

Teachers' Decisions About Mathematics Tasks When Planning

Peter Sullivan

Monash University

<peter.sullivan@monash.edu>

Doug Clarke

Australian Catholic University

<doug.clarke@acu.edu.au>

David Clarke

University of Melbourne

<d.clarke@unimelb.edu.au>

Anne Roche

Australian Catholic University

<anne.roche@acu.edu.au>

At some stage when planning, teachers make decisions about the mathematics tasks they will pose and how they will structure lessons. It seems, though, that these decisions are complex, and that this complexity has been underestimated by curriculum developers and teacher educators. The following is a report of data collection that simulated some of these planning decisions. The results suggest that teachers may need support in matching tasks to curriculum content statements, in articulating the purposes of tasks, and in considering how tasks might be used to address differences in student readiness.

The development of the new Australian Curriculum: Mathematics (AC:M) provides an opportunity to reflect on the types of support that teachers might need in the process of converting a curriculum into classroom actions. The following is a report of some data collected as part of a larger research project¹ exploring aspects of the implementation of this new curriculum.

A fundamental assumption in the AC generally, and this project in particular, is that curriculum documentation has the potential to assist teachers in improving their planning, teaching and assessment. Sullivan, Clarke, and Clarke (2012a) reported results of a survey of primary and secondary teachers in which the planning of units of work (or lesson sequences) starts with four preparatory actions, specifically: reading curriculum documents to identify the important ideas in the concepts to be covered; examining resources and textbooks for task suggestions; meeting with colleagues to draw on their experiences to inform planning; and making judgments based on their own assessments of student readiness. Based on the outcomes of those actions, teachers then identify tasks and sequence those tasks. Subsequent to the selection of tasks, teachers plan their lessons. Yet all of these steps are complex. The following is a descriptive account of data that were collected under conditions that aimed to simulate the decisions teachers make when planning their teaching, specifically the link between tasks, as suggested in available resources, and decisions teachers make about their use.

Teacher choices about classroom tasks

Fundamental to teachers' decisions about tasks is their view of what mathematics they expect students should learn. If, for example, teachers emphasise student fluency with procedures, then the choice of tasks is straightforward. If, on the other hand, teachers seek

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to develop understanding, problem solving and reasoning, using the language of the proficiencies from the AC:M (ACARA, 2011) then the choice of task is both more important and more complex.

Anthony and Walshaw (2007), for example, in a synthesis of research results that inform practice, concluded that “in the mathematics classroom, it is through tasks, more than in any other way, that opportunities to learn are made available to the students” (p. 96). Yet teachers are busy and the task of teaching is demanding so teachers mostly draw on resources developed by others rather than creating tasks for themselves. So identifying the potential of tasks and articulating their purpose are key skills for teachers.

As described by Stein and Kaufman (2010), classroom tasks create the potential for students’ learning, but whether or not tasks achieve their potential is dependent on the ways in which they are enacted by teachers. Stein and Kaufman argued that the choice and use of tasks should be informed by the demands they place on teachers as well as the opportunities for teacher learning contained in the curriculum materials.

When choosing tasks, the first decision for teachers is identifying the potential of the task and the extent to which it matches their curriculum goals. This is informed by their appreciation of the inherent mathematical ideas (see, e.g., Ball & Bass, 2000; Chick, 2007), as well as the pedagogical actions needed to implement the task (see Hill, Ball, & Schilling, 2008). Connected to this is identifying the purpose of the task both to inform their interactions with students and also to articulate that purpose to the students. At the time of writing, in many schools in which we work it is common for teachers to be asked by school principals and others to identify learning intentions for each lesson and to prescribe criteria for student success. This is based on findings of the comprehensive meta-analyses by Hattie (2009) which identified feedback as a factor that is likely to produce learning gains in students. Earlier, Hattie and Timperley (2007) listed the key elements of feedback as being that teachers anticipate the rhetorical student questions of “where am I going?”, “how am I going?”, and “where am I going to next?” Clearly to answer these questions, teachers need to know and be able to articulate the purpose of tasks.

A second decision is whether the demand of the task is appropriate for the students. Sullivan and Mornane (in press) argue that tasks that are challenging are more likely to prompt students to build connections between the mathematical ideas that are foundational to learning mathematics. Yet it has been argued that teachers are reluctant to pose challenging tasks for fear of negative student reactions (Tzur, 2008). Decisions on task demand are also a function of the diversity in students’ cultural backgrounds, language fluency and readiness to learn (see Delpit, 1988),

A third decision relates to the potential of the task for catering for the diversity of student readiness. Given the spread of achievement of Australian students on international comparisons (Thompson, De Bortoli, Nicholas, Hillman, & Buckley, 2010), and the evidence that the major differences are those within schools (Rothman & McMillan, 2003), it can be assumed that all classrooms are diverse and teachers need to be able to use tasks that can be adapted to cater for that diversity.

The data collection process reported here was intended to simulate each of these decisions. In connection with the first decision above, teachers were asked to indicate the important ideas underpinning a given topic and to state the purpose of a set of mathematical tasks that were offered to them. In terms of the second decision, teachers rated which tasks they would be likely to use. In exploring the third decision we asked teachers to indicate which tasks would be suitable for catering for all students.

The impetus for the data collection

The data collection was prompted by the responses of teachers to an earlier online survey on aspects of the use of curriculum documents, and the AC:M in particular. One of the items in the earlier survey invited teachers to indicate which of a range of content descriptions matched some mathematics tasks that were presented (see Sullivan, Clarke, & Clarke, 2012b for a full description of the items). Two of the tasks which were proposed as relevant for Year 4 students were as follows:

- Oranges:* How many oranges have been cut up here? (this was accompanied by a photo of a plate of 14 quarter oranges)
- Numbers between:* What are some numbers between $\frac{1}{4}$ and 0.8?

The teachers were presented with a table that included the six content descriptions from the AC:M that mentioned fractions and decimals for years 4 and 5 and were invited to indicate a match for as many of the content descriptions as they wished. An example of the content descriptions is as follows:

Year 4: Recognise that the place value system can be extended to tenths and hundredths.

While around half of the responders to the *Oranges* task identified the same content description as we would, others chose descriptions that seemed quite different from the task such as “Compare and order common unit fractions and locate and represent them on a number line”. Likewise, while for the *Numbers between* task around half of the responses matched with our choice, there were substantial numbers of teachers who chose descriptions such as “Count by quarters, halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line”. Similar tasks and content descriptions were presented to secondary teachers and there was also variation in their responses. While it is possible that the content descriptions are ambiguous, and that teachers read more into the task statements than we do, we were interested in exploring this apparent anomaly in more detail. In particular, we sought to explore further the ways teachers might indicate the potential and purpose of tasks. The questions that guided this data collection were:

1. To what extent are the descriptions of the purpose of tasks given by teachers clear enough to communicate to students the important idea or mathematical purpose?
2. Does it make a difference to the clarity of their descriptions if teachers have worked through and discussed the task with other teachers?
3. What is the variability of teachers’ preferences for particular tasks, the levels at which they consider the tasks to be applicable, and their indication of potential of the tasks for addressing the diversity of student readiness?

The methods, data collection and results

To establish a context for the gathering of data that could reveal insights into the above questions, we set up a simulated planning exercise for teachers who were participating in teacher professional learning sessions connected to the AC:M. The teachers were invited to respond to prompts related to a set of six brief descriptions of lessons on the topic of division that were intended to act as teaching suggestions on the topic. The teachers were presented with the relevant content descriptions from the AC:M and were asked to identify the important ideas in teaching division, although these responses are not reported here. There were two groups of teachers, with 26 primary teachers and 17 secondary teachers altogether. Given the nature of the programs in which these teachers were participating, it

could be anticipated that these teachers were more aware of related curriculum issues than the average teacher. Note that not all teachers completed all the prompts and so there is variation in the numbers of responses presented in the data below. The intention was to simulate the process of teacher planning and so teachers were allowed time to consider the suggestions, and to discuss their reactions with peers although they were asked to respond individually. It is noted that, while the number of responses overall is small, the data are detailed and the intensive nature of the process required two hours of the teachers' time in which they discussed, then formulated, their responses. Due to space limitations responses to the prompts for only three of the lesson suggestions are presented below although the responses to the others lessons are similar. The three lessons not reported here involved suggestions for multiplicative partitioning, a set of division exercises for practice, and a game involving a die. The teachers were asked to consider that they were planning:

a hypothetical sequence on division for an upper primary or junior secondary class (ages 11 to 14)

One lesson suggestion, *Missing number division*, was based on the following task:

I did a division question correctly for homework, but some numbers got washed away. I remember it looked like

$$_ _ 4 \div _ = _ 4$$

What might be the numbers that got washed away? (give as many answers as you can)

In this case, teachers worked through the task as though they were students, including discussing their responses in small groups (readers are also invited to work through the task). There was a plenary review in which teachers discussed a range of different approaches to the task.

To gain insights into what teachers described as the purpose, they were asked

What would you say to the students in your hypothetical upper primary/junior secondary class is the mathematical purpose of this lesson?

Their individual responses were typed and categorised as either “adequate” for communicating the purpose of the lesson to students in ways that would focus the students’ thinking without giving away possible solution strategies, or “inadequate”. We are aware that our interpretations of responses are subjective and highly inferential. To address this, we discussed our interpretations in detail before making judgments. We defined an “adequate” response as one that we consider would communicate the purpose unambiguously to the students. Illustrative examples of such responses to *Missing number division* were:

To use this number problem to see the relationship between multiplication and division and how our multiplication facts can help us work out division problems.

The purpose of this lesson is for you to recognise the relationship of multiplication and division and have knowledge of place value and patterns. Also to not give up when you find one.

Use knowledge of multiplication to help solve division problems. Recognise patterns in numbers.

Such responses assure us that these teachers could communicate the purpose adequately to students. Examples of responses which we coded as inadequate because they were incorrect or vague were:

Division problems can be written in different ways.

Understanding there are many ways to reach the same goal/answer.

To figure out what these numbers are and be able to show how you get the answers.

The responses of 33 out of the total of 43 (18 out of the 24 primary teachers) were coded as adequate. Having worked through the task, three quarters of the teachers described the purpose of the task in such a way to give us confidence they would describe it clearly to the students. Indeed the adequate descriptions are insightful. On the other hand, one quarter of the teachers did not describe the purpose well, even after completing the task and participating in a discussion of different solution approaches.

To get an indication of the extent to which teachers might choose to use the lesson, we asked them to indicate the lesson out of the six they were most, and least, likely to use. Out of 33 responses, six (three of them primary teachers) indicated it was the lesson they were most likely to use, and eight (seven of them primary) out of 31 said it was the one they were least likely to use. As with the other prompts, there was substantial variation in the ways that teachers interpreted the lesson suggestion. We also asked the teachers to respond to the following prompt:

Which of the lessons would be most suited for maximising the learning of all students? Why?

Only two teachers chose *Missing number division* meaning that fewer teachers considered this suitable for catering for all students than any other of the lessons posed. Although the task is challenging, we feel it is accessible to students from different levels of readiness since students can respond in various ways, and it is not difficult for even normally low achieving students to find at least one solution. It is also easy for teachers to create a similar but simpler task if necessary. This is discussed further below.

A second lesson suggestion (*Handfuls*) was presented to the teachers as follows:

Players take a handful (or cup) of counters (more than 30 but less than 100). Teacher asks questions like “how many groups of 7 (say) do you have?” or “divide your counters into 7 (say) groups, how many in each group?” Students first estimate then work out the answer.

The teachers did not have the opportunity to play the game. We see this lesson as useful for alerting students to the different forms of division. This might take a whole lesson in younger grades but would be simple for upper primary and junior secondary students, the level at which the prompt was proposed.

Illustrative examples of descriptions of the purpose of *Handfuls* that we consider adequate for communicating the purpose to students are as follows:

Differentiating between types of division and understanding terminology division/quotient, recognising the difference in questions in terms of the meaning (literacy focus)

Different ways of forming groups and different aspects of division. Remainders

Show different ways of modeling division

Thirteen out of 43 teachers gave such responses. It could be noted that another eight mentioned the partition/quotition aspect which indicates that they see the purpose of the task but we do not consider those terms by themselves as suitable for describing the purpose to students which is what they were prompted to do. In other words, only a minority of teachers described the purpose of this lesson with any clarity.

Out of 33 responses, eight (6 of them primary teachers) indicated it was the lesson they were most likely to use, and six out of 31 (2 of them primary) said it was the one out of the six lessons they were least likely to use. Again the variation in teachers’ preferences is evident.

Eight out of 28 teachers, the equal most frequently chosen response, chose this as the lesson most suited to catering for the needs of all students. Some representative explanations of this choice were as follows:

A good beginning lesson and hands on for primary students

Would be easy to adapt to different levels and then further adapted to extend students. It is also useful in incorporating different aspects of dividing e.g. remainders, factors, worded problems etc.

Depends on the level! I think (this lesson) gives a very open way of demonstrating how division works!

We do not agree that it is a good lesson for catering with diversity other than in levels much lower than upper primary. It is possible that these teachers considered the lesson suitable for all students because it is easy.

The third lesson, *Division with remainders*, was proposed as follows:

Students solve each of these problems. They can use a calculator. They can be told that all the answers are different in some way.

We need to hire buses to take all the students in the school to a concert. There are 1144 students and each bus can take 32 students. How many buses do we need to hire?

We are making up packets of chocolates. Each packet must have exactly 32 chocolates. If I have 1144 chocolates, how many complete packets can we make?

Our basketball club won a prize of \$1144. The 32 members decided to share the prize exactly between them. How much money will each of them get?

etc

The teachers did not have an opportunity to work through this task, but did have the opportunity to discuss it with peers. Illustrative examples of responses to *Division with remainders* that we consider adequate for communicating to students

To demonstrate an understanding of the process of division and how an answer can be arrived at (i.e. inverse relationships within a real life context); identifying the maths to be used

To see how the same mathematical question can be applied to solve many different real problems – what does remainder mean?

There were 19 out of 44 teachers (11 out of 26 primary teachers) who gave such responses. This is fewer than for the task that the teachers worked through. Examples of responses which would not communicate effectively the potential of the task to students were:

Knowledge of units of measure can affect the answer

Division

Eleven out of 33 (8 of them primary teachers) indicate this as being the one they would be most likely to use, making it the most popular task, even though it was chosen by only a third of the responders. Two teachers considered it the one they were least likely to use.

There were also eight out of 28 teachers, the equal most frequently chosen response, who suggested this was the one most suited to addressing diversity in readiness. Some representative explanations of this choice were as follows:

Not only involving division, students need to reason on the suitability of their responses. Covers other topics such as whole numbers, rounding speed and distance

As it caters the most for different ability levels, offering some challenging questions. Helps students see real life importance of dividing whole/rounded numbers. Lesson incorporates aspects of all proficiencies including reasoning

In contrast with the similar number of teachers who chose the previous lesson, these teachers have described a more elaborated purpose for the potential of this lesson. We agree with these teachers that this is a suitable lesson for catering for all students.

Conclusion

As part of extended professional learning sessions, teachers completed a planning exercise in which they were presented with six illustrative lesson suggestions and invited to respond to prompts about those suggestions. The process was intended to simulate the planning of a sequence of lessons and teachers had the opportunities to discuss the lesson suggestions, although they responded individually to the prompts. The responses of the teachers to three of those lessons were presented above. These lessons, which are representative of the others, address respectively an open-ended exploration that reveals an interesting pattern, a game that illustrates different forms of division, and a lesson that explores the meaning of remainders in context.

We had led the teachers through the *Missing number division* task, including orchestrating a group discussion of the experience. More teachers were able to describe the purpose with sufficient clarity that would give confidence that they would use the task productively than there were for other tasks that teachers did not work through. Teachers seemed to overestimate the level of this lesson, nevertheless, and did not consider it suitable for mixed ability teaching. Even so, it seems that there are advantages for teachers in working through tasks before using them, and also in discussing tasks with colleagues.

The *Handfuls* lesson is based on a simple game, included in the set of lessons because we considered it to be too simple for the level at which the hypothetical planning exercise was proposed. Quite few teachers were able to describe the purpose in a way that would be helpful for students. We suspect that the teachers who rated this lesson as the one that best catered for all students to have made that judgment because it was simple. In our view lessons that cater for all students are those that are challenging with potential for variations to be posed for those who need them. Lessons that are easy are unlikely to extend more advanced students.

The *Division with remainders* lesson is the one we prefer but there was only a minority who agreed. Some teachers were able to describe the purpose but most were not. We consider this lesson suitable for catering for all students but quite few teachers see this as the lesson that is most suited to this purpose.

Perhaps the most significant finding is the variability in all aspects of the responses. Different teachers liked different lessons, there was no consistency in the adequacy with which they described the purpose, there was a significant range in the levels for which teachers considered the lessons appropriate, and wide diversity in what lessons teachers considered suitable for mixed ability teaching.

We acknowledge that the simulation itself and our interpretation of responses is high inference and subjective, but we argue that the exercise closely aligns with aspects of planning processes in which teachers engage and the responses and our interpretations can be taken on face value. We suspect that describing the purpose of tasks is more complex than is commonly recognised, that teachers' preferences for tasks are diverse, and they have differing opinions on what constitutes mixed ability teaching. These aspects can be productively emphasised in teacher professional learning sessions, especially those associated with the implementation of the new AC:M.

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