Designing mathematical modelling tasks in a technology rich secondary school context

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The potential of digital tools to enhance student learning is well researched, however, the potential of technology to promote students' engagement with mathematical modelling tasks has received limited consideration. This paper draws on a research study that aimed to investigate the possibilities that exist for student learning when teachers from six secondary schools designed tasks that anticipated for the use of digital tools within mathematical modelling tasks. The paper describes and analyses the collaboration which took place in identifying principles of design for such tasks.

Keywords: Mathematical modelling and Applications; Digital tools; Technology

Introduction

While there are strong research traditions in the fields of mathematical modelling and applications and the use of digital tools in mathematics classrooms, few studies have explored the potential of the nexus which exists between these two powerful approaches to thinking (Geiger, Faragher and Goos, 2010). Mathematical modelling is often described as a process involving the formulation of a mathematical representation of a real world situation and then using mathematics to derive results, interpret the results in terms of the given situation and if necessary, revising the model. The purpose of models is to interpret real world situations and/or make predictions about the future or past states of modelled systems (English, Fox, & Watters, 2005).

There is now a large corpus of literature devoted to the way in which digital tools can enhance teaching and learning opportunities in mathematics classrooms. Studies, however, have tended to report on advantages to instruction in mathematical thinking and learning within content specific domains such as number (e.g., Kieran & Guzmán, 2005), geometry (e.g., Laborde, Kynigos, Hollebrands & Straesser, 2006), algebra and calculus (e.g., Ferrara, Pratt & Robutta, 2006) or social aspects of classroom practice such as collaborative investigative practice (e.g., Beatty & Geiger, 2010). Thus, there is little research on how digital tools can be used in tandem with mathematical knowledge to work on problems that exist in the real world, as Zevenbergen (2004) observes:

> While such innovations [ICTs] have been useful in enhancing understandings of school mathematics, less is known about the transfer of such knowledge, skills and dispositions to the world beyond schools. Given the high tech world that students will enter once they leave schools, there needs to be recognition of the new demands of these changed workplaces. (p. 99)

Given this identified need for students to be provided opportunity to use digital tools when working on real world problems consideration needs to be given to the nature of the learning experiences, and the tasks at the centre of these experiences, students should encounter within school mathematics classes. The aim of this paper is to explore an approach to the design and implementation of tasks which focus on the a mathematical modelling approach to teaching and learning that is supported by digital tools. In doing so, the paper will address Theme A, Tools and Representations, through the following research question.

What are the principles of design for technology rich modelling and applications tasks that result in effective learning experiences for students?

Artefacts as mediators of mathematical learning

In developing principles of design for technology integrated modelling and applications tasks the role of artefacts, in this case the task and the digital tool(s), must be examined. Verillon and Rabardel's (1995) iconic work on the distinction between an artefact and an instrument provides insight into the role of artefacts in mediating learning by distinguishing between an artefact, which includes both physical and sign tools that have no intrinsic meaning of their own, and an instrument in which an artefact is used in a meaningful way to work on a specific task. Different tasks make different demands on the user and their relationship with the artefact. The development of this relationship, and thus how the artefact is used, is known as instrumental genesis. Instrumental genesis is a complex process in which, firstly, the potentialities of the artefact for performing a specific task are recognised which transforms the artefact into an instrument (instrumentalisation), and, secondly, there is a process that takes place within the user in order to use the instrument for a particular task (instrumentation) (Artigue, 2002). Instrumentation generates schemas of instrumented action that are either original creations by individuals or pre-existing entities that are appropriated from others. An instrument, therefore, consists of the artefact and the user's associated schemas of instrumented action. The process of instrumental genesis is also dynamic between the instrument and the user as the constraints and affordances of the artefact shape the user's conceptual development while at the same time the user's perception of the possibilities of the artefact during instrumentation can lead to the use of the artefact in ways that were not originally intended by the designers of a tool (Drijvers & Gravemeijer, 2005).

Instrumental genesis has been used to explain how digital tools are transformed into instruments for learning through interaction with teachers and students (e.g., Artigue, 2002). A teacher's activity in promoting a student's instrumental genesis is known as instrumental orchestration (Trouche, 2005). This process recognises the social aspects of learning as it allows for the sharing of schemas as of instrumented action that individuals have developed within a small group or whole class. A teacher can facilitate the appropriation of these schemas by other students by making the nature of these schemas explicit by orchestration classroom interaction around the schemas through careful and selective questioning

More recently, others have attempted to extend our understanding of an instrumental approach to the role of artefacts in mediating learning by recognising that the genesis of an artefact into an instrument takes place within highly interactive environments, such as school staff rooms or mathematics classrooms, where a number of artefacts are used simultaneously. Gueudet and Trouche (2009) extend the definition of artefact by introducing the term *resources* to encompass any artefact with the potential to promote semiotic mediation in the process of learning. *Resources* include entities such as computer applications, student worksheets or discussions with a colleague. A *resource* is appropriated and reshaped by a teacher, in a way that reflects their professional experience in relation to the use of resources, to form a schema of utilisation – a process parallel to the creation of a schema of instrumented action within instrumental genesis. The combination of the resource and the schema of utilisation is called a *document*. The process of documental genesis is an ongoing one as utilisation schemas will be reshaped as a teacher gains more experience through the use of a resource.

A modelling task oriented research project

Six teachers were recruited from six secondary schools; three from each of two different Australian states. Schools were drawn from across different schools systems (government and non-government) and were representative of a range of socio-economic characteristics. Teachers were invited into the project because of their reputations as highly effective teachers with particular skills in the use of digital tools in mathematics learning and their commitment to improving the learning outcomes of their students. The project was managed by two university based researchers – one in each state. The researchers were primarily responsible for the: conceptual development of the project; classroom data collection including lesson observations, teacher and student interviews, and collection of student samples. Teachers were primarily responsible for the development and implementation of technology demanding mathematical modelling tasks. Researchers played a vital role in providing feedback about the effectiveness of tasks trialled in teachers' classrooms. Together teachers and researchers developed principles of design for effective tasks based on their shared experiences while trialling tasks in mathematics classrooms.

This paper reports, specifically, on the work of one teacher and on his students in a Year 11 (15-16 years of age) mathematics class. The curriculum context in which he taught mandated the teaching, learning and assessment of mathematical modelling as a key objective of a state-wide syllabus (educational authorities are state based in Australia). The use of technology in mathematics teaching and learning was also prescribed in the Mathematics B program (incorporating the study of functions, calculus and statistics) in which his students were enrolled. Students had almost unrestricted access to digital technologies including: powerful handheld digital devices with mathematical facilities such as data and function plotters and Computer Algebra Systems; computers with mathematically enabled applications; the internet; and electronic white boards.

The research design consisted of three components: (1) two whole day teacher professional learning meetings which took place at the beginning and middle of the project; (2) three classroom observations for each teacher; and (3) a focus group interview near the end of the project that involved all teachers. The detail and purpose of each of these activities is outlined in Table 1. Further detail on the research methodology can be found in (Geiger, Faragher and Goos, 2010).

Time	Activity
Sept-Dec Year 1	Teacher workshops in each state: research team outline the aims of the project; offer prototype tasks; discussion of principles which underlie prototype tasks.
Jan-April Year 2	Lesson observations ; teacher and student interviews; collection of student work samples; feedback on effectiveness of trialed tasks in relation to modeling and the use of digital tools.
April-June Year 2	Lesson observations; teacher and student interviews; collection of student work samples; feedback on effectiveness of trialed tasks in relation to modeling and the use of digital tools.
July Year 2	Teacher workshops in each state: teachers share exemplars of digital tool and modelling tasks; discussion on principles which underlie teacher developed tasks; research team offer accounts of practice from classroom observations.
Aug-Sept Year 2	Lesson observations; teacher and student interviews; collection of student work samples; feedback on effectiveness of trialed tasks in relation to modeling and the use of digital tools.
Oct-Dec Year 2	Final project meeting and focus group interview in each state; teachers share exemplars of modelling and digital tool tasks; further discussion on principles which underlie teacher developed tasks.

Table 1: Research design

Principles of task design in technology demanding modelling tasks

The teacher (the co-author of this paper) who is the focus of this paper, proved to be an effective designer of technology demanding modelling tasks while, at the same time, demonstrated keen insight into his own design processes and how these developed through the duration of the project. This teacher, in particular, contributed to the development of principles for designing modelling tasks. These principles and their descriptions are presented in Table 2. While these are useful insights they confirm rather than extend what is widely accepted as approaches to designing effective modelling tasks or general advice on good teaching practice.

Principles	Description
Syllabus compliance	The task must meet the requirements of the syllabus for content knowledge and the dimensions related to applications and technology.
Authenticity and relevance	Tasks must be set in an authentic or life-related context. The task must be of interest to the teacher and be of potential interest to the student.
Open-endedness	The mathematics necessary to solve the problem set up in the task should not be immediately apparent. The task must be open-ended in nature providing for opportunity for multiple solution pathways.
Connectivity	Ideally the task must make links to different content areas within the syllabus.
Accessibility	The task must provide opportunity for students to link to their previous learning. There should be provision for multiple entry and exist points. The task should allow for the introduction of scaffolding prompts or hints.
Development	The task must provide challenge and so encourage students to go beyond what they presently know and can do through the modelling process. Students' engagement with the task should provide feedback to the teacher about the development of their understanding.

Table 2: Characteristics of effective modelling tasks

The teacher also provided valuable input into the role technology played in the design of modelling tasks, and indicated that digital tools served as an enabler of each of the identified principles. He provided comment on the role of digital tools in relation to each principle of design.

The use of digital tools is a mandatory element of the state-wide senior secondary mathematics *syllabuses*. Genuinely *authentic* problems are mathematically complex. The representational capabilities of digital tools allow students to accommodate this complexity and thus provide *access* to *authentic* problems that otherwise might be considered beyond the scope of their capabilities.

> If we didn't have the CAS calculators we couldn't do half the stuff that we do. From my perspective it is the integration of the whole lot together. We have a set of data and we try and build a model from that. We do a scatter plot and we make decisions about the model. We build a model and make some sorts of predictions.

Digital tools also provide the means for students with gaps in their content knowledge to *access* challenging problem scenarios.

> Lower achievers may be struggling with differentiation or integration at that particular point in time…but they can still have access to the problem. My lower achieving kids can still engage in the problem and still make some meaningful contributions. If they don't get caught up in all that manipulation they can still be thoughtful about it.

The nature of *authentic open-ended* problems means there is no clear solution pathway and students need to evaluate options as they progress toward a solution. The teacher argued that digital tools offer facilities that are essential for exploring possible solution pathways. Technology also provides the means for *connecting* different types of mathematical knowledge, for example, data representations and functional relationships that modelled patterns in the data.

> Selecting authentic, open tasks to model generally implies the students will need to make use of technology. Even if the teacher has scaffolded the task to facilitate access to the context, there is a requirement that the task be sufficiently open for there to be multi-representations of the solution and perhaps different solutions.

The *authenticity* and *open-endedness* of a problem is enhanced if students are required to collect data relevant to a problem from an original source; a capacity provided by digital tools in his classroom.

> There is often a need to collect data and then to determine whether a relationship exists within that data. Students may need to collect primary data, through the use of probes, or from a video that is then analysed using the technology or use secondary data collected from a newspaper, magazine, web site or some other source.

Used effectively, digital tools provide immediate feedback to students about their initial attempts to build models and solve problems thus progressing students' understanding of the underlying mathematics at the core of the task and hence their mathematical *development*.

> Technology has a significant role to play in the provision of feedback to the student in the first instance, about the models they have built and how well they fit the context being investigated. In mathematical modelling it is important to look for consensus between the mathematics and the context, hence, it is necessary to consider the validity of the conclusions in terms of the context.

Exemplar task and commentary

The principles for design of technology demanding modelling tasks are evident in the following description of a task developed and then implemented by the teacher in his Year 11 mathematics classroom – the *Algal Bloom Problem* outlined in the Figure 1. In developing this task, the teacher had expected his students to build a mathematical model for these data by first creating a scatterplot using their CAS active calculator. A plot of this data suggests a piecewise function (one part linear and one part power function) would be appropriate. The teacher anticipated that students would then use the plot to determine the general form of the functions that would best fit the data and, in due course, develop an equation that would best fit the data. Students were then expected to use the model they had created to respond to the question at the end of the task and also to list any assumptions they made in developing their model and also comment on any limitations they believed were inherent in the repose they provided.

In observing the lesson in which this task was used, the researcher noticed that while every student was able to produce a plot of the data using their handhelds, few had drawn the conclusion that a piecewise function was necessary to model the data. Most students attempted to model the data using a single function, generally by trying to generate a model for the data using the digital handhelds regression model facility. When their single functions were plotted on their screens with the original data points it was obvious that their various functions were a poor fit. When students asked the teacher for assistance he simply encouraged them to have a closer look at their data and explore a wider range of possibilities for fitting a model to the data. After a period of time, two students, working together near the researcher, attempted to fit a piecewise function to the data, and after performing fine adjustments to each part of their function were happy with the result. Their success prompted a subdued celebration by the two students which attracted the teacher's attention. After discussing their conjectured model with the teacher students went on to complete the task. A short period of time after his discussion with these students, the teacher called for the attention of the class and asked them about their progress. The two students near the researcher volunteered and were asked to outline their attempt at the task. When they announced they had decided to make use of a piecewise function, sections of the class responded in different ways. A small number of students indicated agreement with the approach the pair of students were proposing even though the details of the functions other students had used differed. Most students, however, expressed exasperation that they had not noticed what was now an obvious feature of the plotted data. These students then returned to the task and were able to develop a piecewise function that fitted the data for themselves. A small minority of students needed more direct help from the teacher but were also able to develop a model based on a piecewise function by the end of the lesson. The lesson concluded when the teacher asked the students to work further on their assumptions and limitations for homework.

> The CSIRO has been monitoring the rate at which Carbon Dioxide is produced in a section of the Darling River. Over a 20 day period they recorded the rate of CO_2 production in the river. The averages of these measurements appear in the table below.
>
> The CO_2 concentration [CO_2] of the water is of concern because an excessive difference between the [CO_2] at night and the [CO_2] used during the day through photosynthesis can result in algal blooms which then results in oxygen deprivation and death of the resulting animal population and sunlight deprivation leading to death of the plant life and the subsequent death of that section of the river.
>
> From experience it is known that a difference of greater than 5% between the [CO_2] of a water sample at night and the [CO_2] during the day can signal an algal bloom is imminent.
>
> Rate of CO_2 Production versus time

Time in Hours	0	1	2	3	4	5	6	7	8	9
Rate of CO_2 Production	0	-0.042	-0.044	-0.041	-0.039	-0.038	-0.035	-0.03	-0.026	-0.023

Time in Hours	10	11	12	13	14	15	16	17	18	19
Rate of CO_2 Production	-0.02	-0.008	0	0.054	0.045	0.04	0.035	0.03	0.027	0.023

Time in Hours	20	21	22	23	24					
Rate of CO_2 Production	0.02	0.015	0.012	0.005	0					

Is there cause for concern by the CSIRO researchers?
Identify any assumptions and the limitations of your mathematical model.

Figure 2 : Algal Bloom Problem

Discussion and conclusion

This task satisfies each of the principles developed during the project for mathematical modelling tasks and for the use of digital tools within tasks. The use of modelling tasks and digital tools are consistent with mandatory requirements of the relevant state *syllabus*. As national scientific bodies monitor the blue-green algae in the various river systems because of the effect on aquatic wildlife this represents a task set in a near *authentic* life-related context. The task is *open-ended* in that a variety of mathematical models are plausible and the use of different models will lead to different, but still valid, responses to the problem. The available digital tools provided the facility to trial a range of functions to fit a complex underlying pattern and offered immediate feedback on the appropriateness of a conjectured function allowing students to develop specific solutions from a wide range of possibilities. Different types of mathematics were necessary to explore the data (data representation, different forms of function) and so, students were expected to make *connections* to different types of mathematical knowledge. The available technology provided the option of viewing different types of mathematical representations (e.g., scatterplots and function graphs) on a screen at the same time, so enhancing the connecting between these types of mathematical knowledge. Students found the task to be *accessible* as it linked to mathematical knowledge they had studied in previous classes and the teacher made use of progress made by other students to provide a prompt when many were experiencing difficulty. The opportunity to trial a function against the data and receive immediate feedback provided an entry point to most students and so made the problem *accessible*. As the task required students to make use of mathematical knowledge they had already studied in previous lessons within an unfamiliar context it provided opportunity for students' *development* in mathematical knowledge and their capacity to apply this knowledge in real world contexts. Here, digital tools acted as a catalyst for this development by providing feedback which indicated students' first single function conjectures were not consistent with the data.

As outlined above, there is an inseparable interplay between the task and digital tools. The teacher has created the task by drawing on principles for developing effective technology active modelling tasks. These principles are based on the potentialities of both types of resource – the task and the digital tool. In implementing the task, the teacher anticipated how students would interpret the potentials of the task for learning and of the digital tool to act as a resource. The relationship between student, teacher, task and digital tool represents a documental genesis as each element within this genesis transforms the other in some way. The task is transformed, from the perspective of the students when they realise they need to make use of a piecewise rather than a single function in order to model the data presented in the problem. This transformation occurs as a result of an attempt by the students to use a single function and receiving feedback via the digital device that this was an inappropriate model. The use of the digital tool changes from that of a device that provided a specific solution for students once they had made a decision on the general form of the function to model the data into a tool used to explore the data and eventually find a model that fitted the data to their level of satisfaction. Students' learning is also transformed during this same process as they realise the purpose of the task and the digital tool is not to algorithmically implement prior learning but to apply their knowledge and understanding in an original way. The teacher had to transform his approach to the lesson when students took a path he had not anticipated – attempting to fit a single function to the data. He changed his approach by orchestrating the resources at his disposal, in this case the two students who had eventually solved the problem, to provide an insight into the problem other students were yet to see.

At the same time, nearly all of the teacher's principles of design, the characteristics of effective modelling tasks, acted as enablers of the process of instrumental genesis of both digital tools and of the task. The principle of *authenticity and relevance* requires students to recognise the potential of the available digital tools to assist them in exploring and solving the problem described in the task from within both purely mathematical and real world contexts. There was a necessary duality about the schemas of instrumented action required to accommodate the purely mathematical and contextual demands of the task. Students needed to recognise that the real world context demanded the development of a piecewise rather than single function to model the inhalation and exhalation of CO_2. This required a specific use of the digital tool that was different from the development of a single function to model the provided data. Having decided that two functions were needed to model the data, a specific instrumentation of the digital tool was needed to find the most appropriate functions for each section of the piecewise function. This second process takes place within a purely mathematical context.

The *open-endedness* of the task placed students in a position where they were challenged to make choices among multiple potential solution pathways. Thus, students were required to make choices among existing schemas of instrumented action or to generate new schemas. To generate new schemas students must firstly recognising the potential of the digital tool for meeting the challenge defined by the task and then, secondly, develop processes for the use of their digital tool that are specific to the set task.

The principle of *connectivity* designed into this task required students to generate schemas of instrumented action that were inclusive of different types of mathematical content. The CAS active calculator students used while working with the task included the capacity to link statistical plots with the graphs of specific functions, and these functions could be developed using the regression facility of the calculator. With these facilities available, students needed to find ways of taking advantage of the capabilities of their digital tool in engaging with the demands of the task and pursuing a solution. This is a type of instrumental genesis in which the potential of an artefact is only realised through its instrumented action.

The task was designed to link the demands of the activity to students' previous learning as the separate functions required to build an appropriate piecewise function had been studied and applied to real world contexts in earlier classes. Thus, the task was created to be *accessible* to students but, at the same time, required students to apply this previous learning in a more complex context – one in which multiple functions were needed to model a phenomena rather than a single function. This meant that students' existing schemas of instrumented action required adaptation in order to accommodate a more complex scenario. The CAS enabled calculator was the tool the teacher believed would mediate this adaptation through the provision of a medium that provided for the representation of multiple functions against complex data.

The *development* aspect of the design is most apparent in the way the way the teacher invited the pair of students who had found that an appropriate solution required a piecewise function to offer their solution to the whole class and the subsequent realisation by most of the class that this was an insight they had missed. This revelation changed both the ways in which these students used the available digital tools and also the way they viewed the task. In this circumstance the teacher *orchestrated* changes in students' schemas of instrumented action related to both the digital tool and also the task

The episode included in this paper demonstrates it is possible to design for effective technology demanding mathematical modelling tasks, and so the approach offers direction for curriculum designers, teachers and teacher educators. While the teacher had designed an engaging task based on principles developed during the project, students took an approach that was not anticipated by their teacher. The teacher, however, was able to take advantage of students' original but inappropriate approaches, generating a dynamic learning environment where students' knowledge of using mathematics within real world contexts was transformed. This raises a challenge for teachers in how such triggers can be deliberately embedded in planned learning experiences in a way that provides space for the type of documental genesis described in this paper. This also indicates that further research is necessary to investigate how to take advantage of unanticipated events in a well planned lesson and in turn for how teacher educators provide advice about task design and implementation in pre-service and in-service programs.

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