LIKE A BRIDGE:
SCAFFOLDING AS A MEANS OF ASSISTING LOW-ATTAINING STUDENTS IN MATHEMATICS DURING COGNITIVELY CHALLENGING TASKS

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Statement of Sources

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Signature:

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ABSTRACT

The present study examined the case of two upper primary teachers and two low-attaining target students in each of their classes. This study was a component of a large study examining three specific types of mathematics tasks aimed at building conceptual understanding. The two classes observed and described in the present study were engaged in using these cognitively challenging types of mathematics tasks. The study aimed to explore the impact that particular scaffolding practices had on the cognitive and affective responses of the target students during their work on these challenging tasks. The three scaffolding practices focussed on during this study were the use of discussion; the use of manipulative materials and visual representations; and explicit attention paid to concepts.

This study aimed to build a rich and detailed description of this case and data were gathered in three phases. The purpose of the initial phase of data collection was to examine the participants' beliefs about mathematics, about themselves as mathematics teachers or learners and, in the case of the teachers, about teaching mathematics to low-attaining students. During this phase, data on the target students' understanding of the mathematics concepts to be taught during the observed lessons were gathered through use of an assessment task. The second phase of data collection involved the observation of six mathematics lessons in each classroom. From these lessons, eight tasks from each classroom, aligned with the task types from the larger study, were selected for further analysis. The focus of lesson observations was the extent and use of the three scaffolding practices, the cognitive and affective responses of the target students, the choice of tasks used by the teachers, and the ways in which they were used. The third phase of data collection occurred one year after the lesson observation period concluded. These data were gathered using semi-structured interviews with all the participant teachers and students. During these interviews, emerging themes and patterns from the data were shared with the teachers, and their reactions sought.

Data were analysed using a coding framework developed under the categories of the three scaffolding practices. There were also codes relating to different types of tasks and how tasks were implemented in the classroom. Codes were devised from the

literature in each of these areas. Patterns and themes were explored using these pre-determined codes but also those which emerged through interviews and observations.

The teachers in this study represented differing approaches to teaching mathematics to low-attaining students. One teacher consistently maintained the high level of cognitive demand of all tasks with no variations for low-attaining students. The other teacher used less cognitively demanding tasks with the class most of the time with some shorter tasks maintaining a higher level, and occasionally varied the demands of the task for the target students. The responses of the target students also varied, with two of these low-attaining students struggling to cope with the higher level of cognitive demand for half of the tasks and therefore completing little mathematical activity. The other two target students demonstrated clear progress in their ability to carry out teacher devised procedures but did not appear to develop in their understanding of the concepts underlying such processes.

The findings from this study have implications for teachers of low-attaining mathematics students. First, most whole class discussions were not effective in scaffolding the learning of low-attaining students, whereas individual "scaffolding conversations" with teachers were generally effective. The use of manipulative materials and visual representations was effective in providing scaffolding to the low-attaining students if the mathematical concept focussed on was illustrated through the use of the materials. However, manipulative materials or representations that were intended to trigger prior knowledge or link knowledge sets were less effective. Effective use of the third scaffolding practice, explicit attention to concepts, relied on the use of the more effective elements of the previous two scaffolding practices. That is, using scaffolding conversations and materials or representations that illustrated concepts, were most effective in drawing out the underlying concepts.

This case study has the potential to offer classroom examples of scaffolding; a complex and little understood practice, which has been under researched in mathematics classrooms. There is also little research that has combined the study of conceptually challenging mathematics tasks and low-attaining students, creating the need and space for this study. This research also adds to the literature on effective teaching of low-attaining students in mathematics, and prompts further worthwhile research topics for investigation.

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Chapter One – The Heart of the Issue

Introduction

One of the biggest challenges faced by teachers is how to respond to diversity amongst their students be it cultural, economic, gender-related or in academic performance. Research indicates that there exists an "achievement gap" in mathematics, some assert as much as seven years (Cockcroft, 1982), between students operating at expected levels of attainment and those who are low attaining. Low-attaining mathematics students represent a group deserving attention by teachers and researchers, and a much greater focus is needed on how this gap may be reduced. At the same time, mathematics teaching advice current at the time of the present study advocated the importance of all students learning mathematics with understanding, engaging in challenging, interesting and meaningful tasks, discussing and communicating in mathematical ways and building positive attitudes and dispositions toward mathematics and themselves as mathematics learners (National Council of Teachers of Mathematics, 2000; Numeracy Review Panel, 2008). It has been argued that to deny students the opportunity to learn important mathematics with understanding is a matter of equity as mathematics is often considered to be a "critical filter" vital to an individual's future ability to operate as an informed, critical citizen (Ma & Johnson, 2008). Furthermore, practices such as streaming or tracking have also been considered inequitable due to problems with self-fulfilling prophesy (Brophy, 1983), less breadth in the curriculum offered to lower streamed classes and increasing disengagement and low self-esteem among low-attaining students (Boaler, Wiliam & Brown, 2000; Slavin 1987). It appears then that teachers are being urged to teach low-attaining students within heterogenous classrooms, without changing content and maintaining a focus on learning mathematics for understanding. This is no easy task for many teachers and more research is needed to explore appropriate pedagogies to support low-attaining mathematics students whilst maintaining a focus on conceptual understanding.

One such pedagogy that has emerged is "scaffolding". Scaffolding has been described as "a process [that] enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts" (Wood, Bruner and Ross, 1976, p. 90). Rosenshine and Meister (1992) also added that scaffolding does not involve the provision of "explicit steps" but that the student is supported to learn the skill. In addition, scaffolding is temporary and increasingly the student is encouraged to do more while the teacher "fades" their assistance. Scaffolding has been proposed as an appropriate way to assist students during conceptually challenging tasks in a way that does not reduce this challenge significantly. The notion of scaffolding will be explored in detail throughout this dissertation.

The present case study sought to explore the experiences of four Year 5 students who were low attaining in mathematics and two teachers. This study adopted a case study approach that aimed to build a detailed view of the interplay between task, pedagogy, and the cognitive and affective responses of low-attaining students. It had the twin purpose of examining the use of particular types of mathematics tasks aimed at conceptual understanding and the use of particular forms of scaffolding. I sought to build a rich description of the experiences of both the teachers and students. Though this was a case study of two teachers and two low-attaining students in each class, it has the potential to illuminate issues that have wide ranging implications particularly regarding scaffolding, which is a complex but potentially important part of teaching mathematics when using cognitively challenging tasks.

The age of low-attaining students in mathematics is an important consideration. While the early years are a vital time for identifying difficulties and intervening in the education of low-attaining students, during the middle years of schooling low-attaining students may experience a widening of the gap between their performance and that of their average or above-average peers (Cockcroft, 1982; Thomson & De Bortoli, 2007). In Australia, The National Curriculum Board (2009) noted that, "although there are challenges at all years of schooling, participation is most at threat in Years 6–9. Student disengagement at these years could be attributed to the nature of the curriculum, missed opportunities in earlier years, inappropriate learning and teaching processes, and perhaps the students' stages of physical development" (p. 9). These factors make these "middle years" of schooling a crucial juncture in the education of low-attaining students.

The purpose of this case study arose from concerns regarding the social, affective, and economic consequences of persistent low attainment in mathematics, which will be explored in the following discussion. However, this research also stemmed from personal experiences, beliefs and interests. In case study research, the role of the researcher is particularly vital as the researcher acts both as describer and interpreter of the case. Therefore it is important the reader understands the researcher's personal interests and motivations, in order to judge potential biases. As the researcher, it was important that I also bore in mind my own opinions, feelings and biases throughout the course of this case study so that findings and interpretations could be carefully checked to ensure they reflected as closely as possible the reality of the case.

As a mathematics student at school, I considered myself to be low attaining, and I believed most of my teachers did also. I formed this view during middle to upper primary school, which I have since found is a common age when students begin to disengage from mathematics. My disengagement with mathematics continued until my teaching degree when a sympathetic and skilled mathematics education lecturer finally explained the reasons behind the mysterious "steps" and formulae with which I had struggled. Early in my teaching career, I had the great privilege of being a teacher participant in the Early Numeracy Research Project (Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough et al. 2002), which again reinforced learning and teaching mathematics for understanding that I found particularly engaging. As a former "low-attaining" mathematics student, I have a personal interest in engaging and teaching low-attaining students well. As a teacher, I believe in the importance of all students learning mathematics with understanding. These beliefs certainly impacted on the formation of the research questions and the purpose for this study, as did the wider issues that form the following discussion.

In this chapter I explore the difficulties associated with disengagement in the middle years of schooling. I then introduce the literature regarding mathematics tasks, including how tasks are described in the curriculum documents of various countries. I then outline scaffolding and, in particular, the three scaffolding practices that were of focus for this study. These practices will be elaborated in Chapter 2. Next I introduce the literature regarding low-attaining students in mathematics, which will also be explored in detail in Chapter 2. To conclude this chapter, I will give a brief overview of the methodology of my study.

'THE MIDDLE YEARS'

The middle years of schooling, often considered in Victoria to be Years 5 to 8, can represent a turbulent time in the education of students. *Beyond the Middle* (Luke, Elkins, Weir, Land, Carrington, & Dole et al., 2003) reported that there was a "noticeable dip" in the performance of Year 7 Australian students in mathematics during the transition from primary school to secondary school. Furthermore, Luke et al. warned that recovery of performance was often delayed until Year 9.

The report on the *Student Alienation in the Middle Years of Schooling Project* (Australian Curriculum Studies Association, 1996) showed a corresponding decline in the students' attitudes toward school and learning. The report used terms such as "estrangement", "detachment", "fragmentation", "isolation", and "powerlessness" to describe the feelings of middle years students at school. Studies of the affective domain show that attitude and learners' views of themselves impact strongly on achievement levels (Ames, 1992; Dweck, 1986; McLeod, 1988). These studies are examined in Chapter 2.

More recently in Australia, the National Numeracy Review (2008) discussed the issue of student disengagement and motivation. This review examined the report *Maths? Why Not?* (McPhan, Morony, Pegg, Cooksey, & Lynch, 2008) which examined the reasons why Australia's most able students chose not to continue with mathematics as a subject in upper secondary school. This research revealed that motivation and engagement with mathematics relied on interesting and challenging mathematical tasks being offered in the classroom. This highlights the link between the kind of teaching and tasks offered in mathematics classrooms and students' self image, motivation, engagement and, ultimately, achievement in mathematics. Considering this important role that mathematics tasks play, I now discuss mathematics tasks including an examination of the types of tasks advocated in curriculum documentation from various countries around the time of the present study.

MATHEMATICS TASKS

There is a growing body of research that recognises the importance of mathematics tasks. For example, Henningsen and Stein (1997) asserted:

> The nature of tasks can potentially influence and structure the way students think and can serve to limit or to broaden their views of the subject matter with which they are engaged. Students develop their sense of what it means to "do mathematics" from their actual experiences with mathematics, and their primary opportunities to experience mathematics as a discipline are seated in the classroom activities in which they engage. (p. 525)

The tasks that teachers plan and implement for students to engage in during mathematics lessons are crucial for student learning. Not only do these tasks create opportunities to learn important mathematics, but also the nature of classroom tasks sends messages to students about mathematics itself, that is, about what is important, what is valued, and about their own place as "doers" of mathematics. By studying the kinds of tasks teachers choose and how they use them with students, research is attempting to define the link between teaching and learning, between what teachers intend the students to learn, what students have the opportunity to learn, and what students may actually learn (Hiebert & Wearne, 1993; Peled, Clarke, Clarke, & Sullivan, 2009; Stein & Lane, 1996).

Mathematics curriculum documents at the time of the present study advised the teacher about the kinds of mathematics tasks students should experience in mathematics lessons. I will now examine this advice, drawing on mathematics curriculum documents from various countries. The countries included in the following discussion represent a small sample of countries, including Australia. This is not an extensive review of international mathematics documents, but rather a snapshot of the current mathematics curriculum advice at the time of the present study.

The National Curriculum 2007 (Qualifications and Curriculum Authority, 2007) in the United Kingdom used the term "task" frequently. It described how students should work on "sequences of tasks that involve using the same mathematics in increasingly difficult or unfamiliar contexts, or increasingly demanding mathematics in similar contexts," "work on open and closed tasks in a variety of real and abstract contexts," "work on problems that arise in other subjects and in contexts beyond the school," and "work on tasks that bring together different aspects of concepts, processes and mathematical content" (Qualifications and Curriculum Authority, 2007, p. 148). It also described affective goals for students such as developing confidence, and working collaboratively. The key processes were identified in this curriculum as “representing, analysing, interpreting and evaluating and communicating and reflecting” (p. 148).

The National Council of Teachers of Mathematics (NCTM) in the United States emphasised the importance of “worthwhile tasks” in mathematics. The *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) advised that the teacher’s role in choosing worthwhile, quality tasks was paramount to delivering a robust mathematics curriculum aimed at higher order thinking. The motivating aspect of appropriate challenge, was a central point in this document.The kinds of tasks which the NCTM advised teachers to use were described as “tasks… connected to the real-world experiences of the students” (National Council of Teachers of Mathematics, 2000, p. 19), and tasks which could be approached in many different ways. It was stated that tasks should challenge students and require hard thinking. The Qualifications and Curriculum Authority (2007) also mentioned this idea of a challenging curriculum using phrases such as “increasingly difficult” and “increasingly demanding” (p. 148), as I described above.

In a similar way to *The National Curriculum* (Qualifications and Curriculum Authority, 2007), the affective aspect of mathematics learning is discussed in the *Principles and Standards for School Mathematics*. This document emphasises the importance of student engagement in choosing tasks stating that “well-chosen tasks can pique students’ curiousity and draw them into mathematics” (National Council of Teachers of Mathematics, 2000, p. 18).

The *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) also pointed to the significant role the teacher plays in determining aspects of the task to emphasise, organising the work of the students, deciding on questions to probe students’ understanding, challenging students’ thinking and providing support without taking over the thinking required to complete the task. Such a point is not found in the United Kingdom’s mathematics curriculum document though perhaps is implied through its description of the kinds of tasks in which students should engage.

Given Japan’s high performance in international mathematics studies such as TIMSS (Hollingsworth, Lokan & McCrae, 2003), many countries, including Australia, have examined how mathematics is taught in Japan. Shimizu (2006) reported that mathematics in Japan aimed to deepen students’ understanding of concepts, principles and regularities in mathematics. It also focused on the ability to represent and analyse mathematically, and view and think mathematically. Japanese mathematics teachers often organise lessons around just a few problems. Students work through four general phases of the lesson:

- Phase one - The teacher presents the problem.
- Phase two - Students attempt to solve the problem individually first and then with others.
- Phase three - Some students present their solutions to the class for discussion.
- Phase four - The teacher summarises the main points of the lesson.

This whole-class problem solving process was found commonly in Japanese mathematics classrooms (Sekiguchi, 2006).

In Australia, the Australian Curriculum was being developed at the time of the present study. A framing paper, *Shape of the Australian Curriculum: Mathematics* (National Curriculum Board, 2009) stated:

> It is assumed that teachers will use a variety of mathematical task types including those that give students choice of approach and those for which there is an optimal strategy; those for which there are various possible solutions and those which have a single correct answer; those that prompt the development and use of mathematical models; those that incorporate ideas across content strands; and those that require thinking in more than one discipline. (p. 14)

This document discussed specific types of mathematical tasks. The types of tasks described appear to include open-ended tasks with more than one solution or a choice of approaches, tasks using mathematical models and those which incorporate other strands of mathematics or more than one discipline, often referred to as "interdisciplinary" tasks.

To summarise, the mathematics curriculum documented advice from these countries regarding mathematics tasks is that

- tasks should be challenging and increasingly difficult;
- tasks should use real world contexts;
- tasks should be engaging and motivating;
- tasks should centre on student-devised strategies;
- students should communicate mathematically, to justify their ideas and explain their thinking;

- tasks should be open either in strategies or multiple solutions;
- tasks should use mathematical models; and
- tasks should connect more than one area of mathematics or link to other disciplines.

The present study was part of the *Task Types and Mathematics Learning* project (TTML, Peled et al., 2009). In an effort to learn more about the appropriate use of tasks in mathematics during the middle years of schooling, the Australian Research Council funded the TTML project, which studied three task types in mathematics: tasks that use models, representations or tools that exemplify the mathematics; tasks that situate the mathematics within a contextualised practical problem; and open-ended tasks which investigate specific mathematical content.

These types of tasks were considered potentially engaging for middle years students and also thought to have the capacity to support the teaching of important mathematics. These task types also featured in many of the curriculum advice documents examined previously. Each of the TTML task types is described and discussed in Chapter 2. All three types of tasks were aimed at conceptual understanding and were designed to be cognitively challenging. An issue explored in this study was how to support low-attaining students in mathematics while using the types of tasks found in the curriculum documents discussed and examined in the TTML project. I will now discuss scaffolding, which has been proposed as an appropriate means to offer such support.

SCAFFOLDING

Scaffolding is a complex interaction between teachers and students. First, a clear understanding of the students' current knowledge is essential in order to target the next step requiring scaffolding. This is where Vygotsky's (1978) "Zone of Proximal Development" comes to the fore so that scaffolding can be aimed at the very edge of the student's current understanding. Next, the manner in which scaffolding is provided needs to be determined. Materials, visual representations, careful questioning and conversation, grouping or pairing with more able peers or task variation are some of the options. Finally, scaffolding is a departure from traditional forms of teaching in that it is less teacher–directed. Traditional teaching involving careful teacher explanations, and passive student listening followed by guided practice is not considered scaffolding (Bliss, Askew, & Macrae, 1996). It is clear from the literature that both researchers and teachers have sought for many years to define more clearly the characteristics of scaffolding in the classroom. This will be discussed in detail in Chapter 2.

There are many ways in which teachers can scaffold student learning. For this study I chose to focus on three practices in particular. These were the use of whole class and individual discussion; the use of manipulative materials and visual representations; and explicit attention to concepts. These practices were chosen as they represented common ways in which teachers scaffold student learning, or ways that might be recommended particularly for low-attaining students. This is the case particularly with the first two practices of focus. The final practice, explicit attention to concepts, reflects this study's purpose in examining mathematics tasks that aim for conceptual understanding. These three practices are examined in detail in Chapter 2. Of course, drawing attention to underlying concepts cannot occur in isolation. Discussing concepts, illustrating concepts with materials or representations or via some other means is necessary. This points to the overlapping nature of the three scaffolding practices for the present study. This study sought to examine each practice and any relationships or interactions that occurred between the three scaffolding practices.

Drawing together the previous discussions, the literature also describes the characteristics of low-attaining mathematics students that interact with particular forms of support and types of mathematics tasks. I will now introduce this literature, which will be detailed in Chapter 2.

LOW-ATTAINING MATHEMATICS STUDENTS

The literature on low-attaining students uses two main categories for such students. First, low-attaining students are described as students requiring Special Education; that is, those with diagnosed learning disabilities. Second, the literature describes students for whom there is no explanation provided by way of learning disability or physical or intellectual impairment for their lack of attainment or inadequate progress. Low-attaining students may qualify for remedial assistance, or additional tuition in some form, may be placed in classes designed for low-attaining students only, or may be catered for in regular mathematics classes as part of a heterogenous group. My study identified more with the second description of low-attaining students in that I did not examine cases of intellectually impaired or learning disabled students but rather of low-attaining students whose history of difficulty in mathematics could not be explained by a pre-existing condition. However, in Chapter 2 I will examine some of the literature from Special Education as the issues raised in this research have implications for how low-attaining students in general cope in mathematics classes.

It is recognised that there are a number of other important factors, such as cultural or economic background, which impact on the learning of low-attaining students (Hollingsworth et al., 2003; Zevenbergen, 1997). A limitation of this study is that these factors did not form part of this research. A case study must have bounds, a factor examined in the following discussion. In this case study, the bounds were the three particular task types, three kinds of scaffolding practices and the responses of the target students. Figure 1 illustrates the focus for this study and indicates some of the other factors, not under detailed consideration in this study, that nevertheless impact on the learning of low-attaining students.

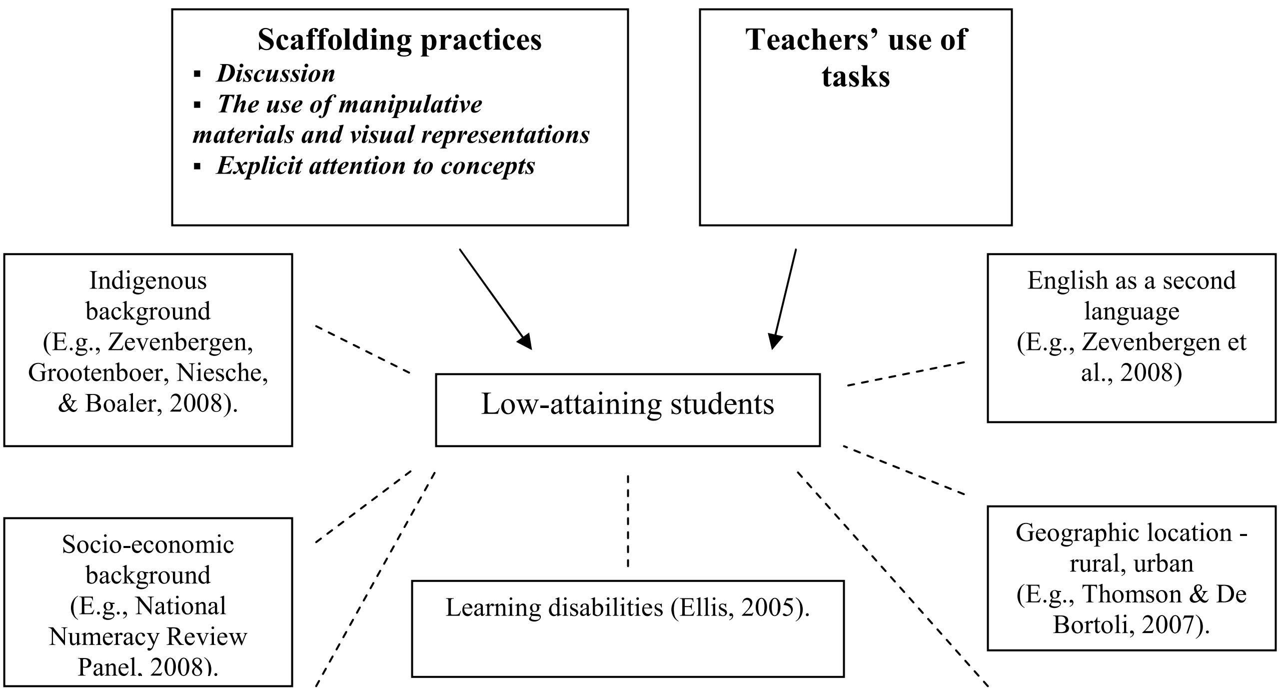


Figure 1. A range of factors which impact potentially on the learning of low-attaining students.

As Figure 1 shows, the two main influences on the learning of low-attaining students examined in the present study were the teachers' use of three particular scaffolding practices and the use of tasks. In Chapter 2, the literature on responses of low-attaining students to the types of tasks studied in this research and the three scaffolding practices will be examined.

The discussion in the present chapter has introduced the three spheres of research that the present study sought to examine, that is, the literature on mathematics tasks, on low-attaining mathematics students and on scaffolding. These spheres form the discussion in the following chapter. I now discuss how the purpose of the present study arose before summarising the methodology adopted.

PURPOSE OF THIS STUDY

In Australia, there is a paucity of literature about low-attaining students and their experiences in mathematics classrooms where the teacher aims for conceptual understanding. This study enters the debate on effective teaching approaches for low-attaining students while using tasks intended to build conceptual understanding. Studies of mathematics tasks have explored the general effect on student learning, teacher beliefs and pedagogies, the effect on the learning of different groups in society (class, racial, gender issues), and the justice or injustice of using particular types of tasks with all students. However, there has not been a study in Australia at the time of writing that examined how low-attaining students respond to particular task types in mathematics. As such, this study provides additional insight into the mathematics learning of low-attaining students. In a review of the literature on low-attaining students we will see that numerous studies (e.g., Knapp, Shields, & Turnbull, 1995; Ridlon, 2004; Silver, Schwan Smith, & Scott Nelson, 1995) have identified that an approach that focuses on higher order thinking and strategies can be successful for low-attaining students. However, little analytic attention has been paid to the experiences of low-attaining students in Australian schools.

I intended to address this issue by exploring how a group of low-attaining middle years students responded to the scaffolding their teacher provided and the use of three types of mathematics tasks. The scaffolding practices of the teachers were examined to build up a rich description of the interplay between task, scaffolding, affect and mathematical learning. A case study methodology is appropriate for such description.

My primary research question was:

How does a teacher's use of particular scaffolding practices, while using specific mathematics tasks, impact on low-attaining students cognitively and affectively?

Sub-questions were:

1. Do any of the scaffolding practices appear to be more effective in scaffolding the learning of low-attaining students than the others and in what ways?
2. Do low-attaining students have a preference for the kind of scaffolding their teachers offer and what are the reasons given for these preferences?
3. Do low-attaining students have a preference for the kinds of tasks used in mathematics lessons, and what are the reasons given for these preferences?
4. Do teachers show preferences for using particular types of mathematics tasks with low-attaining students and in what ways?

METHODOLOGY

The issues explored in this study are complex and inextricably linked to the students and teachers themselves – their backgrounds, personalities and previous mathematical experiences. As such, a case study enabled me to explore the "messy" reality of being a teacher and a low-attaining student in a mathematics classroom using certain task types.

This study used a subjective ontology and constructivist epistemology (Neuman, 2003). A case study methodology was appropriate for such an epistemology because case studies use an inductive approach described by Neuman as "beginning with detailed observations of the world and moving toward more abstract generalisations and ideas. When you begin you may have only a topic and a few concepts but as you observe you refine the concepts" (Neuman, 2003, p. 51).

This study was based on a social constructionist orientation in that it was "assumed that the interactions and beliefs of people create reality" (Neuman, 2003, p. 63). The reality of the mathematics classroom for the low-attaining students in this case study was the reality they described. The reality for their teachers was as the teachers described it. As researcher, observer and collector of artefacts, I described reality as I saw it. These descriptions were compared and blended to form a description of the social reality of that mathematics classroom.

Case studies put a human face on data. They allow the researcher to examine in detail the lived experiences of a few cases in order to gain a clearer picture of the larger field. This study permitted the researcher to "select key cases to illustrate an issue and analyse it in detail" (Neuman, 2003, p. 33). A case study of the experiences of low-attaining students and specific task types in a mathematics classroom was a suitable methodology as it provided insights into areas for further research. As this field is somewhat under-researched in Australia thus far, case studies can highlight the issues as they arise in these cases rather than pre-empt what the experiences of these teachers and students may be. In this way, this case study widens the scope of what can be explored and discussed without narrowing the research to researcher-predicted phenomena.

In the next chapter, I discuss the literature regarding mathematics tasks, characteristics of low-attaining students and scaffolding in the mathematics classroom. Teacher beliefs about low-attaining students and their view of learners and learning will also be examined in light of how this affects the opportunities offered to low-attaining students. I also examine studies of low-attaining students and their responses to mathematics teaching approaches and tasks, particularly those types of tasks represented by the TTML project.

Chapter Two – Review of Literature: Three Spheres of Research

Introduction

This study attempted to marry three spheres of research, being tasks, low-attaining mathematics students and scaffolding**.** As such, this study sought to describe the use of specific task types, and particular scaffolding practices and the impact this had on low-attaining students. To follow, literature from each of these fields will be examined. Of particular focus for this study was scaffolding as a potential "bridge" between the use of cognitively demanding mathematics tasks and the learning of low-attaining students.

The results of the Program for International Assessment, PISA 2006, clearly identified a group of students who had not reached standards deemed necessary for future success in mathematics or in the workforce. The data presented in this report (Thomson & De Bortoli, 2007) pointed, with a real sense of urgency, to a need for more research about low-attaining students. The ultimate objective of such research is to discover what it is that teachers, schools and education systems can do to narrow this achievement gap and maximise the success in mathematics of low-attaining students.

As mentioned in Chapter 1, part of the answer may lie in the tasks with which low-attaining students are engaging in mathematics classrooms. Tasks have been described as "important vehicles for building student capacity for mathematical thinking and reasoning" (Stein, Grover, & Henningsen, 1996, p. 455). Doyle (1988) asserted that tasks define what students learn and also how students come to regard the subject matter itself, and themselves as learners of that subject. Tasks have been considered the mediating factor between what is taught and learnt (Stein et al., 1996). A number of researchers have examined the tasks of mathematics classrooms (Christiansen & Walther, 1986; Doyle, 1988; Hiebert & Wearne, 1993; Stein & Lane, 1996). These studies will be amongst those examined in this chapter.

A critical point is that the tasks themselves do not ensure that students gain in their understanding of mathematical concepts. There is significant evidence in the literature that tasks can undergo “metamorphosis” in the hands of teachers (Tzur, 2008). It appears that even more critical than the task itself is the way in which a task is enacted in the mathematics classroom. It is through the teachers’ actions, and the students’ responses that the potential for learning arises from the task. Research has shown that, particularly when using tasks aimed at conceptual understanding, it is the actions of the teacher that ultimately determine the level of thinking students can potentially achieve by engaging in the task (Henningsen & Stein, 1997). Factors that contribute to how a task is implemented in the classroom will be examined in this chapter.

Conceptual tasks are characterised by the use of non-routine tasks, problematic tasks, or tasks that primarily aim for growth in the understanding of students in the concepts of mathematics (Hiebert et al., 1997; Kazemi & Stipek, 2001; Stein & Lane, 1996). In contrast would be routine tasks that aim for memorisation, or practice through repetition of processes where the underlying concepts may not be explicitly explored (Stein et al., 1996). There is evidence in the literature that routine tasks are most commonly offered to low-attaining students (e.g. Anderson, 1997). The literature reports that some teachers believe that many of the characteristics of low-attaining students make the use of conceptual tasks inappropriate for such students (Zohar, Degani, & Vaaknin, 2001). Characteristics of low-attaining students have also been the subject of research identifying particular difficulties low-attaining students might experience (Ball, 1993; Baxter, Woodward, & Olson, 2001; Baxter, Woodward, Voorhies, & Wong, 2002; Beswick, Watson, & Brown, 2006; Lawson & Chinnappan, 1994), affective issues (Ames, 1992; Middleton, 1995) and the thorny question of how to effectively teach mathematics to such students (Cobb et al., 1989; Ellis, 2005; Kroesbergen, Van Luit, & Maas, 2004; Lambdin, 2003).

If teachers do wish to use conceptually challenging tasks with low-attaining students, teachers need to provide a form of assistance that allows for such challenge to remain; hence the emergence of “scaffolding”. Scaffolding (Wood et al., 1976) has become a popular way to describe a kind of assistance teachers offer students that fits with a constructivist philosophy to learning (Anghileri, 2006). Linked to Vygotsky’s Zone of Proximal Development (Vygotsky, 1978), scaffolding aims to bridge the students’ current understanding and their as yet unknown understanding. Scaffolding is particularly suited to tasks aimed at conceptual understanding (Anghileri, 2006) and as such, was well suited to the types of tasks examined in this study. The notion of scaffolding will form a significant part of the discussion in this chapter. To begin, I will describe the context and purpose of the present study.

CONTEXT AND PURPOSE OF THIS STUDY.

The focus of this study was twofold: it investigated three forms of scaffolding and examined teachers' use of three types of mathematical tasks. It sought to explore and describe scaffolding in the mathematics classroom as a bridge between the task and the learning of low-attaining students. To this end, the perspectives of both teachers and low-attaining students were examined. Specifically this study sought to investigate the following question.

> How does a teacher's use of particularly scaffolding practices, while using specific mathematics tasks, impact on low-attaining students cognitively and affectively?
>
> Sub-questions were
>
> 1. Do any of the scaffolding practices appear to be more effective in scaffolding the learning of low-attaining students than the others and in what ways?
> 2. Do low-attaining students have a preference for the kind of scaffolding their teachers offer and what are the reasons given for these preferences?
> 3. Do low-attaining students have a preference for the kinds of tasks used in mathematics lessons, and what are the reasons given for these preferences?
> 4. Do teachers show preferences for using particular types of mathematics tasks with low-attaining students and in what ways?

This study's focus was on three types of mathematical tasks researched in the *Task Types and Mathematics Learning* (TTML) project (Peled et al., 2009). These mathematical task types were discussed in Chapter 1 and can be summarised as tasks that use models, representations or tools that exemplify the mathematics; tasks that situate the mathematics within a contextualised practical problem and; tasks that use open-ended tasks to investigate specific mathematical content. This project and these tasks types will be discussed later in this chapter. These types of tasks are not necessarily typical of mathematics classrooms and do not represent all the types of tasks possible. However, they do represent types of mathematics tasks that aim for conceptual understanding, higher order thinking and engagement for students. As mentioned, studies have shown that aiming for higher levels of thinking in tasks does not ensure such a level is maintained during implementation (Henningsen & Stein, 1997), hence the TTML project's and the present study's focus on teachers' use of these tasks. Furthermore, some researchers have found such tasks are not used often with low-attaining students or the cognitive demand is reduced for low-attaining students (Anderson, 1997; Zohar et al., 2001). Such issues will also be discussed in this chapter.

There are many influences that can impact upon how a teacher implements tasks in the classroom. Teachers' beliefs about the nature of mathematics and about teaching and learning mathematics (Thompson, 1992) have a significant impact on implementation of tasks and this will be explored in this chapter. Due to this study's focus on low-attaining students, teachers' beliefs about the teaching and learning of low-attaining students will also be examined (Anderson, 1997; Beswick, 2005a). Teachers' mathematical content knowledge and their knowledge of teaching mathematics are married in the concept of pedagogical content knowledge (Shulman, 1987) and mathematical knowledge for teaching (Ball, Hill, & Bass, 2005). This particular type of knowledge affects how tasks are used in classrooms. Teachers' own background in mathematics and their feelings toward mathematics impacts on the kind of mathematics teachers they become (Bibby, 1999). These influences are also discussed in this review.

In addition, this study sought to examine the impact that particular scaffolding practices had on the cognitive and affective responses of low-attaining students while engaged in challenging types of mathematics tasks. The scaffolding practices of focus for this study were

- the use of discussion (e.g., Baxter et al., 2002);
- the use of manipulatives, concrete materials, representations or tools (e.g., Sowell, 1989); and
- explicit attention to concepts (e.g., Hiebert & Grouws, 2007).

There are many forms of scaffolding found in the literature. Some of these will be discussed as well as a detailed examination of the three scaffolding practices chosen for this study.

The final part of my research question concerned the low-attaining students themselves. This study sought to investigate the impact on target low-attaining students of the teachers' use of specific scaffolding practices, and the use of particular task types in mathematics. Specifically, I sought to examine the cognitive impact or impact on learning, and the affective response of the target students. Some studies have examined the possible learning and growth in understanding resulting from particular teaching approaches (Boaler, 1998; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989), and some studies also focussed on low-attaining students (Baxter et al., 2001; Bottge & Hasselbring, 1993). Less prevalent are studies that examine the affective responses of low-attaining students to particular types of teaching or tasks. Both these cognitive and affective issues are examined in this chapter.

The literature underpinning this study is drawn primarily from three spheres of research; research on mathematical tasks, the literature regarding low-attaining mathematics students and studies on scaffolding in mathematics teaching. The present study endeavoured to draw together these spheres of research and explore the nature of their influence on each other through the examination of a particular case. Each of these spheres, that is, tasks, low-attaining mathematics students and scaffolding, will now be examined in turn. These spheres do not exist independent of each other and there will be evidence of the overlapping nature of these areas of research with some studies spanning more than one sphere. I will begin by exploring the literature surrounding mathematics tasks. I will then detail research concerning low-attaining students, encompassing common characteristics and affective issues. I will also examine how teachers and schools have attempted to cater for low-attaining students through intervention programs, streaming or particular teaching practices. Finally, with tasks and characteristics of low-attaining students in mind, research regarding scaffolding practices in the mathematics classroom will be examined.

MATHEMATICAL TASKS

Classrooms are complex environments with "a myriad of cognitive, affective and social elements to which teachers and students alike contribute" (Beswick, 2008). There are many factors influencing the teaching and learning that occurs in the classroom. Educational researchers are interested in studying classrooms in an attempt to answer questions about the nature of teaching and learning (Carpenter, Fennema, &

Franke, 1996; Cobb et al., 1989). The kinds of tasks used in mathematics classrooms provide a link between the teacher's intended learning and the potential mathematics students learn. Many studies have examined tasks in mathematics in an attempt to understand more about this link between teaching and learning.

Studies in mathematics education have had various meanings for the term "tasks". Doyle (1988) described "academic tasks" as what the students are expected to produce, how they are expected to produce it and with what resources. Hiebert and Wearne (1993) discussed "instructional tasks" and their effect on student learning, through an analysis of classroom tasks and classroom discourse. Christiansen and Walther (1986) defined the task as what the teacher has in mind for the students but "activity" as what the students make of this task, which can differ from the task as the teacher intends it to be. Sullivan, Mousley and Zevenbergen (2006) offered a similar definition stating that the task is the prompt for students' work and activity is the thoughts and actions students engage in when responding to this prompt. I will assume these definitions in my uses of the term "task" and "activity", encompassing the teachers' intended task and the students' work on this task, in what follows.

Studying tasks is complicated by their multi-layered nature. There is the task that the teacher proposes to the students, and the task as the students interpret it and then the task that students actually engage in. Additionally, tasks have a variety of purposes in mathematics classrooms. Some are focussed on memorisation or practice, whereas others can promote higher order thinking. Tasks can be as diverse as completing exercises, chanting times tables, working through a problem, discussions with peers or communicating strategies and findings.

The importance of choosing quality tasks is paramount. Tasks provide the link between what the teacher wants the student to learn and what the student actually has the opportunity to learn. The National Council of Teachers of Mathematics (2000) advised that teachers choose "worthwhile tasks" that teach important mathematical ideas, engage the students and challenge them intellectually. Studies have analysed and categorised task types in various ways. Some studies sorted tasks according to the kind of thinking they could afford students (Doyle, 1988; Henningsen & Stein, 1997) and others by the ways in which the tasks are presented and solved (Hiebert & Wearne, 1993). I now examine these studies.

Doyle (1988) studied two average ability mathematics classes, one seventh and one eighth grade. The types of tasks and work the students engaged in and produced were analysed along with the teachers' perceptions of the tasks. Doyle proposed that tasks served as a context for students' thinking and that the work students did during tasks influenced how they came to think about the content and understand it. For example, if the usual mathematics tasks are solving equations for unknown values where the objective is to complete as many as possible and get as many as possible correct, students may perceive mathematics as repetitive, where speed is paramount, with little relevance to their lives. Conversely, tasks that encourage creative thinking, understanding, working with others and links to everyday life for example, communicate that such elements are an important part of mathematics.

Doyle (1988) defined lower cognitive level tasks as those based on memory, formulas or "search-and-match", which he described as finding key words in worded problems then applying ready-made algorithms. Higher cognitive tasks were defined as requiring students to identify the knowledge needed for the task, select a strategy and interpret problems and solutions. Doyle also divided tasks into familiar and novel tasks. Familiar tasks included "routinized, recurring exercises … in which relatively standardised operations or algorithms are used to generate products" (p. 173). The outcomes were clear and procedures to follow set down by the teacher. In contrast, novel tasks involved students in choosing the procedure or solution path and commonly included problem solving or open-ended tasks. There was a higher chance of anxiety for students as such tasks were less predictable and required higher levels of thinking.

The type of task also affected the workflow in the classroom, a factor that could impact on task-choice by the teacher. Doyle (1988) acknowledged that familiar tasks often flow smoothly with the teacher firmly in control, students all working reasonably quietly, work being mostly completed at the same time and a sense of order maintained. In contrast, novel tasks often have a less predictable, "bumpy" work flow, with work being completed at different levels and times, some students feeling threatened and frustrated, and the teacher having a less prominent role in maintaining order. In an attempt to smooth out such lessons, teachers often reduced the cognitive load of novel tasks by simplifying them or providing a highly familiarised task environment where procedures were provided or more difficult parts of the problem were actually done by the teacher. Researchers studying reform mathematics classrooms, where novel problematic tasks were used, have also reported similar issues of workflow. Ball (1993), Lubienski (2000b) and Boaler (1997) reported that some students felt frustrated and anxious and that teachers felt pressured to simplify tasks to offer reassurance.

In a similar way to Doyle (1988), the QUASAR project (Silver et al., 1995) categorised hundreds of tasks used in the project's mathematics classrooms based on the kinds of thinking the tasks could provoke in students. This resonates with the work of the Cognitively Guided Instruction project (Carpenter et al., 1996) which proposed that teaching does not directly influence student learning, but it influences students' thinking, which impacts on learning.

Stein, Smith, Henningsen and Silver (2000) studied the tasks from the QUASAR project. They proposed four levels of mathematical tasks. Two levels are linked to a higher level of cognitive demand and two with lower cognitive demands. At the highest level of cognitive demand were tasks termed "doing mathematics". Such tasks involve non-algorithmic thinking, and exploring mathematical relationships, require self-monitoring behaviour, and may result in higher levels of anxiety for students, as these tasks are unpredictable in nature. The second higher-order thinking category was "procedures with connections" that focussed students on procedures with the aim to deepen understanding of mathematical concepts. Such procedures are broad general procedures that link closely to the underlying mathematics rather than teacher directed algorithms, but may not show a clear relationship to the concept. These tasks often use multiple representations and concrete materials to link procedures with concepts and attempt to make meaning of procedures. They require some cognitive effort, as students need to understand the underlying concepts in order to succeed at the task. The first lower cognitive demand task type was described as "procedures without connections". These tasks are algorithmic and the procedure students need to use is clear. Students solve equations for unknown values usually all focussed on the same algorithm, and there is no direct link to the concepts underpinning the procedure. The focus is on correct answers rather than understanding. Most students would probably find such tasks familiar and relatively easy with little thought required. The second lower cognitive demand task was "memorisation". These tasks have a focus on speed and memory, for example, completing a set of questions requiring only the recall of previously learned facts. They also involve committing formulas, facts or definitions to memory. Students know what is expected and need only to use their memory to produce answers. These tasks do not link to underlying concepts or procedures. Stein et al. found that tasks did not always remain at the cognitive level throughout implementation with many tasks having the level of cognitive demand lowered by the teacher as Doyle (1988) also identified. This issue and the study of Stein et al. (2000) will be explored further during the discussion of teachers' use of mathematics tasks in the classroom.

Both Doyle (1988) and Stein at al. (2000) provided ways of categorising tasks according to the thinking such tasks provoked. Hiebert and Wearne (1993) also studied the nature of mathematics classroom tasks focussing on other aspects such as time, discourse and the use of pictures, physical aides and written symbols.

Hiebert and Wearne (1993) studied six second grade classrooms to examine instructional tasks and classroom discourse. Though the students were in mixed ability classes for other subjects, for mathematics they were assigned to either high track (two classes) or average to lower track (four classes). Four "control" classes continued with their usual form of instruction that was nearly all textbook based. The way in which two "experimental" classes, one higher track and one lower track class, were taught differed from the conventional program in that these classes did not use the textbook and teaching emphasised understanding of procedures and concepts. Hiebert and Wearne examined three elements of the problems or tasks presented to students. First, the number of problems presented and the average time spent on each problem was examined. Second, the kinds of problems presented were analysed, and third, the physical materials available for solving the problem was described.

The five types of problems identified in this study were

- Problems presented and solved using written symbols only.
- Problems presented using pictures or diagrams.
- Problems solved using physical aides.
- Problems presented through a story and solved using pencil and paper only.
- Problems presented through a story and solved using physical materials.

A key difference in the instruction of the experimental classes compared to the other classes was that the experimental classes worked on fewer problems and spent more time on each problem. In fact, these classes worked on about half the number of problems as the other classes yet their results were significantly better. They spent more time on whole class discussions and in explaining solution strategies than on individual seatwork.

The assessment data from this study emerged from a test designed by the researchers on place value and addition and subtraction at the beginning and end of the year. The lower achieving experimental class achieved results on the post-test that far exceeded the other three low track classes. In fact their results began to resemble the results of the higher track control class. This lower track class actually surpassed the higher track control class on items involving story problems by the end of the year.

This study indicated that more time spent on fewer problems that are rich and truly problematic for students could have a positive impact on student learning, whether in the higher or lower track. Additionally, the use of story situations, pictures or physical materials also seemed to enhance learning. More time on whole class discussions and less time on individual seatwork also had positive effects.

The studies on tasks I have discussed so far have focussed primarily on the cognitive effects of particular types of mathematics tasks. As I outlined in Chapter 1, the type of tasks and associated teaching students experience in mathematics classrooms also impact on motivation and engagement. To broaden this discussion, I now examine the literature on tasks and teaching to further explore issues of affect. Given that the target students in the present study were in Year 5, these studies are focused on middle years mathematics classrooms.

NATURE OF TEACHING AND TASKS

One possible factor in the disengagement of middle years students may be the type of teaching and mathematics tasks they experience during these years. Research has indicated that teaching may become more “traditional” during the middle years and that there is a greater emphasis on conventional written algorithms and following procedures. For example, Anderson, Sullivan and White (2004) studied 162 Australian teachers and categorised them as having *traditional* tendencies or *contemporary* tendencies. Traditional teachers for example, agreed strongly with statements such as “students should learn algorithms before they are presented with applications or unfamiliar problems” (p. 41). Contemporary teachers agreed with statements such as “students can learn most mathematical concepts by working out for themselves how to solve unfamiliar or open-ended problems" (p. 41). This study found that teachers became increasingly traditional as student progressed through to upper primary grades. Their study found that 74 per cent of teachers in Years 3 to 6 could be categorised as traditional. These authors postulated that the increase in traditional teaching might be due to the curriculum's emphasis in middle to late primary school on written algorithms, basic fact recall and mental strategies. Yates (2009) reported similar findings based on interviews with 15 middle school teachers. These teachers were representative of the 154 teachers surveyed, in their views about teaching and learning mathematics. Yates found these teachers held "back to the basics" beliefs about teaching mathematics stating, "Their strongly held beliefs that students must have a firm foundation in basic skills before they can undertake more challenging instruction and that this foundation can only be attained through a focus on procedural skills in mathematics are deeply rooted, (and) resistant to change" (p. 624).

Dole (2003) painted a similar picture of mathematics education of middle years students when she described the curriculum in early secondary schooling as a "revisitation of topics covered in primary school" (p. 279). She argued that from Years 7 to 9, much of the mathematics covered is revision and forces many students to "stand still" and "mark time" (p. 279). Such a curriculum could easily be viewed as boring, repetitive and pointless for students, and could heighten their sense of frustration and disengagement. In addition, Battista (1999) reported that mathematics teachers of middle years students in lower secondary years often have no specific mathematics education training. This compounds the problems discussed above for, with little mathematics knowledge, let alone pedagogical mathematics knowledge, Battista reported that such teachers tend to revert to the way they were taught mathematics, usually the kind of traditional approach discussed previously.

This 'traditional' style of teaching that emphasises 'basic facts', written algorithms and following memorised procedures has been variously described by researchers as "instrumental mathematics teaching" (Skemp, 1972) "traditional teaching" (Boaler, 1998) and "explain and practice" (Wheatley, 1992). These and other studies into how students respond to such teaching will be discussed further in later chapters. It is sufficient for now to say that such teaching styles are often cited as a factor in student disengagement.

Other vital factors for student engagement are the kinds of tasks teachers choose and use in the mathematics classroom. Mathematics curriculum documents across many countries have shown evidence of change during the late 1990s to the first decade of the 21st century (Wong, Hai, & Lee, 2004). This change, though proposed many years before (National Council of Teachers of Mathematics, 1980; Stacey & Groves, 1984), is thought to be a response to the perception of a rapidly changing workforce and to the growth in new technologies (Wong et al., 2004). This requires students to learn skills to deal with novel situations and problems, rather than content that is subject to change or redundancy. International testing such as Program for International Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS) may have also provided the impetus for reform in some countries (Dindyal, 2006). As such, tasks that both develop and require certain skills and characteristics of students such as problem solving skills, communication skills, thinking skills such as self-reflection as well as attitudes and behaviours such as persistence, curiosity and confidence, are necessary to meet the needs of today's students.

Of course, there is more to mathematics education than the curriculum stated in documents. Such advice or guidelines goes through many filters before it is ultimately realised in the mathematics classroom. Schools and teachers, sometimes through textbooks or other curriculum support materials, interpret the curriculum (Hargreaves, 1994). This interpretation forms the basis of the planned mathematics curriculum for students, which is also influenced by factors described by Ball and Cohen (1996) such as school, parental or student expectations, or the limitations of time, student ability, materials or teacher knowledge. The final filter is the students themselves who reinterpret the teachers' planned curriculum into the mathematics curriculum they actually experience and from which they potentially learn. Thus, "curriculum" can simultaneously hold many meanings.

Researchers have described these various meanings of curriculum. Robitaille, Schmidt, Raizen, McKnight, Britton, & Nichol (1993) described the "intended" curriculum, the "implemented" curriculum and "attained" curriculum. Similarly the curriculum has been described as "planned", "enacted" and "experienced" (Gehrke, Knapp, & Sirotnik, 1992) and also the "ideal", "adopted", "implemented", "achieved" and "tested" curriculum (Burkhardt, Fraser, & Ridgway, 1990). For this study I will be describing the intended curriculum when using the term the "current mathematics curriculum”. By this, I am referring to the curriculum that was contained in current curriculum documents at the time of writing. I will also examine the implemented or enacted curriculum when describing mathematics tasks that teachers and students actually do in their mathematics lessons during the observed lessons. As the intended and enacted curriculum are sometimes quite different, this is an issue I will explore further in this chapter.

As discussed, teaching style and tasks have a considerable impact on student engagement and on student achievement in mathematics. Research regarding the types of mathematics tasks that might be more engaging and how teachers might implement such tasks was the focus of the *Task Types and Mathematics Learning* project of which this case study was a part.

The *Task Types and Mathematics Learning* (TTML) research project (Peled et al., 2009) examined types of mathematics tasks used in upper-primary to lower secondary school, or “middle years”, mathematics classrooms. This project studied students in these years because, as I have discussed, this has been identified as a critical time, often characterised by disengagement that can limit future participation in mathematics (Luke et al., 2003; McPhan et al., 2008). The TTML project proposed that teaching mathematics through engaging, motivating and potentially rich tasks might address some of these issues. The TTML project built on previous studies that examined the use of task types in mathematics. This project differed from the studies on tasks previously discussed in an important way. Instead of studying a range of tasks, some of which were aimed at lower levels of thinking, the project explored with teachers three specific types of tasks, all aimed at higher-level thinking. This meant that this study was not a comparative study of tasks such as those by Doyle (1988), Stein et al. (2000) or Hiebert and Wearne (1993) but rather an extension of this research. These previous studies suggested that certain kinds of tasks improved student higher order thinking skills in mathematics. TTML took these successful types of tasks and studied the factors that could help or hinder the success of using such tasks in the classroom. Essentially, the project took the view that if tasks such as those chosen for TTML have been found to be successful in teaching mathematics, teachers should be using such tasks in the majority of their mathematics lessons. If this is the case, the TTML project asked, what might be the constraints, opportunities and appropriate pedagogies associated with the use of such tasks?

The three task types studied in this project were designed to encourage the kind of higher order thinking and communication skills called for by current mathematics curriculum advice (National Council of Teachers of Mathematics, 2000; Numeracy Review Panel, 2008; Victorian Curriculum and Assessment Authority, 2006). The first type of task described by the TTML was when "the teacher commences with an important mathematical idea, and proposes tasks which involve models or representations or tools, which help students to understand the mathematics" (Clarke, 2009, p. 719). Manipulatives and concrete materials are often advised for use by low-attaining students (Grouws & Cebulla, 2000), although the success of such use on the mathematical understanding of the students can vary. Using concrete materials as an answer-getting device for example has been found to have little effect on building mathematical understanding (Andrew, Beswick, Barrett, Swabey, & Bridge, 2004), whereas judicious use of manipulatives can build powerful mental models (Sowell, 1989).

The TTML described the second type of task studied, stating "teachers situate mathematics within contextualised practical problems where the motive is explicitly mathematics" (Clarke & Roche, 2009a, p. 721). Such "realistic" problems have been extensively studied and are advocated by many curriculum documents (National Council of Teachers of Mathematics, 2000; Victorian Curriculum and Assessment Authority, 2006). Boaler (1994) identified three key benefits in the use of context problems. First, such tasks may help students make the connections between their own everyday lives and mathematics. Second, they may aid in transfer of "school mathematics" knowledge to real world applications. Third, they may help to demystify mathematics from being seen as arbitrary numbers and disconnected procedures to more understandable and useful concepts of mathematics for real life. As such, mathematics in context may be more motivating and interesting for students. However, using context problems effectively can be complex. Poorly chosen contexts can actually diminish rather than enhance students' understanding (Sullivan, Mousley, Zevenbergen, & Turner-Harrison, 2003). Careful consideration of whose 'real world' is represented in the context is also required. Real world applications such as budgeting, shopping and banking, for example, are "real" for adults but not necessarily for students in the middle years (Boaler, 1994).

The third type of task studied in the TTML project was described as "content specific open-ended tasks (that) have multiple possible answers, they prompt insights into specific mathematics through students discussing the range of possible answers" (Sullivan, 2009, p. 727). These tasks have multiple solutions and/or there are multiple strategies with which to solve them. The benefits of this task type are claimed to be in students valuing and learning from each others' strategies, and in the emphasis being on the exploration required to find solutions and not finding only a single answer (Sullivan et al., 2003). There is evidence in the literature that many students find open-ended tasks motivating and engaging (Boaler, 1998; Ridlon, 2004). Research has also raised some concerns about the performance of low-attaining students during the use of open-ended tasks. One objection is that open-ended tasks by their very nature – the openness of the tasks, the multiple strategy paths and multiple answers – simply confuse these students who may require more explicit teaching in fewer strategies and skills (Ellis, 2005; Kroesbergen et al., 2004).

These three task types are representative of "potentially successful task types" (Sullivan, Clarke & Clarke, 2009, p. 88) for teaching mathematics that also align with the recommendations from curriculum documents examined previously. For the purposes of the TTML project, the task types were separated, as in the professional learning program teachers focussed on a particular task for an extended period of time before moving on to the next one. However, all three tasks share some common elements.

An examination of the verbs used in the task descriptions for TTML points to the elements on which all these tasks are based. Through these tasks students are invited to "inquire", "investigate" and "engage" with the tasks. Teachers' actions are described as "using a model", "exemplifying mathematical connections and principles", "situating the mathematics", and "fostering interdisciplinary perspectives". Student actions are not described as memorising, practising, or following processes, but rather are focused on the development of conceptual understanding. The teacher is not described as overtly "teaching" these tasks through explanation, drill, correcting or lecturing. Rather, teachers are most often directed to use the tasks for a mathematical purpose where the learning will come through the use of the tasks, which are designed to be cognitively challenging. This brings to mind the principles of constructivist learning described by Ridlon (2004).

> The constructivist model asserts that the teacher's role is to continually present students with problematic situations that are designed to meet defined classroom goals. By creating goal-appropriate tasks, the teacher

creates the opportunities children need to construct an experiential body of knowledge in the most personal, significant manner. (p. 3)

The role of the students in this construction of knowledge is mirrored in the verbs describing student actions in each of the TTML task types – “inquiring”, “investigating” and “engaging”.

A constructivist epistemology is a theory of knowledge that asserts that knowledge is within individuals, “a belief that all knowledge is necessarily a product of our own cognitive acts. …We construct our understanding through our experiences, and the character of our experience is influenced profoundly by our cognitive lenses” (Confrey, 1990, p. 108). Knowledge can be built and can grow but only as part of the individual and not from some exterior factor such as a book or a teacher. The book or teacher can influence the creation of knowledge within the individual but only if the learner can use the book or teacher in their own way to enhance the knowledge they possess. A constructivist philosophy of learning is that people construct their own understanding and knowledge. New knowledge or understandings build on and link to what is already inherent in each person’s mind. Knowledge cannot be passed on, given or transmitted by other people or tools unless the individual person constructs it within what they already know and understand (Simon, 1995).

Constructivism has become a widespread term in education circles but there is some evidence that understanding what constructivism is and is not has been confused (Clements, 1997). As stated, constructivism is a theory of learning not a method of teaching. Of course, learning and teaching are directly related so a teacher may adopt a constructivist approach to teaching. Such an approach would reflect the beliefs of constructivism such as the need for students to construct their own knowledge. Therefore the teacher may strive to get to know more about what individual students know so that experiences can be planned to build on this knowledge. A constructivist teaching approach would recognise the role of the teacher in providing experiences to build on or challenge the knowledge and understanding each student already has, rather than view students as being able to absorb knowledge by transmission of knowledge from the teacher. Constructivism has become an approach to education that is not without controversy, particularly when it comes to low-attaining students (Ellis, 2005). This debate will be examined in this dissertation. I will use the terms “constructivist approach to teaching” or “constructivist principles” against the background discussed here, namely that constructivism is a philosophy of learning.

The constructivist movement in mathematics has spawned many calls for change in mathematics teaching. In the United States, the National Council of Teachers of Mathematics advised a constructivist approach to teaching mathematics in documents such as *An Agenda for Change* (National Council of Teachers of Mathematics, 1980) and *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000). This major reform in mathematics teaching led to reform initiatives, reform teaching programs and many studies of the effectiveness of reform approaches to teaching mathematics (Knapp et al., 1995; Silver et al., 1995).

This reform approach encompasses potentially all the TTML task types. Reform mathematics teaching is described widely in the literature. Schwan Smith (2000) stated that in a reformed mathematics class, "classroom instruction is characterised by its emphasis on student thinking, reasoning and communication in rich problem solving situations" (p. 352). Ball (1993) also provided descriptions of her work in a reformed mathematics classroom in which she used types of mathematics tasks similar to those described in the TTML project. I am proposing that the use of mathematics tasks described by TTML is in harmony with constructivist learning principles, and the reform curriculum in the United States. In addition, the TTML task types were designed to be cognitively challenging and focused on conceptual understanding.

The use of cognitively challenging types of mathematics tasks presents a dilemma to teachers and mathematics educators of low-attaining students. Many believe such a curriculum may not be suitable for low-attaining students and recommend a different approach for such students (Ellis, 2005; Kroesbergen et al., 2004). Some assert fervently that low-attaining students deserve to be taught the same content using similar teaching approaches as their more able counterparts. (Cooper & Dunne, 1998; Knapp et al., 1995; Silver et al., 1995). Others go further to suggest that to deny them this is unjust and inequitable (Delpit, 1988; Moses, Silva, Rivers, & Johnson, 1990). The main elements of this debate are appropriate tasks and content for low-attaining students, and appropriate pedagogies for teaching such students effectively. My study focused on effective teaching of low-attaining students in mathematics and on the use of specific task types in mathematics. Teachers' chosen pedagogies, particularly scaffolding practices, was the focus of this study and will be discussed in detail in the following chapters. Mathematics curriculum advice current at the time of this study advised the use of tasks such as those from the TTML project for effective teaching of mathematics. One question under consideration in this study was whether the use of cognitively challenging mathematics tasks is just as appropriate for low-attaining students as for others.

I will now turn my attention from the tasks themselves to the teaching that surrounds such tasks when enacted in the classroom. As mentioned previously, the type of mathematics task selected and planned by the teacher can differ from the task as it is implemented. There are many factors that influence a teacher when implementing tasks. These are discussed below.

TEACHERS' USE OF TASKS

As mentioned in Chapter 1, the curriculum can be expressed as the "intended curriculum", found in curriculum documents, and the "implemented curriculum" (Robitaille et al., 1993), which is the enactment of such curriculum in the classroom. Researchers have often found a mismatch between the intended and implemented curriculum (Henningsen & Stein, 1997; Tzur, 2008). Therefore, by only examining the intended curriculum, the actual mathematics teaching presented to students may not be found. It is through studying the implemented curriculum, that is, what is being done in classrooms, that insights into the reality of mathematics for students can be described. The implemented curriculum can be examined through studying mathematical tasks, both as the teacher intends them and as the students find them.

The role of the teacher is crucial in how tasks are implemented in the classroom. This was characterised by Hargreaves (1994) who stated, "teachers don't merely deliver the curriculum. They develop it, define it and reinterpret it too. It is what teachers think, what teachers believe and what teachers do at the level of the classroom that ultimately shapes the kind of learning young people get" (p. ix). The teacher decides when, how and why to use certain mathematics tasks with their classes. Although the current mathematics curriculum documents advise the more frequent use of some task types over others, such as those that encourage higher-level thinking, the final decision lies with the teacher. First, I will examine the evidence in the literature regarding the kinds of tasks teachers use in mathematics lessons. To follow, I will discuss the influences on the teachers' decision to use these types of tasks. These influences include those posited by Hargreaves (1994) such as teachers' beliefs about the nature of mathematics, beliefs about learning and teaching mathematics and beliefs about low-attaining students in mathematics.

One way to examine the kinds of tasks that mathematics teachers are using with their students is to analyse the results of the TIMSS 1999 Video Study. This study videoed Year 8 mathematics lessons in seven countries across a year with about 100 lessons taped from each country. Hollingsworth et al. (2003) described the types of tasks which Australian mathematics students were seen to engage in during this video study. This provided a snapshot of what happens in a general sense, in Australian mathematics classrooms. Therefore these results may give valuable information about the kinds of mathematical tasks students were experiencing across the nation around the time of the present study.

This study found that Australian students spent more time per lesson working on sets of *concurrent problems* than on *independent problems*. Concurrent problems were assigned as a set and worked on individually. Independent problems were single problems solved as a class but with some private work time. Concurrent problems, or exercises to practise the same algorithm or formula, were used 54 percent of the time in Australian classrooms whereas independent problems were used 26 percent of the time. Hollingsworth et al. (2003) analysed the time spent on a single problem to shed more light on the kinds of thinking tasks may have afforded students. Tasks were divided into those that took less than 45 seconds to solve and those that took longer. Australian teachers used tasks longer than 45 seconds 55 percent of the time whereas Japanese teachers in this study used longer tasks 98 percent of the time.

Hollingsworth et al. proposed that the relatively short time spent on each problem in Australian classes indicated that it was unlikely that students would be solving challenging problems, examining mathematical relationships in detail or discussing solutions and strategies with teachers and peers. Evidence supporting this view can be found on further analysis of the type of problems Australian students worked on. Hollingsworth et al. reported that 77 percent were repetitions of problems solved earlier in the lesson and were of low-procedural complexity. This description resonates with Doyle's (1988) "routine tasks" and "low cognitive demand tasks" described by Stein et al. (2000). It would seem then on the strength of this evidence, that the higher level cognitive tasks were not being used most of the time in Year 8 Australian mathematics classrooms. Conversely, this meant that the lower cognitive tasks such as procedure practice and memorisation were being used the majority of the time. Indeed Hollingsworth et al. (2003) stated that there was "a syndrome of shallow teaching, where students are asked to follow procedures without reasons" (p. 21). The TIMSS results (Hollingsworth, 2003) suggest that teachers were not using cognitively challenging tasks and therefore it is important to examine why.

As mentioned previously in this discussion, there is evidence in the literature that tasks aimed at a higher cognitive level can fail to achieve this level of thinking for the students (Henningsen & Stein, 1997). The reason lies in the implementation of such tasks. While tasks can be set up and intended to target higher level thinking for students, the demands of these tasks can be reduced during implementation until the outcome for students actually becomes lower level thinking. This is a major constraint of using higher cognitive tasks. These tasks are difficult to maintain at this higher level while it is relatively easy to maintain the cognitive intention with the cognitive outcome when using low-level tasks (Stein & Lane, 1996). The risk of lowering the cognitive demands of a higher level task might be exacerbated when the students are low attaining (Beswick, 2005a). Teachers may feel that simplifying the task is more warranted for the students they consider low achieving.

The *Quantitative Understanding: Amplifying Student Achievement and Reasoning* project (QUASAR, Silver et al., 1995), mentioned previously in this chapter, had access to 144 tasks at different schools as well as data on student achievement. The students in this project were from poor, urban schools with a large number from minority and disadvantaged groups. A high number of these students were considered low attaining. Stein and Lane (1996) studied the tasks at QUASAR schools with different levels of student gain. Observations and videotapes of 620 sixth, seventh and eighth grade lessons were observed and coded. Using the levels of cognitive demand described earlier in this chapter (Henningsen & Stein, 1997), Stein and Lane found that tasks that were set-up for lower cognitive demand often remained at this level throughout implementation whereas "over half of the tasks that were set up to encourage the doing of mathematics declined during the implementation phase into procedural thinking, nonmathematical activity, or unsystematic and/or nonproductive exploration" (p. 60). Classes that achieved the highest student gains were those where most of the tasks were set up for multiple solution strategies, for the use of a variety of connected representations and for student explanations. In these classes most tasks remained at a high level of cognitive demand throughout implementation. In contrast, classes that achieved the lowest student gains were those where the majority of tasks were set up for low-level cognitive demand such as those requiring a single solution. Stein and Lane (1996) summed up the relationship between tasks and assessed student gains by stating that

> classroom instruction that primarily focussed on tasks with high-level cognitive demands was associated with the most gain in student learning. Conversely classroom instruction that primarily focussed on tasks with lower-cognitive demands was associated with the least gain in student learning. (p. 71)

Henningsen and Stein (1997) analysed the factors that caused tasks to either decline in their cognitive demand or maintain a high level of demand during implementation. The factors that most influenced a decline in the level of cognitive demand during task implementation were the "removal of challenging aspects of the task" (p. 535); "shifts in focus from understanding to the correctness or completeness of the answer" (p. 535); "inappropriate amounts of time allocated to the tasks" (p. 535); "inappropriateness of the task, including low levels of student motivation, lack of student prior knowledge, and lack of suitably specific task expectations" (p. 537); "lack of accountability for high-level products, (where) students are not expected to justify their methods and their unclear or incorrect explanations were accepted" (p. 537); and "classroom management problems" (p. 537). Henningsen and Stein noted, "The one major factor that occurred across the three types of decline was inappropriate amounts of time. Thus, it appears as though planning for appropriate amounts of time and being flexible with timing decisions as the task implementation phase unfolds are extremely important in order to avoid declines of all types" (p. 538).

Henningsen and Stein proposed that five factors influenced a task maintaining a high level of demand throughout implementation. These factors were that tasks built on students' prior knowledge, scaffolding was used to support students, there was an appropriate amount of time for the task, there was modelling of high-level performance, students engaged in self monitoring, the teacher drew conceptual connections between mathematical ideas and there was a sustained press for explanation and meaning (p. 534).

Kazemi and Stipek (2001) also examined this press for explanation and meaning in their study of four upper elementary classrooms. They proposed that there were four characteristics of lessons that were high press for conceptual thinking. These were expressed as sociomathematical norms that stated "(a) an explanation consists of a mathematical argument, not simply a procedural description; (b) mathematical thinking involves understanding relations among multiple strategies; (c) errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, and pursue alternative strategies; and (d) collaborative work involves individual accountability and reaching consensus through mathematical argumentation" (p. 59).

These characteristics described by Henningsen and Stein (1997) and Kazemi and Stipek (2001) are not easily implemented by teachers and involve quite complex understandings of mathematics, the teaching of mathematics, and how students might best learn mathematics. Hill, Shilling and Ball (2004) described this knowledge as Mathematical Knowledge for Teaching (MKT). Elsewhere Ball, Hill and Bass (2005) proposed that mathematical knowledge for teaching involved "figuring out where a student has gone wrong, explaining the basis for an algorithm in words that children can understand and showing why it works, and using mathematical representations" (p. 21). Ball et al. asserted that MKT involved "mathematical reasoning as much as it does pedagogical thinking" (p. 21) and in this way differentiated MKT from Shulman's (1987) "pedagogical content knowledge".

Charalambous (2008) examined how teachers' MKT influenced how tasks "unfolded" in mathematics classrooms. Charalambous completed case studies of two teachers, one identified as "high MKT" and one identified as "low MKT". Each teacher taught fifth grade, with 21 and 15 lessons respectively observed and coded. Charalambous found that time allocated to challenging tasks and less challenging tasks differed between the two teachers. The high MKT teacher spent even amounts of time overall on challenging and less challenging tasks while the low MKT teacher spent more time, 81 percent of lesson time, on less challenging tasks. Most of the tasks in the high MKT teacher's class were categorised as "procedures with connections" whereas most of the tasks in the low MKT teacher's class were categorised as "procedures without connections", using the levels of cognitive demand described by Stein et al. (2000). The unfolding of tasks appeared to be influenced by each teacher's own mathematics knowledge and ways of solving problems. In an example given by Charalambous, the low MKT teacher looked immediately for a formula to solve a mathematical problem while the high MKT teacher drew pictures and diagrams to prove the answer. Thus, Charalambous suggested that MKT is a factor in the unfolding of tasks in the mathematics classroom.

It seems imperative that teachers consider what kind of thinking the tasks they choose may afford students. It is therefore necessary that teachers are equipped with the knowledge of how to find, adapt or devise tasks that support higher-level cognitive demands if students are going to have the opportunity to learn at this level. Arbaugh and Brown (2005) proposed that by assisting teachers to analyse tasks and the type of thinking they could afford students, teachers would become better equipped to choose quality tasks. They engaged seven high school mathematics teachers in analysing tasks using the levels of cognitive demand (LCD) from the QUASAR project (Silver et al., 1995) described previously in this chapter. Arbaugh and Brown found "that engaging teachers in learning to examine mathematical tasks using the LCD criteria supports both a growth in pedagogical content knowledge (ways of thinking about mathematical tasks) and a change in practice (choosing mathematical tasks)" (p. 527).

The *Task Types and Mathematics Learning* (TTML) project (Peled et al., 2009), to which the present study was attached, also engaged teachers in analysing the types of mathematics tasks they used in their classrooms. This project differed somewhat from Arbaugh and Brown's (2005) study in that only potentially high-level cognitive tasks were examined. This project created opportunities for teachers to recognise, use, devise, and adapt three types of quality mathematics tasks; tasks that use models, tools and representations, tasks using a real world context and tasks which are open-ended. The TTML project found that teachers became more aware of the different types of mathematics tasks possible for use in their mathematics lessons and "actively looked for opportunities to use all three task types" (Clarke & Roche, 2010, p. 160).

Teacher beliefs play an important role in the selection, implementation and evaluation of the tasks teachers offer students in their mathematics classrooms. The following section addresses teacher beliefs including beliefs about the nature of mathematics, beliefs about teaching and learning mathematics and beliefs about the learning of low-attaining mathematics students.

TEACHER BELIEFS

The literature points out that the beliefs teachers have about mathematics influences the ways in which they teach mathematics in the classroom. Thompson (1992) asserted that "teachers' beliefs about mathematics and its teaching play a significant role in shaping the teachers' characteristic patterns of instructional behaviour" (pp. 130-131). The role of the teachers' beliefs about mathematics, and about teaching mathematics and learning mathematics all influence the types of tasks teachers choose to use in mathematics lessons. Ernest (1989) asserted that beliefs are also central to changing instructional practice stating, "change cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change" (p. 1). In addition, teachers' beliefs about low-attaining students of mathematics effects the tasks given to these students, the expectations teachers hold of low-attaining students and ultimately the kind of learning to which low-attaining students are given access (Anderson, 1997; Beswick, 2005a; Zohar et al., 2001). Clearly a teacher's beliefs are an important influence on task choice and task use in the mathematics classroom.

In the literature, teacher beliefs often fall into two categories – beliefs about the nature of mathematics and beliefs about the teaching and learning of mathematics. I will now discuss each of these types of beliefs in turn. Given the focus of my study, I will then discuss the literature about teacher beliefs regarding the teaching and learning of low-attaining students of mathematics.

Many researchers have offered categories of beliefs that teachers may hold about the nature of mathematics. Ernest (1989) defined three kinds of beliefs; a dynamic, problem solving view of mathematics, a Platonist view of mathematics as a static body of knowledge, and an instrumentalist view that mathematics is "like a bag of tools" (p.10) made up of rules, facts and skills. Lerman (1983) suggested two additional conceptions of mathematics in an "absolutist" view, that all mathematics is based on "absolute foundations" of an unchanging and certain nature, or a "fallibilist" view that mathematics develops through conjectures and proofs and is uncertain and changeable. Skemp (1972) added to this literature the idea that the kinds of understanding teachers strive for in teaching mathematics relates directly to the kind of mathematics they teach. Teachers holding the view that "relational understanding", "knowing both what to do and why" (p. 9) constitutes mathematics understanding would strive for the development of "conceptual structures that enable the possessor to construct several plans for performing a given task" (Thompson, 1992, p. 133). In contrast, a view that "instrumental understanding", "rules without reason", is mathematics would lead teachers to encourage "knowledge of a set of fixed plans for performing mathematical tasks" (Thompson, 1992, p. 133).

One of the problems in researching teacher beliefs is that it is "quite conceivable, indeed probable, for an individual teacher's conception of mathematics to include aspects of more than one of [these view of mathematics]" (Thompson, 1992, p. 132). Indeed, in Thompson's (1984) case studies of three teachers, she found that one teacher simultaneously held conflicting views about mathematics as both creative and uncreative. Thompson posited that this teacher had not examined these conflicting beliefs but held each separately, thereby allowing the incongruity to continue.

Teachers' views and beliefs about the nature of mathematics itself have a significant impact on their teaching of mathematics. Beliefs about teaching and learning mathematics, though connected to the nature of mathematics itself, also impact on the kinds of opportunities for learning students are afforded in mathematics classrooms.

Thompson (1992) found that teachers' beliefs about teaching and learning mathematics centred around their views about "appropriate locus of control" either more student or more teacher controlled, what teachers viewed as evidence of mathematical understanding in students and the purpose of planning lessons. Clark (1988) claimed that it is the implicit theories teachers hold about their students, the subject matter and their roles and responsibilities that impacts on beliefs about learning and teaching. Other researchers found that such beliefs are formed during the teachers' own schooling and are shaped by their experiences as students of mathematics (Bibby, 1999).

Kuhs and Ball (1986) identified four main views of how mathematics should be taught. These are *learner focussed* where mathematics is focussed on the students' own construction of knowledge and understanding; *content focussed with an emphasis on conceptual understanding* where mathematics teaching is driven by the content but focussed on understanding underlying concepts; *content focussed with an emphasis on performance* where mathematics teaching is driven by the content but focussed on student performance and mastery of rules and procedures; and *classroom focussed* where mathematics teaching relies on effective classrooms.

According to Kuhs and Ball, "learner focussed teaching" relates to a constructivist approach where the learner constructing their own understanding forms the basis of instruction and the teacher facilitates this construction. Thompson (1992) suggested this approach would also link to a problem solving view of mathematics offered by Ernest (1989). A "content focussed emphasising understanding" view would follow from a Platonist view of mathematics (Ernest, 1989) where teaching originates with the mathematical content but emphasises student understanding of the concepts of such content. "Content with a focus on performance" differs from the first two views of mathematics learning in that performance and mastery of rules and procedures takes precedence over understanding. This would align with an instrumental view of mathematics (Skemp, 1976). In the final view, classroom focussed model of teaching, the teacher "skilfully explains, assigns tasks, monitoring student work, provides feedback to students and manages the classroom environment, preventing or eliminating disruptions that might interfere with the flow of planned activity" (p. 26). The students' role is to listen attentively and co-operate with the teacher's instructions.

Anderson (2003) used the terms "contemporary" and "traditional" to describe the beliefs of the teachers in her study. Anderson studied 162 teachers' choice of tasks and how this related to their beliefs about mathematics teaching and learning. Anderson defined tasks as the questions posed to the students. She theorised that the type and frequency of tasks teachers chose for their students revealed their beliefs about mathematics and learning. Anderson proposed that in order to change the type or frequency of task types, teachers' beliefs must first be addressed.

Anderson found that teachers chose tasks for different purposes. About 70 percent of the teachers for example, often chose exercises that "required the application of a known fact or mathematical procedure and would typically be used for practising skills" (p. 73). Teachers indicated that they felt exercises were more appropriate for lower achieving students and children in lower grades. They also believed exercises could be motivating for students as they allowed success and were in a format that was familiar and easily understandable for students. Teachers also felt confident using exercises as tasks as they were familiar with the pedagogy required and such exercises were readily available in textbooks.

Teachers responded quite differently to the use of open-ended or unfamiliar problems, with just 20 percent often choosing open-ended tasks and 11 percent often choosing unfamiliar problems. These teachers indicated that they chose these tasks because they wanted to encourage higher-level thinking, they wanted to challenge and motivate students, particularly high achieving students, and they wanted to teach problem solving strategies.

The reasons teachers gave for not choosing such tasks often were also illuminating in examining more closely the relationship between teachers' beliefs and task choice. Many teachers believed that open-ended tasks and unfamiliar problems were challenging and therefore only suitable for gifted or highly able students. They also identified that such tasks required persistence and led to a degree of frustration for students. Additionally teachers mentioned their own inexperience in teaching such tasks and a resulting lack of confidence to use these in their classrooms. This has implications for low-attaining students. If teachers believe some task types, such as open-ended tasks or non-routine problems are not suitable for such students, it follows that the students may not be given opportunity to experience such tasks.

Anderson, White and Sullivan (2005) examined how teachers perceived the use of problem solving in mathematics classrooms. They proposed two differing views of such use. Firstly, problem solving can be seen by teachers as an "ends", the type of task offered to students after instruction on concepts or practice of skills. This is usually after what the teacher deems as an appropriate proficiency has been achieved in such skills. Secondly, problem solving can be seen as the "means" to teach skills or concepts, that students can learn these through solving problems. In this way, the motivation to find a solution encourages engagement with the mathematical concepts underpinning the problem. Each of these views influenced the teachers' choice of tasks for mathematics. A teacher who held an "ends" view of problem solving would emphasise tasks such as exercises and practice of basic skills which would fall into Stein, Grover and Henningsen's (1996) lower cognitive demand tasks category. Alternatively, a teacher who held the view that problem solving was the "means" by which students could learn mathematics would choose problem-solving tasks more often which require a higher level of cognitive demand (Stein et al., 1996).

The implications for low-attaining students is that with an "ends" view of problem solving, such students would rarely be able to experience problem solving as the teacher would first need to judge that a sufficient level of proficiency in basic skills had been reached. As many low-attaining students often struggle with automatically recalling some basic facts, this would preclude them from being deemed proficient. Many teachers in this study made statements that indicated their belief that problem solving was not appropriate for any but the highest achieving students they taught. For example, one teacher stated that "only the top ten or so in my class can handle unfamiliar problems" (Anderson et al., 2005, p. 24).

This study also found that teachers chose tasks for academic and non-academic reasons. They chose tasks they believed students require for learning mathematics and tasks students required for feeling positive and confident about mathematics. They also considered classroom management issues, school and parental expectations and standardised assessment. The decision ultimately rested on what the teacher perceived to be "learning mathematics". If they believed learning mathematics is remembering procedures and applying them in the correct circumstances or answering questions quickly and accurately, perhaps an instrumental view of mathematics (Skemp, 1972), their task choice will reflect this. Conversely, if teachers believed learning mathematics was devising strategies, testing these, finding a number of solutions, communicating with classmates and reflecting on thinking, a relational view of mathematics (Skemp, 1972), tasks would reflect this instead. In this way, beliefs about the nature of mathematics is linked to the teachers' task choice.

Putman, Heaton, Prawat and Remillard (1992) conducted case studies of four fifth-grade mathematics teachers to examine the interrelationship between teacher knowledge and beliefs and their instruction. Just as Anderson et al. (2005) found, Putman et al. asserted that it was the teachers' knowledge and beliefs about teaching and learning mathematics that acted as "important lenses or filters through which teachers perceive and act on various messages to change the way they teach" (p. 213). These cases also highlighted the interaction between beliefs and knowledge with one teacher believing strongly in the importance of making mathematics enjoyable but, without the necessary knowledge of mathematics, mathematical content was overlooked or misrepresented (Heaton, 1992).

The *Improving Attainments in Mathematics Project* (Watson & De Geest, 2005) intended to investigate how teachers could improve attainment for low-attaining students through the use of practices aimed at improving student thinking. This project studied how nine teachers changed from traditional teaching of low-attaining students that involved re-teaching the curriculum, to teaching in ways to encourage mathematical thinking. Successful changes teachers made to their practice included allowing greater wait time for both answering questions, and to complete tasks, sometimes extending tasks over several lessons. Teachers were also more mindful of making connections between mathematical ideas and between mathematics and the students' everyday lives. Other features of their teaching entailed planning to respond to students so that the lesson's direction and emphasis was influenced by how the students were responding, and varying the kinds of tasks. However, Watson and De Geest noted that while these changes in practice were successful in teaching these low-attaining students to think more mathematically, it was primarily the underlying principles these teachers held that proved to make the difference. While their individual practice varied greatly, their deeply held beliefs about mathematics and the learning of mathematics was a common element and one that Watson and De Geest believed held the key to improving the attainment of these students. Such beliefs were "that all students could learn mathematics, that mathematics is intrinsically interesting, and that it is the teacher's job to support learner's approach to mathematics as it is, with all its complexities" (p. 225). Watson and De Geest noted that such beliefs were "at odds with usual practices for low-achieving students" (p. 225).

Zohar, Degani and Vaaknin's (2001) examined the beliefs junior high school and high school teachers held about learning and their students, particularly low-attaining students. They found that some teachers held a "hierarchical view" of learning in that some skills were considered at a lower level of complexity and that this gradually increased in complexity to higher order thinking and reasoning. These teachers also held a corresponding hierarchical view of their students with only 20 percent of the teachers in this study reporting that higher order thinking skills were just as appropriate for low-attaining students as students who were higher-attaining. In effect, these teachers viewed learning as a ladder with students only ascending when the previous rung was mastered. The issue for low-attaining students is that teachers with such views would prevent these students from reaching all but the lowest rungs on the learning ladder.

The teacher's view of intelligence itself can underpin a hierarchical view of learning and curricula. Dweck (2000) describes two possible views of intelligence: entity view and incremental view. An "entity" view of intelligence holds that as intelligence is largely genetic, it is also relatively static and therefore people can do little to change their level of intelligence. Teachers who hold this view would believe that low-attaining students are not "maths people" and therefore there is little a teacher can do to change this. An "incremental" perspective views intelligence as an evolving entity that people can change and improve with effort and the right experiences. In this case, a teacher holding these views would believe improvement and change is possible for all students if teachers and schools can provide the right conditions (Dweck, 2000). Such beliefs influence the ways in which teachers behave toward their students and the expectations they hold for their success or failure.

Holding low expectations of students has a number of pedagogical implications. Teachers with low expectations of low-attaining students may limit student opportunities for challenging mathematics, not call on low-attaining students to answer questions or contribute to discussions, and sometimes treat these students in a less friendly manner than higher attaining students (Brophy, 1983). It is likely that teachers with low expectations of their students' abilities to learn will present low-level content and believe rote and memorisation are important (Sullivan et al., 2006). Furthermore, teachers with low expectations may engage in protective behaviour that prevents these students from having to struggle through tasks independently (Schwan Smith, 2000).

Brophy (1983) used the term "self-fulfilling prophesy" to describe the effect teacher expectations can have on student learning and achievement. Brophy's theory is that because teachers treat low-attaining students in ways that restrict their opportunities for hard thinking or challenging tasks, these students are not able to succeed at such tasks. Therefore the teacher's view of them is validated and the cycle continues. In contrast, students who the teacher sees as high attaining are challenged and expected to improve, are given more opportunities to do so. In response, these students often show improvement and progress, confirming their status in the teacher's eyes of being high-attaining.

Brophy (1983) identified research studies that described the way teachers responded differently to different students. Some of these ways of responding included shorter wait time for low-attaining students to answer questions as they are passed over for more able students or given the answer by the teacher rather than the teacher having to wait. Teachers also called less on low-attaining students, had less friendly verbal and non-verbal contact with these students, and demanded less of them cognitively.

Good (1981) also described some of the behaviour teachers with low-expectations of low-attaining students exhibit. Amongst these were giving low-attaining students less content than they could handle, requiring less effort of these students, calling on them less, and giving less useful feedback. Good warned that this could lead to a passive learning style where low-attaining students reduced their efforts and took a less active role in their learning.

Although Brophy's (1983) study reported less friendly contact between teachers and low-attaining students, there are many caring teachers whose concern for their low-attaining students is clear (Schwan Smith, 2000). In some ways, the behaviours described by Good (1981) whilst ultimately disadvantaging low-attaining students, could easily be attributed to the teachers' overall efforts to shield these students from the shame or humiliation of not knowing the mathematics. This over-kindness may stem from Dweck's (2000) entity view of intelligence where teachers believe there is nothing they can do about low-attaining students' progress except protect their self-esteem and afford them as much success as possible by presenting them only with tasks or questions which ensure this.

Pogrow (1988) warned that by protecting the self-image of under-achieving students through giving them only "simple, dull material" (p. 84), teachers actually prevent them from developing self-confidence. He maintained that it is only through success on complex tasks that are valued by the students and teachers that such students can achieve confidence in their abilities. There will be an inevitable period of struggling while the students begin to grapple with problems but Pogrow asserted that this "controlled floundering" is essential for students to begin to think at higher levels. Similarly, Askew, Brown, Rhodes, Wiliam and Johnson (1997) found that effective teachers offered *all* students a challenging curriculum whereas less effective teachers tended to offer challenge to high attaining students but strived to keep low-attaining students "feeling they've either understood something or done something well" (p. 44) by removing the challenge.

In summary, the expectations teachers have of low-attaining students can have a considerable impact on their learning. Low-expectations, low-level content and protecting students from productive struggling can all have a negative effect on the progress of low-attaining students.

In the discussion of mathematics tasks so far, I have discussed the importance of tasks as vehicles for learning (e.g., Stein et al., 1996); types of tasks that were most commonly used in Australian mathematics classrooms (Hollingsworth, 2003); studies of tasks that were categorised in various ways including according to cognitive demand (Doyle, 1988; Stein et al., 1996); the issue of changes in the cognitive demand of tasks by teachers during implementation (Henningsen & Stein, 1997; Tzur, 2008); and the impact of teacher beliefs on the types of tasks they choose to use during mathematics lessons (e.g., Anderson, 1997).

The literature regarding teachers' use of tasks has identified factors that affect the level of cognitive demand throughout implementation of a task. Factors that support maintaining a high cognitive demand during mathematics tasks include

- tasks build on students' prior knowledge and connect with what students know and understand (Henningsen & Stein, 1997);
- modelling high-performance (Henningsen & Stein, 1997);
- appropriate use of scaffolding (Henningsen & Stein, 1997);
- appropriate amounts of time for tasks (Henningsen & Stein, 1997);
- an emphasis on meaning (Kazemi & Stipek, 2001);
- explicit connections made between mathematical ideas (Henningsen & Stein, 1997);
- teachers who "press" students for explanations and to demonstrate understanding (Henningsen & Stein, 1997; Kazemi & Stipek, 2001);
- an attitude toward errors as important vehicles for learning (Kazemi & Stipek, 2001);
- a high level of mathematical knowledge for teaching (Ball et al., 2005); and
- teachers' ability to analyse, select and implement tasks according to the level of thinking the task affords (Arbaugh & Brown, 2005).

Factors that can lead to tasks declining in the level of cognitive demand through implementation include

- pressure from students to lower cognitive demand due to the uncertain nature of non-routine tasks (Doyle, 1988);
- teachers attempting to create a smooth and ordered workflow by reducing uncertainty and increasing familiarity (Doyle, 1988);
- teachers' perceptions of students' prior knowledge being a barrier to completing the task (Tzur, 2008);
- teachers being overly concerned about students' self-esteem and feelings about mathematics so that frustration and struggle is decreased or eliminated (Watson, 2002);
- inappropriate amount of time for the task (Henningsen & Stein, 1997);

- a “drift away from a focus on meaning and understanding toward an emphasis on accuracy and speed” (Henningsen & Stein, 1997, p. 526)
- a lack of alignment between tasks and students’ prior knowledge, interests and motivations (Henningsen & Stein, 1997).

These factors will be discussed in more detail during an examination of the literature regarding explicit attention to concepts. I now discuss the effect that students have on the implementation of tasks in mathematics.

THE INFLUENCE OF STUDENTS ON TASK IMPLEMENTATION

It is not just the teacher who determines how tasks play out in the classroom. Students also have an influence on how tasks are implemented in the mathematics classroom. Tzur (2008) pointed out that it is in response to perceived student needs that teachers alter tasks. This alteration could occur either at the planning stage or during implementation when teachers interpreted students’ responses to the tasks. This resonates with other studies that found students themselves demanded more direction from teachers and felt frustrated with having to investigate mathematics more independently. Lubienski (2000b) for example, found resistance to her problem centred approach to mathematics teaching came from the low SES seventh grade students in her class who found this style of teaching frustrating and intimidating. These students wanted the teachers to tell them the answer and have rules to follow. Lubienski warned that these students were “feeling increasingly mathematically disempowered” (p. 476) by her open, challenging mathematics problems.

Similarly, Robert (2002) reported on her struggle to use open-ended tasks and problem solving with fifth grade students from low-socio economic backgrounds who were considered persistently low-attaining. These students were initially hostile, disruptive and lacking in self-confidence. They found Robert’s use of open-ended or problem-solving tasks humiliating and frustrating as they felt such tasks were too difficult. They were not accustomed to mathematics being anything more than rote learning and this translated into disruptive behaviour as these students rebelled against these kinds of tasks. Robert admitted that she considered returning these students to learning basic skills through rote learning as such tasks were familiar and non-threatening. However, she decided to persist with problem solving but chose to scaffold their problem solving skills by constraining the openness of the tasks, modelling problem solving strategies such as making tables and lists, and gradually building up the students' confidence in their own ability to solve problems. It was this confidence, Robert asserted, that was the key to successful use of problem and open-ended tasks with these students who were so used to failure. After experiencing success on problems with higher levels of teacher direction and modified tasks, these students were able to tackle more open-ended problems and grew more confident about their abilities as solvers of problems.

Schwan Smith's (2000) case study described a similar dilemma faced by a teacher as she attempted to balance her long-held beliefs about her role as a teacher with the demands of the new mathematics curriculum and pedagogy introduced through a reform mathematics research project. She expressed concern that her students must have "felt bad" when they began this new curriculum as they had little previous experiences in mathematics to prepare them. As a result she sought to eliminate any potential sources of struggle or discomfort for them. By attempting to alleviate her students' struggles, this teacher also closed down the opportunity for higher order thinking and independent exploration of the concepts. Doyle (1988) also described how students "sometimes respond to the ambiguity and risk involved in novel work (non-routine problems) by negotiating directly with teachers to increase the explicitness" (p. 174) of mathematics tasks. In addition, Doyle pointed out that teachers themselves often reduce the cognitive demands of tasks in order to "smooth out possible workplace tensions" (p. 174) in the classroom. Similarly Watson (2002) suggested that a practice she termed "path smoothing" was more prevalent when students were low attaining. Watson stated that teaching mathematics to low-attaining students in high school "often involves the simplification of the mathematics until it becomes a sequence of small smooth steps which can be easily traversed" (p. 462). Watson reported that teachers took students through a task until students filled the gaps with the numerical answer or engaged in low-level recall of facts. However, Watson warned that this "path smoothing" practice did not lead to sustained learning because the opportunity for new learning was eliminated by keeping students within the bounds of what they could already do.

Many teachers find it difficult to allow students to flounder. Smith (1996) described the tension between a teachers' sense of efficacy and the practice of "not telling" which is in line with the current relevant mathematics curriculum advice. Smith asserted that many teachers believe that their role is to tell students how to do mathematics and show them clearly through demonstrations and explanations how to succeed in mathematics. When their students then demonstrate improvement, teachers feel this is due to them, that is, to their explanations and demonstrations, and this leads to a strong sense of efficacy. However, the mathematics curriculum that emphasises student led learning and students building their own knowledge without direct teacher input, threatens this efficacy. Some teachers may feel they do not have an effect on such learning and may begin to question their role. Smith (1996) asserted that teachers could find efficacy in such a mathematics curriculum. The teachers' role remains vital with the emphasis simply shifting from what content can be taught to what experiences teachers can offer to promote learning. According to Smith, elements such as choosing quality problems, predicting student responses, generating and directing discussions and judicious telling can all provide opportunities to feel effective as teachers.

Chazan and Ball (1995) were critical of this advice for teachers to "not tell" when teaching mathematics. They asserted that it was unhelpful for teachers to be advised that they should not tell students answers, solution strategies and the like if they were not also given advice on what they *should* do. Chazan and Ball asserted that there are instances in teaching when it is vital that the teacher takes an active role and does tell students things such as acceptable modes of communication, when their idea contradicts another students' idea or is contrary to correct mathematics. In contrast to the passive model of a "not telling" teacher, Chazan and Ball stressed the very active role a teacher has in a reform mathematics classroom. They must guide, monitor, provoke, orchestrate and actively involve students in discussion. Far from their role being diminished, these researchers believe the teachers' role is amplified and complex, requiring thoughtful planning and strategic thinking. Teachers who are feeling their sense of worth is threatened by current mathematics teaching advice need to find new ways to identify themselves as good teachers. Perhaps, as Chazan and Ball (1995) asserted, mathematics teaching advice needs to go beyond advising teachers "not to tell" but instead focus on the active role teachers can continue to play.

The students' level of attainment can also affect the type of mathematics tasks teachers use in the classroom. There is evidence in the literature that teachers deliberately choose tasks which are not cognitively demanding for low-attaining students with some task types and content in mathematics denied to such students. Some research has found that types of mathematics tasks such as problem solving and open-ended tasks (Anderson et al., 2004), and content areas such as algebra (Moses et al., 1990) have been withheld from low-attaining students.

Anderson, Sullivan and White‘s (2004) study of teachers’ beliefs about teaching problem solving found that teachers believed problem solving was “too hard” for lower streamed classes but that they would probably teach it more in higher streamed classes. One teacher reported that “she seldom uses problem solving approaches in her teaching since the students need drill and practice in basic skills and generally find problem solving difficult” (p. 44).

The *Quantitative Understanding: Amplifying Student Achievement and Reasoning’* project (QUASAR, Silver et al., 1995) aimed to allow all students at the poor, urban schools in the project to have access to quality, meaningful mathematics learning without being tracked out of this opportunity. These researchers held that the mathematics curriculum in QUASAR schools went beyond what they termed “the usual diet” of middle school mathematics, which was a focus on teacher-driven procedures and rote learning, and instead supported the goals of thinking, reasoning and problem solving.

Knapp et al. (1995) also sought to provide children from high poverty settings access to important mathematics. This was a two-year study of 140 classrooms ranging from first to sixth grade at fifteen elementary schools that had a large number of low-income families. Knapp et al. studied the effect a changed mathematics curriculum could have on these students. The former curriculum emphasised basic skills, tightly teacher controlled content, and inflexible ability grouping. In contrast to this, the researchers introduced a mathematics curriculum that emphasised teaching for meaning and understanding. For example, the curriculum made explicit links between content areas, concepts and the students’ lives. It was aimed to be academically challenging. There was little to no attention given specifically to basic skills. This study found that students improved their knowledge of advanced mathematics skills such as reasoning and problem solving. Furthermore their basic skills did not suffer. In fact they improved on testing more so than students in classrooms where teachers devoted most of the time to such skills.

The preceding discussion has explored the dissonance evident in the literature regarding appropriate ways to teach low-attaining students. The debate appears based around the view that low-attaining students have particular needs and characteristics that might demand particular ways of being taught mathematics in order for such students to succeed. I will now discuss the needs and characteristics of low-attaining students and explore in more detail the kinds of measures teachers, schools and educational researchers have advised for low-attaining students in mathematics.

LOW-ATTAINING MATHEMATICS STUDENTS

There are many studies in the literature that have sought to address the issue of effective teaching of students who are low attaining in mathematics. Some research has studied the characteristics of low-attaining students and their responses to learning mathematics which includes both their academic (Ball, 1993; Baxter et al., 2001; Baxter et al., 2002; Beswick et al., 2006; Lawson & Chinnappan, 1994) and affective responses (Ames, 1992; Middleton, 1995). Research has studied teachers' expectations of low-attaining students (Brophy, 1983; Good, 1981), their beliefs about appropriate content for such students (Anderson, 1997; Schwan Smith, 2000), and their views on learning and intelligence (Dweck, 1986; Zohar et al., 2001). Studies have also examined pedagogical issues such as the teaching approaches (Cobb et al., 1989; Kroesbergen et al., 2004; Lambdin, 2003), or the grouping arrangements (Boaler, 1997; Linchevski & Kutcher, 1998) schools make for students.

I begin by discussing the literature on some of the characteristics of low-attaining students; both cognitive and affective issues will be explored. I will then discuss the ways school systems have attempted to address the needs of low-attaining students.

LOW-ATTAINING STUDENTS AND COGNITIVE CHARACTERISTICS

As examined previously in this discussion, mathematics curriculum advice current at the time of this study emphasised tasks that prompt higher order thinking such as problem solving and communication. Some of the essential skills students need to thrive under such a curriculum are the ability to participate in and understand discussions, communicate thinking and strategies, work with others to solve problems, choose and use appropriate strategies, and be able to determine through self-monitoring and reflective thinking when such strategies are unsuccessful. For low-attaining students, issues such as difficulty in accessing appropriate prior knowledge, using inefficient or immature strategies, a lack of fluency with basic facts and a working memory deficit interfere with their ability to succeed in mathematics. I will now discuss these cognitive issues in more detail before moving on to a discussion of the affective issues.

Baxter et al. (2001) studied low achieving students across five third grade classes and examined their participation in whole class discussions. Baxter et al. found that the participation of these low achieving students was almost non-existent. In addition, the students were often off-task during discussions indicating that they were not participating as listeners either. Baxter et al. asserted that low-attaining students lacked the cognitive and social skills to participate effectively, as either speakers or listeners, in class discussions.

The study of Baxter et al. (2001) also reported on how low-attaining third grade students worked in pairs. They found that while the engagement of the low-attaining target students seemed high when working in pairs, an analysis of what low-attaining students were actually achieving found that the majority of these students were copying their partner's work and organising the materials. They engaged mainly in non-mathematical, low-level functional tasks and left the substantive mathematics to their more able partner. When the pair was made up of two low-attaining students, the time spent on the task was significantly reduced due to their poor organisation of the materials required to begin the task and their inability to recall instructions. This led to a marked reduction in time on-task for these low-attaining students and therefore limited their capacity to learn the mathematics from the task. This study will be examined more closely later in this chapter.

Another important factor in the performance of students during their work on problems is the ability to access appropriate knowledge to assist them in solving a problem. Lawson and Chinnappan (1994) found that high achieving students were able to recall knowledge that was relevant to the problem. Furthermore, they were able to freely recall such knowledge independently. They found that high achieving students had more and longer episodes of generative activity than low achieving students That is, for higher achieving students, one cue or piece of knowledge led to another then another, which led to an overall solution. In contrast, low-attaining students tended to have access to a smaller knowledge set initially but also were less effective at activating the knowledge that was available to them. Lawson and Chinnappan discussed the lack of organisation and connections between knowledge sets for low achieving students that made an effective search of available knowledge difficult for these students.

Prawat (1989) explored the literature on this issue of access to knowledge and found that although students may possess knowledge relevant to the task at hand, recognising they have it and activating its use, particularly in the case of low-attaining students, does not necessarily follow. Prawat asserted that it is a combination of organisation of knowledge and the reflective awareness of the student that influences a student's ability to effectively access knowledge.

Pegg and Graham (2007) pointed out that low-attaining students typically used "slow and effortful" (p. 2) strategies during mathematics. Gersten, Jordan and Flojo (2005) also described the "immature and inefficient" counting strategies which low-attaining students continued to use when it became inappropriate to do so. Baroody (2006) described three phases students progress through to master basic number combinations. These are "counting strategies, using fingers, blocks or other markers; reasoning strategies using known information and relationships; and mastery which is characterised by quick and accurate production of answers" (p. 22). Students considered low attaining in mathematics often fail to progress past phase one and continue to demonstrate "a heavy reliance on counting strategies" (p. 27). It has been proposed that this lack of automaticity with recall of basic facts overloads the working memory, slows down work on problems and therefore hinders low-attaining students in being able to complete problems that use higher order thinking (Gersten et al., 2005; Pegg & Graham, 2007).

The cognitive characteristics of low-attaining students described here have been focused on the deficits in the mathematical approaches of these students. However, Watson and De Geest (2005) examined the mathematical thinking possible for low-attaining students through the *Improving Attainment in Mathematics Project*, which was described earlier in this chapter. Importantly, Watson and De Geest warned that assuming a constant deficit model of low-attaining students did not recognise that "some potentially powerful mathematical talents of these students (could be) unrecognised and unused" (p. 20). In addition, Gervasoni (2004) found that, far from being a homogenous group with similar characteristics, low-attaining students represented a range of "diverse learning needs" (p. 253) and that teachers need to be aware that difficulty in one area of mathematics does not necessitate difficulty in other areas.

In a curriculum that emphasises higher order thinking and communication, some low-attaining students feel frustrated and anxious. This may be due to less teacher control and feeling overwhelmed by needing to construct their own strategies and solutions (Lubienski, 2000b). Apart from cognitive characteristics such as those discussed, low-attaining students, and indeed all students, have affective characteristics that impact on their learning. I will now discuss affective issues and the mathematics learning of low-attaining students.

LOW-ATTAINING STUDENTS AND AFFECTIVE ISSUES

As discussed in Chapter 1, motivation and engagement play a vital role in student achievement. It can be a challenge for low-attaining students to become engaged and maintain motivation for mathematics learning when they struggle to understand the content, and may feel inferior to their classmates. There is substantial evidence in the literature that a mathematics curriculum with less repetitious textbook work and with group or partner work and interesting problems to investigate, can be more motivating for students (Boaler, 2002; Ridlon, 2004; Silver et al., 1995). In contrast, other evidence points to an increase in confusion and anxiety for students, particularly low-attaining students, with such a curriculum (Baxter et al., 2001; Ellis, 2005). This may be due to a diminishment of the teachers' role as judge and knowledge-imparter, placing more responsibility on students to create knowledge. Solving problems and devising their own strategies can also be more threatening for students than completing a set of questions for which the procedure required is made explicit (Lubienski, 2000b).

The low-attaining students in the present study experienced particular types of mathematical tasks aimed at conceptual understanding. Their affective response to these tasks and their teachers' actions surrounding the task formed an important part of this research. Though academic issues such as those raised in the preceding discussion are clearly areas of concern, the literature also identifies the importance of motivation and engagement for effective learning of mathematics. In the following discussion I explore the literature regarding emotions, motivation and engagement and beliefs of students toward mathematics with a particular focus on research pertaining to low-attaining students.

There are various definitions of *affect* found in the literature but during this discussion, affect will include emotions, attitudes and beliefs (Gomez-Chacon, 2000) each of which will now be discussed. The role of affect in mathematics education has been widely researched. Indeed Maker (1982) insisted “it is impossible to separate the cognitive from affective domains in any activity … The most important thing is that there is a cognitive component to every affective objective and an affective component to every cognitive objective” (pp. 30-31).

Gomez-Chazon (2000) suggested that affect has an impact on “how pupils learn and use mathematics” and that affect can be an “obstacle … to effective learning” (p. 149). McLeod (1988) proposed that emotions such as frustration, panic, joy, and satisfaction have significant impact on students’ ability to solve mathematical problems. McLeod also suggested that emotions during mathematics problem solving could be characterised by four variables: the magnitude or intensity of positive or negative emotions; the duration of the emotion; the level of awareness students had of their emotions; and the level of control students had over their emotional states. For example, if a student had a strong negative emotional reaction to solving a problem that reached “panic level” (p. 138), that student would be likely to stop processing the problem and concentrate on controlling their emotions. At this stage, mathematical thinking is all but impossible. McLeod suggested that those students with persistent negative emotions about mathematics, including mathematics anxiety, were at greater risk of experiencing this type of “shut down” when solving mathematics problems.

Beswick, Watson and Brown (2006) described the variety of attitudes toward mathematics including “confidence or anxiety, liking or disliking mathematics, an inclination to engage in or avoid mathematics, beliefs about whether one is good or bad at mathematics, and beliefs that mathematics is important or unimportant, useful or useless, easy or difficult and interesting or uninteresting” (p. 68). The findings of Beswick et al. suggested that attitudes toward mathematics declined with grade level for most students.

Kloosterman, Raymond and Enemaker (1996) conducted a three-year study focused on elementary school students’ beliefs and feelings about mathematics. Twenty-nine students were interviewed each year regarding their feelings about mathematics, themselves as learners of mathematics and their enjoyment of mathematics. Student achievement was also studied through standardised and researcher-developed problem solving tests. The results of this study indicated that the students’ beliefs centred around four themes: views on the usefulness of mathematics; the value of group work versus individual work in mathematics; the relationship between confidence and ability; and the tendency to like mathematics more as it became more difficult. Students in younger grades had few consistencies between confidence and ability; that is, they were confident in their abilities regardless of their achievement levels. However, as students moved into older grades the link between their achievement and confidence became more pronounced with average and above-average students reporting high self-confidence but lower achievers having less confidence in their abilities. In fact these older students nearly always agreed with the researchers about their level of achievement indicating "by the end of second grade most students had formed an accurate picture of their performance in mathematics" (p. 51). These students' beliefs were relatively stable over the three-year period.

Walls (2007) offered a sobering view of ten students' views of "doing mathematics" over an eleven year period from 1996 to 2007. She found that almost every year these students reported that doing mathematics involved working alone at a desk from a book or worksheet. "For most of these children the isolation, tedium and inaccessibility of written mathematics tasks experienced on a daily basis … have been sufficiently off-putting to produce profound feelings of alienation and inadequacy" (p. 764). More encouraging were the findings of Franke and Carey (1997) who interviewed 36 first grade children from six Cognitively Guided Instruction (Carpenter et al., 1996) mathematics classes about their views of mathematics. Seventy-eight percent of these children reported that mathematics was solving problems, including word problems, sixty-four percent mentioned the use of manipulatives and thirty-nine percent said mathematics was about communicating different strategies to their classmates. "Our data suggests that within these classrooms that where problem solving is valued, … and where teachers have knowledge of children's mathematical thinking, children hold different perceptions about what it means to do mathematics from those traditionally held by students" (p. 23). This resonates with Doyle's (1988) assertion that the tasks assigned in mathematics form the students' view of the subject itself.

Boaler and Greeno's (2000) study revealed that another major influence on affect appeared to be the type of mathematics teaching students experience. This study found that student attitudes and views about themselves as mathematics learners arose from their experience of mathematics teaching in the classroom. Students who experienced "didactic teaching and received knowing" believed mathematics was "a series of procedures that needed to be learned" (p. 183) whereas students in "discussion-based teaching and connected knowing" classrooms "identified more positively with mathematics … because they were able to be thoughtful and to develop connected, relational understanding” (p. 188).

Ridlon (2004) studied 52 low-attaining sixth grade students and their responses to either textbook “explain-and-practice style” teaching (control group) or teaching through problems whilst working in small groups (experimental group). She found that the students’ attitudes were markedly different between the control and experimental groups when the nine-week intervention concluded. Students in the control group showed little difference in their attitude toward mathematics – if it was positive before the intervention it remained so, and the same was true if their initial attitude was negative. The majority of these students continued to describe mathematics as boring, repetitive and uninteresting. In contrast, the experimental group reported a change in their attitude with many of these students reporting a change from negative to positive attitudes. These students described doing mathematics as fun and enjoyable.

In a similar way, Boaler (1997) compared the effect of two different styles of teaching at secondary schools in the United Kingdom – one “traditional” (Amber Hill) and one “progressive” (Phoenix Park). Amber Hill was a school where teachers taught from the textbook, students were streamed, teachers taught in a transmission style and students were expected to work alone. The other school, Phoenix Park, was a progressive school. This school taught using open problems where students were in mixed ability groups, worked in small groups, where the teachers’ role was as a facilitator, and students were expected to construct their own learning. Boaler found that the students at Phoenix Park, when asked to describe their mathematics classes, used words like “noisy”, “a good atmosphere” and “interesting” (p. 50). In contrast, the students from Amber Hill said mathematics was “difficult”, “something related to their teacher”, and “boring” (p. 50). Boaler noted that students from this school “demonstrated a marked degree of disinterest, uninvolvement and boredom with their work” (p. 50).

It is important to note that this study found some mixed results in the engagement of the students at Phoenix Park. About one-fifth of the students studied reported that they did not like the openness of their mathematics classes or the freedom they were afforded. They reported preferring to work from textbooks. Boaler (1998) offered an explanation for this response when she identified that open tasks require more flexible, creative and deeper thinking which certainly entails more effort on the part of the student, and that some students preferred a more immediate answer or explanation. This resonates with the findings of other studies that found students themselves demand more direction from teachers and feel overly frustrated with having to investigate mathematics more independently. This is possibly more so with low-attaining students. Lubienski (2000b) for example, found the greatest resistance to her problem-centred approach to mathematics teaching came from the low socio-economic status (SES) seventh grade students in her class who found this style of teaching frustrating and intimidating. They wanted the teachers to tell them the answer and have a rule for them to follow rather than to work it out for themselves. Lubienski warned that "in contrast with the reformer's rhetoric of 'mathematical empowerment', some of my students reacted to the more open, challenging mathematics problems by becoming overly frustrated and feeling increasingly mathematically disempowered" (p. 476). As previously discussed, Robert (2002) also found her low-attaining students reacted negatively to her open approach to teaching mathematics.

The key for teachers is therefore to find a balanced combination of offering challenge without overwhelming low-attaining students, encouraging persistence without causing frustration, giving assistance without taking over and providing empowerment with support. Having low-attaining students feeling anxious and frustrated is obviously not the intention of the teacher, however this is not justification to abandon problem solving or open-ended tasks. As discussed previously, it is through success on challenging tasks, not simple tasks, that low-attaining students may begin to feel proud of their efforts and confident in their ability to succeed in the future (Pogrow, 1999).

As discussed, student affect can be influenced by factors such as the type of teaching and tasks experienced in the mathematics classroom. I will now examine how teachers and schools have attempted to cater for the academic needs of low-attaining students.

CATERING FOR THE EDUCATION OF LOW-ATTAINING STUDENTS

I have discussed some characteristics of low-attaining students that may inhibit their ability to operate successfully in mathematics classrooms that reflect current mathematics curriculum advice. I now discuss the literature on effective teaching of low-attaining students and examine the issue of withholding mathematical content or types of tasks from low-attaining students, which is also tied to the practice of streaming or tracking. I will then discuss the advice that direct instruction and strategy instruction are the most effective teaching approaches for such students. First, I will discuss the responses of schools to teaching low-attaining mathematics students in Australia around the time of the present study.

The Australian Government commissioned a major study into primary school students, commonly aged between five and twelve years, with learning difficulties in literacy and numeracy. This study, *Mapping the Territory* (Louden, Chan, Elkins, Greaves, & House, 2000), gathered data from 20 case study schools, several national surveys and reviewed relevant literature in an attempt to identify the programs and strategies Australian schools were implementing for low-attaining students.

This report found that while support structures for students experiencing difficulty with literacy were systematic and widespread, supports for numeracy were more piecemeal. To illustrate this point, the report has two chapters entitled *Policies and Practices: Students with Literacy Difficulties* and *Patterns of Support: What do Schools do to Support Children with Difficulties in Learning Literacy* (Louden et al., 2000). There is one chapter specifically regarding mathematics simply called *Numeracy* but no extensive discussion of policies or patterns of support are included. This is probably due to the fact that, at the time of gathering data in 1998, many schools indicated that they were moving toward a focus on numeracy but that policies and supports were not yet in place both in terms of funding and in practical resources, such as appropriate assessment tools or programs for intervention.

Results from a survey of 377 schools found that while 72 percent of schools assessed students in numeracy, only 14 percent had programs to support students found to be low attaining. These programs were most often reliant upon the support from staff with a special interest in mathematics who designed assessments, in some cases taught withdrawal groups of low-attaining students, ran mathematics lessons outside of school hours, mentored teachers in effective mathematics teaching or assisted teachers in developing Individual Learning Plans for low-attaining students. Some schools were able to fund this work, while others could not do so.

In her analysis of these results, Milton (2000) identified three key factors in the lack of identification of students having numeracy difficulties, especially in the later years at primary school. First, a lack of suitable assessment material for mathematics was one factor with most schools using teacher-made tests and assessments. Milton highlighted that the problems of relying on such testing were that such tests might identify students who had not learnt the mathematics content taught in the class but would be less helpful in diagnosing individual problem areas. Also teacher-made tests rely on the expertise and knowledge of the teacher regarding important concepts and possible misconceptions of the students across a wide range of mathematics topics. Results are also difficult to compare across classes and may not lead to the development of appropriate interventions for students having difficulty.

Secondly, many schools and teachers also identified numeracy problems as literacy problems. Milton reported that "several teachers in the case studies indicated that they believe there is a strong link between literacy and numeracy and by focussing on literacy they are also helping the child's numeracy" (2000, p. 117). However, Milton warned that "if numeracy is not seen as requiring separate and different skills to literacy, then potential numeracy difficulties will not be addressed" (p. 131). The third factor, which Milton found "worrying" in this case study, was an acceptance by many teachers that some children "cannot do maths" (p. 116). This normalisation of numeracy difficulties has serious consequences for low-attaining students of mathematics as it validates low expectations of such students, the problems of which I have discussed previously. Milton pointed out "some teachers may accept difficulties in numeracy that would not be accepted in literacy" (p. 118). She went on to warn that "if numeracy difficulties are accepted as normal in some children, then teachers may not take steps to identify where the difficulties lie or try to remediate them" (p.118). Milton advised that key elements in supporting low-attaining students in numeracy were funding for mathematics intervention programs or appropriately qualified mathematics specialists, appropriate assessment strategies and tools and more professional development for teachers in order to develop their understanding of numeracy and numeracy teaching.

As Milton (2000) alluded, and the discussion previously in this chapter highlighted, low expectations by teachers toward low-attaining students leads to a number of negative outcomes. An issue which lies at the heart of the equity debate is that some teachers and educational researchers believe withholding content is unjust while others assert that low-attaining students do not need to be taught more complex mathematical concepts when they require mastery of basic skills to help them cope with everyday life. As discussed, *The Algebra Project* (Moses et al., 1990), *QUASAR*

(Silver et al., 1995) and the study of Knapp et al. (1995) focussed on disadvantaged low-attaining students and asserted that denying these students important mathematics content and particular task types was unjust.

Sometimes the reasons for low-attaining students not being given the opportunity to access these areas of mathematics are due to a practice called "streaming", "setting" or "tracking". This is where students are divided into "same ability" classes for mathematics. The rationale is that by minimising the diversity of abilities in a class, each student's needs could be more easily met by the teacher. Higher track classes may be taught problem solving, algebra and reasoning, however lower tracks are often taught lower level skills of memorisation and practise of basic facts (Silver et al., 1995).

Streaming, setting or tracking

Schools engage in streaming, setting or tracking in an attempt to cater for differences in student learning needs. These terms refer to the practice of creating classes in which students are considered to be of similar ability or level so that there might be a high-ability, a middle ability, and a low ability class, for example. Often these classes remain in these "streams" for all subject areas (streaming). The rationale for streaming or tracking is that teachers are more able to teach to student needs if those needs are similar to the rest of the class rather than cater for the range of needs in a heterogenous classroom (Slavin, 1987). Streaming or tracking can be linked to a hierarchical view of learning (Zohar et al., 2001) which I discussed previously, where students are taught at the level teachers feel is appropriate. An entity view of intelligence (Dweck, 2000) also connects to ability grouping of students as this practice intimates that ability is innate and unlikely to change.

In a best-evidence synthesis of research on ability grouping in elementary schools, Slavin (1987) pointed out that between-class ability grouping had "few if any benefits for student achievement" (p. 293). Slavin described two main arguments against streaming. The first argument is that streaming creates classes of low-achieving students who are "deprived of the example and stimulation provided by high achievers" (p. 296), are labelled as a low group "which communicates low expectations … which may be self-fulfilling" (p. 296), and can suffer from "behavioural contagion" (Polansky, Lippitt, & Redl, 1950, p. 319), where time off-task is the norm (Felmlee & Eder, 1983). The second major reason Slavin offered for eliminating streaming is that it does not lead to improvements in student achievement. "Evidence from 17 comparisons in 13 matched equivalent and one randomized study clearly indicates that assigning students to self-contained classes according to general achievement or ability does not enhance student achievement in the elementary school" (p. 328). In addition, Slavin asserted that streaming is contrary to the values of equity and equal opportunities for all students regardless of cultural or economic backgrounds. With many low-ability classes dominated by particular cultural or social groups, high-ability classes can be considered as the "academic elite" (p. 297) and exacerbate inequality and "increase divisions among class, race and ethnic group lines" (p. 297). Supporting this view are LeTendre, Hofer and Shimizu (2003) who reported that tracking should be seen as undemocratic in the United States and "a major mechanism through which inequality of educational opportunity is transmitted and maintained" (p. 78).

The disadvantages of streaming or tracking, particularly for low-attaining students, are well documented. Boaler, Wiliam and Brown (2000) explored how streaming influences students' attitudes towards, and achievements in mathematics. In this study 943 students in Years 8 and 9 were surveyed, 72 students interviewed and 120 hours of classroom observations were analysed. Boaler et al. reported that students in the high streams or sets experienced "high expectations and high pressure" (p. 635) and were taught at a pace that was "incompatible for understanding" (p. 631). One student in a high set class remarked, "you don't even get time to think in the maths lessons" (p. 636). In an earlier study, Boaler (1997) found that students in top sets were more negative than students in other sets or mixed-ability classes. Forty-three percent of the top set students responded that they "never" or "not very often" enjoyed maths lessons. In addition, these students had the highest proportion of responses indicating that "remembering was more important than thinking" in mathematics. This is in contrast to other studies that have found that streaming or tracking advantages students in higher sets but have no benefits for students in lower groups (Linchevski & Kutcher, 1998).

For students in lower sets, Boaler et al. (2000) reported that these students were receiving "low expectations and limited opportunities" (p. 637). They warned that "the repercussions are … severe and damaging" for low set students. Low set students experienced "frequent change of teachers, the allocation of non-mathematics teachers to low sets and a continuous diet of low-level work that the students found too easy" (p. 637). In addition, this study found that teachers used a more restricted range of teaching approaches with homogenous classes than they would with heterogenous classes. In heterogenous classes, teachers tended to vary the pace, provide differentiated tasks and allowed students to work at their own pace. In contrast, homogenous classes were taught all at the same pace, given identical work and expected to finish tasks at the same time. Boaler et al. posited that in setted classes, teachers appeared to believe that students were not just similar, but "identical in terms of ability, preferred learning style and pace of working" (p. 640).

On an international level, results from studies such as the Second International Mathematics Study (Schleppenbach, Flevares, Sims, & Perry, 2007) showed that opportunity to learn, by experiencing a range of mathematics topics and concepts, and the degree to which students are taught in mixed ability classes were the two most important factors in determining the success of a country's mathematics education. The Australian National Numeracy Review (2008) noted that countries that do not engage in streaming or tracking of students in mathematics tend to perform better than countries that do.

The alternative to homogenous grouping arrangements, is inclusive models where classes are a mix of abilities. This also has its disadvantages. The demands for teachers teaching a group of students with vastly different achievement levels are a consideration when examining the merits of streaming or tracking. Ellis (2005) stated that "the inclusion of students with diverse educational needs in the regular classroom is proving to be an extremely difficult and complex task for many teachers" (Ellis, 2005, p. 2).

Possible disadvantages of mixed classes have been acknowledged by Mousley, Zevenbergen and Sullivan (2005). They warned that having students with different levels of achievement in the same class can actually emphasise their differences for the teacher and this can have negative effects. For example, high achieving students are given more challenging tasks and higher expectations are placed upon them whereas, as discussed previously, low-attaining students are singled out for lower-level tasks and given "repeated explanations and demonstrations – perhaps more loudly or slowly" (Mousley et al., 2005, p. 201). Furthermore, Diezmann, Faragher, Lowrie, Bicknell and Putt (2004) found that heterogeneous grouping might actually disadvantage high-attaining students who require a faster pace.

Inclusive models also have many benefits for low-attaining students. As discussed previously, Slavin (1987) found that discussion with peers takes on a new depth when there are students with differing levels of attainment present. Task persistence also seems more prevalent in mixed classes. This was also illustrated by an Israeli study which found that many students in an homogenous low-ability mathematics class failed to even attempt a relatively easy pencil-and-paper test whereas low-attaining students from the mixed ability class found the test easy to complete as they were used to more challenging tasks (Linchevski & Kutcher, 1998).

However, streaming or tracking itself may not be the problem. Diezmann et al. (2004) argued that grouping low-attaining students together and providing quality teaching and unrestricted content could be beneficial for low-attaining students. Gamoran (1993) reported on the success of some low-track classes where the teaching was high quality with well qualified teachers who had high expectations of their low-ability students. Unfortunately this is not often the case, as Linchevski and Kutcher (1998) reported that "low-ability setting leads to low-quality teaching… [which is] characterised by a low status, non-academic curriculum, valuable class time spent on managing student behaviour, most class time devoted to paperwork, drill and practice and low expectations" (p. 535).

Perhaps then, the issue lies more in the quality of teaching provided for low-attaining students, regardless of the existence or lack of streaming or tracking. As Diezmann et al. (2004) asserted "whether or not classes are streamed, if students experience unsupportive teachers, low expectations, disrupted lessons and a restricted curriculum, their learning environment would appear unlikely to yield high quality student outcomes" (p. 182). Other steps schools and education systems have taken to support the learning of low-attaining students in mathematics include the use of intervention programs in which students are withdrawn from their usual class for specialised assistance. I will now examine the literature on such programs.

Withdrawal intervention programs

Research on withdrawing low-attaining students from their regular mathematics class for further mathematics instruction, reports varied success. The interventions themselves can vary from whole groups of low-attaining students receiving extra mathematics lessons to further explanations either in a separate class or during in-class teacher “pull-out” groups. Intervention may also take the form of the students working separately from the rest of the class, either one-to-one or in small groups with a specially trained teacher.

Knapp et al. (1995) reported that in high poverty classrooms, students who did not keep up with the class in mathematics were often taught in supplementary instructional programs. They reported that these students did show gains in basic skills of mathematics but were “short-changed” when it came to higher order thinking skills. Pogrow (1999) described the teaching in withdrawal programs in a similar way indicating that “supplemental drill and content instruction” was the norm. The effect of such withdrawal programs on student outcomes was that, while there was evidence of individual improvement on the standardised testing, this gain was not sufficient to begin to “close the gap” between low-attaining and average attaining students’ achievement (Heid, 1991).

Dole (2003) reported that middle years students in withdrawal classes seemed to develop skills but did not experience the richer mathematical environment an inclusion model could provide. In her study of middle years classes, Dole compared an inclusive class with mixed abilities to withdrawal classes for students experiencing difficulty. The inclusive class worked on an open-ended task in groups of their own choice. The teacher stated that students having difficulty could work with her and some did. Concrete materials were made available for students to use as they wished. The class all worked on the same problem and discussed strategies and solutions at the conclusion of the lesson. Dole observed rich language, a community of learners and an emphasis on conceptual understanding. In the withdrawal classes, low-attaining students worked on basic facts often supported by individual computer work or sets of exercises worked on alone and at a pace set by the student. Dole reported that these withdrawal classes had tenuous or no links to real situations and students seemed to be engaged in lower level cognition. However, the students were happy and felt positive about their involvement in such classes. Dole concluded that the picture of mathematics in the middle years classes in this study was both alarming and positive. The withdrawal students were happy, experienced some success and showed progress in basic skills. It was concerning for Dole that these students had little experience in applying these skills in real situations. They were also denied the rich language and open-ended tasks found in the inclusive class. It seems then that withdrawal programs can be beneficial in terms of low-attaining students learning basic skills, but that students in withdrawal programs may not as successfully attain higher order thinking and problem solving skills.

As I have discussed, there are many teachers who would assert that learning these basic skills is essential for low-attaining students if they are to access higher order thinking or simply achieve a level suitable for everyday life (Anderson, 1997). Researchers have also opined that learning basic skills can free up the brain for more complex thinking. This is the premise of programs such the QuickSmart program (Graham, Bellert, Thomas, & Pegg, 2007).

Graham et al. (2007) developed the QuickSmart program in an attempt to address the needs of low attaining middle years students, mainly from Indigenous, non-English speaking, or rural backgrounds. These students were performing at or below Australian National Benchmarks for Literacy and Numeracy in Years 5 or 7. This program entailed two students working with a teacher or teacher aide using a specially constructed teaching program mainly using computer-based resources. The QuickSmart program was aimed at increasing fluency and automatic recall of basic skill information, strategy uses and incorporated timed strategic practice. Its major objective was to free up the working memory so that retrieval times for routine tasks could be shortened, thus leaving students more able to undertake higher order thinking or problem solving. Graham et al. reported that the QuickSmart program was "very much in tune with how many researchers consider students with LD (learning difficulties) can be most usefully supported" (p. 417), citing proponents of direct instruction and strategy instruction such as Ellis (2005). Ellis' work will be examined more closely in the subsequent parts of this discussion. The QuickSmart program reported some early success. For example, a school where 60 percent of Year 7 students were not meeting National Benchmarks found that after one year using the program, the entire cohort was performing above the benchmarks. In addition, data from interviews with parents and students in this program showed an increase in enjoyment, motivation and satisfaction in mathematics learning.

More recently, Bellert (2009) conducted a study of 12 students, six Year 5 and six Year 7 students who were identified as having learning difficulties in mathematics. These students participated in the QuickSmart program, which occurred three times a week for 20 to 22 weeks. Bellert compared the speed and accuracy of these students to eight average attaining students while answering "basic skills" questions. She found that the QuickSmart students did appear to be "closing the gap" between themselves and their average attaining peers on these tasks. The performance of the students with learning difficulties became more like the performance of the average attaining students in that retrieval times for the participants decreased and accuracy levels increased. Bellert warned "alarmingly, there are indications that current Australian school mathematics curricula, especially in the primary school years, advocate constructivist instructional approaches that have limited benefit for students with learning difficulties" (p. 172). Bellert concluded that the success of QuickSmart was due to "the superior effectiveness of cognitive strategy instruction and the combination of direct instruction and strategy instruction … that have informed the development of the QuickSmart intervention" (p. 172).

Though the students from Dole's (2003) study and those in QuickSmart (Pegg & Graham, 2007) reported higher engagement and confidence in their mathematical abilities, Sullivan et al. (2006) warned that withdrawing low-attaining students into small groups to work with the teacher may be unhelpful, as these students will not be experiencing the same activity as the rest of the class and are not benefiting from discussion with more able students. Withdrawal into small groups is also potentially embarrassing and may lower students' self esteem. Perhaps this depends, in part, on what happens in the withdrawal program. It makes sense that initially students may have reservations about being singled out, but if the program is successful, this may counter their negative feelings.

The Extending Mathematical Understanding withdrawal program (EMU, Gervasoni, 2002) involves children in their second or third year at school who are assessed via a clinical interview as being "vulnerable" to "poor developmental outcomes" in mathematics. This program focuses on building up rich understanding of "big ideas" in mathematics and, in contrast to programs such as QuickSmart (Graham et al., 2007), does not emphasise speed and accuracy with basic facts. A specialist trained EMU teacher teaches individual students or groups of three or four for 30 minutes every day for 10 to 20 weeks. The sessions focus on "10 minutes of activities focusing on counting and place value, 15 minutes of rich learning activities focusing on problem solving (often with an addition and subtraction, or multiplication and division focus), and 5 minutes reflection about the key aspects that were covered in the session" (Gervasoni, 2001, p. 268). Research on the results from EMU has reported significant gains in mathematical understanding for the low-attaining children on the program (Gervasoni, 2001).

Maths Recovery (Wright, 1994) is a withdrawal program that is similar to the popular and widely used Reading Recovery program (Clay, 1985). The organisation of Maths Recovery is similar to that of the EMU program and features "intensive, individualized teaching of low-attaining first-graders by specialist teachers for teaching cycles of up to 20 weeks; (b) an extensive professional development course to prepare specialist teachers, and on-going collegial and leader support for these teachers; (c) use and further development of a strong underpinning theory of young children's mathematical learning and (d) use of especially developed instructional activities and assessment procedures" (Wright, 1994, p. 31). The results of two cohorts of students that received Maths Recovery showed that these students progressed by two to three Stages in their learning of Number after participating in the program (Wright, Cowper, Stafford, Stanger, & Stewart, 1994). Wright et al. stated "we claim that the progress of the participants … is, on average, quite outstanding" (p. 713).

It appears there are some varied results reported in the literature about withdrawal interventions. The interventions described here that seem to be effective are for younger children, have specialised teachers and high expectations of the children to deal with mathematics that is challenging for them. In contrast, other withdrawal programs appeared to limit low-attaining students' progress in higher order thinking, limit their access to rich discussion with higher attaining classmates and focussed mainly on automaticity with basic facts.

I have examined the use of streaming or tracking and withdrawal intervention programs for low-attaining students. I now examine more general advice about teaching approaches for low-attaining students in mathematics.

A "BALANCED APPROACH"

The *Adding It Up* report (Kilpatrick, Swafford, & Findell, 2001) made recommendations on the kind of best practice "quality teaching" for mathematics. Though this report was not exclusively about low-attaining students, the recommendations would also apply to such students. The central recommendation was for teachers to take a more active instructional role in teaching mathematics. This active role meant that the teacher should directly instruct students with a focus toward a specific learning goal rather than solely using a more constructivist approach to learning where students constructed their own knowledge without explicit teacher intervention. Indeed, this report recommended that teachers use a variety of teaching approaches that would include explicit, focussed instruction on specific mathematics concepts, as well as open, problem solving tasks. Other advice was that teachers should be equipped with knowledge about students' misconceptions and plan to address these throughout the lesson. Anticipating students' possible responses allows teachers to be prepared to react to these by providing challenge or by scaffolding the understanding of low-attaining students.

Providing low-attaining students in mathematics and other curriculum areas with teacher aides or learning support assistants is also a common way of supporting low-attaining students within heterogenous classes, often without withdrawing these students from the classroom. However, recent research has called into question the effectiveness of this practice. Muijs and Reynolds (2003) studied primary school students aged 5 to 7 years who were low-attaining in mathematics. This study compared the effect of receiving support from trained teacher aides in mathematics for 180 students with 180 similar students who did not receive this support. This study concluded "pupils who had received … support did not make more progress in mathematics than those who had not" (p. 227). Muijs and Reynolds hypothesised that, more important than receiving teacher aide support, was the quality of the classroom teacher with highly effective teachers improving results for students of all attainment levels, with or without teacher aides. More recently the Deployment and Impact of Support Staff (DISS) project in the United Kingdom examined the effect of teacher aide (TA) support on 8,200 students across seven year groups "and found that those who received the most support from TAs made less progress than similar pupils with less TA support" (Webster, Blatchford, & Russell, 2010, p. 1). Webster et al. warned that the more support students received from teacher aides, the less involved the classroom teacher was in planning and implementing learning experiences for these low-attaining students. In effect, the teacher aides took over much of the instructional role from the teacher, yet of course had less training and education in pedagogical issues and mathematical knowledge than a qualified teacher. Interactions between teacher aides and pupils were found to be much less effective than interactions between teachers and pupils, though these interactions declined as teacher aide support increased.

Ellis (2005) examined the literature on effective teaching of students with learning difficulties in both reading and mathematics. She found that in examining meta-analyses of studies on low-attaining students, two approaches seemed to stand out. The results from eight substantial meta-analyses, across literacy and numeracy, found that using direct instruction and strategy instruction were the most effective approaches in teaching low-attaining students.

A meta-analysis of studies examined by Ellis (2005) was conducted by Baker, Gersten and Lee (2002) who reported on research regarding best practice specifically for teaching mathematics to low-attaining students. They analysed 15 studies that used an experimental or quasi-experimental design. Some elements analysed through these studies were student and teacher feedback, peer assisted learning, explicit teacher-led instruction, and concrete feedback to parents. Overall, Baker et al. found that low-attaining students benefited from regular and specific feedback on their performance and setting goals for future learning. Teachers also benefited from feedback about individual students, as well as whole class performance, although it was unclear whether providing instructional direction along with such feedback was significantly beneficial or not. Peer assisted learning had positive effects for the low-attaining students in these studies. Working with others increased their task persistence and also led to improvements in computation. Overall, the effect of explicit instruction in these 15 studies was positive. The conclusions of this extensive synthesis of research on low-attaining students found that four elements had the most positive effect on computational and general mathematics achievement for such students: feedback on performance to the student; peer or partner assisted learning; support and feedback provided to their parents; and the implementation of principles of direct instruction or explicit instruction.

On the basis of these studies and others, Ellis (2005) recommended a "balanced approach" when teaching students with learning difficulties. This resonates with the advice from Kilpatrick et al. (2001) that teachers use a variety of teaching approaches. Ellis warned that constructivist approaches to teaching alone are not adequate in teaching students with learning difficulties. Ellis suggested teaching that used some elements from a constructivist teaching approach but also drew on direct instruction and strategy instruction principles. Ellis pointed out that the teaching approach selected by the teacher should match the type of task that is presented to students and match student needs. Ellis noted that direct instruction lends itself more naturally to tasks that can be broken down into its requisite parts, those requiring a step-by-step process. Ellis recommended that teachers move toward direct instruction approaches when the content is new, when students are falling behind due to too little teacher direction, for students who learn more slowly than others, for students who are losing confidence or interest when working independently or for students with auditory learning styles. Teachers are advised to use more constructivist approaches when the concept or strategy can be learnt easily through student exploration, when the concept does not need to be learnt within a specific time, when the concept is not a prerequisite for other concepts students need to know about, for students who successfully learn independently, for students who lose interest by having to listen to explanations of concepts they have mastered, or for students with visual or tactile learning styles.

When examining Ellis' advice, it seems that direct instruction approaches are being targeted to low-attaining students for whom falling behind, losing confidence and learning more slowly than others could be seen as characteristic. Higher attaining students might be more likely to learn easily through exploration, are independent learners and have mastered concepts. Ellis does not specifically state that direct instruction should be used for low-attaining students in favour of more constructivist approaches. Rather she advises a balance of these approaches where the decision lies with the teacher who makes careful and considered choices about teaching approaches based on the task and student needs. However, as I have discussed earlier in this chapter, teachers often hold views about the nature of learning and intelligence that influence their perception of what low-attaining students need. Often this perception could lead teachers to adopt a more direct instruction approach as they would believe low-attaining students could not learn independently (Zohar et al., 2001), would become anxious without teacher direction (Schwan Smith, 2000) and would need concepts to be broken down into smaller, more manageable pieces (Anderson, 1997). Choosing constructivist teaching approaches, while holding such views, would be unlikely.

Woodward and Brown (2006) studied the effect of combining some elements of direct instruction whilst teaching higher order thinking to low-achieving students. This comparison study of two classes of low-achieving sixth grade students incorporated many of the principles of direct instruction, which this study termed "special education principles" such as distributed practice, visual models and aids, and high expectations. This yearlong study compared a class taught using National Council of Teachers of Mathematics standards-based mathematics curriculum materials that incorporated special education principles and a class using a similar curricular but without special education principles. This study was mainly a comparison of pedagogy rather than mathematics content or tasks. Both classes used similar mathematics programs that, according to Woodward and Brown emphasised conceptual understanding, problem solving and discussion. The difference was in the teacher's delivery of the program. The intervention teacher incorporated some special education principles described above such as distributed practice, visual models and aids, and high expectations. The comparison class' teacher did not explicitly do this.

Woodward and Brown (2006) analysed the students' results on standardised-testing, their own devised test and on an Attitude Measure. They found that intervention students had better achievement results and more positive attitudes than those in the comparison class. Woodward and Brown advised that a mathematics program with "a mix of relatively easy tasks, particularly distributed practice activities and more challenging application and problem-solving activities" (p. 158) may be the most effective way of teaching low-attaining students in mathematics. In other words, the kind of balanced approach recommended also by Ellis (2005).

I will now provide an examination of studies on direct and explicit instruction, and strategy and cognitive instruction, which Ellis (2005) recommended were most effective in teaching low-attaining students.

Direct, explicit and strategy instruction

Engelmann and Carnine's "Direct Instruction" (1982) was a specific educational model that Carnine and many other researchers proposed is an effective way to teach low-attaining students. It was usually recommended for teaching reading and mathematics. This teaching approach was composed of three main elements. First, there was a specific curriculum; second, a specific way of teaching; and third, teacher and student performance was carefully monitored. Engelmann and Carnine posited that all students could learn mathematics if the lessons are designed so that students can understand what is being presented, if there is adequate practice with "corrective feedback" and if there is regular assessment of progress. Direct Instruction was originally based on intensive small group teaching of about 30 minutes per student per day with the teacher or trained para-professional. During such lessons, the students were explicitly taught rules and procedures for solving problems. Each step and new skill was explicitly taught, and then the teacher assessed the students' understanding of this before moving on to the next step. This draws on the mastery principle where students need to have 85 to 95 percent accuracy before moving on to the next lesson. The students were not asked to infer a rule or devise their own strategies or make generalisations. Another principle of Direct Instruction was the use of advance organisers, when the teacher linked the current lesson to ideas or lessons the students have encountered in the past, outlines the objective for the lesson and gives a rationale for learning this skill. In addition, teacher modelling, guided and independent practice, and corrective feedback are key elements of Direct Instruction.

In many respects, this model is at odds with contemporary mathematics curriculum advice from around the world that emphasises students using their own strategies, forming their own hypotheses, and conjecturing and testing without being taught teacher-devised rules or procedures (Dindyal, 2006; National Curriculum Board, 2009; National Council of Teachers of Mathematics, 2000). It would seem then that Carnine and other proponents of Direct Instruction might have been advising a different method of teaching mathematics to low-attaining students than that which is found in the literature about "worthwhile tasks" (National Council of Teachers of Mathematics, 2000), which was explored earlier in this chapter.

Although Direct Instruction was devised about 30 years ago, it has continued to influence the teaching of low-attaining students with many researchers drawing on this research. Some of these studies have given rise to teaching approaches such as explicit instruction (Ellis, 2005; Kroesbergen et al., 2004). Others have incorporated some elements into mathematics teaching that is more in line with current curriculum advice such as problem solving and strategy training (Ellis, 2005; Woodward & Brown, 2006).

In the Netherlands, Kroesbergen et al., (2004) studied 265 low achieving students aged 8 to 11 years from both regular elementary schools and special schools. These students were taught in small groups using either explicit mathematics instruction (EI) or constructivist instruction (CI). Explicit instruction was a teaching approach where the teacher directly taught a strategy, often using manipulatives or visual aides. The teacher pointed out the benefits to students, guided them through practising using the strategy then allowed them some individual practice of the strategy. The constructivist teaching intervention began with a brief introduction by the teacher simply stating the focus of the lesson followed by a discussion with the students about this concept. Problems were given but no strategy was directly taught and students' own strategies became the main emphasis of discussion. Each group of students received 30 minutes of constructivist or explicit mathematics instruction twice weekly for five months. Pre and post-tests were given and results also compared to a control group who received neither treatment. The performance and motivation of the low achieving students in each group were part of the analysis.

Students in both of these interventions produced better results than similar ability students in the control group. This is not surprising given the small groups and intensive teaching these students received for five months. There were some interesting similarities and differences in the results of the two experimental groups. Students in both constructivist and explicit instruction groups improved in automaticity to almost the same degree. Kroesbergen et al. expressed some surprise at this result given that explicit instruction could be seen as relating more closely to automatic response. Explicit instruction (EI) was found to have a slightly more positive effect than constructivist instruction (CI) for problem solving strategies. This, asserted the researchers, is evidence that explicit instruction in strategy use was beneficial for low-attaining students in developing effective problem solving skills. However, they did note that constructivist teaching produced almost as positive results and certainly showed more progress than the control group. Kroesbergen et al. speculated that the difference might lie in the fact that, although students in CI experienced many different strategies, as these were student generated, they also encountered incorrect strategies. In comparison, the EI group only encountered correct and effective strategies, as these were teacher-led.

The teaching approach Ellis (2005) recommended as effective in teaching low-attaining students was termed "strategy instruction". Strategy instruction commonly occurred in the following phases:

- The teacher describes each step of the strategy, provides a rationale for the steps and explains how the steps are to be used to cue important thinking behaviours.
- The teacher models the strategy using think-aloud, dialectal and scaffolding techniques and then guides the students through a number of practice examples.

- The teacher gradually reduces their control and directs the students to think about when to use the strategy, how they can remember to use it and how to evaluate its success (p. 35).

Strategy instruction differs from direct instruction in that the focus is on generic or global strategies rather than the acquisition of specific skills. This approach was aimed at higher order thinking skills such as problem solving, evaluating strategies and generalising strategies to unfamiliar task. Tournaki's (2003) study compared the use of direct instruction, "drill and practice" with strategy instruction when teaching 42 second grade "general education" students and 42 learning disabled (LD) second grade students, one-digit addition facts. The strategy taught was "counting on" where the child starts with the first addend and counts on the second addend to reach an answer. This study compared the effect of drill and practice with strategy instruction and also compared the results of each intervention to a control group who received no treatment. Drill and practice and strategy instruction were taught to both LD students and general education students.

This study found that the LD students who received strategy instruction improved to the greatest degree from pre-tests to post-tests with the mean score of accuracy increasing from 59.01 percent to 96.16 percent. LD students receiving drill and practice showed much less improvement moving from 60.80 percent accuracy to 76.16 percent. However, LD students in both these groups improved significantly more than their peers in the control group who showed minimal improvement over the course of the study. For general education students without learning disabilities, both teaching approaches demonstrated similar significant improvement in accuracy. Students who had received strategy instruction, both LD and general education students, outperformed both groups of students who had received drill and practice instruction. It would seem therefore that strategy instruction is more effective, particularly when transferring knowledge to slightly different and difficult tasks. Tournaki stated "one can recommend that teachers dedicate time teaching strategies either to all (as all students appear to be benefiting from it) or just to the students with LD" (2003, p. 455).

Ellis (2005) categorised strategy instruction into cognitive, metacognitive or self-regulatory strategies. Evidence in the literature suggests that learning a combination of all these strategies is most beneficial to low-attaining students.

Montague, Applegate and Marquard (1993) studied the effect of teaching 72 low-attaining middle years students, aged 13 to 14 years, a series of cognitive and metacognitive strategies for mathematical problem solving. The cognitive strategies included reading and comprehending the problem, paraphrasing the problem, visualising the problem situation, planning solution paths, estimating an answer, computing the answer and checking its validity and accuracy against the original problem. The metacognitive strategies were "self-instruct" about possible strategies and their uses, "self-question" about the strategy chosen, and "self-monitor" about the success of the strategy both during and after solving the problem.

Montague et al. (1993) found that a combination of cognitive and metacognitive strategies was effective in improving results on one, two and three-step worded problems for these low-attaining students. The results after the 14-day intervention were that the low-attaining students' progress had improved so that they were now operating at a comparable level to their average ability peers. Maintenance of the strategies was an issue with results decreasing over a three to five week interval after the intervention. Montague provided a "booster" session to remind students of the strategies and this proved effective in maintaining the students' abilities to remember and utilise them.

The advice from the literature appears to be that constructivist teaching approaches alone are not effective for teaching low-attaining students (Bellert, 2009; Ellis, 2005). Strategy instruction has emerged from the literature as being beneficial for teaching low-attaining students (Montague et al., 1993; Tournaki, 2003). Woodward and Brown (2006) found teaching low-attaining students higher order thinking by incorporating elements of special education such as distributed practice, visual models and aids, and high expectations was successful. Kroesbergen et al. (2004) found explicit teaching with concrete materials and visual aides was also effective for teaching low-attaining students. The three task types under examination in this study were aligned with a constructivist view of learning. The literature examined would suggest that, in order for the use of such tasks to be effective with low-attaining students, other pedagogies such as strategy instruction, the use of visual or concrete aides or the kind of "special education principles" described by Woodward and Brown (2006) may need to be employed.

This discussion so far has found that streaming or tracking low-attaining students into homogenous groups appears to be largely unsuccessful and withdrawing middle years students for remedial mathematics teaching also has its problems. However, the low-attaining upper primary aged students in the present study experienced mathematics as part of heterogenous classes and were not part of withdrawal programs or streamed classes. Teachers need to know how to support the learning of low-attaining students within this mixed-ability setting. In addition, teachers are being advised to teach mathematics using conceptually challenging tasks as the best way to support the development of conceptual understanding in students. If such tasks are purported to be effective, as a matter of equity they must be used with all students, including those that are low attaining. "Scaffolding" describes a particular form of assistance teachers can offer that maintains the conceptual challenge of tasks while supporting the learning of students, including low-attaining students. In the next section, scaffolding will be examined regarding the history of practice, the debate about the term "scaffolding", features of effective scaffolding and studies of how teachers have used scaffolding in mathematics. The three scaffolding practices that were the focus of the present study will then be examined.

SCAFFOLDING

Scaffolding holds promise for assisting students, including low-attaining students, particularly when teaching and tasks are consistent with a constructivist philosophy of learning (Anghileri, 2006; Cobb, Yackel, & McClain, 2000). As the three task types under examination in this study were aligned with a constructivist view of learning, it was reasonable therefore to assume that an appropriate form of assistance teachers could give when using such tasks was scaffolding. The present study focused on teachers' scaffolding practices while using the task types and examined the effect that this had on low-attaining students' learning and affective responses. As scaffolding can take many forms, I chose to focus in particular on scaffolding through:

- the use of discussion (Hiebert & Wearne, 1993; O'Connor & Michaels, 1993);
- the use of manipulatives, representations and tools (Maccini & Ruhl, 2000; Sowell, 1989); and
- explicit attention to underlying mathematical concepts (Groves & Doig, 2002; Hiebert & Grouws, 2007).

These scaffolding practices form the conceptual frame through which the data from this study were viewed. In the following discussion, each of these practices is examined and their inclusion in this study justified. Initially, I discuss scaffolding and its origins as a metaphor for assistance provided to students. I then examine the various elements of scaffolding discussed and proposed in the literature. To conclude this section, I return to the three forms of scaffolding framing the present study.

Wood, Bruner and Ross (1976) were the first to describe the assistance teachers offer students as "scaffolding". They described scaffolding as a "process [that] enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts" (p. 90). Subsequently, Rosenshine and Meister (1992) defined scaffolding as

> forms of support provided by the teacher or another student to help students bridge the gap between their current abilities and the intended goal … Instead of providing explicit steps, one supports, or scaffolds, the student as they learn the skill. The support that scaffolds provide is both temporary and adjustable. (p. 26)

This expanded the definition of Wood et al. (1976) by explicitly including peers as potential providers of scaffolding. It also brought to light the idea of the "gap" between current knowledge and new knowledge. This led other researchers to turn to Vygotsky's (1978) work on the Zone of Proximal Development (ZPD) in an attempt to describe this gap (Cazden, 1979). ZPD is described as the "space within which learning can be expected under supportive conditions" (Hiebert & Grouws, 2007, p. 379). It is now often the case that a discussion of scaffolding in the literature draws links to ZPD as scaffolding becomes the "supportive conditions" under which learning can take place (Bruner, 1986; Wood, 1988).

The other important additions in Rosenshine and Meister's (1992) definition is that scaffolding does not involve the provision of "explicit steps." This alludes to the notion that scaffolding is not "showing and telling" but rather is "responsive guidance in developing pupils' thought constructions" (Anghileri, 2006, p. 33). In essence, scaffolding is aimed at assisting the students themselves to build understanding by bridging current understandings and new understandings. This is informed by a constructivist view of learning where "students actively construct meaning" (Cobb et al., 2000, p. 21). Scaffolding is suited therefore to a constructivist perspective on learning. Furthermore, Rosenshine and Meister stated that scaffolding was temporary.

This highlights an important element of scaffolding that can set it apart from other supports offered by teachers. Scaffolding should be withdrawn gradually as the student begins to progress leaving increasing responsibility with the student and reducing the role of the teacher.

The metaphor of scaffolding has not been without its critics. Stone (1993) questioned the utility of using the term “scaffolding”. He warned that the metaphor of scaffolding could be misleading in that it could place the child in a passive role as the “building” while the teacher is active in scaffolding this building. This also implies that the building is to be built in ways the teacher has pre-determined. In this model, scaffolds support the workers, which are the teachers, rather than the building, representing the students. Goos (2004) also asserted that scaffolding as a metaphor could imply a transmissive, rather than responsive, view of learning.

In contrast, Palincsar (1998) noted that scaffolding was an “accessible” term that could be easily understood by teachers and researchers. She argued that the term scaffolding was useful in that “it captures multiple dimensions reflective of teaching/learning processes, providing an instructional context that is at once supportive, flexible enough to accommodate individual differences among learners, and designed to cede increasing responsibility to the learner” (p. 373). However Palincsar also posited that the term scaffolding has become such a familiar term that “we have stripped from the word its subject and object” (p. 370). Myhill and Warren (2005) also warned that “like all words which suddenly gain a common currency in any sphere, the term ‘scaffolding’ is in danger of becoming a vague word for every activity engaged upon in the classroom (perhaps because most classroom activities are intended to support learning?)” (p. 57).

In the first instance, Wood, Bruner and Ross (1976) examined the tutoring process wherein the adult or “expert” assisted someone “less adult, less expert” to “solve a problem, carry out a task or achieve a goal” (p. 90). They claimed that scaffolding involved the adult “controlling those elements of the task that are initially beyond (the learner’s) unassisted efforts” (p. 90). To illustrate the scaffolding interaction between the expert and novice, Wood et al. described the individual tutoring of thirty 3, 4 and 5 year old children involved in constructing a three-dimensional pyramid using blocks. The tutor, Ross, followed a reasonably uniform procedure with each child beginning with verbal suggestions before moving to physical modelling, inviting imitation, depending on her perception of the child's needs. This study identified the following key elements of the tutor's scaffolding:

- *recruitment,* where the teacher involves the student in the task;
- *reduction of the degrees of freedom* through modification of the task;
- *marking critical features* by drawing the student's attention to important parts of the task; and
- *direction maintenance and frustration control,* which keeps the student on-task and monitors their emotional state.

This study found that the prevalence of each of these elements was dependent on the age of the children. For example, the 3-year-old children needed to be "lured" into the task *(recruitment)* and their interest and concentration maintained through a "deployment of zest and sympathy" (p. 98) *(direction maintenance and frustration control).* For the four-year-old children, *marking critical features* became more important as their involvement and persistence in the task was more assured. Wood et al. found that the 5-year-old children required little scaffolding on this task beyond the initial recruitment apart from checking the correctness of their actions with the tutor from time to time. Wood et al. found that each of these scaffolding practices was effective in the tutoring of these children, however the extent to which each was used was dependent upon the needs of the child. Wood et al offered no advice regarding the relative effectiveness of these features for this reason.

In contrast, Anghileri (2006) described a hierarchy of scaffolding in classrooms ranging from minimal teacher input to a highest level of scaffolding requiring significant teacher interaction with students. The first level, *environmental provisions,* is provided with minimal teacher interaction with students. These scaffolds are artefacts found in the learning environment that may assist students such as posters, displays, concrete materials, tasks or puzzles and mathematical tools. Such provisions also include organisational elements such as seating and grouping arrangements, and pacing and sequencing of events. The teachers' role at this level of scaffolding lies in the planning and organization of these environmental scaffolds, however the students interact with the artefacts themselves rather than directly with the teacher. At this level of scaffolding, the students learn relatively independently. By recognising environmental scaffolds, Anghileri differed from Wood el al.'s (1976) study in which the *interaction* between the tutor and tutee was the sole source of scaffolding. Wood et al. did not appear to regard the children's use of the blocks as providing scaffolding *per se,* but rather the actions of the tutor that surrounded the block problem.

Anghileri's (2006) second level of scaffolding parallels closely scaffolding as described by Wood et al. (1976). Anghileri characterised this by two distinct patterns of interaction. The first described the teacher as *elaborating, reviewing or restructuring* akin to *recruitment* and *marking critical features* (Wood et al., 1976). The second involved more traditional teacher interactions such as *showing and telling* and explanations. Anghileri argued that the first pattern of interaction was more effective as it built on the student understanding and was more cognisant of the contributions of the student to the discussion. In *reviewing*, for example, the teacher involves the student in *looking, touching and verbalising* the mathematics in which they are engaged. Often through such interactions, the student can self-correct their errors or recognise the way forward in the task without the teacher telling them how to proceed. The teacher's role is to analyse the student actions, use probing questions or model a similar task for the student. In contrast, Anghileri warned that more traditional interactions in which teachers *show and tell* or present are often one-sided explanations, and can increase confusion for the student if the explanation does not fit with the student's thinking. These interactions effectively shut down the student's input to the discussion as the teacher takes control of imparting knowledge or understanding while the student's more passive role is to listen.

The third level of scaffolding, *conceptual discourse* involves the teacher and student making connections between knowledge sets and understandings. The student is encouraged to make generalisations, propose conjectures and develop transferable skills. The teacher also scaffolds the making and use of representational tools such as images, words or symbols by notating a student's response, for example. While Wood et al. (1976) made no mention of representational tools, *marking critical features* aligns with this third level of scaffolding in that the teacher attempts to scaffold the important ideas of the task.

In a study of the barriers to mathematics learning, Sullivan et al. (2006) suggested that to cater more effectively for a range of abilities including low-attaining students, teachers needed to incorporate four elements in their mathematics planning and teaching – planning a set of tasks leading to a goal task, planning for scaffolding and enabling prompts, making classroom norms explicit and providing tasks to extend understanding. Here I will discuss scaffolding and enabling prompts but in later discussions, the recommendations of Sullivan et al. regarding making classroom norms explicit will be examined.

Planning scaffolding or enabling prompts assists those students who are not able to progress with the tasks. Sullivan et al. asserted that such prompts should be given only after all students have a chance to begin the task and perhaps struggle with it themselves. This *controlled floundering* (Pogrow, 1988) is an important step as it communicates high expectations and does not attempt to pre-judge students' performance on a task.

According to Sullivan et al. (2006) enabling prompts or scaffolding are not different tasks, re-explanations or withdrawal from the main group for time with the teacher. Such prompts should be hints about how students may proceed in the task. They may take the form of visual cues, adjusting numbers, providing materials or careful questioning from the teacher. The reasons for using enabling prompts are twofold. It respects the students' dignity and allows all the students to complete the same task. This is important as it builds a sense of a community of learners and also allows all students to participate in the discussion of the task.

There is evidence in the literature that supports the view that scaffolding can assist students in the mathematics classroom. However, there is also evidence that scaffolding presents a challenge for teachers. The difficulties lie in the complexity of effective scaffolding that requires strong teacher knowledge of both content and how students learn mathematics, and in the fact that such assistance is often a departure from more traditional teacher roles. Studies of teachers' use of scaffolding are discussed now.

TEACHERS' USE OF SCAFFOLDING

Bliss et al., (1996) study on scaffolding involved 13 teachers of students in Year 5 and 6 over the course of a school year in science, design technology and mathematics. The initial finding was that very little scaffolding was actually taking place during the lessons. Teachers were observed giving hints or clues, but their goals for the lesson were relatively obscure.

Upon further analysis by the researchers, this study became much more a study of the *absence* of scaffolding and identified four categories for episodes where scaffolding was absent. The first category described by Bliss et al. was when teachers' directive behaviour precluded the use of scaffolds. The teacher did all or most of the talking, explaining and showing, leaving no room for students to demonstrate their understanding or misunderstandings of the content. Conversely, the second category was when teachers handed over all the initiative for the task to the students, giving little or no direction about how to proceed. Thirdly, the teachers involved students in *pseudo-interactions or bypassing* where teachers and students interacted, perhaps in an attempt to provide scaffolding, but in fact the teacher misinterpreted or re-worded a student's response in such a way that it actually became the teachers' ideas. Also, interactions were observed where the student and teacher had different agendas that did not converge. Each party continued to follow their own line of thinking without regard to the thinking of the other. The final category was when scaffolding was required and the need obvious, yet the teacher did not notice or chose not to provide such scaffolding.

Where scaffolding did occur, this study described some instances as *unsuccessful scaffolding* when the student's response was unexpected as the student misinterpreted the scaffold when it did not connect with their thinking. In these cases, the teacher did not attempt to redress this misunderstanding perhaps, as suggested by the researchers, because they were unwilling to highlight the misconception or felt pressed for time. It is also possible that the teachers' lack of confidence also led to their reluctance to pursue a path seemingly unrelated to their initial goal. Other instances of scaffolding were described as *unintended scaffolding* where the teacher inadvertently provided scaffolding to students with both positive and negative outcomes. Teachers also engaged in a practice Bliss et al. described as *hints and slots scaffolds* where the teacher narrows the question down until there is only one answer possible to fill the slot in a statement. When *actual scaffolding* took place, this was usually emotional or affective feedback such as encouragement, or organisational prompts.

Bliss et al. (1996) suggested that scaffolding was difficult to implement for two main reasons. First, "much school knowledge initially exists as part of the teacher's (but not the pupils') knowledge. It is abstract, takes time to communicate and thus is hard to scaffold" (p. 58). Bliss et al. recognised that the kind of interaction essential for providing the kind of "dialogue and diagnosis" that characterises scaffolding requires interaction by the teacher with small numbers of pupils at a time which “slow down lessons and require time and patience” (p. 58). Second, teachers find this dialogue and diagnosis difficult if their own content knowledge is not sufficient to interpret a student’s thinking and know how to support the student to progress. The researchers posited that the teachers’ lack of subject knowledge and confidence in the subject was the main cause for this lack of focused scaffolding. Mathematics seemed the subject that most caused such discomfort for the teachers in this study. Teachers reported that they were able to plan for scaffolding during a lesson but implementing it was more difficult. Similarly, Verenika and Chinnapan (2006) found that pre-service teachers had difficulty in connecting the theory of scaffolding with implementing scaffolding in the classroom.

McCosker and Diezmann (2009) reported on a teacher’s attempts at scaffolding students while they worked on a mathematics investigation. They reported that ineffective scaffolding occurred as a result of four characteristics of the interactions between the teacher and students. The teacher did not “press” students for explanations of their thinking; students were not supported in understanding what the problem was about; the teacher offered praise and encouragement rather than supporting mathematical thinking or reasoning; and task expectations were not made clear or explicit. McCosker and Diezmann concluded, “at the core of judicious scaffolding is awareness of and responsiveness to students’ thinking” (p. 8).

Williams and Baxter (1996) found the teacher in their study used social scaffolding far more often than analytic scaffolding of mathematical concepts, as providing mathematical explanations was at odds with her view of effective “reform” mathematics teaching. As a teacher in a reform classroom, she felt it was the role of the task, the manipulative or other students to provide mathematical scaffolding rather than have her impose her knowledge on the students. This produced mixed results for students who sometimes felt frustrated and just wanted to know *what to do.* The students were sometimes incapable of providing analytical scaffolding for each other and so in some instances mathematics learning was restricted. This would be a major constraint for low-attaining students in such a classroom, as they may need more explicit scaffolding of the mathematics than can be provided by classmates or manipulatives (Ellis, 2005).

McClain and Cobb (2001) also found that the teacher in their study was uncomfortable with scaffolding mathematical understanding by drawing attention to the fact that strategies offered by the students could be more and less efficient. This teacher felt that this conflicted with her "non-impositional educational philosophy" (p. 238). However, over 12 weeks working with the researchers this teacher was able to develop sociomathematical norms with students such as mathematical difference, and sophisticated and efficient solutions in a way that "transcended this dichotomy" (p. 263).

Siemon and Virgona (2003) reported on teachers' use of scaffolding practices that mirrored mainly Anghileri's (2006) third level of scaffolding in which the teacher engaged in quality discussion and interaction with students in an attempt to probe and build understanding. The teachers in this study observed each other teaching using a Behind the Screen method where a teacher taught a group of children behind a two-way mirror while other teachers observed. They identified interaction patterns such as modelling, guiding, focussing, drawing attention to, reviewing and asking students to justify their answers or explanations. Teachers in this study were able to identify and label 12 scaffolding techniques evident in the observed teaching sessions. Over the course of the study, more of these scaffolding techniques became part of the teachers' interactions with the students. It seems that teachers were able to both recognise and use effective scaffolding techniques when engaged in professional development. Perhaps this intensive mentoring or peer-supported learning is the kind of assistance teachers require to adopt effective scaffolding and overcome the difficulties identified by Bliss et al. (1996).

McMahon (2000) examined the scaffolding that occurred during *Maths Recovery* (Wright, Martland, & Stafford, 2000) sessions, where a low-attaining mathematics student works individually with a specially trained teacher. She described the "micro adjustments" that took place during these sessions as the teachers "kept the students at the cutting edge of his/her competencies" (p. 423) These teachers were able to scaffold these low-attaining students through careful observation and maintaining the task's challenge while minimising frustration.

It seems, on examination of the literature, that many teachers are more comfortable with scaffolding the emotional states of their students through encouragement, non-specific praise and approval than scaffolding mathematical understandings. This is a traditional teacher practice and a familiar part of the teachers' role. The research on scaffolding within a whole class situation shows little success (Bliss et al., 1996; Williams & Baxter, 1996), although there is research on successful scaffolding during whole class discussions (Meyer & Turner, 2002;

O'Connor & Michaels, 1993). Descriptions of teachers effectively scaffolding classes through careful task choice and sequencing (Lampert, 2001; Simon, 1995) are useful, but these descriptions depict the mathematics classrooms of teacher/researchers with considerably more education in the teaching and learning of mathematics than most teachers. In many ways it is not surprising that scaffolding students within small groups or individually appears to be easier for teachers because the teacher can “micro adjust” (McMahon, 2000) to a smaller range of needs than exists in a whole class. It is possible for effective scaffolding to take place within a whole class but there is a need for more research in this area, to which the present study can contribute. The literature seems to reflect conflicting evidence that teachers are able to use scaffolding effectively to enhance learning. Perhaps providing scaffolding is a far more complex notion for teachers than is generally understood. The study of Bliss et al. (1996) certainly suggested that the teachers’ idea of what constitutes scaffolding differed from the descriptions in the literature. The key elements to successful use of scaffolding seem to be sound teacher content knowledge and knowledge of how students learn, while the main constraints seem to be directive or transmissional styles of teaching. The present study therefore has a contribution to make to the discussion on scaffolding through examining particular cases of teachers and low-attaining students.

As stated, scaffolding can be realised in the classroom in many forms. I have chosen to focus on three aspects of scaffolding that potentially support low-attaining students’ understanding of mathematics concepts; the use of discussion, the use of manipulative materials and visual representations and explicit attention paid to concepts. Table 1 provides a summary of the main studies on scaffolding examined in the previous discussion by noting the similarities and differences between the studies and the relationship between this previous research and the three scaffolding practices that are the focus of the present study. It also forms the basis of data analysis by providing possible codes for analysis, which will be discussed in Chapter 3. Table 1 further explains and justifies the inclusion of these particular forms of scaffolding. It also situates this research within the field of previous research on scaffolding. The table is organised chronologically, however it does not seek to provide an exhaustive overview of the literature on the development of scaffolding, but highlights those studies most pertinent to the present study. For example, there have been many studies on scaffolding involving children and their parents (Rogoff, Mistry, Goncu, & Mosier,

1993) or other areas of the curriculum such as literacy (Hobsbaum, Peters, & Sylva, 1996). These are not included in Table 1.

Table 1

Scaffolding literature and the framework of this study

	Context of study	Discussion	Manipulative materials and visual representations	Explicit attention to concepts	Other scaffolding practices
Wood, Bruner & Ross (1976)	3, 4 and 5 year old children with one researcher attempting to make a block pyramid. Individual sessions.	Recruitment where the teacher involves the student in the task. Reduction of the degrees of freedom through modification of the task, Marking critical feature by drawing the students' attention to important part of the task. Direction maintenance and frustration control that keeps the students on-task and monitors their emotional states.		Marking critical feature by drawing the students' attention to important part of the task.	Affective scaffolding. Maintenance and frustration control, which keeps the students, on-task and monitors their emotional states.
Williams & Baxter (1996)	Studies of reform classrooms in mathematics (QUASAR). Whole class.			Analytic scaffolding - Scaffolding the mathematical ideas of the task.	Social scaffolding - Classroom norms.
Bliss, Askew & Macrae (1996)	Studied the teachers' use of scaffolding in science, design and technology and mathematics in Year 5 and 6 classrooms. Whole class.	Hints and slots scaffolds.			Emotional and motivation scaffolding.
McMahon (2000)	Scaffolding by two teachers occurring during Mathematics Recovery sessions. Individual sessions.	Keeping students at the cutting edge of their abilities.			Micro adjusting to respond to the child's needs. Moment to moment adjustments.
Siemon & Virgona (2003)	"Researching numeracy" project Behind the Screen	Guiding, focussing, drawing attention to, reviewing and asking	Modelling.	Drawing attention to asking students to justify their answers or	

	activity- teachers watching each other scaffolding with small groups in mathematics. (primary aged) Small groups.	students to justify their answers or explanations.		explanations.	
Sullivan, Mousley & Zevenbergen (2006)	"Barriers to mathematical learning" project, middle years mathematics students. Whole class.	Scaffolding and enabling prompts.	Scaffolding and enabling prompts.		Planning a set of tasks leading to a goal task, making classroom norms explicit and providing tasks to extend understanding.
Anghileri (2006)	Coltman, Petyaeva and Anghileri (2002) study of young children (4 to 6 years) moving from free play to experimental play with 3D blocks Some with adult interaction, some without. Individual sessions.	Showing and telling- level two scaffolding. Elaborating, reviewing or restructuring	Environmental provisions – level one scaffolding Posters, displays, concrete materials, tasks or puzzles and mathematical tools. Such provisions also include organisational elements such as seating and grouping arrangements, and pacing and sequencing of events.	Conceptual discourse – level three scaffolding. The teacher and student talk to make connections between knowledge sets and understandings.	

Table 1 situated the three scaffolding practices of focus in the present study within the literature on scaffolding. To follow I examine each of these three practices in detail.

DISCUSSION

Teachers have been advised to encourage communication during mathematics classes in order to build a discourse community and enhance the understanding of mathematics for students (Silver & Smith, 1996). "Discussion" in the present study was defined as the conversations, discussions and talk that occurred between students and the teacher. Discussion between students was not examined as this study was concerned with teacher scaffolding and not potential scaffolding which students might provide for each other. I have chosen to use the term "discussion" to encompass both public, whole class discussions and more private conversations between the teacher and students. Discussion can provide scaffolding for students, and is one of the scaffolding practices under examination in this present study. The literature has examined how teachers use discussion and how students, including low-attaining students, have responded to such use.

Discussion is often the primary way that teachers scaffold students' learning. Teachers scaffold by talking to the whole class, and by students listening to their peers' responses. Teachers also scaffold pairs, groups or individuals through interacting during work on the assigned task. This occurs through questioning, making links to previous tasks or experiences, listening to student responses in order to determine their current level of understanding, having students articulate what they are thinking to pick up misconceptions, or for students to clarify their thoughts through putting them into words. Teachers organise and plan tasks that incorporate the opportunity for discussion such as allocating time for whole class discussions, directing students to work with others or encouraging students to explain and justify their thinking. Much of the research on scaffolding has focused entirely on discussion between teachers and students (Bliss et al., 1996; Siemon & Virgona, 2003; Wood et al., 1976). Other research, whilst examining other forms of scaffolding, was also concerned largely with what teachers say to scaffold student learning (Anghileri, 2006; Rosenshine & Meister, 1992; Tharpe & Gallimore, 1988). The scaffolding students potentially provide for each other through discussion has also been the focus of research on peer tutoring, and group work (Noddings, 1985). For the present study, this kind of discussion between students will not be examined, as the research question is concerned with the teacher's use of scaffolding practices and not potential scaffolding from other students.

Discussion in the classroom between the teacher and students generally takes two forms; whole class discussion or discussion with fewer participants such the teacher and small groups or just the teacher and one student. The literature regarding each of these forms of discussion is now explored.

Whole class discussion

Mathematics teaching advice for a number of years now has exhorted the importance of whole class discussion. In the United Kingdom for example, the national curricular documents have called for whole class interactive teaching (DfEE, 1998) in which teachers are encouraged to engage in more whole class teaching in which student contributions were to be "expected, encouraged and extended" (English, Hargreaves, & Hislam, 2002, p. 12). In the United States, the Professional Standards for Teaching Mathematics (National Council of Teachers of Mathematics, 1991) asserted, that "students' opportunities to learn mathematics are a function of … the kinds of tasks and discourse in which they participate" (p. 21). In Victoria, Australia, where the present study took place, teachers were advised to "engage students in discussion … throughout sessions to extend their thinking by building on their contributions and questions and to resolve misconceptions" (Department of Education and Early Childhood Development, 2010, p. 11).

The Victorian Department of Education and Early Childhood Development advised specific times for whole class teaching and suggested that teachers "use a range of flexible student groupings … including whole class focus, small groups, independent activities and whole class reflection" (Victorian Curriculum and Assessment Authority, 2006). This lesson structure of beginning and ending tasks or lessons with whole class discussions can be found throughout the literature. The purposes for these beginning and ending discussions have also been explored and I will now discuss each in turn.

Olson and Barrett (2004) suggested that whole class discussions at the beginning of the task should be a brief presentation of the problem of around five minutes where the teacher does not provide solution strategies for students. For example, Smith (2004) found that a Japanese teacher was effective in encouraging multiple student strategies at the beginning of a lesson by presenting and modelling the problem situation rather than offering a strategy. Erickson (1999) described the beginning of a problem-based mathematics lesson as the teacher "presents the task in the large group, models the level of thinking required and the expectations for the written work and the final product" (p. 518). Sullivan, Mousley, Zevenbergen and Turner Harrison (2003) suggested that teachers needed to establish classroom norms and explicitly state the ways students are expected to work and respond to the tasks. Therefore purposes for these beginning discussions could be to set up the task or problem, provide instructions for how students were to work, demonstrate or model an example, introduce the context, representations or materials for the task.

Stigler and Hiebert (1997) described end-of-task whole class discussions in Japanese mathematics classrooms where the purpose was for students to share their solutions, for the teacher to review or highlight aspects of student strategies or solutions and for the teacher to summarise the major point for the day. Cheeseman (2003) also described the various purposes for concluding discussions as "gathering evidence, summarising, reviewing the focus, sharing common discoveries, celebrating learning, learning from each other, encouraging students to reflect on what they had learned, extending thinking, and building positive attitudes" (p. 20). Cheeseman (2003) also reported that The Early Numeracy Research Project proposed "good lesson endings" should "focus on the mathematical learning and may explicitly address the 'big mathematical idea' of the session, develop with students a sense of completion; and are short and targeted" (p. 21).

The literature describes the various benefits of whole class discussion. For example, Pirie (1988) proposed that mathematical ideas and concepts are not only understood through experiences but also by interpreting and sharing them with others. Chapin, O'Connor and Anderson (2003) supported this view and suggested "putting thoughts into words pushes students to clarify their thinking" (p 22). Listening to the thoughts of other students is also potentially beneficial for a student's learning. In addition, Chapin et al. proposed that teachers could learn about their students' thinking more readily if such thinking is articulated. Yackel, Cobb and Wood (1991) asserted that learning is an inherently social activity and therefore discussion a necessary component of learning. Hiebert and Wearne (1993) claimed that “the cognitive and social act of expressing one’s thoughts (explaining, describing, questioning, etc.) may be a key in understanding how classroom discourse relates to learning” (p. 420).

Despite these potential benefits of whole class discussion, researchers have raised concerns that the level of dialogue required in some mathematics classrooms is confusing and difficult for low-attaining students. Baxter et al. (2001) studied low achieving students across five third grade classes. They found that “during our 34 observations we noted only three occasions when low achievers volunteered to speak during class discussions. Moreover when they did volunteer, they offered one-word answers or remained silent while a peer spoke” (p. 536). Baxter et al. noted that the low achievers were usually the audience rather than participants in class discussions and that “students who listen have the opportunity to compare their thinking with the comments of others. Both speaker and listener are potentially productive roles for students. Unfortunately, as listeners in class discussions, the target students were often “off-task”. “In every one of our observations we found patterns of non-engagement by the target students” (p. 537). Baxter et al. identified two main challenges for low-attaining students with classroom discussions. First, low-attaining students could not follow the arguments or strategies presented by fellow students and second, not all students could be represented in the discussion, which often concluded with more students wanting to speak than time allowed. Low-attaining students often limited their responses to short recounts of the arithmetic used in the solution to the task posed.

Baxter et al. (2002) offered some suggestions for how teachers could improve the participation of low-attaining students in discussions and also how to make such discussions beneficial to learning mathematics. One such suggestion is that the teacher provided some kind of summary statement after students have shared strategies, which highlighted two or three of these strategies. This might aide low-attaining students to recall the strategies discussed and to compare these. Another suggestion was that low-attaining students participated in some kind of mini-lesson after the main lesson concluded. During this time the teacher or teacher-aide could discuss with these students two of the strategies explained in the whole class discussion. The students could be guided to reflect on the similarities and differences of these strategies.

Ball (1993) described the complexity of effective whole class discussion when examining her own practice as a teacher/researcher. Ball illustrated the tension between respecting children's emerging mathematical thinking and her role as a teacher who is responsible for the children's learning of mathematics as a discipline. Though Ball spent about half the lesson engaging in whole class discussions with students, she admitted, "there are many days in which I ask myself whether this is time well spent" (p. 393). Though Ball believed that the children could learn valuable mathematics from each other, discussions could also be a "stimulus for confusion" (p. 394). Similarly Lampert (1990), also a teacher/researcher, described her role as teacher to "follow students' arguments as they wander around in various mathematical terrain and muster evidence as appropriate to support or challenge their assertions" (p. 41). Rittenhouse (1998) described the complexity of whole class discussions by examining Lampert's roles as the teacher in creating effective mathematical dialogue during whole class discussions as a "participant" and as a "commentator". Rittenhouse described Lampert "stepping in" to the discussion with the class as a participant by listening and asking questions. As a commentator, Rittenhouse asserted Lampert "stepped out" in order to provide instruction to students on the rules and norms of participating in mathematical discussions. It is reasonable to question how a teacher, without Ball or Lampert's experiences in additional study and research, could be expected to facilitate effective whole class discussions if these teacher/researchers admitted difficulty. Indeed Clarke (1997) wondered "whether the creation of a discourse community is a reasonable, attainable goal for most teachers" (p. 291).

A number of studies have explored the use of whole class discussion in the mathematics classroom. Hiebert and Wearne (1993) examined tasks and classroom discourse in six second-grade classrooms across nine lessons. Discourse was analysed regarding who was doing the talking, the teacher or the student, and the kinds of questions the teachers asked. These question types were categorised into four groups; *recall* questions requiring recall of facts, procedures or prior work; *describing* questions that asked students to describe a strategy or alternative strategy; *generating* questions that asked students to generate problems or stories; and questions to examine underlying features such as why a procedure works and how it differs from other strategies. Hiebert and Wearne found that the teachers did most of the talking in all six classrooms ranging from 77 percent to 91 percent of the total words spoken for the lessons during whole class discussions. Student responses were most frequently one to two words in length, however in two classrooms, responses of six words or more occurred 35 percent of the time. These longer responses were related to the kinds of questions these teachers asked. Teachers in the two classrooms where students gave longer responses "asked more questions that would require more elaborate verbalizations, either to describe solution strategies that had been used or to explain why they worked" (p. 410).

Other studies have found similar results with teacher dominance during whole class discussion leaving little room for student participation. Burns and Myhill (2004) reported from the TALK (Using Talk to Activate Learners' Knowledge) project that "not only did the teachers occupy far more of the utterance slots, but they also talked for longer in each utterance … an average four word utterance for pupils was obtained. It is clear, then, that the opportunity for interaction in terms of pupil response is being closed down by teacher dominance of the talk arena in whole class teaching" (p. 43). The TALK project (Myhill, 2006) also studied the participation of the children in Year 2 and Year 6 during 54 teaching episodes. The project examined which children were participating in discussions according to achievement level in mathematics and gender. They found that "high achievers are more involved and participatory in these teaching episodes than low achieving peers: … the low achievers are more likely to be engaged in more negative interactions such as being off-task" (p. 31). Myhill and Burns suggested that the "learning benefits" of whole class discussion was therefore different for different groups within the class in that "positive observable interaction behaviors, most likely to support learning, are engaged in most frequently by high achievers and girls, whereas being off-task is associated more strongly with underachievers and boys" (p. 33). Myhill and Burns concluded "the findings from this study suggest that, in general, teacher discourse in whole class teaching provided limited opportunities for pupil learning" (p. 24).

The ability to comprehend classroom discussions can be inhibited by cultural or socio-economic factors. Lubienski (2000a) found that the higher socio-economic seventh-grade students in her study were able to use discussion to clarify, question and potentially advance their mathematical understandings. In contrast, low SES students found such discussions only exacerbated their confusion and led to further feelings or disempowerment. These students in this study stated that teacher-directed discussion was easier for them to understand. Sullivan et al. (2003) also warned that students with a lack of familiarity with standard English were less likely to be able to "crack the code" of language used in whole class mathematics discussions. This literature would suggest that, as low-attaining students are often from disadvantaged groups such as different cultural or low SES backgrounds, their ability to both access and participate productively in whole class discussions could be compromised.

It has been advised that teachers need to scaffold discussions to ensure all students can follow the explanations of classmates and understand the main points of the discussion. Sullivan, Mousley and Zevenbergen (2004) advised the teacher record the main points of the discussion for display and rephrase the responses of students to enable low-attaining students to access and engage in the discussion.

Another factor in using whole class discussions with low-attaining students is their motivation to participate. This is particularly the case with older children and adolescents. "During adolescence, students are often reluctant to do anything that causes them to stand out from the group, and many middle-grades students are self-conscious and hesitant to expose their thinking to others. Peer pressure is powerful, and a desire to fit in is paramount" (National Council of Teachers of Mathematics, 2000, p. 268). Jansen (2006) studied 15 seventh grade students' responses to whole class discussions in two mathematics classrooms. She found that the students' motivational beliefs influenced their participation in whole class discussions. The teacher of each classroom in turn influenced the students' formation of such beliefs. For example, one teacher was seen by students as the "primary evaluator" and held relatively short discussions. Five out of the seven students studied in this class "voiced a belief in a high degree of risk associated with classroom discussions and in the importance of paying attention rather than participating in order to learn" (p. 424). The other teacher asked students to evaluate each other, allowed students to set the course of the discussions and spent longer on discussions. Five out of the eight students studied in this class "communicated the importance of participating in order to learn mathematics" (p. 425). However, it is not just the teacher who influences motivation to participate. Ridlon's (2001) case study of a seventh-grade student found that family and cultural background impacted on this student's participation in class discussions. The teacher's intention of student-led discussion and participation conflicted with this student's beliefs about the teacher as a figure of authority and his role as a student to accept that authority passively.

Apart from studying how much teachers talk during whole class discussions, other research has examined what it is that teachers say during such discussion. As discussed previously, Williams and Baxter (1996) analysed whole class discussions in one middle school mathematics teacher's classrooms and described two kinds of instructional scaffolding possible in such discussions, *social scaffolding* and *analytic scaffolding.* This built on the work of Edwards and Mercer (1987). Social scaffolding was described as the scaffolding of "norms for social behavior and expectations regarding discourse" (Williams & Baxter, 1996, p. 24). An example would be asking students for explanations or attempting to include as many students as possible in classroom discussions. Analytic scaffolding differed from social scaffolding in that it was primarily concerned with mathematics. Examples of analytic scaffolding might be the teacher re-describing student contributions to make the language more mathematically precise or highlighting contributions that have the potential to advance the mathematical understanding of the students.

Speer and Wagner (2009) examined the knowledge required by a teacher to provide analytic scaffolding during classroom discussions in undergraduate mathematics classes. Speer and Wagner found that "despite having extensive teaching experience and possessing strong content knowledge" (p. 530) the instructor in this study still found it difficult to "move whole class discussions toward a lesson's mathematical goals" (p. 530). Speer and Wagner suggested that there is a complex mix of knowledge required for effective analytic scaffolding to take place during whole class discussions involving content knowledge, pedagogical content knowledge and mathematical knowledge for teaching. In support of this view, McCrone (2005) asserted "developing rich and valuable mathematical discussions is a complex process that involves choice of tasks, nature of classroom environment, nature of questions, and growing communicative competence among all participants" (p. 131).

The literature does provide some examples of teachers using whole class discussions effectively in that such discussions allow students opportunities to encounter important mathematical ideas and potentially advance their understanding of mathematics. Manoucheri and Enderson (1999) found that a teacher of a heterogeneous seventh grade mathematics class was able to use whole class discussions in such a way that changed the roles of students and the teacher from those traditionally experienced in mathematics classes. The students engaged in arguing, conjecturing, proving and exploring important mathematical ideas with each other and largely without using the teacher as the conduit for their contributions. Mathematics "became a social activity, which allowed a shift from random individual attempts at problem solving to a systematic group inquiry" (p. 18). The teacher's role seemed diminished by these student-to-student interactions, however Manoucheri and Enderson asserted "a careful examination of the episodes highlights her crucial role in the flow of discourse in the classroom" (p. 18). The teacher in this study asked questions that encouraged students to elaborate their thinking, expanded students' explorations, made connections between students' strategies and conjectures, and invited multiple forms of representation to illustrate, prove or highlight strategies or discoveries.

Wood, Cobb and Yackel (1991) also described a teacher who became increasingly adept at orchestrating productive whole class discussions. This second grade teacher began the year-long teaching experiment experiencing "a conflict between letting children express their thinking or providing them with the official procedures" (p. 608), a similar conflict to that described by Ball (1993). However, the teacher's participation in a research project, which aimed at examining children's development of key mathematical ideas, highlighted the complex strategies children have for solving problems. This allowed the teacher to reconceptualise her role as a teacher from providing children with "official procedures" to one of recognizing and exploring the children's strategies as an important window into their emerging mathematical understandings. Wood (1999) described another teacher in this project who "created an environment in which children were expected to participate in the examination, critique, and validation of their mathematical knowledge through reasoned discourse" (p. 188). This second-grade teacher's thorough establishment of social norms for disagreeing along with the kind of careful questioning described by Manoucheri and Enderson (1999) allowed whole class discussion with these younger children to experience mathematics as a "social activity" and "group inquiry" just as the seventh grade students did in Manoucheri and Enderson's study.

In response to the difficulties teachers have with whole class discussion in the mathematics classroom, a number of studies have attempted to address the issue of how teachers might learn to facilitate effective discussion. McNair (2000) proposed that to maximise mathematical learning during discussions, teachers should ensure such discussion is focused on mathematical subjects and purposes and conducted within a "mathematical frame" which "involves a search for patterns and consistency, efforts to generalize and formalize procedures, efforts to make connections within the system, and to develop logical arguments that can be used to prove and to share the results of these efforts" (p. 201). Stein, Engle, Smith and Hughes (2008) proposed that in order to "use students' responses to instructional tasks in ways that advance the mathematical learning of the whole class" (p. 314), teachers could employ five key practices. These practices are described as "anticipating, monitoring, selecting, sequencing and making connections between student responses" (p. 314). As discussed previously, Baxter et al. (2002) suggested that the teacher provided some kind of summary statement after students have shared strategies, which highlighted two or three of these strategies. Similarly, Sherin (2002) described one teacher's "filtering" approach to student contributions as

> multiple ideas are solicited from students … Students are encouraged to elaborate their thinking, and to compare and evaluate their ideas with those that have already been suggested. Then, to bring the content to the fore, the teacher filters the ideas, focusing students' attention on a subset of the mathematical ideas that have been raised (p. 205).

Lampert (1990) also described how she wrote each student's strategy on the board with the student's name so that these could be remembered and discussed by the class. This kind of scaffolding may be an important factor in improving the effectiveness of whole class discussion for low-attaining students.

Other researchers have examined the use of *revoicing* by teachers as a way to scaffold learning through discussion. Revoicing occurs when the teacher reiterates a student's contribution, making subtle changes in language to make the utterance understood by the class, aligning and contrasting students' points of view or adding emphasis to highlight key mathematical ideas. O'Connor and Michaels (1993) asserted that revoicing allows teachers to "revisit and clarify the academic content" (p. 325) and "lends power and authority to the student's relatively weak voice (p. 327)". They also believed that through revoicing, discussion becomes more accessible to low-attaining students.

Evidence from the literature would seem to suggest that whole class discussions are difficult, and perhaps too difficult, for many teachers to implement effectively in order to advance the class' mathematical understanding (Ball, 1993; Clarke, 1997). In addition, low-attaining students often do not participate satisfactorily, can find whole class discussions confusing and therefore miss the benefits of discussion that may be afforded to higher attaining classmates (Baxter et al., 2002; Burns & Myhill, 2004). However, whole class discussion is only one form of discussion used in mathematics classrooms. Often more private, one-on-one or individual conversations take place between the teacher and students. I will now examine the literature on these individual discussions.

Individual discussions

Kyriacou and Issit (2008) described the purpose of teacher-pupil dialogue as "the teacher seeks to explore through a purposeful conversation with the pupil (or pupils) their understanding" (p. 3). Kyriacou and Issit's review of research concerning teacher-pupil dialogue for promoting conceptual understanding in mathematics revealed that the literature contained evidence that some teachers were using individual discussions with pupils during mathematics lessons to support students as they worked on problems. Kyriacou and Issit noted that "surprisingly little research is reported on the dialogue during such interactions" and that "this is a pity as more needs to be known about the characteristics of high quality dialogue during such private interactions" (p. 8).

Cheeseman (2009) examined the conversations four highly effective teachers of mathematics had with their young students. She reported that these teachers had the kind of purposeful conversations Kyriacou and Issit (2008) described with "a handful of students" each mathematics lesson. These conversations occurred in "interlinked strings" where the teacher periodically left the child to complete further thinking or tasks then returned to continue their mathematical conversation. These conversations were characterised by the teacher requiring the child to "demonstrate, model, explain, calculate, justify, generalise, transfer, connect, and describe their mathematical thinking" (p. 119). Elsewhere, Cheeseman and Clarke (2007) reported that young children were able to describe these conversations they had with their teachers during the lesson often with some detail. In fact, 43 percent of the 53 children aged five to seven years were able to describe the thinking they had done while they spoke to their teacher. Sullivan et al. (2006), reporting on upper primary school students, also commented that "one-on-one interactions between teachers and pupils … seem to be more personally significant for students than general interactions with the class as a whole" (p. 496). Clarke (2004) also noted positive student responses to their teachers' use of *kikan-shido*, "walking between the desks", that created opportunities for one-on-one exchanges.

Anghileri (2006) described conceptual discourse as when the teacher "goes beyond the explanations and justifications … by initiating reflective shifts such that what is said and done in action subsequently becomes an explicit topic of discussion" (p. 49). In this way, the teacher and student engage "in the communal act of making mathematical meanings" (p. 49). Conceptual discourse then seems to be characterised by the teacher listening carefully and observing closely what the student says and does in order to gain insight into their current understandings, described by McCosker and Diezmann (2009) as "awareness and responsiveness to the students' thinking" (p. 33). This also resonates with Cheeseman's (2009) description of the interactions between a highly effective teacher and young student as discussed above.

An important element of these individual discussions is the teacher listening. Davis (1997) offered three categories of teacher listening. The first Davis described as "evaluative" when the teacher is "listening *for* something in particular rather than *to* the speaker" (p. 359). The second type of listening, "interpretive", involves the teacher listening to students and interpreting what is said to understand more about student thinking. The third category of listening Davis described was "hermeneutic" listening. This type of listening involves the teacher in a more "negotiated and participatory manner of interacting with learners" (p. 369) where student responses are interpreted as well as acted upon. Empson and Jacobs (2008) also described three types of teacher listening. The first type, "directive listening" occurs when teachers listen to students "in a way that seeks to determine whether it matches an expected response and actively tries to elicit that expected response even when it is inconsistent with the child's understandings" (p. 257). This resonates with Davis' "evaluative" listening. The second type of listening was described as "observational" listening when teachers seek to uncover and understand a student's thinking but not to actively respond to such thinking, as is the case with Davis' "interpretive" listening. The third type of listening Empson and Jacobs described was "responsive listening". This type of listening involves the teacher in not only attending to a student's thinking, but responding in order to extend that thinking. This resonates with McCosker and Diezmann's (2009) description of scaffolding, described previously, suggesting that responsive listening is an important element of scaffolding. It also appears to align with Davis' "hermeneutic" listening. Empson and Jacobs pointed out that responsive listening is difficult to master and might take teachers years of listening to student thinking before responsive listening becomes part of their teaching repertoire. However, as this literature points out, the manner in which teachers listen is important as it influences the kind of responses students typically offer. Evaluative or directive listening encourages short responses giving the answer students think the teacher is seeking while responsive or hermeneutic listening encourages more extended and explanatory responses as the student strives to help the teacher understand their thinking.

As Kyriacou and Issit (2008) found, there is a paucity of literature regarding the individual discussions between teachers and students in mathematics. Often studies on classroom discussion may touch on these conversations, but they are rarely the focus of the study. For example, Bliss et al. (1996) recognised that the kind of "dialogue and diagnosis" (p. 50) necessary for effective scaffolding would require teachers to interact with small numbers of pupils at a time. As Kyriacou and Issit also asserted, I believe such conversations are a potentially rich and effective means for teachers to scaffold student learning without the potential pressure of a public discussion. It allows the teacher to individualise the discussion to meet the needs of a particular student. My study will add to the literature on individual or private mathematical conversations by providing descriptions from classrooms with a particular focus on the responses of low-attaining students to such scaffolding.

Drawing on the previous discussion, the main points for the use of discussions in mathematics classrooms are that

- Whole class discussions have benefits to students as articulating thinking and strategies supports understanding (Pirie, 1988), students have the opportunity to learn from each other, teachers can find out about students' learning and thinking through student explanations (Chapin et al., 2003), and learning can be considered a social activity (Wood, 1999).
- Whole class discussion have been found to be less effective for low-attaining students who have difficulty following the arguments and explanations of classmates, who rarely participate, including as listeners, and offer lower level responses when contributions are made (Baxter et al., 2002; Burns & Myhill, 2004). Whole class discussions appear to be more effective for higher attaining students and possibly girls (Burns & Myhill, 2004). Non-participation can be due to cultural issues (Ridlon, 2001), socio-economic background (Lubienski,

2000a), peer influence and the teacher's perceived role as primary evaluator where participation is risky (Jansen, 2006). Teachers often talk much more than students during discussions (Myhill, 2006).

- Whole class discussions are difficult for teachers to orchestrate well as teachers need to balance student contributions with the need to present mathematically important ideas (Ball, 1993), to encourage participation by as many students as possible (Baxter et al., 2002), and to ensure the discussion is sufficiently mathematical (McCrone, 2005). This requires a complex mix of knowledge (Speer & Wagner, 2009) and a reconceptualisation of the role of the teacher and students (Manoucheri & Enderson, 1999; Wood, 1999).
- The literature describes strategies for teachers to improve the productivity of whole class discussions. Stein, Engle, Smith and Hughes (2008) suggested teachers engage in "anticipating, monitoring, selecting, sequencing, and making connections between student responses" (p. 314) in order to improve whole class discussions. Sherin (2002) offered "filtering" as a way that teachers could make connections between student contributions and mathematical ideas. O'Connor and Michaels (1993) described how "revoicing" by teachers could help students to understand each other's contributions. Recording the main points of the discussion for students to refer to was also advised (Lampert, 1990; Sullivan et al., 2004).
- Discussions between teachers and individual students have been under-researched (Kyriacou & Issitt, 2008), but have been found to be important to students affectively (Sullivan et al., 2006) as well as having potential for teachers to monitor and develop understanding (Anghileri, 2006; Cheeseman & Clarke, 2007).

I will now re-focus my discussion of scaffolding to examine the use of manipulative material and visual representations and tools, a form of scaffolding that does not rely solely on linguistic structures but draws on physical or visual aides.

MANIPULATIVE MATERIALS AND VISUAL REPRESENTATIONS

There are various definitions in the literature of manipulatives and visual representations. Some refer to manipulatives as concrete materials in that they have a "sensory nature" so that they can be physically touched and manipulated (Clements, 1999). Boulton-Lewis and Halford (1992) described such materials as representations but made the distinction between representations that are “concrete embodiments of mathematical concepts and processes” and those that are “inherent in the discipline of mathematics (e.g. number lines and symbols)” (pp. 1-2). Goldin and Shteingold (2001) refer to such materials as external representations with internal representations being mental models. Goldin and Shteingold differentiate between types of external representations in a similar way to Boulton-Lewis and Halford who stated,

> Some external systems of representation are mainly notational and formal. These include our system of numeration; our ways of writing and manipulating algebraic expressions…. Other external systems are designed to exhibit relationships visually or spatially, such as number lines, graphs. Words, sentences, written or spoken are also external representations (p. 4).

Sowell (1989) used the terms “pictorial” if “students watched animated audiovisual presentations, observed demonstrations with concrete materials, or used pictures in printed materials” and “concrete” where “students worked directly with materials such as beansticks, Cuisenaire rods, geoboards, paper folding or other manipulative materials” (p. 499). Perhaps the key defining element of manipulative materials and visual representations is “that it can stand for (symbolise, depict, encode or represent) something other than itself” (Goldin & Shteingold, 2001, p. 3).

For the present study, I have chosen to describe representations as manipulative materials for those materials students can explore primarily through the sense of touch. Examples of manipulative materials might be counters, Cuisenaire rods, or Multibase Arithmetic Blocks (MAB). I have also chosen to differentiate between manipulative materials and visual representations where the students primarily use sight to explore and interact with the representation. Examples of visual representations might be written explanations, pictures or diagrams, graphs or charts. Such representations rely on students looking at them to engage with the mathematical idea they are intended to portray. These visual representations include the “notational and formal” (Goldin & Shteingold, 2001) aspects to mathematics such as numbers, symbols and expressions that are written or drawn. Defining representations as manipulative materials or “things students can touch” and visual representations, “things students can look at” would seem to encompass many of the definitions found in the literature and reflect the two ways representations are customarily used in mathematics classrooms.

I will now discuss the literature regarding the use of manipulative materials and visual representations in mathematics. With my research questions in mind, I will focus on such use with low-attaining students, and issues with teacher use of manipulative materials and visual representations as effective tools for scaffolding student understanding of mathematical concepts and processes.

The use of manipulative materials and visual representations is frequently discussed in studies of teaching low-attaining students (Butler et al., 2003; Cass et al., 2003; Sowell, 1989). This is due primarily to the belief that students often need to move from concrete to abstract when learning mathematical ideas. This idea that students need to have experiences with concrete materials before moving on to more abstract concepts emerged from researchers such as Piaget (1952) and Bruner (1960). For example, Bruner asserted that concrete materials are important because to internalise a concept, firstly students need to act on concrete objects, then form images of concrete situations and finally adopt symbolic representations. This may also explain why some teachers believe the use of concrete materials is more appropriate for younger students rather than older students (Howard & Perry, 1997). However, some research has questioned the sequence from concrete to abstract. Lesser and Tchoshanov (2005) for example found that students develop understanding by being introduced to a concept in abstract form before experiencing a concrete representation. It is also thought that students may need physical embodiments of mathematical ideas (Stacey, Helme, Archer, & Condon, 2001) and that such physical or visual tools can potentially become mental models for students (Goldin, 1987).

Cass et al. (2003) studied the effect of using manipulative materials when teaching three low-attaining adolescent students concepts in area and perimeter. Geoboards were used as a visual aid to link concrete examples of perimeter and area with abstract ideas contained in written questions. Cass et al. reported a rapid improvement in these three students' understanding of perimeter and area as a result of these sessions using Geoboards. Furthermore, they were able to solve abstract written perimeter and area problems correctly after the intervention.

Butler et al. (2003) studied the use of manipulative materials and visual representations with a larger sample. This study examined the development of fraction concepts in 50 adolescent students. These students were in four mathematics classes and taught as a whole by a special education trained teacher. Two classes were taught using the Concrete-Representational-Abstract method using fraction circles, beans and student-made fraction squares. The other two classes used the Representational-Abstract method that used drawing of pictorial representations to support learning. The differences in the results from the two intervention strategies were minimal but the Concrete-Representational-Abstract group performed slightly better than the Representational-Abstract group on all the subtests of fractions. However, the findings of this study were that the low-attaining students in both interventions obtained results on a Fractions Mastery test similar to a comparison group of mathematics students of average attainment levels. This suggests that this study adds to Cass et al's (2003) findings in that even the use of visual representations, without manipulative materials, had positive effects for low-attaining students.

Maccini and Ruhl (2000) studied the use of a graduated instructional approach that moved from concrete aides to representations of these aides and finally abstract representations in teaching of three learning-disabled secondary students. This study differed from Cass et al. (2003) and Butler et al. (2003) in that it combined strategy instruction with a graduated instructional approach when teaching algebra concepts. This study found that the students increased their accuracy in representing the problem using the concrete materials and drawing the materials. This also translated into a higher percent of accuracy in solutions to these problems. The small sample size for this study makes these findings difficult to generalise however the study suggests that combining strategy instruction and concrete aides could be effective for teaching students with mathematics learning difficulties.

There is a note of caution in the literature that using manipulative materials or visual representations does not inevitably lead to improved understanding for students. Such materials can prove to be a distraction from the task, particularly perhaps for low-attaining students who can spend their time organising and rearranging the materials, that is, undertake "non-mathematical subtasks", without actually engaging in mathematics learning (Baxter et al., 2001). Hiebert and Wearne (1992) found that the materials themselves could actually hinder rather than assist in learning mathematical ideas as the children engage in the problem of how to manipulate the materials rather than the mathematical problem. For example, in their study, some children were confused about the use of bean sticks of ten beans and single beans and used single beans rather than the bean sticks to represent ten.

Boulton-Lewis and Halford (1992) also contended that using concrete representations could increase the cognitive load required of children. Their study, of 29 children in Years 1, 2 and 3 during lessons on place value, found that “some representations … impose an unnecessary processing load which can interfere with conceptual learning” (p. 1). Bolton-Lewis and Halford argued that the processing load for children increases if “children are encouraged to use materials that they do not know well … (and) if teachers do not succeed in matching their representations and strategies as closely as possible to those that children are using” (pp. 20-21).

Ball (1992) pointed out that in many cases, the student needs to have prior knowledge of the concept in order to see how the manipulative relates to it. Teachers have knowledge of place value for example, so they can appreciate how an abacus might illustrate place value ideas. However, Ball stated that for students who have not formed a clear understanding of place value, an abacus is of little use and may in fact, add to their confusion. Similarly, Holt (1982), when discussing the use of Cuisenaire rods, wondered “if we hadn’t known how numbers behaved, would looking at the rods enabled us to find out?” (p. 139). This suggests that teachers should be mindful that manipulative materials may not make sense to the students with their developing understanding even if, for an adult, the relationships are clear. Stacey et al. (2001) studied this idea and described this as the *transparency* of the manipulative, that is the extent to which the manipulative allowed students to “see through it” to the underlying mathematical idea, and the extent to which understanding the manipulative itself limited the students’ access to the mathematics. For example, they found that using Multibase Arithmetic Blocks (MAB) to illustrate concepts about decimal numbers caused confusion for the Year 5 and 7 students in their study. Stacey et al. hypothesised that the volume relationships in MAB and its prior use to model whole numbers caused confusion for the students. In contrast, the Linear Arithmetic Blocks (LAB) appeared to support the students’ developing understanding of decimal numbers perhaps due to LAB being a length, rather than volume model and that LAB seemed more able to represent ideas such as density of decimal numbers.

These findings suggest that a key component of effective use of manipulative materials and visual representations is careful selection of such materials by the teacher. Lesser and Tchoshanov (2005) suggested that teachers need to choose more than one form of representation to illustrate a mathematical concept. In their study, Lesser and Tchoshanov found that high school students who used one form of representation to explore geometry concepts became "stuck" on this form whereas students using multiple forms of representation appeared to show more flexibility in their strategies. Hiebert and Wearne (1992) used multiple forms of manipulative materials and visual representations to teach first-grade students about place value. They reported that teachers who aimed to teach for understanding helped students to make connections and consider the relationship between the various representations because "it can be argued that building connections between external representations supports more coherent and useful internal representations" (p. 99).

In contrast, Boulton-Lewis and Halford (1992) warned that using multiple representations for place value with young children in Years 1 and 2 particularly, could increase the child's processing load so that learning of mathematics was hindered. They advised that although "it is probably important for motivational reasons to allow children to use a wide range of materials to represent numbers … they should be encouraged to use a particular representation of sets of ten and units regularly so that, with practice, that representation becomes well mapped in to the place value of numbers" (p. 20). Boulton-Lewis and Halford argued that with this more automatic "mapping" from the representation to the concept, children could then use the representation without increasing their processing load.

After teachers choose the materials and representations to use during mathematics lessons, the next crucial component is how such materials are used. As Clements and McMillen (1996) pointed out, manipulatives can be used in a rote manner. Many years earlier, Fennema (1972) found manipulatives used in a rote manner had detrimental effects on student learning and, in such cases, groups of students who used no manipulatives demonstrated a higher capability for transferring understanding of mathematical concepts than students who had used manipulatives. Cohen (1990) vividly described the rote use of manipulatives by one teacher who believed she was teaching for understanding in line with reform mathematics teaching. This teacher appeared to think that simply by using materials the children would understand the mathematics. "The ways that she used these materials – insisting, for instance, that all the children actually feel them, and perform the same prescribed physical operations with them suggested that she endowed the materials with enormous, even magical instructional powers" (p. 318). In a similar vein, Ball (1992) remarked, "although kinaesthetic experiences can enhance perception and thinking, understanding does not travel through the fingertips and up the arm" (p. 47).

Ambrose (2002) suggested that it was not only teachers but also students who could develop the belief that using materials ensured, almost magically, that mathematics answers would be correct. In Ambrose's study, she described the actions of one girl who developed an over-reliance on using manipulative materials at the expense of developing mental strategies. "Her modelling strategies were so ingrained that she used them most of the time … Her mathematical thinking did not progress as expected" (p. 18). Fennema, Carpenter, Jacobs, Franke and Levi (1998) supported this finding with their earlier longitudinal study of first, second and third grade students. This study found that far more girls than boys used concrete strategies for solving multi-digit addition and subtraction problems and that more boys than girls then progressed to using invented abstract strategies. Most of the girls using concrete strategies for a longer period of time then used the standard algorithm even though invented mental strategies were encouraged in their classrooms. Ambrose proposed that the problem with using only concrete strategies and then the standard algorithm is that students could operate on "automatic pilot" as both these methods can be used without reflection or understanding of underlying concepts. Again this was illustrated by the study of Fennema et al.(1998) which found that students using the standard algorithm were largely unsuccessful when asked extension problems. Ambrose advised the strategic withdrawal of manipulative materials so that mental strategies could be encouraged and developed.

Teachers' beliefs about the use of manipulative materials also effects how they are used the classroom. Moyer's (2001) study of ten middle school mathematics teachers found that some teachers used manipulatives as a "change of pace" or a "break in the routine of mathematics", to "make it more fun" (p. 185). Others viewed manipulative use as a reward or a privilege and would remove manipulatives from students if student behaviour was deemed inappropriate. Other teachers used manipulatives to provide a visual model, or to reinforce and provide enrichment for concepts. Some of these teachers also expressed concerns about maintaining control while using manipulatives. Moyer suggested, "using manipulatives deviates from the US teaching script, perhaps so much so that it incites student discussion and interaction with peers and materials, causing a disruptive environment as opposed to one in which students are working with paper-and-pencil problems in an orderly fashion" (p. 180). Moyer found two distinctions about mathematics arose from interviews with these teachers, "fun math and real math". Fun math was more often associated with manipulative use while real math was rules, procedures and algorithms. Moyer asserted that this "seems to be making several powerful statements about the manipulatives: they are not essential to real math instruction; they mean playing and not working; and they do not lend themselves to being part of the structure of a mathematics class" (p. 188).

Beswick (2005a) studied 22 teachers ranging from the early years of schooling to middle school, Years 7 and 8, regarding their beliefs about their students before and after a professional learning experience. These teachers were asked to respond to statements about teaching mathematics and the goals of mathematics on two Likert scales, one for all students and one for "students with mathematics learning difficulties" (MLD). Before the professional learning, the teachers were less inclined to believe that conceptual understanding was an appropriate goal for students having difficulty and agreed with the statement that concrete materials were for "supporting answer getting rather than the development of understanding for these students" (p. 142). After the professional learning experience the teachers were "more inclined … to reject the notion that students with MLD should use concrete materials as a substitute for thinking to get answers" (p. 143). For the teachers in this study, the professional learning they engaged in began to change the way they viewed the use of manipulative materials with low-attaining students.

Howard and Perry (1997) found that it was not only the students' level of attainment that influenced their teachers' use of manipulative materials, but also the age of the student. Howard and Perry studied 249 primary school teachers and found that teachers decreased their use of manipulative materials as children progressed through the primary school. This meant that by upper primary most students were experiencing manipulative materials in mathematics only occasionally. More recently, Swan and Marshall (2010) conducted a study that "mirrored" that of Howard and Perry (1997) by surveying 849 primary school and middle school teachers about their use of manipulatives in mathematics. They found that the results also mirrored Howard and Perry's with manipulative use decreasing as grade levels rose with 82.6 percent of Prep teachers using manipulatives daily compared to 9.1 percent of Grade six teachers using manipulatives daily. In addition, Howard and Perry's finding that many upper primary students had the perception that using manipulatives is "babyish" was also a comment made by many teachers in Swan and Marshall's study.

Sowell's (1989) analysis of 60 studies on the use of manipulative materials found that materials that were used consistently for a school year or longer "gave positive effects of moderate to large size in elementary grade studies" (p. 504). In addition these positive effects encompassed a variety of mathematics topics and concepts. Another important finding from this analysis was that the positive effect of using manipulative materials was not limited to particular grade levels but rather appeared to be effective for the range of ages of students found in schools. Sowell identified two factors that were essential for the use of such materials to be effective in student learning. First, the materials needed to be used over the long-term, such as a full school year or more. Second, teachers needed to be knowledgeable about their use.

Drawing on the previous discussion, to effectively use manipulative materials and visual representations, it seems that teachers should

- use manipulative materials and visual representations with the goal of creating useful internal representations of mathematical concepts (Goldin & Shteingold, 2001),
- choose a variety of materials to illustrate the same concept (Lesser & Tchoshanov, 2005) though perhaps with younger children, ensure that one effective representation is used consistently so that children can use them confidently without increasing their processing load (Boulton-Lewis & Halford, 1992),
- be aware of "automatic pilot syndrome", perhaps particularly for girls, and do not insist on the use of manipulative materials at the expense of developing abstract mental strategies (Ambrose, 2002),
- sequence the use of manipulative materials carefully (Lesser & Tchoshanov, 2005),
- make connections between the various materials (Hiebert & Wearne, 1992),
- use materials as an integral part of teaching mathematics making their use important to students and not just a part of "fun maths" (Moyer, 2001),
- use materials to support understanding rather than using materials in a rote manner (Cohen, 1990),
- make sure that the use of the materials helps students to understand more about the mathematics and not more about how to use the materials (Cohen, 1990),

- use materials to support thinking rather than to replace thinking as an answer getting device (Beswick, 2005a),
- use materials with older students as well as younger students (Howard & Perry, 1997; Swan & Marshall, 2010),
- use the materials consistently for long periods of time (Sowell, 1989), and
- be knowledgeable about the manipulative materials to be used (Sowell, 1989).

Using concrete materials and representations is a method of scaffolding that I examined in the present study. As I have discussed, there is evidence in the literature that such use has positive effects on learning mathematical concepts for low-attaining students. However there is also evidence that the use of manipulatives decreases in upper primary mathematics classrooms (Howard & Perry, 1997) and warnings that such use can be "little more than window dressing" (Stein & Bovalino, 2001). More research in this area is needed to explore these issues and the present study has a contribution to make to this discussion.

I will now discuss the third scaffolding practice forming the framework of this study. This is the practice of scaffolding student understanding of underlying mathematical concepts. As the three types of mathematics tasks under examination in the present study are aimed at building conceptual understanding rather than procedural fluency, it is essential that students be scaffolded to build such an understanding via the teachers' explicit attention to mathematical concepts.

EXPLICIT ATTENTION TO MATHEMATICAL CONCEPTS

The scaffolding practices I have described so far, the use of discussion and the use of manipulatives, representations and tools, could be present when teaching solely for procedural proficiency in mathematics. However, the three types of mathematics tasks under examination in the present study are intended to build understanding of important mathematics concepts. Hiebert and Grouws (2007) suggested that attending explicitly to concepts in the mathematics classroom meant "treating mathematical connections in an explicit and public way" (p. 383). "We conclude that when teaching attends explicitly and directly to the important conceptual issues, students are more likely to develop important conceptual understandings" (p. 385). This is one of the scaffolding practices framing my study. The extent to which teachers attend to mathematical concepts, through using materials, teaching problem solving skills, discussion or other means will impact on low-attaining students' ability to develop conceptual understandings. In some ways, this form of scaffolding for understanding underpins the other scaffolding practices under examination. There is an overlapping and intertwined aspect to the three scaffolding practices framing this study. However, the nature of their relationship and the overall effect on the learning and feelings of the target students adds an important dimension to this research and enhances its potential to contribute valuable data to the literature on scaffolding.

Explicit attention to mathematical concepts is often referred to in the literature as teaching for understanding (Fennema & Romberg, 1999) or focussing on conceptual understanding (Cobb et al., 1989). "Understanding" has been defined in various ways in the literature. Hiebert and Grouws (2007) described understanding as "mental connections among mathematical facts, procedures and ideas" (p. 380). Carpenter and Leherer (1999) asserted that "understanding is not an all or nothing phenomenon … it is more appropriate to think of understanding as emerging or developing rather than presuming someone does or does not understand" (p. 20). Furthermore, Hiebert, Carpenter, Fennema, Fuson, Wearne and Murray et al. (1997) alluded to the complexity to examining understanding in classrooms. "One of the reasons that it has been difficult to describe understanding in classrooms is that understanding is very complex. It is not something that you have or do not have. It is something that is always changing and growing" (p. 4). Similarly, Hibert, Wearne and Taber (1991) described as a "myth" the idea that understanding was "an all-or-nothing experience" (p. 321). Rather, Hiebert et al. asserted that understanding was marked by "unpredictable small changes that make the whole process appear halting and erratic" (p. 322).

Much of the curriculum advice in mathematics current at the time of this study emphasised the importance of student understanding (DfEE, 1998; National Council of Teachers of Mathematics, 2000). The Australian National Numeracy Review (Numeracy Review Panel, 2008) stated that "the rush to apparent proficiency at the expense of the sound conceptual development needed for sustained and ongoing mathematical proficiency must be rejected" (p. xi). In Victoria Australia where this study was conducted, effective mathematics teacher were expected to "develop numeracy understanding through strategic questioning and feedback by the teacher and explanation of reasoning and methods by the student" (Department of Education and Early Childhood Development, 2010, p. 11).

Understanding is important for many reasons. First, the nature of the world in which students of today will be living and working in the future is likely to be vastly different to that of the past. No longer are skills such as calculating long strings of numbers neatly in a ledger book required. Problem solving skills are required, as the problems themselves are not known. As Hiebert et al. (1997) pointed out, "understanding is crucial because things learned with understanding can be used flexibly, adapted to new situations and used to learn new things" (p. 1). Furthermore, Hiebert el al. pointed out the affective response to understanding, saying, "Understanding is important because it is one of the most intellectually satisfying experiences and, on the other hand, not understanding is one of the most frustrating and ultimately defeating experiences" (p. 2).

Carpenter and Lehrer (1999) proposed that to learn mathematics with understanding there are five "forms of mental activity"(p. 20) that need to be present. These are "constructing relationships; extending and applying mathematical knowledge; reflecting about experiences; articulating what one knows; and making mathematical knowledge one's own" (p. 20). Putman, Heaton, Prawat and Remillard (1992) asserted that "teaching mathematics for understanding cannot be reduced to using the right textbook, having students work in groups, using manipulatives or using mathematics activities in real-world settings" (p. 214) but rather is a result of the learning environment which teachers created. Hiebert and Grouws (2007) described teaching for understanding as

> discussing the mathematical meaning underlying procedures, asking questions about how different solution strategies are similar to and different from each other, considering the ways in which mathematical problems build on each other or are special (or general) cases of each other, attending to the relationships among mathematical ideas and reminding students about the main points of the lesson and how this point fits within the current sequence of lessons and ideas (p. 383).

Compared with discussion and manipulative materials and visual representations, there is less evidence in the literature that explicit attention to mathematical concepts is a common form of scaffolding for low-attaining students. Many research projects have examined the use of particular mathematics programs (e.g. Baxter et al., 2001; Riordan & Noyce, 2001) or teaching approaches (e.g. Boaler, 1998; Carpenter et al., 1996; Lambdin, 2003) that aim to teach for understanding.

Whilst many make no differentiation between the attainment levels of participating students, others, explored previously in this chapter, have specifically explored how teaching for conceptual understanding impacts on disadvantaged and low-attaining groups (Knapp et al., 1995; Ridlon, 2004; Robert, 2002; Silver et al., 1995).

There are constraints associated with examining how teachers attend explicitly to mathematical concepts to scaffold low-attaining students. Evidence from the literature, discussed previously in this chapter, indicates that some teachers do not believe that tasks aimed at conceptual understanding such as problem solving, are even appropriate for low-attaining students (Anderson et al., 2004). Understanding is not always considered possible for low-attaining students and understanding is sometimes thought to have to wait until processes or skills are mastered. An additional constraint is that some teachers lower cognitive demands of tasks to accommodate low-attaining students either in response to pressure from the students themselves or through their concern for the emotional wellbeing of students, which was also discussed previously in this chapter.

Hiebert, Wearne and Taber (1991) described low-achieving fourth grade students' gradual construction of understanding about decimal fractions. This study was somewhat of a rarity in that it described the use of tasks aimed at building understanding used with low-attaining students. Hiebert et al. proposed that understanding demonstrated by these students was gradual and existed in a state of "change and flux and reorganisation" (p. 339) where regression was evident. Hiebert et al. suggested that a "monotonic" (p. 339) view of growth in understanding was inadequate as this does not account for the "back and forth movements" (p. 339) of coming to understand for these low-achieving students.

Putman (1992) reported on the case of one teacher who held the belief that understanding could come after learning procedures, the kind of hierarchical view (Zohar et al., 2001) of learning described previously in the chapter. The teacher in Putman's study stated, "I think sometimes that (learning why) starts to more confuse them than help them.... I believe that they should learn just the procedures sometimes and then later when their mind is more mature, they can understand the whys of why you do what you do" (p. 165). This view that learning must progress from learning procedures and that understanding must wait, would certainly impact on the extent to which teachers pay attention to underlying concepts.

Kazemi and Stipek (2001) compared two classrooms they described as “low press” for conceptual understanding with two that were considered “high press”. The teachers providing a higher press for conceptual understanding were characterised by their establishment of four sociomathematical norms missing from lower press classrooms. These norms were that explanations needed to consist of mathematical argument and not descriptions of procedures, that mathematical thinking entails comparing multiple strategies and their relationship to each other, that errors are opportunities for learning and that individuals are accountable for their learning and for providing mathematical argumentation to others. Classroom episodes were explored to illustrate the differences between the high and low press teachers and these provide clear examples that highlight these important differences. For example, a low press teacher asked students to describe their strategies but these descriptions were summaries of the steps taken. A high press teacher would engage in similar initial actions, asking students to describe their strategy, but would press students to explain why such steps were taken. The low press teacher seemed to be primarily interested in students describing the how whereas a high press teacher persisted in finding out why students chose particular strategies. Another important difference was how these teachers asked students to respond to each other’s explanations. Low press teachers were observed asking for “global, superficial nods of agreement or disagreement” (p. 69) by asking the class generally if they agreed with the solution. The class’ chorused responses were accepted without the teacher exploring the reasoning behind these. In a high press classroom, students were also asked to respond to each other’s strategies but were asked for reasons for their agreement or disagreement. A final crucial difference between low and high press teachers was that low press teachers seemed reluctant to push students’ understandings, sometimes giving students an “out” such as “or is that part going to make you too stuck?” (p. 69). High press teachers maintained the mathematics at a challenging level. Kazemi and Stipek’s study highlighted that the mathematics classrooms of both low and high press teachers look and sound remarkably similar with students describing their strategies, and teachers eliciting many student responses. However, to create a classroom supporting conceptual understanding the small differences between the high and low press teachers proved pivotal.

Kazemi and Stipek (2001) proposed that the difference between a high press and low press for conceptual thinking in students were subtle variations in teacher actions. These differences were the degree to which teachers “emphasized student effort (e.g., encouraging students to work through difficult problems and find multiple solutions); focused on learning and understanding (e.g., emphasizing the development of better understandings, asking students to explain their strategies); supported students' autonomy (e.g., encouraging student self-evaluation, giving students choices, encouraging personal responsibility); and de-emphasised performance (e.g., getting answers right)” (p. 61). High press teachers require “students to give reasons for their mathematical actions, focussing their attention on concepts rather than procedures” (p. 68).

Kazemi and Stipek (2001) reported that the teacher in their study who taught in a way that encouraged a “high press for conceptual thinking” used mathematical errors as an opportunity to explore misconceptions. This teacher resisted correcting students and instead asked other students to comment on errors so that the class could all learn from these. In contrast, the teacher in this study who was described as exhibiting a “low press for conceptual thinking” often supplied conceptual thinking to her students when they made errors. Borasi (1994) supported this finding and posited that “the behaviorist view of learning that informs much of traditional schooling is not likely to invite students and teachers to see errors in a positive light” (p. 170). Borasi explained that this was due to the fact that, in a behaviourist view, learning should receive positive reinforcement while errors should be punished or ignored to avoid being reinforced. “Within this framework, paying explicit attention to errors in class is even considered by many as dangerous, since it could interfere with fixing the correct result in the student's mind” (p. 170).

The “press” for thinking that Kazemi and Stipek (2001) described was also part of Henningsen and Stein’s (1997) five elements of classroom teaching that impact most on the extent to which students are “doing mathematics”, the highest cognitive level of a task. This was as opposed to lower cognitive activities such as memorising, recalling or practising. The five elements were “tasks that build on students' prior knowledge, scaffolding, appropriate amount of time, modeling of high-level performance and sustained press for explanation and meaning” (p. 534). Scaffolding in this case was defined as “a teacher or more capable peer provides assistance that enables the student to complete the task alone, but that does not reduce the overall complexity or cognitive demands of the task” (p. 527).

Silver, Mesa, Morris, Star and Benken (2009) examined portfolios of tasks submitted to the National Board of Professional Teaching Standards in the area of adolescence and mathematics by 32 teachers. These teachers were seeking "certification of highly accomplished teaching" (p. 505) by this board. Silver et al. examined the level to which the submitted tasks were cognitively demanding. These researchers defined low and high demand tasks as "low-demand tasks exclusively involve recalling, remembering, implementation or applying facts and procedures, which is in contrast to high-demand tasks, which require students to analyze, create, or evaluate facts, procedures, and concepts or to engage in metacognitive activity" (pp. 509-510). Silver et al. found that "about half of the teachers in our sample failed to include in their portfolio entries even a single task that was judged to be cognitively demanding" (p. 520). These researchers concluded that "these teachers did not use such tasks in their instruction, that they did not consider mathematical demand to be a characteristic of highly accomplished mathematics teaching or that their definition of demanding tasks was related to pedagogical rather than cognitive features" (pp. 520-521). This suggested that teachers perhaps lacked knowledge of what cognitively demanding tasks were and how they would be implemented.

Conceptual understanding is just one of the five strands of mathematical proficiency described by Kilpatrick et al. (2001). These strands also include procedural proficiency. Kilpatrick et al. recognised that

> Procedural fluency and conceptual understanding are often seen as competing for attention in school mathematics. But pitting skill against understanding creates a false dichotomy … the two are interwoven. Understanding makes learning skills easier, less susceptible to common errors, and less prone to forgetting. By the same token, a certain level of skill is required to learn many mathematical concepts with understanding, and using procedures can help strengthen and develop that understanding (p. 122).

Einsenhart, Borko, Underhill, Brown, Jones and Agard (1993) also discussed procedural and conceptual knowledge of mathematics. They found that, although both procedural and conceptual knowledge are "necessary aspects of mathematical understanding" (p. 10), conceptual knowledge often "falls through the cracks" in many American mathematics classrooms. Eisenhart et al. described a student teacher's difficulty in focusing on conceptual knowledge due to her supervising teachers' lack of attention to concepts, and her own struggle to understand the concepts underpinning procedures, despite her university course instructor's efforts to focus on concepts. In addition, the student teacher appeared to teach more for conceptual knowledge if the students were "high ability". "She believed that higher-ability students, particularly honours students, were more interested than other students in higher-order questions and discovery-oriented activities ... She never stated that average or low-ability students should not be taught the conceptual underpinnings of mathematical procedures. However, she clearly indicated, on more than one occasion, the importance of repetition and extensive practice for these students" (p. 20).

In contrast, some researchers have argued that scaffolding understanding of underlying concepts is perhaps even more crucial with low-attaining students than it is with higher attaining students. As I have discussed previously in this chapter, low-attaining students often struggle with memory (Prawat, 1989), accessing knowledge (Diezmann et al., 2004) and transferring knowledge to new situations (Lawson & Chinnappan, 1994). Learning with understanding has the potential to combat these difficulties. Lambdin (2003) asserted that understanding is motivating, promotes more understanding, helps memory, enhances transfer, influences attitudes and beliefs and promotes the development of autonomous learners. Learning with understanding actually creates less need to remember unrelated pieces of information, as information or knowledge is linked to other ideas. As Watson (2002) pointed out "it is a very difficult learning task to learn, recall and use facts and procedures which are unconnected to anything about which the learner feels secure" (p. 4). Teaching basic skills through memorisation and repetitive practice might be more likely to fail with low-attaining students who have difficulty with memory, retrieval and transference. In addition, teaching for understanding does not sacrifice proficiency in basic skills (Knapp et al., 1995; Silver et al., 1995). In fact understanding often enhances such skills.

On the basis of the preceding discussion, explicit attention to concepts could involve

- explicit discussions about "why", that is, the underlying concepts of procedures (Hiebert & Grouws, 2007);
- making connections between tasks, the real world, previous experiences or constructing relationships between concepts (Carpenter & Lehrer, 1999);

- expecting explanation and justification of strategies and thinking (Kazemi & Stipek, 2001);
- discussion about the relative efficiency of strategies (Kazemi & Stipek, 2001);
- articulating what is known (Carpenter & Lehrer, 1999);
- viewing errors as important tools for advancing learning (Hiebert & Grouws, 2007; Kazemi & Stipek, 2001);
- an emphasis on understanding and a de-emphasis on finding answers; and
- reflecting on experiences, solutions and strategies (Carpenter & Lehrer, 1999).

Conclusion

In this chapter I have examined the three spheres of research concerning the present study. The first of these spheres examined was mathematical tasks including categories for tasks and the ways in which tasks can undergo change when implemented by teachers. I then examined the literature on low-attaining students in mathematics, discussed the debate about how best to teach such students, and summarised the recommendations in the literature regarding the most effective teaching practices for low-attaining students. This led to my discussion of scaffolding as a potential bridge between conceptually challenging tasks and the learning of low-attaining students. In particular scaffolding through discussion, the use of manipulative materials and visual representations, and explicit attention to concepts were examined as these formed the focus of the present study.

The following model (Figure 2) gives a diagrammatic overview of the present study. This model arose from the review of literature discussed in this chapter, and reflects the main research question of the study. The model has three parts that mirror the three spheres of the literature which underpin this study and were discussed in this chapter. The first part is the tasks which are "filtered" through the teacher, the second is the notion of scaffolding as a bridge between the tasks and the low-attaining students and the third part of the model is the low-attaining students and their cognitive and affective responses to the tasks and scaffolding offered by their teacher. I will now discuss each of these parts of the model and the relationships between them that the model intends to illustrate, beginning with "tasks".

The part of the model regarding tasks can be seen to move from the task as "planned", before any influence by the teacher, to "enacted" (Gehrke et al., 1992) after the task has been filtered through the "teacher" part of the model. As discussed tasks can change from the task as intended or planned, to the task that is enacted with students (Henningsen & Stein, 1997; Tzur, 2008). This occurs mainly through the actions of the teacher, as outlined previously in this discussion. The task is filtered through the teacher. The teachers' beliefs and mathematical knowledge for teaching influence how tasks are enacted in the classroom. Other external factors may also influence how the task is implemented such as professional learning in which the teacher engages (e.g., Clarke et al. 2009; Peled et al. 2009), parental expectations and involvement (Englund, Luckner, Whaley, & Egeland, 2004), the current curriculum expectations (National Curriculum Board, 2009), or standardised testing (Victorian Curriculum and Assessment Authority, 2006). These external factors are acknowledged but were not studied in this case study that was concerned with the personal case of two teachers.

The next part of the model illustrates the kind of scaffolding potentially offered to students. This scaffolding is the intended bridge between the task and the low-attaining students and allows these students access to the task and the possible thinking the task could provoke. In the present study, three types of scaffolding were the focus: discussion, manipulative materials and visual representations, and explicit attention to concepts. These are featured on the scaffolding "bridge". There are other types of scaffolding discussed in this review of the literature (e.g., Baker, Gersten & Lee, 2002; Ellis, 2005). These are acknowledged but were not examined in detail during data collection in this study.

The low-attaining target students form the third part of this model. These students have internal factors that impact on their response to tasks and scaffolding such as beliefs about themselves as learners of mathematics and the nature of mathematics itself. Other external factors, such as socio-economic (e.g., Thomson & De Bortoli, 2007) or Indigenous background (e.g., Zevenbergen, Grootenboer, Niesche, & Boaler, 2008), English as a second language (e.g., Zevenbergen et al., 2008) or learning disabilities (e.g., Woodward & Montague, 2002) would also impact on the responses of low-attaining students to scaffolding and tasks. These influences were identified in Chapter 1 but were not the focus of this case study. The arrow on the scaffolding bridge in the model shows that scaffolding is a potential bridge between the task and the teacher to the low-attaining students. The low-attaining students respond to the task and scaffolding, both cognitively and affectively. This response has the potential to feedback to the teacher, shown by the second arrow, and influence the scaffolding the teacher offers. Conversely, this response might not result in action by the teacher and so the cycle is broken. However, it would seem from reviewing the literature that effective scaffolding would involve this "feedback loop" (Anghileri, 2006; Bliss et al., 1996; McCosker & Diezmann, 2009) and that responsive scaffolding relies on the teacher responding to the students' thinking or feelings.

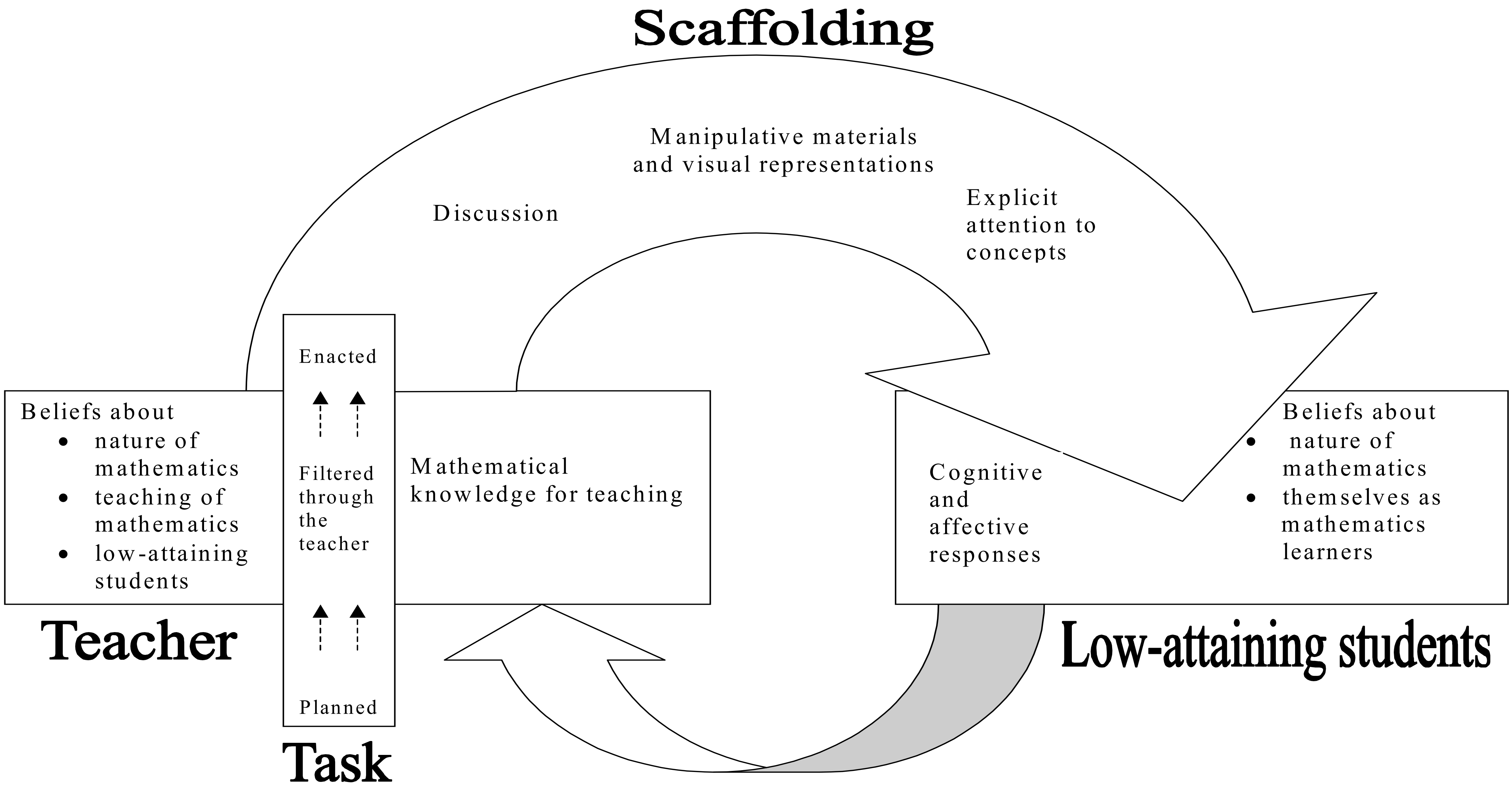


Figure 2. Model of this study.

This model explains and justifies the present study's place in the literature discussed so far in this thesis. The model also illustrates the areas in which data needed to be collected and analysed so that the main research question of this study was addressed. The following chapter will focus on the methods employed to collect and analyse such data.

To conclude, the study of tasks provides teachers with valuable information about how to create opportunities for particular thinking and learning for students. In addition, it provides warnings about the difficulties and potential pitfalls of using certain task types. Effective teaching of low-attaining students is an area of concern for teachers especially as there is evidence in the literature of some conflicting advice. It seems of interest then, to investigate the relationship between these two areas. If certain task types create opportunities for higher level thinking by students, it is pertinent to ask about the effect for low-attaining students. This creates the space for my study that offers some insights into how low-attaining middle years students respond to three particular task types in mathematics, and how their learning can be enhanced by the support of appropriate scaffolding.

CHAPTER THREE – METHODOLOGY

INTRODUCTION

"The critical skill in research design is deciding upon a question that is important and then choosing research methods that will answer that question as unambiguously as possible" (Slavin, 1992, p. 14). In this study I sought to answer questions that are primarily concerned with human behaviour, teaching and learning. The reason I chose to ask such questions is based upon my personal belief that it is important to study human behaviour, that social interactions represent reality (Neuman, 2003), and that studying these teachers and students is intrinsically interesting, as well as being instrumental in advancing understanding about how best to support low-attaining students in mathematics (Stake, 1995).

To summarise the discussion thus far, in Chapter 1, the issues surrounding this study were explored including a discussion on the achievement gap between high and low-attaining students and the particular challenges arising in the middle years of schooling. The three spheres of the literature that underpinned this study were also introduced. In this first chapter, the first sphere, mathematical tasks, examined mathematical tasks found in mathematics curriculum advice from a range of countries followed by a description of the three task types in the *Task Types and Mathematics Learning* (TTML) project. The next sphere, scaffolding, was introduced in Chapter 1 as was the third sphere, low-attaining mathematics students. The participants in this study were also introduced in Chapter 1, two teachers and four low-attaining Year 5 students, two in each teacher's class. The present chapter will provide more detail about the participating teachers and students.

Chapter 2 extended the discussion centred on the three spheres introduced in Chapter 1 and began with examining mathematics tasks. The results of TIMSS Video Study (Hollingsworth, 2003) illustrated the kinds of tasks that are typically implemented in middle years mathematics classrooms, particularly the differences between Australian classrooms and those of other countries. Next, an examination was made of the influences on teachers' use of tasks including beliefs and the role of pedagogical content knowledge. To follow was an analysis of the literature on cognitive and affective characteristics of low-attaining students and teaching approaches recommended for such students. This was followed by an examination of scaffolding and scaffolding practices, in particular the use of discussion, the use of manipulative materials and visual representations, and explicit attention to concepts. This analysis illuminated the issue of implementing cognitively challenging tasks with low-attaining students, and the role of particular scaffolding practices. It also brought to light the paucity of research that has combined the study of conceptually challenging types of mathematics tasks with low-attaining students and studies that illustrate real cases of scaffolding in mathematics classrooms. This created the space and need for my study.

As a reminder of questions under research, I restate the research questions discussed in Chapter 1 and explored through a review of the relevant literature in Chapter 2. The main research question was

> How does a teacher's use of particular scaffolding practices, while using specific mathematics tasks, impact on low-attaining students cognitively and affectively?

Sub-questions were:

1. Do any of the scaffolding practices appear to be more effective in scaffolding the learning of low-attaining students than the others and in what ways?
2. Do low-attaining students have a preference for the kind of scaffolding their teachers offer and what are the reasons given for these preferences?
3. Do low-attaining students have a preference for the kinds of tasks used in mathematics lessons, and what are the reasons given for these preferences?
4. Do teachers show preferences for using particular types of mathematics tasks with low-attaining students and in what ways?

RESEARCH PARTICIPANTS

As this study was a case study, the role of the participants or the case under research was paramount. The teachers and students in this study were all from Victoria, Australia in the area near Melbourne, Victoria's capital city. The teachers each taught a combined class of students in their final two years at primary school, Years 5 and 6.

These students were aged 10 to 12 years old. Typically in Victoria, students move to secondary school in Year 7.

The student participants were all in Year 5 at the time of this study. These participants were low attaining in mathematics. The two teachers had received professional learning either in the use and implementation of three particular types of mathematics tasks during the *Task Types in Mathematics Learning* project (TTML, Peled et al., 2009) or in the *Contemporary Teaching and Learning of Mathematics* project (Clarke, Downton, Roche, Clarkson, Scott, McDonough, Horne et al., 2009). This study was part of the TTML project that examined the use of three types of mathematics tasks. The project defined these task types as tasks that use models, representations or tools that exemplify the mathematics, tasks situated within a contextualised practical problem, and open-ended tasks to investigate specific mathematical content (Peled et al., 2009).

As outlined in previous chapters, two teachers and four students, comprising two target low-attaining students from each of the two classrooms, were chosen for this study. The teachers volunteered and each then nominated two low-attaining mathematics students to be part of this study. Each teacher and their two target students were observed for six lessons occurring over a three-week period.

The first teacher, Ms B, had five years teaching experience, mostly teaching Year 5 and 6 students. Ms B nominated two boys, Carl and David, from her class to be part of this study. According to Ms B, Carl and David were both operating at least 12 months behind their peers in mathematics. Carl and David had no pre-diagnosed learning disabilities. Both boys were in Year 5 in a Year 5 and 6 combined class.

The second teacher, Ms L, had eleven years teaching experience, mostly with middle primary and upper primary classes. Ms L nominated Sophie, a Year 5 girl, and Riley, a Year 5 boy, to be part of this study. According to Ms L, Sophie and Riley were both operating at least 12 months behind their peers. Sophie and Riley had no pre-diagnosed learning disabilities. Both students were in Year 5 in a Year 5 and 6 combined class.

The beliefs, opinions, personalities and behaviour of these participants will be explored in the following chapters. To follow, the purpose of this study will be reiterated and examined.

THE PURPOSE OF THIS STUDY

The research questions were focussed on the experiences of low attaining middle years students with regard to specific types of mathematical tasks and particular scaffolding practices. The purpose of this study was to gain a deeper understanding of the responses, both affective and cognitive, of low-attaining students to three task types in mathematics and three particular forms of scaffolding. Such an understanding will add to the literature on scaffolding, on effective teaching of low-attaining students, and to research on the experiences of middle years students with mathematics, during a period of schooling having been shown to be problematic in the education of many students (Luke et al., 2003). From this study, teachers and researchers may come to know more about effective teaching of low-attaining students, and in particular the notion of scaffolding. In addition, insights may be gained into low-attaining students' feelings and attitudes toward mathematics learning in relation to three types of tasks and the scaffolding practices of their teachers.

I believed my research questions were important as they sought to understand the perspectives of low-attaining students and their teachers in relation to particular task types and scaffolding practices. There was a gap in the literature in that, the responses of low-attaining students to the kinds of mathematical tasks that are recommended by many mathematics curriculum documents across the world (National Curriculum Board, 2009; National Council of Teachers of Mathematics, 2000; Shimizu, 2006), had not been widely examined. In addition, the literature revealed that the use of such tasks could be problematic for teachers (e.g., Henningsen & Stein, 1997). Scaffolding was also a relatively unexplored form of pedagogy and one in danger of becoming "a vague word" that has come to mean almost everything teachers do in the classroom (Myhill & Warren, 2005).

To answer my questions as "unambiguously as possible" (Slavin, 1992, p. 14), the perspectives of the low-attaining students and their teachers were paramount. As such, there was no hypothesis to be tested or theories to prove or disprove thus an inductive approach was adopted. Underpinning my research questions were my beliefs about the goal of this study. Primarily my goal was to understand the experiences and perspectives of the teachers and students in their classroom while engaging in cognitively challenging mathematics. Such a goal is aligned with an interpretive paradigm, which I will now discuss.

INTERPRETIVE PARADIGM

"The central endeavour … of the interpretive paradigm is to understand the subjective world of human experiences" (Cohen, Manion, & Morrison, 2011, p. 17). This study sought to understand the world of the mathematics classroom through the human experiences of the teachers and students. Interpretive research is concerned with "getting to know a particular social setting and seeing it from the point of view of those in it" (Neumann, 2003, p. 76). Merriam (1988) offered support of such an approach for educational research stating, "I believe that research focussed on discovery, insight, and understanding from the perspectives of those being studied offers the greatest promise of making significant contributions to the knowledge base and practice of education" (p. 3).

Interpretive research also forms theories or hypothesis largely after data collection has begun rather than seeking to test preconceived theories. Interpretive researchers "begin with individuals and set out to understand their interpretations of the world around them. Theory is emergent and must arise from particular situations; it should be 'grounded' in data generated by the research act" (Cohen et al., 2011, p. 17). Anderson (1987) linked interpretive research with qualitative methods "which emphasises inductive, interpretive methods applied to the everyday world which is seen as subjective and socially created" (p. 384). As such the research method for this study was largely qualitative, providing "thick description of something else … (that) gives the reader a feel for another's social reality" (Neumann 2003, p. 79). Thus the characteristics of interpretive research align with qualitative research methods that allow for emergent theories, inductive approaches and investigation of socially constructed worlds.

"Qualitative research is an umbrella concept covering several forms of inquiry that help us understand and explain the meaning of social phenomena with as little disruption to the natural setting as possible" (Merriam, 1998, p. 5). Qualitative research originates with the view that there are multiple realities (Merriam, 1998) depending on the personal, cultural or social view from which it is observed. As well as taking the multiple realities of the participants into consideration, a qualitative study also considers the social interactions between participants. In fact, such interactions for a qualitative researcher constitute reality (Neuman, 2003). A qualitative approach allows the researcher to analyse these social interactions and socially constructed realities within the "rich psychological soup of a classroom" (Shuell, 1996, p. 726). It is unapologetically subjective in that the personal opinions, experiences and behaviours of people form much of the data (Slavin, 1992). Instead of seeking to measure phenomena, qualitative researchers seek to interpret the data and shed light on the perceptions and beliefs of people. The main characteristics of qualitative research are the pursuit of understanding of complex interactions and social settings, its necessarily interpretative nature, and the vital role the researcher plays in both data collection and interpretation. Gillham (2000) summarised qualitative research simply as "focussed primarily on the kind of evidence (what people tell you, what they do) that will enable you to understand the meaning of what is going on" (p. 10).

Merriam (1988) offered three main characteristics of qualitative research. First, qualitative research is concerned with the process rather than the outcome. That is, qualitative research asks about how things happen rather than seeking the final outcome. Second, qualitative research is interested in meaning over results. It seeks to ask about the meaning of people's lives, how they make sense of their experiences and how to interpret these experiences. Third, in qualitative research "the importance of the researcher … cannot be overemphasised" (Merriam, 1988, p. 19). Qualitative research relies on the researcher to record, analyse and interpret the data whereas in quantitative research data collection tools are designed to minimise the influence of the researcher. The quantitative researcher strives to remain impersonal so that results can be as free from the researcher's interpretations as possible. In contrast, the qualitative researcher is constantly interpreting what they see, hear or perceive.

One way to add to the definition of qualitative research is to compare it to quantitative methods. Though there has been debate over qualitative versus quantitative research, it is now accepted that each has an important role to play in understanding our world more fully (Peshkin, 1993). A quantitative researcher may seek to quantify the world, test measurable outcomes and view the results as a reflection of a single objective reality (Merriam, 1988). Stake (1995) also discussed important distinctions between qualitative and quantitative research. He described as "the distinction between explanation and understanding as the purpose of inquiry" (p. 37). Quantitative research seeks explanation of phenomena whereas qualitative research seeks to understand the complexity of it. Stake went on to explore how explanation can lead to understanding, but that the researcher when framing research questions makes the important distinction of which is more prominent in their study. In this study, understanding the experiences of the participants and seeking an explanation for their responses were both important, indicating that this study was not of the kind of purely qualitative nature that Stake described.

Qualitative and quantitative research both depends upon the interpretation of the data by the researcher. Quantitative research may provide “tables and charts with numbers … and a researcher gives meaning to the numbers and tells how they relate to hypotheses” (Neuman, 2003, p. 148), but the qualitative researcher “weaves the data into discussions of their significance” (Neuman, 2003, p. 148). Qualitative researchers rely on data in the form of words from interviews, observation notes, descriptions and the participants themselves. These data are constantly interpreted by the researcher who decides what to emphasise and de-emphasise, what to bring to the fore and what to ignore (Stake, 1995). These decisions are inextricably linked to the researcher themselves; their background, their beliefs, their biases, their expertise and their knowledge.

This study aligned with an interpretive paradigm. I sought the view of multiple realities (Merriam, 1988) by exploring the multiple realities of the teachers and low-attaining students in mathematics classrooms. In addition, the interactions of the participants with each other formed part of the reality I sought to capture and examine. I believed these realities made meaning of this experience for the participants. I also sought to study these students and teachers in their natural setting, their mathematics classroom. All care was taken to disrupt natural occurrences as little as possible. I sought to understand the complex relationships between teachers, learners and tasks. I also sought to explain what I found through describing the case in all its complexity. The research methods for this study were mainly qualitative. It is important to note however that almost every study has some quantitative elements to it. For example, as will be explained, on occasions, I measured and reported on the length of conversations, the time given to whole class discussion at the beginning of the lesson compared to the end of the lesson, and so on. I will now examine case study research, a common qualitative research method and the method chosen for this study. Cognisant of the advice from the literature, the development of data collection and analysis tools will be described to conclude this chapter.

CASE STUDY RESEARCH

Case study is a form of qualitative research that has "recently come into its own" owing to increased interest in studying human phenomena (Gillham, 2000, p. 2). The first step in using a case study approach is to define the case. Smith (1979) described a case as a "bounded system." Stake (1995) elaborated this definition describing a case as an "integrated system" to recognise the relationships between aspects of a case. Others described the case as a "unit of analysis" (Yin, 1984) and advised selecting a unit based on " what it is you want to be able to say something about at the end of the study" (Patton, 1980). Given that my study was focussed on the experiences of low-attaining students in mathematics, the case or "bounded system" was four middle years low-attaining mathematics students and their teachers who were participating in professional development regarding contemporary teaching of mathematics and types of mathematics tasks. Boundaries are important in case study research as they provide a focus for the study, centred around the research questions, and prevent the study becoming too unwieldy or vague (Stake, 1995). Furthermore, the relationships between the students and their teachers, and the interaction between the tasks, students and teachers, and particular scaffolding practices formed the focus for this study. In this way, Stake's integrated system can be used to describe this case study also.

According to Merriam (1998), through case studies, readers have the capacity to discover something new about a phenomenon. This discovery may not be a completely new piece of information but perhaps a new way of viewing a familiar phenomenon. The reader may discover that their previously held beliefs or theories are validated or brought into question by the case study. Merriam stated that a case study can "explore reasons for a problem … explain why an innovation worked or failed to work … can discuss and evaluate alternatives not chosen … can evaluate, summarise, and conclude" (p. 31). This case study has the capacity for readers to discover something new about the ways in which low-attaining students react to certain mathematical tasks and scaffolding practices. The point of view of low-attaining students has rarely been studied and so this case study will allow readers to discover, from these students themselves, what engaging in such tasks and experiencing particular scaffolding was like for them. It has the potential to offer teachers examples and descriptions of other teachers scaffolding student learning in mathematics, a phenomenon also not often examined. This study broadens the research on these task types to consider the views and responses of low-attaining students. It also re-examines theories on scaffolding for low-attaining students. It has the potential for evaluating, summarising and concluding (Merriam, 1998) about task use and scaffolding and therefore can impact on teachers' use of such tasks and practices with low-attaining students.

Case studies are inductive because they "begin with detailed observations of the world and move toward more abstract generalisations and ideas" (Neuman, 2003, p. 51). The opposing approach would be deductive where a pre-conceived hypothesis or abstract idea is the beginning and the researcher sets out to prove or disprove this theory. As discussed previously, qualitative case studies find generalisations and hypotheses through the examination of the case and do not hold specific theories prior to data collection. Merriam (1988) asserted that "discovery of new relationships, concepts and understandings, rather than verification or predetermined hypotheses, characterises qualitative case studies" (p. 13).

The research questions for this study supported this "discovery" view of case studies. The research questions asked broadly what the impact was for low-attaining students, cognitively and affectively, of the use of specific mathematics task types and particular scaffolding practices. This study did not seek to prove a theory about how such students may respond but rather sought to discover this through the process of studying cases. However, some "tentative working hypotheses" (Merriam, 1988, p. 13) were expressed during Chapter 2 about possible responses to certain tasks as found in the literature. The three scaffolding practices and the task types provided the lens through which the responses of the target students were analysed and filtered. Though this does not constitute a pre-conceived hypothesis, it does move this case study beyond a grounded theory approach (Glaser & Strauss, 1967) in which "theory is built from data or grounded in the data" (Neuman, 2003, p. 146). These working hypotheses were held prior to data collection and provided a possible framework of responses which these cases may display but do not constitute a preconceived hypothesis to be tested.

Another important feature of case studies is their *particularistic* nature. Merriam (1998) asserted that case studies must focus on a particular situation or phenomenon. This is supported by Stake (1995) who stated that "the real business of case study is particularisation, not generalisation. We take a particular case and come to know it well" (p. 8). A major difference between a qualitative case study and a quantitative research design is that case studies are focussed on one particular case and not on a large sample or cross-section of participants (Slavin, 1992). The findings of a case study can be seen as difficult to generalise to other populations for this reason. However, the purpose of a case study is to examine, in depth, one case so that the particular features of the case may be recognisable to someone in a similar situation. Some elements may be common to their experience and some may differ. Wehlage (1981) stated "the consumer of the research, not the author does the generalising … It is up to the consumer to decide what aspects of the case apply in new contexts" (p. 216). The generalisations made in case studies are generalisations to that particular case, patterns of behaviour evident in observation or interview data analysis. Stake (1995) asserted that such generalisations could be termed *petite generalisations* in order to draw a distinction between these case-specific generalisations and generalisations transferable to a different and perhaps larger, cohort.

The findings of the present study were concerned with a particular case. The responses of the low-attaining students and their teachers gave rise to *petite generalisations* regarding these students, these teachers and this case. However, it is likely that readers of the study will recognise similar responses in their settings or experiences. As Stake (1995) stated, "most find a commonality of process and situation. It startles us all to find our own perplexities in the lives of others" (p. 7). These petite generalisations potentially make the findings of this study more accessible and relevant to other cases in other settings.

Both Merriam (1998) and Stake (1995) used the term *thick description* to explain a critical element of case study research. Merriam (1998) stated that it is the "complete and literal description of the incident or entity being investigated" (p. 30). This study relied on detailed descriptions of the experiences of target students, and their teachers during mathematics lessons primarily using the three task types under research. These descriptions were given depth by interviews, observations, documents, artefacts and the audio recordings of every observed lesson. Such description was filtered through the conceptual categories of the three scaffolding practices, the tasks and the cognitive and affective responses of the students. This allowed the researcher and readers of this study to notice important moments, verify or dispute theories, and build a picture of what learning and teaching mathematics was like for these case study students and teachers.

Stake (1995) offered two types of case studies: *intrinsic* and *instrumental*. The distinction comes from the purpose of the study. An intrinsic case study seeks to understand the particular case due to the researcher's intrinsic interest and curiosity about this case. It does not seek to understand other cases through its study of one case but instead strives to understand one particular case. An instrumental case study on the other hand, seeks to understand a "puzzlement". In the present study, this puzzlement was the effect on low-attaining mathematics students of using particular types of mathematical tasks and specific scaffolding practices. Instrumental case studies take a broader view and try to gain insight into a more general issue by studying a case.

Each of these distinctions has relevance to this study. The experiences of the low-attaining students were intrinsically interesting to me as a researcher and a teacher. As a teacher, my concern for effective ways to teach the low-attaining students in my classroom impacted on the development of this study, as I described in Chapter 1. In these ways, this study can be seen as an intrinsic case study, concerned with the particular students and my personal interest in their education. The notion of instrumental case study is also relevant to this study in that, through studying cases of low-attaining students, a broader understanding of effective scaffolding of mathematics learning and the use of particular tasks was sought. In addition, developing an understanding of the experiences of the low-attaining students studied was an important purpose for this study. The experiences of the case study students illuminated the issues of task use, and effective scaffolding of low-attaining students. The cases were studied not just for their own sake, but also for a greater understanding of wider issues. For these reasons, this study was more an instrumental than intrinsic case study.

In addition to Stake's (1995) intrinsic and instrumental types of case study, Merriam (1988) provided three categories for case studies : *descriptive, interpretative* and *evaluative.* Though this study provided descriptive accounts of the cases under analysis, its purpose was not simply to describe the case. The purpose went beyond description and sought to interpret the responses of low-attaining students and their teachers to particular types of mathematical tasks and scaffolding practices. Evaluative case study progresses from description, to interpretation of this description and finally to making judgments about the effectiveness of an intervention (Merriam, 1998). Making judgements is the primary purpose of an evaluative case study. As this study did not seek to make substantial judgements, it was not primarily an evaluative case study. This study did seek to interpret the responses of the case participants and therefore it is congruent with the definition of an interpretative case study.

Merriam (1998) describes the purpose of interpretative case studies as "containing rich, thick description … [that is] used to develop conceptual categories" (p. 38). This description fitted the purpose of my research. The scaffolding practices and task types provided the conceptual categories that were developed prior to data collection and arose from an examination of the literature. These categories were a lens through which data were gathered, analysed and viewed. Rather than a purely descriptive case study that has no predetermined hypotheses, this case study intended to examine the theories on effective scaffolding of low-attaining students within the context of three particular types of mathematical tasks.

It may seem that interpretive research is flawed in its design as it is unable to be checked or validated – the reader must trust the interpretation of the researcher. However, Mousley and Sullivan (1998) argued that "in the interpretive paradigm, knowledge and notions of validity, cogency and fruitfulness are considered to be relative to individual contexts. Findings are not expected to be replicable … thus while beliefs and interactions of people in specific contexts can be captured, described and interpreted, the aim is not to measure these" (p. 229). The key to interpretive research is the thick description that captures and describes the data in such a way that the reader can see how the researcher came to interpret it the way they did. This measure can alleviate concerns about interpretive research.

Interpretive research relies heavily on the researcher. Stake asserted, "the ongoing interpretive role of the researcher is prominent in qualitative case study" (p. 43). Merriam (1988) warned, "the importance of the researcher in a qualitative case study cannot be overemphasised. The researcher is the primary instrument for data collection and analysis" (p. 19). In qualitative research, the interpretative nature of the research places the researcher at the centre of data collection and analysis. The study relies principally on the researcher rather than on results from experiments or tests as found in more quantitative research. Merriam (1988) pointed out some important advantages to using the researcher as the primary data instrument, stating that "the researcher as instrument is responsive to the context … can adapt techniques to the circumstances … the human instrument can process data immediately" (p. 19). Stake (1995) offered that the case study research was like "placing an interpreter in the field to observe the workings of the case, one who records objectively what is happening but simultaneously examines its meaning and redirects observation to refine or substantiate those meanings" (p. 8). The advantages of the researcher as primary data collection tool is their ability to change, adapt and redirect data gathering in response to the subtleties of the human, social situation of the case in all its complexity. The challenge for the qualitative researcher is refining skills, much like refining questionnaires, tests or experiments, so as to deliver the most reliable results possible.

Merriam suggested three ideal investigator characteristics of a case study researcher. The first she described as *tolerance to ambiguity*. Case study research does not follow a step-by-step procedure but rather adapts and develops as the data are collected and suggests new paths of investigation or new questions to be explored. This lack of direction may unsettle the researcher but Merriam asserted that the ideal case study investigator "must enjoy searching for pieces to the puzzle and tolerate uncertainty for an indefinite period of time" (1988, p. 37). The second characteristic described by Merriam was *sensitivity* to the context and variables of the case including human behaviour and the information being gathered. The researcher must also be sensitive to their own personal biases and how these may impact on the study. "All observations and analyses are filtered through one's worldview, one's values, one's perspective" (Merriam, 1988, p. 39). The case study researcher must also be a *good communicator* according to Merriam, who empathises with participants, establishes a good rapport, asks good questions and listens intently.

Yin (1984) warned that "the demands of a case study on a person's intellect, ego and emotions are far greater than those of any other research strategy … In fact … the skills required for collecting case study data are much more demanding than in experiments and surveys" (p. 56). He then outlined five skills required of a case study researcher: asking good questions; listening carefully and without bias; being adaptable and flexible; having a firm sense of the purpose of the research; and not holding preconceived notions of the outcome of the study or judging reactions of the participants. These attributes are "remediable", Yin asserted, and could be developed through practice. This "practice" for the present study took place via the completion of a pilot study to trial all data collection and analysis tools and most importantly, to hone my skills as a researcher.

Lincoln and Guba (1985) also recommended being immersed in the situation to gain experience and exposure and working with an experienced qualitative researcher. As both supervisors for this study are experienced and skilled qualitative researchers, I intended to learn from their skills by seeking their advice and guidance. I also had the opportunity work alongside other experienced qualitative researchers as part of data collection for the TTML study. Furthermore, the purpose for my pilot study was the kind of experience and exposure suggested by Lincoln and Guba (1985). The pilot study involved lesson observations of an upper primary school teacher using the task types from the TTML. Students from the pilot study class as well as other upper primary students and teachers not involved in TTML were asked to trial some data collection tools. The pilot study will be described in more detail during the following discussion. The purpose of the pilot study was to practise, refine and build up my skills as a case study researcher so as to ultimately deliver a more robust and higher quality study.

I have explored the ways in which this case study compared to elements of case study discussed in the literature. I will now describe the development of data collection tools and analysis that took place in this study, drawing on the literature as discussed. Data collection took place in three phases. Each phase will be described. The discussion to follow will focus on reliability and validity issues of case studies generally and the measures taken in the current study to address these issues.

DATA COLLECTION

Merriam (1988) described three main sources of data for case studies: observation, interviews, and document analysis. Yin (1984) offered six sources of "evidence" for case studies, these being documentation, archival records, interviews, direct observations, participant-observation and physical artefacts. Gillham (2000) proposed a similar list, but drew a distinction between a "detached" observer and a participant observer. For this study I drew mainly on observation including audio recordings of the lessons and semi-structured interviews. Document analysis, photographs and physical artefacts were also used for data gathering and analysis, however the primary sources of data for this study were from the low-attaining students and teachers, gathered from interviews and observations.

Each of these data sources has something different to offer the case study researcher. Observations of the case provide the researcher with the makings of a thick description of the case in a naturalistic setting (Merriam, 1998; Stake, 1995). Such observations are not controlled or influenced by the researcher as much as this is possible. Data from observations can be compared to data from interviews because, as Gillham (2000) pointed out, “a common discrepancy is between what people say about themselves and what they actually do” (p. 13). Despite this, interviews remain an important source of data for case studies. There can be as much to be learnt from the discrepancies between what people say and do as from the congruencies. Semi-structured interviews afford the researcher the opportunity to direct questions toward the research purpose and explore issues relevant to the study. Studying documents and artefacts relevant to the case allows for the opportunity to examine data that are untainted by researcher influences. All these sources of data are important in a case study to build a rich description of the complexity of the case. In addition, it is important to use multiple sources of data, which will be discussed in relation to reliability issues. For this study, a combination of these data collection methods was employed. They are explored in more detail now.

Data collection took place in three phases. The first phase occurred before observations of lessons began. The teachers and the low-attaining students were interviewed and students completed an assessment task during this initial phase of data collection. The data collected were collected to provide some background information about the teachers and students, and to begin to build a picture of their beliefs, experiences and feelings about teaching and learning mathematics. Data collected during this phase could then be compared to data collected during the second phase, the observation period.

The second phase of data collection was the observation period. Six sequential lessons were observed over two to three weeks in each classroom. These lessons involved between one and four tasks each. Teachers were interviewed prior to lessons regarding their intentions for the lesson, the mathematics they hoped students might learn more about and how they anticipated the low-attaining target students might respond to the lesson. During the lesson, the teachers wore a mobile recording device that captured the lesson from beginning to end, including instructions to the whole class and conversations the teachers had with individual students during the lesson. In this way, every observed lesson was audio recorded from the point of view of the teacher rather than the researcher. During lessons I was present at all times and took observation notes that focussed in particular on the target low-attaining students. I noted times and events in the lessons so that the audio could be compared to my observations of these students. I took photos of student work samples and the class whiteboard to add details to the story of each lesson. I spoke to the low-attaining students during lessons, though not every lesson, in order to find out what they were doing, what they thought about what was happening or to check their understanding of the task. I did not assist these students to complete tasks though as their familiarity with me grew, some of these students asked for this assistance. I was seeking through my lesson observations to gain data on the target students' affective and cognitive response to the tasks and their teachers' attempts to scaffold their learning. Target students were interviewed after each observed lesson regarding their feelings about the lesson, their teachers' support for them and their learning from the task. A short assessment task was also given at this time. Teachers were also interviewed after each observed lesson regarding their feelings about the success of the lesson for the class and for the target low-attaining students. Issues that teachers might address in future lessons and plans for the next lesson were also discussed. These interviews were semi-structured and so the teachers largely determined the direction of the interview by discussing issues that they raised themselves. Details about interviews with teachers and students will follow in this discussion.

The third phase of data collection was a "postscript" interview with both teachers and the target students, which occurred one year after the observation period, phase two, was completed. These interviews allowed the opportunity to ask more about themes and early findings that had emerged, to assess how the teachers thought about low-attaining students, tasks and scaffolding one year after the observed lessons, to gauge target students' current feelings about mathematics and themselves as mathematics learners, and to assess their performance on cognitive assessment tasks also given the previous year. I asked also about the impact of being part of this study.

Table 2 and Table 3 outline the data collection tools used for teachers and those used for target students, respectively. In addition, the purpose of these tools is summarised and any piloting of the tools described. These tools and their purpose will be discussed further in this chapter.

Table 2

Data collection tools for teachers

Collection tool	Purpose	Piloting
Interview prior to lesson observations.	To gain background information regarding beliefs about teaching mathematics, about what mathematics is, and beliefs about teaching low-attaining students. The teachers' personal mathematics history was also sought.	Piloted with three teachers of varying years of experience.
"Maths is" word wheel.	A visual task that aimed to prompt more about beliefs regarding the nature of mathematics.	Piloted with 5 teachers.
PPELEM for teachers.	A drawing task that aimed to gain insight into how the teachers viewed their mathematics classroom, their role and the role of students, and the physical environment (materials, computers, grouping arrangements etc.).	Piloted with 40 teachers.
Teacher beliefs questionnaire (Anderson et al., 2004).	A questionnaire with a Likert scale to gauge how "traditional" or "contemporary" the teachers' stated beliefs were.	Piloted with two teachers.
Tasks questionnaire.	A questionnaire that gathered more data regarding the teachers' beliefs about the most appropriate types of tasks for low-attaining students.	Piloted with two teachers.
Mathematics planning documents for 3 weeks prior to data collection period.	Analysis of the planning documents was aimed to build up the picture of how mathematics was taught in each classroom prior to observations so that similarities or differences could be observed.	Not piloted.
Interview prior to lessons.	This interview aimed to gain information regarding the teacher's intention for the lesson, and their perception of the possible responses of the low-attaining students.	Piloted during pilot study.
Observation notes for each lesson. Audio record teacher during lesson.	These tools collected data on the lessons as they took place.	Piloted during pilot study.
Interview after lessons	This interview asked the teachers about their perception of how successful they felt the lesson was, how it matched their intention, the responses of the low-attaining students, and possible future lessons as a result of the lesson and students' responses.	Piloted during pilot study.

As these tables show, most data collection tools could be piloted substantially, enabling greater skill and confidence in their subsequent use during the final study.

Table 3

Data collection tools for target students

Collection tool.	**Purpose**	**Piloting**
"Draw a mark on this line showing how good you feel you are at maths".	Students' perception of themselves as mathematics learners.	Piloted with one class of Year 5/6 students.
"Maths is" word wheel. "Fractions" or "Multiplication" word wheel.	Students' perceptions about the nature of mathematics and knowledge of the focus topic for lesson observations.	Piloted with one class of Year 5/6 students.
Assessment task on concept of the unit Multiplication – "Fish task" (Clark & Kamii, 1996) OR Fractions – draw 1/3, 2/5 and 3/2. Indicate the order of these fractions from smallest to largest.	To gain some information about the level of understanding the students had about the mathematics that would be the focus for the lessons observed.	Not piloted.
Learning Mathematics Survey (10 mins)	Asked about student preferences for the kinds of mathematics tasks they like and dislike and the features of lessons they like and dislike (e.g., "I like it that, when I am working, the teacher comes round to help"	Students in TTML project completed this survey.
PPELEM – drawing task (McDonough, 2002b) "Draw a time when you were learning maths well"	Student perceptions of helpful factors when they are learning mathematics and about their perception of mathematics.	Piloted during pilot study.
Observed during lessons.	Students' responses to the tasks and their teacher's actions.	Piloted during pilot study.
Emoticons scale. Reflection task after the lessons.	Students' affective response to the lesson.	Piloted during pilot study.
Interview after the lesson. "What was today's lesson all about? What did you learn more about? What did your teacher do today that helped you? … Didn't help you?"	Students' affective and cognitive response to the lesson and their teacher's actions.	Piloted during pilot study.
Assessment task for target students after the lessons.	Potential to gauge students' cognitive response to the lesson and developing understanding of the concepts addressed in the lesson and overall unit of study.	Piloted during pilot study.
"Star Rating task". Tasks from the lessons are written up in a brief description. Students place a gold star on favourite task, silver on second favourite, green on the task they learnt most from, red on the task that was most difficult / confusing.	Student preferences for tasks, most popular and least popular tasks, and perceptions of tasks in which most was learnt.	Students in TTML completed this task.

These tables illustrate the range of data collection tools employed in this study in order to build up a thick, rich description (Merriam, 1998) of this case. This range allowed for triangulation of data, which will be examined further in the discussion of reliability and validity issues later in this chapter. First, the pilot study that preceded the final study will be discussed before a more detailed description of the data collection tools found in Tables 2 and 3.

PILOT STUDY

As discussed previously in this chapter, in qualitative studies the researcher is the main research tool and therefore needs to be operating as effectively as possible (Merriam, 1988; Stake, 1995). Such effectiveness is improved by training, practice and experience (Lincoln & Guba, 1985; Yin, 1984). As such, piloting the data collection tools, including the observation schedule, was a vital step in this study as the researcher was relatively inexperienced in formal observations. Through this process, skills in noticing important events, effective note taking and writing detailed observation notes were enhanced, thereby increasing this study's validity.

"A pilot test gives the researcher an idea of what the method will actually look like in operation and what effects (intended or not) it is likely to have … a pilot test enables you to avert … problems by changing procedures" (Slavin, 1992, p. 123). A pilot study was undertaken for this research to test the data collection tools, refine data analysis tools and to hone my skills as a qualitative researcher. As established previously, the role of the researcher as the primary data collection tool is paramount (Merriam, 1988; Stake, 1995). Due to my minimal prior experience as a case study researcher, I felt it particularly pertinent that I conduct a pilot study to enhance my skills and ultimately, to increase the validity and reliability of the study. As Yin (1984) stated, "good preparation begins with adequate skills on the part of the case study investigator" (p. 55).

The pilot study trialled the intended procedures for the final study (Ferguson, 2009). The pilot study was conducted with a classroom that was similar to those intended for research in the final study. The similarities are such that comparable responses to the data collection tools were likely between the pilot participants and research participants. Though it was not possible to predict exactly how different people would respond, the pilot study gave an idea of the kinds of responses that were likely and the opportunity to refine data collection tools. “The pilot site … assumes the role of a laboratory for the investigators, allowing them to observe different phenomena from many different angles or to try different approaches on a trial basis” (Yin, 1984, p. 74).

“In general, convenience, access and geographic proximity can be the main criteria for selecting the pilot cases” (Yin, 1984, p. 74). I considered such factors in the selection of my pilot study site. I knew the site and the participants were somewhat familiar with me owing to their involvement in the TTML research project. Access to the site was unproblematic for this reason. The participant, a teacher of upper primary school aged students and the students themselves were more likely to be “unusually congenial and accessible” (Yin, 1984), owing to their familiarity with the researcher.

The pilot study teacher had four years of teaching experience all in the upper primary school. The school and this classroom were involved in the TTML research project. The teacher had committed to teaching TTML tasks at least once a week and writing up some of these lessons for inclusion in data for this larger study. She had attended a number of days during which professional development had been provided by TTML project staff on use of the task types. The students in the pilot study class were in Years 5 or 6, their final two years at primary school before moving on to secondary schooling in Year 7.

The pilot study followed similar procedures as those in the final study. The teacher was asked to select two students she believed to be low attaining in mathematics. Then, the teacher was asked to complete a questionnaire about the main sources used for choosing mathematical tasks for the class, her beliefs regarding particular types of tasks and their appropriateness for low-attaining students, and some strategies she currently used to cater for low-attaining students in mathematics. The responses to this questionnaire were discussed during an interview with the teacher. Before each observed lesson, the teacher was asked about her intentions for the lesson and after the lesson she was asked how these intentions were met or not met by the lesson.

Next, the pilot study target students identified by the teacher were observed for four lessons, with the predominant task of each lesson being a different one of the three TTML task types. Observation schedules and note taking methods were refined and modified during this process. In particular, the problem of capturing private exchanges between the teacher and students during lessons arose which led to the use of the mobile recording device in the final study. In addition, the target students were interviewed after each lesson to gauge their reactions to the lesson in affective terms as well as their understanding of the mathematical content of the task. This provided the opportunity to test interviewing tasks and refine the selection of such tasks for the main study.

In addition to this pilot study, other data collection instruments were piloted with other groups of willing teachers and students as outlined in Tables 2 and 3. I will now elaborate further the phases of data collection for this study and examine the data collection tools, which drew on the literature, which was used during each phase.

PHASE ONE: PRE-OBSERVATION INTERVIEWS

Interviews are an important source of data for the case study researcher to deepen knowledge and understanding of the case first gleaned through observations (Merriam, 1988). Interviews allow the researcher to gather alternative views on the case by exploring the perceptions of key participants. Interviews also afford the opportunity to discover things that cannot be seen in observations. Feelings, opinions, decisions, and interpretations of events can be explored through an interview but are not always apparent during observations. Interviews have the potential to flesh out the case and provide a rich source of data for the researcher (Gillham, 2000).

Dexter (1970) stated there were three variables in every interview situation. These are "the personality and skill of the interviewer, the attitude and orientation of the interviewee and the definition of both of the situation" (p. 24). These variables affect the kind of information the interview may provide. However, Merriam (1988) offered strategies for dealing with these variables. Firstly, the interviewer must strive to be neutral and non-judgmental; secondly, the interviewer must be mindful of verbal and non-verbal messages; and thirdly, the interviewer must be a good, reflective listener.

Interviews can be unstructured or completely structured, though most case study researchers opt for interviews which fall somewhere in the middle of this continuum (Merriam, 1988). Semi-structured interviews provide the framework of the interview without limiting the interview's scope with tightly prescribed questions

(Yin, 1984). The interviewer has some questions pre-written and perhaps some key issues they wish to have addressed but the impetus of the interview rests with the interviewee and the issues they raise through their answers. In a semi-structured interview, the researcher has the opportunity to pursue issues of interest, clarify points and allow the interviewee to have some control over the direction the interview takes. Thus stated, it is important to note that while the researcher has little control over the events during an observation, an interview gives the researcher more control over what is discussed and emphasised (Merriam, 1988).

Researchers have offered advice on maximising the value of interviews through careful questioning. Strauss, Schatzman, Bucher and Sabshin (1981) offered four types of questions that can be fruitful for interviews. They described these as hypothetical, devil's advocate, ideal position and interpretive questions. The advantages of such questions are that the responses from respondents can be more detailed and descriptive. Hypothetical questions, for example, give respondents the chance to respond to a situation removed from themselves and the reality of their experiences. However, "responses are usually descriptions of the person's actual experience" (Merriam, 1998, p. 77). Devil's advocate questions can depersonalise a potentially sensitive subject by placing the question with "others" such as "some people might say that …" or "some teachers might believe that …" (Merriam, 1998, p. 77). This gives the respondent the opportunity to respond to the issue without risking personal embarrassment or criticism. Ideal position questions ask the respondents to voice their opinion about the best possible situation or solution. These questions are helpful to evaluate positive and negative aspects of the issues. Interpretive questions allow the researcher to check tentative theories or emerging themes in the data. In effect, such questions gauge if the respondent is saying or expressing the information or feelings the researchers thinks they are, and can confirm or challenge emerging hypotheses.

Merriam (1998) also warned that some types of questions should be avoided in an interview. Multiple questions are unhelpful as such questions contain more than one part, therefore it is difficult to ascertain which part of the question the respondent is answering. Furthermore, multiple questions are confusing for the respondent who is not sure which part of the question to focus upon. Leading questions should also be avoided as these questions reveal researcher bias or assumptions that may not match those held by the respondent. Such questions allow for the respondent to accept the researcher's opinion or point of view rather than revealing their own. Finally, yes-or-no questions should not be used during interviews due to the lack of depth possible in responses. Such questions offer respondents "an easy way out". Moreover they can "shut down or at least slow the flow of information from the interviewee" (Merriam, 1998, p. 79).

The limitations of semi-structured interviews are that their success depends largely on the interviewing skills and analysis of the interview by the researcher, human factors such as personality, rapport and attitudes, and the frank honesty of the participants (Yin, 1984). Participants may not be able to provide sufficient detail in their answers due to factors such as age or intellectual abilities. The advantages are that the participants can reveal aspects of the case during interviews that are not always exposed during observations such as opinions, feelings and perceptions. This information is highly valid as it is directly from the source. The researcher has the opportunity to clarify, expand on or summarise the data. "The interview is the best way to find out what is in and on someone else's mind" (Patton, 1980, p. 196). It was with these points from the literature that interviews for teachers and students were developed. I will now discuss the initial teacher interviews, which took place in phase one of data collection, before the observation period commenced.

Initial teacher interview

Teachers of target students were interviewed before the classroom observation period began. The purpose of this pre-observation interview was to gain some insight into the kind of mathematics classroom the target students were experiencing prior to observations. As I identified in Chapter 2, key to understanding the nature of the classroom is the teacher as "it is what teachers think, what teachers believe, and what teachers do at the level of the classroom that ultimately shapes the kind of learning that young people get" (Hargreaves, 1994, p. ix). The purpose of the pre-observation interview was to shed light on the first two of these points – what the teacher thinks and believes about mathematics, about teaching mathematics, about low-attaining students of mathematics and their role as a teacher.

As discussed during the literature review, teacher beliefs shape the learning experiences and classroom environment that students encounter at school (Thompson, 1984). Beliefs impact on the types of tasks teachers choose to use, the way in which tasks are implemented, and ultimately the kinds of thinking students engage in

(Thompson, 1992). Beliefs regarding low-attaining students have also been found to be crucial to these students' learning opportunities. Studies have shown teachers hold lower expectations for such students and may present lower cognitively demanding tasks (Zevenbergen, 1997; Zohar et al., 2001). This often results in less access to higher order thinking tasks such as problem solving (Beswick, 2005b).

As discussed, the initial interview sought information on four main aspects of teacher beliefs – beliefs about the nature of mathematics itself, beliefs about teaching mathematics, beliefs about low-attaining students in mathematics, and beliefs about teaching low-attaining mathematics students. Table 4 outlines the questions in the initial teacher interview that address each of these areas.

Table 4

Initial teacher interview questions regarding beliefs

Aspect of beliefs	**Interview questions**	**Type of question**
Beliefs about the nature of mathematics.	*What words would you use to describe what mathematics is? ("Mathematics is" word wheel task)*	Visual cue from word wheel allows time for reflection before responding in writing.
Beliefs about teaching mathematics.	*Tell me about your background in mathematics.* *How do you think this impacts on your teaching of mathematics?* *What is the main thing you would like students to gain from mathematics in your classroom?*	Ideal position question (Strauss et al., 1981)
Beliefs about low-attaining students.	*What would you say if someone said that there are some children who will always have difficulty with mathematics?* *Why do you think some students have difficulty progressing in mathematics?*	Devil's advocate question (Strauss et al., 1981)
Beliefs about teaching low-attaining students.	*What do you think are the best ways to help low-attaining students learn mathematics?*	Ideal position question (Strauss et al., 1981)

The design of the initial teacher interview drew on these studies of teacher beliefs. During piloting for example, it became apparent that the term "goal" in the question "What is your overall goal for mathematics learning in your classroom" was problematic. My meaning of the word "goal" was of an overall, global, overarching and important goal for mathematics learning in a general sense. However, the teachers I interviewed during the piloting of this interview did not define goal in this way. For them, “goal” was a shorter-term aim for a specific grade level or mathematics unit. Teachers were accustomed to using “goal” to describe outcomes in their planning or assessment of mathematics. Therefore this question was changed to “What is the most important thing you want your students to gain as a result of your mathematics teaching?” (See Table 4). Upon piloting of this new question, teachers offered more global long-term aims for their mathematics teaching such as “engagement”, “success”, “challenge”, “use a range of appropriate strategies” and the “hope that children develop a love of mathematics”. Such responses were more in line with my intention for this question that was to ascertain teachers’ beliefs about teaching mathematics.

The other question that changed most during piloting was “What words would you use to describe what mathematics is?” Teachers in the piloting of this question indicated that it was difficult to answer because they had not considered such a question before. In an effort to allow teachers sufficient time to reflect on this question, I drew on the data collection tool “mathematics word wheel” (see Figure 3). McDonough (2002a) used this task with young children to allow them time and space to adequately reflect on their perception of maths. By allowing teachers time to write and think about their response, a more considered answer could be expected. It is also noted that purely verbal responses are not always appropriate for young children. It is therefore reasonable to assume that teachers may also require time for rehearsal via writing. This word wheel for mathematics was piloted with volunteer teachers and found to be a more effective way for the teachers to reflect on what mathematics meant to them than giving a purely verbal response.

In addition to these questions, the initial interview sought background information about the teacher such as teaching experience and a description of a usual mathematics lesson in their classroom. These questions were aimed at building up a picture of the mathematics classroom the low-attaining target students may experience. During observations, the teachers’ description of a usual mathematics lesson could be compared to the researcher’s observations of the lessons. In this way, interview data could be triangulated with other data sources. Similarly, statements of belief could be compared to observation descriptions and discrepancies could be explored. Table 5 provides an overview of these questions.

Table 5

Initial teacher interview questions regarding mathematics background

Background	Interview questions
Personal mathematics history.	*Tell me about your background in mathematics.* *In what ways does your background in mathematics impact on your teaching of mathematics?*
Current teacher practice.	*Can you describe how a typical mathematics lesson looks in your classroom?* PPELEM (McDonough, 2002b) style drawing task. *"Draw a time you were teaching maths well".*
Target low-attaining students.	*Tell me about Student A (low-attaining target student). Difficulties, background, personality, ways of working, persistence etc.* *Tell me about Student B (as above)*

These questions about the teachers' personal background with mathematics, their own perception of their teaching of mathematics and their impressions of the low-attaining target students all allowed me as the researcher to come to know these participants better. Emerging themes, patterns and conclusions could be compared to these data to support or question theories that arose over the course of the study.

It has been recognised that using a variety of media for interviewees to respond can be beneficial in accessing information (Patton, 1990). With this in mind, the teachers completed two tasks that included visual elements rather than purely written responses. One of these was the "Mathematics is" word wheel (see Figure 3), in which the teachers were invited to write words or short phrases that described what they perceived mathematics to be. The other visual task was an adaptation of McDonough's (2002b) PPELEM task. This task asked the teachers to "draw a picture of when you are teaching maths well". These tasks allowed for further evidence to be collected regarding the teachers' beliefs about the nature of mathematics and about the teaching of mathematics.

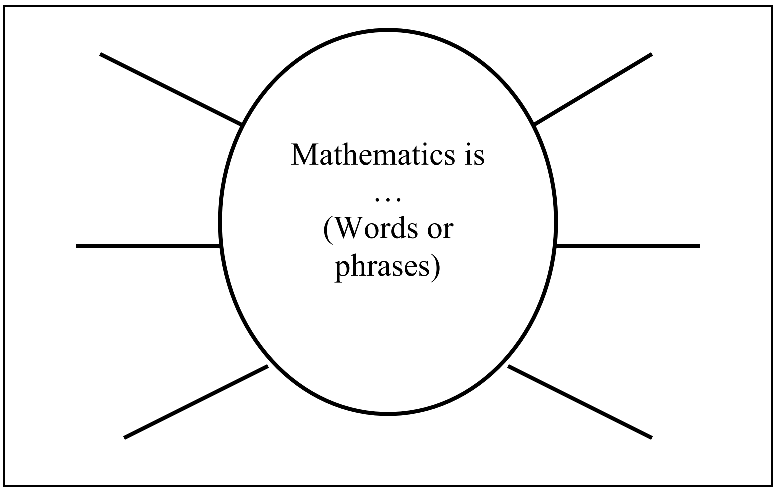


Figure 3. “Mathematics is” word wheel task for teachers.

In addition to the initial teacher interview, the teachers also completed two questionnaires. Teachers completed a “Tasks questionnaire” which asked about the main sources the teachers used for mathematics tasks, and their beliefs about the types of tasks most appropriate for low-attaining students. In addition, teachers were asked, in an open response item, to list four to five strategies they used or could use, that would cater for their students’ differing levels of ability.

Teachers also responded to six items regarding teacher beliefs drawn from the work of Anderson, Sullivan and White (2004). This questionnaire asked teachers to respond to a number of statements on a Likert scale from “strongly agree” to “strongly disagree”. These statements were aimed at gauging how “traditional” or “contemporary” teachers were according to Anderson, Sullivan and White’s description, as discussed in Chapter 1.

A discussion of the findings from the initial teacher interviews with Ms B and Ms L will follow in Chapter 4. I will now outline the Phase one data collected from the target students, which will also be discussed in Chapter 4.

Initial target student interview

Target students were interviewed before the observation period commenced. These initial interviews had a similar purpose to the teacher interview in that I was aiming to gain some insight into the students’ beliefs regarding themselves as mathematics learners, and beliefs about what they considered mathematics to be. For student interviews I took a different approach than the mainly verbal responses that I sought from the teachers. Recognising the target students were low attaining and had not experienced interviews of this kind before, I designed student interviews to allow for rehearsal, a variety of ways to respond and visual cues. These are discussed now.

As touched upon, interviews can pose problems for the researcher if the participants are unable or unwilling to provide sufficiently detailed or thoughtful responses. This is a consideration for this study, particularly regarding interviews with students. The participating students, though upper primary school aged 10 or 11, were low attaining and this meant it was likely that reflecting and communicating their thinking or feelings might have been problematic. For these reasons, I chose to employ "creative interviewing techniques" (Patton, 1990), commonly used for younger children, in an attempt to gain the most information and insight into the thinking of the target students.

The purpose of interviewing target students was twofold – firstly to familiarise the researcher with these students and then to gain their perspective of the tasks observed in the classroom. These interviews will be described during the discussion regarding Phase two of data collection. Familiarisation interviews, which took place before the lesson observation period, were aimed at providing information about the target students' attitudes toward mathematics, toward themselves as mathematics learners, their preferred types of mathematics tasks and factors which the target students believed help and hinder their ability to complete tasks successfully in mathematics.

The student interviews drew on "creative interviewing techniques" (Patton, 1990) as it is recognised that younger children or low-attaining students might not possess the language skills necessary to articulate their thoughts and feelings adequately during traditional interviews. Creative interviewing techniques were used by McDonough (2002a) in her study of eight and nine year old children. McDonough used visual, verbal and text-based tasks in her study about beliefs regarding mathematics and learning held by these children,. This proved to be a reliable method for allowing these younger students to express their beliefs. By using a variety of methods to ask about similar concepts, McDonough was able to draw reliable findings about the mathematical beliefs of these children. I used visual and verbal tasks to interview the low-attaining 10 and 11 year old target students in the present study. It has been shown that many low-attaining students in mathematics also have difficulty with literacy skills such as reading, writing or expressing themselves verbally (Ellis, 2005). Therefore, it is likely that written or purely verbal responses may be difficult for the target students in this study. Indeed many students and even adults might struggle to describe their thinking and feelings accurately through a solely verbal response. As such, I utilised different means to gain insights into the thinking and feeling of these students, including the kind of visual tasks used successfully by McDonough (2002a).

I believed that for students to talk about their experiences and feelings about mathematics, they would require some rehearsal, a chance to reflect before producing a response. The "familiarisation" student interview firstly asked students, looking at a line with "not good at all" to "really, really good" at either end, to "make a mark to show how good you feel you are at learning maths". In piloting, all students were able to complete this task with little difficulty observed. By first making a mark, the students were given the opportunity to reflect on themselves as mathematics learner via a visual cue before being asked to respond verbally to "why did you put your mark there?"

In the same way as the teacher interviews, the target students were asked to use the word wheel to write short responses to the question "what is mathematics?" Bearing in mind the findings of McDonough (2002a) that students can view "mathematics" as being different from "maths", I discussed this with students before they completed this word wheel (see Figure 3). In an extension of this task, students were also asked to describe on the word wheel what they considered their current area of mathematics was, for example, "what are fractions?" Again, the students' written responses were then used as a springboard for discussion during this initial interview.

The target students were asked to complete the "Learning Mathematics Survey" developed by the TTML project (Sullivan, Clarke, & O'Shea, 2010). This survey asked students to consider the types of mathematics tasks they liked and disliked and to consider teacher actions that they felt helped or hindered them in learning mathematics. Parts of this survey were short answer written responses and other parts required students to respond using a Likert scale.

This pre-observation data from students will be discussed in Chapter 4. I will now describe the second phase of data collection, the lesson observation period.

PHASE TWO: LESSON OBSERVATION PERIOD

The lesson observation period collected data via observation or field notes, an audio recording of each lesson, interviews with the teachers before and after lessons, interviews with the target students after lessons, post-lesson assessment tasks with target students, and, on one occasion, the PPELEM task and Star Rating task with all students in each class. The audio recording device was worn by the teachers and captured the entire lesson including any discussions the teachers had with individual students during the lesson. In the pilot study, it had become clear that it was difficult to capture the private conversations the teacher had with students without intruding by standing over the teacher's shoulder. This mobile recording device allowed these exchanges to be recorded without the need for the researcher to hear them while they took place. Literature regarding data collection via observation was considered before the observation period began.

Observations or fieldwork are a mainstay of qualitative case study research (Gillham, 2000). Observations allow the researcher to gain firsthand knowledge of the setting in which the case is situated, the people who operate within this setting and the case itself. "As an outsider an observer will notice things that have become routine to the participants themselves, things which may lead to understanding the context" (Merriam, 1988, p. 88).

The extent to which an observer is involved in the situation under observation can vary. Junker (1960) described this continuum of participation by the observer in four stages moving from "complete participant, participation as observer, observer as participant to complete observer". Gillham (2000) offered "detached observer" and "participant observer". Each of these descriptors may pertain to the same researcher at different stages of research. Researchers may begin by being a complete participant in the case by immersing themselves in the context and situation of the participants, and then move gradually towards becoming a complete observer, enabling the researcher to stand back and take more objective observations of the case. A researcher may operate primarily in one or two of these categories.

There are advantages and disadvantages of each of these roles. A complete participant for example, can give a firsthand insider's view of the case, however a complete or detached observer, can take a wide-view of the case, as she/he is not distracted or preoccupied with becoming involved in the situation themselves. One of the difficulties of observations in the field is the extent to which the researcher becomes involved in the case under observation. There is balance required whereby the researcher "usually participates but not to the extent of becoming totally absorbed in the activity. At the same time as one is participating, one is trying to stay sufficiently detached to observe and analyse" (Merriam, 1988, p. 94).

The researcher must also be mindful of the effect their observation might have on the situation studied. An objective of observation is to observe in its natural state, the situation and people under research. If this state is altered by the presence or influence of the observer, this objective is compromised. Kazdin (1982) offered three reasons why the presence of an observer may change the situation being studied. First, if the participants are apprehensive about being criticised or judged, they may change their behaviour. Second, if the participants believe their actions are being assessed they may behave differently and third, participants may change their behaviour in response to feedback from the observer such as when they observe notes are taken or when the observer seems particularly attentive to certain behaviours. The onus is on the researcher to minimise these factors, perhaps by forging a relationship of trust with the participants (Gillham, 2000) and avoiding the overt taking of notes (Yin, 1984). However, Frankenberg (1982) disputed the issue of the observer changing the situation by their presence and asserted that the activity of a researcher is not likely to change customs and practices built up over a long period of time. Patton (1980) also pointed out that, as much as the observer could prompt change in the situation observed, the situation itself can also effect change in the researcher. This, asserted Patton, is what "gives naturalistic inquiry its perspective" (1980, p. 192). The line a participant observer treads when collecting data via observations can be complex in that the observer needs the participants to feel comfortable and at ease in their presence, yet the presence of the observer should have minimum impact on the observed phenomena. A participant observer needs to be non-threatening without being overly familiar, impartial without being cold and judgmental (Stake, 1995).

For this study, I was a participant-observer insomuch as I maintained the ability to talk to students and the teacher briefly during the lessons to clarify points, ascertain what was happening, check on student work and thinking or discuss students' feelings or attitudes. Minimal notes were taken during observations but detailed notes were made later using the audio recording of the lessons in an effort to provide an "*incontestable description* for further analysis and ultimate reporting"

(Stake, 1995, p. 62). These data provided categories or key events on which to focus and record episodes to "fashion a story" of the case (Stake, 1995). Parts of the lesson that directly pertained to the target low-attaining students, such as all whole class discussions and discussions between the teacher and the target students, were transcribed in full. The audio recording also recorded the time that particular events took place, allowing for correlation between this recording and events I noted during lessons. Data collected through the audio records would not have been available to me as an observer in the classroom without the danger of changing behaviour by standing over the shoulder of the teacher or students which may have been viewed as intimidating, judgemental or critical (Kazdin, 1982). Private conversations between students and the teachers could be collected while more general observations were being made in real time during lessons.

The response of the target students to aspects of the task or their teacher's actions during the observed lessons was crucial to answering my research questions. The aspects I chose to focus on during observations were the affective and cognitive responses of the target students during lessons. Detailed notes telling the story of the target students and the observed lesson were made after each observation using both observation notes and the audio recording of the lesson. This is in keeping with advice from Merriam (1988) and Stake (1995) who emphasised the importance of a full description of the observation, without copious note taking during the observation period.

Interviews were conducted with each teacher before and after each observed lesson. These interviews were usually fairly brief and semi-structured. Before lessons I would usually ask questions like "What are the students going to be doing today? What do you hope students will learn more about as a result of today's lesson? How do you think the target students will go in today's lesson?" In post-lesson interviews I had the opportunity to clarify events that had occurred during the lesson such as changes from the planned lesson, the responses of students or reactions from the teacher. I also asked about what the teacher intended to do, or think about next, as a result of the lesson.

After each observed lesson, the low-attaining target students were interviewed regarding their feelings toward the lesson. In order to prompt discussion, after each lesson the target students completed an "emoticon scale" visual task shown in Figure 4. This task asked students to consider how they felt about the lesson just completed by drawing an arrow to indicate their feelings. It was explained that the emoticon scale worked like kitchen scales where the arrow moves around from one side to the other or like an altimeter on a car in which the arrow moves as the car changes speed. The first time the target students saw the emoticon scale they were asked to name some of the feelings depicted on the emoticons. The words these students used varied but all used positive emotions to negative emotions in the way I had intended.

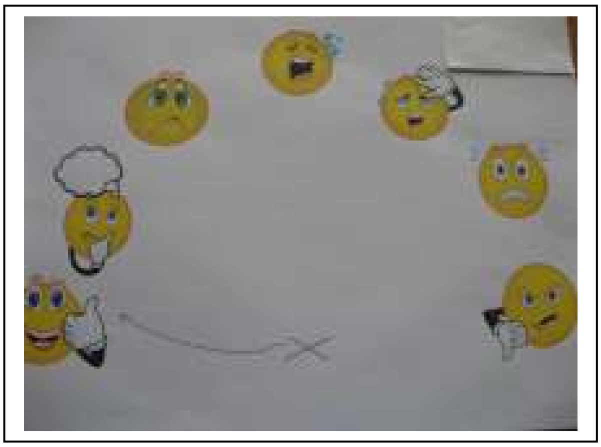


Figure 4. “Emoticons scale” student interview task.

After completing the Emoticons Scales, the target students were asked to talk about where they had placed the arrow and why. These interviews used some questions that were pre-planned but others were asked to delve deeper into issues raised by the students. These questions were cognisant of the advice from the literature, discussed previously, on good questions and types of questions to avoid. Furthermore, the probes and prompts used also took into account the advice from the literature. In this way, data on the target students’ affective response to the lesson and their teachers’ actions were collected after each lesson.

The target students completed a short assessment task after each lesson that aimed to assess the students’ understanding of the concepts contained in the lesson. I designed or chose these tasks based on the concepts I believed the lesson was attempting to address or the overall concepts of the sequence of lessons. These post-lesson assessment tasks formed some of the data collected on the target students’ possible cognitive responses to the lessons. These post-lesson assessment tasks will be described in Chapter 5 during the discussion of data from the observation period.

Two tasks, PPELEM and “Star Rating task” were given to the whole class including the target students in each of the two classrooms. This allowed for comparison between the target students and other students in their classes in regards to preferences and perceptions about learning mathematics well.

The Pupil Perceptions of Effective Learning Environments in Mathematics (PPELEM) task asked students to “draw a time you were learning maths well”. This interview task, developed by McDonough and Wallbridge (1994) initially as a whole class activity, provides an insight into how students perceive successful mathematics experiences and the factors contributing to this success. Students are asked to imagine a time when they were learning mathematics well, then draw this including anything that helped them. In her later work, McDonough (2002a) adapted this task to include students reflecting on a time when learning mathematics was difficult. This task was successfully used with younger children and also older students in the middle years of schooling, years 5 to 8. Its use in this study is intended to gain insights into the possible helpful factors low-attaining students perceive in mathematics learning as well as factors making mathematics difficult for them.

The “Star Rating task” asked students to consider all the tasks observed at the end of the observation period and indicate the task they liked the most, the task they liked the second most, the task they liked the least, and the task they felt they learnt most from. Coloured stars were used to indicate the students’ responses. This data collection tool allowed for data from target students to be compared to those from other students. It also provided a point of comparison with other data regarding the target students’ preferences and responses to tasks.

Other data were collected using photographs, students’ written work and teacher planning notes. These documents were used to clarify, confirm or question themes and patterns emerging from observations and interviews.

The third phase of data collection was interviews with the participants, both teachers and the four target students, one year after lesson observations concluded. These interviews are described now.

PHASE THREE: POSTSCRIPT INTERVIEWS

The postscript interviews with teachers, which occurred during phase three of data collection, were conducted one year after lesson observations concluded. The questions for this interview fell into the three categories of scaffolding practices, the teaching of low-attaining students and tasks. The interview asked the teachers about any changes in their teaching that may have occurred in the year since lesson observations concluded. One question was repeated from the initial teacher interview, "What is the most important thing you would like students to gain from your mathematics teaching?" This interview allowed for emerging themes and patterns to be examined, and observations from the previous year to be compared to the teachers' reported practices one year later.

The postscript interviews occurred one year after lesson observations when the target students were in Year 6. At that time all the students had different teachers from the previous year. This interview asked questions regarding the target students' perceptions of themselves as mathematics learners and used the line with "not good" at mathematics to "really good" at mathematics that was also used the previous year. Students were also asked about any changes they perceived in their ability in mathematics from the previous year. The postscript interview asked target students about the kinds of mathematics tasks they had experienced recently, the actions they thought teachers could take that would help them in mathematics, and what they thought were the best and worst thing about mathematics lessons. To gain information about the learning of the target students, I asked them to "Tell me something you have learnt in mathematics lately" and also repeated an assessment task completed the previous year. The findings from the postscript interviews are described in Chapter 5.

Data were viewed via the conceptual categories of this study, which were the three scaffolding practices and three mathematics task types, along with data regarding the affective and cognitive responses of the target students. Episodes or instances from the data were sorted according to their relevance to these categories in an effort to reveal data relevant to my research questions. Themes, patterns and critical episodes were found and examined in relation to the research questions. These provided categories for data analysis, which I will describe in the following discussion.

DATA ANALYSIS PROCEDURES

"All research is a search for patterns, for consistencies" (Stake, 1995, p. 44). In qualitative case study research such patterns or themes are found through analysing the data provided by the thick and rich descriptions of the case. The case study researcher sifts through interview and observation notes, perhaps comparing these with documents or artefacts, and searches for common themes or patterns. It is important to heed Merriam's (1988) warning however, that such data analysis should be concurrent with data collection. The advantages of on-going analysis are that data collection can be constantly refined and tailored, that questions arising from the data can be addressed during the collection period, that the research questions are being addressed and that the researcher is not left with an overwhelming amount of un-analysed data to examine when the collection period is complete.

The process by which the researcher analyses the data and finds patterns and themes can vary. Merriam (1988) advised a simple sorting of the data initially, either chronologically or in broad themes. Following this, the researcher begins to recognise key issues, patterns or themes arising from the data. More specific categories are created and the data sorted again into these categories. This process is ongoing until the researcher decides on the key themes on which to focus that are directly related to the research questions. This differs somewhat from the approach I took in that key themes were identified prior to data collection, though other themes emerged during data collection and on-going data analysis.

Stake (1995) suggested that the two main ways in which a researcher finds meaning about cases is through *direct interpretation* or *categorical aggregation.* Direct interpretation is taken from individual instances whereas categorical aggregation occurs when such instances can be grouped according to some kind of similarity. Intrinsic case studies, Stake asserted, have more use for direct interpretation as the purpose is a deep understanding of the case, whereas instrumental case studies, such as the present study, seek to understand phenomena revealed through the case and may have more need to use categorical data. The three scaffolding practices, the tasks and the cognitive and affective responses of the target students were examined so that the experiences of this case of two teachers and four target low-attaining students could be explored.

The bounds of this study, the task types and scaffolding practices, provided the framework of the study. The conceptual framework for this study was a result of a review of the literature regarding tasks, low-attaining students and scaffolding in mathematics, as explored in Chapter 2. Miles and Huberman (1984) claimed that a conceptual framework "explains, either graphically or in narrative form, the main dimensions to be studied – the key factors, or variables – and the presumed relationships between them" (p. 28). Figure 2 at the end of Chapter 2 illustrates graphically the main dimensions of this study. In this study, the main dimensions to be studied were the use of conceptually challenging tasks, three scaffolding practices and the cognitive and affective responses of the target students to these practices. In addition, the teachers' actions and factors surrounding such actions such as beliefs, knowledge and pedagogy were explored. The relationships between these factors, the tasks, the scaffolding practices, and student feelings and learning, formed the framework of data collection and analysis.

The literature describes how conceptual frameworks have been used in educational studies to organise data either for collection, analysis or both (e.g. Borko, Eisenhart, Brown, Underhill, Jones & Agard, 1992; Mousley & Sullivan, 1998). The terms theoretical framework and conceptual framework are sometimes used interchangeably (Merriam, 1998). Merriam (1998) described a theoretical framework as "the structure, the scaffolding, the frame of your study" (p. 45). However Neiswiadomy (1998) offered different definitions for the terms theoretical and conceptual framework, describing a theoretical framework as a "broad, general explanation of the relationships between the concepts of interest generally based on one theory" and a conceptual framework as "the linking of concepts selected from several theories or previous research" (p. 96).

This study aligns with Neiswiadomy's (1998) description of conceptual framework, as the categories for the conceptual framework emerged due to an examination of the literature. This study links these concepts from a variety of studies under the umbrella of this conceptual framework. These categories guided the collection of data and provided a frame to analyse data. The establishment of a conceptual framework for qualitative studies has advantages for the researcher in that it provides clear bounds for the research, maintains focus on the research questions and can increase the study's rigour and reliability (Merriam, 1998). This was an important consideration as the magnitude of data that can be collected from one student or even one lesson in a classroom can be overwhelming. Furthermore, the use of a conceptual framework creates the possibility that future research could build on findings of this study by using this same framework in other settings and with other participants.

In essence, this study tested the extent to which and the ways in which teachers used the task types from this study, used the three focus scaffolding practices with low-attaining students and the impact this had on the target students cognitively and affectively. Also central to my research was the question of the responses of low-attaining students to the scaffolding of their teachers and the types of tasks. I anticipated that if elements of effective scaffolding were found during the implementation of the task types, the benefits and advantages of their use for low-attaining students could be enhanced. The three scaffolding practices formed a lens through which the data from this case study was viewed and analysed. The responses of the low-attaining students would be influenced by the teacher's use or lack of use, of these practices. The framework for this study, shown in Figure 2 in Chapter 2, comprised of teacher beliefs, teacher knowledge for teaching mathematics, the use of conceptually challenging tasks, the three focus scaffolding practices, the target students' beliefs and the target students' cognitive and affective responses to the tasks and scaffolding. This framework was used to frame interview questions, explore interview responses, organise observation notes and narratives, analyse documents and artefacts and report on the data analysis and findings. The strength of the framework was its foundation in the literature on scaffolding. It is reasonable to contend that this framework could be used to examine scaffolding practices and task use of different teachers, other low-attaining students and in other situations.

The three scaffolding practices examined in this study formed some categories of data analysis. However, each of these practices contained many other elements and this formed the sub-categories and codes under each of the three overall categories for scaffolding practices. The coding framework consisted of three levels. The first level was the overall scaffolding categories of "discussion", "manipulative materials and visual representations" and "explicit attention to concepts". The second level broke these overall categories into sub-categories that outlined the main features of each scaffolding practice. The final level broke these sub-categories down further into individual codes that described more specific scaffolding occurring within that sub-category. I will now explore these categories, sub-categories and codes for the scaffolding practices so that the rationale for their inclusion might be clear to the reader.

CODING FOR DISCUSSION

Discussion was a feature of much the mathematics curriculum advice at the time of this study, which I described in previous chapters (National Curriculum Board, 2009;

National Council of Teachers of Mathematics, 2000). Teachers of mathematics were encouraged to generate and facilitate discussion in the classroom to enhance student learning through communicating mathematical thinking, justifying, proving, explaining and reflecting on the thinking and strategies of others. Discussion is also an important feature of the types of conceptually challenging tasks explored in this study. Teaching mathematics for understanding often describes the teacher leading, or orchestrating discussions about the mathematics of the task with students.

There are many ways in which teachers can use discussion to scaffold learning. The coding framework sought to identify the various features of discussion found in the literature to be effective for scaffolding learning. These features became the sub-categories under the umbrella of "discussion". There were six sub-categories. These were: recruiting and orienting; monitoring emotional states; inviting and interpreting student responses; questioning; explaining; and establishing social and classroom norms. The codes were the next layer and attempted to tease out the individual features of the sub-categories found in the literature that may be part of using discussion for scaffolding. These are detailed in Table 6.

Table 6

Coding framework for discussion

Discussion	Literature
1. Recruiting / orienting	
1.1 Excavating – uncovering what is known	
1.2 Orienting –contextualising, reminding	Siemon & Virgona, 2003
1.3 Recruiting – inviting and encouraging involvement in the task	Wood, Bruner & Ross, 1976
1.4 Maintaining focus of the task	
2. Monitoring emotional states	
2.1 Reducing frustration by keeping the task at an appropriate level of difficulty	McMahon, 2000
2.2 Encouragement, approval	Anghileri, 2006
3. Inviting and interpreting student responses	
3.1 Revoicing student responses for clarification or elaboration	O'Connor & Michael, 1996 Anghileri, 2006
3.2 Probing questions– students expand on thinking	
3.3 Student explanations –students explain their thinking or strategies	
4. Questioning	
4.1 Funnelling/ prompting questions – leading questions to guide	Wood, 1995

students toward a pre-determined solution procedure	
4.2 Focussing questions – questions that draw attention to critical aspects of the problem, leaving students to decide on solution procedures	
5. Explaining	
5.1 Providing short, focussed explanations	Sullivan, Mousley & Zevenbergen, 2006
5.2 Showing and telling – providing explanations that follow the teachers' planned activity with little student input	Anghileri, 2006
5.3 Reviewing – recounting, summarising	
6. Establishing social and classroom norms	
6.1 Ways of working	Sullivan, Mousley & Zevenbergen, 2006
6.2 Ways of responding to tasks	

Episodes from observation notes, audio and interviews regarding discussion were coded using the NVivo (QSR International, 2005) software with these codes. Some episodes were assigned more than one code due the overlapping nature of the overall categories as well as within categories. Using NVivo, patterns could be identified and analysed across lessons, teachers and tasks. Of particular interest were the codes used most often and least often for each teacher as these data helped to build up a picture of discussion within the classroom and during task use.

I will now discuss the second scaffolding practice, manipulative materials and visual representations, and the sub-categories and codes in this part of the coding framework.

Coding for manipulative materials and visual representations

As I have discussed in Chapter 2, manipulative materials and visual representations have long been considered an effective way to scaffold learning, particularly with students who are identified as low attaining in mathematics. However, the literature demonstrates that using manipulative materials and visual representations effectively can be problematic if teachers are unfamiliar with their use (Sowell, 1989), if students rely on materials instead of developing mental strategies (Ambrose, 2002), if such use overloads the processing abilities of the students (Boulton-Lewis & Halford, 1992), or if students become distracted by manipulating the materials rather than exploring mathematical ideas (Baxter et al., 2001). The literature warns that using manipulative materials does not guarantee understanding by the students of the mathematics such materials are intended to show (Stacey et al., 2001). Some manipulative materials or visual representations are abstract symbols or formal notations of mathematics whereas others are "concrete embodiments" of mathematical concepts (Boulton-Lewis & Halford, 1992). While the use of manipulative materials and visual representations holds promise for scaffolding the learning of mathematics, the ways in which teachers and students use such materials determines the success of this kind of scaffolding.

Under the category of manipulative materials and visual representations, there were four sub-categories that characterised how this form of scaffolding could be used by teachers and students in mathematics classrooms. Each sub-category was then broken down into individual codes that, in a similar way to the category of "discussion", were drawn from the literature. Table 7 outlines the sub-categories and codes for manipulative materials and visual representations.

Table 7

Coding framework for manipulative materials and visual representations

Manipulative materials and visual representations	**Literature**
1. Environmental scaffolds	
1.1 Environmental provisions – posters, displays, concrete materials, tasks or puzzles and mathematical tools	Anghileri, 2006
2. Structuring knowledge	
2.1 Providing a starting point for the task	
2.2 Illustrating processes	
2.3 *Illustrating concepts*	Boulton-Lewis & Halford, 1992
2.4 Connecting written to physical	
2.5 Creating or elaborating a mental model	Butler et al., 2003
2.6 Practising skills or concepts	
2.7 Assisting with calculations or procedures	Beswick, 2005a
3. Communicating	
3.1 A basis for discussion	
3.2 Communicating strategies or solutions	

	Siemon et al. 2004
4. Modelling	
4.1 Modelling the problem or task –demonstration, offering behaviour for imitation	

These codes allowed for instances where manipulative materials and visual representations were used to be analysed, such use compared both within and between the classrooms and the most and least used codes to be identified for each task, lesson and teacher. I will now discuss coding related to the third scaffolding practice, attention to concepts.

CODING FOR ATTENTION TO CONCEPTS

In order to teach for understanding using conceptually challenging tasks, teachers and students must pay attention to concepts (Hiebert & Grouws, 2007). The task types in this study were all aimed at conceptual understanding and therefore the extent to which teachers scaffolded underlying concepts was an important focus. This category is the one that most overlaps with the other two, as attention to concepts cannot occur in isolation but must occur via a medium such as discussion or using materials, for example. Conversely, the use of manipulative materials and visual representations or discussion could be focussed on other aspects of mathematics, such as procedures or processes, rather than on concepts. Therefore, many of the episodes that were coded under the category of "attention to concepts" were also coded under the other two categories of scaffolding. The following table outlines the sub-categories and codes for explicit attention to concepts and the research from which these codes were drawn.

Table 8

Coding framework for attention to concepts

Attention to concepts	Literature
1. Reducing complexity	
1.1Reducing complexity by one conceptual level	Sullivan et al., 2004
	Wood et al., 1976
2. Marking critical features	
2.1 *Drawing students' attention to underlying concepts*	McMahon, 2000
2.2 Noticing – *highlighting, drawing attention to, valuing, pointing to*	Kazemi & Stipek, 2001
2.3 Exposing and discussing misconceptions or errors	

	Hiebert & Grouws, 2007
3. Making connections	
3.1 Between knowledge sets and understandings	
3.2 To previous tasks	
3.3 To real world experiences	
3.4 Between concepts and processes	
	Wheatley, 1992
4. Reflecting	
4.1 Strategies	
4.2 Concepts	

These categories, sub-categories and codes were one way in which data were analysed. Detailed lesson observation notes, interviews and analysis of artefacts such as photographs or student work samples were also analysed to find patterns, and aberrations to these patterns, and to build up a rich description of this case. It is through such description that the reader can judge the interpretations of the researcher and the trustworthiness of the study. I will now discuss the measures taken in this study to address the reliability and validity of the findings.

RELIABILITY AND VALIDITY ISSUES

All research is subject to scrutiny, and issues of reliability and validity must be considered carefully. Case studies, according to Yin (1984), have been criticised for their "lack of rigour" resulting from "the case study investigator [being] sloppy and [having] allowed equivocal evidence or biased views to influence the direction of the findings and conclusions" (p. 21). A case study researcher must carefully consider the ways in which their study can be more rigorous by addressing external and internal validity issues. The key point, according to Merriam (1988), is that "all research is concerned with producing valid and reliable knowledge in an ethical manner" (p. 163). Such knowledge may be used to change the way phenomena are viewed, prompt further research in the area, bring into question long-held views, or change practice in the field. It is therefore crucial that the results of research can be trusted to be as clear, unbiased, honest and thorough as possible.

In qualitative research, the main research tool is the researcher themselves (Merriam, 1988; Stake, 1995). Due to the more prominent role of the researcher, researcher bias, knowledge and skills have the potential to play a larger part in qualitative research than in quantitative research. Qualitative research relies on written descriptions more than numerical results and the researcher brings a degree of interpretation to the findings. All research, including quantitative designs, involves some interpretation. As Ratcliffe (1983) asserted "data do not speak for themselves; there is always an interpreter or translator" (p. 149). Interpretation of data can be a source of disagreement or contention for readers of research. However, there are strategies that the qualitative researcher can use to increase the validity and reliability of their research that are explored in the following discussion.

INTERNAL VALIDITY

At the heart of *internal validity* is the degree to which the findings match reality. Internal validity is a term often associated with quantitative research. Some qualitative researchers such as Lincoln and Guba (1985) used the term *truth value* in place of internal validity to describe this issue in qualitative research. That is, the qualitative researcher attempts to reflect the truth of the issue under research as honestly as possible. Merriam (1998) offered these questions regarding internal reliability: "Do the findings capture what is really there? Are investigators observing or measuring what they think they are measuring?" (p. 167). Qualitative research assumes that reality is "holistic, multidimensional and ever changing; it is not a single, fixed, objective phenomenon waiting to be discovered, observed and measured" (Merriam, 1988, p. 167). Thus qualitative case study research offers a glimpse of this complex reality, as the participants themselves perceive it to be. It does not seek to give the whole picture of the situation but rather a "slice of life" (Lincoln & Guba, 1985, p. 377). It is the responsibility of a case study researcher to represent this reality as faithfully as possible and present "a more or less honest rendering of how informants actually view themselves and their experiences" (Taylor & Bogdan, 1984, p. 98). Stake (1995) talked about "finding good moments to reveal the complexity of the case" (p. 63). Moments from the experiences of the target students and teachers in their mathematics classrooms were described so that the reader has the opportunity to reflect on the interpretations made and ask whether they believe the researcher's interpretations reflects reality.

Goetz and LeCompte (1984) offered four elements of case study research that illustrated its high internal validity. First, such research often involves continual data analysis that strengthens the link between constructs, findings or themes and the participants themselves. Second, interviews are closely aligned with the experiences of the participants and are less abstract or arbitrary than perhaps tools used in other research designs. Third, the data collection is carried out in naturalistic settings "that reflect the reality of the life experiences of participants more accurately than do more contrived or laboratory settings" (p. 221). Finally, the case study researcher is continually self-monitoring and re-evaluating the research activity.

For this study, the use of a pilot study allowed data analysis to test the strength of the conceptual framework by examining the relationship between the key factors, the possibility of new factors and others being omitted and how these elements related to the experiences of the students and teachers in the pilot study. The piloting of data collection tools and the analysis framework strengthened the final study. The pilot study was carried out in the naturalistic setting of the low-attaining students and their teachers. As a researcher, I continually monitored the validity of the framework for the study and re-evaluated the progress of the research.

Merriam (1988) proposed six strategies researchers could use to strengthen a study's internal validity. These were

- Triangulation – using multiple researchers, multiple sources of data or multiple methods of identifying emerging findings.
- Member checks – taking data to the participants themselves for verification and discussion.
- Long-term observations or repeated observations – gathering data occurs over a period of time or many observations are taken of the same situation or case.
- Peer examination – asking colleagues for comments and discussion on emerging findings.
- Participatory modes of research – involving participants in all phases of the study from beginning to end.
- Researcher biases – clarifying the researcher's views, opinions, background and theoretical orientations that impact on the study. (pp. 169-170)

Multiple sources of evidence including questionnaires, observations, audio of the lessons, interviews using creative interviewing procedures, documents and artefacts such as photographs were used for this study. This allowed for triangulation of conclusions drawn about the experiences of the target students and teachers. Data gathering took place over a sustained period of time with repeated observations and interviews with the participants. In addition, the participants were interviewed one year after the initial data collection period. My supervisors fulfilled the role of peer examination by providing a sounding board for emerging theories, and critical questioning of results. Researcher bias was addressed in Chapter 1 when I outlined my views, theories and opinions regarding the research questions. I did not use participatory modes of research due to the age of my participants in the case of the students, and the possibility of altering the case through their involvement in the case of the teachers. My study intended to research the experiences of the target students and their teachers within their natural setting. My role was to maintain the integrity of reality for these students and teachers, and take the steps discussed previously, not alter it by my involvement.

EXTERNAL VALIDITY

External validity is the extent to which the finding of the study can be generalised to other situations (Merriam, 1998). Case studies have traditionally been difficult to generalise as by their very nature, they are researching a particular case and as such, are not seeking to be transferable to other cases (Merriam, 1998). Some would argue this is a limitation of case study research, however others argue that it is through studying a particular case in detail that generalisations can emerge (Erickson, 1986). Merriam (1998) asserted that "in qualitative research, a single case … is selected precisely because the researcher wishes to understand the particular in depth, not to find out what is generally true of the many" (p. 208). That being said, cases do not exist in a vacuum. There are often points of commonality in human behaviour and social situations that can be transferred to other cases with varying degrees (Stake, 1995).

Merriam (1998) contended that issues of generalisation needed to be reconceptualized for qualitative research. For example, generalisations could be more appropriately termed "working hypotheses" (Cronbach, 1975). Cronbach asserted that, as human behaviour and social situations are constantly changing, any generalisation made would be subject to such change and therefore perceiving generalisations as working hypotheses recognises this state of flux. Readers of the research can determine the extent to which these hypotheses are true for their situation and use this "context-bound information" (Patton, 1980, p. 283) to guide their decisions.

For Erickson (1986), generalisation is not an appropriate goal for interpretative research. He asserted that such research does not aim to find "abstract universals" by applying findings from one population to another but rather seeks to find "concrete universals" that are revealed through the study of particular cases. Concrete universals are commonalities found through studying particular cases but supported by findings of other cases. Erickson, according to Merriam (1988), believed that "the general can be found in the particular" (p. 175). In other words, there are elements of certain social situations or human behaviours that can be found across many cases, settings and participants. Such elements are concrete universals. The framework for this study consists of the concrete universals found within other studies of low-attaining students, teaching, teachers and scaffolding. These are commonalities of other research that this study seeks to examine in the light of this case.

Stake (1995) preferred the term "naturalistic generalisations" to describe the generalisations possible from case study research. He asserted that the researcher uses tacit knowledge, intuition and personal experiences to form naturalistic generalisations about their world and the world of their case study. Much like Cronbach's (1975) working hypotheses, such generalisations can guide actions but do not govern them. Through studying a particular case, the researcher can see similarities between the case and other contexts.

Another way to view generalisations is to place the responsibility of transferring findings to the hands of the reader or user of the study. Through the researcher providing detailed descriptions of the case, the reader of the study can draw conclusions about the similarities or differences to their own situation. Walker (1980) stated that "it is the reader who has to ask, 'what is there in this study that I can apply to my own situation and what clearly does not apply?'" (p. 34). Such a view relies on clear, detailed description so that such conclusions may be drawn. "The description must specify everything that a reader may need to know in order to understand the findings" (Lincoln & Guba, 1985, p. 125).

Merriam (1998) offered the following strategies for enhancing external validity:

- Providing rich, thick description which gives enough detail to ensure someone else can evaluate the relevance of the findings for their own situation.

- Establishing how typical the case is compared to others to allow for comparisons to be made by the reader to their particular situation.
- Conducting cross-site or cross-case analysis. (pp. 177)

The present study used thick description so that readers can compare this case to other studies and their own situation. This study involved two different schools and teachers to allow for some cross-case analysis. The description of the participants, and the mathematics classroom as they experienced it also allows readers to judge how typical this case was and its relevance to their particular situation.

RELIABILITY

Reliability is the extent to which the findings of the research can be replicated. In social sciences, this can be problematic as human behaviour and social situations are not static (Merriam, 1988). Furthermore, the researcher as an individual will affect the participants' responses even if care is taken to reduce this influence (Stake, 1995). People respond differently to different people according to a myriad of factors that cannot be controlled or even quantified. In qualitative case study research, Merriam (1998) asserted that "achieving reliability in the traditional sense is not only fanciful but impossible" (p. 206). However, Merriam did propose that reliability in qualitative case studies could be applied to the instruments for data collection. The primary instrument in a case study is the researcher themselves who must accurately and with detail describe the case and setting, analyse these descriptions, recognise significant themes or patterns and propose reasons for their existence. Such an instrument can become more reliable through training and experience.

Lincoln and Guba (1985) offered that reliability of qualitative research should be viewed in terms of consistency or dependability. That is, rather than expect other researchers to find the same results in subsequent studies, the qualitative case study researcher seeks to establish for the reader the consistency and plausibility of this case. The reader should be able to deduce that, on the basis of the data collected, they would draw similar conclusions to the researcher; that the results are consistent and dependable. Merriam (1988) offered the following techniques for enhancing dependability:

- The researcher should clearly explain the assumptions and theory behind the study, their position regarding the participants, the basis for their selection, a description of the participants and the social context in which the data were being collected.

- The researcher should use multiple methods of data collection and analysis (triangulation).
- The researcher should "leave a trail of the study" (Lincoln & Guba, 1985) by describing data collection, how categories, themes or patterns were derived and how decisions were made throughout the study. Essentially, another researcher should be able to use this as a guidebook for conducting a similar study. (p. 172)

Multiple sources of data were used for this study including audio recordings, observations, interviews, documents and artefacts from both the target students and the teachers. These tools allowed for triangulation of emerging themes. Similarities and differences with the categories of the framework across various sources were also analysed.

The coding categories contributed to providing a trail of this study. By describing how these elements were selected for examination and how decisions regarding the categories were made through the pilot study and final study, other researchers should be able to use these coding categories with other cases. The strength of these categories is that it allows others to examine the plausibility of this study and illustrates that it is firmly grounded in the studies that went before.

Table 9 illustrates the ways in which issues of reliability and validity were addressed in this study. It provides a summary of the points made in the literature discussed previously and links these to the current study.

Table 9

Reliability and Validity Issues in this Study

Issue of reliability or validity	**Strategies for addressing this issue in this case study**	**Data collection tools or procedures which address this issue**
1. Internal validity The extent to which the findings of this study can be matched to reality.	• Observations are conducted over a period of weeks. • Repeated observations are made of the target students and teachers • Interviews occurred one year after observations as well (Merriam, 1988). • Data analysis was on	• Semi-structured interview • Observation notes • Audio of each lesson.

	going and concurrent with data collection. • Researcher was constantly self-monitoring and re-evaluating the progress of the study (Goetz & LeCompte, 1984). • Interviews are tailored to the participants such as creative interviewing tasks. • Observations and interviews are conducted in naturalistic settings (Goetz & LeCompte, 1984). • Multiple data collection tools are used (triangulation) (Merriam, 1988). • Colleagues and supervisors discuss emergent findings and progress of study (peer examination). (Merriam, 1988) • Researcher biases are examined and discussed (Merriam, 1988).	• Researcher's journal • Observation notes • Semi-structured interviews • Observation notes • Observations • Interviews • Document analysis • Audio of each lesson. • Supervisors' comments • Details about researcher addressed in Chapter 1
2. External validity The extent to which findings can be generalised to other cases.	• The establishment of working hypotheses (Cronbach, 1975). • Researcher cross-analyses the target students and teachers with each other. (Merriam, 1988).	• Detailed in Chapter 2 • Observation notes • Audio of lessons • Cross-examination of student work samples. • Interviews with teachers.
3. Reliability The extent to which the findings of the study can be replicated by other researchers.	• Researcher was trained in the use of observation tools and interviews (Yin, 1984). • Researcher piloted the use of observation tools and interviews (Yin, 1984) • Researcher stated clearly the assumptions and theory behind the study (Merriam, 1988). • Researcher described clearly and explained selection of target and case study students,	• Pilot study conducted prior to final study. Data collection tools trialed during this pilot study • TTML provided some training in the use of interviews and in classroom observations • Detailed in methodology • Detailed in methodology

	and described these participants. (Merriam, 1988).	
	• Data collection tools cover multiple sources of data.	• Observations, interviews, documents and artefacts (responses to creative interview tasks)

SUMMARY OF KEY ELEMENTS OF METHODOLOGY

Case study research is an effective way of researching important questions in education (Merriam, 1988). The classroom provides rich data for studying students, teachers and learning. Through studying cases, such as the low-attaining students in this study, it is possible to learn more about the teaching and learning of mathematics, about the perspectives of low-attaining students on particular task types and ultimately, about effective ways of teaching mathematics to low-attaining students.

Data collection tools such as observation, semi-structured interviews and analysis of documents and artefacts provided a rich description of this case. The use of a variety of tools increased the reliability of this study and will allow readers to make their own conjectures and observations as well as test the reliability of those made by this research. The moments and episodes found in the descriptions of the case were filtered through the conceptual categories of three scaffolding practices of this study. This provided a lens through which data were gathered, analysed and reported.

In this chapter, I outlined the data collection phases that included pre-observation interviews with teachers and students, the observation period of six sequential lessons in each class, and the postscript interviews that occurred one year after the observation period. The data collection tools used were those commonly found in case study research, namely interviews, observation and analysis of artefacts. These were described including the creative interviewing tasks used with both teacher and target students. Data analysis occurred through the sub-categories and codes of the three scaffolding practices using NVivo (QSR International, 2005), which was outlined in this chapter. Other data from observation notes, audio of the lessons, student and teacher interviews and artefacts were then used to search for patterns (Stake, 1995). The findings from this analysis will be detailed in following chapters, beginning with data from Phase one of this study, the pre-observation interview data from teachers and target students in Chapter 4.

CHAPTER FOUR – SETTING THE SCENE

INTRODUCTION

As discussed in previous chapters, students who are low attaining face serious difficulties in progressing in mathematics both in their future school lives and their life beyond school, including future employment opportunities (Thomson & De Bortoli, 2007). Students in the middle years of schooling, Years 5 to 8, are particularly at risk of disengagement with mathematics (Luke et al., 2003). This can exacerbate the cognitive difficulties low-attaining students may have, leading to further disengagement and the negative cycle continues. A possible strategy for dealing with this situation is that teachers choose to use tasks with the twin purpose of motivating students as well as encouraging higher order thinking aimed at conceptual understanding (Sullivan et al., 2005). The motivation for such tasks is understanding, and, as Lambdin (2003) pointed out understanding is inherently engaging.

However, as also discussed earlier, tasks with high levels of cognitive demand are complex for teachers to implement (Henningsen & Stein, 1997). Often in the implementation of such tasks the cognitive demand is lessened, particularly for students the teacher views as low attaining (Good, 1981). Some researchers have asserted that cognitively demanding tasks are not appropriate for low attaining students (Ellis, 2005; Engelmann & Carnine, 1982), while others suggest that success for low attaining students with such tasks is contingent upon the teachers' use of particular pedagogies to maximise such success (Sullivan et al., 2006).

The focus of the present study was to examine the classroom interactions of teachers and target low-attaining students while using tasks aimed at conceptual understanding. The aim was to explore the interplay between the teachers' use of such tasks and the teachers' use of particular scaffolding practices. The responses, both cognitive and affective, of low-attaining target students were examined.

There are many influences on how teachers use tasks and implement scaffolding for low-attaining students in mathematics. Beliefs about the nature of mathematics, teaching and learning mathematics, and teaching and learning for low-attaining students of mathematics (Zohar et al., 2001) will all influence the way a teacher behaves in the classroom (Thompson, 1992). Beliefs held by teachers, both consciously and unconsciously, have been shown to have a significant impact on the learning experiences offered to students and the way such experiences are implemented in the classroom (Peterson et al., 1989). It is for these reasons that beliefs of the teachers in this study were sought prior to observations.

Teachers' own personal background in learning mathematics is often where beliefs about mathematics are formed (Bibby, 1999). Such histories influence the kind of mathematics teacher they become. Exploring the teachers' mathematics histories in this study also provided important information through which observations could be filtered.

The low-attaining students in this study also held beliefs about themselves as learners of mathematics, about the nature of mathematics and about teaching and learning mathematics, which were sought prior to observations. These data provide important information about these target students as the impact of teaching and tasks would be influenced by these beliefs (Kloosterman et al., 1996). In addition to exploring affective influences, the target students' cognitive understanding regarding the proposed content of observations was investigated. The researcher administered an initial assessment task with each target student prior to observations aimed at particular mathematical ideas to be covered during observed lessons. These data were intended to provide a baseline of the target students' understanding of a mathematical concept. Therefore these data provided a point of comparison to understanding demonstrated during lesson observations.

This chapter provides a description of the participants in this study. The data drawn on for the discussion during this chapter were collected prior to the observation period; data and findings regarding lesson observations will be discussed in subsequent chapters. The pre-observation data are intended to provide background information and insights into the participants' feelings and beliefs. In the case of the target students, data were also gathered regarding cognitive aspects.

PARTICIPANTS

As outlined in previous chapters, the participants for this study were Ms B and Ms L, both Year 5 and 6 teachers of heterogenous classes in Victoria, Australia. The target students chosen by Ms B were Carl and David, both Year 5 boys. Ms L chose Sophie, a Year 5 girl, and Riley, a Year 5 boy. All the target students were identified by their teacher as operating about 12 months behind their peers in mathematics.

Ms B had five years teaching experience at the time of this study, most of this experience in the upper primary school, Years 5 and 6. Ms B's description of her experiences of mathematics revealed a confidence in her own mathematical ability. "I got by (in maths). I don't think it was ever a problem". There did not appear to be any significant negative experiences that shook this confidence. Ms B stated that she had "no memories of learning maths at school – especially primary school". She did remember an instance in secondary school when she asked her mathematics teacher why a formula worked and the teacher replied, " 'cause that's the formula", which Ms B stated she interpreted as "you don't need to know why, just do it." This incident influenced Ms B's teaching of mathematics. She stated that as a result of this comment, she is "extremely gung ho about making sure kids know why things work".

Ms L had eleven years teaching experience with about five years teaching Year 5 and 6. At the time of this study, Ms L was the "Maths Leader" for the Year 5 and 6 area of her school, leading four other teachers. In contrast to Ms B, Ms L's background in mathematics during her own schooling seemed to be characterised by negative feelings.

> "I hated maths … I don't remember doing maths until Grade 6. I know I was in trouble because I didn't know my times tables. Grade 6 we had maths every morning except Fridays so I loved Fridays. That's when I was made to learn my times tables. It's not a bad memory; it was just hard. Secondary school, same again. I'm not a formula person; I'm not a maths brain. I don't see … I don't always see it."

When it came to learning how to teach mathematics at university, Ms L described her anxiety. "Uni was just … you had to do it. But also when I had to do teaching rounds, I hated … I was always nervous they were going to give me fractions and decimals". Ms L summarised her mathematical history saying, "so not the greatest memories of maths. I've never been confident in it, never particularly liked it". However, Ms L described a change in her enjoyment of teaching mathematics.

> "I started enjoying teaching it about six years ago. I think we were starting to make it a bit more hands on, playing games you could see a bit more success in the kids more immediately than in Literacy because you could get an immediate result, you've taught them something new. Whereas in

> Literacy … Maths is developmental too obviously, but certain topics kids would get it whereas in Literacy it's an on going thing. You don't necessarily go "Oh that kid's got that" but in maths you see a result. And you can see the kids actually getting something and it clicks and they're excited".

It would seem that Ms L's sense of efficacy about successfully teaching mathematics and her students' excitement at learning mathematics were major influences in this change to enjoyment in teaching mathematics. Ms L also seemed to believe that mathematics could be more "black and white" than Literacy and results could be seen more clearly and immediately in mathematics. This might indicate a Platonist (Ernest, 1989) or absolutist view of mathematics as unchanging and static knowledge to be learnt (Lerman, 1983). Describing herself as never being confident in mathematics but being the Maths Leader for her area of the school points to a significant change in attitude. Ms L now seemed more confident in not only teaching mathematics but also leading other teachers to do so. However, it is interesting to note that Ms L still described herself as "not a maths brain". This seems to contradict her role as Maths leader. Perhaps this illustrates the dichotomy described by Bibby (2002): "While their personal, non-teacher identity may wish to declare itself innumerate and distance itself from mathematics, the professional one cannot" (p. 717).

While these personal histories gave some insights into Ms B and Ms L's backgrounds in mathematics, further data were sought regarding the beliefs these teachers held. Sources for such data were a Teacher Beliefs questionnaire taken from Anderson, Sullivan and White (2004), a tasks questionnaire developed by the researcher, the initial teacher interview, responses to the "Mathematics is …" word wheel and responses to a drawing task "a time when I was teaching maths well", adapted from the PPELEM procedure of McDonough (2002b). These data collection tools were described in Chapter 3. Specifically, Ms B and Ms L's beliefs regarding the nature of mathematics, on teaching and learning mathematics and on teaching and learning of low-attaining students in mathematics were sought. The teachers' responses are discussed in turn below.

BELIEFS ABOUT THE NATURE OF MATHEMATICS

In order to gain insights into the beliefs Ms B and Ms L held about the nature of mathematics, the following data collection devices were used:

- Initial teacher interview
- "Mathematics is" word wheel in which the teacher filled in words to complete the phrase "Mathematics is …"
- Teacher beliefs questionnaire (Anderson et al., 2004)

Ms B strongly agreed with statements on the teacher beliefs questionnaire (Anderson et al., 2004) such as "mathematics lessons should focus on problems rather than on practise of algorithms" and "it is essential for students to explore their own ways before being shown the teacher's methods". This suggested Ms B might have held a problem solving view of mathematics. On the "Mathematics is …" word wheel task, Ms B described mathematics as "complex" and "a way to understand the world better". She described aspects such as "shapes, area, temperature, visualising, calculating, understanding and transferring". She also used the words "logical thinking", "number sense" and "transferable". In the initial interview Ms B stated the most important thing she wanted students to gain from her teaching of mathematics was "The ability to transfer the strategies to any aspect of maths, to know why things work and therefore to understand it and know when to apply it not just when they're faced with an equation". This would seem to indicate that she might have held a reasonably static view of mathematics albeit interconnected and logical, a more Platonist view (Ernest, 1989). Perhaps more obvious were the beliefs Ms B rejected. She responded to the statement "Students should learn algorithms before they do application and unfamiliar problems" with strongly disagree. To the statement "mathematics lessons should focus on practising skills" she indicated disagree. This indicated that she rejected an instrumental view of mathematics (Ernest, 1989).

Ms L also strongly agreed with the statement "Mathematics lessons should focus on problems rather than on practise of algorithms". Ms L tempered her response to "It is essential for students to explore their own ways before being shown the teacher's methods", agreeing with this statement but not strongly. A variety of themes arose from Ms L's response to the "Mathematics is…" word wheel. One of these was the role mathematics plays in everyday life revealed in statements such as "all around us – cooking, shopping, driving". Another theme centred around problem solving –

"looking for strategies, asking questions, solving problems, estimations, making generalisations and conjectures". Ms L also listed some topics of school mathematics, "shape, numbers (decimals, whole, fractions) and measurement". The other statements Ms L wrote were concerned with the affective side of mathematics and its relative difficulty. These were "easy for some", "often viewed negatively", "seen as hard", "fun when presented with enthusiasm", and "challenging to teach and learn". In the initial teacher interview Ms L said that what she wanted her students to gain as a result of her teaching of mathematics was "knowledge, confidence, see the connections to real life. I want them to see the importance of it; that it's not just something that they have to do at school and then when they leave school it's over. And be able to apply it to their real life to make generalisations to what we do at school to real life".

It would seem from some of these responses that Ms L held a problem solving view of mathematics (Ernest, 1989). However, other statements seemed to point to a more static idea that mathematics was "knowledge" that could be thought of as the topics of the school curriculum, indicating a more Platonist view (Ernest, 1989). For example, Ms L's comments discussed previously, compared mathematics with Literacy and seemed to suggest that mathematics was a more "cut and dried" subject. It would appear that none of Ms L's responses pointed to mathematics as a changing or evolving entity requiring discovery, a fallibilist view (Lerman, 1983). Her responses indicated also that she believed mathematics was more than rules and formulas, reflected by the instrumental view (Ernest, 1989). Suffice to say, Ms L appeared to hold some aspects of the Platonist view, however she also appeared to reject an instrumental view.

It seems then that both Ms B and Ms L held views of mathematics that fell somewhere between a problem solving view and a Platonist view. Certainly neither appeared to hold an instrumental view of mathematics. Ms B talked about understanding and transferring knowledge and Ms L specifically mentioned the real world applications of mathematics. Ms L also raised issues about affect in mathematics, which Ms B did not. The mixed nature of Ms B and Ms L's beliefs about mathematics is consistent with Beswick's (2005b) findings in which "very few teachers appear to hold beliefs that are exclusively consistent with a problem-solving view" (p. 64). Thompson (1992) also asserted that it is "quite conceivable, indeed probable, for an individual teacher's conception of mathematics to include aspects of more than one of [these views of mathematics]" (p. 132). As well as data regarding Ms B and Ms L's beliefs about mathematics itself, data were sought about these teachers' beliefs about the learning and teaching of mathematics.

BELIEFS ABOUT LEARNING AND TEACHING MATHEMATICS

In order to gather data about Ms B and Ms L's beliefs regarding learning and teaching mathematics, the following collection tools were used:

- Initial teacher interview.
- Drawing task adapted from Pupil's Perceptions of Effective Learning Environments in Mathematics (McDonough, 2002b) in which the teachers were asked to draw and label "a time when they were teaching maths well".
- Tasks questionnaire developed by the researcher.
- Teacher Beliefs questionnaire (Anderson et al., 2004).
- Teachers' mathematics planning notes for a period prior to observations.

Data from the Teacher Beliefs questionnaire showed that Ms B indicated strong agreement and Ms L agreement with the statement that "students can learn most mathematical concepts by working out for themselves how to solve unfamiliar and open-ended problems". Ms B strongly agreed and Ms L agreed with the statement that "it is essential for students to explore their own ways before being shown the teacher's methods" (Anderson et al., 2004).

An examination of Ms B and Ms L's mathematics planning notes revealed information about the way in which each teacher planned for mathematics. Ms B did not provide evidence that she based mathematics lessons or topics on the children's mathematical development. Ms B seemed to feel the need to adhere to her detailed and pre-written planning documents made up of a collection of tasks, proceeding from task to task over the course of the topic much in the way Thompson (1992) described when the locus of control is with the teacher. In contrast, Ms L's planning process appeared to be that the topic of study was planned loosely, with blank spaces for flexibility, so that students' responses to each task could influence future tasks and lessons. This practice seemed to resonate more with Thompson's (1992) description of a classroom where the students have the locus of control.

Ms L talked about a change in her teaching from a focus on processes to a focus on understanding. "I used to put the sums on the board; yep the kids can do it but they couldn't really explain it. Now I'm concerned about the kids explaining it to me, telling me, showing me how they did it, proving it to me, doing it in different ways. The (research project) has brought that out, understanding rather than just the doing". This perhaps indicated that Ms L had adopted a more cognitively based perspective (Peterson et al., 1989).

Ms B used the phrase "a way to understand the world better" (Word wheel task) to describe what mathematics is. In her other responses, Ms B seemed more focussed on mathematics as a school subject than part of everyday life. For example, she said that the most important thing she wanted her students to gain from her teaching of mathematics was "the ability to transfer the strategies to any aspect of maths" (initial teacher interview). The belief that mathematics should be linked to real life seemed to be more strongly expressed by Ms L who recognised the importance of showing the relevance of mathematics to real life. This seemed to stem from her personal experiences somewhat; "'cause I went through the whole 'where do we use maths in real life?' " However she believed that "making it more meaningful" was important for teaching mathematics effectively. "We use it everyday and it's all around us but I make that quite explicit to them. I want them to see the importance of it; that it's not just something that they have to do at school and then when they leave school it's over. And be able to apply it to their real life to make generalisations to what we do at school to real life".

Ms B's illustration of a mathematics lesson in her classroom showed students working in groups discussing, students using manipulatives, students recording their mathematical thinking and learning and the teacher "listening, questioning, assisting and moving around" (teachers' drawing task). Ms L also drew the students prominently in her drawing task with the teacher "asking good questions" to one small group while the other students interacted with each other. This would suggest Ms B and Ms L believed in an active role for students and the teacher's role as responding to the students rather than controlling the lesson more tightly.

According to the results of the Teacher Beliefs questionnaire (Anderson et al., 2004), Ms B and Ms L fell into the "very contemporary" category of teachers due to their disagreement with traditional statements, such as "mathematics lesson should focus on practising skills", and agreement with contemporary statements such as "mathematics lesson should focus on problems rather than on practice of algorithms". The study of Anderson et al. found that contemporary views of teaching mathematics seemed to decline as the year level increased. For example, 70 percent of the contemporary teachers taught year levels K-2 and only 10 percent taught in years 5 and 6. As "very contemporary" upper primary teachers, it would seem that Ms B and Ms L fell into this minority.

These pre-observation data seems to reveal certain aspects of Ms B and Ms L's stated beliefs about teaching and learning mathematics. The locus of control seemed to be mixed between the teacher and the students (Thompson, 1992) and each of these teachers seemed to be learner focussed (Kuhs & Ball, 1986), emphasising students developing their own understanding. From these data on stated beliefs, it could be expected that Ms B and Ms L's mathematics lessons would centre around student strategies, problems, and real world situations, the latter perhaps particularly in the case of Ms L who made several references to mathematics in everyday life. The students would be actively learning while the teacher would take on a facilitating role. Planning in Ms L's class might change day-to-day depending on student responses. A focus on processes, rules and algorithms would not be expected, but rather the focus would be on understanding of underlying concepts.

Having examined Ms B and Ms L's stated beliefs about the nature of mathematics and the learning and teaching of mathematics, data were gathered about the teachers' beliefs regarding effective teaching of low-attaining students in mathematics. Given the present study's focus on low-attaining students, it was important to understand the underlying beliefs that Ms B and Ms L had about teaching low-attaining students as this would impact on the experiences of the target students in their classrooms.

BELIEFS ABOUT EFFECTIVE TEACHING OF LOW-ATTAINING STUDENTS

Teachers' beliefs about student learning are connected to their overall view of intelligence. As discussed in the literature review, Dweck (2000) provided two views of intelligence: entity and incremental. An entity view of intelligence believes that intelligence is largely genetic and static. Such a view would lead to the belief that some students cannot learn all mathematics concepts regardless of teacher actions. An incremental view of intelligence stems from the belief that intelligence evolves and that people can improve their intelligence through effort and the right experiences.

This view would lead to teachers who believe all students can learn mathematics given the right experiences and personal effort.

Ms B responded to the interview prompt “some people say that some children will always have difficulty with mathematics” initially with agreement. She stated “It’s easier for some kids than others. Ultimately Ms B seemed to display an incremental view of intelligence highlighted by her assertion that certain teacher actions could be effective in improving the learning of low-attaining students. “Put in time and effort to find out what kids are engaged by. Everyone can learn the basic skills that they need to function in the world”.

Ms L appeared to hold slightly conflicting views of intelligence with some statements indicating an incremental view: “I don’t think that it’s impossible for them (low-attaining students) to overcome it. I think with the right support, the right activities they can. They just need to do it in a different way” (initial teacher interview). Ms L believed that difficulties students had with learning mathematics were largely a result of inadequate previous experiences or teaching. “But there’s also something that has been missed along the way to give them those opportunities to lessen those gaps” (initial teacher interview). In other responses, Ms L appeared to move more toward an entity view with statements such as “I think because I found it hard, I tell the kids I found it hard, it’s not easy, it doesn’t come easily to everyone” (initial interview), “I’m not a maths brain … I don’t always see it” (initial interview) and “[mathematics is] easy for some” (word wheel task).

In the initial teacher interview Ms L talked about manipulatives, links to real life experiences, specific questioning and adequate time as being key to teaching low-attaining students mathematics. Ms B also talked about time and using materials, but highlighted discussion as a main strategy for assisting low-attaining students. “Sometimes they need to be supported by talking through a strategy with other kids or you. Discussions that you have at the end of a lesson are valuable [for low-attaining students] to be a part of to hear what strategies others are using. [You need] more structure in the way you [the teacher] approach a task, questions you ask them, helping them to get started”.

In response to what is most appropriate for teaching low-attaining students, both Ms B and Ms L indicated strong agreement with the statements “tasks using concrete materials or manipulatives”, “discussing the task and thinking with other students”, “engaging in whole class discussion about the mathematics of the task” and "scaffolding from the teacher while working on the task". Ms B also strongly agreed that "tasks where the mathematics is made explicit", "problems taken from real-world contexts", "investigations using other subject areas as well as maths" and "scaffolding from other students while working on the task" were most appropriate for low-attaining students.

Ms L clearly believed that having the same task for all students was important although this represented a change in her teaching. "I used to have pull out groups [but] now I have open-ended tasks, I have mixed ability groups. [The teacher needs] directed questioning for that child. Not a different task but a different prompting question" (initial teacher interview). She also indicated that "working directly with the teacher on a different, easier task than the rest of the class" (tasks questionnaire) was least appropriate in her opinion for low-attaining students. Ms B also indicated disagreement with this statement but qualified this by writing "sometimes needed but as a last resort".

Ms L indicated that part of effective teaching for low-attaining students involved addressing affective issues. One of her goals for teaching mathematics was that her students would find mathematics "enjoyable. I don't want it to be monotonous for them. Like when I'm planning I'm thinking about how to make it interesting for them. I want them to have the perception that maths is good and fun. I don't want them to think of maths as boring or scary … You need to latch onto the things they can do to build their confidence" (initial teacher interview). In fact she believed that affective issues play a part in students having difficulty with mathematics. "If they've experienced trouble early on, it's quite hard to come out of that lack of confidence and think "I can do this" (initial interview). As Thompson (1992) suggested, it could be the case that this belief stemmed partly from her own experience as a mathematics student. "I think because I found it hard, I tell the kids I found it hard, it's not easy, it doesn't come easily to everyone. It makes me think about what I'm giving to the kids to learn it themselves, what didn't I have when I was at school – support" (initial interview). On the "Mathematics is …" word wheel, two of Ms L's responses were about affective responses to mathematics, "fun when presented with enthusiasm" and "often viewed negatively, seen as hard".

In contrast Ms B focussed less on feelings in her responses to initial data collection tools but rather on issues of engagement. For example, regarding low-attaining students, Ms B said "If a child's not interested it's going to make it difficult.

[The teacher should] put it time and effort to find out what kids are engaged by …” (initial interview). Ms B seemed to think low-attaining students could progress given “encouragement and [if they are] supported in enjoying it and understanding the processes…” (initial interview).

When characterising the low-attaining target students in their class, each teacher focussed on different aspects of these students. Fitting with the previous discussion, Ms B tended to focus on engagement and motivation issues “[He is] lacking confidence in most areas of the curriculum. Has trouble with reading. Half the time tries really hard, half the time pretending to try really hard, learned strategies to cover for the fact that he doesn’t really know what’s going on. Main part is confidence … he disengages pretty easily, hard to keep him motivated” (initial interview). Ms L focused more on academic difficulties “There are a lot of gaps now coming through … Some of the basics are just actually not there. She doesn’t make connections; she can do some things on their own but it’s not crossing over … I’ve noticed when we do number work he’s not quick; he doesn’t always understand the task. He had to see it, can you explain it again? He’ll be the first to put his hand up. He seems to be struggling with times tables” (initial interview).

Both Ms B and Ms L expressed the belief that low-attaining students in mathematics should be given time, materials, the opportunity for discussion and the same task as their classmates. They also recognised the important part that affective aspects, such as confidence, and motivation and engagement play in the learning of these students.

Having discussed the teachers, I will now move on to discuss the target students from Ms B’s class and then Ms L’s class in more detail, reporting on the students’ responses to initial data collection from firstly an affective perspective and then considering cognitive factors.

TARGET STUDENTS

Data from the target students collected prior to lesson observations were gathered in an interview that used the following tasks as springboards for discussion:

- “Draw a mark on this line showing how good you feel you are at Maths”.
- “Mathematics is” word wheel.
- “Fractions” word wheel for Carl and David.

- "Multiplication" word wheel for Sophie and Riley.
- Pupils Perception of Effective Learning Environment in Mathematics (McDonough, 2002b) drawing task and description of "a time I was learning maths well".
- Learning Mathematics Student Survey, adapted from *Task Types and Mathematics Learning* project student survey (Sullivan et al., 2010).
- Cognitive assessment task – draw 3/4, 2/5 and 3/2 and order these from largest to smallest for Carl and David and multiplicative thinking "Fish task" (Clark & Kamii, 1996) for Sophie and Riley.

The vital role of beliefs, feelings and attitudes toward mathematics and mathematics learning has been highlighted in the literature review (Gomez-Chacon, 2000; McLeod, 1988). Hence, the present study was focused on the affective as well as cognitive responses of the target students in this study. I will discuss the two low-attaining target students from Ms B's class, Carl and David, and then Sophie and Riley from Ms L's class. For each student, I will begin examining data for affect followed by cognitive data.

Carl was aged 10 years for the major part of this study and in Ms B's Year 5 class. Carl seemed to be similar to most of the boys in his class. He was interested in sport, and enjoyed talking and playing with his friends. Carl was a fairly extroverted boy and was usually happy to talk with the researcher. On one occasion Ms B had cause to reprimand Carl just prior to a post-lesson interview and for this interview he appeared less enthusiastic than on most occasions.

Ms B described Carl as "lacking confidence in most areas of the curriculum". She also described his avoidance behaviours and the ways in which Carl attempted to appear to understand when he was actually confused. "Half the time he tries really hard, half the time he's pretending to try really hard. He has learned strategies to cover for the fact that he doesn't really know what's going on" (initial teacher interview).

In his responses during the pre-observation interview, Carl showed that his feelings were foremost in his mind when considering mathematics and his learning of mathematics. The "Mathematics is" word wheel task was an open-ended prompt that invited the students to offer words that described how they perceived the nature of mathematics. For this task Carl responded purely with emotions writing "boring if you can't understand it", "fun", "exciting", "excellent", "sometimes not fun" (Mathematics is word wheel). When responding to the subsequent task, a word wheel focussed on the current unit of work the class was working on, "Fractions", Carl repeated this response writing "fun", "exciting" and "excellent". It appeared that Carl did not interpret the question in the manner in which it was intended, that is, to gather data on what he saw as the nature of mathematics. Despite this, Carl's responses are revealing. For Carl, how he felt about mathematics was perhaps more important than what he was learning. His feelings about mathematics appear inextricably linked to his learning of mathematics. The strongly positive words Carl chose were interesting. There might have been an element of "pleasing the researcher" behaviour at play but observations of Carl during lessons would gauge if this positive response to mathematics were consistent with his responses to mathematics lessons.

Carl's response to another task, "Draw a mark on this line showing how good you feel you are at Maths" also focussed on his feelings about mathematics. His mark was approximately halfway along the line from "not good" to "really, really good". His comment that "sometimes I don't like it but sometimes I like it" supported his placement, indicating a mixed feeling about learning mathematics though perhaps does not address the question of this task that focussed on his perception of his ability as a mathematics learner. When further questioned about when he likes mathematics and when he doesn't, Carl responded that he liked "playing games" but didn't like "sitting down". The type of mathematics task had a strong bearing on how Carl felt about learning mathematics. Active tasks like "playing games" seemed to be viewed positively by Carl whereas more passive "sitting down" drew negative responses.

In order to examine Carl's potential learning, data were collected from an assessment task before the observation period began. This task assessed concepts that were to be addressed during the observation period. Concepts surrounding fractions, then decimals and percent were the focus for Carl's class during the observation period. The initial assessment task asked the following:

Draw 3/4, then 2/5 and then 3/2.

Nominate the largest or biggest fraction, then the second largest and the smallest fraction of these three fractions.

These fractions were chosen as they represented non-unitary fractions and improper fractions. More specifically, two fifths was chosen because the students would be unable to use a "halving" strategy (Empson, 2003) to solve this and would need to attend to the number of pieces in the whole or the denominator. The second part of the task assessed the student's ability to compare fractional amounts and order fractions.

Carl confidently drew a circle, divided it into four roughly equal parts and coloured three of these parts in response to "draw three quarters". I then asked him to label it. Carl seemed uncertain and said "Just three q?" I said "Do you know how to write it in the fraction number?" Carl was silent, so I told him to go ahead and label it the way he thought and he wrote "3Q".

The next part asked Carl to draw two fifths. He drew a rectangle, divided it in half then into five parts making ten altogether. Then he stopped and said "oh". He scribbled out five of the tenths then coloured two of the remaining five parts. I prompted him to label it and he wrote "2 5ths".

I then asked Carl to draw "three halves". He hesitated and said "Like three equal bits?" and I said "Mmm. Whatever you think." Carl drew a circle and divided it into three roughly equal parts. He did not shade any of the parts. He then wrote "3 halfs" [*sic*] beside it.

Carl was able to draw and shade the fractional parts for 3/4 and 2/5. Labelling each of these fractions proved difficult for Carl. He did not write conventional fractions with numbers and a vinculum but instead wrote words or abbreviations. This task revealed that Carl did not know how to write fraction numbers, though the expected standard for the end of Year 4, for Carl six months earlier, stated that "[students] develop fraction notation" (Victorian Curriculum and Assessment Authority, 2006). These results also showed that Carl did not understand improper fractions, though according to the state-wide curriculum, this was not expected until the end of Year 6 (Victorian Curriculum and Assessment Authority, 2006). Carl did know that fractions were equal sized parts of a whole and he did attend to both the denominator and the numerator in 3/4 and 2/5.

The second part of the task asked Carl to indicate which of the fractions he had drawn was the largest. It was an expectation of the state curriculum that students by the end of Year 4 should be able "to compare simple common fractions such as 3/4 > 2/3" (Victorian Curriculum and Assessment Authority, 2006). Carl hesitated and said, "So like two fifths is higher than all of them?" Again I made a non-committal sound so he marked 2/5 as the largest. He then put 3/2 as the next largest and 3/4 as the smallest fraction.

It is difficult to draw conclusions about Carl’s thinking on this ordering task. It would seem that he might have been using *whole number thinking* (Steinle & Stacey, 1998) by saying, “two fifths is higher than the others”, and perhaps reasoning that five is larger than three or four. It is unclear why he marked 3 halves as the next biggest as whole number thinking would say that four (quarters) is larger than three. As stated above, on the “fractions” word wheel task, Carl wrote the same words to the “Mathematics is” work wheel – “fun, exciting, excellent” which again, were about his feelings and not the content or concepts of fractions. Carl did not have well developed concepts of fraction notation, improper fractions or ordering fractions. This would be expected to make an impact on his ability to succeed in the fraction tasks within the lessons to be observed. I will now discuss David, the other target student from Ms B’s class.

David was aged 10 years during the major part of the study. He was a quieter student than Carl during lessons, although was described as “quite extroverted” by his teacher. David seemed to enjoy talking and working with a select few of his classmates. When placed in a pair with a student with whom he was not particularly friendly, David was more reticent and less likely to discuss the task than when he worked with a friend. David seemed to enjoy the attention of talking with the researcher and was quite eager to participate in post-lesson interviews, often asking the researcher if he was going to “do the thing with the faces” at the beginning of a lesson.

Ms B described David as “low in most areas of the curriculum, has problems with reading” (initial teacher interview). She believed David lacked confidence describing how he “avoids sharing at all costs” and “puts up a barrier” to learning mathematics. She identified the importance of scaffolding David in the initial stages of a task and talked about “having to ask basic questions to help him get started”. She also believed that David lacked perseverance and avoided mathematics learning at times saying, “he disengages pretty easily, it’s hard to keep him motivated. If he can get away with not doing it, he will”.

David’s response to the “Mathematics is” word wheel suggested that to some extent, he held a narrow view of mathematics. He wrote “divided by”, “division”, “fractions”, “multiplying” and “measuring” around his word wheel. The word “measuring” suggests a slightly broader view of mathematics than purely number concepts although the majority of his responses were focussed on division and multiplication operations.

David seemed to believe that he was not a proficient learner of mathematics. However, he was not completely negative about his ability to learn mathematics. The task asked him to "draw a mark on this line showing how good you feel you are at maths" on a line that read "not good" at one end to "really, really good" at the other. David's mark was about one third along the line closer to "not good". David spoke about specific concepts of mathematics when discussing his response to this task. He said, "I feel good about taking away, sharing and fractions but bad about times (multiplication tables) and measuring weight".

David was given the same assessment task as Carl about fractions:

Draw 3/4, then 2/5 and then 3/2.

Nominate the largest or biggest fraction, then the second largest and the smallest fraction of these three fractions.

David had trouble drawing 3/2, drew two thirds, writing "2/3" when he labelled this drawing. He also did not shade parts to show the fractions 3/4 or 2/3 but just drew the whole and parts. David did shade 2 parts out of 5 for 2/5. He incorrectly ordered the fractions from 2/3 (supposed to be 3/2) to 3/4 and then 2/5 when asked to order from largest to smallest. Of course the 3/2 is the largest fraction but David didn't draw this, he drew 2/3. However, in just comparing 2/3 and 3/4, David indicated that he thought 2/3 was the larger. As discussed with Carl's results, state wide expectations at the end of Year 4, six months earlier for David, indicated that students should be able to "compare simple common fractions such as 3/4 > 2/3" (Victorian Curriculum and Assessment Authority, 2006) which David couldn't do in this instance. Having said that, the data of Clarke and Roche (2009b) indicated that many Victorian students at this stage would not be able to give a reasonable explanation for why one of these fractions was larger than the other.

On the word wheel task, "fractions", David wrote "a quarter of something, a half of something, a third of something, a fifth of something, fraction numbers" and drew a circle in seven parts, shaded 3 and wrote "3/7". This demonstrated some understanding of fractions as part of "something" and used fractional language.

These results show that David may understand that fractions are one whole partitioned equally as shown by his drawings. He might have been operating with fractions at a unitary level. This may explain his ordering with thirds being the largest, then quarters then fifths. This shows that he may understand that fractional parts are larger if the denominator is smaller and the whole is partitioned into fewer parts. However, David did not seem to attend to the number of these parts shown in the numerator. David reversed the digits of 3/2 to make a proper fraction, 2/3, indicating perhaps his lack of understanding of improper fractions. These misconceptions would be expected to make an impact on David's ability to succeed in the fraction tasks on which the students worked during lesson observations.

I will now discuss the target students from Ms L's class who, it will be recalled, were at a different school to Carl, David and Ms B.

Sophie was a Year 5 student in Ms L's class. She was described by Ms L as "Bubbly, lovely, sweet … She won't be shy. She's got a few self-esteem problems" (initial teacher interview). Confirming this description, Sophie was always happy to talk to the researcher and indeed seemed to thoroughly enjoy the attention. She worked hard in class and genuinely wanted to succeed in mathematics. She was a quiet member of the class and rarely spoke during discussion either, whole class or when in pairs.

As reported earlier, Ms L described her difficulties saying, "There are a lot of gaps now coming through. Mum has been doing some work with her at home. Some of the basics are just actually not there. She doesn't make connections. She can do some things on her own but it's not crossing over" (initial interview).

Sophie put her mark closer to "not good" on the "how good you feel you are at Maths" line task. She said "I put myself there because I'm not that good at maths. I get confused very easily". Sophie also disagreed with the statement that "I like the maths questions to be hard enough that I don't know how to do them straight away" on the Learning Mathematics survey.

Sophie's response to the "Mathematics is" word wheel was the four operations, "numbers and symbols" and measurement. This perhaps revealed her view that mathematics is school mathematics and primarily about numbers. Sophie's response to "multiplication is" word wheel was "times tables, the symbol x, 6x6=36, timesing [*sic*] 1,2,3,4,5,6,7,8,9,10,11 and 12, doubling". Three of these responses are about times tables or multiplication facts. Times tables is clearly an aspect of mathematics in the forefront of Sophie's mind and were certainly a major focus in her classroom. This was also revealed by her response to the cognitive task for multiplicative thinking.

Sophie was given the following multiplicative thinking assessment task developed by Clark and Kamii (1996) as the lesson observations with her class were about multiplication of whole numbers:

> There are three different sized fish for this task. Fish A is 5 centimetres long, Fish B is 10 centimetres and Fish C is 15 centimetres. Due to each fish's size, fish B eats two times as much as fish A and fish C eats three times as much as fish A.

This task asks students to find the number of pieces of food (counters) each fish eats compared to the other fish when the number of pieces of food changes. For example, if fish A eats 2 pieces of food, how much does fish B and fish C eat? Counters and three cardboard fish of the correct size were provided for this task for students to use if they chose.

Sophie used times tables language from the beginning of the task even though I was careful not to use such language. In the first question fish A eats one and Sophie said B would eat 2 and C would eat 3 "because 1 times 1 is 1, 2 times 1 is 2 and 3 times 1 is 3". In subsequent questions, Sophie used this language again, "3 times 2 is 6. It goes 3, 6, 9, it's the 3 times tables, 3 doubled is 6, 6 add another 3 makes it 9". This emphasis on multiplication facts probably stemmed from the emphasis placed on learning times tables in her class. Sophie was able to answer all five questions on this task correctly with only one error.

For the second question, "if B eats 4 chips, how many does A and C eat?" Sophie said A = 2, B = 4 and C = 8 because "Half of 4 is 2, B is doubled A's food, C is 3 times bigger than A which makes it 8". This appeared to be a calculation error with what 3 times more than 2 is, and Sophie did not make this error again.

Sophie used the counters for every question, carefully laying them in a line to the corresponding fish even though in her explanations she identified patterns in the numbers that could solve the questions. "If there's 12 that would make it in 4 times tables, 4, 8, 12 … C is of course 3 times bigger than A, 3 times 7 is 21. B is number 2 which would count as 7 times 2 which is 14". It is not clear whether Sophie was relying on modelling the situation with the materials before she could then make observations about patterns or if she believed she was expected to use them even if she didn't require them. I used the cardboard fish and counters to illustrate the task but did not indicate that Sophie or Riley were required to use them. The materials were on the table and both chose to use them.

Sophie's performance on this task would place her between Clark and Kamii's (1996) "Level 4 A- multiplicative thinking but not with immediate success" and "Level 4B – multiplicative thinking with immediate success". Students at these levels are able to demonstrate multiplicative thinking with increasing success. Clark and Kamii found that 47 percent of the fifth grade students in their study were at this level.

Riley was the other target student and was in Year 5 in Ms L's class. He was new to the class having joined them seven weeks prior to this study beginning. Riley was a friendly boy who enjoyed working with classmates. He tried hard to complete tasks well and meet Ms L's expectations. Ms L said this about Riley: "I've noticed when we do number work he's not quick; he doesn't always understand the task. He had to see it, (he says) can you explain it again. He'll be the first to put his hand up. He seems to be struggling with times tables" (teacher interview).

For the task "draw a mark on this line showing how good you feel you are at Maths", Riley put his mark towards "not good". He explained, "Sometimes I need to think for about ten minutes about what is … like on worksheets". For the "Mathematics is" word wheel, Riley wrote, "something to help in everyday life, subtraction, plus, times tables". Riley disagreed with the statement "I like the maths questions to be hard enough that I don't know how to do them straight away" and strongly disagreed with the statement "I prefer to work on maths questions by myself" (Learning mathematics survey).

In the multiplicative thinking "Fish task" (Clark & Kamii, 1996) Riley answered the questions correctly although self corrected the second question that he originally answered additively.

For the first question, Riley explained his answer; "Fish A can only eat 1 so this one can eat 2 times more so it can eat one more this can eat three times more so it can eat one more out of all of them". This explanation suggests a more additive approach with phrases such as "eat one more". This may help to explain Riley's error with the second question, "if fish B eats four fish, how many would fish A and fish C eat?" Riley gave three counters to A and five counters to fish C. His explanation was; "Because if that (fish A) can eat one times more that's 3 so it can eat one more (B) … I made a mistake. I think that would be that (A = 2) 'cause that can eat double his amount so that would be that (B = 4). How much was there? (4) So he (fish B) should be able to eat 8 and fish C would be able to eat 3 times what A can eat so that is 9". It is clear that Riley is somewhat confused about this question and initially used the same approach as question one saying A eats one more (should be less) and giving A three fish for B's four fish. He then self corrected and realised that B eats "double" what A eats so then assigned A two fish which is correct. However, then Riley doubled fish B's amount of four to eight then again used additive thinking to give fish C one more, that is nine fish.

By this time Riley appeared confused so I took away the counters and asked the question again, giving fish B four counters. This time Riley immediately gave fish A 2 and said, "Because that's double what he can eat (fish B)". He then asked, "Does C need to eat double what B eats?" I restated that fish C eats three times as much as fish A. Riley then gave fish C six counters because "that's three times two equals six". This showed that Riley had some difficulty at least initially, in using multiplicative rather than additive thinking. For the remainder of this task however, Riley used multiplicative ideas and his explanations for the last three questions were centred on "patterns". "C has 9 pieces of food and B has 6 pieces of food and A has 3 pieces of food so it goes 3, 6, 9 in a pattern". In the final question Riley linked times tables identifying the pattern as the seven times tables. The class was focussed on learning times tables at the time of this study so this response was not surprising although Riley's late realisation was in contrast to Sophie who immediately used times table type language from the beginning of this task.

It appeared that Riley achieved similar results to Sophie using Clark and Kamii's (1996) descriptions of multiplicative thinking. Riley's responses indicated he was operating between "Level 4 A- multiplicative thinking but not with immediate success" and "Level 4B – multiplicative thinking with immediate success". As stated previously, 47 percent of the fifth grade students in Clark and Kamii's study were at this level.

Conclusion

It would seem that the target students in Ms B's class, Carl and David, confirmed her assessment of them as being low attaining. Both Carl and David had difficulty representing fractions accurately and showed a lack of knowledge of improper fractions. Ms L's target students, Sophie and Riley, demonstrated multiplicative thinking on the Fish task and did not illustrate their low-attaining status as clearly. This is perhaps a limitation of the task, which was perhaps a little easy, or the use of materials enabled them to achieve success. However, it is worth noting that both Sophie and Riley had moments of confusion and, in the case of Riley, demonstrated additive thinking albeit briefly.

Affectively, all four students identified themselves as having difficulty learning mathematics. Riley used as evidence, that he took a long time to answer mathematics questions, “about ten minutes”, Sophie said she “got confused”, David felt confident about some aspects of mathematics but not about “times tables or measuring weights” and Carl talked about liking some aspects of mathematics, such as games, but not others such as “sitting down”.

Ms B and Ms L both professed beliefs about mathematics being problem solving and also defined by static topics studied at school such as the operations and calculating. Each also identified the role mathematics plays in everyday life and Ms L gave examples such as shopping. Ms L identified affective responses to mathematics such as “fun” or “difficult to learn and teach” while Ms B described mathematics as “understanding, logical thinking”. Each felt that problem solving, discussion, using manipulatives, open-ended tasks and students devising strategies were essential elements of teaching and learning mathematics. In regards to teaching low-attaining students in particular, Ms B and Ms L emphasised time, using materials, working with others and discussion as key for these students. Ms L also talked about supporting low-attaining students’ self esteem and confidence.

What emerges from this pre-observation data is a picture of two dedicated teachers, striving to teach mathematics for understanding. Ms B had always enjoyed mathematics learning and teaching and was confident about her ability to do so effectively. Ms L spoke about her own negative experiences as a student of mathematics and a change in her enjoyment and confidence in teaching mathematics due to her involvement in action research projects and professional development about contemporary mathematics teaching.

The picture of the four target low-attaining students is varied. All of these students demonstrated reasonably optimistic views of themselves as mathematics learners as none rated themselves as “not good at all” at learning mathematics. Even the reasons these students gave for their difficulties could be considered reasonably transitory, for example, taking time to answer or having trouble with only some aspects of mathematics. All professed to enjoy at least some mathematics lessons, with Sophie and Riley citing examples of their favourite mathematics lessons from their current teacher. However, Carl and David demonstrated substantial confusion with fraction concepts that would be considered to be foundational for future progress. Sophie and Riley were able to use multiplicative thinking during a relatively simple task and so their abilities with more difficult multiplication tasks to be used during observations were of great interest. The following chapter describes these observations and examines in detail the observation phase of data collection and the postscript interviews. With the background knowledge of the professed beliefs of the teachers and students, and information regarding cognitive issues that this chapter has outlined, data from observations can be viewed with this in mind.

CHAPTER FIVE - SCAFFOLDING PRACTICES, TASKS AND THE IMPACT ON LOW-ATTAINING STUDENTS

INTRODUCTION

In Chapter 1, I introduced the issue this study sought to explore – teaching low-attaining students in mathematics using conceptually challenging types of tasks. The importance of research regarding effective teaching of low-attaining students is critical with international studies such as the Program for International Student Assessment (PISA) 2006 identifying an achievement gap between low-attaining students and students of average attainment (Thomson & De Bortoli, 2007). This gap widens increasingly over the years of schooling, contributing to less students studying mathematics at higher levels including tertiary study (McPhan et al., 2008).

The issue of disengagement was also discussed, as upper primary to lower secondary years of school, termed "the middle years", are critical times when students can disengage in mathematics (Luke et al., 2003). Using types of mathematics tasks which are engaging, challenging and potentially encourage higher order thinking has been proposed as a way to combat disengagement while improving the quality of mathematics teaching (Clarke, 2009; Henningsen & Stein, 1997; Hiebert & Wearne, 1993). The TTML project (Peled et al., 2009) investigated the use of three types of tasks in middle years classrooms that were both potentially engaging and challenging mathematically: tasks using manipulatives, representations or tools, tasks based on real world contexts and open-ended tasks.

The present study is part of this larger project but focussed on the experiences of low-attaining students when taught mathematics using the types of tasks from the TTML project. Through a review of the literature in Chapter 2, scaffolding emerged as a possible "bridge" between these challenging tasks and the learning of low-attaining students. With some research calling for more directive approaches to teaching low-attaining students (Ellis, 2005; Gersten & Carnine, 1984; Kroesbergen et al., 2004), I sought to explore the question of how low-attaining students would respond to these potentially more challenging tasks while their teachers used particular scaffolding practices. Scaffolding can take many forms but I chose to focus on three forms that were directly relevant to this study. The first form of scaffolding of focus in the present study was discussion, as this is a common way teachers offer scaffolding (O'Connor & Michaels, 1993; Siemon & Virgona, 2003). The second scaffolding practice was the use of manipulative materials and visual representations (Cass et al., 2003; Maccini & Ruhl, 2000), as this is often advised for low-attaining students. The third scaffolding practice addresses the conceptual nature of the task types and is where the teacher pays explicit attention to underlying concepts (Hiebert & Grouws, 2007).

As discussed in Chapter 3, the present study is a case study. Case studies can offer detailed descriptions of a situation and allows the researcher to study a particular case and "come to know it well" (Stake, 1995, p. 8). By coming to know a case well, phenomena can be explored, described and reflected upon, as the case exists in a particular situation. However, in the particular, general observations can also be found as readers "find a commonality of process and situation" (Stake, 1995, p. 7). By conducting a case study of two mathematics classrooms, two teachers and two low-attaining students in each classroom, the phenomena of scaffolding during conceptually challenging mathematics tasks could be explored. In addition, there are few descriptions of scaffolding by teachers in heterogenous classrooms in the "messy reality" of a class of approximately 25 students engaged in challenging mathematics tasks. Even more scarce are descriptions of low-attaining students' experiences in such classrooms and their responses to their teachers' efforts to support their learning. Though this study aimed to build a "thick rich description" (Merriam, 1998, p. 38) of a particular case, it is hoped that the readers will be able to recognise elements of this case within the sphere of mathematics classrooms more broadly.

In Chapter 4 I explained that the teachers chosen for this study had not been identified as highly effective and did not have postgraduate qualifications, although both teachers had been recently involved in mathematics education research projects. The teachers varied in their years of teaching experience. Each teacher volunteered to be part of the research. In this way, relatively "normal" classrooms were studied with relatively "normal" dedicated teachers teaching mathematics the best way they could. Similarly, the two low-attaining students from each classroom were not diagnosed with particular disabilities or pre-existing conditions that accounted for their low attainment. They were students that their teachers identified as low attaining informed by their own observational assessment and data from national testing. Studying two classrooms allowed for comparison, for similarities and differences to be identified and explored. Exploring the responses of four low-attaining students allowed for the experiences of a more diverse range of students to be studied because, although all were low attaining, each was unique and had idiosyncratic needs and preferences (Gervasoni, 2004).

The purpose of this chapter is primarily to describe, in as rich detail as possible, the case under research so that readers may make interpretations of the data, evaluate the observations I make as researcher and perhaps recognise something of their own experience in the experiences of these others. In this chapter, I will describe the mathematics lessons of the two teachers, Ms B and Ms L, and in particular their use of the three scaffolding practices that formed the focus of the present study: the use of discussion, the use of manipulative materials and representations and explicit attention paid to mathematical concepts. In addition, the teachers' use of specific types of mathematics tasks: tasks using manipulatives, representations and tools; those using a realistic context; and those that are open-ended, will be considered. Perhaps most importantly, these aspects of mathematics teaching, scaffolding and tasks, will be analysed regarding the impact such use had on the learning and affective responses of two low-attaining students from each classroom. To begin, I will describe the overall form that mathematics lessons took in each classroom before describing each scaffolding practice in terms of the teacher's use, and the cognitive and affective responses of the low-attaining students.

THE MATHEMATICS CLASSROOMS OF MS B AND MS L

During the lesson observation period, Ms B's lessons were focussed on rational number including fractions, decimal numbers and percent while Ms L's lessons focussed on multi-digit multiplication.

Prior to lesson observations, Ms B and Ms L were each asked to describe a typical mathematics lesson in their classrooms. These descriptions from the initial teacher interviews were given in the previous chapter. Both descriptions followed a similar structure for a mathematics lesson; that is they began with a whole class focus, moved into small groups, pairs or individual work then concluded with the whole class. The *Victorian Essential Learning Standards* (Victorian Curriculum and Assessment Authority, 2006), the main source of curriculum advice for Ms B and Ms

L, advised such a structure, stating that teachers should use whole class, small group or independent activities and whole class reflection at the end of lessons. This structure was also reflected in the United Kingdom's National Numeracy Strategy (NNS) that advocated numeracy lessons should have three parts: mental or oral work as a whole class, a main teaching session in groups or individually, and a plenary with the whole class at the end of the lesson (DfEE, 1998). Stein, Engle, Smith and Hughes (2008) summarised a similar "vision" of mathematics as, "in this vision, students are presented with more realistic and complex mathematical problems, use each other as resources for working through those problems and then share their strategies and solutions in whole-class discussions that are orchestrated by their teacher" (p. 315). However, there were also differences in the structure of Ms B and Ms L's mathematics lessons.

Ms B described three main stages in her mathematics lessons: setting up the problem "making sure their head's in the right space", students working on the problem, and discussion of the problem as a class at the end of the lesson. The lessons observed in Ms B's classroom followed this structure reflected in planning notes containing "Stage one", "Stage two" and "Reflection focus". Four out of six lessons comprised one task that formed the focus for the whole lesson. The remaining two lessons were each comprised of two tasks. The observed mathematics lessons in Ms B's classroom averaged 82 minutes in total. Ms B's planning notes for mathematics prior to lesson observations commencing indicated that this format and length of mathematics lessons was typical of Ms B's mathematics program.

Ms L's mathematics lessons differed in that three lessons contained four tasks, two lessons contained three tasks and one lesson contained two tasks. As Ms L described in her initial interview, the first task was usually a game focussed on quick recall of multiplication facts. These tasks averaged 7 minutes. This was followed by a reasonably short task, for an average of 12 minutes that focused on multiplication, and then the main task or "Learning Activity", comprised the most time in the lesson, on average 18 minutes. Ms L's planning notes reflect these stages with her planning proforma containing the following sections: "Counting warm up (3 mins), Tools Session (10 mins), Tuning In/Whole Class and Learning Activity". On average, mathematics lessons in Ms L's classroom were 70 minutes long. Similarly to Ms B, Ms L's mathematics planning notes indicated that this format and length of mathematics lessons was indicative of her usual program.

The main focus of my observations and analysis were the major task or tasks of the lesson. I have not focused on the short games Ms L played at the beginning of many lessons as they formed a small part in the overall lesson. In addition, Ms B did not conduct such games and therefore comparison was not possible. Furthermore, an important part of the present study was concerned with the teachers' use of particular task types as part of its inclusion in the TTML project (Sullivan, Clarke, & Clarke, 2007). For this reason, tasks that fell into the three categories of task types from this project were chosen as the main focus of analysis.

The following tables summarise the tasks from which data were gathered and analysed, from Ms B's classroom (Table 10) and Ms L's classroom (Table 11), respectively. These tables outline briefly each task, and which of the TTML task types each task would be classified as. The TTML task types often overlapped so I have categorised tasks by their initial classification understanding that many tasks could be considered "using manipulatives, representations and tools" as well as "open-ended", for example. The class organisation or how students mainly worked, such as whole class, small groups, pairs or independent work, and the total time for the task is also outlined in Tables 11 and 12. These tables will show the variations in the observed tasks such as task type, total time and class organisation. The variations in the tasks are such that direct comparisons between tasks may not be possible and the teacher's actions or students' responses must be considered with such differences in mind. These specific variations between the tasks used by Ms B and Ms L are discussed after the tables.

Table 10

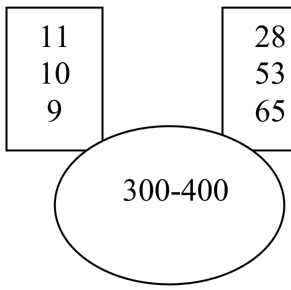
Tasks from Ms B's classroom

Task	Task type	Class organisation	Time
Fraction Rods Using Cuisenaire rods, students answer six questions that ask them to compare fractions and the whole, fractions and fractions and wholes and fractions.	Using manipulatives, representations or tools	Independent	58 minutes
Fractured Figures Each group has pieces of a puzzle that makes four identical shapes. Each piece is a different size or fraction of the whole puzzle. Groups need to make the shapes then name the fraction for each piece.	Using manipulatives, representations or tools	Small groups	69 minutes

Sticky Numbers and Number line. Students have a number on their back, either a fraction or a decimal. They must ask questions in order to find out what their number is. These numbers are then ordered on a number line.	Using manipulatives, representations or tools	Whole class	47 minutes
Sorting Cards Students are given cards to sort out in any way they choose initially then into those representing the same value. The cards are a mixture of fraction numbers, decimal numbers, fraction times, decimal weights, and pictures.	Using manipulatives, representations or tools	Pairs or groups	41 minutes
Dominoes Students are given a set of dominoes with fractions, decimals and percentages on them. They are asked to match them to form a circle and then to design their own dominoes.	Using manipulatives, representations or tools	Pairs	65 minutes
Fruit and Vegetables Students are asked to imagine they work in a grocer's shop. They need to put the fruit and vegetables into bags so that the bag weighs less than 1 kilogram. Some are fractions and some are decimal parts of a kilogram.	Real world context	Pairs	64 minutes
Decimat Students play a game to colour one whole using a six-sided dice and a six-sided decimal dice with */10, */100 (x2) and */1000 (x3). They use the decimat, a rectangle divided into tenths, hundredths and thousandths to play.	Using manipulatives, representations or tools	Pairs	69 minutes
Construct a Sum Using the digits 1, 3, 4, 5, 6 and 7 and two blank fractions, students must construct a sum where two fractions add to just less than 1.	Open-ended	Independent	21 minutes

Table 11

Tasks from Ms L's classroom

Task	Task Type	Class organisation	Time
Missing numbers 12 X__ = 48 64= __ X___ 26= (__X___) + ___	Open-ended	Whole class	14 minutes
Find the Pairs Each player has 10 cards. They put these cards into 5 pairs. They multiply each pair to make a product. Their partner is given only the products and must find the possible factors to find the pairs.	Using manipulatives, representations or tools	Pairs	33 minutes
Target Multiplication Choose a number from each box that multiplies to fall within the target range. 	Using manipulatives, representations or tools	Whole class	12 minutes
Highest Product X □ X □ □ Using numbers 9,8 and 7, place these to make the highest possible product.	Open-ended	Independent	39 minutes
Veggie Patch I have 60 plants to plant in equal rows in my veggie patch. How could I plant them?	Real world context	Whole class	11 minutes
Help Julia Julia is having trouble multiplying 45x26. What advice or strategies can you give her that might help?	Real world context	Independent	46 minutes
Array of 48 I have made an array using 48 tiles. What might it look like?	Using manipulatives, representations or tools	Whole class	10 minutes
Athletics Day	Real world context	Independent	45 minutes

Today is Athletics Day. 13 schools are going. Mr Grimes in an organiser and wants to know how many children are going. There are about 75 children from each school. (Other questions also about Athletics Day involving multiplying and dividing)	

As shown in the above tables, the tasks from each classroom differed in the time allocated by Ms B and Ms L. For Ms B, the shortest task, Construct a Sum was 21 minutes long and the longest task, Decimat, went for 69 minutes. In Ms L's class, Array of 48 was the shortest task at 10 minutes long, while Help Julia was the longest task at 46 minutes. These differences in time are important to consider. For example, Hiebert and Wearne (1993) found that classes that worked on fewer problems during mathematics lessons, therefore spending more time on each problem, were more effective in increasing student learning than classes where more problems, of a shorter duration, were completed. Henningsen and Stein (1997) cited inappropriate amounts of time, both too long and too short, as a major factor in tasks declining in their level of cognitive demand. Time for the task potentially influences the amount of scaffolding the teacher can provide and the type of scaffolding, the length and depth of discussion that is possible, opportunities for low-attaining students to engage and work on the task as well as opportunities to disengage.

Classroom organisation also differed across tasks and between Ms B and Ms L's classrooms. One task was completed as a whole class Ms B's class, whereas Ms L used whole class organisation for four tasks. These tasks involved the whole class completing a task together, on some occasions with short periods of individual work, and with teacher-led whole class discussion. This potentially affects the scaffolding possible for the low-attaining students, as during a whole class task it would seem more difficult to provide scaffolding to particular students. In addition, tasks the whole class completed together were shorter than tasks where students worked independently or in pairs in Ms L's classroom. Ms B organised students into pairs to complete tasks on four occasions, whereas Ms L organised pairs on one task. Again, this would affect the scaffolding that low-attaining students could potentially receive with pairs perhaps offering support. Working individually occurred about the same in each classroom, twice in Ms B's room and three times in Ms L's room.

Regarding the types of tasks observed in each classroom, Ms L used a wider variety of the three TTML task types than Ms B who used mainly tasks using manipulatives, representations and tools. It is interesting to note that although Ms B was involved in the TTML project that examined specifically the use of a variety of tasks, she chose to mainly use one type. Ms L, who was not part the TTML project but a project aimed more broadly at contemporary teaching of mathematics, used all three types of tasks almost equally. Ms B discussed the frequency of her use of tasks using manipulatives, representations and tools and explained that when teaching fractions, she felt tasks using physical and visual models were most appropriate.

> Fractions particularly … having that model, I think it simplifies it. They have to have a better understanding. I think it's tricky to come up with, like if I had to think of a real life [task] for fractions … So much of it does have to be modelled.
> (Post lesson interview, Sorting Cards)

Ms L mentioned the use of all three of the TTML task types during her initial teacher interview when asked about the best ways to help low-attaining students.

> They need concrete materials, they need real tasks, real life situation tasks so it needs to be made real to them … I have open-ended tasks too.

This was reflected in the lessons observed where Ms L planned three tasks with a real world context, two were open-ended tasks and three tasks used manipulatives, representations or tools. However, the way in which Ms L enacted these tasks in the classroom could differ from the way tasks were planned, and this will be explored further in this chapter.

As stated previously, in each classroom, six lessons were observed and audio taped with eight tasks selected for data analysis, remembering that Ms L's short games were not analysed. The teachers were interviewed before and after each lesson. Low-attaining target students were interviewed after each lesson regarding their feelings about the lesson. They also completed a short assessment task I designed to assess their understanding of the content or concept covered during the lesson. The lessons occurred over a three-week period in each class with two lessons observed each week. As I mentioned previously, all the observed lessons focussed on one mathematical concept – rational number, including fractions, decimals and percent in Ms B's class and multi-digit multiplication in Ms L's class. The teachers planned and implemented tasks with minimal input from the researcher. Occasionally, I made suggestions about tasks in response to the teachers' questions. For example, Ms B asked about representations or models for decimals and I showed her the Decimat task. As seen by her response during a post-lesson interview quoted above, Ms B also said that she had difficulty finding real world tasks for decimals and fractions and I showed her the Fruit and Veggies task. I gave Ms L a brief verbal description of the TTML task types when explaining my part in the TTML project. Ms L asked about visual aides for multi-digit multiplication and I showed her tasks using arrays to show distributive law, but she chose not to use these.

The observed lessons were analysed regarding the three scaffolding practices investigated in this study: discussion, manipulative materials and visual representations and attention to underlying concepts. Each lesson was coded using the coding framework described in Chapter 3 in which episodes were categorised under the three headings of the scaffolding practices, then the sub-categories and the individual code for that sub-category. One episode or event could be coded under more than one code due to the overlapping nature of the scaffolding practices. The low-attaining target students' responses were gathered via classroom observations and an interview after each lesson. These responses were also coded where appropriate. Other data from interviews or observations notes were used to identify patterns as well as instances that contradicted these patterns. The various data sources and analysis were used to build up a detailed picture of the case.

I will now discuss the focus scaffolding practices, firstly in Ms B's classroom and then in Ms L's classroom. This discussion will include how each teacher used the scaffolding practice, the response of low-attaining students from each class and evidence of the effectiveness of such scaffolding for these students, both cognitively and affectively.

DISCUSSION

In Chapter 2, I examined the literature regarding discussion in the mathematics classroom. Discussion often forms a major part of the scaffolding teachers provide either during whole class discussions (e.g. O'Connor & Michaels, 1993) or during more private individual discussions (e.g. Cheeseman, 2009). However, not all discussion can be described as scaffolding (McCosker & Diezmann, 2009). The purpose of some discussions is to set up social norms (Yackel & Cobb, 1996) or acceptable behaviour whilst others provide emotional support or encouragement (Ames, 1992). These are not the discussions that will form the focus of this chapter.

Furthermore, while I recognise that students can provide scaffolding for each other (Yackel, Cobb, & Wood, 1991), any discussion that occurred only between students was not the focus of the present study, which was concerned with the teacher's use of scaffolding practices.

The discussions that occurred in Ms B and Ms L's classrooms that had the potential to scaffold the learning of the low-attaining target students were those in which the whole class was expected to participate and those occasions when the teacher spoke more privately to the low-attaining students. Accordingly, the following discussion will be divided into two parts – whole class discussion (both beginning and end of the lesson discussions), and individual discussions between the teacher and target students. Each teacher will be discussed separately, followed by the response of the low-attaining students and then similarities and differences between the teachers and students will be explored.

BEGINNING OF TASK WHOLE CLASS DISCUSSIONS

As discussed in the review of the literature, there are various purposes for discussions at the beginning of lessons. The following is a summary of this discussion. Beginning of lesson whole class discussions

- involve the teacher in exploring the problem with students but not showing strategies of solutions paths (Smith, 2004);
- should be a brief presentation of the problem of around five minutes (Olson & Barrett, 2004); and
- should establish classroom forms such as how students are to work and respond to the task (Sullivan et al., 2003).

I will now discuss the whole class discussions that took place at the beginning of lessons in Ms B's classroom.

Beginning of task whole class discussions – Ms B

Whole class discussions at the beginning of lessons averaged 6 minutes in Ms B's classroom. Table 12 outlines the time taken for each beginning whole class discussion in Ms B's classroom, the apparent purpose for each discussion, and observations regarding the two low-attaining target students, Carl and David, during these discussions.

Table 12
Beginning of task whole class discussions – Ms B

Tasks	**Beginning of task whole class discussions**	**Carl and David**
Fraction Rods	4 minutes Instructions for task. A student demonstrated an example.	Both watched the teacher and demonstrating student.
Fractured Figures	5 minutes Instructions for task. Instructions for how students are to work.	Both watched the teacher.
Sticky numbers and number line	2 minutes. Instructions for task. Instructions for how students are to work.	Carl contributed to discussion. David swung on his chair, playing with his pencil case.
Sorting Cards	2 minutes Instructions for task. Instructions for how students are to work.	Both watched the teacher.
Dominoes	8 minutes The teacher demonstrated examples with students.	Carl put his hand up. David had his head down. When the teacher prompted him, he looked at the board.
Fruit and Veggies	10 minutes Introduction of the context.	Both watched the teacher.
Decimat	12 minutes Introduction of the representation for the task with student participation. Demonstration by teacher of game (one round only)	Both sat watching the board. Both contributed to discussion.
Construct a Sum	2 minutes Instructions for task. Instructions for how students are to work.	Both watched the teacher.

As we can see from Table 12, it appeared that Ms B's purposes for these whole class discussions were to provide instructions for the task, to demonstrate an example with student contributions, to introduce the context of the task, to introduce the representation to be used, and to give instructions about how students were to work.

Ms B did not appear to use beginning of task discussions to talk about what students were expected to produce. In fact, the students in Ms B's class were not often asked to record anything during any stage of their mathematics lessons apart from labelling fraction pieces in one task (Fractured figures, lesson observation) and a suggestion to "take a piece of paper to scribble your number on" in another task (Sticky Numbers, lesson observation). This seems to match Ms B's stated purpose for beginning-of-task whole class discussions that focussed on student thinking rather than student actions.

In one instance, Ms B demonstrated part of the task using student contributions such as this example at the beginning of the Fraction Rods task.

> Ms B: Okay so fairly straight forward. You are going to answer these questions. What fraction of the orange rod, remember that the browny one, is the yellow rod. So if we find the yellow rod [holds up the two rods, one yellow and one orange]. So what's that asking you to find? Chad?
>
> Chad: What fraction, like a half.
>
> Ms B: Okay in that case your theory is that the yellow rod is half the orange rod? Okay so you are going to need to prove it. Okay, have a go, if you're really stuck put up your hand and I'll come and talk to you or talk to the people on your table. Is everybody okay with what to do? Have a go, just have a go.
> [The students began the task].
>
> (Fraction Rods, lesson observation)

For another task, Dominoes, the whole task was demonstrated with students contributing in one of Ms B's longer beginning-whole-class discussions. The other more lengthy discussion occurred when Ms B introduced the context of the Fruit and Vegies task. Ms B spoke for most of this discussion painting a picture of the problem in a real world example.

> Ms B: I want you to imagine that you are in high school and you get a part time job at a greengrocer selling fresh fruit and vegetables. This is an environmentally aware shop that uses paper bags for customers to take their produce home in. It's your job to pack their purchases in these bags but each one can only hold one kilogram. Any more than one kilogram and they will simply fall apart. They always put at least two things in the paper bag. The scales that you use to measure weight are faulty and sometimes it gives some weights in decimals and some in fractions. You have to work out what to put in each bag so it doesn't weigh more than a kilo. You can't have one thing in a bag. It must be two or more.
> [The students are all fairly quiet, watching the teacher.]
>
> (Fruit and Vegies, lesson observation)

This example was the longest utterance by Ms B without student participation of all the beginning-of-task discussions observed. However, Ms B seemed to believe that her domination of this discussion was warranted in order to engage the students with the context of the task.

> I could tell they were hooked into the story. They were engaged with that from the beginning so they were able to get started straight away. I was generally pretty happy with how they tackled it.
> (Post lesson interview, Fruit and Veggies)

During the shorter discussions such as Fractured Figures, Sticky Numbers and Sorting, all around two minutes long, Ms B talked exclusively and in all these examples brief instructions were given about the task and how students were to work. For example,

> Ms B: If there is a piece you need, you are not allowed to take them you are only allowed to give pieces to others in your group to complete their shapes. You are not allowed to take; you're only allowed to give. You are going to need to work as a team to complete your puzzle. It's not a case of "I've finished now I'll just sit back". There is more than one way to put the puzzles together so sometimes you might need to give away some of your pieces. So sometimes you have to go "hang on a minute, I might need to give some away".
>
> (Fractured Figures, lesson observation)

These discussions seem to fit the "problem presentation" purpose for beginning a lesson (Olson & Barrett, 2004), where discussion is brief and the teacher gives no strategies or solution paths. Also the "social norms" (Yackel & Cobb, 1996) for working together were of focus here.

I have outlined how Ms B used whole class discussions at the beginning of tasks. However, the focus of this study was on the impact these teacher actions had on the learning and feelings of the target low-attaining students In Ms B's class they were Carl and David.

Responses of Carl and David

Carl and David were observed beginning tasks with no apparent confusion about what they needed to do or how they were to complete the task. However, in Ms B's classroom, students usually worked with at least one other student and were instructed to do so. These other students may have provided support for Carl and David to get started on the task if the whole class discussion did not. Perhaps Ms B's use of manipulative materials for tasks also provided some scaffolding in beginning tasks for Carl and David.

David usually remained silent during beginning whole class discussions. He made one contribution when Ms B specifically called on him and asked a closed question:

> Ms B: David, how many parts altogether?
>
> David: One thousand.
>
> (Decimat, lesson observation)

Earlier, during the same lesson, David called out "millions!" to Ms B's question about how many parts there are altogether if the parts are thousandths. Otherwise David was observed watching the board or teacher. Twice David appeared to disengage from these discussions by putting his head down or playing with his pencil case.

In contrast, Carl, also in Ms B's room, was observed calling out responses at the beginning of two lessons, putting his hand up to offer responses, and asking questions.

> Students guessed number like 64.8. Carl said out loud "Do we have to say the decimals straight away? … Carl was watching the board and had his hand up. He suggested "44"
> (Sticky numbers and number line, lesson observation)

Ms B:	So if one of these is zero point one (pointed to tenth part of the rectangle), what's this? (Pointed to the hundredth part)
Carl:	Twelfths
Student:	Isn't it one fifth?

(Decimat, lesson observation)

Here we can see Carl spontaneously participating, albeit incorrectly in one case, to the whole class discussion, indicating that he must have been listening and attending to the discussion at least to some extent.

During the whole class discussion at the beginning of the Decimat lesson, both Carl and David were observed being actively involved in the discussion. Both boys also revealed in the post-lesson interview that they viewed this lesson positively.

> Thumbs up because it was really fun colouring in the squares, playing. [Carl]
>
> Really happy face because it was fun and we were learning lots of stuff about decimals like how to write them. [David]

Ms B tended to use beginning-of-task whole class discussion for a brief introduction to the task or problem. Carl participated more than David in these discussions but both boys were observed watching the teacher or the board during these introductions to the lesson. Both Carl and David appeared to be able to begin tasks with little confusion following these whole class discussions.

I will now discuss beginning of task whole class discussions in Ms L's classroom and the responses of her two target low-attaining students, Sophie and Riley.

Beginning of task whole class discussions – Ms L

Ms L's discussions were similar to Ms B's in that on some occasions her purpose was to set up the task or problem, provide instructions for how students were to work, demonstrate or model an example, and introduce the context, representations or materials for the task. However, Ms L also differed in her purpose for beginning discussions in that she used these discussions to remind students about prior knowledge or to define key terms that could assist them with the task. In addition, Ms L made “global overview statements” of the lessons that linked them to previous lessons, to the real world and explained each lesson’s part in the overall sequence or development of the class’ mathematical ideas. This was similar to descriptions of “advance organisers” (Engelmann & Carnine, 1982). In Ms L’s class beginning whole class discussions averaged 4 minutes. Table 13 outlines the beginning whole class discussions in Ms L’s classroom, their apparent purpose and observations regarding the two low-attaining target students, Sophie and Riley, during these discussions.

Table 13

Beginning of task whole class discussions – Ms L

Tasks	**Beginning of task whole class discussions**	**Sophie and Riley**
Missing Numbers	3 minutes Instructions for task. Instructions for how students are to work. Reminding students about prior knowledge (factors).	Both watched the teacher.
Find the Pairs	6 minutes Global statement. The teacher demonstrated an example with students.	Both watched the teacher.
Target Multiplication	2 minutes Instructions for task. Instructions for how students are to work. A student demonstrated an example.	Riley watched the teacher. Sophie drew on her whiteboard (unrelated to the task).
Highest Product	5 minutes Global statement. Instructions for task. Instructions for how students are to work. Definition of key terms e.g., product The teacher demonstrated an example with students.	Both watched the teacher.
Veggie Patch	5 minutes Introduction of the context. Reminding students about prior knowledge and key terms needed to complete the task.	Both watched the teacher.
Help Julia	8 minutes	Riley asked a question. Sophie was

	Introduction of the context. Instructions for how students are to work.	looking down most of the time and drawing or writing on her whiteboard (unrelated to task).
Array of 48	30 seconds Instructions for the task	Riley watched the teacher. Sophie was absent.
Athletics Day	6 minutes Global statement. Introduction of the context. Instructions for how students are to work.	Riley contributed to the discussion.

As Table 13 shows, Ms L gave students instruction on how they were to work on five out of seven occasions, demonstrated an example three times (twice with student participation and once without), introduced the context three times and twice defined key terms or reminded students about prior knowledge they could use to complete the task.

Ms L began three of the observed mathematics lessons with a "global overview" statement including an explanation of what students were going to do during the lesson and why this would be the focus.

> What we are going to do today guys is start looking at multiplication. Last week I noticed that we'd forgotten a little bit about factors and products. So we're going to do a little bit of revision on that today and that will help us with more difficult multiplication later in the week. (Find the Pairs, lesson observation)
>
> Okay. Today we are going to continue looking at factors and products. We are going to look at estimating calculations involving multiplication. So today we're not looking at being exactly precise but on how we can develop our idea of estimation because remember we've talked about estimation being the majority of the work we do in maths in our everyday life. When we go shopping, we estimate. We don't often walk around with a calculator doing the whole amount. So that's why that's quite important.
> (Highest product, lesson observation)
>
> Yesterday guys, we were looking at strategies, and we did a really good job. I was really, really impressed actually. I think all of us have a strategy that we can rely on. Whether it's the grid method, whether it's partitioning or whether it's lattice method and you might even have another method or strategy that you used. A couple of people were using the grid method and the partitioning method and I'm wondering if actually they are very, very similar and just a different way of recording so that's something we might need to think about. What I wanted us to think about today was how we actually use multiplication in our everyday lives. (Athletics Day, lesson observation)

These episodes illustrate Ms L's habit of placing the task or lesson within the context of the class' unit of work or the wider world.

Ms L stated that her purpose for these beginning of the lesson discussions was to introduce what students were to do (initial teacher interview). This purpose seemed borne out by observations of these initial discussions where Ms L was quite specific about student actions. For example, she occasionally instructed students on where to write their answers (Find the pairs, lesson observation) and to write the learning focus at the top of the page (Highest Product, lesson observation). Ms L was also explicit about how students were expected to work. For example,

> Our activity is called highest product. Which combination will give you the highest product? You need to estimate first. You need to use the digits 9, 8 and 7 set out like this [showed _ _ x_]. I would like you to try and solve these problems without the calculator. How would *you* be a calculator? I really want you doing this on your own. I don't mind you discussing this with the person next to you because we share our learning
> (Highest Product, lesson observation).

Ms L seemed to feel that students needed to be cued into accessing prior knowledge to assist them in completing tasks. During lessons she was also observed asking "what knowledge" students could use. However, particularly in the Veggie Patch example to follow (see Figure 5), the number of suggestions and questions Ms L made could well have contributed to, rather than alleviated, confusion. Ms L suggested four different strategies: arrays, multiplication facts, factors and products. She also pointed out that there were many possibilities and that only whole numbers would be acceptable.

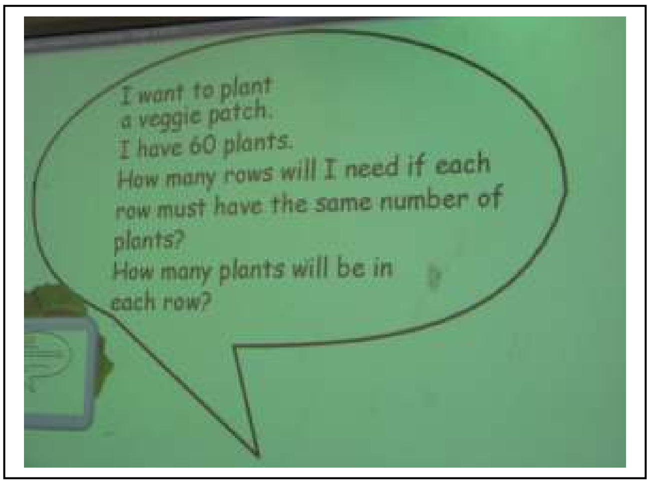


Figure 5. Veggie Patch problem on interactive whiteboard.

> I'm going to hand out a whiteboard and a marker and I want you to have a go at showing me the answer to this problem [see figure 5]. What strategies can you use? Could you use an array to help you? Would an array be good to use? Could you do it just by using multiplication facts? Could there be more than one possibility? How many possibilities are there? … Remember last week we looked at products and factors? How could the factors that we were working on last week help us? So the product is my 60 plants but do I only have one possibility for how I set out my veggie patch? I can't cut my plants in half so we're only using whole numbers.
> (Veggie Patch, lesson observation)

During this part of the discussion, Ms L asked nine questions and made two statements without input from students. On other occasions students were asked to respond to questions requiring a short answer such as "Is there more than one possibility? (Missing numbers)", "Do we need to answer the questions in order? (Athletics Day)" and "Who can remind me what product means?" (Highest Product) On one occasion, one of Ms L's questions was answered by Riley though Ms L had not been seeking a student response but simply reading the question from the worksheet.

> Ms L: How many teachers are needed? In total there needs to be one teacher for every ten children at the athletics park. If each school has taken 3 teachers, are there enough teachers at the athletics?
> [Riley was shaking his head.]
> Why are you saying no now Riley?
>
> Riley: Cause there'd be more, if there 75 children from each school going, even 75 is a lot so if there'd only be like 30 teachers…
>
> Ms L: Okay so at the moment you're basing it on how many are there. I still want you to work that out and find out how many teachers need to be there and are there enough. Well done, good on you.
>
> (Athletics Day, lesson observation)

This represented a more substantial contribution than a one-word answer and also seemed to indicate that for this task at least, Riley was listening to and thinking about the discussion. It also demonstrated that Riley was able to estimate in a multiplicative situation in a logical and reasoned way that might not be expected from a low-attaining student. Ms L interrupted Riley's explanation because it would appear that she wanted him to construct an answer while working on the task and not during this whole class discussion. However, she showed her approval for Riley's willingness to think about his response to this question.

Ms L asked more open questions twice during these whole class discussions that did require a more detailed explanation from students.

> Ms L: You're looking a little confused Joe. You don't know what to do? Okay. Who'd like to explain what we need to do here?
>
> Student: We need to like, example you need to times 11 by 28 … [interruption]
> So you have to estimate like if you wanted to multiply 10 by 53 you have to estimate what it might be …
>
> Ms L: Hopefully you would be able to work out 10 by 53 in your head. We have talked about multiplying by 10 and 100. Okay we choose one number from that box and one number from that box, Joe that will give us an answer of between 300 – 400.
>
> (Target multiplication, lesson observation)

In this example, although Ms L asked a student to explain the task, she stepped in and repeated her initial instructions rather than allow the student to continue the explanation. A similar example can be found in another lesson.

Ms L: Do you think it would matter whether we put the single digit number first or last? [A student said yes.]
Hands up if you think it will matter. So if I changed this box to here, would it make a difference to my product? (Ms L moved the box on the IWB)
Student: Say you did 4 times 5 it equals 20 and if you did 5 times 4 it equals 20 so it doesn't matter …
Ms L: Okay but where we put the 9,8 and 7 will make a difference. It doesn't matter whether you put it that way (one by two digit) or that way (two by one digit) you'll still get the same answer.
(Highest product, lesson observation)

Again, Ms L interrupted the student's explanation, even though the student appeared to be raising an important mathematical idea about commutativity. Although she made the point that multiplying by the single digit first or second would not change the answer, she also stated that the position of the digits would make a difference. This was potentially confusing for students. These facts were stated quickly and the lesson moved on. It would seem that these were important ideas to explore when considering how to make the highest product and yet Ms L did not highlight them or explore the student's contribution fully.

In both these cases, the question Ms L asked required students to construct responses that were more than one word and potentially involved them in articulating thinking to the class. However, also in both these cases Ms L interrupted student explanations and student thinking was not explored or articulated fully. It would appear then that Ms L believed her role during beginning whole class discussions was to ask questions, prompt prior learning and drive the discussion to a timely conclusion in order for students to begin the task. I will now examine the responses of the target low-attaining students in Ms L's class, Sophie and Riley.

Responses of Sophie and Riley

For Sophie and Riley, there were four occasions where they seemed confused at the beginning of tasks just after whole class discussions. For example, during Missing Numbers, neither Sophie nor Riley attempted to answer the third question "26 =(_x _) + __". Similarly in Target Multiplication, neither Sophie nor Riley initially had anything written on their whiteboards. Riley drew nothing for the Array of 48 task until solutions were shared (Sophie was absent for this task). In Find the Pairs, Sophie appeared to not understand how to begin the task.

Ms L: (to Sophie) Have you sorted your cards into pairs? So where are they? Sort them into pairs so they're clear. You can only use each card once. You don't want

> [your partner] to see them because he's going to be guessing your pairs
>
> (Find the Pairs, lesson observation)

Ms L volunteered during the post-lesson interview that there had been some confusion about how to play Find the Pairs.

> She [Sophie] struggled to understand the actual activity at the beginning and if I had more time I probably should have played it together first so they could have seen it (Post-lesson interview, Find the Pairs).

This statement would seem to highlight Ms L's concern with time taken for whole class discussion at the beginning of tasks. It would appear that Ms L was able to identify that modelling the task would have provided scaffolding for Sophie to understand the task. Conversely, Ms L seemed to think that such modelling could have only occurred "if I had more time" emphasising the tension between providing appropriate support and time constraints.

In Ms L's classroom, Sophie was often silent during beginning of the lesson whole class discussions and she was observed watching the board or teacher or drawing on her whiteboard. Sophie put her hand up to respond to questions three times and was called on twice. One of these responses was to identify what the teacher had drawn, "an array", and one was to add to the factor tree on the board. Riley responded twice during these discussions, on one occasion to offer his hypothesis about the Athletics Day task as discussed above, and to clarify if working in partners was permitted.

Overall beginning of task whole class discussions in Ms L's classroom were short, tightly controlled discussions with minimal student contributions. Ms L's main purpose for these discussions appeared to be instructions for the task, including how to work, suggesting the prior knowledge that may assist in completing the task and giving global statements that placed the task and lesson within the wider unit of study or its relevance to every day life.

SUMMARY

It would appear that Ms B used beginning of the task discussions more effectively than Ms L as Carl and David did not show the confusion about starting the task that Sophie and Riley did for half of the observed tasks. However, as discussed, Ms B used partner or small group work during all observed lessons that may have provided scaffolding for Carl and David in starting tasks if these beginning discussions did not.

In addition, Ms B often used manipulative materials as the basis for tasks that may have also assisted Carl and David in getting started.

Ms L's habit of asking her students to record using a whiteboard or their books perhaps made it more obvious when Sophie and Riley had not responded or were unsure of how to answer, whereas Ms B's class rarely recorded during mathematics lessons. Social interaction was part of both classrooms, although in Ms B's room the students seemed to talk to each other more than in Ms L's classroom which was often quiet. Ms L was observed asking students to keep their voices down whereas Ms B was not. It is interesting that even though Ms L's beginning of task discussions focussed on the details of how students were to complete tasks, Sophie and Riley displayed more confusion than Carl and David in Ms B's class, where the discussion focussed more on the concepts of the tasks and less on these details.

I will now examine the other form of whole class discussion that occurred in both Ms B and Ms L's classrooms, which were end-of-task discussions.

END OF TASK WHOLE CLASS DISCUSSIONS

End-of-task discussions were conducted in Ms B and Ms L's class at the end of every observed lesson. In the review of the literature, various purposes for end-of-task whole class discussions were described. The following is a summary of this discussion. End-of-task whole class discussions are for

- students to share their solutions, for the teacher to review or highlight aspects of student strategies or solutions and for the teacher to summarise the major point for the day (Stigler & Hiebert, 1997);
- gathering evidence, summarising, reviewing the focus, sharing common discoveries, celebrating learning, learning from each other, encouraging students to reflect on what they have learned, extending thinking and building positive attitudes (Cheeseman, 2003);
- anticipating, monitoring, selecting, sequencing, and making connections between student responses (Stein et al., 2008); and
- "filtering" student contributions so that ideas are elaborated, compared and evaluated to bring to the fore mathematical ideas (Sherin, 2002).

I will now describe end-of-task whole class discussions in each classroom including responses of the target low-attaining students to these discussions.

End of task whole class discussions – Ms B

Table 14 outlines the end of task whole class discussions in Ms B’s classroom including the purpose of each discussion, time taken and observations of Carl and David.

Table 14
Ms B – End of task whole class discussions

Tasks	**End-of-task whole class discussions**	**Carl and David**
Fraction Rods	15 minutes Some students shared their solutions. Teacher made connections between student responses. Class worked through the most difficult question together.	David asked to share his strategy.
Fractured Figures	8 minutes Teacher summarised the main point of the lesson and identified commonalities with previous lesson. Teacher made connections between student responses.	Both watched the teacher.
Sticky numbers and number line	17 minutes Students shared their solutions.	Both called out responses. Later David threw an object up and down.
Sorting Cards	10 minutes Some students shared their solutions and teacher filtered by highlighting a misconception. The teacher wrote in the class journal.	Carl continued to place cards in piles then flicked a card in his fingers while watching the teacher. David was blowing the cards on the table. Later David was under the table and Carl had his head resting on the table watching the board.
Dominoes	10 minutes Students shared their solutions. Identified commonalities with previous lesson. The teacher wrote in the class journal.	Carl watched the board. David rocked on his chair and looked at his dominoes.
Fruit and Veggies	15 minutes Students shared their solutions.	David was playing with his cards. Carl had his head down on the table.
Decimat	5 minutes Students were asked to reflect on what they learned. The teacher wrote in the class journal.	Carl asked to share what he learnt today.

Construct a Sum	8 minutes Some students shared their solutions. Two possible solutions were discussed.	David was tearing small pieces of paper and throwing them at a classmate and Carl was colouring the rest of the decimat in green texta.

Ms B spent almost double the amount of time on these concluding whole class discussions than on beginning of task discussions. Concluding discussions averaged 11 minutes whereas, as identified in an earlier discussion, beginning discussions in Ms B's room averaged 6 minutes.

All the end of task whole class discussions observed in Ms B's classroom centered on students sharing their solutions or strategies. These were typically quite lengthy exchanges where Ms B would ask students to explain their thinking. These contributions were "filtered" (Sherin, 2002) by Ms B who, on two occasions, made connections between student strategies.

Student: In the end, when we worked it out, we got all the little triangles and we used eight for each one.
Ms B: Now this was something that you clued on to pretty quickly wasn't it Chris? What did you tell me right at the beginning?
Chris: That you use the small triangles to measure how many there was.
(Fractured Figures, lesson observation)

Ms B: Now it got a little bit trickier here didn't it? What fraction of the blue rod is the dark green rod? Can you talk us through your strategy Greg? 'Cause that was quite interesting. It was quite similar to the strategy that John was doing.
Greg: I got the blue rod and the ones of the MABs then actually put the blue rod with it and it was nine little rods of the MABs then I measured the dark green one and it was six, so I knew it was nine.
Ms B: Okay so you actually worked out that nine of the little individual blocks made up one of the wholes so you knew each of parts had to be a ninth didn't you?
Greg: Yep
Ms B: How many ninths … obviously it would be nine ninths in the whole, how many ninths was the green one?
Greg: Six
Ms B: So there were six ninths in the green one. Now John what did you do after that? Remember we talked about it didn't we?
John: Then you break them up into three groups of three, then they're thirds.
(Fraction Rods, lesson observation)

The other occasion when Ms B filtered student responses was to highlight a misconception held by a group.

Ms B: Kurt can you explain how your group sorted yours this time?
Kurt: Well we had three groups, one was a whole, another was a half and one with quarters.
Ms B: Okay one group of whole, one group with halves and one group with quarters and what did we decide in the end?
Kurt: They could all be in one group!
Ms B: If you're going to sort them that way they could all end up in the same group.

> Because what Kurt and his group had, was … they had all the ones that were worth one quarter and all the ones that were worth three quarters they decided that they'd call them things that were broken into quarters, basically, and put them all in one group. Then they have things that were broken into halves so you had a mixture of halves … So they called that "things that were broken into halves" but then we decided that if you were going to call all those things that were broken into halves we could call them broken into quarters too couldn't we?

(Sorting cards, lesson observation)

As there was a relatively high level of student involvement in end of task discussions in Ms B's classroom, this often resulted in Ms B revoicing (O'Connor & Michaels, 1993) student contributions for clarity or to emphasise a more mathematically correct way of using language. The following excerpt is a continuation of a discussion quoted above.

John:	So then it's two thirds
Ms B:	So you knew then that if there were three groups of equal number, they were the same size so therefore they're thirds. Two of those thirds made up the green rod so therefore it was two thirds. Quite a nifty strategy that one wasn't it? They broke it down to the simplest level then built it up again.

(Fraction rods, lesson observation)

Here we can see Ms B elaborating a student's strategy of making ninths, dividing these into three groups of three and naming two of these groups two thirds. This was quite a complex strategy but Ms B used her drawing on the board and her revoicing of John's words to emphasise important mathematical ideas to the class; that is that three groups of equal number are thirds.

On another occasion Ms B used revoicing as a way to slow down the discussion so that students could follow what was being explained. She also elaborated on the student's words in order to clarify what the strategy was.

Ms B:	How do you know that 30% and 3/10 are the same thing?
Braydon:	Because 10, 20, 30 all the way to a hundred is ten.
Student:	Yeah and three tenths is thirty
Ms B:	Hang on.
Braydon:	And it was three to count to 30.
Ms B:	Okay if we counted up to 100 by 10s there's 10 lots of them. There's 10 tenths in a hundredth so one tenth would be what then?
Braydon:	Ten percent
Ms B:	One tenth is ten percent so three tenths must be thirty percent. Do you follow his logic?

(Dominoes, lesson observation)

Ms B seemed to think that this strategy could be difficult for some students to follow though it appeared another student knew what Braydon meant. Ms B slowed the discussion ("hang on"), and reiterated the strategy of counting by tens so that other students could understand it. In total, 23 episodes were coded as revoicing from Ms B's discussions. Given the high rate of student contributions, this is not surprising, as most of Ms B's end of task discussion involved her eliciting and responding to what students said.

End-of-task whole class discussions in Ms B's classroom also involved the class recording main points from these discussions in the "class journal" after three lessons. These entries were "0.33 = 33/100 = 33%" (Sorting Cards, lesson observation), a table of equivalent decimal numbers, fractions and percents (Dominoes, lesson observation), and observations about the relative size of decimal parts (Decimat, lesson observation). Recording the discussion has been suggested as a useful tool in scaffolding students to understand other students' contributions (Baxter et al., 2002) as often only listening to other students can prove difficult to interpret for students. Many of these discussions were potentially confusing for students as shown in the previous examples. This was perhaps particularly so for Carl and David. With this in mind, I will now discuss how Carl and David responded to these largely student-driven discussions.

Responses of Carl and David

The low-attaining students in Ms B's classroom were each asked once to contribute to the end-of-task whole class discussions. David was invited to explain his strategy after the Fraction Rods activity.

Ms B: David you had a really good strategy for working it out.
David: I did…um
Ms B: Can you remember?
David: :I… I….
Ms B: You found the one that fit on the end to make the whole. What did you discover about this rod?
David: It was a third… no… half
(Fraction Rods, lesson observation).

Here we can see that Ms B was attempting to involve David in the discussion by inviting him to share his strategy. However, David seemed to find articulating his strategy difficult and so Ms B provided the explanation for him. This issue was also raised with the findings of Baxter et al. (2002) who described the efforts of a teacher to encourage participation of low-attaining students to whole class discussions, only to find that they were unable to articulate clearly what they had done or thought.

Carl was also invited to participate in the whole class discussion when Ms B asked him to share what he knew about tenths, hundredths and thousandths.

Ms B: Carl, tell me one thing that you've learnt about tenths, hundredths and thousandths today.

Carl: That thousandths are smaller
[Ms B wrote this into the class journal]
(Decimat, lesson observation)

This was a short response though did perhaps highlight something Carl had learnt as a result of the task. Ms B did not ask Carl to elaborate on his answer by explaining how he knew thousandths were smaller for example. Rather she accepted his brief reply to record in the class journal.

Apart from these two contributions, Carl and David were observed engaging in off-task activities during four end-of-task discussions. This included colouring the answer sheet green or tearing up paper (Construct a Sum, lesson observation), putting their heads down (Fruit and Vegies lesson observation), throwing objects (Sticky numbers and number line lesson observation) or playing with cards (Sorting cards lesson observation). Twice Carl was observed watching the teacher or the board (Sorting cards and Dominoes) and once both David and Carl watched the teacher (Fractured Figures). This suggested that for about half of the concluding whole class discussions, David and Carl were not participating either as listeners or speakers. Again, this is similar to findings of Baxter et al. (2002).

It is difficult to attribute learning to end of task whole class discussions when they formed only part of the lessons observed in each classroom. However, there were instances in Ms B's class where the end-of-task discussion touched on an area that also formed part of my post-lesson assessment tasks with David and Carl. For example, after the Dominoes lesson, I asked David to match some fractions, decimals or percentages and explain how they matched. David correctly matched one quarter and 25 percent, which had formed part of a conversation he and his partner had had with Ms B during the lesson. However, he could not match 75 percent though this was discussed as a whole class.

Emma came to the board and matched 3/4 to 75%.
Ms B: So you've got 3/4 and 75%. How do you know?
Emma: Well the reason I know, I've got three 25% in 75
Ms B: Okay and we know that 25% is one quarter so therefore 75% must be three of those quarters. Fantastic. Really good.
(Dominoes, lesson observation)

On the same assessment task, Carl was not able to match one quarter with an equivalent decimal number or percent. He matched 0.7 and 70% but his explanation was unclear; "because 7/10 goes into 7 'cause that's a tenth (7) so ten sevenths makes

70%" (post-lesson assessment task, Dominoes). The whole class discussion at the end of this lesson had just discussed a similar example.

Ms B:	What is 80% as a decimal Sean?
Sean:	Zero point eight
Ms B:	Eight tenths because zero point eight is the same as eight tenths because that's the place that it holds yeah? Which is the same as eighty out of a hundred okay? Yeah?

(Dominoes, lesson observation)

In addition, neither Carl nor David could tell me the decimal number that is equivalent to half during a post-lesson assessment task. This was despite the fact that this question was discussed twice during whole class discussions.

Ms B:	All right lets have a quick look at the dominoes you started off with. I'm going to put three columns here fractions, decimals, percentages (drew this in the class journal) Lets start with an easy one. What's half as a decimal?
Student:	Zero point five.
Ms B:	[Wrote zero point five] Which is what? What does that five mean John?
John:	The five? Half, half of ten.
Ms B:	Half of ten yeah so it's five tenths?
John:	Yeah.

(Dominoes, lesson observation)

Ms B:	Okay let's do one together. There's some confusion. Megan, tell me how many tenths you've coloured in?
Megan:	A half
Ms B:	Okay so you've coloured in 5 tenths. What would I write for the decimal?
Megan:	Zero point five
	[Ms B wrote 0.5 on the board]

(Decimat, lesson observation)

In all these examples, Ms B moved quickly over the point and in one instance David was observed with his head down (Dominoes lesson observation). However, these discussions took place just before my interviews with Carl and David and yet they were not able to draw on them to answer my questions. For these students these end-of-task whole class discussions did not provide scaffolding that allowed them to complete this assessment task successfully. This would suggest that end-of-task whole class discussions might have been confusing and difficult to follow for Carl and David, just as Baxter et al. (2002) found for the low-attaining students in their study.

End of task whole class discussions – Ms L

Ms L conducted whole class discussions at the end of every observed lesson. Table 15 shows that the purpose of these discussions appeared to be to check answers, for students to share solutions, to define key terms, to assign homework, to point to future directions in mathematics learning and to gauge students' feelings about the lessons.

The average time spent on these concluding discussions was 7 minutes. Table 15 also describes what Sophie and Riley were doing during these discussions.

Table 15

Ms L – End of task whole class discussions

Tasks	**End-of-task whole class discussions.**	**Sophie and Riley**
Missing Numbers	7 minutes Some students shared their solutions.	Riley put up his hand. Sophie looked down. Later Sophie had her hand up.
Find the Pairs	9 minutes Discussed a definition of factor and product. Assigned homework. Asked about how students felt about the lesson (traffic lights).	Riley sat with his hand down. Sophie was on the computer answering the class' Wiki survey questions.
Target Multiplication	2 minutes Some students shared their solutions.	Both watched the teacher.
Highest Product	7 minutes Students invited to share something they learnt. Student offered a conjecture about multiplying.	Both watched the teacher.
Veggie Patch	4 minutes Some students shared their solutions.	Riley contributed to the discussion. Sophie was still writing on her whiteboard to complete the task.
Help Julia	8 minutes Checked answers. Some students shared their solutions. Teacher made connections between student responses.	Both watched the teacher.
Array of 48	8 minutes Students shared solutions and teacher wrote these on the board. Discussion about division as the inverse of multiplication.	Riley contributed to the discussion. Sophie was absent.
Athletics Day	11 minutes Checked answers. Discussed a definition of division. Indicated the direction of future lessons.	Riley contributed to the discussion.

In these end-of-task discussions, Ms L defined key terms with students, such as factor, product and division, and pointed to future directions, lessons or homework, as well as discussing the task just completed.

> Ms L: 974 divided by 48 [wrote on the board] And we're going to come back to that now tomorrow maybe Friday and we are going to look at how, a few different ways, of solving that. Now I heard a few people talking about this saying "I find this really hard, I find division confusing." What I'd like you to do tonight is do a bit of an investigation into how you would

divide. Think about some of the strategies that maybe you've experienced in the past about dividing. What are we doing when we are dividing?
(Athletics Day, lesson observation)

Ms L also used these discussions to gather information about students' feelings about the lesson just completed. The "traffic lights" were explained by Ms L as red indicating considerable confusion, yellow some confusion and green no confusion. She asked students to indicate their feelings using traffic lights once during observations.

Ms L: Does anyone have any burning questions for me to think about for tomorrow? If you're a red traffic light today, hands up. Yellow? Green? [Some students put up hands for all colours] Okay, good honesty.
(Find the Pairs, lesson observation).

Ms L asked about "burning questions" at the end of two of the observed lessons, Find the Pairs and Highest Product. On both occasions, no students offered such a question.

In the main Ms L did not seem to have selected students to contribute to end-of-task discussions, as recommended by Stein et al. (2008), but rather asked for volunteers to explain how they arrived at an answer. Students shared solutions but these exchanges were often brief replies to the teachers' requests.

Ms L:	So lets go on to that last one. What did you have as a possibility?
Student:	12 times 2 plus 2
Ms L:	Good anyone do it in a different way?
Student:	2 times 5 add 16
Ms L:	Okay you could do it that way. Different way?
Student:	6 times 4 add 2
Student:	10 times 2 plus 6
Student:	5 times 4 plus 6
Student:	0 times 0 plus 26
Ms L:	Okay I'm going to put that one there but do you really need to do zero times zero? No, but I'll put it there.
Student:	26 times 1 add zero
Ms L:	Okay it could look like that
Student:	3 times 8 add 2
Ms L:	What we've seen is that we can have many possibilities. Were we using factors of 26 to answer that?
Student:	What we did multiplication but then we added
Ms L:	Okay 'cause we brought in another operation didn't we?

(Missing numbers, lesson observation)

This exchange is more like Kazemi and Stipek's (2001) description of a "low press" teacher where contributions are shared but not filtered by the teacher. There were many answers shared in the excerpt above, but Ms L did not comment on the mathematics of these, draw any similarities or differences about these solutions or ask students to elaborate on how they arrived at these answers.

Ms L:	Is there any we could eliminate straight away that we could say is not in the target range if I multiply that?
Student:	10 times 78
Ms L:	Any others?
Student:	10 times 69
Ms L:	So we can eliminate those two. What else could we eliminate from over there then?
Student:	11 times 91
Student:	Okay, why?
Student:	It would be too much.
Ms L:	How did you work that out quickly?
Student:	I used addition.
Ms L:	You used addition to do multiplication of…
Student:	No I did 91 times 10 then I added 91
Ms L:	Okay and that took you over the thousand.

(Target multiplication, lesson observation)

In this exchange, Ms L did ask a student to explain how they worked out that 11 times 91 would be too much. The explanation was brief, "I did 91 times 10 then I added 91", and Ms L did not elaborate on this but simply stated, "okay and that took you over the thousand". This did not make clear the strategy the student used or how this took them over the thousand by only one. Earlier in this task, Sophie had 11 times 91 as her solution and Ms L had pointed out this was incorrect but not elaborated on why. This may be why in the whole class discussion this particular example was chosen by Ms L to be further explained. The brief and ambiguous nature of the explanation both by the student and Ms L would suggest that it is probable that Sophie would have remained unclear on why her solution was incorrect despite it being the subject of the whole class discussion. In addition, the task was made more difficult due to the fact that 11 times 91 was so close to 1000 that it required actual calculation rather than estimating a range.

On one occasion, Ms L compared two students' strategies.

Ms L:	Okay, I asked Erin to set it up in a more systematic way and this is what Erin has come up with. You might have had some different strategies to this as well. [Erin came to the board and wrote 45×26 vertically with each step in brackets written beside.] I know Chris got there in a slightly quicker way so I'm going to ask Chris to come and share his strategy. Okay. So Erin has just… starting to look a little bit more like our traditional algorithms and that's another strategy we can use. So we've used partitioning, we used our knowledge of place value. Chris did it in smaller steps. [Chris came to the board and wrote 45×6, 45×20 and 270+900.] So he kept his 45 he has then partitioned his 26 into 20 and 6. Okay so he got to the same answer. So he has partitioned one of the numbers. But when we come to, remember what Georgia said as one of the rules? We need to make sure that you're actually multiplying all the numbers together in the right way. We're going to look at this again tomorrow.

(Help Julia, lesson observation)

In this exchange, we can see that Ms L was filtering student responses, commenting on similarities and differences in their strategies. This was more congruent to Kazemi and Stipek's (2001) description of actions a "high press for conceptual thinking" teacher might engage in, although Ms L was not explicit about how these strategies would help in "multiplying all the numbers together in the right way". In addition, though the students were asked to share their strategies and wrote these themselves on the board, the verbal explanation was actually provided by Ms L.

Responses of Sophie and Riley

In Ms L's class, Sophie was not observed contributing to any of the end of lesson discussions. Riley contributed by suggesting the class use their list of mathematics vocabulary to help define division. Riley also explained to the class that halving one side of a multiplication expression necessitated the other side "going up".

Ms L invited students to share their solutions. Riley put up his hand up.

Riley:	I did two. 6 times 10…
Ms L:	Okay can you give it to me in rows? Because we are planting my veggie patch here.
Riley:	6 rows on top and 10 down.
Ms L:	Hold on. Can I have 6 rows on top and 10 down? What do you mean?
Riley:	Like this [showed board]
Ms L:	Yeah do you think it's enough to just say 6 rows of 10?
Riley:	Yeah and my other one is 20 by 3.
Ms L:	So 20 rows of 3.
Riley:	Yeah because I just half 6 it would be 3 but when I halve 6, that needs to go up [pointed to 10]
Ms L:	Ooh. Excellent. We can put that on another rule. What Riley just said was I have six rows of 10 so if I halve my 6 to 3 the amount of plants in each row will go up. So he's doubled that.

(Veggie Patch, lesson observation)

This was another episode, similar to Riley's estimation at the beginning of the Athletics Day task described previously, in which Riley demonstrated sound reasoning and mathematical thinking. There were no other instances from this lesson or others observed, of Riley or Ms L exploring this concept of doubling and halving.

This episode was the longest utterance during a whole class discussion observed by the target low-attaining students in both classrooms and across all observations. Just as Baxter et al. (2002) found, all other contributions were not more than four words long. This was true of Carl, David, Sophie and Riley, though other students contributed longer responses.

Ms L tended to use the discussions to preview future lessons, to gauge affective responses or to assign homework more than to discuss the concepts covered in the lesson. Owing to this, similar conclusions as those drawn for Carl and David cannot be drawn for Sophie and Riley in terms of potential learning from these discussions as the assessment tasks I gave Sophie and Riley did not match up with the points covered by Ms L in her end-of-task whole class discussions.

SUMMARY

Ms B spent more time on end-of-task whole class discussions than Ms L with the average time spent on such discussion being seven minutes in Ms L's classroom and eleven minutes in Ms B's classroom. The overall structure of each teacher's mathematics lesson contributed to this duration difference. Ms L's lessons often contained five sections and up to four tasks, leaving less time for concluding discussions. Ms B's lessons contained three sections, mostly around one task therefore allowing more time for the concluding discussion. As well as this difference in time, the content of the concluding whole class discussions also differed. Ms B's discussions were more based on student strategies, whereas Ms L used these discussions more for teacher explanations or to highlight the future directions for the class. Student contributions potentially take more time than teacher-led explanations, perhaps accounting for some difference in the average time for these discussions.

The worth of whole class discussion for providing scaffolding in mathematics for the four low-attaining target students was questionable. Each student contributed very little to such discussions and, when they did contribute, were called upon to answer rather than volunteering to contribute. Their contributions were short with the longest utterance being 44 words while all other were 4 words or less. Of course, as Baxter et al. (2002) pointed out, listening during these discussions can potentially be just as productive. During beginning of the lesson discussion, the target students seemed to be listening more often than during end of lesson discussions. The length of these concluding discussions was about 10 minutes, perhaps supporting Baxter et al's (2002) proposition that this short amount of time does not allow for many students to participate. Ms L in particular, with the many parts contained in her lessons, might have felt pressured to use this time "wisely" by highlighting correct and well-explained strategies or methods rather than engage in time consuming and potentially confusing dialogue about less efficient strategies or thinking. It would certainly appear that the low-attaining target students participated less than higher attaining classmates, which supports the finding of Burns and Myhill (2004) and Lubienski (2000a). In addition, there was evidence that Carl and David could not relate information from whole class discussions to related post-lesson assessment tasks suggesting that such discussions were not beneficial for their learning, a finding which resonates with that of Burns and Myhill (2004).

I will now discuss a different form of discussion, the private discussion that took place between each teacher and the target low-attaining students in their class. These discussions were much more conversational and individualised, with the potential to provide the kind of conceptual discourse described by Anghileri (2006). These conversations were recorded on the device worn by the teacher, allowing the students' voices to also be recorded.

Individual teacher and student interactions

Both Ms B and Ms L moved around the room habitually while students worked on tasks, observing and talking to students, similar to Clarke's (2004) description of the Japanese practice *kikan-shido* or "walking between the desks" (p. 8). The nature of the individual conversations that Ms B and Ms L had with students was varied. Some conversations centered more on social norms (Yackel & Cobb, 1996) such as reminding students about instructions for how to work, repeating task instructions, encouraging students to engage in the task, redirecting students who may be off-task, or giving additional tasks. Other conversations focused on sociomathematical norms (Yackel & Cobb, 1996) such as students explaining their solutions or strategies, highlighting errors or misconceptions, exploring the concepts surrounding the task, using manipulative materials or representations to illustrate aspects of the task, or simplifying or extending the task. It is these conversations about mathematics, as opposed to organisational prompting, that I have classified as "scaffolding conversations" (Ferguson & McDonough, 2010). Scaffolding conversations are those interactions that took place between the teacher and individual students where the focus was mathematics, where the teacher showed an awareness and responsiveness to the students' thinking (McCosker & Diezmann, 2009), and where the teacher attempted to facilitate the development of understanding of mathematical concepts

(Anghileri, 2006). In addition, the scaffolding conversations I will describe sometimes occurred with the teacher “stepping in and out” of the conversation. My use of the terms “stepping in” and “stepping out” differs from Rittenhouse’s (1998) use of these terms, which she used to describe Lampert’s (1990) role as the teacher during whole class discussions. This was described in Chapter 2. My definition of stepping in and out is that, when engaged in individual discussions with students, the teacher leaves the student to continue thinking or working alone before returning to continue the conversation. This definition is comparable to Cheeseman’s (2009) description of “interlinked conversation strings”.

As discussed in Chapter 2, there is little research on the individual conversations between teachers and students in mathematics classrooms although the importance of such conversations is recognised (Kyriacou & Issitt, 2008). Cheeseman (2009) examined the “interlinked conversations strings” between highly effective teachers and their young pupils. During such conversations, teachers asked the children to “demonstrate, model, explain, calculate, justify, generalise, transfer, connect, and describe their mathematical thinking” (p. 120). Though the nature of the discourse, be it individual or whole class, is not specified by Anghileri (2006), Cheeseman’s findings resonate with Anghileri’s description of conceptual discourse where “the teacher goes beyond explanations and justifications … by initiating reflective shifts such that what is said and done in action subsequently becomes an explicit topic of discussion” (p. 49). Bliss et al. (1996) alluded to the importance of teachers interacting with “smaller groups of pupils” in “dialogue and diagnosis” (p. 58).

I will now discuss scaffolding conversations firstly between Ms B and David and Carl, followed by conversations between Ms L and Sophie and Riley. I will also describe an area of potential cognitive growth for each low-attaining student that formed part of the scaffolding conversations they had with their teacher. As mentioned, the conversations between Ms B and Ms L and the low-attaining target students in each class were captured using a mobile recording device worn by the teacher that allowed the student’s voice to also be recorded. Evidence will be drawn from post-lesson assessment tasks, interviews and lesson observations. Any evidence of an affective response from the low-attaining students to these conversations with their teacher will also be described.

Scaffolding conversations – Ms B

When describing a typical lesson in her classroom, Ms B stated that after setting students to work on the task, she would spend time talking to students to check understanding and ask questions (initial teacher interview). This was reflected in Ms B's lessons in which she routinely walked around the room while students were working on the task, spending time talking with groups or pairs of students. The amount of time Ms B spent with a pair or group varied but often she would spend about five minutes with one pair or group of students. This part of the lesson, when students worked independently or in groups on the task, averaged 40 minutes, therefore spending 5 to 10 minutes talking to a small number of students represented a large portion of this time. As she stated, Ms B felt that this was her role as the teacher, to spend time with one to four students discussing in detail the mathematics in which they were engaged. Occasionally, she expressed frustration in post-lesson interviews that she did not have the opportunity to speak to Carl or David during particular lessons. However, given the time she spent with students individually, it would not be possible to spend this with all students every lesson. Table 16 illustrates the time Ms B spent with Carl and with David during each lesson.

Table 16

Time spent with Carl and David during tasks

Task	Time spent with Carl	Time spent with David
Fraction Rods	5 minutes in total (1 minute, and 4 minutes)	9 minutes in total (1 minute and 8 minutes)
Fractured Figures	3 minutes in total (Spoke to the small group)	3 minutes in total (Spoke to the small group)
Sticky Numbers and Number line.	Whole class discussion only.	Whole class discussion only.
Sorting Cards	None	6 minutes in total. (2 minutes, 2 minutes, 2 minutes to small group)
Dominoes	3 minutes	15 minutes in total. (3 minutes and 12 minutes. Spoke to David and partner)
Fruit and Vegetables	7 minutes (1 minutes and 6 minutes)	None
Decimat	2 minutes	1 minute
Construct a Sum	None	None

This table shows that Ms B sometimes spoke to Carl and David on more than one occasion during a lesson. Ms B spent 12 minutes with David and his partner during the Dominoes task, which was the longest amount of time she spent with one pair or group during the observation period. On average, Ms B spent three minutes talking to Carl each lesson and five minutes talking to David. For two tasks each, Ms B did not speak directly to Carl and David.

The following episode represents a scaffolding conversation Ms B had with David. This conversation was the longest interaction just between David and Ms B that occurred during the lesson observations. David's response, cognitive and affective, will also be described.

Scaffolding conversations with David

The following conversation occurred between Ms B and David during the Fraction Rods lesson. We can see that Ms B used "interlinked conversations strings" (Cheeseman, 2009) by stepping in and out of the conversation. The conversation seemed to be a purposeful attempt (Kyriacou & Issitt, 2008) by Ms B to explore and to build up David's understanding that the number of parts in one whole names the fractional piece.

Ms B: What's the first question?
David: [Reading from the question sheet] What fraction of the orange rod is the yellow rod?
Ms B: So what fraction of the orange one is the yellow? How could you compare them?
David: Two of these
Ms B: So what fraction is this [held up yellow rod] of this? [Held up orange rod]
David: Two halves, a half?
Ms B: Why?
David: Because, look [Held two yellow against the orange rod]… a half
Ms B: What are you telling me?
David: One of these [yellow rod] is half of this [orange rod].
Ms B: So how many of these would you need to make an orange one?
David: Two
Ms B: That makes sense doesn't it? Exactly, yeah. So you can answer the first question can't you?

Now Ms B left David, or "stepped out" of this conversation and allowed David to continue working. She returned seven minutes later.

Ms B: Can I just ask you to come back to here? What have you written down here?
David: Two halves
Ms B: So what fraction of the orange rod is the yellow rod?
David: Two halves
Ms B: It is two halves on its own?
David: No
Ms B: What is it on its own?

David: A half
Ms B: What's the yellow rod?
David: A half
Ms B: How many halves?
David: One
Ms B: One half. So we can answer that question "what fraction of the orange rod is the yellow rod". It's one half isn't it?

Here Ms B addressed an error that David had made. Rather than naming one part, he had named the parts into which the whole was divided. Although these fractions were not written but found by manipulating the rod, David seemed to be attending to the denominator part of the fraction but not the numerator. Ms B then stayed with David and continued talking about subsequent questions for the next eight minutes, beginning by scaffolding more about the idea that the number of parts names the fraction.

Ms B: What fraction of the brown rod is the red rod? [Reading the question] Ahh [looked at David's written answer] So how do you know it's a quarter?
[David began to use the red rods and placed them beside the brown rod.]
What are you checking to see?
David: What it is. I'm kind of measuring it. [He was moving small red rods along the longer rod]
Ms B: Yeah and what are you checking to see with those red ones?
David: See if it's a quarter or not.
Ms B: How would you know if it is a quarter?
David: An ordinary guess
Ms B: No, you're absolutely 100 percent right. I'll tell you that. But how did you know that?
David: Because I went like this … [He was moving small red rods along the longer rod]
Ms B: How many parts were you checking to see? … How many parts fit?
David: Four.
Ms B: So how do you know that's quarters?
David: Cause there's four pieces.
Ms B: There's four pieces in the whole. We know that's quarters don't we. Okay, well done. So is it four quarters or one quarter?
David: A quarter!

Ms B emphasised the number of pieces, not just the size of the pieces by asking David "is it four quarters or one quarter?" This reinforced the conversation string they had earlier regarding two halves or one half. During this part of the exchange, Ms B also appeared to be listening to make sense of the strategy David used and to elicit his thinking about the task. These are features of "responsive listening" (Empson & Jacobs, 2008) as Ms B attempted to link the four parts in the whole to David's labelling these parts "quarters". However, Ms B also asked some leading questions such as "So how do you know that's quarters?" She also concluded this exchange by stating "There's four pieces in the whole. We know that's quarters don't we". This was her statement, not David's, so it was not clear if David knew this.

In these first two questions, David devised a strategy and explained it to Ms B. For the next two questions it appeared that Ms B increasingly led David to solutions.

David: We've got those three …
Ms B: Yeah, we've got the parts that fit now so those together make up whole, make up the blue one so how can we use that now to help us work out how big this one is? You had a really good strategy working for you before with those red ones [rods].
David: That is about … these… so… if you add these together it would be three of …it would be … a third? [He was moving and comparing rods as he spoke]
Ms B: So that's one third isn't it? The little one. So if that's one third what's the big green one?
David: Two thirds

This exchange showed that David was using his strategy of finding out how many of a particular rod fitted into the whole to find the fractional name, as he did with the red rods in the example given previously. Ms B reminded him of this successful strategy, "You had a really good strategy working for you before with those red ones". It would appear that David knew that "four pieces" meant quarters and, in this example, that "three of it would be a third".

The final question asked David to find the whole knowing which rod was two thirds. Having just found two thirds by doing one third twice, Ms B seemed to believe that David would understand about thirds and in particular, that three thirds makes one whole. However, from the following exchange we can see that David does not appear to understand this.

Ms B: Okay so what are we going to do now?
David: Three thirds
Ms B: That's exactly right. So what's three thirds the same as?
David: Three thirds is the same as…
Ms B: What number is three thirds the same as?
David: Same as a quarter?
Ms B: Is it? What is this altogether? [Pointed to the rods]
David: Three thirds
Ms B: What is three thirds altogether? What are we trying to find?
David: [Reading the question] if the pink is two thirds, what is the whole?
Ms B: What does that mean for this one?
David: This isn't a whole?
Ms B: So what does that mean if there's three thirds to a whole?
David: This isn't whole
Ms B: It *is* a whole, isn't it? Because if you've got three parts that go together to make the whole, then that's the whole isn't it?

Ms B initially resisted telling David three thirds was the whole though her questioning narrowed the answers down. However, eventually Ms B had to correct David's assertion that three thirds wasn't the whole and she gave a brief explanation. It seems that Ms B might have stayed with David a little too long during this task. David worked through two questions himself after some brief discussion with Ms B but the next two questions were done with Ms B present. The longer Ms B stayed, the less the task and thinking appeared to be controlled by David.

David appeared to appreciate the time his teacher spent with him during this lesson. In the Fraction Rods post-lesson interview he said,

> I was happy because that was pretty easy until I got up to number 6 'cause I knew it all. Ms B helped me a little bit. She told me the parts, if I was doing good she'd say you're on the right path, you are doing good.

It seemed that David recognised that Ms B had assisted his mathematical understanding about "parts" or fractional pieces though he rather ambitiously said he "knew it all". In addition, David was positive about the emotional support Ms B provided him through encouragement and praise. Ms B also recognised that David needed this kind of support. She described her conversations with him during the Fraction Rods in the post lesson interview.

> David got into it quite quickly. When I went over to him, he'd actually nearly done the first one. He did the first three, I questioned him but he did them. He needed reassurance I think he needs to know that is what you're supposed to be doing.

Ms B demonstrated in this comment that although she "questioned him" she believed David did the work for this task. She described her role as more one of providing reassurance. The examples discussed above certainly show Ms B did reassure David but perhaps in the latter stages of their conversation this support was more than merely affective and became more directed at hinting and leading David to solutions.

This conversation was built up over 9 out of the 40 minutes allocated for students to work on the Fraction Rods task, about 25 percent of the overall time for the task. Ms B invested a considerable amount of time and effort into this conversation with just one student. It is worth exploring the potential impact this conversation had on David's understanding of fractions, specifically that the number of parts names the fraction and, that if the numerator and denominator are the same, for example three thirds, this fraction is equal to one whole.

Over the course of the assessment tasks I gave to both Carl and David, David showed some growth in his understanding of fractions and decimal numbers. Prior to the Fraction Rods lesson described previously, was the "Draw me the whole" assessment task (Clarke, Roche, Mitchell, & Sukenik, 2006), as shown in Figure 6. "Draw me the whole" asked students to consider whether, if a rectangle with no partitions marked, represented 3/4, they could draw what the whole would look like.

Then another rectangle, again with no partitions marked, was to represent 4/3 and the students were once again to show the whole. The questions were read out so interpreting the written fraction was not assessed in this task.

If this is 3/4, show one whole. If this is 4/3, show one whole.

Figure 6. "Draw the whole" assessment task.

For the first part of this task, David appeared initially unsure. After I repeated the question, he drew one more part on the end of the rectangle. I asked "what have your drawn?" and David replied "Another piece, a third". This demonstrated that David knew one more part was needed to make 3/4 into one whole however he did not name this part correctly. In the second part of the task, again David seemed hesitant to begin but then drew another piece, in the same way as the first question, and named this "a quarter". The improper fraction confused David and he couldn't show that three thirds out of four thirds was a whole.

After the Fraction Rods task, the next day's assessment task was a modified "Fraction Pie" task (Clarke et al., 2006), and shed more light on David's growing understanding of fractions. The target students were shown a circle partitioned into two quarters (A and B), a sixth (C) and a third (D). The students are then asked to identify what fraction each part is of the whole circle and explain, "how they know". David correctly identified A as a quarter and said he remembered it from the task he had just done (Fractured Figures). When asked to prove it was a quarter, he proceeded to divide A into four pieces. When asked again to explain why A was a quarter, he then indicated that there would be four As in the whole circle. This may relate to the conversation David had with Ms B, quoted above, regarding the fact that three thirds is one whole. Although David did not understand this during the conversation, perhaps a day later he was beginning to form an understanding shown in his response to this assessment task. In addition, though incorrect, David's reasoning for why C would be a twelfth revealed similar thinking.

> David was quick to identify C as a sixth but then said there were six Cs in a half so there were 12 in the whole. He then changed his answer to a twelfth. (Assessment task, Fraction pie)

The following week, another assessment task, a shortened version of "Fraction Pairs" (Clarke & Roche, 2009b), required David to indicate the larger fraction of each of the following pairs: 3/8 and 7/8, 1/2 and 5/8, 4/7 and 4/5 and 2/4 and 4/8. For the first pair, David said that 3/8 was larger because "larger the denominator smaller the pieces and there's already 8 in 3 pieces of something" (assessment task, Fraction Pairs). Although correct in saying a larger denominator indicates smaller pieces, this did illustrate David's confusion about the role of the numerator and denominator. Later he correctly identified that 4/5 was larger than 4/7 and said "what I said before, the larger the pieces the smaller the denominator and this is like how much someone has eaten because there's two pieces left, no there's one piece left".

These results are interesting when compared to the results of 323 Year 6 students who completed the Fraction Pairs task as part of Clarke and Roche's (2009b) study of rational number. This study found that 77.1 percent of the 323 Year 6 students at the end of the school year could correctly identify with explanation the larger of 3/8 and 7/8, whereas David's answer for this pair was incorrect. On the question comparing 4/5 and 4/7, only 37.2 percent of students in the rational number study answered this correctly with an appropriate explanation, yet David's answer was correct.

It is possible that perhaps David was actually comparing the two numbers in each fraction to each other, that is the three to the eight in 3/8 and the seven to the eight in 7/8 and thinking that the difference between three and eight is greater than the difference between seven and eight. Looking at David's response to 4/5 and 4/7, he said there was one piece left perhaps meaning the difference between four and five is one whereas the difference between four and seven is three. Clarke and Roche (2009b) described this as "gap thinking" which is "really a form of whole number thinking, where the student is not considering the size of the denominator and therefore the size of the relevant parts" (p. 129).

It would seem that Ms B's conversation with David might have been partially successful in that he seemed able to confidently relate the number of pieces with fractional names in subsequent assessment tasks. In addition, David seemed more able to recognise that the same number on the numerator and denominator, or 12 twelfths for example, was equivalent to one whole. However, David continued to use "gap thinking" when comparing the size of two fractions and improper fractions continued to be an area of confusion.

I will now discuss scaffolding conversations Ms B had with the other low-attaining target student in her class, Carl. These differed somewhat to the conversations Ms B had with David in that the conversations with Carl took place over a number of lessons, though often centred on a common concept.

Scaffolding conversations with Carl

Ms B had conversations with Carl over the course of the observed lessons regarding the concept of half and the equivalent decimal number to one half. These began during the first observed lesson, Fraction Rods.

Ms B: If the blue is one and a half, what each of these green ones worth?
Carl: A third
Ms B: A third *of the blue rod* because the blue rod is worth one and a half. How many halves in one and a half?
Carl: Three
Ms B: Yes why?
Carl: Cause … we just did it then. [Carl's voice is quiet and sulky]
Ms B: No but I'm asking you. How many halves in one and a half?
Carl: I don't know.
Ms B: How many halves in a whole?
Carl: Four
Ms B: How many *halves* in whole?
Carl: Two
Ms B: Have you switched off?
Carl: Yes
Ms B: Switch back on please. How many halves in a whole?
Carl: Two

The conversation continued with Ms B pressing Carl to explain that there are three halves in one and a half. Carl appeared confused about the fact that three green rods fitted into one blue rod and yet the green rods were not thirds. As the blue rod represented one and a half, an improper fraction, the green rods were worth one half of course. Given Carl's inability to recognise improper fractions during my initial assessment task, the cognitive demands of this task were too great for Carl. Ms B persisted in trying to scaffold his learning.

Ms B: Three okay so if this is one and a half, and we know there are three halves in one and a half, what are the green ones worth?
Carl: A quarter no a half
Ms B: Why?
Carl: Because, no it's one and half
Ms B: Think about it. The blue is one and a half, what's each of these green ones worth?
Carl: Four quarters

Ms B talked to Carl about this question for four minutes and eventually Carl was able to find a rod that was equal to one whole. The question asked was "if the blue rod is one and a half, which rod is two thirds?" After a struggle to have Carl identify that one and a half has three halves, then to find the whole, Ms B decided not to continue with Carl to find two thirds.

Ms B:	Okay so if that's a half and that's a half what's this?
Carl:	A half a whole
Ms B:	So could I find one that's a whole?
Carl:	[Carl moved the rods around.] The green
Ms B:	Okay so that's the whole what do we have to find out though? [Carl said nothing.] Okay [Ms B clapped for attention from the class.]

This conversation involved Ms B in continuing to press Carl to explain his thinking and not to refer to his partner or "what we said before" as the source of his answer. There was quite a degree of cajoling and encouragement involved in this exchange as Ms B attempted to maintain Carl's concentration ("switch back on please"), and confidence ("right, beautiful, 'cause you're good at this").

It is clear that Carl was confused about the task, probably due to misconceptions about improper fractions. It also appeared at times that he was feeling negative about being pressed by Ms B to continue to engage in the task. However, in the post-lesson interview Carl reported, "it was pretty fun 'cause it was about fractions. I got to build stuff". This perhaps indicated that Carl held no on-going negative feelings towards Ms B pressing him to engage in this task. In fact, in his initial interview, Carl said that mathematics is "boring if you can't understand it". He then elaborated on why mathematics could be boring and said, "if the teacher doesn't really explain it to you and you don't really know what to do". This appeared to indicate that Carl believed the role of the teacher was to explain mathematics so that he could understand it and that he valued such explanations. However, it is not clear if Carl viewed Ms B's questioning and prompting him to explain his thinking as fitting with his view of what it means if the teacher is explaining. In addition, the task was beyond Carl's current understanding so it is likely that no amount of explanation from Ms B would have been enough for Carl to have understood this task.

A conversation during a lesson the following week also centred on the concept of half.

Ms B:	So that's 3/4 and that's point five. What's point 5 equivalent to? What fraction?
Carl:	Four?
Ms B:	What's that 5 actually mean? What's that place value?
Carl:	Point five
Ms B:	What comes after the decimal point? Tenths, hundredths…So this is 5?

Carl: Tenths
Ms B: Tenths. And five tenths is the same as what fraction?
Carl: Half
Ms B: Okay so it's half a kilogram and 3/4 of a kilogram in the same bag together?
Carl: No.
Ms B: So you're going to need to have a look at that.
(Fruit and Veggies, lesson observation)

This exchange revealed that Carl was able to name the fraction that is equivalent to five tenths though he seemed unsure of what "point five" meant until Ms B led him to "fill the gaps" with five tenths. It appeared that Carl was comparing equivalent fractions, five tenths and one half, but was not connecting five tenths to "point five", a decimal number. For example, Carl was able to answer what fraction was the same as five tenths when this question was reversed, but when asked to name the decimal number equivalent to one half, he showed some confusion. This was revealed by the next part of this conversation that occurred about ten minutes later.

Ms B: More or less than half? What's a half as a decimal?
Carl: Zero point two, zero point oh two?
Ms B: What half of a tenth? How many is half of the tenths?
[No response.]
In a whole, how many tenths?
Carl: Ten.
Ms B: What's half that?
Carl: Five.
Ms B: Five tenths as a decimal? Zero point? …
Carl: Zero point zero one?
Ms B: Five tenths same as a half. Which place value tells us about tenths?
Carl: Zero point one zero
Ms B: What is that one worth?
Carl: Zero ... One.
Ms B: One what?
Carl: Tenth.
Ms B: We want to say five tenths so what do we put here?
Carl: Five.
Ms B: Same as a half? You knew that didn't you? You do now.
(Fruit and Veggies, lesson observation)

Again Ms B resorted to "fill the gaps" questions, "zero point...?" Carl clearly showed in this exchange that he does not know the decimal equivalent to one half and demonstrated a common misconception by offering that "point two or point zero two" might be equivalent to half taking the number two from the denominator of one half and applying this to decimal numbers. In addition, Carl was unable to express five tenths as a decimal number. I will now discuss evidence from assessment tasks and other data of Carl's understanding of these concepts.

One assessment task asked the target students to identify the decimal number equivalent to one half and explain why. The task was completed two days after the

Fraction Rods lesson in which Ms B and Carl had the conversation described earlier but before the Fruit and Veggies task also described. Carl appeared quite confused by the question "what is the decimal number that is the same as one half" and said:

> Fifty point zero is the same as half … Does it have to be a decimal? … Five point five? … I don't get it … Maybe about thirty point five? (Equivalent decimal to half task)

The following day during the Dominoes task, Carl and his partner were observed during the following exchange.

> Carl and his partner have written some of their own dominoes with most being written by the partner. Carl wrote one and his partner called out "Carl! 1.2 isn't a half!" Carl said, "Yes it is!" After walking around the room, Carl returned to his partner and said "5 out of 10 is a half". (Dominoes, lesson observation)

This exchange offers further evidence that Carl was thinking of one half as equivalent to another fraction or amount, "five out of ten" but could not name five tenths as a decimal number. The final lesson observation underscored this point.

> Carl: [Calling out] Ms B! I've coloured in a half!
> Ms B: Good! How many tenths is that?
> Carl: Five.
> Ms B: Five tenths, yeah, half the tenths.
> (Decimat, lesson observation)

It would appear that Carl's understanding of one half as a decimal did not improve appreciably although Ms B had many conversations with him about this concept. Carl repeatedly connected five-tenths, an equivalent fraction, to one half yet remained unable to express five tenths as a decimal number. However, as Steinle and Stacey (1998) reported, this continued confusion could be expected with "many students throughout schooling and indeed many adults have difficulty understanding the notation used for decimal fractions" (p. 548).

These episodes characterise the "halting and erratic" nature of coming to understand a concept (Hiebert et al., 1991). Carl appeared to know that half is the same as five tenths and did not know how to express this as a decimal number. It would appear then that the scaffolding conversations Ms B had with Carl were not successful in scaffolding Carl's understanding that 0.5 is the same as one half. However, these conversations could have aided Carl in coming to understand that five tenths or five out of ten is the same as half, which still represents progress in his mathematical understanding.

The scaffolding Ms B provided to David and Carl through her conversations with them during lessons seemed to be somewhat effective. David showed some growth in understanding that the number of parts names the fraction and Carl was developing a clearer understanding that five tenths is equivalent to one half. As illustrated by Table 16 at the beginning of this section, Ms B spent on average three minutes per lesson talking to Carl and an average of five minutes per lesson talking to David. However, on two occasions her total interactions with Carl were less than two minutes and during one lesson Ms B did not speak to Carl. For David, during one lesson Ms B spoke to him for less than one minute and for one lesson Ms B did not speak to David at all. To summarise, for Carl there were three out of six lessons where Ms B did not have any substantial discussion with him and for David two out of six lessons saw him have little or no discussion with Ms B. Ms B did have sustained conversations, of five minutes or more, with both Carl and David during two lessons. The impact of this time spent, or not spent, with Carl and David will be explored further later in this chapter.

I will now shift the focus of this discussion to describe the scaffolding conversations between Ms L and her target students, Sophie and Riley. Again, an area of potential cognitive growth for Sophie and Riley will be described along with their affective responses to discussions with Ms L during mathematics.

Scaffolding conversations – Ms L

In a similar way to Ms B, Ms L spent time while students were working on tasks to talk to individual students. This was reflected in her description of a typical mathematics lesson in her classroom in which she described her role during student independent activities as roving and asking questions without telling students the answer or how to do the task. In all the observed lessons, Ms L moved around the room talking briefly to students individually including the target students, Sophie and Riley. As Table 17 shows, Ms L's conversations with Sophie averaged six minutes and with Riley the average was four minutes. Some of these conversations consisted of more than one exchange. These short "interlinked conversations strings" occurred over the course of the 30 – 40 minutes students were working independently on the main task of the lesson. Other tasks were completed as a whole class. These tasks were typically shorter tasks averaging 12 minutes. During these tasks, there were whole class discussions only.

Table 17

Time spent with Sophie and Riley during tasks

Task	Time spent with Sophie	Time spent with Riley
Missing Numbers	Whole class discussion only	Whole class discussion only
Find the Pairs	1 minute	2 minutes
Target Multiplication	Whole class discussion only.	Whole class discussion only.
Highest Product	6 minutes in total (1 minute, 2 minutes, 2 minutes, 1 minute)	4 minutes
Veggie Patch	Whole class discussion only.	Whole class discussion only.
Help Julia	10 minutes	5 minutes
Array of 48	Absent	Whole class discussion only.
Athletics Day	6 minutes in total (1 minute, and 5 minutes)	3 minutes in total (1 minute and 2 minutes)

Table 17 shows that for three tasks, Ms L did not speak directly to either Sophie or Riley. For Riley, during four tasks Ms L did not speak directly to him though Sophie was absent for this fourth task which was Array of 48. Therefore, for half the analysed tasks Ms L was not available to speak to individual students, including Sophie and Riley, due to the task being completed as a whole class group. I will now discuss Ms L's conversations with Sophie, which occurred over the remaining four tasks.

Scaffolding conversations with Sophie

Ms L spent on average six minutes talking with Sophie during the four tasks that were not completed as a whole class. During the Help Julia task, Ms L spent 10 minutes with Sophie, which was their longest exchange, while during the Find the Pairs task Ms L only spoke to Sophie to re-direct her or give organisational prompts.

The scaffolding conversations Ms L had with Sophie involved strategies for multiplying multi-digit numbers. The following scaffolding conversation focussed on a strategy for multiplying by nine. This was to multiply by ten then subtract one set. It occurred in small chunks of one to two minutes over the course of the Highest Product task. This task asked students to arrange the digits 9, 8 and 7 into a two by one digit multiplication expression in order to make the highest product. The conversations began with Sophie's own strategy for multiplying.

Ms L: Can you tell me what you're doing here? [Sophie has 98×7 written on her whiteboard]
Sophie: Well first I said that 7 plus 7 was 14 and 14 plus 14 is 28 so I did that twice and that was 56 and I added the 7 to that.
Ms L: For 98 times 7? What … where have you gone 98 times 7 can you explain that to me again?
Sophie: Well I'm doing … I mucked it up. It's supposed to be 9 times... [Wrote on her whiteboard] I made a mistake. It was meant to be 9 times 7 and I did 8 times 7.
Ms L: Would it be 9 times 7 though? What do we need to use?
Sophie: It would be ninety
Ms L: [To the class] Do you think guys that place value will be important here? [Students said yes.]

Here we can see Sophie was attempting to calculate seven multiplied by nine, which was actually 90. She was working up from two sevens twice (four times seven), to give 28, then again to make eight times seven then added one more seven to make nine times seven. This was quite complicated and inefficient though a valid strategy that would have led Sophie to the correct answer, providing she then "added a zero" to this answer. It would appear that she did not know seven times nine without such a strategy. Ms L did not appear to recognise what Sophie was doing here and although she asked for clarification ("can you explain that to me again?"), in the end she focussed instead on the fact that it is ninety and not nine that Sophie needed to multiply by. Within one minute, Ms L returned to talk to Sophie.

Ms L: You have to use three digits Sophie. Your digits are 9, 8 and 7 to give me the highest possible product. It needs to be set out like a two digit number times a one digit number. At the moment you've only got 9 times 7. You've got three digits but look at that [pointed to the board] it's one digit times two.
Sophie: Ohhhh! Woops.
Ms L: So if I put the 9 there and I said that's the nine now use the other two numbers to give me the highest possible product, how would you arrange the other two numbers, in there to give me the highest product? Is that the only way to do it though? So I put the nine there but what would happen if I put the nine here or if I did that one and I put the nine there, how would I arrange the other numbers and which one of those would actually give me the highest product out of those three? And how would you work it out? So record it in your book. Show me how you would work these out.

Ms L did not ask Sophie to explain why she calculated 9×8 but assumed that Sophie had not understood that she needed to have a two digit by a one-digit expression. Ms L used the rest of this exchange to re-explain the task. This explanation was quite complex involving five questions and two statements. Ms L then left Sophie for four minutes before returning. The first part of this exchange involved Ms L drawing Sophie's attention to the fact that her equation "9 × 87 = 202" could not possibly be correct. Then Ms L discussed how Sophie could calculate 9 times 87.

Ms L:	So now, how can you tell me what 9 times 87 is? What can we keep doing here?
Sophie:	Keep adding on 87
Ms L:	Until you've added it?
Sophie:	Until we've got 9.
Ms L:	Okay that will give you the right answer so that's one strategy because addition, multiplication is when we keep adding the same number over and over and over again. So keep adding on for that please.

Ms L invited Sophie to use the adding strategy Sophie had been using at the beginning of the lesson to calculate 7 times 9, saying, "what can we keep doing here?" This would appear to show that Ms L might have now been attending to and responding to the strategy that Sophie had decided to use herself, illustrating the kind of "awareness of and responsiveness to the students' thinking" described by McCosker and Diezmann (2009) or "responsive listening" (Empson & Jacobs, 2008). Ms L returned in eight minutes for the final part to this conversation.

Ms L:	How are we going here Sophie? Okay if you think, this is pretty time consuming isn't it? So let's look at, if it's 9 times 87 do you think perhaps we could use our knowledge, how do we multiply by 10? So 10 times 87 which would be what?
Sophie:	870
Ms L:	Okay, spot on. But we only want to multiply 9 times so what do we have to take away from 870 to make it correct?
Sophie:	Ahhh… 87?
Ms L:	Because we've multiplied one extra. So you do 870 take away 87. See if that will help you. That will be a quicker … if that's going to help you because that's a quicker way of doing it, isn't it?

Now Ms L offered Sophie an alternative strategy that would be more efficient. Importantly she did so after allowing Sophie to experience an inefficient strategy, perhaps adding to Sophie's motivation for finding a quicker way. Sophie appreciated the efficiency of the "multiply by ten and take one set" strategy for multiplying by nine and commented in an interview after the lesson.

> I was confused with trying to find the answers. I got confused trying to find the answer to 87 times 9. Then I used subtraction. If you do 10 times 87 it will make 870. If you minus 87, it gets to 783. If I kept adding 87 to my answer it would've taken a long time. It was Ms L's idea.
> (Post lesson interview, Highest Product).

Ms L described in some detail the scaffolding "moves" she made throughout the lesson with Sophie, culminating in introducing her to a more efficient strategy for multiplying by nine. The following is an excerpt from the post-lesson interview with Ms L.

> Sophie in the end, I got her from adding … she was still [adding], when she was multiplying doing single by single … So we need to go right back to basics … So we eventually got her to, we said okay so we're doing that [adding 87 nine times], it's taking up … it's not an efficient

method. What if we rounded our 9 up to 10 and she did give me that it is going to give me 870 but I need to take away 87 ‘cause I did one more
(Post-lesson interview, Highest Product)

Two weeks later, Sophie was still initially using her old adding strategy for finding answers to multiplication facts she didn’t know, however she did recall the strategy for nines in this instance.

Sophie was counting on her fingers. She told me she was counting two more nines from 18 (2×9) to get an answer for 4×9. Then she was trying to work out 9×7. After a minute she said “I could just do 10 times 7 and take off 7 to get the answer”. (Lesson observation notes)

It would seem that the strategy of repeated addition was still part of Sophie’s repertoire but that the “multiply by ten and take one” strategy was co-existing with this less sophisticated strategy. In potentially taking up this new strategy, Sophie appeared to be having a period of time where the two strategies were “overlapping” (Siegler, 2000). Overall, it would seem that Ms L’s scaffolding conversations with Sophie about a strategy for multiplying by nine were effective in that this strategy was used by Sophie spontaneously two weeks after her conversation with Ms L. The extent to which Sophie continued to use this strategy in the weeks after the present study concluded would give a clearer picture of the total effect of these scaffolding conversations.

I will now discuss Ms L’s conversations with Riley, which differed somewhat from those she had with Sophie. Again the content of these conversations were centred on multiplication.

Scaffolding conversations with Riley

Ms L’s scaffolding conversations with Riley centred around strategies for multiplying, both written strategies such as the grid method and mental strategies like building up from a known fact to calculate the product. The conversations with Riley on three out of four occasions involved one exchange. This differed from Ms L’s conversations with Sophie that occurred over four exchanges in one lesson and two in another. It seemed that with Riley, Ms L was less likely to return and continue the conversation, illustrated in this example from the Highest Product task.

Ms L: Do you have any strategies you can use to work out the answers to multiplication problems?
Riley: Not sure
Ms L: If I asked you to, in your book, work out the answer to that problem how would you show it to me? What would you do? So let’s write, I’m going to write that [wrote 9 ×87 horizontally on Riley’s whiteboard]. That’s what you’re telling me that’s what

> it should look like. So I'm going to go away, and I'm going to come back and I want you to show me on here how you would work out the answer to that problem. Okay? So if you were asked to solve these problems and give me an accurate answer, what would you do? What strategy would you use?
>
> (Highest Product, lesson observation)

Riley was still working on 9×87 until the lesson ended. Ms L had said, "I'm going to come back" and then did not return to continue a scaffolding conversation with Riley. Ms L's initial invitation to "show her a strategy" might have been an attempt to attend to Riley's own strategy before imposing one of her own. However, as we can see from Riley's whiteboard shown in Figure 7, he struggled to use the traditional algorithm he had chosen to calculate an answer.

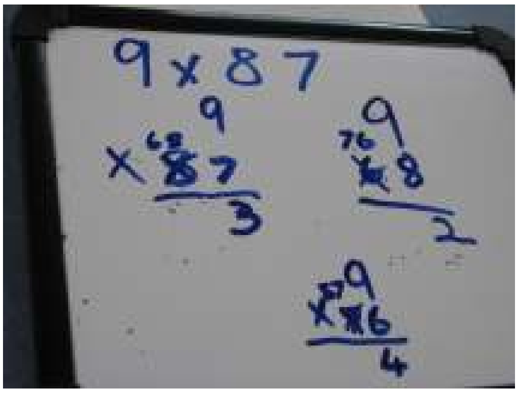


Figure 7. Riley's whiteboard for Highest Product.

Without another "string" in this conversation, Riley was left to struggle for the remainder of the lesson. Ms L recognised this and commented in the post-lesson interview "I didn't actually get back to Riley so I don't know what he did with his strategy in the end". Despite his lack of success in completing the task, Riley spoke positively about the lesson in the post-lesson interview.

> I was trying to think of an answer on my board for 9×87. It's [the arrow] in between happy. I was enjoying this lesson. I was trying to get an answer. It was a good lesson.
>
> (Post-lesson interview, Highest product)

Riley did not appear to react negatively to Ms L's failure to return to him. It seemed he enjoyed "trying to get an answer" and still viewed the lesson overall as a good lesson.

During the next lesson, Ms L attempted to scaffold Riley's understanding of multiplying by multiples of ten. This began with Riley demonstrating how he used the traditional algorithm to calculate 45×26.

> Riley: I'm not sure. 5 times 6 is 30 …
>
> Ms L: Can we write each step? So we can see exactly what you've done.

[Riley wrote on his whiteboard. Ms L got the flip video to film his explanation.]

Riley: I go 30 then I lift up the 3 so then it would be 34 times 2 equals 8, wait 34 times 2 equals … wait

It is clear that Riley is having difficulty with the traditional algorithm for multi-digit multiplication. Ms L recognised that and the conversation continued.

Ms L: I'm going to stop you there. Lets not worry about this carrying because I think that gets really confusing for people. If we're thinking about place value, what can you do? You've done 5 times 6, what do you need to do now? If we just think about the 6 for a moment.

Riley: Umm…

Ms L: What could we do? If we just think about 45 by 6 and 45 by 20 would that help us? If we broke it up to two smaller problems?

Riley: I don't really get it.

Now Ms L suggested another strategy for breaking the question into two smaller parts but Riley said he didn't understand this so Ms L pursued his original strategy. This demonstrated Ms L's responsiveness to Riley's thinking during this conversation.

Ms L: Okay. So you've told me. I'm going to show you what you've told me so far. Basically 6 times 5 is 30 so you've multiplied that one but you've only done it by the five. So what would you do next? What would you multiply the 6 by now …

Riley: Is it 4 times 6?

Ms L: Is it 4 times 6?

Riley: Or 6 times 4?

Ms L: Is it 4? If we're thinking place value what is that actual number?

Riley: Umm...

Ms L: What's our place value? If we've got 5 as a unit what is the four?

Riley: Umm, the tens?

Ms L: So how do we actually say that number?

Riley: Umm, 40 … five?

Ms L: Yep, no it's 40 because you've already said that.

Here, Ms L was pressing Riley to articulate the place value of the numbers he was going to multiply next. In a traditional algorithm it is often considered that this would be 6 times 4 and the place value of the four is not specified. However, Ms L persisted in pressing Riley to understand that the four was actually forty. In the end she resorted to "fill the gaps" questions in order for Riley to say that the four represented forty.

Ms L: So what's 6 times 40?

Riley: It's ...

Ms L: If you know your multiplication, if you know your four times tables how can you use that to help you with multiplying a number by 40?

Riley: Would you be able to do 4 times 6 …

Ms L: Yeah lets try and read it in the order that it's written.

Riley: 6 times 4 …

Ms L: Which is?
[Riley was silent for 10 seconds]
What's 6 times 4?

Riley: Umm …

Ms L: If you know 6 times 2 …

Riley: Is 12. Oh! 24

Ms L: Right. Okay so you know… remember that rule we just talked about? If 6 times 4 is 24, what's 6 times 40? Thinking about that other rule we just spoke about?

Having successfully given Riley strategies for working out 6 times 4 which he couldn't answer, Ms L now attempted to draw his attention to the "multiplying by ten" rule discussed as a whole class earlier in the lesson.

Riley: It is … I could, would it be 400?
Ms L: No, if you've got 6 times 4 is 24, what's 6 times 40 if we think about that rule that Chris just said with multiplying by 10?
[Riley was silent for about 30 seconds.]
Right if you know 6 times 4 is 24, 6 times 40 … we said it makes it 10 times bigger didn't we? [Ms L wrote on the whiteboard, drawing a place value chart and writing 24 in the tens and units place] So that's our hundreds, tens and units, if we make it ten times bigger, what do we do with our numbers?
Riley: Ten times bigger, you would times that by…
Ms L: So where can I move it? [The 2]
Riley: You can move that to hundreds
Ms L: Uh hummmm… and what do I do with the four?
Riley: Move it to 10
Ms L: And what do I put here? [Units place]
Riley: Zero
Ms L: So answer is?
Riley: 240
[Ms L wrote this on Riley's whiteboard.]

To scaffold Riley's understanding of multiplying by ten, Ms L drew a place value chart to illustrate the numbers increasing to the power of ten. Riley then offered his interpretation of what multiplying by ten means.

Ms L: So if you know that then …
Riley: This is what I was thinking. Umm adding a zero here, that's what I was thinking but then I accidentally did it wrong, I added a zero …
Ms L: Okay. So 6 times 4 is 24, 6 times 40 is 240, what would 6 times 400 be?
Riley: Umm…
Ms L: Same thing, think about place value, it's getting 10 times bigger again so 6 times 400 is 10 times bigger than 240
Riley: Can I use the … [pointed to the place value chart]
Ms L: You can use that to help if you want. I'll come back in a moment
(Help Julia, lesson observation)

Ms L seemed to have decided that Riley needed to spend time practising multiplying by ten so that even though 6 times 400 was not in the original task, he was left then to use the place value chart to complete this unrelated calculation. Figure 8 shows Riley's whiteboard and his working for 6 times 400.

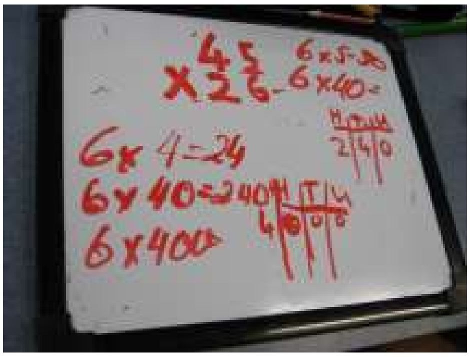


Figure 8. Riley’s whiteboard for Help Julia.

While Riley was working on 6 times 400, I asked him about what he was doing.

> I spoke to Riley about his work. He was working on 6×400 having worked on 6×4 then 6×40 with Ms L. He talked about how the numbers “Moved up” the place value chart and it was like adding a zero but it changes the whole number. I asked what 6×400 really meant and he wasn’t sure. He said “it’s like timesing [*sic*] by 10 the whole way”.
> Then he decided to halve the number and did 200×2, 400×2 and 800×2. I asked him why he did this and he wasn’t sure. Then he said, “I just realised it’s 2400 (6×400)” I asked him why he was trying to work out 6×400 as it wasn’t in the question. He said, “I’m not sure”.
> (Help Julia lesson observation)

This would seem to indicate that Riley at this time did not see the connection between calculating 6 times 400 and the multiplying by ten rule as he attempted to build up to this by working out 200 times 2, 400 times 2 etcetera. Again, although Ms L said, “I’ll come back in a moment,” she did not return to Riley. Instead, Riley approached Ms L.

> As the students began to pack up and come to the floor, Riley approached Ms L.
> Riley: I left out the answer but I just realised it was that (2400)
> Ms L: For six times four hundred? Okay so you’re starting to find a …?
> Riley: Pattern
> Ms L: Well done.
> Riley: So if I do 6 times 4000 I know what it will …
> Ms L: Okay but what’s happening each time? We’re not just adding a zero. We’re making it…
> Riley: I know, a whole different number
> Ms L: Because we’re moving it along. We can’t just add a zero because when we work with decimals and things, just adding a zero doesn’t work.

Though Riley did not complete the original task of 45 times 26, Ms L indicated in the post-less interview that she was satisfied that the lesson still assisted Riley’s understanding by highlighting place value and multiplying by ten patterns.

> With Riley we didn’t actually get anywhere near the answer but I wanted him to get into place value but at least he came to an understanding of pattern when multiplying by ten so that was good.
> (Post-lesson interview, Help Julia)

In contrast to the Highest Product lesson, Riley's response to the Help Julia task was less positive.

> In between question and sort of … alright. Sort of like really tiring 'cause I had to keep going over and it got sort of annoying. Keep going over … It wasn't the best lesson.

I also asked Riley about what he had learnt during the lesson about patterns for multiplying by ten that Ms L believed Riley had come to understand.

> I found out ways how to times 6 times 4 to 6 times 40 like patterns. I learnt patterns. Just that … ummm … did you say what I learnt? I learnt how the pattern … I learnt how like, 'cause I didn't know what the last answer would be and I just got the answer just by thinking …

He appeared to be unsure about what exactly Ms L had intended him to learn and could not articulate the multiplying by ten pattern that Ms L said he had "come to an understanding of".

The next time Ms L had a conversation with Riley about multiplying by multiples of ten was during the final lesson observation when Riley was attempting to calculate 975 by 7 using the grid method.

> Riley: You go 7 times 7 then I go 5 times 7
> Ms L: Yeah okay. Is that seven? That one?
> Riley: That's seventy
> Ms L: Okay so it's really important that you write that isn't it? Because if you haven't thought about the values in that then you're going to make a simple error there. Now you need to work out 7 times 70. We looked at that yesterday didn't we? Of having to work out 7 times 70 if you know…
> Riley: [Interrupting] I just need to do 7 times 7.

Riley showed that he knew that he could use 7 times 7 to find an answer for 7 times 70, which is similar to the conversation discussed previously that he had with Ms L during Help Julia about 4 times 60. It would seem that Riley had begun to adopt the strategy for multiplying by multiples of ten. However, Riley did not know 7 times 7. Ms L gave him a valid strategy by building up from a known fact of 7 times 5 and adding 14, two more sevens. Unfortunately, Riley did not understand this method and appeared confused about why 35 and 14 would be added.

> Riley: Are we timesing [*sic*] 13 and 35?
> Ms L: Pardon?
> Riley: Are we timesing 13 and 35 to get?
> Ms L: No we're adding it! 14 and 35 to get 7 times 5, 7 times 7 sorry. What's 7 times 7 then?

This appeared to be an example of a "scaffold misfire" (Bliss et al., 1996) in that the scaffolding Ms L offered of building up from a known fact, did not connect with Riley. It appeared Ms L and Riley were on “different wave lengths” (Bliss et al., 1996, p. 47). Riley did not complete this calculation during this lesson.

After this lesson Riley reported, “I learnt how to do it, use the grid with 3 digit numbers. I asked Ms L and she gave me a hand with making the grid longer. I got it straight away, quicker than I usually do it” (post-lesson interview, Athletics Day). Again, even though Riley did not complete the task or his calculation of 7 times 70, he felt positive about the lesson.

These scaffolding conversations between Ms L and Riley seem to illustrate her belief that an important part of her role as teacher was not to “tell them the answers” (initial teacher interview). Even though Riley did not complete tasks because he could not answer reasonably simple multiplication facts, Ms L did not tell him the answers. On all four main tasks, Find the Pairs, Highest Product, Help Julia and Athletics Day, Riley did not complete the task and remained stalled on single digit multiplication expressions. Though this seems dispiriting, there was evidence of growth in Riley’s ability to use written methods to solve multiplication problems.

During an assessment task after the second lesson observed, Riley was asked to tell a story or draw a picture to match 20×19 and to describe how he might solve this question. Riley could not tell a story to match the expression, an issue discussed later in this chapter. He attempted to use the traditional algorithm to solve the expression and wrote “20×19 = 20” vertically. He then looked at this answer and said, “no, that can’t be right. I had 20 to start before I multiplied by 19”. It would appear that Riley multiplied 9 by 0 and wrote zero in the ones place then multiplied 1 by 2 and wrote two in the tens place. Clearly he is unable to use the traditional algorithm successfully but also had no other strategy to use in its place.

A later assessment task, given after the sixth lesson, asked Riley to find an answer to 14 multiplied by 16. Riley was able to use the grid method, shown by Ms L during lessons, to successfully complete this calculation.

These results demonstrate some improvement in Riley’s ability to solve multi-digit multiplication problems using a written method. In the first week of observations, Riley was attempting to use the traditional algorithm but was unsuccessful. By the third week of observations, Riley was able to confidently and accurately use the grid method shown to him by Ms L to calculate multi-digit multiplication expressions. Riley’s difficulties in knowing answers to single digit multiplication facts were still present in the final lesson observation. Ms L was attempting to scaffold Riley by offering strategies such as building up from a known fact, doubling a known fact and multiplying by ten. Riley remained confused about these strategies as shown in his question regarding 7 times 5, "are we adding 14 and 35 or multiplying?" It would appear that while Riley's written methods showed improvement, his understanding of the underlying concepts of multiplication might still be developing.

Ms L used scaffolding conversations quite differently with Sophie and Riley, although she spent on average three to five minutes with both students over six lessons observed. With Sophie on two occasions, Ms L had "strings of conversations" (Cheeseman, 2009) where she would return after Sophie had worked a little more to continue to scaffold her learning. With Riley on four occasions, Ms L began a conversation, asked Riley to complete some work and then did not return to speak to him. Ms L spoke to both Riley and Sophie during five out of the six lessons observed. During the other lesson, Sophie was absent and Ms L spoke to Riley briefly over the lesson about organisation rather than mathematics. Both Sophie and Riley demonstrated some growth in understanding of the relevant content over the lesson observation period. In particular, Sophie became more proficient with using a strategy for multiplying by nine and Riley was able to use a written method to reliably calculate multi-digit multiplication expressions.

SUMMARY

There were many similarities in the way that Ms B and Ms L used scaffolding conversations during mathematics lessons. Each teacher spent about the same amount of time, an average of three to five minutes per lesson, talking with the low-attaining target students. Both Ms B and Ms L had sustained exchanges of more than five minutes during two of the observed lessons with each of their low-attaining target students. The longest conversations with each target student were about eight minutes for both classrooms. Both teachers did not speak directly to the target students for about half of the analysed tasks, in Ms L's class due to half the tasks being whole class tasks and in Ms B's class because she spent extended time attending to a small number of students each lesson. Both teachers initially held back from telling the students answers or strategies. Both teachers pressed these low-attaining students to explain their thinking and to persist in the tasks. Both teachers appeared to be seeking the kind of "purposeful conversation to explore the pupils' understanding" described by Kyriacou and Issit (2008).

There were also differences in the scaffolding conversations in each classroom. While both teachers spent an average of five minutes talking to the target students, in Ms B's class this was more likely to be at least five minutes for one exchange whereas Ms L would split the five minutes by spending one or two minutes with these students, moving on, then returning. Ms L spoke to Riley and Sophie during all the tasks that were not whole class tasks. Ms B did not speak to Carl at all during three of six lessons or David for two of six lessons.

When considering the impact the scaffolding conversations had on the learning of the target students in each classroom, it would appear that these conversations were potentially effective with the target students demonstrating modest gains. David began to consolidate that the number of parts names the fractional piece, Carl grew in his knowledge of half and the equivalent fractions, Sophie began to use a strategy other than repeated addition for multiplying by nine and Riley was able to use a written process to accurately calculate multi-digit multiplication expressions. Affectively, Sophie and Riley in Ms L's class and David in Ms B's class seemed to both notice and welcome the teachers' discussions with them. Sophie pointed out that the strategy for multiplying by nine was "Ms L's idea" and that it saved her time. On another occasion she stated that the task was hard but that with Ms L's input it became more manageable. Riley also identified that Ms L had helped him use the grid method. David stated that Ms B helped him and offered him encouragement by telling him when he was correct. Only Carl made no mention of his teacher, Ms B, during interviews but focussed instead on the features of the task or the students with whom he had worked.

CONCLUSION

Both Ms B and Ms L used discussion as a significant and regular part of their mathematics lessons. This was in line with recommendations from NCTM in the United States (National Council of Teachers of Mathematics, 2000), the National Numeracy Strategy in the United Kingdom (DfEE, 1998) and the National Numeracy Review from Australia (Numeracy Review Panel, 2008). However, the extent to which such discussion provided scaffolding to the low-attaining target students in their learning of mathematics was part of the focus of the present study. The two types of discussion with the potential for the teacher to scaffold the target students' learning were whole class discussions and individual "scaffolding conversations".

It is well documented that low-attaining students can have difficulty in understanding and participating in whole class discussions (Baxter et al., 2002; Burns & Myhill, 2004). Some researchers have offered strategies for improving the effectiveness of whole class discussions for low-attaining students. These have included summarising the student strategies and explicitly teaching an efficient strategy as a class or in a mini-lesson (Baxter et al., 2002), and recording the discussion in some way to provide a visual reference for what has been said (Lampert, 1990; Sullivan et al., 2006). Neither Ms B nor Ms L were observed highlighting a particular strategy offered by students or explicitly teaching such a strategy. Some recording of whole class discussions was observed. Ms B used the manipulative materials from the task in whole class discussions and wrote in the class learning journal, but student strategies were not recorded in the way Lampert (1990) described, with each contribution written with the student's name beside it. Ms L was observed showing student strategies, and recording vocabulary. Some of these examples will be explored further in the following part of this chapter concerning manipulative materials and visual representations.

The effectiveness of whole class discussion for the target low-attaining students could be questioned. Carl and David in Ms B's class could not answer questions in post-lesson assessment tasks that were part of the whole class discussion that had occurred just minutes before. During whole class discussions, all four target students contributed minimally and on many occasions were observed engaging in off task behaviour. Of the four students, Riley spoke the longest single utterance of 44 words while all other contributions by these students during whole class discussions were four words or less. Carl contributed most often with Sophie and David making two brief contributions.

The individual scaffolding conversations between Ms B and Ms L and the low-attaining students held more potential to scaffold student learning. There was some evidence provided to suggest that the target students progressed in their understanding of concepts and processes discussed during these conversations. In addition, with the exception of Carl who made no comment, the target students recognised and appreciated their teacher talking to them individually.

The individual and whole class discussions that took place in Ms B's and Ms L's classrooms are one aspect of scaffolding that had the potential to impact on the learning and feelings of the target students. I will now focus on the second aspect of scaffolding in this study, the use of manipulative materials and visual representations.

THE USE OF MANIPULATIVE MATERIALS AND VISUAL REPRESENTATIONS

Studies have shown that the use of manipulative materials and visual representations can be successful for developing low-attaining students' understanding of concepts such as area (Cass et al., 2003) and fractions (Butler et al., 2003). However Baxter et al. (2001) found that the low-attaining students in their study spent time organising and arranging the manipulative materials rather than engaging in the mathematics of the task. Other studies warned that using concrete materials could confuse children further if they did not have the mathematical understanding to connect the materials to the relevant concept or skills (Ball, 1992). Ambrose (2002) found that some students, in particular girls, can become "stuck" on using materials instead of developing more sophisticated strategies. Boulton-Lewis and Halford (1992) suggested that some use of representations increased the cognitive load of students, thus inhibiting progress. Sowell (1989) suggested that a key to the successful use of concrete materials was long-term use of the materials and the teachers' knowledge about using the materials. Stacey, Helme, Archer and Condon (2001) also suggested that the transparency of the materials in illustrating clearly the intended mathematical concept was critical to their successful use.

Both Ms B and Ms L indicated that they felt "tasks using manipulatives" were appropriate for low-attaining students (Tasks questionnaire). However, Ms B and Ms L differed in their use of manipulative materials and visual representations. In the following discussion, each teacher's use of manipulative materials and visual representations as scaffolding tools will be described. In addition, the response of Carl and David in Ms B's class and Sophie and Riley in Ms L's class to such use will be explored.

MANIPULATIVE MATERIALS AND VISUAL REPRESENTATIONS – MS B

All the lessons observed in Ms B's classroom used manipulative materials and visual representations. The manipulative materials comprised

- cuisenaire rods;
- puzzle pieces or shapes to make circles or rhombuses; and
- the decimat, a rectangle divided into a grid of one thousand equal parts with bold lines showing one tenth and one hundredth of the rectangle.

Visual representations included

- number cards;
- a number line;
- cards with a variety of decimal numbers, fractions and percentages depicted as numbers or pictures of objects, in time or weight situations; and
- pictures of fruit and vegetables and written numbers showing decimal and fractional parts of one kilogram.

Manipulative materials and visual representations in Ms B's class were organised such that each student, pair of students or small group of students were given the materials or representations at the beginning of the task. The use of such aides was an integral part of the lessons. All students were expected to use the materials and all students did so.

Ms B was enthusiastic and seemed committed to using manipulative materials and visual representations in her mathematics lessons. She spoke particularly about the importance of having a model when teaching fractions during a post-lesson interview which I quoted earlier in this chapter. Ms B commented positively on the use of the Decimat which I introduced her to, describing how the Decimat filled a "gap" in her teaching in that it illustrated to students the relative size of the decimal parts (post-lesson interview, Decimat).

It would appear that Ms B used manipulative materials and visual representations as an integral part of her mathematics lessons and not as something for "fun" or to break the routine as described by the teachers in Moyer's (2001) study. The way in which Ms B used the materials would suggest that she wanted to build student understanding of the mathematical concepts. She did not appear to use materials in the kind of rote manner described by Cohen (1990), nor did the students appear to be using the materials only as "answer getting" devices (Beswick, 2005a).

Upon coding lesson observations, the data indicated that Ms B used manipulative materials and visual representations mainly for three purposes. These purposes were to connect physical materials to written symbols for which there were five examples, as a basis of discussion with four examples, and to illustrate underlying concepts with three examples. An episode where Ms B used materials to connect to written symbols occurred during the Fraction Rods lesson.

Ms B:	So there were six ninths in the green one. Now John what did you do after that? Remember we talked about it didn't we?
John:	Then the you break them up into three groups of three, then they're thirds
Ms B:	So they were all joined together [Drew a diagram on the board of one whole, wrote nine ninths beside it then circled three ninths]
John:	Then three thirds was the whole of one green.
Ms B:	Okay so those together made the green one if you stuck them together [circled six ninths]
John:	So then it's two thirds
	[Ms B wrote "2/3" on the board.]

(Fraction Rods, lesson observation)

In this example, we can see Ms B was connecting the materials which were Cuisenaire rods, to the writing of the fraction "2/3". The Decimat task and Fractured Figures tasks also provided instances of Ms B connecting how to write decimal numbers for example, to the physical representation of the decimat.

The coding of data revealed overlap between the codes. The example above could also have been coded as "using materials as a basis of discussion". However, due to the fact that Ms B did specifically link the written notation of fractions to the materials during this discussion, it was also coded as "connecting manipulative materials to written notation". An example of data coded as "using materials as a basis of discussion" occurred during the Fractured Figures task when Ms B spoke to a small group of students including Carl.

Student:	Six these make one whole.
Ms B:	Ooh how did you find that out? That's fantastic.
Carl:	Cause that's three [covering a half]
Ms B:	Right!
Carl:	If you go up… they're sixths.
Ms B:	You've worked out one of the trickiest ones to work out! Now if you know 6 of those fit, and you know that three fit in there so that would be what size?
Carl:	Half

(Fractured Figures, lesson observation)

Here the students showed Ms B how they arrived at an answer by using the pieces of the puzzle to illustrate their solution strategy. The manipulation of these materials became the basis of the discussion between Ms B and this group of students about how many parts were in the whole, and how many sixths were in one half.

Examples discussed previously from the Fraction Rods task have shown how Ms B used the rods to illustrate the concept, for example, that the number of equal parts in one whole names the fraction. Other examples of Ms B using materials to illustrate underlying concepts were found during the Decimat task. In this task, Ms B spent some time at the beginning of the task constructing the Decimat with students by dividing a rectangle firstly into ten, then one hundred and one thousand equal parts to demonstrate decimal numbers. Ms B linked this representation to the concept of equal parts becoming smaller by one tenth.

> Ms B: So a thousand equal parts, hundred equal parts, ten equal parts, one whole. Okay? [Ms B pointed to each part of the decimat while saying this.]
> We know anytime we multiply anything by ten there is ten times more of it. In this case they're ten times smaller, ten times smaller.
>
> (Decimat, lesson observation)

In this example, the representation of the Decimat played an integral part of Ms B's illustration of the concept of decimal numbers and their relative size.

The ways in which Ms B used manipulative materials or visual representations can be examined through the coding allocated to episodes, as discussed, but also in a broader sense. Some of the manipulative materials and visual representations Ms B used were "concrete embodiments" (Boulton-Lewis & Halford, 1992) of mathematical concepts. For example, the Cuisenaire rods used during the Fraction Rods task allowed students to answer the questions about relative sizes of rods primarily through the use of the rods. The rods themselves illustrated the concept that two yellow rods fitted together to make an orange rod for example. Students could solve the task by manipulating the materials and also gain insight into the relationships between parts and the whole through this manipulation. The puzzle pieces in Fractured Figures performed a similar function. The Decimat model also allowed students to see the relationships in decimal place value through their interaction with this representation. Aside from knowing the fractional or decimal names to give parts, prior understanding was not essential for students to be successful in these tasks. The materials themselves could both provide an illustration of the concept and prompt new insights.

In contrast, the visual representations Ms B used acted as prompts for prior knowledge or provided a medium through which students could draw together sets of knowledge. The representations themselves did not illustrate the concepts. The number line for example, provided a visual representation on which students could order various rational numbers on cards but the number line and number cards did not illustrate how such an order could be constructed. Students were required to have some prior understanding about equivalent rational numbers and their relative size and the number line was the vehicle through which they could demonstrate this understanding. Similarly, the cards used in the Fruit and Veggies task showed a number, either decimal or fraction of one kilogram, and a picture of the fruit or vegetable. There was nothing on these cards that showed the relative mass of the fruit or vegetable, they were merely rational numbers that required students to have some prior understanding about their relative size. The effect of these different roles of manipulative materials and visual representations on the learning and feelings of the target students, Carl and David, are examined now.

RESPONSES OF CARL AND DAVID

Both Carl and David responded positively to the three tasks that used materials that were more like "concrete embodiments" of the mathematical concepts. For example, interviews with Carl and David after the Fraction Rods task demonstrated that this task was well received by both boys. Carl said, "It was pretty fun 'cause it was about fractions. I got to build stuff", while David said, "I was happy because that was pretty easy until I got up to number 6 'cause I knew it all. Ms B helped me a little bit. She told me the parts, if I was doing good, she'd say you're on the right path, you are doing good". On the other two tasks that used materials that were "concrete embodiments", Fractured Figures and Decimat, Carl and David also reported positive feelings.

Fractured Figures:

Carl: I liked today's lesson better than yesterday 'cause you got to make puzzles.
David: We worked it out pretty quick, pretty easy. Today's easier than yesterday's 'cause I already knew it, my brain already knew it.

Decimat

Carl: Thumbs up because it was really fun colouring in the squares, playing
David: Really happy face because it was fun and we were learning lots of stuff about decimals like how to write them.

In contrast, Carl and David did not report such positive feelings about other tasks that required students to use materials or representations to activate and organise prior knowledge, such as Sorting Cards or Fruit and Veggies.

Sorting Cards

Carl: The cards was [*sic*] alright. It wasn't great; it was a little bit boring. We had to sort them into groups. It was easy but boring. It was a bit harder today.

David: I didn't really like that lesson. It was really hard. The cards we had to sort them out 'cause you had to find out time and percentages and all that and we had to put in

> quarters and halves and stuff. I didn't work it out; we all took a turn of working it out. I just felt like it was really hard.

Fruit and Veggies

> Carl: It was pretty boring and I'm happy that it's over. There was challenge. [Partner] knew what to do but didn't do it all. Didn't like it because it was challenging.
>
> David: I put it on the thinking 'cause I had to think really hard and unlike yesterday I got heaps of help from my partner. It was a bit more challenging because it wasn't' written in kilograms just in halves and points.

It seemed that Carl and David preferred the tasks using concrete embodiment type materials as opposed to the tasks using representations to prompt prior knowledge. I will now explore Carl and David's potential learning that arose from Ms B's use of materials in each of these ways by closely examining two tasks –Sorting Cards as an example of a prompting prior knowledge task; and Decimat as an example of a concrete embodiment task. The assessment tasks given after the Sorting Cards and Decimat lessons also provided some data about Carl and David's learning. These were "equivalent decimal number to half" after Sorting Cards and "ordering decimal numbers", which was completed after the Decimat task.

The Sorting Cards tasks involved Carl and David in sorting cards with a variety of rational numbers and representations of rational numbers on them. For example, one card had "25%" on it, another had "1/4" and had a picture of a circle divided into quarters with one shaded. Ms B asked the students to sort the cards in any way they liked, but that they had to be able to "justify why you've done it that way" (Sorting Cards lesson observation). Ms B's goal for this task was that students would be "able to make that connection between a fraction and decimal and what that really means" and that the task was primarily about "equivalence, that they have the same value" (pre-lesson interview, Sorting Cards).

At the beginning of the task, Carl and his partner began by putting "all the points" (decimals) together, the words, the times, the pictures and the "lines" (fractions). Carl pointed to a picture card and said "that's one and a half". The card showed one whole apple and a half. The boys were not attending to the value of the numbers on the cards but simply their appearance. They were not using correct mathematical language to describe the types of rational numbers referring to decimal numbers as "points" and fractions as "lines". In terms of Ms B's goal for this task of students exploring equivalence, Carl and his partner were not attending to equivalence.

Ms B instructed Carl and his partner to sort the cards a different way. They decided to put "all the quarters" together. Carl said that 0.25 and 0.75 were quarters because "everything with a 5 or a zero is quarters". He said 0.75 was the same as 75% because "the zero doesn't mean anything so this is 75 (0.75)" The boys discussed whether a fraction with the vinculum on an angle meant it was "one of those" (pointed to the percentages). Carl referred to decimal numbers using whole number language such as 0.5 as 50 and 0.75 as 75 (Sorting Cards lesson observation). Carl and his partner were grouping the cards loosely according to their value. However, the reasoning for these groupings was not mathematical. By chance, 0.25 and 0.75 are both multiples of quarters but Carl's description that "everything with a five or a zero is quarters" was not entirely accurate. Similarly, Carl's reasoning as to why 0.75 and 75% were equivalent was not mathematical, though it did result in a correct matching. The fact that Carl referred to decimal numbers using whole number language indicated that he might not be aware of the relative size of decimal numbers.

This method of sorting, along with the incomplete reasoning behind it, was how Carl and his partner ended the lesson. Ms B did not speak to Carl during the lesson apart from the brief instruction to sort another way. The forms of scaffolding available to Carl for this task were the cards themselves, which were mainly numbers but with some pictures, or his partner's knowledge which appeared to be at a similar level to Carl's.

In the assessment task after this lesson, Carl was asked to name the decimal number that is "the same as one half". As reported earlier, Carl's responses demonstrated his confusion with equivalent decimal numbers and fractions.

> Carl: Fifty point zero is the same as half … Does it have to be a decimal? … Five point five? … I don't get it … Maybe about thirty point five?
>
> (Assessment task, equivalent decimal number to one half)

It is possible that Carl was not sure what the whole was in answering this question. For example, if the whole were 100 then 50.0 would be half. However, the level of confusion about the question indicated that the lesson, with Ms B's intended focus on decimal and fraction equivalence, was not successful for Carl in being able to identify the decimal equivalent to one half. It would appear that the cards themselves did not scaffold his learning effectively and that his partner was also unable to provide Carl with the necessary support for this task.

In a similar way to Carl and his partner, David's group initially grouped their cards into types of rational numbers, pictures or topics such as money. Ms B spoke to them about their sorting strategy early in the task for two minutes then returned and asked them to sort the cards another way. David suggested that the decimal numbers could be made into fractions.

David: These can be into fractions. [Held up the card with "0.5" on it]
Ms B; Ahh. So you're saying that might match with some of the fractions. What's that the same as?
David: Point five.
Ms B: That's what it says, point five, but what's that the same as? What fraction is that the same as?
David: One fifth!
Student: Three fifths!
Ms B: It's the same as five out of ten, isn't it, five tenths? So what's five tenths the same as?
Student: It's the same as ten twentieths.
Ms B: Which is the same as which simpler fraction?
Student Fifty hundredths!
Ms B: You're getting more and more complicated. Which simpler fraction is the same as five tenths?
Student: One two?
Student: One half!
Ms B: Right try and sort the rest that way.
(Sorting Cards, lesson observation)

There appeared to be considerable confusion amongst the students in this group, including David, about what fraction is equivalent to 0.5. Only after some discussion did the group arrive at the answer of one half.

David potentially received scaffolding from Ms B during this task as she spoke to David's group on three occasions for two to three minutes each time. This differed to the support that Carl received in that, as discussed previously, Ms B did not speak to Carl or his partner during the task. Despite this additional scaffolding, David was still unable to answer the assessment task question of "what decimal number is the same as one half".

David: Zero point two is the same as one half because the denominator is a 2 same as half.
(Assessment task, equivalent decimal number to half)

David displayed a common misconception when converting decimal numbers to fractions and vice versa which Clarke and Roche (2009b) also found. It would appear that the cards, the group and even Ms B's direct discussion with David's group was not successful in scaffolding his understanding of equivalence in this case. The materials used were intended to prompt prior knowledge. David's responses to assessment tasks would suggest he did not have enough understanding of fractions, let alone equivalent percent and decimal numbers, to complete the task. The task itself appeared beyond David's grasp and the materials could not offer the necessary support for David in this task. I will now discuss the Decimat task in which materials were used to illustrate concepts rather than prompt prior knowledge.

During the introductory discussion of the Decimat task, Carl showed some confusion when Ms B was constructing the Decimat with students (see Figure 9).

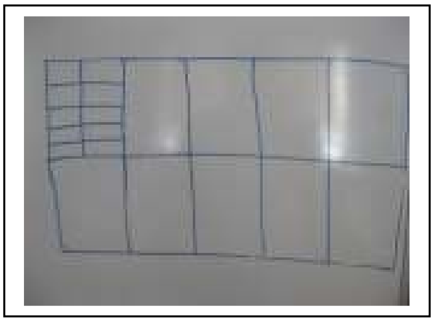


Figure 9. Decimat drawn on whiteboard by Ms B.

Ms B divided the whole rectangle into ten (roughly) equal parts and labelled one of these parts with "0.1". She then divided one of the tenths into ten.

> Ms B: So if one of these is zero point one [pointed to tenth part of the rectangle], what's this? [pointed to the hundredth part.]
> Carl: Twelfths?
> Student: Isn't it one fifth?

Carl's confusion may have arisen from the fact that Ms B only divided one tenth into ten making 19 parts in the rectangle and not one hundred equal parts. Later in the discussion both David and Carl were not sure how many parts there would be in the whole rectangle if it were divided into thousandths.

> Ms B: Imagine if we got all of these, cut all the tenths up into hundredth then cut all the hundredths up into thousandths, how many part would we have?
> David: Millions!
> Ms B: If I've cut them into thousandths?
> Carl: Ten thousand! Two thousand!

After more discussion regarding the decimat and the various sizes of decimal numbers, David and Carl appeared to be clearer on what the completed decimat was representing.

> Ms B: Have a play with this for a minute. How many parts do you think might be here? David?
> David: One thousand
> Ms B: There are one thousand parts here so each one of these parts is one one thousandth of the whole. Find the tenths, and hundredths and that will make it easier when we play the game" [The students cheer at the word game].

	How many thousandths will you have to have coloured in to have coloured one whole hundredth? [Carl put up his hand and the Ms B called on him.]
Carl:	Ten?
Ms B:	Ten. Well done, because there's ten thousandths in a hundredth.

Following this discussion, David and Carl played a game where each student had a copy of the decimat with thousandths, hundredths and tenths all shown. After playing the game for about five minutes, Ms B modelled an example on the board. Carl and David were called on to contribute to the discussion.

	[Ms B coloured nine hundredths on the decimat on the board.]
Ms B:	How would I write that as a decimal? Carl, have a go?
Carl:	Zero point nine?
Ms B:	Zero point nine? What would …
Carl:	No zero point zero nine
Ms B:	Yes because zero point nine would be 9 tenths wouldn't it? Now how many thousandths?
Student:	Six
	[Ms B coloured six thousandths on the decimat.]
Ms B:	How would I write that as a decimal? David?
David:	Zero point zero zero six
Ms B:	Zero point zero zero six. Super star

It would appear that Carl and David's initial confusion about the name and size of the parts of the decimat might have been alleviated somewhat. This example showed that both were able to link the colouring of the decimat parts with the written notation for decimal numbers. Later in the lesson, I spoke to David and Carl as they played the Decimat game and made the following observations;

> David wrote 0.001 and 0.06. He told me that he had coloured 1 thousandth and 6 hundredths.
> Carl wrote 0.09 and was able to say that he had coloured 9 hundredths.

Again, David and Carl seemed to be linking the visual representation of the decimal parts to the written notation. Over the course of this Decimat task, David and Carl showed progress in their ability to write and compare the sizes of decimal parts. As discussed previously, at the end of the lesson Ms B asked Carl to "tell me one thing that you've learnt about tenths, hundredths and thousandths today." Carl replied, "That thousandths are smaller".

Following the Decimat lesson, I gave David and Carl an adaptation of the assessment task "Ordering decimal numbers" (Roche & Clarke, 2004). The set of decimal numbers to be ordered was 0. 0.01, 0.10, 0.9 and 1, a smaller set than the original task. These decimal numbers were on cards that were placed randomly on the table in front of the students. This task asks students to put the numbers in order from the smallest to the largest. David's response was 0, 0.01, 0.9, 0.10 then 1. His verbal explanation was

> David: That's small [0] that's big [1]. Because 9 is just under the ten [0.10]. 0.9 is one ninth.
>
> (Assessment task, Ordering decimal numbers)

David showed a developing understanding of decimal numbers in this task. He made one common error (Clarke & Roche, 2009b) by viewing 0.10 as ten rather than one tenth. David was also unable to name 0.9 as nine tenths and reverted to fractions by naming it ninths.

On the same task, Carl placed the cards from largest to smallest. His answer was 1, 0.10, 0.01, 0.9 then 0.

> Carl: Because 1 is actually 1 whole, that is 10 (0.10) which is bigger than a hundredth, (0.01) that (0.01) goes there 'cause it's bigger than a ninetieth, no a ninth"
>
> (Assessment task, Ordering decimal numbers)

It is clear that Carl had some misconceptions and some emerging understandings about decimal numbers. Firstly, by placing 0.10 as larger than 0.9 Carl showed that he was thinking of 0.10 as a whole number; this was also revealed by his words "that is 10". Carl's inability to identify 0.9 as nine tenths indicated that he had not formed a clear understanding of the place value of this decimal number. However, contrary to this he was correct in identifying 0.01 as a hundredth. He was also able to identify that a tenth ("ten") was larger than a hundredth. This perhaps indicated a beginning understanding about decimal place value. Similarly to David, Carl made an error with his placement of 0.9.

Both boys demonstrated some understanding of the relative size of decimal numbers that had been the focus of the Decimat task that preceded this assessment task. This was in contrast to the "Equivalent decimal to half" task after the Sorting Cards lesson where neither David nor Carl was able to name the decimal number equivalent to one half, even though this had specifically arisen during their sorting of the cards. David and Carl's understanding of the relative size of decimal numbers was by no means complete or robust as they both became confused by the addition of a zero on the end of 0.1 and were unsure of what 0.9 represented. However, they demonstrated a growing understanding of decimal numbers. Their ability to understand equivalent decimal numbers and fractions did not appear to be supported by tasks such as Sorting Cards as the boys' performance in post-lesson assessment tasks revealed. It would appear that the Decimat, a concrete embodiment of the concept of relative size in decimal numbers, was more effective in scaffolding David and Carl's understanding than was the Sorting Cards, which relied on prior knowledge and was mainly abstract symbols. This is perhaps not surprising in light of Prawat's (1989) observation that low-attaining students often have difficulty in activating and accessing prior knowledge. In addition, evidence from lesson observations and post-lesson assessment tasks showed that Carl and David held misconceptions about rational number that made many tasks too difficult.

I will now examine Ms L's use of manipulative materials and visual representations and the responses of Sophie and Riley, the low-attaining target students in Ms L's class. I will then compare the use of manipulative materials and visual representations as scaffolding from each classroom.

MANIPULATIVE MATERIALS AND VISUAL REPRESENTATIONS – MS L

Ms L was not observed using manipulative materials during the observed lessons. Visual representations were used and these included

- playing cards;
- a graphic of a target and two boxes with numbers;
- a vegetable patch of 60 plants, visualised and drawn by students only;
- an array of 48 tiles, visualised and drawn by students only; and
- written processes for multiplication that included the grid method, partitioning and extended notation.

Ms L did not distribute manipulative materials to any student. It appeared that Ms L believed that if students wished to use them, they could find the materials themselves. Ms L's drawing of teaching mathematics, a pre-observation data collection tool, demonstrated this belief when Ms L drew and labelled "manipulatives available at all times". Ms L indicated that her classroom did have manipulative materials such as Multibase Arithmetic Blocks (MAB) but the students did not use such materials during any observed lesson. On one occasion, Ms L reminded students that the MAB materials were available and said, "you might want to use concrete materials. If you need to use them or would like to use them, please feel free. We've got the MAB at the back. If you think that's going to help you, get it out. You know that. You don't need to wait to be told okay?" (Help Julia, lesson observation).

Further evidence regarding Ms L’s beliefs about the use of manipulative materials can be found in this statement. Though Ms L appeared to encourage students to use manipulative materials, but on closer examination it could have actually had the opposite effect for students. Firstly, MAB would not be a useful tool for multiplying multi-digit numbers, as the materials would be cumbersome to use for the expression 45×26. It is not immediately obvious how to use the MAB to evaluate this expression unless students were making 45 groups of 26 or vice versa. Certainly the students’ written methods explored with Ms L such as partitioning, the grid method or extended notation would not lend themselves to the use of MAB meaning the “mapping” (Boulton-Lewis & Halford, 1992) from the strategy to the materials would not match. Secondly, Ms L said, “if you need to use them” and “if you think it’s going to help you”, which suggested that only students having trouble might need to use the materials. As older primary students, it is unlikely that students would be comfortable to admit in front of their peers that they were having difficulty (Jansen, 2006; National Council of Teachers of Mathematics, 2000), and getting out concrete materials would potentially be socially embarrassing as it would highlight the fact that they were struggling.

Ms L raised this issue in the first post-lesson interview when discussing Sophie’s progress.

> I think what happens is she wants to do the top stuff and she doesn’t want to go to concrete materials. I think she sees that as a baby thing and I think that’s across the board. They think if you’re telling them to get concrete materials out, that it’s saying they can’t do it. And I feel that that’s how she is.
> (Post-lesson interview, Find the Pairs)

Though she attributed these feelings to Sophie, there is some evidence to suggest that Ms L also felt that concrete materials were not appropriate for all students and that most students in her upper primary class should not have to use them. Firstly, Ms L’s use of language with phrases like “she doesn’t want to *go* to concrete materials” seemed to indicate that using manipulative materials would signal a departure from the norm. Using materials was also positioned as the opposite of the “top stuff” Sophie wants to do. Although Ms L said that Sophie thought manipulative materials were a “baby thing”, Ms L’s behaviour could also be interpreted as supporting this view. First, the fact that manipulative materials were not used could send the message to students that using manipulative materials is not something normal for upper primary students. Second, Ms L only ever did offer manipulative materials or visual representations to students who were struggling such as Riley and this was done privately. Third, although Ms L talked three times about the potential positive effect of using manipulative materials in reference to Sophie and Riley, she did not deviate from her lessons emphasising written processes to use such materials. For example, in a post-lesson interview Ms L said

> I think we need to go back step by step showing it pictorially as much as possible, how can I use … I want to have a look at what you sent me with the arrays to try that.
> (Post-lesson interview, Veggie Patch)

However the next day, Ms L shied away from using grids and arrays by suggesting that using these visual representations could confuse students and in fact, may have confused her.

> I might actually show them the other one, drawing the grid and showing them as well. So they shade in the 50 I was going to show that but I thought that might be too confusing. I think with a lot of kids the visual can confuse them more. I looked at it and thought "Oh I don't know how that's helping me" but kids are different I suppose.
> (Post-lesson, Array of 48)

Ms L was not familiar with how to use grid and arrays for multi-digit multiplication, as it was a suggestion I had made. This unfamiliarity with the materials could have been a large part of the reason why Ms L did not use them in the observed lessons. For example, Moyer and Jones (2004) found that teachers who were inexperienced with using manipulative materials tended to use them less often in mathematics lessons. It would appear then that Ms L believed that using manipulative materials was not appropriate for most upper primary school students and that such use could cause confusion. However, she did seem to believe using manipulative materials and visual representations was more appropriate for low-attaining students.

The coding of data regarding Ms L's use of manipulative materials and visual representations showed two main purposes for such use. Four examples were coded as using manipulative materials or visual representations for assisting with calculations while two examples were coded as using materials and representations as a basis of discussion. In the case of using representations for assisting with calculations, two of these examples involved only Riley, one only pertained to Sophie, while the remaining example involved the whole class. Ms L suggested to the whole class that factor trees might help them during the Find the Pairs task and Sophie was then observed using a factor tree. All examples of using representation as the basis of discussion involved the whole class. The fact that nearly half the total examples of Ms L using visual representation as scaffolding involved only Riley, and in one instance

Sophie, supports the conjecture that Ms L felt that using representations was more appropriate for low-attaining students than for the whole class.

Ms L suggested representations to Riley on two occasions to help him calculate a multiplication expression. For example, she suggested the use of arrays to Riley.

> Ms L: Might need to check that one Riley I think. 9 times 8 you need to check it again. Where could you go to help you? Could you draw an array perhaps to help you answer that? Did you use arrays at your last school?
> Riley: No. I've gone blank
> Ms L: You've gone blank. Do you know what an array is? Did you use them? So if I had here, 2 times 2 is 4 I'm going to go right, 2 and 2 [drawing dots in rows and columns on the whiteboard] If I count them now I'm going to have 4. So can you use an array to help you work out 9 times 8? So what would you do?
> Riley: I've never used those …
> Ms L: You've never used those at your old school. Okay so how many dots would you draw along here?
> Riley: Nine
> Ms L: Okay nine.
> [Riley drew nine dots up.]
> Okay so how many am I going to have in each row?
> Riley: Eight
> Ms L: Okay. So how am I going to work out the answer to that problem?
> Riley: Is that including with my eight? [pointed to the first of the nine dots in the column]
> Ms L: Of course, yep. And what will you do then?
> Riley: Count by 2s or …
> Ms L: Find the most efficient way of counting. So you can always use an array to help you.
> (Find the Pairs, lesson observation)

In this case, Riley was not able to use the array effectively and reached an answer of 75. For Riley the task of drawing the array to calculate 9 multiplied by 8 took most of the time allocated for Find the Pairs game and he did not play the game with his partner. After the lesson, Ms L reflected on her attempts to scaffold Riley's learning.

> Riley hadn't seen an array and even when I showed him he said he'd never seen it. They'll [Sophie and Riley] need a lot of work just on basic multiplication. Riley didn't know nine times eight. Even when I asked him to do the array and he still came to me with the wrong answer. We'll need to spend a bit of time on that … I know that they are going to have to do a lot of arrays stuff before we move on to anything two by two digit for those two (Sophie and Riley).
> (Post-lesson interview, Find the Pairs)

This also illustrated that Ms L believed that both Sophie and Riley would benefit from using materials like arrays.

The other example of Ms L suggesting a representation to assist Riley with a calculation occurred during the Help Julia task, when Riley was attempting to calculate 45×26. On this occasion, Ms L used a place value chart to demonstrate how multiplying numbers by a multiple of ten makes the number ten times larger, as described previously in this chapter.

Ms L used representations as a basis of discussion on two occasions with the whole class. However, the way Ms L used the representation was as a mental model that she asked students to visualise and draw rather than a physical representation drawn or shown to the class. Ms L's task, Veggie Patch asked students to visualise then draw and these drawings became the basis of discussion. Ms L's instructions for this task were

> So I want to plant a veggie patch and I have 60 plants sitting in the back of my truck. How many rows if I need to have the same number of plants in each row? What might my veggie patch look like? Is there more than one possibility for the way my veggie patch will look? I'm going to hand out a whiteboard and a marker and I want you to have a go at showing me the answer to this problem.

The Array of 48 task was another example of a task that aimed to link multiplication with the representation of an array, but during implementation became listing multiplication and division facts about 48. The task began with Ms L asking students to visualise an array made from 48 tiles and then to draw on whiteboards "what they saw". During this initial stage, Ms L did not give students any additional instructions, hints or reminders but just re-stated, "so I made an array of 48 tiles and now you're drawing what you saw" (Array of 48, lesson observation). No tiles or arrays were shown and students were then left to work individually for around 30 seconds with just these instructions. After this time, Ms L asked students to share a solution, which she wrote on the whiteboard as "rows of" as shown in Figure 10. No comments were made about the solutions offered unless they had already been said. Relationships between expressions such as "2×24" and "24×2" appeared to be recognised by some students but Ms L did not comment on this. After all possible multiplication expressions had been listed, Ms L asked for division expressions leading students in a discussion that identified division as the inverse of multiplication.

> Ms L: Okay looking at those I want us to write up all the multiplication and division facts that we can get from these arrays. Why am I saying can we list division facts from this array?
> (Array of 48, lesson observation)

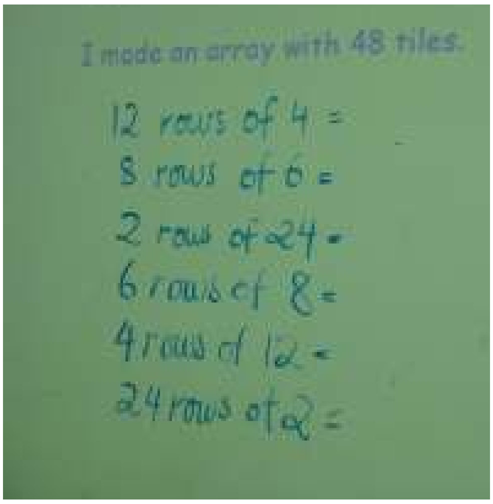


Figure 10. Ms L's recording of student solutions for Array of 48.

Although Ms L referred to the facts "we can get from these arrays", the arrays themselves were not drawn as we can see in Figure 10 but just written. The linking of multiplication and the representation of an array, the explicit attention to this underlying concept, appeared to be lost in the implementation of this task.

There were important differences in the scaffolding using manipulative materials and visual representations that Ms L offered Riley compared with Sophie. Ms L stated that she felt that Sophie would "need a lot more visual with multiplication" (post-lesson interview, Find the Pairs) and along with Riley would also benefit from using manipulative materials such as arrays or visual representations. However, despite stating this, Ms L was not observed offering the use of visual representations such as arrays, at all to Sophie. The reasons for this can only be speculated, but perhaps Ms L was respecting her perception that Sophie thought that using manipulative materials was a "baby thing" as discussed previously. Sophie was also a student that Ms L knew well having taught her the previous year. Perhaps Ms L felt that the manipulative materials used then meant that Sophie should now be able to operate without them. Riley was a new student to the school and appeared not to have experienced the use of representations like arrays. It appeared that Ms L felt he needed to use them and therefore encouraged the use of arrays with Riley.

I will now discuss the responses of Riley and Sophie to Ms L's use of manipulative materials and visual representations. Data regarding cognitive responses were drawn from the post-lesson assessment tasks as well as from lesson observations. Affective data were gathered through the post-lesson interviews with

Sophie and Riley using the Emoticons scale along with their verbal explanations. Lesson observations also yielded affective data at times.

RESPONSES OF SOPHIE AND RILEY

During the two tasks in which the whole class was asked to consider visual representations, Riley responded quite differently. The first of these tasks was the Veggie Patch problem. Riley appeared to be confident in using arrays to solve this task and quickly drew two different arrays for 60 on his whiteboard, 6 rows of 10 and 3 rows of 20. In addition, he was able to provide a detailed explanation about doubling and halving when sharing his solutions, which, as discussed earlier in this chapter, was the longest utterance of any of the low-attaining students in this study during a whole class discussion. It is not clear why Riley was so successful on this task. Perhaps this was a good example of the idea that Watson (2002) proposed that low-attaining students can have "some potentially powerful mathematical talents … that are unrecognised and unused" (p. 20). A possible explanation could be that the model of arrays was one that Riley could understand and that arrays had potential to become a useful mental model for him. However, the next day during the Array of 48 task, Riley sat looking confused and did not draw anything initially. Perhaps the real world context of the vegetable patch assisted him in the first instance while the slightly more abstract "48 tiles" caused some confusion.

Further evidence about Riley's response to the representation of an array was found during a post-lesson assessment task. This assessment task asked Riley to "tell me a story for 14×16. Find an answer using a written method and then use the grid paper to draw an array". Riley quickly grasped the idea of using grid paper to show a multiplication expression as an array. He was then able divide up the larger array into manageable pieces in order to find a total. After showing the outline of the array "14 × 16", Riley said "It would be much easier if it was 14 times 10 'cause I can go 10, 20, 30, 40 … I could break it down again 'cause I'm not well [*sic*] at my 6 times tables. I can break it down again. 1, 2, 3, 4, 5 I can go 5, 10, 15, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75 and then I go down so then I can go, then I know that is 14 so I've got the answer". Though he made an error in counting by fives by omitting 20, this strategy was effective and showed a distributive strategy for multi-digit multiplication. This was quite a sophisticated and logical strategy which seemed to again point to some hidden "mathematical talents" (Watson, 2002) in Riley. I believe Riley's ease at using the grid and array showed that arrays was a useful mental model for him.

Evidence of this also emerged during this same assessment task when Riley was asked to "tell a story" to go with 14 × 16 = 224. Riley responded,

> um … there were 224 people … sorry I made a mistake. These people were trying to think how they could get 224 people onto the boat. They were seeing how they could fit. The people counted … they tried using the space then they did … they times … they knew there was 14 people in 16 groups. Each group had 14 people and they found out that … they timesed [*sic*] it all so they knew there was 224 so they could go onto the boat".

Riley seemed to be using the array as a mental model for his boat story when he said, "They were seeing how they could fit". It is possible that Riley was imagining the rows of people although he used "groups". On the same day as this assessment task, the class had experienced a lesson about fitting people into a standing-room-only space at a football match by standing in rows. This was not one of my observed lessons, however Ms L had reported that Riley was enthused by the lesson and showed good understanding during this class. He may have identified well with the context or perhaps previous experiences became clearer due to this lesson as it seems to have influenced his response to this assessment task.

Affectively, Riley had various responses to Ms L's use of representations with him. Riley's response to the Find the Pairs game was that he found the lesson "Good because it was something different and we got to use playing cards and do the maths questions" (post-lesson interview, Find the Pairs). This was despite the fact that Riley spent the majority of time attempting to find an answer to 9×8 as discussed previously. It would appear that, although he did not play the game as such, Riley viewed the use of the playing cards as something positive and engaging. Riley's response to Help Julia in which Ms L drew the place value chart and asked Riley to continue the pattern of 6×4, 6×40, 6×400, was less positive as reported previously. For this task it seemed that Riley was not clear on the pattern that Ms L wanted him to learn about by pursuing a different task to the one she set the class. In addition, Riley reported that he found this task somewhat repetitive and "annoying".

There is less data about Sophie's responses to the use of representations because, as previously discussed, Ms L did not suggest their use to Sophie as she did with Riley. The only whole class use of representations for which Sophie was present was the Veggie Patch problem. She was observed spending some time writing the question on her whiteboard before drawing a rectangle and dividing in half then into eleven parts which she then labelled “2×30”. This is shown in Figure 11.

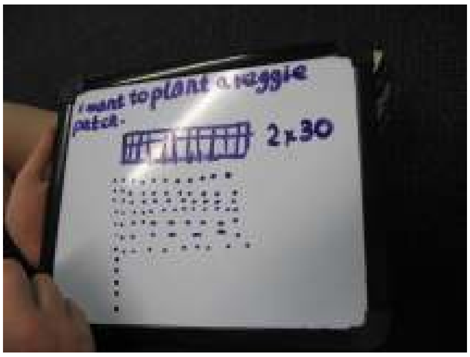


Figure 11. Sophie’s whiteboard for Veggie Patch.

Although her expression was correct for an array totalling 60, Sophie’s drawing did not reflect this. In addition, it would appear that her next attempt at drawing an array of 60, also shown in Figure 11, was not correct. The dots Sophie drew were eleven by eleven. It seemed that Sophie might not have understood arrays as a representation of multiplication.

The post-lesson assessment task previously discussed for Riley was also given to Sophie. The task was “Tell me a story for 14×16. Find an answer using a written method and then use the grid paper to draw an array”. Sophie had solved this expression using the partitioning method before using the grid.

Interviewer: Show me 14 times 16 on this grid.
Sophie: Yep …
[Began to colour one row of 14 and 16 down- partial array]
Times 14 by 16?
Interviewer: Using the square, what could you do?
Sophie: Like what I did before, 4x6, 10x6 …
Interviewer: Show me on the grid.

Now Sophie drew a 4 by 6 array, then she drew a 10 by 6 array using the same squares she used for the 4 by 6 array. I asked Sophie, “What do we do now?” Sophie stopped using the grid and said “Times it all together so you go 40 and 60 is a hundred plus another hundred is two hundred plus 24 is 224”. This mirrored what Sophie had done during her written calculation using the partitioning method.

It appeared that Sophie was unable to use the array on grid paper to find an answer to the expression and that she preferred her written process which she was able to use quite competently. In this case, it did appear, as Ms L predicted, that the use of this representation did cause Sophie some confusion. Sophie made no comments about her affective response toward the use of visual representation or manipulative materials.

CONCLUSION

An indication of the difference in Ms B and Ms L's approach to manipulative materials and visual representations is that while Ms L drew and labelled "manipulatives available at all times" in her PPELEM-style (McDonough, 2002b) drawing of teaching mathematics well (drawing task); on the same task Ms B drew and labelled "students using manipulatives – cards, dice, games, counters". In Ms B's class, manipulative materials and visual representations were distributed to every student and formed an integral part of the tasks. Ms L had manipulatives available but their use was dependent upon the students making the choice to use them with little guidance from Ms L. Neither the tasks themselves nor the teacher required such use. These differences may be due partly to the mathematical content covered by each teacher. It is possible that fractions, decimals and percent concepts lend themselves more to the use of manipulative materials and visual representations in a way that multi digit multiplication does not. However, it would appear then that the difference in Ms B and Ms L's use of manipulative materials and visual representations centred around whether their use was a planned part of the task or an optional extra.

This difference in the use of manipulative materials and visual representations meant that the low-attaining target students had varied experience of using such materials. Carl and David used the materials or representations for each task, as did all their classmates. Riley and Sophie's experiences were different in that Ms L specifically asked Riley to use representations such as arrays and place value charts but did not require this of Sophie. The rest of Ms L's class did not use materials or representations, making Riley's experience somewhat separate to that of his classmates.

The four low-attaining target students varied in their response to the scaffolding their teacher offered via manipulative materials and visual representations. In Ms B's class, Carl in particular seemed to enthusiastically embrace tasks where the materials were a concrete embodiment of the mathematics and he talked about being able to "build stuff" or "do puzzles" when reflecting on tasks after lessons. David also responded more positively to these types of tasks. The use of manipulative materials and visual representations as concrete embodiments also appeared to have a positive impact on the learning of Carl and David as previously discussed. In Ms L's class, Riley indicated a positive response toward the use of playing cards for one task although his understanding did not seem supported by using the cards in that case. Sophie made no comments either positive or negative about the use of manipulative materials or visual representations. In addition, Sophie appeared unsure about how to draw or use representations such as arrays to illustrate or calculate a multi-digit multiplication expression.

The coding of the data from each classroom regarding the use of manipulative materials and visual representations revealed an interesting dichotomy. "Connecting written symbols and physical materials" was the code used most often for Ms B's data while Ms L had no data for this code. Conversely, Ms L's most used code was "assisting with calculations" whereas Ms B had no data for this code. This underscores the fundamental differences in the purpose of each teacher's lessons. Ms B was interested in her students connecting concepts of rational number with symbolic representations while Ms L's goal was that her students would perfect a written strategy for multi-digit multiplication. Again, the mathematical content covered in each classroom might have impacted on Ms B and Ms L's decisions about the use of manipulative materials and visual representations.

Ms B and Ms L indicated on the Tasks questionnaire their belief that using manipulative materials and visual representations was effective in scaffolding student learning. In their pictures of teaching mathematics well, as discussed previously, both teachers drew and wrote about such use. Ms L wrote on an open question that the "use of concrete materials/ manipulatives/ visuals" was a strategy for catering for the different ability levels of her students.

As suggested by the literature, espoused beliefs can often differ from practice (e.g. Cooney, 1985; Shield, 1999). Sullivan and Mousley (2001) argued that context could constrain beliefs becoming enacted in practice. Thompson (1992) also suggested that teachers could hold conflicting beliefs simultaneously. It appears that Ms B aligned her espoused beliefs with practice as she used materials and representations for all students and all observed lessons. Ms L used representations to scaffold Riley's learning but not as a whole class strategy, perhaps indicating her belief that materials and representations are more appropriate for low-attaining students than other students. Contextual issues that potentially impacted on Ms B and Ms L's use of materials and representations were the perceived ability levels of the students, the mathematics content, and familiarity with and the availability of appropriate materials or representations for this mathematics content. Another contextual issue that perhaps impacted on Ms L's use of manipulative materials was reflected in Moyer's (2001) claim that teachers are concerned that using manipulative materials causes a "disruptive environment" and they prefer an "orderly" classroom without the use of materials.

To conclude, it appeared that Ms B and Ms L used manipulative materials and visual representations in quite different ways. The purpose for such use also differed from illustrating concepts to assisting with procedures. The third scaffolding practice of this study, explicit attention to concepts, also draws out some of these points regarding manipulative materials and visual representations and draws on the issues raised in scaffolding through discussion. As discussed, the three scaffolding practices were not completely separate entities. There were overlapping themes, examples and observations that occurred between the practices. With this in mind, I will now focus on explicit attention to concepts.

EXPLICIT ATTENTION TO CONCEPTS

The third scaffolding practice examined in this study, explicit attention to concepts, was in some ways an overarching category that encompassed aspects of the other two scaffolding practices. When paying explicit attention to concepts, the teacher must use a vehicle to draw attention to concepts, which often might be through discussion or the use of materials or representations. Episodes where the teacher's purpose appeared to be drawing out underlying concepts were coded under the category of "attention to concepts" as well as the appropriate codes under "discussion" or "manipulative materials and visual representations".

Hiebert and Grouws (2007) stated that in order for students to learn mathematics with understanding, teachers needed to allow students to struggle with important mathematical ideas and teachers and students must pay explicit attention to underlying concepts. They believed it was obvious that in order for students to learn mathematics with understanding, they must be taught by attending to the concepts of mathematics. As described in the review of the literature, this explicit attention to concepts can also be described as teaching for understanding (Fennema & Romberg, 1999), for conceptual thinking (Kazemi & Stipek, 2001), or for relational as opposed to instrumental understanding (Skemp, 1972).

Ms B and Ms L were familiar with cognitively demanding types of mathematics tasks due to their involvement in research projects that examined such tasks. However, the ways in which tasks are implemented is key to the level of cognitive demand that is realised in the classroom (Henningsen & Stein, 1997). To examine the attention paid to concepts or the extent to which Ms B and Ms L taught for understanding, the level of cognitive demand of the tasks as planned and then implemented will be explored. In addition, I will examine how each teacher scaffolded the learning of the target low-attaining students through the attention paid to underlying concepts. In keeping with the previous sections in this chapter, I will first discuss Ms B's use of attention to concepts then Carl's and David's responses followed by a discussion of Ms L and Sophie and Riley. To conclude, comparisons of each teacher's use of attention to concepts and the responses of the low-attaining students in each classroom will be given.

Explicit attention to concepts – Ms B

Ms B expressed her belief during the initial teacher interview that focussing on concepts was an important goal of her mathematics teaching mentioning several times that she wanted students to know "why things work" (initial teacher interview). When discussing her goals for lessons, Ms B often talked about concepts such as "find the fraction of a whole without measuring it, comparing the size of a fraction" (Fraction rods), "idea of the fraction being part of a whole, finding a fraction of a whole without any cues except how big the pieces might be" (Fractured Figures) and "so today we're looking to introduce decimals and percentages and some of the other lingo (language) about rational number. Hopefully they'll cue into that there are numbers between whole numbers" (Sticky Numbers and Number Line, and Sorting Cards). It appeared that Ms B's expressed beliefs were borne out by the tasks she set for her students. The observed tasks about fractions, decimal numbers and percent were all centred on exploring concepts, features of highly cognitively demanding tasks (Silver et al., 2009). Only once was a procedure mentioned and this was for calculating the sum of decimal numbers, which was touched on briefly during the Decimat task.

Many aspects of Ms B's implementation of tasks maintained students' attention on the underlying concepts of mathematics. For example, Ms B often asked students "why". On average, Ms B asked "why" nine times per lesson. This requirement that students justify and explain their "mathematical actions" is a feature of teaching with a "high press for conceptual understanding" (Kazemi & Stipek, 2001). Explanation was also a feature of high demand tasks identified by Silver et al. (2009). By requiring students to explain their thinking, Ms B drew attention to underlying concepts of fractions, decimals and percent as shown in the following example.

> Ms B: What's the blue rod worth again?
> [Student called out one and a half.]
> Okay what's another way we can put one and a half so we can deal with this? Fred?
> Fred: 3 halves
> Ms B: Why would that be easier to deal with in this particular situation?
> Fred: Cause three halves is one and a half
> Ms B: Okay if you know that one and a half is the same as three halves, what can you do? Can I find the whole if I know that there's three halves in one and a half? What would be the smartest thing to break one and a half into?
> Student: Three parts
> Ms B: Why would I break it into three parts? If I break this into thirds, what's one of these thirds actually worth?
> Student: A half.
> Ms B: Why?
> Student: Cause you're breaking one and half into thirds, 'cause three thirds is a whole.
> Ms B: So you're saying each of those thirds is actually worth a half because the whole rod is worth one and a half.
>
> (Fraction Rods, lesson observation)

In this episode, Ms B pressed students to explain their thinking and drew attention to the concept of the relationship between improper fractions and mixed numbers. Ms B did not accept students re-telling their solutions but asked them to explain why and how they arrived at their answer.

Making connections is an essential part of conceptual understanding (Hiebert & Grouws, 2007). Four codes in the coding framework described in Chapter 3, focussed on the kinds of connections teachers might make to draw attention to concepts. These were drawing connections between concepts and processes; between knowledge sets or understandings; to previous tasks; or to real world experiences. Ms B connected concepts and processes once during the Decimat task as described above. Ms B made connections between knowledge sets during the Dominoes task by connecting decimal numbers, fractions and percent.

Ms B:	Hmmm. Let's try this one first. [1/4] Why would that be easier than that?
	[A student said something inaudible]
	Yep 'cause if you know one quarter, you can work out three quarters. So 25% or 0.25, 25 out of 100, two tenths, five hundredths. What about three quarters then Kurt?
Kurt	Zero point seven five.
Ms B:	What would the percentage be Kurt?
Kurt	Seventy five percent.
Ms B:	Okay [Ms B wrote this in the class learning journal] 75 out of 100, 75 hundredths.

(Dominoes, lesson observation)

Ms B appeared to be connecting an established piece of knowledge, that 1/4 is equivalent to 25 percent and 0.25, in order to scaffold students in finding out what the equivalent decimal number and percent is for 3/4. However, the response of David to this particular part of the whole class discussion was described in the section of this chapter that describes end of task whole class discussions. As mentioned earlier, David was unable to match 75 percent and three quarters in the post-lesson assessment task just after this discussion took place.

Ms B made connections to previous tasks on four occasions. For example, during the concluding whole class discussion to the Fractured Figures task, Ms B asked students to consider how this task was similar to the Fraction Rod task of the previous day.

Ms B:	Yesterday's task, using those rods, do you think that might have helped you today?
	[Students called out "nope" and "yep"]
	What do you think Emma?
Emma:	Yes because it's the shapes and smaller, there's eighths and stuff.
Ms B:	Are there similarities between the tasks? What were you doing?
Student:	You were comparing
Ms B:	What do you mean comparing?
Student:	The different sizes to make a whole.
Ms B:	What was the key that you needed to know?
Student:	The whole

(Fractured Figures, lesson observation)

Ms B also drew connections between a fraction game and the Decimat game, Construct a Sum and Fruit and Veggies and Dominoes and a previous task about sharing a chocolate bar.

Ms B connected real world experiences with the mathematics tasks students were engaged in also on four occasions. During the real world context task from Ms B's class, Fruit and Veggies, obviously connections were made to fractions of one kilogram for purchasing fruit and vegetables, as this was central to the task. On other occasions Ms B drew connections to real world experiences, although the tasks themselves did not centre on a real world context. For example, Ms B drew connections between sharing a cake into four parts and quarters, and asked students to suggest a "real life way we might use 250%" during the Dominoes task.

Two episodes from Ms B's classroom were coded as "drawing attention to underlying concepts". One of these occasions has been described previously in this chapter and occurred when Ms B used manipulative materials to draw attention to the concept that the number of parts in one whole names those parts when talking to David during Fraction Rods. The other occasion occurred during a whole class discussion at the end of the Sorting Cards task when Ms B drew out the underlying concept of percent.

> Ms B: Point three three we say, three tenths three hundredths. Now what does percentage actually mean? If I say ten percent, what does that actually mean? What does percent actually mean?
> Student: Like ten percent of whatever you've got.
> Ms B: Per cent. What does that actually mean?
> Student: How many out of 100.
> Ms B: Out of 100. So if I'm talking about percentages I'm talking about out of every hundred (wrote this in the class journal). So if I've got 200 and I want to find 10 percent of 200 I have to take 10 out of every hundred. 10 out of the first hundred, ten out of the second. So 20 is 10 percent of 200.
>
> (Sorting Cards, lesson observation)

The code in the category of "attention to concepts" used most often for data from Ms B was "exposing misconceptions". As discussed in the review of the literature, Kazemi and Stipek (2001) pointed out that using mistakes and misconceptions as opportunities to further understanding is a characteristic of teaching for understanding and that using mathematical errors is an opportunity to explore misconceptions. Furthermore, Borasi (1994) stated that, under a constructivist view of learning, errors are an "inevitable and integral" (p. 170) part of learning and an opportunity for teachers to find out more about student thinking. From the data collected from Ms B's classroom, seven episodes were coded as exposing misconceptions. An example of a private exchange discussing misconceptions occurred during the Dominoes task between Ms B and the target student, Carl.

> Ms B: Are you done? Ooh! You've got some mixed numbers in here. I love that! What does 1.2 match with?
> Carl: 120
> Ms B: Should that have a percentage sign on it?
> Carl: No
> Ms B: Think about it logically. The number 1.2 and 120 do they match? What symbol does this need to have with it to make a match?
> Carl: Point? Line? Decimal? Percent?
> Ms B: Yeah we might have 6 things and we might have 120% of 5 things. Do you know what I mean? 1.2. You've got to have a percentage because you can't say 120 and 1.2 matches. That doesn't make sense!

(Dominoes, lesson observation)

In this exchange, Ms B attempted to draw Carl's attention to the fact that his 1.2 represented more than one whole and therefore matched with 120%, which represents an equivalent amount. Carl appeared unsure why he needed to make 120 into 120%, indicated by his guessing of all the possible symbols. Ms B led Carl throughout this exchange into "filling the gap" with the correct answer. A similar occurrence happened during the whole class discussion of this same task. A student struggled to match equivalent rational numbers and Ms B again appeared to lead this student to the correct answer by narrowing down the possibilities.

Ms B: Got some tricky ones left. 4/5, 3/5, 2/5 and 40%.
[Lily matched 40% with 80%].
40% and 80% are a match? Do you think? What do we think?
Lily: I'll just put that back [she moved her domino away]
Ms B: Hold on. Stay. So we've got 80% on the end here. What information do we have here that can help us out? There might be some information here you can look at.
[Ms B underlined some matches including 1/5 = 20%]
'Cause it's going to have to be a fraction. We've got 2/5, 3/5 and 4/5 so we're going to be looking at a number of fifths.
(Dominoes, lesson observation)

Although Ms B drew attention to errors and attempted to expose misconceptions, she also rescued students by narrowing down the possible correct answers. This occurred in four out of the seven episodes coded as "exposing misconceptions and errors". It would appear that in these instances, while Ms B did not intend to ignore errors, she also wanted students to be able to resolve them reasonably quickly which is consistent with Kazemi and Stipek's (2001) description of a "low press" teacher.

The remaining three episodes were slightly different. The first occurred between Ms B and Greg, the student working with David during the Dominoes task. The following exchange took place at the beginning of a discussion between Ms B and these two students that lasted for 12 minutes of this lesson.

Ms B: Okay what does this mean? [pointing to 1/4.]
Greg: I dunno, one out of four?
David: One quarter!
Ms B: What does the four mean? That's how many parts make up the whole yeah? [Pointing to the denominator] So if I had one item, let's say it's a cake, then there would be four parts in that whole wouldn't there? What does the one mean?
Greg: One whole thing
Ms B: One thing? What do you mean? I think you've got it.
Greg: One object, I don't know.
Ms B: One of the parts? Yeah.
Greg: So the answer would be three?
Ms B: What do you mean the answer is three?
[Greg said nothing else. Ms B then asked him to get out some paper.]
(Dominoes, lesson observation)

This example showed Ms B exposing Greg's misconception about what 1/4 meant. Ms B appeared to begin by providing the answer of "one of the parts" and rescuing Greg in a similar way to the other examples discussed. However, Greg clearly did not understand, shown by his response "so the answer would be three". It is possible Greg was looking at the difference between one and four, or one part of the four taken away, leaving three, so called "gap thinking" (Clarke & Roche, 2009b). Ms B did not appear to recognise this and chose to move on to a more detailed explanation of fractions by drawing and talking with Greg and David for many minutes.

The next two examples differed from Ms B's usual habit of resolving errors reasonably quickly by narrowing down answers or hinting. In these examples, during whole class discussions, Ms B allowed the discussion to continue and concluded the lessons leaving some mathematical questions unresolved. The first example occurred during the Sticky Numbers and Number Line task when Ms B asked students how many numbers there were between 32 and 33.

> Ms B: How many dots would you have to fit in here? [32-35]
> Student: 50.
> Ms B: Why?
> Student: Because there's ten in between each.
> Ms B; So there's only 10 possible numbers between 32 and 33? [To the class].
> Students: Yes!
> Ms B: Okay
> (Sticky numbers and number line, lesson observation)

The second example was during the Construct a Sum task.

> Ms B: 1/6 more and we've got 1/7. How tricky! We need a quarter and we've got a fifth, we need a sixth and we've got a seventh.
> Student: It's the same
> Ms B: Is it? Not going to be exactly the same is it? What would I need to do to check which one is closer?
> [The students are silent]
> Could I look for a common denominator? What would I do? Chad
> Chad: Umm….
> [The students are silent]
> Ms B: We might come back to that tomorrow
> (Construct a Sum, lesson observation)

In these examples, Ms B resisted telling the students answers or providing hints. The first example seemed to illustrate that at least some students held a strong view that there are ten decimal numbers between consecutive whole numbers. Ms B chose not to confront this view at that time. Perhaps she was hoping that subsequent tasks would reveal to students that this view was problematic. In the second example, the students did not seem to know how they might check whether 5/6 plus 1/7 or 1/5 plus 3/4 was closer to one whole. When Chad, who appeared to be high attaining, was unable to find an answer, Ms B seemed to realise the question may be too difficult and chose to end the lesson with this problem unresolved.

Reflection on teaching and learning by examining learning and strategies is an essential part of conceptual understanding (Hiebert et al., 1997; Wheatley, 1992). Ms B was observed reflecting on learning three times. On these three occasions, Ms B also wrote in the class mathematics learning journal during the end of the lesson discussion. After the Decimat task, Ms B wrote an observation by a student that "10 thousandths make up a hundredth, 10 hundredths make up a tenth and ten tenths make up a whole" (Decimat lesson observation). At the end of the Sorting Cards task, Ms B wrote a definition of percent in the journal and after Dominoes drew a table linking decimal numbers, fractions and percent in the class journal. These occasions were opportunities for students to look back on what they might have learnt during the task. Hiebert et al. (1997) maintained that this kind of "stopping to think carefully … is almost sure to result in establishing new relationships and checking old ones" (p. 5), which has the potential to lead to a deeper understanding of the concepts.

Ms B habitually asked students to explain their strategies and elicited many student strategies after tasks. However, Ms B less often reflected on these strategies by comparing and contrasting, or drawing out similarities and differences. In fact, Ms B was observed reflecting on strategies only twice which was discussed during the end of task whole class discussions section in this chapter. On both these occasions, after Fraction Rods and after Fractured Figures, Ms B drew out the similarities between two student strategies.

In summary, Ms B often aimed for tasks that drew attention to the underlying concepts of fractions, decimal numbers and percent. The tasks remained at the level for which they were set in that Ms B did not reduce the cognitive demand during implementation for any students. However, David and Carl did not always engage in the tasks at the cognitive level that Ms B intended, an issue I will explore in the following discussion.

RESPONSES OF CARL AND DAVID

With all eight tasks in Ms B's class aimed at conceptual understanding, the responses of Carl and David varied in how productive they were during these tasks. I have defined more productive responses as occasions when Carl or David actively engaged in the mathematics of the task and appeared to gain in their mathematical understanding. I have categorised less productive responses as those occasions when the mathematics of the task was not engaged in sufficiently by Carl or David to support gains in their understanding of mathematics. On these occasions, the level of cognitive demand was too high for Carl and David, resulting in their disengagement. To examine this conjecture, I will now discuss data from post-lesson assessment tasks in order to establish Carl and David's understanding of the concepts surrounding the observed tasks, to illustrate how the cognitive demand of the tasks were often too high for these students.

Figure 12 shows the post-lesson assessment tasks that Carl and David completed after each lesson. As stated previously, these assessment tasks were aimed at exploring the concepts covered by the lesson. Some of these tasks were more aligned with the lesson tasks than others but all examined concepts about fractions, decimal numbers and percent, which was the mathematics of the observed lessons. In addition to these tasks an initial assessment was given before lesson observations began that asked Carl and David to draw 3/4, 2/5 and 3/2 and then to order these fractions.

Assessment task 1	**Assessment task 2**	**Assessment task 3**
Draw the Whole If this rectangle is 3/4, draw what the whole would look like. Then another rectangle represents 4/3 show the whole.	Fraction Pie Identify the fractional parts of one circle divided into different fractions (1/4,1/4, 1/6 and 1/3)	Equivalent decimals. What is the decimal number that is the same as one half?
Assessment task 4	**Assessment task 5**	**Assessment task 6**
Dominoes assessment. Choose dominoes that match and explain this match.	Fraction pairs Identify the larger fraction from each pair. 3/8 and 7/8 1/2 and 5/8 4/7 and 4/5 4/8 and 2/4	Ordering decimal numbers 0 1 0.01 0.10 0.9

Figure 12. Post lesson assessment tasks for Carl and David.

Carl's performance on these tasks indicated that, by the end of lesson observations, Carl had a partial understanding about the size of proper fractions in relation to the whole and a beginning understanding about the relative size of decimal parts. Carl was unable to identify the larger fraction except from 3/8 and 7/8. He appeared to be attending to each number in the fraction as a whole number rather than the size of the fractions and also showed signs of using “gap thinking” (Clarke & Roche, 2009b). Carl was unable to name the decimal number equivalent to one half and said “Fifty point five? Thirty five point five?” as reported earlier. Carl also could not match equivalent decimal numbers and percents, though he initially chose 0.7 and 70% as matching, his explanation was not clear; “because 7/10 goes into 7 ‘cause that’s a tenth (7) so ten sevenths makes 70%.” When asked to match 1/4 to another number, Carl chose 1/10 and said “because 1 goes into 10 ten times so it’s a quarter” (Dominoes assessment task).

David’s response to these tasks indicated that he also was able to operate more successfully with unitary fractions, did not understand improper fractions, and was unable to name the decimal equivalent to one half saying, “0.2 is the same as one half because the denominator is a 2 same as half” (Equivalent decimal assessment task). David was beginning to link common fractions and decimals. For example, he identified that 1/4 was equivalent to 0.25. However, as discussed, David could not identify what 3/4 was equivalent to. David could identify the larger fraction in one pair but in others his explanation indicated whole number or “gap thinking” (Clarke & Roche, 2009b). David showed some beginning understanding of the relative size of decimal parts only confusing 0.9 with “ninths”.

It would appear from these tasks that Carl and David did not have an understanding of what fractions were representing unless they were unitary common fractions and that they did not know how decimal numbers, fractions and percent relate to each other, which were the concepts Ms B attempted to explore in the observed lessons. I will now examine the eight tasks and Carl and David’s responses to Ms B’s attempts to explore the underlying concepts of fractions, decimal numbers and percent through these tasks.

For six of the observed tasks, the cognitive demand appeared to be too high for Carl. During two tasks Carl’s partner took over the task. For example, after Fraction Rods, Ms B said, “Carl was actually the one who went along with the partner. When I talked to him about the last one he really didn’t have any idea so I think he probably piggybacked his way through that one a bit. His partner is one of my higher attaining grade fives”. During another task, Dominoes, Ms B recognised that working with a partner for that task was not conducive to Carl engaging in thinking about the task.

> Today he was working with someone who could do it and he didn't have to do any thinking again. I don't like ability grouping but in a situation like that there's no point in being with someone who is just going to do it. I guess that's the battle – I don't like ability grouping but sometimes you have to in order to get them to do the thinking you want them to do. Because when I asked him why does that match that he said "Oh Braydon told me".
> (Post-lesson interview, Dominoes)

Even Carl acknowledged the fact that in this case working with a partner meant that he did not have to engage fully in the task saying, "working with Braydon was good 'cause you didn't have to do all the work" (post-lesson interview, Dominoes). Dominoes used a "prompting prior knowledge" use of materials, which was explored earlier in this chapter.

The degree to which Carl attended to concepts during these lessons was minimal due to his partner either taking over much of the task or being unable to scaffold Carl in the underlying concepts. The latter occurred during the Sorting Cards task when Carl worked with a student of similar ability. Both boys were unable to engage cognitively in the task of finding equivalent rational numbers. Affectively, Carl was not positive about this Sorting Cards task, which used "prompting prior knowledge" materials and was described earlier in this chapter.

> It wasn't great. The cards was alright, it wasn't great it was a little bit boring. We had to sort them into groups. It was easy but boring.
> (Post-lesson interview, Sorting Cards)

Although Carl said he found this task "easy", the task, as described earlier in this chapter, was intended to focus on equivalent rational numbers. Carl and his partner grouped the same types of rational numbers together for example, the fractions, decimal numbers and the percents. Overall, it appeared that working with a partner did not scaffold Carl's learning effectively, which was also so for the low-attaining students in the study of Baxter et al. (2001).

For the other three tasks, the cognitive demand remained too high for Carl and he was unable to engage in the mathematics of the task. On these occasions, Carl would sometimes begin the task then become confused and unable to continue. When he did not receive scaffolding to enable him to continue, Carl chose to disengage with the task and engage in non-mathematical activity as shown in the observations from the Fruit and Veggies task.

50 minutes into the lesson	Carl moved around the room with a group of boys laughing and throwing things at each other.
57 minutes into the lesson	Carl remained at his seat for a minute or so after Ms B left then began to move around and throw things again.

61 minutes into the lesson	Ms B called out "Carl!" and he stopped playing with the other boys and returned briefly to his seat.

During Construct a Sum, Carl coloured his Decimat with green marker (Construct a Sum lesson observation). For both Fruit and Veggies and Construct a Sum, Carl needed to know equivalent decimal and fraction numbers and be able to estimate to make a sum close to one whole. As we have seen from Carl's responses to post-lesson assessment tasks, he did not understand these concepts.

Both Fruit and Veggies and Construct a Sum used representations that were formal notations and mathematical symbols. The materials for these tasks could not scaffold Carl's understanding of equivalent rational numbers or the relative size of such numbers. Without scaffolding from the materials, Carl had to rely on Ms B to provide scaffolding. During Fruit and Veggies Ms B spoke to Carl for one minute then six minutes about equivalent decimal numbers to one half, as reported earlier, but this did not result in Carl completing the task. For Construct a Sum, Ms B did not speak to Carl at all. In both these tasks, where the cognitive demand was too high, Carl quickly became completely disengaged with the task. Carl's affective response to Construct a Sum was negative. Carl gave Construct a Sum a "thumbs down" because it was "a bit more challenging" (post lesson interview, Construct a Sum). Carl's negative response to Fruit and Veggies and Sorting Cards have been described previously.

Sticky Numbers and Number Line was the other task in which the cognitive demand was too high for Carl but he responded quite differently. During this task, Carl appeared to be trying to understand the task and did not disengage. This task was a 17-minute whole class task placing a mix of decimal numbers and fractions on a number line from 0 to 3. Ms B did not scaffold Carl's learning by addressing him directly although other things said to the whole class might have done so such as "is it bigger or smaller than one whole? How many more would you need to make one whole?"

> Carl said to himself "mine's not on there" [Carl had the number "5/8"]
> Carl said again to another student, "mine's not up there"
> Carl came up to the board at the same time as many other students. He wrote 5/8 between 2 and 3, closer to 3.
> (Sticky numbers and number line, lesson observation)

Ms B did not notice that Carl said he did not believe his fraction could be represented on the line from 0 to 3. In addition, when he did place his number on the line, Ms B was beginning to move on to the next task and did not notice that he placed 5/8 near 3, perhaps indicating he was attending to the difference between 5 and 8, exhibiting gap thinking which was also apparent from Carl's responses to the Fraction Pairs assessment task. It is not clear why Carl remained reasonably engaged in this task but perhaps the shorter time for this task might have been a factor considering that during the longer tasks when the cognitive demand was too high as discussed above, Carl became disengaged. Again, in this task, the visual representation was intended to prompt and link prior knowledge using formal notations and symbols.

All eight tasks were set up by Ms B to attend to concepts. For example, Dominoes and Sorting Cards linked equivalent rational numbers, and Fraction Rods illustrated the relative size of fractional part to the whole. While in the latter case the materials also demonstrated the concept explicitly, the other two tasks used symbolic materials aimed at activating prior knowledge. It is clear from lesson observations and from post-lesson assessment tasks that Carl did not have sufficient understanding of decimal numbers, fractions and percent to access six of these eight tasks. When unable to use the materials for scaffolding, Carl could have relied on his partner though this usually resulted in the partner completing the majority of the task for Carl. The other alternative was discussion with Ms B who spent, on average, three minutes talking to Carl individually per lesson. It would appear that, unless the materials used could support Carl's understanding, as occurred in the two productive tasks for Carl, the cognitive demand was too high and resulted in his disengagement. Indeed, it would seem that for Carl materials and representations that illustrated the concepts were more important than scaffolding conversations with Ms B. Fruit and Veggies for example, used formal notations as representations but Ms B spent 7 minutes talking with Carl during this task. Carl did not complete the task and remained mostly engaged in non-mathematical behaviour, which was described in the previous discussion. Perhaps this indicated that the Fruit and Veggies task was simply too difficult for Carl, beyond his "zone of proximal development" (Vygotsky, 1978), and no amount of time spent talking with Ms B would have been sufficient.

The data about Carl that does not fit with other data was his response to the Fraction Rods task. This task used materials that illustrated fraction concepts, and Ms B had a five-minute conversation with Carl during this task. Despite the presence of the two factors that have emerged as being most effective for scaffolding learning, Carl could not explain how to find one third if you know the size of two thirds, for example, and had great difficulty answering how many halves in one and a half, a conversation reported previously in this chapter. Ms B expressed her belief that Carl had “piggybacked” during this task by relying on his higher attaining partner to do most of the task. This was an observation I also noted during this task. This might account for Carl’s lack of success on a task which otherwise appeared to have the elements of effective scaffolding practices. It is important to note that this task became increasingly difficult and David’s “success” was due largely to Ms B’s hints and leading questions. Given Carl’s confusion on assessment tasks about improper fractions, this task was too difficult for Carl and therefore even the most effective scaffolding might not have been sufficient.

Both Carl and David demonstrated a positive affective and cognitive response to the tasks Fractured Figures and Decimat. Both these tasks used manipulative materials or representations that could be considered to be “concrete embodiments” (Boulton-Lewis & Halford, 1992) of the mathematical concepts of relative size of fractional parts to the whole and relative size and relationships between decimal parts.

During Fractured Figures, Carl and David worked in a group of four students whom Ms B considered were of similar ability. The materials for this task were shapes divided into three unequal parts that would fit together like a jigsaw puzzle to make one whole. The pieces could be placed on top of each other to determine the relative sizes and then given fractional names. The task was to find out what fraction each puzzle piece was equal to. Ms B spent about two minutes talking to Carl and David’s group over the course of the hour-long lesson. As the following example shows, both Carl and David appeared to be engaged in the task and also in the mathematical concepts underpinning it, namely that the number of equal parts in one whole names the fraction and equivalent fractions.

Student: Six of these make one whole.
Ms B: Ooh how did you find that out? That’s fantastic
Carl: ‘Cause that’s three [covering a half]
Ms B: Right!
Carl: If you go up… they’re sixths.
Ms B: You’ve worked out one of the trickiest ones to work out! Now if you know 6 of those fit, and you know that three fit in there so that would be what size?
Carl: Half
Ms B: Are you labelling it on here? I would be labelling it on here. So we know that’s a half and B’s a sixth. Now if you know B can you work out A?
David: It’s two of those.
Ms B: Okay so if it’s two of those, what size is it?

David: A half
Ms B: If B is a sixth
David: That is a half!
Student: Two out of six
Ms B: Two sixths. Excellent. Two sixths. What fraction is the same as two sixths?
Carl: Four eighths
Student: Two thirds!
(Fractured Figures, lesson observation)

Ms B was scaffolding the concepts of fractions during this task via the use of concrete embodiment materials and supported this through her questioning of Carl and David in their group during the task.

Apart from the materials that could have scaffolded Carl and David's learning during Fractured Figures, the group in which they worked for this task also appeared to have a positive effect. Ms B indicated before the lesson that she was going to "loosely ability group" (pre-lesson interview) and she chose the groups. Ms B discussed the effect of this decision in the post-lesson interview.

> Today he was working with people he could work *with* rather than being told what to do or what the answers are, I think today he felt a lot more confident with the maths. He was able to explain to some of his group members what some things were and he was able to have it explained to him on the same level by some of his group members why it didn't work. David is feeling a little bit more keen to have a go at things. I think he got a bit of a boost yesterday when he could do the first few on his own. They were able to have a conversation on a level that everyone could understand.
> (Post-lesson interview, Fractured Figures)

Carl also talked about the advantages of working in a group for this task.

> I work better in a group because you get to like, have a talk and like, when you're done you can have a bit of a rest and the other person does it. I liked today's lesson better than yesterday 'cause you got to make puzzles. (Yesterday's lesson) Sometimes I get bored with partners because when you don't get a fair turn. We got to do what we wanted (today) then the other two got to do what they wanted.

David found working in a group motivating for different reasons.

> I had to think about it really hard because we were working with a group 'cause if I get it wrong they might be disappointed. We worked it out pretty quick, pretty easy. Today's easier than yesterday's 'cause I already knew it, my brain already knew it.

Although both boys demonstrated misconceptions during this task, their engagement was clear and in the post-lesson assessment task, Fraction Pie, each demonstrated some understanding of fractional parts and relationships to the whole, which might have been influenced by their participation in the Fractured Figures task.

The Fraction Pie task used a circle divided into two quarters, one sixth and one third. Carl and David were asked to identify each part and explain their answers. This task was very similar to the Fractured Figures task the boys had just completed as both used circles, sixths, thirds, halves and quarters. David's response to this task has been described previously but to summarise, he made only one error by changing his answer from one sixth, which was correct, to one-twelfth during his explanation. Carl's response to this task demonstrated some understanding as well. He correctly identified one quarter explaining that "four would fit into the whole" and one third because "three of them make the whole". He was incorrect in identifying a sixth and a twelfth but his explanation showed some logic.

> For B, also a quarter, Carl said it was a sixth because there would be three in one half, so 6 in all. C he said was a twelfth because it was smaller than the sixth and because 12 would make one whole.
> (Post-lesson assessment task, Fraction Pie)

The results of this assessment task showed that the task Fractured Figures appeared to have supported Carl and David's understanding of fractions. This task was productive for both boys as they were engaged and they demonstrated some understanding. The other task for which both Carl and David were productive was the Decimat. Due to the overlapping nature of the scaffolding practices explored in this study, Decimat was described in some detail in the "Manipulative materials and visual representations" section of this chapter. Suffice to say that the concrete embodiment of the concepts of the relative size of decimal parts and notation of decimal numbers that the Decimat provided appeared to scaffold Carl and David's emerging understanding of these concepts.

For David, three out of the eight tasks were unproductive. During these tasks, Fruit and Veggies, Construct a Sum, and Sticky Numbers and Number line, Ms B spent no time talking directly to David. It appeared in these instances that David did not understand the mathematics required to complete the tasks and the cognitive demand was too high. In each of these cases, materials used were symbolic and abstract. During Fruit and Veggies, the following observations were made.

> David and his partner have made some "shopping bags" however when I discussed it with them, his partner pointed out that 0.7 and 1/3 was more than one kilogram. David could not say what 0.01 of a kilogram of mint was. He suggested it was one whole or one tenth. His partner told him it was one hundredth. I asked if one tenth was more or less than one hundredth and David couldn't say.
> (Fruit and Veggies, lesson observation)

Without the knowledge of the relative sizes of the fractions and decimal parts of one kilogram, the task was too difficult for David to complete. This task preceded the Decimat task after which David appeared to have a greater understanding of the relative size of decimal parts. Despite the lack of progress David made on the Fruit and Veggies task, his affective response was quite positive.

I put it on the thinking 'cause I had to think really hard and unlike yesterday I got heaps of help from my partner. It was a bit more challenging because it wasn't' written in kilograms just in halves and points. I enjoyed that we were nearly …[interruption]
(Post-lesson interview, Fruit and Veggies)

A similar lack of progress was observed during Construct a Sum.

David spent time tearing out each digit card. He had 1/6 + 5/7 = 3/4. I asked him about this and he said, "I think that adds to that".
(Construct a Sum lesson observation)

David's affective response to this task was negative.

It's like a flashback to yesterday. I didn't enjoy it because it was hard.
(Post-lesson interview, Construct a Sum)

David's assessment of his own performance on this task was accurate. The task was hard, as David did not understand the relative size of fractions in order to add them to make a sum close to one. It appeared that he was also unclear that the sum needed to be close to one and that he did not need to make a fractional answer.

The other task in which Ms B did not speak directly to David was Sticky Numbers and Number Line. As in the case of Carl, other things she said to the whole class may have assisted David, although his final answer demonstrated his confusion about equivalent decimal numbers and fractions.

David continued to throw his object up and down. He then called out "I know where mine goes!" David went up to the board to put up his 1/3, which he placed on the line above 0.8.
(Sticky numbers and number line, lesson observation)

In a similar way to Carl, Ms B did not comment on where David placed his number.

These three unproductive tasks for David had some common factors. First, in all three tasks Ms B did not speak directly to David. One task, Sticky Numbers and Number line, was a whole class task. The other two tasks, Fruit and Veggies and Construct a Sum, were not whole class tasks but Ms B did not speak individually to David during these tasks. All three tasks used materials or representations that were aimed at prompting and connecting prior knowledge. As discussed previously in this chapter, this use of materials and representations was not effective for Carl and David, as they appeared not to have the prior knowledge necessary to complete such tasks. This illustrates again the overlapping nature of the scaffolding practices of this study.

Two tasks, Dominoes and Sorting Cards, appeared to be partially productive for David in that he displayed some understanding of the concepts of the tasks though also retained some misconceptions. Both of these tasks used representations that were formal notations and aimed at linking prior knowledge of equivalent rational numbers.

During the Dominoes task, Ms B spent 12 minutes talking to David and his partner Greg. This was the longest period of time Ms B spent talking to a pair of students during the observation period. Though Ms B said that this discussion was more for Greg, David seemed engaged and followed the discussion. Ms B illustrated the relationship between 1/4 and 25%, between 0.75, 3/4 and 75%, between 1/2 and 50% and 0.5, and tenths and 10% using diagrams and discussion. David mimicked what Ms B drew for Greg in his response to the assessment task that asked him to match two dominoes and explain how they matched.

> David chose 1/4 and 0.25. I asked him why they matched and he said "because, here I draw a quarter here, [drew on paper a rectangle divided into quarters] each one holds 25 but a half of it is 50" I asked again why then did 1/4 and zero point two five match. David said "because there's 0.25 in a quarter".
> I then asked him to find a match for 75%. He was silent for 20 seconds. When I asked "not sure?" he said "yeah".
> (Post lesson assessment task, Dominoes)

It seems from this response that David gained some understanding from the time Ms B spent with him and his partner in this lesson. However, during the Dominoes lesson, after Ms B left David and Greg to continue working, the following observation was made.

> David worked on the new set of dominoes by himself. I asked him about some of his matches – 10% and 1/10 "because 10% is 1/10 out of 100", 1/5 and 3/10 "because 1/5 out of 3/10", 1/2 and 25% … Ms B clapped for attention before this match could be explained.
> (Dominoes, lesson observation)

This demonstrates that although David had some understanding of some equivalent percent, decimal numbers and fractions, he was still unsure about others, even common equivalent numbers such as half and 50 percent. Interestingly, despite Ms B spending such an extended time assisting David and his partner, David's affective response to this task was negative.

> (The arrow is) pointing to the kind of sad bit because I had to pick the cards out. I had to do the cards 'cause I hardly got no help. I did the cards and he was just explaining to the teacher.
> (Post-lesson interview, Dominoes)

It appeared that David believed that Ms B's extensive explanations and diagrams were not intended for him and were not helpful for his learning but all aimed at Greg leaving David to complete the activity alone.

For the Sorting Cards task, David worked in a group of three. This task was aimed at using the cards to activate prior knowledge of rational numbers and as a manipulative materials task was not as successful for David as a task that used concrete embodiment materials. This was explored during the discussion of manipulative materials and visual representations in the chapter. David did display some attention to underlying concepts in this task when he identified that decimal numbers could also be fractions. However, his knowledge of this was incomplete as he proposed that 0.5 would be one fifth as a fraction.

It would appear that for David to progress in his understanding of underlying concepts, scaffolding conversations with Ms B might have been more important for David than concrete embodiment materials. As described, two tasks on which David was at least partially productive included materials and representations that aimed at prompting prior knowledge but also featured conversations with Ms B. However, the three tasks on which David was most productive, Fraction Rods, Fractured Figures and Decimat, did include materials that illustrated the concepts. In addition, during Fraction Rods and Fractured Figures Ms B had conversations with David though during Decimat Ms B only spoke to David for one minute. It appears that scaffolding conversations, followed by materials that illustrate concepts, were important for David's learning whereas for Carl, the materials appeared more important than conversations with Ms B.

To summarise, for three tasks the level of cognitive demand seemed too high for David. On these three tasks Ms B did not spend any time talking with David, perhaps further evidence that interaction with Ms B was key to David's learning. For two tasks, David showed partial attention to concepts though retained misconceptions. During one of these tasks, Ms B spent considerable time with David and his partner, which David drew on during the post-lesson assessment task. Despite this, David rated this task poorly, indicating that he believed Ms B was helping his partner but leaving him to complete the task alone. The other task saw David work in a group, which he viewed as positive and he showed some awareness that decimal numbers could also be expressed as fractions. For three tasks, David appeared to show progress in his understanding of fractions and decimal numbers. All these tasks used materials that illustrated such concepts and for two of these productive tasks, also included scaffolding conversations with Ms B.

Ms B did not lower the level of cognitive demand during implementation on any of the observed tasks. However, the level of demand was often too high. For Carl this occurred for six out of the eight tasks and for David on three to five tasks. Many tasks appeared beyond the understanding and prior knowledge of Carl and David, shown by their responses to post-lesson assessment tasks. These tasks could be considered inappropriate, as they did not build on students' prior knowledge, a factor Henningsen & Stein (1997) proposed was vital for tasks maintaining a high level of cognitive demand throughout implementation. An important factor in effective scaffolding is that tasks take into account prior knowledge of students and challenge students to go just beyond what is known. Tasks that are too difficult might not be possible to scaffold as they fall beyond the students' "Zone of Proximal Development" (Vygotsky, 1978).

When the cognitive demand was too high, Carl tended to leave much of the thinking to his partner where possible or to engage in non-mathematical behaviour. David often persisted with these tasks but did not reach a satisfactory solution. Ms B paid attention to the concepts of rational number that underpinned the observed tasks by planning and implementing tasks focussed on such concepts and by not lowering the cognitive demand throughout implementation. For Carl and David this meant that demand was too high for many of the tasks. When without materials, suitable peers or interaction with their teacher to scaffold their learning, Carl and David disengaged from these tasks. Ms B's habit of trying to talk to each student every lesson meant that there were long periods of time in which Carl and David were without her assistance. Over the 6 lessons, averaging 82 minutes, Ms B spent an average of 3 minutes talking to Carl and 5 minutes talking to David making the scaffolding that she could offer minimal. Three tasks enabled scaffolding in the concepts to be drawn from the materials themselves, which illustrated such concepts. These tasks were more appropriate for Carl and David in that the level of difficulty was, for the most part, within their grasp. These tasks appeared the most successful for Carl and David in terms of engagement and learning of the concepts. The data suggest that the most effective scaffolding for Carl and David occurred when Ms B used materials to draw attention to the underlying concepts, along with scaffolding conversations, and the use of tasks with appropriate levels of cognitive demand. I will now discuss the extent to which Ms L paid explicit attention to concepts. This discussion will then examine the responses of Sophie and Riley, both cognitively and affectively.

EXPLICIT ATTENTION TO CONCEPTS – MS L

In interviews with Ms L it was apparent that she supported the idea of teaching mathematics for understanding. Ms L stated that the most important thing she wanted students to gain as a result of her mathematics teaching was "knowledge, confidence, the ability to see the connections to real life" (initial teacher interview). Ms L described a change in her teaching of mathematics from students "doing it" to "understanding it".

> Now I'm concerned about the kids explaining it to me, telling me, showing me how they did it, proving it to me doing it in different ways. The (research project) has brought that out; understanding rather than just the doing.
> (Initial teacher interview)

Despite saying this during the initial teacher interview, it appeared Ms L may have held a different view in post-lesson interviews, particularly when talking about the low-attaining target students, Riley and Sophie. Rather than using words such as "understanding", "connecting", or "explaining" as in the initial teacher interview, Ms L used words like "basics", "back", "step by step" and "repetition". In every post-lesson interview, Ms L used these words when talking about Sophie or Riley. For example, Ms L said "basic" and "back" in three out of seven interviews.

> They'll need a lot of work just on basic multiplication.
> (Post-lesson interview, Find the Pairs)
>
> So we need to go right back to basics.
> (Post-lesson interview, Highest Product)
>
> I think again, they needed to be taken right back to basics. Riley needed to go right back to place value to help him, so did Sophie … I think we need to go back step by step.
> (Post-lesson interview, Help Julia)

This suggests that Ms L might have held the kind of "back to basics" belief reported by Yates (2009), where basic facts must be mastered before more challenging tasks can be attempted.

Ms L used the word "understanding" once to describe Riley's work on the pattern for multiplying by ten during the Help Julia task. This was described earlier in this chapter in the discussion regarding visual representations when Ms L drew a place value chart and asked Riley to multiply 6 × 4, then 6 × 40 and then 6 × 400. It will be recalled from the "manipulative materials and visual representations" discussion that Riley was unable to give a clear description of this pattern suggesting that he had not reached the understanding Ms L believed he might have.

It would seem that Ms L believed in the importance of students understanding mathematics as she expressed in the initial teacher interview. However, as found in the literature, teachers' espoused beliefs can sometimes differ from the enactment of such beliefs in the classroom (Thompson, 1992). Factors such as her own mathematics background, her understanding and experience in linking appropriate models and representations to multiplication and her opinions about the students' capabilities would impact on Ms L's enactment of her beliefs (Bibby, 1999). The literature also shows many examples illustrating how difficult teaching for understanding can be (e.g. Cohen, 1990; Eisenhart et al., 1993; Kazemi & Stipek, 2001; Olson & Barrett, 2004) and that some teachers believe teaching for understanding is not appropriate for students regarded as low attaining (Anderson, 1997; Beswick, 2005a; Zohar et al., 2001). Abstractly, and removed from actual classroom episodes, Ms L supported understanding as a goal of her mathematics teaching. However, in the "messy reality" of her classroom, such beliefs might not have been easy to enact. I will now examine more closely Ms L's teaching during the observed mathematics tasks to attempt to tease out the extent to which student learning was scaffolded through her attention to concepts.

One way to examine the extent to which Ms L paid explicit attention to concepts can be examined by focussing on the level of cognitive demand for the tasks she planned and if this level was maintained throughout implementation (Stein et al., 1996). Ms L planned some tasks that appeared to be aimed at exploring underlying concepts of multiplication. For example, Missing Numbers was an open-ended question of "26= (? × ?) + ?" and Find the Pairs game asked students to find the possible factors to a given product. In fact, according to Ms L's mathematics planning documents, all eight tasks were planned to require students to find multiple solutions, choose or devise an appropriate written process, explore underlying concepts, estimate and connect multiplication to real world situations - features of tasks designed for high cognitive demand.

It has been recognised that maintaining a task's high level of cognitive demand is challenging for teachers with many tasks failing to maintain their initial high cognitive demand throughout implementation (Stein et al., 1996). For example, Doyle (1988) pointed out that using novel tasks that challenge students often results in a "bumpy workflow". This sometimes prompts teachers to reduce the demand of tasks to keep the class more orderly with a "smoother workflow". In Ms L's lessons, four out of eight of the tasks set up for higher cognitive demand diminished in the level of demand during implementation. This decline was from exploring concepts into finding a solution or answer, also described by Henningsen and Stein (1997). Three of

Ms L's tasks were set up and remained at a high level of cognitive demand throughout. One task, Find the Pairs, differed in the level of cognitive demand across the class with students working at various levels. I will now discuss examples from Ms L's classroom that illustrate tasks that began as high in cognitive demand but were lowered during implementation with the whole class.

Highest Product was a task that asked students to arrange the digits 9, 8 and 7 into a two-by-one-digit multiplication expression in order to find the highest possible product. The task itself appeared to be designed to encourage students to consider the relationship between the multiplier and the multiplicand in achieving the greatest possible product. Strategies such as "guess and check" may be employed and students might also produce a statement that explains why their solution produces the highest possible product. In her planning notes, Ms L wrote that students were going to be asked to "estimate first" and her assessment focus was "estimations made – are they reasonable? Do children use knowledge of place value to make reasonable estimations?" (Planning documents).

During implementation, the estimation stage was not emphasised by Ms L with students. Instead at the beginning of the task, Ms L led students in a discussion that defined "product". Ms L then instructed students to calculate all the possible combinations in order to prove which resulted in the highest product.

> Ms L (to class): Do you think you could work out all the combinations of numbers first and then that might help you to work out which one will give you the highest product?
> (Highest product, lesson observation)

In this way, students were involved in the lowered cognitive task of practising many examples using written processes for multiplying. There was little worth in estimating and justifying a solution, as students were required to calculate every possible combination. In this way the task appeared to decline in cognitive demand to a focus on the solution (Henningsen & Stein, 1997). At the conclusion of the task, when the combination of the digits producing the highest possible product was found, there was little discussion of why this might be so. Indeed, Ms L privately admitted that she hadn't worked through the task herself and that she was surprised at the result.

> Ms L: Who would like to share something they found out today, something they learnt, something they discovered because I think we all made an assumption.
> Student: For my first one I did 9 times 87 then I realised there were more possible ones I could do and the one I ended up, I noticed if you do something times 9, and I realised that if you do the numbers from biggest to smallest it gives you a different number.

Ms L: So you think 9 times 87 gives you the highest possible product. Emily can we come back to what you said? Alright so 9 times 87 [wrote on the IWB] is 783. And then Emily what did you say gave you the biggest product? 87 times 9 which is the same because it's just the way we positioned it. Remember we said before, 87 times 9 …
Student: I'm just thinking that the smaller the number you're multiplying by the larger your answer will be?
Ms L: What are you saying? So you're saying …
Student: Say 87 the smaller the number you're multiplying by the larger your answer will be.
Ms L: So you're saying if we multiplied 87 by 6 we'd get a bigger product?
Student: No! From out of those.
Ms L: Show me what you mean.
Student: So I'm saying 87 times 9 = then if I did 98 times 7 it would equal 686.
Ms L: Ah. But what's important there? Do you think where you place the nine is important? The nine is the highest individual digit we have so that's going to make a difference.
(Highest Product, lesson observation)

The student in this exchange appeared to be attempting to articulate an important point about the task, that the multiplier holds the key to the largest product. Ms L did not seem to understand what the student was trying to say or recognise the importance of the concept he was talking about. She did state that nine was the "highest individual digit" and therefore would "make a difference" but how or why this was so was not explored.

For the students, the Highest Product task was a lower level of cognitive demand in that the task required them mainly to practise a written process for multiplication. As I will illustrate, this lowered level of demand was still too high for Riley. Ms L immediately went to Riley after setting students to work on the task.

Ms L: So you've got 78 times 9 and 87 (times 9). Which way do you think those two numbers should be?
Riley: 9 times 87
Ms L: Okay so out of those two which one would give you the highest product?
Riley: This one. (9 × 87)
(Highest product, lesson observation)

Riley's response was interesting in that he appeared to have already formed a correct opinion of the combination of digits, which would result in the highest product. Ms L did not question Riley as to why he had estimated 9 × 87, which may have indicated something about Riley's understanding of multiplication. Instead, Ms L asked Riley to proceed with calculating the product of 9 × 87 for which Riley attempted, unsuccessfully, to use the traditional algorithm as I reported earlier in this chapter.

For Sophie this task was perhaps at a more appropriate level of demand than for Riley in that she reached a solution by exploring her own less efficient strategy, of adding 87 nine times, before Ms L showed her a more efficient strategy for multiplying by nine. This was described previously as a scaffolding conversation Ms L had with Sophie. In considering Ms L's attention to concepts in this lesson, it would appear that Ms L and the students did not explicitly explore the concept of the relationship between multiplier and multiplicand. Instead the task became practising processes for multiplying.

During three other tasks the level of cognitive demand was lowered during implementation, resulting in less attention being paid to underlying concepts. In two tasks this lowered demand was for all students. Array of 48 has been discussed in the manipulative materials and visual representations part of this chapter. This task began with imagining an array of 48 tiles and what it could look like but then became generating all the multiplication and division expressions possible with a product of 48. Help Julia was the other task for which the attention on concepts was reduced for all students.

Help Julia started as a task about helping a fictitious child find a strategy for two-by-two digit multiplication but became, for Riley, an attempt to do the traditional algorithm with no success then completing 6 × 40, 6 × 400 at Ms L's request which was discussed in "manipulative materials and visual representations".

Sophie's response to this task was to attempt to complete an extended notation process to find an answer. She had written 5 × 6=30, 40 × 20=80, 30 + 80=110, and 10 × 40=400. After pointing out Sophie's error with 40 × 20, Ms L spent 10 minutes working through the extended notation method, with Sophie firstly by breaking the problem down into 45 × 6 and 45 × 20. Sophie's response to multiplying 6 by 40 was to do 6 × 10 four times, continuing with her preferred repeated addition strategy. Ms L asked her not to use this strategy and instead directed her to multiply 6 by 40 by adding a zero to 6 × 4. The discussion centred on which numbers to multiply together and Sophie showed some confusion about the whole process when she had reached her answers to 45 × 6 and 45 × 20.

Ms L: What do you do now? So you've worked out now 45 times 40 'cause you worked out 40 times 20 and 5 times 20 gives you 900. You worked out 45 times 6 so what can you do with those things now to help Julia with her original question of 45 times 26?
Sophie: Do 900 times 6?
Ms L: Why are you going to times it by 6? You've found out two answers already. What can you do with them?
Sophie: Add 270 to 900
Ms L: Can you do that?
Sophie: Yep

The focus of the discussion was on the steps to take to multiply two digit numbers. There was no discussion about the concepts that underpin such processes. Sophie's strategy for calculating 6 by 40 by adding 6 × 10 four times was more meaningful for Sophie than Ms L's strategy for adding a zero to 24. In implementation this task became focussed on finding the solution to the question and not investigating strategies or explaining strategies to Julia. Ms L indicated this during the post-lesson interview and that she felt finding an answer was important for Sophie affectively, though the task may have been too difficult.

> The numbers were probably still a bit too big, a bit too daunting but I think she felt happy that in the end she got the answer and that's why I said let's share the answer so she could see that she has the right answer.
> (Post-lesson interview, Help Julia)

For her part, Sophie also recognised that the task was difficult but that she appreciated Ms L's assistance with the written process.

> It was hard but when Ms L came around it started to get very easy. What she did was went through with it and what she did was … she put umm … she put a sum that would end up as 270 then she did umm … the normal one which was 900 and then I had to add 900 to 270 which was 1170.
> (Post-lesson interview, Help Julia)

Interestingly, Sophie also focussed on the answer in this post-lesson interview, perhaps proving Ms L's theory that the answer was important to Sophie. This might have also been in response to Ms L's interest in finding an answer. This response also demonstrated Sophie's on-going confusion with the process Ms L worked through with her during this lesson.

The final discussion of Help Julia involved three students writing the methods of calculating on the board with little discussion about these methods. This was discussed earlier in this chapter. To conclude the lesson, Ms L asked students, "what would you tell Julia now?" A student offered the advice "multiply all the numbers by each other." Ms L summarised the task and said, "okay and if we can't multiply those bigger numbers we can break it down and use our times tables to help us." (Help Julia lesson observation). This did not appear to match the task's original intention of explaining strategies.

The other task in which the level of cognitive demand declined was Find the Pairs. Find the Pairs began as a task about finding all the possible factors for a given product. This task differed in that most students continued to focus on the concept of possible factors for a given product throughout the task. Sophie attempted to do this but appeared a little confused using addition as well as multiplication to make the given products.

However, again the demand for Riley was reduced in this task as he spent the time for the task unsuccessfully attempting to find the answer for 9x8 using an array at Ms L's suggestion, which has been discussed previously.

Ms L was observed maintaining the task's focus on concepts for three tasks, Veggie Patch, Missing Numbers and Target Multiplication. The three tasks were all relatively short tasks running for an average of 12 minutes. According to Ms L's planning notes, these tasks were for the "tuning in" section of the lesson. Ms L sustained the high level of cognitive demand of these tasks for all students, including Sophie and Riley. I will now describe Target Multiplication to illustrate how Ms L maintained this task's higher cognitive demand throughout implementation.

The goal for Target Multiplication was described in Ms L's planning notes as "to estimate calculations involving multiplication". The task asked students to select a number from each of two boxes that, when multiplied, would fall within the range of numbers shown on the target (see Figure 13).

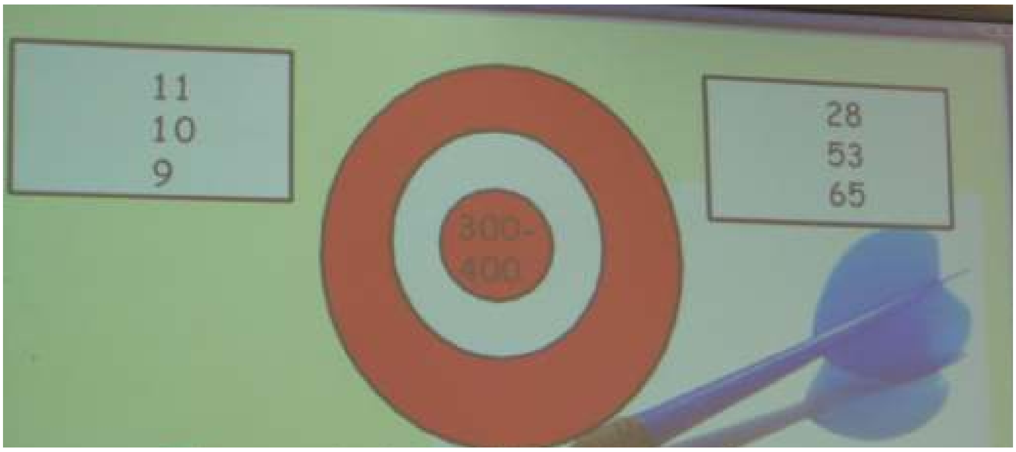


Figure 13. Target Multiplication graphic shown to students.

In setting up this task, Ms L was explicit about her goal of having students estimate the outcome of multiplying two digit numbers.

> Ms L: This is called Target Multiplication. At the moment we are not looking at working out the exact answer. We're looking at estimating. Choose a number from this box and a number from this box so that the answer will be somewhere between 300 – 400, the middle of the target. Now you need to use your knowledge of multiplication to help you, you need to use your knowledge of rounding to help you because we can't just take two random numbers. We need to think carefully about the two numbers we choose.

Ms L then asked a student to explain the task again in response to her belief that at least one student was confused by these instructions, This student chose to explain that 53 and 10 could not be chosen as the product of these numbers would not fall within the target range of 300 to 400, though the student did not give the answer. Ms L reminded students to think about multiplying by ten to help them and then students began the task.

In discussing student solutions to this problem, Ms L asked students to explain their answers and thinking. Through these explanations Ms L was able to draw out the concept of estimation using strategies such as multiplying by ten.

Ms L:	Caleb which two numbers did you choose?
Caleb:	11 and 28.
Ms L:	Why did you choose 11 and 28?
Caleb:	Well if you choose 11 and 53 it would be 5 hundred and something which would be over and if you choose 65 it would be over so I chose 28.
Ms L:	Okay but why did you choose… that's good estimation what you've done there because you've already eliminated 53 because it was over so therefore 65 is over. Why did you choose 11 and not 10 or 9?
Caleb:	Umm… if you did 10 it would be … 280. That's under the 300- 400.
Ms L:	Okay and then you eliminated the nine as well.

(Target multiplication, lesson observation)

In this exchange, Ms L pressed Caleb to explain every aspect of his solution asking "why" twice. This resonates with Kazemi and Stipek's (2001) description of "high press for conceptual learning" exchanges between teachers and students. Similar exchanges occurred after Ms L changed the target number range to 800 to 1000.

Ms L:	What else could we eliminate from over there then?
Student A:	11 times 91
Ms L:	Okay, why?
Student A:	It would be too much.
Ms L:	How did you work that out quickly?
Student A:	I used addition
Ms L:	You used addition to do multiplication of..
Student A:	No I did 91 times 10 then I added 91.
Ms L:	Okay and that took you over the thousand.
Student B:	I did 11 times 78 because I did 10 times 78 was 780 and I knew another 78 would be over 800.
Ms L:	Okay so what he's done there is he has said 10 times 78, we know how to multiply by 10, we know how to do that, it makes it 10 times bigger and then he's added another 78 which takes him over 800.

(Target Multiplication, lesson observation)

Ms L asked these students to explain why they chose numbers and also "revoiced" (O'Connor & Michaels, 1993) their strategies to emphasise the concepts of multiplying by ten and estimating. It would appear that throughout this task Ms L maintained her initial goal of asking students to estimate. She also pressed students to explain and prove their solutions, a feature of high cognitive demand tasks. During this task, Ms L appeared to maintain the focus on the underlying concepts of multiplying by ten and estimating multiplication.

Examining the coding for "connections" for Ms L's lessons revealed no data coded as "connections between concepts and processes". As Ms L's lessons examined processes for multiplication during five out of the six lessons observed, Ms L taught these processes without explicit attention to the concepts that underpin them. This was described by Stein, Grover and Henningsen (1996) as "procedures without connection to concepts" and was categorised as a lower level of cognitive demand type of task. Ms L often asked students to show examples of written procedures but did not discuss the concepts underlying such procedures. She compared the procedures in a minimal way, pointing out in Help Julia for example, that Chris' procedure was smaller steps than Erin's. Both Ms L and students used the word "partitioning" in discussions but this term was not explained or linked to the concept of partitioning numbers to multiply them or why this would be a valid strategy. Ms L reiterated Georgia's "rule" of "make sure you multiply all the numbers together in the right way" during discussions but did not explore what this meant. In this way during four tasks Ms L taught the procedures for multiplying without attending to the concepts that underpin them.

On closer examination of these lessons, this focus on procedures without connections only occurred for part of the overall lesson. In fact, in all four lessons where procedures without connections were used, they were described as "learning activity" in Ms L's planning notes and formed the longest part of the lesson averaging 18 minutes. However, the "tuning in" tasks of about 12 minutes that came before these "learning activities" focussed on concepts of multiplication such as multiplying by ten, real world examples, arrays or estimation. It would appear that Ms L focussed on concepts more during the shorter parts of the lesson, the "tuning in" tasks. For the longest part of the lessons, the "learning activities", Ms L tended to focus on procedures without connections. This resonates with the findings of Charalambous (2008) who found that a teacher with low "mathematical knowledge for teaching" tended to spend less time on conceptually challenging tasks and more time on procedural tasks.

Ms L made connections between knowledge sets and understandings on two occasions linking multiplication and division during Array of 48 and reminding students about the order of operations during Missing Numbers. The most frequently coded type of connection that Ms L made was to real world experiences, which occurred five times. Ms L connected the shape of her vegetable patch to the number and length of rows, she compared the grid method to reading the street directory, and she connected estimation to shopping and multiplication to children on buses. During one lesson, Ms L spent some time explaining how she and other teachers used multiplication to organise the recent class sleepover.

> It's like when we organised the sleepover, us teachers. We had to first know how many children we had and we had to know how much everything costs so we could calculate how much it was going to cost for each of you to come on the excursion and stay at the sleepover. So that's where we were using our multiplication. So we've got 88 children in our year group we need to cover this amount of money how much will each one need to pay? So it was using a bit of division as well but we were using multiplication to check our answers.
> (Athletics Day, lesson observation)

This would appear consistent with interview data in which Ms L talked about the importance of students recognising how mathematics was part of their everyday lives.

On three occasions in Ms L's classroom, students made errors during whole class discussions. In the first instance, we can see that Ms L encouraged the students to elaborate on their response and therefore discover and correct the error themselves.

Ms L:	What's happening here? What's our pattern here?
Student:	8 times 6
Ms L:	Is it 8 times 6?
Student:	Cause half of 16 is 8 and that's adding 2 so it's 6.
Ms L:	Is it adding 2 though? I wonder could anyone see a pattern happening here?
Student:	We are dividing each number by 2.
Ms L:	We're dividing each number by 2?! Erin?
Erin:	Well 64 times 1, we're halving 64 to get 32 and doubling 1 to get 2, then we're halving 32 to get 16 and doubling 2 to get 4…
Ms L:	Okay so have we found all the possibilities or based on the pattern that Erin's just pointed out is another possibility?

(Missing Numbers, lesson observation)

Although in this case the student did not correct their own error, Ms L allowed another student to articulate the correct pattern rather than providing it as the teacher. Ms L did not explore the misconception with the class, although using an additive approach in multiplicative situations may have been a common misconception with these students.

In the next example, Ms L uncovered a lack of understanding about what division really meant, but did not examine the concept. Instead she focussed on the correct way to notate a division expression.

Ms L:	48 children per bus for the 975 children. So how would I write that as a number

expression?
[A student suggested 48 divided by 975. Ms L wrote this on the board.]
Everyone look at what I've written here. What do you notice about it?

Student: How much can fit on the bus

Ms L: Would you write it that way? Hand up if you'd write it a different way?
[No hands went up]
I'm just curious, when we are dividing, do we start with the smaller number or do we start with the bigger number? If I'm dividing something what am I doing?

Student: Putting them into groups.

Ms L: Okay starting with what?

Student: The bigger number

Ms L: The bigger number okay. So what would be a better way of writing that division problem?

Erin: 975 divided by 48

Ms L: 974 divided by 48 [wrote on the board] And we're going to come back to that now tomorrow maybe Friday and we are going to look at how, a few different ways, of solving that.

(Athletics Day, lesson observation)

During this exchange it seemed clear that many students were confused about the actual nature of division, with no students appearing to note the error with 48 divided by 975. When Ms L questioned what it meant to divide, a student offered "putting them into groups". Ms L did not continue to focus on the meaning of division but instead asked a closed question, "starting with what?" A student then provided the response she was waiting for, "the bigger number". By the end of this discussion, the expression was written correctly and Ms L noted that more work needed to be done about division. Ms L did not use this error to explore the students' understanding of division though, as this occurred at the end of the lesson observation period, Ms L may have spoken more about the meaning of division in subsequent lessons not observed. On another occasion, Ms L largely ignored the following incorrect voluntary contribution during a whole class discussion about multiplying 45 and 26.

Ms L: What is it Murphy?

Murphy: He hasn't done 40 times 5

Ms L: Why would he have to do 40 times 5? He wouldn't because it's 45.
Okay, I asked Erin to set it up in a more systematic way and this is what Erin has come up with.

(Help Julia, lesson observation)

Though Murphy pointed out what he believed was an error, his misconception was not explored. This misconception concerning which numbers to multiply in a multi-digit multiplication was noted by Lampert (1986) and evident with the target students, as well as other students, during lessons. Ms L had highlighted the importance of "multiplying all the numbers" and this might have been an opportunity to turn this contribution into an exploration of what this "rule" really meant. These examples illustrate that Ms L seemed to want to deal with errors in a timely way during whole class discussions rather than as a “springboard” to explore concepts as described by Borasi (1994).

When dealing with errors in private exchanges with Sophie or Riley, Ms L behaved quite differently. In these instances, Ms L confronted Sophie and Riley with their error and spent time working through correcting it with them. For example, during Highest Product, Ms L drew Sophie’s attention to her error that 9 × 87 was 202 by asking her to work out 3 × 87 by repeated addition then asking if 9 × 87 is 202 “made sense”. Similarly in Help Julia Ms L noticed that Sophie had 40 × 20 = 80 and asked Sophie if this “made sense”, pointing out that 40 × 2 is 80. In Find the Pairs, Ms L noticed that Riley had incorrectly answered 9 × 8 but instead of providing the answer, Ms L suggested he use an array to help him. It would appear that in private exchanges Ms L was more willing to spend time exploring errors than during whole class discussions. The reasons for this can only be speculated but perhaps this was due to time constraints because of the number of tasks Ms L usually tried to include in her mathematics lessons. Ms L also talked about the importance of students feeling “confident” and might have resisted publicly drawing attention to errors for affective reasons. Another possible explanation is that Ms L lacked confidence to deal with complex mathematical issues publicly.

To summarise, all of the eight tasks Ms L planned for her lessons exploring multi-digit multiplication were centred around exploring the concepts of multiplication through open tasks, estimation, real world examples, arrays and exploration of strategies for multiplying. In implementation, five of these tasks were reduced in cognitive demand for the low-attaining students and the focus on concepts diminished. Four of these were “learning activities” and comprised the longest tasks of the lesson with one “tuning in” task, Array of 48 declining in its level of demand. The main focus of these tasks became developing accurate and efficient written processes for multi-digit multiplication. For three tasks the focus on concepts continued throughout the task. These were “tuning in” tasks and were shorter than the learning activities. These tasks focussed on estimation strategies, doubling and halving and real world examples using arrays. Ms L attended to concepts for about 12 minutes out of an average lesson time of 70 minutes. However, this time was not as long as the 18 minutes on average spent per lesson on developing written processes without exploring underlying concepts. I will now examine the effect this had on Sophie and Riley, the low-attaining target students in this class.

RESPONSES OF SOPHIE AND RILEY

For five out of the eight tasks in Ms L's class, Sophie and Riley were practising written processes or attempting to calculate multiplication facts along with their classmates. Three tasks maintained their focus on concepts such as real world examples of multiplication, arrays and estimation throughout implementation. Riley demonstrated some success on one such task, Veggie Patch whereas Sophie showed some confusion. Both Sophie and Riley struggled during the other two conceptually focussed tasks, Target Multiplication and Missing Numbers. I will now discuss these tasks to examine the extent to which Ms L scaffolded Sophie and Riley's understanding of the concepts behind multiplication.

During Veggie Patch, students were asked to consider how to arrange 60 plants into equal rows for a vegetable patch. Students had small whiteboards each to draw and write their responses. As discussed previously, Riley was reasonably successful in this task in that he was able to generate two solutions, 10 rows of 6 and 20 rows of 3, and explained his strategy of doubling and halving. Sophie spent some time at the beginning of this task writing the question on her whiteboard before drawing a rectangle that she divided into 2 rows of 11 but wrote "2 × 30" beside it. She did not generate another possibility.

Target Multiplication has been discussed previously and shown to be a task where Ms L maintained a conceptual focus on estimating with multi-digit multiplication. Both Sophie and Riley struggled during this task suggesting that the cognitive demand might have been too high, as demonstrated by the following observations:

> Ms L: I'm going to give you 35 seconds to choose two numbers from there, estimate 2 numbers from there that are going to give you a target range of 300 – 400 but you need to be able to explain how you chose your numbers and how you prove to me that you're right. Go.
> [1 minute and 9 secs passed.]
>
> Riley sat looking confused then wrote 53, 78, 11, 10, and 28 on his board. Sophie hadn't started and had a blank board.
>
> Ms L: Okay, your 35 seconds is up.
>
> (Target Multiplication, lesson observation)

Neither Sophie nor Riley had an answer to this first part of the task. Ms L then introduced a new range, 800 to 1000. Riley wrote "11 × 91" on his board and seemed to be looking at other students' boards. Sophie also had 11 × 91. This fell just out of the range with 1001. For this part of the task, Ms L noticed Sophie had written 11 × 91 and questioned her about this answer.

> Ms L: How did you choose your numbers? How did you choose 11 and 91? Why did you?
> Sophie: I did one times one is one …And then 11 times 9 which is 99 and 9 times 1 which is 9 so…
> Ms L: Okay but we're working with 11 times 91 not 11 times 9. [to the class] Alright let's share.
>
> (Target Multiplication, lesson observation)

Ms L did not draw Sophie's attention to the concept of multiplying by ten and one more for 11 by 91 or the concept of estimation. Sophie appeared to be trying to complete an algorithm to find 11 × 91, though this was not Ms L's intention for this task. During the end-of-task discussion with the class, 11 × 91 briefly emerged as a calculation that fell outside the range, but this was brief and not fully explored. It would appear that Riley and Sophie did not explore the concept of estimation during this task. Given their lack of success, it would appear the cognitive demand was too high for Sophie and Riley without additional support being offered.

The third task that remained focussed on concepts was Missing Numbers. This was a series of three multiplication expressions with missing numbers, the first a closed question but the other two questions had multiple possibilities. Ms L's instructions at the beginning of this task were "I'm going to give you your own whiteboards to work out these problems on your own at this point. So we might have more than one response for the second two. You might show me more than one possibility. Show me everything you know" (Missing numbers lesson observation). Sophie wrote no solutions on her whiteboard to these three expressions. Riley wrote one solution to the second question after a student shared an example. Neither Sophie nor Riley wrote any solutions for the final expression.

Target Multiplication and Missing Numbers were two tasks in which Sophie and Riley did not succeed in attending to the concepts these tasks were attempting to explore. This lack of success may be due to a number of factors. First, the tasks themselves were reasonably complex considering the prior knowledge Riley and Sophie demonstrated during lessons and also in post-lesson assessment tasks. As with Carl and David in Ms B's class, these tasks appeared to be inappropriate in that they did not build on Sophie's or Riley's prior knowledge or understanding. Second, scaffolding offered during these tasks was minimal. Ms L only spoke briefly to Sophie during one task and in both these tasks students were expected to work alone.

Each of these tasks used numbers only with no other manipulative materials or visual representations that could assist Sophie or Riley in understanding the concepts. Lastly, time for these tasks was also short with Missing Numbers running for 14 minutes and Target Multiplication for 12 minutes, meaning Sophie and Riley might not have had sufficient time to "grapple with the important mathematical ideas contained in the task" (Henningsen & Stein, 1997, p. 537).

Riley's success with the Veggie Patch task is interesting to examine when compared with his lack of success with these other tasks. Veggie Patch used a real world context that Target Multiplication and Missing Numbers did not and perhaps this real world context assisted Riley. Similarly, Riley demonstrated his ability to estimate during the beginning of task discussion for Athletics Day, also a task set in a real world context. Ms L invited students to draw during Veggie Patch whereas the other two tasks did not use drawing. These may have been factors that assisted Riley during Veggie Patch. For the remaining two conceptual tasks, the cognitive demand appeared to be too high for Sophie and Riley. I will now examine their responses to post-lesson assessment tasks in order to explore their understanding of multiplication concepts.

The extent to which Sophie and Riley attended to the concepts of multiplication can be examined through their responses to post-lesson assessment tasks. I designed these tasks to address the concepts covered in observed lessons and also to provide opportunities for Sophie and Riley to demonstrate their solution strategies for multi-digit multiplication. Figure 14 summarises the assessment tasks given to Sophie and Riley after each lesson. Sophie was absent for assessment task 4.

Assessment task 1. Fish task If fish B eats twice as much as fish A and fish C eats three times as much as fish A, how much will each fish eat when A eats x, when B eats x etc.	**Assessment task 2.** Tell me a story about 6 × 3.	**Assessment task 3.** Tell me a story or draw a picture to match 20 × 19. How might you solve this?
Assessment task 4. Use a strategy you saw today to solve 12 × 13 then tell a story to match. Strategies – grid method, partitioning or lattice method.	**Assessment task 5.** In which range of numbers would the answers to these questions be; 63 × 99, 72 × 11, 51 × 50 Possible ranges 8000 – 9000,	**Assessment task 6.** 14 × 16. Tell me a story for 14 times 16, find an answer using a written method and then using the grid paper to draw arrays.

	6000- 7000, 2000- 3000 700- 800 200- 300	

Figure 14. Post lesson assessment tasks for Sophie and Riley.

Two main observations came out of the results of these assessment tasks. First, both Sophie and Riley demonstrated improvement in their proficiency in using written processes to calculate multi-digit multiplication. Second, creating a multiplicative situation to match a multi-digit multiplication expression proved difficult for Sophie and Riley. I will now discuss each of these findings.

The focus of many of the observed lessons was investigating and practising three different written strategies for multi-digit multiplication. These were the lattice method, extended notation and the grid method (see Figure 15).

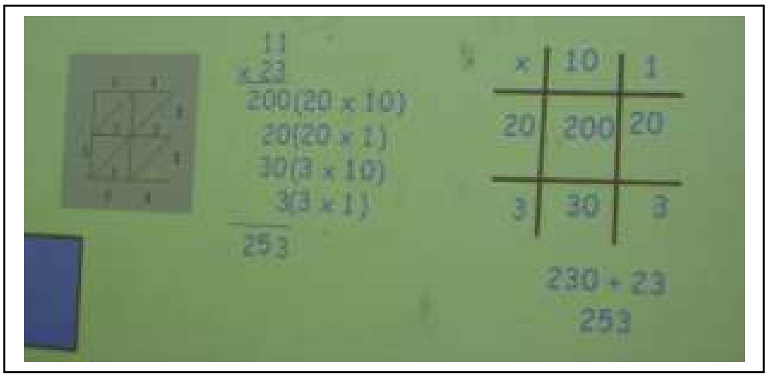


Figure 15. Three written processes for multi-digit multiplication.

I will now briefly review how proficiency in these processes was developed over the observed lessons. The first lesson observed that used a written process for multiplying saw Riley struggling to use a version of the traditional algorithm to calculate 9 × 87, which I have described previously. He was not successful in reaching a solution. In the same lesson, Sophie used repeated addition initially to calculate the same expression and then the “multiply by ten and take one set” strategy Ms L showed her, also described earlier in this chapter. The next lesson Riley attempted to show Ms L how he would calculate 45 × 26 again, using a version of the traditional algorithm. Ms L suggested he break the question down. Riley then struggled to answer 6 × 40 as he didn’t know 6 × 4. When 24 was established, Ms L then asked Riley to fill in the place value chart with 6 × 4, 6 × 40 and 6 × 400, as discussed previously. For the same question, Sophie had written 5 × 6 = 30, 40 × 20 =

80, 30 + 80 = 110, 10 × 40 = 400 on her whiteboard. In the final lesson of the observation period, Riley showed proficient use of the grid method (see Figure 16).

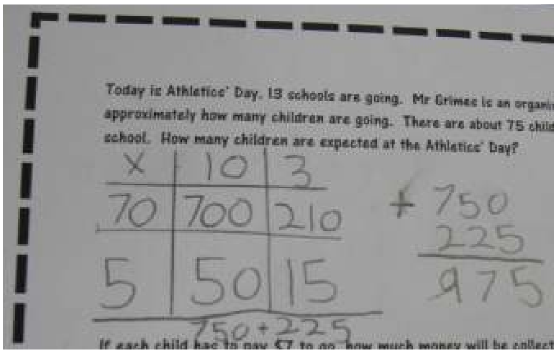


Figure 16. Riley's grid method solution to 13x75.

Similarly, Sophie demonstrated improvement in her use of written processes for multiplication shown in Figure 17.

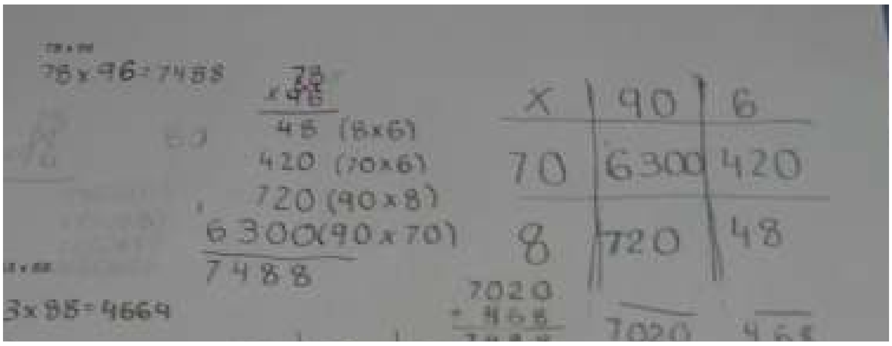


Figure 17. Sophie's written solutions using extended notation and grid method.

Results from the post-lesson assessment task also showed progress, with Sophie and Riley's use of written procedures for multiplying. Part of assessment task 3 required Sophie and Riley to calculate 20 × 19 using any method they chose. As reported earlier, Riley wrote 20 × 19 vertically then multiplied 0 × 9 in the ones place and 1 × 2 in the tens place to give an answer of 20. Riley recognised this answer could not be correct and commented, "no that can't be right. I had 20 to start before I multiplied by 19." Sophie calculated 20 × 19 by saying "20 × 10 is 200, 0 × 9 is 0, 20 × 9 is 1800 add these so the answer is 2000." Clearly in both these cases, Riley and Sophie did not have accurate written methods for multiplication. In assessment task 4, Riley used the lattice method and with some assistance was able to calculate 12 × 13. Sophie was absent for this lesson. For the final assessment task Riley and Sophie were asked to calculate 14 × 16. Riley did this successfully using the grid method and Sophie was successful using the extended notation method. The results of these tasks supports the results from the lessons; that Sophie and Riley progressed from having no reliable written method to calculate multi-digit multiplication to confidently using at least one method with accuracy. Both Sophie and Riley expressed their sense of satisfaction with this progress.

> Sophie: Multiplication was easy and I got both the answers correct … My multiplication is helping me get it right. It's fun and it's really easy. It's easy because all you have to do if you multiply big numbers by big numbers is multiply it and if it's a big number, all you have to do is multiply the two small numbers then times it by 10 or 100 it's making it 10 times or 100 times bigger. It's moving along the place value.
> (Post-lesson interview, Athletics Day)

> Riley: I learnt how to do it, use the grid with 3 digit numbers. I asked Ms L and she gave me a hand with making the grid longer. I got it straight away, quicker than I usually do it.
> (Post-lesson interview, Athletics Day)

This positive affective response should not be underestimated. Sophie and Riley clearly knew, and were proud of the fact, that they could now do something that they had previously been unable to do. Carl and David in Ms B's class did not express this level of satisfaction probably due to the nature of their progress, which was in understanding rather than processes, and therefore more minimal and less obvious (Carpenter & Lehrer, 1999).

The second part of most of the post-lesson assessment tasks asked Sophie and Riley to describe a situation to match a multi-digit multiplication expression. Assessment task 2 asked Sophie and Riley to "tell me a story about 6 times 3 is 18". Sophie was able to tell a story to match the expression saying "A girl had 6 marbles and she got 3 times the amount for her birthday. How many did she have? 18". Similarly Riley was successful saying "There was 6 cars then 6 other cars came to the car shop then 3 weeks later 6 other cars came. At the end of the week there were 18 cars at the shop". His insertion of "3 weeks later", probably taken from the 3 in the question, perhaps indicated a little confusion but nevertheless, his story was multiplicative. Assessment task 3 asked Sophie and Riley to match a situation to the expression "20 × 19". Sophie showed some confusion saying, "There were 300 seats. There were 20 children in each grade and 19 teachers. How many seats were left over?" This response potentially showed multiplication but this is not clear. Riley could not answer this question and said, "I don't have a story, I don't know". Riley responded to assessment task 4, tell a story to match 12 × 13, with "There was 12 bears. Two weeks later another 12 came. Four weeks later, another bear came. Four weeks later a panda came. Ten weeks later 5 bears came then another two weeks later 3 bears came. This makes 156 bears". Sophie was absent for this task. For the final assessment task for 14 × 16 Sophie said, "Amelia went to the shop to buy some food for a party. The chips were all $14 and the chocolate was $16. How much money did she spend altogether?"

Clearly these examples showed that Sophie and Riley were unable to match a situation to a multi-digit multiplication expression, although they were successful with single digits. This confusion with larger numbers was identified by Lampert (1986) who stated that "when larger numbers come into play it is less likely that children will see multiplication as simply a way of counting the total number of objects in situations where those objects are arranged in groups and all of the groups contain the same number of objects" (p. 315). However Riley's response to this final task showed improvement. As reported earlier, in response to "tell me a story about 14 × 16", Riley said

> Um … there were 224 people … sorry I made a mistake. These people were trying to think how they could get 224 people onto the boat. They were seeing how they could fit. The people counted … they tried using the space then they did … they times … they knew there was 14 people in 16 groups. Each group had 14 people and they found out that … they timesed [*sic*] it all so they knew there was 224 so they could go onto the boat.
> (Assessment task 6)

The ability to relate a multiplicative situation to an expression assesses understanding of what it means to multiply rather than completing written processes accurately. Sophie was unable at the end of the observation period to describe a multiplicative situation to match a multi-digit multiplication expression and gave an additive situation instead. Riley was able to devise an appropriate multiplicative situation for the final assessment task, something he was unable to do in previous tasks. Though his response was a little confused and hesitant, it demonstrated at least an "on the way" understanding of what multiplying larger numbers means.

In summary, Sophie and Riley showed improvement in their computational knowledge of multiplication (Lampert, 1986). Riley demonstrated a growing awareness of what it means to multiply by multi-digit numbers through his ability to tell a multiplicative story. The improvement in Sophie and Riley's computational proficiency was probably due, at least in part, to the time given to developing written processes during lessons with more time per lesson, an average of 18 minutes, devoted to practising procedures compared to time given to exploring concepts, with an average of 12 minutes. Ms L did not attend to the underlying concepts of multiplication as much as to completing accurate written processes. When she did set tasks that focussed on concepts, these were shorter, whole class tasks, completed alone and without the use of manipulative materials or visual representations. Sophie in particular reported feeling positive about her progress in being able to correctly complete multiplication questions using a written process.

CONCLUSION

Ms B planned all eight tasks to focus on underlying concepts of fractions, decimal numbers and percent. This focus was maintained throughout implementation with no processes being taught and no tasks adjusted to lower the cognitive demand for any students. The reasons why Ms B taught this way could be traced to her assertion in interviews that she was "gung ho" about students "knowing why things work". In addition, Ms B's own experience as a student being told by a teacher that she "didn't need to know why" might have influenced Ms B's teaching. It is possible that Ms B perceived that good mathematics teachers should "not tell" (Smith, 1996) students information in mathematics. Her comment about assisting David during fractions rods might also support this theory when she said, "I questioned him but he did them". However, although Ms B did not specifically tell Carl or David information, she often narrowed down her questions or provided increasingly pointed hints so that the students themselves "filled the slots" (Bliss et al., 1996) with the correct responses. There were also instances during whole class discussions when Ms B did not tell students information but preferred to either draw the discussion out, sometimes providing hints, until a student contributed the correct information. Alternatively, as in the case of the students' assertion that there are only ten decimal numbers between two whole numbers, Ms B chose to end discussions with some information not relayed at all.

Ms L planned all eight tasks to focus on the concept of multi-digit multiplication but in five of these tasks, during implementation, the focus became developing written processes for multiplying without making connections to underlying concepts. The level of cognitive demand could vary across Ms L's class with some students working on higher cognitive tasks while others completed tasks at a lower cognitive level. Again, the reasons why Ms L so often reduced the level of cognitive demand are mostly speculative. It is possible that Ms L believed that she was not in fact lowering the demand of tasks. Ms L named the various written procedures that students practised as "strategies". Her aim that student explore many "strategies" could therefore be seen to be fulfilled. In addition, interview data revealed that Ms L was more focused on student affect than was Ms B. Ms L often said that she wanted students to be "confident" and to "enjoy maths". Perhaps Ms L "smoothed the path" (Watson, 2002) by reducing the cognitive level of tasks in her attempt to ensure that students felt confident. Finally, Doyle (1988) pointed out that novel, cognitively challenging tasks have a "bumpy workflow" whereas more routine tasks, such as written processes completed mostly alone as occurred in Ms L's class, were smoother and more orderly for teachers. It was observed that Ms L's classroom was usually quiet and ordered and that Ms L often reminded students of the need to work quietly, so this might have been a factor in lowering the cognitive demand of tasks.

For the low-attaining target students, these variations meant each had a different experience in their mathematics classroom. For Carl and David in Ms B's class, the level of cognitive demand was too high on five to six out of the eight tasks. It would appear that Ms B had not chosen appropriate tasks that built on the prior knowledge of Carl and David, a main factor in maintaining the level of cognitively demanding tasks throughout implementation (Henningsen & Stein, 1997). When the materials in the task did not appear to scaffold Carl and David's understanding of the concepts, they tended to disengage from the task as peers were also ineffective in providing scaffolding. Ms B was a source of scaffolding for Carl and David but her habit of trying to talk to every student every lesson meant that she spent three to five minutes on average with Carl and David per lesson, out of a typical lesson of 82 minutes. This meant that there was a lot of time when Carl and David were unable to continue in the task as they did not have the necessary prior knowledge, and without assistance from Ms B, engaged in off-task behaviour instead. This is a limitation of scaffolding in a classroom setting where the teacher is responsible for many students and is unable to spend large amounts of time with low-attaining students, even if this time was spent in short exchanges where the teacher steps in and out. The two or three occasions that were most successful in drawing attention to underlying concepts for

Carl and David were when the materials used were concrete embodiments of the mathematical concept or Ms B engaged in scaffolding conversations with them during the task.

The tasks in Ms L's class were often lowered in cognitive demand through implementation, sometimes for the whole class, sometimes for Riley and less often, for Sophie. Of the three tasks that maintained focus on concepts, one was successful for Riley whilst the demand was too high for both Riley and Sophie on the other two tasks. On all the main learning activity tasks, the longest tasks in the lesson, Riley was unsuccessful in completing any of these tasks while Sophie reached an answer for all of these tasks. Both Sophie and Riley demonstrated considerable improvement in their procedural proficiency with written processes for multi-digit multiplication. However, as Ambrose (2002) warned, the use of standard algorithms could mean students operated on "automatic pilot" and did not reflect on underlying concepts. For example, Riley could devise a multiplicative situation to match a multi-digit multiplication expression by the end of the observation period but Sophie was unable to do this. This appeared to be the key to whether or not Sophie and Riley could attend to the concepts that underpinned multi-digit multiplication.

Focussing on concepts is an essential part of teaching mathematics for understanding. Ms B focussed exclusively on concepts and yet for about half the observed lessons, Carl and David engaged largely in non-mathematical behaviour due to the mismatch between the task expectations and their level of understanding, and their lack of regular interaction with the teacher. Ms L focussed much less on concepts and more on procedural proficiency and Sophie and Riley demonstrated distinct progress in this area. For Carl and David, materials that were concrete embodiments appeared beneficial. Sophie and Riley did not use materials at all in their lessons so the potential benefits for them cannot be tested.

In more than half the observed tasks the level of thinking for the low-attaining students in both classrooms declined to the level of completing procedures without understanding underlying concepts or to non-mathematical behaviour. The time that each teacher spent on tasks appeared to have had an impact on the level of thinking that the low-attaining target students demonstrated. Seven out of eight tasks in Ms B's class were more than 40 minutes long whereas only two out of eight tasks in Ms L's class were longer than 40 minutes long. Henningsen and Stein (1997) found that "the one major factor that occurred across the three types of decline was inappropriate amounts of time" (p. 538). They found that when tasks declined into using procedures without connections to concepts, there was often "too little time for students to grapple with the important mathematical ideas contained in the task" (p. 537). This resonates with the observations of Riley and Sophie in Ms L's class in which often tasks became focussed on completing procedures rather than exploring the potentially richer elements of the tasks. As discussed, Ms L completed as many as four tasks per lesson, which averaged a total of 70 minutes long. All of the "learning activity" tasks that comprised most of these lessons averaging 18 minutes, declined into procedures without connections. Perhaps the time Ms L allocated to such tasks was not sufficient to allow students to explore the concepts.

In contrast, Ms B's lessons that were on average 82 minutes long, tended to include two tasks at most. However, Henningsen and Stein (1997) also found that too much time was a factor in students declining in their mathematical thinking. In particular, tasks where thinking declined into "no mathematical activity" were often judged to be too long. "When students are observed not to be making discernable headway toward constructing and understanding key ideas, additional time by itself (i.e. without the introduction of additional support factors) appears to exacerbate the situation" (p. 537). This often appeared to be the case with Carl and David.

If understanding is as halting and erratic, with back and forth characteristics as described by Hiebert, Taber and Wearne (1991), then growth in understanding over a two or three week period is likely to be minimal. Progress in written procedures may be quicker and is more readily observable due to their written nature. Both procedural proficiency and conceptual understanding are important in mathematics learning however, as Eisenhart et al. (1993) pointed out, often it is conceptual understanding that "falls through the cracks" and the focus becomes mainly on procedural proficiency as was the case in Ms L's classroom. However, Ms B's reluctance to lower the cognitive demand of any tasks resulted in Carl and David spending significant amounts of time engaged in non-mathematical activity.

This study in part involved teachers' use of particular types of mathematics tasks examined in the larger project, TTML (Peled et al., 2009). In the following section I will discuss data regarding the TTML task types. To follow, the three scaffolding practices in the classrooms of Ms B and Ms L will be examined together as a summary of the previous sections of this chapter.

THE THREE SCAFFOLDING PRACTICES IN MS B AND MS L'S CLASSROOMS

The mathematics classrooms of Ms B and Ms L differed in organisational aspects. For instance, Ms B had one to two tasks per lesson whereas Ms L had two to four tasks per lesson. Consequently the time spent on tasks also differed with the average time per task in Ms B's lessons being 54 minutes while in Ms L's class, the average time per task was 26 minutes. The primary goals for the observed lessons also differed, with Ms B focussing on building conceptual understanding and once briefly mentioning a procedure, while in Ms L's class the longest tasks were focussed on developing procedural proficiency with shorter tasks directed to conceptual understanding. There were also similarities between the classrooms such as the overall time spent on mathematics lessons, which was 70 to 80 minutes, and the overall format of these lessons that followed the "whole class, small group, whole class" approach advised in curriculum documents at the time (Department of Education and Early Childhood Development, 2010). Examining the three scaffolding practices of this study in Ms B and Ms L's classroom also revealed similarities and differences, beginning with those related to discussion, then manipulative materials and visual representations, and finally explicit attention to concepts. I will now discuss these.

Ms B and Ms L conducted whole class discussions at the beginning and end of each observed lesson. Beginning discussions averaged 6 minutes long for the 8 tasks observed in Ms B's room and 4 minutes over the 8 tasks observed in Ms L's room. Ms B used these discussions to give brief instructions about how to complete the task or to provide a demonstration of the task or the representation to be used. Ms L used beginning whole class discussions in similar ways but also defined key terms, or gave global overview statements or an "advance organiser" (Engelmann & Carnine, 1982) about the task in relation to the class' unit of work or the wider world. The four target low-attaining students usually appeared relatively attentive during these introductory discussions though rarely contributed.

End of task whole class discussions were longer than beginning discussions in Ms B's room, as five out of the eight tasks had concluding whole class discussion of 10 minutes or more. During these discussions, Ms B drew extensively on student contributions about strategies and solutions. These whole class discussions were characterised by quite lengthy and at times potentially confusing exchanges between

Ms B and students as they attempted to explain their thinking. For large parts of half of these whole class discussions Carl and David in Ms B's room were observed engaged in off-task behaviour. Each made one contribution in total during the eight concluding discussions observed. Perhaps, as Baxter et al. (2002) suggested, the difficulties that particularly low-attaining students may have in interpreting other students' contributions, could have led to this disengagement by Carl and David. These whole class discussions may have in fact become for Carl and David, "a stimulus for confusion" as described by Ball (1993).

In Ms L's class, end of task whole class discussions were an average of 7 minutes long, shorter than those in Ms B's class. The longest concluding discussion in Ms L's class was 11 minutes long. Ms L's purpose for these discussions differed from Ms B's purpose in that student contributions did not feature as heavily and Ms L did much of the talking. Ms L reviewed the lesson, occasionally asked students about their feelings regarding the lesson using the "traffic lights" protocol, pointed to future directions or lessons and briefly asked some students to share their solutions. Sophie and Riley usually appeared attentive. Sophie was not observed contributing to these end-of-task discussions while Riley contributed twice over the course of the six lessons. Perhaps the shorter time for these whole class discussions contributed to Sophie and Riley's ability to be more attentive than was observed with Carl and David. However, it is difficult to conclude whether Sophie and Riley were listening or simply waiting quietly for the discussion to end. In addition, it has been suggested that discussions that draw heavily on student contributions, as was the case in Ms B's concluding discussions, can be confusing and difficult for low-attaining student to follow (Baxter et al., 2002; Burns & Myhill, 2004). Perhaps Sophie and Riley found their more teacher-directed discussions easier to follow than did Carl and David, in whose class student contributions were emphasised more.

It would appear that in both classrooms whole class discussions were generally ineffective for scaffolding the learning of the low-attaining target students, as Baxter and colleagues (2001; Baxter et al., 2002) also found. However, Hiebert and Wearne (1993) found that increased time on whole class discussions was a characteristic of the classes that performed best in their study for improving student learning. In the present study, the data showed that all four target students contributed rarely to such discussions with most of their contributions being four words or less. The target students in both classrooms were observed engaging in off-task behaviour during these discussions, particularly Carl and David in Ms B's room. Data revealed that Carl and David could not answer questions about specific content discussed during whole class discussions in post-lesson assessment tasks. As discussed previously, specific content was not the focus of whole class discussions in Ms L's class so was not possible to find comparable data for Sophie and Riley regarding their ability to recall such content. Considerable time was spent on these whole class discussions in each classroom, averaging 17 minutes out of 82 minutes in Ms B's room and 12 minutes out of 70 minutes in Ms L's class. As such, Ms B's class were engaged in whole class discussion for about 25 percent of each lesson, in Ms L's room this was about 20 percent of the lesson. This meant that a large proportion of each lesson might have been ineffective for scaffolding the learning of the low-attaining students.

Individual conversations that occurred in both classrooms between the teachers and individual target students appeared to be more effective in scaffolding learning than whole class discussions. In addition, these scaffolding conversations appeared most successful when the teachers stepped in and out of the discussion with the students and the conversation occurred in "strings" (Cheeseman, 2009) over the course of a lesson or task. This practice would also concur with an important element of scaffolding; that support is temporary and adjustable (Rosenshine & Meister, 1992). For example, Ms L's conversation with Sophie about 87 times 9 occurred in strings where she allowed Sophie to explore her inefficient strategy before returning to suggest a more efficient strategy of multiplying by ten and taking one set. Ms B's conversation with David at the beginning of "Fraction Rods" also illustrated the effectiveness of beginning the conversation, leaving David to do some thinking or work independently and returning to discuss his strategies and thinking. This same conversation became less effective when Ms B stopped "stepping out". This resulted in Ms B leading David to answers by the end of the exchange. Conversely, Ms L stopped "stepping in" during three conversations with Riley when she did not return to Riley to continue these scaffolding conversations. In these cases, Riley could not continue with the task. Nevertheless, the students viewed individual conversations with their teacher favourably which was also found by Sullivan et al. (2006). Three of the four target students spoke specifically about these conversations with their teachers in a positive way.

A point of difference between the scaffolding practices of Ms B and Ms L arose with the use of manipulative materials and visual representations to scaffold learning. While Ms B used manipulative materials or representations in every observed lesson, Ms L did not use manipulative materials at all. Ms L's use of the array as a visual representation occurred mainly mentally with no physical materials used, but students asked to visualise arrays. Ms B reported finding it difficult to think of tasks for fractions, decimal numbers and percent that did not include the use of materials. In addition, she stated that such use was important for teaching these concepts. In contrast, Ms L did not appear to be aware of what materials or representations might be effective for teaching multi-digit multiplication. When shown some examples of possible visual representations, Ms L did not use these and stated that she felt such use might be confusing for students. The content that each teacher taught during the observation period could be a contributing factor to the use or lack of use of manipulative materials and visual representations. It may be that teachers are more familiar with appropriate materials and representations to use for teaching rational number but not as aware of materials and representations to teach multi-digit multiplication.

There is some evidence in this study that Ms L held the belief that materials and representations were not appropriate for upper primary school aged students, as also evident in data from Howard and Perry (1997) and Swan and Marshall (2010). However, it seemed Ms L might have believed such use was appropriate for low-attaining students. This appeared evident in that Ms L suggested the use of arrays to Riley, but was not observed suggesting this to Sophie or other students. In addition, although Ms L said there were materials available, these were not visible in the classroom and no students were observed using materials. This was despite Ms L instructing students during one lesson to use materials "if you need to".

The type of materials used arose as an important finding for the learning of Carl and David in Ms B's class. Materials or representations that were concrete embodiments (Boulton-Lewis & Halford, 1992) of mathematical concepts were effective in scaffolding Carl and David's understandings of fractions and decimals. Materials or representations that aimed to activate or connect prior knowledge were less effective, as they did not illustrate the concepts but were more abstract symbols or mathematical notations. Without the prior knowledge to be activated or connected, Carl and David required other scaffolding to be available in order to learn from tasks that used materials in this way.

The ways in which Ms B and Ms L paid explicit attention to concepts also differed. Ms B planned tasks that explored underlying concepts for all the observed lessons and was not observed lowering the cognitive demands of tasks. Ms B used manipulative materials and visual representations to focus on concepts such as the relative size of fractions and decimal parts for example. Ms B asked students to explain their thinking and strategies often asking “why?” Ms L planned shorter tasks that focussed on concepts and did not lower the demand of these tasks. The main tasks in Ms L’s classroom, the longest task of the lesson, were often planned as tasks with a high cognitive demand but throughout implementation became focussed on the development of written procedures for multi-digit multiplication. This was described by Henningsen and Stein (1997) as “decline into using procedures without connection to concepts, meaning or understanding” (p. 535).

The effect on the target students in these classrooms also differed. In Ms B’s room, Carl and David were consistently expected to work on the task at the level it was presented. This meant that the level of cognitive demand was too high for Carl and David to access except for two tasks. Without appropriate scaffolding for these tasks, Carl and David were often observed engaging in non-mathematical behaviour. In Ms L’s class, Riley and Sophie were mainly unsuccessful during the shorter conceptual tasks but did demonstrate considerable improvement in their ability to use a written procedure to calculate multi-digit multiplication, which was the focus for longer tasks. Sophie was not able to describe a situation that matched a multi-digit multiplication expression by the end of the observed lessons while Riley was able to do this on the final assessment task. It would appear that Sophie did not yet understand this underlying concept and Riley might have been just beginning to grasp this concept by the end of the observation period. Both Ms B and Ms L appeared to struggle to support the learning of the target students when tasks remained focussed on concepts. These findings suggested that tasks which did not build on student’s prior knowledge and understanding, and were beyond the target students’ “zone of proximal development” (Vygotsky, 1978) were counter productive. In these cases, it is likely that no amount of scaffolding would be effective as the task demand was simply too high. This also suggests that teachers might need support in how to maintain a high, but appropriate, level of cognitive demand for tasks that build on students’ prior knowledge.

In Chapter 2, I proposed that each of the scaffolding practices of focus in this study could overlap with “explicit attention to concepts” existing only as an overlap of one or both of the other two scaffolding practices. There were instances of discussion that occurred without attention to concepts such as Ms L’s discussions regarding procedures for multi-digit multiplication. There was also data from both teachers that showed they used discussion to draw attention to concepts. There were no instances of using manipulative materials and visual representations that did not explicitly draw attention to concepts such as for answer-getting only or procedures. There were episodes in which Ms B used manipulative materials or visual representations to draw attention to concepts with minimal discussion between the teacher and target students. Instances in which the teachers used both discussion and manipulative materials and visual representations to draw attention to concepts, had two examples from this study occurring during Fraction Rods and Fractured Figures in Ms B’s classroom.

Explicit attention to concepts has emerged as the most important scaffolding practice examined in this study. Without drawing attention to concepts, the other scaffolding practices were not enough to build conceptual understanding for the target students. In addition, the two most effective scaffolding practices that emerged, scaffolding conversations and materials and representations that illustrate concepts, occur within the “overlap” between the other two scaffolding practices. In many ways this is not a surprising result remembering Hiebert and Grouws (2007) assertion that, in order to learn with understanding, teachers and students must pay explicit attention to concepts. The findings of the present study appear to support this assertion.

A REFLECTION ON TTML TASK TYPES AND THIS STUDY

It is difficult to comment on the effect each of the TTML task types had on the learning and feelings of the four low-attaining students in this study due to factors occurring in Ms B and Ms L’s classrooms when these task types were implemented. Ms B planned and implemented mainly tasks using tools, models and representations, with only one contextualised task and only one open-ended task. Ms L planned two tasks using materials or representations, three contextualised tasks and three open-ended tasks. However, during implementation in Ms L’s class, the tasks did not always remain true to their type. For example, some tasks did not use physical materials or representations but students were asked to visualise these instead. For the two contextualised tasks, one became practising written procedures although one, Veggie Patch, remained focussed on the context of exploring real world examples of arrays. Two of the three open-ended tasks remained open during implementation while the other was changed during implementation to practising written procedures. These issues represent a limitation of case studies. By intending to observe the case as it occurred "naturally" with minimal influence as a researcher (Stake, 1995), I did not attempt to change the factors that led to the lack of data on the task types of TTML for this study.

The types of tasks that the four low-attaining students experienced in their mathematics lessons did appear to have an impact on their learning and affective responses, though the task type categories from the TTML project seemed not as significant as other aspects of the tasks. For example, although Carl and David experienced mostly tasks categorised as "tasks that use models, tools, and representations", their responses to these tasks differed. It seemed that the type of materials or representations used and the purpose for such use had more impact than the type of task *per se*. So, while both Sorting Cards and Fractured Figures incorporated the use of representations of percent, decimal numbers or fractions, the response of Carl and David to these two tasks differed. As described earlier, it was tasks in which the materials or representations were concrete embodiments of the mathematical concepts rather than symbols or formal notations, which had the most positive impact on the learning and feelings of Carl and David.

For Riley and Sophie, there was a more equal mix of the TTML task types planned by Ms L. However, due to the changes that often occurred during implementation, many of these tasks did not remain true to the original task type as planned. The data show that the two open-ended tasks that remained open during implementation in Ms L's class, Missing Numbers and Target Multiplication, were also two of the tasks in which both Sophie and Riley were unsuccessful. Riley's success during Veggie Patch, a contextualised task, might suggest that these types of tasks could have been effective for Riley. The only other evidence for this was the comment by Ms L about Riley's success during a lesson not observed that used the real world context of fitting spectators into a football ground.

The target students themselves showed preferences for certain aspects of tasks but these did not necessarily align with the features of the TTML task types. Data regarding the low-attaining students' task preferences were drawn from post-lesson interviews using the "Emoticons", the PPELEM task (McDonough, 2002b) and the Star Rating task that asked students to consider all the tasks observed and indicate the task they enjoyed most, the task they enjoyed second most, the task they believe they learnt the most from and the task they enjoyed least. These data collection tools were described in Chapter 3.

Carl particularly enjoyed tasks involving manipulating materials, but did not like all tasks that used materials or representations. On the "Star Rating" data collection task, Carl wrote that working with a group or a partner were the reasons why he liked the Decimat task the most and also why he believed he learnt most from this same task. David appeared to most enjoy tasks where Ms B spent time with him or when he worked in a group of students and the workload was shared. David wrote that he learnt maths well when "somebody explans [*sic*] it to me like a teacher" (PPELEM task). Similarly to Carl, David wrote that Fruit and Veggies was his favourite task and the task he believed he learnt most from because he "got to work with my best friend" (Star Rating task). Both Carl and David responded negatively, indicated during post-lesson interviews, to tasks when they spent large amounts of time engaged in non-mathematical behaviour or tasks where their partner took over the task for them. David also responded negatively to a task during which Ms B spent 12 minutes talking to him and his partner. These data did not appear to fit with other data that suggested David enjoyed and was most successful on tasks where Ms B spoke directly to him. In the post-lesson interview, David said he believed Ms B was talking only to his partner during this time, although David drew on the conversation in the post-lesson assessment task.

In Ms L's class, Riley and Sophie reported their feelings of satisfaction that they were able to use a written process for multi-digit multiplication. On the Star Rating task, Riley wrote, "I had a try of the grid method and I liked it" as the reason for his favourite task which was also the task he believed he learnt most from. Riley was reasonably successful during the Veggie Patch task which used a real world context. He was able to explain his strategy to the class of finding multiple solutions by doubling and halving. This explanation was the longest utterance of any of the low-attaining students in this study during a whole class discussion. This was the only task in which Riley was successful when the cognitive challenge was maintained. Interestingly, Riley rated Veggie Patch as the task he enjoyed the least because "it was a little bit too easy for me" (Star Rating task). Sophie rated Helping Julia as the task she enjoyed the most. During the next lesson, Sophie spontaneously spoke to me about how she was feeling about mathematics saying, "I'm getting really good at Maths now. My Mum's really proud of me. I told her how I got all of my Helping Julia right yesterday" (Athletics Day, lesson observation). Clearly, "getting it right" was important to Sophie and she enjoyed tasks where she had reached a correct solution. Sophie's proficiency in using the grid method and extended notation was developed by the tasks that Ms L implemented because, as discussed, most tasks became focussed on practicing the use of these procedures. Sophie responded positively to this aspect of the tasks in Ms L's class.

To summarise, the particular task types from the TTML did not appear to impact on the responses of the low-attaining target students *per se.* While Ms B used mainly tasks that used materials, tools and representations, Ms L planned a mix of the TTML task types. However, during implementation of these tasks by Ms L, the type of task was sometimes altered, that is the "intended" task differed from the "implemented" task (Robitaille et al., 1993). The TTML task types still provided an important context for this study as they prompted teachers to plan cognitively challenging types of tasks for students, including the target students. In this way, the use of tasks intended for cognitive challenge with low-attaining students could be examined in the present study, thus responding to a clear gap in the research literature.

CONCLUSION

The experiences of these low-attaining students can be interpreted as Dole (2003) described, being "both alarming and positive" (p. 6). Affectively, the target students appeared usually positive about their mathematics lessons and their teachers. In lessons when Carl and David completed little mathematics, they did not view these lessons positively. They appeared at least somewhat aware that they had not achieved success mathematically and were therefore not satisfied despite appearing to enjoy themselves. These students indicated perhaps, that understanding remains the most motivating aspect of classroom experiences (Hiebert et al., 1997; Lambdin, 2003), as Carl and David viewed most positively those lessons where understanding could be seen to be progressing. Riley and Sophie clearly knew that they were able to do something at the end of the observation period that they were not able to do at the beginning, in using a written procedure to calculate multi-digit multiplication. This was a source of satisfaction for them.

Cognitively, the target students all showed some progress over the two to three weeks of observations. For Carl and David this progress appeared as gradual and small gains in understanding. Sophie and Riley showed clear, and in some ways considerable progress from having no reliable way to calculate multi-digit multiplication to having at least one accurate method. The progress of the target students from each classroom would appear to represent two of the strands of mathematical proficiency described by Kilpatrick et al. (2001) as "conceptual understanding" and "procedural fluency" (p. 116). Kilpatrick et al. stated that these strands, and the three other components of mathematical proficiency, are "interwoven and interdependent" and that "mathematical proficiency … cannot be achieved by focussing on just one or two of these strands" (p. 116). Thus the procedural proficiency developed by Sophie and Riley would be an important part of their overall mathematical development, providing conceptual understanding grew from this. Conversely developing proficiency in procedures might have supported even the relatively small increments in understanding demonstrated by Carl and David.

In some ways, the target students in each classroom showed progress in the very area of mathematics that was their teachers' primary goal for mathematics learning. In the pre-observation interview, Ms B talked about understanding "why things work" (initial teacher interview), and then planned and implemented tasks aimed at building understanding, often using materials and representations. It also appeared that she was prepared to hold back and wait for students to build this understanding. Correspondingly, Carl and David were gradually building understanding in small increments over the course of the observed lessons.

In the pre-observation interview, Ms L talked about a recent change in her mathematics teaching focus from "putting sums on the board" and having students "do it" to more of an emphasis on understanding which Ms L described as students "explaining, proving, doing it a different way" (initial teacher interview). The students in Ms L's class were exploring three different written procedures for multiplying multi-digit multiplication from which they were encouraged to find the one they were most "comfortable" in using. They were observed explaining the steps they had taken in the procedure to reach a solution, and they compared briefly various written procedures. The concepts of multi-digit multiplication, such as real world applications, arrays and estimation, were offered, almost as an aside, during shorter "tuning in" tasks at the beginning of lessons. None of these occurrences contradicted Ms L's stated goal for her mathematics teaching considering her description of "understanding". The students were explaining, using multiple strategies and making choices about which procedure to use. Whether they were also building understanding of the concepts of multi-digit multiplication might be debated, particularly given that Riley and Sophie were mostly unsure about relating an appropriate multiplicative situation to a multi-digit multiplication expression.

It appeared that Ms L had taken her previous mathematics teaching practice of "putting sums on the board" and overlaid it with her new focus on "understanding". She retained her emphasis on students being able to "do it" via procedures, but assimilated this with some new practices in which students were asked to explain the steps in procedures, had an element of choice and were able to use more than one process to solve an expression. Ms L also focussed for short parts of the lessons on more conceptual tasks, which would appear to represent a change from her previously stated practice. Cohen (1990) described this overlap between previous beliefs and practices and new beliefs and ways of teaching mathematics saying "as teachers try to find their way from familiar practices to new ones, they cobble new ideas on to familiar practices" (p. 312). Cohen posited that this was an inevitable stage of teacher change.

Ms L taught using a mix of conceptual tasks and tasks focussed on practising her chosen written procedures for multi-digit multiplication. On the surface, it would appear that Ms L's approach was more congruent than was Ms B's, with the advice of Kilpatrick et al. (2001) that teachers use a variety of teaching approaches including explicit, focussed instruction on specific mathematics concepts, as well as open, problem solving tasks. On closer analysis, Ms L's focussed instruction was mainly on carrying out teacher-mandated written procedures rather than on mathematics concepts. Open, problem solving tasks were used but, as I have described, did not always remain open or problematic for students throughout implementation.

It is interesting to consider the teaching of Ms B and Ms L in light of the advice from the literature that a "balanced" (Ellis, 2005) or "variety" (Kilpatrick et al., 2001) of teaching approaches is most effective for low-attaining students. It would seem neither Ms B nor Ms L were entirely successful in teaching Carl, David, Sophie or Riley, though the nature of "success" over a three week period would be difficult to describe. Ms B appeared to take a largely constructivist approach to teaching Carl and David in that she offered tasks and materials that were designed to assist student to build their own understanding of rational number. She did not teach procedures or suggest strategies that students then practised. Strategies came from the students themselves. Ms B did not change tasks to lower cognitive demand. Ms B expected students to explain and articulate their thinking. Carl and David demonstrated minimal progress in their understanding and often the tasks appeared too difficult for them. It would appear that Ellis' (2005) warning, that tenets of constructivism are incompatible with the learning needs of low-attaining students, might be true for Carl and David, at least to some extent. While a "balanced" and "variety" of approaches has been recommended for teaching low-attaining students in mathematics, the complexity of what such a "balance" might look like in a classroom remains, as this study illustrated.

The picture of two mathematics classrooms that emerged from this study was as complex as the "rich psychological soup of the classroom" described by Schuel (1996, p. 726). The teachers, Ms B and Ms L, appeared dedicated to supporting their students to learn mathematics. Each was concerned for the development of the target low-attaining students and was eager to see these students make progress. The ways in which these teachers scaffolded the learning of the target students varied, with Ms B relying on manipulative materials and visual representations to build understanding and Ms L focussed on students developing proficiency in at least one of a range of written procedures. As is the case with any research involving people and their social interactions, there were examples and counter-examples from the teachers and students of various behaviours at any given time. The "grey areas" of the study are, of course, characteristic of a case study. Humans don't behave the same way on every occasion. The patterns of behaviour that emerged almost always had at least one case where this pattern was contradicted, making Merriam's (1998) statement that qualitative researchers needed "tolerance to ambiguity" (p. 37) quite pertinent.

However, general patterns did emerge such as Ms B's consistent use of materials and representations, and maintenance of a high level of cognitive demand throughout tasks, and Ms L's focus on written procedures for multi-digit multiplication. Positive patterns that emerged for the target students were Carl and David's success with concrete embodiment types of materials and representations, and Riley and Sophie's success in using a written procedure for multiplication. Less positive was Carl and David's unproductive floundering for more than half the observed tasks, Riley not completing any of the longer tasks observed and Sophie remaining unable to relate a real world example to a multi-digit multiplication expression. The three forms of scaffolding of particular interest in this study were observed in both classrooms, although manipulative materials and visual representations only minimally in Ms L's class. Each of these practices appeared to hold promise for effectively scaffolding the learning of low-attaining mathematics students.

The discussion to follow will describe data collected from "postscript" interviews which occurred with Ms B, Ms L, Carl, David, Sophie and Riley one year after lesson observations concluded.

ONE YEAR ON – A POSTSCRIPT

As stated in Chapter 3, one year after the lesson observations ended in Ms B and Ms L's classrooms, I returned to interview the teachers and four low-attaining students. The purposes of these interviews was to check the validity of the findings from the observations, provide an opportunity for the key participants to comment on the scaffolding practices and task types that were the focus of this study, to gain insight into any changes in teaching and learning mathematics that might have occurred since the study, and to access any reflections participants might have had regarding the key issues of the study after the lesson observation period concluded.

One year on, Ms B was again teaching Years 5 and 6 at the same school although she no longer taught Carl and David. Ms L was teaching a Year 3 class at the same school as one year earlier, and as such did not teach Sophie and Riley. Carl, David, Sophie and Riley were now all in Year 6, their final year at primary school, attending the same schools as they did during the observation period.

The data from these "postscript" interviews with Ms B and Ms L will be discussed under the headings of the three scaffolding practices, teaching of low-attaining students and task types. The target student interview data will then be discussed, beginning with affective data followed by cognitive data.

Discussion

Ms B's stated purpose for the beginning and ending whole class discussions concurred with the purposes identified from observations a year earlier. Ms B stated that beginning discussions were to introduce the task, "hook" or engage students in the task or to provide a demonstration or model for students. End of lesson discussions, Ms B said were to "draw out learning or understanding", discuss strategies and for students to explain their solutions or thinking. Ms B thought that whole class discussions were important for low-attaining students as these students could be exposed to alternative or more sophisticated strategies than they had used themselves during such discussions, a view she also expressed the previous year. Ms B also thought that students explaining their strategies and thinking was important for both their own learning and that of their classmates. Ms B believed that students benefited from whole class discussions as they "clarified their thinking" and "cemented ideas just starting to form". She also acknowledged that some students "quietly ignored" these discussions and that low-attaining students were more likely to "tune out". Ms B thought that engaging tasks led to more engaging discussions and that concrete materials helped students to explain their thinking. These statements were interesting given that one of the key findings from my study suggested that whole class discussions were not effective for the target low-attaining students.

Ms B stated that her favourite question was "why?" which was also found in lesson observations the previous year in which Ms B asked "why?" an average of nine times per lesson. She said that her students knew that they would be required to explain and not just give an answer and they were now anticipating her "why" question and formulating an explanation before this was asked.

Turning now to Ms L, her stated purposes for beginning and ending whole class discussions also appeared the same as those purposes identified the year before. Ms L stated that she believed that the ending discussion "should be the most important time" because it served as a means for her to assess the students' understanding and therefore helped plan subsequent lessons. Ms L felt that students gained from these ending discussions as they had improved in the language they were able to use and their ability to elaborate. Also, these discussions allowed students who "felt great" about the lesson to "share" their success.

In terms of the benefits to the learning of low-attaining students of whole class discussions, in a similar way to Ms B, Ms L believed that these students benefited from “hearing from the higher attaining students”. The previous year Ms L had expressed similar beliefs about the importance of whole class discussions, particularly for low-attaining students. Ms L also recognised that low-attaining students could find these discussions difficult if they were too long or too “deep”. She believed that if the class had found something new and exciting, a “wow moment”, low-attaining students were more likely to remember it. She also thought that the features of the discussion could affect low-attaining students’ ability to access it, for example, students talking most, her talking most or using visuals. Of these, Ms L thought that visuals and students talking were most effective for low-attaining students because other students used “child speak” which was sometimes easier for other students to understand than the teacher’s way of explaining. This contrasts to findings from the literature, which suggests that low-attaining students in particular often find other students’ contributions confusing and difficult to follow (Baxter et al., 2001; Baxter et al., 2002; Lubienski, 2000a).

Ms B and Ms L continued to conduct whole class discussions at the beginning and end of lessons, reporting much the same time allocated to these discussions and similar purposes as those observed one year earlier. It was interesting that both teachers continued to express the belief that end of task whole class discussions were particularly beneficial for low-attaining students. The reasons these teachers gave for this belief were that it was important for low-attaining students to hear alternative or more sophisticated strategies than they had perhaps used or thought of and that other students could explain things using language that might be more accessible to low-attaining students than the teacher’s explanations. Affective benefits were that students could feel satisfaction about sharing successes. The belief that whole class discussions were beneficial for low-attaining students is in contrast to my finding, and the findings of others (Baxter et al., 2002; Burns & Myhill, 2004), that whole class discussions are largely ineffective for low-attaining students. Both teachers did note that low-attaining students could tend not to participate and could “tune out” of these discussions. Neither of the teachers described particular ways they scaffolded low-attaining students’ participation or ability to comprehend these discussions, though memorable findings or concrete materials were offered as being potentially beneficial. Both teachers were able to describe the benefits and purposes of whole class discussions and continued to structure lessons around the model used the year before of "whole class, small groups, whole class".

Ms B said that the individual discussions she had with low-attaining students during tasks were often quite lengthy. Ms B wanted to stay with students until she was satisfied that they were "off and away" and understood the task. The importance Ms B placed on these one-on-one interactions with students was a characteristic of Australian mathematics lessons also noted by Clarke (2003) who noticed "the thorough and extensive one-on-one interaction between the teacher and virtually every individual student at some point during … individual or collaborative mathematical seatwork" (p. 3).

Ms B acknowledged that these lengthy individual exchanges could be problematic as other students needed her help as well during this time. She also thought that she might need to "butt out" and allow students to work more independently. This resonates with my finding that scaffolding conversations were more effective if the teacher stepped out rather than remaining with the student for most of the task. In addition, I observed that Carl and David did need Ms B on occasions when she was spending extended time talking to other students. This was a major contributing factor to Carl and David's lack of success during more than half the tasks observed the year before.

Ms L believed that individual discussions with low-attaining students were more beneficial if the teacher spoke regularly to these students in short segments rather than an extended discussion. She felt that extended discussion caused low-attaining students to lose interest. Ms L thought that asking a question or being told something then having a period of time for students to "have a go" by themselves and have the teacher return later was most effective. Ms L was observed engaging in this practice during lesson observations the previous year. This description concurs with my finding that scaffolding conversations where the teacher "steps in and out" and the conversation occurs in "strings" (Cheeseman, 2009) over the lesson are more effective for learning than extended discussions in one block of time. However, there were also examples of when Ms L did not return to check Riley's progress, which led to his non-completion of the task.

I will now discuss Ms B and Ms L's responses to postscript interview questions regarding manipulative materials and visual representations.

One year on, Ms B appeared to have continued her practice of using concrete materials often during lessons. She also stated her belief that such use is "just as important" for upper primary students "as in the junior years". She reported that no students had showed resistance to using materials. Ms B continued to distribute materials to all students at the beginning of most tasks. She also spoke about how students were invited to choose the materials they would like or need to use from the Maths Trolley. Ms B gave students a "tour" of this trolley at the beginning of the year to highlight the materials and tools available. Ms B thought that her low-attaining students would probably use the materials more often or for longer periods than her higher attaining students who could "visualise". Ms B said she did not insist students use materials when they no longer needed to. These statements corresponded to findings posited from lesson observations that Ms B believed strongly in the importance of materials and representations and acted on these beliefs.

According to Ms L, she used concrete materials much more often with her younger students than was observed the previous year. Materials and visual representations appeared to be part of many lessons (money, shapes, counters, spinners, attribute blocks) and also available to students if they chose to use them which is how Ms L described her use of materials the year before. According to Ms L's responses during the postscript interview, it would seem that the younger students were more likely to "grab them" than I observed with the Year 5 students the previous year. This might support the finding from lesson observations that Ms L believed materials and representations were more appropriate for younger or low-attaining students.

Ms L believed that the major influences on when she used concrete materials were the mathematical content being taught and the level of the students. Ms L used materials more frequently with these younger students and stated this was because these students were "operating with smaller numbers". However, Ms L stated that she believed that she did use materials the year before but it was more dependent on the topic, as I have previously conjectured. For example, with fractions, Ms L reported that with Year 5 she used materials such as Cuisenaire rods, fraction squares and representations such as the Fraction Wall. For multiplication the previous year, the focus of the unit I observed, she stated, "some of my lower ones would have been using counters to make arrays". It is interesting that Ms L's recollection was that she would have used materials, as I did not observe this. Again, her statement that "some of my lower ones" would have used materials supports the finding that Ms L believed that, at the Year 5 level, materials were most appropriate for low-attaining students.

Ms L reiterated her belief from the previous year that the older students thought using materials was not appropriate for them and that such use had a "stigma" for students. Ms L thought that when she wanted to use materials, due in part to her involvement with the CTLM project (Clarke et al., 2009) these older students had already been "pushed out" of using materials and saw it as a "step backwards". In addition, Ms L thought that materials were inefficient to use with larger numbers and that students, particularly high attaining students, wanted a pencil and paper strategy to use. This concurs with Howard and Perry's (1997) and Swan and Marshall's (2010) finding that teachers of upper primary students tended to use materials less than did teachers of lower primary students. Furthermore, Swan and Marshall found that teachers in their study tended to believe, as it appeared Ms L did, that manipulatives were best used for "concept formation and hence should be abandoned when the mathematics becomes more complex" (pp. 16-17).

To conclude, Ms B's use of concrete materials and representations appeared to be unchanged from the previous year while Ms L claimed to be using such materials more often. I will now discuss the teachers' responses to questions regarding attention to concepts.

EXPLICIT ATTENTION TO CONCEPTS

During the postscript interview, Ms B reported that her teaching remained primarily focussed on concepts rather than procedures although she now felt that she might need to spend more time on procedures. Ms B reported that, since the lesson observation period, professional development had introduced her to a way of teaching a procedure that was also linked to the underlying concept through the use of materials. Ms B appeared satisfied with this way of teaching procedures as it did not contradict her belief in students understanding why and how such procedures work. Ms B stated that she thought focussing on concepts was particularly important for low-attaining students, as they needed "grounding" in a concept in order to be able to reliably "apply it".

Ms L identified a dilemma in her teaching for understanding. Although she believed that understanding “why you’re doing it” was essential for mathematics learning, she believed her low-attaining students struggled to understand. Therefore Ms L focussed more on “how to” or procedures so that these students could “solve it and move on”. This indicated a more “instrumentalist” (Skemp, 1972) view than Ms L expressed in interviews the previous year. Ms L recognised the contradiction between her belief in the importance of children understanding mathematics and this practice of teaching “how’ without “why”. She believed that her ability to teach in line with her belief in the importance of learning with understanding was impeded by her low-attaining students appearing to be confused by underlying concepts – “they are just staring at me blankly”. This resonates with the stated beliefs of the teacher in Putman’s (1992) case study who was concerned that children could be “bogged down” by learning “why” and that perhaps such understanding could come later, “when their mind is more mature” (p. 165). One year on, Ms L could articulate the disparity between her stated beliefs and the practice I had observed the previous year. Thompson (1984) asserted that not recognising contradiction between one’s own beliefs and practices meant such incongruence could continue. Perhaps by recognising this incongruence, Ms L was in a better position to examine and change her practice to align more with her beliefs.

The scaffolding practice of explicit attention to concepts represents the most fundamental difference between Ms B and Ms L. While each professed to value learning mathematics with understanding, the extent to which this belief was carried out in regard to low-attaining students was varied. Ms B stated that understanding concepts was perhaps even more vital for low-attaining students than for other students who might grasp “basic concepts” more easily. This resonates with Watson’s (2002) statement that it is difficult to learn facts and procedures that are unconnected. In contrast, Ms L worried that, without teaching low-attaining students how to complete procedures, they would not “move on”. It appeared Ms L believed understanding concepts might be too difficult for low-attaining students and that procedural proficiency could come first with understanding having to wait. This teacher belief has been widely reported in the literature (Anderson et al., 2005; Beswick, 2005a; Knapp et al., 1995; Putman, 1992). Both procedural proficiency and conceptual understanding are clearly important (Kilpatrick et al., 2001). However, the debate continues about what might be most effective and beneficial for low-attaining students (e.g. Ellis, 2005).

Teaching low-attaining students

Both Ms B and Ms L described a change in the way they approached teaching low-attaining students in mathematics since the initial data collection period. A year later, both teachers were teaching at least part of their mathematics lessons using “focus groups”. This meant teaching a small group of students, usually with similar learning needs or considered to be of similar ability, while other students completed work more independently.

Ms B stated that she was basing her focus group teaching on a model introduced to her through professional development with the Mathematical Association of Victoria (MAV). The model had been described as “Teaching will be for a concentrated 20 minutes or so, and then the teacher will supervise the rest of the class. Instructions on the board will inform the other groups of what they are to do” (Lowe, 2007). Ms B’s class was divided into three groups according to their achievement reported in school reports and standardised testing. Ms B was taking a teacher focus group for a portion of some lessons and this was shown in her planning notes. Ms B expressed some hesitation with this way of organising students, as she still felt somewhat uncomfortable with ability grouping for students. However, she stated, “The small groups work well to target concepts. I don’t use it every week. Ability groups seem to work depending on the task. Sometimes a really high and low kid works well together. Small groups appreciate the time spent with the teacher and it gives me a chance to give more individual assistance”.

The year before students were “loosely ability grouped” for one task that I observed, Fractured Figures. Carl and David spoke positively about working in this group and Ms B admitted that the boys benefited from being able to talk to students at a similar level to them. However, her comment on ability grouping the previous year was “I don’t like ability grouping … I guess that’s the battle – I don’t like ability grouping but sometimes you have to in order to get them to do the thinking you want them to do” (post lesson interview, Dominoes). It appeared that one year later Ms B was yet to resolve her beliefs about ability grouping but had seen benefits to using such grouping for at least some of her mathematics lessons.

Ms L had changed her teaching of low-attaining students in that she stated that she now too had focus groups more often than a year earlier. This was because she felt her current students could not be models or "teachers" for each other whereas some of her Year 5 or 6 students from the previous year could be. Ms L's organisation of these groups differed from the model used by Ms B. Ms L stated that she chose the focus group on a "needs basis", usually during the beginning of the lesson whole class discussion when she noticed students who may struggle with the task when working independently. These students were then asked to remain on the floor while the other students began the task and Ms L worked intensively with this focus group for 10 to 15 minutes. Ms L explained that concrete materials were often used with these focus groups, as was "reiterating, rephrasing, bringing it down a level, showing and modelling". When Ms L's teacher aide was in the room during maths, the aide usually taught the focus group. Ms L admitted that having focus groups had meant that she was not able to do so much "wandering around" speaking to individual students during the task. A particularly effective scaffolding practice observed in Ms L's class the year before was her use of scaffolding conversations. It appeared that she had less time for these interactions a year later. Ms L felt these focus groups were beneficial for low-attaining students as she was able to tailor tasks to better meet the needs of these students by providing more individualised assistance. It was interesting that Ms L appeared to have returned to the "pull-out groups" she said in her initial teacher interview the year before that she "used to have".

There were differences between Ms B and Ms L in regard to teacher focus groups. Ms B had teacher focus groups with all students placed in ability groups and these groups were decided on before lessons. The model Ms L chose was more ad hoc with the teacher focus group, which was usually the low-attaining students, "chosen" during the beginning of the lesson when she identified students she thought might need extra assistance. The "teacher focus group" model was certainly a well-known teaching practice in Victorian schools both the previous year and at the time of the postscript interview. It is possible that during the initial data collection period whole class tasks were more of a focus for the research projects in which each teacher was involved. However, it was interesting that a year after the lesson observation period, both teachers appeared to be teaching, for at least part of their mathematics lessons, using different tasks for particular groups of students. I will now discuss the teachers' responses to postscript interview questions regarding the task types from the TTML project.

TASK TYPES IN MATHEMATICS

Ms B and Ms L did not have many comments to make about the task types from TTML (Peled et al., 2009). Ms B was involved with a new professional development program that focussed on differentiation, as described previously. She admitted that this had caused her to shift her focus from task types. However, Ms B said she "wanted to get back into it" and that task types was "still in the back of my mind". She spoke about how she evaluated units of mathematics by identifying the task types and attempting to include a mix of the TTML task types when planning.

Ms L said that she and her colleagues planned open-ended tasks regularly and that these tasks often used models. In addition, real world situations or problems were incorporated where possible such as students organising an imaginary canteen during the unit on money.

It appeared that task types had become part of Ms B's reflections on her mathematics units even if it wasn't the same level of focus as the previous year when she was part of the TTML project. Being part of this project had influenced the way Ms B analysed the tasks she gave students even though the project had finished. Ms L had continued with planning the types of tasks observed the previous year and this included a mix of the TTML task types.

I would now like to turn my attention to the responses of the target low-attaining students to the postscript interview.

TARGET STUDENTS ONE YEAR ON

The four low-attaining target students were all in their final year of primary school at the time of the postscript interview. All of the students attended the same school as the previous year but none had the same teacher. I will begin by describing how the target students appeared to feel about themselves as mathematics learners, that is, affective issues, their descriptions of their mathematics classes and their preferences for task types. I then discuss the evidence from this interview in relation to cognitive issues, in particular those that also arose the previous year.

Carl, David and Sophie all stated that they felt more confident about themselves as mathematics learners than the previous year. Riley said he felt he had improved but also said he was “not that good” at mathematics.

Sophie in particular expressed more than once how much more confident she felt about learning mathematics and how much she had improved since Year 5. Sophie now worked with a tutor once a week and described these sessions. “He goes through everything with me so I feel a lot more confident. We do adding and subtracting and division so I actually feel a lot more confident. We still go over the things we did last year so that actually gives me a better understanding”. It seemed that Sophie’s tutor revised procedures for the four operations and focussed on Sophie remembering how to carry out these procedures. Sophie’s current teacher appeared to support this extra assistance as Sophie said, “I get sheets to take home to study with the tutor from my teacher”. Of her current teacher, Sophie said “she was one of them that really helped me, like if I got something wrong she’d go over it with me. She thinks I’ve improved since the start of the year”. At the time of the interview, Sophie’s class had a new teacher. In Sophie’s descriptions of mathematics lessons it would appear that this teacher focussed on speed and carrying out procedures. For example, Sophie described a task where she and a partner rolled dice to generate a three-by-two digit multiplication and the first to find the solution won. Another task Sophie described was a game called “Bang Bang” where two students tried to be the first to say the answer to a multiplication fact then say “Bang Bang” to the other student in order to win. Times table tests and quizzes also featured in Sophie’s descriptions of her mathematics classes one year on from my lesson observations. Sophie could not recall using materials or visual representations except for the dice previously mentioned. She stated that “we don’t use MAB anymore ‘cause a lot of people play with them. I don’t know why we haven’t used them yet but we haven’t used them”. She did not think the class had done any mathematics tasks that either used a real world situation or problem or that was open-ended and had many answers. Sophie couldn’t state clearly the kinds of tasks she felt she learnt best from.

David and Carl both had the same teacher in Year 6, Mr C. Mr C was described by Carl as “sporty” and it was clear that Carl admired Mr C as he said, “I know him outside of school ‘cause he plays cricket at the club I do. I try a bit harder this year”. Carl professed to have learnt “a lot more maths this year” and said that Mr C did maths more often than Ms B did the previous year. Carl thought Ms B taught maths “about twice a week” whereas Mr C taught maths “every day”. Of course, from my observations of Ms B and her planning notes, mathematics was taught daily the year before and it is interesting to speculate why Carl might feel he was having more mathematics lessons in Mr C’s class. Carl’s description of Mr C’s mathematics lessons was quite sparse but he did say that tasks were often completed alone and that whole class discussions were for Mr C to correct work. Carl said the class had not used materials much except MAB for modelling numbers once. Carl could not recall any tasks using a real world problem or that had many answers. Carl thought he learnt best from tasks using materials because “if it’s got hands on stuff, it’s easier to work it out”. In contrast to the positive emotive words expressed by Carl the previous year, such as “fun, exciting and excellent” regarding mathematics, one year later he appeared less effusive.

David also had Mr C as his teacher and was able to offer more detail than Carl about mathematics lessons. David described a typical mathematics lesson with Mr C as, “he explains what we’re doing, he write some challenges which we write up in our books and he corrects it at the end. Then we do them then he goes through them and explains how you do it”. Clearly David appreciated Mr C’s explanations of mathematics and from David’s descriptions these explanations appeared to mainly be about “how you do it” or how to carry out procedures. David proudly stated he now knew about “putting down the zero” in the multiplication algorithm but was less clear about what this meant. “If there’s two numbers at the top you have to put a zero after the first one because otherwise it will be the same amount and you have to make it bigger”. David professed to have learnt more in mathematics than the previous year. “I’m smarter this year, I’ve been listening more. Mr C explains it more, explains it better”. When asked what his teacher could do that would best help him, David said, “What he is doing now! Just explaining it good”. David recalled using materials such as measuring tools like trundle wheels and tape measures as well as the MAB Carl mentioned. David could not recall using real world problems or completing open-ended tasks. David stated that he would learn best from tasks that used materials because “you’re actually using objects and say you were writing on a sheet of paper, you’d have to picture it”.

Riley appeared to be the least confident of the four target students one year on. This may have been due to the current topic of decimals and fractions which his class was studying as he stated, “we’re starting to do decimals and fractions. I’m not good at decimals and I'm not good at adding fractions". Riley said he thought his current teacher would probably think that he was not good at mathematics, "'cause I have to ask quite a lot of questions like how to do stuff, (so) he probably thinks I need more improvement". Riley appeared to equate success in mathematics on being "quick" at answering questions and not needing to ask questions. Riley's least favourite aspect of his mathematics lessons were the quizzes. "He has to read it out and you have to write the answer. I don't like the quick ones". It is interesting to note that, in the initial interview the previous year, Riley stated that he was not good at mathematics because "sometimes I have to think about it for ten minutes". It appeared Riley still believed that speed was synonymous with being successful in mathematics. Riley stated that the class used materials or representations "not much" and couldn't recall tasks using real world problems or open-ended tasks. Riley said he learnt best from tasks that used materials. "I like a hands on thing, like something you actually work with".

With the exception perhaps of Riley, the target students appeared to think their teachers in Year 6 were reasonably successful in teaching them mathematics. In both Sophie and David's descriptions, the teacher "explaining" was emphasised as a positive aspect of their teaching. All the low-attaining students' new classes appeared to be more focussed on procedures and speed than was observed the previous year. From the students' descriptions it seemed that mathematics classes were quite traditional with the teacher setting work, students working individually, then the class or teacher correcting work. The task types from the TTML project (Peled et al., 2009) appeared to be underrepresented with minimal recollection by the students of tasks using materials and no descriptions of real world contexts or open-ended tasks. The three boys in this study stated clearly that they preferred to learn mathematics with tasks that used materials, yet in their classes, materials seemed to be seldom used.

I will now discuss evidence of the low-attaining students' learning with particular focus on areas of potential learning identified a year earlier. In each postscript interview with the low-attaining students, I asked a mathematics question that targeted an area of potential learning identified during the lesson observation period. Other occasions during the interview also shed light on the low-attaining students' understanding of mathematics concepts and I will also report on these data.

Sophie professed that she was "a lot better at maths than I used to be". This she attributed to her tutor "going over things" and her teacher. "If I get stuck he comes back and helps me he goes through it more carefully and once I understand he goes and I do the rest". However, Sophie demonstrated some misconceptions about multiplication, the unit taught during lesson observations the year prior. In response to a task observed in the previous year, Array of 48, Sophie, in the postscript interview, used additive rather than multiplicative thinking saying "it could be 24 by 24 … could make one straight down, could make 5 across and 5 down and whatever else, it would be 43 so 43 by 5". More heartening was Sophie's response to "tell me a story to match 19 times 20" which, it will be recalled, posed significant difficulties for Sophie the year before. She said "Mum wanted to get 20 items that each cost $19 dollars. She added them all up and what did she get?" In addition, Sophie was able to show successful use of a written procedure for multiplication, partitioning, and was also able to talk about the grid method.

Riley had more trouble than Sophie in relating a situation to 19 times 20. He initially said he couldn't think of a story and took about one minute to say, "There was 20 and 19 buses … 19 buses and 20 people on each bus. Would that be one?" It would appear that Riley had progressed only minimally in interpreting multi-digit multiplication by relating them to real world situations. It appeared he was approximately the same as the previous year in his ability to do this. However, Riley was able to successfully show the lattice method to calculate a three by two digit multiplication expression.

David stated that he had "learnt a lot more strategies" and gave the examples of equivalent fractions, for which he wrote "1/2", and improper fractions and wrote, "3/4" then corrected this to "5/4". When I asked David to name 5/4 then draw what this represented, he had difficulty. He drew a circle, divided it into quarters then divided one of the quarters in half giving five parts in all. He said that the smaller part, the eighth, was not a quarter. David then said, "You can't have five quarters. You can't have that fraction". It would seem that his "learning" of improper fractions was somewhat incomplete. David also wrote a decimal number, and read it correctly but was unable to represent it. I asked David the same question I had asked the previous year, "What is the decimal number that is the same as one half?" David wrote "50/100". I restated that I was looking for a decimal number and David said "Isn't it five or two? I think it's five. This is hard!" The previous year David had said, "zero point 2 is the same as half". It appeared from his response one year later that David might not have progressed much in his understanding of equivalent decimals and fractions.

Carl was also reasonably adamant that he had “learnt a lot more” in particular about fractions “like how to add them and stuff”. When asked for an example Carl wrote “1/2 + 1/2 = 1” and his next example was “2/3 + 1/2 = 3/5”. This demonstrated a common misconception about adding fractions. When asked what decimal number is the same as one half Carl said, “I don’t know ... Five maybe? … I don’t know … fifty?” The previous year Carl had said, “Fifty point zero is the same as half … Does it have to be a decimal? … Five point five? … I don’t get it … Maybe about thirty point five?” It would appear that Carl also had not made much progress in his understanding of this concept.

In many ways the four low-attaining students demonstrated that their level of understanding of the focus concepts of the year before had remained the same. Carl and David were still unable to name the decimal equivalent to one half. David did not appear to understand improper fractions and Carl showed misconceptions about adding fractions. Sophie had some trouble using multiplicative thinking but was able to relate a situation to a multi-digit multiplication, which she was unable to do the year prior. Riley was also able to do this but was hesitant and unsure. Both Sophie and Riley could still successfully use a written procedure to calculate multi-digit multiplication. Despite Carl and David’s belief that they had learnt more mathematics with Mr C than the previous year with Ms B, each continued to display worrying misconceptions about rational number. Sophie’s new found confidence might have been warranted in her ability to carry out procedures but it also appeared that her understanding of multiplication might not match this procedural proficiency.

CONCLUSION

At least three out of the four target students reported feeling more confident, and learning more mathematics in their new classes. These classes appeared to be focussed on procedures, speed, competition and traditional ways of teaching such as working alone on practising procedures demonstrated by the teacher. Despite feeling confident about learning more mathematics, the target students did not demonstrate considerable improvement in their understanding of the mathematical concepts assessed in this interview. It is also interesting to note that Riley, Carl and David all reported that they felt they learned mathematics best through the use of manipulative materials and yet also reported that such materials were used minimally, if at all, in their current classrooms.

It is perhaps not surprising that these low-attaining students might have preferred the structure and predictability of their more traditional classes to the more contemporary, open approach observed the previous year. Boaler (1998) reported that one fifth of the students at the progressive school in her study rejected an open approach to teaching mathematics because "they did not want freedom or choice; they "couldn't be bothered to find anything out"; they "needed to know straightaway" (p. 51). Similarly Lubienski (2000b) found some students preferred to follow rules rather than solve problems more independently and Ridlon (2004) described the hostility and resistance to her open approach to teaching mathematics by some students. There may be something comforting about following a procedure and completing many examples involving the same process, particularly for low-attaining students. This may have been what the target students in this study were expressing although it should be noted that these students were mainly positive about their lessons when interviewed the previous year.

Ms B and Ms L appeared to have maintained many facets of their teaching of mathematics present in lessons observed the previous year. Both teachers' lessons followed the same organisational format with whole class discussions at the beginning and end of lessons and independent or group work on the task in between. Ms B and Ms L identified that whole class discussions were an important part of their teaching of mathematics. In addition, both teachers expressed the belief that whole class discussions were particularly beneficial for low-attaining students. They also recognised that some students, perhaps particularly low-attaining students, "quietly ignored" these discussions and could tend to "tune out". Ms B stated that she liked to stay with students while they worked on tasks, often for extended periods, so that she felt they had grasped the task. She admitted that this left other students without her help. Ms L thought that stepping in and out of individual discussions with students while they worked was more effective and gave students a chance to work independently in between teacher interventions. Each teacher had incorporated ability-based focus teaching groups more in their mathematics lessons, though these were selected in different ways. Ms B remained dedicated to teaching underlying concepts though had begun to recognise the importance of teaching procedures. Ms L continued to focus on procedures but recognised a contradiction between students knowing how to carry out procedures without understanding why and her belief that understanding mathematics was important. Ms L reported using materials more often than the previous year but stated this was due to her teaching younger students. Interestingly, Ms L stated that she "would have used materials" the previous year but this was not observed. Ms B continued her strong focus on using materials and maintained her belief that such use was a vital part of learning mathematics for upper primary students. Ms B still used the task types from the TTML project as a basis of reflecting on the mix of task types in her mathematics units. Ms L continued to plan many open-ended tasks and look for opportunities to link mathematics to real world situations. Both teachers felt that being part of this study had caused them to reflect on their teaching of low-attaining students in mathematics.

CHAPTER SIX – KEY FINDINGS, LIMITATIONS AND IMPLICATIONS FOR MATHEMATICS TEACHING AND LEARNING OF LOW-ATTAINING STUDENTS.

INTRODUCTION

This case study sought to investigate the important issue of teaching low-attaining mathematics students while using conceptually challenging types of mathematics tasks. Specific scaffolding practices provided by teachers during such tasks were explored along with low-attaining students' responses, both cognitive and affective, to this scaffolding and specific types of mathematics tasks they encountered in their mathematics classrooms. In particular, the following research questions were examined.

The main research question was

How does a teacher's use of particular scaffolding practices, while using specific types of mathematics tasks, impact on low-attaining students cognitively and affectively?

Sub-questions were

1. Do any of the scaffolding practices appear to be more effective in scaffolding the learning of low-attaining students than the others and in what ways?
2. Do low-attaining students have a preference for the kind of scaffolding their teacher offers and what are the reasons given for these preferences?
3. Do low-attaining students have a preference for the kinds of tasks used in mathematics lessons and what are the reasons given for these preferences?
4. Do teachers show preferences for using particular types of mathematics tasks with low-attaining students and in what ways?

In Chapter 1, I identified the issue of the achievement gap between low-attaining students of mathematics and their peers (e.g. Thomson & De Bortoli, 2007). This gap has been claimed to be as much as seven years by the middle years of schooling (Cockcroft, 1982). The problem is often exacerbated by the age at which low-attaining students reach the middle years of their schooling, that is, Years 5 to 8 (Luke et al., 2003). During adolescence, self-esteem, motivation and engagement are often brought to the fore with low-attaining students finding this time particularly difficult due to a number of factors. Some of these are beyond the teacher's control such as the physical, social and emotional effects of adolescence, but disengagement with mathematics is something schools and teachers can address. Engagement is a crucial element in success in mathematics for all students (Sullivan et al., 2005). This may be particularly so for low-attaining students, who often need to overcome negative attitudes toward mathematics and themselves as mathematics learners (Yates, 2002). Mathematics tasks that engage and challenge students are more likely to lead to growth in mathematical understanding, which is also inherently engaging (Lambdin, 2003). The *Task Types and Mathematics Learning* project (Peled et al., 2009) investigated the use of engaging and cognitively challenging types of mathematics tasks in middle years mathematics classrooms. These types of mathematics tasks included tasks that use models, tools and representations to explore mathematical ideas; tasks that use a real world context; and tasks that are open-ended. The present case study sought to examine the responses of low-attaining students to their teachers' use of tasks, and the scaffolding practices teachers employed in order to support low-attaining students to learn from such tasks.

The review of literature drew together three spheres of research – mathematics tasks, the teaching and learning of low-attaining students and scaffolding in mathematics. Through examining the literature on mathematics tasks, issues arose such as lowering task demands during implementation and the difficulty for teachers in implementing highly cognitively challenging tasks (Henningsen & Stein, 1997). Research on scaffolding revealed a small number of studies (Anghileri, 2006; Bliss et al., 1996; McCosker & Diezmann, 2009) that focussed on scaffolding in mathematics classrooms, illustrating an area to which the present study can contribute. In addition, three scaffolding practices emerged that became a focus in this study. These were discussion (Hiebert & Wearne, 1993; O'Connor & Michaels, 1993); the use of manipulative materials and visual representations (Maccini & Ruhl, 2000; Sowell, 1989); and explicit attention to concepts (Groves & Doig, 2002; Hiebert & Grouws, 2007). Discussion has been shown to be a common and effective way that teachers scaffold learning (Siemon & Virgona, 2003). The use of manipulative materials and visual representations is often considered particularly appropriate for low-attaining students to support their development in mathematics (Butler et al., 2003; Cass et al., 2003). As all the task types under investigation were aimed at conceptual understanding, scaffolding through explicit attention to concepts was also chosen for examination in this study (Hiebert et al., 1997). The research regarding low-attaining mathematics students within heterogenous classrooms was also relatively sparse, highlighting another contribution the present study can make to the field. Studies did reveal that low-attaining students are often given less challenging, lower level tasks (Zohar et al., 2001), are expected to memorise basic facts before being perceived to be capable of problem solving (Anderson, 1997) and continue to fall behind their peers. There were few studies that examined low-attaining students' responses to higher-level tasks or teachers using higher cognitive tasks with low-attaining students. The studies that have examined the use of higher-level tasks with low-attaining students (Knapp et al., 1995; Lane & Silver, 1994) demonstrated that such tasks were effective for the mathematics learning of such students. This dissonance suggests the use of higher-level tasks with low-attaining students requires further examination, which the present study aimed to do.

In Chapter 3, the various methods of data collection were described. Case study methods were employed in order to gain a detailed picture of the experiences of the main participants, two teachers, Ms B and Ms L, and four upper primary school-aged low-attaining students, Carl, David, Sophie and Riley. Six lessons were observed in each of the two classrooms. These lessons were audio taped via a mobile recording device worn by each teacher throughout every lesson. As the researcher, I was present during all lessons and took photographs and wrote observation notes particularly regarding the behaviour and responses of the four target low-attaining students, two from each classroom. Semi-structured interviews were conducted with both teachers and target students prior to the lesson observation period, before and after each observed lesson and one year after the observation period concluded. In addition to collecting data on the target students' affective responses to the tasks and teacher scaffolding, these interviews also included an assessment task to collect data regarding the possible cognitive impact of the lesson, in relation to the mathematical focus of the day. Detailed accounts of each lesson were written after each observation and parts of the lesson directly pertaining to the target students were transcribed in full, including whole class discussions and individual teacher discussions with these students.

Data were analysed using the three scaffolding practices as focus categories, primarily for teacher actions. Student data were analysed according to data for affective or cognitive responses. *NVivo* (QSR International, 2005) software was used to code episodes from the observed lessons under the three scaffolding practices and their sub-categories or codes. Data were also viewed holistically with a view to identifying patterns and contradictions to these patterns. Analysis of data aimed to build up a thick, rich description (Merriam, 1998) of the mathematics classrooms and the experiences of the two teachers and the four low-attaining students.

The literature would suggest that the experiences of both the teachers and target students would be influenced by their beliefs (Beswick, 2005b; Ernest, 1989; Kloosterman et al., 1996). The interviews with the teachers and target students before the observation period began were aimed at gathering data regarding beliefs. The teachers' beliefs about the nature of mathematics, the teaching and learning of mathematics and the teaching and learning of low-attaining students in mathematics were explored in Chapter 4. This chapter also explored the target students' beliefs about the nature of mathematics, themselves as mathematics learners and how they might best be supported to learn in mathematics. This background information helped to set the scene for the lesson observations and began to paint the picture of the main participants in this study prior to the commencement of the observation period.

Chapter 5 described the lessons observed, interviews with the teachers and target students and cognitive data from the post-lesson assessment tasks. During this chapter, the main themes of this study were drawn out through the various data sources, including the rich data that the audio recordings of the lessons provided. Chapter 5 also contained a description of the data from all the main participants gathered during the postscript interviews that occurred a year after the lesson observation period. The three scaffolding practices of the study each led to some interesting findings, which are explored in the following discussion.

SUMMARY OF FINDINGS

This summary of the main findings of this study will be discussed by examining the research questions for this study outlined at the beginning of this chapter. The main research question has three parts. The first part of the main research question concerned the teachers' scaffolding practices. Data regarding scaffolding will be discussed under the three scaffolding practices that formed the focus of data collection and analysis. To follow, findings regarding the task types, the focus of the second part of the main research question, will be discussed. Finally, data regarding the affective and then cognitive responses of the target students, the third part of the main research question, will be examined by drawing together all three parts of the main research question. Data concerning the sub-questions will be woven into this discussion where appropriate.

DISCUSSION

The main findings regarding the teachers' use of discussion to scaffold low-attaining students' learning are as follows:

- Whole class discussions did not seem to be as effective for the target low-attaining students as one-to-one discussions with the teacher. The low-attaining students' contributions to whole class discussions were usually less than four words long and often these students did not participate actively. In addition, it would appear that the target students had difficulty following the discussion and understanding other students' contributions. This finding also addresses the first sub-question that asked if any of the scaffolding practices appeared more effective in scaffolding the learning of low-attaining students than others. These data would suggest that whole class discussions are not as effective as other scaffolding practices. This finding concurs with the findings of other studies into the effectiveness of whole class discussions with low-attaining students (e.g., Baxter et al., 2002; Burns & Myhill, 2004).
- Target students reported that they would contribute to whole class discussions when they felt confident that they knew the answers or could comment on other students' answers. They did not feel whole class discussions were a time for asking questions. This resonates with the findings of Jansen (2006).
- Scaffolding conversations that occurred one-to-one with the teacher appeared to be effective in scaffolding learning for low-attaining students. Short, frequent conversations appeared particularly effective. This was when the teacher stepped in and out of the conversation leaving the low-attaining student time to think or work on their own, but then returning to check their

progress and resume the conversation. This finding resonates with Cheeseman's (2009) reports from the mathematics classroom of much younger students. It also concerns the first research sub-question about the relative effectiveness of some scaffolding practices over others. Scaffolding one-to-one conversations appeared to be more effective than whole class discussions in supporting the learning of low-attaining students.

MANIPULATIVE MATERIALS AND VISUAL REPRESENTATIONS

The main findings regarding the teachers' use of manipulative materials and visual representations to scaffold low-attaining students' learning are as follows:

- Low-attaining students viewed using manipulative materials and visual representations positively, which was also found by Sowell (1989). Materials or representations that illustrated concepts were viewed most positively and also appeared most effective in supporting learning. Materials or representations aimed at prompting prior knowledge or linking knowledge sets were not as effective or viewed as positively. This finding addresses the third sub-question regarding low-attaining students' preferences for types of scaffolding. These data suggest that the target students preferred scaffolding via the use of materials and representations that illustrated concepts (Boulton-Lewis & Halford, 1992).
- Three of the target students reported that their preferred way to learn mathematics was by using manipulative materials, which also addresses the third sub-question.
- Ms B stated that manipulative materials and visual representations were just as important for junior and upper primary students and that she had not encountered any resistance from students in using such materials. Ms L reported that there was a "stigma" attached to using materials for upper primary students who saw such use as immature or evidence of failure in mathematics. This appeared to contradict the finding described above in which three out of four target students expressed a preference for using materials in their mathematics lessons. Ms L appeared to believe that using materials was more appropriate for younger students and low-attaining students, a teacher belief also reported by Howard and Perry (1997). The second sub-question

involved teacher preferences for task types. It would appear from these data that Ms B preferred tasks that use manipulatives, representations and tools whereas Ms L appeared to feel these tasks were less appropriate for older students.

EXPLICIT ATTENTION TO CONCEPTS

The main findings regarding the teachers' attention to concepts to scaffold low-attaining students' learning are as follows:

- In Ms B's classroom where the focus remained on concepts and the task demands were not altered, low-attaining students relied on the materials to scaffold their understanding or on one-to-one discussions with their teacher. Due to time constraints, Ms B was not necessarily able to spend time with all the students and so the role of the materials became paramount. In many cases the cognitive demand of the tasks was too high and did not connect with Carl and David's prior knowledge or current understanding.
- In Ms L's class, low-attaining students reported feelings of satisfaction and pride that they could perform written procedures accurately to obtain a solution. However, conceptual understanding about such procedures was less evident. As discussed previously in this thesis, both are well recognised as important (Kilpatrick et al., 2001). Tasks in Ms L's class were also often too difficult and not at an appropriate level for Sophie and Riley.
- Ms B stated that she thought that understanding concepts was very important for low-attaining students so that they could have a chance of being able to apply concepts to other situations. Ms L thought that understanding could come after knowing how to carry out procedures for low-attaining students, so that they could at least "solve it and move on". This contradicted the recommendation of the National Numeracy Review (2008) in Australia which stated that "the rush to apparent proficiency at the expense of the sound conceptual development needed for sustained and ongoing mathematical proficiency must be rejected" (p. xi). Ms L acknowledged that teaching procedures without understanding contradicted her belief in the importance of understanding mathematical concepts.

- Growth in understanding for Carl and David in Ms B's class appeared just as "halting and erratic" as Hiebert, Wearne and Taber (1991, p. 322) described, whereas proficiency in written procedures seemed to occur more quickly for Sophie and Riley in Ms L's class.

AFFECTIVE RESPONSES OF TARGET LOW-ATTAINING STUDENTS

The main findings regarding the target students' affective responses to their teachers' scaffolding, the tasks and mathematics generally are as follows:

- The low-attaining students nearly always viewed the lessons positively and appreciated the efforts of their teachers. None of the low-attaining students reported negative feelings about their teacher. In particular, conversations they had with their teachers could often be re-told with considerable detail and were viewed positively by the low-attaining students. This was also found by Sullivan et al. (2006) and by Cheeseman and Clarke (2007) with younger students. This addresses the third research sub-question and suggests that the low-attaining students in this study did show preferences for particular scaffolding practices; in this case, scaffolding conversations.
- Three out of the four low-attaining target students mentioned the use of materials as being more enjoyable and helping them to learn mathematics. Again, these findings concerns sub-question three in that three low-attaining students preferred scaffolding through the use of materials. This resonates with Sowell's (1989) finding that using materials in mathematics improved student attitude toward the subject.
- Low-attaining students were generally happier about lessons or tasks where they could see that they had made mathematical progress in some way. This was particularly evident for Sophie and Riley in Ms L's class who demonstrated satisfaction in being able to carry out procedures for multi-digit multiplication. Tasks in which time was spent "off task" or in which the student struggled to make progress were regarded negatively.

COGNITIVE RESPONSES OF TARGET LOW-ATTAINING STUDENTS

The main findings regarding the target students' cognitive responses to the tasks, teacher scaffolding and their progress in the units of mathematics studied during lesson observations are as follows:

- Sophie and Riley in Ms L's class made clear progress in that they were unable to solve multi-digit multiplication expressions using a written procedure at the beginning of observations and were able to do this successfully by the end of the observation period. In addition, the postscript interviews one year later showed that Sophie and Riley were still able to carry out written procedures for multiplication. On the other hand, understanding underlying concepts of multiplication appeared to be somewhat lacking for Sophie and Riley during observations and continued to prove difficult a year later during postscript interviews.
- Carl and David in Ms B's class made some limited progress during the observed lessons but this was difficult to identify. This might have been partly due to Ms B's focus on understanding the concepts of decimals, fractions and percent with no procedures taught. As mentioned previously, growth in understanding is more complex, slower and "comes gradually in small advances" (Hiebert et al., 1991, p. 322). However, David showed progress in his understanding that the number of parts in the whole names the fraction and Carl in his understanding of the equivalent decimal number to one half. Having said this, David and Carl's understanding of these concepts at the time of the observations appeared to be less than robust and still developing. This also remained the case one-year later as demonstrated by Carl and David during postscript interviews.

TASK TYPES

The main findings regarding the teachers' use of the three task types of interest in this study and the students' responses to these task types are as follows:

- The low-attaining students did not appear to have preference for any of the particular task types from the TTML project (Peled et al., 2009). Although three of the target students reported that their preferred way to learn mathematics was by using manipulative materials, this did not relate to all tasks categorised by the TTML project as tasks that used manipulatives, representations and tools. Indeed for some tasks in this category, the low-attaining students reported negative reactions and appeared to gain little in terms of cognitive growth. Rather than preferring a particular TTML category of tasks, the low-attaining students reported most positively about tasks using materials that illustrated concepts.

I will now discuss the implications of the findings of this study for teachers of low-attaining mathematics students.

IMPLICATIONS FOR TEACHERS OF LOW-ATTAINING STUDENTS

Though this study examined the case of just two teachers and four low-attaining students, the findings have implications for other teachers and educational researchers. Stake (1995) pointed out that, in the particular, more general aspects of the wider world might be recognised. The findings of this case, whilst being particular to the context in which it was examined, might afford other teachers insights into their own teaching of mathematics and the learning of students, especially low-attaining students. In addition, the feelings of the target students in this study can allow teachers a window into the possible feelings and thoughts of their own students or prompt them to find out more about the affective responses of their students to their teaching.

The implications for teachers will be examined under the headings of the three focus scaffolding practices of this study. I will begin by examining the use of discussion as scaffolding.

DISCUSSION

Whole class discussions formed a large part of the observed lessons in both Ms B and Ms L's mathematics lessons in that whole class discussions occurred at the beginning and end of each of the lessons. Both teachers expressed their belief in the importance of whole class discussions, and thought whole class discussions had particular benefits for low-attaining students. However, whole class discussions appeared ineffective for scaffolding the low-attaining students, who rarely contributed more than four words. In addition, the target students could not answer questions in post-lesson interviews, the content of which had been addressed during such discussions.

This has implications for teachers who are advised to use whole class discussion to build a “discourse community” (National Curriculum Board, 2009; National Council of Teachers of Mathematics, 2000; Silver & Smith, 1996; Yackel & Cobb, 1996). If teachers seek to follow this advice, much of mathematics lesson time will be devoted to whole class discussion. However, there is evidence of dissonance in the literature regarding the worth of whole class discussions. While the findings from this study concur with Baxter et al. (2002) for example, who found that time spent on whole class discussion was not effective for low-attaining students, there is other evidence in the literature that suggests whole class discussion was beneficial to student learning (Hiebert & Wearne, 1993). Whole class discussions are difficult for most teachers to orchestrate effectively (e.g., Burns & Myhill, 2004; Clarke, 1997; Myhill, 2006). Based on the findings of this study and others (Baxter et al., 2002; Burns & Myhill, 2004), I would suggest that less time should be spent on concluding whole class discussions during mathematics lessons and more time be devoted to engaging students in scaffolding conversations. Whole class discussions should be focused on two or three key strategies (Baxter et al., 2002) and these should be represented visually if possible (Sullivan et al., 2006) to assist students in following their classmates’ contributions. In addition, all students should be explicitly taught about the purpose of whole class discussions and their role in such discussions (Sullivan et al., 2003).

Individual scaffolding conversations between the teacher and the target students in this study, captured via the audio recording device worn by the teacher, proved to be more effective than whole class discussions. Indeed, scaffolding conversations emerged in this study as one of two most effective scaffolding practices identified in this study. Scaffolding conversations appeared particularly effective when the teacher stepped in and out, leaving the student to work independently before returning to check their progress. In addition, the low-attaining students often recalled these conversations with their teacher spontaneously and in some detail during post-lesson interviews. The target students expressed mostly positive feelings about these interactions with their teacher and appeared to appreciate both the attention and support given.

The implication for teachers are that scaffolding conversations should be actively pursued during mathematics lessons with low-attaining students to scaffold their learning. However, as Clarke (2004) noted “if the teacher is devoting significant proportion of class time to interacting with students either individually or in pairs or small groups … then certain assumptions are implied as to the capacity of the other students to work independently of teacher whole-class direction" (p. 15). As seen in Ms B's class, students often became disengaged with the task when Ms B was involved in scaffolding conversations with individual or small groups of students for lengthy periods. This illustrates the importance of the teachers' practice of stepping in and out conversations with students.

Effective scaffolding conversations from the present study can be compared to Anghileri's (2006) highest level of scaffolding "conceptual discourse", described as "the teacher goes beyond explanations and justifications … by initiating reflective shifts such that what is said and done in action subsequently becomes an explicit topic of discussion" (p. 49). This is clearly a complex idea and not easy for teachers to implement. Many studies have shown that in practice scaffolding is a difficult task for teachers (Bliss et al., 1996; McCosker & Diezmann, 2009). As part of the Researching Numeracy Teaching Approaches in Primary Schools project, Siemon and Virgona's (2003) study offered twelve scaffolding practices teachers had used during conversations with students about mathematics. These practices, such as "noticing", "modelling", "reviewing" "drawing attention to" and "focusing" might be useful for teachers to bear in mind when talking to students. Sullivan et al. (2006) suggested that teachers plan before lessons, the "enabling prompts" they will offer students who are having difficulty with a given task, to assist them with tasks in ways that do not take over the thinking for the student. This also appears to resonate with the "stepping in and out" during scaffolding conversations that I have described which allowed the student independent space to think. Examples of possible enabling prompts include explaining language or semantics of the problem, providing short-focused explanations, asking questions and giving reminders.

The use of discussion to scaffold the learning of low-attaining students in this study had varied success. An implication of these findings is that teachers should spend less time on less effective, and difficult practices, such as whole class discussions, and more time on more effective practices such as scaffolding conversations. The implications of the findings for the use of manipulative materials and visual representations will now be discussed.

MANIPULATIVE MATERIALS AND VISUAL REPRESENTATIONS

The main finding regarding the use of manipulative materials and visual representations was that the type of thinking the use of the material or representation was intended to prompt resulted in different levels of cognitive effectiveness and different affective responses from the target students. Materials or representations that were intended to prompt prior knowledge or link knowledge sets were not effective for the target students who may not have developed such knowledge, and not have the ability to make connections between knowledge sets or activate appropriate prior knowledge (Lawson & Chinnappan, 1994; Prawat, 1989). In addition, target students did not view tasks that used materials or representations in this way positively. Along with scaffolding conversations, the use of materials and representations that were concrete embodiments (Boulton-Lewis & Halford, 1992) of mathematical concepts emerged as one of the most effective scaffolding practices found in the present study. Through manipulating the materials or representations during a task, students could solve the task and develop in their understanding. Materials and representations used in this way were viewed positively by students and appeared to result in progress in their understanding of mathematical concepts.

The implication is that teachers need to consider the thinking or learning that using particular materials or representations might prompt when deciding how to use such materials in mathematics lessons. Teachers need to identify when the materials or representations they are using are intended to prompt prior knowledge or connect knowledge sets and reflect on whether students have this knowledge sufficiently to carry out the task (Prawat, 1989). For the benefit of low-attaining students in particular, teachers should more often use materials or representations that illustrate the concepts. This means that low-attaining students can be scaffolded in their learning through manipulating the materials or representation and therefore may be less reliant on the teacher providing scaffolding conversations. In addition, many of the target students identified that "hands-on" activity assisted their learning suggesting that the use of concrete embodiment materials and representations might be an effective way of supporting these students. Again, the suggestion of Sullivan et al. (2006) to carefully plan enabling prompts ties in with this use of materials. By planning for scaffolding learning, in other words for what to do when students struggle, teachers are more likely to reflect on the role of materials as scaffolding.

This study concurs with many others (e.g. Ball, 1992; Moyer, 2001; Stacey et al., 2001) that have shown that materials and representations are not all equally effective and teachers must give careful consideration to how and why they are used in mathematics lessons. This study suggested that concrete embodiment materials were more effective for the learning of low-attaining students than materials for prompting prior knowledge.

The final important finding in this area is that using materials and representations was an effective and engaging tool for teachers of upper primary students. The students in Ms B's class, who habitually used materials, had no resistance to their use whereas Ms L, who was not observed using materials, reported that she felt there was a "stigma" for upper primary students in using materials. This was contrary to the findings of Sowell (1989) who found in her meta-analysis of 60 studies regarding the use of materials, that student attitudes toward mathematics were improved when materials were used by teachers who were knowledgeable about their use. Howard and Perry (1997) reported that the decline in the use of materials by teachers as students become older was quite common and Swan and Marshall (2010) supported this finding twelve years on with a similar survey for teachers.

The way in which Ms B used materials was to distribute them to all students and to require students to use them. In this way, no students were singled out and the choice of using materials was not given to the students. In contrast, Ms L invited students to use materials "if they needed to" with the implication being that students who struggle would need to do so. If teachers were to adopt Ms B's method of using materials with upper primary students by planning tasks that necessitate their use, distributing them to all students and requiring students to use them, the stigma described by Ms L could be alleviated. The implications of using materials and representations in Ms B's way are that teachers need to believe that using materials and representations is appropriate for upper primary students, that the materials are actually essential for the task in hand, and that teachers are knowledgeable about materials and representations suitable for teaching various mathematical concepts (Sowell, 1989). At the same time, teachers need to be mindful of Ambrose's (2002) warning that some students can become overly dependent on the use of materials and begin to operate on "automatic pilot" rather than thinking more deeply about problems.

One of the two most effective scaffolding practices in this study was the use of materials and representations that illustrate concepts. Many other studies have also found the use of materials and representations is effective for the learning of middle years low-attaining students in mathematics (Butler et al., 2003; Cass et al., 2003; Maccini & Ruhl, 2000). This points to the need for teachers of upper primary students to be knowledgeable and consistent in their use of materials with these older students, realising that the use of materials and representations is as vital for learning mathematics with older students as it is with younger students. The implications for teachers in light of the findings for scaffolding via explicit attention to concepts will now be examined.

EXPLICIT ATTENTION TO CONCEPTS

Attention to concepts was the most complex practice to observe and analyse in this study. Each teacher approached drawing attention to concepts in different ways. Ms B appeared to believe that using materials and representations and talking extensively and intensively to individual students were most effective in building understanding of concepts. This was successful with Carl and David when the materials and representations illustrated concepts and when Ms B spent time engaging in scaffolding conversations with them. For about half the tasks, these conditions were not present and the level of cognitive demand was too high for Carl and David. Ms L chose to draw attention to underlying concepts of multiplication for shorter parts of overall lessons with the majority of time devoted to developing procedural proficiency. The tasks addressing concepts were not linked to the procedures taught in the remainder of the lesson and Sophie and Riley appeared unable to explain underlying multiplication concepts in post-lesson interviews. It seemed Ms L saw the development of procedural proficiency as more important than developing concepts or that understanding such concepts could wait until procedures were mastered.

One of the implications for teachers regarding explicit attention to concepts is that the teacher must endeavour to provide scaffolding without lowering the demand of the task. However, though maintaining a task's high cognitive challenge is desirable, if the challenge is too great for low-attaining students, they can struggle unproductively, as shown in this study. A key finding of this study is that tasks must be at an appropriate level for students in terms of prior knowledge and understanding.

Henningsen and Stein (1997) posited that one of five factors essential for maintaining a high level cognitive demand for a task was that it built on students' prior knowledge. Similarly, Vygotsky (1978) described scaffolding occurring within the students' "zone of proximal development" where the known meets the unknown. Tasks that are too difficult are not possible to scaffold because the student is not working within their known understanding and reaching just beyond. Tasks that provide an attainable and attractive challenge have been found to promote persistence and motivation for students so teachers should strive to set tasks that are attractive and challenging for students (Ames, 1992). Sullivan et al. (2006) described a set of tasks leading to a goal task, where the initial tasks provide *enabling prompts* for students that provide scaffolding leading to the goal task. The teacher plans these tasks and other enabling prompts to scaffold student learning on tasks appropriate to their level of understanding and prior knowledge. Given that many tasks in this present study were too difficult for the target students, teachers should be aware of students' prior knowledge when planning tasks and plan enabling prompts or alternative tasks that build on this prior knowledge.

The other issue that arose in this study and which has implications for teachers is teaching for proficiency in procedures and teaching for understanding of mathematical concepts. While both Ms B and Ms L expressed their belief in the importance of teaching mathematics for understanding, in practice, during the observation period, Ms L mainly taught written procedures without making explicit connections to underlying concepts (Stein et al., 1996). Shorter tasks were focused on the concepts of multiplication proving that teaching for conceptual understanding was not absent from Ms L's lessons. Furthermore, in the postscript interview, Ms L voiced a dilemma faced by many teachers of low-attaining students. She stated that she was aware that she taught low-attaining students "how" to do procedures without teaching the "why" of underlying concepts. While Ms L identified this as a contradiction to her belief that knowing why was important, she stated that she wanted low-attaining students to be able to "solve it and move on". Furthermore, the target students in Ms L's class demonstrated considerable satisfaction that they could carry out written procedures for multi-digit multiplication.

Kilpatrick et al. (2001) included both procedural proficiency and conceptual understanding in their description of the five strands of mathematical proficiency, indicating that both are important. Procedural proficiency can be developed before or simultaneously with conceptual understanding so it is not a case of one or the other. However, Eisenhart et al. (1993) warned that conceptual understanding can often “fall through the cracks” in favour of procedural proficiency. The implication for teachers is that procedures are important but can be taught and learnt in connection with underlying concepts. Indeed, tasks where students learn procedures with connections to underlying concepts were categorised by Stein, Grover and Henningsen (1996) as higher-level cognitive tasks. If procedures are taught this way, students can gain procedural proficiency without compromising conceptual understanding. This requires teachers to know how procedures are underpinned by concepts and effective ways of illustrating this connection for students.

To build understanding of mathematics concepts, drawing attention to those concepts is the key. It has emerged from classroom data in this study that this scaffolding practice underpinned the other two practices that were the focus of this study. The two most effective practices – scaffolding conversations and materials that illustrate concepts, were effective in successfully drawing attention to concepts. Considering Ms L’s teaching of multi-digit multiplication, teachers need to be knowledgeable about the concepts underpinning the mathematics they teach, about appropriate materials or representations to illustrate such concepts and about how concepts underpin procedures for calculations. Teachers also need to have knowledge of common misconceptions, such as “gap thinking” in the case of Ms B, in order to scaffold student learning effectively. Building understanding takes time and, ideally, teachers in subsequent years should build on the concepts taught in previous years rather than replace conceptual understanding with a focus on procedures, speed, and learning by following the teacher’s instructions only.

I will now discuss the limitations of this case study followed by an examination of possible areas for future research that arose from the findings and limitations of this study.

LIMITATIONS OF THIS STUDY

It is important to recognise the limitations of any study in order to aid readers in evaluating the soundness of the findings and their transferability to other situations (Merriam, 1998). In addition, acknowledging the limitations of the study points to future research opportunities.

The first limitation is that the present case study examined just two teachers, two classrooms and two low-attaining students in each of these classrooms. Though case studies are not intended to be generalised to other contexts or wider populations (Stake, 1995), it is important that the reader has as much information as possible in order to judge the relevance of the findings to another setting. Merriam (1998) said that the extent to which the case is "typical" must be revealed to readers. In this case study, the two teachers were both involved with university research projects at the time of the lesson observations. Ms B was part of the *Task Types and Mathematics Learning* project (Peled et al., 2009), of which this study was part, and Ms L was involved in the *Contemporary Teaching and Learning of Mathematics* project (Clarke, 2009). Each received professional development in the use of cognitively challenging mathematics task types and contemporary methods of teaching mathematics, respectively. In this way, these two teachers cannot be viewed as typical of teachers in Victoria at the time, as most teachers would not have been part of these projects, although it is likely most teachers would have been involved in some kind of professional learning in mathematics in recent years. It could be assumed that Ms B and Ms L might have been more familiar with teaching mathematics using cognitively challenging mathematics tasks and were more contemporary in their approach to teaching mathematics than would have been typical of most upper primary mathematics teachers at the time.

The four low-attaining students demonstrated idiosyncratic responses to their teachers' scaffolding attempts, the mathematics tasks and how they viewed themselves as mathematics learners. Like any group of students, they represented diverse achievements, knowledge, attitudes and dispositions (Gervasoni, 2004). Having said that, it is possible that teachers will recognise characteristics of students they teach within the detailed descriptions of Carl, David, Sophie and Riley.

The low-attaining students were chosen by their teachers to be part of this study. Ms B chose Carl and David while Ms L chose Sophie and Riley. The instructions I gave to the teachers regarding choosing the low-attaining students were that the students must not have a pre-diagnosed learning disability and should be willing and capable of talking with me about their feelings and knowledge about mathematics. The teachers chose the target students based on standardised testing, their own class or school assessments and the teachers' individual judgement of the students' progress in mathematics. While I believe teachers are dependable and reasonably accurate in evaluating their students, it is possible that had I (or others) assessed the students in the class, different target students might have been identified. Additionally, both Ms B and Ms L stated that there were other students they identified as low attaining whom they considered choosing for inclusion in this study. Ms B and Ms L cited factors such as school attendance, the likelihood of parental permission, the students' ability to communicate and the teachers' desire to know more about certain students, as impacting on their selection of the target students. Baker et al. (2002), in their synthesis of 15 studies on low-attaining mathematics students, reported that this method of asking teachers to nominate low-attaining students for research was a common way for researchers to identify such students.

The lesson observations occurred over a two to three week period in which six lessons from both classrooms were observed. These lessons in each class focussed on one particular area of mathematics; in Ms B's room the class was focussing on fraction, decimal and percent concepts while in Ms L's class, students were focussing on multi-digit multiplication. The units of mathematics were chosen, planned and taught by the teachers and as the researcher, I had no influence over what unit would be taught during the lesson observation period. This was in keeping with the notion that case studies explore the participants in a naturalistic setting with the researcher striving to have as little impact as possible (Erickson, 1986). However, a limitation of this study is that the two units taught in each class were different, allowing for little direct comparison of the scaffolding the teachers took when teaching the same mathematics concept. The teachers were asked about how they might or did teach the content taught by the other teacher but without classroom observations these data were limited to the teachers' perceptions. In addition, each teacher and their target students were observed only during one particular unit of mathematics. In both classrooms this was in the area of Number. Data regarding teacher actions and low-attaining students' responses might have been different if other units in mathematics, such as Measurement or Geometry, were also observed.

This study explored three scaffolding practices, which were the use of discussion, the use of manipulative materials and visual representations, and attention to concepts. These three scaffolding practices arose from an examination of the literature on scaffolding and were pertinent to the use of cognitively challenging tasks. However, there was also evidence in the literature of other potentially effective scaffolding practices that were not chosen for focus in this study. Peer tutoring or group work (Noddings, 1985), strategy instruction (Montague et al., 1993), metacognitive strategies (Kramarski, Mevarech, & Arami, 2002), and feedback (Baker et al., 2002) for example, have all been shown to have some success in scaffolding the learning of low-attaining mathematics students. Due to the necessity that case studies have clear boundaries (Stake, 1995), these were not chosen for inclusion in this study. However, these scaffolding practices remain potentially successful and it was a limitation of this study that these were not examined in this case.

The use of manipulative materials and visual representations was one of the scaffolding practices examined in this study. Whilst Ms B used materials and representations in her mathematics lessons habitually, Ms L used them much less. This limited the ability to compare substantially the teachers' use of this scaffolding practice.

This study sought to explore the use of the task types from the TTML project (Peled et al., 2009), but the data regarding these specific task types was sparse. In an attempt to maintain researcher neutrality, I did not specify the types of mathematics tasks to be taught during lesson observations. As reported earlier, I did talk to Ms B about the mix of task types in the fractions, decimals and percent unit she taught as all the tasks had been tasks that used models, tools or representations. In response she taught one task that used a real world context, and one open-ended task. This did not allow for data from particular TTML task types to be compared as the mix of tasks in Ms B's class was heavily weighted toward tasks that used models, tools or representations. In Ms L's classroom, although there was an even mix of the TTML task types, in implementation many tasks did not remain true to the task description intended by the project. This imposed further limitations on being able to collect and analyse data about the TTML task types.

The postscript interviews with the teachers and target students a year after lesson observations concluded emerged as a rich data source and confirmed many of the earlier findings in relation to teachers' stated beliefs and stated and observed practices. However, these data were based solely on semi-structured interviews. No lessons were observed at that time. A limitation of these data would be that teacher and student reported actions or beliefs could not be compared to what actually took place in the classroom at that time. These limitations draw out potential areas for future research, which I will now discuss.

FUTURE RESEARCH

Future research into the area of effective teaching and learning of low-attaining mathematics students is vital for creating critically numerate citizens and increasing the equity of schooling for low-attaining students. As discussed in Chapter 1, the achievement gap between low-attaining students and others can be as much as seven years (Cockcroft, 1982). With practices such as streaming or setting having been shown to lead to worse rather than better results for low-attaining students (Boaler, 1997; Zevenbergen, 2001), heterogenous classrooms have become more desirable. In addition, teachers are being asked to teach students, including low-attaining students, using cognitively challenging types of mathematics tasks (Department of Education and Early Childhood Development, 2010; National Council of Teachers of Mathematics, 2000; Numeracy Review Panel, 2008). Scaffolding is a promising strategy for teachers to use that allows low-attaining students to work on higher-level cognitive tasks without having the cognitive demands lowered (Anghileri, 2006). In addition, scaffolding has the potential to allow all students in a heterogenous classroom to access the same task creating a more equitable learning community.

In Chapter 2, I identified the lack of research on low-attaining mathematics students in general, and in particular, while engaged in challenging tasks within a heterogenous mathematics classroom (Empson, 2003; Hiebert et al., 1991). Case studies such as the present study can offer a starting point for research as they contribute findings, issues and observations that arise from the participants' experiences (Merriam, 1998). In this way, categories and areas for other research can be established. Case studies can offer teachers and educational researchers real examples of classroom experiences. Classroom examples of scaffolding in mathematics lessons, for example, are rare. Future case studies could use the scaffolding practices and the associated coding framework from this study to explore cases in other contexts such as with younger children, with secondary school students, with a wider group of teachers or in other countries, for example. In particular, the two most effective scaffolding practices of the present study, scaffolding conversations and the use of materials and representations that closely match concepts, could be tested in other contexts. The data collection tool of the audio recording of teacher and student interactions brought richness to the data in the present case study and could also be replicated by future studies. This study also points to further research on using materials and representations with upper primary students. Studies comparing "concrete embodiment" type materials with "prompting prior knowledge" materials, would add to the literature on the use of materials and representations and might also assist teachers in using this scaffolding practice more effectively.

I believe that the most pressing area for future research that arose from this study was the issue of whole class discussions and scaffolding conversations. At the time of this study, whole class discussion was widely recommended for teachers in mathematics, particularly discussion at the end of tasks or lessons. However, this study and others have found that such discussions can be ineffective for low-attaining students (Baxter et al., 2002; Burns & Myhill, 2004), and are difficult for teachers to orchestrate well (Ball, 1993; Clarke, 1997; Lampert, 1990). Scaffolding conversations, the kind of "conceptual discourse" identified by Anghileri (2006) or the mathematical conversations Cheeseman (2009) described, appear to hold promise for effective scaffolding in mathematical understanding for low-attaining students. Yet, there is little research on these interactions in mathematics classrooms (Kyriacou & Issitt, 2008). Future research could entail an action research study investigating the use of shorter, more focused whole class discussions while creating more opportunities for teachers to engage low-attaining students in regular, relatively brief scaffolding conversations.

CONCLUSION

It is hoped, as I stated in Chapter 3, that through reading this case, readers have had the capacity to discover something new (Merriam, 1998) about the learning and teaching of low-attaining students experiencing challenging types of mathematics tasks. Perhaps this discovery will not be a completely new piece of information but a new way of viewing the learning and feelings of low-attaining students, of mathematics tasks and of scaffolding in the mathematics classroom. Readers may find their previously held beliefs or theories about challenging tasks and low-attaining students have been validated or brought into question through reading this case.

The two most effective scaffolding practices that emerged from this case study were scaffolding conversations and the use of materials or representations that illustrated the concept. It was interesting to note that the target students identified using materials and talking with their teacher, where the teacher "sits down with them" as being the most effective ways they felt their teachers could help them. In effect, the students identified scaffolding conversations and materials themselves as the most beneficial for their learning. Other data support the finding that tasks where the teacher employed one of these two scaffolding practices were more successful for the low-attaining students than tasks where conversations were either not conducted or not completed, or tasks where materials were prompts for prior knowledge rather than embodiments of the concept. The most successful tasks for the target students, when the task was completed and when they demonstrated growth in understanding important mathematical concepts, employed both these scaffolding practices. The paucity of literature on low-attaining students in mathematics, and on scaffolding, essentially points to the need for further research in this area. Scaffolding remains a confusing and difficult practice for many teachers who would benefit from reading cases of other teachers who are attempting to scaffold learning in their mathematics classrooms. Low-attaining students continue to linger behind their classmates in mathematics, with the danger of growing ever more disengaged and disillusioned. Teachers, schools and the educational research community ignore their plight at the students' and their country's peril. These are the future citizens of our increasingly technological world. Effective ways of teaching and supporting low-attaining students to grow in their understanding, engagement and motivation to learn mathematics needs to be an essential part of future research and practice in mathematics education.

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