'GET DOWN AND GET DIRTY IN THE MATHEMATICS': TECHNOLOGY AND MATHEMATICAL MODELLING IN SENIOR SECONDARY

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Applications and mathematical modelling have been a distinctive part of the senior secondary curriculum in Queensland for over two decades. Findings related to technology use from an on-going longitudinal study of this initiative are reported. Twenty-three teachers and curriculum figures from across the state were interviewed and artefacts related to technology use were collected from teachers. Teachers' understanding of the nature of modelling and the potential for technology to be used at various junctures in the modelling cycle affected the extent of technology use in teaching and assessment. The culture of the classroom was perceived as being very different by teachers who made significant use of technology during modelling. Technology was also seen as being essential for the future successful teaching of applications and modelling.

Introduction

With the wisdom of hindsight it seems obvious in 2011 that a plethora of technological devices is relevant to the teaching of applications and mathematical modelling at all levels of schooling but particularly at the senior secondary level. The use of technology appears relevant whether modelling is seen as a vehicle for teaching other mathematics or as part of mathematical content to be taught and learnt in its own right. Both of Hußmann's "central tasks of the technology that supports [sic] independent concept formation" (2007, p. 348) are relevant to either approach—"the *function of construction* by contributing to building ideas, and on the other hand, … the *function of irritation* by initiating a change of concept" (p. 348). Indeed we have found both operating in modelling classrooms where technology rich teaching and learning environments were being researched (Stillman, in press; Stillman, Brown, & Galbraith, 2010). Nevertheless, the question needs to be asked what is the reality across the spectrum of classrooms in a context where applications and mathematical modelling have been promoted at an educational system level for a considerable time? As an example of what has transpired in everyday classrooms we consider the implementation of applications and mathematical modelling within senior secondary mathematics curricula in Queensland, where the initiative was first introduced in 1989 (e.g., Queensland Board of Senior Secondary School Studies [QBSSSS], 1989).

Background

A long lasting ideological legacy of the 1960s and 1970s which saw marked changes in many countries in the Western world has been a desire among young people to be convinced of the efficacy of any activities in which they are asked to engage rather than being expected to be willing participants who follow directions given by others in authority (Niss, 1987). At the secondary and tertiary levels of education students began questioning the relevance of the mathematics they were studying; "and right from the beginning relevance was interpreted by students, teachers and educationalists as *applicability*" (Niss, 1987, p. 491). At the same time there was employer dissatisfaction with mathematics departments of universities (see McLone, 1973) because of the scarcity of mathematics graduates who appreciated the applicability of mathematics in other fields and who could model real problems and readily communicate results to non-mathematical clients. Educational reforms such as those flagged in *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (National Research Council, 1989) identified modelling as one of the "distinctive modes of thought" (p. 31) offered by mathematics and mathematics was said to play a special role in education because of "its universal applicability" (p. 31). From this milieu of influences came the impetus to change the Queensland senior mathematics syllabuses from purely abstract approaches to teaching and content to ones incorporating an emphasis on applications and mathematical modelling as a distinctive characteristic. According to the current Mathematics B syllabus, mathematical modelling is "the act of creating a mathematical model, which may involve the following steps: identify assumptions, parameters and/or variables; interpret, clarify and analyse the problem; develop strategies or identify procedures required to develop the model and solve the problem; investigate the validity of the mathematical model" (Queensland Schools Authority, 2010, p. 44).

The advancement of technological devices and the beginnings of the manufacture of such devices for dedicated teaching purposes in school and university settings serendipitously coincided with the development of the new syllabuses (Stillman & Galbraith, 2009). However, the importance of technological devices to the work of applied mathematicians who engage in mathematical modelling of real situations and to teachers and students teaching and learning through applications and mathematical modelling quickly became apparent. "These devices provide not only increased computational power, but broaden the range of possibilities for approaches to teaching, learning and assessment" (Niss, Blum, & Galbraith, 2007, p. 24). Niss, Blum and Galbraith also warned of the possibility of "associated problems and risks" if these devices were not used and incorporated in the teaching/learning environment in an appropriate manner.

The use of technological devices as tools to carry out repetitive or difficult processes in the solution of a mathematical model has been recognised for some time but several researchers (Confrey & Maloney, 2007; Galbraith, Stillman, Brown, & Edwards, 2007) have seen the potential for technology in the inquiry/reasoning processes that occur throughout the modelling cycle. Recently, Geiger, Faragher, and Goos (2010) confirmed that "student-student-technology related activity takes place during all phases of the mathematical modelling cycle" and that, in particular, technology plays a role in "the conceptualisation of the model" not just the solving process (p. 64). This is

consistent with what happens in workplaces where modelling is conducted. Ekol (2010), from a study of 10 applied mathematicians teaching in university but also working as modellers in industry, concluded that "technology plays a big role in fostering exploration towards discovery, also in sustaining interest in the modeling process" (p. 196). In particular, his interviewees believed that modellers needed to be able to make the appropriate choice of which technology to use and when to use it and also to use technology in a playful way during modelling "for meaningful exploration" (Ekol, 2010, p. 194) of the situation being modelled and the mathematics being applied.

Research methods

Queensland syllabus and review documents from the late 1980's up to the latest syllabuses implemented in 2009 were examined. In addition semi-structured interviews were conducted with 23 interviewees. Samples of 5 key curriculum figures [QKCG] (e.g., non-teacher members of expert advisory committees, curriculum officers of the state education department, or statutory board or authority officers overseeing syllabus implementation), 6 secondary mathematics teachers in key implementation roles [QKTG] (e.g., state or district review panel chairs or state review panel members), and 12 secondary mathematics classroom teachers [QCTG] were purposefully selected (Flick, 2006) as being relevant to the purposes of the study (Richards, 2005, p. 41). These teachers were representative of several school districts and of the state, Catholic and independent schools systems. A series of interview questions covering the period of introduction, and later periods of widespread implementation and modification were asked. In addition, practising teachers provided artefacts, usually in the form of tasks, which typified their use of real world applications and modelling in teaching and assessment, and their use of technology in these contexts.

In order to identify emergent themes within the interview responses, and the teaching and assessment artefacts, these data were entered into an NVivo 8 database (QSR, 2008) and analysed through intensive scrutiny of the data from a particular interviewee and across the corpus of the data from all interviewees to develop and refine categories related to these themes (Richards, 2005). Specifically this paper will address emergent themes related to responses to the following interview questions:

1. To what extent have you incorporated the use of technology when exploring real-life situations that require investigative, modelling or problem-solving techniques?

2. The syllabuses require a balanced assessment plan that includes a variety of techniques such as extended modelling and problem-solving tasks and reports. What types of task do you use in your alternative assessment? To what extent do these use real world contexts? To what extent do they also incorporate the use of technology? How?

3. How is the culture of the classroom influenced by the presence of technological devices in a classroom environment promoting both technology and applications and mathematical modelling?

4. What possible implications does technology have for the future successful teaching and assessment of applications and modelling within upper secondary mathematics?

Findings

Implications of developing understanding of nature of modelling and potential of technology use

As the affordability and quality of technology allowed it to be freely able to be used in the classroom, it was soon realised by some teachers that a classroom rich in technology would serve to facilitate the implementation of the syllabuses particularly those aspects pertaining to mathematical modelling and applications. It allowed the messiness of real world data to be dealt with as this teacher points out:

> I think that has been a big driving thing, the fact that you have the technology that you can then explore real-life situations and the kids can actually get down and get dirty in the mathematics rather than everything being really nice and neat because up until that stage, like in the old syllabus, because they didn't have that facility, everything was always pretty much nice. (QKTG3)

This potential has not been realised in all schools, however, with the uptake of technology being described as "patchy" by some (QKCG4; QKCG5) especially with respect to the extent of how it is used mathematically in exploring real world situations.

> Some people use technology really well and all the time and others, because the syllabus says you have to use it, they will use it just to do more calculations or just to draw graphs or things like that. (QKCG4)

Extent of technology use in teaching

As teachers' understanding of (a) what mathematical modelling entailed increased, (b) how it differed from mathematical applications became clear and (c) what technology offered to teaching and learning, the necessity for modelling and technology to be an integral part of the teaching/learning environment became more accepted. Thus, modelling and technology came to enjoy a symbiotic relationship in the classrooms of these teachers where technology is "just natural, you don't even think about it that it is there. Kids pick it up and just use it" (QKTG2). Although technology was seen as ideal for demonstrating by the teacher, it also had a pivotal role to play in the hands of students who were allowed to play and explore models and emerging ideas when modelling.

> I think you need to be able to engage people more immediately in what's going on there so I think technology being used to demonstrate and for students to play with as well as illustrate mathematical concepts generally I think is very important and also for modelling and problem solving as well. (QCTG12)

Technology allowed timely access to modelling or exploring of situations for which students were yet to learn more sophisticated mathematics to model.

> So at this stage in the course [end of semester 2 in year 12] I am actually revisiting the same problem and employing the algebraic approach and differentiating and saying, "Okay, that's how we do that at this stage in the course" even though earlier in the course we were prepared to let the calculator do most of the work for us. (QCTG2)

Others saw this as a means to extend the sophistication of the modelling their students were able to do with one teacher stating: "The increase in technology we can get our hands on now means we can tackle increasingly sophisticated modelling" (QCTG7). This was seen as an underpinning reason for using technology.

Models become quite sophisticated quite quickly and then kids can't take them any further but technology offers you an opportunity of scaffolding around that. (QKTG2)

Not all teachers used technology to a significant extent in teaching about applications or modelling, with some reserving it "mostly [for] alternative assessment" (QCTG5) although they were "quite happy to go to the computer labs" and work on computer investigations from textbooks.

Extent of technology use in alternative assessment

"Assessment techniques other than traditional written tests or examinations" (QBSSSS, 1992, p. 40) became known as alternative assessments. These were required to be included in a school's assessment program at least twice yearly. Some teachers spoke of using technology almost exclusively in their alternative assessment although some, but not all of these, also used technology in teaching when exploring and investigating real world situations. For many the motivation was not that they believed using technology when exploring real situations to be good pedagogy or essential but rather it was "because it is mandated" (QCTG5).

In assessment, well we can't use computers in exams so we try to see if we have their alternative assessment task, their one per semester, try to have something there where they would be using the computer. … (QCTG1)

How students used the technology seemed to resonate with the teachers' view of modelling. Those teachers who saw modelling as no different from mathematical application designed assessment tasks that provided opportunity for using technology only as a tool in solving.

It is just making use of the technology to do the number crunching more than anything else and then being able to interpret what you have at the end of that. (QCTG5)

Others saw alternative assessments as providing the ideal forum to show evidence of meaningful technology use when assessing applications and modelling.

We look to our assignments as the main evidence that our students use technology because in the supervised exams they certainly use technology to draw graphs, to do calculations, find mathematical models…but what is the proof of it really but it is evident in the assignments. (QCTG7)

Classroom culture in an environment promoting technology and application and modelling

Most teachers who had embraced technology spoke of their classroom culture being "very different to what we used to do way back in time. Absolutely we couldn't do the sorts of things that we do if we didn't have the technology" (QKTG3). This was partly in response to teaching a generation of students who are technologically knowledgeable in certain respects reacting in quite different ways to students of the past:

I think having an internet generation has meant that the way that students interact with each other has obviously changed and [as] learners has become different and I think students need more immediate gratification these days. They need to see a dynamic situation happen in front of them. They don't have patience to sit there and graph things manually. (QCTG12)

Elements of the classroom culture that were said to be enhanced were also elements of what researchers have identified as integral to conducting modelling successfully in the

classroom such as the technology rich environment becoming a "vehicle for opening up ideas" (QKTG1) and "more discussion amongst them" (QCTG4; QKTG1). The increased discussion was seen by some teachers as helping students' mathematical understanding (QKTG1). The classroom was also seen as becoming "a little bit more collaborative" (QKTG1) with a "bit more [group work] because even though they have got their own [calculators] they still compare" (QCTG4).

Several teachers pointed out that it was not just having the access to the technology that was the key to the changed culture. It was very much dependent on the approach taken to teaching modelling.

> Oh, yeah, very definitely changes the way you teach because the tedium of the algebra or whatever it is, the calculation is taken away and the answer … to that stage will come up very quickly and the kids are more interested, much more engaged. It's not just technology but yes it does help. … it depends on what you do in the classroom too. (QCTG1)

Some acknowledged that an enquiry approach was called for.

> It is not just the fact that technology is there. It is the way it is used. And it is the way the teacher uses it and the type of culture they build themselves. So if they build a culture that is about enquiry and mathematical modelling and all that sort of thing and incorporate technology into that then you can really kick on. (QKTG2)

However, these teachers were still limited in their view of the potential of technology in a modelling environment as technology was seen as being of assistance only in the solving phase of the modelling cycle and not as a means of enabling model conceptualisation or decision making at all phases throughout the modelling.

> If you're going to introduce technology in there it is just likely going to be used as a number cruncher and not much more. So you have to build in the other stuff as well and it is not just technology alone that does it. (QKTG2)

Implications for the future for successful teaching and assessing of applications and modelling

Some teachers saw technology as essential to successfully implementing the intentions of the syllabuses in the allowable time.

> I think it is essential because I think you have got to, for the limited time that we have that we can spend in assessment you can't have them not using the technology. It is too time consuming to do all that without the technology… as long as they know what the technology is doing and I think that is the idea. (QCTG1)

Others spoke of it enriching the whole experience that was the perceived intention of the syllabus with technology playing an essential role in exploration of real life situations mathematically enabling students to confirm their own understandings.

> I don't think you can teach mathematics successfully without technology to be honest with you. You can teach mathematics but you can't build an understanding of those real life situations. (QKTG3)
> I just think it is enriching the whole process, the whole experience. It is giving kids other ways of confirming the learning that they have. (QCTG9)

One of the key curriculum figures took a futuristic "learning community" approach considering the classroom as borderless with students being willing to share ideas with

others within their classroom and across classrooms which could be co-located or geographically distant.

> I am interested in Rudd's idea of providing every student with a laptop … What it seems to me is that it would provide the opportunity for kids to form learning groups and to share things and to see how other people work on things. Now I think this would be more powerful than anything, if a teacher here who was working on mathematical modelling, one of their [groups] could some how or other share what they were doing and let the others see what they were doing and thinking about and how this group was thinking about it. You would get a lot of "Ahas". What I am saying is the technology if it could provide that sort of networking then you could really pick up the pace in the mathematical modelling side of it. …The learning community stuff is still pie in the sky, I suppose, but it is still exciting even to someone who is past exciting. (QKCG3)

Discussion and conclusion

With respect to the responses of participants in relation to the extent of technology use in teaching and alternative assessment involving real world contexts some teachers clearly had welcomed the opportunity to expand their repertoire of teaching and assessing practices with respect to applications and modelling that technology brought. Others saw technology providing little more than a computational device to remove the tedium and potential inaccuracies of repetitive calculations or graphing associated with the solution of a mathematical model. In the latter instance this usually was related to a view of modelling as being no different from using mathematical applications and opportunities for use of technology being more prominent in assessment than in teaching.

In classrooms where technology was said to play a significant role in teaching applications and modelling the classroom culture was said to be very different as the "internet generation" was more engaged by immediate feedback and dynamical displays available by teaching with technology. The constructive function of technology in concept formation (Hußmann, 2007) was acknowledged by these teachers. Hußmann's "function of irritation" was less obvious in the responses but could perhaps be inferred as being present in communities of inquiry or when students were said to be using the technology to confirm their learning. Exploration, sustaining interest and engagement, and playing with the mathematical ideas and the situation being explored as identified by Ekol's (2010) applied mathematicians were all mentioned as elements of the classroom culture where technology was readily available and expected to be used. Again the teacher's view of modelling limited the perceived potential and promoted use to the solving phase or expanded it to pervade the modelling cycle along the lines promoted by Confrey and Maloney (2007).

Finally, some saw the use of technology as essential to successfully fulfilling the intentions of the syllabuses with respect to modelling. Even though several saw this as clearly enriching the whole teaching/learning experience as intended by the syllabus writers, there was mention of the unfulfilled potential of a borderless learning community providing networking amongst modelling groups across distances and geographical boundaries further enriching that experience.

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