Number sense or no sense: pre-service teachers learning the mathematics they are required to teach

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Statement of sources

This thesis contains no material published elsewhere or extracted in whole or in part from a thesis by which I have qualified for or been awarded another degree or diploma.

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This thesis has not been submitted for the award of any degree or diploma in any other tertiary institution.

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Candidate's signature . . . .

Date . . . .

Abstract

As a result of two years working with the pre-service primary teachers in a College in Fiji I became aware of the difficulty many of the students were having understanding the primary school mathematics they would be required to teach. During that time I had attempted to help them overcome the difficulties by using different teaching approaches and activities but was far from satisfied with my efforts. Hence I decided to make a concerted effort to help the students by planning, implementing and partially evaluating a mathematics education unit, known as the Teaching Program for the first semester of their course. This work formed the basis of my study.

For the Teaching Program I chose a constructivist teaching approach with number sense as the underlying theme. To examine the aspects of the Program I used my observations and those of the students especially ones reported in their mathematics journals. To evaluate the effectiveness of the Teaching Program I collected and analysed quantitative data from traditional testing of the class of forty students as well as data from case studies of six of the pre-service teachers in the class. To determine what features of the Teaching Program were linked to positive changes my main source of data was the case studies, especially entries from their journal writings.

The findings suggested that a significant development of the cognitive aspects of the students' number sense did occur during the time of the Teaching Program but not as much as was hoped for. As a result of the analysis of the data I came to a greater realisation of the importance of the non-cognitive aspects of number sense and the necessity for a greater consideration of them in the development of a Program. I also realise now that a major development that did occur was in my understanding of the knowledge and learning of mathematics. My ideas of a teaching paradigm of social constructivism had not guided me sufficiently to incorporate activities and procedures to develop the non-cognitive aspects. I suggest that a paradigm which extends the theory of social constructivism to give greater consideration of these aspects of learning in general, and hence numeracy and number sense in particular, was needed. As a result of this study, my introduction to the theory of enactivism appears to be giving me some direction in this search at this stage.

Acknowledgements

Throughout my journey for this study there have been many people who have helped me along the way, the students and staff of the College and others I met in Fiji, the staff at the University of ACU Victoria, my family and new friends in Ballarat.

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Assoc. Prof. Doug Clarke and Dr. Marj Horne from ACU set me on my journey for the first two years and then Assoc. Prof. Phil Clarkson helped me sieve through the data I bought home from Fiji, analyse it and complete this study. Because of his thorough editing I have come a long way in improving my writing skills. Others, both academic and support staff of ACU, especially those at the Aquinas Campus, offered invaluable services. Also many thanks to those, including Dr. Carolyn Smith and Prof. Alan Bishop who offered further helpful advice.

Though many have being a support during various stages of the study, my family have been there through it all. I am very grateful to my mother and Les who have given me a home when I was on leave from Fiji, and also Margaret and Paul who welcomed me on my many visits to Melbourne for my study. My sisters, Mary and Noreen gave me advice in the writing of the thesis, and Peter, Joan and Bryan were always interested in how my study was progressing and so offered me the encouragement I needed to persevere with it.

Lastly but not least, thanks to my new friends in Ballarat, especially Anne Griffin, Dr. Robyn Pierce, Robyn Brandenburg and colleagues at SMB who, during the last two years have patiently supported me and have been a sounding board for my ideas.

List of tables

List of figures

CONTENTS

CHAPTER 1 UNDERPINNINGS OF THE STUDY

> Numeracy is a key life skill. Without basic numeracy skills our children will be disadvantaged throughout life. That is why we set a target of 75% of 11-year-olds reaching the standard of mathematics expected have for their age by 2002. We recognise the crucial importance of giving teachers the support they need to raise standards. But the success of both the Literacy and Numeracy Strategies depends on the commitment and enthusiasm of teachers themselves. (Department for Education and Employment, 1999, Foreword)

One of the major concerns in many countries is education and a major concern within education itself is mathematics and science education, perhaps partly because of governments' interests in technological development over recent decades. The above quote was written by the Secretary of State for Education and Employment in United Kingdom (Department of Education and Employment, 1999). What he has to say highlights a number of the concerns I had when I began my study. He voices the need to be able to apply mathematics in daily life and indicates that teachers need support in their role in the process of helping raise the standard of numeracy. As well as recognising the support they need, he states that success of the teachers' work depends on their commitment and attitudes. When I began my study I also wanted to support the teachers in their role of helping their pupils improve their numeracy and, although I was not worried about their commitment to their work, I had doubts about their attitude and whether it was an appropriate one.

Other governments and educational bodies have written similar documents advocating reforms in mathematics education by encouraging a rethinking of how mathematics might be taught and learned in schools (Department of Employment, Education and Training, 1989; Cockcroft, 1982; National Research Council, 1989; National Council of Teachers of Mathematics, 1989). The Ministry of Education, Victoria (1988) discusses how to help people make sense of mathematics and be able to use it meaningfully. The Ministry states that there is an acute awareness that mathematics in schools does not serve its clients well. It also notes the need for teaching practices and beliefs to change, saying that there is a need for classroom approaches which encourage all students in their mathematical learning. The Ministry suggests that teachers need to reconceptualise a view of mathematics so it can make sense when it is linked to what students know or want to know. The overall impression is created that teachers must help build students' mathematical knowledge out of the students' interests, experiences and the environment created in the classroom.

In Fiji, the setting for the research for this study, there has been discussion about the poor performance in mathematics especially by the indigenous Fijian secondary students. At all levels of the Fijian Education System, over the last few decades administrators and teachers are asking for change to help students think logically and to use mathematics with understanding (Metcalfe, 1978; Prasad, 1988; Muralidhar, 1989; Bakalevu, 1997a). The discussion in these papers will be elaborated on later in the literature review.

How the initiation of reform is to take place is not made obvious in the government documents from across the world. Most educators would probably agree that to bring about the change that is advocated is far from a simple process. Many questions need to be answered to determine the most effective ways to carry out the reforms including questions concerning where to initiate the necessary changes. Do we begin in primary or secondary schools, or in tertiary education institutions or in all three simultaneously? Tobias and Oakes (1997) suggest that implementing changes in mathematics in schools has necessary implications for mathematics teacher education in the tertiary sector. The initiating of reform in whatever place, involves not just putting into place the latest policy, but it means changing the cultures of the classrooms, the schools and the universities (Fullan, 1991). Fullan suggests that reform cannot be done *to* others but the only fruitful way ahead is to carve out one's own niche of renewal and build on it. He continues that if reforms are to be successful individuals and groups must find meaning concerning *what* should change as well as *how* to go about it.

This lends support in part to my small effort to create change by attempting to implement some reform in mathematics education in one classroom in a primary teachers' College in Fiji during the final of my three years in Fiji. It was to be my final year because after two years in Fiji I had come to realise that my time there would need to be limited because of the then unstable political situation, racial tensions and lack of acceptance of expatriate workers by many Fijians. Therefore after collecting the data, the analysis of the research and writing of the thesis was completed in Australia, my home country.

Two episodes illustrate my interest to study how to help the pre-service teachers. After a few months of working with students in this College I was concerned with the difficulties pre-service teachers were having doing the mathematics that was of primary school standard. For example, I was alerted to the pre-service students' apparent evidence of poorly-developed understanding of number at a College money-raising event. I noticed it as I sat with the second year students who were aiming to reach a target of $1000. With their total takings at that stage above $900, one student asked the group how much more did they need to reach the target. There was a call for pen and paper and someone proceeded to do the necessary subtraction algorithm - correctly. Then when a further donation of $10 was passed in another pen and paper subtraction was done.

Another experience which motivated the formation of this study also happened in my first year of lecturing at the College. I was fascinated by the delight and enthusiasm of my students when they were doing simple problem solving after a few words of instruction by me. With the freedom and encouragement to use their own strategies to find the answers they appeared to discover an unknown ability they possessed to do mathematics. I observed students deciding on which strategy to use depending on the numbers and context of the problem. This sign of the facility to adapt the method used is possibly an indicator of a development in their understanding of number, often referred to as number sense, a theme picked up again in some detail later in the literature review. In other areas of their learning of mathematics the students appeared to have a poorly-developed number sense, hence my fascination at the unexpected evidence of the development of number sense. I asked myself why was it not evident in other areas of their mathematics. I kept this experience of a positive outcome seen in problem solving activities in mind as part of a possible answer as to how to bring about change in mathematical understanding. I watched for other activities that I and the students felt were helpful in improving their mathematical understanding. Without realising it at first, I had begun an investigation of how to help the students with their mathematics. Thus a study had begun in practice but it had not yet been theorised.

The Fijian context

This section in which I describe the context of the study is divided into three sections. Firstly, I will briefly describe Fiji, the country in which the study was completed. Secondly, I will give an overview of the structure of education in the country. Then finally, I will briefly describe the relevant facts about teacher education in Fiji.

Fiji

Fiji is an archipelago of 300 islands with a central position in the south west of the Pacific Ocean (Figures 1.1 and 1.2, maps adapted from Jones & Pinheiro, 1997, and de Ishtar, 1994). This places it approximately 3000 kilometres east of Townsville in Queensland and 1000 km north of New Zealand. Two of the hundred inhabited islands are much larger than the others with most of Fiji's population of 800,000 living on them. About 50% of the population are of indigenous Fijian origin and the other 50% are mostly of Indian origin. The latter are either the descendants of the Indian indentured labourers who were imported at the end of the 19th Century, or the descendants of immigrants from India who run most of the businesses in the urban areas. A small percentage are of European, Chinese or other Pacific Islander origins (International Development Program, 1990).

The main island is Viti Levu where 75% of the population live. Suva, the capital city is on the south-east corner of this island and is the centre for Government, commerce, and the main campus of The University of the South Pacific. The large international airport is on the other side of the island near the second largest city, Lautoka.

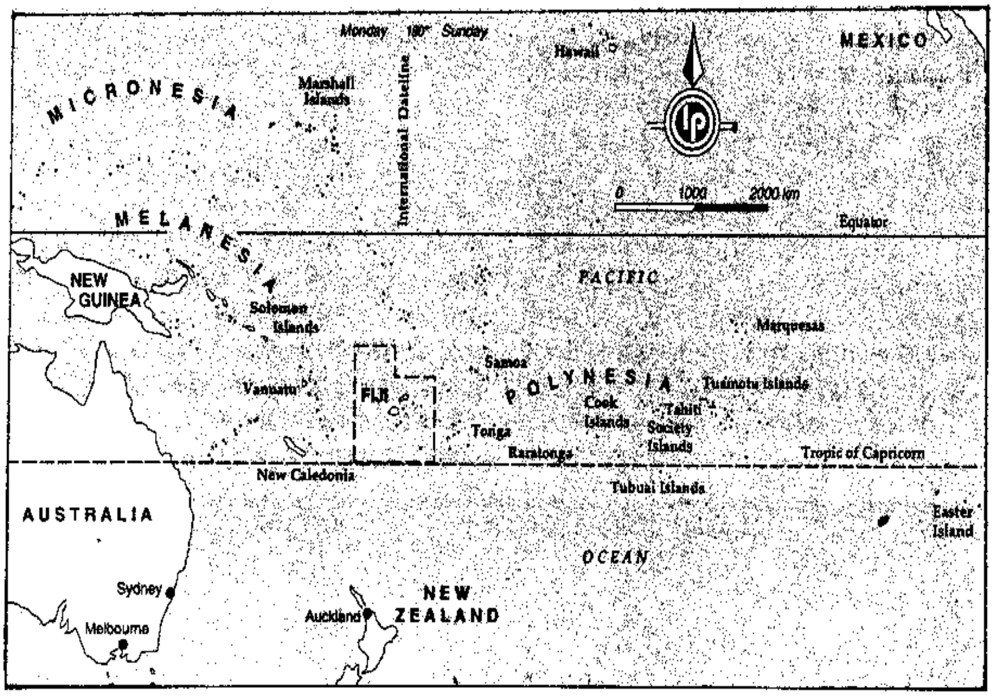

Figure 1.1 Map showing the position of Fiji in the south Pacific (Adapted from Jones & Pinheiro, 1997, P.9).

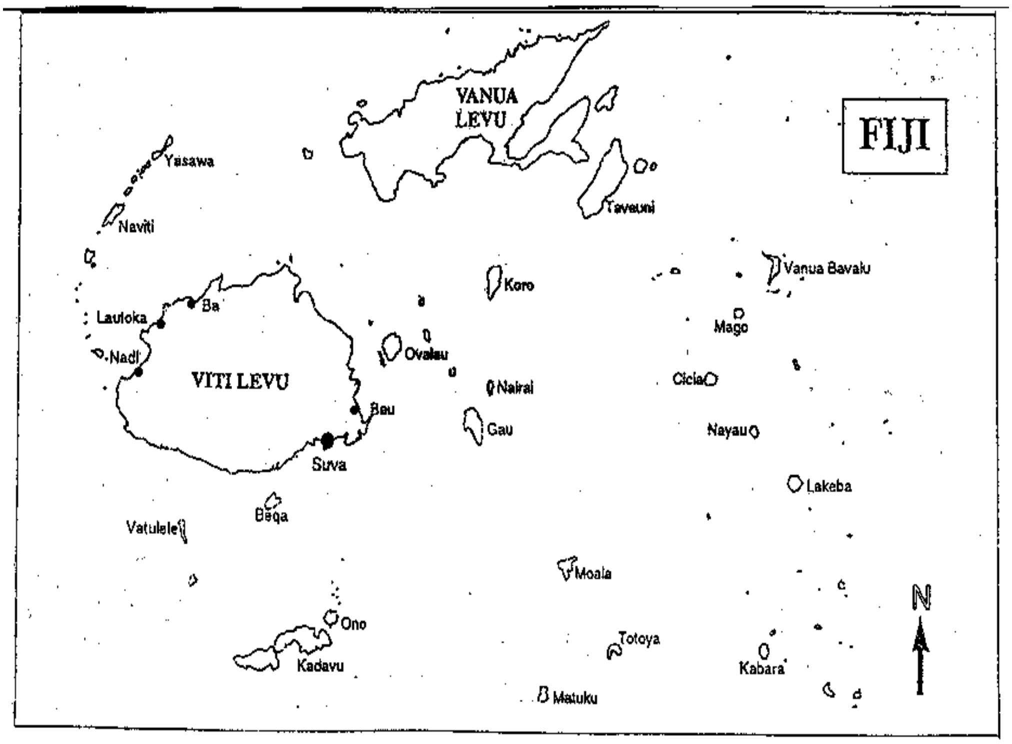

Figure 1.2. Map showing the main islands and cities of Fiji (Adapted from de Ishtar, 1994, P.120).

Structure of education in Fiji

In this section I will describe an overview of the current education system with a brief mention made of the place of mathematics education. Later in the literature review where cultural issues are discussed, I will give an outline of history of the system of education in Fiji. The facts for the overview in this and the next section are largely drawn from two sources, National Office of Overseas Skills Recognition (NOOSR) (1995) and International Development Program (1990).

Table 1.1 provides a summary of the structure of the current system of education. Though not compulsory, most children enter formal school at about the age of six. There are private pre-schools in the larger urban areas but anecdotal evidence suggest that generally indigenous Fijian children do not attend because parents neither see a need to enrol their children in them nor can they afford to pay the fees. At the moment many primary schools are undergoing a structural change in that they are continuing for eight years of schooling instead of the six. This means that some secondary schools are phasing out Years 7 and 8, the first two years of secondary schooling, and taking in the new Grade 8 leavers from the primary schools.

The number of levels in a secondary school generally depends on the population of the school. This in turn is generally determined by where the schools are situated, such as an urban coastal area with larger population, and smaller populations in remote inland places or on one of the smaller islands. Many schools of a few hundred students have Year 10 as the top class. The bigger secondary schools with enrolments of nearer to a thousand continue until Year 12 of schooling and a limited number, mostly in Suva or other major centres have Year 13.

What has been the situation of mathematics education in this structure? Looking specifically at what officially guides the type of mathematics taught in schools, we need to look to the policies and publications of the Ministry of Education. The authors of the Mathematics Curriculum have been generally influenced by their overseas education in either New Zealand or Australia. The staff in the Curriculum Development Unit (CDU) of the Fiji Department of Education are generally familiar with and accepting of the major directions in the international mathematics scene. They appear to work conscientiously to do what they see is necessary to promote good mathematics education with a number of them doing all the preparation work for the publication of revised editions of text books. They are constrained by insufficient time, facilities and money to fulfil many of their functions of helping the teachers effectively educate the school students in Fiji. At present anecdotal evidence suggests that the transmission mode of approach to teaching in the schools with much rote learning in preparation for examinations is very common. In time the influence of the CDU on what is happening in the mathematics lessons in the schools in Fiji will probably begin to take effect.

Table 1.1

Structure of education provided in Fiji

Level	Institutions	Providers	Education provided
Pre-school		Private	
Primary	Grades 1 – 6 or 1 - 8	Government, Church, Community	Schooling for pupils of age six until eleven or thirteen years.
Secondary	Schools with Years 7-13, 7-12, 7-10, 9-12, 9-13, 11-13	Government, Church, Community	Main subjects taught: English, mathematics. science, geography, economics
Tertiary	University of the South Pacific	Governments of the South Pacific region	Foundation Year, courses for Degrees, Diplomas, Certificates, Primary and Secondary teaching education courses
	Four teacher institutions	Government	Teacher education for lower secondary teaching
		Government	Primary teacher education
		Catholic	Primary teacher education
		Seventh Day Adventist	Primary teacher education
	Other vocationally-oriented institutions	All the above	Medicine and other medical courses, trades, technology, theology, agriculture

Fiji Teacher Education

In Fiji, teacher education, which is especially relevant to this study, is available for primary and secondary pre-service teachers, and for in-service teachers who are upgrading certificates and diplomas to degree or post graduate courses. There are three primary teachers' colleges: the large Government-funded College in Lautoka and two smaller non-government colleges. The Lautoka College has four hundred students and conducts a two-year certificate course. Corpus Christi Teachers' College (CCTC) in which this study is based, is one of the smaller colleges, with about a hundred students. It is located in Suva and administered by the Catholic Church. It conducts a three-year certificate course. The students who enter the courses provided by these two institutions generally have twelve years of schooling of a similar standard to those of equivalent years in developed countries. The qualifications gained in these institutions require an upgrading course of one year to be recognised by the Australian Government.

The Seventh Day Adventist Mission administer the other smaller College and about twenty teachers graduate from it each year. As yet their qualification is not recognised by the Ministry. Since 1999, University of the South Pacific (USP) has included in its education courses a Bachelor of Education (Primary) but at this stage it is only available to experienced primary teachers to upgrade their qualifications to a degree. The courses provided by all these teacher education institutions are comparable to those of developed countries having units similar in content and structure and staff generally trained in Western countries. Some differences include the allowance made for students' lower standard of English and a narrower range of units. As yet these education courses do not include much that is related specifically to the traditional cultures of the students but there are tentative moves to do so.

The site and purpose of the research study

The College in which this research was begun is a primary teachers' College operated by the Catholic Archdiocese of Suva with the goal to prepare teachers for schools in Fiji, but primarily for their own system of schools. It was opened in 1958 and staffed originally by expatriate missionaries. Later some of the Fijian ex-students joined the staff. In the nineteen ninety's when there were few foreign missionaries remaining in the country, there were not sufficient qualified, local or expatriates who were prepared to staff the College. As the College was not supported financially by the government it was unable to provide salaries equivalent to the Public Service. Also necessary improvements to the College such as the upgrade of library and technology were not happening. As there was a serious lack of mathematics teachers in the country finding suitable mathematics educators was especially difficult. I was fortunate to hear about the search for a mathematics and science lecturer and so was able to join the staff two years before I began this study. During my time at the College I also lectured part-time at the University of South Pacific to the primary and secondary pre-service and in-service teachers doing first degree and postgraduate teacher education courses.

When I arrived at the College it appeared to me that little had been done recently to develop an appropriate three-year mathematics program in the College. As described earlier and in the assessments that students completed for me in my first two years at the College, it appeared that they generally had little understanding of basic mathematical concepts such as the values of numbers greater than a hundred and rarely did simple computations mentally (Hanrahan, 1999). It would appear that such students were lacking an ability to use numbers with ease, the so-called quality of *number sense*. Reys and Yang (1998) say that it is number sense which typifies the theme of learning mathematics as a sense-making activity, an activity which it is hoped most primary school teachers would firstly possess themselves, and then help develop in their pupils.

As considered earlier, it is an open question as to whether teacher education is the most appropriate place in the education system to concentrate initiation of reform, but on a personal level it was the only one open to me. When it came to introducing appropriate change in mathematics education units at the College I was fortunate because what I chose to do in the mathematics education units in the course was generally my own responsibility. The only major controlling issues for the content of the units were the need to have the students prepared to teach weekly lessons in nearby schools and the content of the primary school mathematics prescription designed by the CDU for Classes 1 to 8. I saw the need to familiarise the students with basic teaching strategies and the contents of the prescriptions early in the course.

The main goal of the study became an investigation of how pre-service teachers in a College in Fiji could be helped to improve their mathematics learning during the first semester of their teacher education course. Because the study was completed in a developing country with factors such as limited finance, lack of personnel to support the study, serious political tensions because of racial strife and limited time for implementation, the study has special limitations. Similar studies have been completed in educational institutions in other countries. However although suggestive, such studies have few implications, as noted in the literature review, that could be applied to the situation in Fiji.

Research question

Before outlining the research questions, it would be helpful to give definitions of numeracy and number sense, terms which are used in the questions. Of the variety of meanings given to numeracy in school education the one that seems most appropriate for this study describes *numeracy* as the ability to have and be able to use appropriate mathematical knowledge, understanding, skills, intuitions and experience whenever they are needed in everyday life (Education Department of Tasmania, 1995). It is more that being only able to manipulate numbers which is closer to the definition of number sense which can be considered as a component of numeracy and is discussed in detail later. A workable definition at this stage is that *number sense* is the ability and inclination to understand and use numbers and their operations in flexible ways, to make mathematical judgements, and to develop useful and efficient strategies for managing numerical situations (McIntosh, Reys & Reys, 1997).

With the purpose of the study and these definitions in mind my four research questions prior to review of the relevant literature were as follow:

1. What initial mathematics education unit could be devised to best help pre-service teachers at the College gain the knowledge of numeracy they need to become effective teachers?
   In particular,

a. would the content need to be specifically designed to include all the areas of numeracy or would the development of number sense as the underlying theme suffice,

b. would the teaching approach need to have features of social constructivism to create the learning environment that would help the pre-service teachers develop their number sense?

2. How can the effectiveness of such a unit be evaluated?
3. Can the development of number sense be used to measure an improvement in numeracy?
4. If the answer to 3. is yes, then to measure the development of number sense, what elements does number sense have, which speak to the essence of numeracy?

Structure of the thesis

The sequence of the chapters follows a traditional progression of five sections from the introduction, to the literature review, followed by the methodology, the results of the research and finally the conclusion. The structure in this sequence has evolved during the study to become seven chapters including two chapters for the literature review and two for the results as explained below.

Initially I did not appreciate fully the need for much review of literature about culture and about pre-service teachers, and their conflicts and other experiences. As the study progressed I realised that there was much more to knowing mathematics than the cognitive knowledge aspect of numeracy. For *knowing,* the whole person is involved. For the participants of this study their whole person included them as persons of their traditional culture and as pre-service teachers doing the mathematics education unit. This realisation helped me to develop a framework, used in some cases for whole sections such as the literature review and sometimes for subsections such as the individual case studies. The three parts of the framework are related to the questions: 'who? what? and how?'. *Who* were these pre-service teachers as individuals, and Fijians as part of their environment? *What* was the numeracy that they were 'knowing'? *How* was the process of change occurring for them as they began their journey in becoming teachers of mathematics? As a result of considering these questions my framework consists of three sections: cultures and enculturation, understanding numeracy and the pre-service teachers as emerging teachers of mathematics and lifelong learners.

Thus the literature review first elaborates on the background and context of the study by discussing the cultural aspects of mathematics education both in the international and in the Fijian scene. Second it will discuss issues related to the current situation in mathematics education. Special attention will be given to ideas such as some theories of mathematics learning, the meaning of number sense, instructional issues related to number sense and computation skills. Lastly in the literature review the mathematics education of pre-service teachers will be examined. This will include topics such as mathematics knowledge, affective and metacognitive issues, and initiation of change. The literature review is structured with these three sections but for the appropriate balance of the chapters the review is divided into two chapters. The first considers the broad pictures of mathematics and culture, and mathematics education. The second chapter considers more specific issues related to number sense and mathematics teacher education.

The major research methodology used in this study is the multiple case study. This provides a rich source of data to help explain why changes occurred. These case studies are supported by other data collected from the class using research instruments including assessments, tests, interviews, questionnaires and journal writing. In the first section of the methodology chapter I will describe the instruments used and how the data produced was collected and analysed. Then in the second section I will outline my rationale and procedures that I have used for the multiple case study methodology to answer the research questions.

The reporting of the results of the research is divided into two chapters. The first of the two chapters describes the findings related to the planning and implementation of the Teaching Program. The second chapter, using the data collected from the instruments and the multiple case studies provides answers to the research questions about the changes that occurred and possible reasons for the changes. Conjectures from these results, other speculations and the limitations of the study are presented in the final chapter. Throughout the thesis the research questions will be redefined in the light of the discussion and content presented.

With an overview of the thesis in mind I now invite the reader to share my review of the literature related to mathematics education relevant to this study. As mentioned the first of the two following chapters discusses the broad pictures of mathematics and culture, and mathematics education.

CHAPTER 2 CULTURAL ISSUES AND MATHEMATICS EDUCATION

This study is set in Fiji, a small country in the South Pacific that has a culture considerably different from that of developed Western countries. As described in the last chapter, my aim in this study is to help some pre-service primary teachers in Fiji overcome difficulties related to their understanding of the mathematics that they will be required to teach. If one wishes to lessen difficulties related to the teaching and learning of mathematics in such a country, consideration may need to be given to appropriateness of the so-called Western mathematics with its teaching approaches, as well as to issues related to the traditional culture of the participants. In the first section of this chapter I will cover these considerations. After having investigated issues related to mathematics and culture, in the second section of this chapter consideration will be given to some aspects of mathematics education in the contemporary international scene. This consideration is necessary to help provide some answers to what teaching and learning strategies are the most appropriate in Fiji for the development of mathematics understanding.

Then, with some insight into the ideas of and other influences on teaching and learning described, in the next chapter I will review more specific issues for the development of understanding of mathematics. In the first part of the next chapter the issues related to number sense will be discussed with special attention given to computations. In the second part I will investigate how these issues are thought to affect pre-service teachers becoming effective mathematics teachers.

General cultural issues

In this section I will divide the literature review related to cultural issues into two main areas; culture and mathematics education, and Fiji and mathematics education. First, the consideration of culture and mathematics education includes some ideas concerning the development of culture, the features of mathematics that are considered to be ethnomathematical and those considered universal including mathematical activities, processes, and values. Considered in the discussion of the universality of aspects of mathematics is the acknowledgment of enculturation processes within mathematics into the various cultures, related to groups of people with some common feature such as homeland or occupation.

The second major section of this literature review of culture and mathematics will describe issues related to Fijian mathematics education because this study includes an investigation of an aspect of mathematics in a Teachers' College in Fiji. The few studies related to mathematics education in Fiji will be reviewed. One such study that has had an influence on my study will be discussed in some detail. This is an investigation of Fijian traditional mathematics and the possible insight it provides into difficulties of present-day mathematics learning in Fiji. Then, this and other studies that give insight into the background of the College participants will be reviewed under headings related to the difficulties in teaching and learning of mathematics in Fiji. The headings include language, home support for schooling, non-cognitive issues and the call for change in school learning.

Culture and Mathematics education

To consider mathematics education research in a non-Western context, the ideas of mathematics that are thought to be universal, and those considered culturally-bound, need to be distinguished. Today in any discussion of these two aspects of mathematics, the term ethonomathematics is used and so requires some explanation here. Ethnomathematics and related anthropological aspects of mathematics have been a growing interest in recent times in the area of mathematics education. Some of the relevant works for this study are the following: D'Ambrosio, 1983; Carraher, Carraher, & Schliemann, 1987; Bishop, 1988; Lancy, 1983; Harris, 1991; Pinxten, 1994; Lerman, 1994; Clarkson, & Leder, 1984; Barton, 1996; Bakalvu, 1997a. As cultural aspects of mathematics education are not the major theme of this study, only a limited discussion of the writings of these authors will be given. Once a useful meaning of ethnomathematics is established, in the next section the mathematics activities, processes and values that are considered universal and those considered culturally-bound are described. Possibly the emergence of the distinction between the universal and the culturally-bound areas of mathematics is partly the consequence of the conflicts that have arisen between the two and so finally, related aspects of conflict and the enculturation process will be discussed.

Hence in this section the review of the literature on ideas about culture and mathematics education is divided into five sections. These are the development of culture, ethnomathematics, mathematics activities and processes, mathematical values, and enculturation in mathematics education.

Development of culture

D'Ambrosio (1985) describes the concept of culture as the result of hierarchisation of behaviour from individual behaviour, through social behaviour, to cultural behaviour. He notes that, in advances in theories of cognition, culture and cognition have been shown to be strongly related. These advances have included a holistic recognition of the interpenetration of biology and culture leading to the opening up of a fertile ground of research on culture and mathematical cognition.

Bishop (1988) also sees culture as the result of a development, but for him he says the process begins with effects from the environment. He considers that people in a cultural group are those for whom the values of that culture act as a 'glue' binding them together. He then describes cultural development as the process in which the environment becomes more complex resulting in the culture responding by developing new ways to handle the new complexity. Then as the people develop new ways, these become subsumed into their environment and so the process continues. In this process Bishop suggests that cultural learning is a re-creative act on the part of every person. He suggests that every young person and every new generation of young people re-create the cultural symbols and values of their culture, their lives, and validate them within their lifetime. They also engage with the next generation, who in their turn re-create, redefine, and therefore re-live the symbols and values. In this age of globalisation Bishop considers that the process of cultural learning is complex because of the influences near and far that affect it. These ideas will be discussed in the context of learning generally, and for the learning of mathematics in Fiji in a later section

Smagorinsky (1995), who claims a neo-Vygotsky's perspective, believes there is a need to include cultural elements within formal education. When Smagorinsky is discussing Vygotsky's notions of zone of proximal development (ZPD) he says that, when there is little or no congruence between formal instruction and students' prior culturally-fostered tool use, instruction will fail. Smagorinsky therefore believes that there needs to be the provision of forms of mediation that involve students in problem solving experiences using appropriate cultural tools rather than using tools outside their cultural repertoire for tasks that do not build on prior problem solving experience.

Baba (1985) who was Head of the School of Education at the University of the South Pacific (USP) considers that, in the convergence of Fijian and Western culture in schools there are a number of issues to be faced by all those concerned with Fijian education and culture. He suggests that children must be made aware of the problem and so helped to make an effort to retain the culture they cherish bearing in mind their own needs and the national goals. Even though these ideas were presented in 1985 they have not lost their relevance:

> … we are living in a multicultural society and the education of young Fijians must prepare them for the world they find themselves in. They need to be aware of the modern ways of living and the linguistic variety that has become part of the Fiji society. This does not mean they have to lose their identity and a lifestyle they cherish, but due cognizance will have to be given to these modern elements while they retain some elements that are unique to Fijians. This requires an understanding of their needs and the goals in the context of the national goals, and a concerted and conscious effort to realise this. (Baba, 1985, p. 30)

Traditional cultural ways of thinking and learning must be considered but might it also be considered that learning and understanding is unique to the individual? Gunstone and White (1992) and other Western constructivists suggest, among other ideas which will be discussed in a later section, that learners construct their own meanings for the knowledge they acquire. The patterns of episodes, images, and other elements that result in the construction of knowledge will differ even if the persons have been together for a long time. In Fiji much of the learning gathered through incidental events would come from the traditional life in the village. Therefore, even though a group of students are from one cultural background, for each his or her learning might be seen to be unique. But this learning would have similarities with the learning of other students who are learning in the same culture.

From this literature related to development of culture and learning, it appears that authors do agree that individuals respond to their surroundings whether it is to their own biology, other people, or the environment. The question that is left unresolved is how much is one's behaviour influenced by one's traditional cultural group, outside influences such as schooling, one's biology, and in this day and age, by globalisation. For this study the influence of culture on mathematics which is known today by some educators as ethnomathematics needs to be considered, and so is discussed in the next section.

Ethnomathematics

One issue that has been debated in recent decades is whether different cultures develop their own mathematics. D'Ambrosio (1985) suggests that since people believed in the universality of mathematics, little has been done in the cultural anthropology of mathematics. He describes ethnomathematics, in contrast to the academic mathematics taught in schools, as the mathematics which is practised among identifiable groups such as national-tribal groups, children of a certain age bracket, labour groups or professional classes. He presupposes a broad concept of 'ethno' to include all culturally identifiable groups with their jargon, codes, symbols, myths and even specific ways of reasoning and inferring. He says that the focus of ethnomathematics depends largely on focuses of interest, on motivation and on certain codes and jargon. For these cultural manifestations and practices which constitute ethnomathematics, D'Ambrosio gives examples such as ciphering and counting, measuring, classification, ordering and modelling.

Bishop (1988) asked if there is one mathematics appearing in different manifestations and symbolisations, or if there were different mathematics being practised which have certain similarities. However from a mathematical educational perspective he believed that the concerns of mathematics are generally focussed by the implications of the differences between the mathematical knowledge of different cultural groups. He says that mathematics needs to be understood as culturally-based knowledge with cultural *forms* of mathematics. These will be discussed in the next section. He used the word Mathematics, with an uppercase M to designate Western mathematics which is the mathematics that is the result of developments both within cultures and between cultures throughout history. He describes this Mathematics as a particular line of knowledge development which has been cultivated by certain cultural groups until it has reached the particular form known today in the Western world as mathematics.

When Barton (1993) discusses the meanings of ethnomathematics described above and other similar attempts to define it, he suggests that ethnomathematics is much more pervasive than concentrating on objects of mathematics or the way we talk about it. Because he views mathematics as a process and action, he believes that ethnomathematics will be culturally determined to the extent to which a community of mathematicians share cultural characteristics, e.g. language, values, perceptions. Hence Barton would describe mathematical processes as being ethnomathematical rather than trying to define ethnomathematics.

These three authors agree that in different cultures there are aspects of mathematics that are specific to the cultures but whether they consider these aspects as part of ethnomathematics differs. Broadly these aspects include the manifestations and practices, the forms of mathematics, and the processes. Some of these activities and processes are discussed in the next section.

Mathematics activities and processes

Nearly two decades ago D'Ambrosio (1985) suggested that research carried out by anthropologists showed evidence of practices which were typically mathematical, such as counting, ordering, sorting, measuring and weighing and that these were done in radically different ways to those done in schools. At the time he was identifying within ethnomathematics a structured body of knowledge. He did this by collecting examples and data on practices of culturally differentiated groups which are identifiable as mathematical practices, hence ethnomathematics. He was trying to link these practices into a pattern of reasoning and to create a body of knowledge from a systematic organisation of the practices.

A few years later with the help of the available cross-cultural evidence, Bishop (1988) proposed the hypothesis that there are six universal activities and that these were the significant activities for the development of mathematical aspects of a culture. He attached the corollary that all cultures develop mathematics, that is, that mathematics is a *pan-cultural* phenomenon. He has chosen six mathematical activities and processes because he believed they conceptualised and defined the field of mathematics. These six activities and processes are counting, measuring, locating, designing, playing and explaining.

In this study the activities and processes directly connected to number sense: counting, measuring, playing and explaining are considered though not all directly in these terms. The study aimed to help students improve their skills and understanding of the mathematical processes and computations used in daily life and taught in primary school. Hence this study is limited to an area of numeracy which does not specifically include the spatial structuring activities of locating and designing. Bishop claimed that the six activities he described were part of the universality of mathematics but in describing what he proposed as the values of mathematics he did not attach the same universality. This is considered in the next section.

Mathematical values

Bishop (1988) believes that the mathematical cultural approach must attend explicitly and formally to values of mathematical culture. Bishop chose six sets of ideals and values which relate in complementary pairs to three components: ideological, sentimental and sociological. These values are:

1. Ideology: rationalism, objectism
2. Sentiment: control, progress
3. Sociology: openness, mystery

The following is a summary, illustrated with examples from Bishop (2001), of his descriptions of the values. *Rationalism* is the heart of mathematics with its focus on deductive reasoning as the only true way of achieving explanations. It is the main value that people commonly associate with mathematics as it involves logical and hypothetical reasoning. A person with this value would seek a relational understanding of the standard algorithm and so investigate the procedure to determine why it is a correct method. *Objectism* attempts to characterise a world-view dominated by images of material objects, not a subjective view of reality. Essentially the ideas about the material objects provide the intuitive and imaginative bases for the rationalism ideas. It is the power of objecting those abstractions which enables them to be handled so precisely. When pre-service teachers choose to make a concrete representation of a number such as 560 by using the appropriate number of straws bundled in tens and hundreds, they are enacting the value of objectism.

Control is concerned with a feeling of control and security as in the rote use of algorithmic procedures. The quest for information, explanations of natural phenomenon and the ability to predict facts are powerful knowledge. Knowledge is about control in this sense. Mathematics is not like other subjects which offer authoritative opinions. The complement to control is *progress* which is related to the dynamic feeling in change and development. The knowledge gained can be checked and verified by later generations. For example, in the classroom a student who has the value of progress will not be satisfied to use only a developed process in examples similar to the initial cases given, but will seek out other situations in which the process could be applied.

Openness is concerned with mathematical truths, propositions and ideas which are generally open for examination by all. The value of openness is promoted in a classroom in which the teacher encourages students to justify their answer to the class. Whereas *mystery* is related to questioning where mathematical ideas come from and who generates them. The value of mystery is promoted by a teacher who suggests to students to investigate interesting findings such as all the patterns they can find in the Pythagorean triplets and why the name Pythagoras is connected to them.

Bishop (1988) admits that, though values may not be taught explicitly, this does not mean that no values are taught. The mathematics education in past decades has developed a balance of the values that, in his view has over-emphasised objectism, control and mystery. He believes that a redress of the balance must be undertaken and he argues for the explicit attention to all the values in the curriculum. Therefore he suggest a curriculum that allows rationalism to be stressed more than objectism, where progress can be emphasised more than control, and openness can be more significant than mystery so as to redress the imbalance. The development of a deep understanding of the values of mathematical culture is one of the five principles that Bishop (1988) looks for in the education of teachers. However discussing values with teachers has not proved to be easy (Clarkson, Bishop, FitzSimons & Seah, 2000a, 2000b, 2001). Bishop suggests proper use of project work is a way to assist this development through the analysis of society's present uses of symbolisation and conceptualisation of mathematics.

The broadness and variety of the understandings of the nature of mathematics make it difficult to structure an investigation of students' ideas related to its nature. However I consider that the important aspects of the nature of mathematics can be related to each of Bishop's values. Therefore I will propose at a later point that students' ideas of the nature of mathematics can be examined using a framework structured on Bishop's six values.

Enculturation in mathematics education

Bishop (1994) says that up till the mid-eighties mathematics was generally considered to be a culture-free knowledge. For example, Piaget (1958, cited in Lancy, 1983) assigned minimal importance to culture in performance of conservation-of-length tasks. Even in some of Piaget's studies where a difference was found for the effect of education, it was not significant for children below the age of fourteen years. However in Papua New Guinea (PNG), Lancy (1983) found that for the performance of conservation-of-length task, education and culture effects overshadowed age effects. Bishop believes cultural consonance has been assumed because of the lack of understanding of mathematics as cultural knowledge and the lack of awareness of any values underlying mathematical knowledge. Bishop (1988) believes that educating people mathematically is much more difficult, and the problems and issues more challenging, than just teaching some mathematical content. He considers a mathematics education is concerned with 'a way of knowing' and this speaks to him of a cultural perspective on mathematical knowledge.

Bishop (1994) believes that difficulties and anxieties in learning mathematics have been contributed to the societal pressure to learn, to only psychological explanations, and to pedagogical teaching and materials that emphasise the precise following of procedures. He believes that all formal mathematics education is in part a process of cultural interaction and that every child experiences some degree of cultural conflict in the process of enculturation into the school mathematics culture. In particular he believes that many people experience a dissonance between the cultural tradition of schools, and those outside schools, such as those in the home. This means that educational decision-making in mathematics takes on a new order of complexity.

Because of the dissonance between in-school and out-of-school cultural norms, it is very unclear what the educational task should be. Some of the conflicts Bishop discusses are concerned with language, calculations, procedures, symbolic representations, logical reasoning, attitudes, goals and cognitive preferences, and values and beliefs. For example, the procedures for rounding off when approximating in school requires one to choose the closest whether below or above, whereas if finding the approximate money left to spend one approximates to the need to have the money closest above.

On consideration of conflicts during enculturation in mathematics education classes, Bishop (1994) notes that there are different interpretations and responses to conflict. He classified his approaches to cultural conflict into five groups: culture-blind traditional view, assimilation, accommodation, amalgamation and appropriation. The title of the group *culture-blind traditional* describes its view of culture. *Assimilation* includes remediation for children with difficulties due to cultural differences. In *accommodation* there is a re-structuring of curriculum allowing culture to influence the education with a modified teaching style of one preferred by children with the support of different languages. For a*malgamation,* aspects such as the bilingual and bi-cultural are incorporated with the culture's community. And for *appropriation,* structures such as a special early-primary-years educational system conducted at a village level are implemented. As a temporary lecturer in a teachers' college I am not in the position to introduce much change, so the approach I considered appropriate for me was akin to *accommodation* which includes some restructuring of the curriculum and occasional use of different languages.

Summary of culture and mathematics

As a foreigner to Fiji I considered that to plan and implement a mathematics education unit in a College in Fiji would require a knowledge and understanding of the relationship between mathematics and culture, both culture in the general sense and traditional culture. From the literature I noted that some aspects of mathematics knowledge and its learning are thought to be culture-bound. There are values, activities and processes that are thought to be basically universal, but their manifestations and practices are ethnomathematical so will vary with the culture in which they exist. In formal education conflict is believed to arise during enculturation into school mathematics and so there is a need to consider how to lessen any harmful effects where possible.

Fijian culture and mathematics

As this study is set in a Fijian educational institution in which the students are generally Fijians, foreign to me the researcher, the Fijian cultural background in relation to personal histories and educational context needs to be considered. Also as I am investigating possible solutions for overcoming the difficulties the pre-service teachers are having in their learning of mathematics, understanding the background of their mathematics education is important. Therefore in the first part of this section after briefly considering possible problems related to conducting research in Fiji, I will describe a brief history of education in Fiji that relates to aspects of this study. Next an important study which investigated Fijian traditional mathematics will be examined in some detail. Then other research related to mathematics learning in Fiji will be discussed under the headings: language in the mathematics classroom, home support for schooling, affective issues and calls for change in learning strategies.

Problems related to educational research in Fiji

To conduct research in a country foreign to me I believe it is important to do a relevant and sensitive study of cultural issues related to the research. I see myself in a situation similar to Harris (1991) who completed a study of the teaching of mathematics to the Australian Aboriginal children. She did not want her research to be seen as another form of exploitation of the indigenous people. Instead she hoped that her investigation of the work of the teachers of Aboriginal children had the function of raising the teachers' awareness of the types of questions that needed to be asked in reference to their teaching of mathematics. In my study I was also interested in the teaching of mathematics and specifically sought to improve pre-service teachers' understanding of mathematics. I do not expect to produce the answers for how to help Fijian pre-service teachers learn mathematics, but am hopeful that my study may provide some questions that may lead to helpful answers from future researchers. (I also believe on another level, that the students will benefit immediately.) The participants of my study are Fijian students in whose country I had lived for only three years. Compared to a researcher who belonged to the culture of the participants I therefore have a limited knowledge and understanding of the culture and hence have a greater need to study the literature related to the culture.

I was conscious that I had come from a comparatively large Western country to a small developing Pacific country. Therefore in applying the suggestion of Clarkson and Leder (1984) when discussing education in PNG, I needed to be more aware of the differences between my new students and Westerners in relation to learning styles and attitudes. To help me do this I reviewed the relevant literature that would better enable me to make choices to plan and implement a teaching program. In this teaching program I allowed for and encouraged the learning styles which would best suit Fijian students so that they did not feel as if they were losing part of their identity whilst learning their mathematics.

Taylor (1997), who did research similar to mine in another Fijian primary teachers' college but in the area of science, used teaching strategies which were largely derived from Western research. Interestingly he found that these strategies produced considerable success in the context of Fiji with the strategies appearing to be effective in improving the understanding for students from different cultural backgrounds. Taylor suggests that cultural influences may be minimal in the construction of science conceptions and suggests that this outcome may be important when considering those involved in teacher education and curriculum development in other developing countries. He concludes that more research is necessary to determine the truth of the suggestion. Such a conclusion was some source of comfort. However my experience in

Papua New Guinea suggested that perhaps it was not as straight forward as Taylor concluded.

Brief history of education in Fiji

The first 150 years of European influence in education did not specifically allow for any elements of Fijian culture to intrude except at times the use of indigenous languages. Missionaries came to the islands in about 1830 and, as well as their church activities, they provided basic European schooling. The first Methodist Mission set up "village schools at which children could learn to read and write their own language and at the same time gain some proficiency in arithmetic" (Mann, 1935, p.23). Fiji became a British colony in 1874 and remained so until it was granted independence in 1970. According to the National Office of Overseas Skills Recognition (1995) the first Government school was opened in 1881 and this was followed by other Government schools whose number has never been great compared to that of church schools. In 1924 New Zealand became responsible for education in Fiji. This lasted until the mid 1960's. The New Zealand curriculum was used in both primary and secondary schools until the Fiji Curriculum Development Unit (CDU) was established in 1969. Then the CDU devised curriculum which gradually replaced the New Zealand one from that time. Secondary students however continued to sit for the New Zealand examinations until the late 1980's.

The present perceived need for change in the mathematics education in Fiji is part of the Government's call for reform in education in general. Throughout the Twentieth century there have been attempts at reform in education by the governments of the day. Mann (1935) notes that the Government set up Education Commissions in 1909 and 1926 which resulted in major changes, such as the Fijian Colonial Government taking control of country's education and a system of grants-in-aid was instituted to provide Government funds to non-government schools. Following an Education Commission in 1969 a further major movement in education came with the setting up of the Ministry of Education after Independence in 1970. In 1971 the leading Fijian teachers of mathematics participated in a workshop funded by UNDP / UNESCO which produced guidelines for curriculum development. Five years later the objectives and recommendations of the workshop led to the introduction of the Fijian Curriculum for Forms 1 to 4 mathematics (Muralidhar, 1989).

According to hearsay during the last ten years whichever government has been in power has been motivated by the dissatisfaction of the standard of education of school leavers and the failure rate of students in the first year at University, to reform the education system. The Government is looking for an education for the children of Fiji that would be relevant and productive. In 1997 it authorised a group of experts from both within and outside the country to produce a reform document "Education 2020". But that Government lost power before the document was published. The new Government in 1999 set up an Education Commission with some of the same experts on the Commission to examine the state of education in Fiji. The results of the Commission have been completed and are to be promoted in the near future. All these recent moves show that there is at present a perceived need for change in the education system.

From the Government's action it can still be assumed, what Mann concludes in his book *Education in Fiji* (1935), that there is an urgent need for reforming the curriculum so what is taught will more fully serve the life-interests of the people of the country. He believed that the imposition of a traditional European education in Fiji was unsuccessful, and therefore people expert in their field and familiar with the racial, social, political and economic conditions of Fiji need to be the ones researching and creating the curriculum material. As yet there are few 'experts in their field' who have the opportunity to undertake much study. Perhaps this is because the more highly educated students leave the country to seek an economically-better life in nearby Western countries. But there are a number of Fijian researchers in the area of mathematics education who have played their part in doing what Mann suggests. These are Metcalfe (1978), Prasad (1988), Muralidhar (1989), Bakalevu (1997a) along with a number of Masters students in the University of South Pacific who are at present carrying out research in mathematics education. The study by Bakalevu, as well as discussing the present mathematics education in Fiji, examines the traditional mathematics of the indigenous Fijians. In the next section Bakalevu's study will be reviewed in some detail. And then in the section following, it and other studies will be reviewed within a framework of some important cultural issues for Fijian education.

A research study of Fijian traditional mathematics

In this literature review so far I have discussed cultural issues in general to prepare the context for an examination of the specific Fijian cultural issues that I expected to influence the work for this study. As mentioned earlier there are a limited number of studies completed in Fiji. However these do have importance because of their location and will be now discussed, beginning with the major study undertaken by Bakalevu (1997b).

The study by Bakelavu is as yet the only formally researched source of information about the mathematics that is part of the indigenous Fijian culture. Through extensive interviews, observations and review of archival material Bakaleveu has examined the Fijian notions of education and learning as well as identifying Fijian mathematical ideas normally hidden in cultural and everyday activities. A broad outline of the some of the knowledge that she gathered will be given here. This cultural knowledge is divided into two sections: traditional notions of education and traditional mathematics including counting and the number system.

Bakalevu used as the framework and starting point for her research and analysis, Bishop's identification of the six activities which conceptualise and define the field of mathematics developed in any culture. Then she concentrated on the features of the mathematics that are important practices that differentiate culturally the Fijian society such as counting. Educators such as D'Ambrosio (1985) and Bishop (1988) would considered these features as ethnomathematics.

Bakalevu (1997b) says that the traditional learning process is part of everyday living with no specific instructions given and no designation of teacher or student roles. This is indicated in the dual meaning of *vuli* as *teach* and *learn*, although in today's speech in Fiji this notion has developed to take into account defined roles and actions in formal schooling.

Another notion of education is given in the Fijian words used for the different stages of a child's development. The words are used to describe how Fijians consider the possibilities of learning at each stage. Up to the age of five a child is *tawa vakayalo* (without spirit) and the five to nine year-old child, *yalowai* (of watery spirit). These are common terms used to excuse a child's mistakes and justify an adult's firm hand. Bakalevu notes that these terms are not related to the learning that a child does as he or she learns at school but applies to traditional learning of skills from family members. Some Fijian parents have a belief that it is useless to give children schooling until the child's spirit has become stable, hence they are not interested in pre-schools and kindergartens for their children. Also some people believe that a certain new spirit is needed with school learning as this learning occurs in an environment very different from the traditional culture and it comes with stress and anxiety for the child. From this it can be seen that Fijians see school learning as a difficult experience.

The attaining and becoming, in Fijian society is *yalo-matua* (mature spirit). This is suggestive of an intense spiritual being and is differentiated from *kila* (know), which is a mental capacity and normally associated with school. Bakalevu discusses a traditional value which could be transferred to learning in the classroom, of cooperative effort and interdependence. Fijians see chiefs as gifted to lead and they are looked up to for appraisement, judgement and direction. People esteem a strong hand and are implicitly submissive to the word of elders and chiefs. Bakalevu noticed during interviews that the informants used the words *we* and *our* more than the singular pronouns. A Fijian finds it difficult to work alone, but becomes enthusiastic and energetic when with company.

The second section of the traditional knowledge outlined in Bakalevu's research discusses the ideas about mathematics, especially counting. Counting is not part of the everyday fabric of traditional Fijian society. It does not interest the Fijian and they see no reason to count. In their cultural setting they do not collect but continually share except for special ceremonies. It is culturally inappropriate to touch as one counts as finger pointing is considered rude. The Fijian words for the index finger is *dusidusi ni turaga*, the chief's pointing finger, because it is his prerogative to point. Counting to a Fijian suggest a calculating inclination and precision that they do not need. Bakalevu noted an interviewee who said that if ten men were needed for a job it is not much different to them if thirteen or fifteen turn up.

Bakalevu (1997b) says that indigenous Fijians use qualitative evaluations rather than numerical, such *as many, few, full, empty, high pile*. This use of descriptive terms is due to the inhibition to measure using culturally inappropriate counting. Rather than count how many mats there are among the gifts at a presentation the people describe what the mats look like. This is more true for women, than for men who are impressed with the quantity thus raising a question about whether this difference correlates with a gender aspect in the understanding of number and may be an area for further research.

Bakalevu noted that the complete Fijian number system is only used in traditional presentations. The framework in which this counting takes place is primarily social. In these ceremonies the Fijians employ an elaborate counting system that uses base ten. They have specific words for ten objects such as ten pigs (1 rara), 10 turtles (1 bi) and 10 mats (1 sasa) and they have words for 100 coconuts (1 koro) and for 1000 coconuts (1 selavo). Sometimes a word is not for an accurate number of the items, such as a high pile of taro is 1 duludulumata. Even though this counting is done, sensitivity, respect and modesty would see parties under-rate and depreciate their contributions.

As mentioned earlier, Bishop (1994) considered how cultural conflicts arise in mathematics learning. Bakalevu (1997b) found conflicts that are contributing to problems created because there is a cultural mismatch for Fijian students. Some of the mismatch is between the home and the school environment, the language spoken at home and English used in school. In the Western-style life at school Bakalevu notes, students find it hard to find links between school and home knowledge. She believes this calls for change in mathematics education. Her solution is to widen the perspectives in the classroom to include mathematics with cultural elements. She suggests that the mathematics learning of Fijians can be helped by finding the common elements and differences between traditional and Western culture. One change she offers could be the using of the numerous qualitative terms that are present in Fijian conversational forms to construct benchmarks and number ideas.

For this study the important outcomes of Bakalevu's research (1997b) are those related to mathematics learning in Fiji, specifically to number sense development. In the summary of her findings she suggests that her research shows that mathematics is a culture-bound phenomena and that the traditional mathematising by the indigenous Fijians is probably a reason why they have an apparently poorly-developed number sense. Some examples from Fijian culture that she considers are not helpful in developing number sense are that quantification is normally given in qualitative terms, and that large numbers are not part of the everyday culture. Other more general changes that she suggests will be noted in a later section where hers, along with others' ideas about mathematics schooling in Fiji are discussed.

I highlight these ideas about number sense and Fijian learning because they give me insight into the difficulties that I aimed to help the pre-service teachers overcome. Also, I believe that if the pre-service teachers study her research, the findings may encouraging them to devise ways to implement Bakalevu's suggestions to assist them to develop their own number sense and to become more effective mathematics teachers.

Further research into mathematics education in Fiji

The present study is concerned with the mathematic education of students in a teachers' College. For information on the background of the students it is important to review the limited amount of research completed about mathematics learning in schools in Fiji. Using headings related to mathematics education, this section discusses various problematic aspects discussed in the research. The main sources of the material relevant to this study are from some studies completed in the last few decades. Until a couple of decades ago there had been very little research related to the way in which Fijian children develop mathematical concepts (Hopkins, 1978). At the time this was being said, Metcalfe (1978) was doing some pioneering work in this area. Two other studies (Muralidhar 1989; Prasad, 1988) have examined how students were performing at their mathematics in high schools. I will now discuss these studies, Bakalevu's (1997a) and other South Pacific studies using the headings of language, home support for schooling, non-cognitive issues in learning and finally, the call for change in learning strategies.

Language in the mathematics classroom

In Papua New Guinea (PNG) the interaction between language and mathematics was an issue that Clarkson (1991) notes as among the issues of great concern for education. At the time he suggested that it was time for PNG to devise its own particular teaching styles to tackle the difficulties inherent in such a multi-lingual context. Bishop (1979) when discussing language and mathematics, also in PNG, believes that the problems were often the result of the vocabularies of local languages which were not designed to accommodate recently introduced areas of formal education. He gives examples of terms used in geometry such as *above, top, over* and *up* which in one local language have the same word.

Along with PNG, Fiji is a developing country in which the students are generally completing their schooling in a language other than their first language. The official form of the indigenous Fijian language, the Bauaun dialect, is spoken as the first language by only 15% of the population. Vocabulary differences as high as 40% are found in the dialects of the main island. With teachers coming from various parts of the country many Fijian children are often taught in a vernacular quite distinct from their own (Moag, 1978).

Bakalevu (1997b) believes that language difficulties in mathematics in Fiji need attention. She sees the vocabulary of mathematics language as causing problems. Many of the words used in mathematics have different meanings when used in everyday speech including word combination, for example, *leaves* and *simple interest*. Bakalevu also notes that the texts for the senior students were written for overseas students. Students progressing through secondary school have to cope with new meanings for words in mathematics and continually face more technical language and master increasingly complex symbolism often resulting in rote memorising and recall. Bakalevu believes that this situation needs urgent attention to avoid having frustrated students whose learning is inhibited through no fault of theirs or their home background. A suggestion she offers is that there needs to be an effort made to assist children to move naturally and effectively into technical and abstract language, and the symbolism of numbers and operations.

But it is not just a different vocabulary that is cause for concern. Muralidhar (1989) who examined some aspects of how students were performing at their mathematics in high schools found, as have other educators mentioned below, that the students had language difficulties with word problems and with symbolic notations. The students were unable to recognise which operations were needed to solve the problems. For example, words such as *take away* in a word problem was thought by many to require a division sum. She recommends that word problems be drawn from a variety of real-life situations relevant to the local setting. She suggests that students should be given the opportunities to construct situations for expressions that involve number operations. Even though the present junior secondary textbooks were written for Fijians, Muralidhar notes that the language is difficult for them and contains many exercises and only a few word problems or investigations.

Prasad (1988) who explored the major influences on the mathematics achievement of the Indian Fijians compared to indigenous Fijians also saw that an important area of learning that needed attention was the language competence. He thought any effort made to improve it would very likely improve the students' achievement in many subjects including mathematics especially for indigenous Fijians.

Being bilingual is not all bad news in mathematics. Clarkson (1992) concludes in a study done with primary school students in PNG that bilingual students, competent in two of their languages scored significantly higher on two different types of mathematics tests compared to colleagues who had low competence in their languages. This was so when other factors such as home background and cognitive development were controlled for. He also suggests that the results indicated that the use of the student's original language could be used to good effect in accessing the mathematical ideas of local cultures in the local language without the fear of disadvantaging students in their learning of mathematics.

In summary the issue of language is one that can adversely impact on Fijian students learning mathematics. There is a need to explore further Clarkson's assertions to see whether they also apply in the Fijian context.

Home support for schooling

As well as a mismatch between the mother tongue and the English language a number of researchers note a mismatch between the importance given to education at school, especially in mathematics and the support it receives at home. Muralidhar (1989) notes that the subject of mathematics itself earns high respect in the community and this raises issues on a number of fronts. Many employers want a good mathematics report as an entry requirement to their jobs. Parents' belief in the importance of the subject leads them to thinking it is important that children score high marks in mathematics irrespective of the children's ability and that a good mathematics teacher obtains a high number of passes. But when it comes to parents' support, especially towards better achievement in mathematics, Muralidhar comments that there is not always encouragement and motivation provided by the parents.

Prasad (1988) also discusses the need to encourage home-based support. He says Fijian parents do not emphasise reasoning, independence or responsibility in their children and that these are the factors likely to influence their achievement in secondary school. He also sees a need to investigate constructs such as achievement motivation and independence training. Prasad cautions that, when implementing the necessary changes in teaching strategies, the methods used should not violate traditional values. He suggests using the traditional values of cooperation, affiliation and group goal-setting, and instilling responsibility to improve mathematics achievement.

Even though he gives no research to support his statement, Baba (1982) also says he believes that in Fiji, values and personality aspects such as motivation levels, aspiration levels, and loci of control are shaped by their socialisation practices at home. Fijian parents foster co-operation rather than competition in their children. He also believes that the indigenous Fijians are less competitive academically and have lower educational and occupational aspirations. Hence he says that these factors that lessen motivation to excel, are particularly important in explaining lower achievement of the Fijian students.

Although these statements were made by Baba about two decades ago, anecdotal evidence and the findings from the other research mentioned continue to support these statements. In this study the pre-service teachers are students who in most cases may have lacked home support for their academic studies during secondary school. The effects of this has probably made the enculturation into mathematics education at the College difficult for some students as they may often lack the self-discipline to complete the requirements of the units.

Non-cognitive issues

In attempting to discuss the non-cognitive issues, including affective issues related to mathematics education in Fiji, it is difficult to separate the affective from the metacognitive issues. Since feelings greatly affect the process of learning, here the discussion of both are combined. From the literature about the needs of students who are having difficulties learning mathematics in Fiji, one recommendation appears a number of times, but described in different terms. It is that there is a need to develop confidence in the learning of mathematics both as an affective issue as well as a metacognitve one. Bakalevu (1997b) believes that an intuition about numbers and the ability to move flexibly and intuitively through the world of numbers is the confidence that Fijians lack and desperately need. She also believes that this issue is one for the teachers of these students, suggesting that teacher education courses need to foster renewed confidence and consciousness in pre-service teachers of mathematics. She says there is a need to build in teachers, confidence in their knowledge and their practice because in these times of major changes and greater specialisation, expertise in mathematics teaching is needed.

Muriladhar (1989) notes that the mathematics curriculum of 1980 was the outcome of the objectives and recommendations of the UNDP / UNESCO workshop which stressed that students be encouraged to participate in discussions to help them to become more self-confident. This workshop lead to the introduction five years later of the Forms 1 to 4 mathematics course. But she continues by suggesting that the majority of teachers, who have little say in curriculum matters and have pressures of large classes and heavy teaching loads, see their main role to teach the content prescribed and not 'waste their time' in trying new methods of teaching. She also suggests that parents and teachers have a role to play in encouraging pupils to develop healthy attitudes towards mathematics.

Prasad (1988) recommends that trying to help the students develop a sense of self-efficacy may contribute to their achievement. He adds that teaching them to attribute success and failure to effort and the use of achievement-oriented strategies for learning, may prove fruitful. Fijian society is authoritarian and Prasad believed that this is likely to foster dependence in the young. This in turn would make the indigenous children less suited to the handling of abstractions or principles which form the basis of mathematics and partially account for some of the ethnic disparity with the Indo-Fijians in mathematics. His ideas follow from the results of his questionnaire specially adapted to measure locus of control within the area of mathematics. His results suggested that indigenous Fijians were likely to believe that reinforcements are under the control of external forces because of their home environments which are usually authoritarian. Also he notes that the students need to be helped to develop a sense of self-efficacy which may contribute to their achievement. But much more research needs to be done first on home socialisation and child-rearing practices. Even if the studies suggested were done, Prasad warns that there would still be the difficulty of using the results in such a way that traditional values are not violated.

Professor Baba (1982) believes that personality aspects such as motivation level are particularly important in explaining the lower achievement of the indigenous Fijians students. Kishor (1982) did research in Fiji in this area of affective issues. He investigated the locus of control of 540 Year 9 Fijian students' by administering a general questionnaire, the Nowicki Strickland Scale. He found that the ethnic Fijians were significantly more *external* than the Indian Fijians. As well, the locus of control scores had a much stronger relationship with a composite measure of the achievement scores for the Fijian students (0.57) than for the Indian students (0.41). It appeared to him that the greater externality of the indigenous Fijians emerged as a considerable handicap for their academic success.

As for examinations, Pajares and Schunk (1997) discuss issues related to the completion of them. They comment that self-efficacy and self-concept researchers agree that some factors work to destroy the fragile self beliefs of those who are ill-prepared or less academically talented. These factors include the competitive grading practices and the encouraging of students to compare their achievements with that of their peers. In Fiji for the past few decades there have been five of the thirteen years of schooling that have had external examinations. So Fiji students have been described (and could still be more than twenty years later) as among the most-examined upper primary and secondary students in the world (Hopkins, 1978). Pajares and Schunk (1997), when discussing academic motivation research in the international scene, say there is ample evidence that both self-efficacy and self-concept beliefs are related to, and influence academic achievement. They describe self-efficacy beliefs as revolving around questions of 'can' whereas self-concept beliefs reflect questions of 'being' and 'feeling'. They suggest that many students have difficulty in school, not because they are incapable of performing successfully, but because they believe they are incapable of *believing* they can perform successfully. Pajares and Schunk recommend that children be provided in school with challenging tasks and meaningful activities that can be mastered and that they also receive support and encouragement for their efforts in order to help them develop a robust sense of self-worth and self-confidence. They continue by stating that school is the primary setting in which cognitive capabilities are cultivated and evaluated.

Calls for change in learning strategies

In the past decade anecdotal evidence indicates that continual staffing of high schools with untrained teachers has contributed to a plateau effect in the achievement levels of students in mathematics. Many of the students are taught by indigenous secondary teachers who usually have a tertiary qualification which does not include teacher education. This lack of trained teachers was mainly due to the emigration of a large number of teachers from Fiji after the 1989 Coups.

But research has shown that the teaching by untrained teachers may only be part of the reason why these students are now experiencing difficulties in mathematics. Metcalfe's (1978) results suggest that conservation of number, length and area were not established among the majority of children at ages when the school syllabus in Fiji required manipulation of number symbols. He comments that it was therefore not surprising that the students resorted to rote learning when they were presented with work which they were not equipped to handle with understanding. He suggests that his study indicated that the then primary school mathematics learning was a shaky foundation upon which to build the secondary learning. There is little evidence to suggest that much has changed in the last twenty years in mathematics teaching in many schools so the foundation is possible still 'shakey'. Prasad (1988) later concluded that the Fijian students' lack of competence in primary school was likely to be the main cause of their difficulties in mathematics in secondary school. He admits that little had been done to further investigate this hypothesis. Such comments seem to indicate that primary teaching institutions need to become more involved in improving the mathematics of students in Fiji, perhaps with among other things the help of research. If the pre-service teachers in teacher educational institutions are assisted in developing learning strategies which they are able to pass on to their pupils to help them with their mathematics, the number of students with major difficulties may decrease.

Another study (Nabobo & Teasdale, 1995) found that cooperative work was an effective learning strategy in class. The research was a case study in a Fijian teachers' college where cultural studies units were being introduced. The authors of the study found that indigenous Fijians preferred to work collaboratively in small groups and that they were more successful in making oral presentations than they were in producing written work. In the school setting, Nabobo and Teasdale found that the greatest opportunities for cultural education was in the rural isolated schools through the medium of music and dance. This last strategy would appear to have a limited application in mathematics, but with the indigenous Fijians' creative ability it may have possibilities.

In a teachers' college this teaching strategy of cooperative work could be partly utilised by giving the students tasks in which they work together to present work to their peers. This could include small items such as acting out a story to correlate with a graph such as in the RIME activity 'Tell me a story' (Lowe & Lovitt, 1984). With encouragement and guidance they could progress to conducting seminars on topics that they have researched. Another activity suitable in a teachers' college involving performing, is peer teaching in which short primary-school mathematics activities could be presented such as five-minute Warm-up activities as used in PNG (Gough, Lilburn, Rawson, & Sullivan, 1991).

As discussed earlier, Bakalevu (1997b) sees an important need to link cultural and school learning. She believes that the traditional mathematical ideas used at home could be used as stepping stones and as critical pathways to make sense of the formal concepts difficult for the child when they come to school mathematics. With regard to the idea that Fijian children have a poor number sense she suggests that instruction must begin by helping the children to develop a good number sense which she saw as a process that is complex and takes time. Along with the previous suggestions Bakalevu believes there is a need to make mathematics fun. The present mathematics text books in Fiji have investigations and other interesting activities in them but from cursory observations I notice that few are used in the classroom as teachers are too anxious to do questions similar to the examination ones, and hence resort to 'chalk and talk' strategies.

When Muralidhar (1989) compared the numeracy skills and knowledge of the Fijians to a British sample using a standard test for understanding fractions, Chelsea Diagnostic Mathematics tests, the British had 60% of the sample on the highest level whereas the Fijian' sample had only 5% of the sample at this level. Muralidhar describes some areas of difficulty experienced by students in classes 7 and 8 as: understanding the meaning of the basic number operations, recognising the appropriate operation in word problems and constructing of a story problem for an operation, including recognising fractions and their equivalence. She made some recommendations related to fractions including the following which have implications for learning strategies that Fijian students presumably employ: provide opportunities for students to come to recognise the different representations of fractions, help students to recognise the equivalence of fractions in a word setting and in a symbolic setting, guide students to see fractions as single entities rather than as two numbers, and encourage students to justify their decisions through discussions in a variety of situations involving sizes of fractions.

Muriladhar (1989) also offers suggestions about change related to attitudes to mathematics education in the schools. She notes that schools strive to achieve high examination scores and do not worry about the understanding of mathematics. She believes that a major change that is necessary is to lessen the over-emphasis on learning for examinations. The prescriptions of the latest Fiji mathematics courses for schools recommends a discovery approach be used related to mathematics of everyday situations as far as possible, but she comments that many of the schools do not appear to follow these guidelines. Muralidhar suggests that the students should be given problems and investigations as a means of promoting mathematical activity with emphasis placed on understanding rather than on rote learning. She continues by commenting that most teachers of Fiji's latest mathematics curriculum were not involved in its formation and therefore this is a possible reason why they have not changed techniques or approaches but continue to drill for exams rather than 'waste time' on new ideas. When she herself attempted to use new teaching approaches, such as playing an appropriate mathematics game her students thought it was a waste of time as the game was not in the syllabus. But after discussion with them they appreciated the benefit of playing the game. Among the changes Muralidhar recommends in the classroom that have not been described already, is the need for more opportunities for practical activities and use of materials in order to provide useful dialogue between students and the teacher. Also students need to be able to discuss their strategies for arriving at their solutions. Their unacceptable responses or mistakes need to be investigated to see what in their thinking and understanding has led to their errors or misconceptions.

Summary of findings related to Fiji mathematics education

This study aimed to help find solutions to the problems pre-service primary teachers at a teachers' College in Fiji are having learning the mathematics that are required to teach. The literature highlights the difficulties the learners of mathematics in Fiji schools are having and it sheds some light on possible reasons for these problems. The researchers suggest that the reasons are related to problems caused by the mismatch of traditional culture and Western-type school culture. Another cause is the serious lack of confidence in the doing of mathematics by indigenous Fijian students. These student attribute lack of achievement to external factors of ability and environment, blaming such factors as untrained teachers and a lack of ability. A third cause suggested is the ineffective learning skills which include rote learning and examination-oriented procedures of memorising. The suggestions for improving the learning of mathematics from the different studies have some common elements such as the recommendations for including cooperative group learning, discussion and reflection. The domination of learning by assessment procedures of many external examinations needs to be lessened and other techniques for evaluation used such as group presentations. Problems related to non-cognitive issues include negative attitudes to mathematics and poor learning strategies in the mathematics classroom. The solutions for these difficulties in the affective areas are not easy to find as many problems are believed to be the result of traditional values cherished by the Fijians. There are suggested learning strategies included in the Ministry of Education curriculum documents and text books that could be helpful if used, but these strategies have not been implemented. These include such things as the use of hands-on materials and the use of discovery methods by doing investigations and problem solving and other activities.

The previous paragraphs contains many strategies and ideas for teaching mathematics that could be included in a teacher education mathematics unit to help pre-service teachers overcome difficulties in learning mathematics. In this study some of these have been specifically targeted because of their perceived importance and their practicability. These include metacognitive activities, raising of awareness of importance of traditional mathematics, co-operative learning, use of manipulatives and alternative assessment procedures.

Ideas about the teaching and learning of mathematics

It was stated in the Australian National Curriculum (Australian Education Council and Curriculum Corporation, 1991) that there is no definite correct approach or style for the teaching of mathematics. However they state that generally, teaching should be informed by a thorough understanding of how learning occurs and of the nature of mathematical activity. The teaching of any particular concept will be influenced by the nature of the concept itself and by the abilities, attitudes and experiences of the students.

As described by Boero, Dapueto and Parenti (1995) mathematics has greatly changed during the last century. A century ago mathematics was considered to be a set of elementary tools needed in everyday life as well as a specialised domain of investigation. Even though it is a study that has existed for millennia, mathematics today more than ever, appears to be a pervasive component of the Western culture. It has become deeply embedded in technology and in many aspects of the viewing of natural and social phenomena. Capra (1996) in his description of the latest social, scientific and philosophical thought says that the recent scientific breakthroughs were only possible with the development of mathematics and its related language. This has lead to an increased need for the development of mathematics education and the need for new competencies on the part of the teacher. The traditional teacher-centred approaches that were used in the early part of the last century are generally no longer considered helpful in the learning of mathematics, and yet they are still used in many schools throughout the world.

In this section the ideas of teaching and learning that are considered relevant to today's needs and to this study will be reviewed. I will first review the traditional view with which the participants of the study were familiar. Then in more detail the constructivist view will considered on which many curriculum developments in the last twenty years have been based. As learners of mathematics in this study are adults I will then review literature related to the learning of mathematics by adults and lastly present a brief consideration of some ideas of the emerging theory of enactivism. The specifics of numeracy, including number sense and mental computations are discussed in the following chapter along with pre-service students and their study of mathematics education.

Traditional teaching and learning of mathematics

In the current study the participants had experienced mathematics learning that involved much rote learning and practice for external examinations. These examinations as mentioned earlier were held at the end of five of their twelve years of schooling. Bishop (1988) describes this method of teaching mathematics as one based on a technique-oriented curriculum of procedures, methods and skills, rules and algorithms which portrays mathematics as a 'way of doing' rather than a 'way of knowing'. He noted that mathematics taught in this way was not portrayed as a reflective subject. Thinking was seen as limited and constrained, related to adopting the appropriate procedure, using the correct method of solution, following the rules and obtaining the correct answer. It required practice to perfect the skills, with examples to be emulated and exercises to be carried out. Bishop commented that this learning of mathematics cannot help develop understanding, cannot develop meaning, and cannot enable the learner to develop a critical stance either inside or outside mathematics. In his opinion this learning is not an education but instead, at the best it can only instruct and train. Otherwise it is disastrous for a child's overall education.

Cobb (1988) states that the central assumption of the traditional or transmission view of teaching and learning is that meaning is inherent in the words and actions of the teacher and of the objects in the environment. He believes that this situation works well in many everyday situations, particularly those involving successful communication between adults about topics of mutual interest. But this approach according to Cobb is generally no longer considered helpful in the learning of mathematics. Yet it is still the approach used in many schools throughout the world, and I would suggest, including in Fiji.

This situation has directed me, when looking to help change the situation in Fiji, to make a study of some of the more recent developments in the theory of learning mathematics so that I would be more confident about the choice of the framework for my Teaching Program as foreshadowed in Chapter 1. I came to believe that many aspects of the theory of constructivism were an appropriate theoretical framework on which to build my teaching approach and these aspects are discussed at some length in the next section.

Constructivist teaching and learning of mathematics

In this study the constructivist perspective was the philosophical paradigm on which the teaching program was planned and implemented. During the decade previous to the study I had been studying and applying many of the ideas of the theory of constructivism in my experience as a secondary mathematics teacher, and as a lecturer of primary and secondary pre-service teachers. I had come to believe that, by applying the ideas emerging from the development of this theory, some of the difficulties that were being experienced in mathematics education might be overcome.

Various mathematics educators have elaborated on the ideas of constructivism, creating specific forms of it. Two major forms that have emerged are radical and social constructivism. In this section I will outline the ideas of the theory of constructivism as applied by many mathematics educators, ideas of radical constructivism promoted by writers such as von Glasersfeld, and social constructivism with ideas drawn from the writings of Piaget, Vygotsky and more recent mathematics educators. Then how the ideas of constructivism may be, or have been applied in instruction and in the learning processes will be discussed. Next these theories and ideas will be compared with further developments that the theory is currently undergoing with the emergence of the theory of enactivism.

Constructivism is the name given to a theory related to knowing and knowledge and hence is readily applicable to instruction and learning. The theory emerged in the latter decades of the twentieth century. Many mathematics educators see its origin in the work of Piaget. Lerman (1993) says that Piaget launched the notion of constructivism in development psychology and that it is the direct consequence of two fundamental insights of Piaget. First, that cognition produces conceptual structures by *reflective abstraction* from material that is available within the system and from the operations that are carried out with this material. Second, that the function of cognition is adaptive in the biological sense. “Applied to cognition this means ‘to know’ is not to possess 'true representations' of reality but rather to possess ways and means of acting and thinking that allows one to attain the goals one happens to have chosen" (von Glasersfeld, 1991a, p. 16, cited in Lerman, 1993). Lerman continues by suggesting that unless, or until one's concept and one's actions conflict with experiences, one's knowledge is said to be adapted to the environment in which one functions.

Kamii (1996) believes Piaget's theory of constructivism is the only theory that explains children's construction of knowledge from birth to adolescence and that the understanding of Piaget's distinction of the three kinds of knowledge should change the way we teach. She describes them as physical, social and logico-mathematical kinds of knowledge, though she admits this distinction is not clear-cut. Physical knowledge is knowledge of objects in external reality and it can be partly known empirically by observation such as the knowing the colour of a block. The word *partly* is used because Piaget said that classification from logico-mathematical knowledge is involved in coming to know both physically and socially. Social knowledge has its source partly in objects and partly in man-made conventions. In mathematics the knowledge gained by teaching of rules such as 'carrying' and 'borrowing' is social knowledge. Logico-mathematical knowledge consists of relationships created by individuals. The source of this knowledge is in the child's mind with the child making new relationships out of previously created relationships, as in the coming to know that 3 x 4 = 12. Kamii's application of these ideas as applied to teaching are discussed later.

On the other hand with regard to Piaget's contribution, Osborne (1993) states that constructivism has its roots in a reaction to the naive inductivism and deterministic Piagetian development stage-model of cognitive growth. Yet in listing the following key postulates he gives a description of it that contains the main ideas given above. His key postulates state that the learners' existing ideas influence what use is made of the senses and in this way the brain can be said to actively select material. The learner's existing ideas will influence what sensory input is attended to and what is ignored. The input selected or attended to by the learner has no inherent meaning. Finally, the learner generates links between memory store and sensory input to actively construct new meaning (Osborne & Whittrock, 1983).

For von Glasersfeld, Piaget is essentially the original constructivist and in his work, von Glasersfeld interprets Piaget's ideas (Smith, 1997). In Smith's review of

Radical Constructivism by von Glasersfeld (1995), he summarises von Glasersfeld's three premises which underlie the coherence of radical constructivism. The first premise says that radical constructivism focuses on the individual as a receptor of and actor on experience. It is a model of how an individual's experience forms a basis for knowing, knowledge and communication. The second premise needs the existence of an ontological world that is separate from the knower, thus maintaining a dichotomy between the knower and the source of knowledge. The third premise for radical constructivism is that it is a model that focuses on rational knowledge and therefore excludes metaphysics, mystical and revealed knowledge. Reason is elevated as the means by which humans come to know.

As well as summarising these premises for radical constructivism Smith outlines von Glasersfeld's two principles. The first states that knowledge is not passively received either through the senses or by way of communication but is actively built up by the cognising subject. The second states that the function of cognition is adaptive in the biological sense of the term, tending towards fit or viability and that cognition serves the subject's organisation of the experiential world, not the discovery of an objective ontological reality. Smith describes how von Glaserfeld applies his ideas to the learning of mathematics by suggesting that the role of the mathematics teacher is to provide an appropriate environment such that the student will be intrinsically motivated to find solutions to the problems encountered.

Even though Smith praises the work of von Glasersfeld he admits, and describes how von Glasersfeld also acknowledges, that many mathematics educators challenge von Glasersfeld's ideas. Smith suggests that these people include those working in a Vygotskian frame and the social constructivists. Smith says that those who hold the Vygotskian view would question the emphasis on isolated individual construction of knowledge primarily as a result of reflection and experiential perturbation. The linguists among them would view the individual as essentially part of a language community and others would view the individual as intrinsically situated in a social world in which the individual is not independent of his or her environment, while others might question radical constructivist's sole focus on rationality. These social constructivists, suggests Smith (1997) would thus question the three premises of von Glasersfeld's radical constructivism.

For example, Lerman (2000) describes (without quoting a reference) how Vygotsky emphasises the social aspect of knowing. Firstly he says Vygotsky assumed a natural line of development of society from primitive to advanced and therefore did not problematise the differing sociocultural experiences of social groups. He examined how the social nature of people comes to be different in different economic and social situations, which in psychological terms amounts to the question of how culture and social life shape the individual's consciousness. Vygotsky describes that, in all learning, individuals are developing their identities in their zones of proximal development. Lerman states that Vygotsky proposed a theory related to learning and teaching that locates development as taking place in social practices such that meanings are defined within them. Also the theory includes the ideas that the individual, driven by goals and needs first imitates, then performs with assistance, and finally performs alone. Thus Lerman would see Vygotsky's ideas as not upholding the premises of von Glasersfeld, as he does not believe that individuals construct their knowledge independent of the environment of which they are part, and that knowledge gained is not all the consequence of rationality.

The ideas of constructivism described by Cobb (1988) form the basis of social constructivism in which students actively construct knowledge in a way that satisfies the constraints inherent in instruction. Cobb (1988) says that constructivists agree that the teacher's actions and instructional activities are of crucial importance in that they are the potential sources of problematic situations for students. Therefore he says the analyses of the situation needs to include the teacher's and students' beliefs about the nature of mathematics, their beliefs about their own and each other's roles, and their forms of motivation while doing and talking about mathematics. Hence he asserts that research indicates that non-cognitive goals that pertain to beliefs, affects and metacognition must be considered.

Yackel, Cobb, Wood, Wheatley and Merkel, (1990) further develop the view of mathematics as a creative human activity with the social interaction in the classroom playing a crucial role as children learn mathematics. They say that both the interaction between teacher and child and the interactions among the children influence what is learnt and how it is learned. They see the teacher playing a crucial role in developing the environment in which children feel free to talk about their mathematics and where this is accepted as the norm. As well as mathematical concepts Yackel, et al suggest children also develop beliefs about mathematics, beliefs about their own and their teacher's role, and develop a sense of what is valued as well as attitudes and forms of motivation. They suggest that beliefs and values are to be fostered rather than just allowed to develop. Among the beliefs and values to be fostered are that the process is more important than the correct answer, mathematics activities are meaningful. Most importantly mutual trust and respect between teacher and students are to be fostered as well as cooperation and negotiation rather than competition and conflict.

These applications of the theory contain factors about learning that involve people's life-experiences and reflections on them, and their actions in responses to these reflections. Hence the learning is related to people's culture as defined previously. As noted earlier, D'Ambrosio (1985) says that culture and cognition have been shown to be related in the developments in the theories of cognition. This suggests that constructivism is not as simple and clear-cut but as it had been proposed by some it may need to be broadened to include more ideas about the processes of cultural development.

Constructivism and the teacher

Although there are many references to constructivist approaches in the reform curriculum documents both in Australian and elsewhere (Australian Education Council, 1991; Board of Studies, 1999; Cockcroft, 1982) practical suggestions on how to implement these approaches were initially limited especially in the tertiary sector (Cooney, 1985). But since then many mathematics educators (Cobb, 1988; Ernest, 1994, July; FitzSimmons, 1993; Kamii, 1996; Mousley, 1993; National Council of Teachers of Mathematics, 1989) have considered the process of instruction in relation to the theory of constructivism.

One of the most influential documents has been *Curriculum and evaluation standards for school mathematics* (National Council of Teachers of Mathematics, 1989), a document which describes aspects of the role of the mathematics teacher. These include: creating an appropriate environment, setting the necessary tasks to achieve the teacher's goals, stimulating and managing classroom discourse to promote understanding, and analysing learning in order to plan action. These factors in themselves are not all features of constructivism. But if they help develop the environment in which successful communication in instruction takes place, then effective learning is more likely to be promoted as Cobb (1988) believes this environment is crucial for constructivist learning.

Cobb (1988) also believes that the fundamental goal of instruction should be to help students build structures that are more complex, powerful and abstract than those they possessed when instruction commenced. He continues that one of the primary responsibilities of the teacher is to facilitate profound cognitive restructuring and conceptual reorganisation. An example of such a situation is when the teacher 'Socratically' questions a student resulting in the student successfully completing a mathematics task. In this way Cobb asserts that the student has actively constructed the mathematical knowledge, even though in doing so the student has figured out what the teacher might have had in mind all along. In a similar vein Cobb suggests that the practice of discussing the limitations of the student's current methods and suggesting alternatives is compatible with constructivism. Even more direct forms of instruction such as lecture and direct discussion can be effective with older students who can conduct an internal, reflective dialogue (Siegel, 1981, cited in Cobb, 1988). Constructivism as applied in the area of adult education will be expanded on in a later section.

Cobb (1988) sees a serious problem for teacher education because of what is expected of a teacher in mathematics instruction that follows a constructivist approach. He believes that constructivism provides a rationale for teaching by negotiation, which requires far more of the teacher. He lists the requirements of the teacher: to possess a deep relational understanding of the subject matter, to be knowledgeable about possible courses of conceptual development in specific areas of mathematics, and to continually look for indications that students might have constructed unanticipated, alternative meanings. In short the teacher should have undergone a conceptual revolution of his or her own.

Kamii (1996) also suggests pedagogical ideas in that it is the understanding of the three kinds of knowledge described by Piaget outlined above that will drastically change the teaching of mathematics. As had Dienes (1971) in the 1960's, Kamii developed an approach to primary school mathematics based on Piaget's constructivism including his understanding of logico-mathematical knowledge. With these understandings in mind she tested a hypothesis that children can invent their own procedures for the four arithmetical operations. As well as confirming her hypothesis she made an unexpected find. This finding suggested that the algorithms now taught in most schools in United States are harmful to children's development of numerical reasoning for two reasons. The first states that when compromise with their teachers' methods is not possible, children have to give up their own thinking to use rules such as 'carrying' and 'borrowing'. The second reason states that these algorithms unteach place value and prevent children from developing number sense. These findings and other notions of Kamii support the idea that constructivist teachers must allow their students to develop their knowledge by helping them use their own strategies in mathematics, discouraging the teaching of algorithms where possible. Though Kamii discourages algorithms and I can understand why, I suggest they do have some place in mathematics education as discussed in a later section because of, among other factors, their efficient procedures.

Ernest (1994) offers certain pedagogical emphases that come from his ideas of the theory of constructivism. These emphases include the following ideas about knowledge: it is not passively received but actively built, it is based on previously constructed knowledge, it is individual and personal, and finally it is constructed via language and social interaction. These ideas, among other things are echoing the fundamentals of social constructivism as outlined earlier. He suggests that the teacher needs to help the learners use metacognitive processes and so help them be more responsible for his or her own learning. On the part of the teacher there needs to be an awareness of the importance of goals for the learner and of the dichotomy between the learners' and the teacher's goals. The teacher needs to continually attempt to come to know the learners' knowledge, understanding and metacognitive processes. There also needs to be an awareness of the social context of the concept, such as the difference between folk and school mathematics, with the former exploited for the latter. With all these unknowns Ernest suggests that there is no one road to truth, or near to it, so methodological approaches need to be much more circumspect and reflexive.

At the time of writing her paper Mousley (1993) believed that the ideas of constructivism had begun to influence practice, even if superficially with the use of students' own strategies and the discussion of ideas. She includes the practices that have an emphasis on the teacher allowing students to express and use personal constructs and a focus on problem solving and problem posing activities. She herself suggested that in mathematics classrooms, teachers and students should communicate using verbal and written texts to share their understandings, as well as to make and clarify meanings.

Constructivism and the learner

Ideas about constructivism and the learner may be gleamed from the above section which deals with the pedagogical issues for constructivism. But there are some writers such as Yackel, Cobb, Wheatley and Merkel (1990) that specifically deal with learning and the learners. They suggest that a chief feature of an instructional approach that is based on a constructivist view is that instructional activities should give rise to problems for students to resolve. The situations that students find problematic differ owing to wide differences in their knowledge, experiences and goals. Yackel, et al see that this can be an advantage in that it is a means of individualisation with students at differing conceptual levels not only using different solution methods but interpreting tasks in different ways. This links to the notion embedded in constructivism that real learning occurs when students encounter perturbations and are intrinsically motivated to find solutions (Smith, 1997).

In a similar vein Burton (1992) suggests that if mathematics education adopts a constructivist approach then the learners need to be given certain opportunities. They should have the chance to interpret, to negotiate meaning, to be challenged and as a result construct some new understandings of their own which might or might not be matched by those of other learners in the same class. A constructivist view she believes requires rich opportunities for reflection by the learner which, among other advantages provides the teacher with the necessary information about what has been understood.

The learner's reflection may be seen as part of the process of metacognition. Mildren (1993) believes that the constructivist environment provides the opportunity for metacognitive development because it respects the right of an individual to engage in the active construction of mathematical thought. The type of situation in which this could be developed would be in regular reflexive writing such as writing in journals as well as to a lesser degree, in class discussions, or in smaller groups doing cooperative learning.

Further ideas of the learner and constructivist ideas that are linked with metacognition and an approach that is reflexive, are given in *A National Statement on Mathematics for Australian Schools* (Australian Education Council and Curriculum Corporation, 1991). It states that learning is likely to be enhanced by feedback. For learning to occur when existing conceptions are challenged Burton (1992) also suggests that feedback is essential. Little is learnt if students or the teacher simply mark errors wrong. Burton suggests that richer feedback than this may take many different forms coming from the physical, social or mathematical environments. She sees the role of the constructivist teacher as critical in this feedback process. Often appropriate feedback can help students recognise inconsistencies in the structure of their own thinking. In order to use the inconsistencies to further their own understanding Burton believes that students have to believe that mathematics makes sense and that it is not just a series of arbitrary rules but that they, the students, can work it out logically. This is related to the notions of rationality embedded in a constructivist view of teaching and learning as noted earlier.

Along with these processes a teacher will hopefully be attempting to become more aware of a student's construction of his or her knowledge by using other reflexive strategies and approaches such as journal writing. Oaks and Rose (1992) believe that any change in conception must originate, not from an external source but from within the student. They suggest that writing can help people reflect on their experiences allowing them to structure meaning. Clarke, Stephens and Waywood (1989) found that journal writing was a unique data source of information about the way in which students construct meanings. Borasi and Rose (1989) commented on the wealth of information about students and their mathematics unit which journal writing provided for the lecturer. McLeod (1992) suggests that it seems more natural to use qualitative rather than quantitative techniques to obtain data about issues of attitudes and beliefs in mathematics. These last few comments suggest that journal writing is also a valuable source of information for researchers as well as teachers.

Constructivism and learning numeracy as an adult

At this stage in the thesis most of the ideas about becoming numerate and constructivism have been taken from papers discussing the ideas from the perspective of the students being children. When adults are the ones doing the learning the situation has a number of important differences. In this study, the learners of numeracy are pre-service teachers and hence adult learners. In fact some of them had left school for a number of years before commencing the College course. They have been through a schooling of six years of primary and generally six years of secondary in which they were taught mathematics on most days of those twelve years. Yet, as discussed later, on entering the Teachers' College, they appeared to have a need to relearn many of the numeracy skills they attempted to learn in primary school. Given this context it is useful here to review the literature related to adult numeracy education. Therefore in this section I will consider issues related to learning as an adult, the development of numeracy skills for adults, and the content and the constructivist approach in numeracy courses in adult education.

Adults as learners

FitzSimons (1993) notes a number of recommendations as consistent with constructivism for teaching adults, in her case women. These include emphasising connected knowing, understanding, acceptance and collaboration, allowing time and respect for knowledge that comes from first-hand experience, and encouraging students to involve their own patterns of work. She also discusses the positive effects of a constructivist mathematics classroom on a women that was a participant in her study. In particular, she mentions that this woman's self-concept was strengthened and she was able to constructively assist her children who also gained in self-confidence in mathematics. This form of instruction is in some ways very demanding for the teacher.

FitzSimons (1994) reports on a further study which set out to investigate factors which influence persistence of adults with study, and to show that the teaching of adults returning to study mathematics could be enhanced by adopting a constructivist perspective. One factor she found was that the adults returning to study have a high motivation to learn. They have given of their time and money to pursue a goal. She says that probably for many of them their previous mathematics learning was something that was suffered or even avoided where possible. It was during their prior experience of learning mathematics that the adults' feelings, attitudes and beliefs were first developed. FitzSimons believes that a previous enjoyment of mathematics classes can generate positive attitudes towards the subject and a high self-esteem in the learner. But on the other hand threats of physical violence, humiliation or ridicule may have resulted in feelings of anxiety. She suggests that under-achievement may result from peer-group pressure, and discouragement from negative parental and community attitudes towards success in mathematics. However a constructivist approach with its emphasis on the individual constructing knowledge built on past learning within an appropriate learning environment will help counteract any repetition of a past negative learning situation.

FitzSimons (1994) further discusses the problem of mathematics anxiety with regard to adults. She says that it is a major factor influencing whether an adult will enrol and persevere with attendance at mathematics classes. She discusses her research which addresses the effects of constructivist learning environments and suggests that more research is needed to determine whether constructivist learning environments prevent or ameliorate mathematics anxiety. Her constructivist learning environment was influenced by the ideas of Burton (1987) who also anticipated that adults are highly motivated but anxious about learning mathematics. Burton designed a course with a constructivist perspective that would enable the students to change their views of mathematics and their own feeling in relation to it. FitzSimons' and Burton's work are further discussed in the pedagogy section below.

Hembree (1990) found in a study of the nature and effects of mathematics anxiety developed during school education that anxiety appears to comprise a general fear of contact with mathematics, including classes, tests and homework. He states that it often depresses performance and is related to test anxiety. The study also suggests that anxiety is a learned condition and hence special courses designed to later enhance students' competence often fail to reduce competence levels.

Adults and numeracy

When adults return to study numeracy, they come with much mathematical experience (FitzSimons, 1994). As described above Bishop (1988) considers six mathematical activities to be the basis of working mathematically and as FitzSimons comments, most adults will have spent a significant amount of time on each of these activities in their life experiences, although they are often not conscious of the mathematics involved. Adults would continually have used mathematical skills when they had to perform activities such as budgeting, shopping, cooking, gardening, travelling and playing sport.

FitzSimons and Sullivan (1993) have shown that adults can overcome the effects of lack of practice, cope with the introduction of new curriculum content and be receptive to new ideas. In my experience anecdotal evidence suggests that adult students have low self-confidence in their mathematics learning partly because they believe the contrary of these findings. Though these findings relate to older adults than the majority of the participants in this study. But they could be a source of encouragement to the students who feel they are disadvantaged in their learning of numeracy skills because they had left school some years earlier than those who had come directly from secondary school.

Rose (1998) discusses the differences between street and school mathematics, with the former the mathematics used by most adults. Rose says that the numeracy knowledge of adults involves high levels of both cultural and social knowledge. He suggests that much knowledge is quite prevalent in the community. For example, the knowledge to calculate the telephone bill requires a different usage model from the electricity one, because of the procedures for costing.

On a more specific note about numeracy skills, Mikuliak (1998), an adult numeracy teacher wished there were other concrete materials like money with its associated well-understood concepts. She found that decimals that deal with tenths and hundredths are fine because they relate to money, but thousandths cause confusion and panic because the majority of the class come with no idea of what place value meant and with the understanding of large numbers as a difficulty. She therefore included a unit on place value at the beginning of her units. Another area of concern she encountered was estimation. When Mikuliak introduced a section on estimation she was surprised at the dismay of students who were not at ease in using the formal rules. They had their own strategies which appeared to be at odds with the rules.

Overestimating is the norm in everyday life when one is calculating if there is enough time or money in a particular situation.

Teaching approach for adults

FitzSimons (1994) and Burton (1987) based their teaching of adults on constructivist ideas. Some other educators, who did not name similar strategies as constructivist but used them for teaching adults, were Brougher (1997); Rose (1998); McGee (1998); Owens, Perry, Conroy, Geoghegan and Howe (1998); and Mikuliak (1998).

FitzSimons (1994) believes that the skills of the teacher are critical and adopting a constructivist perspective can inform the many decisions that are made by the teacher before, during and after each class. As mentioned above, FitzSimons' teaching is influenced by a course designed by Burton (1987). In Burton's course use was made of such constructs as learner autonomy, personal knowledge, inquiry-based learning and collaborative organisation. Activities were used that helped the students to be active investigators, constantly posing problems, conjecturing and solving problems. Burton hoped that these activities would change the image of mathematics from one of being always 'closed and correct'. It provides the learners with the perturbations that von Glasersfeld says are necessary for real learning (Smith, 1997).

In attempting to overcome the problems associated with numeracy learning, Rose (1998) suggests that one aim is to move beyond memorisation to a more inquiry-based approach with the emphasis on computation being replaced by one on problem solving. Rose sees this movement that focuses on the learner and on problem-solving as especially important for adult educators. She, at this point seems to be arguing for conditions where the adult student will know that they will be creating their own knowledge. She notes that these ideas are dependent on what the learner knows and how mathematical problem-solving can be related to real-life situations. Adults, more than children enter educational settings with a wealth of experience and they have already developed mathematical strategies. Rose suggests that an awareness of this fact is important. This approach is based on some of the constructivist ideas described by Smith (1997) that cognition serves the subject's organisation of the experiential world and that learning requires some problem solving to be effective.

Also Rose (1998) believes that adult mathematics education must consider the importance of preserving the cultural meaning of situations when transferring real-life situations for problem solving into the classroom. She sees the need for the problems to be part of a decision-making process and to be realistic. Being realistic involves cultural sensitivity and knowledge of the particular on the part of the teacher. The more recent emphasis on discovery must be culturally and socially connected if the problems are to be realistic. Rose believes that all learning must take into account the knowledge the learner brings to the classroom. Adult learners possess vast stores of mathematical knowledge involving abstract thinking. She notes that the central problem for the teacher of adults is to tap into what is already known while increasing the complexity of the students' new learning and attempting to overcome general resistance to mathematics study.

In their mathematics classes for adults Owens, et al (1998) also used a problem solving approach as they considered it a necessary element in the constructivist view to facilitate the mathematics learning of students in the pre-service teacher education course. They found that the approach developed the students' confidence in their own abilities to at least get started on mathematics problems. Their research illustrated the importance of affective variables in learning. Owens, et al saw that over a period of time, learning was assisted by increasing success and positive feelings associated with a supportive classroom. They believed the catalyst for change was embedded within areas such as positive feelings about teaching primary school mathematics, the fun and support of the classroom, students becoming consciously aware of their use and ability with mathematics and students being able to use skills such as language, reasoning, group skills, or humour to break from the negative affect pathways.

Brougher (1997) when discussing Gardner's theory of multiple intelligences suggested that when teaching adults, learning could be much more enjoyable and productive when the classroom environment resembled in some manner an elementary classroom. In such a classroom learners are fully engaged in the creativity and excitement of learning. By participating in an environment that is nourishing for all the multiple intelligences she believed adults begin to experience a richness and enjoyment in learning they thought they had outgrown or in many cases never experienced. Ideas of how she applied the theory include many that I have found are applicable in a mathematics education classroom. Brougher includes constructivist strategies such as the use of journal writing, oral presentation, songs, rhythm, graphic images, mind-mapping, role-playing, working in small groups and reflective writing. Also pre-service teachers, after a time of initial awkwardness, are generally willing to assist in developing an environment for themselves that resembles an elementary classroom. Therefore in mathematics education Brougher's ideas are very applicable.

When teaching adults it seems more obvious that each student would have developed different learning styles. Hence a constructivist teacher attempts to be sensitive and responsive to these differences. In response to these differences Mikuliak (1998) in her teaching of numeracy to adults strives to create a *student-driven* learning experience using unconventional ways that are more meaningful to a particular student. Mikuliak realises that allowances must be made for the needs of tactile learners. She found that cutting up cakes for fractions and stringing beads for place value helped these learners even if they were adults. She believed that for some students there were reasons why times-tables were not able to be memorised so, if after trying all the normal strategies a student had not mastered the basic facts she would leave the memorising of them.

It is believed that the teacher of adults can help students by taking time to discuss the jargon used in mathematics. In teaching numeracy to adults McGee (1998) suggests that big words become less threatening when association is made with already familiar words, such as *centimetre* with *cents*. She suggests that this can also be done for new content by the use of analogy from everyday experiences wherever possible.

Summary of ideas about constructivism and the learning of numeracy for adults

The theory of constructivism was the paradigm for the planning and implementing of the teaching program for this study. Therefore the main idea, on which I based my decisions related to the Teaching Program, is that pre-service teachers construct their own knowledge. To construct knowledge they gather new information from their environment and attempt to incorporate this information into the construction of knowledge they already possess. This may involve some reconstruction and even sometimes discarding of past knowledge in order to accommodate the new ideas. To assist the pre-service teachers' learning, the ideas of constructivism suggest that the lecturer needs to be aware of the knowledge the student already possesses and provide the learning environment and activities that will enable the construction of knowledge. The learners need time and processes to reflect on the information that the environment provides and procedures to help them incorporate the new facts into their current knowledge in order to further construct knowledge. This learning process needs to be monitored by the teacher, not by formal written examinations but by activities both short and extended such as challenging tasks, problems, investigations and projects. How I applied these and other ideas of constructivism in this study are also described in Chapter 6.

The discussion on the teaching and learning of numeracy by adults has considered mainly adults who have returned to study after years away from schooling but it has many ideas applicable to those who have entered tertiary studies directly from school. For teaching adults in a constructivist mode it seems that the following issues need to be considered. When an adult is working mathematically, on the positive side they have a high motivation, a wealth of life's experiences and possess self-taught skills to incorporate into their learning. On the negative side they often come to a mathematics class with a high anxiety level and a low self-confidence in their numeracy knowledge and skills. Mathematics teachers of adults who are social constructivist would attempt to be aware of these factors and prepare the classroom environment and activities appropriately. Also the provision of a learning environment that is part of a language community and not foreign to the learner is embedded in the constructivist approach. This could be achieved by adapting the problems to the individuals and ensuring that there are opportunities for interaction between students, and between teacher and student.

The discussion about culture and mathematics learning by Rose (1998) I believe is very important. In my study I suspected that in the research I would see the need to preserve the cultural meaning of the situations when transferring the real-life context into problem solving tasks. Rose commented that, while researchers are exploring the study of mathematics as a culturally defined entity, mathematics teachers are just beginning to consider alternative ways of teaching to accommodate cultural influences.

Beyond constructivism, the ideas of enactivism

Since my introduction to the theory of constructivism I have been aware that there have been critics of it, e.g. Begg, 1999; Davis & Sumara, 1997; Klein, 1997; Solomon, 1994. While in the early stages of my analysis of data for this study I talked with Andy Begg about the teaching of mathematics in Fiji because of his considerable knowledge and understanding of mathematics education in Fiji. This meeting led me to read a paper of his in which he has expresses concerns about the ideas of the theory of constructivism and exposed his thinking about theories of teaching and learning in a discussion of the ideas of enactivism.

For Begg (1999) enactivism is an alternative theory to constructivism. It is a theory about knowing, learning and teaching, that helps him make sense of what happens in the classroom. He says that constructivism is concerned only with cognitive knowing and does not explain a number of issues including unformulated or subconscious knowledge, how things are known intuitively or instinctively and how emotions are constructed and their role in learning. Secondly he suggests that in constructivism there does not seem to be explicit links made between constructivism and the learning theories developed from ideas embedded in brain-science or neural biology. Lastly he notes that constructivism has numerous forms which include dichotomies with respect to an individual or to social foci. He suggests that there is a need to consider these dichotomies and decide what is meant by *individual* and *social*.

Begg directs the reader to the interpretation of enactivism given by Davis and Sumara (1997). They state that the enactivist theory of cognition requires teacher and teacher educators to reconceive the practice of teaching by blurring the lines between knower and known, teacher and student, school and community. Begg suggests that the theory coherently ties together some of the research about knowledge and learning and links it with teaching and the curriculum. This ongoing exploration of other ways of knowing will help as the emphasis in schools changes from a transmission approach. Begg writes that such a change might involve a shift from teaching to learning or give critical thinking more emphasis. He suggests that the task of educators might be to ensure that schools interpret such shifts in a cross-cultural way that involves notions from both the East and the West. Begg concludes his paper by suggesting that the future challenge of educators is to continue to explore other ways of knowing and learning.

Davis, Sumara and Keiren (1996) summarise the ideas of enactivism saying that they are concerned with how the learner-and-learned, knower-and-known, self-and-other co-evolve and are co-implicated. The *context* of learning does not contain the student and teacher but rather they literally are *part* of the context. The teacher participates with the student in bringing forth a world of understanding. Davis, et al discuss the school curriculum and point to the futile attempt it makes to select a path. They suggest that the best that can be done is to participate in the continuous process of learning which they suggest is like the laying down of a path. The teacher works from good learning activities but must anticipate different ways that the lesson might move in response to the students' interactions. Yet the teacher must still link the lesson with the major ideas that underpin the particular curriculum.

Although my teaching approach in this study is based on constructivism I am open to the possibility that the ideas of enactivism may need to be considered when analysing the data. My findings may suggest to me that my teaching may have been more effective in developing the students' understanding if I had given more consideration to the participation of the whole person, the community of learners and their environment rather than to emphasising the person's cognitive facilities. As Smith (2000) suggests, concepts such as enactivism appear to offer new frameworks and directions for considering learning.

Summary of cultural issues and mathematics education

Because this study is set in a developing country, issues of culture need special consideration. For the cultural aspect of this study I investigated a little of the ethnomathematics of Fiji including some of the symbols, and specific ways of reasoning and inferring that are described in research completed in Fiji. In this day and age the ethnomathematics of Fiji is not isolated but greatly affected by global influences. Therefore, as a mathematics lecturer in an educational institution in a country foreign to me, I see the need on the one hand to play my part in safeguarding the cultural aspects of mathematics as learnt in the village community. But on the other hand there is a need to help students consider the effects of globalization on their culture as well as helping them compete in a global community. The Western mathematics that is taught in schools in developing countries appears to be necessary if a country wishes to be part of the global community and to trade with other countries for what they desire.

For mathematics in all countries Bishop (1988) describes six mathematical activities and processes that he sees as part of the pan-cultural phenomenon of mathematics. In this study I considered number sense which includes the activities of counting, measuring, playing and explaining, four of the six universal activities proposed by Bishop. Bishop also writes about culture and mathematics, and discusses in some detail aspects of conflict. It is perhaps because of the problems and conflicts that have emerged, that a growing interest in culture and mathematics has developed. In developing countries such as Fiji some of these conflicts originate from the colonial legacy, and in particular from the differences between traditional village life and school life. Bishop classifies the conflicts according to interpretations and responses to them. In this study I would classify my approach to cultural conflict according to Bishop's analysis as one similar to *accommodation.* I will show how I re-structured the curriculum in as much as my knowledge and the contributions from the students' knowledge of the culture allowed. I sought to incorporate the teaching style that the pre-service teachers I had previously lectured preferred, from what they indicated in discussions and in journal writing. Hence it will be noted later for example that students used their home language when they felt the need, such as when they were unable to explain ideas to their peers in English. The mathematical words and terms of home languages were incorporated wherever appropriate.

As a further part of my quest for knowledge of the culture I have investigated the origins of the present education system from a historical viewpoint. Then to examine the context of the study and the background of the participants I have studied the research into mathematics education that has been completed in Fiji. The major piece of research I have discussed here is one that studied the traditional mathematics and ways of mathematising of the indigenous Fijians (Bakalevu, 1997a). From this work I believe I have a deeper insight into some aspects of the number sense of the participants as I have come to know what mathematics they have learnt and use in their village life.

In the review of the research in Fiji there are indications of some serious needs as regards development of the self-esteem of the students. Changing a person' beliefs about self-efficacy does not happen overnight or even in three years – the length of the education course at the teachers' College, but hopefully a beginning can be made. As suggested the Fijian pre-service teachers need to be given challenging tasks and meaningful activities that can be mastered. With support and encouragement this mastering will begin to improve their sense of self worth and self-confidence without hopefully interfering negatively with their cultural traditions and practices. Thus if all these ideas are incorporated into a mathematics education unit, pre-service teachers may be helped to overcome the problems they are having in learning the mathematics they are required to teach.

In the second half of this chapter I have discussed some of the considerations found in the literature related to ideas and theories about teaching and learning of mathematics relevant to my study. These issues were divided into three areas: a brief description of the so-called traditional approach, a more extensive review of the literature of the constructivist approach, adults as learners of mathematics and the recent ideas about enactivism. For the section on the traditional approach I have given a summary of the descriptions of teaching and learning of mathematics that is thought now to be inappropriate because generally it did not help students to develop a relational understanding of their mathematics learning. Too much of such teaching led to instrumental understanding such as students memorising rules and procedures without developing underlying concepts. The teaching did not help students to become independent learners of mathematics.

In the later part of the last century this traditional approach has been gradually replaced by a constructivist approach. This constructivist approach sought to develop relational understanding in contrast to instrumental understanding, with the mathematics learners building their own personal construct of knowledge upon previously-constructed knowledge. This approach suggested that the teacher provide a learning environment in which the learners are given opportunities to discover and develop concepts, using activities that requiring manipulatives, discussion and reflection.

Much of the literature I used for the above discussion was centred around the learning of mathematics by children. Hence I also sought to investigate whether adults learn mathematics in different ways. The major differences are related to the life experiences of the adults before they return to study. For the non-cognitive aspects of the differences many adults have developed emotions, attitudes, beliefs and values related to mathematics and the learning of it which are negative and so produce initial obstacles. On the positive side adults are generally highly motivated. For the cognitive aspects adults come with many experiences of using mathematics in their daily life and they often have self-taught and meaningful strategies of doing computations. If these positives are used to best effect the negatives can be lessened and a new approach to learning mathematics generates new learning and attitudes.

Adults have been found to appreciate a constructivist teaching approach in helping them learn mathematics. Even though it has many strengths the constructivism has in turn had its critics. Many questions about learning and teaching are left unanswered and so other theories are sought to satisfy those searching for answers. One of these is the theory of enactivism. Enactivism suggests that learning is not just produced by a rational process by the individual learner or by a group of learners but that it requires participation on the part of the everyone and everything in the environment of the learners.

Generally the teaching approach that the participants of this study experienced in their past schooling was the traditional transmission approach. So, when planning a teaching program for this study which had a constructivist approach, I needed to understand the former education of the pre-service teachers. In Fiji the mathematics education of past decades was generally not student-centred but often ignored the needs of the students so that the students tried in vain to accommodate the mathematics teaching they had received. In the constructivist approach the teacher considers the individual backgrounds of the students and allows them individually to construct their understanding of mathematics, rather than to rote learn facts and procedures the lecturer might present. Therefore I believe that, with this approach the student, especially as adults may develop an appropriate and useful sense of number.

In the following chapter I will review literature related to more specific aspects of mathematics education of pre-service teachers. This will include two major sections, the first dealing with numeracy, number sense and the other section considering mathematics education of pre-service teachers.

CHAPTER 3 PRE-SERVICE TEACHERS AND MATHEMATICS EDUCATION

So far in this literature review issues related to two main areas of mathematics education have been discussed, namely cultural issues and the ideas and theories of mathematics learning and teaching. In this, the second chapter of the literature review, first an area of mathematics known as number sense is investigated. Then, as this study seeks to examine how to begin to improve the numeracy of a class of pre-service primary teachers, issues related to mathematics education in pre-service teacher education courses are considered.

Number sense

My motivation for completing this study was the apparent poorly-developed numeracy of pre-service teachers in a Teachers' College in Fiji. As the study was centred around the mathematics education unit for Semester one not all aspects of numeracy could be considered. Hence I chose to concentrate on the essence of numeracy, number sense. The rationale for this will be discussed following clarification of terminology. At this stage it is sufficient to say that without a reasonably developed number sense a person has difficulty understanding and applying the numeracy skills that are required to be used in daily life. Therefore, for pre-service teachers number sense is an essential quality that they should develop in their pupils.

What number sense is and why it is important will be reviewed, as well as a discussion of teaching programs related to it. Further sections will consider the area of number sense, mental computation and estimations in some detail because of its importance in number sense. But before it is investigated, the different but related feature of standard procedures of the basic algorithms will be examined. This will highlight the problems resulting from the traditional teaching and learning of these procedures, often without much thought given to their understanding, an essential feature of number sense development. Then the importance of mental computations will be investigated, as well as some ideas and research about instructional issues for mental computation and estimations.

Definitions and descriptions of number sense

A sense of number could be said to be as old as number itself but it is only in recent decades that the term 'number sense' has been used in mathematics education literature. As a consequence, definitions of the term have not been refined to produce a common definition. A few decades ago in mathematics education literature, number sense was discussed under the term 'quantitative intuition' (Carpenter, Coburn, Reys & Wilson, 1976). Since then descriptions or definitions of number sense seem to be as nume+rous as there are authors referring to it. Some authors give a definition in a comprehensive statement, such as Carpenter et al. Their statement says number sense is a well-organised conceptual network that enables one to relate number and operations, and to solve number problems in flexible and creative ways. Reys, Suydam, Lindquist and Smith (1998) concisely describe it as an intuitive feel for numbers and their various uses and interpretations. Reys and Yang (1998) refer to it as a person's general understanding of number and operations. Sowder (1992a) suggests that because it is such a large phenomenon, we should focus on pieces of it until we understand better how the pieces fit together. She also says that the definitions that have been written do not provide guidance for instruction or assessment.

Sometimes writers leave one to infer what is meant by the term. The meaning of number sense used by the National Council for Teachers of Mathematics (1989) can be inferred from their document *Curriculum and Evaluation Standards for School Mathematics*. In it children with a good number sense are described as those who thoroughly understand number meanings, have developed multiple interpretations and representations of number, recognise the relative and absolute magnitudes of numbers, know the relative effects of operating on numbers and have developed a system of developed referents for quantities and measures of common objects and situations in their environments.

But is it possible to give a clear description of number sense? Van de Walle, Bowman and Watkins (1993) believe that the facets of number sense are intricately connected and rely upon a strong conceptual development. They see the developing of number sense as a way of teaching rather than as a body of knowledge and skills to be taught. This is because they see it as having an interconnected, multi-faceted and highly conceptual nature. They consider that prior to Grade 3, a child is not ready to learn specific things such as computation estimations. In their idea of a number sense way of teaching, a major ingredient is a high degree of open-ended discussion in the classroom.

McIntosh, Reys and Reys (1992) see number sense as highly personalised and related to what ideas about number people have established, as well as how those ideas were established. For their definition of number sense they include the NCTM ideas given above as well as adding an understanding of the applications of numbers and operations as a result of an expectation that numbers are useful and have a certain regularity. They describe the aspects of it that are important in everyday living which are described below. Reys and Yang (1998) similarly comment on the personal use of number in describing number sense adding that it reflects an inclination and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information.

Resnick (1989) discusses personal aspects of number sense when she describes number sense as having the following two characteristics. Firstly, it is self-evident to the persons who have it, and secondly it is easily accessible for them because it is linked in the memory to specific situations. She considers number sense as higher-order thinking in mathematics and therefore suggests that it is difficult to set objectives that educators can consider but that it is relatively easy to list some key features of number sense when it occurs. Resnick proceeds to list the following features: non-algorithmic; complex; often yields multiple solutions each with its appropriateness; involves uncertainty, nuanced judgements and interpretations; self-regulating of the thinking process; imposes meaning by finding structure in apparent disorder; and involves thinking that is effortful.

Some mathematics educators have attempted to help students and teachers by separating number sense into well-defined areas. McIntosh, Reys and Reys (1992) have devised a framework for the areas of number sense (see Table 3.1), which is discussed in some detail in a later section. The framework is abridged by the omission of the extensive list of activities related to each strand which, as listed is not considered in this study. For this discussion here it is sufficient to describe the framework as having the three main sections of number; their meanings and relationships, their operations and their applications. In this framework they include a definition of *number sense* as the propensity for and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information. They also state that number sense results in an expectation that numbers are useful and that mathematics has a certain regularity, that is, it makes sense.

Table 3.1

Framework for number sense (Abridged McIntosh, Reys & Reys, 1992.)

Categories of number sense	Six strands of number sense
Knowledge of and facility with *number*	Absolute and relative magnitudes of numbers
	Multiple relationships of number
Knowledge of and facility with *operations*	Relative effects of operating on numbers
	Equivalent expressions
Applying knowledge of, and facility with, numbers and operations to *computational settings*	Computing and counting strategies
	Measuring benchmarks

These descriptions of number sense include many common features. They contain both cognitive and affective aspects. The cognitive aspects are the understanding and use of numbers, their operations and applications. The affective aspects are included in the words such as 'intuitive feel', 'flexible', 'creative', 'propensity' and 'expectation'. Then a combination of both kinds of aspects need the foundation of an appreciation of the nature of mathematics which includes an understanding of the nature, and a possession of the values, of mathematics. What I understand by the nature of mathematics and its values were discussed in the previous chapter. From this review of the literature, the definition of number sense that I have come to consider most appropriate for my study states that *number sense is the* *inclination and ability to understand and use numbers, their operations and applications with ease and to understand and appreciate the nature of mathematics*.

Instructional issues of number sense

How can teachers themselves teach in a way that promotes number sense in their pupils if they arrive at teacher education institutions without good number sense? Is there some appropriate process or program to assist them prepare to help their future pupils develop number sense? In the early planning stages of my study I wondered whether to teach number sense directly as the content of a unit or expect that its development will occur if I use the teaching strategies recommended by constructivists.

Greeno (1991) is one mathematics educator who believes it is important to consider all activities of mathematical instruction as potential contributors to students' development of number sense. He suggests the designing of specific activities for the growth of number sense but he also encourages mathematics educators to think about how the current activities of the curriculum can be redesigned and reorganised so that they contribute to the growth of number sense. Reys and Yang (1998) also suggest that number sense is better not taught in isolation. They say that it is best nurtured if the focus on its development is consistent over time and occurring frequently within each mathematics lesson. Similarly this continuous evolution of number sense was discussed by Reys (1991) when she stated that it is not an finite entity that a student has or has not, but rather a process that develops and matures with experience and knowledge. She suggests that helping students develop number sense requires appropriate modelling, posing process questions, encouraging thinking about numbers, and in general creating a classroom environment that nurtures number sense.

Other mathematics educators, for example Sowder and Kelin (1993) strongly recommend that all mathematics instruction should emphasise making sense of mathematics and that helping students develop a good number sense can go a long way towards reducing the frustration of both students and teachers. The instruction they recommend includes teaching that has open-ended verbal interaction which encourages children to talk and share ideas. Van de Walle and Bowman Watkins (1993) also advocate open-ended discussion. They believe that the teaching of number sense almost always involves a high level of teacher-student and student-student interaction. They suggest that encouraging children to talk and share their ideas of number sense helps them to make connections for themselves and their peers. Within an environment of open-ended discussion they suggest that children do the following: create relationships about numbers, devise methods of solving problems, and share strategies for mental computations. This sharing in the classroom is also seen as important by the National Council of Teachers of Mathematics (1989). In the section *Standards Grades K-4* it is considered that communication plays an important role in helping children construct links between the informal intuitive notions, and the abstract languages and symbolism.

Sowder and Kelin (1993) suggest that an exception to the non-formal instructional approach should be used for some instruction in the development of mental computations and estimation. Even so, they discourage formal teaching of strategies for it. Instead they suggest that teachers provide students with carefully structured situations leading them to the discovery of rules and the invention of algorithms. In a similar vein, Trafton (1989) suggests that pursuing number sense itself might not be as useful as pursuing *how* children process number in computational situations because, as mentioned earlier, Trafton believes that number sense is something that unfolds rather than something that is taught directly.

For the early stages of pre-number and for counting Reys, Suydam, Lindquist and Smith (1998) have produced instructional material. These authors in partnership with others were involved in many research projects related to number sense that are cited throughout this review and so they have a research basis for the creation of written teaching material. Their material implies that actual counting practice can be carried out in such a way as to use high level and logical thinking. Reys et al consider that counting experiences can contribute to estimation skills, developing benchmarks, and place value concepts which are components of number sense. Counting on and skip counting help with the development of the operation of addition and subtraction. The trouble spots of bridging numbers such as 10, 20 and 100 need special attention so that children will not have difficulty with them in later mental computations. They outline many other similar-type activities and open-ended tasks that they believe help students develop good number sense. McIntosh, Reys and Reys (1997) have also published number sense activities because they believe that number sense can be developed through exploring patterns, developing mental computational skills, understanding benchmarks, recognising reasonableness of answers and acquiring estimation skills. Though these activities are for the early years of school there is a place for them in an educational unit so that pre-service teachers who are having difficulties with numeracy have the opportunity to both relearn the concepts and skills and think about the teaching of them.

When students are doing some of these activities such as solving problems mentally, context plays an important role (Van de Walle & Bowman Watkin, 1993). Various settings that children are familiar with such as using money, seem to keep children's attention focused on number meanings in a problem. When children resort to pen and paper in a classroom setting, they tend to focus on individual digits and lose the meanings involved in the computation.

In summary then it seems that informal instructional approaches are favoured in the literature when it comes to developing a number sense in children. In the main this is because number sense is viewed more as a state of mind rather than just a specific set of identifiable skills.

Studies related to instructional program for number sense

With the popularising of the term *number sense* studies related to the number sense of pre-service teachers have been conducted. Some of these studies attempt to research the area number sense as a whole, while others look at specific components of it. Some studies investigated the development of general number sense as a result of instructional programs (Kaminski, 1996; Clarke & Clarke, 1996; Majdalani, 1993; Civil, 1990; Quinn, 1997; Ebby, 1997). Other studies examined aspects of number sense as a result of general instructional programs rather than number sense generally (Jones, 1995; Rusch, 1997; Ferguson, 1993).

A number of authors believe that number sense cannot be directly taught, but that it must permeate the curriculum and therefore an appropriate method of teaching must be found (Trafton, 1989; Van de Walle & Bowman Watkins, 1993). These authors do not provide much practical help to teachers who themselves do not understand or possess number sense. How can teachers themselves teach in a way that promotes number sense in their pupils if they have a poorly-developed number sense?

Is there some appropriate process or program to assist them to prepare to help their future pupils develop number sense? This section describes programs for pre-service teachers which aimed to help the students improve their understanding and skills for numeracy.

Mathematics education unit and number sense development

The most influential study for the planning of the Teaching Program and the assessment of number sense was the one completed by Kaminski (1996). Although this study was completed in Australia, a developed world country, it was one that was directly aimed to developing number sense in pre-service teachers. It is this important feature that made it so relevant to the present study. Kaminski worked with first year pre-service teachers who wished to be primary teachers but who appeared to lack the knowledge and confidence to teach mathematics. In discussing his program he suggests that often past schooling of his participants promoted the view of mathematics as too abstract, irrelevant to real life, and consisting of rules and formulas that required memorisation and practice. He notes that his students appeared to have had little opportunity to construct their own knowledge base and lacked the flexibility to modify their mathematical procedures due to the over-practising of them. In his study Kaminiski aimed to develop, implement and partially evaluate a number sense program for pre-service teachers. He was attempting to promote the student teachers' development, understanding and use of mathematical knowledge. His program incorporated socio-cognitive perspectives and constructivist approaches to the learning and understanding of mathematics with a special emphasis on promoting the development of reflective practices in mathematics.

For his research methodology he adopted a qualitative approach. He obtained descriptions and interpretations of the pre-service teachers' experiences in participating in the number-sense program. He also obtained data about how reflective practices in mathematics may have been promoted by the program. Early in the program six students were interviewed. They were asked a series of questions and completed a number of mathematical exercises. From this data collection it was possible to get, not a comprehensive but a useful perspective on the student teachers' understanding and use of mathematics in their past schooling. Kaminski suggests that the analysis of these results indicated that the students had competent and proficient use of written calculations for whole numbers but not for rational ones. Also the analysis of the initial interviews indicated that when calculating, the students seldom explored or made use of their understanding or relations between numbers or processes, beyond specific solutions given. They appeared unable or reluctant to use any intuitive knowledge. Little use was made of estimation. Initial computations were in standard written formats with all students experiencing difficulties in using mental computations. Their solutions were rarely reviewed for multiple or alternative solutions. Most experienced high levels of anxiety about working with mathematics and generally lacked confidence in their ability to interpret results and provide effective solutions.

Kaminski suggests that to have student teachers reconsider their views of, and beliefs in, mathematics taught in school would require that they have opportunities to review and reflect upon them. At the beginning of the course the students indicated that they had not previously participated in reflective activities such as considering the reasonableness of their solutions. They appeared to have had little experience in orally justifying or communicating their mathematical thinking. They also revealed a preference for direct teacher instruction, routine practice of procedures and reference to textbooks when difficulties arose.

The students were questioned again, both at the mid-point of the semester and at the end of it. The results showed that the information gained at the end of the semester was a further development on the positiveness of their views of mid-semester. Kaminski suggests that results indicated that they valued and were positive about the number sense program and were actively involved as groups in exploring mathematical ideas. With what they saw as supportive learning in a challenging environment, the students appeared willing to share their growth in understanding and admit to their difficulties. This may have been a factor in the development of communication among students in the small groups as well as the larger class group. This improvement in communication was illustrated by the fact that, although some still grappled with the understanding of concepts and processes, they were no longer afraid to make errors and reveal misconceptions or ‘gaps’ in their knowledge. With a greater understanding of, and confidence in the mathematics covered in semester unit there was a strengthening of their belief that they were able to make sense of this mathematics. When given the occasion to justify their thinking and opinions, the pre-service teachers' beliefs about mathematics generally emerged often implicitly, so that students heard differing views which stimulated reflection based on these externalised beliefs.

Kaminski's study suggests ideas that I was able to incorporate into my study. Although I chose not to include a program for teaching number sense directly, because the literature I reviewed did not generally promote it, there are features of Kaminski's research including his interviewing techniques and reflexive practices that I considered could be beneficial if adapted for my study. In his research methodology he found that the interviewing in which students did exercises and explained their understanding was helpful to gauge the development of their number sense. It also gave him insight into the depth of understanding of the algorithms and their previous mathematics knowledge. The reflecting by the students was important for the development of mathematics understanding and for evaluating its development. Kaminski's research found that his Australian students developed their number sense using his program. By adapting some of his strategies for the special circumstances in Fiji I hoped to achieve similar outcomes.

Mathematics education unit including problem solving

Clarke and Clarke (1996) describe another mathematics unit that has been designed to help pre-service primary teachers improve their understanding of the mathematics taught in school. The content was not exclusively the development of the students' number sense as other topics with which the students appeared to have difficulties were included, such as algebra and geometry. One of the aims of the unit was to present mathematics as a key to solving worthwhile and meaningful problems and thus lessen the negative attitude that considers school mathematics to be 'rules without reason'. The unit was built around cooperative problem solving. The teaching strategies *used* problem solving rather than *taught* problem solving. Considerable use was made of problems from newspaper articles and current events so that students began to see that part of the power of mathematics lies in its ability to help make sense of the world, to evaluate arguments and answer important questions.

Clarke and Clarke note that a preliminary study of the outcomes of the unit produced overwhelmingly positive responses. The students' evaluations of the unit suggested that, over the twelve weeks many students seemed to improve in their confidence and their competence with mathematics. So Clarke and Clarke believe it should help their future pupils to view mathematics as enjoyable, a key to problem solving and an important and useful way of making sense of the world. They also believe that the program's evaluation suggests that the doing of problem solving exercises is a means of bringing about change in attitudes so that students have a more positive approach to mathematics seeing it as enjoyable and making sense.

In a related study Clarkson (1998) studied the performance of students at another campus of the same University who were studying a mathematics education unit which followed the problem solving unit referred to by Clarke and Clarke (1996) From his analysis Clarkson suggests that the recommendation in the Speedy (1989) report, which implied that an extra unit of mathematics in teacher education would bring all students up to an acceptable achievement level, did not appear to do so. Clarkson considers that some students were helped, but that there were a number who have such deep misunderstandings that the completion of one such unit, although helpful in some ways, was simply not enough to help them come to grips sufficiently with their long term problems. These students needed more time to upgrade their skills with the help of skilled practitioners who were able to enthuse and instil confidence in the students. Thus the pre-service teachers would be empowered to fill other gaps in their knowledge of mathematics.

If the cycle of poor teaching of mathematics producing poor teachers of mathematics is to be broken in Fiji by the development of number sense in pre-service primary teachers, perhaps the use of problem solving exercises may be one way of tackling the problem. However, as Clarkson's finding shows, the problem solving strategy as used by Clarke and Clarke (1996) is not the ultimate solution.

As mentioned above in the early planning stages of my study I wondered whether to teach number sense directly as the content of a unit or expect that its development will occur if I use the teaching strategies recommended by constructivists. After investigating the relevant literature I have found that instructional programs specific to developing number sense are not generally recommended. Instead teaching mathematics that is permeated with number sense activities is advised. Other relevant studies and work done by teachers with school students are discussed in the next sections. They are related to the major areas of number sense in the operations and applications of number, namely computations, both written and mental with the related topic of estimations.

Computations

In the last section literature related to instructions that are said to promote number sense were discussed. In this section literature related to computations which are an important area of number sense is considered. The well-developed skill of doing mental computations and estimations requires considerable ability and an inclination to use numbers, their operations and applications. That is, it requires a well-developed number sense. As most mathematical activities use some mental computations and estimations, the development of these skills and abilities are possibly the major and most important part of the development of number sense. This is why I have given a considerable proportion of this part of the literature review to research computations. It must be remembered that for the Teaching Program in this study, number sense is the central theme.

In this section computations are divided into two types, algorithms and mental calculations. Mental computations are usually paired up with estimations because the latter also requires mental computations but using simplified numbers. Algorithms are considered first because of the emphasis placed on them in most primary school mathematics curriculum in contrast to that on mental computations and estimations. Also a number of issues related to algorithms affect the development of number sense in general, and mental computations in particular. Hence algorithms and their importance are discussed first, and then the meaning and importance of mental computations as well as instructional issues related to them will be reviewed.

Algorithms

For the past decade in mathematics education a growing attention has been given to the study of this so-called 'number sense' as a reaction to the over-emphasis on computational algorithmic procedures (McIntosh, Reys & Reys, 1997). The algorithms are still very much part of mathematics education in primary schools and therefore, with the attention given to number sense, the place of algorithms in the curriculum and their relationship, or lack of it, to number sense must be considered. I was interested in this matter since I needed to incorporate the algorithms in an effective manner into the Teaching Program that I was going to devise since they were part of the stipulated curriculum in Fiji. I was also interested in what findings might emerge in this study related to changes in the skills in using written algorithms, as well as the development of number sense using a constructivist teaching approach.

In this section literature concerning the relationship between number sense development, and skills in and understanding of algorithms is considered beginning with a discussion of the understanding and importance of algorithms, and then issues related to their uses. This prepares the context, in the following section for their relationship to mental computations and hence to number sense.

Understanding and importance of algorithms.

Algorithms are generally defined as routine procedures for computations in mathematics when using pen and paper. They are efficient, automatic, symbolic, general and contracted as they reduce several steps to one (Plunkett, 1979). Two very different kinds of cognitive processes can be involved in executing routine procedures - automatisation and reflection (Heibert, 1990). In the former, procedures are practised over and over until they are executed without much thought and so are performed with instrumental understanding. In the latter a person consciously thinks about what is happening and why it is happening that particular way and so they are performed with relational understanding. Skemp (1978) describes an advantage of automatisation of algorithms as that they are usually easier to follow than the processes of relational mathematics and therefore the rewards are more immediate and more apparent. Also, for the teachers of instrumental mathematics to have their pupils succeed in getting good marks in examinations, the kind of teaching that helps pupils gets correct answers efficiently is compelling. The curriculum in mathematics is often overloaded with content and so teachers have trouble covering the work. Therefore as processes of instrumental mathematics appear to cover topics more quickly they are the preferred strategies. Lastly Skemp suggests that the teaching of instrumental mathematics avoids the need for many teachers to overcome the psychological difficulty of re-structuring their own existing and long-standing schemas.

Heibert (1990) states that the efficient execution of a procedure is a benefit of automization. The more efficient a procedure is executed, the less mental effort is required. Yet he comments that in some cases an estimate is all that is required to solve a problem. Another factor he suggests supporting the idea that algorithms are not as important as they have been considered, is the availability of calculators and computers to do the laborious and time-consuming calculating.

Issues related to the use of algorithms.

The routine procedures reduce the mental effort, as mentioned above but, when promoted as the only way to do mathematics, they do not allow students to become independent and build up their own schema (Skemp, 1978). He says that there is less need for the re-learning of procedures for algorithms with relational understanding, as once a concept is grasped it is easier to remember ideas and hence in the long term mental effort is reduced.

Heibert (1990) also says that the learning of algorithms do not contribute directly to the development of meaningful mathematical knowledge. He suggests that perhaps the mental effort saved could be used to search for relationships and to create meaning. But there is no guarantee that students will use the saved time and effort to do other mathematical work which would contribute indirectly to the development of mathematical competence. Procedures are probably better remembered if time is spent making sense of them. Heibert believes that if we want students to remember procedures we need to ask them to step back and reflect on the procedures rather than to practise more of them. The conventional form of the procedures is quite compact and hides many of the patterns that exist. He suggest that a teacher could help the process of reflection by discussing alternative, more-elaborate formats that make the patterns more accessible, e.g. 37 x 42 = 2 x 7 + 2 x 30 + 40 x 7 + 40 x 30. He also believes that recognising the patterns does much more than help students make sense of the algorithms, as it help them make sense of mathematics.

McIntosh, Reys and Reys (1997) include reasonableness of answers as an important consideration when pen and paper, as well as calculator algorithms are used. Sowder and Kelin (1993) would also support this statement for they comment on students' apparent lack of the concept of reasonableness of answers shown by otherwise good mathematics students who are well versed in the algorithms. They suspect that this lack of thinking about the solution process is the result of earlier training to do large sets of problems quickly.

Mental computation and estimation

As most mathematical activities use some strategies and skills of mental computations and estimations, the effective use of their processes is closely related to a well-developed number sense (McIntosh, Reys & Reys, 1997). Therefore in this study I have incorporated the learning of these skills and strategies in the hope that it may assist the development of number sense. In this section I will first describe the meaning and importance of mental computations and estimations. Then issues and research related to their instruction that are relevant for the planning and implementation of a teaching program of the pre-service teachers will be discussed.

Meaning and importance of mental computations and estimations.

Before discussion of some of the issues of mental computations and estimations, I will consider briefly what mathematics educators mean by each of the terms. Sowder (1992a) says that the popular meaning of mental computations is the doing of problems in the head quickly but suggests that a view advanced in literature also requires the invention of procedures that are appropriate to the particular problem. Mental work requires understanding all the way through the calculation and the use of the complete number rather than individual digits (Sowder & Kelin, 1993). Estimation is defined as forming an approximate opinion of size, amount or value that is sufficiently exact for a specified purpose (Smart, 1982). Hall (1984) notes that estimation and approximation are not synonymous and explains that estimation is usually a mental exercise, whereas approximating usually requires first the use of a tool such as pen and paper or calculator.

Mental computations and estimations are related to number sense as they are seen as two major components of it (McIntosh, 1989). Many mathematics educators strongly promote the development of mental computing because of its importance as a necessary life-skill for dealing with numbers, and therefore included it in the development of a number sense (Carraher, Carraher & Schliemann, 1987; Cockcroft, 1982; French, 1987; Maier, 1980; Plunkett, 1979). Until relatively recently, curriculum developers had not placed importance on estimation (Sowder, 1992b). It was not until 1977 that the National Council of Supervisors of Mathematics included estimation and approximation within the ten basic-skill areas in need of development. This was followed by the *Agenda for Action* (National Council of Teachers of Mathematics, 1980) in which there was a call for incorporating estimation activities regularly and extensively into the programs.

Wandt and Brown (1957) sought to determine the relative importance of mental and 'pen and paper' mathematics in the solutions of problems encountered by adults in everyday non-occupational usage. They found that even then 75% of the uses reported were mental and one suspects that this percentage has increased through the intervening years. This suggests that school students, especially those in the primary classes, need to learn more about the development of mental computations than about the skill of doing the written algorithms. A further indication of the need to develop number sense when teaching standard algorithms comes in the form of a warning from Hope (1987). He believed that some able children may lose part of their ability to calculate mentally as they begin to learn the algorithms taught in school. He stated this in his conclusion to a paper about a case study in which he had investigated the estimation strategies of a thirteen year-old girl who had been highly-skilled at mental calculations. Hope noted that, like children in other studies, the girl's skills deteriorated and suggested that one reason for this may have been coincidental learning of written algorithms at school. He continues that children should be encouraged to look for calculation short-cuts and to use pen and paper or calculator only when absolutely necessary. He says that because most written computational algorithms seem to require a different type of reasoning than mental algorithms, an early emphasis on written algorithms may discourage the development of the ability to do mental computations.

Instructional issues for mental computations and estimations.

As noted in the last section, possessing good number sense seems to be a necessary requirement for being proficient at mental computations and estimations which are considered a necessary life-skill. A question for mathematics educators to consider is what procedures and skills related to number sense, and mental computations and estimations in particular should and/or could be included in mathematics programs. Sowder (1992b) strongly argues that instruction in mental computations and estimations can provide an avenue for developing number sense. She especially notes, commenting that some may disagree, that instruction on estimation and mental computation should be part and parcel of any instruction on arithmetic operations. She also suggests that instructional programs on estimation should take advantage of students' spontaneous mathematical intuitions and of their development. A warning she gives is that educators should not be in too much of a rush to teach estimation procedures in case the same mistake is made as has been made in the teaching algorithms, resulting in the rote learning of the processes.

Sowder and Kelin (1993) suggest it would be helpful to look at self-taught good estimators for ideas as these estimators' techniques would be natural and could form the basis for instruction. Hope and Sherrill (1987) list some of the techniques that skilled mental calculators use: factoring, distributive law, avoidance of carry operation, keeping of a running total and the tendency to work from left to right rather than from right to left as taught in the standard algorithms. Further, they describe the procedures within these techniques as being: variable, flexible, active in that the person calculating selects his or her own methods, holistic because complete numbers are used rather than single digits, and constructive as pieces of computations will be completed and then other parts added or subtracted.

Computational estimation is highly dependent on an understanding of the operations, knowledge of basic facts, place value and the ability to often work with the powers (Sowder & Wheeler, 1989). Van de Walle and Bowman Watkins (1993) state also that the mastery of basic facts is essential for effective mental computation and estimation. They state that thinking strategies for basic facts and other forms of mental computations must build directly on children's understanding of number and not be taught as mindless procedures. French (1987) suggests that the variety of methods that children and adults use in mental calculations is very great. He suggests that a discussions of these in the classroom is valuable, not necessarily in producing the best method but in encouraging a flexible approach and making explicit the advantages and insights that come from considering alternatives. French also suggests that much of the value of mental mathematics arises through discussing the methods used. If children are just given a set of questions and then told the answers, a valuable opportunity for learning has been lost if nothing more is said. Sowder and Kelin (1993) also believe that exposure to a variety of mental computation techniques such as the use of 'nice' number pairs that sum to ten and adding multi-digit numbers from left to right is necessary. The development of skills for simplifying calculations needs to begin early in a student's mathematics learning as it would encourage flexibility in thinking. Many of the ideas mentioned here appear worthy of being incorporated into a mathematics education unit that is attempting to develop the pre-service teachers' numeracy and to educate future primary school teachers.

As mentioned above the learning of algorithms can negatively affect the relational understanding of computations. Hence if and when they are introduced to students needs serious consideration especially in this day and age of increasing use of calculators, even in developing countries such as Fiji. With understanding and well-established skills of mental computations and estimations students are equipped to do most of the computations used in daily life, otherwise it is appropriate to use a calculator.

Summary of issues related to number sense

So far in this chapter I have discussed some of the considerations found in the literature related to number sense and relevant to my study. As can be seen in the definitions and descriptions of number sense, there are various aspects of number sense considered by mathematics educators. From this review I have collated what I consider the most important aspects of number sense for this study. I will consider *number sense* as *the ability and inclination to understand and use numbers and their operations and applications with ease, and to understand and appreciate the nature of mathematics*. A well-developed number sense as defined would be more that could be expected of all pre-service teachers. However the question may be raised as to what aspects of the descriptions are the minimum components necessary for teachers to possess to enable them to satisfactorily help develop their pupils' numeracy. If the key role of a teacher is to encourage students to make sense of situations in learning mathematics then number sense, which is essentially sense-making is fundamental (McIntosh, Reys & Reys, 1997). Therefore in any mathematics education unit for pre-service primary teachers a suitable program would need to include to some degree all the important features of my description of number sense, the understanding of number, its operations and applications, and an understanding and appreciation of mathematics.

The next area of the review in this chapter was computations, both written and mental and, as the discussion outlined it is an important area of number sense and therefore rather crucial in this study. Many of the mathematics activities used in a constructivist approach which help to develop number sense are related to written and especially mental computations. In the discussion of computations the cost and benefits of learning algorithms were discussed. This review led into mental computations and their increasing importance relative to the written algorithms, as well as their importance in everyday mathematics and hence in the school curriculum.

One problematic issue today is the teaching approach generally used for helping children learn the algorithms. Children are often not helped to understand the processes involved when they are taught the algorithms and hence they need to rote memorise the procedures. Research has shown that the development of number sense can be stifled when rote learning of algorithms is taught. The literature suggests that development of number sense is greatly enhanced if students are helped to develop strategies for mental computations and estimations.

Some of the strategies that are suggested in the literature that would be helpful in the Teaching Program for the pre-service teachers to improve their skill in computing mentally, and hence develop their number sense, include using activities and discussion that have been suggested for primary school students. These include an investigation into the base ten number system including the bridging numbers and discussions of the meaning of the operations. From anecdotal evidence gathered in the past previous two years of lecturing at the College (Hanrahan, 1999) I believed that these basic investigations into aspects of number sense were needed and beneficial. Research tends to favour brief discussion of strategies used by the students incorporated into regular instructions or formal discussion for a few minutes each class rather than being a structured component of the teaching program with full lessons assigned to it.

So far in this chapter I have reviewed the literature related to the place of number sense in mathematics education. In the previous chapter I reviewed the literature related to culture and mathematics. In the next section I will focus the review on a consideration of the mathematics education of pre-service teachers in the context of the discussion so far.

Pre-service teachers and mathematics education

The primary question in this study is how to help pre-service teachers in a primary Teachers' College in Fiji know and understand the mathematics they will be required to teach. In this section I will examine the literature related to how the issues related to culture, number sense and mathematics education that have already been discussed affect pre-service teachers and their mathematics education. The section is divided into four areas: cognitive issues; non-cognitive issues; lifelong learning; and aspects of mathematics teacher education.

Cognitive issues

In this section of the literature review the ideas considered will be related to mathematics generally rather than to number sense which is central in this study. This is because the literature does not specifically discuss number sense when considering the issues for pre-service education. The first part of this section reviews the level of mathematical understanding required to teach mathematics. Then the importance of an understanding of the nature of mathematics is considered because as the discussion indicates, both these understandings are considered important in this study as they affect the development of number sense.

The knowledge of concepts required.

There are differing opinions about the amount of knowledge teachers of mathematics must possess in order to help their pupils learn mathematics effectively. It is suggested by the Commission on Education of Teachers of Mathematics (1981) that a teacher's knowledge and understanding of mathematics should extend substantially beyond that which they are expected to teach, to include mathematics that which their pupils will learn in the future. Yet Fenema and Franke (1992) cite research from Eisenberg (1977) that has provided little support for a direct relationship between a teachers' formal knowledge of mathematics and their pupils' learning. But as Fenema and Franke note the studies that they reviewed measured teachers' knowledge according to the number of university-level mathematics units that had been successfully completed. The researchers did not attempt to measure what the teachers knew about mathematics nor ascertain which mathematics was covered in the university units the students had studied. So these studies may be considering a factor of teachers' knowledge that has minimum relevance to the knowledge necessary to teach mathematics effectively at school level. Another study by Erickson (1986) infers that much formal mathematical knowledge may not be necessary, but suggests that the level does influence the decisions a teacher makes about classroom instruction. His study found that in the area of mathematics in which the teacher was more knowledgeable, the teaching and subsequent learning was richer.

While the teachers' need for extra knowledge in order to help their pupils achieve may be a matter of debate, it is believed that for their own confidence they need to feel that they know enough to be effective teachers. Pengelly (1990) says that many teachers lack confidence in their wider knowledge of mathematics and mathematics thinking. For them it is difficult to venture from conventional methods such as teaching for rote learning. Pengelly comments on the mathematics learnt by pre-service teachers and how it is learnt. She believes that the learning needs to be such that it will lead teachers to believe that they know sufficient mathematics to teach it. This is especially true for first year students. The early stage of their development as teachers is characterised by a concern for self. It is a time when students worry about personal adequacy and survival in the classroom rather than later years when the task of giving a lesson is more their concern.

Ball (1990) also comments on the mathematics knowledge of students completing teacher education courses. She says that subject matter preparation of teachers is rarely the focus of any phase of teacher education. In her report of a longitudinal study she discussed the mathematics knowledge of pre-service teachers. The research investigated the understanding of mathematics held by 252 pre-service teachers at five teacher education institutions. In the study all the college students completed a questionnaire at repeated intervals during their mathematics course. As well, a sample of students was closely followed throughout their pre-service program and into their first year of teaching. The results of the study suggest that the pre-service teachers' understanding of mathematics is rule-bound and compartmentalised. This, it is asserted, is inadequate for teaching mathematical concepts and procedures in a meaningful way. Ball says that in order to teach mathematics, teachers must not only be able to describe the steps for following an algorithm but also discuss the judgements made, the meanings of, and reasons for, certain relationships in the procedures. Teachers must be able to generate explanations or other representations, often on the spot in response to student's questions. Summarising her findings, Ball describes three criteria for the kind of subject matter teachers of mathematics need. Firstly, their knowledge of concepts must be correct. Secondly, they should understand the underlying principles and meanings of the procedures. Lastly teachers must appreciate and understand the connections between mathematical ideas. Ball concludes by saying that the data suggest that the mathematical understanding that prospective teachers bring to their job needs to deepen in order for them to teach mathematics for understanding. She notes that, although the teachers claim to learn subject matter from teaching mathematics, this is an empirical question as yet unanswered.

As well as the understanding of concepts of mathematics another important understanding for pre-service is that of the nature of mathematics and their conceptions of it. This is also related to affective issues for the pre-service teacher as well as the cognitive issues and so is an important consideration for pre-service teachers' number sense development especially for those who are lacking in confidence and knowledge and so is discussed in the next section.

The knowledge of the nature of mathematics required.

Thompson (1992) defines teachers' conceptions of mathematics as their conscious or subconscious beliefs, concepts, rules, mental images and preferences concerning the discipline of mathematics. Although no research is given to support the idea it is said that the crucial point about teachers' conceptions of mathematics is that they affect their choice of ways of learning and teaching mathematics (see also Wright & Tuska, cited in Brown & Borko, 1992).

Ball (1990) also believes that teachers need knowledge about what mathematics is, as well as subject matter knowledge. From questionnaires and interviews with pre-service teachers she found that it was widely believed that mathematics was a collection of arbitrary rules to be memorised. Ball sees a need for pre-service teachers to understand the nature of mathematics and mathematics as a field. She says this would enable teachers to establish the validity of an answer, know what ideas in mathematics are arbitrary and convention, and know what is derived from logical methods. She concludes by saying that mathematics educators need to understand more than they currently do about how to help pre-service teachers develop their understanding of the nature of mathematics.

Because these writers suggest that the understanding the nature of mathematics is necessary for effective learning and teaching it is important for pre-service teachers to possess it. Comments by Ball (1990) indicate that its understanding is in contrast to thinking of mathematics as a collection of arbitrary rules. Therefore she seems to be suggesting that this understanding of mathematics is closely related to a well-developed number sense as defined earlier. For a teacher of mathematics to enact his or her understanding of the nature of mathematics they need to know they do possess values of mathematics and what these values are. Values of mathematics as described by Bishop (1988) are considered in the previous chapter. In this study for the investigation of the pre-service teachers' understanding of the nature of mathematics, I use the classification of Bishop's values' for my framework: rationalism, objectism, control, progress, openness and mystery.

Non-cognitive issues

In the initial stages of this study I came to the realisation that the cognitive aspect of number sense was far from the total picture in the development of number sense. Therefore I planned to also include in the study how the affective as well as the cognitive aspect was involved. In this section I review the literature concerning affective issues in the education of pre-service teachers. For this discussion the literature from three areas are reviewed. First, ideas about mathematical beliefs and attitudes, then the processes involved in changing these beliefs and attitudes, and lastly the need for these changes to set in place processes for lifelong learning.

Mathematical beliefs and attitudes.

During the past decade many mathematics educators have researched issues related to the understanding and beliefs of, and attitudes to, mathematics of pre-service teachers in educational institutions (Mcleod & Ortega, 1993; FitzSimons, Jungwirth, Maab, & Schloeglmann, 1996; Thompson, 1992; Bobis & Cusworth, 1997; Ernest, 1988; Brown & Borko, 1992; Lortie, 1975; Martin & Tobias, 1997; Furinghetti, 1996; Sullivan, 1987; Nisbet, 1991). This change is related to a shift from studies of behavioral conceptions of teaching to studies from the teachers' and the learners' perspective (Thompson, 1992).

What is meant by mathematical beliefs and attitudes? FitzSimons, Jungwirth, Maab and Schloeglmann (1996) in discussing adults' beliefs and attitudes to mathematics say that the beliefs and attitudes adults hold cannot be completely distinguished but the terms are useful. They state that beliefs are indicators of people's ideas of mathematics, and attitudes are indicators of their emotional relationship to it.

Furinghetti (1996) begins her explanation of beliefs with a description of mathematics knowledge as a wide network of concepts, images, and intelligent abilities possessed by human beings. She describes beliefs as part of that knowledge, as incontrovertible personal 'truths' held by everyone, deriving from experience or from fantasy with a strong affective and evaluative component. For mathematics teachers, Furinghetti considers the beliefs of mathematics are focused and specific, and rely on quantitative and rational components. She says that they are often unconscious but the teacher is able to reflect on them and to make them explicit through statements and examples hence making them conscious. She defines the conception of mathematics as the set of these beliefs.

Mcleod and Ortega (1993) say that there is the general meaning for attitudes that includes beliefs about mathematics and about self, but they themselves use attitudes as referring to affective responses that involve positive or negative feelings that are relatively stable. They suggest that attitudes towards mathematics develop in two different ways. First they may result from the automatising of a repeated emotional reaction to mathematics. And a second source of attitudes is the assignment of already existing attitudes to a new but related task. For example, a student who has a negative attitude to proofs in geometry may attach the same attitude to proofs in algebra.

A number of studies have investigated the origins of attitudes and beliefs. Results of one study (Wright and Tuska, cited in Brown & Borko, 1992) indicated that early experiences exert a powerful influence on the images future teachers have of teaching and teachers, and that these images continue to influence them in their own teaching careers. Wright and Tuska suggest that, unless formal teacher education can change these images, teachers will teach using the methods that their teachers used which is usually far removed from the roles envisaged by the authorities such as the National Council of Teachers of Mathematics as described above. Also Thompson (1992) suggests formal teacher education programs must sometimes bring about change by helping students to 'unlearn' as well as to learn, discarding some of the knowledge, beliefs and attitudes about mathematics and teaching that they bring to the programs.

Other educators have also argued that the main influences on teaching are the experiences teachers have prior to their teacher education. Lortie (1975) investigated the source of teachers' belief from a sociological viewpoint, and found that teachers' beliefs and practices were strongly influenced by their experiences prior to training. He interviewed teachers from a range of socioeconomic backgrounds with a focus on the thinking of teachers.

Evidence described from research and the opinions of many is that a majority of prospective teachers hold negative attitudes towards mathematics. Sullivan (1987) found that the majority of pre-service teachers have negative attitudes to mathematics and to the teaching of it. Their beliefs about teaching are largely the product of their own learning experiences and therefore usually consistent with a traditional approach. Bobis and Cusworth (1997) found that these kinds of attitudes can change. They completed a five year longitudinal investigation into the attitudes of pre-service teachers towards mathematics and towards the teaching of the subject. They found that teacher education can improve the attitudes of pre-service teachers towards mathematics and that this shift in attitudes can continue well into the first year of teaching.

It is important that teacher educators seriously consider this issue of negative attitudes of pre-service teachers (Cockcroft, 1982; Department of Employment, Education and Training, 1989; Ernest, 1988). The Department of Employment, Education and Training (DEET, 1989) suggest that teacher education courses should pay special attention to attempting to turn negative attitudes to positive ones. This is because DEET had consistently received reports that students enter primary teacher education courses with feelings of fear and anxiety, and hence negative attitudes to mathematics. Cockcroft (1982) says it is a must that those who train teachers take responsibility for the task of helping pre-service teachers establish positive attitudes to mathematics.

As many of these writers suggest, negative attitudes towards mathematics are developed long before students arrive at a teacher education institutions. For the pre-service teachers in this study in Fiji their attitudes were formed during a schooling that included much rote learning in preparation for the frequent external examinations. The processes to change their emotions and attitudes related to mathematics, is not easy or quick but requires special teaching approaches in which there is some ‘unlearning’ of inappropriate attitudes. Some ideas from the literature about the processes of change are discussed in the next section.

Processes for change.

Change is not usually considered easy to implement especially changes involving attitudes, beliefs and understanding in mathematics education. I am optimistic, and so believe that if it is to happen anywhere in the life of a teacher, the time in which they are preparing to become teachers seems an opportune time. Pre-service teachers generally come to the institutions with the aim to be educated and learn about becoming teachers. Then ideally the pre-service teachers graduate from their education institutions after experiencing appropriate and necessary changes, and equipped with many skills and a willingness to continue the learning process. With this in mind, my study aimed to help bring about some necessary changes in numeracy, including to their beliefs and attitudes of the pre-service teachers by implementing a special mathematics education unit. In this section, some of the relevant literature on the processes of change will be considered.

In teacher education courses, the educators are usually hopeful that their students might implement what they discover, hear and are shown about teaching while attending their institutions. Thompson (1992) is rather pessimistic about the possibilities for change in teachers and the lack of strategies for bringing about change in pre-service teachers during their courses. She says that research has shown that such things as teachers' practices and conceptions are deeply rooted. Even when new ideas are enthusiastically embraced they are often interpreted through old mindsets. Change should be regarded as a long-term process. One useful strategy she suggests is for pre-service teachers resulting from the teachers to trial alternatives in their classrooms. Thompson says that whether reflections on such activities as the discussion of case studies of teaching practice have an effect depends on the extent to which teachers accommodate their existing conceptions to the new ideas and how, if at all, those ideas are translated into action. Therefore she says mathematics educators must investigate ways of helping pre-service teachers reflect on the ideas that they are asked to interact with. The educators need to help them develop the intrinsic motivation to consider alternatives to their current practices and to develop their own personal reasons for justifying their actions.

Others have also argued for similar approaches. For example, Fullan (1991) believes that for the process of change to be successful, the students themselves must be engaged in it. He suggests that those responsible for innovations would be well-advised to consider how an innovation will be introduced to students explicitly, and how students' reactions will be obtained at the initial point as well as periodically throughout implementation. Fullan continues by saying that innovations that require new activities on the part of students will succeed or fail according to whether students actually participate in these activities. Students will participate to the extent that they understand and are motivated to try what is expected. He found that one method to promote intrinsic interest in learning tasks was cooperative learning.

Also, when discussing the difficulties of change in mathematics education, Furinghetti (1996) discusses the problem of how to help teachers be aware of their conceptions and of the influence the conceptions have on their teaching. She found that the processes of metacognition regarding their practical knowledge appeared to help the teachers. She believes that this kind of metacognition is particularly important for preparing teachers to accept curricular changes. She says it is especially true when the proposed changes are in contrast with the existing beliefs and when there would need to be a reshaping of teachers' conceptions.

Most of these writers see the need for change to occur during a teacher education course. Especially in mathematics the change needed is often one of attitudes towards, and conceptions of, mathematics. Because these changes are in deeply-rooted attitudes and beliefs they are difficult to change. Among the strategies for change that are suggested are to involve the students as much as possible in the activities for the change process, metacognitive processes including reflection on such activities as the active and critical reading of case studies, and cooperative group work. All these processes are not only suitable for teacher education courses, but pre-service teachers should be encouraged to continue them as part of their lifelong learning process especially if permanent change in their beliefs and attitudes is looked for. Consideration of this lifelong learning for pre-service teachers is undertaken in the next section

Lifelong learning

Probably pre-service teachers in Fiji would imagine that on entering the College they were embarking on the last phase of their education. They would most likely expect that the course they were beginning would equip them with the knowledge and skills they needed to become and remain effective teachers until they retire. The Teaching Program that is the central feature of this research was conducted in the first semester of their course and was planned and implemented with the aim of helping the pre-service teachers make a renewed effort to develop their numeracy, in particular their number sense and to a lesser degree to learn to teach. As both these processes need to be ongoing, I planned that the learning involved in the mathematics education unit would encourage the students to continue to develop their metacognitive thinking and skills so, in particular the development of number sense and the learning to teach would continue for the rest of their lives. In the following discussion I will review literature which supports the need for learning to be lifelong. First I will consider its meaning, then the learning process and finally lifelong learning and its relationship to teacher education.

Meanings of lifelong learning and its processes.

To some people lifelong learning is seen as no more than adult education but Longworth (1997) believes in a more all-encompassing view of it as “a philosophy and set of methodologies for developing the human potential in everyone” (Longworth, 1997, p.3). He spells out his definition by describing education as a holistic process with its focus on the satisfaction of the learner. He sees this type of education as lifelong as it continues from birth to death. It is ‘learning’, as it gives the learners the tools by which they can learn according to their own learning styles and needs. It is ‘for all’, pro-actively creating the conditions in which learning develops creativity, confidence and enjoyment at each stage of life. The Nordic Council of Minsters (1995) believe that in the learning process for lifelong learning the most important incentives are motivation and joy. The learning should reinforce the learner’s self-confidence, capacity for taking initiative and sense of own worth. They also say that learners must have the opportunity of assuming responsibility for their own work and to set their own goals while incorporating their own experience of life in the learning. These authors are suggesting that lifelong learning has a role to play in personal development. Therefore it would be important in the development of pre-service teachers as considered in the next section.

Lifelong learning for pre-service teachers.

Longworth (1997) considers that the change of paradigm from teaching to learning emphasised in the lifelong learning literature will demand a radically different approach to the education of pre-service teachers. Though he is speaking about European teacher education there is little reason to believe it would not be true in other countries, including Fiji. In his guidelines for educational institutions Longworth includes development of creative, rewarding, enjoyable and productive learning programs which most importantly stimulate a permanent habit of learning. In speaking about effective educational organisations he believes the administrators should integrate work and learning and inspire their students to seek quality, excellence and continuous improvement in both.

Chapman (2000) also sees a need to assist teachers in promoting lifelong learning. She says that teachers need to be helped to recognise the metacognitive dimensions of classroom learning and involvement. In what she describes as a learning revolution she sees the need for the teachers to promote, among other things communication skills, learning how to learn, learning how to solve problems and learning how to research. She says an important part of developing learner competence involves interaction between motivation, goal-orientation and interest on the students' part as they progressively acquire the skills of self-regulation, self-monitoring, self-regulated engagement in class, and self-regulated knowledge-building. This clearly has implications for pre-service teacher courses where students need to become independent learners.

The Nordic Council of Minsters (1995) say that in the field of adult education, lifelong learning requires the development of new methods and that new tools need to be applied. They also say that the educators themselves must learn how to acquire new knowledge and skills and how to reshape old roles such as communicating via information technology. To do this, learning environments and relevant structures must be developed with the building of bridges between learning in the workplace and in the classroom. For example, they describe combating unemployment by promoting a more appropriate allocation of the individual citizen's lifespan, allowing for periods of paid work, education/training and other activities

These requirements of teachers suggests that the educational institutions need to include in their units, modelling of such processes so that pre-service teachers would consider that lifelong learning was an important part of their own lives and indeed their teaching. Then with reflection and discussion on these processes which will help to make features of such learning explicit, the students would be helped to see, for example, the need to develop their own and eventually their students' metacognitive abilities.

Aspects of mathematics teacher education

While in the literature there may be controversy about the amount of mathematics knowledge necessary for teaching in a primary school, there appears to be no question about the necessity of courses to provide units for the learning to teach mathematics effectively. But there are questions about what is considered necessary to be included in these units and how they are to be presented. In planning my Teaching Program for this study these were two crucial questions that I asked myself, and sought assistance from the literature. Throughout this literature review many ideas about what future teachers of mathematics need to learn have been identified. In this section the few remaining issues which seem critical that are more specific to teacher education are reviewed.

Content of mathematics education units.

Helping pre-service teachers to be prepared for their roles as teachers is the function of teacher education programs. But becoming a teacher does not all happen in the teacher education institution as it is a life-long process. In fact Brown and Borko (1992) describe the process as beginning long before the formal education course and continuing after graduation as teachers continue to learn and change throughout their careers. They say that ideally the pre-service teachers would graduate from their education institutions equipped with many skills but more importantly a willingness to continue the learning process. In commenting on teaching mathematics to pre-service teachers they suggest that there is limited research about good mathematics education programs.

In a study completed by Sullivan and Leder (1992) he investigated the classroom practices of beginning teachers teaching mathematics so as to determine what changes might be necessary in teacher education. In the observations and review of beginning teachers it was found that the pupils' responses to instruction were foremost in the thinking of the novice teachers. The influence of the pupils tended to lead the teachers to be more teacher-directed, to give more explicit instructions, to emphasise completion of work over comprehension, and to avoid open-ended or problem-solving tasks. Sullivan saw this as having serious consequences for trying to teach mathematics with understanding. He suggested that teacher educators address ways that pre-service teachers could identify and improve their pupils' attitudes towards learning. Pre-service teachers should be introduced to the use of assessments that are not predominantly recall of facts and facility with skills. This would mean that in turn their pupils would be less likely to reject higher order skills and problem-solving. He suggested that teachers could improve their pupils' attitudes to mathematics by their own reflection on what happens in the classroom. This would consequently result in an improvement in their pupils' understanding of mathematics. He also inferred that the ability and practice of teachers to reflect on their own teaching was a possible strategy for new teachers to come to recognise the influence students have on the strategies they use, inferring perhaps that teacher educators instil the practice in the pre-service teachers' learning skills. His assumptions were that these ways of teaching needed to be promoted first of all in pre-service courses.

Comiti and Ball (1997) believe teachers need assistance to deepen their understanding of and skills in mathematics. As regards specifics, they see the new visions in mathematics education as promoting a shift from an emphasis on speed and accuracy to an emphasis on reasoning, and from an emphasis on memorisation and procedures to an emphasis on conceptual understanding. This is a general move away from efficiency in using processes to learning to understand the processes used in numeracy. Calculators can replace the former.

Teaching approaches for mathematics education units.

Becker and Selter (1997) believe that elementary teachers, with some exceptions, are not provided with enough mathematical learning experiences during their education to adequately equip them to teach mathematics in a way that promotes the sense-making nature of mathematics. They believe that courses for pre-services teachers need to be organised in such a way that the learning experiences provided by the lecturer will be applicable to what the pre-service teachers want their students to experience. Thus in teacher education they suggest that a culture of teaching be established which is similar to a favourable culture of learning in school classrooms. Though not always acknowledged, Becker and Selter believe that an activity-oriented conception of education is at the heart of much present-day thinking about re-organising elementary school practices. They give some general principles for important guidelines for organising teaching. These included learning actively, individually and co-operatively, and learning the content in strands and in contexts which involve constructing knowledge towards a structured entity.

Comiti and Ball (1997) believe that teacher educators have to simultaneously help pre-service teachers develop know-how, knowledge, methods, attitudes and habits. Mathematics teacher educators must recognise teachers as learners and realise that they need time to reconstruct their ideas and to construct new ones. They also see a need for a general commitment to the idea that learning to teach is a constructive process so classes need to be organised in ways that encourage group interaction, co-operative learning and group discussion.

Bishop (1988) lists principles which summarise to a certain extent what has been suggested in this section about the role of teacher educators and the content and teaching approaches of their mathematics units. He states that these principles should be included in the education of mathematics teachers. For his first principle he states that, during their education the pre-service teachers should develop a broad understanding of mathematics as a cultural phenomenon. He describes teachers as mathematics enculturators which incorporates more than just the teaching of the subject matter. Secondly he says that they should develop a deep understanding of the values of the mathematics culture. As well their education should improve their competence in, and understanding of the symbolic technology of mathematics, and it should develop their knowledge and understanding of the technical level of mathematics culture. Lastly the education of the teachers should develop a strong metaconcept of the mathematical enculturation process generally. In this present study, the enculturation processes were included in the investigation into the effectiveness of the Teaching Program as I was interested in what and how it occurred as the First Year pre-service teacher adapted to learning in the mathematics education classroom and developed their number sense.

From this section of the review it can be seen that teacher educators have a number of overlapping aims for their students such as to help them develop knowledge, methods, appropriate attitudes as well as good habits. In their teaching, the literature suggests teacher educators need to have a general commitment to the idea that learning is a process. Their own teaching should provide and model learning experiences that pre-service teachers would like to practise. Finally teacher educators are advised to address ways of helping their students reflect on their abilities, practices, and how they can identify and improve their pupils' attitudes along with their knowledge.

Summary of pre-service teachers and mathematics education

In this chapter of the literature review the discussion has been related to pre-service teachers and mathematics education. This review is central for this study because the study is centred around the mathematics education unit during the first semester of a pre-service teachers' course. It appears from this review that teacher education needs to attend to two major aspects. It needs first, to have teaching programs that aim to improve the understanding, both of the concepts and the nature of mathematics, and second, to aim to help bring about changes in inappropriate beliefs and attitudes of pre-service teachers leading to development of lifelong learning skills. In fulfilling these aims one of the consequences would hopefully be the improvement of numeracy, of which the development of number sense is central and is the focus of this study.

The review of literature about teachers' mathematics knowledge was completed with the aim of helping me to determine what knowledge content in a mathematics education unit was considered necessary for pre-service primary teachers. Although in some research the extent of the mathematics knowledge of teachers has not been found to influence the achievements of their pupils, it does appear to affect their teaching strategies as well as their own confidence in teaching mathematics. From my personal cursory observations of pre-service teachers that I had taught during the four years previous to the commencement of this study, I would tend to agree. Hence I had a concern for the participants in my study, the first year pre-service teachers who appeared to be lacking in understanding and knowledge of the mathematics they had to teach. There was an apparent need to help them develop their own knowledge of mathematics so that they felt confident that they had gained sufficient knowledge to teach mathematics effectively. Not only is the understanding of subject matter a consideration but also the understanding of the nature of mathematics is seen as important to help understand concepts and broaden their view of mathematics which in turn will contribute to their development of number sense. This understanding also improves the quality and depth of the teaching.

Mathematics educators in the last decade are turning their attention more to affective issues in the area of the pre-service teacher education. When pre-service teachers arrive at educational institutions they often have inappropriate beliefs about, and negative attitudes to, mathematics. Mathematics education lecturers are faced with the problem of how to bring about change in these attitudes and beliefs, so that their actions as mathematics teachers will be influenced by appropriate attitudes. It is believed that there is a link between attitudes and actions of teachers. Because learning experiences provided by the teachers can result in the construction of knowledge for the students it seems reasonable to assume that teachers' actions and consequently attitudes and beliefs have a major influence on numeracy and the development of number sense of their students. In this study one of my questions is concerned with what elements of number sense speak to the essence of numeracy. This literature highlights the importance of attention given to affective factors in the education of pre-service teachers.

For teacher educators, bringing about changes by guiding students to become effective primary teachers is a challenge as the students often have deeply ingrained negative attitudes to mathematics and lack confidence in their ability to become affective teachers of mathematics. One of the suggestions offered in some papers includes the promotion of constructivist strategies for learning. Along with these strategies, the studying of research such as case studies of teaching practices is suggested in the hope that with reflection, pre-service teachers will be prompted to consider changing their practices. A further suggestion is the need to use some empowering processes such as including the participation of the students as much as possible in the change process. Thus to bring about change, challenging pre-service teachers' existing thinking and acting via challenges and external stimuli, reflection and discussion is suggested as the way to help the students face and consider the differences between their own and others' ideas of teaching.

The above discussion for mathematics teacher education includes the realisation that teacher education is the beginning of a life-long process. Therefore the education of pre-service teachers must consider its responsibility towards promotion of lifelong learning in its courses. This would include the opportunity to help the students become more independent and confident learners and to develop metacognitive thinking and skills. Also, the pre-service teachers are the educators of the next generation and so they will be associated with helping their pupils participate in the economic, social and political activities of life which will require lifelong learning. In implementing a mathematics education unit I can attempt to play my part in helping the pre-service teachers with their own and their future pupils' lifelong learning.

In the education of pre-service teachers mentioned above, Brown and Borko (1992) comment that not enough research has been done into what constitutes a good mathematics education program. They therefore see a need to design, implement and study programs for teachers so that documentation of the results will provide insights into the process of becoming a teacher. Hopefully this study can be a small part of the response to this comment.

The research questions revisited

The various summaries throughout these two chapters of the literature review partly answer the questions posed at the end of the Chapter 1 and thus the questions need to be revised. The new questions are:

1. Can a mathematics education unit be designed for Fijian students in this College using:
   a) number sense as the underlying theme with an emphasis on the areas of:
      i. numbers,
      ii. their operations,
      iii. their applications in everyday living, and in particular,
      iv. estimations and mental computations within these three areas;
   b) a social constructivism approach and hence in particular an emphasis on:
      i. acceptance of each student as they are,
      ii. encouragement of construction of one's own knowledge of mathematics,
      iii. use of materials and everyday contexts,
      iv. reflection and communication to improve learning?

The literature review suggests that the effectiveness of the Teaching Program can be deduced from the design of the mathematics unit, from the responses of the students to the Program , and my reporting of, the implementation of it. There is much material in the literature review which describes teaching strategies and content used in mathematics education units that produced development of number sense. Such development was recognised in the responses of the students and the reporting by the lecturers concerned.

Hence my second question now becomes:

2. Can the effectiveness of such a unit be evaluated by examining:
   a) the design documents of the unit,
   b) the responses of the students to its implementation, and
   c) my reporting of its implementation?

The literature review suggests that the answer to Question 3 is 'yes' because other research studies have used it effectively. Also the literature review suggests that there are two main aspects of number sense which can be used to determine that numeracy has improved. Thus Questions 3 and 4 become:

3. What development of numeracy occurred for students who took this unit? In particular,
   a) what changes occurred in the cognitive aspect of number sense?
   b) what changes occurred in the non-cognitive aspect of number sense?

Question 4 related to Question 3 also arose from the review of the literature which will be important to ask in the Fijian context.

4. If changes in the development of numeracy do occur for the students, what brought about these changes?

The next chapter will turn to the methodological issues of the study. A description of the instrumentation, data collection, data analysis and the multiple case study approach used will be presented and discussed.

CHAPTER 4 METHODOLOGY AND METHODS

This chapter outlines the methodological design for this study, the instruments used, the procedures related to the instruments and finally the reasons for the analyses that were used to examine the data. However to begin the chapter the revised research questions are shown here in Figure 4.1 for convenience.

The first of these questions relates to the development and teaching of the Teaching Program. This will be dealt with at length in Chapter 5. The discussion in this chapter will elaborate further on matters 2 to 4. They will be revisited at the end of the chapter and elaborated so that they are clearly attuned to the data collection and analysis process. Discussion in this chapter will elaborate further on matters related to these questions so that they become operational and hence give direction to the choice of data, its collection and analysis. Then the questions will be restated at the end of the chapter.

Design of study

The major element of the context of the study is the class of First Year students at a Fiji Teacher's College, the participants in this study. There were initially forty students in the class. This number decreased by two because of personal reasons. Nearly all of the students were highly committed to becoming effective primary school teachers. They largely came from rural or island communities to which they hoped to return to teach after graduating. An unwelcomed expectation of the staff of the College was that the pre-service teachers' standard of numeracy was generally poor compared to their fellow students in other tertiary institutions, indicated by the poor mathematics examination results shown on their application forms for entry to the College.

Table 4.1 summarises the data collected, the ideas investigated and a brief rationale for their use. Further details about each of the instruments and other data collection procedures are given later in this chapter.

Research questions

1) Can a mathematics education unit be designed for Fijian students in this College using:
    a) number sense as the underlying theme with an emphasis on the areas of:
        i) numbers,
        ii) their operations,
        iii) their applications in everyday living, and in particular
        iv) estimations and mental computations within these three areas;
    b) a social constructivist approach and hence in particular, an emphasis on:
        i acceptance of each student as they are,
        ii encouragement of construction of student's own knowledge of mathematics,
        iii use of materials and everyday contexts,
        iv reflection and communication to improve learning?

2) Can the effectiveness of such a unit be evaluated by examining:
    b) the design documents of the unit,
    c) the responses of the students to its implementation, and
    d) my reporting of its implementation?

3) What development of numeracy occurred for students who took this unit? In particular,
    b) what changes occurred in the cognitive aspect of number sense?
    c) what changes occurred in the non-cognitive aspect of number sense?

4) If changes in the development of numeracy did occur for the students, what brought about these changes?

Figure 4.1. The research questions.

As discussed in the literature review the cognitive aspect of number sense as described by McIntosh, Reys and Reys (1992) includes the ability to understand and use numbers, their operations and applications in computational settings. This number sense was the central issue chosen to investigate the efficacy of the cognitive aspects of the learning during the Teaching Program, the measurement of its development was used to help determine if effective change had occurred. Pre and post testing of the students' number sense was used to investigate this. The instrument, Assessment of Number sense (see Appendix A) was used for the testing of the cognitive aspect of number sense. As an outcome of the discussion in the literature review, the measurement of any improvements in the skills of doing traditional procedures for number operations was thought might also indicate to a lesser degree, a change as the result of the Teaching Program. Hence a test of algorithms for number operations (see Appendix B) was used to measure this skill and knowledge associated with number sense.

Table 4.1.

Summary of details for the data collection

Process	Issue	Rationale
Assessment of number sense	Development of cognitive aspect of number sense	To measure change in number sense
Test of algorithms for number operation	Skill at performing algorithms for number operations	To relate any change in skill at performing algorithm for number operations to any number sense change
Questionnaire on beliefs about mathematics	Beliefs about mathematics	To investigate changes in beliefs and their relationship to any number sense change
Interviews	Changes related to number sense	To help determine reasons for any changes in number sense
Journal writing by students	Changes related to number sense	To help determine reasons for any changes in number sense

But then the question arises as to why any development occurred. Was the number sense development largely the result of the constructivist approach or because of one or more other factors? Data from the responses to a questionnaire on beliefs about mathematics (see Appendix C), semi-structured interviews and their answer sheets (see Appendices D and E) of a sample of ten students, and journal writings kept by students throughout the Teaching Program and the following semester were used as data banks which would link to this issue

To measure any change, all the testing and interviewing were completed before and after the first semester Teaching Program, and then later in the year towards the end of the second semester. Journal writing by the students was completed continuously throughout the year. Table 4.1 gives an outline of the sequence of the events of the data collection.

Table 4.2

The sequence of events for data collection

Data collection	Feb.	Semester 1	Jul	Semester 2	Oct.
Census Information	*				
Assessment of number sense	*		*		*
Test of Algorithms for number operations	*		*		*
Questionnaire on beliefs about mathematics	*		*		*
Interviewing of the sample of ten students	*		*		*
Implementation of the adapted mathematics unit		*			
Continuous journal writing		*		*	

Before the administering of the instruments the students were asked to complete two forms (see Appendices F and I). The first explained the research to the pre-service teachers, and requested their permission to participate in the study by signing the form at the end of it. The first part of the second form contained eight instructions for the participants to read before beginning the *Assessment for number sense* instrument and the other part was in a form for the collection of census type data. It gathered personal information that I thought might be needed in the analysis and discussion of the results. This form requested the participants to give demographic information about identification of age grouping, gender, years of mathematical schooling, and cultural and regional background.

This section has introduced the elements of the design of the study. The following sub sections give details of each of the data gathering instruments and other associated administrative procedures.

Assessment of number sense

The central research questions for this study required information about the development of number sense of the pre-service teachers during the study. Hence an instrument, *Assessment of number sense* was created and administered three times to give a measure of the participant's number sense at the three particular stages of the year. A copy of the instrument is given in Appendix A.

The instrument was designed to investigate the aspects of number sense as outlined in the framework shown in Table 4.3 which is a modification of the detailed categorising by McIntosh, Reys and Reys (1992). My modification is the third column of the table which is the classification of the major activities of the Teaching Program that I planned for this study. The three main areas of the cognitive aspect of number sense are considered by McIntosh, Reys and Reys to be knowledge and facility with number, knowledge and facility with the number operations, and application of knowledge and facility with number and their operations.

Table 4.3

Framework for number sense (Modified from McIntosh, Reys & Reys, 1992)

Categories	Strands	Associated activities
Number – Knowledge of Facility with it	Absolute and relative magnitudes of numbers	Counting Bridging Formation of patterns Recognising size and order of numbers Study of whole numbers and fractions Study of base 10 and place value, Finding patterns in apparent disorder Recognising regularity in numbers Recognising representations of numbers
	Multiple relationships of number	Study of: Fractions as decimals, %, etc. Expanded form of numbers Number line Bases other than 10 Decomposition and recomposition Referents-mathematical and personal
Operations- knowledge of and facility with them	Relative effects of operating on numbers	Understanding the meaning and effects of each of the four operation Understanding basic number facts Estimation using operations Determining reasonableness of answers
	Equivalent expressions	Understanding commutative, associative and distributive laws Development of strategies Bridging in addition and subtraction
Computational settings - knowledge of and facility with numbers and operations applied to them	Computing and counting strategies	Creating mental models of numbers Use of number and their operations in daily life Make mathematical judgements Problem solving and posing Tackle problems even if effortful Counting, bridging, skip counting Mental computations and estimation skills Appropriate use of written algorithms Calculator use
Computational settings - knowledge of and facility with numbers and operations applied to them (cont.)	Computing and counting strategies (cont.)	Creating flexible and appropriate strategies Developing reasonable speed doing basic number facts Using times tables appropriately and efficiently, Self-regulating of thinking process Possess, communicate and interpret numerical information
	Measuring benchmarks	Use of benchmarks- personal, non-standard, standard Ability to estimate measurement Visualise and use of standard units Possess and use personal referents

The instrument is composed of twenty questions which were chosen and adapted from a number sense item bank of a hundred questions developed by McIntosh, Reys, Emanuelsson, Johansson and Yang (1996). The designers of the item bank hoped its use would help provide insight into the thinking of the students related to the three areas of number sense as shown in their framework. For the purpose of developing a number sense assessment McIntosh, Reys and Reys (1992) divided each of these areas into two strands to give a total of six strands. McIntosh et al used interviews for trialing many items to create an item bank of hundred questions. They selected items that required thoughtful introspection and that relied on conceptual understanding rather than on immediate recall of known facts. The hundred chosen test items were framed as questions in non-routine contexts in order to illicit strategy generation based on understanding rather than use of standard procedures. In choosing the 20 items from the hundred for this instrument I have used the classification of the six strands and have chosen three or four items from each strand to ensure the inclusion of a variety of questions. The conceptual knowledge-base level chosen was roughly approximate to Fiji's Grade 6 standard.

The questions were generally either multiple choice or short-answer type. An answer sheet was provided to allow for ease at filling in answers by participants and ease of correction by the lecturer. The wording of the questions was adapted to the Fijian culture by the inclusion of such things as local names and terminology. The test was trialed on a student from circumstances similar to participants and of low standard of mathematical ability to check for ambiguities and lack of understanding of the wording of the questions. As a result no changes needed to be made to the test. Using Cronbach's alpha measure, the reliability coefficient for the instrument was calculated to be 0.73. This is an acceptable level of reliability for such a test.

I administered the *Assessment of number sense* instrument (Appendix A) during class periods three times during the year, pre and post the Teaching Program in first semester and as a delayed testing at the end of the second semester. Before the students completed the instrument the first time I read aloud to them the instructions at the top of the page and allowed time for questions about them to ensure that were noted and understood. The students were allowed fifteen minutes to complete the test. They filled in their answers in the spaces provided or circled the 'best response'. To encourage students to answer each item based on reflection rather than by a computation a time-pacing scheme was used. To adhere to a suggested pace I prompted slow workers to move along in the test rather than spend much time on any one question by using statements such as ‘by now you should be working on page 2”. The tests were corrected with a score of one given for a correct answer and zero for an incorrect one.

Test of algorithms for number operations

As well as measuring the understanding of number, its operations and applications, a short test was designed to investigate changes in the pre-service teachers' skills of the written algorithms which they had been taught at school (see Appendix B). Knowledge of these skills is often required if people want to apply their understanding of number sense efficiently. Importantly for this study, the skilful execution of these procedures is believed to be helped if it is supported by an understanding of number sense (Heibert, 1990). Thus I considered that a development of number sense might be reflected in an improvement in the accurate completion of these test questions. Yet as Pesek and Kirshner (2000) suggest, the students receiving instrumental instruction prior to relational instruction, as was the case for the participants of this study, may *not* be helped. It is suggested that such a process may effectively block students from achieving the relational understanding sought. In similar research completed by Moss and Case (1999) an interesting finding was that no difference was found in conventional computation between two groups, one of which was taught using traditional approach and one taught using more of a constructivist approach. These findings suggest that care is needed in the analysis of the results for this study.

Pesek and Kirshner (2000) also found that the students in their study who had received both of the instrumental and the relational instructional treatments said they had learnt more from the instrumental instructions. The authors suggest that the finding was probably the result of the greater similarity of the instrumental teaching to their regular classroom instruction and it did not constitute a threat to what they had already managed to learn. Hence I used non-worded questions similar to those given in the Department of Education (1981) Grade 6 Mathematics text to design the test. The test consisted of nine questions in order to include the different algorithms for the four operations, and a variety of kinds of numbers such as whole numbers, decimals and common fractions. It was also hoped that in completing the test the students would feel comfortable answering questions with which they were familiar and so feel that the research testing gave them an opportunity to show what they had learnt. Cronbach's test of reliability was used and an alpha coefficient of 0.69 was obtained which is an acceptable level of reliability for such a test.

I administered this test three times during the year during a class as close as was possible in time to the administering of the other instruments. The students wrote their solutions to the nine questions in the spaces provided on the question sheet and were allowed as much of the class time as they wished to complete their solutions. Their answers were corrected with a score of two for each fully-correct answer and one for a partly-correct one.

Questionnaire for beliefs about mathematics

When the mathematics education literature considers measuring number sense, it does not discuss in detail any parallel change in the non-cognitive aspects such as the learner's beliefs. In fact little is mentioned about non-cognitive aspects of number sense even though the definitions given in the literature include non-cognitive terms such as 'inclination', 'ease' and 'natural tendency'. As I was interested in the non-cognitive aspect as well as the cognitive I wished to consider any affective issues that might have been a component of the development number sense. It has been noted that Nisbet (1991) believed that the role of attitudes occupies a deservedly important place in the study of teaching and learning of mathematics because of the posed link between attitude and behaviour. I was interested to measure how any changes in cognitive number sense development might relate to changes in beliefs about mathematics that are consistent with a constructivist's approach. To investigate this I chose to use data from the responses to a questionnaire on beliefs about mathematics (see Appendix C).

The questionnaire was originally designed by Mayers (1994) to assess the degree to which pre-service students' beliefs about mathematics as a subject were consistent with constructivism. An example of an item which was deemed to relate positively to a constructivist approach is: "Mathematics problems can be solved in different ways". This instrument is a 22-item self-report questionnaire with a five-point Likert scale for responses. Some of the items were taken from a questionnaire in a study undertaken by the International Association for the Evaluation of Educational Achievement (Department of Education, New Zealand, 1987). Other items were composed by Mayers in collaboration with two experienced mathematics educators. It was first administered to First Year pre-service teachers in a teacher education institution in New Zealand where 11% of the participants were Pacific Islanders. Therefore possibly some of them had a similar culture and education to the participants of this present study. The alpha coefficient for the questionnaire of 0.72, indicating a moderately high level of internal consistency.

I administered the questionnaire three times, and each time within a few days of the other instruments. The first time the students completed the questionnaire I read aloud the instructions at the top of the page and allowed time for questions related to the completion of it. Then the students were free to take as much of the class time as they wished to read and to ask questions before completing the twenty-two items. But they were encouraged to choose a response without too much time spent considering their choice. Spending too much time may have led them to make judgements about their beliefs and so influence their responses.

Interviews

As discussed in the section on cultural issues, in Chapter 2, traditional ways of thinking and learning must be considered, but how much consideration needs to be given to an individual's uniqueness of aspects of learning and understanding. Gunstone and White (1992) suggest that learners construct their own meanings for the knowledge they acquire. They see a person's understanding develops as new elements are acquired and linked with the existing pattern of association between elements of knowledge. This construction can occur in three ways, reflecting on knowledge, through incidental learning and through guidance by the teacher. Although there would be similarities in the structures of the knowledge formed in the minds of others each of those minds is a different receptor. The patterns of episodes, images, and other elements will differ even if the persons have been together for a long time.

In Fiji much of the learning gathered through incidental learning would come from the traditional life in the village. Even when a group of students are from one cultural background, the learning of each individual may be considered unique but with strong similarities to others in the same village, and less with those of the same cultural group from another village. Gunstone and White continue by suggesting that this uniqueness has implications for assessing and testing as understanding is so complex and no tests score could satisfactorily represent a person's complete understanding. Among the strategies for assessing they suggest one should use diverse probes. A semi-structured interview situation could be considered such a strategy where students are required to explain their thinking as they complete mathematics activities (Kaminski, 1996). It also has the advantage of attending far more to the process of learning, rather than the product of learning as the three instruments previously described do.

With this in mind I chose to use such semi-structured interview as one of the instruments for this study. For a copy of the interview activities and the answer sheet used see Appendices D and E. The interview procedures for February and July were composed of activities and related questions that investigated concepts from the six strands of the number sense as classified in framework (Table 4.2) and described by McIntosh, Reys, Emanuelsson, Johansson and Yang (1996). Some of the activities were original while others I chose from a range of sources including Kaminiski (1996), Reys (1991), and Sowder and Kelin (1993). For the interviews a roughly-stratified sample of ten students was chosen. As the first interviews were completed before any teaching had begun the stratification was based on evaluations made at the time of interview of prospective students a month earlier. Three of these ten students were thought to be among the higher achievers in mathematics in the class, three among the lower achievers and the others in between.

The interviews were conducted with each of the ten students individually. Each of the twenty activities (see Appendix D) was on an individual card. For the interview, students were presented one at the time with an activity card and materials they could use to help them complete the activity. They received an answer sheet, part of which is shown in Appendix E, on which to fill in their answers and show any working. As they completed each activity I asked them to explain verbally if possible, how and why they arrived at the answer they gave. Each interview took about an hour and a half and was audio-taped. These tapes were transcribed later.

The interviews conducted at the end of the second semester were structured differently as it was thought that the questioning in the previous interviews could not be satisfactorily be repeated as much probing and discussion leading to satisfactory answers in most cases. Instead the probing questions in the interviews in October were related to any incorrect responses which the interviewees had given in the *Assessment of number sense* and the *Test of algorithms for the operations*.

Journaling

Journal writing by the pre-service teachers was part of the data collecting for the study for two reasons. It was a method of learning and a tool for research. As has been noted in the literature review, activities requiring reflection are consistent with a constructivist approach to teaching and learning. It was also noted that journal writing was a valuable source of information for teachers and researchers. As I was seeking to investigate what caused the changes in their number sense or beliefs about mathematics I considered that journals were a good tool for research from which information might be obtained for *why* changes were occurring for the students.

A personal reflection a few times a week by writing in their Thinkbooks, a termed coined to describe the student's journals, was used to help the pre-service teachers consider their learning during class and the reasons for any difficulties they were having. They were encouraged to comment on their learning, feelings and attitudes to the mathematics learning and teaching. For the journal-writing assessment task a considerable amount of preparation and on-going support was needed as the exercise was foreign to the students as it required writing about mathematics, something unheard of for them.

During the first week of the semester I discussed the benefits of writing in a mathematics class and journal writing using examples of work that I had collected from students during the previous two years. I then gave the students some opportunities to write about their mathematical thinking in class. I presented each of the pre-service teachers with a small exercise which I called a *Thinkbook*, and a few requirements and a list of guidelines for how to use their journals (see Appendix H). These guidelines were open to negotiation. The directions about the entries included that they were to be completed a few times a week, of approximately 100 words, and the content to be their reflections and ideas on the mathematics work done during the week.

For assessment purposes marks were credited mainly for consistency in making entries and to a small degree, for quality. I attempted to read all the Thinkbook entries at least twice a month and write a few words in response in them before returning them. After about a month in which these requirements were not being followed and the work was generally being done in a manner that was not helpful to the students' learning or to my assessment of their response to my teaching, we negotiated a further set of guidelines. These adaptations contained some organisational items such as that the length was shortened to fifty words, each entry was to be dated but more importantly ideas were to be their reflection of work done in class. I encouraged them to write reflections related to their response to the work rather than such things as the performances of their peers' Warm-up presentations, or summaries of what was done in class. Insights and application of the ideas for teaching that they derived from class discussions were encouraged. A sample of some of the writing can be seen in Appendix I. At the end of the year I asked students to volunteer to give me their journals for research purposes. About half the class, including all ten of the students whom I had interviewed, submitted them.

Data analysis

In this study I was seeking, among other things, to determine what, if anything in the Teaching Program was effective in bringing about development of the pre-service teachers' knowledge and understanding of the mathematics they were required to teach on completion of their course. In particular I needed appropriate methods of analysis to use the data as described above to help find answers to the research questions as summarised in Table 4.1.

1. Can a unit be designed for Fijian students in this College incorporating the following design elements:

a. -using number sense as the underlying
b. -using a social constructivism approach?

2. Can the effectiveness of the Teaching Program be determined?
3. What development of numeracy, for both the cognitive and non-cognitive aspects, occurred for students who took this unit?
4. If changes in the development of numeracy did occur for the students, what brought about these changes?

The analyses I used to seek answers to these questions are described in the following sections.

Assessment of number sense

A significant change in number sense would indicate an improvement in numeracy. It was planned to analyse any changes in the data from the results of the instrument, *Assessment of number sense,* that may have occurred for the class as a whole. This was completed by calculating the means and standard deviation for each of the three sets of results and using a t-test analysis on paired results. The paired results of interest were the sets of results obtained at the beginning and end of the first semester, the beginning and end of the year and the beginning and end of the second semester. The significance of the t test scores indicated if any significant change had occurred, with the 'p' value set at 0.05. For these calculations and all others for this study I used the computer program *Statistical Package for Social Sciences (SPSS),* (Nie, Hull, Jenkins, Steinbrenner & Bent, 1999).

As well as investigating the change in the students' total scores for the number sense assessment in this manner, the results of the average scores for each item for each of the three assessments were examined. Any changes for item averages between assessments that were notable were examined to see if patterns for the averages for items which emerged was related to specific elements of number sense.

Test of algorithms for number operations

To a minor degree, the second measure of development of number sense used the results of the *Test of algorithms for the operations.* Changes in performance were again investigated by calculating the means and standard deviation for each of the three sets of results and using a t-test analysis on paired results. For the paired results I chose the same three pairings as for the *Assessment number sense* data. The significance of the t test scores indicated if any significant change had occurred, again with a 'p'value set at 0.05.

Questionnaire for Beliefs about mathematic

To investigate whether there was any change in the students' beliefs, an investigation of the responses to the questionnaire *Beliefs about mathematics* was made. Item responses were based on a five-point Likert scale with the scoring ranging from 1 (strongly agree) to 5 (strongly disagree). Half the items were positively worded and the others negatively worded. The scoring of the negative items was reversed. For the scores for the responses to each item I calculated a class mean and compared these in a tabulated form.

Prior to deciding upon this method of analysis of the students' responses a factor analysis was considered. But this was rejected because a) the population number was small and although this did not technically prevent such an analysis the quality of the result on balance did not warrant this approach, b) there was some doubt about the robustness of the data given that this was the first time some of the students had filled in a questionnaire of this type and c) the analysis that was eventually chosen gave the needed insight into the ways in which these students' beliefs were changing hence there was no need for the more extensive factor analysis.

Interviews

In completing these activities the students did rough working on the answer sheets and gave brief explanations which were audio-taped to determine any correlation of changes or developments with particular aspects of the Teaching Program. The interview scripts contain very little dialogue for analysis especially the first set of them but the written work on the answer sheet provided useful information about the students' understanding and working of the mathematical activities.

Journaling

I analysed the journal entries of about half the pre-service teachers in the class including the ten students interviewed and ten others who volunteered their journals promptly when asked to do so at the end of the year. From comments made it was apparent that the other students would have been happy to submit their Thinkbooks but during the end of the year festivities they forgot to do so, or perhaps for a few, chose to forget. The hand written entries were retyped into computer files. Sentences or groups of sentences were then classified and rearranged into three sections where possible. These three sections related to the three parts of the framework for this study, namely enculturation, mathematical understanding, and pre-service teachers' lifelong learning processes. For some sentences all sections were relevant and multiple classifications were made. This provided me with data in a form that I was easily able to draw upon for the case studies (see later) and other parts of the analyses such as for the implementation of the Teaching Program (see next chapter).

Rationale for collective case research design

From the analysis of the quantitative data related to the two assessments of number sense, indications of the development of number sense for the group were obtained, but it was to be expected that it would produce little evidence to explain *why* the changes happened. Clearly many factors besides the implementation of the Teaching Program were at work between February and July. For example, some major factors were the students' adapting to a new environment of teacher education, living in a Teacher's College far from where they lived the previous year, and the strangeness of the first set of instruments of the research compared with the second time the students read and completed them.

As I was interested in investigating *what* helped students to develop their number sense so that it could be exploited in future teaching, I needed a process that could help answer a '*why'* question. According to Yin (1989), research that is looking for explanations needs the potential of case study research design. He says that in general, case studies are the preferred strategy when *how* and *why* questions are being posed. Also case studies are the choice in examining contemporary events and usually include direct observations, documents and systemic interviewing and its unique strength is in its ability to deal with a full variety of evidence such as interviews, documents and observations.

Stake (1994) suggests that when a researcher wishes to inquire into a phenomenon, population or general condition, a study of a number of joint cases, namely a *collective case study,* also referred to as a *multiple case study,* is appropriate. The individual cases are not of primary interest but play the supportive role of facilitating understanding of some issue or refinement of a theory. Therefore as I was examining what was happening for the students in my mathematics education class I considered that the collective case study was the appropriate research design.

Yin (1989) suggests that the evidence for research with multiple case design is considered more compelling and the overall study is therefore regarded as being more robust than individual case studies. Each of the cases should serve a specific purpose within the overall scope of the inquiry. He suggests that each case must be carefully selected so that either it predicts similar results or that it produces contrary results, but for predictable reasons. For collective case studies the choice of cases is made to advance an understanding or perhaps to theorise an issue (Stake, 1994). For choosing the cases, Stake believes that selection by sampling of attributes should not be a high priority but balance and variety are important, with opportunity to learn, of primary importance. This may mean that some desirable types have to be omitted if we cannot learn important things from them because of the lack of information they provide. These ideas of Stake influenced my choice of cases as discussed in the next section.

My choice of the six cases

In the class of originally forty students in my study an interview sample of ten were selected including students with a range of mathematical abilities from poor to good. Then, when deciding to investigate in some detail the effects of the Teaching Program using a collective case study, six were chosen from the ten for whom I had interview data. In the six there were two groups of three with sufficient variety of contrary features as I will now describe. Initially in making the choice I wanted to investigate the contrary situation of half the cases who appeared to show much number sense development during the Teaching Program and half who did not using the data described earlier in this chapter. From this, a better understanding of some of the factors that may have influenced the development or lack of it may have emerged. But on examining the data from the two numeracy tests for the sample of ten students it was found that *all* ten showed a marked improvement as did most students in the class. So instead I chose three who began with an apparent poor sense of number and poor numeracy skills, Vita, Wili and Jo, and three who had a much better number sense and numeracy skills at the beginning of the year, Ana, Lisa and Wani.

Even though the choice of the six was limited I chose students so that there was a variety in factors such as gender, age, background schooling, and number sense (see Table 4.3). This was because, as Yin (1989) suggests one wants to learn as much as one can with each case serving a purpose of giving specific information within the overall scope of the inquiry. I attempted to ensure that I had at least two of each gender, the males were Wili and Jo, and some cases who had not come directly from school, Wani, Vita and Jo. It was not possible to get all variations such as both a male and female who were older as Wili and Jo were the only older ones available within the ten interviewees. Students who came straight from school were usually in the 17-19 age range. I chose not to be more specific about age as I knew from experience the students for cultural reasons, would prefer not to disclose their exact ages. There was little variation in the number of years during which the pre-service teachers studied mathematics within this group, except for Lisa who did more than thirteen years and Wani eleven years. Of the ten students, these six because they were among those who wrote most in their journals. As Stake (1994) said it is of primary importance to choose cases who are able to provide the opportunity to learn much. The names used for the students are pseudonyms.

Table 4.3
Information and quantitative data for the six students

	General information					
	Wani	Ana	Lisa	Vita	Wili	Jo
Gender	F	F	F	F	M	M
Age range	17-19	17-19	17-19	17-19	20-25	20-25
Yrs – maths	11	13	13+	13	13	13

Validity and reliability

Generally in research, concerns about validity both internal and external, and reliability need to be considered. I wished that my readers view my results trusting that they are reliable and valid within the limitations of the methods I used. Only if this is the situation can my work be understood and applied in further research or action.

Internal validity according to Merriam (1988) deals with the questions of how one's findings match with reality. Qualitative researchers are interested in perspectives rather that truth per se. She says that the view of the reality that researchers obtain is of how their participants see themselves and their experiences. Merriam suggests basic strategies an investigator can use to ensure internal validity. In my research I used some of the strategies that she suggested including triangulation, long term observation, and early in the study clarification of my assumptions, worldview and theoretical orientations.

For triangulation my study included tests, questionnaires interviews, observations and journal writing. My multiple sources of data did not confirm one emerging finding, and at times produced data that were inconsistent. But as Hubermam and Miles (1994) say triangulation is less a tactic than a mode of inquiry. They suggest that in the use of multiple sources and modes of evidence, the researcher will build the process into ongoing data collection. My formal observations and data collection was to some extent long-term as it continued for nearly twelve months with the support of informal observations and data-gathering processes during the two years previous to the study. These longer term observations helped to increase the validity of the findings. The validity is also increased by the clarification of my ideas and paradigm in what I have included in this thesis.

At another level the issue of validity was also taken seriously. As had already been discussed each specific instrument used in the study has face validity. As well when possible the questions used in the interview and the style used for journals were checked with sympathetic College staff for comment.

Reliability is traditionally concerned with the extent to which one's finding can be replicated. Merriam (1988) states that in education achieving reliability research in the traditional sense is not only fanciful but impossible. This is because what is being studied is highly contextual, multifaceted and in flux. Because reliability in the traditional sense seems to be something of a misfit, Lincoln and Guba (1985, cited in Merriam, 1988) suggest thinking about *dependability* and *consistency*. They list several techniques that an investigator can use to ensure that his or her results are *dependable*. It is not surprising that these include the strategies of Merriam (1988) listed above as she states reliability and validity are inextricably linked in the conduct of research. One of Lincoln and Guba's notions that is not in Merriam's list which I employed in this study especially in the case studies is an audit trail. For each journal entry I will include the pseudonym name for its author and the date it was written, and so allow the original sources of this data to be trailed.

External validity refers to the extent to which the findings of one study can be applied to another that is, how generalisable the results are. Stake (1994) says the purpose of a case study is not to represent the world but a case. He says cases are of value in refining theory and suggesting complexities for further investigation as well as helping to establish the limits of generalisability. Merriam (1988) suggests that generalisability of findings in the traditional sense can be enhanced with the use of many case studies to study the one phenomenon where sampling, predetermined questions and specific procedures for coding and analysis are employed. To a limited degree I did as Merriam discussed because I used six specially selected cases and followed a structured framework to describe the findings of the case studies. Yet I will refrain from making any generalisations of my findings. I did not seek to know what was generally true but rather sought to provide a perspective on the particular situation of the study. Merriam suggests other ways of improving generalisability of results include providing a rich source of information for others to do the transferring and appropriate judging, and to establish in the descriptions of the findings the typicality of the case and so allow readers to make comparisons. In my study I believe I have provided the reader with a rich and deep source of data and sufficient descriptions of the various cases to allow comparisons to be made with other pre-service teachers in similar circumstances.

Ethical considerations

First, the conduct of the study was discussed and approved of by the authorities of the College in Fiji. The Fiji Ministry of Education and Technology was notified of the proposed research data collection and permission was granted from the Permanent Secretary to extend the research into the Government Teacher Education Institutions (see Appendix J). Then a formal written permission was requested from each of the participants to use any of the information they supplied during the course of the study. This permission was obtained at the beginning of the first day of administering the instruments using a form distributed to the participants (see Appendix F). This form described the research to be conducted, the rationale for it and a permission slip to be signed and returned if the students was agreeable to participate in the research. All the students in the first year class signed the permission slip and filled in all the personal information requested.

With regard to the anonymity for case studies it is nearly impossibility to protect the identity of either the case or the people involved (Merriam, 1988). The names I have used for the students are pseudonyms but by definition case studies are intensive investigation of a specific phenomenon of interest and so recognisable by persons close to the situation of the study. Yet as Merriam emphasises, researchers always need to remember their ethical responsibilities should there be exposure through publication and consider all the ramifications for the participants.

Restatement of the research questions

To inform the reader of the research questions as they are now structured as a result of the design of the methodology I will present them here and outline how in the next two chapters these questions are answered.

1. Can a mathematics education unit be designed for Fijian students in this College using:
   a) number sense as the underlying theme with an emphasis on the areas of:
      i. numbers,
      ii. their operations,
      iii. their applications in everyday living, and in particular
      iv. estimations and mental computations within these three areas,
   b) a social constructivism approach and hence in particular, an emphasis on:
      i. acceptance of each student as they are,
      ii. encouragement of construction of one's own knowledge of mathematics,
      iii. use of materials and everyday contexts,
      iv. reflection and communication to improve learning?

Chapter 5 contains an analysis of the documents related to the design of the Teaching Program, anecdotal details from my teaching notes and other material which were part of the planning process and which pertain to this question.

Because of the design of the methodology the second question to be operational has now become:

2. Can the effectiveness of such a unit be evaluated by examining:
   a) the design documents of the unit,
   b) the responses of the students to its implementation, and
   c) my reporting of its implementation?

   To elucidate this question the following framework of headings drawn from the review of literature will be used.

- Enculturation into mathematics education classroom
- Constructing knowledge of concepts and the nature of mathematics
- The lifelong learning for the future teachers

The second half of Chapter 5 contains this analysis.

The third and fourth questions of the study address the changes in development of number sense that occurred. Question 3 analyses *what* were the changes and the related fourth and final question analyses the *why* for the change and so these questions overlap.

3. What development of numeracy occurred for students who took this unit? In particular,
   a) what changes occurred in the cognitive aspect of number sense?
   b) what changes occurred in the non-cognitive aspect of number sense?

   Addressing question 3,. data from both the class as a whole and from the six case studies will be used.

   For question 3. a) the following will be taken as indicators of change in the cognitive aspect of number sense:

   i. Changes in the scores of for their *Assessment of number sense*
   ii. To a much lesser extent, changes in scores for their *Test of algorithms for the number operation*
   iii. Comments in the students' Thinkbook entries related to a change in number sense
   iv. Evidence of this change from the Interview data.

   To address question 3. b), the following will be taken as indicators of change in the non-cognitive aspect of number sense:

   i. Changes in the mean scores for item responses to the questionnaire of *Beliefs about mathematics*
   ii. Comments in their Thinkbook entries related to the non -cognitive aspects of number sense
   iii. Changes in their understanding and resulting appreciation of the chosen elements of the nature of mathematics
   iv. Development in their learning to learn and learning to teach

This analysis is found in Chapter 6 where the results of the class as a whole and the six case studies are discussed.

Question 4 deals with the causes of any change and reads:

4. If changes in the development of numeracy do occur for the students, what brought about these changes?

The implication is that some of the change was linked to the Teaching Program developed for this group of students. To elucidate my response to this question the following data sources were employed:

a. Thinkbook entries
b. Responses to the questionnaire
c. Assessment of number sense
d. Test of algorithms for number operations
e. Interviews

The above data banks were developed from the six case studies described earlier in this chapter and the analysis of such is found in Chapter 6.

The next chapter, Chapter 5, recounts the development of the Teaching Program and the response of the students to its implementation. Following this, Chapter 6 will focus on the group data from the class as a whole and then the six case studies are dealt with in depth.

CHAPTER 5 THE TEACHING PROGRAM

This study is centred around the learning that occurred during the implementation of the initial mathematics education unit of the three-year primary teacher education course at the Teachers' College in Suva. The planning and implementation of this unit is referred to in this study as the Teaching program. This chapter is the first part of the results of the study. It is a description of the Teaching Program and is divided into two parts, the planning of the program and the implementation of it. The planning is further divided into two sections, the theoretical notions for the Teaching Program and the practical issues that required attention in the planning. The theoretical notions will include ideas about the constructivist approach to teaching and learning, some important values of mathematics and number sense. Following these theoretical notions I will discuss the practical issues for the planning during in part on the literature already reviewed in Chapters 2 and 3. This will include an outline of the context and constraints of the Teaching Program, the content of the unit of work and the teaching approach.

For the second part of the chapter I will describe results of the implementation of the Teaching Program. To do this I will use the three sections of the framework used throughout this thesis. These headings relate to who are the participants, what knowledge we are considering and how might this knowledge and their participation affect the lifelong learning and teaching of participants. The implementation is situated here at the beginning of the results because it constructed in part the situation in which the rest of the data were collected. The very construction of the Teaching Program, as well as its implementation, were an outcome of the plan of action on which this study is premised. In summary then this chapter directly addresses the first two research questions.

The planning

During the two years prior to the Teaching Program I had been presenting mathematics units to the First and Second Year students at the College. For two years prior to these, I was lecturing at another South Pacific Teachers' College in PNG, and earlier still I was teaching mainly in mathematics classrooms in secondary schools in Africa and Australia with a couple of years in primary school classrooms. Therefore I came to the planning of the Teaching Program with much practical knowledge and experiences gained during these years. In the following description of this planning I will first outline the relevant parts of the theoretical framework that I have reviewed in Chapters 2 and 3 and then I will discuss the practical issues that need to be considered.

Relevant theoretical notions

In this section concerning the theoretical issues that were considered important in the planning I will draw on the literature review from Chapters 2 and 3, but now situate the discussion specifically for teaching in this Fijian College. Because of the various aspects mentioned in the research questions this section includes discussion of the constructivist approach to teaching and learning, values of mathematics, and number sense.

The constructivist approach to teaching and learning

The constructivist teaching approaches as developed by authors such as Cobb (1988), Burton (1992), Ernest (1994), Kamii (1996), NCTM (1989) and AEC (1991) have been discussed in the literature review. To summarise the approach I will use the brief outline of the theory given by Begg (1999) as it describes succinctly the understanding of it that I had during the planning stage of the Teaching Program. He simplifies the common elements of constructivism into four ideas: a relativistic view of knowledge, the importance of prior knowledge and culture (especially language) to learning, learning not being transmitted one to another but being constructed by the learner, and each person's construction being unique and representing their view of reality.

Hence at the core of a constructivist approach to teaching numeracy, teaching is student-centred and requires of the teacher to help the student to learn by creating a learning environment that promotes the elements given above. This includes providing mathematical activities and tasks that stimulate reflection, discussion and understanding. These tasks often involved the use of manipulatives because they help embody the concepts which, hopefully would result in the abstraction of the ideas. Problem solving tasks drawn from the context of daily life have been found to often help students to build on their own knowledge in an environment that is related to their culture and language (Clarke & Clarke, 1996). This problem solving is also seen as a method for encouraging the students to use their own strategies for computations.

In discussing constructivism, Lerman (1993) says that it is through peer questioning that students develop sophisticated forms of explanation and argumentation which are influenced by the social interaction of the participants. Also a teacher needs to provide the students with opportunities and activities that are designed to encourage metacognitive thinking via reflection and discussion, for example, cooperative group work and writing in mathematics journals.

Values of mathematics

In devising the Teaching Program and its implementation it was clear that values embedded in the teaching and learning of mathematics would be inevitably taught. My reading of the literature related to numeracy had led me to include *the ability to understand and appreciate the nature of mathematics* in the definition of number sense used in this thesis. Without this ability a student's number sense would be lacking a basis on which to continue its further development during a student's lifelong learning.

To investigate the students' understanding of the nature of mathematics I am using a framework based on the values of mathematics developed by Bishop (1988). He has chosen six sets of ideals and values which relate, in complementary pairs, to the three components ideological, sentimental and sociological. These values are: ideological- rationalism and objectism: sentimental- control and progress; sociological- openness and mystery. Bishop (2001) give examples of situations in class where these values are promoted by the teacher and these are described in the literature review. For my framework for investigating the students' understanding of the nature of mathematics I have used these six categories.

Number sense

Another theoretical issue directly related to the planning concerns was that of the number sense aspect of numeracy. From the discussion in the literature the definition of number sense used in this thesis is that number sense is that *number sense is the inclination and ability to understand and use numbers, their operations and applications with ease and to understand and appreciate the nature of mathematics*. It is these ideas of *number sense* that I and the other staff members of the College hoped the First Year students would develop and hence it became an important part of the Teaching Program and the study. The consideration of the other aspects of numeracy were not seen to be as central and urgent for this particular situation.

From this definition it can be seen that the cognitive area of number sense has been divided into three categories, namely number, the basic operations on number, and applications of number and their operations. The extended framework for number sense (see Table 4.2) that I use in this study is the one devised by McIntosh, Reys, Emanuelsson, Johansson and Yang (1996). In the framework number itself is again divided into the two strands of absolute and relative magnitudes of numbers, and the multiple relationships of number. The two strands of operations on numbers are their effects and their equivalent expressions. The section on applications of number and its operations includes the two strands of computing and counting strategies, and the understanding and use of benchmarks in measuring.

In the literature review about development of number sense the issue of direct instruction has been discussed. From this review it was concluded that there was no consensus as to whether to teach an intervention program directed specifically to number sense development, or to teach a mathematics curriculum that is permeated with number sense learning. However as the latter appeared to be the popular option I considered it the more appropriate one for the Teaching Program.

As well as the cognitive aspect of number sense there appear to be non-cognitive features in relation to the terms in my definition such as the *inclination* to use numbers, and the terms *with ease* and *to appreciate the nature of mathematics*. These words and terms are related in some manner to the feelings, attitudes, beliefs, belief systems and values in mathematics education. It is this aspect of number sense that is hard to define and to research, but in this study I hoped to investigate some dimensions of it. The main source of information that was devised for the investigation was the students' journal writing during the year. From the previous two years' experience it was to be expected that the students would often described their feelings, attitudes and beliefs about mathematics and so provide insight into the issue. Also the questionnaire into the beliefs about mathematics was chosen to provide further data about the students' non-cognitive aspects of number sense.

Practical issues

With the theoretical framework as described in the previous section in mind I proceeded to the practical planning of the Teaching Program. This involved considering the opportunities and constraints of the context of the Teaching Program, the content of the mathematics education unit and the teaching approach to be used, all of which I will describe in the following this section. In the context section I will mention briefly features of the College, its mathematics education, the participants of the study and minor organisational matters, noting any constraints for the planning that emerged from these. For the content of the mathematics education unit I will present the list of topics, including the pedagogy and content knowledge, and the assessment for the unit. Finally I will discuss the intended teaching strategies.

Context

In describing the context of the Teaching Program I will not repeat all the information about the College, its mathematics education, and the participants of this study that are included in other parts of this thesis. Rather, to assist in the understanding of the discussion in this chapter I will remind the reader of some important relevant facts.

The College

The College is a private Catholic Teachers' College with a student body of a little more than a hundred, established to provide an education for the future teachers of Catholic primary schools in Fiji, though the graduates would also be qualified to teach in any Fijian primary school. The education provided by the College is a three-year course which includes units in the basic subject areas usually covered in primary teacher education courses in a Western country. The course also contains units in religious studies, and some other specialised units depending on the availability and expertise of staff. The College relies on financial support from the small population of Catholics in Fiji and so is limited by lack of staff, a small library and a shortage of other technical resources.

Mathematics education at the College

When I arrived at the College, two years prior to commencing this study, the six semesters of mathematics education units in the College were not as yet organised into a coordinated program. The details for First Year mathematics education units that were available to me were very brief in that they consisted of a few sentences. This included the objective that the students be helped to improve their understanding of the mathematics concepts and skills outlined in the Fijian primary school mathematics curriculum in preparation for their effective teaching of mathematics in primary schools. This lack of documentation was partly because some of the recent mathematics lecturers, who worked sessionally, came and went rapidly and it had not been their practice to submit unit descriptions. As a lecturer on a limited overseas' work-permit I knew I would probably be another one of these *fly-by-night* lecturers, but I wished to leave something more permanent and informative as to the content and effectiveness of the units I conducted.

The previous unit I devised and implemented in the two years prior to the study included topics similar to that of elementary schools in the second half of the last century in Western countries. During those two years lecturing at the College, I had gathered worrying anecdotal evidence which suggested that the pre-service teachers had not been deriving much benefit from the completion of their first few mathematics education units. At the end of two years, their assessment tasks and journal entries appeared to indicate that they had a poorly developed number sense. They were still lacking understanding of some of the concepts of number, especially rational numbers, and of the meaning and skills related to the four basic operations.

Bearing in mind this situation and the general requirements of the Teachers' College course, I was given the authority to plan the First Year unit with the objective of helping the development of the pre-service teachers' understanding of primary school mathematics At the time of the study other teacher education institutions and organisations in the South Pacific region were interested in research being done to help bring about development and improvements in education courses. Our College had been awarded a research grant by a regional organisation to do such work in the area of mathematics education. This study became part of that research.

The participants of the study

The major element of the context of the study is the class of First Year students, the participants in this study. Information about these pre-service teachers has already been described in the methodology chapter so here I will just remind the reader of a few important details. For most of the year they were thirty-eight in number. These students were generally highly committed to becoming effective primary school teachers. They mostly came from rural or island communities to which they hoped to return after graduating. Poor examination results in the past as evidenced on their entry forms indicated that their standard of numeracy was generally poor compared to their fellow students in other tertiary institutions.

Organisational features of the class

Other important features of the context to note here are of an organisational nature. The unit consisted of four forty-five minute periods a week for eighteen weeks. The classes were usually held in the mornings and in classrooms furnished with individual desks for each student which were arranged in rows. The desks were easily moveable for cooperative group work or other activities that required a large table or open space. After six weeks of the first semester the students began their first teaching experience in the nearby primary schools in the city of Suva. This continued for one morning a week for six weeks. During the mornings each student gave a short lesson, observed their peers giving lessons and participated in a discussion time about the features, both positive and negative of the teaching that they had observed or which they had participated. I accompanied one group of six students whom I helped prepare lessons in various subject areas.

Summary of context

From the above discussion of the context of this study, the opportunities and constraints for the planning of the Teaching Program are probably clearer to the reader. On the positive side, the staff of the College were most supportive of my study and so allowed me to proceed with my ideas of helping the students develop their number sense. The education course in place at the College was not a constraint for me in creating a new mathematics education unit since, before I came to the College there had been very little structure for the unit in place. Also for the study the First Year students were very willing and open participants who had a need and a desire to be helped. The major constraint for the study was the limitation of time that my temporary position at the College imposed on the planning and implementation. The time of the implementation of the study was my third year at the College, that due to other circumstances, had to become my final year. The inclusion of the teaching experience six weeks into the semester was another constraint which did not prove to be a problem for the planning. Other factors that one might see as constraints such as lack of facilities, students who were poor at English and who came from a schooling in which the constructivist method of teaching was foreign, could be rather seen as challenges to be approached creatively.

The content of the mathematics education unit

This study was also my attempt to begin to produce an official program of mathematics education units for the College. As mentioned in the last section I wished to leave documentation for the mathematics units I had conducted and also information as to the content and effectiveness of the units that had been completed.

Besides providing the College with a formal and trialed unit, I also wished to investigate in this study if and how the development of number sense of pre-service teachers occurs as the result of them completing the unit. Therefore, for the study I planned and implemented a program which would monitor the learning, specifically the development of number sense, that occurred. I chose number sense because I had been given the brief, as mentioned above, that the goal of the first unit of mathematics education was to help students improve their numeracy, and the number sense area of numeracy appeared to be the most in need of attention. The main elements of number sense included have been discussed above. As well as, and included in the number sense development, I also hoped to help the pre-service teachers develop affective aspects of numeracy such as an appropriate attitude to mathematics and the corresponding values of it. In the long term I hoped that changes would occur which would result in the pre-services teachers becoming confident and effective mathematics teachers and lifelong learners.

For a copy of the unit description used in the Teaching Program, code MS101, refer to Appendix K. The unit outline is not available as the College did not as yet require them for any of its units. As can be seen from the unit description the main components of the content of the unit consisted of method knowledge and content knowledge.

Method and content knowledge

The method component of the mathematics education unit was designed to serve as an introduction to teaching in preparation for the pre-service teachers' practical experience in the first semester of the course. In this component I planned to discuss basic teaching strategies and some class management skills that might assist the pre-service teachers in their presentation of weekly lessons in neighbouring primary schools after six weeks of lectures.

The documentation of the content knowledge of the unit uses much terminology associated with number sense. In the light of the review of mathematics education literature and given the circumstances of my study, it seemed more appropriate *not* to create a unit that was specifically designed to develop number sense. Instead this unit is one which includes the ideas of number that caused difficulties for the pre-service teachers. These included the many aspects of number sense outlined in the framework composed by McIntosh, Reys and Reys (1992) as shown in Table 4.3. The third column in this table is my adaptation as it includes the list of topics I collated to be used in the Teaching Program as explained in Chapter 4. The activities are classified according to the areas of number sense given in the framework. The proposed weekly topics and activities can be seen in Table 5.1. Ideas for the activities I used were obtained from a wide variety of sources. Most of these sources are referred to in the literature review where number sense is discussed. For rational numbers, some of the major resources which I used without much adaptation were lessons from Reality in Mathematics Education (RIME) by Lowe and Lovitt (1984). I had found in previous years that the activities in these lessons helped the pre-service teachers deepen their understanding of the concepts involved and were enjoyed by the students.

As well as the outline of the content of the unit, other issues need to be noted. From my review of the literature mental computations seemed to be an important area of numeracy and so it became a major component of the program. After giving an introductory class on the meaning of mental computations and their importance, I planned to spend some time in many classes investigating a concept or skill related to the topic. I would then encourage the use of the skills whenever the opportunity arose. Estimation and approximation were to be included in the unit in a similar way.

It also needs to noted that I would concurrently be conducting a science unit with the same class. Three weeks of this unit was to be spent investigating the history, organisation and use of the metric system. Therefore much of the work for measurement in the curriculum for primary mathematics would be covered, and so this unit would probably contribute directly to some numeracy learning.

Table 5.1

Weekly teaching program for Semester 1

Week	Semester 1 - Topics and activities
1	Orientation program
2	Introduction to mathematics education unit
	Research methods and first collection of data for the study
3	Warm-up activities and related teaching strategies
	Outline of a lesson plan. Production of teaching materials, digit formations
	Use of teaching materials
4	Communication in a mathematics class, including Thinkbook writing
	Problem solving and use of own strategies
	The importance of mental computations, estimations and approximations
5	Theory of constructivism
	Cooperative group work
	Personal history of learning numbers
	Numbers: cardinal, ordinal and nominal
6	Counting principles, conservation of number, and counting on
	Discussion and production of dot number cards
	Traditional Fijian numbers and counting
	Counting in languages other than Fijian
7	Number ten, ten frames and *nice* numbers
	Family facts and family fact cards
	Roman numerals, base ten
	Base 10 materials re *quantity*: Straws bundled in tens, MAB materials, coins, metre ruler
	Base 10 materials re *place valu*e: spear cards, abacuses, place value cards, expanders
8	Times tables: skip counting, strategies for learning the 7, 8 and 9 times tables
	Times tables RIME lessons: Circle patterns,
	Number lines: count the points and/or the intervals?
9	Number chart to 100 for adding and subtracting 10
	Mental computations: simple strategies for addition and subtraction
	Benchmarks and bridging
	Effects of the basic operations: Game of the year
10	Number chart to 100 for adding and subtracting 9, 99, 19
	Addition and subtraction using base ten material
11	Multiplying and dividing by multiples of 10, very large numbers
	Multiplying and dividing using base ten materials
	Examination of base 5
12	Examination of times tables, RIME lessons:N7#7 Check maths, N8# Clock patterns
	Mental computations for multiplication and division: operating using 9, 99, 5, 25,
13	RIME lesson: N7#3 Multiplication by 99
	Estimation of numeracy, measurement, and computations
	Approximations: formal and informal strategies
14	Reasonableness of answers: number of digits, order of magnitude, relationship to one
	Understanding the meanings of rational numbers
	Fractions, RIME lessons: F7#1 Clock fractions, F7#2 Compass fractions, F7#5 Fractions of whole numbers,
15	F7#5 Fractions by comparing lengths, F7#6 Order of fractions
	Fraction charts and equivalent fractions
	Games revising equivalent fractions and addition of fractions.
	Diagrams and creative strategies for operating with fractions
16	Percentages their meaning and applications in everyday life.
	Decimals: computations with money, lengths and number line
	Calculations of decimals with MAB materials
	Decimals operations using calculators- The Maze Game
17	Reasonableness of decimal answers: placement of decimal point
18	Revision and Examinations, Second administration of the instruments for the study

Some discussion of basic research skills which might normally be included in a later year was included in this semester unit. This was because I wished my students, as participants in the study, to understand something of the processes of the research in which they were participating. As part of this I planned to review the study by Bakelevu (1997a) about traditional Fijian mathematising that I have discussed in the literature review. This also had the advantage of highlighting quite forcefully that Fijian mathematics was worthy of study in its own right. Also I planned to discuss the processes of data collection that I would be using. The instruments for the data collection of the study were to be administered during classes of a week at the beginning and at the end of the unit.

The assessment

The assessment tasks were also designed with a constructivist approach in mind to complement the learning processes where practicable, except for a short traditional final examination which would relate to what they were familiar with. Other than this examination the tasks included ongoing assessment via a number of activities. The tasks planned consisted of production of a portfolio of mathematics teaching materials, regular journal writing, peer-teaching involving a short presentation of a mathematics activity called a Warm-up, and the writing of a guided essay. This differs from the unit outline in that the group presentation is interchanged with the Warm-up activity of the second semester. The theme of the essay was "Teachers teach as they were taught themselves". It was *guided* in that I gave prompts such as to describe their own mathematics schooling, the schooling they hope to give their own pupils, and how they might attempt to ensure that they would not reproduce any ineffective teaching they experienced themselves. Tests which were part of the data collection for the study were not included in the formal assessment.

Semester two

The mathematics education unit delivered in semester two is involved in the study to a lesser degree. This is because the instruments used to investigate the effects of the first unit, which were administered at the beginning and end of the Teaching

Program, were also administered at the conclusion of the second unit. Also students continued to write about issues related to the first unit in the Thinkbooks during the time of, and hence in the context of, the second unit. The main topics covered during the shorter second semester were the Fijian mathematics curriculum materials for the lower and middle primary years, further consideration of teaching materials, and the use of calculators in primary schools. For a copy of the second semester unit description see Appendix L.

Teaching approach and strategies

This section will briefly outline the teaching approach and the planned strategies of the mathematics education unit, drawing upon the constructivist approach already reviewed. Because I was employing a constructivist approach, one feature that was planned for, and promoted by Kamii (1996), was to advocate that students use their own strategies, particularly when doing activities in a problem solving context. Cooperative group work, beginning with working in pairs and then progressing to groups of four or so students was also to be used, following Cobb (1988) who believes that group work is necessary for analysing the situations of what is learned and believed. As Ernest (1994) suggests, metacognitive thinking and communication were to be encouraged in the classroom, so I planned problems and other activities that would facilitate reflection, discussion and writing. Also Burton (1992) believes that a constructivist view requires reflection. The pre-service teachers generally had no previous experience of writing reflections in a mathematics class, so I planned to begin with a few lines at the end of the classes early in the year, with the expectation that the strangeness of the exercise would hopefully evaporate. This writing exercise was also to be the introduction to journal writing, which was to be an important part of the learning process and the assessment for the mathematics unit.

As well as the students having a variety of different numeracy needs they often had a poor attitude to mathematics learning, inappropriate beliefs about it and a high mathematics anxiety. Therefore in my planning I needed to consider in my methods how to develop a more appropriate attitude to mathematics, to bring about a change in beliefs and to lessen the fears the students had of mathematics. My hope was that the use of a constructivist approach would contribute to these developments and changes because of its student-centred learning, the expected challenges and enjoyment in doing the tasks, and the reflection and discussion of the work attempted.

The first part of this Chapter has outlined the background and planning processes for the Teaching Program including a description of the content of the unit and the teaching approach to be used. In the second part I will describe the implementation of the Teaching Program. This implementation followed closely the prepared plan as nothing untimely or untoward happened immediately before or during the semester. Minor changes, additions and omissions were made to the sequence of topics for various reasons such as a need to respond to questions that required considerable time for an appropriate response.

The implementation

In describing the outcomes of the implementation of the Teaching Program I have employed headings from the framework that I have used throughout this thesis. These headings are: Enculturation into the mathematics education classroom, Constructing knowledge of concepts and the nature of mathematics, and Lifelong learning for the future teacher. Under these headings I will describe the features of the teaching program that were mentioned by the students and that appeared to produce changes in the students' learning and teaching of mathematics especially for number sense. Thus I will be describing features of the Program that were part of its design and implementation that were possibly related to the students' development of numeracy and hence the effectiveness of the Teaching Program.

To help describe the implementation I have used quotes from student entries in their mathematics journals called 'Thinkbooks'. The students continued to make entries in these for the semester following the Teaching Program. Following each quote a pseudonym for the author and the date of the entry are given in parenthesis. The students who are quoted in this chapter are not the students who are in the case studies in the following chapter although they too made similar comments as the students commented here. As the discussion of the data from the administering the instruments of the study will be part of the content of the following chapter it not be included in this description.

Enculturation into the mathematics education classroom

Enculturation into the Teachers' College and the mathematics eduction classroom was a major phenomenon in this, the pre-service teachers' first semester of their three-year course. It would involve a major shift in their general lifestyle and education. The main issues that I noticed that emerged during the enculturation process are discussed in this section. These included aspects related to affective issues, motivation to study, mathematics in every day life, use of teaching materials, cooperative group work, language issues, assessment and communication problems. Clearly they are not discrete issues, and from time to time in the following discussion the same quote may be used more than once in different sub sections. Such is the overlapping nature of the issues.

Affective issues

In facing a new mathematics education culture the students were generally very motivated to learn, with the few who were not, quickly becoming so in the prevalent atmosphere of general enthusiasm. It was a pleasure to teach the students because they appeared to be enjoying the classes and finding them so helpful. Students wrote about the relevance of mathematics, its learning, understanding and discovery such as the structure of the number system to 100 for Ria which were causes of the enjoyment:

> "Wow it is wonderful and right now I have really fallen in love with maths. I want to discover new things and to learn to understand easier ways to learn maths. Now I know that maths will be part of my life and I will definitely enjoy living with maths" (Ria, 30/3).

> "I am not only beginning to discover but I am enjoying what I have learnt so far" (Joni, 26/5).

Yet being new to the College and to each other, many of the students were unsure of themselves and quiet. Being quiet many of them did not participate much in class discussion but, as shown in their entries they communicated with me by writing in their Thinkbooks:

"However learning to become a teacher is not an easy task and in here I am challenged to face the crowd of students teaching them things that need to be taught" (Kela, 18/2).

"This is how I did it but I was too afraid to explain it because I thought I could be wrong" (Kela, 8/6).

Even for students who had performed poorly at mathematics examinations in the past, their enthusiasm for learning mathematics education was high at the beginning of the year. Many acknowledged later that they were anxious about their ability to cope with what was required of them to become teachers of mathematics in primary schools:

"I have to study so that I can pass with a good mark and also mathematics is a compulsory subject and we have to pass in order to get our certificate" (Liku, 19/5).

"The test showed I had forgotten some maths problems because I left school two years ago" (Keca, 18/2).

Except for a few pre-service teachers, all the students had in general experienced a schooling of at least eleven years with mathematics classes on most days. The teaching approach with which they were most familiar was a traditional one where transmission of facts was the normal method. Rewards and punishments were often used and in many cases negative attitudes were developed:

"When time came for mathematics I could not think straight in primary school. Primary school was full of adding, subtraction, division, etc. and I am a slow learner and a maths hater" (Ria, 30/3).

"The method we used was to study by heart our times tables and if we don't remember we get punished and that makes learning much harder. So I learnt through fear of being punished" (Sisa, 20/5).

The newness of the experiences of the Teaching Program was somewhat daunting for some of the pre-service teachers yet for most it appeared to be an enjoyable one.

Motivation to study

One of the problems for the students was the lack of support during their past education for developing self-discipline in the area of study habits. We spent some time in a few classes towards the end of semester discussing development of good study habits. The pre-service teachers during the Teaching Program easily became discouraged when they experienced difficulties studying because they were not externally motivated or pressured:

> "… it seems that now all people are serious in doing their Warm-ups because some of the marks come from it" (Liku, 21/7).

> "I think Ms F should start giving revision test (short ones) weekly or every two weeks. This will motivate some of us to study and learn" (Keca, 25/2).

Although Keca asked for regular testing she was prepared to help herself by preparing study plans and taking notes in class:

> "I think it is helpful to write up plans or study timetables" (Keca, 19/5).

> "Ms F. just talks but it is my responsibility to take down notes that are useful to me. Now I realise that we have got to be organised and look forward to every day's teaching" (Keca, 29/6).

Ria appeared a little unrealistic in her determination to study.

> "I am going to spend at least an hour every night in revising my maths" (Ria, 29/6).

Because of anecdotal evidence during the previous two years I did not expect the Fijian students to quickly develop good study habits but I was pleased to see some progress during the semester. Many students made timetables for revision for examinations and attempted to adhere to them. In summarising their achievements and lack of them at the end of the year many acknowledged their need to improve in disciplining themselves to study.

Mathematics in everyday experience

During the Teaching Program I attempted to develop an awareness of the social context and historical development of the pre-service teachers' own mathematics learning. The differences between traditional and Westernised school mathematics was investigated via a study of the research completed by Bakalevu (1997a). This study of the traditional ways of mathematising led to fruitful discussions on knowledge, language and experiences of their own mathematising in their villages.

With regard to relating the mathematics concepts learnt in class with their own uses of it in their daily life the students had plenty of examples to offer once they were encouraged to do so. I remember the exercise early in the unit where I asked for examples of groups of ten that they could note from everyday living. The first few examples were slow in coming but then the students flooded the discussion with ideas, and even added more of their own in their Thinkbook entries later. Most commented on how interesting they found this exercise yet Keca was one who found the exercise difficult:

> "There are many things that are grouped in tens, toes, fingers, decades. I only knew two groups and it was very hard for me to think of some others" (Keca, 17/3).

Use of teaching materials

By relating their mathematics to objects around them via regular use, collection and production of manipulatives, the pre-service teachers appeared to construct their own knowledge while deepening their understanding of concepts. The use of materials also helped them learn strategies suitable for the teaching of primary school mathematics. The most commonly used materials were the Multi-based Arithmetic Blocks (MAB) because understanding of place value was a major topic in the unit. Also the Fijian Government, using foreign aid had supplied sets of MAB to the primary schools that had Years. 7 and 8 classes. Because of the high cost of these materials to other institutions including the College, the pre-service teachers produced their own MAB using cardboard. Many of the pre-service teachers mentioned in their journals that they especially liked the class in which they produced the blocks from the careful drawing of the nets of cubes:

> "The whole process of making is very important to me because I was motivated by the fact that, how much I put into the making of the blocks counts a lot towards presentation and understanding of blocks" (Sisa, 14/4).

By constructing the material themselves the students appeared to have fun as well as consolidate their understanding of the relationships between the pieces. They said the act of cutting up a flat to produce ten longs, and a long to produce ten cubes emphasised the base-ten ideas for them. Other materials were produced and used such as straws bundled in sets of tens and hundreds, fraction discs, and clocks.

Cooperative group work

Social interaction during learning is promoted in the constructivist approach by mathematics educators such as Cobb (1988). I incorporated the process in the Teaching Program via the students working in pairs, cooperative group work, group presentations and class discussions. Even though students were not familiar with cooperative group work and did not know each other prior to the beginning of the unit they needed little encouragement to participate in it. The group work was very much appreciated:

> "I liked working in groups because I discovered my groups mates' ideas of ways of explaining the Dienes' blocks [name used in Fiji for MAB]" (Sisa, 5/7).

To most students the experience was new, especially for mathematics classes. They appeared to enjoy as well as to learn during the social interaction in the mathematics classes.

Language issues

For the students of the College the mathematics unit of work needed to be implemented mindful of the difficulties related to language which were discussed in the literature review. The participants of the study were mostly among the weaker students graduating from Fijian secondary schools and some of their problems included a poor understanding of English. Also the mathematics language and symbols used in class and in the text books were problematic for many. To give the pre-service teachers opportunities to understand and use mathematical words and terms that they may have avoided in the past I used writing in mathematics classes:

> “I also found out that certain operations have their own words [for] expressing their answers and [other parts] such as sums or terms in + [addition], product and factors in x [multiplication], quotient or dividend in division. …. Although I know how to deal with these numbers, I did not know there were special words to name them when writing mathematical ideas or facts” (Kela, 7/4).

The process of pre-service teachers learning to teach primary school children was a opportunity to thoroughly investigate the language difficulties children (and pre-service teachers!) might have:

> “Today we were told that when teaching we should translate [mathematical] words into our own words” (Kela, 21/7).

> “Today I learnt a new term in maths *reasonable answers*” (Sisa, 5/7).

Another issue related to language was that many of the pre-service teachers did not know the counting words used in the different languages commonly used throughout Fiji such as the Fijian dialects, Hindustani and Chinese. I alerted them to expect a variety of mathematics words their pupils would know when beginning school learnt in languages other than English such as their home languages. The pre-service teachers and I, had great fun learning the basic mathematics words and symbols including the counting words to ten for the languages spoken in Fiji:

> “I was allocated to make up a poem in the vernacular with the numerals included. It was fun and interesting to do. And I also found it interesting to do because this way I tend to conserve the local numbers in my head without using them since we are used to saying them in English” (Kela, 5/8).

Related to aspects of culture and language were the derivation of words and the place of convention in mathematics. The pre-service teachers were interested in background knowledge for words and symbols, such as the development of the Hindu-Arabic digits and the variations in writing symbols such as the decimal point. I also attempted to help them to become aware of the meanings of, the differences between, and the relative importance of conventions and concepts. They appeared to place great importance on the convention. Quite a few of the students became anxious about how to teach the decimal point after the class in which we discussed the variations throughout the world of writing it:

> "We need to be specific about where the decimal point goes. For example, on the line is not correct and halfway down is correct. We need to teach students correctly so that they can also do work correctly" (Keca, 30/6).

The students realised their need as future primary school teachers to improve their knowledge of mathematical language and symbols and took these opportunities to do so. The learning of counting numbers in other languages was a beginning to what I hoped would be the initial learning of more words and mathematical terms in the main languages used in Fiji. Here again was another example of the importance of developing habits to promote lifelong learning.

Assessment

On-going assessment tasks were not familiar to many of the students. They appeared not to have developed the self-discipline at secondary school to regularly do the required work for continuous assessment. I learnt from experience that students new to the College expected to be continually reminded about completing tasks and expected that late work would be accepted. In fact some of them expected the lecturer to continually follow them up until the work was produced. Coming from a culture where requirements for assessments were strictly adhered to, I found it hard to accept the situation of this apparent lack of seriousness in fulfilling the requirements. I worried about my strict enforcement of requirements and my handling of assessment, but the College had a policy that supported my actions. Kela was one of the few students who appeared to agree with my actions:

> "The student must have not read the instructions [assessment requirements] and instead followed or listened to others" (Kela, 13/9).

Even though details of the assessments were presented to each student in printed form, students had not noted the requirements:

> "… but unfortunately I did not know the marks for the Thinkbook were included in my mid-year exams. … After this I gained confidence [motivation?] in filling in my Thinkbook" (Keca, 10/11).

I believed that the ongoing assessment and my expectation that they fulfil the requirements were supported by constructivist approach as this assessment procedure required student-centred learning. Burton (1992) believes that the monitoring of individual's learning is necessary to determine its rate and content. Also the tasks and the feedback provided were designed to be an integral part of the process of effective communication and for learning to become teachers and I was worried when students did not understand this:

> "To me I really feel embarrassed because I did not expect a lot of corrections on the paper. I think it was good to be corrected but the way the paper was marked really embarrassed me. … then I think I am not good enough to teach maths" (Kela, 23/7).

Laka commented on poor marks after she had made only a few of the required entries in her Thinkbook:

> "Some of us tend to be discouraged about marks given and thus end up not writing anything at all" (Laka, 12/10).

During the three years I was at the College the practice of examinations as the main form of assessment diminished. I included a traditional final short test on teaching methods as a way of accommodating the expectations of the students who recently came from a schooling where it was the only form of assessment. It was a minor part of the assessment and was mostly multiple-choice questions. It included questions about such things as their understanding of the counting principles. The students scored very poorly in this short examination. They said they had not taken notes and so had not revised any of the topics about teaching methods. I had expected that they would have been able to draw on what they had been discussing frequently in class and so they would have remembered and not needed special study to be able to correctly answered the items. From the results of the short examination it appeared to me that the material had not been as well understood as I had assumed. For the second semester both the students and I made a greater effort to ensure that deeper understanding occurred when the ideas were first presented in class.

From this discussion about the assessments it appears that a number of problems emerged in the process of assessment for the mathematics education unit. The students needed assistance in understanding and applying the requirements because in the past their assessment tasks were presented to them with much external motivation from their teachers to perform well. Feedback that included constructive but negative connotations was not always appreciated. In a number of aspects the assessment tasks were created with a constructivist approach as they did allow for a broad range of learning styles and levels of knowledge such as for the journal writing and production of teaching materials. The short examination was not satisfactory for the students to exhibit their understanding and knowledge but it did inform me of the lack of understanding that I had not suspected.

Communication problems

I did not expect everything to be perfect in the Teaching Program. Besides the problems with assessment there were communication problems and lack of understanding of concepts in spite of efforts made on the part of the students and myself:

> "But on the other hand I am not sure of what Ms F was talking about and when I asked her a question she seemed to tell me to ask the other students. This made me down and I lost hope of learning maths" (Kela, 28/6).

But later Kela reported that she found the journal writing gave her an avenue through which she could communicate with me:

> "Writing in my Thinkbook has given me a chance to share my ideas and weaknesses with you especially since I do sometimes feel ashamed to talk it over with you" (Kela, 11/10).

In the case studies in the next chapter more examples of communication problems are described. Being from a country foreign to the students I was not surprised that communication problems arose. I believe that the inclusion of the Thinkbook writing in the Teaching Program did give the students and me an avenue of communication to supplement what took place in class. Even so if I had continued for another year at the College I think I would have taken more time to specifically discuss issues related to communications early in the year and encourage more dialogue in class. I as a teacher with a constructivist approach see good communication as an important aspect in my teaching approach. But generally speaking considering the circumstances, the enculturation process into the mathematics teacher education had begun satisfactorily.

Constructing knowledge of concepts and the nature of mathematics

In a discussion of pre-service teachers' constructing knowledge of concepts and of the nature of mathematics it is difficult to separate these two areas of knowledge. When considering the different aspects of the nature of mathematics examples of successes and difficulties with understanding concepts emerge. Therefore my discussion of the knowledge of concepts related to number sense will be limited to its development via strategies related to the constructivist approach. The strategies especially, the use of the students' own methods, use of materials, cooperative group work and reflexive activities in problem solving and mental computations will be considered in the discussion, and other developments of number sense in the discussion of the aspects of the nature of mathematics.

Knowledge of concepts

A major novelty of the unit for the participants in this study was in its presentation. Rather than the traditional 'lecture, example, then exercises' type of lesson, with its rote learning of facts and procedures, a constructivist teaching approach was used. This meant that among other things I, as a constructivist teacher used teaching strategies such as those that encouraged the pre-service teachers to use their own computation procedures. These strategies which appeared to produce change in the students' understanding of mathematics included the doing of small mathematical tasks which often involved cooperative group work, and the use of materials. Also journal writing, the use of mental strategies and problem solving appeared to deepen their understanding of concepts. Effects of some of these strategies have been discussed in the previous section so here I will briefly consider how they appeared to enhance understanding of numeracy. Students often described how the use of materials and cooperative group work helped them to understand concepts of number sense:

> "The whole exercise was so vivid to me because I used the materials (MAB) to bring out the story of how I understand the value of the numbers and how they are put together to give a number" (Sisa, 26/4).

> "It is always good to divide students into groups as group work is more enhancing and [students] understand better" (Muri, 5/7).

Many students described the benefits of increased understanding for them by the writing in their Thinkbooks. Joni overlooked that it was an assessment task and concentrated on it as more a learning task which to me was a thrill. Here was one student who was prepared for more than rote learning:

> "To me Thinkbooks meant more than having classes. In the future I will try and have Thinkbook for every subject because I find it to be much more effective in my learning and most importantly in my UNDERSTANDING of a certain topic. Furthermore I feel that writing up Thinkbooks for me was much better that doing assignments" (Joni, 18/2).

I encouraged the use of their own strategies via problem-solving exercises early in the unit when I encouraged them to use personal methods of finding the solutions:

> "But I think it is worth trying to figure our how to get this and how to do it. Problem solving not only tells us how to get the answer but also what and why we did a particular thing to get the answer" (Keca, 11/4).

> "I think the most beautiful insight about maths is that I can learn better and faster by discovering my own methods of computing to get the answers" (Sisa, 7/4).

Later in the Teaching Program students continued to believe that use of their own strategies promoted understanding:

> "I must say I agree with what she [myself] said about methods because I feel that children only learn when they are left to work out the answer by themselves and develop their own strategies" (Joni, 7/6).

Most days I included some work involving mental computing including the discussion of strategies to help the students construct ones they were comfortable with. The idea of computing mentally was a new idea for most students:

> "I never really knew about mental arithmetic until today. Before, when adding and subtracting I always did it without realising that I was actually working the facts out in my head" (Joni, 19/3).

Often the entries in the pre-service teachers' journal were discussions about their great interest in the short activity related to mental computations rather than the main topic of the day. One weak mathematics student wrote that he was glad he learnt this topic. I was hopeful that he understood the strategies though from what he says it is not clear:

> "Today I found it so easy … subtraction by 10. I just look at the number and I can say what is the answer" (Liku, 19/4).

And some weeks later he wrote,

> "Multiplication by 9, 99 and 999 was so enjoyable and a lot of thinking has to be put in. … the method was so interesting that I wanted to continue to do maths for the whole day" (Liku, 28/5).

Understanding of the nature of mathematics

As well as developing their understanding of concepts I hoped that the activities would assist the pre-service teachers to better understand the nature of mathematics and so develop appropriate values of mathematics. I have considered the nature of mathematics using six categories related to the six values of mathematics (Bishop, 2001) which are rationalism, objectism, control, progress, openness and mystery. The processes involved in doing the activities promoted one or more of the aspect related to these values. Ria and Keca in their struggle to understand ideas appeared to realise that mathematics was *rational* and *open* to verification:

> "When doing the division, the answer … should be smaller but doing division with decimals you get a bigger number. Why is that? I still don't understand" (Ria, 15/7).

> "If a topic is not explained properly and I don't seem to understand it then it really gives me a heck of a time. … Some topics are understandable while others are really hard to get" (Keca, 18/3).

The value of *control* is concerned with security as in the use of algorithms and for work involving rote learning. Ria experienced as a child the overemphasise on rote learning in mathematics but now had understanding to support her security:

> "Way back in primary school I never learnt to use this method (compilation of nine times tables). I used to cram it [times tables] but never knew what it really meant but now I fully understand the meanings especially the patterns it shows" (Ria, 20/5).

During the Teaching Program I included much use of manipulatives and many of the pre-service teachers demonstrated that they valued *objectism* as they liked to make concrete representation of concepts and processes. They could also see objective mathematics as a stepping stone to abstract reasoning. In the case studies there are a number of entries that illustrate this well and Sisa suggest it here :

> "Working with straws is something I discovered as exciting and effective. The picture of that particular sum, e.g. 125 + 10 stood out clearly as I used straws to classify them according to the bundles of 100 and of 10's" (Sisa, 12/4).

Many students believed in the *progressive* nature of mathematics as can be seen from their creativity in problem solving. Also in response to doing of many of the tasks the student came to realise that they are able to explore and put forth their own ideas:

> "It is good to explore and find out these easy and new ways of dealing with fractions [using fraction charts]" (Ria, 7/6).

Mystery in mathematics appeared to remain a mystery or rather non-existent for most students. From where and from whom mathematical ideas emerged was something that generally they had not yet considered. Only a couple of students, the higher-ability ones chosen for the case studies (see next chapter), mentioned in their Thinkbooks the discussions we had in class about the evolution of the number system, mathematics educators such as Zoltan Dienes and the theories of learning mathematics, including constructivism. Sisa showed possible signs of a developing interest in discovering some of the mysteries in mathematics:

> "To find a number and its multiples using the number chart. I am beginning to learn the art and beauty of times tables" (Sisa, 20/5).

This section is describing the implementation of the Teaching Program and so a detailed description of knowledge of concepts and the nature of mathematics is not to be expected. The next chapter which discusses quantitative data for the whole class as well as the case studies considers the issue in more detail.

Lifelong learning for the pre-service teacher

In the Nordic Council of Ministers' Report (1995) it was noted that nobody knows tomorrow's curriculum and therefore it is important to learn how to learn rather than learn what is considered important today. As noted in the literature review the importance of this for pre-service teachers is clearly evident. Hence one of my goals was to establish a foundation for lifelong learning processes especially in two areas, metacognitive thinking and pedagogy. To do this I included activities that encouraged firstly the development of metacognitive skills for the students' learning especially their own number sense development. Secondly to a lesser degree I aimed to help them begin to learn to develop pedagogical content knowledge and teaching strategies. I would hope that both of these would continue to develop throughout their course at the College and their professional lives and be an important part in their lifelong learning.

Metacognitive thinking

As mentioned earlier, Ernest (1994) in discussing constructivist ideas, suggests that the teacher needs to help the learners use metacognitive processes and so help the students to be more responsible for their own learning. Besides the class discussions the metacognitive activities I included in the program consisted of such things as writing about what learning occurred in their mathematics classes. The pre-service teachers frequently commented in their Thinkbooks on their new-found independence in the process of their own learning:

> "What I have learnt is more than I expected. … and I am happy because I have learnt lots of things from struggling" (Toni, 11/10).

Ria often wrote about the processes occurring in her learning:

> "I feel responsibility for my own learning ability in mathematics" (Ria, 8/3).

> "Testing myself is a good thing" (Ria, 16/7).

After explaining very clearly her understanding of operations with fractions Ria was joyous:

> "It makes me proud of myself because of myself I could use my [fraction] chart and here comes the answer" (Ria, 9/6).

A week later:

> "I am proud to be an expert in dealing with fractions" (Ria, 25/6).

The process of planning and implementing the writing in the pre-service teachers' Thinkbooks has been outlined in the methodology chapter. Here I will remind the reader of a few aspects of its use during the Teaching Program and describe some apparent development in their metacognitive thinking and skills because of it. When the students did make entries they generally wrote freely discussing such things as what they felt, believed, did or did not understand and their plans for future learning and teaching.

Yet many of the students seemed to find it difficult to make entries regularly, especially in the first two months. After this poor start I negotiated the requirements with them and cut back on the number and length of the entries to a few times a week and only 50 words. Interestingly after that, the length of entries was generally 50 words for a short time then doubled in length after a week or so and at the same time the regularity of the entries improved. Some of the following quotes describe this struggle to be consistent about writing in the Thinkbook describing their early problems with the writing:

> "The first impression I got about this idea of Thinkbook was that it was a waste of time writing in it and it was worthless" (Joni, 18/2).

> "The first time I was told about the Thinkbook I was really lost. I did not think to bother filling in my Thinkbook" (Keca, 10/11).

At the end of the year when I asked the pre-service teachers to comment on the use of the Thinkbooks there were both positive and negative comments, for example:

> "I have learnt to reflect and think of the activities and lessons in class. Reflecting is also the main idea of having a Thinkbook. It helps me to evaluate class activities and helps me to think. At first I thought it was just a waste of time, ink and paper. But now I have come to realise how useful and important it is to me" (Keca, 10/11).

I was conscious at this time about the negative comments. A by-product of writing the essay was that it gave me, an outsider to their country and culture some insight to the pre-service teachers' thinking and hence a better knowledge of how to contribute more effectively to their learning environment. I found it sad at times to read of the difficulties, and sometimes suffering, they experienced learning mathematics in school:

> "I remember my teacher in Class 6. Any wrong doings were severely punished with hose pipe on the back. At times he wasn't ready with the lesson. If students got lots of mistakes in the homework they used to get hose pipes and sticks on the backs" (Muri, 24/2).

It was when reading their Thinkbooks that I heard about their struggles and successes in learning to learn, independent of me. From their comments above about a number of experiences such as problem solving activities, cooperative group work, class discussions and Thinkbook writing it can be seen that these experiences contributed to their reflection and metacognitive thinking but the Thinkbook writing appeared to be the most beneficial.

Learning to teach

As well as developing metacognitive skills, another influence on the future lifelong learning for these pre-service teachers is the development of their ideas about and skills for teaching. For this it seemed best for them to reflect on their past mathematics learning early in the Teaching Program. One major activity that helped them reflect on this was the essay, which was an assessment task. The essay, “Teachers teach as they were taught themselves” aimed to help the pre-service teachers reflect on the ideas of teaching mathematics which they had brought with them to the College. Throughout the Teaching Program, perhaps as a result of this exercise, I noticed that they often reflected on what we did and compared it to what happened in mathematics classrooms for them in the past. A few wrote about resolving not to repeat the ineffective or worse methods their teachers used:

> “I would not like my students to go through what I went through” (Sisa, 25/5).

I had hoped that, after writing the essay they would be in a better position to assess the values they held about teaching relative to any new ones I presented. The theme remained in Liku’s mind until her last words in her Thinkbook,

> “I will teach my class but not the same way we were taught” (Liku, 20/10).

Writing in the Thinkbook also helped many of the students reflect on themselves as teachers and on their teaching strategies:

> “The Thinkbook means a lot to me because it makes me think and revise after every class every day. The Thinkbook allows us to communicate our ideas and what we think is right for the teaching of maths” (Laka, 12/10).

During the implementation of the Teaching Program the students thought much about teaching and their commitment to it. When one student was commenting on her collecting of recycled material for producing teaching materials she expressed her commitment:

> “Wow, I love going out looking for bottle tops and asking for egg cartons but some people stare and make fun of me. I ignore people. My first priority is learning and I will make the most of it.” (Ria, 17/3).

Some of the pre-service teachers' ideas of what being committed to becoming a good teacher meant were far from what I expected but I discovered that these were views commonly held by many people in the community:

> "Learning to write [correct letter and figure formations] is something that needs to be acknowledged because I feel that if we are to be teachers, one not only has to be well-dressed but has to have a good presentation" (Joni, 18/2).

The pedagogical content knowledge part of the unit was mostly related to discussion of skills used in the short student-presentations. Generally my classes began with a student engaging the class for five minutes in a mathematics activity called a Warm-up. The students were each given a copy of one of many of these activities that are described in the PNG textbooks (Gough, Liburn, Rawson & Sullivan, 1991). An example of one such activity is the guessing of a secret number known only to the teacher. In this activity when each guess is given by a student it is written in the appropriate column on the chalkboard as being either greater than or less the secret number. The guessing continues until the students narrow their guesses down to the secret number. During the first week I conducted the activities daily. In following weeks each class began with one of the students doing an activity. Generally the presentations were well-prepared by the students with the materials to be used distributed to the class before time. Many students wrote in their Thinkbooks that they were nervous about the activity, but their peers were very supportive knowing that it was the first time most of them had stood in found of a class to speak:

> "[Today] I did the Warm-up with the students in which I acted as the teacher. At first I was a bit nervous and I forgot to tell the class the topic of the [activity]. I thought being a maths teacher was only to do with numbers but then I realised I had to say questions and sentences which related to the problems to be solved" (Kela, 7/4).

> "Presenting my Warm-up to the class was scary. Presenting gives me courage and enables me to talk or teach in front of my class. After my presentation I felt overwhelmed with happiness and gratitude because I know I am ready to teach my Warm-ups to my classes" (Ria, 28/2).

Much of the learning to teach happened after each presentation. When the student conducted his or her short activities I gave each of them the opportunity to comment on their own teaching. This often lead into reflections about the good and the less-then-good features of the teaching that the student was comfortable with being discussed. These activities also became an important part of each class for the pre-service teachers as they found them so motivating:

> "Doing the Warm-up is just great. It makes me lively and enjoy the lesson" (Laka, 25/2).

The students also produced some mathematics teaching materials both in class and for an assessment task and we discussed the uses of some of them. For the pre-service teachers the production of a portfolio of mathematics teaching-materials was a novelty and an enjoyable learning experience. I had expected this as during the two previous years I had received many comments to this effect. Among the materials to be produced were number expanders, family fact cards, flash cards of words such as days of the week, ten frames containing objects. I chose these materials as they were related to what we were doing and using in class. This meant that while working on them the pre-service teachers were hopefully revising and in the constructivist sense, constructing, or re-constructing their knowledge of the work being discussed in class. Some of the pre-service teachers did use them during their lower primary years but others had to be satisfied with whatever was 'on hand':

"Way back in primary school I had to learn using fingers, toes and ruler" (Ria, 30/3).

Summary of the chapter

In this chapter I have presented the first part of the results of this study. It was focussed by the research questions (see Figure 4.1). The chapter includes a description of the context, planning and implementation of the mathematics education unit that was central to this study. The planning was constrained to a small degree by a number of aspects of the context such as the financial position of the Teachers' College and the past mathematics education of its students. The planning was influenced by my past experience as a lecturer in mathematics teacher education and my study of mathematics educational research. The content of the education unit was somewhat focused on development of number sense though not confined to that area of numeracy and it also included some study of pedagogy. The paradigm for my teaching approach was a constructivist one.

The implementation of the Teaching Program was generally conducted according to my planning and my expectations from the previous two years' experience. The students' enculturation into the program had its positives and negatives. Once the students were familiar with the variety of activities I used they enjoyed participating in them. Their understanding of concepts deepened and they learnt much about how to teach mathematics from them. But past learning experiences of the students made fulfilling the requirements of the assessments difficult and there were a number of communication problems with my presentation of the instructions for them. For the construction of knowledge of the concepts and the nature of mathematics the pre-service teachers appeared to be assisted by the constructivist teaching approach. They also appeared to develop skills for metacognitive thinking and teaching practice and so would be better equipped for a life of learning as primary school teachers. For me the implementation of the planning was a pleasurable and an informative experience.

This chapter contains the first part of the results of the study as it outlines the planning and implementation of the Teaching Program. In the next chapter the rest of the results are described. First the quantitative data from the administering of three instruments is presented. Discussion of this quantitative data will demonstrate the need for the last part of the results which is the data provided by the six case studies. Then with all three parts of the results I will be in a better position to interpret and analysis the findings, and then discuss what has developed and been learnt as a result of the planning and implementation of the Teaching Program.

CHAPTER 6 THE CHANGES AND THEIR CONTEXTS

This chapter presents the second part of the results of this study. The first part given in the previous chapter was the description of the planning and implementation of the Teaching Program. In this chapter I describe changes that occurred probably as a result of the Teaching Program and investigate possible causes of them.

Originally I had planned that this study would use statistical research to determine the outcomes resulting from a teaching program that I would plan and implement. But in the process of further planning, a collective case study method with the support of the quantitative data seemed more appropriate. This was mainly because I wished to investigate which features of the Teaching Program were of benefit to the development of numeracy of the students. I believed that in determining the *why* for any change that occurred, the qualitative data contained in the students' journals and other material were a more valuable source of information than the quantitative data available. These data will be discussed in the case studies after the analysis of the quantitative data which gives the context within which the case studies were situated.

Data for the whole class

The quantitative data for the class were obtained from the responses to the administering of the instruments, namely the Assessment of number sense, Test of algorithms for the number operations and the Beliefs about mathematics questionnaire. The major part of the data collection, which took place at the beginning and the end of the Teaching Program, appeared not to impact in any significant way on the implementation of the unit as most students did not refer to it in their Thinkbooks. The instruments, their data collection processes and the analysis procedures have been described in Chapter 4. The means, standard deviations and the results of the t-tests comparing the class results at the beginning of the unit, the end of the unit and at the end of the following semester for the Assessment of number sense and the Test of algorithms for the number operations, are given in Table 6.1. This summary indicates that for each of the two instruments there were significant changes in the means of these two tests during the Teaching Program which was the time between February and July. Also for the pairs of results obtained from the July and the October administration of the instruments there were changes. The change was significant for the Test of algorithms for the number operations but not for the Assessment of number sense, although there was still a small increase for this instrument but a decrease in its standard deviation.

Table 6.1

Quantitative data for First Year class

Instrument		Number sense assessment		Test of algorithms	
No. of students		36		37	
Max. possible score		23		18	
Statistics		Mean	SD	Mean	SD
	Feb	8.42	3.70	10.41	3.95
	Jul	12.36	4.01	12.22	3.61
	Oct	12.60	3.66	13.08	3.31
Significance	Feb-Jul	*		*	
	Jul-Oct	NS		*	
	Feb-Oct	*		*	

* indicates significance: $p < 0.05$
NS indicates non-significance: $p > 0.05$

Assessment of number sense

In this section I will discuss the results of the scores for the *Assessment of number sense* and the *Test of algorithms for the operations*. To more closely examine the results of the former (see Appendix M) I will consider the percentage scores for individual items for results for February. Table 6.2 shows the items listed in order of

Table 6.2

Ordered percentage correct in Feb for Assessment of number sense (N=36)

Item	Content of item	Feb
13	Money calculation	88.9
	Explanation of answer	44.4
1	Estimation of area -fractional	86.1
8	Estimation multiplication	80.6
5	Verifying subtraction of money	69.4
16	Estimation for addition -fractions	50.0
11	Effect of addition -whole numbers -	50.0
9	Percentage	41.7
6	Place value -money	38.9
12	Estimation of time -days and years	38.9
	Explanation of answer	19.4
14	Estimation -multiplication	33.3
19	Application -rearranging of groups	30.6
2	Estimation of numeracy	27.8
3	Effects of subtraction	22.2
18	Inverse operations	22.2
17	Decimals -effects of multiplication	22.2
7	Estimation of distance -fractional	19.4
10	Decimals -effect of operations	19.4
4	Number- place value	16.7
15	Decimals -continuous nature	13.9
	Explanation of answer	5.6
20	Estimation – computations	2.8

percentage correct in February. These are divided into three groups. The first four items had average scores of more than 69%. These included the first easy question about estimation of the fractional area shaded, the estimation of the largest answer to multiplication of whole numbers and two questions which involved money calculation items. For the other nineteen questions only 50% or less of the answers were correct, with nine of them, in the third group with 23% or less correct. These nine poorly answered questions included four about decimals, three requiring understanding of the effects of operations on whole numbers, a place value question, and finding a third of the distance around a square.

For the July results for the *Assessment of number sense* (see Table 6.3) there was a remarkable improvement with a little over 50% of the answers correct for half the items. Table 6.3 shows the items listed in the order of the greatest change in the number of correct answers for each item during the Teaching Program. The pairs of items, with calculations followed by an item with a required explanation are not separated. All the items are divided into two major groups with Item 1 on its own as it had 100% correct in July, the maximum possible change.. For the questions other than Item 1 there was a gradual variation in the percentage change with a greater gap after the first six questions thus providing the dividing line at this point for the two groups. In the first group there are six questions with more than a 27% increase in the number of correct answers. These questions included three that examined the effects of the operations, one requiring computational estimations, one an estimation of numeracy, and the other an explanation for an answer to a money calculation. The items for which there was little change in the percentage of students with correct answers included four with rational numbers, an estimation of time and an explanation for an answer. The rational number ones were two about decimals, a percentage, finding a third of the distance around a square. These results will be discussed later.

Table 6.3

Ordered changes in percentage correct, Jul-Feb number sense assessment (N=36)

Item	Content of item	Feb	Jul	Change Jul - Feb
1	*Estimation of area -fractional*	*86.1*	*100*	*13.9*
10	Decimals -effect of operations	19.4	58.3	38.9
3	Effects of subtraction	22.2	58.3	36.1
11	Effect of addition -whole numbers -	50.0	83.3	33.3
2	Estimation of numeracy	27.8	58.3	30.6
20	Estimation - computation	2.8	33.3	30.6
19	Application -rearranging of groups	30.6	58.3	27.8
4	Number- place value	16.7	38.9	22.2
5	Verifying subtraction of money	69.4	88.9	19.4
6	Money-place value	38.9	58.3	19.4
18	Inverse operations	22.2	38.9	16.7
17	Decimals -effects of multiplication	22.2	36.1	13.9
14	Estimation –multiplication	33.3	44.4	11.1
16	Estimation for addition -fractions	50.0	58.3	8.3
8	Estimation -multiplication	80.6	88.9	8.3
12	Estimation of time -days and years	38.9	47.2	8.3
	Explanation of answer	19.4	25.0	5.6
9	Percentage	41.7	47.2	5.6
13	Money calculation	88.9	94.4	5.6
	Explanation of answer	44.4	75.0	30.6
7	Estimation of distance -fractional	19.4	25.0	5.6
15	Decimals –continuous nature	13.9	13.9	0.0
	Explanation of answer	5.6	5.6	0.0

As already discussed there was not a statistically significant increase in the overall class score for the assessment between July and October but there was an overall improvement (see Table 6.1). Table 6.4 shows the results for the changes that occurred during the semester following the Teaching Program. For this delayed testing in October nearly two thirds of the items had greater than 50% of answers correct. There were no major changes in the percentage of correct answers for each of the items from the July assessment except for estimation of multiplication of whole numbers. Among the third of the items with less than 50% of answers correct were six items for which the percentage correct actually dropped a little from the July tests. This drop may have been the result of questions for which some pre-service teachers were unsure about in July but had correctly guessed, but then had wrongly guessed the answers in October. These included questions about estimation of time, two explanations of answers, three about decimals and another for which two students made a minor error.

The question which required an understanding of the third decimal place remained the most difficult for the students. This may have been because many had come to deepen their understanding of decimals by using its relationship to money notation and so had not considered thousandths. Another problematic question was related to finding a third of the distance around a square. McIntosh, et al (1996) said that students he had worked with, answered such an item poorly because they had a lack of a benchmark with which to compare and make judgements about the fraction, one third. This was possibly also true in this study as many of the pre-service teachers marked a third, midway between ½ and ¼.

It appears from these results that, before the Teaching Program at the beginning of the year, computations involving money and multiplication of whole numbers posed little difficulty for the majority of pre-service teachers whereas decimals and computational estimations proved to be problematic. However the results suggest that the Teaching Program may be partly responsible for a notable improvement in estimation of computations for the four operations. As discussed in the previous

Table 6.4

Ordered changes in percentage correct, Assessment number sense, Jul-Oct (N=36)

No.	Content of item	Jul	Oct	Change-Jul-Oct
14	Estimation -multiplication	44.4	83.3	38.9
19	Application -rearranging of groups	58.3	69.4	11.1
20	Estimation - computation	33.3	44.4	11.1
6	Money-place value	58.3	66.7	8.3
2	Estimation of numeracy	58.3	66.7	8.3
8	Estimation -multiplication	88.9	94.4	5.6
9	Percentage	47.2	52.8	5.6
3	Effects of subtraction	58.3	61.1	2.8
11	Effect of addition -whole numbers -	83.3	86.1	2.8
16	Estimation for addition -fractions	58.3	61.1	2.8
7	Estimation of distance -fractional	25.0	27.8	2.8
13	Money calculation	94.4	97.2	2.8
	Explanation of answer	75.0	55.6	-19.4
5	Verifying subtraction of money	88.9	88.9	0.0
18	Inverse operations	38.9	38.9	0.0
4	Number- place value	38.9	33.3	-5.6
1	Fraction -estimation of area	100	94.4	-5.6
15	Decimals -continuous nature	13.9	8.3	-5.6
	Explanation of answer	5.6	11.1	5.6
10	Decimals -effect of operations	58.3	50.0	-8.3
17	Decimals -effects of multiplication	36.1	22.2	-13.9
12	Estimation of time -days and years	47.2	33.3	-13.9
	Explanation of answer	25.0	19.4	-5.6

chapter much time and effort was given to this topic and the students often commented on how helpful and interesting for them were the activities related to it. It appeared that the unit was not responsible for much improvement in estimation of time, fractional distance, percentages and the understanding of decimals. Apparently estimation of time required special attention and there were no activities in the unit were related to it. The other topics were all related to rational numbers. Although, as mentioned in the Teaching Program time was devoted to various aspects of rational numbers and the pre-service teachers had demonstrated basic knowledge of facts, they were unable to apply them in questions that required other than simple statement of facts. It appears that more time needs to be given to the consideration of the concepts of rational numbers in further mathematics education units of their course.

Test of algorithms for the operations

The *Test of algorithms for the operations* was not designed to directly assess the number sense development of the students. Instead it was introduced into the study to investigate changes in the students' performance of the written algorithms to see if an improved understanding of number sense was supported by a more skilful execution of the algorithms. The algorithms had been previously studied in school and were not formally taught during this study.

Under these circumstances an improvement in such a test would generally indicate a change in development of number sense or some such similar development. A decrease or no change would generally be an indication of no development of number sense but, as the literature suggested it is not necessarily the case. One study suggests that many other factors need to be considered such as feelings of familiarity and security when using such procedures and resistance to change using newly understood concepts. These can prevent the development of number sense being reflected in other numeracy work. Thus analysing results of performance in these algorithms required careful analysis.

In Table 6.1 the analysis shows that the improvements in total score were significant at the end the Teaching Program and again at the end of the year. As described in Chapter 4 the results for the individual questions could not be analysed using the same method as for the *Assessment of number sense* because each of the questions required a written answer to which a mark of 2, the maximum mark, 1 or 0 was assigned. The table of results gives the class average marks for each of the nine questions (see Table 6.5). For discussion purposes the questions have been divided into three groups because of the similarity and difficulty of the questions. In the first group are the easiest questions, addition, subtraction and multiplication of whole numbers, the second group, division for whole numbers and the third, question for the operations on rational numbers.

Table 6.5

Mean marks, Test for algorithms for the operations (N=37, Max mark 2)

No.	Content	Feb	Jul	Oct
1	Addition of whole numbers	1.95	1.95	1.86
2	Subtraction of whole numbers	1.86	1.87	1.86
3	Multiplication of whole numbers	1.79	1.68	1.95
4	Division of whole numbers	1.41	1.22	1.65
9	Long division of whole numbers	0.57	0.95	1.32
5	Addition of fractions	1.03	1.16	1.11
6	Division of fractions	1.11	1.49	1.38
7	Multiplication of decimals	0.81	0.97	1.08
8	Division of decimals	0.43	0.95	0.92

Before the teaching program the average marks for the questions of the first group indicated that these algorithms were generally well executed. Closer inspection of the written work and interviews generally suggested careless mistakes were the reasons for the less than maximum mean marks. For the second group, the two divisions had lower class means than the other questions with whole numbers. The short division had the difficulty of requiring a zero in the answer and the long division was the least well-performed of all the questions before the Teaching Program. At the end of the year mean marks for both divisions were much higher with the most remarkable improvement for the long division. This is discussed at some length in the case studies.

For the rational numbers which were performed poorly at the beginning of the Teaching Program there was an improvement during the Teaching Program but this improvement decreased a little during the last part of the year except for multiplication of decimals. Generally these mean marks were well below those for the whole numbers.

The results for the whole class for the algorithm tests indicated that there were significant improvements for the July and the October tests. The most notable changes were for the mean marks for the questions on division of whole numbers. As discussed later this appears to be related to number sense development as students completed the questions using their own meaningful procedures. The questions involving rational numbers did improve but at the end of the year their mean marks continued to indicate a lack of ability to perform the algorithms. Unlike for division, the students did not use their own strategies but attempted to perform the poorly remembered standard algorithms. The algorithm for long division, even though it was included in the primary school mathematics curriculum was rarely remembered, and it appears that students resorted successfully to using their own methods whereas for rational numbers they attempted procedures which resembled incorrectly the standard procedures. It seems that partly remembering an algorithm may be more of a hindrance to devising one's own strategies that lack of knowledge of them. One feature which suggested a development of number sense was that for many of the questions, the written work in the tests after the Teaching Program showed answers that were corrected by the students when verification processes highlighted mistakes.

Changes in beliefs about mathematics

The results from the questionnaire, *Beliefs about mathematics* are shown in Appendix N. As described in the methodology chapter the scores for the individual items range from 1 to 5 with one indicating a high agreement with a constructivist approach (for brevity I use 'positive agreement' for this end of the scale, and 5 a decided lesser agreement). I realise that my interpretations of items as being 'positive towards a constructivist approach' is somewhat subjective and so to lessen this subjectivity I discussed briefly with the students my interpretations of the items.

I will now consider the mean scores for some of the individual items for the *Beliefs about mathematics* from two different aspects. First I will describe the scores for the responses for the items the pre-service teachers gave in February, before the Teaching Program. Then I will investigate the notable changes in the scores for the individual items for the February-July scores. Any changes suggest that they may have occurred partly as a result of the Teaching Program.

For this first discussion refer to Table 6.6 which lists the items in rank order of their mean scores in February. I have divided the items into three groups. My classification is, firstly a mean score approximating to 4 or greater indicating a less than positive approach to constructivism for the item, secondly, a mean score approximating to 3 indicating that students are generally unsure of their stand for the item, and thirdly, a mean score approximating to 2 or less indicating a belief in a more positive approach to constructivism for the item. Six of the twenty-two questions are included in the first of these categories. These items that indicated a less than positive approach in constructivism were those for which the pre-service teachers generally believed strongly in dependency on the knowledge and memorisation of facts, rules and procedures (items 1, 3, 11, 21 and 13). Their responses were also less than positive for the items that state that questions can be tackled once they are explained (item 19).

For other items that indicated a dependency on the memorisation of facts, rules and procedures their response was generally unsure (items 22, 10, 5, 7 and 9). They were also unsure about the importance and place of mathematics in daily living (items 4 and 8). Another item for which they did not take a stand was the importance of the need to estimate (item 17).

Table 6.6

Ordered Feb mean scores for responses, Questionnaire for Beliefs (N=36)

No	Item	Feb
1	*Mathematics consists of a set of rules and procedures.	4.4[a]
3	*There is always a rule to follow in doing a mathematics calculation .	4.3
11	*Doing maths means following standard procedures precisely.	4.0
19	*In maths it is important that students are able to tackle a question once it has been explained.	4.0
21	*In maths it is important that students are able to follow routine instructions.	4.0
13	*In maths it is important that students remember facts.	3.9
22	*In maths it is important that students are able to memorise rules.	3.4
10	In maths questions can be answered without using rules.	3.4
5	*Learning maths involves mostly memorising rules.	3.3
7	*To be successful in mathematics it is important to memorise rules.	3.1
9	*There is little place for originality in doing maths calculations.	2.9
4	Maths is useful in solving everyday problems.	2.8
17	In maths it is important that students are able to estimate.	2.6
8	*Maths is not needed in everyday living.	2.5
16	In maths it is important that students are able to do calculations without a calculator.	2.2
20	In maths it is important that students are able to show initiative and think creatively.	2.1
15	In maths it is important that students are able to be flexible in their thinking.	2.1
12	There are many different ways to do most calculations.	2.0
14	In maths it is important that students are able to answer unfamiliar questions.	2.0
18	In maths it is important that students are able to see patterns and relationships.	1.9
6	A maths calculation can be solved in different ways.	1.9
2	Understanding the process is more important than correct answers in mathematics	1.4

[a] Low score indicates positive attitude to constructivist ideas.

* Items negatively worded for constructivist approach: original scores have been reversed

For all other responses in February the mean scores showed a positive approach towards constructivism. These included beliefs that it is important when working mathematically that students show initiative, answer unfamiliar questions, be creative and flexible in their thinking (items 20, 14, 15 and 12) and that there are different ways to calculate, it is important to be able to work without a calculator and to see patterns and relationships (items 6, 16 and 18). The item for which they were the most positive was that understanding the process was more important than the correct answer (item 2).

The changes given in Table 6.7 are given as the decrease in means, that is the July means minus the February means. After the Teaching Program in July the mean scores for all the items except one became more positive towards a constructivist approach. For the February scores there were six out of the twenty-two items with a mean score approximating to 4 or 5, whereas in July this had decreased by three items for which three items, the changes were among the four greatest. Although admittedly some changes were only by 0.1, the fact that 21 of the 22 mean scores moved in the same direction can be taken as an important indicator of overall change.

For the rank order of changes there are no obvious cut-off points to use in this discussion. So for convenience the first group of items are those with a change greater than 0.5 as they generally all have the high scores for February, except for two items. Seven of these items with the greatest change are related to dependency on rote learning in mathematics (items 5, 11, 3, 7, 10, 1, 22). Other items for which the mean scores indicated a considerable change including, that it is important to be able to estimate, that it is important that students are able to tackle a question once it has been explained and that mathematics is needed in everyday living. (items 17, 19 and 8).

The two items that in February had scores approximated to 4 or greater that are in the second group with lesser changes were also related to memorisation in mathematics. These stated that it is important that students are able to follow routine instructions and that it was important to remember facts (items 21, 13) for which there were changes in scores of 0.4 and 0.1 respectively. Also in the second group are the items 18 and 16 with changes of 0.4 and 0.5. These items state that it was important that students are able to do calculations without a calculator and that it was important to see patterns and relationships.

Table 6.7

Ordered changes in mean scores for Questionnaire, Feb-Jul (N=36)

No	Item	Feb	Jul	Change Feb-Jul
11	*Doing maths means following standard procedures precisely.	4.0 [a]	3.0	1.0#
5	*Learning maths involves mostly memorising rules.	3.3	2.3	1.0
3	*There is always a rule to follow in doing a mathematics calculation.	4.3	3.4	0.9
17	In maths it is important that students are able to estimate.	2.6	1.9	0.7
19	*In maths it is important that students are able to tackle a question once it has been explained.	4.0	3.3	0.7
7	*To be successful in mathematics it is important to memorise rules.	3.1	2.5	0.6
8	*Maths is not needed in everyday living.	2.5	1.9	0.6
10	In maths questions can be answered without using rules.	3.4	2.8	0.6
1	*Mathematics consists of a set of rules and procedures.	4.4	3.8	0.6
22	*In maths it is important that students are able to memorise rules.	3.4	2.8	0.6
18	In maths it is important that students are able to see patterns and relationships.	1.9	1.4	0.5
16	In maths it is important that students are able to do calculations without a calculator.	2.2	1.8	0.4
21	*In maths it is important that students are able to follow routine instructions.	4.0	3.6	0.4
20	In maths it is important that students are able to show initiative and think creatively.	2.1	1.8	0.3
2	Understanding the process is more important than correctanswers in mathematics	1.4	1.1	0.3
12	There are many different ways to do most calculations.	2.0	1.7	0.3
15	In maths it is important that students are able to be flexible in their thinking.	2.1	1.8	0.3
4	Maths is useful in solving everyday problems.	2.8	2.6	0.2
6	A maths calculation can be solved in different ways.	1.9	1.8	0.1
9	*There is little place for originality in doing maths calculations.	2.9	2.8	0.1
13	*In maths it is important that students remember facts.	3.9	3.8	0.1
14	In maths it is important that students are able to answer unfamiliar questions.	2.0	3.1	-1.1

[a] Low score indicates positive attitude to constructivist ideas,
Positive indicates a change to a more positive approach
* Items negatively worded for constructivist approach; original scores have been reversed.

An exceptional change of -1.1 was for the item which stated that it is important that students are able to answer unfamiliar questions. From my knowledge gained when teaching the students I suggest that some student may have considered that unfamiliar questions referred to unfamiliar context for questions. The class still remained divided in their stand on this statement for responses given in October. Two other results that surprised me were the small changes for the items 4 and 9 that stated that mathematics is useful in solving everyday problems and that there is little place for originality in doing mathematics. Also my knowledge of the students' language in July suggest that 'everyday problems' do not refer to problems in numeracy and 'originality' may have been related to creativity for the context of the problem. But for both these questions there was a further change later in the year of 0.3 for the October result which was among the greatest change at that stage (see Appendix O) indicating perhaps that they had come to my understanding of the terms and a more positive constructivist view.

In summarising the results of the responses to the questionnaire this analysis suggests that there were notable changes in the pre-service teachers' beliefs. After the Teaching Program they generally no longer believed that they needed to depend on learning facts, rules and procedures. They also came to believe that estimations, patterns and relationships are important in mathematics. In July the pre-service teachers continued to hold the beliefs that they held in February which indicated a positive approach to constructivism. For all the items for the delayed questionnaire at the end of the year the mean scores were either more positive than the July score or, for a few they remained unchanged. All these beliefs generally reflected what they wrote about in their Thinkbooks or discussed in class, as will be discussed later in this chapter. The lessening of their dependency on rules and routine procedures is well illustrated in their answers on the algorithms' test, as also discussed later in the case studies.

These results appear to suggest that the Teaching Program may have helped the pre-service teachers change many of their beliefs. In particular they seem to be what would be expected for a Teaching Program with a social constructivist approach. The emphasis to encourage one's own construction of knowledge and the use of everyday contexts as has been outlined in the previous chapter appear to have influenced the pre-service teachers' beliefs. These changes do not directly indicate a change in numeracy but it may be a necessary prelude for a development in it.

The foregoing analysis indicates significant changes but it does not explain *why* the changes occurred or the finer details of the changes. To focus on these issues I turn now to a series of six illustrative case studies.

The case studies

The rationale for and the details of, the procedures I employed for the multiple case study approach have been outlined in Chapter 4. For choosing the cases Stake (1994) believes that selection by sampling of attributes should not be a high priority but balance and variety are important, with opportunity to learn, of primary importance. This may mean that some desirable types have to be omitted if we cannot learn important things from them because of the lack of information they provide. For the ten students whom I interviewed one of the major reasons why I chose the particular six was because they were among the ones who wrote the most in their journals and were therefore able to provide more information for analysis.

The general information for each of the case studies is summarised in Table 4.2. Wili and Jo were the two males, and were the only ones of the group of ten interviewees who were a few years older than most in the class. Students who came straight from school were usually in the 17-19 age range. Wani and Vita were in that age range but, along with Jo, they had not come directly from school. There was no variation in the number of years during which those within this group studied mathematics except for Lisa who did more than thirteen years and Wani only eleven years.

Data from the instruments, the *Assessment of number sense* and the *Test of the algorithms for the operations* are shown in Table 6..8 The three cases considered of higher mathematics ability at the beginning of the year based on the information provided in their application forms to enter the course, were Wani, Ana and Lisa. The other three, Vita, Wili and Jo were among those in the class who were apparently of lower ability. The data in Table 6.8 tends to support this division. More detail about each of the six cases will be given below.

Table 6.8

Quantitative data for the six students of case studies

Instrument				Scores for case studies					
	Mth	Max	Class mean	Wani	Ana	Lisa	Vita	Wili	Jo
Number	Feb	23	8.4	9	10	13	9	7	7
sense	Jul	23	12.4	15	14	17	12	12	14
assessment	Oct	23	12.6	17	19	16	12	11	12
Test of	Feb	18	10.4	12	12	16	9	9	8
algorithms	Jul	18	12.2	16	17	16	9	10	13
	Oct	18	13.1	15	17	18	11	14	10

Structure of the report of the case studies

Stake (1994) states that the case content evolves in the act of writing. The researcher is the one who decides what is necessary for an understanding of the cases. The criteria for selecting content are many, including who prospective readers may be and what represents the case most comprehensibly. Stake as a constructivist believes that knowledge is constructed therefore the researcher needs to assist readers in the construction of knowledge. He suggests that to help the researcher and reader, the ideas need to be structured, highlighted, subordinated, connected, embedded in context and with illustration, and expose confidence and doubt. The structure I have used to help the readers construct their knowledge of each of the case studies is based on my framework for the study.

The structure for my writing of the report for each of the case studies will include a brief descriptions of the student, the quantitative data I have for them for the instruments they completed, and the experiences and reflections that have emerged as a result of the Teaching Program. These experiences and reflections will be described under the three areas which form the framework for this study. Who are the student as they are experiencing the enculturation into the mathematics education class? What knowledge related to mathematics have they constructed or had problems in constructing? How have they learnt, and will learn and teach in the future?

Hence my first heading is *Enculturation into the mathematics education classroom*. This section will include affective issues including the joys, difficulties and conflicts the student experienced. The second heading is *Knowledge of concepts and of the nature of mathematics.* This will describe any apparent success or struggle in development of the pre-service teachers' understanding of mathematical concepts, and of their understanding of the nature of mathematics. The aspects of the nature of mathematics that are considered are those related to each of the values considered and discussed in the literature review as described by Bishop (1988, 2001). For example the mathematics value of rationalism is related to nature of mathematics as being rational. These include sets of ideals and values which relate in complementary pairs to three components ideological, sentimental and sociological. These values are for ideology: rationalism and objectism; sentiment: control and progress; sociology: openness and mystery. The last heading is *Lifelong learning for the future teacher*. This section will describe the pre-service teachers' development of their metacognitive skills and their ideas about teaching. It will note any change in feelings, attitudes to and beliefs of their own learning especially of mathematics and the teaching of it.

These experiences will be illustrated using material from Thinkbooks, interviews, notes of workings shown on test papers and other less major sources such as my observations. The same technical details of the Thinkbook quotes as were used in the last chapter will also apply in this chapter, except of course the name of the author for each entry will not be included in parenthesis as only one student is considered in each case.

For a number of reasons the cases studies which consider the experiences of the students, Lisa, Wani and Wili are included in Appendices V, W, and X. One reason for placing these there was to shorten the length of the thesis and another was because three cases were considered sufficient to give the reader general insight into the students in the class. The choice of the three is rather arbitrary except that I chose to keep a male student, a student who was among the stronger and another from the weaker three in numeracy skills. The summary sections for each of the other three cases in the appendices are kept in the body of the thesis following the reports of the case studies.

This is to give the reader the background that may need in comprehending the discussions in Chapter 7. Also the findings for the six case studies are tabulated in Appendix Y. This table was used in the process of composing the reports for the case studies. The orders in which the three studies are reported in this chapter and in the appendices have no significance except that the student of apparent higher ability is considered first. For a copy of the original data for a case study, in this case Ana's data, see Appendix Z.

The learning experience for Ana

Ana

Ana described her culture as indigenous Fijian and her nationality as being partly Chinese. She comes from a family that has a great interest in primary education. Her mother is a teacher and a graduate of the College. She was named after her mother's friends, two women who have become important educationalists in the field of primary education in Fiji.

Ana came to the College directly from secondary school and so had no experience of the workplace other than time spent in the classroom in which her mother taught. She was in the age-range of 17-19 years, which was also true for 60% of the class, and she had thirteen years of schooling that included learning mathematics each year. She was a very conscientious student who tried to do all that was expected of her by those in authority. She worked quietly in class but was not lacking in confidence nor assertiveness and was prepared to speak when she occasionally saw the need.

Her mathematics ability was above average for the successful applicants to the College according to the entry data. The data for Ana from the administering of the two mathematics tests is shown in Table 6.9. As can be seen from the scores for the number sense assessment, Ana is well above average in all three assessments especially for the October result. For the test of algorithms her score is also well above the average at each time the tests were taken. A score of 17/18 for the last two tests indicated one incomplete answer. Hence one would not be surprised that Ana coped well with the mathematics that she was expected to do and learn.

Her first set of responses to the questionnaire of Beliefs about mathematics show that her scores were generally much more negative towards constructivist ideas than the class mean scores but she became considerably more positive on the later administrations of the questionnaire.

Table 6.9

Quantitative data - Ana

Instrument		Number sense assessment			Test of algorithms		
Month		Feb	Jul	Oct	Feb	Jul	Oct
Score	Max	23	23	23	18	18	18
	Mean	8.4	12.4	12.6	10.4	12.2	13.1
	Ana	10	14	19	12	17	17

Enculturation into mathematics education

In this section I will discuss how Ana experienced the culture of the mathematics education classroom during the Teaching Program. This discussion will include her descriptions of the positive and negative experiences of it and how she responded to them.

Initially Ana found the unit was not exactly as she expected. The culture of the College mathematics classroom was a little slow and boring perhaps compared with what she had recently experienced in secondary school:

> "It's like a whole replay of being in primary school again. Some of the warm-ups took a bit too long. So maybe in the future we do shorter warm ups so that when time comes to do the actual thing the children would not be bored because of too much of an activity of the same kind will slow down the interests of the students" (18/2).

A fortnight later Anna was still saying similar things:

> "I thought it would have been presented more interestingly. Too much time was taken up" (8/3).

Also she sounded unimpressed with some aspect of the mathematics classes. She was surprised at spending time discussing such things in class as the formation of digits. At first she did not mention enjoying any of the work until we discussed an algebra method for solving a problem.

> "I enjoyed doing this as algebra was one of my favourite topics in Maths in high school" (3/3).

Then it was the very next day that she gave the first of many comments about enjoying the experiences of primary teaching:

> "Actually I enjoyed it - playing with the bottle tops and stones. It was like being in class 3 and 2 [and] 1 again" (4/3).

But the more significant change in her attitude to the unit appeared to occur when she began her assignment of preparing teaching materials and then when she experienced teaching children in a classroom:

> "Well [for] maths class is one thing I find myself collecting a lot of 'rubbish' good rubbish. I have now come to realise how useful these cartons, bottle tops and such are useful for methods. Now when I open a packet of something I try and not to spoil it. I try and preserve it" (11/3).

> "Now I see the efficiency [efficacy?] of doing warm-ups. It doesn't only have to apply to the maths but also other subjects. My reason for pointing this out is that today our first practical teaching practice I went to Marcellan Primary and though I didn't do maths I applied the concept of doing warm-ups and the class actually enjoyed it" (25/3).

Once she felt comfortable being a pre-service teacher she often commented on her enjoyment in doing activities and this included doing tests:

> "The clock-making too was fun. I enjoy making things like clocks. This makes us use things around us. To make use of things to the fullest" (12/4).

> "Today we worked with dominoes. This was the first time I've ever handled dominoes and I found it fun working in groups playing the game so I'm sure the children would enjoy it very much" (7/7).

> "I found it a very pleasant test and thus finished before time" (19/7).

Ana was ready to take all opportunities to learn and discover what she could, and she often mentioned insights both into issues related to understanding and to attitudes:

> "But when Ms Fran first distributed the blocks there was immediately free play among us. We did this unconsciously. It was only when Ms Fran asked us if we noticed something, did we realised that it was happening. This is how children would feel if they were given things to play with" (19/4).

> "Maths lessons don't have to be so solemn but can be fun" (19/4).

Ana had probably been a good pupil in school where the transmission mode was used in teaching and so she had learnt her algorithms well. Therefore when she came to experience constructivist teaching methods she had to struggle a little:

> "The Game of the Year for 1999 was used [forming expressions using the operations and the digits in 1999]. I've been trying to come up with many possible combinations to fill up my paper. But it's frustrating me when I do something different I sometimes get the same answer. Anyway but it was fun for me. I'm still trying to get as many sums as possible" (16/3).

> "Writing my thoughts this year about maths was a new experience. At first I found it hard because I did not know what to talk about. Even towards the end of the term this year I am still sometimes unsure. Writing my thoughts every day was a bit of a hassle for me. I found myself forgetting at times to fill in my Thinkbook but on the other hand I feel it has benefited me somehow" (11/10).

Ana's response to the ideas about the constructivist approach were also investigated by means of the questionnaire for *Beliefs about mathematics*. A complete list of Ana's responses are given in Appendix P. Her responses at the beginning of the year were among the most negative in the class towards a constructivist view but these changed markedly at the end of the Teaching Program and changed again at the end of the year. In Table 6.10 the most notable changes for item responses over time are given.

Table 6.10

Notable responses to Questionnaire – Ana

	Feb	Jul	Oct
Always strong agreement with constructivist view			
Understanding the answer is more important the correct answers in mathematics	SA [a]	SA	SA
In maths it is important that students are able to do calculations without a calculator.	SA	SA	A
Shift in agreement towards constructivist view			
*There is little place for originality in doing maths calculations.	A	A	D
*Doing maths means following standard procedures precisely.	SA	A	D
*Learning maths involves mostly memorising rules.	SA	A	U
*In maths it is important that students are able to follow routine instructions.	SA	A	U

[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.
*Items are worded negatively

In summarising Ana's responses to the questionnaire on *Beliefs about mathematics*, Ana strongly believed in the importance of understanding mathematics. She came to believe in the possibility of being able to do calculations using different and original methods rather than rules and procedures. Of these, the greatest change was in coming to disagree that doing mathematics meant following standard procedures. Yet at the end of the year she still believed, though to a less degree, that rules had a major place in mathematics and the learning of it. Part of the reason for her adherence to the important place of rules might be because she realised, that for her, they were very helpful as she continued to use them to get correct answers and had not had much experience of creating her own strategies. In general these findings probably show that Anna's beliefs were moving more to a constructivist position. For the other items not mentioned here such as mathematics used in daily life, and the place of patterns and relationships, Anna believed in their importance but not strongly on each of the three occasions.

Ana seemed to practice her belief in following rules, not only in mathematics but the rules imposed by authority. A few times she was somewhat impatient at the difficulties and shortcomings of less conscientious students. She noted a number of the reminders I gave the class about requirements not being met:

> "Today we've got to re-read our Thinkbook instructions. Probably some of us are not following the instructions. That's why Ms Fran has asked this of us" (6/4).

> "The person rostered for the warm-up wasn't prepared and neither were the following others" (3/3).

> "All we did at maths today was the warm-up from Mr M. He could have a better control or coordination to what he taught but unfortunately it wasn't a success. He could have done better" (20/4).

I was always conscious that some of what was written in Ana's and the other students' Thinkbooks may have been what they thought I wanted to hear rather than what they thought themselves. Therefore I considered that entries which included negative comments required some courage to write and so were worth my serious consideration. Ana was probably more open than most of the students and she spoke freely in many of her entries about my actions and sometimes addressed me directly:

> "Anyway later on in class, the discussions listing benefits to the students and teachers of a Thinkbook, was more to my liking [than ideas about Warm-ups]" (24/2).

> "Nothing much was done in class today except that we used our place value mats and used the base ten objects to work out the sums written on the board. It's exactly the way we used the straws" (23/4).

This last remark also shows that Anna understood the isomorphism embedded in the two activities.

But being a very gracious person any criticism she made was discussed very diplomatically. There were times when I did not make my intentions as clear as I thought I had:

"She [Ms Fran] gave us a short test [*Test of algorithms*] on multiplication, addition and so on but I wasn't sure what it was for. I think it was to help her evaluate how she's gonna plan her other lessons" (11/3).

"Today Ms Fran asked us to produce our charts. Well I hadn't done it because I thought we could use our actual number chart and that was done on brown paper. When I discovered this wasn't so, I got so frustrated. I was cursing within me. But then I did the chart as quickly as possible. And when I got down to the exercise with Mika, I really enjoyed myself and got to overcome my frustration" (7/4).

From the above discussion it appears that Ana was not at first, easily enculturated into the mathematics education classroom. She took some weeks to adjust to being a pre-service primary teacher and not a secondary student, and so she saw primary school mathematics activities at first to be slow and boring. She needed to have all the details of the organisation of what she was required to do very clearly explained to her as shown on a number of occasions when she had misunderstood what I explained about requirements. Such confusion is understandable in the light of anecdotal evidence I had which suggested that secondary teachers continually remind students how to complete work, and follow them until it is submitted. But once Ana felt at home with primary school materials and curriculum she enjoyed the work done in class. She noted new insights about her own understanding of concepts and about the teaching of mathematics. A major turning point in her enculturation process appeared to be her first experience of teaching practice.

It appeared that Ana's confidence in her mathematics enabled her to be assertive enough to discuss critically the activities of the Teaching Program and how they conflicted or conformed with her feelings, her thinking, her activities and the cultures of her environment. She was generally conscientious about doing what was required of her in class yet in the first semester her responses to the mathematics unit were not all positive. She had struggles with some aspects of the Teaching Program that were part of my teaching approach. In particular, she mentioned the activities: having to think through ideas, use of trial and error methods to answer open-ended problems and writing about mathematics. Also she felt impatient with her peers who were not fulfilling requirements or who had poor teaching strategies. She was also sometimes frustrated by what she saw as the requirements of the unit whereas they were often the outcome of unsuccessful communication.

In the next two sections I will discuss further some of the issues touched on in this section. First I will describe what understanding of mathematics she appeared to gain during the Teaching Program and then I will examine how all of these phenomena in the mathematics education unit with its constructivist approach to teaching, produced learning for Ana both of her own learning and her learning to teach.

Constructing knowledge of concepts and the nature of mathematics

This section deals with some of the evidence of the apparent knowledge Ana already possessed concerning mathematics and what she constructed the time of the Teaching program. This knowledge will include both the understanding of concepts and the understanding of the nature of mathematics. To examine the conceptualisation of the nature of mathematics I will use six categories related to six values of mathematics as described by Bishop (2001), rationalism, objectism, control, progress, openness and mystery.

Understanding of concepts

To investigate Ana's knowledge of the concepts of mathematics I have limited my study to number sense. The first source of data that I will examine is her scores for the *Assessment of number sense* as can be seen in Table 7. Her score was 10/23 before the Teaching Program and this increased to 14/23 after the Teaching Program and finally to a high for the class of 19/23. All these scores were well above class average especially the final one.

In the initial assessment Ana attempted every question. Four questions she correctly answered involved single operations for whole numbers, another four were worded questions involving money and the other two were estimations. In the second assessment at the end of the semester she did not give answers for five of the questions, one of which she had given the correct answer in February and yet she still obtained a score of 14/23. Three of the five answers missing were at the end of the assessment suggesting that she did not complete the test quickly enough to attempt all questions.

The improvement in her score was the result of apparent improved understanding of the meaning and effects of the operations on numbers as she correctly answered the estimating of answers to multiplication of whole numbers, the division of decimals, the addition of fractions and two easy questions which she admitted later she had misread the first time.

At the end of the year, even though with no further work directly on these topics, she attempted all questions and obtained a score of 19/23. An interview about the four questions incorrectly answered revealed that she had misread the direction of the path around the square in one item, and in another question about decimals misread the word *different* for *difference* but ignored further explanation in the question. In looking at the reasonableness of answers she considered 0.95 as approximately zero expecting near zero for the answer. On reflection she realised the error and quickly chose the correct answer. These results suggest that her number sense had improved. However there were still aspects of decimals which remained as the main cause of problems, and she was still slow in working mentally shown by the fact that she said she did not have time to check answers as others students said they did.

Ana's first *Test of the algorithms for number operations* indicated a good knowledge of number operations (score of 12/18) except for the multiplication and division of decimals. When completing the test for the first time Ana placed the decimal point incorrectly in her answers in these questions. The long division was correct except for how she wrote the decimal answer. On her paper, the workings at the side contained a multiplication of 46 x 2 and *long* multiplications by 11 and by 12. Her working for 1.2 x 0.02 included a line of zeros. For the next two on this test on algorithms she correctly completed all questions except for not simplifying an improper fraction to a mixed number as required (scores both 17/18). Her workings at the side for these two tests were much less but still with a long multiplication of 0.02 x 1.2 containing many zeros. In the second completion of the test for the question 6.9 divided by 0.30, the decimal point was correctly placed after estimating the answer by writing a division of 600 by 30. These results for the algorithms suggest an initial poor understanding of decimals and a poor skill of computing mentally but both these problems decreased markedly by the end of the Teaching Program.

It is interesting to note that for the final test she chose to use her own strategies where possible. For example, in the final completion of the test the long divisions were correctly calculated using a method similar to long division but less efficient as it had more subtractions. But it showed good understanding of the process even though in the second completion of the test she accurately used the standard algorithm. Also at the end of the Teaching Program in her Thinkbook she described how she attempted to use her own strategies but admitted that she resorted to the 'old ways' or 'long forms', only using mental methods to check reasonableness of answers. She noticed improvement especially in speed:

> "Anyway I found that doing the addition and multiplication were a bit easy although I still used the long form. Anyway one division sum that I had taken long to do was the last one. I had to do long division though roughly. I knew the answer had to be in the 200's. The fraction sum was one that I could not do just by looking at it. I had to go back to the old way of doing it. I did picture the parts but couldn't pick out the answer accurately. But on the whole I did the sums a lot faster than I would have done when school began [beginning of the Teaching Program]" (23/7).

An indication that she used her approximation to the answer of 200 was that her first crossed-out try had left out a factor of ten to which she had later obviously added the missing zero. Also she probably attempted to mentally picture the fractional parts as there is no evidence of diagrams which she did draw when she completed the fraction questions in the interview activities.

In doing the activities in the *Test questions for interview* (see Appendix F) Ana's initial difficulties in numeracy were more obvious. For the questions attempted during the interview at the beginning of the semester Ana was generally unable or lacking in confidence to verbalise reasons for either correct or incorrect answers. This may have been largely because the interviews were conducted only a couple of days after arrival at the College and before I had begun to teach her. The question which caused the most difficulties for Ana, and in fact all students, was Question 9. The students were to choose the two numbers with the smallest difference from a group of eight numbers ranging from 23 to 1234 which they had all correctly arranged in order of magnitude. Ana did not consider the correct pair which was 194 and 237 but tried others such as

237 and 299, and decided upon 456 and 511 after a written subtraction algorithm showing the working for rearranging. Among the other difficulties that emerged for Ana were the inability to: apply the distributive law, estimate measurement and numeracy, use reverse operations for back-tracking, find a fraction between pairs of fractions and find alternative methods for long multiplication.

For the second interview at the end of the Teaching Program, Ana was often prepared to discuss in detail how and why she obtained answers. Many of the same questions still proved difficult for her but with some improvements in the processes of getting to the answers. She showed improvement in backtracking, estimating numeracy and measurement, and in the use of strategies for mental computations. Special difficulties still remained with computing mentally, relationship of big numbers and understanding common fractions. The developments in mental computations that did occur were that Ana more often used special strategies such as for multiplying by 99, and for subtracting 286 from 1000 for which she mentally subtracted the hundreds first, and in many other minor parts of computations for which she had previously used short written algorithms. She still had a problem finding the two closest numbers in a group of numbers greater than one hundred. For fractions it appeared that the inability to use or consider using equivalent fractions prevented her from completing the questions involving finding a fraction between fractions such as 2/5 and 3/5.

Even though these two sets of assessment highlighted problems for Ana, her journal entries appeared to illustrate that she had gained some deeper insights into numeracy during the Teaching Program. As well as mentioning concepts that she came to understand better herself, such as the meaning and effects of operations in mental strategies, she often discussed how the activities attempted in class would help the children to understand them. I wondered if this was partly because she came to understand the concepts better herself while doing the activities. For example, early in the Teaching Program we discussed personal histories of learning various sets of numbers and mental computations:

> "I feel that this activity not only puts kinds [of] numbers that I learnt in order but it helped me realise that without learning some numbers we won't really understand the concept of the others. And this is what I must always bear in mind [when] I am teaching maths in the future" (8/3).

> "Anyway today we learnt an easier way to add 9, 99, etc., …. This has allowed to me think and it'll do the same for the other students [her peers]. And with practice children can become mentally fit to calculate any sum" (6/4).

The two areas of difficulties, as described above, that Ana struggled with during the whole year were mental computation and decimals. She found that applying decimals to money values was helpful for operating on them and mentioned it a few times:

> "… we did some mental computations on division but then working with dollar signs. I found it easier than without the dollar sign. I think I will always use this [dollar sign] when I do division" (7/7).

> "I never really liked working with decimals before. But now it has become more interesting thus changing my point of view especially when changing decimals to money value[s]. This has helped me to work through it more easily" (19/7).

So she seemed to have come to a deeper understanding of decimals and would hopefully move beyond the need to depend on the use of the money context. As for the mental computations, not only did she consider her own recent development of mental computational skills but saw possibilities for using them with her pupils. Towards the end of the year when discussing the use of calculators Ana was talking about her pupils doing mental computations instead of using the calculator:

> "When they know they can't do the sum mentally then it [calculator] will be useful" (4/10).

In this section Ana's understanding of concepts have been discussed. There did not seem to be a major development in her understanding but there appeared to be a deepening of understanding of many ideas with the resulting ability to understand and do mental computations and to use and verify answers for standard algorithms. This discussion has, as one would expect, suggested also some of her understanding of the nature of mathematics, but I will consider this understanding specifically in the next section

Understanding the nature of mathematics

Much has already been described in the last section about Ana's ideas of rationalism in mathematics where her understanding of concepts was discussed, but I will give a further example which illustrates the importance she gave to logical reasoning in mathematics:

> "Today more importantly Ms F stressed the importance of … thinking questions in order to carry out the activity. He [fellow student] could have made the activity more active in the sense of getting the children more involved from the very beginning. The children would not have grasped the full understanding of the activity" (9/3).

Objectism in mathematics developed as Ana became comfortable with primary school activities in the mathematics classes and as she valued the use of materials to represent concepts and the application of mathematics in real life situations:

> "We continued using fractions by using discs. These are all strategies I would definitely use in the future. It allows free play and using models to express equations. I think the students would learn a lot because they are actually familiarising themselves with the materials and thus the maths concept" (10/5).

> "Maths is practical and many objects can be used to show things" (23/4).

Control as a value in mathematics had probably always been a value for Ana as indicated by her first set of responses to the questionnaire of *Beliefs about mathematics* in which she believed in such features as the importance of rules and procedures. Ana had succeeded in the past with the efficient use of algorithms. As described in the last section the importance of control seemed to lessen for her during the Teaching Program as she came to see the many possibilities for using her own procedures.

This new insight led her to see *progress* as an aspect of mathematics as she explored and produced ideas related to numeracy. A month into the Teaching Program she attempted to devise her own strategies rather than use algorithms:

> "This method of solving [using algebra] is a strategy that can be used and is very efficient but follows certain rules. … I would have to make up my own strategies and go through trial and error in order to form equations. This would

also allow me to reflect on the method and to think deeper of what is at hand. And so to make the method as simple as possible in order for the children to understand" (3/3).

She was open to development and change in mathematics and prepared to make an effort:

"I'm not really keen on the method for mental computation although I know it is very effective. Sometimes I can't get my brain working fast enough. I've got to try harder I know I can do it if I put a lots of effort into it" (10/5).

"Today we did something new. Well not exactly but a different way of doing multiplication. …. I've learnt that multiplication can be taught in many ways that would interest the students" (25/5).

"Maybe in the next few years the primary school classes will be using calculators. Calculators may very well help children with their work but they do not have to use it for every single problem" (4/10).

She appeared to come to know better, mathematics as knowledge that has an *openness* such that it could be verified in public. Before the Teaching Program she was unable to justify her answers to me in the interview but after it she readily explained her answers. She mentioned a few times about checking answers as in the discussion of algorithm test above and the following:

"Discussing the strategies on 'checking answers' has given me a better equipment in order to check answers with very little use of the calculators" (12/10).

Mathematics for Ana did not seem to be knowledge with much *mystery* attached to it. She did not often mention issues in mathematics teacher education that were new for her as much as other students did. This may have been because her mother was currently teaching in a primary school. One example where she did comment was in discussing MAB material.

"I've only heard about [MAB material], but only found out its meaning today and who founded it. It's the same [as straws in bundles of ten] or deals with base ten materials" (14/4).

In this section I have discussed to some degree the extent of Ana's knowledge as illustrated in her Thinkbook and the other data. Ana was one of the few pre-service teachers in the class who appeared to have a good grasp of the knowledge of the primary school mathematical concepts, skills and some aspects of the nature of mathematics. It is probably the combination of the understanding of concepts and of the nature of mathematics that has helped Ana to develop her number sense. Yet the analysis appears to indicate that there is still room for improvement to become an effective mathematics teacher in a primary school. Hopefully she will develop her knowledge and skills for computing mentally and deepen her understanding of rational numbers especially decimals and, with more reflection on her learning, as described in the next section, she will come to understand more fully the nature of mathematics.

Lifelong learning for the future teacher

In the two previous sections I have described, first Ana's enculturation into the pre-service teachers' classroom with its pleasures and problems for her, and second her knowledge of the concepts and the nature of mathematics. Within those sections I have already considered much about her knowledge of her own learning and her ideas about teaching mathematics. In this section some of those ideas will be referred to in different contexts. First I will discuss the metacognitive issues that emerged for Ana during the Teaching Program. Then I will consider her learning of pedagogical concept knowledge and teaching strategies.

Learning to learn

Ana appeared to have some good metacognitive skills as she was well able to reflect on her own learning in her writing with comments such as the following:

"We worked in pairs, which helped a lot. I prefer working in pairs. It allows me to discuss what I may not know or what I may know … with my partner" (12/4).

“Recalling what kind of numbers I first learnt was a bit difficult at first but I unfolded my memory and got out quite a few results” (8/3).

“Sometimes I can’t get my brain working fast enough. I’ve got to try harder. I know I can do it if I put lots of effort into it” (10/6).

“By doing all these work-outs [worksheet on fractions] it has helped me get through my test” (19/7).

In her Thinkbook entry after the discussion of gender issues in mathematics education Ana noted that

“… girls (not all) have to think of how a teacher did a particular sum and not to their own thinking” (1/7).

I wonder if the ‘not all’ was said because she did not include herself?

Although Ana commented on the difficulties she had writing about her learning of mathematics she came to realise how she benefited by the reflection entailed:

“When I actually flip through my maths notes to see what we had been doing a particular day, it [Thinkbook] helped me revise. I guess it also gave me a sense of deeper thought of what I was actually doing. It made me think about why and how I may encounter things in the future. Maybe that’s the whole point of why Ms Fran calls it a Think book” (10/11).

Another difficulty Ana thought she had was her inability to verbalise ideas. When discussing the myth that ‘using calculators in primary school is bad’ Ana noted:

“I had the idea [about myths about calculator use] in my mind but could not put it into words. This is one of my weaknesses that I observed during this particular part. I hope to improve” (5/10).

Ana not only said she hoped to improve but she made resolutions to develop her learning. In the preparation for the examination Ana outlined a her calculated study plan:

“I’ve got to write a study plan or revision plan. But I’m not sure whether or not I will be tested on Warm-ups. (I’ll have to ask her in the morning.) Anyway I’ve

> got to study at least 16 topics altogether. So therefore it is 4 topics per week" (19/5)

She noted mistakes and resolved to learn form them:

> "I did not really understand the 5th question in the paper and therefore got it wrong. But now the concept is clear after going over the paper with Ms Fran. Probably I did not spend enough time thinking of the questions of what I got wrong" (23/5).

> "Maybe I hope that if our new lecturer [next year] or any lecturer gives us a Thinkbook to write in that I'd do better and learn from my mistakes" (10/11).

From this discussion it can be seen that though Ana had some struggles with her learning she used her reflections to attempt to improve it. This included learning through reflecting on mistakes and through deeper thinking as a result of writing. In the next section where Ana's ideas of learning to teach are considered there are also many references to activities that Ana thought assisted the learning process.

Learning to teach

Ana often discussed ideas about teaching. In this section I will describe her consideration of strategies to help the students learning processes, their understanding of concepts and their enjoyment of mathematics. She appeared to take time to reflect on the strategies I used, their effectiveness and often was able to consider other uses for the methods:

> "We continued doing the division sums today with the use of the base ten materials. This gave me an idea of how I would use the same materials to work through divisions, multiplications and additions and subtraction sums" (29/4).

> "This has given me ideas in which I could [integrate] with other subjects" (9/3).

> "Today I learnt that to memorise numbers children or anyone picture a set of items in their minds and associates them to the total number. We form mental processes and this is known as conservation of numbers. I learnt that what we show children must be constant so that they'd memorise it [arrangements on dot

cards]. When they finally learn the concept then the order of shapes or ideas can be changed" (29/3).

> "Doing the activity [questions to assist investigation of new texts] has given me a better sense of looking for things/texts in future. It would not only apply to Maths but other subjects. Thus given one a better understanding of how to conduct my lessons" (20/7).

She often mentioned ideas that she considered would assist students to learn. These included cooperative group work, mentioned above and strategies for motivation to revise times tables:

> "Today I learnt more of how I could get my class to learn more effectively their times tables [strategies to generate understanding]. It has given me a new outlook on what I have already learnt in order to make my students understand, not only recognise their times tables" (21/5)

> "Ms Fran I definitely use this strategy [investigation of patterns in the multiples] in future for revision in class" (25/5).

This last comment was the only time in Ana's Thinkbook where she addressed me directly so she must have been impressed. She was also impressed with other issues such as the realisation that affective issues were to be considered when discussing strategies that could be used such as materials especially colourful ones, videos and games:

> "Maths lessons don't have to be so solemn but can be fun [played a game]" (19/4).

> "Ms Fran showed us how to make a bead frame. It must be very colourful in order to attract the children's attention" (15/9).

> "… watched Jurassic Park video. Just the thought that mathematics can be presented in all sorts of ways" (26/3).

> "Today we worked with dominoes. This was the first time I've ever handled dominoes and I found it fun working in groups playing the game so I'm sure the children would enjoy it very much" (7/7).

Ana often discussed the use of other activities working with fraction discs and fraction boards and how their use can lead to concept formation. She was able to imagine difficulties her pupils might have:

> "I realised today how the multiplication [sign] could be very confusing for a child because it could also be an incorrect sign [a cross for wrong work] 'x' " (4/10).

Her teaching ideas did not only apply to mathematical concepts but to pedagogical knowledge. During the Teaching Program she was focused on learning *how* to teach mathematics effectively and hoped she would become an effective teacher. She discussed the importance of using real life contexts rather than exercises from text books:

> "Maths - teachers should associate it [mathematics concepts] with the real world and not just in black and white print. I hope I don't become one of these teachers" (9/9).

> "So Ms Fran has given me some insight into how to explain multiplication better so that it wouldn't be confusing to the children and I hope that I do teach it well. I wouldn't want them to be confused that's for sure" (6/9).

I imagine Ana will be a very organised teacher as she commented on the lack of organisation of one of her peers and occasionally discussed small organisational matters:

> "But we must emphasise the fact that the calculator must be looked after very carefully for they are fragile" (4/10).

> "I never really had thought much of the digit formations I did, or how perfect it was" (18/2).

Even though these entries were written by a First Year pre-service teacher there are indications that this future teacher, Ana will be a lifelong learner. She will most likely reflect on her own learning and mathematics understanding, and her teaching methods as she has made a good beginning to do so. She sees the benefits of metacognitive thinking with its reflecting, discussing in groups and writing. She described many plans to apply the strategies used in class in her future teaching. Her reflections on how they were used in class are hopefully an indication that she considers it a necessary part of teaching in the long term.

Summary

Although the mathematics education class was a very new culture for Ana she did not avoid facing its challenges for her. On coming to the College she had recently experienced a schooling in which the transmission mode of teaching and instrumental learning were often the norm, but she adapted to the constructivist learning environment with little difficulty. She took more time than many other students to become enculturated into the primary teacher education aspect of it but she responded to the new learning environment by making an effort on her part to learn all she could about teaching mathematics. Her interest in learning to teach overcame her wish for the past learning of more challenging concept such as in algebra.

In the development of her number sense in her first semester of the course, which is under investigation in this study, some of the quantitative data showed she seemed to improve in the cognitive area. The qualitative data such as the interviews and writings in her Thinkbook indicated that she had deepened her understanding of estimations, the meanings and effects of the operations and hence Ana was better able to use mental computations. She continued to struggle with the understanding of some areas of rational numbers such as decimals, and the use of mental strategies sometimes.

The data from the later tests and responses to the questionnaire*,* also showed a change related to the non-cognitive area of number sense. This analysis indicated changes in her beliefs about mathematics. She came to believe in the place for originality in mathematics and that she could use her own methods for mathematics computations but she still held to the importance of memorising rules and procedures by students when doing their mathematics.

There was also a development in her understanding of the nature of mathematics. Although she believed that mathematics was about standard procedures and the understanding of them, Ana showed an interest in knowing more about them, developing her own strategies and ideas in mathematics and being able to explain her understanding. Thus her understanding of the control and rational aspects of mathematics were becoming more balanced by her understanding of the objective, progress, openness and mystery aspects of the nature of mathematics.

When Ana considered that she needed to change she did not avoid facing challenges. Some of these challenges for her included computing mentally, deepening her understanding of decimals and common fractions, and participating in and conducting activities such as Warm-ups. She learnt new strategies of learning such as thinking, talking and writing about mathematics. She was prepared to consider new methods of learning mathematics especially when she believed it would help her pupils understand. Ana applied these new strategies for teaching during presentations and teaching practice. Hopefully all these developments would allow here to accept more change in her own learning and understanding of mathematics, and establish a lifelong process of learning, and learning to teach, mathematics. With two more years at the Teachers' College she has the opportunity to continue the renewed constructing of knowledge that has begun during her First Year.

The learning experience for Vita

Vita

Vita is a Fijian national of the indigenous culture. She was in the age range of 17-19 years old and has learnt mathematics for each of her thirteen years of schooling. which was typical for a First Year student at the College. She grew up and attended school in the inland part of Fiji that was not always accessible in wet weather because of poor roads. Because of the remoteness of the school, she had limited access to communication in English outside of school and grew up with little experience of urban culture. Anecdotal evidence from discussion with teachers suggest that there were many good teachers in country areas of Fiji, but often thse inland schools were among the last to be staffed and were often left without qualified teachers, especially mathematics teachers. In class Vita was quiet but attentive. I noticed that outside class she had plenty to say with her friends but not using English, so perhaps she did not contribute in class because she felt her spoken English was inadequate. Yet when it came to writing, her Thinkbook entries were more numerous and longer than anyone else's in the class, though the grammar was not as good as most other students.

Among the six case studies, I considered Vita was one of the three with lower mathematics ability according to the entry data provided when she arrived at the College. The data gathered for this study appear to confirm this lack of ability or at the best average ability (see Table 6.11). Her results for the *Assessment of number sense* were, for the first assessment, slightly above average but below for the other two assessments. Her results each time for the algorithms test were less than class mean and this difference increased each time.

Vita's responses for the questionnaire *Beliefs about mathematics* showed generally a much more positive agreement with the constructivist view than for the mean average scores for each item compared to her peers (see Appendix S). Her responses changed little at the end of the Teaching Program, but were generally even more positive at the end of the year. These data will be discussed in more detail in the following sections.

Table 6.11

Quantitative data - Vita

Instrument		Number sense assessment			Test of algorithms		
Month		Feb	Jul	Oct	Feb	Jul	Oct
Score	Max	23	23	23	18	18	18
	Mean	8.4	12.4	12.6	10.4	12.2	13.1
	Vita	9	12	12	9	9	11

Enculturation into the mathematics education classroom

In coming to the College after a life distant from urban living, Vita settled into working in class quickly. Yet in the first few weeks she did express surprise at some of the new ideas she encountered at the College and she wondered why she had not heard about or seen them in her thirteen years of learning mathematics:

> "During my years at high school, I never come across a teacher who gives Warm-up activities at the beginning of each lesson. …. I find writing a maths essay very surprising since it was the first time for me to write an essay in maths" (18/2).

This comparing of the learning and teaching she experienced during her own school days when schooling was apparently difficult continued throughout the year:

> "Compared to my last 13 years of schooling these topics on decimals and fractions were some of the topics that I hated because I found it difficult. But now I am glad that I can now work with decimals and fractions without finding difficulties" (19/7).

> "If the teachers introduce the things all at once, students would have no idea of what they are learning and I have experienced that during my last thirteen years of schooling . When teachers put up charts and never explain them we really

don't know what it is all about. What I used to do is just read it and never understand what it means. And that really makes learning very difficult" (24/8).

As a result Vita wanted her teaching to be different in some respects from how she had been taught. From the start of the Teaching Program she was keen to learn about the profession of teaching:

> "I find teaching Warm-up[s] very interesting because it gives me an idea of what a maths class is going to be like" (23/2).
>
> "Sometimes it comes to a stage when I am tired and fed up of doing these things [producing teaching materials]. But then I realise that I have no choice but to make them and I can't sit there for the rest of the period doing nothing. So it really challenges me to work. It really gets me to know what a teacher's job is like" (21/4).

How Vita viewed mathematics itself when she arrived at the College is indicated by her responses to the questionnaire for *Beliefs about mathematics* (see Appendix S). As noted earlier there were indications that she was more of a constructivist than her peers. For the questionnaire administered before the Teaching Program her item responses indicated, a more constructivist view of mathematics compared to that of her peers. Her view was the same at the end of the Teaching Program and was much more constructivist at the close of the year. Perhaps the presumed lack of teaching meant she became more of an independent learner and hence developed a view of mathematics which was more constructivist. However later discussion suggests she was not as independent a worker in numeracy as might be supposed which calls into questions this hypothesis.

The responses for Vita that are notable as indicated by the headings and described here are given in Table 6.12. In this table her responses for nearly half the items throughout the year were strongly constructivist. These items were statements mostly about the broad view of the nature of mathematics. They also included items for which she strongly disagreed such as ideas related to mathematics involving much memorising of rules. Statements indicating a change to a more constructivist view by

Table 6.12

Notable responses to the questionnaire by Vita

	Feb	Jul	Oct
Strongly held constructivist view			
Understanding the process is more important than the correct answers in mathematics	SA [a]	SA	SA
In maths it is important that students are able to be flexible in their thinking.	SA	SA	SA
In maths it is important that students are able to do calculations without a calculator.	SA	SA	SA
In maths it is important that students are able to estimate.	SA	SA	SA
In maths it is important that students are able to see patterns and relationships.	SA	SA	SA
In maths it is important that students are able to show initiative and think creatively.	SA	SA	SA
*Learning maths involves mostly memorising rules.	SD	SD	SD
*In maths it is important that students are able to memorise rules.	SD	SD	SD
*To be successful in mathematics it is important to memorise rules.	SD	D	SD
Change to agreement with a constructivist view			
*There is always a rule to follow in doing a mathematics calculation.	SA	SA	D
*Doing maths means following standard procedures precisely.	SA	A	D
*In maths it is important that students remember facts.	SA	A	D

[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.
*These items are worded negatively towards a constructivist view.

the end of the year were items which she came to disagree with such as the need for rules, procedures and memorising, in doing mathematics. Half her other responses not included as notable indicated a constant but not strong agreement with the constructivist view, and the other half a lessening of disagreement with it.

As for her experience of a constructivist approach during the Teaching Program Vita found that new methods of learning, including cooperative group work and group presentations helped her and she readily agreed with ideas such as that people all think differently doing mathematics:

> "During this maths class we carried out discussion in our group work. Working in groups like this is also helpful especially when sharing ideas. You might learn something from others that you don't know" (25/8).

> "I wouldn't have known some of the things they talked about since we all think differently. And that is the advantage of working in groups [presentations]. We learn a lot of things" (30/8).

Vita also found some metacognitive skills such as drawing concept maps and writing in her Thinkbook helpful:

> "Also we drew a concept map on rational numbers. It was a good way for revision for me because as I try to draw I try to recall all the things that I learnt in decimals, fractions and percentages" (13/7).

> "Writing Thinkbook is like evaluating your own self about what you learn or think about an idea. It is really interesting and very useful since it really helps me to see how I am doing in each maths topic" (9/11).

Vita was not the type of person that complained, criticised or enthused about experiences to me so I may not have heard about issues that were a problem for her during the Teaching Program. Besides those already mentioned she did mention some activities she found difficult:

> "When reading the instructions given on the board it looked easy but when carrying out the activity it was a bit complicated at some stage specially when dealing with big numbers [greater than a hundred]" (21/4).

Perhaps because she and her fellow school friends all had problems related to learning in a second language in school she saw the learning of other languages as not much more than developing a practical skill. The cultural aspects of the lives of others was not a consideration for her at this stage of her education:

> "Also we learn to write figures in Hindustani. It was interesting indeed and at the same time it was difficult to write them correctly. But anyway it is good to

learn other languages because you don't know which school you will teach in" (2/8).

In summary Vita appeared to become enculturated easily. She did comment with surprise at the differences she noted between her past and present experiences wondering why she had not learnt this or that and noting that mathematics had sometimes been difficult to understand. She noticed new strategies helpful for her learning such as group work, and the metacognitive skills of creating concept maps and writing reflections in her Thinkbook. She was very keen to learn how to teach mathematics. However coming from a rural school environment to life in the mathematics education class in the city of Suva, Vita must have experienced a big adjustment. The only problems she commented on were about trying to understand some of the work that she would be required to teach.

Constructing knowledge about concepts and the nature of mathematics

This section examines first Vita's understanding of some mathematical concepts and then her understanding of the other aspects of the nature of mathematics.

Understanding of concepts

For the *Assessment of number sense* instrument Vita's scores (Table 6.11) were initially similar to the rest of the class and remained so after the Teaching Program and for the end of the year assessment. She gave correct answers to sixteen of the twenty-three questions over the three times with only six of them correct each time. She suggested a number of times in her Thinkbook that her understanding comes and goes and so this tentative hold on understanding may be the reason for the lack of consistency for correct answers. Wild guessing is another possibility, but my knowledge of Vita suggests that this was highly unlikely. The questions that she had correct in all tests were the questions about money and whole numbers which left those that were incorrect mainly involving rational numbers and estimations. After the Teaching Program the only extra question she correctly answered on the next two occasions was one involving understanding the effect of the operation of subtraction. When commenting in her journal on her performance in the assessment after the

Teaching Program she herself felt confident about having a deeper understanding of the questions as well as some concepts:

> "The reason why I said it was easy was that I understood most of the questions and was able to solve them. After doing this test I realise that I now understand my work on decimals, percentages and fractions" (19/7).

Her score does not indicate an improvement in choosing the correct answer, perhaps the assessment could not show the development in *understanding* of the questions that she said had occurred.

In the interview after the administering of the third assessment Vita was able to choose the correct answers to all the questions with the help of leading questions which involved simplifying of some of the wording of the questions. It appeared to me that Vita had some difficulty understanding the questions and also she was only prepared to think through one-step questions. This was more clearly illustrated in her responses to the activities and problems in the interview at the beginning and end of the Teaching Program. In the second interview Vita had sufficient knowledge of the numeracy facts to answer correctly any question that I asked her, but she could not apply her knowledge unless the linking of ideas were suggested to her. For example, she needed to be prompted to consider first what ¾. 0.4 and 60% were as common fractions, before being able to arrange them in order of magnitude.

Another area of working mathematically that Vita apparently found difficult initially was computing mentally. In the first interview Vita rarely used any mental computing and instead she wrote in detail all her workings including all the 'carrying' digits. For Question 19 (Appendix A) concerning the straws bundled for base 10 she showed the working for such computing as 100 x 10 = 1000 and the vertical addition of the sum of two 1000s and four 100s. In the interview after the Teaching Program she showed that she had begun to use some mental strategies because she quickly gave all the correct answers without the need for written work. Also her improvement was shown in Question 3, where, for a string of factors to multiply, she chose the 2 and 5, and the 4 and 25 first to simplify her calculations. She had commented on this strategy earlier in her Thinkbook:

> "What I usually do is just multiply the numbers in the order in which they appear, not knowing that there are some easy ways of multiplying numbers" (26/5).

It is not obvious what she means by saying "not knowing" strategies for working mentally. Her responses to the questionnaire suggested that she believed in showing initiative and flexibility and creativity in thinking but it seems that she did not always put her beliefs into practice. It appears that she had never been encouraged to work mentally nor did she consider it, but when we discussed strategies in class she was pleased to use them.

Another area of working mathematically that Vita felt insecure about was working with big numbers, that is for her, numbers greater than one hundred. For Question 9 of the interview activities (Appendix D) which required choosing the smallest difference between a pair of numbers from a group of numbers less than 1300 she gave the same incorrect answer each time. She used the pair of number with the same number of hundreds, 237 and 299, for the closest pair rather than the correct pair that bridged two hundred, 195 and 237. After a number of subtractions she was able to choose the correct pair.

Besides not understanding numbers greater than a hundred and her lack of having being introduced to mental computation skills, Vita believed another of her problems was in the lack of understanding of rational numbers. In the three different instruments she had difficulties with the questions involving rational numbers. For example, in the interview after the Teaching Program she had great difficulty with Question 15, arranging a set of rational numbers in order of magnitude. Of all the methods I suggested to her for arranging the rational numbers, the ones she best understood and used was the changing of the numbers into tenths when possible. Also visualising fractions of objects such as pieces of fruit helped her to distinguish the relative sizes of the fractions. Yet as mentioned above, she was able to answer simple question about any of the individual rational numbers. The one exception was after the Teaching Program when she continued to think that there were no fractions between 2/5 and 3/5, and she made no attempt to think about it when I suggested she might like to take time with her response. Interestingly her perceived increased understanding of equivalent fractions did not suggest to her to link her knowledge of them and use it in cases such as these.

During the Teaching Program Vita had been able to use her understanding of equivalent fractions to compose equations using fractions with different denominators, such as ½ = ¼ + 1/8 + 1/8 and she was amazed that she did it without using her learnt algorithm considering the lowest common denominator. She found it interesting working out these answers using a fraction chart:

> "We did activities on adding fractions and it was really interesting because we learnt a different way altogether as compared to the lowest common denominator method which we learnt while at primary schools. I found out the new method was just simple and we don't even have to do calculations. And as we go on I seem to just picture the fraction chart and try to work out things mentally which was easy for me. As compared to using the old method which usually takes me a while in order to finish a problem" (9/6).

From how she discusses the rewriting of a fraction as a string of other fractions here she appears to see the new process as yet another procedure to learn which replaces for her a more difficult one. But there is a suggestion of visualising the fractions as they are seen on the chart and so at least some thinking about the reasonableness of the answer is probably involved. Yet a firm understanding of the underlying concepts is not evident. She concluded this entry with her belief that she had come to a better understanding of equivalent fractions and then a few weeks later described her new understanding of percentages:

> "From this activity I was able to figure out that different fractions on different fraction charts [parts of the chart] can have the same value or are equivalent to other fractions" (9/6).

> "In this way I came to realise the different ways of writing percentages as before I never realised it. For example 7% would be the same as 7/100" (29/6).

Then after doing the number sense assessment at the end of the Teaching Program she believed she had improved:

> "After doing this test I realise that I now understand my work on decimals, percentages and fractions" (19/7).

This was written during a class prior to administration of the instrument for the study after the Teaching Program. For some reason her belief in an improvement in understanding was not substantiated by an improved outcome in the fraction questions in the July testing when she did not apply her new understanding. Maybe there was some element lacking in the testing that might not have been able to verify this perceived improvement. This may be also true for another topic, the operations on numbers, for which Vita stated a greater understanding but results did not show it. She appeared to understand the processes of multiplication and division because in her Thinkbook she described different methods that could be used:

> "In this class I was able to learn that divisions have three different meanings. As before I never realised that. And I found out that the three meanings are to cut up something into equal parts, to share something equally, and the third meaning is grouping equally" (23/4).

Then again it may be that in the limited time and the added stress of doing a test, Vita either chose to rely on her older trusted methods, or was not able to bring to mind efficiently the new methods she had been shown.

So far in this section I have described Vita's rational understanding of mathematics and lack of it in some areas. She described the many times in which she believed that her understanding had improved, but the data from the testing did not substantiate a considerable improvement. The interviewing did verify that there was some improvement in knowledge of and skill in using mental computations.

Understanding the nature of mathematics

In this subsection Vita's understanding of the nature of mathematics is discussed using the six categories Bishop (2001) using for mathematics values. Similar to determining Vita's understanding of the concepts of mathematics, finding evidence of understanding of the nature of mathematics is more difficult than seeing the lack of it. In this section I will attempt to illustrate the limited view she had of mathematics.

In the above subsection I commented on Vita's degree of understanding of the *relational* aspect of mathematics. It appears that in her past learning, understanding was not automatically included but she now wanted it to be:

> "I see that teaching these principles [ways of expressing multiplication] would really help them to learn maths very easily. Not only that, [but] they would also be able to understand what they learn" (13/9).

Vita comments discussed aspects of *objectism* as she planned to help her pupils work using manipulative materials. For the development of an understanding of an abstract idea Vita saw a need to use hands-on materials first:

> "This gives me an idea that children should be taught with concrete things. Things that they can see. So that when they learn numbers they will have a mental picture of that number" (30/3).

Vita appeared to understand that *control* was present in mathematics when one knew the rule and procedure to be applied in doing a computation. I will consider in some detail this aspect as I suggest that her view of mathematics is largely related to her need for the security of a rule or procedure when computing. But she was prepared to discontinue using a procedure if a more efficient one was to be found such in her adapting to the use of strategies for mental computations. I suspect that when Vita learnt some of these strategies, more instrumental understanding was involved than relational understanding. For example, in the interview after the Teaching Program she calculated 234 x 99 as 234 x 100 –1. Earlier in her Thinkbook she attributes her lack of knowledge of this strategy to not having been taught it:

> "It was interesting to me because I noticed some easy way of multiplying by 9, 99 with other numbers. In my thirteen years of attending primary and secondary schools I never heard of this because I was never taught about it" (28/5).

Another example where it was difficult to categorise her understanding was when Vita said that, for the relationship between numbers less than 100, using a number chart helped her with her additions and subtractions of nine:

> "I was able to use some of the rules we learnt previously and this is to add 10 first to the given numbers and then subtract 1" (20/4).

It appeared that she continued to learn rules blindly to help her determine the correct answers even though she used words such as “understanding” and “figuring it out”. At the end of the Teaching Program in her responses to the questionnaire she said she continued to believe that mathematics was about following rules and standard procedures and remembering facts. This is probably why she had difficulties such as the following:

> “At times I found it easy to distinguish between the two fractions as to which one is bigger. But at some stages when dealing with some particular fractions, it took me a while in trying to decide which one of the two was bigger. One method that I tend to use when carrying out this activity was the method that I learnt in primary school and that is ‘the bigger the denominator, the smaller the fraction’. I tend to use this method when we were given the worksheet. But when it comes to general discussion or time to argue with each other, I found out the method I used was not really true. Because when we argued and tried to figure things out, I learnt there were some fractions which were bigger than others [because they had large numerators] even though their denominators were bigger. So I must thank Ms F for teaching me this and I’ll make sure that I get rid of the method I use and use Ms Fran’s method” (15/6).

This entry illustrates that she can come to recognise that some ideas she hold are not correct. It also appears to indicate, even if only in the beginning stages that she is trying to rationalise the ideas about concepts that she has accepted in the past as true. It was not until the end of the year that she said she no longer believed that mathematics was about following rules and standard procedures and remembering facts.

Yet her knowledge of the standard algorithms was not very good nor did it improve much during the year with results of 9/18, 9/18, 11/18 in *the Test for the algorithms for the operations*. In most cases she used correctly the algorithms for common fractions and for whole numbers except for long division. The lost marks in the final test were because she was unable to correctly answer the questions involving decimals and the long division, not were the common fractions questions completed accurately. For the long division she did not make a reasonable attempt at the standard algorithm the first two times, and at the end of the year she only completed a few correct steps. A few days later after the second algorithm test, she acknowledged that she had difficulties doing operations on decimals and big numbers and promised herself to work hard:

"I still have more work in these areas I am weak in" (22/7).

There were a numerous occasions, some of which are mentioned in the paragraphs above that indicated that Vita was happy when she had a procedure for calculating something especially when if it simplified working. She was also satisfied if she had a definition to learn, knew the accurate conventional use of a symbol such as the decimal point, could check her answer by repeating a computation, and then practise what she tried to learn. These all seemed to indicate that being in *control* when doing mathematics was important to her.

Bishop (2001) commented that the complement of *control* in mathematics is *progress* and that in education these two need to be in balanced. In Vita's case control was not balanced by a desire to explore mathematics and put forth ideas. There were some ideas that Vita came to realise about mathematics but as she acknowledged these were a result of others' explanations rather than her own thinking:

> "Counting principles. It makes me realise that children don't just learn counting anyhow" (?/3).

> "Also we were asked to list down some of the things that turn [through angles]. I was able to list down a few things but when we came to discuss it, I then realised that there are many things around us that turn. Even when the teacher asked us to write down different ways of representing a quarter, I was able to write down only two ways, whereas there are about six different ways" (7/6).

> "We also talked about compass fractions and I was a bit surprised to hear or see that the mariner's compass is divided into sixteen equal parts. I've seen it before but never realised it was divided into 16 equal parts" (7/6).

Openness for mathematics with its expectation that answers can be justified publicly was seemingly not important to Vita. She had difficulty explaining to me why she obtained the answers she did in the interviews. Language problems may have been partly responsible for this. She did verify clearly on the last algorithm test her answers to the divisions by doing multiplications.

The last entries in her Thinkbook given above concerning her inability to think laterally, appear to illustrate that Vita had missed out on the discovery of some of the *mystery* of the mathematics in her environment. Perhaps with time these realisations may inspire her to seek to independently discover the wonders of mathematics.

There was only a limited improvement of the test scores in the testing after the Teaching Program and at the end of the year, yet Vita often commented on her improved understanding. It appears that she used the idea of understanding in the sense of instrumental understanding and therefore this may be the reason why her results did not show a development of number sense. Consequently her general understanding of the nature of mathematics also appeared to be restricted to that of a knowledge that relied on learning procedures and rules. In discussing Vita's understanding of the aspects of the nature of mathematics, I had difficulty in finding entries in her Thinkbook related to them. She appeared to have a narrow understanding of mathematics which was confined to a view of it as able to be controlled in the performance of algorithms, that the use of materials was beneficial for illustrating concepts, and to a small degree that it has a rational aspect. Other aspects, related to the progress, openness and mystery of mathematics were generally outside her thinking about mathematics at this stage.

Lifelong learning for the future teacher

In this section I will discuss the aspects of the Teaching Program that appeared to have an effect on Vita's own learning and learning to teach, and hopefully therefore, on her lifelong learning. In discussing aspects of her learning I will include her learning to learn via her metacognitive thinking and skills, and then I will discuss her learning to teach.

Learning to learn

For a student who apparently had not in the past knowingly used many skills for mathematics learning such as writing reflections to help her metacognitive ability, Vita acquired them quickly. She was keen to extend her knowledge and learn new ways of learning mathematics. Among other activities this is shown by her interest in improving her problem solving skills and her computing mentally:

> "It is very interesting because I get to know that each problem can have more than one method of solving it. I find that this [problem solving] can also apply to our daily lives" (4/3).

> "I try to learn it [mental computations] very well because it will help me to figure out things in my mind" (24/3).

On a number of occasions she used expressions such as 'I now realise' which usually followed some reflection on the work done in class. It appeared to indicate that she had learnt something at a deeper level than the rote learning of other knowledge. For example after we had corrected a test she was able to reflect on the causes of her mistakes:

> "I now realise that I am sometimes careless. Looking through my test paper I found out that I sometimes work fast and never check my work. I tend to rush through my work without reading the questions properly" (23/7).

And with some reflection on the mathematics around her:

> "One thing that was very surprising during this class was when the teacher asks the students to name some of the things that are in tens. From here I came to realise that there are many things that we have are in tens" (18/3).

> "It's interesting looking at the ways they [young children] carry out their measurement and that is using non-standard measures, comparing it to the way we adults do our measurement" (2/9).

> "….to multiply by five and two first and then multiply it with the third number in order to get the total sum [example 5 x 34 x 2]. I find it very surprising that I never realised that before" (26/5).

Among the strategies that Vita found helpful were the drawing of concept maps to revise topics, as mentioned above, and cooperative group work:

> "I think that this type of work [cooperative group work] is good. It helps us to relate ourselves to others. Especially when becoming a teacher because teachers must learn to work with a group of people" (25/8).

She believed that writing in her Thinkbook, as well as summarising the day's work , it helped her to learn:

> "In addition, when writing Thinkbook you're not only thinking about what you have learnt or what you have done different. Thoughts come into my mind and it also makes me think of some ways to improve my particular teaching ways. In other words how you can improve on what you have learnt" (9/11).

No matter how much she wanted to succeed she experienced a sense of failure sometimes. On a number of occasions she thought she understood what was discussed but then discovered she was unable to apply it when on her own:

> "For example a ½ of a 1/3 we had to complete the diagrams by shading. When I looked at the sheet that was given to me it looked quite easy but when I get to do it I found out it was a bit difficult" (15/7).

> "While Ms Fran was explaining it [times tables], it sounded a bit easy. But when I got to do the real work on circle pattern I found out that it was a bit hard. Sometimes I had to go very slowly and think of the multiples of each number" (25/5).

These and other entries indicate that as well as reflecting on the content of the work in class, Vita also reflected on her learning during the Teaching Program. She thought and wrote about what she learnt about her learning, what helped her to learn, and what did not help. Hopefully this is the initial stage of a life of reflecting on her learning and the forming of related good habits that may bear fruit for her lifelong learning including her development of number sense, and her learning to teach, which is discussed in the next section.

Learning to teach

Vita appeared to view learning to learn as only part of the important reason for why she came to the College which was to learn to teach. As discussed above Vita knew she lacked understanding of some concepts that she was required to teach, but she did not appear overly anxious about it. What she seemed more anxious about was learning to teach. She wanted to know the mathematics she needed to teach and how to teach it well:

> "And that is, I have to learn everything that I am going to teach and try to know it well" (23/2).

> "Preparing warm-up activity is very interesting because I get to learn what it is like to prepare an activity or preparing a lesson" (23//2).

> "I find listening to students doing their warm-up activity very interesting because I got to know what it is like to be a teacher. I also learn a lot from the students when they are up in front teaching their warm-up activity. The way I should teach, tone of voice and also I learnt that I should make expression and be steady while teaching" (4/3).

Vita resolved to apply new ideas and avoid what she saw as poor teaching. She wanted to ensure that her pupils understood what she taught them. She did not refer directly to using constructivist strategies. In fact she suggested traditional methods of 'lecture, example and practice':

> "This also gives me an idea that before making children learn anything by words I shall try to show them examples. So when they learn it they will have a rough idea of what it is like. And I think this will make their learning much easier" (30/3).

> "And I should be very careful when teaching maths and make sure that I go step by step in order for the children to understand maths" (13/9).

She mentioned in her Thinkbook many times the different methods we used in class that she might apply in her teaching because the children would understand the work and some of these are quoted above in other contexts. Among those she discussed were strategies to help children learn patterns, the meanings and effects of the four basic operations, and times tables. Other issues she noted in her journal were the use of materials, language such as learning some mathematics words of other languages that the children speak, and simplifying her own language in a mathematics class:

> "And that is not only teaching through words but more importantly through materials since children learn more easily through concrete things" (21/4).

> "I would make sure that the language I use would be simple and suits the level of the class I am teaching" (6/9).

Sometimes she discussed her concerns about teaching generally:

> "I find that if teachers are not careful of the way they teach, they can cause little people to feel confused and inadequate. Also teachers should realise that children are very different from adults and therefore should deal with reality in completely different manner" (10/9).

In summarising Vita's learning I would say that foremost she was very keen to learn how to become a good teacher. Very early in the Teaching Program she had developed the exercise of observing how others taught and she learnt from this. She reflected on the activities we did and how she would apply them in her teaching often noting how not to hinder or harm the children's learning. The knowledge gained will hopefully contribute to the continuation of her learning throughout her professional life as a teacher and to her personal life for as long as she lives.

Summary

I believe that if I had only read Vita's Thinkbook and did not know her or have the results of the other data I collected for her, I may have the picture of a very conscientious student who progressed well in her development of number sense during the Teaching Program. But a more complete picture of her is of a pre-service teacher who was very conscientious but who according to this analysis, showed only a little development of her number sense during the Teaching Program.

In some entries Vita described new understandings of concepts, such as addition of fractions. This seemed to indicate that her number sense had developed . But testing did not verify much development. To me this is a bit of a conundrum as I am undecided about whether or not I can say, in spite of the data from the testing, that there was a development of number sense for her. Perhaps she has made the initial steps and, with more use of her writing to reflect on her number sense and further use of other features of the Teaching Program, she may gradually improve her knowledge and be able to apply her perceived improved understanding in her numeracy.

Also there is some contradiction in Vita's responses to the questionnaire items as they often indicated a very constructivist approach to learning and teaching with a strong agreement with the importance of being able to do such things as to show initiative and think creatively. However she expected to gain her own knowledge through being taught. She believed that she learnt well as a result of the strategies new to her such as cooperative group work. Yet again when describing her future teaching strategies she included strategies of the traditional transmission mode of teaching. Perhaps the solution to these conundrums may lie in her understanding of the questionnaire items because her understanding of the items may not have been what I thought. For example, her use of "creative" in one entry referred to making a problem contextual, motivating and interesting rather than *creating* mathematics itself which was implied in the questionnaire. On the other hand this analysis may be interpreted to show that she had started to move to a constructivist view, but had not fully seen all the implications of such as a change as yet.

The data showed that Vita began the year with a poorly-developed number sense but with skills of doing some algorithms. These improved little during the year, but the interviews suggested some changes such as that she had come to use mental computations more often. Where there was little improvement was in her understanding of rational numbers, and big numbers which to her were numbers greater than one hundred. Although Vita knew the basic facts and other primary school facts about mathematics concepts, I found when interviewing her that she was not able to the link her knowledge in order to answer questions requiring more than a one-step process.

Vita was able to reflect on her past schooling and compare it to what she was experiencing at the College and decide that she would teach differently from how she was taught. It was this learning to teach that she saw as her urgent need rather than a need to improve her number sense. When she learnt new knowledge and skills for number sense she was generally surprised because she appeared to expect that she had learnt all there was to learn about mathematics at school. Perhaps she arrived at the College with the idea that she was only going to learn to teach and not learn any mathematics content knowledge.

Another area of her understanding that appeared limited was her understanding of the nature of mathematics. Vita appeared to believe that mathematics was knowledge that people were totally in control of, and hence answers could be obtained if one knew just what procedure to use. In what she wrote about her improved understanding of rational numbers and her skill at working mentally it is not easy to determine if her ‘understanding’ indicated relational or instrumental understanding. She appeared to believe that there was a possibility, and she showed an interest in, ‘coming to understand’ and to explore new strategies.

Although there may not have been much quantitative data to suggest apparent development in Vita’s number sense, I believe there was development in her metacognitive thinking which may eventually result in other developments in her knowledge such as her numeracy. Perhaps reflecting on her mathematical working will lead to more development of her number sense. She had a great desire to learn and her reflections indicated that she had learnt some aspects of how she learnt and in some cases used this knowledge to develop her numeracy knowledge. It appeared that she had never been encouraged to think independently and so it did not occur to her to examine what she was doing when computing. She expressed surprise at times that she had not thought of some strategies which suggests that she came to realise that to understand more clearly the meanings and effects of numbers and their operations she needed to reflect on what she was doing.

As regards learning how to teach mathematics Vita was even more enthusiastic but whether this will lead to development of number sense is not possible to predict. She used every opportunity to extend her knowledge of pedagogy by observing and reflecting on what was done in class and wrote about how she would apply what she had learnt in her own teaching. With her openness to learn from others, and if she allows her students to be independent learners, perhaps she may reflect on and learn from her pupils when she sees them construct their knowledge and develop their number sense.

I am wary that I may not have heard Vita’s full story including some anxieties and difficulties, because she may have been too afraid to admit them. She gave me the impression that she did not want to impose her problems on anyone. There may be facts that were not revealed that would throw more light on to the contradiction of her lack of development of number sense as indicated by the testing, and her descriptions of an improved understanding. At this stage she apparently does not appear to believe that she herself has the capacity to construct numeracy knowledge. But I wonder if, with time and more reflection, she might come to believe it and develop her numeracy, and hopefully as a consequence become a more effective teacher of mathematics.

Jo

Jo described himself as a Fijian of indigenous Fijian culture. His home region was Nadroga which is a rural area on the main island. His age is in the range 20 – 25 years and so he is older than most of the other students. He attended school for thirteen years in which time he studied mathematics each year. On the sport's field he was popular and greatly valued for his skill at rugby. The academic record provided in his application was such that he was classified as among the weaker mathematics student in the class. In class during the Teaching Program he was very quiet and inconspicuous, rarely answering and never asking questions. He often did not appear to be very interested or involved. Yet he wrote in his Thinkbook spasmodically, and wrote more than many of the other male students.

The quantitative data I collected for Jo from the administering of the research instruments is shown in Table 6.13. As can be seen from the scores for the assessment Jo is below average in the first and third assessments. He was one of the few whose score dropped for the October result and over the three assessments he only had five of the question correct every time time. For the first an d final tests for algorithms he gained a score well below the class average. His July test with a score above class average, showed incorrect working with a correct answer similar to one obtained from using a calculator.

Table 6.13

Quantitative data – Jo

Instrument		Number sense assessment			Test of algorithms		
Month		Feb	Jul	Oct	Feb	Jul	Oct
Score	Max	23	23	23	18	18	18
	Class mean	8.4	12.4	12.6	10.4	12.2	13.1
	Jo	7	14	12	8	13	10

The scores for his responses to *Beliefs about mathematics* (see Appendix U) show that initially he was generally as negative towards constructivist ideas as was the average student but he changed to be much more positive for the two later administrations of the questionnaire.

Enculturation into the mathematics education classroom

In this section I will describe features of the process of enculturation that Jo experienced as he began the first of the two mathematic education units. One might have expected Jo to become enculturated into the College and the mathematics classroom more quickly than other students as he had a brother who was a Third Year student at the College. But if his brother was an influence Jo did not refer to it. However he did appear to have expectations:

> "The teacher does not use the blackboard frequently" (18/2).

> "Well first of all I thought that writing in the Thinkbook was just a new step from secondary to tertiary institution at the beginning of the year. So therefore I did not take much interest in it at the beginning of the year" (7/11).

His expectations may have been raised because of the initial interesting activities:

> "Today's maths class was not really interesting as I had expected. We started up with a warm-up from Miss V. She mentioned something about the days in the week [which day is before …, after …,. between ...]. She asked 7 students to put the names [days of the week] in order. I really could not get what she was doing at the blackboard" (23/2).

But, like some other male students he may not have developed during the first weeks, an interest in primary school mathematics as illustrated by the previous quote.

Sometimes his Thinkbook entries illustrated that he did not understand or listen to the discussion that occurred in class. I remember we listed a number of objectives for doing Warm-ups but they did not include one that he wrote:

> "The warm-ups usually test how fast you think in solving a problem with a limited time given" (18/2).

Other remarks in the same entry suggested that the Warm-ups were motivating and it was this kind of action that he needed to hold his attention. This did not include group discussions:

> "In doing this it keeps my mind alert and to be aware of what is taking place in the classroom. …. We discussed it in groups. So today I expected to be doing more practical which is more lively but it turned [out] the other way" (18/2).

I found it difficult to come to know Jo's attitude to the work we did in class. Perhaps the interest and excitement of his experiences on the rugby field were such that reflection rather than action in the mathematics classroom were seen as boring and not worth becoming involved in. He did note on a number of occasions that he found some experiences satisfying:

> "Ms Fran gave out our essays and I was a bit satisfied with my mark" (23/2).

And some months later Jo noted:

> "Anyway now I am confident about fraction and decimal problems, that I've got easier methods of solving it [applying decimals to money-value context]" (17/7).

I noticed that the young men in this class did not often describe getting as excited about doing the activities as the ladies did. Perhaps young men saw the overt enjoyment of learning as a feminine practice. Also Jo did not use expressive language in his writing that can be found in all other case studies, possibly because he was quiet and unassertive. However there were some glimpses of his enjoyment when he commented once in the first month on how interesting activities were and on two other occasions a few months into the semester:

> "He [his peer] wrote numbers that give up the sum of 19. It was really interesting. I had to find different factors [terms] of those numbers [of 19]. … This was one of the interesting exercises that I did today [Finding the number of chicks and pups when given the number of heads and legs]" (24/2).

> "The class was very exciting as we used various methods to find the multiples of a number 7" (20/5).

"However to me I find all the methods [learning times tables] very exciting and I'm thinking of applying some of them to my students" (25/5).

"I think little children would be so interested in doing this method [finding patterns from multiples, RIME #8. in Lowe &, Lovitt, 1984] because they love drawing and it would be really exciting to them once they know what to do. I think students won't find many difficulties because they just have to count around the clock face to get answers" (25/5).

From entries in Jo's Thinkbook I was able to understand some of the difficulties he had. In the above quote where he commented on his lack of interest in his peer's Warm-up presentation he partly excused his inattentiveness to his position in the classroom:

"I was sitting right at the back of the classroom" (23/2).

A fellow student made much the same comment about the difficulties that he felt were the consequences of sitting in the back seats. These comments suggests that students expect it to be difficult to participate when sitting in the back of a large class, which is understandable. To contribute orally from the back of a row of eight desks students required assertiveness and enthusiasm, which Jo appeared to lack. The students' positions in the room were organised by the Year Level Coordissssnator but changed a number of times during the year with mostly smaller students given front seats. Jo also mentioned later the effort and commitment necessary to do the course and his difficulty in maintaining these:

"Learning maths is dependent on how the students have listened, organised and commit themselves to it" (1/7).

"However it is really a very hard task in trying to arrange the straws in bundles [illustrating the structure of a thousand in base ten]; so it needs sacrifice and commitment" (9/4).

"As the year proceeded Ms Fran kept reminding the class the main things to be done in the Thinkbook but I didn't take any notice about it" (10/11).

He also noted times he found some topics hard and the extra effort that was needed:

> "We were also taught of how to use diagrams [when using MAB material] in subtracting. I got a bit confused over this because we have to draw only one diagram and do the working from there. However I managed to catch up with the class after a few practices" (20/4).

> "But the only thing that I'm a bit slow in is converting from fractions to decimals or decimals to percentages. So therefore this tells me that I need to take notes in class and go over them thoroughly" (17/7).

> "Today I received back my test paper and found my mistakes and weaknesses. But for some questions it shows my carelessness by not reading the questions carefully and even trying to understand it" (23/7).

As a constructivist teacher I attempted to understand his learning, but I experienced difficulties in getting to know him because of his lack of communication. Only a few times did he address me directly in his Thinkbook and that was twice during one week early in the year and, I suspect, addressed me indirectly in his last entry:

> "Ms Fran I found this topic very important because we will be counting objects most of the time" (24/3).

> "Ms Fran, this topic on addition with and without bridging really helps me a lot" (30/3).

> "Sometimes I felt that I had done enough for a day's entry. However I still did not get good marks for it. Maybe I wasn't doing the right thing. Perhaps I sometimes feel maybe Ms Fran does not like me so she usually gives me low marks in what ever I do" (7/11).

The last sentence seems to imply that he felt that his low marks in assessment tasks were because I did not like him rather than for not fulfilling the requirements and the poor standard of his work. In Papua New Guinea this same cultural attitude has also been noted with university level students (Clarkson, 1984). However this comment might also imply that the written requirements were not explanatory enough for him or, as was often the case the students relied on each others' memory of the assessment requirements rather than reading them themselves. Yet besides having difficulties communicating with me he also had similar problem with his peers:

> "I was not brave enough to ask any of my friends what was supposed to be done in the Thinkbook" (7/11).

Jo did not appear to feel at ease in the classroom. I also suspect he was not at ease in doing his academic work outside class such as study at night and therefore did not discuss study with others. He was the only young man in the class who responded to the work in the shy and nervous way that he did. It did not appear to be a cultural family trait as his brother was very responsible and interested in his studies in his first year at the College. I wonder if there was some pressure on him to follow his brother's footsteps to come to do the teaching course.

It is interesting to note that in some of his Thinkbook entries Jo used terms 'we were introduced to', 'We were told' and 'We were given' which give me the impression that I was imposing on him in some way. He does not use these phrases often but in the other students' entries I cannot remember getting the same impression from reading them. They usually identified themselves as being more part of the process using terms, 'we did', 'I tried' and 'It was interesting'. I wonder if this turn of phrase is another sign that Jo did not feel part of the process of learning that was occurring.

As well as being interested in his enculturation into the classroom environment I was interested in how Jo's view of mathematics, related to the constructivist view, may have changed during the Teaching Program. To help me investigate this I examined his responses to questionnaire for *Beliefs about mathematics* (see Appendix U). Before the Teaching Program Jo's responses to the questionnaire for *Beliefs about mathematics* were generally as positive towards a constructivist view of mathematics as the average class member. His responses became much more positive for the next set of responses and remained positive at the end of the year. Jo's responses that showed a shift of at least two categories are given in Table 6.14.

These are notable in a number of ways. They include first, items for which he always indicated a strong agreement with constructivist ideas and second, major shifts in his beliefs to a more constructivist view. These notable responses suggest that Jo believed in the importance of a relational understanding of mathematics, of using patterns, of finding solutions in various ways. Although he always disagreed that learning mathematics involved following rules and procedures he initially agreed with similar items, but then he came to generally disagree strongly with them. Because of his poor ability in English the initial responses may have been because of a lack of understanding of the item in the context of mathematics.

In summary then, this lack of ability in the use of English was perhaps also one of the reasons why he did not write much in his Thinkbook. Jo found some parts of the work satisfying and interesting but from my overall picture of him in class experiencing difficulties was more prominent. I suggest that his difficulties can be condensed into the two critical ones; lack of understanding of some important concepts of numeracy,

Table 6.14

Notable responses to Questionnaire by Jo

	Feb	Jul	Oct
Always strong constructivist view			
Understanding the answer is more important the correct answers in mathematics	SA [a]	SA	SA
A maths calculation can be solved in different ways.	SA	SA	A
In maths it is important that students are able to see patterns and relationships.	SA	SA	A
Major shift towards a constructivist view			
In maths, questions can be answered without using rules.	D	A	SA
*There is always a rule to follow in doing a mathematics calculation.	SA	SA	D
*In maths it is important that students remember facts.	SA	D	D
*In maths it is important that students are able to memorise rules.	A	SD	SD

[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.
*Item is worded negatively

and his poor communication skills. The combinations of these two issues probably made it difficult for him to lessen even one of them. Besides these, he showed less enthusiasm about becoming a teacher than the other students which might have helped explain why he did not always participate well. Yet there was some interest in teaching mathematics as he occasionally described how he would apply what we did in class in his teaching .

Constructing knowledge about concepts and the nature of mathematics

In this section I will discuss Jo's development of understanding of mathematical concepts and of the nature of mathematics.

Understanding of concepts

As mentioned above Jo came to the College with a reputation of weaker numeracy skills than many of his peers. This reputation was confirmed during the year by his assessment results (see Table 6.13). For the assessment administered before the first week of classes he scored 7/23 which was below the class average, and immediately after the Teaching Program 14/23 which was above the class average. However by the end of the year he scored 12/23, a little below the class average. In the second test he correctly answered five items which he did not get correct months later, which may indicate a deepening understanding of these concepts did not occur during the Teaching Program. Jo correctly answered only five of twenty-three questions on all three tests. This suggests quite a restricted area of the numeracy that Jo understood.

Although the students were encouraged to complete the assessment mentally, faint workings could be seen on the side of Jo's answer sheet. Included in the workings at the end of the year were: 10 x 10 = 100 and a vertical addition of 4.15 + 4.15 = 8.30, 365 days = 1 yr, 426/1 x 10/100 = 42.6. This working appears to indicate a lack of practice or ability at working mentally and quite primitive processes for written notes.

Interestingly he rarely mentioned mental computations in his Thinkbook whereas it was a major topic in many other students' journals. One mention he did make of it was in a discussion about bridging the multiples of ten when adding and subtracting. I do not have any explanation for this but his style of writing for this entry is out of character as he addressed me and expressed excitement on his part. Perhaps he felt part of the class and participated well that day:

> "Ms Fran … In doing this [consideration of bridging the multiples of ten] children won't have to spend much time in calculating the result when they will

only have to add or subtract from the bridge (multiples of 10) in order to get their answer. Wow ! It is really interesting" (30/3).

This could have been one of the few times when there was evidence that Jo really understood an aspect of the deeper structure of mathematics. I wonder if there may have been a process that could have enabled this participation to develop instead of him retreating back into his reserved self?

Another mention of mental computations were remarks related to multiplication by ninety-nine. Also he did use his own mental strategies rather than the algorithms later in the Teaching Program. In the interview after the Teaching Program in subtracting 714 from 1000 he subtracted 300 first then 14 to get 286 and also in calculating the change for $10 he subtracted the dollars first and then the cents. This appeared to indicate that he was using strategies he developed himself, as they were different to school taught algorithms which he used once in the interview before the Teaching Program for the same questions.

At the end of the year in an interview Jo was able to choose the correct alternatives to all questions in the *Assessment of number sense* when he had the help of many leading questions. He appeared unable to apply his knowledge by making links between aspects of his numeracy knowledge to answer the questions without prompting, even though he had considerable knowledge of facts both of whole and rational numbers.

In his first year of the course I did not expect to see great changes in Jo's number sense but there were some indication of apparent insights. Early in the year he commented on the structure of the number system with its base ten. At other times he remarked on being able to understand easily how to compute using big numbers and use patterns to help understanding:

> "Well today I have learnt a few other things. I don't think it is new to me. This is by bridging numbers in addition. This bridge refers to the numbers that link one number to another, i.e., multiples of ten or multiples of 100. Today I came to realise that each number has a role to play e.g. multiples of 10 act as bridges. I was very pleased as I started the exercise" (24/3).

"I think one of the most important things students must do is to try and recognise any patterns in the digits. So that when finding the multiples we can just use the patterns which could be easier for some students" (20/5).

"The most interesting thing is that I [in the past] have just learnt how to multiply small numbers by learning the times tables from 1 – 12. But today I feel I taken another step forward because I was able to understand how to multiply big numbers like tens, hundreds and thousands [multiplying by 9 and 99]" (28/5).

He used the strategies of multiplying by 99 and using patterns correctly in the second interview. Another example of improvement in mentally computing was demonstrated in a strategy for long division he used in his test of the algorithms on the paper at the end of the year. He used a form of repeated subtraction in which he took a step to improve the efficiency of the process (see Figure 6.1) he was using and hence he showed a clear understanding of division.

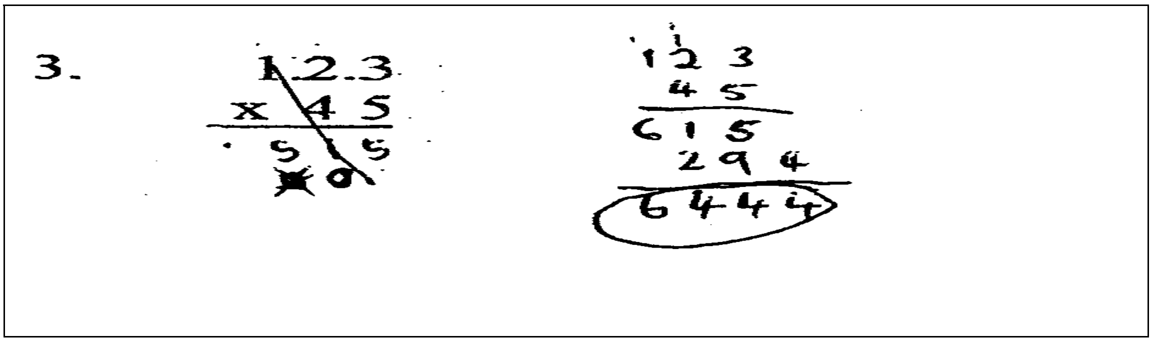

Figure 6.1 Jo's long multiplication method in the algorithm tests.

But there was no improvement in his skill at doing the algorithm for long multiplication. In the three tests his working showed a poor but consistent attempt to use the standard algorithm for long multiplication giving the same unreasonable answer each time. Also there was considerable lack of development in some other areas, such as numerical estimations. An interesting lack of comprehension of strategies investigated in class emerged in the July interview when estimating the number of lollies in the jar for Question 23. Initially he estimated 100 but then, with a few leading questions I encouraged him to look closely at the jar from different sides. He counted

22 in the base and 12 up the side and multiplied these on paper to get 264. Yet he still insisted on his original answer of 100, but was unable to justify his estimate. In the interview in February he had given the estimate of 300 without any explanation.

Fractions was a topic he discussed often in his Thinkbook. He had difficulties in understanding some aspects of the topic. Also in these discussions his use of mathematical words and terms left much to be desired probably because he was having difficulty coping with English as his second language:

> "Fraction is a word that mostly deals with dividing things into equal parts. I think the most important thing in fractions is to know how to divide things into small equal parts of how they might be able to fit into each other e.g., ½ = 2/4 = 4/8 = 8/16 = etc." (7/6).
>
> ¾= 6/8 = 4/6 [?] = 9/12   1/3 = 2/6 = 4/12 = 3/8[?]" (9/6).

From my knowledge of him after the interviews at the end of the year I believe he would be able to see the mistakes in these expressions. Because he found the fraction chart helpful he came to understand equivalent fractions better:

> "In addition, fractions can be added or subtracted by just simplifying [using] equivalent fractions to the simplest one. So when solving fractions problems children can just refer to the fraction chart but after some time children can just calculate equivalent fractions in their heads" (9/6).

Yet he handled common fractions better than decimals as in the algorithm test he changed 1.2 x 0.02 into 12/10 x 2/100 and proceeded to obtained the correct answer. As with many other students Jo was not as confident about percentages as he was with common fractions:

> "Firstly percentages are regarded as numbers upon hundred (100%) This was a bit confusing to me because I used to be slow in counting or dealing with percentages. Today I learnt a lot from the examples and activity given by the lecturer like for instance. 100% = 1 , ½ = ½ of 100 = 50 %. ¼ = ¼ of 100 = 25 %" (13/7).

It appears that he did come to a better understanding of rational numbers. In the interview after the Teaching Program he was easily able to arrange a list of fractions in order of magnitude, whereas he was very muddled about this at the beginning of the year. Jo believed there were no fractions between 2/5 and 3/5, and between 1.52 and 1.53 until he was much prompted. These were common difficulties in this class.

From his Thinkbook entries I noted situations where Jo applied reasoning skills to determine why I used certain activities. He was the only class member who reflected on a game I used. It was played in groups to introduce rearranging of place values when subtracting:

> "After the class I discovered that this could be the easiest way to teach children about subtracting larger numbers as this game would mean a lot to them where they are motivated, about borrowing from other numbers when subtracting larger numbers" (19/4).

Also he came to see the relationship between the numbers the MAB materials represented and their *volumes* which I did not mention specifically:

> "Simple, using the number of cubes in a long, 10, the number of longs in a flat, 10 longs, 100 cubes, and the number of flats in a block, 10 flats, 100 longs, 1000 cubes. This would easily enable the children to understand the pattern and also make the thinking by calculating the volumes for each" (12/4).

The discussion in this section describes all the major ideas about Jo's development of the cognitive aspects of number sense I could see from the data. There is an indication of some, but not a major, development in computing mentally. Jo had difficulty understanding rational numbers, but there was some improvement during the Teaching Program. He appeared to be satisfied with his understanding of equivalent fractions, but was still unable to apply this knowledge in finding fractions between a pair of fractions. He was still confused by percentages, but he was able to operate on decimals if he changed them to money values to visualise the operations. I would like to believe in Jo's ability to eventually understand the concepts he is still struggling with if he takes time to reflect on activities done in class. The insights he described, such as my rationale for doing some of the activities, seemed to indicate that he was using his ability to reason when he reflected on some of the work we did in class.

Understanding the nature of mathematics

In this second part of his development of understanding of mathematics I will describe Jo's understanding of the nature of mathematics using the six categories related to the values described by Bishop (2001). The first category is related to the value of *rationalism* in mathematics. This is already commented on in the previous subsection where I have discussed his struggle and desire to understand the various concepts. Also his desire to help his future pupils to understand illustrates his realisation of the importance of rational understanding of what is taught. In his discussion of using counters, subtraction using MAB material and the fraction chart for equivalence he hoped the children he taught would develop understanding as he showed in the following three quotes:

> "Today we used treasure items [any objects suitable for use as counters] and some other concrete ways, so that children would be able to grab hold of it rather than just knowing the numbers sentence by words. II II II for 3 x 2 = 6" (10/9).

> "Where we use [rearrange] cubes to represent longs when subtracting numbers. … this will make children to think" (21/4).

> "Therefore the doing of a fraction chart would be helpful for the children as they would recognise and understand equivalent fractions" (10/6).

The second value, *objectism* is related to the nature of mathematics that Jo often described through the use of manipulative materials and diagrams to represent ideas of number and the operations on number. It was often the stepping stone for his understanding of a topic, or he suggested it would be so for children. He often noted the presence and benefits of using manipulative materials in a number of other ways:

> "In today's class we were introduced to a new activity. As lower class teachers we should physically show the children the actual objects or any items demonstrating the activity to the children. In this case we used Dienes' blocks for subtracting. We used cubes, longs, flats, as counting objects" (21/4).

> "I cannot really work out the exact answer but by using a diagram I was able to get the answer. So this reveals to me that in doing mathematics that it does not only include writing down figures. Diagrams make work easier" (24/2).

Jo had a near-blind faith in the value of *control* when it came to the use of the standard algorithms. As mentioned above in his responses to the *Beliefs about mathematics,* he originally believed in the importance of rules to be memorised, although this changed by the end of the Teaching Program. Jo had a good memory for how he performed the algorithms for the basic operation on whole and rational numbers. He described his second *Test of algorithms for the operations*:

> "At first I was worried that I wouldn't be able to solve many problems on fractions and decimals. As the test proceeded I began to recall the methods I was taught in class" (17/7).

But his understanding appeared to be an instrumental understanding. This was clearly illustrated by his unusual incorrect method of multiplication which he repeated four times in the data I collected on four separate occasions. His working is shown in Figure 6.1. In the second line of his multiplication where he attempted to multiply 123 by the 4 tens, he multiplied the digits by 4 in the reverse order from the usual and placed the numbers going from left to right to give an answer with the second line of 2940 extending to the right beyond the units column. When I interviewed him he showed how he could correctly multiply by the units and then separately could correctly multiply by 40. He appeared to have developed the method of doing it incorrectly and had never questioned the unreasonable answers he obtained.

I wonder how many ideas we discussed in class he never questioned but just summarised as facts to be memorised such as the following:

> "We were introduced to multiplication and division. These two operations mustn't be separated" (10/9).

Another algorithm he had not learnt correctly which was taught in primary schools in Fiji was the one for long division. On the paper at the end of the year he showed a clear understanding of division by using his own method (see Figure 6.2).

Jo performed correctly the long division using a contracted repeated subtraction in which the steps he chose show an understanding of the process, but then he miscopied his answer. As mentioned before, we had used a similar method for division in class.

I suspect that Jo believed that mathematics *progresses* as demonstrated by his attitude to calculators and the many possible methods to do a calculation:

> "Calculators are a very important device. As time moves on technology also improves therefore sometime in the future young children need to be trained at an early stage" (15/9).

When discussing multiplying by 9 and 99 he was pleased to be using a method new to him:

> "We used a new system which was really exciting to us because I feel I have just learnt part of it during my time in primary and secondary school" (28/5).

Jo appeared to lack an understanding of the *openness* of mathematics as a considerable part of his numeracy skills included mechanical use of algorithms. As well he initially he was unable to explain his working or answers in the interviews. Yet as the year progressed Jo appeared to appreciate to some degree at least that mathematical calculations were open for verification and that concept and answers could be explained. He did explain some of working for his answers in the interview and in his Thinkbook:

> "So when solving fraction problems they could just simplify the equivalent fractions in their minds rather than using the long method. The main idea of doing this is to let the children recognise and understand equivalent fractions" (10/6).

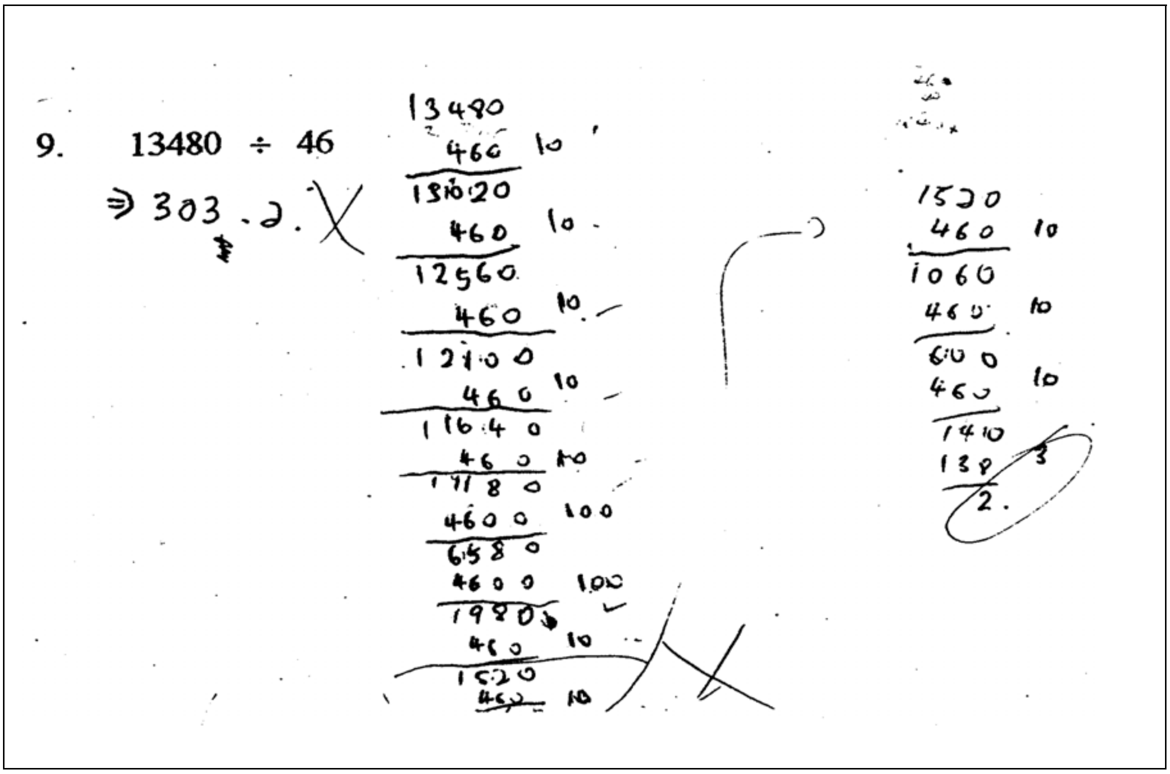

Figure 6.2 Jo's method for long division in October.

Jo did not mention directly aspects of *mystery* and discoveries related to mathematics but what did grab his imagination were teaching materials including MAB materials and the bead frame. He saw possibilities for their use in teaching:

> "Today the maths class was really enjoyable because we were introduced to a new topic. This topic is called Multi-base arithmetic blocks which was founded by a mathematics professor Dr. Zoltan Dienes. We were told how a long is formed from cubes. How a flat is formed form longs and how a block is formed using flats" (12/4).

> "Today we were introduced to another new item. We were given a framework with 10 bars on it containing 10 beads each. This instrument is used for counting by tens (multiples of 10). The instrument is a very useful one which can also be used in addition, e.g., crossing bridges [bridging] and counting to 100" (15/9).

I suggest that by the end of the Teaching Program Jo understood that mathematics was rational and so reasoning could be used, that mathematics could be seen as objective allowing for the use of materials to illustrate ideas. He always saw that with the use of rules and procedures, problems could be solved yet by the end of the year he knew they could also be solved using newly created methods. That mathematics as able to be explained and verified was not as familiar to him as one might wish as he did not check the reasonableness of many of his answers.

Lifelong learning for the future teacher

This study is concerned with the learning of number sense that occurred during the Teaching Program. I was hopeful and expectant that more than a greater knowledge of number sense would emerge. For a lasting benefit of Jo's learning I hoped he would also learn *how to learn* so that he would be equipped to maintain a practice of lifelong learning in the areas of mathematics and pedagogy. In this section I will discuss any evidence of development of metacognitive thinking or skills from the data related to Jo. Also I will discuss what Jo learnt, practiced or planned to apply in his teaching especially in relation to mathematics teaching.

Learning to learn

There is not much direct evidence of Jo's metacognitive thinking. He did not appear to worry about his low level of numeracy and therefore he did not seek strategies to improve it. He did mention low marks in tests, but did not express anxiety about his level of knowledge in mathematics or any frustration in trying to understand concepts. As mentioned above, once he attributed his poor marks to my dislike of him, although at other times he blamed his lack of application. Perhaps one thing that prevented him from knowing about his low standard of numeracy compared to his peers was his lack of examining the unreasonableness of some of his answers and the reasons for getting incorrect answers to his computations.

Rather than reflections on his learning that was encouraged for the Thinkbook writing, many of Jo's entries were a summary of work done in class. He did occasionally comment on some aspects of his learning processes. These included his performance in some tests and his difficulty in disciplining himself to do his study.

Under the entry headed 'Reflection on Test' Jo considered that more revision would help him to remember work done in class:

> "The test gave me a clear meaning of where I am standing at the moment. This shows I need to do more revision on the activities done in class so that it won't consume time by trying to recall my memories about what was being done in class. I should revise my work thoroughly" (17/7).

Jo did try to improve his poor study habits which were possibly a legacy of his years at secondary school. When the first examinations were approaching I encouraged the class to devise a study plan:

> "I can start completing my homework straight after dinner so that I can have more time to study maths and other subjects. I am also thinking of using these coming weekends for revision instead of mixing around with my friends" (19/5).

But his summary of his performance at the end of the year illustrates that he thought he had not studied as required nor made good use of his time:

> "Anyway I have come to realise that this was through my own fault. I did not put much effort into keep[ing] my Thinkbook up-to-date. This was mainly due to my mismanagement of time during the week-ends which I spent in drinking grog [the traditional drink of kava], watching films and relaxing in the hostel" (10/11).

A few other comments describe some of his learning methods such as using his own methods, the need for taking his own notes in class, and the use of patterns:

> "If I know the problem very well I just make up my own methods to get the answer" (4/3).

> "Our maths lecturer [Ms Fran] informed us about making our own notes without depending too much on her" (1/7).

> "Today we learnt another method of learning the times tables by the circle patterns. This was really exciting as we drew lines inside the circle and later we discovered some patterns in them" (25/5).

Something Jo did acknowledge was that the use of teaching materials helped him understand concepts. He described how they helped him and he often resolved to use them in teaching because he has not had them in his learning at school:

> "Today I was very pleased with our new activity in using straws [bundled in base ten system] for counting. I think this would be very helpful to me and even children while counting because they are counting the actual figures in hundreds, tens and in units" (6/4).

> "So I think the students would be more interested in learning the times tables if only they are given the various methods like we learnt today" (20/5).

From the above discussion the metacognitive strategies that Jo thought would help him to learn were making a study plan, taking notes during class, using his own strategies, recognising patterns and using teaching materials. There are many metacognitive skills that other students discussed which Jo did not mention. These include strategies such as cooperative group work, study and revision strategies, reflecting on their knowing by writing in the Thinkbook and by examining why they found learning easy sometimes or difficult at other times.

Learning to teach

In this section I will discuss Jo's ideas on how to teach mathematics. In what follows he is possibly also describing indirectly how he thought he would help his pupils to learn. For the first month there were few comments related to teaching but later there were times when he looked forward in anticipation to what and how he would teach in the future. He appeared to appreciate the difficulties his students would have understanding the concepts and was keen to help them:

> "This time next year I will be using this book [school text] in doing my lesson plan" (19/7).

> "I also realised that in teaching small children, it would be much better for them to calculate or add numbers without bridges, e.g. 3+1, 5+3, 7+2 On the other hand numbers with bridges would be hard for them to add or calculate" (24/3).

> "So to us teachers I think we have got a wide range of methods [examining patterns in times tables] to teach the children and also to learn from. For teaching I think we would be able to choose whichever method is easier to apply and for the children to understand" (25/5).

In his Thinkbook there were a number of other reflections about applying what was completed in class in his future teaching. As already mentioned Jo believed that teaching materials were very helpful for learning:

> "I reckon it would be much easier for children to count objects that are in groups. For example, a group which contains four objects, children would see that as two groups of two, or one group of three and one extra to it to make four. So therefore children would easily recognise numbers when they are arranged or put together in a group" (24/3).

As mentioned above Jo believed that he and his pupils could be helped to understand concepts with the use of teaching materials:

> "We also found the importance of using concrete objects to the students. For addition and subtraction children would be greatly assisted by the use of pictures and diagrams. Therefore I recalled my early years in primary school. I don't think I have covered all these things [use of materials, pictures and diagrams] that we are doing at the moment. So now I will try to apply it to the children whom I'm going to teach" (6/9).

> "From my view children cannot learn much from just reading and listening. Children have to actually see, touch and feel things in order to gain knowledge and experience" (9/9).

This discussion considered what Jo had written about learning to teach. He seemed to appreciate the difficulties children might have and he wished to help them. This appreciation of their difficulties appears to have been the result of his coming to better understand concepts himself. One strategy that he especially mentioned to assist understanding was the use of manipulative materials. However it will also be noted that the emphasis on this theme of learning to teach found in the data related to Jo was far more scarce than for the other students. This may indicate that there were reasons or pressures for why he was doing the primary teaching course other than a commitment to the teaching vocation.

Summary

This study aimed to devise strategies to help pre-service teachers, such as Jo who came to the College with a reputation of having poor numeracy, to develop their numeracy during the implementation of a mathematics education unit. After the Teaching Program there was some suggestion of his development of number sense. Even though Jo did not openly acknowledge a need to improve his sense of number, some of the Thinkbook entries suggest that he was thinking about numbers and their embedded relationships. He sometimes gave detailed description of ideas for teaching which indicated he reflected at times on the meanings of numbers and the operations on them. This in itself may also be considered a development of his number sense. Also numerous comments he wrote about ways of helping his pupils to think and understand concepts using materials, diagram and games were possibly related to a deepening his understanding of numeracy.

His understanding of the nature of mathematics became deeper as the Teaching Program progressed. He appeared to appreciate that calculations could be done using methods other than the standard procedures, and that materials and diagrams could be used to help explain concepts. However though he was able to explain come working he did in calculations, he did not appear to think about such features as the reasonableness of answers or show on tests attempts to verify answers. Only rarely did he describe an interest in, or appreciation of, mathematics.

The changes in his beliefs about mathematics that supported constructivism after the Teaching Program showed that he no longer believed that rules and memorising facts were as important as he used to think. This is also shown in his computations for division in the algorithm test where he used his own strategies. But he used a procedure for long multiplication which was apparently automatic for him, but irrational, resulting in unreasonable answers. Yet he continued to use it all year without questioning it. Perhaps after our discussion of it at the end of the year he will not be as ready to use it again.

From my anecdotal observations during classes Jo did not appear to participate well in activities that required discussion. Yet his Thinkbook entries indicate that he reflected on many activities that happened and he often applied his observations to constructing ideas to be used in his future teaching. He appears to have been doing some metacognitive thinking but as yet has developed few skills to help himself think about such matters as the reasonableness of answers. From his few comments about how he planned to teach he does seem to have reflected on how to help others to learn and therefore indirectly he may be in the process of helping himself more than he did during the Teaching Program.

From this investigation of Jo's experiences there appears to have been for him some initial steps made in the process of development of number sense, and hence the understanding of numeracy which will assist him to become an effective teacher of mathematics. During the Teaching Program and the following semester he had become more interested in ideas about number and the operations on them and was able to develop his own strategy for long division. He came to understand that ideas in numeracy could be explained. He also noticed factors that will help children to understand concepts better such as the use of materials. But without the detailed examination of his workings on tests, the questionnaires, interviewing and reading of his Thinkbook these factors would have been difficult to notice. Yet there appears to be still much room for further development of Jo' numeracy even just to come closer to the average development of number sense of others in the class. Jo appears to need continual encouragement to examine the work he does in numeracy and this would require close monitoring of what he did and needed to do.

Summaries of learning experiences for Lisa, Wani and Wili

As the reports of the case studies describing the learning experiences for Lisa, Wani and Wili are placed in the appendices as Appendices, V, W and X, a summary for each is given in this section. Hopefully the overviews of these three case studies will be sufficient for the reader to follow the discussion in Chapter 7 when the case studies are referred to occasionally.

Lisa

Lisa was a conscientious pre-service teacher who took every opportunity to learn what was offered to her during the Teaching Program. The quantitative data indicated that Lisa's number sense developed. But to help determine whether the actual teaching of the unit was in part a cause of this development, I investigated the other data banks in more detail, including what Lisa wrote in her Thinkbook. From these data there were indications that she gained understanding of the meanings of the relationships in the number system, the meaning and effects of the operations on numbers, and to a lesser degree the applications of number such as in estimations. From her comments it appears that the discussions and reflections on the comparison of past and present mathematics learning and on the class activities, lead to insights and greater understanding of aspects of number sense. Lisa improved her mental computing skills and saw how she could help her students do likewise as she came to appreciate the difficulties they might have. She found the use of manipulatives, the cooperative group work and the writing in the Thinkbook especially helpful.

Lisa's understanding of concepts developed along with her understanding of the nature of mathematics. I suspect that it was this latter increased understanding that produced the interest and surprises that she often commented on. It appears that after the Teaching Program she had a broader idea of the nature of mathematics especially in relation to its progressive, open and mysterious nature.

I had suspected that for any successful outcome in mathematics education more than the specific learning of cognitive knowledge was involved. While the learning took place for Lisa I noted the enculturation process that was involved. Lisa was very motivated to participate to the full in all aspects of the unit. The constructivist approach I used in the mathematics class appeared to free her to think and act in ways she had not previously been or encouraged or able, to do. She came to like and be interested in mathematics and the process of learning and helping children to learn. She was a student who apparently was comfortable with change because she readily discussed positively the ideas that were new for her.

Some of the data that I have collected from Lisa may be biased because culturally in Fiji, students tended to say what they think I wanted to hear. She did not criticise me or anything I did in her writing, as other students occasionally did, but she did suggest ways she would adapt what I suggested in class in her teaching. I believed that she was assertive and open enough to be critical if she thought it was necessary. Of course whether her new ideas will actually be incorporated into her teaching is another matter. In small ways I did observe change in Lisa's actions, such as in the adoption of strategies for mental computations, for estimating, and in her ability to discuss the difficulties children would have learning mathematics.

Lisa did change in many ways but I cannot state with certainty, how much of the change could be contributed to the learning that occurred during the Teaching Program. For Lisa the positive changes include a deeper understanding of some mathematics concepts and a broader understanding of the nature of mathematics. Also she now enjoys mathematics and is motivated to become an effective teacher of it. On the negative side there is still much development of number sense before, according to my definition of number sense, she has *the ability and inclination to understanding and use numbers, their operations and applications with ease*. But as she has developed metacognitive thinking, her skills her learning will hopefully continue. There is still time before she graduates as the Teaching Program was at the beginning of the first of three years of the course that she has to complete before she graduated as a primary teacher.

Wani

In the class Wani was one of the students who entered the College with a higher mathematics ability than most but a low degree of commitment to becoming a teacher. During the first term of twelve weeks there was little development especially in the areas of learning much about her own learning, her learning of mathematics, or her learning to teach. In the second half of the Teaching Program she, after a struggle to commit herself to the requirements of the mathematics education unit, began to show change and progress.

The data collection at the end of the Teaching Program suggested a considerable development of number sense. She improved by being able to answer correctly questions requiring the understanding of the meaning and effects of the operations and therefore computational estimations. She used more mental computations and, during the interview she was able to explain more of her understanding of how she did her computations. Also her skill at doing algorithm for division improved even though we did not consider it directly in class and she was more accurate in use of times tables. She performed the test more carefully and thoughtfully and so had less careless mistakes.

Wani's responses to the questionnaire showed that she had come to agree with items that described a constructivist view which were related to work done during the Teaching Program such as that mathematics can be solved in different ways. This was after the use of personal strategies in problem solving. Yet there appeared not to be much change in her view of the other aspects of the nature of mathematics as she continued to believe, though not strongly, that rules and procedures had an important place in mathematics and that she did not expect mathematics primary curriculum to change.

After the first half of the Teaching Program Wani appeared to be interested in her own learning and was able to motivate herself to learn to become a teacher of mathematics. The entries in her journal suggested that she had become a pre-service teacher who was interested in learning about teaching as she observed how others taught and was able to note strategies she wanted to apply in her teaching.

If this new interest was kept alive and she continued to broaden her ideas about mathematics Wani would probably become an effective mathematics teacher. To further develop her number sense I suggest she would need to continue to be open and often reflect on ideas encountered in mathematics education classes so that the rational, progressive, open and mystery aspects of the nature of mathematics would gradually change her attitudes and improve her numeracy.

Wili

Wili was one of the students who came to the College with problems in understanding of, and use of, skills in numeracy. From what he wrote it appears that he believed that his numeracy had improved much during the Teaching Program both in his development of number sense in the cognitive area, and in its other aspects such as in his attitudes to and ease in understanding and use of numbers and their operations. I myself am less confident about such a development as I have no firm evidence from the data that much development in number sense occurred. His understanding of the nature of mathematics, his self-confidence and assertiveness had developed and perhaps these changes may be the initial steps that are necessary for a development of number sense. It may be that one year of mathematics education units is not sufficient, for students with an apparent poorly developed number sense like Wili, to progress to a certain threshold where the development gains a momentum of its own.

During the Teaching Program he noted a few specific topics that he had had difficulties with in school such as fractions, decimals, volume and division. For each of these, when we worked on activities related to them, he readily noted a deepening of understanding and usually was pleased with his achievements. The evidence from the interviews and his descriptions in his Thinkbook suggest that his understanding of the concepts did improve but the other data of this study did not suggest much improvement. The evidence of improvement was that his mental computational skills had developed and he was able to explain why he obtained some of the answers that he did. Also after completing the test of algorithms, he wrote that he had improved on the performance before the Teaching Program.

Some evidence for improved number sense can be gleamed from his algorithm test results. His score for the test indicated that his skill and knowledge of them had greatly improved but on close examination of his workings on the test, his improved score was largely due to use of his own strategies and strategies of verification of answers. Both of these results are related to elements of number sense and so indications of improved number sense rather than improved knowledge of algorithms.

These improvements that have been described by Wili or myself appear to be related to some aspects of the constructivist approach I used. Often it was after reflecting on one of the many student-centred, problem solving activities that he said his understanding was deepened. The activities requiring social interaction and communication such as cooperative group work, class discussion, peer presentations and journal writing were among those aspects that he said he found most helpful to enabling him to be stand in front of the class with confidence. I also suggest that these helped him to gain confidence to be assertive, to join in discussion and to ask questions in class.

To summarise what happened for Wili during the Teaching Program, I found that data indicated a small improvement in ease and ability to use numbers, their operations and applications but not much measurable improvement in understanding. His self-confidence greatly improved. He made the most of the opportunities he was given during the classes to develop himself, his learning and his knowledge and skills of teaching. There is still a great need for Wili to develop his numeracy further, both the cognitive aspect of it and the affective aspect including his attitudes, and beliefs and his understanding of the nature of mathematics.

CHAPTER 7 DISCUSSIONS, IMPLICATIONS AND MORE

For the two years prior to this study as the mathematics education lecturer, I had watched the pre-service primary teachers in a College in Fiji struggle to understand the numeracy they were required to teach. This led me to consider what I could do to assist them. What initial mathematics education unit could I implement that would help overcome this problem? This study has in general terms been my attempt to answer this question.

A review of the relevant literature and reflections on past experiences resulted in the planning of a mathematics education unit which had number sense as its underlying theme, and a social constructivist approach for the teaching strategy. As well as preparing and implementing the unit I needed a process to determine the effectiveness of the unit. I chose to investigate changes in the cognitive aspect of numeracy by determining the development of number sense of the students using traditional tests, information gathered from their journal entries and observations of students working in class. To gauge what affective changes had occurred in the non-cognitive aspect, I analysed the students' responses to a questionnaire, their comments in their journals and my own observations. As well as obtaining an indication of what changes had occurred I wished to know if these changes were the result of the mathematics education unit and if so, to which particular features of the unit were the effective changes linked. That is, I attempted to discover what helped to make the Teaching Program effective and so have findings that could be a guide for the design of further units.

In this final chapter, I will first consider the findings for the particular research questions. Then I will briefly discuss other findings of the study related to mathematics teacher education. Following this I will review my motivation for the study and discuss my initial goal in relation to the outcomes. In the context of the findings I will suggest some general issues that appeared to emerge from the study related to areas of mathematics education of pre-service teachers in Fiji and perhaps further afield. Then before my final statements, I will comment on the limitation of this research and suggest the need and possibilities, for further research.

Findings for the research questions

In the following section my discursive commentary on the findings for this study considers the answers to the research questions (see Figure 7.1) individually except for Questions 3 and 4 which are combined because, as already noted, the questions overlap.

Research questions

1) Can a mathematics education unit be designed for Fijian students in this College using:
   a) number sense as the underlying theme with an emphasis on the areas of: numbers, their operations, their applications in everyday living, and in particular, estimations and mental computations within these three areas:
   b) a social constructivism approach and hence in particular, an emphasis on: acceptance of each student as they are, encouragement of construction of student's own knowledge of mathematics, use of materials and everyday contexts, reflection and communication to improve learning?
2) Can the effectiveness of such a unit be evaluated by examining:
   a) the design documents of the unit,
   b) the responses of the students to its implementation, and
   c) my reporting of its implementation?
3) What development of numeracy occurred for students who took this unit? In particular,
   a) what changes occurred in the cognitive aspect of number sense, and
   b) what changes occurred in the non-cognitive aspect of number sense?
4) If changes in the development of numeracy did occur for the students, what brought about these changes?

Figure 7.1 Research questions.

Question 1: Designing the mathematics education unit

In Chapter 5 the design and implementation of the mathematics education unit has been described and discussed in some detail. In that discussion I described the theoretical notions and the practical issues as well as the content for the unit which was designed for the first semester of the primary teachers' course. Here I will summarise the main elements of the design that was successfully implemented.

With the experience of teaching in a similar context both in the two years prior at the College in Fiji and in Papua New Guinea, and with the research discussed in the literature review, a unit with an underlying theme of number sense was developed. The unit was the first of six in mathematics/mathematics education that the students would complete in their course, so an overview of the important concepts of primary school mathematics were included leaving a more in-depth treatment of them to be completed in later units. In Table 5.1 the topics and activities completed are listed. As can be seen in Table 3.1 the three major areas of number sense as described by McIntosh, Reys and Reys (1992) in their framework for number sense are; number, their operations, and their computational settings. Mathematics education research influenced my decision to include an emphasis on estimations and mental computations within these three areas to help the development of numeracy.

Social constructivism was the teaching paradigm for the design of the unit. The ideas of constructivism were a major influence on the choice of activities and their implementation. The activities were chosen to encourage students to use their own strategies rather than learnt algorithms. Much emphasis was placed on the production and use of teaching materials and the use of mathematics of their traditional culture and everyday urban living in Fiji. The provision for class discussion and written reflection was made so as to promote communication in the language of mathematics, learning and reflection on the concepts. I considered that the pre-service teachers were preparing for a life as a primary teacher and therefore in my use of a constructivist approach I planned to model the approach so that the students would be inspired to adopt and adapt it in their future learning and teaching.

In designing the unit I also needed to consider the context of its implementation, especially the participant students with their past mathematics education and current attitude to the subject. I allowed for a low level of understanding of concepts that I had noted in the students of previous years, and so included the study of the basic concepts that primary school students learn. To help overcome the fears and lack of confidence I planned that the activities chosen would contribute to the development of a more appropriate attitude because of their enjoyment and challenge elements, and the discussion and reflection components of the unit.

Returning then to the first research question, I was able to design a mathematics unit as envisaged within the resources and cultural context of this Fijian College However it was to be expected that the design of the unit would most probably need revision after reflection on the effectiveness of its implementation. This effectiveness is discussed in the answer to the second question.

Question 2: The effectiveness of the unit

In determining the effectiveness of the unit, the design documents were examined indirectly by noting the elements of the unit that the students, and myself as the lecturer found were beneficial for the development of numeracy of the students. Using the framework for this study, the student's responses and my observations were noted in the three areas of; the students' enculturation into the mathematics education classroom, their construction of knowledge of mathematics, and their lifelong learning thrust of the unit. The unit's effectiveness with regards specifically to the development of numeracy is given in the response to Question 3 in the next section.

One test of the design and measure of the effectiveness of the Teaching Program was partly determined by how well the students were able to be enculturated into the mathematics education unit so that they were better able to benefit from what it had to offer. From their journal writing and anecdotal evidence from conversations about their school days I heard that their experiences in the culture of secondary mathematics classes were often considered as one of dependence on authority and consisted of learning in a transmissive mode. This at times made the process of enculturation into the Teaching Program difficult for some students. Initially there were the difficulties; lack of confidence, mathematics anxiety, communication problems, insufficient helpful study habits and related to this, difficulties fulfilling requirements for ongoing assessments. On the positive side I observed the growing enthusiasm and commitment of the students to developing understanding and interest in the mathematics of their daily life. Two elements that the students described as helpful and enjoyable were the making of teaching materials and the cooperative group work. Coping with language difficulties related to learning English and mathematical terms was an issue for some, whereas the learning of new words and terms related to the common languages in Fiji and to mathematics were seen as interesting challenges for many. Generally the experience of the Teaching Program was a positive one that would probably have stood them in good stead for the rest of their course. They would be familiar with what to expect and what was expected of them, both in the opportunities and the challenges of their mathematics classes at the College.

The discussions of the other aspects of the Teaching Program, mainly using the data of the six case studies have shown that the enculturation was a process with aspects common for most students given that the data collected for the whole class was in line with the detail data of the case studies. Unlike the general population in Fijian secondary schools, these students were motivated to participate in mathematics classes or, in the cases of Jo and Wani, they became so. Most of students expected that the mathematics education classes would be all about learning to teach mathematics to primary school students. Generally they did not expect to learn anything new of primary school mathematics, or to relearn concepts that had been learnt in primary school. So they were in some cases surprised, and in others frustrated initially, when most of the activities were related to primary school numeracy.

For the six students in the case studies there were some variations in responses to the Program. Lisa, a better student of the six, and Vita, one of the students weaker in numeracy, were enthusiastic about the Teaching Program from day one. This seemed to be because they both were so relieved and delighted it was not like their past mathematics learning in secondary school which they did not like. Initially the others experienced difficulties in becoming enculturated into the class. The difficulties they described included lack of assertiveness to contribute orally to discussion, and a lack of good study habits to help them fulfil the requirements of the unit. Also some of the six initially experienced frustration because, in many of the classes they were required to participate in primary school activities instead of the higher secondary school mathematics they had recently studied and with which they were familiar. All these difficulties lessened to different degrees for the individual students as the Teaching Program progressed. By the end of the unit all six, except Jo, were participating fully and were enthusiastic about the work done in the class. It appeared that their personal effort to participate brought about this change. In one case after my specific encouragement there was a major change. However Jo continued to be reticent to participate in open discussion and his difficulties continued. But on a brighter note he did write about applying the ideas discussed in class in his future teaching.

The main areas of the cognitive knowledge which the six students did mention was a deepening of understanding of the concepts about place value and fractions. Besides this understanding of mathematical concepts there was the development of understanding of the nature of mathematics and its possible enactment of related values. They appeared to develop their understanding that mathematics was objective shown by their interest and ease in using manipulatives. The development of understanding of the other aspects of the nature of mathematics was less pronounced.

All six students in the case studies agreed that the construction and use of materials was helpful and they believed that if they use them in their own teaching it will assist their students to understand the concepts. Jo bemoaned the fact that he had not been given the opportunity to use them in his past learning of mathematics. The materials that were the most often discussed were the MAB materials, straws bundled for base ten, fraction discs and other materials related to fractions. These are all related to learning of decimals and common fractions which were often referred to as difficult concepts to understand.

The effectiveness of the constructivist approach was also determined partly by how helpful the pre-service teachers found the combination of strategies I employed. A list of these strategies includes; the use of many organised mathematical tasks, the construction and use of manipulatives, the encouragement to use their own methods for doing calculations and solving problems, the exploration of strategies for mental computations and estimations, the journal writing in their Thinkbooks, the class discussions, and the cooperative group work.

From what the six students wrote the mathematics education unit appeared to be effective in developing the student's lifelong learning skills such as study habits, metacognitive thinking skills, and pedagogy knowledge. The pre-service teachers never mentioned specifically what related to their own future learning processes. This was probably because they were so involved in the extensive amount of new learning in the course. Yet from what I have described in earlier chapters it appears that the pre-service teachers were conscious of developing metacognitive skills for their own learning and knowledge about how they could become effective mathematics teachers. Their understanding of concepts appeared to deepen when they became more confident and willing to acknowledge in class and in their writing that they had not, until that time, understood some of the concepts.

But becoming an effective teacher of mathematics does not happen exclusively at a Teachers' College. These students need firmly entrenched lifelong learning skills, including the metacognitive thinking skills to maintain and enhance the processes of lifelong learning. The establishment of the skills during the Teaching Program was generally rather tentative because it was a new experience for them and probably needed more time to become part of their mathematics education culture and life generally. For this learning to become a lifelong habit a continued emphasis on it during the rest of the three-year course is probably necessary. Therefore this habit requires more time, encouragement, guidance and ongoing monitoring to help the students realise more deeply their need for development of the appropriate skills for being effective lifelong learners.

Perhaps if the students had been initially given more opportunity to consider their own needs and understand what was involved in the completion of the unit, the difficulties and frustrations encountered may have been lessened. This opportunity could have been in the form of negotiation between lecturer and the pre-service teachers of the processes and content to be included in the Teaching Program. Yet perhaps their past experiences in education would have made a process of negotiation so incomprehensible and threatening that it would have been ineffective unless conducted very sensitively and very gradually. Another restraining factor on such a process would have been the somewhat conservative expectations by the College authorities of having a formal program of work fully prepared by the lecturer before the beginning of the semester.

It appears that the major change from their past experience of the transmission mode of teaching to their new strategies of learning during the unit was crucial to their coming to understand their own knowledge and themselves as learners of numeracy. The new experiences of the mathematics education unit required then to manipulate objects and to reflect on what was happening during the activities and again later when writing in their Thinkbooks. Hence during the unit they seemed to realise their need to transform themselves into persons with a more highly developed number sense and an understanding of concepts rather than to be persons with a greater knowledge of the facts, rules and the procedures of primary school mathematics curriculum. This finding suggests that the unit of work developed for these students can be judged in general, as effective.

Question 3 and 4: The development of numeracy and reasons for change

As well as evaluating the effectiveness of the Teaching Program generally as discussed in the last section, Question 3 asks specifically about the its effectiveness in the development of numeracy, in particular number sense. This investigation of number sense included an analysis of quantitative data collected from tests administered to all the students and the descriptions given in the case studies of the students' development in understanding of the concepts of numeracy and the nature of mathematics. The definition of number sense used in this study has a non-cognitive as well as a cognitive aspect, and so in examining its development I consider both. Therefore this discussion has two sections, one for each aspect. In discussing the changes in the development of numeracy, the causes for the changes, which are responses to Question 4, will generally be given.

Development in the cognitive aspect of number sense

Evidence of the development of number sense was investigated by analysis of the results from a traditional test administered three times during the year. Also I sought indications of its development from the data in the case studies of the six pre-service teachers. The sources of this data were their writings in their Thinkbooks, their semi-structured interviews and their test results.

The class results of the number sense assessments indicate that, before the Teaching Program at the beginning of the year, computations involving money and multiplication of whole numbers posed little difficulty for the majority of pre-service teachers whereas decimals and computational estimations proved to be problematic. The results of later tests suggest that the Teaching Program may be partly responsible for a notable improvement in estimation of computations for the four operations. Much time and effort was given to this topic, and the students often commented on how helpful and interesting for them were the activities related to it. But it appears that the unit was not responsible for much improvement in estimation of time, fractional distance, percentages and the understanding of decimals. Apparently estimation of time required special attention and there were no activities in the unit related to it. Also much time was devoted to various aspects of rational numbers and although the pre-service teachers had demonstrated basic knowledge of facts, they were unable to apply them in questions that required other than simple statement of facts. It appears that more time needs to be given to the consideration of the concepts of rational numbers in future mathematics education units of their course. One component of the unit that appeared to be effective in constructing knowledge of concepts and the nature of mathematics for the students in the class was problem-solving. As a result of this process the students were better able to develop their own computational procedures and also produce positive responses to learning mental computational strategies.

Many of the tasks in the unit required reflection and the manipulation of materials to illustrate concepts such as equivalent fractions. For the class these tasks were often described in their Thinkbooks as interesting, enjoyable and as a help to their understanding. This was especially true when cooperative group work was involved. Most of the pre-service teachers said the discussions and arguments during group work helped them to understand the concepts better. They also found that it taught them to work together as a team. Interestingly Wani and Jo, two of the case study students who did not appear to participate easily with others, did not mention group work in their Thinkbooks. The one exception was when Wani referred to her group presentation in a positive manner. Perhaps if these two students had reflected on and written of their experience of the processes of group work it may have resulted in greater participation by them.

Looking at more specific features of the pre-service teachers' numeracy using the case studies a few areas did develop considerably for some of these six students. These included a deeper understanding of rational numbers, the ability to estimate and compute mentally, and the use of their own strategies for computing. Although all six students showed an improvement in decimals and common fractions, the three students weaker in numeracy skills did not appear at the end of the year to be competent enough to satisfactorily compute questions required for the teaching of rational numbers in primary school. The better three students appeared to show in the interviews that they were able to link their areas of knowledge in solving problems. However there were limitations to this in that none of them, without some help, could find a fraction between 2/5 and 3/5.

In three of the case studies it was remarked at least once that division of decimals made more sense when they could visualise the decimals as money values. But this restriction of decimals to tenths and hundredths may have been part of the reason why they were unable to find decimals between 1.25 and 1.35, until they were prompted with leading questions. Perhaps by using other applications of decimals such as measurements they may have more easily extended their understanding to include thousandths. Again, I suggest there seems to be no apparent reason why this initial development in numeracy might not continue to develop if similar class work continued, especially if reflection on the work was encouraged.

Besides the understanding of concepts as part of number sense, the understanding of the nature of mathematics was also considered. As numeracy is embedded in mathematics, it seemed reasonable to expect that in developing number sense, students would also develop their understanding of the nature of mathematics. This understanding could then provide a stronger background and foundation for their activities in numeracy.

During the Teaching Program the pre-service teachers generally experienced a change to a more positive attitude to the doing of mathematics because of their enjoyment in doing the activities. For many of them it was their first experience of enjoying mathematics. So probably because of this, the pre-service teachers resolved to make their mathematics classes interesting, fun and less threatening. As well, they saw the application of everyday experiences in their mathematics teaching as important since this in turn promoted understanding and interest. If a desire to promote a deeper understanding of number sense in their future pupils is a sign of their own development of it, then these pre-service teachers showed this development. Their frequent response to a deepening of understanding for themselves was a resolve to teach with understanding. This was another marker of their better appreciation of the rational nature of mathematics as described by Bishop (2001).

Generally this understanding that mathematics was *rational* improved considerably during the Teaching Program, and they developed a wish to be able to help their pupils *understand* such things as the times tables as well as *know* them. Also at the beginning of the Teaching Program many of the pre-service teachers had not been able to distinguish between concepts and convention in mathematics and so accepted all that they were taught, rarely following rational thought processes. What interested some students, were the possibilities of *progress* of such things as finding patterns and being able to apply knowledge from one area to another. Most of the students came to the College with an understanding that mathematics was a subject that one could master if one was in *control* by using correctly the algorithms that were taught in school. Generally they appeared to like the security that this learning of algorithms had provided. Also during the Teaching Program the *objective* nature of mathematics with its aspect of the importance and benefits of understanding concepts via the use of manipulatives, diagrams and other symbols, developed considerably as they realised the importance and benefits of their use and applied them in their teaching. They appeared to understand the *openness* of mathematics. In the interviews after the Teaching Program they were pleased to be able to explain to me how they obtained their answers. It was clear that the three better students of the six in the case studies were able to verify many of their answers in tests and learnt new methods to do so. The weaker students occasionally commented on discussions we had in class about reasonableness and verification of answers but, other than using multiplication to verify division answers, they often left unreasonable answers unreasonable in tests. *Mystery* as an aspect of the nature of mathematics was a feature that most of the students had never thought about. During the Teaching Program a few in the class reflected on mysteries of the world of mathematicians. These students came to be interested in such aspects as patterns and mathematics of other cultures, and also the work of persons such as Deines and his MAB materials, and the development of the Hindu-Arabic notation.

For the pre-service teachers this opening up to a broader view of mathematics was apparently the result of the Teaching Program since it emerged in most of the Thinkbooks when they reflected on the activities done in class. Insight into the nature of mathematics is not an understanding that can be easily taught, but probably comes as a consequence of a continual reflecting on the minor insights that occur. Hence a development of the understanding of the nature of mathematics can be used in some way as a measure of the development of number sense along with the understanding of concepts. How this can be accomplished is a question for further research. These ideas about the nature of mathematics are the reason why I have included *an appreciation of the nature of mathematics* in the definition of number sense that I have used in this study.

In summarisng this section on the development of the cognitive aspect of numeracy and reasons for this development I will use findings of the case studies as they are generally representative of the whole class. The three students who began the unit with an apparently greater ability in numeracy appeared to develop the most. In the entries in their Thinkbooks, their insights and their greater understanding of the concepts are attributed to the work done in class and hence the processes included in the mathematics education unit. The other three students described an improvement, and the data confirmed it but to a much lesser extent. These latter three students also discussed how much they were interested in and enjoyed coming to an understanding of the concepts. They attributed their interest and improvement to the activities completed in class and to their reflecting on them when writing in their Thinkbooks. However it appears that their ideas of considerable changes in feelings, attitudes and understandings are not reflected in a comparative improvement in number sense at this stage as measured by the tests. They still lacked basic understanding of concepts such as those related to decimals. It would require research to determine whether, with further similar mathematics learning, this initial improvement would lead to sufficient development in numeracy to enable these students to have the necessary understanding and skills to teach mathematics effectively in primary schools.

Development in the non-cognitive aspect of number sense

As I have considered that number sense and hence numeracy has a non-cognitive aspect I needed to consider this aspect in its development. Included in the non-cognitive aspects are: emotions, attitudes and beliefs towards mathematics and the doing of it; appreciation of the nature of mathematics; and the development of ideas about the learning and teaching of mathematics.

The students' journal entries early in the unit suggested a development of a positive attitude to numeracy, as well as to the doing of mathematics for most of the students. The pre-service teachers often commented on their past dislike for mathematics and the difficulties they had experienced in secondary school mathematics classes and compared it to their current work in which they were very interested and found helpful. They described their past schooling as not being helpful as they had learnt much work but had not developed strategies for coming to an understanding of the concepts.

These attitudes to, and beliefs about mathematics that were the result of their past experience also underwent changes. The analysis of the class results of the responses to the questionnaire *Beliefs about mathematics* suggests that there were notable changes in the pre-service teachers' beliefs (see Table 6.7). After the Teaching Program they generally no longer believed that they needed to depend on learning facts, rules and procedures. They also came to believe that estimations, patterns and relationships are important in mathematics. The same was true for the use of their own strategies, flexibility in thinking and originality in doing mathematics. The beliefs which they held in February that indicated a positive approach to constructivism were still held by them in July. For all the items for the delayed questionnaire at the end of the year the mean scores were either more positive than the July score or, for a few they remained unchanged. Their beliefs generally reflected what they wrote about in their Thinkbooks or discussed in class. The lessening of their dependency on rules and routine procedures is well illustrated in their answers on the algorithms' test. These changes appear to be related to the constructivist approach that I used as they are partly aligned to the individuality of constructing mathematics knowledge using the knowledge they had previously constructed. Their attitude also became more positive during their learning of teaching strategies. During this learning the pre-service teachers became very interested in and enthusiastic about teaching.

The responses of the six students of the case studies to the questionnaire generally indicated a considerable change in beliefs during the year, but some variations and, contradictions were difficult to interpret. Their major changes in beliefs were similar to the class results already mentioned. It emerged that a reason for the contradictions appeared to be that the students had different interpretations to mine of a few of the items. More discussion at the time of administering the instrument may have been helpful to avoid this difficulty.

Being encouraged to reflect on the concepts of and the nature of mathematics was a new experience for all the students. Was it the teaching approach used in their past learning which previously prevented them from reflecting on such ideas? I suggest that if they had been given the appropriate environment that allowed them to reflect on what and how they learnt, their numeracy may have developed to a greater degree. As described earlier in examining their understanding of the nature of mathematics, changes were noted in their appreciation of its nature, a non-cognitive aspect of number sense. Some aspects of the appreciation of the *rationality* of mathematics was often commented on as was *objectivism* as related to the use of manipulatives. Because of past rote learning, appreciation of *control* was strong though became somewhat weaker as evidenced by the use their own strategies doing of the algorithm test for some questions in the last test. The appreciation of the other aspects of the nature of mathematics related to *progress, openness* and *mystery* were not as obvious but was present and developed to some degree.

In this study I have suggested that many of the developments of number sense appear to be related to the reflexive activities of the pre-service teachers especially those that resulted in the pre-service teachers reflecting on their learning processes, both learning to learn and learning to teach. Their metacognitive thinking appeared to initiate insights into concepts and processes. It appears that it was not so much the doing of the activities that gave them understanding, but rather the reflecting upon it hours, and sometimes days later. For example, all of the six students in the case studies appeared to reflect on their own learning evidenced by their ability to discuss what helped them and what hindered them in their learning.

The extended times discussing and writing about the ideas were often described as the important processes. Some students considered the possibility of improving their learning by continuing the reflexive writing in the Thinkbook in following units of their course. Perhaps if these pre-service teachers encourage their pupils to do likewise, then their students may not experience the difficulties with numeracy that the pre-service teachers had to contend with. One important issue related to the journal-writing assessment task that needs to be noted was that a considerable amount of on-going support was required during the Teaching Program for many of the students to persevere as the exercise was so foreign to them.

A final aspect of the students' development of the non-cognitive aspect of number sense is in their learning to teach. This experience appeared to bring about some of the changes in number sense that I was investigating. It was possibly because they were in a Teachers' College that they initially considered the learning of teaching skills as the important aspect of the mathematics education unit. Hence their motivation to become effective teachers of mathematics probably led them to a more in-depth consideration of the mathematics concepts and as a result, a development of number sense. Initially for some there was a lack of acceptance in completing mathematics of primary school standard rather than the more advanced mathematics work of secondary school. But in no time studying to become effective teachers gave them an acceptable reason to reflect on and relearn the basic ideas of numeracy and to use the primary school manipulatives they found so helpful in developing an understanding of concepts.

There were other experiences of learning to teach that helped them to learn generally, and so to learn numeracy. They came to realise the importance of being well-prepared and to be able to speak in a manner which enabled their listeners to understand what was being asked or explained. The improvement of these skills helped them to learn to learn as, in Wili's case he became more confident and assertive because of his Warm-up presentation. It appeared to enable him to participate better in class. Also the learning to be well-prepared helped them to be more independent learners and develop useful study skills for the study of numeracy that many of them had been lacking.

From the data collected there were many indications of development of numeracy, both of a cognitive and non-cognitive nature. These indications could often be related to some aspect of the unit such as a teaching strategy, reflexive activity or an organised exercise. With these findings about the effectiveness of components of the mathematics unit, its design could probably be adapted to further strengthen is effectiveness and lessen the weak elements of it such as the lack of understanding of rational numbers and some lack of communication.

Other emergent issues

Although the immediate discussion of results focuses quite rightly on the research questions that helped form this study there were four other issues that are important in the literature reviewed that need extra comments. These are; the use of mental computations and estimations, the understanding of times tables, the use of algorithms, and the need for more than the theory of constructivism.

Use of mental computations and estimations

In the designing of the mathematics unit to help develop the pre-service teachers' numeracy I included a special emphasis on estimations and mental computations within the areas of number sense. From my reading of the literature it appeared important that the processes involved in these two activities needed to be included in a unit that had an underlying theme of number sense.

Although exploration of strategies for mental computations and estimations were areas that we considered generally for only short periods of time in many classes, discussion of them was prominent in the pre-service teachers' Thinkbooks. The pre-service teachers said that computing mentally was a new experience for them. Their number sense was such that they were initially unable, without pen and paper to use numbers with ease. Some found it a considerable strain and were only able to do mental computations very slowly. Yet they all were very interested in and, in some cases excited about the strategies we discussed. The second set of interviews of the six students in the case studies undertaken at the conclusion of the Teaching Program showed a greater use of mental computing. Their reflections on the strategies indicated that the three better students generally understood the ideas we discussed. Even Jo, a weaker mathematics student who only mentioned mental computing a few times, was pleased with how he understood and used the strategies. He said he could see how children would come to see aspects such as the patterns in the multiples to mentally compute answers. But it was difficult to determine if the three weaker students were considering some of the strategies as further algorithms to be rote learned. Hence for the weaker students their understanding appeared to be more instrumental than relational.

Most of the students when completing the assessments at the end of the year were still working long multiplications with rows of zeros instead of doing the multiplications by multiples of ten mentally. Lisa, one of the three better mathematics students of the case studies who worked at using the strategies reverted to written computations for the number sense assessment and so had not enough time to complete the paper. This appears to show a lack of inclination or ability to use numbers and their operations with ease, which I consider are features of possessing a good number sense. Much more discussion and encouragement to use mental computations appears to be necessary but in such a way that the weaker students are allowed more time to come to construct strategies from their own understanding. This will also help build their confidence so that they feel able to use them even in tests.

At the beginning of the Teaching Program the pre-service teachers had little skill at estimating, as shown by the class results in the *Assessment of number sense*. After discussion of strategies for estimating numeracy, measurement and answers to computations, the students showed an improvement in some of these skills in the tests after the Teaching Program. The skill that improved the most was in estimating numeracy, and to a lesser extent, measurement. Generally estimating time did not improve, but estimating distance did although it first required encouragement to reflect on the situation of the problem. The estimating of computations that improved little included those related to decimals and fractions. Computing an estimation requires considerable number sense in the form of an understanding of the meanings and relationships of numbers and the operations. This understanding in the area of rational numbers for the students appeared to require further development of understanding of the numbers before useful links between understanding and the effects of the operations could be established to produce mental solutions. This did happen for the better mathematics students.

Understanding of times tables

The times tables were generally well-memorised by the pre-service teachers but not understood. There were many references to discussions about the construction of the times tables as being very interesting. Early in our study of them I can remember one student in a class discussion making the discovery that the answers to the three times tables increased by three. I withheld my response to her comment and instead noted that others also came to see and understand the pattern she had discovered. In discussing their future teaching of times tables most of the pre-service teachers resolved to teach them with understanding using many of the activities we completed in class. These included construction of the times tables, discovering patterns formed, and interesting and thoughtful exercises to help familiarise their pupils with them. I wonder if these students had understood the times tables when they were introduced to them during their school days, then whether later work such as the operations of multiplications and division may have been more comprehensible to them. Also their ideas about the nature of mathematics may have been developed more appropriately because they would have better understood numeracy as having the aspects of rationalism and progress. Learning mathematics would probably have made more sense to them and have been so much easier.

Use of algorithms

To help determine the effectiveness of the Teaching Program I included a *Test of algorithms for the operations* in the investigation of what changes in the students' mathematical skills and knowledge occurred during the Teaching Program. Beneficial aspects related to the use of students' own strategies could be seen from the examination of the completion of the algorithm tests. The results for the whole class for the tests indicated that there were significant improvements for the July and the October tests (see Table 6.5). The most notable findings were for the mean mark for the question on long division of whole numbers and the questions dealing with rational numbers. The long division result appears to be related to number sense development as students completed the questions using their own meaningful procedures. The results for questions involving rational numbers did improve although at the end of the year their mean marks continued to indicate a general lack of ability to perform algorithms with rational numbers. Unlike for division, the students did not use their own strategies but attempted to perform the poorly remembered standard algorithms. In other words the algorithm for long division, even though it was included in the primary school mathematics curriculum was rarely remembered, and it appears that students resorted successfully to using their own methods. However for rational numbers they attempted procedures which incorrectly resembled the standard procedures. It seems that partly-remembered algorithms may be more of a hindrance to devising one's own strategies than a lack of knowledge of them. Another feature of the algorithm test which suggested a development of number sense was that, the written work for many of the questions in the tests after the Teaching Program showed answers that were corrected by the students when verification processes highlighted mistakes.

In the case studies an interesting finding was the six pre-service teachers' response to the long division problem on the final test of algorithms. In the tests before and after the Teaching Program the three weaker students either made no attempt or a very poor one to complete a calculation for the long division. In the final test Vita was the only one of the three who attempted to use the standard algorithm, and her attempt showed some lack of ability to use it accurately. Jo and Wili, the other two weaker mathematics students used their own strategies of repeated subtraction in a way that showed an understanding of the process. Both of them were able to improve the efficiency of their methods as they proceeded with the subtractions for the division. This reflected some appropriation of the work in the teaching program since we had examined forms of this process in class a few weeks before the test. Also for the short division by five, which Jo and Wili had incorrect in the second test, in the final test they correctly used processes other than the standard algorithm. Wili used the repeated subtraction method and Jo used a strategy which we had mentioned briefly seven months earlier. He multiplied by two and divided by ten to get the correct answer. Interestingly even though Lisa and Ana, two better students had accurately computed the long division using the standard algorithm at the end of the Teaching Program, in the final test they chose not to use it. Instead their processes of repeated subtractions showed a good number sense as it was nearly as efficient as the standard algorithm.

By the end of the year it appeared, that for the algorithm test most of the students understood and felt confident using alternative processes that we had examined in class a few weeks earlier than the test. This surprised me as I thought I presented the test in such a way that to use the standard algorithms was the expectation. Of all the work we discussed in class this brief discussion of division was the most widely applied new strategy. I think that the variations in the methods used for the long division showed a well developed number sense for the meaning and effect of the operation of division. It must also be said that the standard algorithm for division was generally not known, and therefore the alternative method was not replacing a familiar one for some. Hence it was perhaps more readily accepted.

Yet, in contrast to the processes used in division, Jo consistently used an incorrect algorithm for long multiplication which gave an unreasonable answer (see Figure 6.1). He only noticed the unreasonableness of the process when I discussed it with him in an interview at the end of the year. He apparently had never questioned his strategy. Perhaps with the introduction of calculators he was able to complete secondary school without having become aware of his incorrect algorithm. I wonder how many other mistakes the use of calculators 'covered up' for these pre-service teachers?

The pre-service teachers' ability to discover mistakes and helpful strategies independent of my intervention had apparently not developed yet. Perhaps it is unreasonable to expect the students to examine all their numeracy activities so as to become aware of inconsistencies in their processes and to accept more efficient alternatives, in one year, especially if their present processes, even if protracted, produced correct answers. Maybe if I had discussed alternatives to multiplications with rows of zeros, they would have responded with more efficient processes. Perhaps a less directed process such as more encouragement in finding alternative methods and the checking of reasonableness of answers, may have led them to develop their number sense and consequently more efficient strategies.

Need for more than the ideas of Constructivism

In the literature review in Chapter 2 criticisms of the theory of constructivism were discussed. It will be recalled that for Begg (1999) the important issues included:

- constructivism is concerned only with cognitive knowing,
- there does not seem to be explicit links between constructivism and theories that brain science or neural biology have to offer, and
- no one form of the numerous forms of constructivism seems to consider the differences between them.

Criticism such as these leave me looking for a development that provides ideas about knowing, learning and teaching that would have helped me more to understand what was happening in the First Year classroom.

Begg (2002) describes his ideas of theories as ways of making sense of what one believes is happening, with new theories as complementing older ones by enriching one's view of the world. For him, and for me also, the theory of enactivism fits more easily with this way of thinking about theories. While listening to discussions of enactivism and related theories since I began this study, it seemed to me that what was discussed made more sense to me than what I knew of constructivism. At these times I have felt, as Vita did when she described herself as understanding ideas during a discussion, then unable to understand them later when she wanted to apply them. When I have attended seminars in which ideas about enactivism were discussed, and then when I have tried to put the ideas into words later for my colleagues or students, I also felt inadequate to do so. Therefore my attempt in this section to relate my findings to the theory of enactivism may be for the reader and for me, far from satisfactory. However it is my first effort at changing the paradigm from which I have been working.

When I began the journey of this study the philosophical basis for my teaching strategies was the theory of constructivism. But now at the end of this journey I do not feel as satisfied with its ideas. I have noticed that I felt more at ease with the phrase "help the students become effective teachers of mathematics" rather than "help the students to learn about teaching mathematics". Also the term "number sense" and to a lesser extent "development of number sense" seem unsatisfactory for me. I have come to see number sense not so much as an ability that a person has or develops, but as an integral part of a person. Therefore "learning about or developing a number sense" would better be described as "becoming a person of number sense". In a similar vein I noticed that one of my students used a phrase of "a hater of maths" rather than "I hate maths", indicating a more personally integrated approach. These turns of phrase seem to be related to a description of the effects of learning as suggested by Begg (2002). He says that learning affects the entire web of being, thus what one knows, what one does, and who and what one is cannot be separated. This goes beyond constructivism which is more concerned with and seems to be limited to, cognitive knowing.

Davis, Sumara and Luce-Kapler (2000), in drawing on evolutionary and ecological metaphors describe the way that an agent knows as inseparable from what that agent does which in turn is inseparable from what or who that agent is. They describe philosophical perspectives that seek to demonstrate that learning and knowing, whether focused on the level of the individual or the social group, can only be understood when considered in the broader cultural context. These discourses shift the narrow focus of education "from the individual's effort to shape an understanding of the world to the manners in which the world shapes the understanding of the individual" (Davis, Sumara & Luce-Kapler, 2000, p. 69). This made me even more conscious that my influence as lecturer in the mathematics classroom is only part of the influence on the learning and that I, along with the students, are in turn influenced by the broader cultural context.

Begg (2002) also says that for him, for enactivism the dynamic agents of learning and knowing are not limited to the individual and the social group but he views the knowing agents as nested in other complex systems. He suggests that we need to consider individuals as they are influenced by the cultural context which in turn, affect and are affected by emergent environmental circumstances. No matter how much I wished to implement a mathematics education unit that was adapted for a class of students who had much in their environment in common, their individual responses to the teaching was very varied. Most of the factors that affected the learning were related to *who* the students were, rather than to *what* I presented or to the conceptual knowledge they previously possessed. Some of the immediate factors included; attitudes to mathematics and to the learning of it, lifestyle of the previous years, their reason for coming to the College, their private study habits, and their metacognitive skills. I now see that in planning the unit I did not give much consideration to these factors but concentrated more on what I assumed was the cognitive knowledge they possessed. Among other issues already discussed above, more negotiation of the content and processes to be used in the unit may have been beneficial to help each individual student be more open to the knowledge that could transform him or her into persons of appropriate number sense.

Language and culture are now seen as important factors in mathematics education and are significant in the theory of enactivism. I suggest that if I had a greater knowledge of the culture and language of my students much more could have been achieved more quickly. Some of the disadvantages I experienced because of this included: extra time spent building up understanding, trust and communication early in the Teaching Program; my lack of knowledge of daily experiences to help suggest application of ideas; my inability to use words or phrases from their languages to support explanations; and my lack of encouragement of the aesthetic aspect of an education.

For the last of these I noticed, but did little to incorporate the pre-service teachers' ideas, gifts and interest in areas such as the arts. I remember being jolted in my tracks when one student who had a mature outlook on life wrote in her journal:

> "But today was a very promising day of learning maths. I am beginning to appreciate the art and beauty of the times tables" (20/3, Kul).

I regret that I did not discuss with this student what she meant by this statement. I feel I missed an opportunity to understand better some qualities of all the students that I may have been able to draw upon in helping them to become more positive to aspects of the nature of mathematics. Other related aesthetic ideas that emerged during classes were the great interest in colour, patterns, in music and in one assessment, poetry. Perhaps the students may have been better able to give themselves to the learning if more of these areas were immersed in the environment of the learning.

To summarise I would say that for me there is a continual quest to broaden and deepen my understanding of learning and knowing. I have come to see that, to be an effective part of the learning process for my students I need to seek to understand more deeply the cultures in which my students are living and in turn the world in which those cultures are nested. My understanding will be influenced by my cultural context and it will naturally be limited, but my consciousness of this limitation is in itself important. In some ways the realisation of the complex nature of learning is overwhelming, yet the understanding of my contribution to the students' learning and knowing is now clearer to me as one of participation. Rather than feeling at times that I am powerless to help my students learn I need to concentrate more on what I can do to contribute to the possibilities of the learning that might occur.

Goal accomplished?

When I reconsider the initial motivation I had for this study, how satisfied am I that I have found at least some partial solutions to my quest? I was seeking a process to help First Year pre-service primary teachers in a College in Fiji improve their numeracy so that they would be better prepared to teach mathematics effectively. One aspect that is clearer to me now is that *all* these students, not only those whose numeracy appeared to be less than satisfactory, needed some kind of intervention to develop their understanding of concepts and the nature of mathematics, and their attitude to it.

From this study it appears that if the pre-service teachers are empowered to independently examine more closely their thinking and actions when working mathematically then they have a process for developing some, if not most areas of their own numeracy. Thus they would be better prepared to teach mathematics. The activities, reflections and discussions in class and then the delayed reflecting undertaken when writing in a mathematics journal, all appeared to be part of a solution. With this process their understanding of a number of concepts deepened. Also their ideas of the nature of mathematics became more appropriate, and so in turn, empowered them to be more open to the possibilities of what their own study of numeracy could achieve if they independently explored features such as patterns. I had hoped that this would have enabled them to be more independent in developing their number sense but as yet they generally appeared to need the initial stimulus from me as their teacher to examine the processes they were using in each area of numeracy we considered.

Some major areas for which they did deepen their understanding included the structure of the number system up to one hundred with its base of ten, the structure of the times tables, operations on numbers especially addition with bridging powers of ten, and division as repeated subtraction. Their skills that improved included those for mental computations and estimations. But the skills at using the standard algorithms did not so much improve, as that the students developed strategies to help them get the correct answers using variations of the standard methods. Understanding of rational numbers especially common fractions and decimals did improve, but at the end of the study this development was not sufficient to enable most of the six students to understand primary school numeracy in that area. They came to realise that their understanding was helped by the structured activities, the use of manipulatives and the introduction of contexts from daily living.

But what I consider an important realisation, which in most cases was beginning to emerge, was that learning improves if one reflects on one's own learning as well as on what was learnt. The inconsistent results of the students concerning rational numbers has left me wondering how much or little understanding of abstract ideas is necessary to operate with numbers in daily life. It appeared that much of the negative attitude to mathematics in schools was the result of failure to successfully understand except at a very superficial level, rational numbers,. Something must be seriously amiss with its presentation in primary school. Perhaps not much beyond the knowledge of decimals and common fractions that are related to everyday context is needed. The abstract ideas and the operations for rational numbers might be developed in secondary mathematics classes. This may lessen the negative attitude to mathematics that begins to develop in primary school.

All in all I would say that, considering the circumstances and limitations, which are discussed below, my goal has been largely accomplished. For the time allocated to the mathematics education unit that I implemented, the numeracy of the pre-service teachers appeared to improve considerably in a number of important ways. Continued numeracy study using a similar structure appears necessary to enable more of the pre-service teachers to become confident and proficient enough in their numeracy to teach mathematics in primary schools effectively.

Further speculations and generalisations

While I was doing this study I noticed in Fiji positive changes in the professional development of teachers, as well as in the structures and the practices in primary school mathematics education. I will not elaborate on them here but it will probably be some time before these developments manifest themselves in better mathematically-educated pre-service teachers in Fiji. Hence in the meantime mathematics teacher educators in Fiji need to do what they can to help the pre-service teachers with their difficulties in numeracy. I do not expect that the findings in this study would provide answers for pre-service mathematics teacher education in general, but I hope it may provide some helpful information for consideration by mathematics teacher educators in all sectors of education in Fiji and perhaps in similar situations in neighbouring countries.

At this stage it appears that the students who arrived at the College would have benefited if they had had more opportunities during their schooling to discover their own mathematics knowledge, for example, the structure of the times tables. This may have been helped by completing a greater number and variety of mathematical activities, using more manipulatives and having discussions in their mathematics classes both in cooperative learning groups and whole-class groups. But perhaps more importantly I suggest they would have been helped if the students had been encouraged to reflect and write about their mathematical ideas in some genre such as journal writing.

I realise such practical solutions are not easy to implement. Change in mathematics education in Fiji may be difficult to achieve while two major pressures are operating. These are the limited number of qualified teachers and the pressure on the students to achieve good passes in the many traditional external examinations. The latter also puts pressure on teachers to produce high numbers of good results. Then again not much change in assessment procedures will happen while there are limited places in higher classes in secondary schools and in tertiary institutions, and while assessment procedures need to be simple and quick to administer and process. Some assessment tasks that could have a beneficial effect are those that would encourage the development of metacognitive thinking and skills, and those that allow for individual learning styles.

Another difficulty which my students experienced may require little effort to lessen. This difficulty, also highlighted by research of mathematics education in Fiji as discussed in the literature review, was the apparent lack of motivation and/or self-discipline to study independently outside class. For a few of the students it was not a problem but for others it took time for them to organise themselves to regularly fulfil the assessment requirements. For others again they were still bemoaning the fact at the end of the year that they had not been faithful to their study and resolved to do better next year. I know it is already being tackled in some places in Fiji by means of discussion sessions with groups of parents of primary school students, but this or some similar process needs to be extended.

Another answer to this problem was demonstrated to me on two occasions that were particularly noticeable for the engagement of the pre-service teachers and for which they were successful and pleased with their achievements. These were in the production of teaching resources, and in the preparation of the group presentations. Both these activities were completed in group situations and so this working cooperatively may be a key to how the students might effectively work outside class. For regular reflection and writing in mathematics journals, group work is difficult but not impossible. I noted that there were a few journal entries from groups of friends that appeared to follow some private discussions of issues, such as the gender issue in mathematics. If pre-service teachers developed the practice of working in groups out of class there may be some hope they would continue this practice once they had finished their course. This would be invaluable in the continuation of the process of lifelong learning for the effective teacher.

Limitations of the study

The findings for this research obviously cannot be generalised to all pre-service teachers, not even to Fijian ones, because I have used one class of forty students and six case studies, and hence investigated particular experiences. Yet the data was gathered under circumstances that are in some ways similar to the context of many other pre-service teachers in Fiji and neighbouring countries. Some of the speculations discussed above may be useful to consider in further teacher education in Fiji. Case studies in particular have the advantage of being able to delve deeply into aspects of a situation and so uncover special features that would otherwise not have been noted.

To be able to come to any generalisation a much larger sample would have been necessary by using students from other pre-service teacher education institutions in Fiji. Also a longitudinal study would have enabled the process of development of numeracy to be investigated to a more substantial degree. These possibilities were considered in an initial plan for this research project but the political situation in Fiji at the time meant that such a plan was untenable, so a project that involved a short-term-focussed process needed to be devised.

Bearing in mind that the skepticism surrounding the findings of Margaret Mead, the anthropologist who worked in the South Pacific in the 1920's, I am cautious of the validity of interpretation of the facts in the data that students supplied. It is commonly believed that the young people that Mead interviewed did not always tell her the truth, but told her what they thought she wanted to hear. In Fiji where teachers and researchers are held in such high respect, data collecting, especially that which requires self-reporting such as in written questionnaires and journal writing, leaves open the possibility that participants may feel obliged to write and say what they think the researchers want to hear. This phenomenon which I refer to as the '*Margaret Mead effect'* made me wary that some of my students may have felt obliged to say what they thought I would like to hear. I realise this effect could not be eliminated but I hoped that the trust built up lessened it as much as possible. There were signs at times, such as their criticisms of me and my activities, which suggested that this issue was not significant for the case studies.

The situation in which I, the researcher was also the teacher for the mathematics education unit, may be considered by some as a disadvantage because of my subjectivity in viewing outcomes of classroom phenomenon. Yet this situation has an advantage in that I could investigate the experience from the teacher's viewpoint and as one in close continuing communication with the pre-service teachers. My reading of the theory of enactivism has made me more conscious that what I have investigated and written is consequently the result of whom I am. Maturana (1980, as cited in Begg,

2002) says that all our perceptions and experiences occur through and are mediated by our bodies and nervous systems and that it is therefore impossible to generate a pure description of reality independent of ourselves. Therefore if I had been an observer of the teacher, rather than the teacher, this would not have eliminated a subjective view of the findings.

Concluding statements

The mathematics education unit that I planned and implemented in this study was completed with the hope that it would assist the First Year pre-service teachers in the College in formally preparing to become effective teachers of mathematics. Results showed that it was successful in some respects. The numeracy of the pre-service teachers improved, their attitude to the doing of mathematics became more positive and their understanding of the nature of mathematics developed. As well there was some evidence that these changes in part can be contributed to the teaching of the mathematics education unit. All these changes suggest that the students have a more appropriate basis for further developing their numeracy in the next two years of their course before they graduate as teachers. Hopefully they also have developed understanding and skills to empower them to learn to learn and so become lifelong learners of among other things the effective strategies to apply in their teaching of mathematics.

I am also hopeful that the findings of the study may be of assistance to mathematics educators in the primary, secondary and tertiary section of education in the region of Fiji. Perhaps my attempt to help the pre-service teachers develop a numeracy to improve their life skills will suggest to them that they can similarly help their own students by considering using aspects of the teaching approach that I used. I would be very pleased if, in the numeracy of the pupils of the pre-service teachers in the study, I could see evidence of understanding that could be described using the words of Begg.

> "Understanding is not just about facts, results or ideas, but also knowing how to apply these ideas in practical situations and how to relate them to everyday life." (Begg, 2002, p.8).

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APPENDICES

Appendix A Test for Assessment of number sense

Time allowed: 15 minutes

Work through these questions as quickly as possible. You may return to questions you leave unanswered if you have time left.

Avoid spending extra time on questions you find difficult. Some questions are much easier than others.

Do any calculations mentally without the use of written work..

Choose the most appropriate answer by circling the letter if alternatives are provided, otherwise write your answer in the space provided.

1.	<u>About</u> how much of this box is shaded? Give your answer as a simple <u>fraction</u>.	__________________
2.	<u>About</u> how many dots are here?	A 100 B 200 C 500 D 1000
3.	The digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Put two different digits in the spaces so that the answer to the subtraction will be as big as possible.	431 − 2 __ __ = ?
4.	Here are five digits: 2, 6, 3, 5, 1. Arrange the digits to make the number nearest 20 000.	__________
5.	Pius has $50 and then spends $29. He is given $24 in change. Which sum could he do to check if this is the right change?	A 29 + 24 B 24 + 50 C 50 + 24 D 50 + 29

6.	For a long time Julie had been putting only 10 cent coins in her piggy bank. Last night she opened it and counted her money. She had $ 46.70. How many 10 cent coins were in the bank?	__________
7.	You are going to walk once around a square-shaped playing field. You start at the corner marked S and move in the direction shown by the arrow. On the diagram of the field mark with an X where you will be after 1/3 of your walk.	____ S ___________ ___________
8.	<u>Without calculating the exact answer</u> choose the largest product.	A 18 x 17 B 16 x 18 C 17 x 19 D 17 x 16
9.	Mary had $426 and spent 90 percent of the money on clothes. <u>Without calculating an exact answer,</u> choose the best estimate for how much she spent.	A slightly less than $426 B much less than $426 C slightly more than $426 D impossible to tell without calculating it
10.	Without calculating decide which is the greatest?	A 29 ÷ 0.89 B 29 x 0.89 C 29 + 0.89 D Impossible to tell without calculating
11.	When a 3-digit number is added to a 3-digit number the result is:	A always a 3-digit number B always a 4-digit number C always a 5-digit number D either a 3 or a 4-digit number
12.	Vicki is in Class 5 at my school. She says she is 30 000 days old. Is that possible? Say why?	A Yes B No C Maybe Why. _________________ ________________________

13.	Ten bottles of juice cost $7.95 at one store. I can get 5 bottles for $4.15 at a second store. Where is the juice cheaper – at the first or second store?	A First store B Second store Why? ______________________
14.	Which of these two numbers multiplied together give an answer closest to the target number? 4 18 50 37 Target number 1000	_____ and _____
15.	How many different decimals are there between 1.52 and 1.53? Choose your answer, then fill in the blank.	A None. B One. What is it? _____ C A few. Give two: _______ and _______ D Many. . Give two: _______ and _____
16.	<u>Without calculating</u>, which total is more than 1 ?	A 2/5 + 3/7 B 1/2+ 3/7 C 3/8 + 2/11 D 4/7 + 1/2
17.	<u>Without calculating</u>, decide which one of these answers is <u>not</u> reasonable.	A 45 x 1.05 = 39.65 B 45 x 6.5 = 292.5 C 87 x 1.076 = 93.612 D 589x 0.95 = 559.55
18.	93 x 134 is equal to 12 462. Use this to write the answer to the following: 12 462 ÷ 930	_______________
19.	The farmer had stored all his apples in 80 boxes with 40 apples in each box. He now needs to repack all of them evenly into 40 new boxes. How many apples will there be in each new box?	A 2 B 20 C 40 D 80 E 160
20.	<u>Without calculating</u>, choose the expression which represents the larger amount?	A 145 x 4 B 144 + 146+145+144 C 145 x 5 - 144 *D 145 x 3 + 144*

Appendix B Test of algorithms for the operations

Name . . . .

1. 4 5 6
   + 7 8

2. 3 8 7
   - 1 8 9

3. 1 2 3
   x 4 5

4. 5 160 ÷ 5

5. 2/3 + 3/5

6. 5/18 ÷ 5/9

7. 1.2 x 0.02

8. 6.9 ÷ 0.3

9. 13 480 ÷ 46

Appendix C Questionnaire of Beliefs about mathematics

Tick the response that most closely expresses how you think about mathematics
SA= strongly agree A= agree U= uncertain D= disagree SD= strongly disagree

		SA	A	U	D	SD
1	*Mathematics consists of a set of rules and procedures.					
2	Understanding the process is more important than correct answers in mathematics					
3	*There is always a rule to follow in doing a mathematics calculation.					
4	Maths is useful in solving everyday problems.					
5	*Learning maths involves mostly memorising rules.					
6	A maths calculation can be solved in different ways.					
7	*To be successful in mathematics it is important to memorise rules.					
8	*Maths is not needed in everyday living.					
9	*There is little place for originality in doing maths calculations.					
10	In maths questions can be answered without using rules.					
11	*Doing maths means following standard procedures precisely.					
12	There are many different ways to do most calculations.					
13	*In maths it is important that students remember facts.					
14	In maths it is important that students are able to answer unfamiliar questions.					
15	In maths it is important that students are able to be flexible in their thinking.					
16	In maths it is important that students are able to do calculations without a calculator.					
17	In maths it is important that students are able to estimate.					
18	In maths it is important that students are able to see patterns and relationships.					
19	*In maths it is important that students are able to tackle a question once it has been explained.					
20	In maths it is important that students are able to show initiative and think creatively.					
21	*In maths it is important that students are able to follow routine instructions.					
22	*In maths it is important that students are able to memorise rules.					

Appendix D Test questions for interview

(The questions were written on separate cards)

1. Petero dropped a coconut from a tall coconut palm. About how long will the coconut take to reach the ground?
   1 sec.? 5 sec.? 10 sec.? or 15 sec.?

2. What mathematical working can be used to simplify the following:
   i. 4 + 4 + 4 + 4 + 4 =
   ii. How many 35's can be subtracted from 455 before we reach zero?

3. How could these calculations be done in a simple way;
   i. 17 + 17 + 17 + 17 + 13 + 13 + 13 + 13
   ii. 22 x 16 - 22 x 6
   iii. 234 x 99
   iv. 2 x 18 x 5
   v. 25x 62 x 4
   vi. 2.5 x 4 x 0

4. Without calculating decide if <, >, or = completes each expression. Explain your reasoning.

i.	6 x 14 x 25	14 x 25 x 6
ii.	34 + 36 + 39	3 x 36
iii.	101 x 54	54 x 100 + 54 x 1

5. How many digits in the numbers 1 to 100 inclusive?

6. What is the approximate number of students needed to stand shoulder to shoulder to form a line 1 km long?

7. Think of the number 264. What could you add to or subtract from this number to change the 6 into a 0? What other answers could you give?

8. How long would it take to count to a million? Give you reasoning.
(This question was not used in the interviewing.)

9. Consider the following numbers on the cards and answer the questions below:

   456, 23, 237, 194, 854, 1234, 511, 299

i. Choose the smallest and the largest values

ii. Arrange them in order of magnitude, beginning with the smallest

iii. Which pair of numbers is the closest together in value?

iv. What is the difference between these close values?

v. If we counted backwards by tens from 456 to just pass 299, how many tens would you count?

10. a. Subtract 714 from 1000. How could you check your answer?

b. Think of a one-digit number, double it and multiply the result by 5, then divide by 10. Is your result the number you began with? Explain how I knew.

c. Make up a similar trick using addition, subtraction each.

d. Think of a one-digit number. Add 6 to it. Multiply it by 2. subtract twice the number you first thought of. Is your answer 12? Explain how I knew.

11. Can you think of a fraction between each of the following pairs? If so, give an example and explain your reason for the choice of example.

i. 3/10 and 7/10

ii. 1/3 and 1/6

iii. 3/4 and 6/7

iv. 2/5 and 3/5

12.a. If the product 127 x ___ ends in a six what can you say about the missing number ?

b. If the product 127 x ___ ends in a six and you know the number is less than 4000 what else can you say about the missing number?

13. Consider the following number line and the scale marked on it. Approximately what values would the points marked with letters have?

_0_____________A_________1________________________2_

_0______________________1_____D___________C_____2_

14. a. What is the total value of the heap of coins:

b. What is the smallest number of coins for a value of:$3.85?

c. If a $10 note is used to pay for a shirt costing $7.14, what would the change be? Describe your working.

d. What coins need to be given as change if the smallest number of coins is used? Describe your working.

15.a. Name a decimal, a percentage and a common fraction that all have the same value.

b. Arrange in order of size the values on the cards beginning with the smallest:

i. 0.4, 3/4, 60%

ii. 120% 0.4 1/2 3/4 60% 0.09 1/10 1 ¼

16.a. Consider the numbers 43, 69. Decide which of the following numbers is the best approximation of their product.

2800 1100 3000 2400 3500

17. The sum of five two-digit numbers is less than 100. For each of the following statements, decide, explaining your reasoning whether it is:

A always true, B sometimes true, C never true.

i. Each number is less than 20

ii. One number is greater than 60

iii. Four numbers are greater than 20 and one is less than 20.

iv. If two numbers are less than 20, at least one is greater than 20.

v. If all 5 numbers are different, then their sum is greater than or equal to 60.

18. Show by writing four different ways of calculating - 32 multiplied by 14.

19. Take a look at this packaging of straws. Here are less than 10 loose straws. In the each of the envelopes there are 10 straws and in these plastic bags there are sets of 10 full envelopes. The plastic bags of envelopes in groups of 10 are placed in these boxes. Using this packaging system of envelopes, plastic bags and boxes, and loose straws, to answer the following questions:

i. How many straws are in the plastic bags and in the boxes

ii. How many straws are here? (5 envelopes and 3 loose straws)

iii. How many straws are here? (3 envelopes and 2 plastic bags)

iv. How many straws are here? (1 box, 2 plastic bags, 6 envelopes and 8 loose straws)

v. How would the following number of straws be fully package?

a. 45 straw

b. 2305 straws

20. 'The ones that got away'

In trying to estimate the number of fish in a pond, a wildlife officer caught 50 fish, put a brightly coloured tag on each one and returned them to the lake. The next day he caught 30 fish and noticed that 10 of them were wearing the tags. What would be your estimate of the total number of fish in the lake? Explain your reasoning.

22. Estimate how many lollies are in this jar. Explain the reasoning you used to reach your estimate.

Now you are finished. Thank you. You can have a few lollies

Appendix E First page of the answer sheet for interview activities

1. 1 sec.?   5 sec.?   10 sec.?   or   15 sec.?

2. What mathematical working can be used to simplify the following?

i.

ii.

3. How could these calculations be done in a simple way?

i.

ii.

iii.

iv.

v.

vi.

4. Without calculating decide if <, >, or = completes each expression. Explain your reasoning.

i. 6 x 14 x 25   14 x 25 x 6

ii. 34 + 36 + 39   3 x 36

iii   101 x 54   54 x 100 + 54 x 1

Appendix F Permission form for participation

Corpus Christi Teachers' College Research Project
Student Permission Form

Corpus Christ Teachers' College has received a research grant from the Pacific Educator's Research Development Scheme to investigate the implementation of a new unit in the Mathematics Department. This unit will be designed to assist students in their future teaching of mathematics in the rapidly-changing primary school classrooms. It is also anticipated that it will specially benefit those students who are experiencing difficulties and lack of confidence in their study of mathematics. The research is expected to start at the beginning of 1999 and be completed in 2000.

The work of the research by the lecturer will take the form of the production, implementation and evaluation of a new mathematics unit. This work will be little different from the normal process carried out by a lecturer when implementing a new unit. It will differ in that the work has funds to assist the research and a formal report of the work will be prepared for the College and available to those interested in the findings for further research or other educational purposes.

Your participation will not involve anything special other than the expected participation in any other mathematics unit of work. The progress of the unit will be carefully monitored using questionnaires, tests, interviews and normal evaluation procedures, with opportunities for you to present your views via spoken or written comments.

The actual names of the students who are part of the class that is doing the new unit will not be revealed to anyone outside the College, and no reports from the project will enable anyone to identify the work or other contributions of individual students.

We will be most grateful if you could retain one signed copy of the form below and return the other to your lecturer. Any questions or queries regarding this study can be directed to myself.

Thanking you in anticipation

Yours sincerely,

Ms Fran Hanrahan

Lecturer in charge of the research

STUDENT PERMISSION FORM FOR PARTICIPATION IN C.C.T.C. RESEARCH

I . . . . will participate in the research.

I understand that I will not be identified by name in any presentation or published reports regarding the research.

Signature: . . . .

Appendix G Instructions to the participants & Questionnaire for details

- This test is to help teacher educators determine some of the mathematical educational needs of pre-service teachers.
- The test is not part of your formal assessment and will not be corrected by your lecturer.
- The time allowed will probably enable you to complete all the questions in reasonable time.
- The questions cover many areas and levels of number concepts so you will find some questions much easier than others.
- Choose and use any methods you wish to get your answers. School-learnt methods are not always the best methods for a person to use.
- Scribble on the paper near the questions if you need working space rather than on other sheets.
- Do the test carefully, yet as quickly as you can.
- Use short cut methods if they help you to work efficiently.

Personal information

Name or Identification (the results are confidential and not part of your assessment).
. . . . Date .

Educational institution
. . . . .Course or group . . . .

Nationality . . . . Culture . . . . Region . . . . . .

Tick the appropriate information

1. Male . . . . . Female . . . . .

2. Number of years of schooling that included learning mathematics

Less than 10 . . 10 . . 11 . . 12 . . 13 . . 13+ . .

3. Age in years

Less than 17 . . 17-19 . . 20-25 . . . 25+ . .

Appendix H Assessment of Thinkbook

Maximum marks 10

Presentation

1. *Due date: The work will be collected and assessed a number of random times throughout the semester.*
2. Entries are to be made a few times each week.
3. Each entry will contain deep reflections on mathematics ideas and understanding of the work done in class or other work for mathematics.
4. Each entry should be at least 100 words.
5. Marks will be credited for quality and consistency in doing the work.
6. More detail of the writing and its assessment will be given in class.

Appendix I Sample of page from a Thinkbook

(See hard copy in archives of ACU Library)

Appendix J Permission from the Permanent Secretary, Fiji

(See hard copy in archives of ACU Library)

Appendix K Unit description for semester 1

Corpus Christi Teachers' College
MS101 General Mathematics Methods
Unit Description, Year 1, Semester 1, 1999

Objectives

This unit will assist students to:

- Investigate their number sense and explore ways to develop it.
- Revise basic mathematics concepts of number and the four operations
- Produce and use materials to deepen their understanding of basic mathematics
- Practice good teaching skills
- Practice basic research skills

Content

Number sense experiences in –

- Ordinal, cardinal and rational numbers
- Place value and patterns in the number system
- Operations: addition, subtraction, multiplication, division.
- Mental computations
- Estimation

Traditional number sense
Cooperative group work
Production and use of materials to make models of mathematical processes
Principles of action research

Assessment
(For details see sheet attached.)

1. Essay
2. Production of teaching materials.
3. Think Book writing.
4. Group presentation of an activity from the primary mathematics course related to the material being studied.
5. Test.

References

Bakalevu, S. (1997, July*). A different system: notions of Education and ways of mathematising in Fijian society.* Conference paper presented at MERGA 20. Rotorua, New Zealand. The Mathematics Education Research Group of Australasia.

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Gough, J., Lilburn, P. Rawson, P. & Sullivan, P. (1991). *Community school mathematics: teacher's resource books, pupils books for classes 1,2 and 3.* Papua New Guinea, Oxford Press Department of Education.

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Appendix L Unit description for semester 2

Corpus Christi Teachers' College
MS102 General Mathematics Methods
Unit Description, Year 1, Semester 2, 1999

Objectives

This unit will assist students to:

- Study the prescriptions and texts for lower primary classes
- Reflect on the aims and objectives of teaching mathematics
- Design and present activities for lower primary classes through peer teaching and some practical experience in local schools.
- Produce teaching materials

Content

- Lesson planning
- Warm-up activities
- Traditional mathematics and language issues
- Sorting and classifying familiar objects by shape, colour and size
- Ordering and patterns Shape, movement and position
- Counting, Number and numeration, Operations, Measurement, Problem solving
- Assessment techniques of lower primary children
- Technology: calculators in lower primary mathematics

Assessment

1. Production of teaching materials
2. Presentation of warm-up activity
3. File of warm-up activities
4. Thinkbook entries
5. Test

References

Bakalevu, S. (1997, July*). A different system: notions of Education and ways of mathematising in Fijian society.* Conference paper presented at MERGA 20. Rotorua, New Zealand. The Mathematics Education Research Group of Australasia.

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Appendix M Percentage correct for Assessment of number sense (N=36)

No.	Content of item	Feb	Jul	Oct
1	Estimation of area –fractional	86.1	100	94.4
2	Estimation of numeracy	27.8	58.3	66.7
3	Effects of subtraction	22.2	58.3	61.1
4	Number- place value	16.7	38.9	33.3
5	Verifying subtraction of money	69.4	88.9	88.9
6	Place value –money	38.9	58.3	66.7
7	Estimation of distance –fractional	19.4	25.0	27.8
8	Estimation -operation multiplication	80.6	88.9	94.4
9	Percentage	41.7	47.2	52.8
10	Decimals -effect of operations	19.4	58.3	50.0
11	Effect of addition –whole numbers -	50.0	83.3	86.1
12	Estimation of time –days and years	38.9	47.2	33.3
	Explanation of answer	19.4	25.0	19.4
13	Money calculation	88.9	94.4	97.2
	Explanation of answer	44.4	75.0	55.6
14	Estimation –multiplication	33.3	44.4	83.3
15	Decimals -continuous nature	13.9	13.9	8.3
	Explanation of answer	5.6	5.6	11.1
16	Estimation for addition -fractions	50.0	58.3	61.1
17	Decimals -effects of multiplication	22.2	36.1	22.2
18	Inverse operations	22.2	38.9	38.9
19	Application –rearranging of groups	30.6	58.3	69.4
20	Estimation – computations	2.8	33.3	44.4

Appendix N Mean scores for responses to Questionnaire for Beliefs about mathematics (N=36)

No	Item	Feb	Jul	Oct
1	*Mathematics consists of a set of rules and procedures.	4.4[a]	3.8	3.8
2	Understanding the process is more important than correct answers in mathematics	1.4	1.1	1.3
3	*There is always a rule to follow in doing a mathematics calculation.	4.3	3.4	3.1
4	Maths is useful in solving everyday problems.	2.8	2.6	2.3
5	*Learning maths involves mostly memorising rules.	3.3	2.3	2.3
6	A maths calculation can be solved in different ways.	1.9	1.8	1.7
7	*To be successful in mathematics it is important to memorise rules.	3.1	2.5	2.5
8	*Maths is not needed in everyday living.	2.5	1.9	1.8
9	*There is little place for originality in doing maths calculations.	2.9	2.8	2.5
10	In maths questions can be answered without using rules.	3.4	2.8	2.4
11	*Doing maths means following standard procedures precisely.	4.0	3.0	2.6
12	There are many different ways to do most calculations.	2.0	1.7	1.8
13	*In maths it is important that students remember facts.	3.9	3.8	3.8
14	In maths it is important that students are able to answer unfamiliar questions.	2.0	3.1	3.0
15	In maths it is important that students are able to be flexible in their thinking.	2.1	1.8	1.8
16	In maths it is important that students are able to do calculations without a calculator.	2.2	1.8	2.4
17	In maths it is important that students are able to estimate.	2.6	1.9	1.8
18	In maths it is important that students are able to see patterns and relationships.	1.9	1.4	1.6
19	*In maths it is important that students are able to tackle a question once it has been explained.	4.0	3.3	3.1
20	In maths it is important that students are able to show initiative and think creatively.	2.1	1.8	1.6
21	*In maths it is important that students are able to follow routine instructions.	4.0	3.6	3.4
22	*In maths it is important that students are able to memorise rules.	3.4	2.8	2.6

[a] Low score indicates positive attitude to constructivist ideas.

* Items negatively worded for constructivist approach.- original scores have been reversed

Appendix O Change in mean scores for responses to Questionnaire Jul-Oct(N=36)

No	Item	Jul	Oct	Change Jul-Oct
1	*Mathematics consists of a set of rules and procedures.	3.8	3.8	0.0#
2	Understanding the process is more important than correct answers in mathematics	1.1	1.3	-0.2
3	*There is always a rule to follow in doing a mathematics calculation.	3.4	3.1	0.3
4	Maths is useful in solving everyday problems.	2.6	2.3	0.3
5	*Learning maths involves mostly memorising rules.	2.3	2.3	0.0
6	A maths calculation can be solved in different ways.	1.8	1.7	0.1
7	*To be successful in mathematics it is important to memorise rules.	2.5	2.5	0.0
8	*Maths is not needed in everyday living.	1.9	1.8	0.1
9	*There is little place for originality in doing maths calculations.	2.8	2.5	0.3
10	In maths questions can be answered without using rules.	2.8	2.4	0.4
11	*Doing maths means following standard procedures precisely.	3.0	2.6	0.4
12	There are many different ways to do most calculations.	1.7	1.8	-0.1
13	*In maths it is important that students remember facts.	3.8	3.8	0.0
14	In maths it is important that students are able to answer unfamiliar questions.	3.1	3.0	0.1
15	In maths it is important that students are able to be flexible in their thinking.	1.8	1.8	0.0
16	In maths it is important that students are able to do calculations without a calculator.	1.8	2.4	-0.6
17	In maths it is important that students are able to estimate.	1.9	1.8	-0.1
18	In maths it is important that students are able to see patterns and relationships.	1.4	1.6	-0.2
19	*In maths it is important that students are able to tackle a question once it has been explained.	3.3	3.1	0.2
20	In maths it is important that students are able to show initiative and think creatively.	1.8	1.6	0.2
21	*In maths it is important that students are able to follow routine instructions.	3.6	3.4	0.2
22	*In maths it is important that students are able to memorise rules.	2.8	2.6	0.2

[a] Low score indicates positive attitude to constructivist ideas.
 * Items negatively worded for constructivist approach.- original scores have been reversed
Positive indicates a change to a more positive constructivist approach

Appendix P Responses to Questionnaire by Ana

		Feb	Jul	Oct
1	Mathematics consists of a set of rules and procedures.	SA[a]	SA	A
2	Understanding the answer is more important the correct answers in mathematics	SA	SA	SA
3	There is always a rule to follow in doing a mathematics calculation.	SA	A	A
4	Maths is useful in solving everyday problems.	A	A	A
5	Learning maths involves mostly memorising rules.	SA	A	U
6	A maths calculation can be solved in different ways.	D	A	A
7	To be successful in mathematics it is important to memorise rules.	D	D	D
8	Maths is not needed in everyday living.	SD	D	D
9	There is little place for originality in doing maths calculations.	A	A	D
10	In maths questions can be answered without using rules.	D	D	D
11	Doing maths means following standard procedures precisely.	SA	A	D
12	There are many different ways to do most calculations.	A	A	A
13	In maths it is important that students remember facts.	A	A	A
14	In maths it is important that students are able to answer unfamiliar questions	D	A	D
15	In maths it is important that students are able to be flexible in their thinking\	A	A	A
16	In maths it is important that students are able to do calculations without a calculator.	SA	SA	A
17	In maths it is important that students are able to estimate.	A	A	A
18	In maths it is important that students are able to see patterns and relationships.	A	A	A
19	In maths it is important that students are able to tackle a question once it has been explained.	A	A	D
20	In maths it is important that students are able to show initiative and think creatively.	U	A	A
21	In maths it is important that students are able to follow routine instructions.	SA	A	U
22	In maths it is important that students are able to memorise rules.	A	A	

[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.

Appendix Q Responses to Questionnaire bt Lisa

		Feb	Jul	Oct
1	Mathematics consists of a set of rules and procedures.	A	D	A
2	Understanding the process is more important than correct answers in mathematics	SA	SA	SA
3	There is always a rule to follow in doing a mathematics calculation.	SA	A	A
4	Maths is useful in solving everyday problems.	A	A	A
5	Learning maths involves mostly memorising rules.	A	D	SD
6	A maths calculation can be solved in different ways.	SA	SA	A
7	To be successful in mathematics it is important to memorise rules.	SA	SD	SD
8	Maths is not needed in everyday living.	SD	SD	SD
9	There is little place for originality in doing maths calculations.	U	D	SD
10	In maths questions can be answered without using rules.	SD	A	SA
11	Doing maths means following standard procedures precisely.	SA	D	D
12	There are many different ways to do most calculations.	SA	D	SA
13	In maths it is important that students remember facts.	SA	A	A
14	In maths it is important that students are able to answer unfamiliar questions.	A	D	D
15	In maths it is important that students are able to be flexible in their thinking.	SA	A	A
16	In maths it is important that students are able to do calculations without a calculator.	A	D	D
17	In maths it is important that students are able to estimate.	SA	SA	SA
18	In maths it is important that students are able to see patterns and relationships.	SA	A	SA
19	In maths it is important that students are able to tackle a question once it has been explained.	SA	D	D
20	In maths it is important that students are able to show initiative and think creatively.	A	A	SA
21	In maths it is important that students are able to follow routine instructions.	A	A	A
22	In maths it is important that students are able to memorise rules.	SA	D	D

[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.

Appendix R Responses to Questionnaire by Wani

		Feb	Jul	Oct
1	Mathematics consists of a set of rules and procedures.	A	A	A
2	Understanding the process is more important than correct answers in mathematics	SA	SA	SA
3	There is always a rule to follow in doing a mathematics calculation.	A	SA	A
4	Maths is useful in solving everyday problems.	SA	SA	SA
5	Learning maths involves mostly memorising rules.	D	D	D
6	A maths calculation can be solved in different ways.	SD	A	A
7	To be successful in mathematics it is important to memorise rules .	SD	SD	SD
8	Maths is not needed in everyday living.	A	SD	D
9	There is little place for originality in doing maths calculations.	D	D	A
10	In maths questions can be answered without using rules.	U	SA	A
11	Doing maths means following standard procedures precisely.	A	D	D
12	There are many different ways to do most calculations.	SA	SA	SA
13	In maths it is important that students remember facts.	A	A	A
14	In maths it is important that students are able to answer unfamiliar questions.	U	A	D
15	In maths it is important that students are able to be flexible in their thinking.	SA	SA	A
16	In maths it is important that students are able to do calculations without a calculator.	D	A	D
17	In maths it is important that students are able to estimate.	SD	A	A
18	In maths it is important that students are able to see patterns and relationships.	U	A	A
19	In maths it is important that students are able to tackle a question once it has been explained.	D	D	D
20	In maths it is important that students are able to show initiative and think creatively.	SA	SA	SA
21	In maths it is important that students are able to follow routine instructions.	D	D	A
22	In maths it is important that students are able to memorise rules.	A	SD	A

[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.

Appendix S Responses to Questionnaire by Vika

		Feb	Jul	Oct
1	Mathematics consists of a set of rules and procedures.	SA	SA	A
2	Understanding the process is more important than correct answers in mathematics	SA	SA	SA
3	There is always a rule to follow in doing a mathematics calculation.	SA	SA	D
4	Maths is useful in solving everyday problems.	A	A	A
5	Learning maths involves mostly memorising rules.	SD	SD	SD
6	A maths calculation can be solved in different ways.	A	A	A
7	To be successful in mathematics it is important to memorise rules.	SD	D	SD
8	Maths is not needed in everyday living.	D	D	D
9	There is little place for originality in doing maths calculations.	D	A	D
10	In maths questions can be answered without using rules.	A	D	A
11	Doing maths means following standard procedures precisely.	SA	A	D
12	There are many different ways to do most calculations.	A	A	A
13	In maths it is important that students remember facts.	SA	A	D
14	In maths it is important that students are able to answer unfamiliar questions.	SD	D	D
15	In maths it is important that students are able to be flexible in their thinking.	SA	SA	SA
16	In maths it is important that students are able to do calculations without a calculator.	SA	SA	SA
17	In maths it is important that students are able to estimate.	SA	SA	SA
18	In maths it is important that students are able to see patterns and relationships.	SA	SA	SA
19	In maths it is important that students are able to tackle a question once it has been explained.	SA	A	A
20	In maths it is important that students are able to show initiative and think creatively.	SA	SA	SA
21	In maths it is important that students are able to follow routine instructions.	SA	SA	A
22	In maths it is important that students are able to memorise rules.	SD	SD	SD

[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.

Appendix T Responses to Questionnaire by Wili

		Feb	Jul	Oct
1	Mathematics consists of a set of rules and procedures.	A	A	A
2	Understanding the process is more important than correct answers in mathematics	SA	SA	SA
3	There is always a rule to follow in doing a mathematics calculation.	A	D	D
4	Maths is useful in solving everyday problems.	U	SA	A
5	Learning maths involves mostly memorising rules.	D	SD	SD
6	A maths calculation can be solved in different ways.	U	SA	A
7	To be successful in mathematics it is important to memorise rules.	D	U	D
8	Maths is not needed in everyday living.	A	SD	SD
9	There is little place for originality in doing maths calculations.	U	U	D
10	In maths questions can be answered without using rules.	A	A	A
11	Doing maths means following standard procedures precisely.	SA	D	D
12	There are many different ways to do most calculations.	D	A	A
13	In maths it is important that students remember facts.	A	D	U
14	In maths it is important that students are able to answer unfamiliar questions.	A	D	U
15	In maths it is important that students are able to be flexible in their thinking.	SA	U	A
16	In maths it is important that students are able to do calculations without a calculator.	SA	A	U
17	In maths it is important that students are able to estimate.	D	U	A
18	In maths it is important that students are able to see patterns and relationships.	A	SA	SA
19	In maths it is important that students are able to tackle a question once it has been explained.	SA	A	A
20	In maths it is important that students are able to show initiative and think creatively.	A	A	U
21	In maths it is important that students are able to follow routine instructions.	SA	U	A
22	In maths it is important that students are able to memorise rules.	U	SD	D

[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.

Appendix U Responses to Questionnaire by Jo

		Feb	Jul	Oct
1	Mathematics consists of a set of rules and procedures.	A	SA	A
2	Understanding the process is more important than correct answers in mathematics	SA	SA	SA
3	There is always a rule to follow in doing a mathematics calculation.	SA	SA	D
4	Maths is useful in solving everyday problems.	A	A	A
5	Learning maths involves mostly memorising rules.	D	SD	SD
6	A maths calculation can be solved in different ways.	SA	SA	A
7	To be successful in mathematics it is important to memorise rules.	U	SD	SD
8	Maths is not needed in everyday living.	D	SD	SD
9	There is little place for originality in doing maths calculations.	A	D	D
10	In maths questions can be answered without using rules.	D	A	SA
11	Doing maths means following standard procedures precisely.	D	D	D
12	There are many different ways to do most calculations.	A	SA	A
13	In maths it is important that students remember facts.	SA	D	D
14	In maths it is important that students are able to answer unfamiliar questions.	SD	D	U
15	In maths it is important that students are able to be flexible in their thinking.	A	SA	U
16	In maths it is important that students are able to do calculations without a calculator.	U	SA	D
17	In maths it is important that students are able to estimate.	D	A	D
18	In maths it is important that students are able to see patterns and relationships.	SA	SA	A
19	In maths it is important that students are able to tackle a question once it has been explained.	A	A	U
20	In maths it is important that students are able to show initiative and think creatively.	U	A	U
21	In maths it is important that students are able to follow routine instructions.	D	A	D
22	In maths it is important that students are able to memorise rules.	A	SD	SD

[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.

Appendix V The learning experience for Lisa

Lisa

Lisa described her culture as a combination of indigenous Fijian and Solomon Islander. Her age was in the range of 17-19 years. She reported, but did not explain, that she had completed more than thirteen years of schooling involving mathematics learning. Possibly she had begun another tertiary course. In the information gathered for her application to gain a place in the College, she was ranked among the students who had a better than average numeracy standard. In class she participated well, asking and answering many questions. She wrote much in her Thinkbook and often used expressive language about the learning experience.

The quantitative data I collected from the administering of the research instruments is shown in Table 6.15. As can be seen from the scores for the number sense assessment Lisa is above average in all three assessments, though she was one of the few whose score dropped for the October result. For the first two tests for algorithms she gained a score of 16/18 indicating that she had one complete question incorrect, but for the third she obtained the maximum score.

Table 6.15
Quantitative data – Lisa

Instrument		Number sense assessment			Test of algorithms		
Month		Feb	Jul	Oct	Feb	Jul	Oct
Scores	Max	23	23	23	18	18	18
	Mean	8.4	12.4	12.6	10.4	12.2	13.1
	Lisa	13	17	16	16	16	18

Her responses to the questionnaire of *Beliefs about mathematics* show that she had a positive disposition to constructivist ideas at the beginning of the year. This changed considerably to become more positive than the class means for the two later administrations of the questionnaire. Details of all the results are discussed later in the report of this case study.

Enculturation into the mathematics education classroom

In this section I will describe the process of enculturation that Lisa experienced as she began the mathematic education units at the College. What she saw as notable, interesting and different from her past education will be discussed, mainly using entries from her Thinkbook. To examine how her beliefs during the Teaching Program relate to a constructivist view of mathematics learning I will use her responses to the questionnaire of *Beliefs about mathematics* as well as her Thinkbook entries.

Lisa appeared to settle into the mathematics unit quickly with its many new experiences for her during the first month:

> "At the moment I really like going to maths class which is really peculiar for me because back in high school days, maths used to be a fairly frightening class for me" (18/2).
>
> "Sometimes I would say to myself that maths is really an interesting subject. Maths is fun and should be enjoyed because one doesn't deal with words a lot but with numbers" (3/3).

Numerous times in her Thinkbook she described her interest in and surprise in the activities completed in class. These included activities such as writing in a mathematics class, and the classification of numbers:

> "I found that really surprising because never once in my life, I've written a maths essay" (18/2).
>
> "Today's maths class was really really interesting. It was the first time I realised that ordinary numbers are categorised according to their respective meaning [ordinal and cardinal]" (9/3)

This seemed to indicate that mathematics classes in the past did not have such interesting activities. Sometimes the surprise was the result of the discovery of the meaning of known facts and at other times the novelty of experiencing exercises that she had not associated with mathematics such as writing in a mathematics class. Lisa was surprised at being trusted to work unsupervised by the staff and equally surprised that the students responded well. From what was described in the literature review, this experience was in contrast to what was generally the expected behaviour of both teachers and students in many secondary schools:

> "Today's class was ace Ms Fran. Thanks for allowing us to work on our 10 cubes and Deines' blocks on our own. Thanks a lot for the trust. Well everyone sort of caught up on the message (trust /reliability) and started off straight away on cutting cubes, flats, longs, etc. It was wonderful seeing everyone having something to do" (22/4).

Perhaps, also because she compared the work done in secondary school mathematics classes to the work done during the mathematics units she noted that what we did was simple and interesting:

> "Well Ms Fran this is the first time I have seen the different patterns for each multiplication table. It's really interesting to see them. Anyway that is new for me" (26/5).
>
> "While I was going through it [her maths file] the night before, I just made a swift glance of the notes and realised that we have covered quite a lot of topics. Most of these topics were simple yet very interesting" (7/10).

Though Lisa was successful at mathematics in secondary school she described negatively her experiences of school mathematics classes. She had apparently not investigated strategies nor gained many personal insights into the mathematics she was doing. It may have been a lack of reflecting on mathematics and its associated skills that meant she was frightened by mathematics and was weak at doing computations mentally. She was apparently conscious of the difference in teaching methods and her learning at the College compared with her previous experiences at secondary school:

> "Well Fran I really like the style and method of learning you're teaching on the times tables, especially the hard tables, i.e, 7, 8 and 9. Particularly the steps we followed when learning each table [listing the multiples]. I knew all these things but I never thought or realised them" (20/5).
>
> "Often it is the way mathematics is taught rather than the nature of mathematics that creates learning difficulties" (9/9).

> "I was really struck by the story of creativity. [An anonymous poem about a small boy's struggles to be enculturated into different schools]. I feel that most students nowadays are like the boy in the story. Because most teachers still stick to the old method preventing students from creating their individual ideas. Some teachers nowadays should allow their pupils in exploring their skills and talents. This way the students will have the enthusiasm to learn and think for themselves" (10/4).

Two of the teaching methods that were new to her were problem solving and cooperative group work. She noted the importance of problem solving and came to understand early in the Teaching Program that mathematics problems could be solved in many ways. Cooperative group work was also a method that Lisa enjoyed and she saw it as demanding greater participation than individual activities did:

> "We had a lesson on problem solving. I learnt that this topic is not distinct but is present in the entire maths curriculum" (6/9).

> "In this lesson I learnt the various methods one could apply in getting the solution and it is indeed interesting and fun" (24/2).

> "Ms F, I like working in groups because I find it really helpful and easier too. This way those who are slow or weak would be able to get help from their friends, especially if one had a particular doubt on a topic. (21/4).

Lisa's ideas about the teaching approach she experienced during the Teaching Program were also investigated indirectly by means of the questionnaire for *Beliefs about mathematics* (see Appendix C). The questionnaire aimed to examine before and after the Teaching Program, and at the end of the year, her beliefs related to constructivist ideas as expressed in the items (Appendix Q). In Table 6.16 Lisa's most notable sets of responses are given. They include groups of items for which first, her responses indicated a continuing agreement with constructivist ideas, second, a major shifts in her beliefs to a more positive agreement to a constructivist view, and finally two items indicating a move away from a constructivist view.

As for the ideas from the session on the Theory of Constructivism Lisa was the only class member who mentioned it in her Thinkbook after the discussion we had about it in class. From my knowledge of the class it may well be she was the only one who was able to grasp enough of it to commit the ideas in writing in English. But she had shown indications of understanding the ideas some time earlier:

> "I also learnt the construction (sic) topic and that is it is a theory of learning in which it is said that a child constructs/builds their own knowledge that links to their past knowledge" (18/10).

> "I really like the way you said about teachers. They are not there to teach but to help children to learn. I think that is more challenging than teaching" (15/3).

> "Sometimes it is very hard to picture what is going on in a child's mind in maths. But learning the way children think in maths is really interesting, especially the steps one has to follow and the patience one has to have in order to teach them" (2/8).

Even though she experienced a schooling in mathematics that was very different from the one Lisa was experiencing in the mathematics education units she appears to have been open to new ideas. Her beliefs about learning and teaching mathematics appeared to change during the Teaching Program. She reflected on ideas and seemed to develop new beliefs that are more positive towards a constructivist approach. She now believes it is important for teachers to allow students to build their own knowledge on what they already know, using original methods rather than relying on rules and procedures she had been expected to memorise.

Lisa also reflected on many aspects of mathematics education such as gender issues and ones of culture.

> "Learning the counting numbers in vernacular is really interesting, especially the Hindi ones. Before we began this topic I knew how to say the Indian numbers from 1 to 10 but now I can also write them in Hindi which is really good. As a teacher-to-be this would help me understand my Indian children" (4/8).

Table 6.16
Notable responses to Questionnaire – Lisa

	Feb	Jul	Oct
Strongly held constructivist view			
Understanding the process is more important than correct answers in mathematics	SA [a]	SA	SA
In maths it is important that students are able to estimate.	SA	SA	SA
In maths it is important that students are able to see patterns and relationships.	SA	A	SA
*Maths is not needed in everyday living.	SD	SD	SD
Shift towards constructivist view			
*Learning maths involves mostly memorising rules.	A	D	SD
*To be successful in mathematics it is important to memorise rules.	SA	SD	SD
*Doing maths means following standard procedures precisely.	SA	D	D
*In maths it is important that students are able to memorise rules.	SA	D	D
*In maths it is important that students are able to tackle a question once it has been explained.	SA	D	D
*There is little place for originality in doing maths calculations.	U	D	SD
In maths, questions can be answered without using rules.	SD	A	SA
Shift away from constructivist view			
In maths it is important that students are able to do calculations without a calculator.	A	D	D
In maths it is important that students are able to answer unfamiliar questions.	A	D	D

[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.
*Items negatively worded towards constructivist ideas

Lisa's acceptance of different cultures may have been influenced by her own mix of cultures because of her Fijian and Solomon Islander ancestry. This may also have accounted for her previous knowledge of and new interest in numbers in other languages. With the widespread and publicised racism in Fiji that was prevalent during the time of the study this written comment was somewhat out of the ordinary indicating an openness to other cultures. She was also open to ideas about gender in mathematics. After the class in which we considered gender issues in the learning of mathematics there were a great variety of interpretations of the discussion, but Lisa was the only one who had considered girls as better at mathematics.

> "Ms Fran I did not realise about how boys and girls learnt maths. I thought also girls were good in maths (in my school). I did not know that boys learn through discovering" (1/7).

Not only these issues caught Lisa's attention but, as was true for many other students, attractive materials in her classroom environment also stimulated her interest:

> "I really liked the idea of an abacus [home-made counting frame] What caught my attention was the brightly-coloured beads you used. They would enjoy, have fun and learn at the same time" (15/9).

> "Your material [attribute material] is so colourful and it's pretty interesting" (10/6).

> "On top of that it is also enjoyable to draw in different coloured lines" (25/5).

Colourful objects and interesting ideas helped to create for Lisa a pleasant learning environment. Of all the students in the class Lisa appeared to be one of the most satisfied throughout the Teaching Program. The only time she mentioned any difficulty or shortcomings she had was in the writing of her Thinkbook, but even then she reported a happy ending for herself. :

> "Initially Ms Fran when you gave us the idea and introduced writing and having a Thinkbook, I thought of it as boring because to me then it was just like writing a report on every maths lesson I did. I had failed to recognise the real purpose of Think Book writing then. Anyway eventually I began to understand it and I really enjoyed writing my feelings and comprehension in maths down. …. After all, having this Think Book is not a bad idea. In fact it has become a useful one" (7/11).

As mentioned later she found the Thinkbook useful as a method of revision and a way of reflecting on the day's lesson.

Generally Lisa appeared to have become enculturated into the mathematics education environment without any major difficulties. In fact she appeared motivated to participate to the full and was comfortable with learning in a way that was very different to what she had experienced at school. She was very enthusiastic about the novelty of what she did and learnt. As shown by the many insights into the understanding of ideas she mentioned, she must have reflected on what was happening both during class and when writing in her Thinkbook. From all her comments about what she believed was important in teaching mathematics, I would classify her as a constructivist teacher assuming she were to act on her beliefs.

Constructing knowledge of concepts and of the nature of mathematics

In this section I will preview the knowledge Lisa seemed to construct during the Teaching Program and the evidence I have found for this. This knowledge will include both knowledge of concepts and knowledge of the nature of mathematics. As discussed earlier, I will examine the nature of mathematics using six categories related to the six values of mathematics as described by Bishop (2001) of rationalism, objectism, control, progress, openness and mystery. But first I will consider the understanding of concepts and then of the nature of mathematics.

Understanding concepts

To investigate Lisa's understanding of the concepts of mathematics I have limited my study to the area of numeracy of number sense. The sources of data that I will examine first are her scores for the *Assessment of number sense* as can be seen in Table 6.3. The score was 12/23 before the Teaching Program and this increased to 17/23 after the Teaching Program. Her score improved the second time because this time she correctly answered questions involving operations on whole numbers and some on decimals. In the third assessment at the end of the year she correctly answered another two questions, about an estimation of numeracy and the relationships of decimal but Lisa did not give any responses for four of the questions which she had answered correctly in the earlier assessments. She commented that she did not have enough time though the time was the same for all tests. Perhaps she was not as prepared to guess answers. Also I noticed she had quite a bit of working on the answer sheet which was discouraged and it would have used much precious time meant to be used computing mentally. This working included a *long* multiplication of 365 by 11. In the interview after the final assessment of number sense she needed only a few short leading questions to correctly answer her other four incorrect questions involving mostly estimations. From one part of this questioning it appeared that she did not clearly understand operations on decimals even though she believed she had improved in this area. Earlier she addressed me in her Thinkbook::

> "Well Ms Fran at first whenever I see decimal sums I always feel dizzy because they look so complicated. When I worded the questions using money it was easier for me to understand and all the questions were really simple. Now I could simply divide sums. I sort of felt confident about them. Converting them into money sums was really helpful" (7/7).

My second source of information for her knowledge of and development of number sense is from the interviews that included activities (see Appendix D). In the interview before the Teaching Program, Lisa quietly did the activities and problems but said she was unable to explain what she did other than by showing the working. This may have partly been because she was not at ease with me as it was the first time we had talked together. She made a poor attempt at the activities for estimations of numeracy, measurement and computations. She showed little skill in simplifying computations when working without pen and paper.

When doing the activities after the Teaching Program Lisa showed considerable improvement. She was better able to explain all her working which may have been because of her active participation in class during the term but also because she was more at ease in talking with me. Lisa was able to quickly calculate a multiplication by 99 using the strategy of (100-1), and to calculate the total number of digits in all the numbers less than one hundred. She described three different ways of finding the product of 34 and 14 and was able to estimate well the measurement of length, of numeracy but not of time. Estimating was an activity that she had become very involved in during the Teaching Program:

> "First we did estimation *of lengths*, then we took measurements and found the errors. At least from now onwards my estimations would be better compared to the previous ones" (8/6).
>
> "Boy Ms Fran today's lesson was really great [pacing out dimensions of a sport's field to estimate a hectare]" (28/5).

Also in the second interview Lisa's demonstrated that she had not mastered skills to worked mentally because she still needed to do written algorithms for simple computations such as 1/10 x 100 and 1000 – 714 = . Calculating without the use of pen and paper was a new experience for her. Early in the Teaching Program she had commented on it:

> "Well today we concentrated more on mental computations. How fast any mind could work?" (23/3).
>
> "I know them [additions] but I was sort of slow in doing them. But when you introduced this type of idea [thinking about bridging] it really made the calculation easier and fast. This also applied to the lesson we learnt today on subtraction by 9, 99, and 19 especially when applying the subtracting 10 and adding 1 which is the opposite to the working used in addition" (31/3).

Although Lisa was able to arrange a group of rational numbers in order of magnitude she still had difficulty finding a common fraction between a given pair. She did not hesitate to tell me there were no fractions between 2/5 and 3/5 even though she had been able to name decimals between 1.52 and 1.53. Equivalent fractions were known but her knowledge was not constructed in such a way that she was able to make links between aspects of her knowledge of fractions to answer this part or other parts of this question. Similarly, she apparently was unable to link her knowledge of the relationship of numbers with difference of numbers. For example, initially she chose the two closest whole numbers in a group of eight numbers less than 1300 as being the two having the same number of hundreds, 237 and 299 rather than the correct pair, 194 and 237 that bridged two hundred. Once alerted to her mistake she chatted about her lack of awareness of the relationship between numbers on either side of bridging numbers.

Lisa never expressed any anxiety about not understanding the mathematics she had to teach, but she came to understand for herself and to appreciate the difficulties in understanding that her pupils may have:

> "In today's maths class I really like the way we went through the counting principles. At first I didn't realise that counting was a bit difficult. (I mean learning how to count for the first time.)" (15/3).
>
> "At first I thought that these [ordinal numbers] were easy but I did not realise it would be a bit difficult for children to grasp it" (9/3).

Many less significant ideas about mathematics that were new to Lisa she found fascinating and she wrote much about them in her Thinkbook. These ordinal numbers, counting on, conservation of number, the relationship included between the equivalent fractions, times tables, decimals in money and addition with bridging the multiples of tens. (Some of these are mentioned in more detail in other contexts in this case study.) An example of where Lisa gained a new insight during the Teaching Program and then deepened her understanding of it months later is described by her about division:

> "The good thing about it was discussing the different ways on how to divide things, in other words the meanings. One could cut them into equal pieces or by sharing them equally or by grouping them equally. We did the structured play on division and then wrote stories related to number sentences on division which was really overwhelming!!" (28/4).

And then at the end of the year,

> "I did not realise that there were two meanings of division. One is to divide them into groups of a certain number, e.g. groups of nine. The other is to divide or share them into groups, e.g. three groups. The difference is that in the first one we know the number in the group and the second type [number in the group] is anonymous" (18/10).

Possibly Lisa compared her own understanding to that of others in the class and saw that her knowledge was above average and so was satisfied with the situation and not anxious about any shortcoming she had in her knowledge. I wonder if, her coming to understand children's difficulties, was the result of an unconscious deepening of her own understanding?

From this analysis it appears that Lisa came to the College with a considerable knowledge and understanding of concepts as well as the rules and procedures used in mathematics. During the Teaching Program she believed that her knowledge had developed and the data collected supported this belief. The main areas she thought she understood better at the end of the Teaching Program were number relationships, decimals, mental computing and estimations as shown in the interviews. In this analysis even though improvement was noted I have highlighted a number of areas for which Lisa still had a lack of understanding. There was still need for improvement in being able to link her knowledge in answering questions that required such a skill.

So far in this section I have discussed aspects of her understanding of the concepts of mathematics, specifically in number sense. Now I will consider her understanding of the nature of mathematics using the categories related to the six values of mathematics described by Bishop (2001).

Understanding the nature of mathematics

The six categories I will use to examine Lisa'a understanding of the nature of mathematics are rationalism, objectism, control, progress, openness and mystery. With regard to the first of these Lisa understood that mathematics was *rational*. In the last subsection this was often illustrated but I will give some more examples of her search for understanding of concepts. In her discussion of times tables considered in class she mentioned a deepening of her understanding of basic ideas:

> "This lesson deepened my understanding [and] learning for times tables" (25/5).

> "I also found the order of learning the numbers interesting [personal history of learning numbers]. The way it was listed down from the easiest to the hardest. Although I knew these numbers and some of their orders but in today's lesson the ordering we did as a class made it more clear" (8/3).

Lisa appreciated that many concepts in mathematics could be represented using concrete materials and diagrams and so the ideas related to *objectism* in mathematics was not new for her. She realised that the use of materials to illustrate numbers and operations, and to compute answers, helped the learning process:

> "I was really impressed by the usage of straws in counting and grouping them in ones, tens, and hundreds" (14/4).

> "I liked the way you brought about the relationship between cubes, longs and flats; blocks, big blocks, big longs, big cubes. When we get to think about it, it's so simple, but needs thinking" (16/4).

> "I also learnt [after an activity for estimating area of sport's field] that if you understood, knew and did the activity well you learnt something" (28/5).

Lisa did not accept the status quo but saw that *progress* was part of the nature of mathematics. She came to appreciate that in mathematics she was able to explore and produce ideas:

> "Thanks Ms Fran for teaching and showing us that bit on bridging. The same is applied in subtraction but it differs by working backwards" (31/3)

> "Another interesting aspect of maths is that one could apply and have various procedures in order to get to the desired solution" (3/3).

> "Ms Fran you really are correct with the calculation of fractions [operations on fractions are rarely used in daily life]. It is utter rubbish because why do we need to burden student's brain with this kind of calculation when there are other easier methods of getting towards it, such as calculators and computers!!" (7/6).

Control is a value that Bishop (1988) says is overemphasised in mathematics education with the extensive practice of the skills for algorithms as an example of its prominence. Lisa was skilled at using the algorithms for the operations. In the test used for this study her only apparent lack of knowledge was that prior to the Teaching Program she could not divide by a decimal. She correctly converted the divisor into a whole number by multiplication by 10/10 to do the division, but then she divided her answer by ten and so had incorrectly placed the decimal point. But she correctly completed this question for the following two tests. She continually used this knowledge of the algorithms and hence rarely used mental procedures. She did very simple calculations using pen and paper in the interviews as described above. When she did attempt to become more proficient in mental computations she was very interested in the strategies we investigated in class. Even for these I wonder how much was the learning of more algorithms rather than development and use of number sense?

> "The 9 x tables and 99, 999, you name it, etc. was great. At first we approached the long method but after we learnt the drill, i.e. multiplying by 100 and minus-

ing the respective numbers we could do the 99, 98, 101, 999, you name it, times table [multiplications] in 5 seconds" (28/5).

Initially in the interview situation Lisa did not show *openness*. An indication of this was she was unable to justify her answers to me. But this was not as true in later interviews. She did verify 6.9/0.3 in the test at the end of the year but with a long multiplication containing a line of zeros. Probably because Lisa relied on her dependable knowledge of the algorithms she usually did not need to question the reasonableness of her answers by using simple checks. However she knew some ways of checking and used them:

> "We went through "checking a calculation" and there were seven ways. I was familiar with some and it was interesting for to learn the other two …, e.g. odd number + odd number = even number. I never thought of this. In other words I did not take it into consideration" (12/10).

Lisa may have believed that mathematics had a value of *mystery* but I wonder if she ever thought that the mysteries of mathematics could be investigated before she began the education unit. As Lisa was a person who was open to novelty and change she was fascinated by many ideas that came her way during the Teaching Program. To her mathematics had become full of beauty and many mysteries for her to discover:

> "It's kinda funny when we put dots for decimals [compared to other symbols used]" (30/6).

> "It is really interesting to know and learn the different ways or methods of doing things [arrangements of dots for counting]. That is the beauty of mathematics" (19/3).

> "We were also introduced to the rest of the Chinese numbers 1- 10 which had a few weird but very interesting figures. It is a wonder how these people created their number structure" (8/9).

> "I liked learning and finding patterns in their positions [of multiples on the number chart] and digits" (20/5).

> "While I was doing this exercise today I then realised the different pattern for each table [times tables]. That is one of the beauties of multiplication tables" (25/5).

This last statement about the mysterious aspect of mathematics by Lisa indicates how much she had come to appreciate and understand the nature of mathematics.

Throughout the semester she was interested and surprised by many activities and ideas presented that were part of the mathematics education unit. From the discussion in this section it appears that her responses to the activities in the mathematics education unit were indications of her coming to understand better the nature of mathematics especially the aspects of progress, openness and mystery.

Lifelong learning for the future teacher

It would be hoped that as a pre-service teacher Lisa would see the importance of lifelong learning for her own learning of mathematics and for learning to teach it. But perhaps more importantly is the learning of metacognitive skills which would facilitate her lifelong learning generally. In this section I will first discuss Lisa's learning to learn and then her learning of some ideas for teaching, especially teaching mathematics.

Learning to learn

With regard to metacognition, Lisa appeared to find it natural to reflect on and write about her learning. She was prepared to admit when she did, did not, or could not learn:

> "I really like the orderly way she [Ms Fran] puts through the reminders to us also the procedure she used when doing her maths lessons. It really taught me a lesson to always plan my work well because it will always be efficient and easy for me (the learner) … it is very helpful when it comes to forgetful people like me" (8/3).
>
> "At times when I was really lazy or did not have anything to think of [for Thinkbook], I just simply summarised the maths lesson that I just had and that is when I got marks that were not satisfactory" (7/10).
>
> "I feel so refreshed from the holiday after relaxing my body especially [relaxing] my brain from thinking so today it took a little while for me to adapt myself into the learning class or school life" (24/8).

It was not very often that she found it difficult to participate as on the above occasion because she was usually very much mentally involved in the class work:

> "Boy Ms F, today's lesson was really great [estimating how many hectares in the playing field by pacing out the dimensions]" (28/5).

As mentioned a number of times above, Lisa noticed first her lack of mental computing and then, the difficulties she had adapting to using various strategies we discussed:

> "This exercise really got my head working. I also found out that when doing this type of mental calculation it's easier to put the same numbers together or to calculate those figures that are easy to add and leave the hard ones for the last [3+ 7 + 6 + 4 + 9 =]" (23/3).

Two strategies that helped Lisa to learn were co-operative group work and writing in her Thinkbook:

> "Having group work discussion is really good because I'm able to share and learn at the same time from my fellow mates" (5/7).
>
> "Well initially I didn't know there were quite a lot of benefits of writing in a maths class. I thought it was a practice for students to keep writing in their books and to revise them when studying for their exams. But now I know there are more reasons than that" (24/2).
>
> "On the other hand, filling in my Think Book has been sort of a revision for me and also a reflection on the day's lesson" (7/10).

As well as often reflecting on how she learnt or did not learn, Lisa reflected on how she would teach which is discussed in the next section.

Learning to teach

Lisa was a conscientious pre-service teacher who often indicated that she was keen to become a teacher. The learning she expected from the mathematics education unit was what and how to teach mathematics and throughout the Teaching Program she often noted what she wanted to learn, and did learn, about this:

> "Ms Fran I really want to start learning the maths stuff that we're going to teach" (29/3).
>
> "It really fascinates me to see how each student [pre-service teacher] went up in front of the class to present their individual Warm-up activities" (24/2).
>
> "Previously I didn't even realise these steps happening every time or things never got into my head that this was the way little children behave and think. [children needing free play before formal use of new materials]" (14/4).

After reflecting on what we did in class she did not hesitate in adding her suggestions to what was discussed:

> "Telling a story relating to a maths problem [that] one will be presenting. Ms F I suggest that we should stress a bit on that type of teaching too" (19/3).
>
> "Calling out the factors for each multiple of three. But I think that exercise could be improved a little bit …. For example, we could use objects such as bottle tops, shells, stones and so on. While learning, the students would be able to touch them and apply them. Pardon me Ms F but I think students would be able to learn better on concrete ideas and not abstract ones" (25/2).

Lisa also discussed how content could be taught after using her number sense to think about her own learning of topics:

> "Now I know that before I teach the children the 9 times tables I must teach them the ten times table because from their learning and knowing the 10 times, learning and knowing the nine times tables would be a bit easier" (20/5).
>
> "The next new stuff I learnt in multiplication today was identifying the easy method. Boy this was really cool. No wonder most maths teachers could multiply so fast. When I was in primary school I thought they were geniuses. I never realised the easy methods that they adapt themselves [5 x 48 = 5 x 2 x 24 = 10 x 24 = 240]" (26/5).
>
> "The topic that struck me was bridging [the multiples of ten] in addition. Previously I never realised that there were bridge numbers in counting. Probably I never realised it but it is only the students who would realise and find the problem on bridging numbers" (20/3).

Lisa appears to have learnt much about her own learning and learning to teach. To do this she used a number of metacognitive skills such as using writing to help her reflections. She reflected on her learning and noted a number of her needs such as to be organised, prepare reminders and give herself time to adapt to study after relaxing holidays. Processes that she found helpful were cooperative group work, writing in her Thinkbook, doing mathematics activities and learning mental strategies in mathematics. Lisa often noted occasions when she came to understand better the difficulties children have when learning. As already mentioned she discussed the strangeness of new ideas for children, problems with words and symbols, patterns, and concepts such as conservation of number, adding numbers that require bridging and zero. If, throughout her life as a teacher, she continues to reflect on the difficulties that she and her pupils might have in learning and then she acts on the knowledge gained, her lifelong learning will be a great benefit to her and her students.

Summary for Lisa in Chapter 6

Appendix W The learning experience for Wani

Wani

Wani is a Fijian of indigenous culture. The region of Fiji to which her family belongs is an island group. She is in the age range from 17 – 19 years and she had studied mathematics for eleven years of her education. The school she last attended was a girls' private secondary College in Suva which was seen as a progressive one where many of the more affluent and educated parents sent their daughters. In the year prior to coming to the College Wani said she had no formal occupation such as study or employment, and so was strongly encouraged by her mother to apply for a place at the College. She had a bright personality and if she concentrated in class she participated well. When she wrote in her Thinkbook, she did so with openness and enthusiasm.

Among the six case studies, Wani was considered as one of the three with higher mathematics ability because of the results recorded on her application form. The data gathered for this study appeared to confirm her mathematical ability (see Table 6.17). Her results for the *Assessment of number sense* were, for the first assessment a little above average but well above for the other two assessments. For the *Test of algorithms for the operations* her results were, for all three tests well above the class mean but there was a small decrease from the second test to the third.

Table 6.17
Quantitative data - Wani

Instrument		Number sense assessment			Test of algorithms		
Month		Feb	Jul	Oct	Feb	Jul	Oct
Score	Max	23	23	23	18	18	18
	Mean	8.4	12.4	12.6	10.4	12.2	13.1
	Wani	9	15	17	12	16	15

Her responses for the questionnaire *Beliefs about mathematics* were generally more positive towards a constructivist view than the class means for the items in February and especially for the July questionnaire but less so for the October responses.

Enculturation into mathematics education

Wani had not attended school the year prior to coming to the College and this was possibly partly responsible for the difficulties she had settling into her first year of College study. Her Thinkbook entries do not tell us much about her early enculturation into the mathematics class because, during the first half of the semester she wrote only three entries. But during the last weeks of the Teaching Program she wrote more with often a few entries per week. Initially she was enthusiastic as described in her first entry:

> "Math teacher [Ms F] made the subject interesting. She captivated us with her creative mind, so energetic and she encourages us on the subject of maths" (18/2).

She also described her response to the Warm-up activities:

> "Introduction of the Warm-up exercises - this is interesting because, not only does it make the subject interesting but it captures our attention" (18/2).

Towards the end of the Teaching Program Wani commented on her poor performance during the first few months attributing it to a lack of concentration but did not give a reason for it:

> "Last term [first half of the semester] I found myself not really participating in class I was always miles away and in that way I missed some very important points which if I had paid attention would have enhanced my performance to a more respectable level" (29/6).

Her lack of concentration in class was not because of boredom as the result of being over-confident in her mathematical ability because, in spite of her reasonable ability and appearance of self-confidence, Wani expressed fears related to these. She doubted her ability to do the mathematics that she was required to teach:

> "I guess since I am always poor at mathematics I dread the time when I have to teach mathematics problems" (26/5).

She also feared standing in front of a class to teach:

> "One thing these people [peers who presented Warm-ups] had in common was they had the ability to stand up in front of the class and introduce their warm up activities" (3/3).

In both of these cases in later entries she smoothed over her fears with some hope and optimism. When Wani did make a presentation with her partner in front of her peers she was very pleased with the result:

> "I was so excited about the maths class mainly because we were going to present on our topic which was measurement. I couldn't wait to go on and I could see that even the cool appearance my partner was also excited. We planned our work well, we prepared well and we knew we were ready. Measurement of time was our topic and the feedback I got from the class was that our preparation was superb!! The key to the good presentation was good preparation" (1/9).

Wani mentioned the importance of preparing well on a number of other occasions when she thought she had done so. She was probably conscious of her major weakness of lack of commitment to her study. As discussed in the literature review Fijian students expected extrinsic motivations and so Wani probably expected me to motivate her all the time. Halfway through the first term Wani noted her resolve after I had discussed with them the need to be more responsible for their own study:

> "Today when Miss F came to the classroom she reminded us of a very important word RESPONSIBILITY. That word really put me on track because sometimes …. " (4/3) [and that was the end of her last entry for many weeks].

In mid-semester when I reminded them to look ahead to what they had to complete for the rest of the semester Wani again resolved to work:

> "The study plan is a good and useful one. Somehow this kind of organisation by our teacher has inspired me to improve on this particular subject" (19/5).

It was about this time that Wani did begin to improve by applying herself to her study. From this time on she also wrote regularly in her Thinkbook after commenting on how interested and motivated she was:

"Thanks to the maths teacher I am now taking an avid interest in the subject" (26/5).

Perhaps for Wani the classes were not as captivating and motivating as they were for most other students as she did not comment on the teaching methods related to constructivism that I used during the Teaching Program. This may be because, of all the students in the class there was possibly less change for her coming from her urban secondary schooling. Her responses to the questionnaire *Beliefs about mathematics* showed that her responses were generally more aligned to a constructivist view than most of the class. Her second set of responses after the Teaching Program were the most positive of the class (see Appendix R). The notable responses for Wani are given in Table 6.18

. In this Table 6.18 her responses indicated that she always strongly believed in the importance of understanding the processes, showing initiative, thinking creatively and being flexible in her thinking. She also believed that mathematics was useful in solving everyday problems and that memorising rules was not necessary for success.

After the Teaching Program her responses showed that she came to hold a more constructivist view for the items related to work done during the Teaching Program. This included work completed using one's own strategies, estimating, doing mathematics related to everyday living and not having to follow procedures precisely. For the whole year her beliefs which did not, though not strongly, support a constructivist view were that mathematics consisted of rules and procedures to follow, and that memorising facts was important. I can partly understand these responses after examining her skilful use of algorithms. For some of the other items not shown in this table she was undecided at the end of the Teaching Program or end of the year, but tended at some times during the year to favour the constructivist view as she did for all other items. Her responses throughout the year are difficult to summarise in a sentence but generally they indicated a change towards a constructivist view for items related to the activities done in class.

Table 6.18
Notable responses to Questionnaire by Wani

	Feb	Jul	Oct
Strongly held constructivist view			
Understanding the process is more important than correct answers in mathematics	SA [a]	SA	SA
Maths is useful in solving everyday problems.	SA	SA	SA
There are many different ways to do most calculations.	SA	SA	SA
In maths it is important that students are able to show initiative and think creatively.	SA	SA	SA
*To be successful in mathematics it is important to memorise rules.	SD	SD	SD
In maths it is important that students are able to be flexible in their thinking.	SA	SA	A
Shift in agreement towards constructivist view			
A maths calculation can be solved in different ways.	SD	A	A
In maths it is important that students are able to estimate.	SD	A	A
*Maths is not needed in everyday living.	A	SD	D
*Doing maths means following standard procedures precisely.	A	D	D
Constructivist view not held			
*There is always a rule to follow in doing a mathematics calculation.	A	SA	A
*In maths it is important that students remember facts.	A	A	A
*Mathematics consists of a set of rules and procedures.	A	A	A

*Items which are negatively worded towards a constructivist view.
[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.

Other than the responses to the questionnaire Wani commented rarely on ideas related to the constructivist approach. She mentioned on a few occasions towards the end of the Teaching Program that she was interested in activities especially work on mental strategies and times tables:

> "Thank you Ms F for making the multiplication tables learning interesting and creative for me to learn. In all my days of school I have never come across a maths class where we would practice our times table on a clock face. And when we drew lines on the clock face, it formed a pattern" (25/5).

She was impressed by an activity, in a cooperative group learning situation which directed the group to study the introductory pages of the teachers' guides for lower primary mathematics:

> "Our lesson was really unique today. We covered questions in the maths Class 1 textbooks. The questions were different and unlike any other that I have come across. It started off with a question about the permanent Secretary's comments and the beliefs that were included in the introductory text. Questions like these have never been asked before so in a way it was refreshing and also forced us to take more notice of the beginning pages of the maths texts books" (20/7).

She must have let the work done in first term, which I consider was similar to that activity, pass her by without much reflection on it as she asked for more of this kind of activity:

> "And I guess I enjoyed today's exercise. It's time to be original if it is to make us think so please more of this type of question for maths" (20/7).

One difference in approach that she did note between her school mathematics education and that at the College was assessment strategies. Her past experience probably accounts for her beliefs in the overriding importance of rules and procedures in mathematics:

> "We sat a test today and to be honest I wasn't really expecting it to turn out the way it did. I mean it was a test that made us use our common sense [and number sense] more than just to repeat the teacher's method. It really tested our level of understanding about decimals and fractions" (16/7).

Another occasion when she gained some insight into her thinking about the doing of mathematics was when she was asked to review a journal article (Koep, 1987):

> "We had to summarise an article … on mathematics and child development. When reading the article I had this weird feeling that it was written for me because I also went through the experiences [rote learning]. A quote which I found important was 'Often it is the way mathematics is taught rather than the nature that creates difficulties.' I hope when I teach I don't repeat this mistake" (9/9).

In other narratives for these case studies I have usually included a discussion of affective issues and difficulties with communication with both myself and fellow students. But in the data collected from Wani's Thinkbook and other sources she did not mention anything about her enculturation into the class other than what has already been discussed. From my observations of her during classes, especially early in the Teaching Program she appeared to be content, perhaps too content, with what was happening around her instead of becoming involved in the discussions and work with others in the class and so produce for herself challenges, insights, difficulties or conflicts. Her enculturation consisted mainly in becoming more committed to beginning to do the requirements of the unit of mathematics education and in learning to teach.

Constructing knowledge about concepts and the nature of mathematics

It was difficult to pinpoint the evidence for Wani's knowledge of numeracy as so often her understanding was hidden by her lack of concentration and careless work. Her writing was possibly the best means of seeing the true state of her knowledge because it was more difficult to make careless mistakes when discussing understanding. In this section I will be using mainly her Thinkbook entries to describe her construction of knowledge.

Understanding concepts

As noted above Wani thought she was poor at mathematics though the data I collected did not indicate that this was so. Perhaps she was still comparing herself to her peers in secondary school who were publicly acclaimed to have a high level of mathematics ability in relation to the general secondary school population in Fiji.

Wani's score for the first *Assessment of number sense* was average for the class but there was a remarkable improvement with the two later scores were well above average for the class (see Table 6.17). In the second and third assessment she correctly answered a further eight questions which included questions involving operations on decimals, fractions and a numeracy estimation. My interview with her after the last test revealed that most of the incorrect responses she gave were the result of lack of careful reading of the questions. She said she read the word *nearest* as *nearest below, days* as *years* and *different* as *difference,* but in the last case she must have ignored the rest of the question which elaborated on the *different* decimals. Also she thought that 90% of the cost was much less than the full price and she added two zeros when she multiplied by ten.

Wani appeared to compute without much consideration of reasonableness of answers. For the activities completed in the interview situation before any classes had begun, Wani did not account for a factor of zero for simplifying a multiplication, and produced a few unexpected expressions such as a simplification of repeated terms written as powers. In Question 14 for finding the change from \$10 for \$7.14 and another similar subtraction she completed a detailed subtraction algorithm even though I encouraged her to do it mentally. A correct answer that she did give which other students had trouble with, was Question 3 where there was expressions with a string of factors which she multiplied, first using together the factors for 10 and for 100. She also had no difficulty arranging a list of rational numbers in order of magnitude.

After the Teaching Program Wani's initial answers to many of the questions in the interview were the same incorrect ones she gave for the first interview. However with a few leading questions from me she was able to use her skills and reasoning to obtain the correct answer in most cases. She used mathematical language incorrectly at times such as 'divide 35 from 455'. In this second interview she mentally computed the change from \$10 for \$7.14 and the subtraction 1000 – 714. In Question 11 where a fraction between the two given ones was to be chosen, her attempt at that question was the best I was given of any of the ten students interviewed. Whereas in the first interview she had difficulties using alternatives to the algorithms for the operations as in Question 2 and 18, in the second interview she gave alternative methods such as the multiplication of 32 x 14 = 32 x 15 – 32. For estimation of the number of lollies in the jar for Question 23 she promptly said 200. When I asked her for an explanation using leading questions she counted 20 in the base and 10 up the side. She gave the answer as 230 to 250 as she included up to 50 extras for the top which was a little generous. Hence generally during her second interview Wani showed a considerable development in number sense with the improvement in computing mentally, use of her own strategies, and an increased ability to explain her working.

Wani appear not to use her number sense unless prompted to do so. There were times when she appeared to lack concentration and seemed to misunderstand what was happening in class. For example, when displaying an initially covered number chart of 1 to 100, I asked for the multiples of 7 so that they would see the pattern which forms:

> "We gave her the answers and she uncovered the multiples e.g. 7, 14, 21, 28, 35, 42, 49 she had put it in a pyramid-like fashion. In this way it was easy for us to notice the answers in a much simpler way. The funny thing is I never used to emphasise much on the fact that arranging numbers can create a difference … students notice, think, infer the result. I wouldn't hesitate but use this method when I teach if given the opportunity" (20/5).

It appears that she did not realise that the presence of a pyramid pattern was the nature of the seven times table. Or perhaps she did not express herself well as in the following entries:

> "By dividing it into hundredths, e.g. block to a long, and then we had to use our imagination when dividing the long into hundredths" (30/6).

Wani did not appear to be motivated to deepen her understanding of the concepts but she was interested if she grasped a concept:

> "Our minds were rolling we were thinking and I think this is vital when learning maths concepts and problems" (30/6).

> "We had to discuss which topic we were to teach in term 1 and which in term 2 and term 3. I found out that we had to use visual aids thus enhancing a more learned class. It is interesting but it can also exhaust your brain! But then again it is a challenge" (25/8).

Wani' s last comments appeared to suggest that she was not always keen on doing anything that required mental effort in mathematics. But, even though she continued to make comments similar to this she also increasingly made comments about being interested in understanding and teaching the concepts. More will be mentioned about this later in this case study after the subsection in which I will first discuss Wani's understanding of the nature of mathematics.

Understanding the nature of mathematics

Wani appeared to have some understanding of the nature of mathematics as related to the six categories of values devised by Bishop (2001). First, she appeared from the discussion in the last section to show an appropriate understanding of the *rational* nature of mathematics. Another example of where she saw the benefit of understanding mathematics rather than only rote learning was in her entry after we considered gender issues in a class discussion:

> "… boys get higher marks than us because they thought more than girls, who are used to following the teacher's method without really thinking or understanding it" (1/7).

Wani understood *objectism* which is related to the ideas that concepts are able to be represented using materials, diagrams and other images. Wani only commented a few times on the use of materials. Again perhaps this was because, in her past schooling she often used materials in class and therefore it was not the novelty it was for her fellow students. She did mention her use of fingers in calculating three times tables (18/2), and her skill in the use of materials when using MAB:

> "Decimal place value. This lesson today was very creative. We were told to bring our place value mats and to distribute the numbers 0.462 - we would arrange it to 4 flats 6 longs and 2 cubes. This helped me to understand it faster and I was really confident in arranging it. In my mind I was already having a rough sketch of it" (6/7).

As mentioned above Wani believed that mathematics was concerned with rules and procedures indicating a strong sense of the *control* in mathematics. Also her score for the *Test of algorithms for the operations* indicated that she was able to get correct answers because of a good knowledge of the algorithms. She was also exceptional in the class as regards the correct use of the algorithms for operations involving decimals. She used algorithms even if not efficiently, e.g. a long multiplication for 1.2 x 0.02, though this practice of not computing mentally lessened after the Teaching Program. In the interview after the third test she quickly corrected her only two mistakes, a slip in the long multiplication and a subtraction completed instead of the asked for addition.

How much rational understanding she had of the standard algorithms was difficult to determine. The following entry illustrates what I suggest was an instrumental understanding and a satisfaction with this situation:

> "In today's class we learned a very interesting method. In multiplying a number by 99 all we had to do was to move to 100, get the total and subtract the number once and we will get the answer e.g. 43 x 99 = 430 –43 = 387. This method is really interesting because it give the teacher the upper hand in teaching multiplication. I was in awe of how easy it was to find the answer. All we had to do was to trigger the device in the brain that would unlock the numbers. This is what I call an interesting subject. Somehow this makes me want to understand more" (28/5).

Her lack of relational understanding was also illustrated later when she used this method for multiplication by ninety-nine in an activity we completed during the interview at the end of the Teaching Program. At that time she asked me if she used the strategy correctly because she was unsure if she had remembered it correctly.

As well as having strategies that "give the teacher the upper hand" she "did not want to be caught off guard". She wanted to be in *control* and know the answer to mathematics questions including those about conventions:

> "Miss Fran taught us why there is a need for us to be aware of the decimal point. I mean in case a student comes and asks me why we put a decimal point. Not to be caught off guard by that question" (1/7).

Another entry in her Thinkbook gives an example of wanting to teach the 'right thing' without seeming to have an understanding of the concept of the relationship of multiplication and division, even though she knew to use one operation to verify the answer to another:

> "For our lesson today we learnt something interesting, well everything we learn is interesting but this is extra extra interesting because it will help us greatly

> when we are out in the field teaching maths or specifically multiplication & division. These two operations I was told in today's class must not be completely separated" (10/9).

With such a dependence on the security of algorithms and knowledge of other procedures it is not surprising the Wani did not appear to see mathematics or mathematics education as being able to *progress* and so be open to change and development. When we discussed in class possible advances she could not see the possibility of change:

> "Early on in the lesson she asked a question that made me think. She asked us if our parents used fractions like 9/11 or 2/7 in their daily lives. We said no and that made us wonder about the relevance of teaching fractions to students. I guess we don't really have a choice" (7/6).

Wani knew about aspects of *openness* in mathematics because she was able to verify answers when asked by me in the second interview and in test situation although she did little of it in class:

> "This test was to be done in such a way that we were to show our working and even when we rechecked the problem" (22/7).

She did verify her answers and so was able to correct some mistakes. Her short division, in which she omitted a zero in the answer each time was corrected after trying to verify the answer using a multiplication. I noticed she did not correct the division but only changed the answer, which seems to indicate only a need to get correct answers, not to justify all the working she has done.

As for progress, Wani did not appear to consider much *mystery* related to mathematics. She never mentioned issues that had impressed other students such as mathematics in other cultures in Fiji, but she was fascinated by patterns in the time tables:

> "It is really interesting and it makes me curious to find out about the other multiplication tables like 8 x, 9 x, 10 x and the patterns they formed" (25/5).

At this stage of her mathematics education Wani is in an interesting situation. She has the ability to reason out much of the mathematics she does and has begun to do it occasionally. But, because she has always believed in the important place of rules and procedures in mathematics and had only begun to be enthusiastic about understanding, she relies on her knowledge of algorithms until prompted to use her own strategies. This reliance appears to have prevented her from developing a broader view of the nature of mathematics for such aspects as progress, openness and mystery. The combination of the understanding of concepts and of the nature of mathematics appears to have helped Wani to develop her number sense, that is the development of the inclination to understand and use numbers, their operations and applications with ease.

Lifelong learning for the future teacher

When Wani arrived at the College she lacked the commitment to prepare to become a teacher that other pre-service teachers had. Her interest in learning to learn, and learning to teach developed late in the Teaching Program. The following two sections illustrate this happening.

Learning to learn

Wani's major learning experience was in coming to learn how to apply herself to her studies. It took a few months for her to begin to participate in class and to begin to complete the requirements of the unit. She had to overcome an inertia to having to think, but she did achieve this to a considerable extent. During most of the Teaching Program she often made resolutions to do what was required but action did not always follow:

> "My resolution for this term is a simple one. 'Pull up my socks and change my attitude' " (29/6).

Eventually she was successful in motivating herself to study but it appeared that effort included more of the past rote learning. Also she expected immediate success:

> "I was really disappointed with my test results. In a way I was also discouraged. I really felt down because I thought at least I was keeping up and doing my work. In doing my work and writing everything the teacher taught us" (23/7).

After her disappointing feedback from the test she acknowledged she did not think about the questions:

> "I believe I did not think but became a puppet. I guess I will just have to get out of my situation and get back on track" (23/7).

Later in the year she felt very satisfied with herself when she succeeded in her plans in having work finished promptly as required:

> "An important lesson I learnt from today's experience is that good preparation paves the way. If I hadn't done my teaching materials I would be in a panic and everything will be chaotic. So I guess today I feel good because I know at least I am improving" (13/9).

She was not always enthusiastic about being required to think but appreciated the challenges she was given:

> "This not only stretched our brains but it helped us to use it [counters for multiples of three] instead of using ones fingers" (18/2).

> "The test [problem solving using rational numbers] made me think and to really search my mind about the clues that would help me find the answers. I guess a major focus for giving us this type of test is to make us think and be curious about mathematics lessons" (16/7).

> "The test was a challenge of my intelligence and I liked that. It was a test that made me think and I really enjoyed it" (16/7).

Wani appreciated the example and encouragement she gained from me and was proud of herself when she faced challenges and succeeded. It was pleasing to note that she came to see me, not as a teacher with all the answers, but as a guide, an example and a source of inspiration:

> "Also the fact that Miss Fran hovers over us gives us new added confidence because I know she is there to guide us but most importantly encouragement" (3/5).

> “One thing that struck me about this plan [for revision] is that it helped me to organise my work and to revise in an orderly way. I really appreciate that about the maths teacher because she herself is well-organised” (19/5).

> “Even though we did not have maths today I would like to talk about my preparation for maths teaching aids. …. It is challenging but one thing is certain never leave things to the last minute” (7/9).

The experience of the Teaching Program for Wani appeared to have helped her develop a self-discipline for study which, because it came with personal satisfaction, would possibly remain with her and help her to continue learning. She recognised that she had a tendency to be a ‘puppet’ when she was wanting to learn so if she continued to avoid this tendency she may examine her development of such aspects of her knowledge as number sense and work to understand her mathematics better, rather than depend on rules and procedures. Her learning to learn is well summarised in an entry late in the Teaching Program:

> “By letting me write my resolution this term I think what Miss Fran really wants is to see changes. This is what I consider as a challenge and I must commend her for letting me or giving me this term to shape up. I guess all I have to do now is to learn from my mistakes of last term. To be more attentive in class and to participate more with the class” (29/6).

Learning to teach

Wani was interested in how to teach even more than what to teach. She believed that one of the functions of teachers is to be an inspiration to her pupils and that what is learnt, depends on how it is taught. She was anxious about her ability to stand in front of a class and teach and, as described earlier, she did not want to be ‘caught off guard’ or ‘not do the right thing’. She was inspired by those she saw teaching, both her peers and myself:

> “Personally I can’t go in front of the class to do that but to see my colleagues do that inspires me and motivates me a lot” (3/3).

> “… but what she [Ms Fran] showed us today [patterns in times tables] boosted my confidence and I can’t wait to teach this subject out in the field” (26/5).

> “I believe a good teacher who is unique in her teaching is a major draw towards the subject she teaches” (18/2).

> “Often it is the way mathematics is taught rather than the nature that creates difficulties.’ I hope when I teach I don’t repeat this mistake [teaching rote learning]” (9/9).

Wani took longer than most of the pre-service teachers to become interested in learning to teach but when she did, she often noted different strategies that she would apply in her teaching:

> “I learnt a lot from the group’s presentation and I commend them for their preparation, for it certainly made a difference” (3/9).

> “We learnt that it is crucial not to give and show whole charts to students but to spend a day or even a week on one line. In this way the children will understand but most importantly avoid confusion amongst the students” (24/8).

She began to express surprise at ideas new to her related to teaching such as the need to plan and providing a context for mathematics work:

> “The units we were supposed to teach - two weeks per unit. I didn’t have even the slightest idea of that” (20/7).

> “They presented their topic which was based upon problem solving. The format of this is interesting because you tell a story and weave inside the concept of problem solving. This would enable to children to become more interested” (6/9).

Only occasionally did she refer to the teaching of the content in the Teaching Program rather than her future teaching of it. Among some of the teaching activities that impressed her were the use of MAB for place value, devising and using mental strategies, for example, a short method for multiplication by ninety-nine, the assessment tasks that investigated understanding, and the following:

> “I have never thought of finding out or making the teaching of multiplication tables interesting. This has really changed it all. I really am anticipating more creative ones [ideas] thanks to you” (25/5).

> "Today’s lesson was mind boggling, i.e. the way or method of teaching mathematics today was really interesting (*5 x 48 = 5x2 x24 = 10 x 24, etc.*)” (26/5).

> “This kind of active participation [use of MAB] with the teacher helped us to understand more about decimals” (30/6).

After the initial slow start Wani became observant of the teaching she experienced and reflected on how she could apply some of it when she taught. If this process continues she would possibly become an effective teacher because her reflections and resolutions were generally of the kind that would lead to better teaching if applied.

Summary for Wani in Chapter 6

Appendix X The learning experience for Wili

Wili

Wili is a Fijian national of indigenous Fijian culture from the western side of the main island, that is the opposite side to the capital city, Suva. He is in the age range of 20 –25 years because he had left school for three years before coming to the College. He occasionally showed a lack of ready knowledge and skills, since he had forgotten what he had learnt in school. He had attended school for thirteen years completing some mathematics each year. In class at the beginning of the Teaching Program he was a quiet conscientious student but gradually became more assertive by asking and answering questions in class. During the Teaching Program he made many long entries in his Thinkbook which appeared to be his main way of communicating with me for the first few months until he began participating in class discussions.

Among the six case studies, Wili was considered as one of the three of lower mathematics ability based on the results recorded on his entry form to the College. The data gathered for this study appears to confirm his lack of ability (see Table 6.19). His results for the *Assessment of number sense* were for the three assessments slightly below the class average. His results for the *Test of algorithms* were, for the first two times below the class mean, but higher for the last test which was a considerable improvement for him.

Table 6.19
Quantitative data – Wili

Instrument		Number sense assessment			Test of algorithms		
Month		Feb	Jul	Oct	Feb	Jul	Oct
Score	Max	23	23	23	18	18	18
	Class mean	8.4	12.4	12.6	10.4	12.2	13.1
	Wili	7	12	11	9	10	14

His responses to items on the questionnaire, *Beliefs about mathematics* (see Appendix T) indicated a more negative view towards with a constructivist approach to mathematics education than the majority of responses from other class members (see Appendix N). But after the Teaching Program and at the end of the year he was much more positive in his responses.

Enculturation into the mathematics education classroom

Wili was one of the few class members who had considerable difficulty settling into life as a student. This appears to have been because he had been away from study for three years. However after a month he was noting:

> "At the beginning of the term I found it hard to concentrate in class and can't easily get the teacher's teaching but now I have come to know her well and to understand what is mentioned in class" (8/3).

It took Wili time to overcome a lack of self-confidence and so to be assertive:

> "It takes me a long time to raise my hand to ask a question because I am nervous for I might be wrong in saying the answer. I will try my best to get over this and change my attitude" (8/3).

Then he was proud of himself when, after a few months he asked a question in class:

> "You will remember today that I asked these questions and you explained them to me" (2/6).

Not asking questions was a habit he said he had in secondary school and he thought that this was the cause of some of his lack of knowledge of mathematics:

> "Moreover I learnt nothing from this topic [fractions] in secondary school which always made me dislike mathematics in class. I don't know why, but I tend to blame myself for not asking questions of my teachers" (7/6).

As well as a number of other students, Wili commented on the difficulties of participating when sitting in his appointed place at the back of the class at College with seven rows of desks in front of him:

> "At first I wasn't listening down here at the back of the class but I tried my best to recognise what sums of money or where to place the decimal point" (1/7).

This last comment appears to indicate that for Wili there was not an environment in which he could construct his own knowledge in collaboration with others in the class. There were other times when his comments did indicate such an environment. Cooperative group work in class was planned to be such a situation, but he did not mention it during the first few months even though it often occurred. Then when he did comment on it he noted not only that the experience helped his understanding and so was important for him, but that he enjoyed it:

> "I enjoyed today's class very much when we shared our ideas with my group" (26/8).

> "I think the children will understand this well if given good instructions and taught in groups" (?/5).

He also commented on the group class presentations:

> "I understand that I know it well especially when I [taught] and heard from my classmates when they [taught] it in front with their groups. Group work is very important" (8/9).

Another strategy, new to Wili was writing in a mathematics class. Although at first he found it surprising in his last entry he wrote about how it helped him to understand and revise his work:

> "Lastly what surprises me was essay writing because this is the very first time for me to notice that the latter is also done in maths" (18/2).

> "However it [Thinkbook writing] has given me thoughts and further understanding of my work. It also focuses on a day's work or activity" (6/11).

Another strategy that Wili found helped him to learn was using materials such as filling containers with water for volume, a topic he said he hated. The following entry also illustrates how his self-confidence has developed nearer to the end of the Teaching Program:

> "In today's activity [arranging in size containers of different volumes] I started to realise that the volume from smallest to biggest is just that easy. I learnt much of today's lesson and learnt thoroughly because I was given a chance to stand in front [of the class] and arrange the different quantities of water that is held by the bucket, basin, can and a container. By filling them with water it was easy to arrange them" (2/6).

Hence it can be seen that with time Wili changed and came to benefit from his experiences of the teaching approach particularly cooperative group work, writing in his Thinkbook and using materials, by becoming more confident and assertive.

His responses to the questionnaire *Beliefs about mathematics* also indicated a change (see Appendix T). Before the Teaching Program, Wili's responses to the questionnaire were among the most negative in the class towards a constructivist view of mathematics but after the Teaching Program his beliefs were considerably more positive as shown Table 6.20. The responses given in the table are his most notable because of the lack of change or the size of the change of at least two scores. In this table there are indications that he always believed strongly the importance of understanding the process rather than getting correct answers and seeing patterns and relationships. For him memorising rules was not important for learning mathematics.

The shift towards a more constructivist view can be related to work done during the Teaching Program. This included work completed using one's own strategies, estimating and doing mathematics related to everyday living. For the whole year his beliefs that did not support, though not strongly, a constructivist view were that mathematics consisted of rules and procedures, and that questions are to be tackled once they are explained. For most of the other items not shown in Table 20 he agreed with the constructivist view and for the other few he also tended to agree but was undecided at the end of the Teaching Program or end of the year.

One of Wili's greatest fears at the beginning of the Teaching Program was to stand in front of a class. This was something that would prevent his enculturation into the class. It must have been forefront in his mind because Wili mentioned it regularly. He believed that this also was the result of his past education:

"I always get nervous when I am told to teach in front of a crowd. At first I am worried and can feel my heart best very fast. I think it was the teaching that I was brought up with from lower classes and I did not have the opportunities to stand in front of a crowd. I am trying very hard to overcome it by practising a lot especially when given a Warm-up activity. I have usually got a tremble voice and I can feel my body all shooken up with fear" (8/3).

Table 6.20
Notable responses to Questionnaire by Wili

	Feb	Jul	Oct
Constructivist view – strongly held			
Understanding the process is more important than correct answers in mathematics	SA [a]	SA	SA
In maths it is important that students are able to see patterns and relationships.	A	SA	SA
*Learning maths involves mostly memorising rules.	D	SD	SD
Constructivist view – Change in agreement towards			
There are many different ways to do most calculations.	D	A	A
In maths it is important that students are able to estimate.	D	U	A
*There is always a rule to follow in doing a mathematics calculation.	A	D	D
*Maths is not needed in everyday living.	A	SD	SD
*Doing maths means following standard procedures precisely.	SA	D	D
Constructivist view – not held			
*Mathematics consists of a set of rules and procedures.	A	A	A
*In maths it is important that students are able to tackle a question once it has been explained.	SA	A	A

[a] SA strongly agree, A agree, U undecided, D disagree, SD strongly disagree.
*Item is worded negatively to a constructivist view

When his time came to present his short presentation of a Warm-up he was very pleased with his attempt and wanted to repeat the experience:

> "… because I found out that I got more confident when doing it. At first I was against it but it has helped me a lot" (14/4).

A week later, Wili was thinking about standing in front of a class with a seemingly positive frame of mind:

> "It is also interesting because I can view myself standing in front of a class teaching these things [operations using MAB]" (21/4).

Then some months later he was more confident to face the class for the longer group presentation:

> "I enjoyed when we were [organised] to teach with the topics given. I know this will really help me to stand in front of a class for 40 or 45 minutes. This is the stepping stone in my life" (25/7).

This last comment seems to suggest that Wili was very conscious of his need to be able to stand confidently in front of a class. I wonder how much of his mental energy was used in achieving this ability rather than in other mental activities such as discussing the work related to the activities. I say this because after this time it can be seen that he participated more in class discussion.

One anxiety I might have expected Wili to show was in doing tests but in fact, when he did the instruments for this study which included three different kinds of mathematics tests, he sounded pleased with his performances. He believed he had done well for the second administration of the number sense assessment even though results indicated very little change from the score he obtained before the Teaching Program.

> "I enjoyed the test [number sense assessment] so much because I have seen the improvements I have made. The test really gave me a opening to the pathways of doing things correctly within a short period of time. Although I still doubt in some cases, I count myself as one of the best in what I have done" (19/7).

For the test of the algorithm for the four operations his score did show an improvement with most marks lost as a result of getting three division questions incorrect:

> "… well comparing with the last exam, I think there has been some improvement. I enjoyed the test but I will try my best to do a lot of practical work. However dealing with division was not so good for me. I could honestly tell you that I am stuck up in solving division and I'm asking you if you could help me overcome it" (22/7).

Another aspect of confidence which emerged after some months was his enjoyment of the classes. When Wili wrote that he enjoyed an activity it was usually for one that he also thought was beneficial to him:

> "I enjoyed the day's lesson [comparing volumes of household containers] because I learnt so much from this thing" (2/6).

> "I enjoyed today's class very much when we shared our ideas with in our groups. They came up with important facts on what to do with our presentation to the class next week with the topic counting" (28/8).

He was pleased to able to compute mentally and noticed that he now had a positive attitude to mathematics:

> "I learnt that when I know the numbers I quickly know the answers. Now I am getting used to seeing numbers and adding and subtracting them or dividing it by a simpler way. Thanks for being so helpful. I can now say that I like maths" (16/6).

He enjoyed and was impressed with the use of music in class:

> "But [best of] all I liked the introduction of music [ABC Radio Number songs] which was played to the class. [It] tells the children the real words used in maths" (20/7).

In this last entry Wili referred to 'real words' use in mathematics. Wili had problems with the specialised mathematics language along with his lack of fluency in English. He found explanation of terms helpful:

> "The only thing I have to know is to follow and know the terms, i.e., deka, deci, kilo, micro, etc" (2/6).

He sometimes used new terms incorrectly:

> "I think it is a good idea to teach children more on 'conservation of number' [referring to discussion on 'bridging'] because it really shows the method or steps by adding or subtracting" (8/3).

Wili made an effort to be enculturated into the mathematics education class because he was motivated to become a teacher. One major aspect of his enculturation was that, after beginning the year with a real lack of assertiveness and self-confidence, he became as confident as most other students. During the classes he consciously made the effort to be assertive and stand in front of a class with confidence and he succeeded in his goal. A few of his comments and his responses to the questionnaire support that a change in beliefs and action had occurred. I suggest that this may have been assisted by the communication encouraged during the Teaching Program via group work, class discussion and Thinkbook writing. Wili appeared to be pre-occupied with his difficulties as the wider variety of experiences such as attitudes and cultural issues did not get a mention in his Thinkbook. It is difficult to determine if his problems were connected to lack of development of number sense or vice versa yet as his problems faded there was only a minor evidence of change in his number sense but this may have needed more time to emerge.

Constructing knowledge about concepts and the nature of mathematics

In this section first, the knowledge of concepts is considered as it is a major element of the development of number sense. Then second, the understanding of the nature of mathematics is discussed.

Understanding of concepts

The academic results in Wili's application for the course at the College indicated an apparently poor development of the cognitive aspect of number sense. At the same time he came with a lack of knowledge of the facts and skills because as he said he had forgotten what he learnt at school which he had left three years earlier:

> "It seems that I have forgotten numbers like natural numbers, whole numbers, fractional numbers, odd and even numbers, and decimals, but I now know them because it's being discussed in class" (8/3).

For the number sense assessment Wili's scores were well below average for the class (see Table 6.9.). Out of 23 he obtained the scores of 7, 12 and 11. For the seven questions he correctly answered both prior to the Teaching Program and at the end of the Teaching Program, he only correctly answered three of these at the end of the year, Questions 5, 13 and 14 (see Appendix A) which were two questions about money transactions, and one about the product of whole numbers. This suggests there may have been a considerable amount of guessing in choosing the best alternative in the tests. After the Teaching Program and at the end of the year he correctly answered four more questions, Questions 1, 2, 10 and 19 which were questions on estimation of numeracy and fractional length, on relative size of answers to operations on decimals and on re-arrangement of groupings. In the interview after the last test, he said he understood that for Question 4 (see Appendix B), *closest to* would mean the number closest *less than* not closest *greater than*. The College staff verified that this meaning was common in Fiji. But this does not explain why he composed the correct answer from the digits for the first two assessments. He excused his poor score at the end of the year as due to lack of time to check working which, after further discussion, indicated that he wanted to calculate each alternative answer rather than use number sense to determine the correct alternatives. For example, he wished to do many examples to determine the number of digits in the answer to the sum of two three-digit numbers. On the back of the test he had actually worked a few long multiplications instead of using estimates.

In the number sense assessment at the end of the year Wili correctly answered for the first time four questions, Questions 3, 6, 9 and 18, ones on an understanding of subtraction, percentages, an inverse operation including an extra factor of ten and the number of 10 cents in $46.70. In the interview I had with him after this last assessment he was able to choose the best responses, with some help from me in the form of leading questions, to most of the questions he had incorrect. Wili had not answered Question 12 about the approximate number of days old that a Class 5 student would be. When we discussed it in the interview he was able to estimate in days the age of a 10 year-old. It is interesting to note that in the interview he checked his response by doing a long multiplication in three steps of 300x 100 with its two rows of zeros to get the 30 000. Even after many leading questions Wili was unable to understand, for Question 15, that there were decimals between 1.52 and 1.53. Also he said he was unable to attempt Question 7, about the third of the distance around a square.

Wili's results for the two later administrations of the number sense assessment indicated little development of number sense since the first assessment. When choosing the correct response in some cases he appeared to understand the effects of the operations and a factor of ten while for others he was still more inclined to use an algorithm than do a computational estimation. Rational numbers remained problematic throughout the year.

For the interview prior to the Teaching Program in which Wili was asked to do mathematical activities (see Appendix D), Wili either left out questions or had difficulties attempting the questions. He said he was unable to explain reasons for the answers he did give. The questions he did answer correctly with apparent confidence were the first question about the estimating the time for a coconut to drop to the ground from a tall palm and the simple first parts of six of the questions, Questions 2, 4, 9, 10, 11 and 19. These were related to concepts about whole numbers such as subtractions, a fraction between 3/10 and 7/10, positions of fractions on a number line and arrangements of groups of straws using base ten. Question 9 ii. was one question that he did have correct that most students did not. This question dealt with the choosing of the closest pair, 237 and 194 from eight numbers less than 1300 that he arranged in order of magnitude.

In the second interview after the Teaching Program, Wili continued to have difficulties completing the mathematical activities and explaining his methods. For Question 6, estimating the number of students side by side to form a kilometre line, he just said it was 4000 and then changed it to 2000 without being able to explain his answer. He still had difficulty doing many of the other questions especially Questions 10, 11,15, involving inverse operations, and questions involving rational numbers - choosing a fraction between two given ones, arranging in order a list of rational numbers and writing 1/5 as a decimal and as a percentage. There were strategies discussed in class that he was unable to apply correctly such as the one for multiplication by 99. For some questions in which mental computations could be used Wili sometimes did use them, such as in calculating change from $10 for $7.14, but other times he used the algorithm showing all working such as for 1000 – 714 = . As was true for most of the other students this time he did *not* choose the correct closest pair of numbers, 237 and 194 in Question 9 ii. even though he had the first time.

But in this second interview Wili did attempt a number of questions using methods similar to those discussed in class. For example in Question 18 when asked to calculate 32 x 14 using four different methods Wili used methods related to work done for multiplication in class. These included diagrams with skip counting along a number line, an array of 32 columns by 14 rows and repeated addition. For estimation of the number of lollies in the jar for Question 23 he used an interesting strategy similar to the one we had discussed in class. He counted in the base seven lollies on both sides and four in the centre of the base. Then he counted eleven rows up the side and so added 77 + 77+ 44 to get 198 which he increased to 220 because of the lollies in the neck of the jar. Also in this interview for Question 12 he was able this time to chose a factor to multiple by 127 to give a number ending with six.

The responses in the interviewing, as well as from the number sense assessment indicated that rational numbers remained problematic even though there had been considerable discussion of them in class. Other evidence indicated signs of improvement related to other work done in class such as some use of mental computations, the understanding of operations and numerical estimations. Although these were related to the class work his strategies were different enough to indicate that he was not just applying a rote-learnt procedure.

During the Teaching Program he had noted some strategies for mental computation that he had learnt but it is difficult to determine whether he had learnt them with relational or instrumental understanding:

> "In today's activity I noticed that when multiplying and dividing by 10 everything is easy because I just add in zero to the answer especially in multiplying" (26/4).
>
> "In the case of division [by 5] it is by multiplying by 2 then dividing by 10 to find the answer" (26/4).

Wili appeared to understand the ideas involved in using materials such as MAB for base ten at the end of the Teaching Program and was able to express his understanding in words:

> "I learnt that the value of 1 will depend on the value we give to the object. If the cube is one then you will go 10 times bigger to the long or if a flat is one then going back to the long is 1/10 or one tenth smaller. I could easily understand this because of knowing their different sizes and what values will be the one given to an object you may select to start with" (5/7).

One computation Wili completed that I suggest did show some number sense development was the method he used in the algorithm test at the end of the year to find the answer to a long division. In the two earlier test his use of the standard algorithm had been very poor unfinished attempts. In the final test he obtained the correct answer using his own strategy which was a form of repeated subtraction similar to standard long division and only a little less efficient. His use of this strategy showed a considerable understanding of division and also he verified this answer with a multiplication. Hence his improved score in the test of algorithms actually did not only indicate an improvement in skill of using the algorithms but also provided evidence of improved number sense because of the use of his own strategies and verification of answers by using inverse operations. One interesting fact about his verification was that he used a three-step long multiplication of 46 *by* 293 rather that the reverse for which I have no obvious explanation.

From this analysis of Wili's understanding of concepts there appears to be a limited development during the Teaching Program. His estimation skills and understanding of the effects of some operations improved but his understanding of rational numbers and the use of mental strategies for computations improved only a little. In fact a couple of questions he answered correctly either verbally or as short written answers at the beginning of the year he incorrectly answered later, apparently indicating a poorer number sense in those areas of number relationships or lucky guesses the first time or some other influence.

Understanding the nature of mathematics

After thirteen years of mathematics learning, and perhaps because of those years, Wili appeared to have a limited appreciation of the nature of mathematics. He appeared to have little understanding that mathematics had a *rational* dimension and was often satisfied if he had a rote learnt procedure. As can be noted from the previous paragraphs, fractions and decimals were a problem for him and he was grateful for coming to understand some of the work we did but admitted at the end of the Teaching Program that he was still having difficulties:

> "Thank you very much for introducing this topic equal parts or fractions [equivalent fractions] – This was one of the topics I did not understand from my primary school" (2/6).
>
> "I hope you will provide more [classes] for us this semester especially the 'fractions' and 'percentages' in which I am still lacking knowledge" (27/6).

He appeared to understand something of the processes involved in bridging the multiples of ten as he compared it to the ease of non-bridging additions though he was not always able to clearly describe the processes:

> "I understand the bridging method [computations for addition and subtraction] easily if I compare it with 'without bridging'. To get to the answer first we have to bridge it, then add or subtract it from the remainder" (8/3).

As already mentioned, Wili was interested in and understood aspects of representations of number with maniplulative materials and symbolic representations such as in drawings, known as *objectism* (Bishop, 2001):

> "By using visual aids like paper money draws the attention of children, therefore when out teaching I will [concentrate] more on visual aids" (19/4).
>
> "In today's warm-up I learnt that division is well understood if grouping or sharing of objects would be taught to little children. They quickly grab what we want to tell them when given objects to group or share equally" (26/4).

> "I think drawing pictures is a better way of problem solving because it emphasises more to young children" (14/4).

Wili, at the beginning of the Teaching Program liked to have *control* over how computations were to be done but was prepared to use his own strategies later in the year. As mentioned above he initially believed that rules and procedures were most important:

> "Warm-up activity and number formations were the most interesting part of the lesson to me because we have to follow simple rules and instructions which an activity has got" (18/2).

> "By following simple instructions, I noticed that this is the simple way to gain knowledge and learn more from a topic. …. I will try my best to catch up with it because it is easy to solve any question if I know the method properly" (8/3).

In the *Test of the algorithms for the operations*, Wili had no problem in correctly using the algorithms for the addition, subtraction and multiplication of whole numbers and multiplication of decimals. For the division by 5 he left out a zero in the answer for the first two times he did the test. In the third test he correctly used his own strategy which was a form of repeated subtraction similar to long division and he verified his answer using a written multiplication. For the fractions in the test his working showed that he remembered parts of the algorithms for the operations but used them for the wrong operations. For the addition of fractions the first time he used the division algorithm and not because he mistook the addition sign for a division one as he clearly had the + sign. For divisions of fractions in the third test he used an algorithm in which he inverted the first fraction. It appeared that algorithms for fractions were procedures to be learnt and that he did not consider to check for reasonableness of answers. Nor did he link these notions with those learnt about equal parts mentioned earlier. As described above he made little attempt the first two times to do the long division algorithm which was taught in primary schools in Fiji. In the final test he did not attempt to use the standard algorithm but obtained the correct answer using his own method.

Wili realised that mathematics will *progress,* firstly in the area of technology:

> "I once thought that calculators might not be necessary in primary school. However by learning it now here at College I know that calculators will be used in the next few years in lower primary classes. Its advantage is that it gives the answer faster but we cannot work it without brain" (4/10).

Wili also came to *progress* in mathematics on a personal level. As noted above he chose to use alternatives to some of the algorithms he had been taught by developing his own strategies, such as he did for division. Earlier in the year he was developing problem solving skills and extending the application of possible strategies illustrating that his understanding of mathematics was not static:

> "I have learnt that there are various methods or ways to follow to get the answers" (8/3).

> "In the warm-up activity, we learned about higher or lower. I also could use a decimal range as an example like between 0.1 --- 0.2" (21/4).

In the last test for algorithms Wili knew that he could demonstrate the *openness* of mathematics to himself and to me by verifying the answers to his computations. He verified his divisions with multiplications. Also he found in class that checking for reasonable answers was helpful:

> "Reasonable answers. I have enjoyed the work I did today especially when given numbers to answer - with mental computations [inserting decimal points in the answers provided]. I have learnt so much from you Ms Fran to identify or solve a problem given, very quickly by looking at it" (4/7).

The understanding of the *mystery* aspect of the nature of mathematics was not obvious in Wili's case. His difficulties appeared to prevent much reflection on the mystery, wonder and surprise in mathematics.

From this it appears that Wili's understanding of the nature of mathematics has not developed much except for some aspects of rationalism and progress. Control and objectism he probably always accepted and understood. Mystery and openness have yet to be understood and expressed to a more effective degree for a teacher of mathematics.

This limited improvement in the understanding of the nature of mathematics is possibly further evidence to the data collected of his lack of development of understanding of concepts. His learning of mathematics has certainly not been standing still so if the processes that are producing the learning are continued, then the necessary progress to help him become an effective teacher may be accomplished. This will require more mathematical activities to be completed on which he reflects and applies the understanding he gains.

Lifelong learning for the pre-service teacher

After three years away from formal learning Wili attempted to become a learner of his own learning in the process of becoming an effective teacher. He came to the Teachers' College to learn how to teach but it appears that in the process he has come to see that learning how to learn is also an essential aspect of his education.

Learning to learn

Wili did progress in his metacognitive thinking. In what has already been discussed there have been many examples of times when Wili noted *what* he was learning and *how* he was learning. The entry given earlier described how Wili found using materials helped him to learn such as filling containers with water for volume, a topic he said he hated:

> "I learnt much of today's lesson and learnt thoroughly because I was given a chance to stand in front and arrange the different quantities" (2/6).

This entry also illustrated how his confidence developed with regard to standing in front of a class. The actual presentation of an activity to his peers helped him to realise he could do it. Also considered above, he saw cooperative group work as helpful and therefore important for his understanding of a concept.

Wili was able to note what made learning difficult for him. He attributed part of the difficulty and lack of concentration to his absence from study in recent years and partly to his lack of application:

> "I had difficulties in learning because I left school for three years. Listening to you was like being in Class one again. At first I did not know what it was all about but now I have caught up with you" (1/3).
>
> "To be honest I may say I was doing good work [writing in his Thinkbook] when I got those good marks, but I was slack and forgot to write those entries when I got those low marks. So it was my attitude towards writing down those entries in my think book that determines my mark" (6/11).
>
> "At the beginning of the term I found it hard to concentrate in class and can't easily get the teacher's teaching but now I have come to know her well and to understand what is mentioned in class" (8/3).

"Come to know her well" suggests that more than understanding what was taught was necessary for him. He also said it was difficult for him to participate by sitting in the back of a large class from where asking questions would have required much assertiveness. He attributed his lack of learning in secondary school to not asking questions in class. The following entry suggests that he thought effort and practice were required to be successful in what one does:

> "I think that boys should put more effort and practice in their warm-ups because most are not doing well in it but the rest are all right" (19/4).

He was serious about his attempt to study for examinations and devised a study plan which included the foregoing of some pleasures. He also noted how he learns best when revising:

> "Today my teacher told us to write our plans about revising for exams. I have to study every night and cut down time or marking [mucking] around or watching TV or after cocoa time in order to pass the exam" (19/5).
>
> "Firstly I have to prepare my timetable that is the different lessons carried to be studied each night. I left 15 minutes for each lesson and to study each lesson thoroughly and understanding it carefully. As I am not so good in just reading notes, I always note down points and at the same time memorise the facts into my minds. In this way I understand what I am doing. So, with all my efforts I will try to pass the exam" (19/5).

Having time to reflect on work on his own was important to Wili. He liked to puzzle over work in his own time.

> "I had difficulties in solving problems as soon as I saw them but now it is easier if I am given some more time later to do it on my own" (1/3).
>
> "I learned from this [fraction game involving colouring in]. I clearly know what to shade because I understand what is to be done in fractions. Thank you Ms Fran for introducing fractions to me. I hope I will teach my class well if I keep on revising this" (14/6).
>
> "From that [my answer to his query about equivalent fractions] I learnt and understood most of it. But if you could give us in class or myself some more examples of the above topic" (7/6).

He was actually quite excited about his coming to understand equivalent fractions better that day as shown by his closing remark in the entry:

> "When I leave the College I will burst out in flying colours with full understanding of [these] fraction problems" (7/6).

This statement was a rare one because he was commenting on his own development of number sense as an aspect of his education at the College rather than the aspect of learning to teach others which he often made reference to.

Learning to teach

Learning to teach appeared to be uppermost in Wili's mind in his reflections on what he experienced in class. From observing his peers presenting Warm-up activities and from his own presentation he said he learnt much about teaching. He discussed ideas many times in entries in his Thinkbook. I will quote from a few only:

> "I think the warm-up activity could be repeated because I found out that I got confident when doing it. At first I was against it but it has helped me a lot" (14/4).

> "I learned in the Warm-up to stand still speak boldly and clearly and talk to the crowd or children. I should be more careful when preparing the Warm-up and be more optimistic with it" (20/4).

> "I think today's Warm-up was interesting to be honest. This can be shown by the way the students and teacher reacted. The teacher organised [himself] well and all his shape[s] were clearly made up. The teacher understood his work well and I learnt I should prepare myself well before teaching" (26/4).

The Warm-ups were helpful to him in many ways as they gave him confidence to stand in front of a class and he learnt how important it was to prepare well. He also learnt minor strategies such as for the teacher to appear confident and to ensure that the children hear instructions given them.

Wili also believed that the use of teaching materials was important and this included number songs which he saw as a good opportunity to teach a number of ideas:

> "For example, telling the time [using ABC Radio Number songs] or 'when the long hand is pointing to 12' and the short hand to any number 'it is something o'clock'. Moreover the clapping of hands from 1 to 10. That really motivates the children. Especially listening to such music which has so many meanings and helps them to understand" (20/7).

Wili, probably conscious of his own learning difficulties, thought about the how to help his future pupils to lessen their changes of experiencing unnecessary difficulties:

> "… the warm-up activity. Firstly I did not understand what it is all about but now I know it makes the children to think and maths more interesting" (8/3).

> "Class 1 textbook - I enjoyed this, looking at what should be taught to lower classes. I noticed some pictures that would not be understood by the class especially those in rural areas. An elephant for example or an aeroplane would not be good for them. However the pictures of Fijian bure [traditional house] and

the women in traditional clothes will give the children something to reflect on" (21/7).

Wili was very conscientious and availed himself of every opportunity that he could to learn both for himself and for his future pupils. He appeared to have begun, in his first year at the College, to learn many things that would help him in the process of his lifelong learning. He reflected on what he experienced himself as well as what he noted happening for others. He also acted on what he believed needed to done even if it required effort both physically and mentally.

Summary for Wili in Chapter 6

Appendix Y Tabulated summary of Case Studies

Factors	Wili	Jo	Vita	Ana	Lisa	Wani
1. Enculturation						
Settling in	Difficult lack assertiveness confidence Initially afraid to ask qn	Think I did not like him Not read instruction or ask friends Lacked self-discipline & commitment	Surprised at new learning No criticisms	Initially boring, slow Teaching led to interest in the work, appreciate, then enjoy Critical. of others Warm-ups appreciated when applied	Often compared work to past schooling Surprise at writing, trust, activities, Very positive attitude	Need external motivation Fears stand in front Doubt her ability Saw need for preparation Effort to think
Interest in maths	Great interest Well-used Opportunities	Little interest, little improvement	Always interested	Changed from school days to CCTC Great use of opportunities	Hate to great interest and enjoyment	Improved from little to much
Attitude	Improved	Less than positive	Initially negative but great improvement	Improved	Good all time	Improved
Traditional maths teaching v constructivism	Found traditional not helpful	Knew facts Unable to link areas	Knew facts Wanted conventions transmission teaching	Frustrations initially using own strategies and Thinkbook writing Becoming a constructivist	Understand theory of and becoming a constructivist	Surprised at new strategies for assessment Critical of past maths teaching
Activities	Gave understand & enjoyment MAB good	Interesting, helpful strategies for teaching MAB good, timetables activities	Helpful for fractions Interesting Warm-ups Some confusion	Helpful for understanding & fun	Fun, exciting, helpful	Timetables exciting, helpful strategy in teaching
Group work	Helpful	No mention	Strongly agree helpful	Helpful	Helpful	No mention
Materials	most helpful	not use before helpful objects, diagrams, MAB	Believed necessary	Helpful for children	Helpful especially straws	helpful but little mention
Thinkbook	Helpful	Negative attitude Blamed his lack of effort	Helpful	Helpful for thinking	Helpful for revision and thinking	
Beliefs	Big change much development		Very positive always	More negative than class but became less negative.	As for class but became more positive	Generally positive

2.. a. Understanding concepts of mathematics						
Number sense results	Poor, little improvement Wild guessing	Teaching Program - much improved Tests – little	Tests – little improvement.	Much improved	Improved but missed many questions on assessment	Much improved
General understanding of concepts	Improved	Understand structure and uses of numbers better	Belief -know & understand but results not have indicated this Want to be taught	Good and great improvement	Good and great improvement	Good and great improvement later in unit
Mental computation	Enjoy improved a little	Few mentions Did use them	Much improve and interested but instrumental?	Initially used algor. Found helpful improved as learnt and used strategies	Improved from poor but worked at it	Improved, Instrumental?
Fractions	Understanding poor Found very difficult	Understanding poor but some improvement e.g. equivalent fractions helped	Understanding poor Believed improvement in equivalent fractions but little evidence.	Not yet competent but improved	Equivalent fractions understood but not used	Understanding poor Some improvement
Decimals	Understanding poor	Understanding poor Found very difficult	Understanding poor Felt understood Had insight	Improved but still need more, use money values	Not liked Money values help	Improved
Estimations	Improved	No evidence	Poor	Improved	Improved by effort	Much improved

2. b. Understanding of nature of mathematics						
Open Verification	Verify div and mental computation	Very little, answers left unreasonable	Multiplication for division	Learnt new ways	Become able, learnt new ways	Used verification to correct errors
Open Explanations	Always difficult	Very difficult	Improved little	Much improved	Much improved	Much improved
Progress-use own strategies	Believe in use of own strategies Long div good strategy	Calculators good Left answers unreasonable did use own strategy for division	Little use of won strategies	Wanted to use own strategies and came to do so	Improved use of own strategies	Believed no progress possible Little use of own strategies
Control Algorithms	Improve Long division and fraction algorithms incorrect	Improved, but multiplication gave unreasonable answers	Little change	Much Improved decimals and mental Prepared to use new methods	Used often even in assessments and tests rather than mental	Improved in division Over-use of long multiplication
3.Lifelong learning						
Learning to learn	Much reflection on what & how to learn	Frustration, understand his difficulties & see need to improve	Enthusiasm and beginning made Reflection led to insight	Much reflection on her learning, improved	Much reflection on her learning & lack of it	Much reflection on her learning & lack of it saw her and need to change
Teaching	Saw need to help understanding	Applied activities Appreciate difficulties of children, like help others	Great desire to improve understand Prepared well	Prepared to be a constructivist	Appreciate children's difficulties more	Came to desire & improve Help in understanding Like to be in control

Appendix Z Data collected for case study, Ana

Ana's personal information

(See hard copy in archives of ACU Library)

Page 1 of Ana's Thinkbook

(See hard copy in archives of ACU Library)

Typed copy of Ana's Thinkbook entries
18/02/1999
The surprising thing I have learnt in Maths so far is that we are forming digits again. It's like a whole replay of being in primary sch. I never really had though much of the kind of digit formations I did or how perfect it was. I also found that we had to do or form warm-up activities. This was taking up part of our maths lesson which was not so bad in one sense but maybe a waste of time on the other hand. Some of the war-ups took a bit too long. So maybe in the future we do shorter warm-ups so that when time comes to do the actual thing the chn. would not be bored because too much of an activity of the same kind will slow down the interests of the students.
24 / 2
Today was the first Maths day for a student, Vika to actually demonstrate a warm-up method. It gave me some idea of how to do mine when my turn comes. I think everyone has to have a chance in doing warm-up methods. This would help me in my actual teaching but I would consider one important thing – time. I hope when I do my activity it won't take too much time. On the whole I don't really appeal to the idea of having a warm. Anyway later on in class after discussions listing benefits of the students and teachers in a think book was more to my liking.
3/3
As usual our maths class began with a warm-up activity after an unfortunate situation though. The person rostered for the warm-up wasn't prepared and neither were the following others. But Ms Fran gave us a warm-up of her own on fractions. Later we did a bit of algebra figuring out the number of pups and chickens for certain heads and legs given. I enjoyed doing this as algebra was one of my favourite topics In Maths in high school. This method of solving is a strategy that can be used and is very efficient. But follows certain rules. This could help me in the future when I do my own lessons. I would have to make up my own strategies and go through trials and errors in order to form equations. This would also allow me to reflect on the method and to think deeper of what is at hand. And so too to make the methods as simple as possible in order for me to understand.
4/3
The maths class today wasn't as bad as I thought it was going to be. Actually I enjoyed it playing with the bottle tops and stones. It was like being in class 3 of two all ones again. I learnt some effective method of teaching the times table because children would grasp the concept more clearly if they did activities. So this was one warm-up I enjoyed very much.
8/3
Though the warm-up activity was a good one, I thought it would have been presented more interestingly. Too much time was taken up. Anyway besides this the results were written up on the board and there was a prize for whoever could figure out why the '5' the majority got the answer 5. This make me think. This method would help me in the future to get students to participate effectively. I enjoyed this bit. Then there were the numbers. Recalling what kind of numbers I first learnt was a bit difficult at first but I unfolded my memory and got out quite a few results. I feel that this activity not only put kind numbers that I learnt in order but it helped me realise that without learning some numbers we won't really understand the concept of the other. And this is what I must always bear in mind. I am teaching Maths in the future.
9/3

Today most importantly ms Fran stressed the importance of organisation and thinking questions in order to carry out the activity. The warm-up wasn't very practical. He could have made the activity more active in the sense that he got the students involved form the very beginning. I feel that if he (Tevita) had carried out the activity in a real situation the children wouldn't have grasped the full understanding of the activity / doing the clock face. This has given me ideas in which I could co-relate with other subjects.

11/3

Ms Fran came into the class today (well not a full class though, but part of Br. Camillus's class). She gave us a short test on multiplication addition and so on but I wasn't sure what is was for. I think it was to help her evaluate how she's gonna plan her other lessons. I enjoyed doing it though. Well maths class is one thing I find myself collecting a lot of 'rubbish' good rubbish. I have now come to realise how useful these cartons, bottle tops and such are useful for methods. Now when I open a packet of something I try and not to spoil it. I try and preserve it.

16/3

A few days last week we were given a warm-up to do. The Game of the Year and 1999 was used. I've been trying to come up with many possible combinations to fill up my paper. But it's frustrating me when I do something different I sometimes get the same answer. Anyway but it was fun for me. I'm still trying to get as many sums as possible. As for the warm-up, Ms Fran I think the way you correct the activities and yes I can understand the frustration you go through when someone doesn't prepare his/hers warm-up before hand.

25/3

Ever since we've started the year its been warm-up activities since. Now I see the efficiency of doing warm-up It doesn't only have to apply to the Maths but also other subjects. My reason for pointing this out is that today our first practical teaching practice I went to Marcellan and though I didn't do maths I applied the concept of doing warm-ups and the class actually enjoyed it. Like an oral question and answer.

26/3

Today for once we skipped maths and watched Jurassic Park instead. I am not really supposed to write this in my think book but I am anyway. This is just the thought that maths classes can be presented in all sorts of ways.

29/3

Today I learnt that to memorise numbers children or anyone pictures a wet of items in their minds and associates to the total number. We form mental processes. We form mental processes and this is known as conservation of numbers. I learnt that what we show children must be constant so that they'd memorise it. When they finally learn the concept then the order of shapes or ideas can be changed / verified.

6/4

Today we've got to re-read our think book instructions that's why Ms Fran has asked of this. Anyway today learnt an easier way to add 9, 99, etc., (By using the bridging numbers.) this has allowed to me think and it'll doe the same for the other students. Any with practice children can become mentally fit to calculate any sum.

7/4

Today Ms Fran asked us to produce our charts. Well I hadn't done it because I though we could use our actual number chart and that was done on brown paper. When I discovered this wasn't so, I got so frustrated. I was cursing within me. But then I did

the chart as quickly as possible. And when I got down to the exercise with Mika, I really enjoyed myself and got to overcome my frustration.
Well one thing too we've got to continuously do for maths is our aids. Geepas, is there so much to do. Well straws for one thing are due on Friday 9/4. And we need 1200 for goodness sake.
12/4
Today we dealt with straws for the first period of our double class. We were grouping them into sums that were written on the board. We worked in pairs, which helped a lot. I prefer working in pairs. It allows me to discuss what I may not know and discuss with my partner.
The clock making too was fun. I enjoy making things like clocks. This makes us use things around us. To make use of things to the fullest.
14/4
MAB blocks – multibase arithmetic blocks. Was what we learnt today in Maths class. I've only heard about but only found out its meaning weeks ago and who founded it. It's the same or deals with base ten materials. And with the use of these we can get children to be very practical and can picture sums or problems assigned to them. This method of teaching would be very effective to my point of view.
19/4
Today we played a game of I can't. It involved blocks sticks and die, score keeper and a bank. But when Ms Fran first distributed the blocks there was immediately free play among us. We did this unconsciously. It was only when Ms Fran asked us if we noticed something, did we realised that it was happening. This is how children would feel if they were given things to play with. Maths lessons don't have to be so solemn but can be fun
20/4
All we did at maths today was the warm-up from Mr Malakai. He could have a better control or coordination. To what he taught but unfortunately it wasn't a success. He could have done better. The rest of our Maths was taken up by science presentation. Well not the whole class, sorry because after that was subtraction using Dienes' blocks. We had to illustrate how to make things clearer to students when they actually see and understand what's going on.
23/4
Nothing much was done in class today except that we used our place value mats and used the base ten objects to work out the sums written on the board. It's exactly the way we used the straws. Maths is practical and many objects can be used to show things.
26/4
Maths lessons today were about place value mats but with division. Now that we are using our base ten materials I'll be very useful for our children in our future, for now they are experiencing model making. It will be a very effective way for children to learn their division.
29/4
We continued doing the division sums today with the use of the base ten materials. This gave me an idea of how I would use the same materials to work through divisions, multiplications and additions and subtraction sums.
Term Two
19/5

The second day of the term is today and our first Maths lesson of the term. Ms Fran gave us a summary of what is to be done this term. We're are having exams on week 6 and therefore I've got to write a study plan or revision plan. Including all the work done during the two terms. We've all got to study at least a week's work of Maths and science each nite for at least an hour give or take. Topics studied would be figure formation, benefits of writing in a maths class. Different numbers, counting principles, ten frame, bridging conservation of numbers mental computations using materials, MAB blocks numbers place value, Dienes' blocks, etc. But I'm not sure whether or not I will be tested on Warm-ups. (I'll have to ask her in the morning.) Anyway I've got to study at least 16 topics altogether. So therefore it is 4 topics per week. And I could revise too the warm-up then considering I have a bit more time.

21/5

Today I learnt more of how I could get my class to learn more effectively their times tables. It has given me a new outlook on what I have already learnt in order to make my students understand, not only recognise their times table.

25/5

Today we did something new. Well not exactly but a different way of doing multiplication. This strategy I would say would be very effective for the lads. Not only this but they would enjoy it very much, i.e. circle patterns I've learnt that multiplication can be taught in many ways that would interest the students. Ms Fran I definitely use this strategy in future for revision etc in class.

26/5

today we did much the same thing as from yesterday but deeper. Doing multiplications and divisions and actually trying to see patterns and easier strategies in revision or finding answers. this concept will help us in the future. It will make the children really understand and grasp the concept of the times-table. I would use or apply this kind of strategy to other subjects as well.

28/5

Strategies of learning maths can be conveyed in many different ways but the most important factor I've come to realise today is that we must think deeply first and come to understand by actually doing an activity in order to fully grasp the concept. Anyway this was brought up in an activity doing metrices [Metric system] And as too multiplication and other problems the same concept can be applied. I've come to realise how deep maths really is.

7/5

Ms Fran burnt a piece of paper today in it and in it are sums such as 2/3 + 4/9, ¾ x 1 2/11, 1 2/11 divided by 2 2/12, etc. The significance of this action was to show that this method is 'rubbish' so she says. Well personally I don't wee how worthless it is . I found it useful. Well different people have different views on things. Anyway we then did fractions using the compass., then clock faces. Ms Fran says this a much more effective way because in the near future chn will be using calculators which I think is true but then again they'll have to know at least the basics of their calculations. How about if their calculator got lost or malfunctioned during an exam? How then would they calculate a simple sum? So may be both methods should be taught.

9/6

The past few days have been filled with fractions. Fractions using clock face, using charts and discs. Today we started doing the disc and we will continue tomorrow. Fractions are easier to do when putting them down in charts, that's what I think personally. In the future when teaching fractions I'll definitely be doing these strategies taught. I think the students will benefit from it and not only would they know, they'd actually be visualising the chart.

10/9 (6?)
The lesson today began with mental computations. We revised some equations and Ms. Fran suggested that we work in partners to practice the equations. I'm not really keen on the method for mental computation although I know it is very effective. Is sometimes I can't get my brain working fast enough. I've got to try harder I know I can do it if I put a lots of effort into it. We continued using fractions by using discs. These are all strategies I would definitely use in the future. It allows free play and using models to express equations. I think the students would learn a lot because they are actually familiarising themselves with the materials and thus the Maths concept.
14/6
We coloured in fractions today by playing a game of throwing the die, I found today that through equivalent fractions the larger fractions in the end were harder to get rid of. This may be experienced too in the future by the children so I would suggest to them to choose the equivalent fractions they think would be the hardest to shade first. Thus would allow them to revise their addition and equal fractions.
1/7
Ms Fran mentioned at the beginning of the class that girls are less slower than boys in Mathematics. This is so because girls tend to copy everything off the board whereas the boys are distracted easily during the class and then have to think about what was taught. Therefore girls (not all) have to think of how a teacher did a particular sum and not to their own thinking.
7/7
Yesterday all we did or worked with the base ten material doing decimals. So the base ten materials have many uses in Maths I'd use it in the future. Though there're all worked on place value the place value mats are very useful when doing Dienes blocks from either decimals addition or other Maths sums.
Today we worked with dominoes. This was the first time I've ever handled dominoes and I found it fun working in groups playing the game so I'm sure the children would enjoy it very much.
Before doing dominoes we did some mental computations on division but than working with dollar signs. I found it easier than without the dollar sign. I think I will always use this when I do division. We also graded the equations and different students will have different gradings due to their intellectual capacity.
19/7
Last week on Friday there was a test on decimals/ or fractions given to us by Ms Fran. From all the notes and activities where we actually worked with fractions i.e. one worksheets and dominoes. There was actually free play and then an understanding of the equation. By doing all these workouts it has helped me get through my test. I found it a very pleasant test and thus finished before time. I never really liked working with decimals before time. But now it has become more interesting this changing my point of view especially when changing decimals to money value. This has helped me to work through it more easily.
22/7

We received ‘teachers’ as well as ‘pupils’ handbooks from Ms Fran and given a set of questions. Doing the activity has given me a better sense of looking for things/texts in future. It would not only apply to Maths but other subjects. Thus given one a better understanding of how to conduct my lessons.

22/7

The nine sums that we given us today was something Ms Fran probably wanted us to do, to see how we go about. Anyway I found that doing the addition and multiplication were a bit easy although I still used the ling form. Anyway one division sum that I had taken long to do was the last one. I had to do long division though roughly. I knew the answer had to be in the 200’s. The fraction sum was one that I could not do just by looking at it. I had to go back to the old way of doing it. I did picture the parts but couldn’t pick out the answer accurately. But on the whole I did the sums a lot faster than I would have done when school began.

23/5

I did not really understand the 5th question in the paper and therefore got it wrong. But now the concept is clear after going over the paper with Ms Fran. Probably I did not spend enough time thinking of the questions of what I got wrong.

2/8

Beginning maths consists of five stages. Sorting, counting, measuring, numeration and problem solving. Ms Fran says we should familiarise students with certain objects first before actually teaching the concept. This I feel is very important. Free play will allow students to know something a little better. Progress will succeed.

5/8

Reminders -Bring think book every day,

-50 words per class,

-don’t write about warm-ups,

-not quantity but quality.

In maths concentration and coordination is very vital. If a person misses out any stage of learning from the beginning, learning would be difficult for him/her. Therefore a professional occupationist would have to deal with this person in order to correct this handicap.

Term Three

9/9

Term Three. Today we were given a sheet of paper to read concerning psychological effect on children. From this I learnt that children have to be active and passive. And in order to do Maths, teacher should associate it with the real world and not just in black and white prints. I hope I don’t become one of these teachers. Later we did multiplication and division. But first concentrated on the latter, i.e. multiplication. It was a revision for us.

15/9

Continuing on multiplication. Ms Fran showed us how to make a bead frame. It must be very colourful in order to attract the children’s attention. This will be a very good teaching aid, especially for multiplication.

6/9

We continued to do multiplication today. We worked from a class three work-sheet. I realised today how multiplication could be very confusing for a child because it could also be an incorrect sign ‘x’ . So Ms Fran has given me some insight into how to

explain multiplication better so that it wouldn't be confusing to the children and I hope that I do teach it well. I couldn't want them to be confused that's for sure.
4/10
Calculators in Primary schools. For the past two weeks we have been working on calculators for primary students. Maybe in the next few years the primary school classes will be using calculators. Calculators may very well help chn. with their work but they do not have to use it for every single problem. When they know they can't do the sum mentally then it'll be useful. Or when they can do it mentally but not fast enough, this is when they can use the calculator. But we must emphasise the fact that the calculator must be looked after very carefully for they are fragile.
5/10
Today we went over the myths about calculators. I really did not realise that people assumed that calculators were bad. The many reasons discussed for the reasons we had to find like solutions. I could do it for some but children becoming reliant. I had the idea in mind but could not put into words. That is one of the weaknesses that I observed during this particular part. I hope to improve.
12/10
Discussing the strategies on 'checking answers' have given me a better equipment in order to check answers with very little use of the calculator and by listing the advantages and disadvantages I have come to realise that calculators are very useful. Because we listed more good to bad.
13/10
Today's lessons after adding up of calculators was the grouping. Probably icebreakers where students are asked to do sums and we as a class answer the question. It was only short lesson and we continue next week.
5/11
Writing my thoughts this year about maths was a new experience. At first I found it hard because I did not know what to talk about. Even towards the end of the term this year I am still sometimes unsure. Writing my thoughts every day was a bit of a hassle for me. I found myself forgetting at times to fill in my think book but on the other hand I feel it has benefited me somehow. When I actually flip through my maths notes to see what we had been doing a particular day, it helped me revise. I guess it also gave me a sense of deeper thought of what I was actually doing. It made me think about why and how I may encounter things in the future. Maybe that's the whole point of why MS Fran calls it Think book. The only major drawback in writing this think book is that when one actually felt that I've put a load of effort in writing my particular thought that day and how I got an unsatisfactory mark it really weighs me down. Maybe this is for the better but somehow I did not know how to improve because even when I wrote my last pieces of thought I did not get such glorious marks. Maybe I hope that if our new lecturer or any lecturer gives us a think book to write in that I'd do better and learn from my mistakes.

Ana’s Results for the Three “Assessments of number sense”

(See hard copy in archives of ACU Library)

Ana’s Results for the Three “Test of algorithms for the operations”

(See hard copy in archives of ACU Library)

Ana’s Responses to the questionnaire “Beliefs about mathematics”

(See hard copy in archives of ACU Library)

Ana Transcribed interview Feb. 1999, CCTC

The formal structured questions which were read and shown to the interviewee were written on individual cards and where necessary materials were provided. I f Ana had difficulty with an earlier part of a question, later parts were not asked.
Written working and solutions are given on the answer sheet provided for this interview.
Most leading questions were given only written responses as shown on the answer sheet.
F. While we are doing these I will not be telling you if your answers are right or wrong.
1. Read the question and choose what you think is the best approximate answer.
Answered promptly
2. Write down your simplified working
ii. How could you find out this answer another shorter way than subtracting 35 many times?
I want to know how many 35's would be taken away. a short way of getting how many
A. Divide
F. 3. These are similar but you may not be able to think of a shorter way.
Tell me why you have done that. Rewrite if you can't think of a shorter way.
No oral but written responses only to the parts of these questions
Why did you put 5x2 and 25x4 first?
Why did you write 0?
4. Look carefully at the numbers and operations on both sides and then determine what you will fill in the space with
Why?
A. ii I can tell by looking at it.
iii 101 x 54 > 54 x 100 + 54 x 1 The right side goes down.
5. Can you give me an example of a one digit number? Two digit? Three digit?
A. 3, 13, 300
First how many one-digit numbers? Two digit? Where do they start?
F. Three digit?
Answers written 9, 90, 1.
For the two-digit numbers you showed that each has two digits, e.g. 10, 11, 12 gives 6 digits
A. Double it
What is the total number of digits?
6. Think of CCTC students and uni students standing side by side with their shoulders almost touching, how much for each student?
A. Half a metre then with each a half a metre there would be 60 students Okay
How did you calculate that?
7. Write down 264 and set out an addition sum with an answer with 0 in the place where 6 is in this number.
What could you add to get such an answer.
Four
F. Do you mean 4 ones?
A. 40
F. Write it down
Now we will do the same for a subtraction. Are there any other possibilities?
A. No
Question 8 withdrawn form the interview procedure

9. i. Write your choices in the spaces.
ii.. I will call out the numbers in the order you have chosen for you to copy down
iii. Do whatever scribble on the page to help you decide. You have tried two subtractions, do you want to try more?
iv Copy down the difference you have calculated for these two numbers.
v. What would 10 back form 456 be? Another 10 back? Continue for a few. What number just below 299 would you come to?
vi. Do whatever scribble you wish to find the number of tens.
What number would you finish on? How many tens would you count?
A. 29 (Long pause) 6, 16
10. a. Write down your answer You can do working if you need to.
A. 286 (Quick response) You add
Look at the operations I said and see if that helps you decide.
b. Write down what you did. Now look at it and can see why I knew you would be back where you started?
A. Not sure? okay
c. Think of a small number add on a number and now subtract a number so that you end up where you began
d. *Not asked*.
11. If you think there is a fraction write it. It does not have to be in the middle. Do whatever scribble you wish to find the answer.
Quick response to part one and two then long pause and no further solutions given.
12. Do you know what a product is? Write down 127 and set out as you would set out a multiplication.
Why did you decide on 8?
b. Remember the answer can be big but must not be greater than 4000.
Can you think of any other number that would be okay?
Why not choose a bigger one?
13. Do you know what a number line is? Imagine this going on from 2 to 3, 4, 5, and so on.
What value would you give for A, what is its position on the line?
Now B.
Here we have D and E further along the line. Where do think D is? E. is?

14. Now to an easy activity What is the exact value of this money?
b. What large-valued coins would be needed so that only a few coins are choosen – the least number of coins. Not two dollars notes to be included.
c. What is the change? (Quick answer) How did you get that?
15. Choose whatever fraction you wish. Make it an easy one for yourself.
b. i. Now for these three – 0.4, 60% and ¾ arrange in order – find the biggest in value, smallest and hence the middle one
ii. Now I am adding a few more and I want you to arrange these in order of magnitude. Do whatever scribble you wish in this space.
A.I will change them all to fractions because I like them better. 0.09 is less than 0.1, 120% and 1 ¼ are the same. No they are not
I will read out your list and you write them down in the spaces.
16. Think of this sum but do not work it out. 43 x 69 Circle the one in the list that you think would be closest to the answer.

Why did you choose that answer?
A. 2400 because 24 x 60 = 2400 so 2500, no 3000
17. Look at this setting out:
a two digit number + a two digit number + a two digit number + a two digit number a two digit number < 100
Do whatever scribble you wish in this space to help you decide which you think is correct. Would it be always true, only sometimes true or would it not be possible – never true?
i. That is each number is less than 20. Would it be always true, only sometimes true or would it not be possible – never true? Quick response
ii. If one number is greater than 60 that all five would give a number less than 100 always, never or sometimes
iii. Think of four numbers greater that 20 and one less is that what is always the case for getting <100, of never, or sometimes. Long pause
iv. Think of two numbers less than 20 and one number greater and the other two any two-digit numbers.
Would it be always true, only sometimes true or would it not be possible – never true?
v. Would it be always true, only sometimes true or would it not be possible – never true?
18. Do the sum 32 x 14 the way you normally do it. Now get the answer by doing it a different way- use a method that is different from that one in some way. Imagine if you never learnt that method how might you get the answer doing it any way you like.
Okay we will leave that one now.
19. i.
ii. Now look at this group of 5 envelopes and 3 loose straws. If the envelopes were filled with ten straws how many altogether?
v. a. Now it is a bit different.
21. Read this through and find how many fish in the lake? Do whatever scribble you wish to find the answer.
Please explain how you got that answer.
A. 70 fish he catches 50 and tags them next day - 20 with no tags.
22. You can hold the jar and turn it anyway you wish and count anything from the outside but you are not allowed to open the jar. I will give you a short time to work out your estimate.
How did you work out that number for your estimate?
A. 130 lollies. They are tightly packed.

Interview with Ana. Jul., 1999, CCTC
Ana's working and solutions are given on the answer sheet. Her oral responses are given here in italics.
ss refers to see answers written on answer sheet
F. Read the question and choose what you think is the best approximate answer.
A. .Do I need to consider how tall the tree is?
It is a prettty tall one but not the tallest in Fiji but not a baby one. Not a dwarf one.

2. How could you simply work out that one?
ii. If you wanted to simply find out how many 35's by subtracted from 455?
A. Division.
F. What number into what number? ss

3. You might do these differently this time.
i. Have a look along the line and see how you could easily add up these numbers – you don't have to add them up one after the other. If there is no way of simplifying it just put down the same thing.
.A. Approximately? just give the same answer?

F. No it needs to be the exact answer.
i. ss
ii Do you know of a simple way of doing it? Ss
Looking along it you can see? Don't make it more complicated Just to find another way Is there a simpler way? Just before you go I noticed you simplified i. By doing what to get to this line?
ss What did you do? I am interested in what you did there (Second line) ss You changed that one but you are not going to change that one. Do you see any similarities?
.22 lots of 16 and 22 lots of 6 Does it work when you have got subtraction
.A. It is hard if you have got subtraction. ***
***I am interested in that one (ii) Say if that 22 was an x just write it down.
ss What is the short way of doing that? Have you forgotten your algebra? What is a simpler way of writing that?
.two into (Pause) Does it look a bit strange? No worries. Algebra is beyond primary school
iii. ss Why 234 x 100 - 234 ?
iv. Why 2x5 first?
A. It is 10 Write down 10 x 18
v. Why 25 x 4 first? ss
v. 0
4 i. Look at the sides and then say what you will fill in the space with =, <,>
A. .equals
Why?
.A. Same number and all multiplication.
F. Not in the same order. Does that matter?
A. .no
ii. Look at both sides Can you tell by looking at it without calculating? Equal?
. (3x36 greater) Tell by looking? Why 3 x 36 greater?.
A. .approximating these numbers to be 40

Can you do it more exactly though? 3x36 means what?
A. .36+ two sets of 36
Right. Scribble that down ss Have you got that on the other side?
.A That [34] is 2 less and that[39] is 3 more.
So which side is treater if they are not equal?
What is on the right? Is this greater or less or equal to the left? That is 2 less and that is 3 more so what does that come to? Scribble on it to even them out. Ss So now you think this side is greater? ss
iii. Looking at both sides? Can you tell by looking
A. .Same (Pointing to the parts?)
5.
First how many one-digit numbers? .9 Two digit?
Which are two digit? .10 to 99 How many is that? .90
Three digit? .1 Four digit? .none So actually how many digits if you had to write them down by hand.
.100 Let us do it step by step.
A. 9, 180, 3
Just jot those done
6. Now to a sum with some metric work in it. How many would be in the line a kilometre long?
A. More than 1000
Do you know what a kilometre is? 1000 metres
Scribble in the space if you like (Long pause) If you have a 1000 m how many students would that be?
Would it be a 1000 students? Would it be more?
A. one shoulder ½ metre .more than 1000 about 2000 ss
7. Write down 264 and set out an addition sum with an answer with 0 in the place where 6 is in this number.
A. 4
Four ones? .
A. Not greater than 5 there .44
A. Subtraction .63
Could you do the sums again and put in other numbers? Ss Do you think there are a few others?
A. Yes
9. i. ii. Find the smallest and largest numbers and then arrange the others in order in between
Now that you have done it I will tick the spaces on the sheet
iii. Now which of those two are the closest together in value – there is -the least value between them.
A. .These are more than 100. 456 511
What is their difference? Ss
Could you mentally do it in your head if you think of what is in between there – a special number
.A. 500
F. Can you find the difference by going up to 500 and ? .
A. 456 to 500 = 44 500 to 511=11 gives 55
Difference between those two

A. .237 approximately 240 3 more than 237 299 to 300 which is 1 more .62 ss
What about here 194 and 237 What is the jump there?
A..To 200 is 6 and then 37 43
What do you think about that?
A..better

v. Counting by tens down from this would you end up on 299? What number just less than that would you finish on?
A. 296
vi. How many tens would you count from 456 to 299? ss

10. a. 286 You wrote the 2 down first How did you do it?
A. Approximately 300 is needed to get to 1000 but then you have to subtract by 14.
F. How check your answer?
A. Addition
b. What is an example of a one-digit number? What does 'double a number' do to a number?
Do you end up with the number you began with? Try it with another number. Do you think you would always get the same answer?
A. .Yes if you follow the same
F. Why? Ss
A. If you multiply x2 and x5 you get x10 and so divide by 10 you get back.

c. This one is simpler. Again think of a small number, add on a number and now subtract a number so that you end up where you began. Write down what you are doing? ss

d. This one is a bit different and harder. Do it for a couple of numbers. Do you think you would always get 12? Why? Do any scribble you wish.
Where does the 12 come from?
A. The 6 and then multiply by two.
Why has the first number gone? Why take it away twice? (Tape became too difficult to hear.)
11. If you think there is a fraction larger than the smaller and less than the bigger one, write it. It does not have to be in the middle. Do whatever scribble you wish to find the answer?
i. I would like you to tell me why you choose that one?
ii. Do you think there is a fraction between these two? What is it? ss
Why choose that one?
Which is the bigger fraction?
Why did you choose that one?
iii. Which is the smaller of ¾ and 6/7?
Draw any diagram you wish or do scribble to help you.
You are showing them on a drawing of a fraction chart
Which is closer to one whole, 1? And why?
Is there a fraction which would come in between the two parts on your diagram?

12. Do you know what a product is? Write down 127 and set out as you would set out a multiplication.
Why did you decide on 8?
A. 8x7 = 56
Now the second part. If the answer is to be less than 4000 is there another answer. Do whatever scribble you wish. Is there another answer? Remember the answer can be big but must not be greater than 4000. Can you think of any other number that would be okay? Why not choose a bigger one? ss
13. If this diagram is of a number line with 0, 1, 2 in the positions marked
What approximate value would you give for A, what is its position on the line? ss
Now B. ss Leave out C. Here we have D and E further along the line. Where do think D is? ss E. is? ss
14. I don't have the coins to count the money as you found it so easy last time.
b. What large-valued coins would be needed so that only a few coins are chosen – the least number of coins? Not two dollars notes to be included. ss
c. What is the change? How did you get that? (Done quickly without written working.)
15. Choose whatever fraction you wish. Make it an easy one for yourself.
b. i. Now for these three – 0.4, 60% and ¾ arrange in order – find the biggest in value, smallest and hence the middle one
ii. Now I am adding a few more and I want you to arrange these in order of magnitude. Do not need to copy them I will tick the space to say you did them
16. If 43 x 69 Which of those answers would be the closest? Are you doing an approximation?
A. .281, 42 x 70 3000
17. Look at this setting out: ss
a two digit number + a two digit number + a two digit number + a two digit number a two digit number < 100
Do whatever scribble you wish in this space to help you decide which you think is correct
Would it be always true, only sometimes true or would it not be possible – never true?
i. Does that mean each number is less than 20. Always true, only sometimes true or never true? .sometimes.
ii. If one number is greater than 60 what could it be? Ss .61 for two digits .can we have a zero before a digit?
No we don't call that a two digit number. Why what is the smallest thing we could have? .10, C
iii. Think of four numbers greater that 20 and one less.
.21 And 21 fives comes to what? .C
iv. Think of two numbers less than 20 and one number greater.
v. What could the numbers be? Write down an example five numbers. .same tens? Yes but the number is different.
Does it have to get to 60? Ss Could they be smaller ? ss With these 5 numbers is there sum going to be greater or equal to 60?
Is it always true, only sometimes true or would it not be possible – never true? .always true
18. Do the sum 32 x 14 the way you normally do it
Now get the answer by doing it a different way- use a method that is different from that one in some way.

Can you think of other ways? Okay we will leave that one for now.
19. i. ss
ii. Now look at this group of 5 envelopes and 3 loose straws. If the envelopes were filled with ten straws how many altogether?
A. 947
Not what is left but what is here in my hand. .
ss
A.53
v. a. Now it is a bit different. .Back wards one.
b. ss
21. The next one is a bit of a biology story. Ones that got away. Read this through and find how many fish in the lake? Do whatever scribble you wish to find the answer. .70
Please explain how you got that answer of 70?
A. The first day there were 50 caught and then tagged then on the second day there were thirty picked out and 10 of them were ones picked out the first day so only 20 other are yet to be added to the 50 to make 70. .
What is your highest maths? .Form 7
So you would have done a bit of probability and that. So if you had a pond that full of fish and the wild life man came in and only caught 50 and he put tags on these 50 and threw them all back into the lake. The next day he pulled out a random 30 and in that 30, 10 of them were wearing tags. What about the number of tagged ones? If I picked out 30 and ten of them were tagged could you work a proportion of them that had tags on them. .
One is to two.
F.If I had 60? .
20
How many tags altogether? Draw the pond. Many fish but one bundle has what? .50 has tags. Whole lot of fish that haven't got tags. So someone comes in and grabs a pile of 30 and ten of those have got tags so what did you think will be left in the pond how many tagged left in the pond .
A. 40
Draw another pond. How many are there? What about the untagged? Would they all be in the one spot. Would they all be swimming around? Tagged and untagged. You said the ratio 1: 2 so 50, untagged?
A. 100
Are you happy with 100? Yes.

22. The last one. You can hold the jar and turn it anyway you wish and count anything from the outside but you are not allowed to open it. Do whatever counting you like. (Long pause) Have you done any counting yet?
A. The base has approximately 20. Height of the bottle is 10 , 11
F. How many would that make approximately Do a sum if you like. ss (Multiplied the algorithm.) More or what. Estimate in the top. 10 Come to what altogether
A. 210.
F. Would you like a few of the lollies?

Third interview with Ana. November, 1999

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F. Thanks Bessie Ann for coming at this busy time for you when you are packing to go home, etc. This interview for the study is different from the other two as this time I will give you the opportunity to talk about the last test you have just done which were part of the study. You will have the opportunity to explain how you got some of your answers. You have nearly every question correct. Let us look at No. 7. Read the question through and then see if you can tell me why you chose that spot there?
Wait which way does it say to move around the square?
A. I just went that way. I did not think about what the question said.
F. Yes if you go that way that would be the correct answer.
A. I did not read the question properly.
F. For questions 15. Take time to read the question again and explain how you got your answer.
A. The difference in between the two there is 0.01
Is the question asking for a difference?
A. Oh I see - different decimals in between.
Here I will write down the two and leave a space to see if you can write any in between
A. 1.521. 1.522, 1..523 1.524 1.525 1.526 1.527 1.528 1.529
F. (I write them down as she says them) Ana could you have four decimal places?
Could we have 1.521 ?
A. Zero
F. Then?
A. Two
(Write 1.5212) Is that in between?
F. Could I have any others?
A. Yes
F. So which of those answers do you think?
A. Many
F. So before you were looking at a word "difference"
A. Yes
F. Okay . See here Number 17. Without calculating by just looking at the answers, decide which of these two is <u>not</u> reasonable? (Long pause) 45 multiplied by 1.05 (Long pause) Is it more or less than 45?
A. More (Looking at the alternative answers)
F. It is less there isn't it? Let's try the others. Next one 45 by 6.5 you approximated that what would you get?
Get more
50 by 6 what does it come to?
300
F. Too big but the answer is a bit less.
A. Yes
Does it look alright?
Yes
F. Next one. 87 by 1
More than 1
F. 90 by 1? Is 93 more than 87
Yes

F. What about this one? 589 by 0.95?
Should be less
F. So is that one less
A. Yes
F. So which one is the most unreasonable?
F. Which did you put? Do you know why? Did you think of A?
Yes I had that first but then I was thinking D
So you did take that choice. Why did you take D?
I was considering that as zero
F. 0.95 Why? What made you think that?
Zero multiplied by something that is nothing
F. So you expect the whole answer to be nothing?
A. Yes
I think the other test was all right. Have you got the other sheet? Here we are. Only one mistake. Oh. The only mark I took off is here. Can you guess why I took a mark off there?
A. Because I not cancel it down
Has it ever been acceptable for you to pass up an answer like that? Are you surprised that I would take a mark off?
No
How much is that? If I have 2/3 and 3/5. Oh it is one and a bit. It is more normal to say it is one and a bit more.   Okay other wise it is right. Very good. Okay Thank you very much.

Ana's answer sheets for the three interviews

(See hard copy in archives of ACU Library)