Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics Education

Hobart, Australia

July 13-18, 2015

Volume 4

Research Reports

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PME39 13-18 July 2015 Hobart, Australia

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Cite as: Beswick, K., Muir, T., & Fileding-Wells, J. (Eds.) (2015). *Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4). Hobart, Australia: PME.

The Proceedings are also available on-line at http://www.igpme.org

ISSN 0771-100X

ISBN 978-1-326-66435-0

Cover Design and Logo: Helen Chick

Printing: UniPrint, University of Tasmania

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SHIFTING THE EMPHASIS TOWARD A STRUCTURAL DESCRIPTION OF (MATHEMATICS) TEACHERS' KNOWLEDGE

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Despite the wide range of various conceptualisations of (mathematics) teachers' knowledge, the literature is restricted in two interrelated respects: (1) the focus is (almost always) limited to the subject matter content, and (2) the form and nature of teachers' knowledge seem not to have been noticed by researchers working in the field. The paper seeks to address these gaps by (a) broadening the current perspective to include an epistemological, cognitive, and didactical lens on the knowledge base for teaching mathematics, and (b) going beyond what the teachers' knowledge is about to take account of how the knowledge is structured and organised. The theoretical work presented here intends to stimulate discussion about the structural description of this kind of knowledge.

CONCEPTUALISATIONS OF TEACHERS' KNOWLEDGE: MAPPING THE TERRAIN

Over the past decades, several interesting approaches, partly distinct and partly overlapping, in conceptualising the *knowledge base for teaching* have been developed; the majority of them follow Shulman's (1986, 1987) distinction between subject matter knowledge, pedagogical knowledge, pedagogical content knowledge, and knowledge of various aspects of the educational setting (including knowledge of the educational context). The frameworks and models that shape the landscape in research on teachers' knowledge are at various levels of *specificity* – ranging from general to discipline-, domain-, and concept-specific frameworks (see, Scheiner, 2015).

Quite a few *general* frameworks contributed to the field, particularly in (a) shifting the attention to subject matter knowledge *for* teaching (in addition to subject matter knowledge per se) (Shulman, 1987), in (b) providing insights into critically important determinants of what teachers do and why they do it, namely teachers' *resources* (including knowledge), *orientations* (including beliefs), and *goals* (Schoenfeld, 2010), and in (c) highlighting the multiple dimensions of *teachers' proficiency*, including, but not limited to, knowing students as thinkers and learners (Schoenfeld & Kilpatrick, 2008). The latter contribution builds the bridge to discipline-specific frameworks since Schoenfeld and Kilpatrick initially developed the framework of teachers' proficiency in the context of *mathematics*.

A substantial body of research work is located in mathematics education, providing both *discipline*- and *domain*-specific frameworks and models (e.g., Ball, Thames & Phelps, 2008; Baumert et al., 2010; Blömeke, Hsieh, Kaiser, & Schmidt, 2014;

2015. In Beswick, K., Muir, T., & Fielding-Wells, J. (Eds.). *Proceedings of 39th Psychology of Mathematics Education conference*, Vol. 4, pp. 129-136. Hobart, Australia: PME.

Fennema & Franke, 1992; Kilpatrick, Blume, & Even, 2006; Rowland, Huckstep, & Thwaites, 2005; Tatto, Schwille, Senk, Ingvarson, Peck, & Rowley, 2008). These frameworks and models of *knowledge for teaching mathematics* can be understood as elaborating rather than replacing Shulman's (1986; 1987) contribution to the field. The approaches taken, and the conceptualisations of mathematics teachers' knowledge proposed, are not inclusive, nor are the identified dimensions of mathematics teachers' knowledge mutually exclusive. In contrast, the identified dimensions are *complementary*, and provide, taken together, a more *refined* picture of the knowledge base for teaching mathematics (see, Scheiner, 2015).

Notice that, with few exceptions (e.g., Even, 1990), researchers have almost overlooked *concept*-specific frameworks. However, from the author's perspective, investigating teachers' knowledge at the level of specific concepts is an important issue that needs particular attention in future research efforts.

MOVING BEYOND PAST AND CURRENT TRENDS IN RESEARCH ON MATHEMATICS TEACHERS' KNOWLEDGE

As described in detail elsewhere (Scheiner, 2015), several trends can be identified in past and current practices in research on mathematics teachers' knowledge. For the purposes of this paper, the attention is drawn to two particular trends:

(1) Although the discipline-specific frameworks mentioned above differ in detail, many of them converge in efforts to further *extend* and *refine* the construct of subject matter knowledge (SMK) and pedagogical content knowledge (PCK).

(2) With few exceptions, the literature tends to a particular orientation, namely the idea of a teachers' capacity to *unpack subject matter knowledge* in ways that are accessible to their students.

In more detail, the literature suggests that subject matter knowledge (SMK), for instance, can be further extended and refined in *qualitatively different* sub-dimensions such as Bromme's (1994) distinction between school mathematical knowledge and academic content knowledge. However, of particular importance and interest are contributions that reflect the idea that there is *unique* content knowledge for teaching mathematics. For instance, the notion of 'specialised content knowledge' introduced by Ball and her colleagues is described as pure content knowledge "that is tailored in particular for the specialised uses that come up in the work of teaching" (Hill et al., 2008, p. 436). In this sense, and in contrast to Shulman (1986) treating 'SMK *for* teaching' as equivalent to PCK, these considerations lead to the claim that there is pure mathematical knowledge specialised for teaching mathematics. Thus, it seems reasonable to distinguish between *mathematical content knowledge per se* (MCK per se) and *mathematical content knowledge for teaching* (MCK for teaching) (see, Scheiner, 2015).

However, recent approaches in the literature on the knowledge base for teaching mathematics center their focus on the *subject matter content* and articulate the importance of the central teaching task that is making the mathematics content accessible to students. In the literature on mathematical knowledge for teaching, these recent practices are reflected in the metaphor of 'teachers' *unpacking* of mathematics content in ways accessible to their students'. The author argues that this dominating *content-oriented* focus can be traced back to Shulman's (1987) conceptualisation of PCK as the capacity of '*transforming*' subject matter of the discipline to subject matter of the school subject. To put it in other words, most of the contributions in the 'mathematical knowledge for teaching' literature tend to be associated with a particular 'school of thought', namely Shulman's (1987) idea of a teacher's capacity for transformation of the subject matter – the capacity to deconstruct one's own knowledge into a less polished final form where critical components are accessible and visible.

Drawing on recent theoretical reflections on conceptualising (mathematics) teachers' knowledge (e.g., Scheiner, 2015), the work calls to broaden the perspective to include an epistemological, a cognitive, and a didactical dimension (see, Figure 1), in addition to a content dimension.

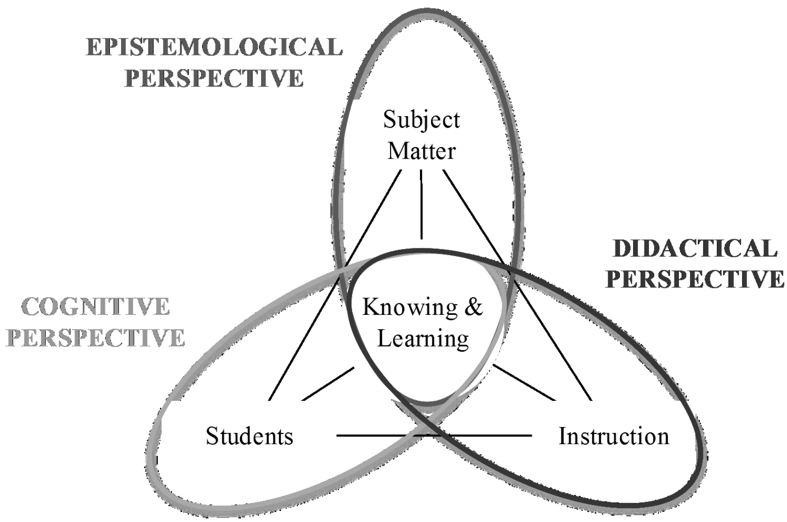

Figure 1: The epistemological, cognitive, and didactical perspective

The *epistemological* dimension refers to knowledge about the epistemological foundations of mathematics and mathematics learning (see, Bromme, 1994). For instance, Harel (e.g., 2008) calls for teachers' knowledge of epistemological issues involved in the learning of specific mathematical concepts including knowledge of epistemological obstacles. The *cognitive dimension* refers to knowledge of students' cognitions (Fennema & Franke, 1992), in particular, knowledge of students' common conceptions, knowledge of students' cognitive difficulties involved in concept construction (Harel, 2008), and the interpretation of students' emerging thinking (Ball et al., 2008). In other words, it includes knowledge of how students think, learn, and acquire specific mathematical knowledge (Fennema & Franke, 1992). The *didactical* *dimension* refers to what Shulman (1986, p. 9) described as knowledge of "the most useful ways of representing and formulating the subject that make it comprehensible to others", including teachers' illustrations and alternative ways of representing concepts (and the awareness of the relative cognitive demands of different topics) (Rowland et al., 2005) and knowledge of the design of instruction (Ball et al., 2008).

These various dimensions (epistemological, cognitive, and didactical) are considered as useful *lenses* in investigating (mathematics) teachers' professional knowledge, in particular, in describing the interconnectedness of knowledge of subject matter, knowledge of students' understanding, and knowledge of instructional strategies. These three resources (subject matter, students' understanding, and instruction) should be directed towards the same goals (i.e., learning goals) and reinforce each other rather than working past each other. However, this is often challenging to achieve. Often what is missing is a central theoretical framework or model about *knowing* and *learning* which guides the process and around which the three resources can be coordinated. From this perspective, a model of cognition and learning may serve as a cornerstone that brings cohesion to subject matter, students' understanding, and instruction (see, Fig. 1).

Bringing these perspectives into focus, several extensions and refinements of Shulman's initial categories of subject matter knowledge and pedagogical content knowledge can be identified, namely (a) knowledge of students' mathematical thinking and understanding (KSU), (b) knowledge of learning mathematics (KLM), (c) knowledge of teaching mathematics (KTM), (d) mathematical content knowledge per se (MCK per se), and (e) mathematical content knowledge for teaching (MCK for teaching).

In summary, the teachers' knowledge base can, and should, be examined from a range of angles using different lenses, including an epistemological lens (knowledge of learning mathematics), a cognitive lens (knowledge of students' mathematical thinking and understanding), a didactical lens (knowledge of teaching mathematics), and a content-oriented lens (MCK per se and MCK for teaching).

A STRUCTURAL DESCRIPTION OF TEACHERS' KNOWLEDGE: THE NATURE AND FORM

In the past, the literature concentrated its focus on what the teachers' knowledge is about. In doing so, the literature limited its attention to the *content* teachers do or should possess. What is missing in the current landscape of the conceptualisation of mathematics teachers' knowledge are efforts in going beyond what the teachers' knowledge is about to include a *structural description* of teachers' professional knowledge. Of course, several perspectives for theoretical reflection on the nature and form of teachers' knowledge can be presented (Scheiner, accepted), including those concerning the *nature* of the knowledge such as

(a) *source* What are the constituent knowledge bases?

(b) *development*	Does the transformation of subject matter knowledge per so to subject matter knowledge for teaching takes place by the individual teacher situated in the act of teaching or is it supported by educators and curriculum?
(c) *specificity*	Is the knowledge general, subject-, domain-, or topic-specific?

as well as those concerning the *form* of the knowledge such as

(i) *degree of integration*	Does the amount of knowledge in each knowledge domain matter most or the degree of integration?
(ii) *size*	Does the knowledge comes in pieces, units, or schemes? Is the knowledge stable and coherent or contextually-sensitive and fluid?

From the author's perspective, the major issues that need better resolution if we are to understand teachers' acquisition of an integrated knowledge base are questions concerning the *nature* and *form* of teachers' professional knowledge. In the following, new avenues for theoretical reflection on these issues are outlined. The objective of such theoretical reflection is evolving – aiming to make new theoretical extensions and innovations.

Teachers' knowledge as a complex system of 'knowledge atoms'

Although the various frameworks and models on the construct of mathematics teachers' knowledge have provided crucial insights on what mathematics teachers' knowledge is about, several of the discipline-specific frameworks represent conceptualisations of mathematics teachers' knowledge by a very general approach that seem ad hoc. The author, by contrast, does not believe in the existence of a general framework on teachers' knowledge but rather thinks that in investigating the form and nature of teachers' knowledge various frameworks may be discovered, which will be quite specific to particular mathematical concepts and individuals.

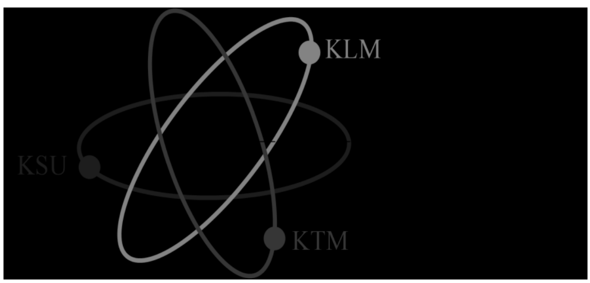

Figure 2: The 'knowledge atom'

The author calls for paying attention to investigating what in this paper is called 'knowledge for teaching mathematics' considered as a pool of personal and private constructed pieces of knowledge that have been transformed along a variety of knowledge bases identified in previous research investigating the multidimensionality of teachers' knowledge. In more detail, this work emphasises to view the professional knowledge for teaching mathematics as the repertoire of '*knowledge atoms*' that have been transformed along (1) knowledge of students' mathematical thinking and understanding (KSU), (2) knowledge of learning mathematics (KLM), and (3) knowledge of teaching mathematics (KTM), taking (4) mathematical content knowledge per se (MCK per se) and (5) mathematical content knowledge for teaching (MCK for teaching) as the cornerstones (see, Fig. 2). Notice that (i) the notion of 'transformation' implies that the constituent knowledge bases are inextricably combined into a new form of knowledge that is more powerful than the sum of its parts (*degree of integration*). (ii) In contrast to Shulman and his proponents' work, it is KSU, KLM, and KTM, together with MCK per se and MCK for teaching that build the knowledge dimensions that serve as the constituent knowledge bases for teaching mathematics (*source*). (iii) The notion of 'knowledge atom' indicates that knowledge is of a microstructure, highly context-sensitive, and concept-specific and has to be considered as of a fine-grained size (*specificity* and *size*). (iv) The notion of 'repertoire' indicates that knowledge is personal and private and that teacher education programs can only provide (as good as possible) rich resources for building up a fruitful repertoire of knowledge atoms (*development*).

The above mentioned considerations draw on the '*knowledge in pieces*' framework developed by diSessa (e.g., 1993), in particular taking the view of knowledge as microstructures coming in a loose structure of quasi-independent, atomistic knowledge pieces. Form the author's perspective, the 'knowledge in pieces' framework provides a rich resource on which to explore these, and related, issues.

NEW PRACTICES IN RESEARCH ON TEACHERS' KNOWLEDGE: MODELING TEACHERS' KNOWLEDGE AT THE 'KNOWLEDGE LEVEL'

As stated in the previous section, with few exceptions, past and current research seems to have skipped describing and characterising the structure and organisation of teachers' knowledge. One of the aims of this work was to progress toward a structural description of teachers' knowledge, and the previous section may have moved in that direction. Since the lack of a theoretical foundation of an adequate description concerning the *form* and *nature* of teachers' knowledge is recognised, research is needed that looks at knowledge (and processes of knowledge development) in fine-grained detail, through which a theoretical framework evolves. A structural description of teachers' knowledge is, at least from the author's perspective, an ongoing process that is always subject to new information and insights. With this, the objective of such research is evolving – by simultaneously developing theory and empirical research. Though a comprehensive theory is targeted, seeking not 'grand theory' but "humble theory" (diSessa & Cobb, 2004) with multiple cycles of revision and extension seems to be appropriate.

Research efforts on the way to a suitable description concerning the *form* and *nature* of teachers' knowledge should take place at the background of well-established practices in research on teachers' professional knowledge describing and identifying what the knowledge is about (concerning *content*). From the author's perspective, it is time to move toward new practices in research on teachers' knowledge that examine in a dialectic way both (1) the nature of certain kinds of teachers' knowledge (theory development, concerning *form*) and (2) what people know of that kind (empirical work, concerning *content*).

Research is needed that aims to *model* (mathematics) teachers' knowledge at the '*knowledge level*', for instance, by drawing on the methodological approach employed by researchers working with the 'knowledge in pieces' framework (diSessa, Sherin, & Levin, in process), namely *knowledge analysis*. Within the wide range of types of methodologies in 'knowledge analysis', in terms of time-scale, empirical and theoretical focus, in particular, microanalytic and microgenetic methods provide a good target for a complex, integrated, and dialectical research design. From the author's perspective, *knowledge analysis* may challenge the boundaries of what is known, and may provide a rich resource for a more complete and nuanced understanding of teachers' knowledge.

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