discussion, students were asked if they had heard the words 'doubling' or 'times two'. Most students agreed that they had heard these terms used before in class with their regular classroom teacher. This language was used to describe the pattern provided by S1. S1 identified that her pattern was doubling each step. From this she made predictions about position 100, and identified that you have to double the position number and thus the answer is 200. As a whole class, S1 participated in the discussion about generalising the pattern using alphanumeric notation. Students were asked, "What if I had a number called *n*, what would I have to do to it?" S1 was

able to respond that you needed to double it or times by two. Though this was a whole class activity, S1 had a high participation level in this lesson.

Lesson 3 Conjecture 3: Providing growing patterns where the variables are embedded and cannot be physically separated from each other, assists students to attend to both variables simultaneously.

It was evident that S1 was now attending to both variables when working with the kangaroo pattern. She was able to express further predictions of the pattern using both the tail and the ears to communicate her understanding. S1 explained to the class that if she had 1 million tails she needed to double the number of tails to determine how many ears there were. She was also able to determine how many tails there would be if there were 10 kangaroo ears (five kangaroo tails). When asked how she worked this out S1 responded, "I did it backwards." S1 was unable to explain that she was halving (or dividing by two) the number of ears to determine the number of tails. As both variables were embedded in the kangaroo pattern, and could not be separated, this assisted S1 to attend to both variables when discussing the pattern.

At the conclusion of lesson 3 of Teaching Episode 1 all previous conjectures and data were considered in order to select patterns and construct questions for the one on one interviews. The clinical interviews provided a one-on-one environment for deeper exploration of the previous conjectures relating to either a mathematical, semiotic, or cultural aspect of the study. The following section presents the 'learning journey' of S1 during the clinical interview. The data from each task (see Figure 6.2) is presented.

6.3.3 Student 1 Learning Through Clinical Interview 1

Task 1 of PCI1(Crocodile Pattern)

For the initial task of clinical interview 1, S1 was presented with the crocodile pattern (See Figure 6.2). The researcher constructed the first three terms of the pattern. S1 was asked, "If I have two crocodile tails how many feet will there be?" S1 successfully responded that there would be eight feet. This was then repeated for five crocodile tails. S1 answered that there would be 10 feet instead of 20. When asked how she worked out there were 10 crocodile feet her response was, "Five plus five equals 10". It was evident that S1 identified that the pattern was doubling, but

was not necessarily attending to the multiplicative relationship between the two variables.

To assist S1 to attend to the structure of the pattern, indexical signs were used in the form of gestures. The researcher explicitly attended to the structure of the pattern by pointing to each foot of the crocodile and counting how many feet there were in total. S1 was then asked to attend to each pattern term and determine how many feet the crocodile would have.

29 R   What about if I have one crocodile how many feet?

30 S1   Four

31 R   Two crocodiles have … [student interrupts]

32 S1   Eight

33 R   Three crocodiles have …

34 S1   Twelve

35 R   Four crocodiles … what would that be?

36 S1   Sixteen

37 R   How did you work that out? [S1 sitting silently head down] You are right. [S1 moving eyes as if she is counting each foot]

38 S1   Moved that one over here [S1 gestured to crocodile in pattern term 1] [S1 has mentally placed four crocodiles in a row]

39 R   Did you count on?

40 S1   Yes

S1 was using an additive process to determine how many feet were in each term of the pattern. It was evident that S1 was not using the direct relationship between the number of tails and number of feet. She was reminded about the work that students had attended to in class that week with the kangaroo tails and ears, in particular her attention was drawn to the multiplicative language used in the lesson (doubling or times by two). S1 was reminded that the rule for the kangaroo pattern was not additive (plus two) but rather it was multiplicative (doubling). After this

discussion, S1 considered the pattern and announced that the crocodile pattern was growing "in fours" (PCI1S1_45[4]).

S1 was then asked to trial a range of rules for the crocodile pattern. It was conjectured that this would determine if she understood the structure of the pattern. S1 trialled 'tail plus three' and then 'tail plus four'. S1 continued using additive language to explain the pattern rule. It was evident that S1 needed further assistance to determine the pattern rule.

S1's attention was once again drawn to the structure, however this time the mathematical language was explicitly introduced in conjunction with gestures from the researcher. This time the words 'group' and 'times' were introduced (students used the mathematical language of 'times' during normal class sessions with their teacher for example, four times two is eight) in conjunction with 'singing/chanting' and gesture. This was intentional and framed the structure to assist S1 to move beyond the additive rule. Once the language was grasped by S1, and the connection was made between the mathematical language and the structure, she was asked to determine what the rule would be if she had any number of crocodiles. S1 responded, "Times four" (PCI1S1_69).

In summary, this initial interview task required more framing of the structure than initially anticipated. The student had difficulty transferring the knowledge of the kangaroo task to that of the crocodile task. Table 6.1 summaries the major teachable actions, with researchers' questions/directions, student responses/engagement, and observations during the interview process for Task 2a.

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[4] PCI1S1_45 – Coding for interview transcripts. Piagetian Clinical Interview 1 (PCI1), Student 1 (S1), Line 45 (45).

Table 6.1

Teachable Actions, Researcher Participation, Student Participation, and Observation During Task 1

Teachable actions	Researcher participation (questions/directions)	Student participation (responses/engagement)	Observation
Researcher constructs pattern		Student observed	
Focus on the structure of the pattern	If I have two crocodile tails how many feet do I have? Repeated for 5 tails	8 10	S1 answered the first question correctly and then provided an incorrect response for the second question
Researcher uses gesture	Researcher gestures to feet and counts how many each crocodile	Student observes	Student gestured to the number.
Student and researcher attend to the structure	one crocodile tail four feet two crocodile tails eight feet	Student continues 3 tails – 12 feet 4 tails – 16 feet Explains that they mentally moved one crocodile on and counted the extra four feet to determine the answer	Student see additive process (plus 4)
Transfer knowledge from previous lesson	Discussed how the kangaroo pattern was not adding two each time but was doubling	Student responds that this pattern is growing by fours	
Trial pattern rule		Student trials additive rules Tail plus three Tail plus four	S1 did not transfer the language of multiplication from the kangaroo pattern to the crocodile pattern
Explicit language and gesture provided by researcher	Discussed multiplicative relationship One times four is four two times four is eight	Student continues trialling multiplicative language Three times four is twelve Four times four is sixteen	
Generalise pattern for any number	What is the rule of any number of crocodiles?	Times four	

It was apparent the key term that assisted S1 see the structure was more than just the selection of a particular pattern type. S1 required explicit teaching of the appropriate mathematical language used to describe the pattern, in conjunction with chanting the structure, accompanied by the researcher's gestures. This requirement prompted the particular approach adopted for the second interview task.

Task 2a of PCI1 (Classroom Pattern)

The classroom growing pattern presented in Task 2a was constructed using blue tiles and number cards. While the researcher was constructing the pattern, she told a story to S1. Student 1 listened and observed the pattern being constructed.

70 R		We need to make a new pattern now. These are going to be desks in my classrooms. This is Grade 2, 3 and 4 (Researcher places number cards onto desk in front of S1). In Grade 2, I have two rows of three children (researcher places two rows of three blue tiles above the second number card). In Grade 3, I have three rows of three children (researcher places three rows of three blue tiles above the third number card). What do you think would be in Grade 4?
71 S1		Four rows of four
72 R		How many in my row here? (researcher gestures to the card and not the pattern)
73 S1		Two
74 R		I have two rows. How many children are in each row sitting at desks?
75 S1		Three
76 R		How many are sitting in these rows? (Researcher gestures to pattern for Grade three)
77 S1		Nine
78 R		Yes it is nine altogether, but I have three there and three there and three there (Researcher gestures to the blue tiles and drags her finger across each row). Four rows of how many?
79 S1		Three
80 R		Good girl. Can you point to the number? How many rows of three in Grade two?

81 S1   Two

S1 was then asked, "What would the next one be if I made Grade 5?" S1 responded correctly, "Five rows of three." She was then given the number card with '1st' written on it and was asked to construct Grade 1. S1 successfully constructed Grade 1. She was then asked, "What do you think my rule is for this pattern?" S1 responded, "Rows of three".

The next part of the interview focused on predictions beyond the pattern presented. S1 was then asked what the pattern would look like at Grade 20 (response - 20 rows of three) and Grade 1 million (response - 1 million rows of three). S1 applied the structure of the pattern to predict quasi-variables. At this point in the interview the researcher and S1 discussed other ways that you could say 'rows of three'. We discussed using the word 'times' and trialled it for each of the pattern positions presented (e.g., Four rows of three is twelve, Four times three is twelve). S1 was then asked to generalise the classroom growing pattern for Task 2a.

111 R   What if I had a class called grade n? What would I have
to do?

112 S1   Times

113 R   By how many?

114 S1   Three

S1 needed to be prompted to generalise the pattern. She was unable to link the alphanumeric notation to the generalised rule. Table 6.2 summaries the major teaching   actions,   with   researchers   questions/directions,   student responses/engagement and, observations during the interview process for Task 2a.

Table 6.2

Teaching Actions, Teacher and student participation and observations in Task 2a of PCI1

Teaching Actions	Researcher participation (questions/directions)	Student participation (responses/engagement)	Observation
Researcher constructs and verbalises the pattern structure	Verbalised structure of the pattern	Student observed and listened to the pattern structure.	
Focus on the structure of the pattern	Student required to identify how many in a row?	Nine	S1 focused on the whole group rather than 3 tiles in a single row.
Student kinaesthetically engages with pattern	Had student gesture to the pattern term numbers under the physical pattern and verbalise the structure	Point to pattern term and say pattern structure 'Two rows of three' 'three rows of three'	Student gestured to the number.
Student creates pattern	Have student continue pattern	Student kinaesthetically engages with pattern and structure	Student continued pattern by placing the blue tiles down to create Grade one
Near generalisation of structure	So if I had grade 20, what would I have to do?	20 rows of three	
Quasi generalisation	What if my class was grade 1 million? What would I have to do?	1 million rows of three	
Pattern rule	What do you think the rule is for this pattern? What do I have to do for any grade?	Rows of three Rows of three	Discussed with the student other ways we could say rows of three – we talked about multiplication and times tables
Alphanumeric generalisation	What if I had a number called n what would I have to do to n?	Times...... three	Prompted for both parts of the rule

Task 2b of PCI1 (Classroom & Teacher Pattern)

Student 1 was then presented with the classroom and teacher pattern that incorporated a variable with a constant (+1), modelled as 'the teacher's desk' (orange tile). The orange tile was placed above the 'Year 1 classroom' (see Figure 6.2) and S1 was asked, "What do you think the pattern rule is now? (PCI1S1_115). Below is an excerpt of the discussion that ensued between S1 and the researcher.

116 S1   One row of four

117 R	Does that work for grade 1?
118 S1	Yes
119 R	What about grade 2?
120 S1	Two rows of four
121 R	I am not sure about that. What do we do when we join something together in maths?
122 S1	Groups
123 R	What do we do when I have two things here and three things here and I want to join them together? What would I say?
124 S1	Three plus two equals five
125 R	So if I have one times three
126 S1	Times four
127 R	Has my rule changed to times four do you think?
128 S1	No
129 R	Times four would be eight here [*S1 gestures to grade 2*]. So it was 1 times three and I am joining this one. What do you think I am doing? What are we doing when we join something together?
130 S1	Plus
131 R	And how many teachers are there?
132 S1	One
133 R	So what do you think this one might be? Two times three
134 S1	Plus one
135 R	Three times three
136 S1	Plus one
137 R	Four times three
138 S1	Plus one

139 R   So what do you think our new rule might be?

Student 1 went on to respond, "Times three plus one". Once the mathematical language had been attended to in the above conversation, the student rapidly went on to providing a quasi generalisation (e.g., the number of desks in Year 100), the pattern rule for any class, and the pattern rule for *n* classes (alphanumeric response). Below are the three responses the S1 provided:

1. Quasi generalisation: '100 times three plus one' (PCI1S1_147)

2. Pattern rule for any number: 'any number times three plus one'(PCI1S1_154)

3. Alphanumeric rule: 'times three plus one'(PCI1S1_152)

It is evident that S1 did not provide an alphanumeric response, as she did not include the *n* in her response. S1's thinking moved rapidly once the mathematical language and structure of the pattern was understood. She quickly moved to being able to provide a quasi generalisation and the pattern rule for any number.

6.3.4 Student 1 Learning in Relation to Conjectures Presented in Teaching Episode 2

In relation to S1, the following section presents the conjectures delineated at the conclusion of Pretest 2 and after each lesson of Teaching Episode 2. It should be noted that S1 had low participation during the three lessons presented in Teaching Episode 2. As mentioned above, S1 is usually a quiet student who requires prompting to answer questions during class time. This was subsequently more pronounced during this data phase, and occasionally when asked questions by the researcher, S1 would not respond. Often it was because she was unsure of the answer and therefore would not answer in front of her peers. With the arrival of S18 in her class, it was evident that the classroom dynamic had shifted. S18 was considered to be the 'new' highest achiever in the class, and demonstrated that she had a good understanding of mathematics. S18 and S2 dominated much of the discussion during lessons presented in TE2. Though S1's participation was low during the lessons, this was not the case for the one-on-one interview.

Lesson 4 Conjecture 1: Exploring growing patterns from environmental contexts assists' Indigenous students relate growing patterns to their prior experiences.

S1 appeared to have a better understanding of the structure of the caterpillar pattern presented in lesson 4, than in the geometric patterns presented in Pretest 2 (this is discussed in lesson 4 conjecture 3 below). S1 had difficulties extending the pattern beyond the terms presented in Questions 3 of pretest 2 (See appendix E). She also had difficulty in Question 4 of Pretest 2. This type of growing pattern (2n) had previously been explored in Teaching Episode 1 (kangaroo pattern) and S2 had little difficulty generalising that pattern.

Lesson 4 Conjecture 2: Explicitly modelling the relationship between the variables in the growing pattern assists students to use the alphanumeric notation to describe the generalisation.

Though S1 had difficulty predicting terms beyond the pattern presented in Questions 3 and 4 of Pretest 2, when asked to write the rule for the pattern S1 wrote 'you can double it, *n*=". This indicates that S1 was beginning to link alphanumeric expressions to generalisations.

During lesson 4, S1 exhibited a level of low participation during the phase that focused on using alphanumeric notation to represent the generalisation of the caterpillar pattern. As mentioned previously, the classroom dynamic had recently shifted, and S18 (new student) was now seen as the 'high achiever' in the class. S18 dominated most of the classroom discussion during the lesson and while this occurred, S1 quietly watched.

Lesson 4 Conjecture 3: Using semiotic bundling, (i.e., using gesture, language and manipulation simultaneously) assists students to identify the structure of the pattern.

When the caterpillar pattern was revisited later in lesson 4 a new rule was applied (3*n*) and the structure was reconsidered (see *Figure 5.12*). The first caterpillar pattern in lesson 4 did not separate the caterpillar length into groups of two (see Table 5.6), and it appears that this contributed to students' recursive thinking. The new caterpillar pattern was structured so groups of three were easily seen. Gestures were made between the number representing the days and the pattern structure. Questioning focused on what the pattern looked like for days beyond what was presented on the board. This is when S1 began to contribute to the class discussion - prior to this S1 was fairly reserved. S1 was able to predict that on day 7

the caterpillar would be 21 counters long. When asked how she worked this out, S1 did not respond. IEO1 stated, "She knows the answer". Before S1 could respond, S2 answered for her, "Seven groups of three". S1 was then asked, "What would the caterpillar look like on day 100?" This time S1 stated, "100 lots of three". From this response it appeared that at this stage S1 could identify the structure of the pattern and work with both variables. Attending to the overall structure through questioning and gesturing to the variables of the pattern assisted S2 to quasi-generalise.

Lesson 5 Conjecture 1: Manipulating hands-on materials that represent the variables of a growing pattern allows students to better attend to the pattern structure.

During lesson 5 S1 created her own growing pattern using hands-on materials. She was able to extend her pattern and explain how it was growing. However, S1 was still only discussing the additive nature of the pattern and was not considering both variables. Later in lesson 5, S1 successfully used hands-on materials to construct the flower pattern.

Lesson 5 Conjecture 2: Exploring pattern structure by attending to both variables will assist students to generalise.

Lesson 5 focused more on the structure of the pattern and attending to both variables. For example, four red centres with four yellow petals around each centre, rather than four red centres and 16 yellow petals. When asked questions about the pattern structure, S1 did not respond. It was therefore difficult to determine if this manipulation of the pattern assisted S1 reach an understanding of the pattern's structure.

Lesson 5 Conjecture 3: Providing growing patterns where the variables are embedded in the pattern and visually explicit ensures that students attend to both variables.

The flower pattern provided in lesson 5 contained variables that were visually explicit and embedded. It is difficult to determine if this assisted S1 as she did not respond to any questions posed to her in the lesson. What could be determined was that she could copy this type of geometric pattern.

Lesson 6 Conjecture 1: Creating a 'story' about how the two variables are related assists students see the co-variational relationship.

The use of story-telling gave many students a context for the geometric pattern presented in lesson 6. This pattern was a revisit of the classroom and teacher pattern presented in PCI1, as not all students had worked with this pattern. S1 was able to identify the constant and then label this as the teacher (green tile), while the other tiles represented students in the class. During this lesson, S1's participation in the whole class discussion was low. However, after further analysis she was discussing the pattern with S6 in the story context. Therefore, it cannot be determined whether this assisted her to see the variables in the pattern.

Lesson 6 Conjecture 2: Exploring multiplicative growing patterns with a constant are more difficult than exploring multiplicative growing patterns without a constant.

It appeared that S1 had little difficulty understanding the notion of a constant. This notion was further explored in more depth during Clinical Interview 2.

Lesson 6 Conjecture 3: Using an iconic symbol (e.g., colour) to represent the constant in a growing pattern assists students in identifying the constant.

S1 identified to the class that the green tile was the object that remained the same in each pattern. It is inferred by S1's comment that, signifying the constant with a different but consistent colour made it easier to identify as the object that remained the same in each pattern term. It is also noted that the position of the tile potentially assisted the student to identify the constant within the pattern.

6.3.5 Student 1 Learning Through Clinical Interview 2

Task 1 of PCI2 (Create a Growing Pattern)

The first task for S1 required her to construct her own growing pattern using tiles. Number cards were also provided. S1 then needed to tell a story about how her pattern was growing (see Transcript PCI 2_S1 10-52). Figure 6.8 illustrates the growing pattern created by S1.

Figure 6.8. Growing pattern created by S1 during lesson 1 of PCI2.

S1 then explained 'on the first day it was four, and on the second day eight, and on the third day it was 12' (PCI2_S1_12). At this stage, S1 had only used the tiles to create her pattern, and the pattern she created was similar to the one presented in the previous clinical interview. The researcher then encouraged her to use the number cards under her pattern (see Figure 6.7). S1 explained that her pattern was growing by 'fours'. When further discussion ensued, it was evident that S1 was only focusing on the additive nature of the pattern. She went on to generalise that you would "Add four more on it" as you continued the pattern. The researcher revisited the types of discussions that had occurred with S1 in Clinical Interview 1. The researcher discussed the structure of the pattern in terms of groups of four in each pattern position. S1 then went on to provide a quasi-generalisation for the pattern: for example, for term 25 she responded that 100 tiles were needed. When asked, "For any number what would I need to do?" S1 responded, "[you] Say the number and then rows of four."

Task 2a of PCI2 (Daisy Chain Pattern)

The second task of PCI2, a pattern with a constant, was presented to the student: the daisy chain pattern. However, this time the constant was not as identifiable as in Task 2b of PCI 1, where the orange tile (iconic sign) was used to represent the constant. The constant was not signified in the daisy chain pattern by an iconic sign; it was part of the pattern. During the discovery of the pattern structure the pattern was deconstructed into separate groups of one red centre with three yellow counters around the red centre. This left the constant as a single entity of the pattern as it was not attached to a red centre. The constant was then highlighted for S1 by using a sticker to signify its position in the pattern.

During this interview, once the pattern had been separated into separated groups by the researcher, S1 was able to identify the pattern rule (times three plus one) much faster than in previous interviews. Table 6.3 summarises the teaching actions, researcher and student participation, and observations made during pattern task 2a.

Table 6.3

Teaching Actions, Researcher and Student Participation, and Observations of Pattern Task 2 of PCI 2a.

Teaching Actions	Researcher participation (questions/directions)	Student participation (responses/engagement)	Observation
Researcher constructs pattern	Researcher places first two pattern terms on the desk	Student watches researcher	
Student continues pattern	Can you make the daisy chain longer?	Student continues pattern to term 5.	Student kinaesthetically engages with pattern and structure
Student verbalises what she is constructing	Can you tell me what you are making?	'Adding on three'	Student has not identified the constant
Student identifies what she 'sees' in the pattern through gesture	Can you point to or circle the groups of three you are seeing?	Student gestures to the pattern and draws loops around the counters.	Student gestures to the pattern
Deconstructing the pattern to see the structure	Researcher separates the structure of the pattern to show the 'threeness'	Student assists to deconstruct the pattern	
Highlighting the constant	Researcher places a sticker on the constant to signify its role in the pattern	'times three plus one'	Student identifies pattern rule with no prompt
Near generalisation	10 flowers	10 groups of three plus one	
Quasi generalisation	100 flowers	100 groups of three plus one	
Generalisation for any number	What would my rule be for any number of flowers?	Groups of three plus one	

Task 2b of PCI2 (Robot Pattern)

Student 1 then continued to complete Task 3 (robot pattern) very quickly. After the initial construction of the first three terms of the pattern, S1 identified what the next term would be in the pattern (three groups of five plus three on top), then provided the 10th term (10 groups of five plus three), then a quasi generalisation (2000 groups of five plus three). S1 provided the pattern rule, "Groups of five plus three." To further extend her thinking, S1 was asked to provide the pattern term number if the structure of the pattern was given. See the excerpt from the interview below:

130 R   What if I had 10 groups of five plus three? What number along would it be in the pattern?

131 S1   10

132 R   Now what if I had 600 groups of five plus three?

133 S1   600

It is clear that by the end of Piagetian Clinical Interview 2, S1 was developing an understanding of how to generalise pattern rules for simple multiplicative patterns and multiplicative patterns with a constant.

6.4   CASE STUDY 2: STUDENT 2

Student 2 (S2) was considered a sound achiever in mathematics as reflected in his score on the RoleM test conducted at the start of the school year (15/30). The classroom teacher and IEOs had, however, identified S2 as a student who was in the high achieving class group for mathematics. Unlike S1 and S6, Student 2 identified himself as Torres Strait Islander. S2 was absent for lesson 1 and 2 of Teaching Episode 1, however, when he was present, S2 displayed high participation levels during lessons conducted in Teaching Episodes 1 and 2. S2 participated in both Pretests in the teaching episodes, but only participated in lessons 3 to 6. S2 attended school on a regular basis. It was these aspects that influenced the selection of S2 for further analysis in the Piagetian Clinical Interviews of the study.

The data are presented in terms of a narrative. The emphasis is on the chronological events of the study, thus assisting to 'tell the story/journey' of student's learning across the Teaching Episodes and Piagetian Clinical Interviews data collection.

6.4.1 General Observations from Lessons

In the course of observations conducted during lessons 3 to 6, it appeared that S2 was a conscientious student and exhibited high levels of participation in class discussions. He was a very confident student, always willing to answer questions. At times, S2 was so keen to provide an answer that he did not listen to what was being asked, and as a result was quite disappointed when he did not answer correctly. S2 often sought the approval of the teacher. Additionally, with regards to mathematical tasks, he worked well with other students, shared his ideas, and could articulate his

ways of working. If he were having difficulty, S2 would always ask for assistance from IEO1 in class. Furthermore, he worked well with other students in class and discussed ideas with them. The classroom teacher and IEO1 concurred that these behaviours were displayed when they were conducting lessons.

6.4.2 Student 2 Learning in Relation to Conjectures Presented in Teaching Episode 1

In relation to Student 2, the following section presents the conjectures delineated at the conclusion of Pretest 1 and after each lesson of the Teaching Episode 1 (see Figure 5.1). The three conjectures posed after Pretest 1 for lesson 1 (see Section 5.4.1) are presented below, together with data collected from the test for Student 2.

Lesson 1 Conjecture 1: Exploring growing patterns from environmental contexts assists Indigenous students to relate growing patterns to their prior experiences.

Pretest 1 results have been used to answer the conjectures posed in lesson 1, as there is no data from the teaching experiments as S2 was absent. S2 showed that he could copy (complete copy) and continue environmental growing patterns. The growing pattern created by S2 in Pretest 1 drew on an environmental context (See Figure 6.9). S2 presented a pattern with a small shark and then a larger shark. He provided no explanation for how his pattern was growing. Figure 6.9 illustrates the growing pattern created by S2 in Pretest 1.
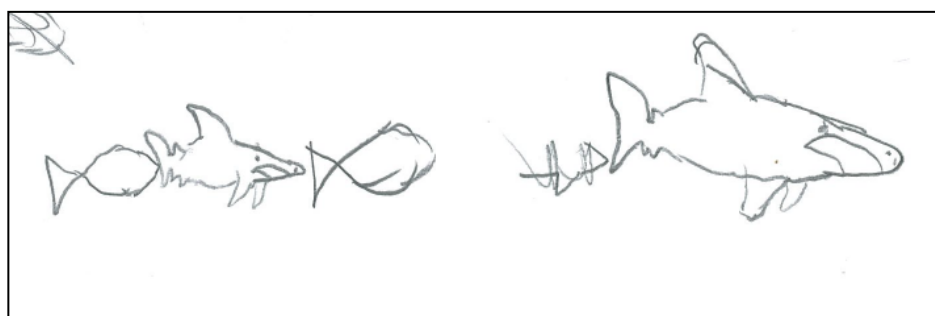
Figure 6.9. Growing pattern created by S2 in Pretest 1.

Lesson 1 Conjecture 2: Making both variables of growing patterns visually explicit assists students to identify the co-variational relationship

S2 demonstrated that he could work with environmental growing patterns and was successful at copying, continuing, completing, and creating the patterns

presented in Pretest 1. S2 could copy the house pattern, attending to both variables (houses and position numbers). However, in Question 6 of Pretest 1 (possum pattern), S2 did not predict the pattern beyond what was presented. S2 incorrectly identified that there were twelve possum eyes for four possum tails. Although both variables were visually explicit, this did not assist S2 to identify the co-variational relationship. It appears that S2 counted all the possum eyes (12) that were present in the pattern provided, rather than attending to just four possums.

Lesson 1 Conjecture 3: Manipulating hands-on materials that represent the variables of a growing pattern allows students to better attend to the pattern structure.

As there were no hands-on materials used in Pretest 1 to explore growing patterns, S2 abilities in relation to this conjecture cannot be determined. The use of hands-on materials is discussed in PCI1.

Additionally, conjectures offered for lesson 2 are not presented as S2 was absent for this lesson.

Lesson 3 Conjecture 1: Transferring mathematical knowledge between patterns with the same multiplicative structure is difficult.

It is to be noted that S2 had minimal input into class discussions in lesson 3. S2 was engaged in the lesson, attentively listening to his peers, and provided some responses to questions posed. In response to conjecture 1 for lesson 3, due to his absences from lesson 1 and 2, it could not be determined if S2 was transferring prior multiplicative structures. Regardless, S2 was able to identify that the rule was doubling. When the class was asked, "How do I work out how many ears there are if I have five tails?" S2 turned to IEO1, and quietly inquired, "Is it doubling?" He did not offer his answer until he had confirmed it was correct with IEO1. S2 identified the multiplicative structure of the pattern, but it could not be determined if this was from prior learning.

Lesson 3 Conjecture 2: Providing students with the mathematical language used to describe multiplicative structures assists students to generalise the pattern.

During lesson 3, students participated in a discussion that focused on the use of explicit mathematical language (doubling or times two). Like S1 and S6, S2 identified that he had used this mathematical language before. S2 was able to use the

mathematical language of 'doubling' to describe how the kangaroo pattern was growing. Similar to the data reported for S1 and S6, students were asked, "What if I had a number called *n*, what would I have to do to it?" S2 responded, "Double it."

Lesson 3 Conjecture 3: Providing growing patterns where the variables are embedded and cannot be physically separated from each other, assists students to attend to both variables simultaneously.

S2 was able to attend to both variables in the kangaroo pattern to assist him explain the relationship between the tails and ears. He was able to predict how many ears there would be if there were 100 kangaroo tails (200), and explained that he was doubling the number of tails to find the number of ears.

At the conclusion of Lesson 3 of Teaching Episode 1 all previous conjectures and data were considered in order to select patterns and construct questions for the one-on-one interviews. The clinical interviews provided a one-on-one environment for deeper exploration of the previous conjectures relating to either a mathematical, semiotic, or cultural aspect of the study. The following section presents the 'learning journey' of S2 during the clinical interview. The data from each task (see Figure 6.2) are presented.

6.4.3 Student 2 Learning Through Clinical Interview 1

Task 1 of PCI1 (Crocodile Pattern)

As Student 2 had been absent for lesson 1 and 2 of Teaching Episode 1, it was decided to do a revision of the kangaroo task from lesson 3 at the commencement of PCI1. S2 recalled that the rule from the kangaroo pattern was, "Times two" (PCI1_S2_10). The researcher then constructed the first three terms of the crocodile pattern for task 1 of the interview. S2 was asked to identify how many feet there would be for two crocodile tails. The video reviewed displayed S2 moving his eyes along the pattern and announcing, "I know how much it is altogether; the legs and tails added altogether...It's 30." He counted, "10, 20, 30" and as S2 was counting he was pointing to two crocodiles at a time. S2 then self-corrected and identified that he should be counting in fours and identified that three crocodiles tails would have 12 feet.

S2 was shown a kangaroo as a prompt to determine if he could transfer any learning from the previous lesson. Below is an excerpt of the discussion:

29 R1:   So, what did we do yesterday with our kangaroos? What did we have to do to the tails to work out the ears?

30 S2:   Count in twos. No times in twos

31 R1:   So what do you think we have to do to the crocodiles' tails to work out the feet here?

32 S2:   Times in twos. Ohhhh timesing the tails by twos

33 R1:   Timesing the tail by...

34 S2:   Four

35 R1:   So if I have two tails what would I times it by?

36 S2:   Time the two by four

37 R1:   Ok what would I do to the three? Times it by?

38 S2:   Four

39 R1:   So what do you think my rule is S2?

40 S2:   Four

41 R1:   Well it is not just four

42 S2:   Times four

S2 was able to transfer the multiplicative concept from the pattern rule used in the kangaroo lesson to the crocodile pattern. Unlike S1 and S6, S2 did not require the same level of scaffolding to determine the pattern rule. There was little gesture used by the researcher, and the structure of the pattern did not need to be attended to as much as for the other students. It appears that S2 was confident using mathematical language, could see the structure of the pattern (fourness), and transferred the multiplicative concept he learnt the previous day.

Additionally, S2 provided quasi-generalisations for position 10, 100 and 1 million of the pattern. He was also able to identify that for position 10, if you multiply it by four it gives you 40, which is the number of crocodile tails (55-58). Though when asked to generalise the rule for position *n,* S2 responded 'four'. Similar to S1, he was unable to link alphanumeric notation to the number.

Task 2 of PCI1 (Classroom Pattern)

S2 immediately engaged with a hands-on approach in the construction of the pattern used in task 2. The number cards were placed on the desk by the researcher. He was shown (by the researcher) how to construct grade 2, 3, and 4 of the classroom pattern. For example, he was told that grade 2 has six desks and was instructed to place the six desks into two rows of three. During this process S2 made computational errors when predicting how many tiles were needed for the next grade (eight tiles for grade three instead of nine tiles). This appeared to be occurring because he was counting on the number of tiles rather than using the multiplicative structure of the pattern. However, he was able to self-correct and continued constructing the pattern. He was also able to identify how many tiles were needed for Grade 1, and what the pattern would look like. He stated, "I just counted in threes" to work out how many tiles were needed for each year. It is evident that S2 was able to see the 'threeness' of the pattern.

S2 was able to identify the rule of the pattern as, "Times three." (PCI2_S2_103). Later in the interview when we revisited task 2, S2 confidently used mathematical language throughout the task. There was no need for the researcher to discuss mathematical language with S2 at any point during the clinical interview, as illustrated by the following excerpt from this interview.

Below is an excerpt of the discussion that ensued:

107 R1: So for grade 2?

108 S2: Two times three equals six [*R separates the pattern into two rows of three*]

109 R1: [*R points to the number card with 3rd written on it*]

110 S2: Three times six equals nine [*R separates the pattern into three rows of three and begins to identify the multiplicative structure*]

111 R1: Three times how many?

112 S2: Three

113 R1: Ok so grade for is four times....

114 S2: Twelve [total number of tiles]

115 R1:   Not four times twelve. What is our rule?

116 S2:   Times three

117 R1:   So four times three is [*Researcher separates the pattern into four rows of three*]

118 S2:   Twelve

119 R1:   Good boy. What would I have to do to find grade 5?

120 S2:   Oh wait if that is twelve then fourteen. Nah [*S2 smiles and turns his head away*] [reverts to adding instead of multiplying]

121 R1:   Good boy I know what you are doing. So what would I have to do to find out the answer?

122 S2:   Five times three equals fourteen

123 R1:   Not fourteen. Fifteen

In summary, throughout this discussion S2 made computational errors when determining how many tiles were in each structure. This suggests that S2 does not know the multiples of three; rather he was counting on three each time and making an error. When S2 was asked to generalise the pattern rule for *n* he stated, "Times it by three." It is apparent that although S2 was able to see the structure of the pattern, on occasions he had issues with computation.

Task 2b of PCI1 (Classroom and Teacher Pattern)

The final task for the clinical interview explored the notion of a constant being added to the classroom pattern (see *Figure 6.3*). It was explained to S2 that the orange tile represented a teacher in the pattern. S2 was asked, "What do you think has happened to my rule?" S2 responded, "Times four. Nah." Below is an excerpt of the ensuing discussion:

137 R1:   Why do you think that doesn't work?

138 S2:   Teacher

139 R1:   So we have one times three [*R points to the first term of the pattern*]

140 S2:   Add four....

141 R1:   Is that adding four [*R pointing to teacher*]. How many teachers are there?

142 S2:   One

143 R1:   So....

144 S2:   One times three add one [*S2 looks uncertain almost in disbelief*] that is 4.

145 R1:   Well let's see if that is true [R writes down 3x1 in notebook]

146 S2:   Four

147 S2:   No three

148 R1:   So three plus one is?

149 S2:   Four

150 R1:   So let's see if we can do it to our next class.

151 S2:   Two times three is six

152 R1:   Ok and what do we have to do to the teacher?

153 S2:   Oh it is the same.

154 S2:   Two times three is six

155 R1:   And what do we do with the teacher?

156 S2:   Add it

157 R1:   How many do I add on?

158 S2:   One

159 R1:   So what does that equal?

160 S2:   Seven

161 R1:   Good boy.

S2 then wrote his own expressions for position three and four of the pattern. Evidently, S2 had to be scaffolded through the activity to 'see' the constant in the structure of the pattern. Once he had established that, he was able to transfer this

knowledge to uncountable situations. Figure 6.10 illustrates S2 written expressions for position three and four.
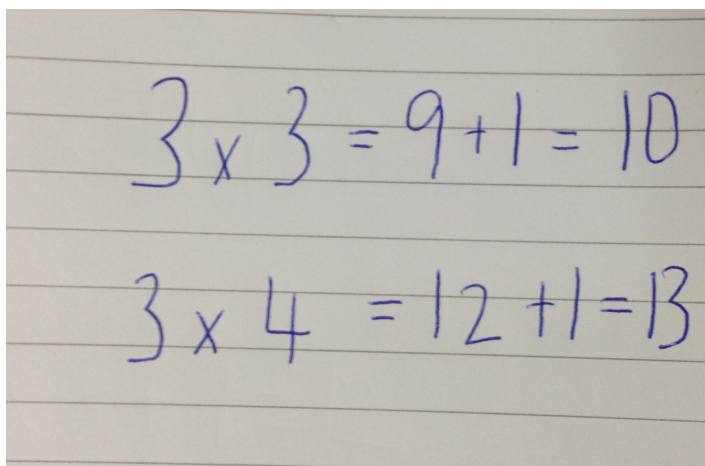
Figure 6.10. Student 2 expressions for position three and four for task2b.

Student 2 was then asked to quasi-generalise the pattern for position 100. He identified that for position 100 it would be, "Times three is 300 plus one is three 301."(PCI1_S2_182). He also identified that the pattern rule was, "Times three plus one" (PCI1_S2_178).

6.4.4 Student 2 Learning in Relation to Conjectures Presented in Teaching Episode 2

The following section presents the conjectures delineated at the conclusion of Pretest 2, and after each lesson of the Teaching Episode 2 in relation to S2.

Lesson 4 Conjecture 1: Exploring growing patterns from environmental contexts assists Indigenous students in relating growing patterns to their prior experiences.

S2 immediately displayed an understanding of the multiplicative structure for the caterpillar-growing pattern. After constructing the first term, S2 called out, "four". He was asked to explain what "four" meant. S2 identified that the next term in the pattern needed four counters and that the pattern was, "Counting in twos" or "Double it." He then predicted the number of counters required to construct term

three, four, and five of the pattern. Like S6, S2 did not use an environmental context to explain how the pattern was growing or to describe the geometric structure of the pattern.

Lesson 4 Conjecture 2: Explicitly modelling the relationship between the variables in the growing pattern assists students to use the alphanumeric notation to describe the generalisation.

In the very early stages of lesson 4, S2 expressed the general rule for the pattern using no alphanumeric notation (double it – doubling the day to determine the length of the caterpillar). Later in the same lesson, when the class was asked, "What would the rule be for n days?", the first student to call out the answer was S2, "Double n." It appeared in this context S2 had little difficulty using alphanumeric notation in generalising the growing pattern.

Lesson 4 Conjecture 3: Using semiotic bundling, (i.e., using gesture, language and manipulation simultaneously) assists students to identify the structure of the pattern.

The caterpillar pattern was accompanied with gestures (researcher) and questioning that framed students to attend to the structure of the pattern. S2 was able to correctly predict the length of the caterpillar on days 30, and 100. He was also able to explain that you times 100 by two to give the answer 200. The class was asked, "What would the pattern look like on day 30?" S2 called out, "Sixty, it's like three plus three... 30 plus 30." When the researcher did not acknowledge his response S2 then called out again, "Its 20 and 20 and 20." It appears S2 was using computation to assist him to identify the structure. Another student (S18) responded correctly stating that it was, "Thirty groups of two." S2 turned to IEO1 and asked if his response of 20 plus 20 plus 20 was still correct.

It was apparent that even though S2 generalised the caterpillar pattern he was not seeing the structure of the pattern displayed. The discussion that followed included the liberal use of gesture and mathematical language to describe the relationship between the day and the length of the caterpillar. Later in the lesson, the caterpillar pattern was revisited with a new rule (3n). S2 had little difficulty seeing the structure of the pattern. He was able to identify that for day seven there needed to be seven groups of three counters. Additionally, S2 discussed with the researcher, if

the pattern was growing by four, day seven would need seven groups of four counters. This highlights that S2 was seeing the structure of the pattern and was able to apply this structure to other multiplicative patterns.

Lesson 5 Conjecture 1: Manipulating hands-on materials that represent the variables of a growing pattern allows students to better attend to the pattern structure.

During lesson 5, S2 successfully used hands-on materials to construct the flower pattern. As soon as he received the hands-on materials, S2 was copying the pattern from the board. S2 was able to use the counters to copy and continue the flower pattern. It appeared that S2 enjoyed working with hands-on materials, and frequently used them to assist him to answer questions throughout the lesson.

Students were asked to explain the structure they could see in the pattern. S2 described that for term three he could see six and six. Figure 6.11 presents the drawing S2 used to explain what he saw.
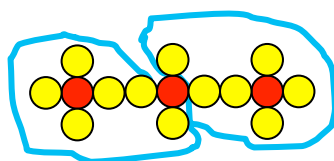
Figure 6.11. Structure seen by S2 for term three of the flower pattern.

S2 continued to use the hands-on materials and called out, "I can see something else. Five, ten, fifteen." It was apparent that S2 was beginning to see the 'fiveness' of the pattern. As mentioned earlier, this was not the intended rule for the flower pattern. The intended rule was $4n$ but it was decided to continue with the rule identified by S2 ($5n$).

All students were then asked to construct term seven of the flower growing pattern. S2 placed all the red counters down first on his desk (red centres) and then placed the yellow counters onto each red centre. He was successful in constructing the seventh term of the pattern. This was the first time that S2 worked with a growing pattern where the two variables were embedded but could be physically separated. Like S6 in lesson 1 and 5, S2 also physically attended to the two variables separately.

The counters were not the only materials S2 used to answer questions about the pattern. He also used the classroom clock. When S2 was predicting patterns beyond the terms presented, he explained that he was using the numbers on the clock to

assist him. He used the structure of the five-minute intervals on the clock to assist him to answer further pattern terms. For example, S2 identified that for 12 red centres there would be 60 counters or 12 groups of five. He was linking the twelve on the clock to sixty minutes and transferring this to the flower pattern. He also identified that if there were 70 counters, you would have pot-plant 14. It appears that S2 was using the clock to support the computational aspect of the task. Additionally, it was evident that S2 recognised the 'fiveness' in the pattern.

Lesson 5 Conjecture 2: Exploring pattern structure by attending to both variables will assist students to generalise.

The focus of lesson 5 was to assist students to consider the pattern structure. Gesturing by the researcher (pointing to structural elements of the pattern) and having students discuss the separate elements (red centres and petals) assisted students to consider both variables of the pattern. S2 had a strong understanding of the 'fiveness' of the pattern and the relationship of the two variables. However, he was not as confident when generalising the pattern. S2 could see the fives in his pattern, but could not express what mathematical operation he was carrying out. He was unsure if he was adding, multiplying, or taking away. Though the researcher was attending to both variables in the lesson, this did not assist S2 to verbalise the generality of the pattern. Even though he could see the general structure and transfer this to other contexts, he was unable to verbalise the generality.

Lesson 5 Conjecture 3: Providing growing patterns where the variables are embedded in the pattern and visually explicit ensures that students attend to both variables.

The two variables in the flower pattern were visually explicit and embedded. The semiotic construction of this pattern assisted S2 to identify the relationship between the number of red centres and the total number of petals, as both counters needed to be used to construct the pattern.

Lesson 6 Conjecture 1: Creating a 'story' about how the two variables are related assists students see the co-variational relationship.

During lesson 6, a story was used to provide a context for the geometric growing pattern. This gave many students an opportunity to discuss the geometric pattern in terms of characters of the story. For example, the blue squares represented

students in the class and the green square was the teacher. S2 identified that the constant of the pattern was the green tile (teacher). S2 was able to identify that in grade 6, six rows of three students were needed to make the pattern and one teacher, and also provided a correct response for grade 10. The story context gave S2 an opportunity to attend to both the pattern position (grade) and the structure of the pattern (rows of three), plus one teacher. S2 was also able to determine the total number of tiles needed for each term (class).

Lesson 6 Conjecture 2: Exploring multiplicative growing patterns with a constant are more difficult than exploring multiplicative growing patterns without a constant.

Like S1 and S6 it appeared that S2 had little difficulty understanding the notion of a constant. This notion was explored in more depth during Clinical Interview 2.

Lesson 6 Conjecture 3: Using an iconic symbol (e.g., colour) to represent the constant in a growing pattern assists students in identifying the constant.

By using the green tile in the pattern to signify the teacher, it is inferred that many students could recognise it as the constant. S2 agreed that the green tile was the same for each term of the pattern presented on the board. He also articulated that the green tile (teacher) was needed for subsequent patterns.

6.4.5 Student 2 Learning Through Clinical Interview 2

Task 1 of PCI2 (Create a Growing Pattern)

The Task 1 of PCI2 required S2 to construct his own growing pattern. S2 selected the red and yellow counters to construct his pattern. Figure 6.12 illustrates the pattern constructed by S2.

Figure 6.12 Growing pattern created by S2 during task 1 of PCI2.

S2 explained that his rule was adding four each time. Both the researcher and S2 trialled the rule, and separated the pattern into groups of four. S2 recognised that he was missing term three of his pattern and then constructed 12 counters between the second term (eight red counters) and the fourth term (16 yellow counters). By separating the groups of four S2 was able to identify that he had made an error. S2 then predicted that he needed 10 groups of four for the tenth term. During the

discussions in this first task, S2 was focused on providing the answer for the total number of counters in each term rather than the structure. This observation was also supported by IEO2. IEO2 explained that S2 was trying to display that he understood the task and was seeking to please the teacher (IEO2_PCI2_S2_32).

Task 2 of PCI2 (Daisy Chain Pattern)

The daisy chain pattern was used to explore the notion of a constant in growing patterns. As soon as the pattern was being constructed by the researcher, S2 assisted with the construction of the pattern. It appeared that S2 enjoyed working with hands-on materials. Immediately after the pattern was constructed S2 announced, "I get it." The excerpt below presents the discussion that followed:

56 S2   Oh now I get it. I know how to make it.

57 R   Ok stop there and we will talk about it. How many red centres do I have?

58 S2   Three

59 R   And how many yellow petals?

60 S2   Twelve

61 R   Ok let's count them

62 S2   Oh ten

63 R   You show me what you see

64 S2   I see four and four. See four plus two and then five and five [*S2 draws imaginary loops around five yellow counter four from the whole flower and one from the centre structure*].

Figure 6.13 illustrates the original pattern structure and the structure S2 distinguishes from the daisy chain pattern. The blue loops signify the 'five and five' identified by S2 (PCI2_S2_64).
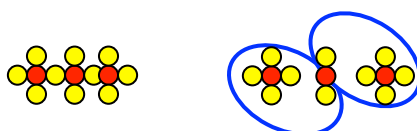
Figure 6.13. The original daisy chain pattern structure and the structure identified by S2.

It is evident that S2 was not seeing the multiplicative structure of the pattern. The pattern was reconstructed to the original daisy chain pattern by S2. It was then separated by the researcher into groups of three with the constant left to the side. When asked what S2 was seeing now, he stated, "three, three, three, three." (PCI2_S2_75) The constant was then signified with a sticker. S2 stated that the rule was. "Adding on three" (PCI2_S2_82).

To assist S2 to see the multiplicative structure, the researcher asked a series of questions so that he attended to the two variables. However, it proved to be challenging, as S2 did not listen to the questions being asked. Below is an excerpt from the discussion:

108 R	How many red centres?
109 S2	Twelve. No that is not Twelve, yeah it is twelve.
110 R	Ok now stop. I just want you to tell me red centres
111 S2	Four
112 R	How many groups of three yellow petals are there?
113 S2	Four
114 R	What would I do to get number five?
115 S2	Add on one more group.
116 R	How many groups of 3
117 S2	Fifteen
118 R	No not the answer
119 S2	Four no Five
120 R	If I need six red centres how many groups of three will there be?
121 S2	Three. One more three? Well it's four with the red.

At this point of the interview it was decided to remove the red centres as they appeared to be distracting S2. This assisted S2 to begin to see the 'threeness' of the pattern. Additionally, S2 was able to identify that the constant (counter with the bee) represented adding one onto the pattern. Though S2 was now discussing the structure

of the pattern, he was intent on showing that he knew the total number in the pattern. This was obstructing S2 from identifying the generality of the pattern. It was not until S2 was explicitly told that he does not need to tell the researcher how many petals there are altogether (PCI2_S2_149), that he began to discuss the pattern structure. This is illustrated in the following excerpt:

147 R	How many groups of three?
148 S2	Eighteen
149 R	No not altogether
150 S2	Oh, six
151 S2	So six groups of three plus one
152 R	So for the seventh one what would I have?
153 S2	Seven groups of three plus one

S2 was able to identify the pattern rule for the daisy chain. S2 stated that the pattern rule was, "Groups of three plus one" (PCI2_S2_172). He could then predict what the pattern structure would be for terms 10, 20, 100, and 200. S2 was not asked to generalise using alphanumeric notation.

Task 3 of PCI2 (Robot Pattern)

The final task for the clinical interview was the robot pattern. S2 again assisted with the initial construction of the pattern. He was confident using hands-on materials and was always thinking ahead. Initially, S2 focused on many other elements of the pattern. The video footage evidenced that it was not until the researcher was pointing to the pattern and gesturing along the pattern, that S2 focused on the structure.

219 R	How many groups of five? [*R points to the first term in the pattern and moves finger along the row of five blue tiles*]
220 S2	One group of five
221 R	And in this one [*R points to the second term in the pattern and moves finger along the two rows of five blue tiles*]
222 S2	Two

223	R	And then [*R points to the third term in the pattern and moves finger along the three rows of five blue tiles*]
224	S2	Three
225	R	So the fourth one how many would I have? [*R points to the desk to indicate where the fourth pattern would be constructed*]
226	S2	Four groups of four.... oh five groups of four.... no I said five groups I got it four groups of five
227	R	So then I have to add on how many? [*R points to the three green tiles*]
228	S2	Three
229	R	What if it was number six in the pattern? [*R points to the desk to indicate sixth term in pattern*]
231	R	How many blue towers would I make for number 6?
232	S2	Six groups of five
233	R	And then I would put on the
234	S2	Yellows
235	R	Which is plus how many?
236	S2	Three

S2 then constructed the seventh term in the pattern; this indicated that he had a good understanding of the structure. S2 then generalised the pattern rule as, "Groups of five add on three" (PCI2_S2_254).

6.5   CASE STUDY 3: STUDENT 6

Student 6 (S6) was considered a low achiever in mathematics; this was based on a mathematics test conducted at the start of the school year (RoleM score 7.75/30). The classroom teacher and IEOs had identified S6 as high achieving student; this meant that he worked in the top mathematics group in the class. S6 had high participation levels during lessons conducted in Teaching Episodes 1 and 2. He often participated in class discussions, sharing his ideas, asking questions, and was

always happy to 'have a go'. Additionally, S6 frequently used hand gestures while working in class and when discussing his mathematical understanding. S6 participated in both Pretests and all lessons presented in the teaching episodes. He attended school regularly. It was these aspects that influenced the selection for further analysis of S6 in the Piagetian Clinical Interviews of the study.

As per case study 1 (Section 6.6.2) and case study 2 (Section 6.6.3), the data are presented in terms of a narrative. The emphasis is on the chronological events of the study that assist to 'tell the story/journey' of student's learning across the Teaching Episodes and Piagetian Clinical Interviews data collection.

6.5.1 General Observations from Lessons

S6 was a conscientious student and had high participation levels and interaction with other students during lessons. S6 was often involved in whole class discussions, and volunteered answers to questions posed during lessons. He was a very confident student. At times, S6 appeared to be working on his own, but also answered questions posed to other students. It was as if he was always listening to what was happening around him in class, and therefore S6 often called out answers to questions during lessons. Furthermore, he worked well with other students in class and discussed ideas with them. The classroom teacher and IEO1 concurred that these behaviours were displayed when they conducted lessons.

6.5.2 Student 6 learning in Relation to Conjectures Presented in Teaching Episode 1

The following section presents the conjectures delineated at the conclusion of Pretest 1 and after each lesson of the Teaching Episode1 in relation to Student 6 (see Figure 5.1). The three conjectures posed after Pretest 1 for lesson 1 (see Section 5.4.1) are presented below with data from Student 6. This data was drawn from both Pretest 1 results and his participation in lesson 1.

Lesson 1 Conjecture 1: Exploring growing patterns from environmental contexts assists Indigenous students to relate growing patterns to their prior experiences.

Student 6 demonstrated that he was able to work with growing patterns presented in an environmental context in Pretest 1. He successfully copied (complete copy) and predicted terms beyond the environmental growing patterns presented.

However, he could not continue a growing pattern (fish pattern). S6 copied this pattern rather than continuing the pattern. It was determined that if S6 extended growing patterns later in Pretest 1, then he was able to continue growing patterns. It is conjectured that the difficulties he was experiencing with continuing growing pattern was related to language, especially the confusion he had between the use of 'continue' and 'copy'. S6 created his own growing pattern (numerical response, see Table 5.3). During a discussion that followed the test, S6 stated,

> It is one block, two blocks, three blocks, four blocks and five. You need to keep putting more blocks into it. Putting more and more stuff into it [*S6 gestures with both hands as if putting things in an imaginary box*]. It is about putting more numbers into it. It just keeps growing and growing.

During lesson 1, S6 copied and continued the butterfly pattern. He identified that 36 wings were needed for nine butterflies. S6 counted each individual wing and gestured (pointed) to each wing as he counted. Later when asked, "If there are five butterflies, how many wings would there be?" S6 counted the wings in twos and again gestured to the wings (using his first and second fingers together to count in twos); however, this time his answer was 30. He identified that this was incorrect and then determined he had counted too many wings. Clearly, S6 made an error counting as all butterflies had four wings in total, thus his answer should have been a multiple of four. Evidently, S6 had difficulty identifying the multiplicative structure ('fourness') of the pattern. From viewing the videos it was difficult to determine if S6 was considering both variables when responding to the questions posed.

Lesson 1 Conjecture 2: Making both variables of growing patterns visually explicit assists students to identify the co-variational relationship

S6 successfully copied the house pattern in Pretest 1, and attended to both variables (the drawing of the house and the corresponding number card). Additionally, S6 was working with both variables in the possum pattern presented in Pretest 1. To predict the 10th term in the pattern, S6 drew the possum tails and eyes and provided a written explanation for his working. Figure 6.14 illustrates S6's working for predicting the 10th term of the possum pattern.
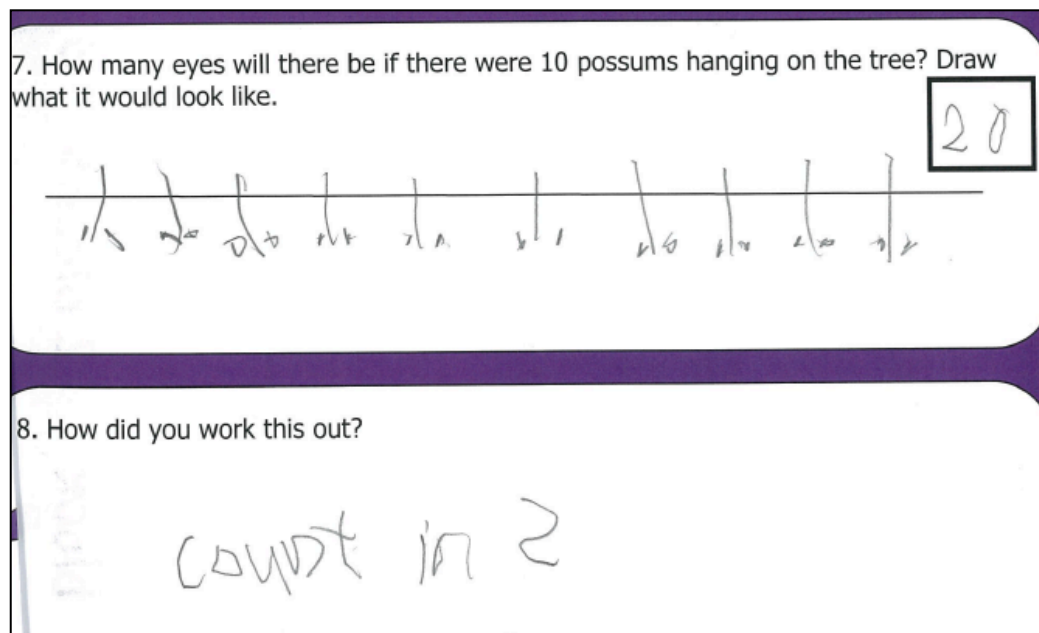
Figure 6.14. Student 6 working for predicting the 10th term of the possum pattern.

Similar to S1, it appears that this particular pattern (possum pattern) made S6 attend to both variables (tail and eyes) of the pattern. Having the variables embedded in the single pattern structure appeared to assist him to 'see' the variables. Unlike S1 however, S6 did not generalise the pattern structure to predict quasi-variables (e.g., he could not answer '376 possums how many eyes?').

During lesson 1, while both variables were visually explicit and embedded in the butterfly pattern, S6 attended to the sign vehicles (matchsticks and counters – iconic signs) separately. One other student in the class also attended to the pattern in this manner. First, he placed an array of matchsticks on the desk to represent the butterfly bodies, and then added the counters (wings) retrospectively. Thus, when constructing the pattern, S6 attended to the two sign vehicles (the iconic signs) sequentially rather than simultaneously. Whether he recognised the co-variational relationship between the two sign vehicles is difficult to determine from the video. It is because of these actions, separating the sign vehicles, that the pattern for lesson 2 was selected (feet and body).

Lesson 1 Conjecture 3: Manipulating hands-on materials that represent the variables of a growing pattern allows students to better attend to the pattern structure.

In summary, during lesson 1, S6 worked with both variables using the hands-on materials provided. Unlike S1, the hands-on materials did not prompt him to

relate the two variables or consider the co-variational relationship between variables. This could be due to the fact that the two variables could be physically separated. This issue is explored further in lesson 2.

At the conclusion of lesson 1, new conjectures were considered and trialled in lesson 2. The following section presents results from S6, collated from lesson 2 in response to the conjectures.

Lesson 2 Conjecture 1: Exploring growing patterns where the structure is multiplicative (e.g., double) assists students to generate the pattern rule.

The pattern used in lesson 2 was the feet and body pattern. The multiplicative structure for this pattern was doubling; doubling the number of bodies (people) to determine the number of feet. When asked, "How many people would there be if there were eight feet?" S6 incorrectly responded, "Eight people." In addition, S6 was unable to predict how many people there were if there were 14 feet (10 people) and 20 feet (23 people). It was evident that he was not seeing the 'twoness' of the pattern. Even though the patterns of two had been taught in mathematics lessons conducted by the regular classroom teacher, in which she had identified S6 as having a good understanding (additionally rote learning of two times tables), S6 did not transfer this knowledge into this pattern context. Though the context provided was accessible to students, this did not assist S6 to see the multiplicative structure of the pattern. This is discussed further in conjecture 2 of lesson 2.

Lesson 2 Conjecture 2: Providing growing patterns where the variables are embedded in the pattern ensures that students attend to both variables.

This pattern was selected as both variables (feet and body – iconic signs) were embedded and could not be separated, thus forcing students to attend to both variables in the pattern. It appears that S6 was not attending to both feet as one variable of the pattern. From the video, his gesturing indicates that he related the number of people to the space on the ladder rather than to the number of feet. As he was counting on one each time, S6 pointed to each ladder space. This indexical sign (pointing to the space) was created by S6.

S6 had created a third sign in the pattern, the ladder spaces. It was anticipated that students would relate the number of people to the feet (two feet) and therefore would count on in twos for each person, like S1. Evidently, the third sign (ladder

space – iconic sign) made it challenging for S6 to see the structure of the pattern. As a result of S6's actions, further consideration was required for the selection of pattern type, and consequently, the selection of the kangaroo pattern in lesson 3.

Lesson 2 Conjecture 3: Embodying the mathematical structure of growing patterns assists students to explain the pattern structure.

S6 participated in the embodiment process of lesson 2 (body and feet pattern). He became part of the pattern by standing on the ladder space acting as a body with two feet. As S6 participated and watched this process he changed his responses. When asked, "If there are 14 feet how many people would be on the ladder?" S6's initial response was 10. As a class we were able to test this answer. Once students embodied the pattern S6 changed his response to, "Fourteen feet would be seven people." Though the use of embodiment assisted S6 to arrive at the correct response, it appears that this did not help him to see the structure of the pattern.

At the conclusion of lesson 2, new conjectures were considered and trialled in lesson 3. The following section presents results for S6 collated from lesson 3 in response to the conjectures.

Lesson 3 Conjecture 1: Transferring mathematical knowledge between patterns with the same multiplicative structure is difficult.

As S6 did not see the multiplicative structure of the pattern presented in lesson 2, he did not transfer the notion of doubling to the kangaroo pattern in lesson 3. However, later during lesson 3, S6 was able to identify the pattern structure for the kangaroo pattern.

Lesson 3 Conjecture 2: Providing students with the mathematical language used to describe multiplicative structures assists students to generalise the pattern.

During lesson 3, students participated in a discussion that focused on the use of explicit mathematical language (doubling or times two). S6 identified that he had used this mathematical language before. All students participated in a discussion about generalising the kangaroo pattern using alphanumeric notation during lesson 3. Students were asked, "If there were five kangaroo tails, how many ears would I have?" While S6 did not provide the answer, another student (S8) answered, "ten". When the other was asked, "How did you work this out?" S6 called across the class, "He doubled it miss." To further explore his response, S6 was asked, "What did he

double?" S6 responded, "He doubled the tails... and it tells you how many ears". It is evident that introduction of the mathematical language assisted S6 to describe the relationship between the kangaroo tail and ears.

However, after describing this to the class S6 had difficulties predicting further terms of the kangaroo pattern. Similar to the data reported for S1 and S2, students were asked, "What if I had a number called *n*, what would I have to do to it?" S6 was able to respond that you needed to double it. Though this was a whole class activity, S6 had a high participation level in this lesson.

Lesson 3 Conjecture 3: Providing growing patterns where the variables are embedded and cannot be physically separated from each other assists students to attend to both variables simultaneously.

S6 was able to attend to both variables in the kangaroo pattern to assist him to explain the relationship between the tails and ears (see lesson 3 conjecture 2). While he explained the general rule, he did not apply it to further terms in the pattern. S6 was asked, "If I have five kangaroo tails, how many feet would there be?" He held up four fingers on his right hand and then four fingers on his left and stated that there were eight ears. Through his gesturing, S6 demonstrated that he saw the doubling structure of the pattern; however, his number sense was preventing him answering correctly. It can be concluded that S6 used the two embedded variables to explain the general rule (see lesson 3 conjecture 2) and therefore saw the general structure, but was unsure how to apply it.

At the conclusion of lesson 3 of Teaching Episode 1 all previous conjectures and data were considered in order to select patterns and construct questions for the one on one interviews. The clinical interviews provided a one-on-one environment for deeper exploration of the previous conjectures relating to either a mathematical, semiotic, or cultural aspect of the study. The following section presents the 'learning journey' of S6 during the clinical interview. The data from each task (see Figure 6.2) is presented.

6.5.3 Student 6 Learning Through Clinical Interview 1

Task 1 of PCI1(Crocodile Pattern)

For the initial task of clinical interview 1, S6 was presented with the crocodile pattern. The researcher constructed the first term and then had S6 construct term two,

three, and four of the pattern. S6 was asked, "If I had four crocodile tails how many feet would I have? *[Researcher points to the four crocodile tails]*". S6 successfully responded 16 and explained he worked this out by counting. Further analysis of the video showed that S6 was moving his eyes to each individual foot as he was counting in his head. It can be assumed that at this stage S6 was counting in ones. S6 was then asked, "What if I had two crocodile tails, how many feet would I have?" He quickly responded, "Eight". S6 explained that he counted in fours and as he did this showed four fingers. It appeared that S6 was seeing the 'fourness' of the pattern and that the context of the pattern was assisting him to see the structure. However, when asked to count in fours S6 responded, "Four, eight, nineteen, no sixteen, then I don't know anymore [*S6 gesturing holding four fingers up on right then four fingers on the left as he was counting*]." While S6 could see the structure of four, it was apparent that his mathematical knowledge (number sense) was weak.

S6 was then asked, "What if I had 12 feet how many crocodile tails would I have?" S6 counted twelve crocodile feet pointing to each one as he counted. Then he held out his thumb and little finger around the three crocodiles and stated, "You'd be having three." This was an indication that S6 was starting to attend to both variables in the pattern; however, he did not apply the multiplicative relationship between the two variables to uncountable contexts. S6 was asked, "If I have 100 tails how would I work out the number of feet?" His response was, "counting in fours... and finish at the last one." S6 gestured four with his fingers and tapped this along the desk from right to left to display how he counted in fours.

It was evident that S6 was beginning to attend to the multiplicative structure of the pattern. However, he was unable to use multiplicative language to express this relationship. Similar to Conjecture 3, Lesson 3, having a pattern where the variable was embedded and unable to be physically separated, assisted S6 to make predictions about the crocodile pattern. Table 6.4 summarises the major teachable actions, researchers questions/directions, student responses/engagement, and observations during the interview process for Task 1.

Table 6.4

Teachable Actions, Researcher participation, Student participation, and observation during Task 1

Teachable actions	Researcher participation (questions/directions)	Student participation (responses/engagement)	Observation
Researcher constructs pattern	Researcher places term one on desk		
Student continues pattern	Researcher guides Student to construct term two, three and four of the pattern – use of gesture indicating the placement of each term	Student places crocodiles in terms two, three, and four.	Student successfully continues pattern
Student asked to identify pattern structure – focus on one variable	Researcher asks student to predict for four crocodile tails how many feet	Student answers sixteen	Student looks at four crocodiles and counts up to sixteen – appears to be counting in ones
Student asked to identify term from pattern	Researcher asks student if there are 12 feet how many tails	Student response 'ten tails'	Student does not respond correctly
Test prediction to assist with justification		Student places ten crocodiles on the desk	Student identifies that there are forty feet
Revisit - student to identify term from pattern		Student correctly identifies three tails	Counts the feet to 12 and then holds his thumb and little finger out over the group and counts to three
Describe pattern structure at term 100	Researcher asks student to describe how he would work out how many feet there were if there were 100 crocodile tails	Student states count in fours	
Transfer knowledge from previous lesson	Discussion with student about rule from the kangaroo pattern focusing on language	Student does not recall the rule (doubling) rather explains it as adding on two	Student having trouble applying the multiplicative structure

Indexical signs were used by both S6 and the researcher throughout the interview. S6 used gesture to assist him to answer questions and to supplement his verbal explanations. The researcher used gesture (pointing) to highlight the two variables (tails and feet) when discussing the pattern with the student. It appears, for

S6, that gesture played a two-fold role. First, assisting S6 to attend to the two variables (tails and feet), and secondly by gesturing (researcher) while discussing the structure of the pattern. In turn, S6 also gestured to the pattern when determining answers. It appeared that the gestures assisted S6 to attend to the two variables. S6 required and also used more gesturing than S1. Second, gestures added another layer of communication to his explanations. Often, it appeared that S6 found it challenging to verbally explain elements of the pattern; however, gestures (hand movements) displayed elements of this thinking. This was also confirmed by IEO1.

Task 2a of PCI1 (Classroom Pattern)

The growing pattern presented in Task 2a was constructed using blue tiles and number cards. Like S1 and S2, the researcher told the story as she constructed term two, three and four of the pattern (see S1 Task PCI1). While, this was occurring S6 also counted the number of blue tiles in each pattern. S6 was then asked to construct Grade 1, the first term of the pattern. He successfully placed the three blue tiles and number card on the desk. S6 was asked, "How did you work that [term 1] out?" And he responded, "Six started from three." The six S6 was referring to were the six blue tiles in term two of the pattern.

S6 was then asked to make grade 5. As he began to construct grade 5, S6 was talking to himself and looking at Grade 4. He muttered, "12 plus 12". Figure 6.15 illustrates the pattern that S6 constructed when asked to construct term 5.
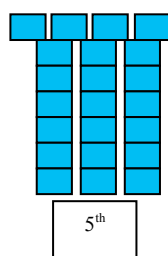
Figure 6.15. Pattern constructed by S6 when asked to construct term 5.

Though S6 placed 22 tiles on the desk for the fifth term, it is suggested that his intention was to place 24 tiles on the desk because of his previous statement (12 plus 12). At this point, it was evident that S6 was attending to both variables physically (tiles and number card) but failed to identify the relationship between the variables.

When S6 was asked to identify how many tiles in each term of the pattern, S6 stated, "three, six, twelve" and then paused. S6 was seeing the pattern as three, six, twelve, twenty-four, a pattern where each term was double the amount in the preceding term. He did not see the nine blue tiles in grade 3, he saw that term as having 12 tiles. The pattern three, six, twelve, twenty four is a number pattern that S6 was capable of exploring. When considering both variables (pattern and term) this pattern is an exponential pattern ($t_n = 3(2)^{n-1}$), therefore identifying the co-variational relationship as too challenging for students this age.

To assist S6 identify the relationship between the two variables, the researcher then explicitly attended to the structure of the pattern. Below an excerpt of the conversation demonstrates how the researcher attended to the structure of the pattern:

81 R   One group of three, two groups of three, three groups of three [*R points to number card*]

82 S6   Nine

83 R1   And four groups of three

84 S6   3, 6, 9, 12, 22

85 R1   Ok so how did you get from 12 to 22

86 R1   Um counted in 12's.

87 S6   Why did you add on another 12?

88 R1   To make it keep growing

S6 was again taken through the structure of the pattern; however, this time there was a heavy focus on language and gesture in an attempt to help S6 identify the structure.

92 R   Have you heard about multiplication? Have you done your times tables?

93 S6   Yes

94 R   What is two times three?

95 S6   Two times three is six

96 R	One group of three or one times three what does that equal? [*R pointing to number card and then pattern*]
97 S6	Six [*S6 shakes his head*]
98 R	One times three or one group of three...[*R gestures to the first term of the pattern*]
99 S6	Three [*R nods*]
100 R	Two groups of three. How many tiles are there? [*R pointing to number card and then pattern*]
101 S6	Three
102 R	[*R Gestures to second term of the pattern*]
103 S6	Makes six
104 R	Three groups of three [*R pointing to number card and then pattern*]
105 S6	Nine
106 R	Four groups of three [*R pointing to number card and then pattern*]
107 S6	Twelve
108 R	What should this be? [*R pointing to number card*]
109 S6	Five groups of three
110 R	So we don't need these do we? [*R removed the extra tiles*] So what would grade 6 be? [*R pointing to where grade 6 would be on the desk*]
111	Six groups of three

Through this process, focusing on language and using gesture, S6 made the shift to seeing the covariational relationship of the pattern, that is the relationship between that pattern quantity and pattern term. He was also able to use the position number to identify the pattern structure. Once S6 identified the structure, it was easy for him to apply this to the quasi-variables. S6 correctly identified the structure for grade 100 (100 groups of three [*S6 gestured the lines of three beside the example*

given]) and grade 1 million (1 million groups of three [S6 *gestured the lines of three beside the example given*]). Though he generalised the structure for grade 100, he was unable to identify the total number of tiles required to construct the pattern.

Finally, S6 was asked to generalise the pattern rule. S6 reverted to his previous thinking and responded, "Keep adding on threes." It is difficult to determine why S6 reverted to explaining the structure in terms of recursive thinking (adding on three), possibly because the question posed did not contain an alphanumeric term to represent the term number in the pattern. So, S6 was asked, "Did we keep on adding threes? Or were we making groups of three?" and he was able to respond that we were making groups of three. S6 was then asked, "What if this [R pointing to the desk as if to indicate another grade] was grade n?" S6 quickly responded, "n times three". From this response it was evident that S6 was able to generalise the structure of the pattern using alphanumeric representations. Also, S6 was using mathematical language 'groups of' and 'times' more flexibly. It should also be noted that the use of this language was heavily supported by the researcher. Table 6.5 summarises the major teaching actions, researchers questions/directions, student responses/engagement, and observations during the interview process for Task 2a.

Table 6.5

Teaching Actions, Researcher Participation, Student Participation, and Observation During Task 2a

Teaching Actions	Researcher participation (questions/directions)	Student participation (responses/engagement)	Observation
Researcher constructs and verbalises the pattern structure	Researcher tells story for the pattern	Student listens to story	
Focus on the structure of the pattern	Ask student how many tiles in each term	S6 response six, nine, and then has to count in ones for 12	Student counts in ones for term four
Student asked to predict next term	Researcher asks how many in term five	S6 response 12 plus 12	S6 is not seeing the relationship between variables. He is creating an additive relationship between the patterns.
Student kinaesthetically engages with pattern		Student constructs term 5.	Student constructs pattern with 22 tiles
Researcher attends to the pattern structure verbally, focusing on mathematical language and using gestures	Researcher takes S6 through terms two, three, four and five. "Two times three is ... [S6 response]" and gesturing to the number card and the pattern	Student observes and provided responses. When S6 arrives at term 5 he is able to identify the structure of the pattern and correct his previous pattern constructed.	S6 watching gestures and contributing answers
Students predicts next term – near generalisation of structure		S6 identifies that term six would be six rows of three	
Quasi generalisation	Researcher asks S6 to predict for grade 100 and 1 million	S6 correctly identifies pattern structure of grade 100 (100 rows of three) and 1 million (1 million rows of three)	S6 gestures the rows of three
Pattern rule		S6 could not identify the pattern rule - adding on three	
Alphanumeric generalisation		n times 3	

Task 2b of PCI1 (Classroom and Teacher Pattern)

S6 did not complete Task 2b of the interview. As mentioned earlier not all students participated in all questions presented in Piagetian Clinical interview 1 and 2.

6.5.4 Student 6 Learning in Relation to Conjectures Presented in Teaching Episode 2

The following section presents the conjectures delineated at the conclusion of Pretest 2 and after each lesson of the Teaching Episode 2 in relation to S6.

Lesson 4 Conjecture 1: Exploring growing patterns from environmental contexts assists Indigenous students in relating growing patterns to their prior experiences.

S6 appeared to have a good understanding of the structure of the caterpillar pattern presented in lesson 4. He also displayed a good understanding of the structure of geometric patterns presented in Pretest 2. S6 did not use an environmental context when explaining how he saw the structure of the caterpillar pattern. While he identified that the rule was doubling, he did not elaborate (to the class) how he knew this.

Lesson 4 Conjecture 2: Explicitly modelling the relationship between the variables in the growing pattern assists students to use the alphanumeric notation to describe the generalisation.

In Pretest 2, S6 identified that the rule for the pattern presented in Question 4 (2n) was "*n* count another *n*". This indicates that S6 was beginning to link alphanumeric expressions to generalisations.

During lesson 4, S6 had high participation during class discussions when focusing on using alphanumeric notation to represent the generalisation of the caterpillar pattern. S6 expressed that the rule for the pattern was, "*n+n=m*". When asked, "Why *m*?", S6 stated, "It's because it's got two...' then he gestured to arcs with his finger in the air, as if he was drawing the letter m. S6 was then asked, "What if it was day *s?*", S6 responded, "Eight." This will be elaborated on in the next chapter.

Lesson 4 Conjecture 3: Using semiotic bundling, (i.e.,, using gesture, language and manipulation simultaneously) assists students to identify the structure of the pattern.

The caterpillar pattern was accompanied with gestures (researcher) and questioning that framed students to attend to the structure of the pattern. S6 was able to correctly predict the length of the caterpillar (2n) on days 5, 30, and 100. This had

been the first time S6 was able to correctly predict patterns in a consistent manner beyond the term presented. Previous to this, S6 was able to predict some terms beyond the pattern but was not always successful. Later in the lesson, the caterpillar pattern was revisited with a new rule (3n). S6 was able to identify almost immediately that it was growing by three each day. He was also able to identify that there were seven groups of three needed to construct day seven. Through this lesson (3n caterpillar), it was obvious that S6 saw the structure but had difficulty applying the rule. For example, on day three of the 3n caterpillar pattern he predicted the length of the caterpillar was 12 counters long, instead of nine. When asked why it was 12, S6 stated, "It's growing in threes." This may have been a computational error. Despite this, it was evident that semiotic bundling assisted S6 to begin to successfully work with both variables in the pattern.

Lesson 5 Conjecture 1: Manipulating hands-on materials that represent the variables of a growing pattern allows students to better attend to the pattern structure.

During lesson 5, S6 successfully used hands-on materials to construct the flower pattern. However, he did not copy the pattern as presented on the board. S6 just placed the flowers randomly over the desk. His actions were seen by the researcher during the lesson, and as a result the researcher discussed with students how each flower pattern was in a flower pot. On the board the flowers were drawn in pots in an attempt to assist students, such as S6, to correctly create the structure of the pattern. Figure 6.16 illustrates the flower pattern drawn for S6 to see the structure.
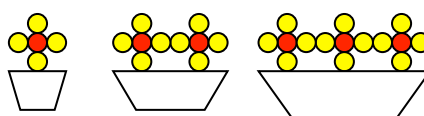
Figure 6.16. Flower pattern redrawn into flowerpots to assist students to see the structure.

The pattern structure was discussed and accompanied with gestures. S6 was able to manipulate the hands-on materials to represent the growing pattern. When asked to make pot-plant seven, S6 counted the red centres, gesturing (pointing) to

each one, then placed the yellow petals around the centres. Similar to the butterfly pattern presented in lesson 1, S6 attended to the two variables separately (placed an array of red counters then placed the yellow counters around the red centre); however, this time he was able to identify the relationship between the two. From this point, S6 started to count each flower in fives. It appears that manipulating the pattern by using hands-on materials assisted S6 to see the 'fiveness' and reach an understanding of the pattern's structure.

Lesson 5 Conjecture 2: Exploring pattern structure by attending to both variables will assist students to generalise

Lesson 5 focused more on the structure of the pattern and attending to both variables, rather than the total number of counters in each pattern term. For example, discussing that pot-plant three has three flowers with three red centres, each flower has five petals, rather than, simply stating that pot plant three has 15 petals. As the structure was explored with the class, gestures (pointing) were used to highlight each variable. When asked questions about the pattern structure, S6 articulated that, "Pot-plant seven had seven red centres" and, "Pot-plant 12 had 12 red centres." Clearly, focusing on the structure rather than the total number of counters in each flower assisted S6 to attend to both variables when predicting terms beyond the pattern presented.

Lesson 5 Conjecture 3: Providing growing patterns where the variables are embedded in the pattern and visually explicit ensures that students attend to both variables.

The flower pattern provided in lesson 5 contained variables that were visually explicit and embedded. S6 was able to copy and extend this growing pattern. This type of pattern assisted S6 to see the relationship between the number of red centres and the total number of petals, as he needed to attend to both the red centres and yellow petals to construct the pattern.

Lesson 6 Conjecture 1: Creating a 'story' about how the two variables are related assists students see the co-variational relationship.

The use of telling a story gave many students a context for the classroom and teacher pattern presented in lesson 6. S6 agreed with S1 that the constant was the green tile and that represented the teacher, while the other tiles represented students

in the class. S6 was able to identify that in grade 3, three rows of three students were needed to make the pattern and there were six rows of three for grade 6. The story context made S6 attend to both the pattern position (grade) and the structure of the pattern (rows of 3), plus one teacher. When asked to generalise the pattern S6 stated, "*n* rows of *m* plus one." It can be seen that S6 was recognising the two variables as he labelled them with different alphanumeric labels. Later in the lesson he generalises the pattern as, "*n* rows of three plus 1."

Lesson 6 Conjecture 2: Exploring multiplicative growing patterns with a constant are more difficult than exploring multiplicative growing patterns without a constant.

Like S1 and S2 it appeared that S6 had little difficulty understanding the notion of a constant. This notion was further explored in more depth during Clinical Interview 2.

Lesson 6 Conjecture 3: Using an iconic symbol (e.g., colour) to represent the constant in a growing pattern assists students in identifying the constant.

S6 identified that the green tile was the teacher in each pattern. He was able to recognise that this green tile did not change in any term of the pattern. By using a different colour tile (green) from the pattern (blue) it signified the constant. This assisted S6 to develop an alphanumeric generalisation (*n* rows of *m* plus one) and identified the teacher as the plus one in his rule. This was the first time S6 had worked with a pattern that had a constant.

6.5.5 Student 6 Learning Through Clinical Interview 2

Task 1 of PCI2 (Create a Growing Pattern)

The first task for the interview required S6 to construct his own growing pattern. S6 selected the blue and green tiles to construct his pattern. Figure 6.17 illustrates the pattern constructed by S6.
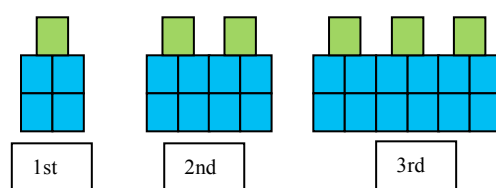
Figure 6.17. Growing pattern created by S6 during task 1 of PCI2.

S6 then explained what day four would look like. Initially, S6 focused on the total number of tiles you needed to construct the pattern (20 tiles). He was then further prompted and stated, "Four fives." Accompanying his explanation, S6 gestured with his hands holding up four fingers. S6 then needed to identify the exact structure of the pattern and explain how it would be constructed: for example, how many green tiles and blue tiles were needed to construct the fourth term? S6 stated, "Four greens and four blues in four." As he was explaining this, S6 gestured the four blues in four. Figure 6.18 illustrates the gesture made by S6 in his explanation.
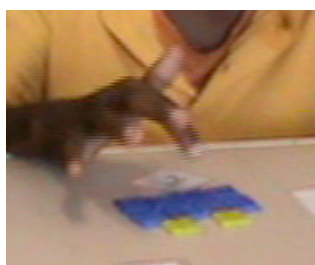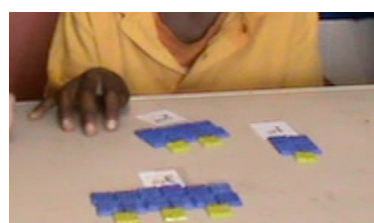
Four greens   Four blues

Figure 6.18 Gesture used by S6 while explain the structure of the pattern.

It was evident that S6 was able to identify the structure of his pattern and describe how to construct it. S6 had semiotically constructed his pattern and embedded both variables in the one structure. For example, term *one* had *one* green tile and *one* group of four, term *two* had *two* green tiles and *two* groups of four. This was considered to be a very sophisticated growing pattern.

Task 2 of PCI2 (Daisy Chain Pattern)

Task 2 of PCI2 presented the daisy chain pattern. S6 observed as the first three terms of the pattern were constructed. S6 was asked to continue the pattern for terms four and five. Figure 6.19 illustrates the pattern S6 created including terms four and five of the pattern.

Figure 6.19. The daisy chain pattern after S6 constructed term four and five of the pattern

It can be seen that S6 placed two petals (yellow counters) between term three and term four. He also repeated this for term four and five. At this point, it was decided to reconstruct the pattern to the original three terms to determine the structure S6 was 'seeing'. Below is an excerpt from the interview of the discussion that occurred.

42 R1   Tell me what you see

43 S6   The flower joining it [*S6 gestures to a yellow counter between the two red counters*]

44 R1   And how is the flower joining?

45 S6   It's one more on each side and then there and there [*S6 points to the yellow counter on top of the red counter and then on the bottom of the red counter*]

46 R1   Can you separate it for me?

47 S6   [*S6 separates the pattern into groups containing one red centre and three yellow petals around the red. There are three groups and one counter left over. S6 has left this to the side of the pattern and has not attached it to a 'flower'*] There is one missing from each side [*S6 points to the side where there is no yellow counter*].

48 R1   So, for four red centres how many yellow petals? [*R points to four red centres*]

49 S6   [*S6 using two fingers to touch the pattern while counting in his head. It appears he is counting in twos*] Thirteen

50 R1   Good boy. Can you tell me how you counted it?

51 S6   Twos. I went two, four, six, eight, ten.

52 R1   How many yellow petals in this flower? [*R gestures in a semicircle around the three yellow counters*]

53 S6   Three

54 R1   And how many here? [*R gestures in a semicircle around the three yellow counters*]

55 S6   Three [*S6 smiles*]

56 R1　　　And how many here? [*R gestures in a semicircle around the three yellow counters*]

57 S6　　　Three and that would equal twelve.

The constant, first yellow counter of the pattern, was then highlighted with a sticker of a bee. S6 was then asked, "Tell me how I would make a daisy chain with seven red centres. What would I do?" S6 responded, "You'd be having seven with three yellows and then I would have to join him on... the bee is sucking all the honey out of it". S6 gestured to the counter that represented the constant. This counter had been signified to the student by placing a bee sticker on the counter after he had earlier deconstructed the pattern. Discussion ensued about the bee and what it was doing in the pattern, and S6 agreed that the bee (constant) was being added to the pattern.

S6 was then asked to predict terms of the pattern beyond what was presented. He successfully identified for 10 flowers you needed 10 groups of three plus one. When asked for position 25, S6 responded, "25 plus 1 equals one." It was evident that S6 was having trouble with the mathematical language in the rule. S6 appeared to be frustrated that he was having difficulty explaining the rule. He started covering his face and looking away from the table. It appeared he knew how to structure the pattern, but had trouble expressing it. To make S6 feel more comfortable the researcher discussed the structure of the daisy chain for position 25. S6 appeared to gain confidence as the discussion continued. From there, S6 was asked to explain how to construct the pattern for any number. S6 responded, "Any number of flowers that you want" he was pointing to the red centres as he was stating this. "Put all the yellows, three yellows around the thing [red centre]....then one more" S6 gestured a semicircle around the red centres. This gesture was identical to the gesture used by the researcher previously as the pattern was deconstructed. Table 6.6 presents the teaching actions, researcher and student participation, and observations made during pattern task 2 of PCI2.

Table 6.6

Teaching Actions, Researcher and Student Participation, and Observations made during Pattern Task 2 of PCI2

Teaching Actions	Researcher participation (questions/directions)	Student participation (responses/engagement)	Observation
Researcher constructs pattern	Researcher constructs the first three terms. Ask the student how many petals needed for each term as the pattern is constructed	Student observes and verbalises with the researcher the pattern	Student successfully identifies the number of petals required for term one, two and three.
Student continues pattern		S6 continues the pattern – however, makes an error	It was as if S6 was constructing the pattern presented in lesson6 of TE2
Pattern is reconstructed by researcher	Researcher constructs the pattern to the first four terms with the correct structure	Student observes	
Student identifies what he 'sees' in the pattern through gesture		Student identifies the groups of three that he can see by gesturing	
Deconstructing the pattern to see the structure		Student deconstructs the groups of three and then leaves the constant on its own.	
Highlighting the constant	Researcher places bee sticker on the constant		
Near generalisation		10 group of three plus 1 25 groups of three and one more	
Quasi generalisation	S6 was not asked for a quasi generalisation as it appeared he was frustrated.		
Generalisation for any number		Three yellow things around it [red centre].....and one more	Added the bee once prompted

Task 3 of PCI2 (Robot Pattern)

The researcher constructed the first three terms of the robot pattern. S6 was able to identify that the three green tiles remained the same in each pattern. He also noticed that each pattern was made up of groups of five, "They are all in a line of five." The excerpt below demonstrates S6 predicting terms beyond the pattern presented for Task 3.

131 R1   What do you think the fourth one will look like?

132 S6   There are four blue tiles five of them [*S6 gestures down the lines of five*] and three green ones [*S6 point to the desk*

> *where the three yellow tiles would be placed on the pattern*]

133 R1   What about number ten what would it be?

134 S6   Ten blue tiles [*S6 gestures down the lines of five*] and 3 around it

135 R1   What about 100?

136 S6   100 blue tiles [*S6 gestures down the lines of five*] and 3 around it

S6 relied on gesture to support his explanation. He had difficulty with the mathematical language, and therefore used gesture to communicate his understanding of the structure. It is important to note that other students also used this communication style when explaining the structure of patterns during the one-on-one interviews (S5). S6 also identified the pattern term if given the structure of the pattern: for example, "If there were 25 rows of 5 blue tiles and three around it, what position would it be in the pattern?" S6 was able to identify that it was position 25. He also did this for position 80. This indicated that S6 was attending to both variables in the pattern.

Additionally, S6 used gesture while generalising the rule for the robot pattern. When asked to provide an explanation for how to construct the pattern for any number, S6 stated, "Any number [*S6 gesture – using his five fingers to make imaginary lines of tiles*] in the blue tiles and three around it". In this explanation S6 is missing the mathematical language of 'groups of five', 'rows of five', or 'times five'. The video data presents S6 gesturing beside a pattern on the desk to show the line of five. Figure 6.20 illustrates the gesture S6 used in his generalisation.
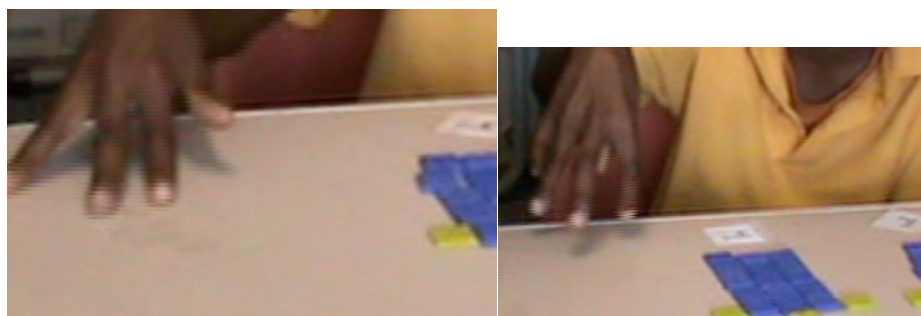
"Lines of 5" [Gesture line up table]   "and three all around it"

Figure 6.20. The gesture S6 used while generalising the pattern structure.

At the end of Piagetian Clinical Interview 2, S6 was able to generalise the structure of patterns with multiplicative structures with a constant. He was also able to create his own multiplicative pattern and explain the structure of the pattern.

6.6   CHAPTER REVIEW

In conclusion, this chapter presented data from teaching episodes and one-on-one interviews with three students to further explore how young Indigenous students generalise growing patterns. The data demonstrated that there are a series of teaching and learning processes that enable students to engage in generalisation tasks. Chapter 7 will discuss these analyses in light of the research literature and context in the areas of mathematics, semiotics and culture.

Chapter 7: Discussion of the Findings

7.1   CHAPTER OVERVIEW

This chapter addresses a full interpretation and synthesis of the findings presented in Chapter 5 and Chapter 6 with reference to the literature. The purpose of this study was to explore how young Australian Indigenous students generalise growing patterns. The conceptual framework for the literature (see Figure 3.5) presents three major themes: early algebra thinking and pattern generalisation, the role of semiotics in pattern generalisation, and Indigenous ways of learning. It was from these themes the research questions emerged:

1. How do young Indigenous students engage in growing pattern generalisation?

2. What teacher actions assist in enhancing young Indigenous students to generalise growing patterns?

3. How does culture influence the way in which young Indigenous students engage in growing pattern generalisation?

In the next sections, findings that emerged from the study are examined and reviewed in light of the literature and the theoretical frameworks. The chapter concludes with the development of a theory and learning trajectory for young Indigenous students' effective engagement when exploring growing patterns. Figure 7.1 presents the overview for Chapter 7.

Chapter 7: Discussion	
7.1	Chapter Overview
7.2	Mathematics: Early Algebraic Thinking and Pattern Generalisation
7.3	Hypothesised Teaching and Learning Sequence for Growing Pattern Generalisation
7.4	The Role of Semiotics When Generalising Growing Patterns
7.5	Semiotic Framework for Teaching Growing Pattern Generalisation
7.6	Influence of Culture When Generalising Growing Patterns
7.7	Cultural Semiotic Learning Model
7.8	Chapter Review

Figure 7.1. Overview of Chapter 7.

7.2 MATHEMATICS: EARLY ALGEBRAIC THINKING AND PATTERN GENERALISATION

How does this differ from past research?

Findings from this study report that young Australian Indigenous students can access high levels of mathematics. This is in contrast to the performance of Indigenous students in both national and international measures of mathematics performance (e.g., PISA, TIMMS, NAPLAN). The results of such measures indicate that Indigenous students perform two years behind that of non-Indigenous students on national numeracy tests (Commonwealth of Australia, 2008; QSA, 2003; Thomson, De Bortoli, & Buckley, 2013). This study has demonstrated that Indigenous students are capable of early algebraic thinking, as demonstrated by their ability to generalise growing patterns, an area where older, non-Indigenous students

have evidenced difficulties (MacGregor & Stacey, 1993; Warren & English, 1995). Therefore, it is argued that these young Indigenous students make comparable progress to their non-Indigenous peers in relation to early algebraic thinking, contradicting their performance on national and international measures of mathematics. The results further suggest that young Indigenous students are ready to engage in early algebraic thinking, disaffirming the notion that young students in general are not cognitively ready to engage in abstract reasoning (Filloy & Rojano, 1989).

Prior research suggests that an understanding of multiplicative thinking is fundamental for older students when generalising growing patterns (e.g., Rivera & Becker, 2011), however, this study suggests prior understanding of multiplicative thinking is not a necessity for generalising growing patterns in early years settings. Young students in general have little understanding of basic multiplication at Year 2, let alone have a strong grounding in multiplicative thinking. Though past research suggests that addition and multiplication of whole numbers are pre-requisites when generalising linear patterns (Rivera & Becker, 2011), this was not necessarily the case for this study with young Indigenous students. Though it should be noted that the linear patterns explored in Rivera and Becker's (2011) study with middle school students had higher levels of complexity in comparison to this study. Despite this, it is argued that in the early years context, linear growing patterns provide a platform for developing an understanding of mathematical operations and arithmetic. In the case of the present study, students initially used the pattern structure to explore additive relationships, but quickly moved to the exploration of multiplicative thinking (see Section 6.5.3). Consequently, while having a strong understanding of addition and multiplication would assist students to generalise the pattern, and definitely deduce the pattern, it was found not to be a necessity for these young Indigenous students (see Section 3.2). What did assist these students to engage with the pattern and develop generalisations, stemmed from the choice of pattern and the teaching actions utilised to support students to access new mathematical content knowledge (see Table 6.1, Figure 5.4 for examples).

While past research has highlighted that mathematical language is a barrier when trying to express generalities for functional relationships, findings from this study indicate that mathematical language is not a necessary requirement. Research

has indicated that older students possess limited appropriate language when trying to express functional relationships (English & Warren, 1998; MacGregor & Stacey, 1995; Swafford & Langrall, 2000). Year 2 students also have limited mathematical language. Often they are confused about how concepts, such as multiplication, can be expressed in traditional ways (e.g., 'groups of', 'multiply', 'times'). Using hands-on materials assisted students in this study to overcome this mismatch. As the mathematical language became less attainable for these students, they used gesture in conjunction with hands-on materials to express the generality (See Figure 6.17). Additionally, hands-on materials allowed students to construct and deconstruct the growing pattern, and this assisted them with identifying and articulating the general structure. It was not necessary for them to have the language 'groups of' as some students gestured this with a circle around the group of tiles or counters to demonstrate what they were inferring at that point in the generalised expression (see Section 6.4.5 Student 6 Task 3 of PCI2). Therefore, it is conjectured that expressing the structure of the pattern in mathematical language is not necessary for students to generalise the structure of the pattern. Additionally, it is suggested that once students have identified the structure in this manner, and expressed it in their own words, the transfer to mathematical symbol is easier.

How does this align and add to past research?

Young Indigenous students are capable of identifying and articulating the general structure of growing patterns. Students were able to identify a general rule that allowed them to deal with any particular pattern term in the growing pattern sequence, regardless of its position. This aligns with past research that has also demonstrated that young non-Indigenous students can engage and generalise 'growth' (Blanton, 2008; Moss, Beatty, Barkin, & Shillolo, 2008; Radford, 2010a; Schliemann, Carraher, & Brizuela, 2007; Warren & Cooper, 2008a). Young Indigenous students were able to identify the relationship between the two variables depicted in the growing patterns, and demonstrate co-variational thinking similar to young non-Indigenous students (Blanton, 2008; Moss, Beatty, Barkin & Shillolo, 2008; Schliemann, Carraher, & Brizuela, 2007; Warren & Cooper, 2008a). In addition, these students demonstrated they were able to identify and articulate the general structure of the pattern in a number of ways.

The ways in which young Indigenous students generalise growing patterns mirrors aspects of how older, non-Indigenous students generalise growing patterns. For example, recursive thinking, where students focused on the additive component of the growing pattern, was prevalent in classroom discussions (Lannin, 2005) (see Section 5.5.2, Section 5.4.1, Section 6.3.2). When predicting near generalisations, (e.g., 10th position) the majority of students provided rules that were defined by a recursive element of the pattern (e.g., adding 2 each time). Past studies with older students have also found recursive thinking to be obvious when exploring growing patterns (Lannin, 2005; Radford, 2008). Consequently, students have difficulties in developing or shifting to *covariational thinking,* that is, identifying the relationship between the pattern term (dependent variable) and pattern quantity (independent variable). Additionally, in the early stages of pattern exploration, these Indigenous students were more concerned with giving the pattern quantity (e.g., total number of tiles required) than focusing on the general structure of the pattern (see Section 6.4.5). Students' providing the explicit rather than the general structure was possibly more pronounced due to their culture, and this is discussed further in Section 7.3.

Young Indigenous students are able to demonstrate varying types of sophisticated generalisations. Students demonstrated movement between *factual*, *contextual* and *symbolic* generalisations (Radford 2003, 2006) as they attempted to determine generalities with new patterning structures and formalise their algebraic thinking (see Section 6.2). Additionally, these young Indigenous students' generalisations aligned with Rivera & Becker's (2011) research with middle school students. They were able to construct *constructive standard generalisations (CSG)*, that is, construct direct formula from the known stages of a geometric pattern, for example saying 'pattern number times 3 plus one more tile tells us how many tiles altogether' (Rivera & Becker, 2011). In their study with middle school students, Rivera and Becker (2011) found that there was a predisposition for them to construct CSG's when considering geometric linear patterns. It is conjectured that, in the present study, these CSG's were more prevalent as a result of students having been directed to the structure of the pattern in the early stage of the interview (see Tables 6.1- 6.6 for examples of students being directed to the structure of the pattern). By attending to these structures, students began to 'grasp' the common features of the pattern (Radford, 2010b) or 'notice' the particular in the general (Mason, 1996). In

turn, this 'noticing' or 'grasping' assisted students to generalise the nth position (unknown position). While it can be argued that some generalisations (i.e. symbolic generalisation, that is the use of alphanumeric notation) are more sophisticated than others (i.e. factual and contextual generalisations), it is suggested that in the early years greater importance lies in the ability to initially determine *contextual generalisatio*n. This involves moving beyond the particular pattern figures and identifying a relationship between the pattern figures and pattern terms. The present study aligns with past research that has identified that older primary schools students (Grade 5 and 6) could identify the relationship between the pattern and pattern term (e.g., Warren, 2006, Rivera, 2006). Additionally, what provides students with the ability to move between these different types of generalisations is of importance and impacts the way in which teachers engage students in the learning experiences.

The use of quasi-variables assists young Indigenous students to generalise growing pattern structures. By using quasi-variables, students were able to observe the general structure of the pattern regardless of the fact that these young students had little understanding of multiplicative thinking. The quasi-variable (e.g., 371th position) pushed students to see the structure of the pattern, as they often found it challenging and unproductive to apply an additive rule to a quasi-variable to determine the pattern quantity. This notion has also been supported in past studies with young students (e.g., Cooper & Warren, 2008; Schliemann, Carraher, Brizuela, 2007; Radford, 2011; Warren & Cooper, 2011). The findings of this present study also highlight that challenging young students to extend beyond their computational knowledge, results in a shift from an arithmetic approach when describing the pattern, to engaging in algebraic thinking (Radford, 2011). Importantly, how teachers educated students to shift from numeric thinking to algebraic thinking is of great importance, and will be considered later in the chapter.

Young Indigenous students are able to use alphanumeric notation to express their generalisations. In the above-mentioned study, Radford (2011) demonstrates how Year 2 students generated factual and contextual generalities for geometric growing patterns. However, this did not include the exploration of how students engage with alphanumeric notion. Past research found young students could engage with algebraic concepts and use alphanumeric notation earlier than anticipated (Blanton, 2008; Carraher et al., 2006; Schliemann, et al, 2007; Warren & Cooper,

2008b). However, scaffolding is required for young students to use alphanumeric notation as they express their generalisations (Blanton, 2008; Moss et al, 2008; Schliemann et al, 2008; Warren & Cooper, 2008a). In this present study with young Indigenous students, the variable did not naturally appear in the case of position *n*. Specific teaching actions were required before students used alphanumeric notation. When these students were pretested 3 months later, at the beginning of Teaching Experiment 2, they were using alphanumeric notion to generalise growing patterns in mathematical contexts (see Section 5.5.1). They were transferring the use of alphanumeric notion from TE1 to the pre-test of TE2, a surprising finding given that alphanumeric notation was not supported in classroom learning experiences between TE1 and TE2. Alphanumeric notation was introduced to these students to explain unknown situations, after a series of teaching actions that explored non-symbolic generalisations; this is further elaborated in the following section.

During the teaching episodes and clinical interviews, students exhibited common issues that have been previously identified from research in relation to the use of letters as variables. These issues have been highlighted in past studies with older students. Past research has delineated many misconceptions students have when using a variable, including: using alphanumeric variables as abbreviations (Stephens, 2005); interpreting the concept of variable as varying quantities (English & Warren, 1998; Lannin, 2002; MacGregor & Stacey, 1995; Swafford & Langrall, 2000); ignoring variables (Kuchemann, 1981); determining variables as objects (Kuchemann, 1981); and misusing symbols when representing rules (English & Warren, 1998; Lannin, 2002; MacGregor & Stacey, 1995). An example of the misuse of variable in this study was when S6 provided the response $n+n=m$ (when trying to determine a rule for $y=2n$) in TE2 (see Section 5.2.2, Section 6.5.4); when asked to explain why, S6 stated, "Because it's got two…. [*gestures to arcs in the air to represent the n*]". This finding aligns with past studies where students have considered the letter as an object (Kuchermann, 1981; Warren & Cooper, 2008b). A reason for this response includes the fact that this was the first time these young Indigenous students had considered an unknown variable in mathematics.

Young Indigenous students can generalise patterns situated in both environmental contexts and mathematical contexts (geometric growing patterns) with the former being the precursor for success in the latter. The type of pattern explored

by Indigenous students influences their ability to identify the pattern structure. While past research highlights the ways in which young students engage with growing patterns, until now little has been considered in regards to the types of patterns that assist young Indigenous students to generalise. In past research, growing patterns have often been explored in a mathematical context (e.g., tiling patterns), where students have to continue, predict, find missing elements, determine the additive rule, and generalise geometric growing patterns (Moss & Beatty, 2006; Rivera, 2010; Warren, 2005). It is argued that these types of patterns are initially challenging for young students (as evidenced in Pretest 2). The results of this study indicate that it is important to consider how the context of the pattern impacts on students' ability to access the structure and relationship between the variables. In other words, how the type of visual used to display the pattern impacts on their ability to generalise the pattern structure. Past studies indicate that the types of geometric growing patterns presented to students did not impact on their ability to extend the pattern (Cooper & Warren, 2011; Leung, Krauthausen, & Rivera, 2012; Radford, 2011; McNab, 2006). In contrast, in the initial stages of the present study, the context of the pattern did impact on students' ability to extend the pattern. For example, students had more success extending and generalising growing patterns that were drawn from their environment (e.g., identifying the relationship between kangaroo tails and ears), than they did extending and generalising growing patterns represented by decontextualized geometric shapes (e.g., items in Pre-test 2). The Indigenous Education Officers supported these statements. They suggested that Indigenous students had better access to mathematics if they could link the mathematics to environmental contexts (see Section 5.4.1, Section 6.4.5).

Young Indigenous students did not need to engage with multiple registers when generalising growing patterns; rather, they needed to engage with multiple representations within the one register. Past studies have highlighted the importance of using and connecting multiple representations when learning mathematics (Cooper & Warren, 2008; Dreyfus, 1991; Duval, 2006; Halford, 1993). Furthermore, Duval (2006) implies that there are four registers of mathematics. He argues that mathematics comprehension results from the coordination or mapping of at least two representation forms or registers. The four multifunctional registers are; natural language, figures/diagrams, mono-functional registers of notation systems (symbols),

and graphs. In this study, students drew on figures and natural language to explore generalisations. However, as the natural language of the mathematics is not fully comprehended by these young Indigenous students at this stage, it is conjectured that rather than focusing on mapping between registers, it is more beneficial for the early years students to experience multiple representations in one register. For example, when exploring growing patterns through the use of hands-on materials, students refined their ability to see the structure of the pattern. Evidently, from this study students did not need to graph or explore notation systems in order to provide non-symbolic algebraic and alphanumeric expressions. The expression of this structure in natural language is about communicating the structure to others. Whether their use of natural language adds to, or distracts from, their 'seeing' the structure needs further investigation. Though it is noted that to have a deep understanding of generalising growing patterns, students will eventually need to shift between and within the registers (Duval, 2006).

Young Indigenous students demonstrate transfer learning and analogical reasoning as they generalised growing patterns with multiplicative structures. Interestingly, as students became aware of the multiplicative structure of the pattern, they were able to use other structures from their own classroom environment to support their use of mathematical computational knowledge. For example, S2 used the structure of the clock to determine the multiples of five. S2 was able to use lateral transfer to generalise the multiplicative structure of the clock and use this in a new situation (Anderson et al., 1995; Bassok & Holyoak, 1993). Additionally, this aligns with the notion of students engaging in analogical reasoning to help them identify similarities between structures when endeavouring to understand a new concept (English, 2004; Goswami, 2001; Halford, 1993). S2's response is a clear demonstration of students transferring general structures from their own classroom contexts, that is, linking the pattern structure to a known multiplicative structure. Student 2 was mapping his learning from structure to structure (English, 2004).

Exploration in growing patterns provides a platform for the exploration of mathematical concepts. Past research highlights the role of engaging in early algebraic thinking as a way to bridge students' understanding and development of arithmetic thinking (Blanton 2008; Blanton & Kaput, 2004; Carpenter & Franke, 2001; Schliemann, Carraher & Brizuela, 2007). Results of this study suggest that

growing pattern activities assisted the development of multiplicative thinking for these young Indigenous students. It was found that the patterning activities provided a construct that supported the exploration and development of multiplication and addition. This finding aligns with studies conducted by Moss and McNab (2011) with young students who had a limited understanding of multiplication. It is conjectured that the visual representation of the pattern, in conjunction with the hands-on materials, provided opportunities for students to construct and deconstruct the pattern. Past research has indicated that constructing and deconstructing figures assists students to generalise (Mason, 1998; Moss et al, 2008; Warren & Cooper, 2008a). Constructing and deconstructing the growing patterns assisted these students to develop an understanding of multiplicative thinking. This in turn, facilitated discussions of the notion of 'arrays' and 'groups of'. Furthermore, students then used this multiplicative thinking when explaining terms in quasi-variable positions.

7.3 HYPOTHESISED TEACHING AND LEARNING SEQUENCE FOR GROWING PATTERN GENERALISATION

The hypothesised learning-teaching sequence presents a framework for students moving from the particular to the general through a series of activities. This teaching sequence resonates with the scaffolding Blanton (2008) used with students in her research, namely, (a) exploring the situation, (b) developing statements that can be true or false, (c) testing the statements, and (d) revising incorrect statements until the mathematical truth is discovered. The hypothesised learning-teaching sequence expands on these findings by presenting teaching actions that support students as they test their conjectures. In this present research, progressions of learning experiences were gradually refined during the teaching experiment and clinical interviews, and evidently there appears to be a series of activities that assist students to generalise. If students were unsuccessful during any task, there was a teaching action that entailed an activity designed to assist students in overcoming the difficulties they were experiencing. Once the teaching action had been completed, the student revisited the learning experience (See Figure 7.2 and Interaction five in Section 7.1.3). Finally, it was noted that the role of the teacher and Indigenous Education Officer is that of mediators between the structures of the pattern, mathematical content knowledge, interpretation and identification of signs, and verbal communication required by young students to express their generalisation.

This interaction is a complex partnership between student and teacher, and is pivotal in the development of students' ability to generalise patterns. This point is further delineated after Figure 7.2. Figure 7.2 illustrates a hypothesised learning-teaching sequence for engaging students in pattern generalisation tasks. The learning experiences are depicted in blue, the green circles indicate a successful response. Additional teaching actions to overcome challenges are in red.
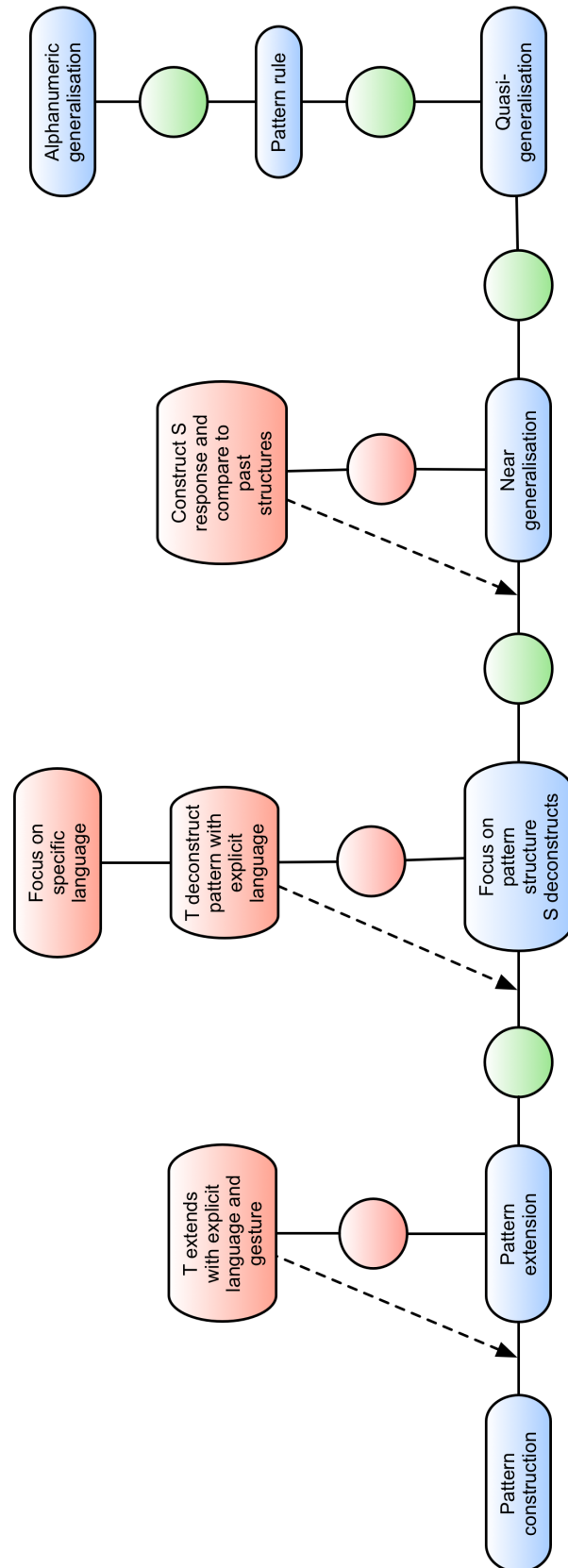
Figure 7.2. Hypothesised learning-teaching trajectory for engaging young Indigenous students in pattern generalisation tasks.

The following section addresses each step of the hypothesised learning-teaching sequences.

Pattern Construction

Teacher/Researcher constructs the growing pattern using hands-on materials. Consecutive terms are constructed, for example, patterns in position two-five. Students observe the teacher/researcher constructing the pattern with hands-on materials.

Pattern Extension

Students extend the growing pattern using the hands-on materials. If students are unsuccessful, the teacher/researcher assists them to extend the pattern. As the pattern is extended, the teacher/researcher uses explicit language and gestures to scaffold students to see the pattern structure.

Pattern Deconstruction

Students are required to deconstruct the pattern structure. As they are deconstructing the pattern, students identify what parts of the pattern remain the same, what parts of the pattern change. During this teaching action, teachers/researchers need to be mindful not to impose how 'they' see the pattern on students' thinking, as there are multiple ways to see the structure. If students are experiencing difficulty, the teacher/researcher deconstructs the pattern structure verbalising what they see. Students then attempt to deconstruct the pattern structure at other pattern positions. Additionally, students may be experiencing difficulty accessing the mathematical language to describe what they are seeing. It is at this stage that a focus on mathematical language can assist students in describing the structure.

Near Generalisation

Students are asked to identify the pattern quantity for a near pattern position. It is essential to use a number that is within their computational knowledge, as this allows them to identify the near generalisation. For example, students are asked to idenitfy a pattern term three or four positions away from the original created pattern (e.g., 10th term). It is important to ask students to explain how they solved this, and what they think the rule is for the growing pattern. If they are unsuccessful and cannot identify a near generalisation, they are then encouraged to construct their

response using hands-on materials, and draw comparisons that are between their near generalisation and the pattern construction.

Quasi-generalisation

This particular teaching action requires students to generalise their growing pattern beyond their mathematical computational knowledge; that is, identify the pattern quantity for a large pattern term such as position 567. Students can nominate the largest number they know, or the teacher/researcher can select their own number. They are asked to explain how they determined their answer.

Pattern Rule

Students are required to express the general rule for the growing pattern. They may use gesture to support their mathematical language as they generalise the growing pattern.

Alphanumeric Generalisation

Students are asked to identify the general rule of the growing pattern using alphanumeric notation (e.g., n^{th} position).

The following section (7.2) presents a hierarchy of the types of patterns to use as these young Indigenous students work through the above teaching sequence (see Table 7.1). In addition, it establishes the role of semiotics within the hypothesised learning-teaching trajectory, and elaborates on the important semiotic interactions that occurred during the teaching experiments and clinical interviews.

7.4 THE ROLE OF SEMIOTICS WHEN GENERALISING GROWING PATTERNS

Current research cogitates the various semiotic resources used within the classroom when exploring mathematical problems related to functions (Arzarello et al 2009; Radford 2009; Radford & Roth 2011; Warren & Cooper, 2009; Warren, Miller & Cooper 2012). While the theory of semiotics has long been established, it is only recently that research in the area of pattern generalisation has considered how semiotics impacts on the learning process. This present study further extends the application and practicality of semiotic theory in the teacher/learning process of pattern generalisation with young Indigenous students.

Semiotics is the theoretical framework that underpinned the research. It provided a model for designing the structure of the growing pattern tasks, and played a pivotal role in the teacher-student, student-teacher, and student-student interactions of the learning process. Semiotics influenced the teaching and learning processes for both young Indigenous students and the teacher when generalising growing patterns in three ways. These are:

1. The initial set up of the pattern tasks

    a. Selection of materials

    b. Choice of representations used to highlight the pattern structures

    c. Utilisation of embedded signs vs separate signs

2. The selection of teaching actions

    a. The role of gesture

    b. The role of questioning

3. The consideration of signs that impacted on how students communicated their generalities

    a. The use of semiotic processes to communicate learning

    b. The use of embodiment to express the general rule

At the end of this section, as young Indigenous students generalise linear growing patterns the teaching and learning framework presented above in Figure 7.2 will be reconsidered in relation to the impact of semiotics in the learning-teaching processes.

7.4.1 Initial Set Up of the Pattern Tasks

Past research suggests that the common issues that arise when students are generalising are potentially due to the way the tasks are presented and taught (Moss & Beatty, 2006). Patterns are often presented that limit students' awareness and accessibility to generalise the pattern structures (Dörfler, 2008; Küchemann, 2010; Moss & McNab, 2011). It is for this reason that consideration was given in the initial set up of each task in the use of semiotic signs and the selection of hands-on materials, as students interpret signs whilst constructing new meaning (Peirce, 1954) and generalising the growing pattern structure. For example, (a) whether both

variables are explicit and how the signs represented them (e.g., colour); (b) if the variables were embedded in the pattern; and (c) if the multiplicative structure could be easily identified in this structure. Of the research conducted with young students engaging in pattern generalisation tasks using hands-on materials (Cooper & Warren, 2011; Leung, Krauthausen & Rivera, 2012; Warren & Cooper, 2008a), few have considered a semiotic perspective (e.g., Radford; Warren & Cooper). This study, with young Indigenous students, further contributes to the literature and suggests that the role of semiotics is essential in assisting students to engage with the pattern and see the general structure of the growing pattern. Within this study, this was achieved by the inclusion of variables as iconic signs represented in the hands-on materials. By using hands-on materials students were physically able to engage with the variables (iconic signs), and manipulate them according to the directions given (e.g., continue growing patterns).

Iconic sign vehicles provide opportunities for dynamic interactions between the student and the pattern. Findings from this study further nuance the importance of, and facilitate the role that dynamic signs play when students physically engage with geometric patterns to construct the general rule (Mason, 1996; Saenz-Ludlow, 2007). Through the use of iconic signs (butterfly bodies and butterfly wings), a geometric pattern created with concrete materials provides opportunities for young students to manipulate both variables, as they examine the pattern structure on their way to constructing generalisations (Cooper & Warren, 2008). This approach differs from geometric patterns that are traditionally depicted in textbooks (as students can not physically manipulate textbook pictures), and it is argued that students do not engage with, or interpret these signs (textbook pictures), with the same intensity. It is conjectured that in this study, it is essential to have dynamic iconic signs that represent both variables in geometric growing patterns, and that students physically engage with these signs.

Selection of Materials

Results from this study suggest that the hands-on materials selected need to explicitly represent both variables, that is, both pattern term and pattern quantity need to be obvious for young Indigenous students, in order for them to generalise. Past research has highlighted an issue that arises from functional situations is the need to coordinate two data sets, and identify the relationship between these sets

(Blanton & Kaput, 2005; Warren, Miller, & Cooper, 2011). Thus, in this present study the growing patterns selected for the tasks were deliberately chosen to ensure that this relationship was transparent. From a semiotic perspective, the signs for each variable were visible and required students to be involved actively in their creation (Warren, Miller, & Cooper, 2011). For young Indigenous students, this was achieved by using hands-on materials where the variables could not be physically separated (e.g., plastic toy kangaroos and crocodiles), pattern term cards, and coloured tiles/counters.

However, in the case where variables can be physically separated, it is essential to signify the pattern term using a pattern term card. Past research has highlighted that students need to be scaffolded in order to recognise the pattern term number (independent variable) in geometric patterns (Moss, Beatty, McNab, & Eisenband, 2006; Moss et al, 2008; Schliemann et al, 2007; Warren & Cooper, 2008a). To overcome this issue, the pattern term card contained a numerical representation, which was an explicit representation of the pattern term variable. Additionally, the use of colour tiles/counters assisted in signifying the variables. For example, the Flower pattern used red and yellow coloured counters (See Table 5.6). The red counters represented the pattern terms, while the yellow counters represented the pattern quantity. It is conjectured that if only a single colour was used, students would have had difficulty discussing the two variables (pattern term and pattern quantity). Additionally, when representing a constant in growing patterns, it was evident that young Indigenous students were able to generalise the pattern structure when this constant was explicitly represented, by using a different coloured tile (counter) to signify its relationship in the growing pattern (Moss et al, 2008). Evidently, when growing patterns are structured in such a way, there is a need for students to co-ordinate between the signs. It is when students begin to co-ordinate between the signs that they shift from recursive thinking to co-variational thinking (Cooper & Warren, 2008).

Representations in Pattern Structures

Iconic signs assist students to move quickly from recursive thinking to covariational thinking. This was achieved by using iconic signs to highlight the two variables. It has been demonstrated in past studies that young students can engage in covariational thinking (Blanton & Kaput, 2011, 2004); however, this study begins to

shed light on the processes that assists students in 'noticing' the relationship between the two variables. Additionally, a recursive approach to solving growing patterns is still a major challenge for both young and older students (Lannin, 2005; Rivera 2006; Rivera & Becker, 2009; Warren, 2006). The results of this present study suggest that this issue relates to the way the patterns are structured, and can be overcome by using iconic signs to highlight both variables in growing patterns, namely, the pattern number (term) and the pattern quantity.

Multiplicative structures need to be explicitly represented in order for young Indigenous students to generalise. One way the multiplicative structure was made evident to students was the pattern's visual representation. For example, the Robot Pattern used in PCI2 was represented in rows of five to highlight the 'fiveness' of the pattern. That is, in pattern term three, the robot was constructed using three rows with five tiles in each row. By structuring the pattern using arrays, students were able to engage with multiplicative structures (Mulligan, 2002). Additionally, the pattern term (e.g., 3rd Robot in the pattern) was being signified by both the pattern term card (which had 3rd written on it) and within the structure of the pattern (3 rows of 5 tiles). When representing the growing pattern in such a way, the representation is both numeric (pattern term cards) and geometric (tiles/counters). It is conjectured that representing growing patterns in this manner assisted in the teaching of multiplicative structures to young Indigenous students.

Embedded Signs vs Separate Signs
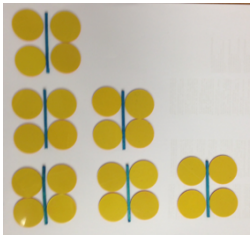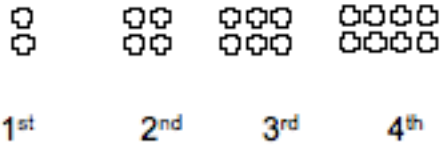
While it is recognised that signs play a central role in the construction of new knowledge (Peirce, 1954; Saenz-ludlow, 2007), literature pertaining to how these signs are represented in pattern generalisation tasks is scarce. This present study contributes to this limited research, and suggests that there are two ways that sign vehicles can be considered when constructing growing patterns: (a) embedding sign vehicles, and (b) splitting sign vehicles. When considering growing patterns the sign vehicles represent the two variables within the pattern (i.e., pattern term and pattern quantity). Embedding both sign vehicles in a single hands-on artefact ensures that students attend to both variables of the growing pattern. This aligns with past research, indicating that students were successful generalising patterns where both iconic signs were embedded in the single structure (Blanton & Kaput, 2005; Leung, Krauthausen & Rivera, 2012; Warren, 2005). Young Indigenous students were

supported in making connections with co-variation, as demonstrated by S1 in TE1 when discussing the general rule for the kangaroo pattern (see page 158). It appeared that the use of *embedded variable patterns* assisted students to attend to both variables: that is, students needed to discuss the pattern attending to the pattern position (tails) and the pattern quantity (ears).

Second, when using *split variable patterns,* the teacher has a significant role in scaffolding students to attend to both variables. Using split variable patterns required interplay between questioning students about the pattern structure, and drawing attention to both of the variables through gesture (pointing to the pattern and the pattern term). These semiotic bundles (Arzarello, 2006; Arzarello et al., 2009) or semiotic nodes (Radford, 2003) work in harmony to achieve knowledge objectification (Radford, 2003). Using these semiotic means of objectification assisted young Indigenous students to attend to both variables in split variable patterns. This adds to past research that suggests that the use of iconic signs such as gesture, questioning, and physical manipulation of growing patterns provides a platform for meaningful mathematical experiences as students embody their experiences (Radford, 2011; Sabena et al., 2005; Santi, 2011). It is these facets, and the push to explore quasi-variables, which contributed to students developing an understanding of covariation rather than recursive thinking between the data sets. It is thus more important to initially engage young Indigenous students with patterns that have embedded variables, than working with patterns where the variables (signs) can be separated. Hence, it is inferred there is a potential hierarchy of pattern structures that need to be considered when introducing growing patterns to young Indigenous students. Table 7.1 displays a hierarchy of patterns that can be used when introducing young Indigenous students to pattern generalisation tasks.

Table 7.1

Hierarchy of Pattern Type when Introducing Young Indigenous Students to Pattern Generalisation Tasks

Pattern Type	Example of pattern
Embedded variable, contextual/environmental pattern with hands-on materials no constant	Kangaroo ears and tails
Embedded variable, geometric pattern with hands-on materials no constant	Geometric circles and matchsticks
Embedded variable, visual pattern with no constant	Drawn pictures of geometric patterns
Split variable, contextual/environmental pattern with hands-on materials no constant	Car wheels and car position
Split variable, geometric pattern with hands-on materials, no constant	Geometric tiles and pattern term cards
Split variable, visual pattern with no constant	Drawn geometric pattern with pattern term

7.4.2 Teaching Actions

The Role of Gesture – Teacher

Indexical sign vehicles are pivotal in the teaching and learning process of engaging young Indigenous students in pattern generalisation tasks. This study adds to past research that suggests that indexical signs (such as gesture, language, and hands-on materials) contribute to making the mathematics apparent for non-Indigenous students (Radford, 2003). To highlight particular signs and structures of the pattern to the young Indigenous students, specific and purposeful gestures were used as the researcher deconstructed the pattern. These gestures were indexical sign vehicles (Peirce, 1954; Saenz-Ludlow & Zellweger, 2012). It was essential to gesture between the variables (pattern term, pattern quantity, and constant) as the pattern was deconstructed. During this process, there was a deliberate coordination between gesture and language (Radford, 2009). An example of the coordination of gesture and language for the first three terms in the 'Classroom Pattern' is presented in Figure 6.15. This learning experience requires students and teachers to coordinate a range of signs as they objectify their understandings.

Communication by Students Through Gesture

Gesture was fundamental to young Indigenous students generalising growing patterns. Its role was two-fold; gesture as embodiment of the task and gesture working as language for communicating mathematical concepts. These roles were consistent with literature. The literature suggests that gesture and language play significant roles in the learning of new mathematical concepts, and these types of body experiences are strongly related to cognition (Arzarello & Edwards, 2005; Lakoff & Núñez, 2000). Within the context of this study it was apparent that there was a natural disposition towards young Indigenous students using gesture during growing pattern activities. From a semiotic perspective these physical processes assisted students to objectify the task and construct new knowledge (Radford, 2003). After students engaged with this process, semiotic contractions occurred as young Indigenous students refined the coordination between the gesture and the language. That is, as students came to an understanding of the relationship between two variables, they refined their gestures and their gestures became more precise. It is also conjectured these processes are strongly associated with cognitive growth, providing a link between experience and cognition (Lakoff, 1987).

Indigenous students used gesture to supplement the language used in communicating their ideas about growing patterns. This resonates with past research, which states that gestures are used by students to communicate their understandings prior to acquiring the specific content terminology (e.g., mathematical terms) (Kendon, 1997; Roth, 2001). The use of gestures appeared more pronounced for these young Indigenous students. It was observed that they used gesture as an adjunct to language for communicating ideas about growing patterns and generalising the growing patterns. It is paramount for teachers to understand student gesture in Indigenous contexts, and provide opportunities for students to gesture as they engage in, and explain their own, mathematical knowledge. The relationship between gesture and language has been described as 'unsplittable' (McNeill, 1992). It was apparent that at times during the lesson, some language was not accessible to these students, or students' home language was at odds with the mathematics (Goldin-Meadow, 2002). It is concluded that when the language commonly used in the mathematics classroom is not available, or is mismatched to their home language, students will use gesture to assist their conversations (Goldin-Meadow, 2002). The less language that they had, the more gestures they used. As Goldin-Medow (2002) claims, gesture may be the first place students display a new thought. Thus, gestures may in fact replace mathematical language when this language is not available to Indigenous students.

7.5 SEMIOTIC FRAMEWORK FOR TEACHING GROWING PATTERN GENERALISATION

Past studies have demonstrated how semiotics is a tool for linking mathematics and culture. Presmeg (1998) demonstrated the progression from mathematics in culture to Western mathematical concepts by using a semiotic chaining process. Considering Presmeg's (1998) model, it is evident that it captures the shift from culture to Western mathematics by embedding the culture within the learning experience. However, it does not capture the complex semiotic processes between and within each step, and how students' culture impacts this learning.

In this present study, students progressed through a series of learning experiences as they engaged in mathematical generalisation tasks. Seeing the 'particular' as termed by Mason (1996) is not apparent for early years students. There needs to be a series of semiotic systems used to ensure students are attending

to, and perceiving the particular, before they can identify 'the general'. As students were deconstructing the signs, they were internalising and forming constructs about these signs to use in subsequent learning experiences. Hence, students were decoding the signs given by the teacher. Additionally, as students engage in the learning activity they too bring their own signs to the task. Figure 7.3 depicts the sequence of semiotic learning process as students move from the particular (immediate object) to the general (real object).
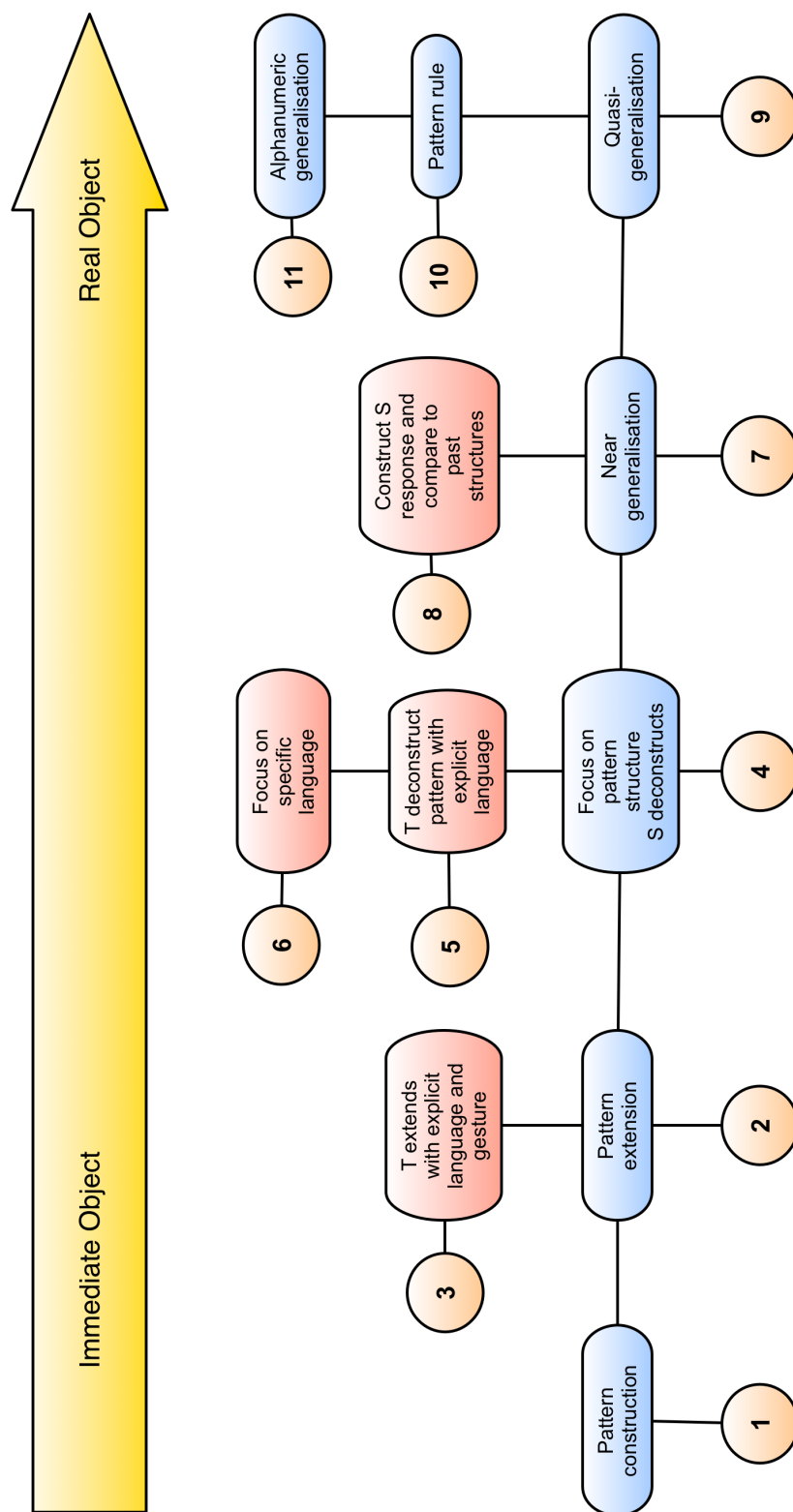
Figure 7.3. Semiotic learning-teaching process as students move from the particular (immediate object) to the general (real object) when engaging in pattern generalisation.

Within the trajectory/learning sequence each one of the orange numbered boxes shows a point of semiotic interaction for the student. The following section addresses each one of the interactions displayed in Figure 7.3.

Interaction 1

Semiotics has two key tenants in interaction 1. First, semiotic consideration is needed when selecting the hands-on materials that best assist young students to see the structure of the growing pattern. Second, the teacher/researcher creates the first three terms of the growing pattern. As the student observes the construction of the pattern, they are immediately decoding and encoding signs (Saenz-Ludlow & Zellweger, 2012).

Interaction 2

Students engage with the pattern as they extend the growing pattern created by the teacher/researcher. Within this interaction, students may mimic initial gestures that the teacher/researcher has used to create the first three terms, that is, they may place the pattern term card and then construct the growing pattern in the same manner as the teacher/researcher. Students continue to decode and encode the signs within the pattern structure.

Interaction 3

If students are unsuccessful in extending the growing pattern, additional teaching actions occur. First the teacher/researcher engages in a conversation with the student about what they see, and how their continued pattern fits with the pattern built by the teacher/researcher. Second, the teacher/researcher highlights key signs (e.g., pattern term cards) using gesture (e.g., pointing to cards) and explicit language (e.g., this is position 3). Third, the student and teacher/researcher re-extend the pattern together. The intention of this additional teaching action is to frame students' eyes to the signs. Additionally, the teacher/researcher is deconstructing the signs made apparent through the student working with the hands-on materials, the language they use to explain their thinking, and their gestures.

Interaction 4

Interaction 4 requires students to focus on the pattern structure. This entails students to decode the signs, and then encode the signs by deconstructing the pattern

structure (e.g., multiplicative structure, pattern quantity in relation to pattern term). Additionally, the teacher/researcher is observing students' gestures as these provide information about how students' perceive the structure of the pattern. For many Indigenous students in the study, it was this juncture where they were beginning to use gesture to support their mathematical language. This interaction is particularly challenging, as students also begin to interact with both pattern variables and coordinate their own signs (e.g., coordinating language and gesture).

Interaction 5

For students who have difficulty during the previous interaction, the teacher/researcher provides additional teaching actions to assist them overcome these challenges in interaction 5. If the student is having difficulty deconstructing the pattern, the teacher then deconstructs the pattern incorporating a heavy use of gesture and language. The teacher/researcher is required to decode the signs made by the student in interaction 4, and encode the signs to assist them to overcome their challenge. Often the challenge is seeing the covariational relationship between the two variables. Teaching actions include pointing to the pattern term card with the student highlighting the position verbally, for example, 'Position four [teacher/student gesturing to pattern term card] rows of four [teacher gesturing to rows of four in the pattern structure] and one more [teacher pointing to constant]'. As the teacher/researcher demonstrates, the student then mimics the gestures and language to assist them to make connections between the two variables. This interaction requires a lot of decoding and encoding by both the teacher/researcher and the student.

Interaction 6

Some students require further support with their mathematical language. Interaction 6 requires the teacher to explicitly teach students the mathematical language used to assist students express the structure. This interaction can be challenging for some young Indigenous students as they are often learning new, specific, mathematical language.

Interaction 7

In interaction 7 students predict a near generalisation for the growing pattern. At this stage, students have refined the semiotic signs, and are able to apply them to

assist in predicting growing patterns that are near the initial constructed pattern (e.g., pattern term 25). While this stage only requires a verbal response from students, often the teacher/researcher can see them coordinating the two variables using hand gestures, eye movements, and self-talk (Warren, Miller, Cooper, 2011).

Interaction 8

If unsuccessful in predicting the near generalisation, students then construct their prediction in interaction 7 and compare this with the other pattern structures. Teachers/Researchers are again required to decode the signs as students construct the pattern using the hands-on materials. Additionally, students need to be encouraged to verbalise their thinking in this stage.

Interaction 9

This interaction aims to assist students to quasi-generalise. Students are required to quasi-generalise the growing pattern for a large pattern term (e.g., pattern position 5673). It is conjectured that for students to be successful at interaction 9 (and interaction 10 and 11) they have needed to decode the signs in the previous interactions.

Interaction 10

Within interaction 10 students identify the pattern rule in their own language. Teachers/Researchers are required to decode the signs as students explain the pattern rule in their own words. Indigenous Education officers play a pivotal role in this, as they assist teachers/researchers to decode the cultural signs.

Interaction 11

Finally, students are introduced to an alphanumeric sign (e.g., *n*) to assist them to express the generalisation. This requires students to accept this new sign as an abstraction of an unknown number.

Throughout each interaction the teacher and student move through an encoding and decoding process as they interpret the signs. As the relationship between the variables become more apparent, students refine their gestures and language to articulate their generalisations. Intra-learning and inter-learning are apparent in all interactions (Saenz-Luldow & Zellweger, 2012). Finally, culture provides an overarching lens on each interaction.

7.6 INFLUENCE OF CULTURE WHEN GENERALISING GROWING PATTERNS

It is apparent that the cultural backgrounds of both the teacher and the student influence the teaching and learning process as young Indigenous students generalise. This study expands upon literature that asserts that culture influences students' mathematical experiences (Ezeife, 2002). Past research suggests that "the cultural experiences of the knower are epistemologically significant because these factors influence knowledge construction, use and interpretation" (Banks, 1993, p 6). While the teacher is the perceived 'knower' of Western mathematics in the classroom context, there is a need for shared knowledge to enrich the learning process. On-going, open dialogue between the 'knowers' was essential to address and enhance cultural differences. Furthermore, this shared knowledge was extended to encompass students' own patterns and these patterns were used as valuable platforms for future lessons. It is in this shared space, where knowledge is freely exchanged with all participants, that empowerment occurs (Denzin & Lincoln, 2008). By building relationships, and valuing Indigenous Education Officers and young Indigenous students' perspective and knowledge, non-Indigenous teachers who lack knowledge of Indigenous people (New South Wales Department of Education and Training, 2004) can be significantly enriched.

Young Indigenous students need the opportunity to make connections to their own contexts when learning new mathematical concepts. Past research has highlighted that linking mathematics to the 'students' known world' assists them in constructing and understanding the mathematical concept (Matthews et al., 2005). Additionally, this approach places value on students' cultural heritage (Matthews et al., 2005). Furthermore, using Indigenous students own life experiences and contexts can assist them in perceiving the relevance of the presented learning, and result in increasing their engagement in mathematics (Howard & Perry, 2005). Initially, when students created their own growing pattern and described how it was growing, many students linked this mathematics to their natural environment (see Section 5.4.1). For example, students drew plants and people when creating their own growing patterns, and described the growth as, "My pattern is growing by eating. The plants are growing by sitting in the sun" (S5). After consultation with the IEO's this became the platform from which growing patterns were explored in the classroom setting. As

evidenced in the interviews and classroom observation, this approach assisted young Indigenous students to generalise. It is to be noted that drawing on a context believed to be accessible to students did not allow them to make immediate links to mathematics. It was only one of the facets that assisted students to generalise. Rather, using students' known world (Matthews et al., 2005) provided them an opportunity to engage with the pattern and use language around the context to express their generalisations.

Although there is a drive to draw mathematics from Indigenous culture, in some conceptual areas these links are difficult to make. It has been acknowledged in past research that Western mathematics fails to include Indigenous culture, and this in turn is the reason why Indigenous students find mathematics difficult to negotiate (Aikenhead, 2001; Howard, 1997; Howard & Perry, 2005). It is argued that even with the best intentions to embed Indigenous knowledge into mathematics, it proves to be challenging to make appropriate links, as this study found. In fact the mathematical construct under consideration may indeed be absent, or if it does exist, may be inappropriate to use. Generalising growing patterns is an abstract concept to both Indigenous and non-Indigenous students. While both Indigenous Education Officers identified kinship models as growing patterns within their culture, it was acknowledged that this would be an inappropriate context for a non-Indigenous teacher to make connections or allusions to. An argument in the literature supports this choice, as it has been identified that Indigenous students experience a set of stereotypes or generalisations about their culture when presented by a non-Indigenous teacher (Person, 2009; Nakata, 2002, 2007). Therefore, it is conjectured that in the instances where it is challenging to link the Western mathematics Indigenous culture, there needs to be more emphasis placed on Indigenous ways of learning (Williams & Tanaka, 2007) to ensure that Indigenous students have an opportunity to value their culture. By using the two facets of the students known world (Matthews et al., 2005) in conjunction with Indigenous ways of learning (Matthews, et al., 2007; Williams & Tanaka, 2007; Yunkaporta, 2009) represents the disparity between two-way learning (Harris, 1990; Pearson, 2009; Sarra, 2003): if this is recognised then the link between Indigenous culture and Western mathematics can be established.

Fundamental to Indigenous students' learning is the opportunity to engage in storytelling. Students demonstrated a natural tendency to use storytelling as a way to demonstrate their thinking about growing patterns and their structure. Storytelling as a means of passing on knowledge is enmeshed in Aboriginal and Torres Strait Island culture (QSA, 2013). Research has highlighted the positive use of storytelling in learning (Matthews, et al. 2007; Williams & Tanaka, 2007; Yunkaporta, 2009). Opportunity for students to create growing patterns by either building patterns using hands-on materials, or drawing growing patterns in their own space, appeared to support students to generalise. Once students had created their own patterns, they then generated their own stories to describe how the pattern was growing. Matthews, et al. (2007), describes this as the process of conventionalisation, a term used within Harré's semiotic model (1983). Within this semiotic model, students' individual and collaborative actions contribute to their own learning and mastery of the subject and to the culture itself. This present study adds to the past literature and suggests that when students are learning about abstract mathematical concepts such as growing patterns young Indigenous students need the opportunity to tell their story in conjunction with hands-on materials. In addition, these young Indigenous students gestured to the hands-on materials as they were storytelling. It was the storytelling together with their gesturing, that evidenced their understanding of the mathematical construct. Again, this widens an understanding of how two-way learning can occur within the classroom context.

Indigenous students displayed a communal nature of learning which is divergent from the Western model of learning. The Western model is grounded more in individual attainment of an answer, as opposed to group participation in working towards a shared truth. Past research has indicated that Indigenous students have a cooperative approach to learning rather than a competitive approach (Nicol, 2008). Findings from the present study add to this literature. Indigenous students would draw upon each other to assist in their understanding of the patterning problem. From the perspective of a non-Indigenous researcher, it was observed that Indigenous students looked to peers whom they perceived as answering questions correctly for further information. Furthermore, through observing footage of the classroom lessons, it was apparent that students supported each other's learning through these

interactions. Teachers need to be aware and supportive of these interactions, and mindful that they can be either verbal or nonverbal cues (Yeatman, 2009).

Young Indigenous students engage in communication that differs from non-Indigenous students (Yeatman, 2009). During the analysis conducted with the Indigenous Education Officers, it became evident that there were particular cultural verbal and non-verbal cues displayed by these Indigenous students. Past literature identifies particular verbal and non-verbal communication styles displayed by Indigenous students (Harkins, 1990; Yeatman, 2009). In the present study, students would call out a variety of answers, and shared answers with one another so that all students were successful in providing an answer to the questions posed. This has also been evident in studies involving young Indigenous students in regional Western Australia (Sullivan, et al., 2013). Notably, eye contact between researcher and students, physical display of shame by students, and a change in manner of participation between the classroom and one-on-one setting were three indispensable outcomes from this present study. Though literature suggests that Indigenous students will avoid eye contact as a sign of respect or politeness (Yeatman, 2009), it was found that while some students had limited eye contact during the interview, there were some students who maintained eye contact. Interestingly, S6 (Aboriginal boy) had more eye contact than S2 (Torres Strait Islander boy). Indigenous Education Officer 2 reported that S2 would not maintain eye contact due to gender difference and respect (see Section 6.2). Within the present study, students displayed shame in response to situations where they answered incorrectly, but unexpectedly for a non-Indigenous researcher some students displayed shame when they were praised for their work.

Non-Indigenous teachers need to be aware that not all Indigenous students disclose their ability in front of their peers. Though this is believed to be the case in the majority of education settings (Commonwealth of Australia, 2008), what is particularly different about this context is that students did not disclose their abilities for cultural reasons. As the Indigenous Education Officers explained, students will not always display an ability or inability to answer questions. It was evident from the study that not all students disclosed their ability in front of their peers. A conjectured reason for this is the notion of shame, a concept that is prevalent in Australian Indigenous cultures (Harkins, 1990; Leitner & Malcom, 2007). Non-Indigenous

teachers need to be attentive to cultural protocols such as shame, as this unawareness can potentially impede student learning (Harkins, 1990; Vallance & Tchacos, 2001). In the present study, to provide opportunity for students to disclose their ability, data was collected from both classroom lessons and one-on-one interviews. Both settings need to be provided for Indigenous students to display their learning.

Building trusting relationships with young Indigenous students is imperative for them to provide answers in class and speak comfortably in a one-on-one setting. Past research suggests that questions/answers are not generally a part of Aboriginal discourse, and that asking Indigenous students direct questions can be considered confronting, particularly questions beginning with 'where', 'how', 'when' and 'why' (Eades, 1995). Young Indigenous students in the present study appeared comfortable in answering direct questions. Students were prepared to provide answers even if they were incorrect, in an attempt to assist the researcher/teacher in finding the answer. It is conjectured that this type of 'risk taking' is possible when young Indigenous students feel safe, and perceive that they are in a trusting environment. Pivotal to building strong relationships and providing a safe environment for Indigenous students, is the relationship they have with the Indigenous Education Officer. That is, in this study the Indigenous Education Officers supported students during lessons and encouraged them to 'have a go'. It was evident that Indigenous Education Officers have a positive influence on students and impact on students' participation in class (MacGill, 2010).

At times, there is a mismatch between young Indigenous students' home language and Western mathematical language used in class. This mismatch has been described as a barrier for Indigenous students when learning Western mathematics (Harris, 1991; Jorgensen, 2011). While there are strategies that have been suggested to overcome these language differences, such as providing bilingual settings (Purdie, 2009) and offering opportunities for students to use their home language when providing explanations in class, challenges still remain. If there is an over emphasis on requiring young Indigenous students to verbalise their mathematics using Australian Standard English, then we may in fact miss out on what these students actually know and understand. Rather, the communication of mathematics should be seen as an embodied process drawing from a combination of gestural cues, manipulation of hands-on materials and language. Students demonstrated that they

had the requisite knowledge to generalise growing patterns, however, their form of communication drew on more subtleties than just communicating in Standard Australian English. This embodied process served to assist students in developing an understanding of the language of mathematics.

Finally, having high expectations, and providing opportunities to engage in mathematical tasks that require abstract thinking, presents a divergence from deficit models of teaching presented to young Indigenous students. This aligns with views expressed in current education reforms for Indigenous students that demand quality education programs in conjunction with high expectations (Sarra, 2012). This study adds to this literature by demonstrating how high expectations can be achieved through exploring early algebraic concepts with young Indigenous students. Furthermore, it demonstrates the opportunity for success of young Indigenous students engaging with mathematical concepts and learning experiences that move beyond basic skill and drill tasks (Baturo, Cooper, Michaelson, & Stevenson, 2008; Jorgensen, Grootenboer, Niesche, & Lerman, 2010). It provides a platform for deeper experiences in mathematics and the potential of instilling a sense of achievement. Additionally, by engaging young Indigenous students in early algebra, there is the potential to overcome related challenges in the later years of school.

7.7   CULTURAL SEMIOTIC LEARNING MODEL

The exploration of the construction of shared knowledge showed that culture played a pivotal role in the learning interactions between the teacher, student, and Indigenous Education Officer. Each person within the class brings their own cultural perspectives. However, when teachers have an awareness of students' culture, they can better interpret the learning and the semiotic interactions. Essentially, teachers need to consider the role of culture when interpreting the semiotic signs, and how students perceive their own semiotic signs.

This study provided first-hand experiences of bringing a non-Indigenous teacher (the researcher) into an Indigenous classroom context to explore elements that aided learning in Western mathematics. Throughout this process, a shift in understanding of what was happening occurred, which involved moving from conveying content to transacting knowledge through shared dialogue. This model poses significant challenges when working in a classroom with contradistinctive

cultures. Figure 7.4 is a depiction of the knowledge interactions that were experienced and observed in the present research.
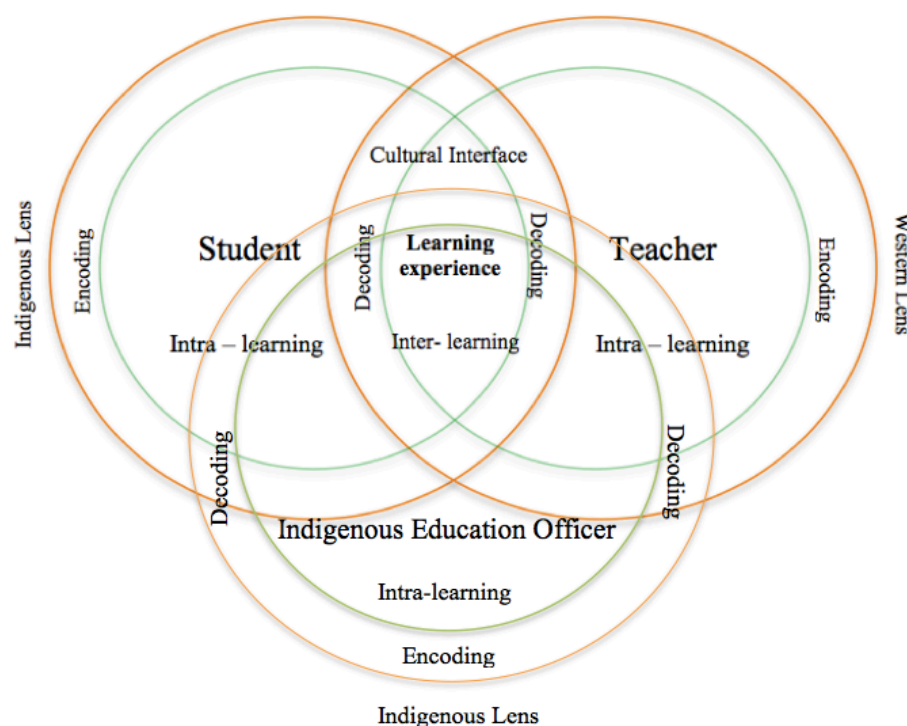
Figure 7.4. Knowledge interactions between Non-Indigenous teachers, Indigenous Education Officers and Indigenous students when engaging in learning experiences.

In considering learning as a shared dialogue and experience, the figure illustrates the teacher arriving at the interaction with knowledge of mathematics largely from a Western perspective. At the same point in time, students bring knowledge from their own life and experiences not heavily dominated by Western mathematical language. The Indigenous Education Officers complete the triumvirate, not by acting as interpreters but rather acting as facilitators for encoding and decoding knowledge from the Western and Indigenous domains in order to create a new and shared knowledge. This created shared space is where empowerment occurs (Denzin & Lincoln, 2008), an essential facet to Indigenous students' learning.

7.8   CHAPTER REVIEW

Within this chapter, findings that emerged from the study were examined and reviewed in light of the literature and the theoretical frameworks, and a learning

trajectory was presented. This trajectory teased out the semiotic interactions that influenced how students engaged with pattern generalisation tasks. Finally, the chapter offered a learning model that encompasses the interactions between the teacher, student and Indigenous education officer. Chapter 8 addresses the research questions, and presents the limitations, recommendations, and future research consideration.

Chapter 8: Conclusions and Recommendations

8.1 CHAPTER OVERVIEW

The concluding chapter reviews the main findings of the study in relation to the research questions, and presents the implications for practice and research. Figure 8.1 presents the overview for Chapter 8.

Chapter 8: Conclusion and Recommendations

8.1 Chapter Overview

8.2 Recapulation of the Research Problem

8.3 Resarch Design

8.4 Research Questions Addressed

8.5 Conclusion of the Study

8.6 Recommendations

8.7 Further Research Considerations

8.8 Limitations

8.9 Concluding Remarks
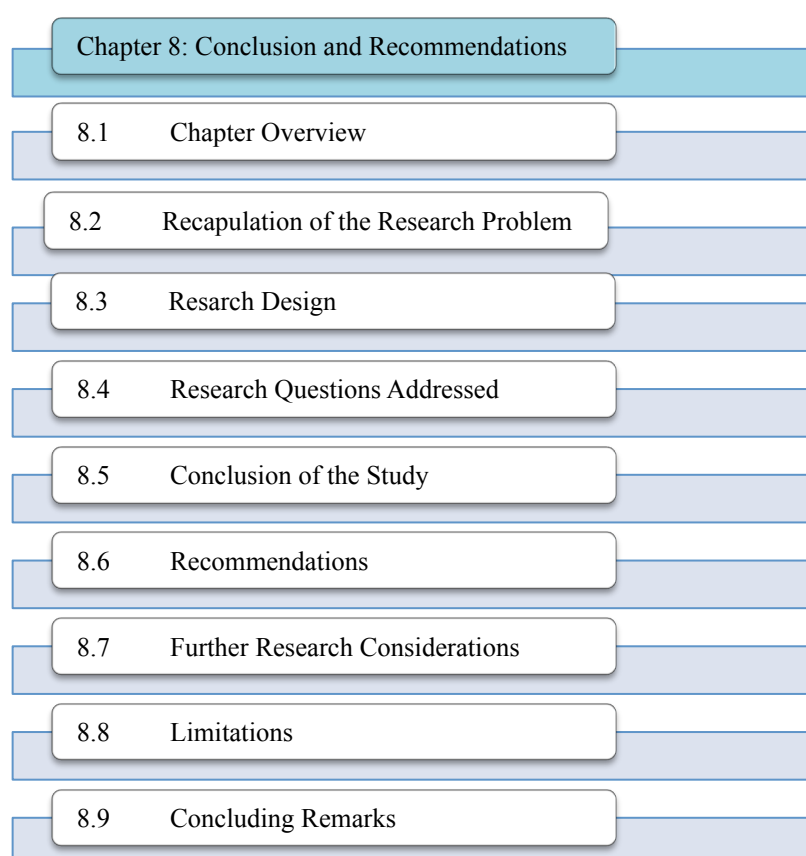
Figure 8.1. Overview of Chapter 8.

8.2 RECAPULATION OF THE RESEARCH PURPOSE

The purpose of this study was to explore how young Indigenous students generalise growing patterns. Semiotics, a system for analysing signs and representations, in conjunction with Indigenous research perspectives, provided a powerful theoretical framework for exploring these phenomena with young Australian Indigenous students. This study was motivated by the limited research literature pertaining to early algebra, and how young Indigenous students could best

engage in mathematical generalisations. More importantly, the study provided a positive story about Indigenous students' achievements in mathematics.

The aims of the study were threefold: (a) to determine how young Indigenous students engage in growing pattern generalisation tasks; (b) to consider the semiotic interactions that are involved in the generalisation process, and how this impacts students' learning and communication; and (c) to identify the role of culture in relation to how young Indigenous students learn mathematics.

8.3 RESEARCH DESIGN

The study contributes to the body of research with regards to young Indigenous students engaging in early algebraic concepts. An interpretive research paradigm was adopted in order to explore how young Indigenous students construct their own knowledge as they engage in pattern generalisation in a naturalistic classroom setting. Additionally, the study offers an opportunity to contribute to our understanding of how Indigenous students conceptualise mathematical growing patterns within the theoretical perspective of semiotics.

The following three research questions provided direction for the design of the study:

1. How do young Indigenous students engage in growing pattern generalisation?

2. What teacher actions assist in enhancing young Indigenous students to generalise growing patterns?

3. How does culture influence the way in which young Indigenous students engage in growing pattern generalisation?

As this study explored the ways in which Indigenous students construct new knowledge while engaging in pattern generalisation tasks, constructionism was adopted as the epistemology of the study. Constructionism considers that knowledge is constructed in social settings and interactions (Stahl, 2003). Thus, constructionism provided the opportunity for deeper analysis of the language, symbolism, culture, and interactions students used when engaging in the learning process.

Semiotics and Indigenous research perspectives were used as the theoretical perspectives of the study. The adoption of both of these lenses provided for a deeper

analysis of the interactions between students and the researcher when considering, (a) the teaching actions that assist young Indigenous students to generalise, and (b) the cultural signs and gestures they use as they generalise. To appropriately account for and celebrate students learning, Indigenous research perspectives as a theoretical perspective were acknowledged.

As the aims of this study were concerned with ascertaining the teaching actions that promote young Indigenous students' engagement with pattern generalisation, conjecture-driven teaching experiments were adopted as the research methodology. Teaching experiments were used in this study for the primary purpose of directly experiencing students' mathematical learning and reasoning in relation to their construction of mathematical knowledge (Cobb, 2000; Steffe & Thompson, 2000).

The research was conducted in one Year 2/3 classroom (7-9 year olds) of an urban Indigenous school in North Queensland. Additionally, an Aboriginal woman (Indigenous Education Officer – IEO1) and a Torres Strait woman (Indigenous Education Officer - IEO2) were consulted during the study, so as to offer cultural information in relation to students' learning. Finally, I was a participant of the study as my role was researcher-as-teacher during the data collection.

To explore how young Indigenous students engaged in mathematical generalisations in a naturalistic classroom setting, the following data-gathering strategies were employed:

1. Initial Classroom Observations.

2. Pretest 1 and Pretest 2 conducted at the beginning of Teaching Experiment 1 and 2.

3. Teaching Experiments with the whole class (N=18), comprising six 45-minute mathematics lessons in total (three in Teaching Experiment 1 and three in Teaching Experiment 2).

4. One-on-One Piagetian clinical interviews with students (n=3) after each teaching experiment.

8.4 RESEARCH QUESTIONS ADDRESSED

In order to address the main overarching research question, 'How do young Indigenous students generalise mathematical growing patterns?', three research

questions were generated. The main findings of the study are addressed with these three research questions in mind.

8.4.1 Research Question One

How do young Indigenous students engage in growing pattern generalisation?

Young Indigenous students engage in generalisation by exploring structures within growing patterns. In order to explore these structures, young Indigenous students engaged in a series of teaching and learning actions (see Figure 7.1). These teaching actions included: extending growing patterns, deconstructing and reconstructing growing patterns, identifying near generalisations, identifying quasi-generalisations, and using alphanumeric notation to express generalisations. Findings from this study have shown that it is beneficial for students to use hands-on materials during each stage of the learning sequence, and that they must physically engage with these materials as they shift from the particular to the general structure of the pattern.

Initially, when young Indigenous students engaged with and created growing patterns, they drew on elements of their own natural environment, and generalised these patterns in relation to these contexts. They created their own stories that assisted them to articulate the structure of the pattern, and which in turn assisted them to express the generalisation.

Students were able to engage in quasi-generalisations and alphanumeric generalisations. However, before students could do this there were a series of teaching actions that assisted them to express these generalisations. The use of quasi-variables assisted students to shift their thinking from recursive to covariational thinking. By doing so this assisted students to express growing patterns in relation to both variables.

It was found that these students did not always use Western mathematical language when generalising growing patterns. When generalising the pattern, some students would use gesture to replicate mathematical concepts such as multiplication. These gestures, in conjunction with hands-on materials, formed part of their expression of the general structure of the growing patterns, and became an essential part in communicating their ideas.

These findings represent a new contribution to the literature in relation to Indigenous students engaging in a specific mathematics concept. Within an Australian context, studies in relation to young Indigenous students focus on broad aspects of numeracy, pedagogical approaches, and learning styles of Indigenous students. There is limited research in relation to young Indigenous students working with early algebraic concepts. Furthermore, it adds to the literature in relation to how young students express growing pattern generalisations.

8.4.2 Research Question Two

What teacher actions assist in enhancing young Indigenous students to generalise growing patterns?

Findings from this study suggest that there are five key factors in relation to teacher actions that assist students to reach generalisations.

First, the consideration of pattern type was essential in assisting young Indigenous students to engage in pattern generalisation tasks. As mentioned earlier, when initially exploring growing patterns, it is important that growing patterns from students' known context are used. Evidently, this alone will not assist students to see the structure of the growing pattern. Iconic signs play a pivotal role in students accessing the structure of the growing pattern. The role of iconic signs is to enhance the representation of variables and the multiplicative structure. This proved to be an important teaching action to consider in enhancing students' ability to see the general structure. Hands-on materials need to represent these iconic signs. Furthermore, it appeared that there was a hierarchy of pattern type that teachers need to consider when engaging young Indigenous students in pattern generalisation tasks (See Figure 7.2).

Second, semiotic interactions occurred at each step within the hypothesised learning-teaching trajectory (See Figure 7.3). This hypothesised sequence drew on the perspectives of early algebra and semiotic theory that support young Indigenous students to generalise. Findings from this study suggest that teachers and students need to physically interact with the pattern in order to assist students to generalise. Teacher interactions need to incorporate indexical signs, including gesture and explicit language, to highlight the structure of growing patterns for students. In turn, as students interact with the pattern, teachers need to be aware of the new signs

students create in relation to the structure of the pattern. As these semiotic interactions occur, students and teachers encode and decode each other's signs.

Third, focused teaching moments are required to assist students to move through learning barriers. As students display difficulty within stages of the learning trajectory (see Figure 7.3), teachers need to provide hands-on experiences where both student and teacher engage with, and deconstruct, the pattern structure. As teachers deconstruct the pattern, teaching actions include heavy use of gesture and explicit language focusing on the structure of the pattern. Students then need to re-enact these deconstructions in order to assist them to identify the general structure. While explicit teaching of language can assist students to develop the Western mathematical language to express generalisations, this is not always necessary, as young Indigenous students can express these structures using gesture. There is a need to incorporate opportunities within the classroom for students to express generalisations in this manner, and use this as a platform to develop their understanding of Western mathematical language.

Fourth, understanding the importance and influence of Indigenous Education Officers in generating trust between students and non-Indigenous teachers is paramount. By doing so, this establishes an environment where young Indigenous students feel safe and take risks. In order to create such an environment, the Indigenous Education Officers provide support to students, and encourage them to answer questions during lessons. The establishment of this important relationship with Indigenous Education Officers can help minimise the impact of cultural protocols that are not necessarily understood or recognised by non-Indigenous teachers.

Fifth, the relationship between the teacher and the Indigenous Education Officer is invaluable in regard to making connections between Indigenous culture and Western mathematics. By building relationships between Indigenous Education Officers and non-Indigenous teachers, learning can be enhanced by making appropriate connections to Indigenous knowledge. Findings from this study suggest that the relationship needs to be ongoing throughout the teaching process, as it is this relationship that assists the non-Indigenous teacher to become aware of cultural gestures and signs. This helps to ensure that the teaching actions and learning experiences that occur in the classroom support young Indigenous students' learning.

These findings represent an extension of the literature, addressing what teaching actions assist young Indigenous students to generalise. While it is established in past research that young students can generalise growing patterns, how students do this continues to be investigated. Findings from this research with young Indigenous students demonstrate that there are a series of learning experiences that students move through as they engage with the structure of the pattern, and this assists them to generalise. It is conjectured that initially students need to engage with hands-on experiences drawn from their known context to assist them to articulate the general structure of growing patterns. Furthermore, this research adds to current literature in relation to how semiotics can enhance students' mathematical learning. Findings from this study demonstrate the important role semiotics plays in: selecting patterns and hands-on materials for students to engage with; the interactions between the student and teacher including indexical signs; and, the encoding and decoding of signs within each learning interaction as students move from the particular to the general.

8.4.3 Research Question Three

How does culture influence the way in which young Indigenous students engage in growing pattern generalisation?

This study suggests that young Indigenous students' culture has a profound influence on how they generalise mathematical growing patterns. It was found that in particular, culture influenced the way in which these young Indigenous students interacted in the learning process, and how they expressed their mathematical generalisations. It is evident that young Indigenous students are required to decode Western mathematics using an Indigenous lens. The implication from this present study is the theoretical construct examining the translation from teacher to learner, and the mechanisms for creating a broad space of overlap or shared knowledge (see Figure 7.4).

Cultural impacts were threefold: the culture of the non-Indigenous teacher and Western mathematics; the Indigenous Education Officer as facilitator; and, the culture of the Aboriginal and Torres Strait Islander students. This view of a triphasic cultural dynamic addressed the perception of Indigenous culture forming an impediment to learning Western mathematical generalisations. By engaging with this

triphasic cultural dynamic interaction, an opportunity to remove preconceptions of the ways in which a 'student' learns was established. This provided the chance for both teacher and students to engage in an open learning space where both cultures can learn from one another, thus reducing cultural bias.

Additionally, culture influenced how young Indigenous students articulated their generalisations. At times, these young students could not access the Western mathematical language required to express their generalisations. Rather, they used gesture to support and express these structures. It was essential to accept these cultural gestures as the language in which these students were articulating their generalisations. These gestural signs were constructed during the interactions between the student and the hands-on materials.

These findings contribute new knowledge literature in relation to culture and learning mathematics. In particular, they contribute to literature pertaining to how young students express the general structure of growing patterns. These findings highlight the need to shift our understanding of how students articulate and communicate their mathematical knowledge, that is, they call us to value a range of new ways in which students express their generalisations. It was evident that not all young Indigenous students communicated their generalisations in Western mathematical terms, but used a combination of gesture and natural language. Thus, an emphasis needs to be placed on the opportunity for young Indigenous students to co-construct language that includes both gesture and their natural language, together with the manipulation of hands-on materials. Furthermore, this study supplements the literature in relation to how non-Indigenous teachers can support and enhance Indigenous students' learning in mathematics. This study provides a model (Figure 7.4) that demonstrates the complex interactions that occur between the non-Indigenous teacher, Indigenous Education Officers, and Indigenous students within the mathematics classroom.

8.5   CONCLUSIONS OF THE STUDY

8.5.1 Conclusion One

Young Indigenous students are undoubtedly capable of engaging in abstract concepts in mathematics, despite poor results in national testing. This study addresses the limited research in relation to the learning of algebra for young

Indigenous students. Patterning has been described as one of central ideas of algebraic thinking (NCTM, 2000), and these young Indigenous students were developing and succeeding in this area. Students demonstrated that they could copy, extend, create, quasi-generalise, and generalise (using alphanumeric notation) growing patterns. Additionally, their exploration of growing patterns led to the development of other mathematical concepts. Thus, these young Indigenous students (Year 2 and Year 3) were demonstrating an aptitude to meaningfully engage with higher-level mathematics.

8.5.2 Conclusion Two

Teacher interactions have a profound effect on enhancing the ability of young Indigenous students to engage in complex mathematical generalisations. These teaching actions need to include experiences where young Indigenous students have the opportunity to construct, deconstruct and reconstruct growing patterns using hands-on materials. Semiotic interactions play a pivotal role in enhancing students' ability to see and generalise these structures. Students and teachers need to be able to decode and encode each other's signs as students construct new knowledge.

8.5.3 Conclusion Three

Culture plays an integral part in these students' engagement with mathematical generalisations. Hence, there is a vital need for shared learning in the mathematics classroom. Consequently, sharing knowledge from divergent cultural starting points provides an opportunity for rich learning. Central to this rich learning is the relationship between the non-Indigenous teacher, Indigenous students, and Indigenous Education Officer, because this provides the mediation and facilitation between teacher and student.

8.5.4 Conclusion Four

Young Indigenous students communicate their mathematical understandings through the use of gesture in conjunction with their natural language. It was found that co-constructing language was vital to the teaching and learning process for these students. To accept these expressions as generalisations however, it is essential to acknowledge gesture as evidence of students' understanding.

8.6 RECOMMENDATIONS

The recommendations arising from exploring how young Indigenous students generalise growing patterns are directed towards teachers and education researchers. The following recommendations emerge from the conclusions of this study. There are two categories of recommendation: (i) Teaching and learning, and (ii) Research.

8.6.1 Recommendations for Teaching and Learning

Young students need to be given opportunities to engage in early algebraic thinking, so as to assist with bridging some of the common misunderstandings about algebraic thinking that older students experience.

The types of growing pattern activities these young Indigenous students engaged in are linked to concepts in the Year 6 Australian national curriculum. In the current Australian curriculum it states that in Year 6 students need to engage in:

> Continuing and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (ACMNA 133) (ACARA, 2012, "Year 6 Content Descriptors", para. 7).

Furthermore, the elaboration states that students need to:

> identify and generalise number patterns; investigate additive and multiplicative patterns such as the number of tiles in a geometric pattern, or the number of dots or other shapes in successive repeats of a strip or border pattern looking for patterns in the way the numbers increase and decrease (ACARA, 2012, "Year 6 Elaborations", para. 7).

Evidently, the Year 2 and 3 students in this study were engaging at mathematics of this level. It is recommended that students can engage in these types of mathematical activities earlier, and this engagement not only can lead to meaningful discussion about addition and multiplication, but also has to potential to help overcome difficulties experienced by older students in relation to algebraic thinking.

Teachers need to be aware that young Indigenous students are capable of understanding higher levels of mathematics and thus have high expectations of these students.

Young Indigenous students are capable of engaging in high levels of mathematics. Teachers in Indigenous communities need to ensure they are providing mathematical tasks that match their students' capability. Breaking the tradition of

offering drill and skill lessons or lessons that provide low-level thinking is essential for young Indigenous students to be successful in mathematics. Additionally, having high expectations for young Indigenous students provides a learning environment that focuses on empowerment and success.

A structural approach is needed when exploring growing patterns from the particular to the general. This approach is essential when assisting young Indigenous students to generalise.

A recommendation is that teachers need to take a structural approach to student learning, that is, attend to, and engage students with, the structural elements of the growing pattern (e.g. exploring the structures in the pattern, highlighting variables and identifying multiplicative structures). From this study, it is evident there are a series of learning experiences that enhance students' ability to generalise. These learning experiences must focus on the structure of the growing pattern. Students need to construct and deconstruct patterns to develop an understanding of the covariational relationship between variables.

Relating growing patterns to Indigenous students' known world, such as an environmental context, can assist them in identifying and articulating the structure of growing patterns.

Generalising growing patterns is an abstract concept in mathematics. Thus, using constructs from students known world provides a platform for them to meaningfully engage with growing patterns. Using examples from environmental contexts means that students can describe the structure of the pattern using language that is familiar to them. Teachers need to consider the types of growing patterns they use when young Indigenous students first engage with this mathematical concept.

Hands-on materials that represent two variables as iconic signs, can engage young Indigenous students in pattern generalisation.

Teachers need to consider how they structure patterns so students can identify variables. These variables need to be represented through the use of iconic signs. This can be achieved by representing pattern terms using different colours or hands-on materials where variables are embedded in the single structure. When variables are separated, pattern terms can be represented using number cards that are placed under the pattern quantity.

There needs to be recognition that high levels of Standard Australian English are not required to communicate mathematical generalisations effectively.

This research has shown that when young Indigenous students generalise growing patterns, they draw on a range of embodied signs when expressing these generalisations. Additionally, there needs to be a shift in thinking, which believes that Standard Australian English is the only form of articulating Western mathematical ideas. Teachers need to provide space, and support students to express mathematics in ways that embrace and value their cultural backgrounds.

Building trusting and safe classroom environments provide teachers with better insights of students' mathematical abilities.

Teachers need to build relationships with Indigenous students that support and encourage their abilities in mathematics. It is not enough to just be a 'nice' teacher. Students need to be comfortable in taking risks in class. Furthermore, teachers need to be aware of the notion of shame, and its impact on how students communicate their learning. Because of this, teachers need to provide an environment that is cognitively challenging with high expectations for their students.

Teachers need to interact with Young Indigenous students in both group and one-on-one settings.

As students rely on one another to enrich their understanding of mathematical tasks, it is imperative that students are given the opportunity to work freely as a whole class and to share their ideas. While this way of learning as a classroom community extrinsically assisted many students, it was also evident that providing students with a one-on-one setting gave different, deeper insights into students' mathematical understandings. Within this one-on-one setting, some students due to cultural protocols disclose more information in this learning environment than in whole class activities. Some students were more willing to 'have a go' when peers were not present and appeared more confortable with taking risks with their learning.

Non-Indigenous teachers need to work closely with the Indigenous Education Officers in class to better understand the cultural aspects and communication styles of Indigenous students.

Indigenous Education Officers are an integral part of the learning experience for both teachers and students. Non-Indigenous teachers need to work closely with

Indigenous Education Officers to ensure that cultural protocols and communication styles of Indigenous students are understood.

Teachers need to consider gestures in young Indigenous student learning and communication.

Teachers need to be aware of the role gesture plays when Indigenous students are communicating their learning, and need to provide learning experiences that enhance these opportunities. Furthermore, during learning experiences, teachers need to be more attuned to gestural cues used by Indigenous students so as to have a more holistic understanding of students' knowledge.

Teachers need to consider their own semiotic interactions that occur in the learning process.

Teachers need to consider semiotic interactions while engaging in the learning process. This will enable to teachers to be more informed about the variety of signs that assist students to build new knowledge and understandings. Subsequently, teachers need to decode students' signs as students build new knowledge, as this will further inform their next teaching action.

8.6.2 Recommendations for Research with Indigenous Students

Qualitative research with young Indigenous students needs to provide opportunities for students to engage in classroom discussion and one-on-one interactions.

It is imperative when researching young Indigenous students, that opportunities to collect data from a number of settings are provided. Multiple data sources allow a holistic appreciation of learning that is taking place. Additionally, some Indigenous students may not disclose their knowledge in front of their peers or family members. By providing opportunities to collect data in whole class settings, small groups and on-on-one environments, in turn respects such cultural protocols, and builds a better understanding of students' abilities.

It is necessary to have a global view of the learning, encompassing all students, teachers and IEO's to better establish how students engage in mathematics.

Research with Indigenous students needs to encompass all members of the learning community. This lends itself to a more global view of the interactions and

the learning taking place. All members playing a role in the research ensures a better understanding of the learning that is occurring and also results in a richer data set from which conclusions are drawn.

Researchers need to ensure that Indigenous Education Officers are aware of the aims of the research prior to data collection phase.

Past studies have highlighted the issues pertaining to Indigenous Education Officers providing answers to students when collecting data. It is understandable why this occurs, as Indigenous students have been negatively depicted in some past research. Thus, it is essential that when conducting research with Indigenous students the Indigenous Education Officers need to be aware of the study's aims. For example, in this study it was explained to both Indigenous Education Officers that this study was exploratory, and was not about students providing the 'right' answers. Rather, it was about understanding how best to teach and engage Indigenous students in mathematics.

8.7 FURTHER RESEARCH CONSIDERATIONS

There are six issues worth pursuing in relation to further research.

First, as this study is clearly case bound, a larger scale study would be necessary to determine if the findings are applicable in other contexts, locations, and cultures.

Second, while young Indigenous students from this study engaged with a series of teaching actions that assisted them to generalise, would these teaching actions remain similar for Indigenous students from different geo-locations? That is, how does geo-location (i.e., urban, rural, remote areas) influence the learning experience and semiotic interactions in Indigenous classrooms? Furthermore, what is the learning sequence for non-Indigenous students as they generalise growing patterns? Is it similar or different to Indigenous students?

Third, besides how young Indigenous students communicated their generalisations, it was apparent that gesture played a pivotal role in the learning experiences. Further research is needed to determine what extent does gesture have in the learning of mathematics. That is, what role does gesture play in the learning of mathematics for young Indigenous students? As an extension of this, what semiotic

interactions influence how teachers teach mathematics? What semiotic interactions are teachers aware of, and how do they influence their teaching of mathematics?

Fourth, considering that young Indigenous students drew from their own environmental contexts as they engaged in growing pattern tasks, what are other growing patterns that exist in Australian Aboriginal and Torres Strait Islander culture? And how do young Indigenous students engage with these growing patterns? Furthermore, is this mirrored in non-Indigenous settings? That is, how do environmental contexts influence non-Indigenous students to generalise growing patterns?

Fifth, it appeared that there was a hierarchy of pattern types that assist students to engage with the structure of growing patterns. Further research needs to consider the proposed hierarchy in relation to larger cohorts, to determine if these assist young students to engage in growing pattern generalisations. Therefore, how does the proposed pattern hierarchy assist students to generalise growing patterns?

Sixth, further research lends itself to exploring other aspects of early algebra with young Indigenous students. How do young Indigenous students transfer the structures of growing patterns to functional relationships in t-tables? What influences this process?

8.8   LIMITATIONS

Limitations of the study are discussed in terms of the design of the study. The study focused on a single school in North Queensland, with a small sample of students. Because of this, the study is bound to both context and time. To overcome this limitation, it is necessary to conduct the same study in 'multiple environments' to provide transferability (Gross, 1998). However, this was not possible in the timeframe of this thesis, as their was a need to have time to build trusting relationships with the participants, and time was also needed for in-depth analysis. Despite this, while the choice of using a single urban Indigenous school implies that the generalisation of the data is limited, it is the rich distinctive cultural perspectives that provide unique perspectives and insights that are valued from this study.

The researcher acknowledges that there would be variation if the study were to be conducted in a rural or remote setting. However, the transferability of findings from this study from one context to another is at the discretion of the reader (Stake,

2005). The study provides a basis for further investigations of young Indigenous students and non-Indigenous students with regards to teaching actions that assist students to generalise growing patterns.

A further limitation to the study is that the researcher is non-Indigenous. Thus, Indigenous Education Officers within the community were consulted to guide the researcher and ensure that best practice was followed. While the study was initially analysed from the researchers' perspective (non-Indigenous), Indigenous Educations Officers were continually consulted to ensure cultural nuances were understood, and an honest overall picture of young Indigenous students learning was reported.

Finally, it is acknowledged that it is not possible to encompass all the findings of all the semiotic theorists (Saussure, Hjelmslev, Barthes, Foucault, Morris, and Eco) within the limits of this thesis. It is anticipated that this has been compensated for by the demonstration of the pivotal role of Peirce's theory of semiotics in the analysis of students' learning within the thesis.

8.9   CONCLUDING REMARKS

Conclusions drawn from this study provide a positive story in relation to young Indigenous students engaging with, and learning mathematics. New insights are gained into the development of early algebraic thinking in Indigenous contexts, and the important roles of the non-Indigenous teacher, Indigenous Education Officer, and Indigenous students in the learning process. Additionally, the study described the teaching and learning processes that enhance and assist young Indigenous students to generalise mathematical growing patterns. Findings presented from this study offer a unique contribution to the role of culture in the learning of early algebraic concepts. Furthermore, this study provides insights in relation to a specific learning area of mathematics, namely early algebra, an area where young Indigenous students are underrepresented in the literature.

Young Indigenous students demonstrated varying levels of sophisticated generalisations for mathematical growing patterns. The semiotics of the teaching and learning interactions, in conjunction with using hands-on materials as students deconstructed and reconstructed the growing pattern, assisted students to generalise. Teachers, Indigenous Education Officers, and Indigenous students need to work

together, to embrace and enhance each other's cultural perspectives to provide richer learning experiences in mathematics.

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Appendices

Appendix A
ACU Human Research Ethics Approval

Human Research Ethics Committee
Committee Approval Form

Principal Investigator/Supervisor: Professor Elizabeth Warren   Brisbane Campus

Co-Investigators: Dr Vincent Geiger   Brisbane Campus

Student Researcher: Ms Jodie Miller   Brisbane Campus

Ethics approval has been granted for the following project:
Young Australian Indigenous students' experiences in mathematics: An exploration in generalisation.

for the period: 1 April 2011 to 31 December 2011

Human Research Ethics Committee (HREC) Register Number: Q2011 15

Special Condition/s of Approval

Prior to commencement of your research, the following permissions are required to be submitted to the ACU HREC:

Shalom Christian College (written permission required)

The following standard **conditions as stipulated in the** *National Statement on Ethical Conduct in Research Involving Humans* **(2007) apply:**

(i) that Principal Investigators / Supervisors provide, on the form supplied by the Human Research Ethics Committee, annual reports on matters such as:
- security of records
- compliance with approved consent procedures and documentation
- compliance with special conditions, and

(ii) that researchers report to the HREC immediately any matter that might affect the ethical acceptability of the protocol, such as:
- proposed changes to the protocol
- unforeseen circumstances or events
- adverse effects on participants

The HREC will conduct an audit each year of all projects deemed to be of more than low risk. There will also be random audits of a sample of projects considered to be of negligible risk and low risk on all campuses each year.

Within one month of the conclusion of the project, researchers are required to complete a *Final Report Form* and submit it to the local Research Services Officer.

If the project continues for more than one year, researchers are required to complete an *Annual Progress Report Form* and submit it to the local Research Services Officer within one month of the anniversary date of the ethics approval.

K. Pashley.

Signed: ....   Date: .... 12.05.2011.....
(Research Services Officer, McAuley Campus)

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Appendix B
ACU Letter of Support from the Indigenous Higher Education Unit

Australian Catholic University
ABN 15 050 192 660
Mc Auley Campus, Banyo
1100 Nudgee Road, Brisbane, 4017

17th March, 2011

The Chair
Human Research Ethics Committee
c/- Kylie Pashley
Research Services
McAuley, Banyo Campus

ETHICS APPLICATION: MASTERS OF EDUCATIONAL RESEARCH
PRINCIPAL SUPERVISOR: PROFESSOR ELIZABETH WARREN
RESEARCH STUDENT: JODIE MILLER

I refer to the above Ethics Application by Jodie Miller and would like to offer this letter of support for the study.

After careful consideration of all information provided within the proposal, I am happy to support this application. The information gathered in the research will be valuable to the Indigenous community.

Trusting that the application is successful, I wish Jodie all the best. I look forward to hearing about its progress in the near future.

If you need to contact me further clarification of my support of this study, please feel free to telephone me on (07) 36237304

Yours Sincerely,

Krishna Heffernan
Academic Coordinator
Weemala
Indigenous Higher Education Unit
ACU National Banyo
PO Box 456
Virginia QLD 4014
Ph (07) 3623 7304
Fax (07) 3623 7311

Appendix C
Research Information Letters

INFORMATION LETTER TO THE PRINCIPAL

TITLE OF PROJECT: Young Australian Indigenous students' experiences in mathematics: An exploration in generalisation

NAME OF PRINCIPAL RESEARCHER: PROFESSOR ELIZABETH WARREN

NAME OF STUDENT RESEARCHER: JODIE MILLER (Masters of Education Research)

Dear Sir/Madam,
RE: Masters of Education Research Project

Your students and staff are invited to participate in a study that will assist in exploring how young Indigenous students generalise mathematics concepts. I anticipate that this project will commence in Term 3, 2011 and finish in December, 2011, and during this time we will investigate a cohort of students from Year 2.

We are asking the parents to consent their child to:
- Take part in patterning diagnostic activities (15 minutes each) involving:
  ➢ making patterns, continuing patterns, completing patterns, copying patterns.
- Take part in teaching episodes (12 one hour lessons over 3 weeks) that focus on patterning activities that promote mathematical thinking
- Take part in a one on one interview (with the Indigenous Education worker present) involving sharing their understanding of mathematical concepts (20 minutes per interview).
- Give permission for their child to be video recorded during the classroom activities and interviews.

We are asking the teachers and Indigenous Education workers to:
- Assist in implementing classroom activities (12 one hour lessons over 3 weeks) which may be video recorded;
- Provide reflective feedback regarding approaches to the mathematics activities;
- Assist with sending consent forms home with the children;
- Assist with pre and post diagnostic activities of children's patterning understandings;
- Provide cultural insight to the students' understandings within the teaching episodes and interviews.
- Give permission to be video recorded and voice recorded during the activities and analysis of the data.

The implementation of video-recording will be utilised and teacher, Indigenous education worker and parental permission will be sought for these activities. It is envisaged that this project will provide no foreseeable risk for participants.

This study will benefit participants by improving the teaching of mathematics to young Indigenous students. Students will be able to find connections between western mathematics models and Indigenous contexts. Additionally, teachers and Indigenous education workers will engage in mathematical activities which are not often presented to younger students and link pedagogical approaches to this type of mathematics.

At any time during the project you are free to withdraw your approval and discontinue participation of your staff and students without giving any reason. Confidentiality will be protected during the conduct of the research, by using coding and pseudonyms, and in any report or publication arising from the research data will remain anonymous.

Any questions regarding this project should be directed to researchers, Professor Elizabeth Warren (telephone 07 3623 7218) or Jodie Miller (07 3623 7405) in the School of Education, McAuley Campus, 1100 Nudgee Road, Banyo, QLD 4014. Results of the research will be provided on request.
This study has been approved by the Human Research Ethics Committee at Australian Catholic University.
In the event that you have any complaint or concern about the way you have been treated during the study, or if you have any query that the Investigator has not been able to satisfy, you may write to the Chair of the Human Research Ethics Committee care of the nearest branch of the Research Services Unit.

QLD
Chair, HREC
C/o Research Services
Australian Catholic University
PO Box 456
VIRGINIA QLD 4014
Tel: 07 3623 7429
Fax: 07 3623 7328

Any complaint or concern will be treated in confidence and fully investigated. The participant will be informed of the outcome.
If you agree to participate in this project, could you please sign agreement form, and return the other copy to Jodie Miller at ACU, Office of the School of Education, PO Box 456 Virginia Q 4014.
Yours sincerely,

Elizabeth Warren   Jodie Miller

ACU

AUSTRALIAN CATHOLIC UNIVERSITY

INFORMATION LETTER TO TEACHERS

TITLE OF PROJECT: Young Australian Indigenous students' experiences in mathematics: An exploration in generalisation

NAME OF PRINCIPAL RESEARCHER: PROFESSOR ELIZABETH WARREN

NAME OF STUDENT RESEARCHER: JODIE MILLER

(Masters of Education Research)

Dear Teacher,

We invite you, as a Teacher to participate in a Masters of Education Research pilot study for a larger project. The study aims to explore how young students engage in mathematical generalising tasks. Data will be gathered from the students in your class by conducting a diagnostic test. This test is scheduled to occur in Term 3 2011. Your participation would be to work in collaboration with the research student during the diagnostic test with the class. Parents will be invited to provide consent for their child to participate in the project. It is envisaged that this project will provide no foreseeable risk for participants.

Your participation would involve the following activities:
1. Provide reflective feedback regarding approaches to the mathematics test;
2. Assist with sending consent forms home with the children;
3. Assist with the diagnostic test of children's patterning understandings;

This study will benefit participants by improving the teaching of mathematics to young students. Additionally, teachers and Indigenous education workers will engage in mathematical activities which are not often presented to younger students and link pedagogical approaches to this type of mathematics.

At any time during the project you are free to withdraw your consent and discontinue participation without giving any reason. Withdrawal will have no adverse consequences with regard to your employment within the system. Confidentiality will be protected during the conduct of the research, by using coding and pseudonyms, and in any report or publication arising from the research data will remain anonymous. This study has been approved by the Human Research Ethics Committee at Australian Catholic University and permission has been granted from all employing authorities to conduct this project in their schools. In the event that you have any complaints or concerns, or if you have any query that the Investigator has not been able to satisfy, you may write to the Chair of the Human Research Ethics Committee care of the nearest branch of the Research Services Unit.

QLD	PO Box 456
Chair, HREC	VIRGINIA QLD 4014
C/o Research Services	Tel: 07 3623 7429
Australian Catholic University	Fax: 07 3623 7328

Any complaint or concern will be treated in confidence and fully investigated. The participant will be informed of the outcome. Any questions regarding this project should be directed to researchers, Professor Elizabeth Warren (telephone 07 3623 7218) or Jodie Miller (07 3623 7405) in the School of Education, McAuley Campus, 1100 Nudgee Road, Banyo, QLD 4014. Results of the research will be provided on request. If you agree to take part in this study please sign the attached agreement to participate.

Yours sincerely,

ELIZABETH WARREN   JODIE MILLER

INFORMATION LETTER TO TEACHER AIDE / INDIGENOUS EDUCATION WORKER

TITLE OF PROJECT: Young Australian Indigenous students' experiences in mathematics: An exploration in generalisation

NAME OF PRINCIPAL RESEARCHER: PROFESSOR ELIZABETH WARREN

NAME OF STUDENT RESEARCHER: JODIE MILLER

We invite you, as a Teacher aide/Indigenous Education Worker to participate in a Masters of Education Research project. The study aims to explore how young Indigenous students engage in mathematical generalising tasks. Data will be gathered from the students by conducting two diagnostic activities, four teaching episodes (12 one hour lessons) and three clinical interviews with the students. These activities will occur from Term 3 2011 till Term 4 2011.Your participation would be to work in collaboration with the research student during the teaching episodes and clinical interviews conducted with the students.

Your participation would involve the following activities:
4. Assist in implementing classroom activities which may be video recorded (12 one hour lessons);
5. Provide reflective feedback regarding approaches to the mathematics activities (voice recorded or video recorded);
6. Assist with sending consent forms home with the children;
7. Assist with pre and post diagnostic activities of children's patterning understandings;
8. Provide cultural insight to the students' understandings within the teaching episodes and interviews (voice recorded or video recorded).

Parents will be invited to provide consent for their child to participate in the project. Children will be involved in the implementation of the planned 12 teaching episodes (4 lessons per week for 3 weeks), three one on one interviews (20 minutes) with the students to develop further understandings of how they interact with the tasks provided. During these sessions video-recording will be utilised and your permission is sought to be part of this. It is envisaged that this project will provide no foreseeable risk for participants.

This study will benefit participants by improving the teaching of mathematics to young Indigenous students. Students will be able to find connections between western mathematics models and Indigenous contexts. Additionally, teachers and Indigenous education workers will engage in mathematical activities which are not often presented to younger students and link pedagogical approaches to this type of mathematics.

At any time during the project you are free to withdraw your consent and discontinue participation without giving any reason. Withdrawal will have no adverse consequences with regard to your employment within the system. Confidentiality will be protected during the conduct of the research, by using coding and pseudonyms, and in any report or publication arising from the research data will remain anonymous.

Any questions regarding this project should be directed to researchers, Professor Elizabeth Warren (telephone 07 3623 7218) or Jodie Miller (telephone 07 3623 7405) in the School of Education, McAuley Campus, 1100 Nudgee Road, Banyo, QLD 4014. Results of the research will be provided on request.

This study has been approved by the Human Research Ethics Committee at Australian Catholic University.

In the event that you have any complaint or concern about the way you have been treated during the study, or if you have any query that the Investigator has not been able to satisfy, you may write to the Chair of the Human Research Ethics Committee care of the nearest branch of the Research Services Unit.

> QLD
> Chair, HREC
> C/o Research Services
> Australian Catholic University
> PO Box 456
> VIRGINIA QLD 4014
> Tel: 07 3623 7429
> Fax: 07 3623 7328

Any complaint or concern will be treated in confidence and fully investigated. The participant will be informed of the outcome.

If you agree to participate in this project, could you please sign both attached copies of the Consent Form, retain one copy for your records and return the other copy to Jodie Miller at ACU, Office of the School of Education, PO Box 456 Virginia Q 4014. If you agree to take part in this study please sign the attached agreement to participate.

Yours sincerely,

Elizabeth Warren   Jodie Miller

INFORMATION LETTER TO PARENT/GUARDIAN

Your child is invited to take part in the following project.

TITLE OF PROJECT: Young Australian Indigenous students' experiences in mathematics: An exploration in generalisation

NAME OF PRINCIPAL RESEARCHER: PROFESSOR ELIZABETH WARREN

NAME OF STUDENT RESEARCHER: JODIE MILLER

Dear Parent/Guardian,

In 2011 your child's teacher and Indigenous Education worker will be part of a mathematics study in your child's class and we are asking your child to participate as well.

We are asking you to allow your child to participate in:

- Two mathematics tests (approximately 15 minutes each) and mathematics lessons (12 one hour lessons over 3 weeks) that focus on patterning activities;
- A one on one interview (10 -20 minutes) involving sharing their understanding of mathematical concepts.

During the project some of the classroom lessons will be video recorded or be recorded using digital photography. This will help the researchers, teachers and Indigenous education workers to look at the children's mathematical learning. All video and digital photography taken will only be used for this research activity and they will not be used anywhere else without your permission.

This study will benefit participants by improving the teaching of mathematics to young Indigenous students. Students will be able to find connections between western mathematics models and Indigenous culture. Additionally, teachers and Indigenous education workers will learn about mathematical activities which are not often presented to younger students and link teaching approaches to this type of mathematics.

The principal, class teacher, and Indigenous education worker at your child's school have agreed to participate in this research.

At any time during the project you are free to withdraw your child's consent, without reason, from participating in the screening tasks and interviews. All students will still participate in the mathematics lessons however data will not be collected from your child if you choose for them not to participate in the data collection. Confidentiality will be protected during the conduct of the research, by using coding and pseudonyms, and in any report or publication arising from the research data will remain anonymous.

Any questions regarding this project, or if you wish to withdraw your child at anytime, please contact the researcher, Professor Elizabeth Warren (telephone 07 3623 7218) or Jodie Miller (telephone 07 3623 7405) in the School of Education, McAuley Campus, 1100 Nudgee Road, Banyo, QLD 4014. Results of the research will be provided on request in writing to this address.

This study has been approved by the Human Research Ethics Committee at Australian Catholic University.

In the event that you have any complaint or concern about the way you have been treated during the study, or if you have any query that the Investigator has not been able to satisfy, you may write to the Chair of the Human Research Ethics Committee care of the nearest branch of the Research Services Unit.

> QLD
> Chair, HREC
> C/o Research Services
> Australian Catholic University
> PO Box 456
> VIRGINIA QLD 4014
> Tel: 07 3623 7429
> Fax: 07 3623 7328

Any complaint or concern will be treated in confidence and fully investigated. The participant will be informed of the outcome. If you agree to participate in this project, could you please sign both copies of the attached Consent Form, retain one copy for your records and return the other copy to your child's classroom teacher.

Yours sincerely
Elizabeth Warren   Jodie Miller

Patterning activities

10. Draw your own growing pattern. Explain how your pattern is growing

9. If there were 376 possums how would you work out how many eyes there would be?

Patterning activities

5. Continue this growing pattern

6. How many possum eyes will there be if four possum tails are hooked on the tree?

7. How many eyes will there be if there were 10 possums hanging on the tree? Draw what it would look like.

8. How did you work this out?

1. Draw your own pattern

2. Use stickers to create your pattern

3. Copy this growing pattern. In the space below.

House 1   House 2   House 3   House 4

Draw here.

4. How is the pattern growing?

Appendix E
Teaching Experiment 2 Pretest 2

Teaching Experiment 2

2011 Growing pattern

NAME: _____

1. Continue this pattern

(a)

2. Using these two shapes create your own growing pattern △ ☺

3. Look at the following pattern and answer the questions below.

Step 1   Step 2   Step 3   Step 4
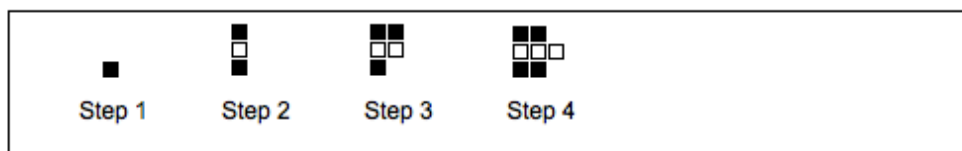
(i) How many □ 's in Step 5_____ Step 6_____ Step 10_____

(ii) How many ■'s in Step 5 _____ Step 6 _____ Step 10 _____

(iii) Which Step has 13 □ and 14 ■ ?   Step _____

4. Fill in the missing steps

1st   2nd   3rd   4th   5th   10th

Write the position rule for this pattern:

_____

_____

Appendix F
Example of In-depth Data Analysis

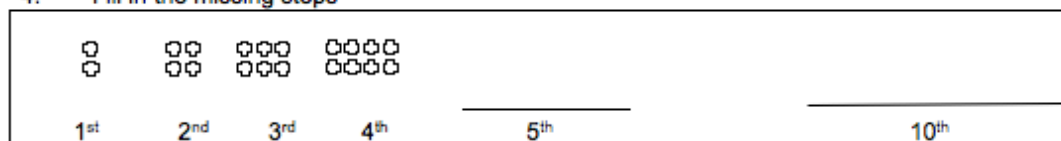| Line		Mathematics	Semiotics	Culture IEO 1 & 2 comments
131	R1	What do you think the fourth one will look like?		He is thinking – looking away and up in his head
132	S6	There are four blue tiles five of them and three green ones	[S6 gestures down the lines of five] [S6 point to the desk where the three yellow tiles would be placed on the pattern]	He knows there are groups of 5 but can not use the words multiply. He is gesturing to show you (IEO1)
133	R1	What about number ten what would it be?		
134	S6	Ten blue tiles and 3 around it	[S6 gestures down the lines of five]	Same as before – gesturing to tell you (IEO1)
135	R1	What about 100?		
136	S6	100 blue tiles and 3 around it	[S6 gestures down the lines of five].	Stills knows what he means - t isn't using the maths words (IEO1/IEO2) Too Deadly.

Appendix G
Example of Cross-Case Analysis

	Teachable actions	Researcher participation (questions/directions)	Student participation (responses/engagement)	Observation
Student 1	Researcher constructs pattern		Student observed	
	Focus on the structure of the pattern	If I have two crocodile tails how many feet do I have? Repeated for 5 tails	8 10	S1 answered the first question correctly and then provided an incorrect response for the second question
	Researcher uses gesture	Researcher gestures to feet and counts how many each crocodile has Researcher gestures to feet and counts how many each crocodile has	Student observes	Student gestured to the number.
	Student and researcher attend to the structure	one crocodile tail four feet two crocodile tails eight feet	Student continues 3 tails – 12 feet 4 tails – 16 feet Explains that they mentally moved one crocodile on and counted the extra four feet to determine the answer	Student see additive process (plus 4)
	Transfer knowledge from previous lesson	Discussed how the kangaroo pattern was not adding two each time but was doubling	Student responds that this pattern is growing by fours	
	Trial pattern rule		Student trials additive rules Tail plus three Tail plus four	S1 did not transfer the language of multiplication from the kangaroo pattern to the crocodile pattern
	Explicit	Discussed	Student continues	

Teachable actions	Researcher participation (questions/directions)	Student participation (responses/engagement)	Observation
language and gesture provided by researcher	multiplicative relationship One times four is four two times four is eight	trialling multiplicative language Three times four is twelve Four times four is sixteen	
Generalise pattern for any number	What is the rule of any number of crocodiles?	Times four	

	Teachable actions	Researcher participation (questions/directions)	Student participation (responses/engagement)	Observation
	Researcher constructs pattern	Researcher places term one on desk		
	Student continues pattern	Researcher guides Student to construct term two, three and four of the pattern – use of gesture indicating the placement of each term	Student places crocodiles in terms two, three, and four.	Student successfully continues pattern
	Student asked to identify pattern structure – focus on one variable	Researcher asks student to predict for four crocodile tails how many feet	Student answers sixteen	Student looks at four crocodiles and counts up to sixteen – appears to be counting in ones
Student 6	Student asked to identify term from pattern	Researcher asks student if there are 12 feet how many tails	Student response 'ten tails'	Student does not respond correctly
	Test prediction to assist with justification		Student places ten crocodiles on the desk	Student identifies that there are forty feet
	Revisit - student to identify term from pattern		Student correctly identifies three tails	Counts the feet to 12 and then holds his thumb and little finger out over the group and counts to three
	Describe pattern structure at term 100	Researcher asks student to describe how he would work out how many feet there were if there were 100 crocodile tails	Student states count in fours	

	Teachable actions	Researcher participation (questions/directions)	Student participation (responses/engagement)	Observation
	Transfer knowledge from previous lesson	Discussion with student about rule from the kangaroo pattern focusing on language	Student does not recall the rule (doubling) rather explains it as adding on two	Student having trouble applying the multiplicative structure
	Researcher constructs pattern		Student observes	
	Connect to last lesson	Do you remember what we were doing yesterday?	Student discusses growing patterns with Kangaroo pattern	
	Transfer from last lesson		Student identifies the rule as timesing the tail by 2	
	Predicting pattern		Student states that he knows the total there – 30	Student is multiplying by 5 instead of 4. He is including the tail Seeking to please.
Student 2	Deconstruct pattern attending to both signs	Researcher deconstructs the pattern gesturing to the tail and the feet	Student observing and gesturing	
	Student revisits pattern		Student can easily identify if there are 2 tails you multiply by 4	
	Student predicts rule		Student predicts rule – multiply by 4	
	Quasi-generalise	Researcher asks student to quasi-generalise 100, 1 billion.	Student can quasi-generalise for 100	
	General rule	Researcher asks student to identify growing pattern rule.	Student identifies as timesing by 4	

Key: Yellow highlight – gesture
Blue highlight – culture
Purple highlight – mathematics/generalising
Red highlight – learning blocks