Mighty knowledge

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Abstract

We often claim to know what might be—or probably is—the case. Modal knowledge along these lines creates a puzzle for information-sensitive semantics for epistemic modals. This paper develops a solution. We start with the idea that knowledge requires safe belief: a belief amounts to knowledge only if it could not easily have been held falsely. We then develop an interpretation of the modal operator in safety (*could have*) that allows it to non-trivially embed information-sensitive contents. The resulting theory avoids various paradoxes that arise from other accounts of modal knowledge. It also delivers plausible predictions about modal Gettier cases.

1 Modal Knowledge

There are many things we don't know. Epistemic modals provide a handy way of communicating our uncertainty. Instead of making a categorical claim about when the train will arrive, we can say things like:

(1) The train $\left\{ \begin{array}{l} \text{might} \\ \text{will probably} \end{array} \right\}$ arrive by 5 o'clock.

We do not just make claims about what *might be* or *probably is* the case. We also claim to *know* such things. For example:

(2) I know it's probably not the time to ask...[1]

(3) We do/don't know that John might have cancer.[2]

(4) I hereby let you know that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.[3]

[1] https://twitter.com/NBTrueFans/status/888181319724916737, Nickelback Fans on Twitter.

[2] DeRose 1991.

[3] Comment in discussion forum on Diptera.info website, quoted in Moss 2013.

How should we understand such modal knowledge?

According to a traditional 'propositionalist' theory of modals, there is no great mystery here. On this view, the content of (1) is just an ordinary proposition—say, the set of worlds in which the contextually determined information leaves open (or probabilifies) the possibility that the train arrives at 5 (Kratzer 1981, 2012; Dowell 2011). And this modal proposition can itself be the object of a propositional attitude like knowledge. So just plug in your preferred account of knowledge and you'll get a theory of modal knowledge.

However, in recent years an important alternative to propositionalism has emerged. According to an information-sensitive semantics for epistemic modals, the semantic content of (1) cannot be modeled with a set of worlds. Instead, it can only be modeled with a formal object that also represents a body of information, for example, a set of world, information state pairs (Yalcin 2007), or a set of probability measures (Moss 2015), or a function from information states to information states (Veltman 1996; Gillies 2006).

A variety of considerations have been used to motivate this information-sensitive alternative. First up: disagreement data. A hearer can disagree with (1) provided that *they* are convinced that the train will not arrive by 5. The hearer need not thereby disagree with the claim that the speaker's information—or the contextually determined information more generally—is compatible with a 5 o'clock arrival. This motivates the thought that the function of asserting (1) is not to assert some proposition about one's own information, but rather to get one's audience to assign a sufficiently high credence to the proposition that the train will arrive by 5.[4]

Second, propositionalism has trouble explaining why a speaker will be disposed to retract (1) upon learning that the train is delayed. After all, it remains true that the speaker's information (at the time of utterance) left open the possibility of a 5 o'clock arrival. So why the need to retract? Information-sensitive theorists have a story: after learning about the train's delay, the speaker no longer intends their audience to assign a high credence to a 5 o'clock arrival. Hence they no longer stand by the update associated with (1).[5]

Finally, propositionalists face an obstacle explaining why 'epistemic contradictions' of the form $\ulcorner \mathsf{A}$ *and might* $\neg\mathsf{A}\urcorner$ seem incoherent even in embedded contexts, unlike their Moore-paradoxical counterparts:

[4]For versions of this argument, see Price 1983; Yalcin 2011; Willer 2013; Moss 2018.

[5]See Knobe and Yalcin 2014 for empirical evidence about retraction data and an explanation along these lines.

(5) ?? Suppose that the train will arrive by 5 but it might not.

(6) Suppose that the train will arrive by 5 but I [/we] don't know it will.

On standard information-sensitive accounts, epistemic contradictions are semantically inconsistent, thereby explaining this contrast.[6]

Of course, these arguments for an information-sensitive approach are controversial; a number of philosophers have argued that propositionalists have the resources to ward off these challenges.[7] For the purposes of this paper, we won't try to settle this fight; we'll simply assume that an information-sensitive approach is worth taking seriously. Our primary question is whether proponents of an information-sensitive approach can make sense of modal knowledge.

On the face of it, modal knowledge looks puzzling for information-sensitive semanticists. After all, knowledge is thought to be a propositional attitude. How could we ever know what *might be* or *probably is* the case, if modal language is non-propositional?

2 The State of Play and the Road Ahead

By now, a number of authors in the information-sensitive tradition have sought to answer this question. But, we will argue, none of the answers currently on offer is fully satisfactory. This paper aims to improve on extant approaches. We develop an information-sensitive account of modal knowledge which is fully compositional and which delivers plausible predictions about a wide range of cases.

Within the current literature, one approach is to adopt what we call the 'transparency theory' of modal knowledge. On this view, one knows the train might arrive by 5 just in case it's compatible with what one knows that the train arrives by 5. The transparency theory is faithful to many of the commitments that motivate information-sensitive theorists. It also follows naturally from combining a Hintikka 1962 semantics for knowledge with an off-the-shelf expressivist or dynamic semantics for epistemic modals. Despite these virtues, we'll argue that the transparency theory faces insuperable difficulties. It is incompatible with certain highly plausible constraints on knowledge. It is also subject to counterexample—for example, in the form of modal Gettier cases.

[6] This contrast forms the backbone of Yalcin's 2007 argument for an information-sensitive approach. For more on this argument, see Willer 2013; Yalcin 2015.

[7] See, a.o., Dowell 2011; Dorr and Hawthorne 2013; Silk 2017; Ninan 2018; Stojnić 2019.

An alternative approach to modal knowledge, due to Moss 2013, 2018, fares better. Moss argues that we can combine an information-sensitive semantics for modals with standard conditions on knowledge from the traditional epistemology literature, such as safety and sensitivity.[8] The key idea is to interpret these conditions in a way that can be applied to information-sensitive contents. To illustrate, a standard safety condition on knowledge says:

(7) **Safety.** A belief amounts to knowledge only if it could not easily have been false.

According to Moss, there is no need to assume that the beliefs in question have propositional contents. As a result, information-sensitive semanticists can say things like:

(8) You know that the train might arrive by 5 only if your belief that the train might arrive could not easily have been false.

We think this is a promising approach; indeed, this will serve as the springboard for our own account of modal knowledge. However, as currently developed this approach has an important lacuna. As it stands, the theory does not tell us how to make sense of claims like (8). What does it mean to say that your belief couldn't easily have been false, when the belief in question is modal?

From a propositionalist perspective, it is clear enough how to make sense of this: (8) says that you know that the train might arrive by 5 only if, for every nearby world w where you believe the proposition *The train might arrive by 5*, this proposition is true at w. And this in turn obtains just in case, for every such w, there's some epistemically accessible world w' where the train arrives by 5. But the information-sensitive semanticist cannot say this; for them, there is no proposition corresponding to *The train might arrive by 5*. So how should they make sense of a modal operator (*couldn't have easily*) embedding some non-propositional content?

This paper develops a solution on behalf of the information-sensitive semanticist. Our main contribution is to incorporate a notion of *worldly information* into information-sensitive semantics. In our theory, every possible world determines a special body of information relevant to knowing epistemic modal claims. With worldly information in hand, we can give a substantive theoretical interpretation

[8] For the canonical defense of sensitivity, see Nozick 1981. For defenses of safety, see Sosa 1999; Williamson 2000, 2009; Pritchard 2005, 2012; Manley 2007; Beddor and Pavese 2020.

of modal conditions like Safety. On the resulting interpretation, modal conditions require that an agent's information tracks worldly information. More precisely:

(9) **Information-sensitive safety.** A belief amounts to knowledge only if, for every nearby world w where the belief is held, the belief is true relative to the worldly information at w.

And this, in turn, can be used to provide a theoretically satisfying interpretation of modal constructions embedding probabilistic vocabulary, such as (8).

The picture of modal knowledge that results has three theoretical virtues. First, it is reductive. Information-sensitive safety does not rely on epistemic modal vocabulary; rather, it explains modal knowledge in non-modal language. Second, it is compositionally serious. We show how to derive this condition compositionally from the independent meaning of epistemic modals and circumstantial modals. Third, the condition is empirically fruitful: it captures our intuitions about a wide range of cases.

Here's the plan for the paper. §3 criticizes the transparency theory of modal knowledge. §4 lays out the safety approach to modal knowledge and the challenges it faces. §§5-6 develop our positive proposal, and §7 addresses two natural objections. In an appendix, we compare our approach with two prominent alternatives in the literature: the context probabilism in Yalcin 2012c and a relevant alternatives theory suggested in Moss 2018.

3 Transparency

In this section, we consider a transparency theory of modal knowledge. While *prima facie* attractive, this theory has serious costs. These costs motivate an alternative approach, to be developed in the ensuing sections.

3.1 Transparency introduced

To introduce the transparency theory, it is helpful to start with modal belief. What does it take for an ascription like (10) to be true?

(10) Ari believes the train might be late.

For propositionalists, it's natural to say that (10) is true just in case Ari believes the proposition: *It's compatible with information i that the train is late.*

Here the default value of i is presumably Ari's own information, though context may sometimes supply other values.

As Yalcin (2011) notes, this analysis makes modal belief into a 'second-order' state. To hold a modal belief is to have a belief about which possibilities are left open by—or probabilified by—some information. Is this analysis plausible?

Information-sensitive semanticists have raised serious doubts on this score. Consider Fido, who believes he might get a bone (Yalcin 2011). Does this mean that Fido believes that it's compatible with his information—or anyone else's—that he gets a bone? This seems like a stretch. It seems more plausible to analyze Fido's belief as a 'first-order' state. For Fido to believe he might get a bone is just for him to be in a belief state that leaves open the possibility he'll get a bone. He need not be aware that he is in such a state.[9]

This motivates the idea that modal beliefs are 'transparent' in that they reduce to first-order beliefs. To make this more precise, let A represent some descriptive (non-modal) claim, B represent the beliefs of an arbitrary agent, and $\Diamond$ represent epistemic possibility. Then:

(11) **Belief transparency.** $\mathsf{B}\Diamond\mathsf{A} \dashv\models \neg\mathsf{B}\neg\mathsf{A}$

On the face of it, the arguments for Belief Transparency generalize to other propositional attitudes, such as knowledge.[10] Fido could know he might get a bone without knowing whether it's compatible with his information that he receives a bone. This motivates:

(12) **Knowledge transparency.** $\mathsf{K}\Diamond\mathsf{A} \dashv\models \neg\mathsf{K}\neg\mathsf{A}$

On this view, Fido knows that he might get a bone just in case his knowledge does not rule out the possibility that he'll get a bone.

Analogous principles can be formulated for other modals. Thus we might hold that an agent believes probably p just in case their credence in p is sufficiently high. Similarly, we might say that an agent knows probably p just in case the probability of p conditional on their evidence is sufficiently high.

[9] As a referee points out, some might wonder whether animals have modal beliefs in the first place. To motivate this, imagine Fido works with three trainers, who always, sometimes, and never (respectively) reward Fido when he does a trick. Imagine that Fido behaves differently in response to each of these trainers, responding eagerly to the first trainer's commands, less eagerly to the second trainer's, and ignoring the third's altogether. A plausible explanation of these behavioral differences is that Fido believes that the first trainer will give him a bone, but he only believes that the second trainer *might* do so.

[10] For further discussion of Belief Transparency, see Fuhrmann 1989, Gillies 2006, and Goldstein 2019a.

Transparency theses are easy to validate in an information-sensitive setting. On an information-sensitive semantics, epistemic modals quantify over an information state which is not determined by the context of utterance. As we will see, this can be combined with a semantics for attitude verbs in the tradition of Hintikka 1962, according to which an agent believes/knows p iff p holds throughout all of their doxastic/epistemic alternatives. To achieve the desired synthesis, we need only add that the attitude verb shifts the information state to the agent's doxastic/epistemic alternatives.[11] (For discussion of the compositional details, see §5.)

So a transparency theory of modal knowledge has a good deal to recommend it. Alas, trouble is just around the corner...

3.2 Theoretical concerns

We begin by showing that Knowledge Transparency leads to absurd consequences when combined with natural assumptions about knowledge. Here we summarize observations already in the literature by establishing three impossibility results.

Yalcin 2012c and Dorr and Hawthorne 2013 observe that Knowledge Transparency, when combined with Belief Transparency, stands in tension with the requirement that knowledge implies belief:

(13) **KB.** $\mathsf{KA} \models \mathsf{BA}$

We can generalize this concern with an impossibility result. The combination of Knowledge Transparency, Belief Transparency, and KB imply that belief and knowledge are equivalent.

(14) **Collapse.** $\mathsf{KA} \dashv\models \mathsf{BA}$

Observation 1. Knowledge Transparency, Belief Transparency, and KB imply Collapse.

Proof. By Knowledge Transparency, $\neg\mathsf{K}\neg\mathsf{A}$ implies $\mathsf{K}\Diamond\mathsf{A}$, which implies $\mathsf{B}\Diamond\mathsf{A}$ by KB, which implies $\neg\mathsf{B}\neg\mathsf{A}$ by Belief Transparency. Contraposing, we have BA implies KA, which leads to Collapse in the presence of KB. □

The second problem for Knowledge Transparency, discussed in Yalcin 2012c,

[11] See esp. Yalcin 2007. Transparency theses are also validated by combining the semantics for attitude verbs in Heim 1992 with the dynamic semantics for epistemic modals in Veltman 1996. For critical discussion, see Mandelkern 2019b.

Dorr and Hawthorne 2013, and Moss 2018, is independent of Belief Transparency. Start with the platitude that knowledge is factive:

(15) **Factivity.** $\mathsf{KA} \models \mathsf{A}$

Combined with Knowledge Transparency, we immediately reach the conclusion that if something is true, everyone knows it is epistemically possible:

(16) **Modal Omniscience.** $\mathsf{A} \models \mathsf{K}\Diamond\mathsf{A}$

Observation 2. Knowledge Transparency and Factivity imply Modal Omniscience.

Proof. By Factivity, A implies $\neg\mathsf{K}\neg\mathsf{A}$, which implies $\mathsf{K}\Diamond\mathsf{A}$ by Knowledge Transparency. $\square$

At first glance, one might respond to this problem by giving up the factivity of modal knowledge. Perhaps an agent can know it might be raining even when it must not be raining, because modal knowledge is somehow nonfactual. But the proof above only relies on Factivity for descriptive sentences. So giving up the factivity of modal knowledge is not sufficient to avoid the problem.

Finally, Mandelkern 2016 observes that Knowledge Transparency poses a special challenge for the full version of Factivity, which applies to modal claims. For suppose we retain KB. In addition, suppose that our agent's beliefs are consistent:

(17) **Consistency.** $\mathsf{BA} \models \neg\mathsf{B}\neg\mathsf{A}$

These assumptions give rise to the implausible result that anything that any agent believes is possible, no matter how outlandish the agent's beliefs may be.

(18) **Skepticism.** $\mathsf{BA} \models \Diamond\mathsf{A}$

Observation 3. Knowledge Transparency, Factivity, KB, and Consistency imply Skepticism.

Proof. Suppose BA. Then Consistency and KB imply $\neg\mathsf{K}\neg\mathsf{A}$. So Knowledge Transparency implies $\mathsf{K}\Diamond\mathsf{A}$, and so Factivity implies $\Diamond\mathsf{A}$. $\square$

3.3 Empirical concerns

We can also argue against Knowledge Transparency by way of counterexample. A descriptive belief can fail to amount to knowledge for various reasons. The belief could be false; it could be unjustified; it could be Gettiered. The same factors can preclude a modal belief from amounting to knowledge. When they do, we'll get a counterexample to the right-to-left direction of Knowledge Transparency: a case where one does not know $\Diamond \mathsf{A}$ even though A is compatible with what one knows.

First, consider a case where a modal belief fails to amount to knowledge due to falsity:

(19) **Mighty Failure.** The exams have just been graded: sadly, Joey didn't make the cut. Not having seen the results, Joey lives in blissful ignorance: he still thinks he might have passed. We, the examiners, know better.

Here it's natural to reject the knowledge ascription:

(20) Joey knows he might have passed.

After all, given what we (the examiners) know, it's not true that Joey might have passed.[12] This gives us a counterexample to Knowledge Transparency: Joey doesn't know he might have passed, even though passing is consistent with what he knows.

Next, consider a case where a modal belief fails to amount to knowledge due to lack of justification:

(21) **Hypochondria.** Hydie the hypochondriac is in the bloom of health. But, being a hypochondriac, she thinks she might get sick at any moment. Unbeknownst to her, someone with a cold has just quietly sneezed in her vicinity. Consequently, she might indeed get sick at any moment.

Intuitively, Hydie's belief that she might get sick does not amount to knowledge. And the reason for this is that her belief—while true—is unjustified. This furnishes another failure of Knowledge Transparency. After all, it is consistent with what she knows that she'll get sick.

[12] This observation is closely related to the observation that we often reject an utterance of $\Diamond \mathsf{A}$ based on our (the assessors') information. For relevant discussion, see, a.o., Egan et al. 2005; Stephenson 2007; MacFarlane 2014; Khoo 2015; Beddor and Egan 2018.

Finally, modal beliefs can also fail to amount to knowledge due to Gettierization. Here we build on Moss (2013, 2018), who develops several modal Gettier cases. Consider, for example, the following probabilistic analogue of the fake barns case (Goldman 1976):

(22) **Fake Letters.** Alice enters a psychology study with her friend Bert. As part of the study, each participant is given a detailed survey of romantic questions about their friend. After the study is over, each participant is informed of the probability that they find their friend attractive. Several disgruntled lab assistants have started mailing out fake letters, telling nearly every participant that they probably find their friend attractive. Alice happens to receive a letter from a diligent lab assistant. Her letter correctly reports that she probably does find Bert attractive. Alice reads the letter and comes to have high credence that she finds Bert attractive. (Moss 2018: 103).

Alice has a justified belief that she probably finds Bert attractive. Moreover, she probably does find him attractive. But intuitively Alice's belief does not amount to knowledge.

Some readers might be inclined to press back on these counterexamples. In **Fake Letters**, some might well ask: What sort of probability is at issue here? Is it *subjective probability*—i.e., Alice's credence that she finds Bert attractive? But Alice could know that she assigns a high credence to this proposition, despite the meddlesome actions of the lab assistants. (Even if she had received a letter from the disgruntled lab assistant, she could still know what her credence is.) Is it *evidential probability*—i.e., the probability that she finds Bert attractive conditional on her evidence? But Alice could know that her total evidence indicates that she finds Bert attractive, despite the nearby possibility of having received a fake letter. Either way, it seems she retains her probabilistic knowledge. Similar points can be, *mutatis mutandis*, using the other cases.

While this is a natural concern, we think it should be resisted. We share Moss' intuition about **Fake Letters** that—in some sense—Alice doesn't know that she probably finds Bert attractive. We take this intuition to be a data point that any complete natural language semantics for modals should accommodate. True, we cannot accommodate this data point on the hypothesis that the sort of probability at issue is either subjective or evidential probability (at least as traditionally construed). But this just shows that we should reject this hypothesis. And we should seek to develop some alternative understanding of the sort of probability at issue which is better-suited to vindicating our pretheoretic judgments.

Indeed, it is worth noting that these pretheoretic judgments provide a fresh line of objection to propositionalist treatments of epistemic modals. Consider the belief report:

(23) Alice believes she is probably attracted to Bert.

We've seen that propositionalists will analyze (23) as saying that Alice believes the proposition: *It's likely, given information i, that I (Alice) am attracted to Bert.* The most natural candidates for i are Alice's credal state and Alice's current evidential state. But then this leads to either the subjective or evidential interpretation of the probability at issue. Hence this approach is unable to accommodate the intuition that Alice's belief does not amount to knowledge.

4 Safety

4.1 In Favor of Safety

The failures of Knowledge Transparency provide support for an alternative approach to modal knowledge, pioneered by Moss 2013, 2018. As noted in §2, Moss proposes that we can shed light on modal knowledge by adopting some of the standard conditions on knowledge familiar from the epistemology literature.

For the sake of concreteness, we focus on a particular modal condition on knowledge: Safety. Our choice is informed by both familiarity and plausibility: Safety is currently one of the most prominent and promising attempts to analyze knowledge in the current literature. Of course, this doesn't mean that Safety is uncontroversial—far from it.[13] But the basic strategy we'll be developing here does not stand or fall with this particular condition. As we'll show in §7, our strategy generalizes to encompass other modal conditions on knowledge.

The key idea behind Safety is that a belief amounts to knowledge only if it couldn't easily have been held falsely (§2). This in turn offers a promising explanation of why Gettiered beliefs do not amount to knowledge. Take the stopped clock case (Russell 1948):

[13] For apparent counterexamples to Safety, see Neta and Rohrbaugh 2004; Comesaña 2005; Kelp 2009; Bogardus 2014. For arguments that the apparent counterexamples are not as damaging as they initially appear, see Dutant 2016; Beddor and Pavese 2020.

(24) **Stopped Clock.** Russ looks at a clock, which reads "3:00". He consequently believes that it is 3:00. Turns out the clock is broken. By a fortuitous coincidence, he happened to glance at the clock at exactly 3.

Safety explains why Russ' belief does not amount to knowledge. He could easily have looked at the clock a minute earlier or a minute later. Had he done so, he would have held the same belief, but it would have been false.

Safety likewise captures intuitions about lotteries:

(25) **Lottery.** Lottie has a ticket in a fair lottery containing a thousand tickets. The winning ticket has not yet bet drawn. Based on the long odds, Lottie believes her ticket will lose.

Safety explains why Lottie does not know that her ticket will lose. Even if she does lose, there is a nearby world where she holds the same belief on the same basis but her ticket wins (Williamson 2000; Pritchard 2005, 2012).

Suppose we accept Safety, at least as a working hypothesis. Moss observes that Safety extends smoothly to capture intuitions about modal Gettier cases. Take **Fake Letters**. As Moss notes, a natural judgment about the case is:

(26) Alice could easily have believed that she probably found Bert attractive, even though she hadn't probably found him attractive.

According to Safety, this is why Alice's belief doesn't qualify as knowledge.

This diagnosis extends to our other examples in §3.2. Take **Hypochondria**. Here it's natural to judge:

(27) It could easily have happened that Hydie believed that she might get sick at any moment, even though it wasn't the case that she might get sick at any moment.

According to Safety, this is why Hydie doesn't know she might get sick at any moment.

4.2 The Challenge for a Safety Analysis

So far, things are looking good for a safety-based account of modal knowledge. However, as noted in §2, a major hurdle remains. The hurdle is this: (26) and (27) are *object-language* claims. That is, they are intuitive judgments that we are inclined to make about these cases. We have not yet given a semantic analysis of these claims. What does it take for (26) and (27) to hold?

To feel the force of this concern, note that (26) and (27) both contain the circumstantial modal *could easily have*, which we will represent with '$\blacklozenge$' hereafter. On a standard analysis, circumstantial modals quantify over worlds: $\blacklozenge \mathsf{A}$ is true at a world w iff there is some nearby world v where A is true.[14] This assumes that A can be assessed for truth or falsity relative to worlds. Yet according to an information-sensitive semantics for epistemic modals, claims about what *might be* or *probably is* the case cannot be assessed for truth or falsity relative to worlds alone, but only relative to states of information.

Indeed, if we combine an off-the-shelf semantics for circumstantial modals with an information-sensitive semantics for epistemic modals, we predict that embedding the latter under the former should have no semantic effect. Specifically, letting '$\blacksquare$' represent circumstantial necessity, we predict:

(28) **Inertia.** $\Diamond \mathsf{A} \mathrel{=}\!\mid\!\models \blacksquare \Diamond \mathsf{A}$

The circumstantial modal will shift the world parameter, to which the embedded epistemic modal is insensitive.

But, as our intuitions about modal Gettier cases reveal, **Inertia** is false: $\Diamond \mathsf{A}$ can be true without $\blacksquare \Diamond \mathsf{A}$ being true. The challenge, then, is to make sense of such object-language modal constructions in a way that vindicates our intuitions (e.g., (26)-(27)), while still preserving the insights of the information-sensitive approach.

Here it is worth pausing to note that this challenge arises for *any* information-sensitive semantics, independent of considerations of modal knowledge. After all, we can and do embed epistemic modals under circumstantial modals, as (26)-(27) show. While such sentences are often clunky, they still seem coherent.[15] Information-sensitive semanticists owe us an account of such embeddings, regardless of whether that account is ultimately used in service of a theory of

[14] Some terminology: following a well-established tradition in linguistics, we will use 'circumstantial modal' to refer to a modal that quantifies over worlds where circumstances are sufficiently similar to the world of evaluation (cf. Kratzer 1981). Circumstantial modals can thus be thought of as restricted metaphysical modals. Whereas metaphysical modals quantify over all metaphysically possible worlds, circumstantial modals only quantify over the nearest metaphysically possible worlds.

[15] Here it is also relevant to consider epistemic modals in counterfactuals, e.g., *If she had won the primary, she might have won the general election.* As we'll see in §7, the theory we develop also sheds light on these counterfactual embeddings.

modal knowledge.[16]

At this point, some might be inclined to say, 'So much the worse for the information-sensitive approach! If it does not play nicely with the standard semantics for circumstantial modals, all the more reason to reject it.'

While this is a natural reaction, we think it is too quick. Here it is worth comparing the embedding problem facing the information-sensitive theorist with the Frege-Geach problem for moral expressivists.[17] According to moral expressivists, the meaning of a moral sentence cannot be modeled with a proposition. This gives rise to the challenge of explaining how moral expressions embed under various operators, including world-shifting operators such as modals and conditionals. In response to this challenge, the past few decades have seen a flourishing research program of increasingly sophisticated expressivist analyses of complex constructions. Now, it is controversial whether any of these expressivist analyses achieves its aims. But it seems to us that the only way to answer this question is to try to identify the most promising expressivist solution to the Frege-Geach problem and assess it on its empirical merits. Similarly, we think that the only way to investigate whether the analogous embedding problem for information-sensitive theorists can be overcome is to try to identify the most promising information-sensitive analysis of constructions like (26)-(27) and assess whether it accommodates the full range of data.

In case some readers remain unpersuaded of the merits of this project, it is worth pointing out that rejecting an information-sensitive framework does not, by itself, solve the problem. It only does so if propositionalists can account for the relevant embeddings. At first blush, it may seem like they are in a good position to do so: since propositionalists allow that epistemic modals can be evaluated for truth or falsity relative to worlds, they avoid **Inertia**. However, there remains a question as to whether they provide a *plausible* account of the relevant embeddings. And here it seems to us that the answer is 'No.' Take **Fake Letters**. Since propositionalists will take Alice's probabilistic belief to be about her own information (either her credences or her evidence), it seems they will predict that (26) is false: even if she had received a letter from the disgruntled lab assistant, her probabilistic belief would still be true. (After all, she still would have had a high credence that she was attracted to Bert, and this would

[16] Moss 2015, 2018 develops an information-sensitive semantics for how epistemic modals embed under other epistemic modals, and also in indicative conditionals. However, she does not provide an explicit semantics for their interactions with circumstantial modals or counterfactuals.

[17] Thanks to a referee for raising both this objection and this analogy.

have been supported by her evidence.) On reflection, this result is unsurprising, since we already noted that propositionalists have trouble accounting for the verdict that Alice does not *know* that she is probably attracted to Bert (§3.3).

The rest of this paper is devoted to solving the embedding problem on behalf of information-sensitive theorists. To do so, we will help ourselves to a notion of *worldly information*. This is some kind of information that is determined by a possible world. This information could either be objective chance or some species of evidential probability, an issue which we will discuss at length later. This notion of worldly probability allows us to give a theoretically illuminating understanding of a safety condition on modal knowledge:

(29) **Information-sensitive Safety (Unpacked).** S knows probably [/might] p only if at any nearby world where S has a sufficiently high credence that p, the worldly probability of p is sufficiently high.

We defend this condition in two ways. First, we show that this condition can be derived compositionally from a natural semantics for the circumstantial modal occurring in Safety. Second, we show that this condition is philosophically fruitful, explaining our intuitions about cases where modal beliefs fail to qualify as knowledge.

5 Semantics

To make sense of modal knowledge, we help ourselves to an information-sensitive semantics for modal expressions.[18] On this semantics, the meaning of any sentence is not a set of worlds, but rather a set of pairs of worlds and information states. While ordinary claims are sensitive to the world component of meaning, modal claims are sensitive to the information state. We start by reviewing standard assumptions within this setting about the meaning of epistemic modals and sentential connectives. With this background established, we turn to our own semantics for knowledge ascriptions.

[18]There are several different kinds of information-sensitive frameworks. For representative work within update semantics, see Veltman 1996, Groenendijk et al. 1996, Beaver 2001, Gillies 2004, and Willer 2013. For other information-sensitive accounts, see Yalcin 2007, Swanson 2011, Klinedinst and Rothschild 2012, Swanson 2016, Moss 2015, Yalcin 2015, Ninan 2018, and Mandelkern 2019a. For useful overviews of the literature, see von Fintel and Gillies 2007 and Willer 2015.

5.1 Background

We interpret a propositional language enriched with epistemic modals: a possibility modal $\Diamond$, a necessity modal $\Box$, and a probabilistic modal $\triangle$. In addition, we add the circumstantial modals $\blacklozenge$, $\blacksquare$, and $\blacktriangle$. Finally, we have two attitude verbs B and K, which for simplicity we do not relativize to different individuals.

Definition 1. $\mathsf{L} ::= \mathsf{p} \mid \neg\mathsf{L} \mid \mathsf{L} \wedge \mathsf{L} \mid \mathsf{L} \vee \mathsf{L} \mid \Diamond\mathsf{L} \mid \Box\mathsf{L} \mid \triangle\mathsf{L} \mid\mid \blacklozenge\mathsf{L} \mid \blacksquare\mathsf{L} \mid \blacktriangle\mathsf{L} \mid \mathsf{BL} \mid \mathsf{KL}$

Since we are interpreting probabilistic modals in addition to quantificational modals, we follow Yalcin 2012a in letting an information state i be a pair of a set of worlds and a probability function over that set. Then an interpretation function assigns each sentence a set of world-state pairs as its meaning. A is supported by i, or true throughout it, just in case A is true at $\langle w, i\rangle$ for every world in i. Finally, we assume that entailment is preservation of support.

Definition 2.

1. W is a set of possible worlds which assign a truth value $w(\mathsf{p})$ to each atomic sentence p.
2. An information state i is a pair $\langle s, Pr\rangle$ where s is a set of worlds and Pr assigns every subset of s a value in $[0, 1]$ as usual, with $Pr(s) = 1$. s_i and Pr_i abbreviate the first and second component of i.
3. An interpretation function $[\![\cdot]\!]$ assigns a set of pairs of worlds and information states to every sentence in L.
4. i supports A ($[\![\mathsf{A}]\!]^i = 1$) iff $\forall w \in s_i : [\![\mathsf{A}]\!]^{w,i} = 1$.
5. $\mathsf{A}_1, \ldots, \mathsf{A}_n$ informationally entails B ($\mathsf{A}_1, \ldots, \mathsf{A}_n \models \mathsf{B}$) iff for every i, if $[\![\mathsf{A}_1]\!]^i = 1$, ..., and $[\![\mathsf{A}_n]\!]^i = 1$, then $[\![\mathsf{B}]\!]^i = 1$.

We now define our interpretation function. Atomic sentences like *it's raining* are not sensitive to the information state: they are true at a $\langle w, i\rangle$ pair if and only if they are true at w. A negation $\neg\mathsf{A}$ is true at $\langle w, i\rangle$ iff A is false there. A conjunction $\mathsf{A} \wedge \mathsf{B}$ is true at $\langle w, i\rangle$ iff each conjunct is true there. A disjunction $\mathsf{A} \vee \mathsf{B}$ is true at $\langle w, i\rangle$ iff either disjunct is true there. Finally, epistemic modals are sensitive to the information state parameter. Possibility and necessity modals existentially and universally quantify over worlds in an information state. The probabilistic modal $\triangle\mathsf{A}$ requires that the information state assigns a high enough probability to the worlds in that information state where A is true.

Definition 3.

1. $[\![\mathsf{p}]\!]^{w,i} = 1$ iff $w(\mathsf{p}) = 1$

2. $[\![\neg\mathsf{A}]\!]^{w,i} = 1$ iff $[\![\mathsf{A}]\!]^{w,i} = 0$

3. $[\![\mathsf{A} \wedge \mathsf{B}]\!]^{w,i} = 1$ iff $[\![\mathsf{A}]\!]^{w,i} = 1$ and $[\![\mathsf{B}]\!]^{w,i} = 1$

4. $[\![\Diamond\mathsf{A}]\!]^{w,i} = 1$ iff $\exists v \in s_i : [\![\mathsf{A}]\!]^{v,i} = 1$

5. $[\![\Box\mathsf{A}]\!]^{w,i} = 1$ iff $[\![\mathsf{A}]\!]^{i} = 1$

6. $[\![\triangle\mathsf{A}]\!]^{w,i} = 1$ iff $Pr_i(\{v \mid [\![\mathsf{A}]\!]^{v,i} = 1\}) > .5$

Having laid the groundwork, let us turn to the semantics for knowledge attributions.

5.2 Knowledge

We will be working with a simple analysis of knowledge, on which it is just true safe belief.[19] This analysis conjoins three conditions:

(30) **Knowledge as true safe belief.** KA iff:

a. A (**Truth Condition**)
b. BA (**Belief Condition**)
c. $\neg\blacklozenge(\mathsf{BA} \wedge \neg\mathsf{A})$ (**Safety Condition**)

Modal knowledge is just a special case of this analysis. So we can analyze knowing what might be as follows:

(31) **Modal knowledge as true safe belief.** $\mathsf{K}\Diamond\mathsf{A}$ iff:

a. $\Diamond\mathsf{A}$
b. $\mathsf{B}\Diamond\mathsf{A}$
c. $\neg\blacklozenge(\mathsf{B}\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{A})$

(And similarly for knowledge of what probably is—or must be—the case.)

To analyze modal knowledge, it suffices to show how to analyze each of these conditions in an information-sensitive setting. The truth condition falls out immediately from the semantics in §5.1. Let us next turn to belief.

[19] As noted in §4, we use this working hypothesis for the sake of illustration. In §7, we show that the approach developed here can be adapted to accommodate a variety of conditions on knowledge.

5.2.1 Belief

For present purposes, we adopt a standard integration of a Hintikka semantics for belief attributions with an information-sensitive semantics for modals (Yalcin 2007, 2012a). First, we introduce the notion of an agent's information state at a world, defined as follows:

Definition 4. For any world w, $Bel^w = \langle B^w, Cr^w \rangle$ is the arbitrary agent's information state at w, where:

1. Cr^w is their credence function at w
2. B^w is their doxastic alternatives at w—that is, the set of worlds consistent with what she believes at w.

Next, we say that for an agent to believe ϕ is for their information state to support ϕ.[20]

Definition 5. $[\![\mathsf{BA}]\!]^{w,i} = 1$ iff $[\![\mathsf{A}]\!]^{Bel^w} = 1$

Combined with our information-sensitive semantics for modals, this validates Belief Transparency. And so a belief report like (10) (*Ari believes the train might be late*) is true just in case there is at least one world consistent with what Ari believes where the train is late. Similarly, Ari believes the train is probably late just in case she assigns a sufficiently high credence to the proposition that the

[20] This account was first proposed by Hans Kamp, and is also defended in Heim 1992; Zeevat 1992; Yalcin 2012b,c; Willer 2013. While we operate with this semantics for the sake of simplicity and familiarity, there are other ways of combining an information-sensitive semantics for modals with a semantics for belief reports that would serve our purposes equally well. See, for example, the proposal in Beddor and Goldstein 2018, which integrates an information-sensitive semantics for epistemic modals with a Lockean theory of belief.

train is late.[21,22]

5.2.2 Safety

Now for our main contribution: analyzing the safety condition. To do this, we develop a semantics for the circumstantial modal *could have*. We propose that circumstantial modals are similar to the belief operator in having two different aspects to their meaning. First, the circumstantial modals quantify over nearby worlds. Second, these modals shift the information state parameter.

Here, we introduce our crucial assumption: every world determines an information state of its own, the 'worldly information'.

Definition 6. For any world w, $i^w = \langle s^w, Pr^w \rangle$ is the worldly information at w, with Pr^w the worldly probability at w and s^w the set of worlds assigned some probability at w.

In the next section, we explore worldly information. To preview, we consider two promising interpretations: objective chance, and a kind of evidential probability. But for now, we simply explore the logical structure of our theory.

With worldly information in hand, we offer a semantics for circumstantial modals. $\blacklozenge\mathsf{A}$ is true at $\langle w, i \rangle$ just in case there is some nearby world v where A is true at the combination of v and the worldly information at v. Similarly, $\blacksquare\mathsf{A}$ requires that A is true at any pair of a nearby world and its worldly information.

[21] Two concerns for Belief Transparency are worth addressing. First, some might worry that Belief Transparency is implausible when A is consistent with an agent's beliefs, but the agent has never even considered the question of whether A. One way to deal with such cases is suggested by Yalcin (2011, 2016), who proposes that believing $\Diamond\mathsf{A}$ additionally requires being sensitive to the question of whether A is true. Of course, this raises tricky issues about how to understand the relevant notion of 'being sensitive to a question'. While a full discussion falls outside the scope of this paper, one promising thought starts with the idea that a question can be modeled as a partition of logical space (e.g., Hamblin 1958). Following Yalcin, we could propose that an agent is sensitive to some question Q just in case Q models logical distinctions that the agent has 'taken note of' (Yalcin 2008: 74; cf. Yalcin 2011, 2016). Whether an agent has taken note of these distinctions might in turn be reflected in their behavioral dispositions.

A second concern comes from the idea that 'belief is weak' (Hawthorne et al. 2016). Ordinary speakers say things like, *I believe the train will arrive by 5, but it might arrive later.* If Belief Transparency is true, such an assertion could never be both true and believed, since the truth of the first conjunct entails that the speaker does not believe the second. In response, one option is to modify our semantics for belief. For example, the semantics in Beddor and Goldstein 2018 (discussed in the previous footnote) invalidates Belief Transparency. But it validates the nearby transparency thesis that one believes $\Diamond\mathsf{A}$ iff one is not certain that A is false. For further discussion of the relation between belief and certainty, see Beddor 2020.

[22] A natural thought at this point (suggested in Yalcin 2007) is that knowledge has a similar quantificational structure. Here we could introduce a knowledge state K^w consisting of the agent's epistemic possibilities paired with their evidential probabilities. Then we could say KA is true at $\langle w, i \rangle$ iff K^w supports A. However, this theory validates Knowledge Transparency, and so has all of the undesirable consequences discussed in §3.

$\blacktriangle\mathsf{A}$ requires that the worldly information at the world of evaluation sufficiently probabilifies the set of worlds where A is true at that world when paired with its worldly information. Finally, we assume for simplicity that the relevant notion of 'nearness' in the above is itself determined by the worldly information at w, the world of evaluation. For example, if the worldly information is simply objective chance, then the nearby worlds to w are just those with positive chance at w.

Definition 7.

1. $[\![\blacklozenge\mathsf{A}]\!]^{w,i} = 1$ iff $\exists v \in s^w : [\![\mathsf{A}]\!]^{v,i^v} = 1$

2. $[\![\blacksquare\mathsf{A}]\!]^{w,i} = 1$ iff $\forall v \in s^w : [\![\mathsf{A}]\!]^{v,i^v} = 1$

3. $[\![\blacktriangle\mathsf{A}]\!]^{w,i} = 1$ iff $Pr^w(\{v \mid [\![\mathsf{A}]\!]^{v,i^v} = 1\}) > .5$

Putting the pieces together, we can finally state our analysis of knowledge precisely:

Definition 8. $[\![\mathsf{KA}]\!]^{w,i} = 1$ iff $[\![\mathsf{A}]\!]^{w,i} = 1$ & $[\![\mathsf{BA}]\!]^{w,i} = 1$ & $[\![\neg\blacklozenge(\mathsf{BA}\wedge\neg\mathsf{A})]\!]^{w,i} = 1$

With this formulation in hand, we now explore some consequences of our view.

5.3 Unpacking the view

5.3.1 Transparency

One consequence of our theory is that while Belief Transparency is valid, Knowledge Transparency is not. In particular, we invalidate the right to left direction of Knowledge Transparency:

Observation 4. $\neg\mathsf{K}\neg\mathsf{A} \not\models \mathsf{K}\Diamond\mathsf{A}$

To see this, note that truth and belief are built into the very definition of knowledge. So we immediately validate both KB ($\mathsf{KA} \models \mathsf{BA}$) and Factivity:

(32) **Factivity**. $\mathsf{KA} \models \mathsf{A}$

Now consider a case where A is true, but someone believes that A is false ($\mathsf{B}\neg\mathsf{A}$), hence does not believe that A might be true ($\neg\mathsf{B}\Diamond\mathsf{A}$). By KB, they do not know A might be true ($\neg\mathsf{K}\Diamond\mathsf{A}$). But since A is true, A is still consistent with what they know ($\neg\mathsf{K}\neg\mathsf{A}$), by Factivity.[23]

[23]Given our semantics for belief, our theory does validate the left to right direction of Knowledge Transparency. Suppose $\mathsf{K}\Diamond\mathsf{A}$. So $\mathsf{B}\Diamond\mathsf{A}$ (by KB), and hence $\neg\mathsf{B}\neg\mathsf{A}$ (by Belief

5.3.2 Flavor Shift

On our theory, circumstantial modals shift the information state parameter. While this shiftiness has important consequences, these consequences only show up when the embedded claim itself contains an epistemic modal. After all, on our semantics the information state parameter does not affect the truth conditions for purely descriptive claims. As a result, our semantics for circumstantial modals is equivalent to a textbook possible worlds semantics when it comes to claims like:

(33) It could easily have rained.

On our semantics, (33) is true relative to some $\langle w, i \rangle$ just in case there is some nearby world v where it rained. This highlights an attractive feature of our semantics for circumstantial modals: it is conservative, in the sense that it only departs from the standard semantics when epistemic modals are also involved.

Having made this observation, let us turn to consider such embeddings. On our information-sensitive semantics, epistemic modals are sensitive to the information state parameter. For this reason, epistemic modals display the unique phenomenon of *flavor shifting*: the flavor of an epistemic modal is sensitive to the operators under which the modal occurs. We've already seen an instance of this sensitivity with modal belief reports. As we saw in §5.2.1, belief reports shift the information state in the index to the believer's information state. Consequently, the embedded epistemic modal $\Diamond\mathsf{A}$ ends up quantifying over the agent's doxastic possibilities. So epistemic modals embedded under B have doxastic flavor.

Something similar happens when epistemic modals embed under circumstantial modals. $\blacklozenge\Diamond\mathsf{A}$ is true at $\langle w, i \rangle$ just in case there is some world v near w where $\Diamond\mathsf{A}$ is true at $\langle v, i^v \rangle$. This means that $\Diamond\mathsf{A}$ has circumstantial rather than epistemic flavor under the scope of $\blacklozenge$, quantifying existentially over the worldly

Transparency). By another application of KB, $\neg\mathsf{K}\neg\mathsf{A}$.

Some might worry that the inference $\mathsf{K}\Diamond\mathsf{A}$ to $\neg\mathsf{K}\neg\mathsf{A}$ incurs skeptical consequences. (Thanks to a referee for raising this concern.) For example, suppose we allow that you can know that you might be a brain in a vat. Applying $\mathsf{K}\Diamond\mathsf{A} \models \neg\mathsf{K}\neg\mathsf{A}$, you don't know that you're not a brain in a vat. One response to this concern is to reject the assumption that you can know you might be a brain in a vat. Indeed, the framework offered here offers a principled way of rejecting this premise. On the most natural ways of fleshing out the notion of worldly information, there is no nearby world where the worldly information is consistent with one's cranial envatment. A potential advantage of this response is that it allows us to explain why conjunctions of the form, *I know that I'm not a brain in a vat, but I also know that I might be one* sound inconsistent. An alternative response would be to modify our semantics for belief along the lines discussed in footnotes 20 and 21. This modification invalidates Belief Transparency and hence blocks the inference from $\mathsf{K}\Diamond\mathsf{A}$ to $\neg\mathsf{K}\neg\mathsf{A}$. This response should appeal to fallibilists who maintain that one can know A even though one recognizes that one has not conclusively ruled out the possibility that A is false.

information at v. This gives us the following equivalences:

(34) a. $[\![\blacklozenge\Diamond \mathsf{A}]\!] = [\![\blacklozenge\blacklozenge \mathsf{A}]\!]$
b. $[\![\blacklozenge\triangle \mathsf{A}]\!] = [\![\blacklozenge\blacktriangle \mathsf{A}]\!]$
c. $[\![\blacklozenge\Box \mathsf{A}]\!] = [\![\blacklozenge\blacksquare \mathsf{A}]\!]$

The phenomenon of flavor shift has further consequences for the logic of circumstantial modals. Suppose that nearness is reflexive, so that any world is close to itself (in that $w \in s^w$). In an ordinary semantics for modals, this would immediately ensure that anything true is possible:

(35) $\mathsf{A} \models \blacklozenge \mathsf{A}$

On our semantics, the reflexivity of nearness does guarantee that (35) holds whenever A is descriptive. But (35) fails when A contains epistemic modals. That is, according to our semantics the reflexivity of nearness does not guarantee the validity of:

(36) $\Diamond \mathsf{A} \models \blacklozenge\Diamond \mathsf{A}$

For suppose that i is compatible with A, but that the worldly information at all worlds near w is incompatible with A. Then $\Diamond \mathsf{A}$ is true at $\langle w, i\rangle$ while false at $\langle v, i^v\rangle$ at all v near w.

Some might balk at the failure of (36). For those who do, a few points are worth considering. First, we should ask ourselves: what are the empirical data motivating (36)? One obvious data point is the infelicity of conjunctions of the form $\Diamond \mathsf{A} \wedge \neg\blacklozenge\Diamond \mathsf{A}$, e.g.:

(37) ?? It might be raining, but it couldn't have easily been that it might be raining.

However, while our theory allows that such conjunctions are semantically consistent, it predicts that they are *unknowable*. By flavor shift, $\mathsf{K}(\Diamond \mathsf{A} \wedge \neg\blacklozenge\Diamond \mathsf{A})$ requires that at any world where the agent believes $\Diamond \mathsf{A} \wedge \neg\blacklozenge\Diamond \mathsf{A}$, the shifted conjunction $\blacklozenge \mathsf{A} \wedge \neg\blacklozenge\Diamond \mathsf{A}$ is true, which in turn requires that $\blacklozenge \mathsf{A} \wedge \neg\blacklozenge\blacklozenge \mathsf{A}$ is true, which is inconsistent with the reflexivity of nearness. Moreover, if we help ourselves to a knowledge norm of assertion, we predict that such assertions are unassertable.[24] On this diagnosis, conjunctions like (37) are similar to

[24] For the classical exposition of the idea that knowledge is the norm of assertion, see

Moore-paradoxical assertions such as:

(38) ?? It's raining, but I don't know it's raining.

In a similar vein, our theory predicts that even more complex embeddings like $\Diamond(\Diamond \mathsf{A} \wedge \neg \blacklozenge \Diamond \mathsf{A})$ are unknowable. After all, the knowledge operator shifts the flavor of the outer $\Diamond$, which in turn shifts the flavor of the $\Diamond$ in its immediate scope. So we submit that our theory explains the linguistic data motivating (36).

For those who find this unpersuasive, another strategy is to introduce further definitions of entailment. For example. we could introduce a notion of worldly truth, where A is true at w just in case $[\![\mathsf{A}]\!]^{w,i^w} = 1$. Then we could define worldly entailment as preservation of worldly truth, and (36) would be valid.[25]

For yet a final response to this worry, we refer the reader to the conclusion (§7), where we show how (36) can be rescued by adopting a more general perspective on our semantics. This more general perspective characterizes both epistemic and circumstantial modals in terms of the notion of an information-sensitive accessibility relation, which relates any world-state pair $\langle w, i \rangle$ to a set of accessible world-state pairs $\langle v, j \rangle$. In that setting, we'll see that (36) can be validated by minimally modifying our present semantics.

5.3.3 How safety is derived

We are now ready to say exactly how the safety condition on knowledge is determined by the meaning of the circumstantial modal. Consider again how our safety condition applies probabilistic modals:

(39) $\neg \blacklozenge (\mathsf{B} \triangle \mathsf{A} \wedge \neg \triangle \mathsf{A})$

We analyze this as saying that the conjunction $\mathsf{B} \triangle \mathsf{A} \wedge \neg \triangle \mathsf{A}$ is false at each nearby world v when paired with its worldly information. However, the belief report $\mathsf{B} \triangle \mathsf{A}$ is not itself information-sensitive, due to flavor shift. So (39) ultimately requires that at any nearby world where the agent's credence in A is high, the worldly probability of A is also high. In short, our semantics allows us to derive our safety condition on probabilistic knowledge:

Williamson 1996, 2000, chp.11.

[25] This worldly notion of entailment sacrifices some of the signature features of more standard definitions: for example, the epistemic contradiction $\mathsf{A} \wedge \Diamond \neg \mathsf{A}$ is now consistent. However, this problem can be avoided by adopting a dynamic meaning for conjunction, where $[\![\mathsf{A} \wedge \mathsf{B}]\!]^{w,i} = 1$ iff $[\![\mathsf{A}]\!]^{w,i} = 1$ and $[\![\mathsf{B}]\!]^{w,i+\mathsf{A}} = 1$, and where $i + \mathsf{A} = \{w \in i \mid [\![\mathsf{A}]\!]^{w,i} = 1\}$. For independent motivations for this dynamic semantics for conjunction, see Heim 1992.

(40) S knows probably p only if at any nearby world where S has a high credence that p, the worldly probability of p is high.

How we understand *this* depends on how we understand worldly information—a topic that we tackle in the next section. If worldly probability is objective chance, this safety condition requires that the agent's credence match the chances at nearby worlds. If worldly probability is evidential probability, the requirement is instead that the agent's credence match the evidential probabilities.

5.4 Taking Stock

Summarizing our discussion, we reach the following theory of probabilistic knowledge:

(41) S knows probably p iff:

a. S has a high credence that p;
b. p is probable; and
c. at any nearby world where S has a high credence that p, the worldly probability of p is high.

Equipped with our semantics, we now turn to consider how to understand the notions of worldly information and worldly probability.

6 Worldly information

This section considers two natural options for developing a theory of worldly information. The first is to understand it as some species of objective chance. The second is to understand it as some form of contextual information.

6.1 Objective chance

A first natural thought is that worldly information is simply objective chance. More precisely:

(42) For any world w, i^w is the pair of the objective chance function Ch^w and the set of worlds s^w assigned some chance.

This gives us a particular interpretation of what it is to know that p might be—or probably is—the case. You know this if and only if at all nearby worlds where you believe might/probably p, the objective chance of p is sufficiently high. Since to believe probably p is just to have a sufficiently high credence in p, this amounts to claiming that at every nearby world where you have a sufficiently high credence in p, your credence matches the objective chance of p.

This interpretation provides a plausible diagnosis of our cases from §3.2. Start with **Hypochondria**. Someone with a cold has just sneezed in Hydie's vicinity. So the objective chance that she gets sick is somewhat high. But there is another nearby world where Hydie believes that she might get sick, but no one has sneezed in her vicinity. At this world, the objective chance of her getting sick is zero, or close enough to zero that it can be appropriately ignored. This explains why we endorse (27) (*It could easily have happened that Hydie believed that she might get sick at any moment, even though it wasn't the case that she might get sick at any moment*). And this in turn explains why Hydie's belief does not qualify as knowledge.

Turn next to **Fake Letters**. Recall (26), our Safety-based diagnosis of this case: Alice doesn't know that she probably finds Bert attractive, because she could easily have held this belief, even though she hadn't found him attractive. What makes this modal judgment true? On the objective chance interpretation, there is a nearby world where there is a lower objective chance that Alice finds Bert attractive. Perhaps this is a world where Alice and Bert have slightly different tastes. Or perhaps this is a world where Alice and Bert have a slightly different rapport, which has squelched any romantic prospects. At this world, Alice still believes she probably likes Bert as more than a friend, since she received the letter from the disgruntled lab assistant. So at this world Alice would have a high credence that she is attracted to Bert, even though the objective chance of attraction is low.

In addition to handling these cases, the objective chance interpretation delivers plausible predictions about **Lottery**. We noted earlier that it's natural to deny that Lottie knows that her ticket will lose. However, the corresponding probabilistic knowledge ascription seems plausible:

(43) Lottie knows her ticket will probably lose.

The objective chance interpretation underwrites this judgment. In cases like **Lottery** three background assumptions seem to be satisfied. First, Lottie defers to the objective chances in accordance with the Principal Principle (Lewis 1980). This means that her credence that her ticket will lose is her expectation of the objective chance that her ticket will lose. Second, we tend to assess the knowledge ascription from an information state that similarly defers to the objective chances. Finally, the objective chances are constant across nearby worlds: at every nearby world, the chance of winning the lottery is the same.

Given these assumptions, the objective chance interpretation predicts that knowledge of the objective chances is sufficient for probabilistic knowledge. First, it is true that Lottie's ticket will probably lose. Second, Lottie has a high credence that her ticket will lose. Finally, at any nearby world where Lottie has this high credence, there is a high chance that her ticket will lose.[26]

Thus the objective chance interpretation delivers the right results in a variety of cases. But it's not all smooth sailing.

An initial challenge comes from modal knowledge of the past, for example:

(44) **Past Coins.** Ari knows a fair coin was flipped yesterday. But she doesn't know the result of the flip.

Intuitively, the following judgment is true in this case:

(45) Ari knows that the coin might have landed heads. She also knows that

[26] On the other hand, the objective chance interpretation does not collapse modal knowledge into knowledge of chance. First, an agent can have modal knowledge without having knowledge of the objective chances. For example, Fido can know that he will probably get a bone, even though he has no views about the objective chance of this happening. But it can even happen to agents who defer to the objective chances. For example, suppose the agent knows that a coin is 50% likely to land heads, but is agnostic about the chances. In particular, imagine that their .5 credence in heads is the result of a .5 credence that the chance of heads of 0, and a .5 credence that the chance of heads is 1. Then the agent does not know that the objective chance of heads is 50%, since they don't even believe this.

A deferential agent can also know the objective chances while lacking modal knowledge. Suppose that the agent knows that the chance of p is 50%. This implies that, at any nearby world where the agent believes that the worldly probability of p is 50%, they are correct. But there could still be other nearby worlds where the chance is different. At those worlds, the agent may be uncertain of the chances (for example, being certain it is either 0% or 100%, and indifferent between these options). In that case, there will be a nearby world where the agent believes that p is 50% likely, even though the chance of p is not .5. As a result, they can know that p's chance is .5 while failing to know that p is 50% likely.

Here, one caveat is required. Safety is often relativized to belief-forming methods (e.g., Sosa 2007), requiring that an agent knows p only if they truly believe p at any nearby world where they form this belief using the same method. Plausibly, this condition allows that an agent can know that p is 50% likely whenever they form this belief (a .5 credence in p) on the basis of their knowledge of the objective chances. After all, suppose the agent knows that the chance of p is 50%, and believes that p is 50% likely on the basis of their belief that the chance of p is 50%. At any nearby world where they use this same method to believe that p is 50% likely, they will believe that the chance of p is 50%. Since they know this latter fact, it will be true at this nearby world that the chance of p is 50%. So the agent will truly believe that p is 50% likely. Thanks to an anonymous referee for help here.

it might have landed tails.

But on most theories of objective chance, the objective chance of any past event is either 0 or 1. For this reason, the theory above seems to predict that (45) is false. Indeed, this is one of the reasons that Moss gives for denying any close connection between modal knowledge and objective chance (2018: 129).

However, we shouldn't write off the objective chance interpretation too quickly. It's widely assumed that objective chances are time-sensitive; indeed, this much is presupposed by the objection. To get around the objection, we could propose that circumstantial modals like *could easily have* shift the time that we use for evaluating the objective chance of an event. This captures the intuition that (45) is true. Given the objective chance function that obtained yesterday, there was a non-negligible objective chance that the coin would land heads, and also that it would land tails.

This response raises the difficult question: How do we select the relevant past time? But perhaps proponents of the objective chance interpretation can just say that context selects some relevant past time, and leave it at that. After all, many semanticists and philosophers are happy to allow that circumstantial modals are context-sensitive in certain respects.[27] Arguably, it is not unduly costly to posit temporal context-sensitivity as well.

A further problem case, due to Moss 2018, is more troubling. Moss asks us to imagine that you are about to toss a fair coin, and a time traveler appears and tells you the 'inadmissible evidence' that it will land heads.[28] Fill in the details in the right way, and it seems you can know—on the basis of this testimony—that the coin will land heads. *A fortiori*, you can know that it probably will land heads. But presumably there are nearby worlds where the objective chance that the coin lands heads is only 50%. Indeed, since you haven't yet flipped the coin, the actual world is one such world!

This case is less easily handled than **Past Coins**. And it motivates the search for an alternative interpretation, to which we now turn.

6.2 Contextual Information

A second option is to explain worldly information in terms of some body of information determined by the context of utterance.

[27] See e.g. Lewis 1973 on the context-sensitivity of counterfactuals.

[28] See Moss 2018: 129. Moss credits the case to Daniel Drucker.

To unpack this idea, let's take a brief foray into the main propositionalist semantics for modals: contextualism. Contextualists say that modals quantify over the worlds consistent with the modal base—a contextually determined function from a world to a body of information (Kratzer 1981, 2012).

In order to extend this contextualist account to probabilistic modals, we can endow the contextual information with probabilistic structure:

Definition 9. The contextually determined information at w (i^f_w) $= \langle s^f_w, Pr^f_w \rangle$, where:

1. f is a function from a world to a set of propositions. (Here we focus on the case where f is realistic, and so $w \in \bigcap f(w)$.)
2. s^f_w is the set of worlds consistent with $f(w)$ ($s^f_w = \{w' \mid \bigcap f(w) \cap w' \neq \emptyset\}$).
3. Pr^f_w is the contextually determined probability, which is a probability function conditionalized on $f(w)$ (and so $Pr^f_w(s^f_w) = 1$).

Of course, our semantics is not contextualist. While we take the extension of unembedded epistemic modals to depend on an information state, this information state floats free from both the world of evaluation and the context of utterance. But this need not stop us from recruiting contextual information to play the role of worldly information. The idea would be that circumstantial modals shift the value of the information state in the index to some contextually determined information state that obtains at a nearby world:

(46) $[\![\blacklozenge \mathsf{A}]\!]^{f,w,i} = 1$ iff $\exists v \in s^f_w : [\![\mathsf{A}]\!]^{f,v,i^f_v} = 1$

To evaluate this approach, let us apply it to our cases.

Start with **Hypochondria**. Consider the context of utterance in which we first told you—the reader—about this case. We told you that at Hydie's world (w) someone sneezed near Hydie. So the contextual information at w includes this fact. However, we also made salient a nearby possibility (v) in which no one sneezed. Relative to v, the contextual probability that Hydie gets sick at any moment is very low. According to the present proposal, this is what makes (27) (*It could easily have happened that Hydie believed that she might get sick at any moment, even though it wasn't the case that she might get sick at any moment*) come out true.

Before moving on, a word of clarification. So far we haven't said much about how to understand the contextual probability function. There are a few options.

One is to think of it as a pseudo-objective probability. In particular, we could say that it has the same priors as the objective chance function, it's just updated with different information. Another option would be to think of it as some sort of evidential probability function, assigning uniform prior probabilities to all worlds. A third option is to think of it as some sort of subjective probability. In particular, we could take it to model some conversational participant's or assessor's credences, which have been provisionally revised to incorporate the contextual information. For present purposes, we need not choose between these options. Indeed, there might be no one-size-fits-all answer; perhaps different contexts select different sorts of probability.

A similar diagnosis applies to **Fake Letters**. Relative to the world where **Fake Letters** holds true, there is a high contextual probability that Alice is attracted to Bert. However, in telling her tale, Moss makes it clear that things could easily have been different. This makes salient a nearby possibility where Alice and Bert had a slightly lower level of romantic compatibility—perhaps for any of the possible reasons mentioned in §6.1—but Alice still formed the modal belief. Plausibly, the contextual information at this world incorporates whatever facts throw cold water on our protagonists' romantic prospects. So relative to this world, there is a lower contextual probability that Alice is attracted to Bert. As a result, (26) (*Alice could easily have believed that she probably found Bert attractive, even though she hadn't probably found him attractive*) is judged true.[29]

6.3 Weighing the Options

We have laid out two possible interpretations of worldly information: the objective chance interpretation and the contextual interpretation. Which should we prefer?

As we saw in §6.1, the objective chance interpretation faces a challenge allowing for (non-extremal) modal knowledge about the past. To deal with this, we anchored objective chances to past times. This in turn raised the tricky question of how the temporal anchor is determined. By contrast, the contextual interpretation avoids this issue. After all, the contextual information will usually not include all the facts about the past.

[29] What about **Lottery**? In our statement of **Lottery**, we didn't specify which ticket won. So, relative to the world at which **Lottery** is true, it's natural to think that the contextual probability assigns equal chances to all the tickets winning, and hence assigns low probability to Lottie's ticket winning. Since at all the nearby worlds the odds are the same, it's natural to think that at all these worlds the contextual probability that Lottie wins is also low. Given these assumptions, (43) (*Lottie knows her ticket will probably lose*) comes out true on the contextual information approach.

At the same time, the contextual interpretation confronts thorny questions of its own. In particular, how does context select, for any given world, a relevant body of information at that world?

While this is a difficult question, three points are worth making. First, contextualists about modals already face this question. And so proponents of the contextual interpretation are in no worse shape on this score.

Second, we should only expect precise answers when the data suggest there are precise answers to be found. But when we look at the data concerning epistemic modals, we see considerable contextual variability. This point is well-illustrated with DeRose's cancer case (1991):

(47) **Cancer** John is undergoing a test for cancer. A negative result means that John definitely does not have cancer. A positive result does not necessarily mean that John has cancer; rather, it means that further tests need to be run.

As DeRose notes, we can imagine John's wife saying:

(48) We don't know whether John might have cancer. We haven't gotten the test results yet.

But, DeRose observes, we can also imagine contexts where it would be natural for her to say:

(49) We know that John might have cancer. That's why we sent him in to get tests.

An adequate theory of modal knowledge should be flexible enough to accommodate both responses. The contextual interpretation of worldly information has no trouble here—indeed, it has no trouble precisely because it is fairly unconstrained. By contrast, the objective chance interpretation struggles to explain how (49) could be true. To do so, objective chancers would need to insist that at all nearby worlds there is a decent objective chance that John has cancer. But, intuitively, this need not be the case.

Third, a growing body of research at the intersection of semantics and pragmatics investigates how speakers and listeners resolve context-sensitivity. We have already alluded to salience as one relevant factor. Others plausibly include the question under discussion and some version of a principle of charity.[30] Perhaps such research could provide further insight into how context selects a relevant body of information at a world.

A final point in favor of the contextual interpretation of worldly information is that it easily handles Moss' (2018) time traveler case. To do so, we need only say that the information provided by the time traveler is part of the contextual information.

Taken together, these considerations favor the contextual interpretation. But for those who are attracted to the objective chance interpretation, it is worth noting that the two interpretations are not necessarily in conflict. Indeed, the objective chance interpretation could be viewed as a special case of the contextual interpretation.

7 Concerns and Conclusion

We conclude by addressing two lingering concerns.

First, some might worry that we have wedded ourselves to a very controversial theory of knowledge, according to which knowledge is just safe belief. In response, we should point out that one advantage of our semantics for circumstantial modals developed here is that it can be used to formulate a variety of different modal conditions on knowledge. Take, for example, a sensitivity condition on knowledge (Nozick 1981):

(50) **Sensitivity** S knows p only if: if p had been false, S wouldn't have believed p.

Our theory generalizes naturally to such conditions. The key idea would be that counterfactuals look to the closest worlds w where the antecedent is true at the pair of w with the worldly information at w.[31] An approach along these lines has independent value, since any complete theory of epistemic modals should explain how they embed in counterfactuals.

Of course, not everyone will grant that knowledge can be analyzed in purely modal terms. For example, one of the leading proponents of Safety, Pritchard,

[30] For relevant discussion, see e.g., Lewis 1979; Roberts 2012; Gualmini et al. 2008.

[31] More precisely, where f is a selection function as in Stalnaker 1968, we could say:

(i) $[\![\mathsf{A} > \mathsf{C}]\!]^{w,i} = 1$ iff $[\![\mathsf{C}]\!]^{v,i^v} = 1$, where $v = f(w, \left\{w' \mid [\![\mathsf{A}]\!]^{w',i^{w'}} = 1\right\})$

now thinks that Safety needs to be combined with an *ability* condition familiar from the literature on virtue epistemology.[32] However, nothing in our framework precludes combining our approach with an ability condition. Fortunately, Konek 2016 has already done the hard work on this front, developing an intricate proposal for making sense of an ability condition on probabilistic knowledge. A full discussion of Konek's approach is outside the scope of this paper. The point is simply that there is no principled obstacle to integrating a condition along these lines into our account.

The second concern for our theory of modal knowledge is that it offers different lexical entries for epistemic and circumstantial modals. But an important theme in Kratzer 2012 and other research on modals is that different kinds of natural language modal expressions have a common semantic core. In closing, we show that our theory provides a unified account of epistemic and circumstantial modals, on which they actually share a common quantificational structure.

Here, we build on work in Goldstein 2019b. On Goldstein's framework, accessibility relations do not simply relate worlds. Rather, they relate a world and information state to a set of worlds. We enrich this proposal by having accessibility relate a world and information state to a new set of world-state pairs.

Definition 10. An information-sensitive accessibility relation R relates a world-state pair $\langle w, i \rangle$ to any other world-state pair $\langle v, j \rangle$ possible from the perspective of $\langle w, i \rangle$. $R(w, i)$ abbreviates $\{\langle v, j \rangle \mid \langle w, i \rangle R \langle v, j \rangle\}$.

We now define modal operators that quantify over accessible points:

Definition 11.

1. $[\![\Diamond \mathsf{A}]\!]^{w,i} = 1$ iff $\exists \langle v, j \rangle \in R(w, i) : [\![\mathsf{A}]\!]^{v,j} = 1$
2. $[\![\Box \mathsf{A}]\!]^{w,i} = 1$ iff $\forall \langle v, j \rangle \in R(w, i) : [\![\mathsf{A}]\!]^{v,j} = 1$

Our earlier semantic clauses for epistemic and circumstantial modals correspond to special instances of information-sensitive accessibility. Epistemic modals correspond to the special case where the information-sensitive accessibility relation is epistemic, which occurs whenever a given $\langle w, i \rangle$ can only 'see'

[32] In particular, Pritchard 2012 offers a case—'Temp'—that he takes to show that safety is not sufficient for knowledge. For a reply, see Beddor and Pavese 2020, who argue that the Temp case can be handled by a safety condition, properly construed. Their construal involves understanding safety in terms of the normal conditions for belief formation. For related discussion of the connections between safety and normality, see Goodman and Salow 2018; Carter and Goldstein 2020.

combinations of some world in i paired with i itself. Circumstantial modals correspond to the special case where the accessibility relation is worldly, which occurs whenever $\langle w, i\rangle$ can only 'see' combinations of some nearby world paired with the worldly information at that world. That is:

Definition 12.

1. R is epistemic iff $\langle w, i\rangle R\langle v, j\rangle$ iff $i = j$ and $v \in s_i$.

2. R is worldly iff $\langle w, i\rangle R\langle v, j\rangle$ iff $v \in s^w$ and $j = i^v$.

Our earlier semantics yields the following equivalences:[33]

(51) a. If R is epistemic, then $[\![\Diamond\!\!\!\cdot\;\mathsf{A}]\!] = [\![\Diamond\mathsf{A}]\!]$.
b. If R is worldly, then $[\![\Diamond\!\!\!\cdot\;\mathsf{A}]\!] = [\![\blacklozenge\mathsf{A}]\!]$.

This more general framework offers a fruitful setting for exploring variants of our earlier semantics. For example, one intriguing variant is to construe circumstantial accessibility as a kind of hybrid between epistemic and worldly accessibility. Say that R is quasi-worldly when it relates $\langle w, i\rangle$ to itself, and to all worlds that are worldly accessible.

Definition 13. R is quasi-worldly iff $\langle w, i\rangle R\langle v, j\rangle$ iff $\langle v, j\rangle = \langle w, i\rangle$ or $v \in s^w$ and $j = i^v$.

Whenever s^w is reflexive, quasi-worldly accessibility agrees with worldly accessibility about which worlds are accessible when paired with some state. They differ only regarding which information states are accessible. Worldly accessibility only allows some $\langle w, i\rangle$ to see the nearby worldly information states, whereas quasi-worldly accessibility also allows $\langle w, i\rangle$ to see the base information state i. This yields yet another way of addressing the worry that the failure of (36) ($\Diamond\mathsf{A} \models \blacklozenge\Diamond\mathsf{A}$) is difficult to stomach (§5.3.2). This is because the inference from $\Diamond\mathsf{A}$ to $\Diamond\!\!\!\cdot\;\Diamond\mathsf{A}$ is valid when accessibility is quasi-worldly. (If $\Diamond\mathsf{A}$ is true at $\langle w, i\rangle$, then A is consistent with i. If R is quasi-worldly, then $\langle w, i\rangle R\langle w, i\rangle$, which in turn implies that $\Diamond\!\!\!\cdot\;\Diamond\mathsf{A}$ is true at $\langle w, i\rangle$.) On the other hand, whenever the prejacent of $\Diamond\!\!\!\cdot$ is descriptive, quasi-worldly and worldly accessibility yield the same predictions (assuming s^w is reflexive); hence they are empirically equivalent when it comes to claims like (33). In this way, the shift from worldly to quasi-worldly accessibility

[33] One open question is how to extend the semantics above to probabilistic modals. The natural option is to enrich accessibility relations so that each $\langle w, i\rangle$ determines not only a set of accessible possibilities, but also a probability measure over those points.

is a conservative revision of our theory that smoothly preserves (36). This is just one of many potential applications of these tools to the larger project of providing an empirically adequate treatment of the interactions between modal operators.[34]

[34]For helpful comments on this material, we are grateful to Dan Greco, Sarah Moss, two referees at *The Journal of Philosophy*, and audiences at the University of Glasgow, the Rutgers Semantics Workshop, and the NUS Epistemology Workshop. Special thanks to Julien Dutant and Matt Mandelkern for discussions that shaped our views on this topic.

Appendix: Comparisons

In this appendix, we compare our account to two other theories of modal knowledge that have been defended in the literature: the context probabilism in Yalcin 2012c and the relevant alternatives framework in Moss 2018.

7.1 Context probabilism

Yalcin 2012c develops a theory of modal knowledge which navigates the impossibility results with which we began. Yalcin defends *blunt context probabilism*, according to which contexts are sets of probabilistic information states, and meanings are rules for updating contexts.

For ease of comparison with our own theory, we suppress some of the details of Yalcin 2012c and focus on the semantics for belief and knowledge. For this reason, we rewrite his theory in terms of earlier framework, where meanings are sets of world-information state pairs.

In that setting, we can interpret belief and knowledge as universal quantifiers over a set of probabilistic information states $\mathcal{B}^w$ and $\mathcal{K}^w$. The former models the possibly imprecise credences of the agent; the latter models the imprecise probabilistic knowledge of the agent. An agent believes A just in case A is true throughout her blunt doxastic possibilities, the information states consistent with her beliefs. An agent knows A just in case A is true throughout her blunt epistemic possibilities, the information states consistent with her knowledge.

Definition 14.

1. $[\![\mathsf{BA}]\!]^{w,i} = 1$ iff $\forall i' \in \mathcal{B}^w : [\![\mathsf{A}]\!]^{i'} = 1$
2. $[\![\mathsf{KA}]\!]^{w,i} = 1$ iff $\forall i' \in \mathcal{K}^w : [\![\mathsf{A}]\!]^{i'} = 1$

The predictions of this theory depend on the constraints on $\mathcal{B}^w$ and $\mathcal{K}^w$. For example, suppose that $\mathcal{K}^w$ is maximal, containing only a single information state i. Then Knowledge Transparency would be valid. After all, the agent would fail to know A just in case A was not true throughout i. And this would obtain just in case $\Diamond\neg\mathsf{A}$ was true throughout i, which would obtain iff the agent knew $\Diamond\neg\mathsf{A}$. One of the goals of Yalcin 2012c is to avoid Knowledge Transparency, and so we assume $\mathcal{K}^w$ contains multiple information states.

As in a Hintikka semantics, the implication from KA to BA corresponds to the requirement that the blunt doxastic possibilities $\mathcal{B}^w$ are a subset of the blunt epistemic possibilities $\mathcal{K}^w$: any information state consistent with the agent's beliefs is consistent with the agent's knowledge.

How should we model Factivity in this framework? Yalcin does not explicitly take a stand on this. However, it seems that in order to capture Factivity for the information-sensitive fragment of the language, we need to take the epistemic possibilities to themselves be information-sensitive, so that $\mathcal{K}$ is a function from worlds and information states to sets of probabilistic information states. Then Factivity requires that $\mathcal{K}^{w,i}$ always include i.[35]

Now consider Safety, that $\mathsf{KA} \models \neg\blacklozenge(\mathsf{BA} \wedge \neg\mathsf{A})$. Suppose that there is a constraint on $\mathcal{K}^{w,i}$ which is equivalent to the validity of Safety, and which is consistent with $i \in \mathcal{K}^{w,i}$ and $Bel^w \in \mathcal{K}^{w,i}$. In that case our own theory would turn out to be a special case of that in Yalcin 2012c, corresponding to an analysis where $\mathcal{K}^{w,i}$ is the union of i, Bel^w, and those states whose inclusion in $\mathcal{K}^{w,i}$ is equivalent to Safety. Interestingly, however, it turns out that there is no such constraint.

Let's see why. A first natural constraint to consider is that the epistemic possibilities include the information at any nearby world, so that $\{i^v \mid v \in i^w\} \subseteq \mathcal{K}^{w,i}$. But this condition is not equivalent to Safety. Instead, it corresponds to the Margin for Error principle that if S knows p, then p could not easily have been wrong:

(52) **Margin for Error.** $\mathsf{KA} \models \blacksquare\mathsf{A}$

This principle implies Safety, but is strictly stronger than it. After all, Margin for Error says that if you know p, then at *all* nearby worlds p is true—not just the nearby worlds where you believe p.

This difference is important, since some have worried that Margin for Error is inappropriately stronger than Safety (Berker 2008; Ramachandran 2009). Consider, for example, the sort of case that features prominently in anti-luminosity arguments (Williamson 2000). An agent is in a borderline state of feeling cold: they are cold, but just barely. Can they know that they are cold? Margin for Error says no: after all, there is a nearby world where the agent is no longer cold. But Safety says yes: after all, it might well be that at all nearby worlds where the agent is no longer cold, the agent correctly believes that they are no longer cold. At least some think that Safety delivers the right verdicts in such

[35] After all, if $\mathcal{K}^{w,i}$ is not information-sensitive, then neither is KA. But then knowing probably p would not be information sensitive. But then knowing probably p could not imply probably p, since the latter claim is information-sensitive.

cases. After all, if the agent's beliefs perfectly tracks the truth of the matter across all nearby worlds, why shouldn't their beliefs amount to knowledge?

Moreover, there is no way to weaken the Margin for Error constraint to reach a constraint that is equivalent to Safety. The problem is that Safety connects knowing p to what happens at nearby worlds where one believes p. So to validate Safety, we would say that whenever S knows p, $\mathcal{K}^{w,i}$ includes any worldly information from a nearby world where the agent believes p. But $\mathcal{K}^{w,i}$ is not itself relativized to a known proposition p. So if we include some nearby worldly information for evaluating whether an agent knows p, we must also include it for evaluating whether an agent knows any other claim q.[36]

In one way this problem is unsurprising. In fact, Safety is already incompatible with a simple Hintikka 1962-style semantics for knowledge even before we consider information sensitivity. For suppose meanings are just sets of possible worlds, and that $[\![\mathsf{KA}]\!]^w = 1$ iff $\forall v \in R^w : [\![\mathsf{A}]\!]^v = 1$. This semantics does not allow for a characterization of Safety in terms of epistemic accessibility. Again, the problem is that we cannot simply let any nearby world be epistemically accessible, because then we validate Margin for Error instead of Safety. Summarizing, then, we have a principled reason to depart from a Hintikka 1962-style semantics for knowledge attributions. Such a semantics cannot validate the kind of Safety principle on knowledge that we use to explain Gettier cases without validating the stronger Margin for Error principle.

We've explored one theoretical difference between the blunt theory of knowledge and our own. There is also a significant empirical difference. Consider:

(53) It might be raining and it might not be raining. But S knows whether it's raining (i.e., either S knows that it is raining or S knows that it isn't raining).

At first glance, (53) is consistent. Our own semantics makes this prediction.[37]

[36] One option here would be to further relativize the set of available epistemic states to contents, replacing $\mathcal{K}^{w,i}$ with $\mathcal{K}^{w,i,\mathbf{A}}$, the set of epistemic states for the agent relative to world w, information state i, and content $\mathbf{A}$. Then Safety would become equivalent to the constraint that $\mathcal{K}^{w,i,\mathbf{A}}$ contain $\{i^v \mid v \in i^w \;\&\; \forall u \in Bel^v : \langle u, Bel^v\rangle \in \mathbf{A}\}$. Then our own semantics corresponds to the definition that $\mathcal{K}^{w,i,\mathbf{A}} = \{Bel^w\} \cup \{i\} \cup \{i^v \mid v \in i^w \;\&\; \forall u \in Bel^v : \langle u, Bel^v\rangle \in \mathbf{A}\}$. But this semantics is a fairly dramatic revision of that in Yalcin 2012c. For example, it gives up the treatment of knowledge as a simple universal quantifier over a set of possibilities, and for this reason there is no guarantee that the resulting theory of knowledge would satisfy the requirements of deductive closure.

[37] For example, let $i = \{w, v\} = i^w = i^v$ with A true at w and false at v. Imagine that $Bel^w = \{w\}$ and $Bel^v = \{v\}$. The conjunction $\Diamond\mathsf{A} \wedge \Diamond\neg\mathsf{A}$ is true throughout i. In addition, the disjunction $\mathsf{KA} \vee \mathsf{K}\neg\mathsf{A}$ is true throughout i. To see why, note that KA is true at $\langle w, i\rangle$,

By contrast, blunt context probabilism predicts that (53) is inconsistent. For suppose that $\Diamond\mathsf{A} \wedge \Diamond\neg\mathsf{A}$ is true throughout i. Then we can show that $\mathsf{KA} \vee \mathsf{K}\neg\mathsf{A}$ is false at $\langle w, i\rangle$, for arbitrary w. After all, Factivity requires that $\mathcal{K}^{w,i}$ include i. Since i is agnostic about A, we know that $\mathcal{K}^{w,i}$ contains a state (i) which implies neither A nor $\neg\mathsf{A}$. So neither KA nor $\mathsf{K}\neg\mathsf{A}$ can be true at $\langle w, i\rangle$. So (53) cannot be true throughout any i, and so (53) is inconsistent.

7.2 Relevant alternatives

Another point of comparison is the theory of modal knowledge in Moss 2018. A direct comparison is somewhat difficult, because Moss does not lay out a semantics for knowledge ascriptions. However, in a number of places Moss suggests a relevant alternatives theory, where knowledge requires eliminating possibilities of error.[38] This section explores one way of developing such an approach.

To illustrate the basic idea, consider a case of 'merely statistical evidence' from Nesson 1979, discussed by Moss. A prison guard is assaulted by 24 of 25 prisoners in the yard. The jury cannot know that an arbitrary prisoner committed the assault. But can they know that an arbitrary prisoner *probably* committed the assault? No, says, Moss:

> The prosecution cannot provide you with knowledge of this content merely by proving that 24 of the 25 prisoners in the yard are guilty, since there is a certain possibility that you cannot rule out—namely, that the defendant is less likely to be guilty than an arbitrary prisoner in the yard. (2018: 213)

This is an intuitive reaction. However, when we try to formulate a relevant alternatives theory of knowledge that applies to probabilistic contents, we face a familiar problem. Canonical relevant alternatives theories such as Lewis 1996 assume that relevant alternatives are worlds, and that the contents of knowledge are propositions which can be assessed for truth or falsity at worlds. So how

while $\mathsf{K}\neg\mathsf{A}$ is true at $\langle v, i\rangle$. To see why KA is true at $\langle w, i\rangle$, note that A and BA are true there. All that's left is to show that $\blacklozenge(\mathsf{BA} \wedge \neg\mathsf{A})$ is false at $\langle w, i\rangle$. $\blacklozenge(\mathsf{BA} \wedge \neg\mathsf{A})$ is true at $\langle w, i\rangle$ iff there is some $v \in i^w$ where $\mathsf{BA} \wedge \neg\mathsf{A}$ is true at $\langle v, i^v\rangle$. Since $Bel^w = \{w\}$ and $Bel^v = v$, this condition fails. So $\blacklozenge(\mathsf{BA} \wedge \neg\mathsf{A})$ is false at $\langle w, i\rangle$, and hence KA is true at $\langle w, i\rangle$. An analogous line of reasoning shows that $\mathsf{K}\neg\mathsf{A}$ is true at $\langle v, i\rangle$, and hence the disjunction $\mathsf{KA} \vee \mathsf{K}\neg\mathsf{A}$ is true throughout i.

[38] See e.g., Moss 2018: 137-139, 141, 213, 227. Thanks to Dan Greco for discussion of these ideas.

can we formulate a relevant alternatives approach that allows for non-vacuous knowledge of non-propositional contents?

One option is to follow Moss 2018 in holding that all knowledge ascriptions are sensitive to a contextually supplied partition over possibilities. The different cells in the partition could then be viewed as comprising the relevant alternatives.[39] On this view, probabilistic knowledge requires that an information state continue to possess a probabilistic property even after being updated with a relevant alternative, i.e., a cell of the partition.

For ease of comparison, we continue to use the semantic architecture from §5, where meanings are pairs of worlds and information states. We let knowledge claims be relativized to a salient partition j. We say that an agent knows A is true at a pair $\langle w, i\rangle$ just in case (i) A is true, (ii) the agent believes A, and (iii) for any cell X in the partition j, updating i with X preserves the information that A is true. Where $\langle s, Pr\rangle + X = \langle s \cap X, Pr(\cdot \mid X)\rangle$:

Definition 15. $[\![\mathsf{K}_j\mathsf{A}]\!]^{w,i} = 1$ iff

1. $[\![\mathsf{A}]\!]^{w,i} = 1$
2. $[\![\mathsf{BA}]\!]^{w,i} = 1$
3. $\forall X \in g(j) : [\![\mathsf{A}]\!]^{i+X} = 1$

In order to make predictions about modal knowledge, we need to supplement this theory with a semantics for epistemic modals. On Moss' view, an epistemic possibility (necessity) claim is true relative to some information state i and some partition just in case, for some (all) cell X in the partition, the result of updating i with X supports the prejacent. Similarly, saying it is probably p is equivalent to saying that one is probably in a cell of the partition where p is true. Translating her proposal into our semantic architecture, this gives us:

Definition 16.

1. $[\![\Diamond_j\mathsf{A}]\!]^{w,i} = 1$ iff $\exists X \in g(j) : [\![\mathsf{A}]\!]^{i+X} = 1$
2. $[\![\triangle_j\mathsf{A}]\!]^{w,i} = 1$ iff $Pr_i(\bigcup\{X \in g(j) : [\![\mathsf{A}]\!]^{i+X} = 1\}) > .5$
3. $[\![\Box_j\mathsf{A}]\!]^{w,i} = 1$ iff $\forall X \in g(j) : [\![\mathsf{A}]\!]^{i+X} = 1$

[39] We should note that while this is consonant with many of Moss' central claims, some of her remarks suggest that she thinks of relevant alternatives not as cells in a partition, but rather as probabilistic contents (e.g., 2018: 137). We will leave as an open question how best to develop these remarks into a general theory of knowledge.

Having assembled the pieces, let's see how this works in the prison yard example. The context makes salient the question of whether the defendant is a model citizen. This induces a partition of logical space into two cells: the worlds where the defendant is a model citizen, and the worlds where the defendant is not. Crucially, the information state relative to which we assess a knowledge claim will not consider the defendant probably guilty once it has conditionalized on the defendant being a model citizen. For this reason, the relevant alternative of the defendant being a model citizen precludes the jury from having probabilistic knowledge that they probably committed the crime.

Having illustrated the theory, let us now turn to assess it. One worrisome consequence is that it predicts widespread skepticism of probabilistic knowledge. Recall our earlier discussion of **Lottery**. We suggested that (43) (*Lottie knows her ticket probably lost*) is intuitively true.[40] But the theory above has trouble delivering this verdict.

To see the problem, consider which partition is relevant to assessing probabilistic knowledge ascriptions in **Lottery**. There are two natural choices. One partition divides logical space into two cells: the worlds where Lottie's ticket is the winner, and the worlds where Lottie's ticket loses. Another partition would divide logical space into 1000 cells: the worlds where ticket 1 wins, the worlds where tickets 2 wins, . . . , and the worlds where ticket 1000 wins. The problem is that Lottie lacks probabilistic lottery knowledge relative to either partition. Both partitions make relevant the alternative where Lottie's ticket wins. But conditional on this alternative obtaining, Lottie does *not* have a high credence that her ticket will lose.

A second concern for the relevant alternatives approach is whether it can validate Safety, and thereby underwrite our diagnosis of modal Gettier cases. Here, we need to analyze circumstantial modals within Moss' framework. As a first pass, we might give circumstantial modals the same quantificational structure and force as epistemic modals, differing merely in choice of partition:

Definition 17.

1. $[\![\blacklozenge_j \mathsf{A}]\!]^{w,i} = 1$ iff $\exists X \in g(j) : [\![\mathsf{A}]\!]^{i+X} = 1$
2. $[\![\blacksquare_j \mathsf{A}]\!]^{w,i} = 1$ iff $\forall X \in g(j) : [\![\mathsf{A}]\!]^{i+X} = 1$

But this proposal is a nonstarter. It implies that for non-modal choice of A, $\mathsf{A} \wedge \blacklozenge\neg\mathsf{A}$ is incompatible, just like an epistemic contradiction. After all, whenever

[40] Moss 2018 seems sympathetic to this verdict (172–3).

A holds throughout an information state, every cell of that information state will continue to support A.

This suggests that it is by no means trivial to develop a semantics for circumstantial modals in this framework. And even if this can be done, it's not clear whether it will succeed in validating Safety without validating the stronger Margin for Error principle. The problem is that the relevant alternatives theory above says that in order to know A, A must be true at a series of relevant information states. But there is no restriction on this quantification to information states where the agent holds the same beliefs as those they hold at the actual world. If this is right, at least one of the worries we raised for context probabilism will resurface.[41]

Summing up, the relevant alternatives theory of modal knowledge developed here faces two difficulties: it predicts skepticism about probabilistic lottery knowledge, and it is unclear how to integrate it with a semantics for circumstantial modals in a way that validates Safety. By contrast, the account of modal knowledge defended in this paper avoids these difficulties.[42]

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[41]Here is one suggestion for how to develop a more promising theory of circumstantial modals in this framework. We might let circumstantial modals quantify over a larger, 'counterfactual' domain. On this proposal, an index is a triple of a world, information state i, and counterfactual state c. We could for example let c be a pair of the set of worlds s_c consistent with the laws of nature and initial conditions, together with the objective chance function Pr_c, and we could require that $s_i \subseteq s_c$. Then we could say:

Definition 18.

1. $[\![\blacklozenge_j \mathsf{A}]\!]^{w,i,c} = 1$ iff $\exists X \in g(j) : [\![\mathsf{A}]\!]^{c+X} = 1$
2. $[\![\blacksquare_j \mathsf{A}]\!]^{w,i,c} = 1$ iff $\forall X \in g(j) : [\![\mathsf{A}]\!]^{c+X} = 1$

With a theory of circumstantial modals in place, we can turn to Margin for Error and Safety, now relativized to partitions. Whether $K_j\mathsf{A}$ implies $\blacksquare_j\mathsf{A}$ depends on the relationship between i and c. This principle and hence Safety is valid whenever $[\![\mathsf{A}]\!]^i = 1$ implies $[\![\mathsf{A}]\!]^c = 1$ for any pair $\langle i, c\rangle$ of an information state and counterfactual state relative to which we assess a knowledge claim. For in those cases, our requirement on knowing A that for every relevant alternative X, $[\![\mathsf{A}]\!]^{i+X} = 1$ will imply that $[\![\mathsf{A}]\!]^{c+X} = 1$. We've assumed that we only ever evaluate knowledge claims relative to $\langle i, c\rangle$ pairs where $s_i \subseteq s_c$. This guarantees that for some parts of the language, $[\![\mathsf{A}]\!]^i = 1$ implies $[\![\mathsf{A}]\!]^c = 1$. For example, whenever A is non-modal, or whenever A is a simple epistemic necessity claim of the form $\Box\mathsf{B}$ for non-modal B, this implication holds. But for example this principle fails in the case of $\Diamond\mathsf{A}$ and $\triangle\mathsf{A}$. So Margin for Error and Safety both hold for some fragments of the language and fail for others.

[42]Rich 2020 develops another theory of probabilistic knowledge by generalizing a standard Hintikka semantics for attitude verbs. On Rich's theory, the meaning of a sentence is a set of pairs of sets of probability spaces and probability spaces. The interpretation is that sentences are true or false at probability spaces, relative to a given probabilistic content. So it may be true or false that Mary knows it is probably raining, relative to the probabilistic content that she probably took an umbrella when she went outside. One major difference from our own theory is that Rich 2020 does not attempt to explain modal Gettier cases. Rather, the goal of that paper is to present a formal framework within which the factivity of modal knowledge can be established. The other major difference is that Rich 2020 offers a new semantic architecture

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