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Research Reports (A-K)

Editors: Mellony Graven, Hamsa Venkat, Anthony A Essien and Pamela Vale

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DESIGNING CHALLENGING ONLINE MATHEMATICAL TASKS FOR INITIAL TEACHER EDUCATION: MOTIVATIONAL CONSIDERATIONS

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Strengthening the personal mathematical capabilities of initial teacher education students is an international priority. In this paper, we report on an aspect of a study that investigated the potential of challenging online tasks to meet this goal. Here, we focus on the analysis of survey data in the form of student feedback on trialled tasks from the perspective of motivation theory. Findings of this aspect of the study are reported along with implications for the design of challenging online mathematical tasks and their implementation in teacher education programs.

BACKGROUND

Engaging learners in tasks that provide genuine challenge has been argued as essential for mathematical development (Sullivan, 2011) and identified as vital in school-focused curriculum documents internationally (e.g. Australian Curriculum Mathematics, 2017; US Common Core Standards, 2010). Despite such support, the implementation of programs that maintain a focus on challenging tasks in mathematics has been problematic.

This paper reports on an aspect of an international collaboration between Australian and German researchers through the project, *Designing Challenging Online Mathematics Tasks*. The purpose of the project was to develop principles of design for tasks implemented in programs aimed at Initial Teacher Education students (ITEs) preparing to teach mathematics. The challenging dimension of the tasks targeted the development of ITEs' personal mathematical capability. It was also intended that tasks would provide ITEs with insight into the nature of such problems, thus developing the confidence necessary for implementation within their own future classrooms. The project also aimed to accommodate the opportunities now available via student online learning systems, acknowledging the rapid movement of initial teacher education programs toward online or *blended* delivery.

The project was initiated by developing principles of design for challenging mathematical tasks through a synthesis of literature. These principles were subject to iterations of revision focused through the generation of initial tasks which were then trialled with ITEs in both Australia and Germany. The synthesis of literature focused drew on: task design in digital environments (Geiger, 2017); the notion of mathematical challenge (Sullivan, 2011); a map of pedagogical opportunities and

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mathematical analysis software (Pierce & Stacey, 2010); and, the quality of online learning environments (Bruder & Sonnberger, 2008). Through the process of iterative revisions, the principles of design were identified as: engagement, transparency, accessibility, challenge and feedforward/feedback. In this paper, we focus on the principle of engagement, particularly the connection between motivational influences and mathematical challenge in addressing the following research question: What motivation influences are evident in primary ITE's engagement with challenging online tasks?

CONNECTING ENGAGEMENT, MOTIVATION AND TASK DESIGN

Engagement is underpinned by aspects of motivation (Martin, 2012), as motivation is the process which serves to energise, direct or sustain activity, with engagement being the outward, observable outcome of this process (Schunk & Mullen, 2012). Motivation is key to engagement and learning achievement, as it focusses the attention of the learner towards specific goals and dictates the energy, effort and persistence that the learner will contribute to engaging in an activity. While energy and engagement are sometimes observable, the motivational influences that act upon engagement are harder to determine (Skilling, Bobis & Martin, 2015) but essential as drivers of engagement.

In the specific context of mathematics education, engagement and motivation are essential considerations in the design of mathematics tasks (Gresalfi & Barab, 2011) as tasks must be designed with an eye as to how they will be valued by students (Hulleman, Durik, Schweigert, & Harackiewicz, 2008). Such valuing has been linked to notions of purpose and utility (Ainley, Pratt & Hansen, 2006). In this view, a task which has *purpose* is "one that has a meaningful outcome for the pupil" (Ainley et al., 2006, p.29), while a task with *utility* addresses the ways in which the mathematical ideas inherent in the task are useful (as distinct from predominantly procedural).

THEORETICAL FRAMEWORK

Expectancy-Value Theory (EVT) provides insight into the engagement potential of a task by identifying individual's motivational influences. EVT proposes that the extent to which individuals *expect to achieve success (expectancy)* and their *subjective valuing of a task (value)* strongly influence their motivation and therefore engagement (Eccles & Wigfield, 2002). Under EVT, *expectancy* of achieving success is determined by prior experiences that shape self-efficacy and self-concept and, in turn, anticipation of progress and challenge. *Subjective valuing of a task* is influenced by *interest or enjoyment value, attainment value, utility value* and *relative cost* of undertaking the task. *Interest value* is differentiated into individual (personal) interest and situational interest. Individual interest being directed towards a specific object or activity and enduring over time (e.g., Hidi & Renninger, 2006), while *situational interest* is evoked by specific features of an activity, such as those that are personally relevant. *Attainment value* is considered the extent to which achieving on a task supports an individual's perception of themselves, for example, as a person, student or mathematician. *Utility value is* the potential of a task to further some future goal, such as a teaching capacity.

More specific to mathematics tasks, the notion of *utility value* relates to tasks in which the mathematical ideas embedded are deemed useful (Ainley et al., 2006). Finally, *relative cost* addresses aspects of a task that may be potentially demotivating, such as the perceived effort required for success. An individual's responses to these aspects of motivation – *expectation of success* and *subjective task valuing*, guide the amount of energy and focussed, sustained attention that is then given to a task. Thus, EVT provides a framework to structure insight into motivational influences which impact pre-service teacher engagement with challenging online mathematical tasks.

METHODOLOGY

Participants

This paper focuses on the survey responses of 43 Australian ITEs and follow-up focus group interview with three members of this group who were undertaking the second mathematics course in a four-year degree leading to a Bachelor of Education (Primary and Early Childhood). The course was concerned with developing content knowledge in mathematics for teaching primary and early childhood contexts. In this instance, the tasks were used as part of a graded assessment portfolio.

Online Challenging Tasks

Three tasks were designed in alignment with the five principles for online challenging mathematics tasks (engagement, transparency, accessibility, challenge and feedforward/feedback) and implemented with the cohort of ITEs. A brief description of these tasks follows:

Task 1: BBQ Task. Students were asked to compare the price of different BBQ packs (both non-vegetarian and vegetarian), for an upcoming social event.

Task 2: CO2 Emissions. Students were asked to make conclusions based on data represented in an interactive graph (generated through www.gapminder.org) about carbon dioxide emissions from different countries over time.

Task 3: Precious Cargo. Students were asked to decide the number and type of bottles that would fit in a triangular prism shaped box used to transport wine.

Data collection

Data sources utilised in this paper include pre-post surveys and a small group, semistructured interview. Data were collected anonymously using a student generated two letter, two number code (for example, AB12).

Pre-post surveys: A pre-post survey was developed in alignment with the previously identified principles of design for challenging online mathematical tasks. Survey questions included both 4-point Likert scaled items and open-ended questions. Pre-surveys (14 statements/questions) were administered one week prior to undertaking the online challenging tasks. The post-survey (34 statements/questions) was administered at the end of the semester after students had competed the online tasks. Included in the pre-surveys were questions designed to ascertain ITEs' beliefs and expectations

regarding challenging online mathematics tasks. The post-survey was designed to match the pre-survey with additional questions aimed at identifying pre-service teachers views on the specific tasks they had undertaken in their course.

Interviews: Semi-structured interviews were conducted with a focus group of three volunteer Australian students via online video conferencing. Interviews were based on questions that were aligned with the principles of task design. The interview lasted for approximately 90 minutes and was audio-recorded and transcribed for data analysis.

Data analysis

Survey data: Duplicate entries were removed where students commenced the survey, withdrew, and then returned to re-do at a later time. The responses to the statements were converted to percentages and examined via graphical representation.

Interview transcripts: Transcripts were analysed through an iterative process using content analysis. This enabled a search for patterns and themes based on a theoretical lens anchored in EVT. Using this grounded methodological approach (Strauss & Corbin, 1998), distinctive themes related to ITEs' *expectation of success* and *subjective task valuing* were identified.

RESULTS

The EVT framework (Eccles & Wigfield, 2002) was employed as the theoretical construct to categorise ITE's responses to pre-, post-surveys and semi-structured interviews in relation to *expectancy of success* and *subjective task valuing*.

Expectancy of success

ITEs' expectancy of success was addressed through the inclusion of two statements on the pre-survey to which students had the option to respond with Strongly Agree (SA), Agree (A), Disagree (D), or Strongly Disagree (SD). The first of these was, "I expect to make good progress with the challenging tasks". The responses overwhelmingly indicated that students anticipated success with 41 of 43 responses indicating agreement (SA or A) with this statement. Forty-two of the 43 respondents also indicated agreement with the statement, "I expect the tasks to be challenging", suggesting that while acknowledging the challenge in the task, they still felt that progress would occur. One of the two students who thought they may not make good progress expressed a desire for *support and clues* when they became *stuck* [EL22]. The other student who anticipated a lack of progress also indicated that the tasks would not be challenging, casting some doubt on the consistency of their responses.

Subjective task valuing

In this section, we present an analysis of survey and interview responses related to the subjective task valuing component for the EVT framework. The four survey questions relevant to the interest aspect of the subject task valuing component and a graphical representation of ITEs' responses (Figure 1) follow:

- 1. The online mathematics tasks were interesting. [Variable: Task Overall]
- 2. The context of the online tasks (e.g., BBQ) was interesting enough to work on until completion: [Variable: Task Context]
- 3. I found the mathematics content (e.g., geometry, statistics) for tasks [all named] to be interesting: [Variable: Maths Content]
- 4. The challenging aspects of the online task kept me interested: [Variable: Task Challenge]



Figure 1: ITE student responses to above questions on the post-survey

Interest or enjoyment value: While the BBQ task had the least total agreement with regards to being interesting enough to work through to completion, it also had the highest 'strong agreement' and was the only task to have students comment on their enjoyment of the context: *The BBQ task...was interesting to see the price differences among different places* [JU27] and *I enjoyed the BBQ task as I enjoyed the investigation of finding different meat prices and found the results really interesting* [TI10]. Relevance to current issues was an additional aspect noted with one student identifying the CO₂ Emissions Task as appealing *because I am interested in global problem CO₂ [as it is] relevant for climate change* [CU24].

The process of mathematical thinking was also identified as a reason a problem was interesting, for example, *The precious cargo task [because it] involved the most mathematical reasoning* [JO14]. There were also students for whom the challenge inherent in the tasks was appealing: *I believed that the CO₂ Emissions task was the best because it was the most challenging* [MI16] and *Precious Cargo...challenged me the most and allowed me to be creative with my answers* [CA37]. While task challenge is included here as a positive aspect leading to reported interest, this was not universal with some students feeling the challenge was excessive. The negative aspects are reported below under relative cost.

Attainment value: The value of obtaining success on tasks was strongly influenced by the assessable portfolio. Hence, ITEs were motivated by an extrinsic aspect (good grades, academic success). This aspect of attainment value was reflected in student

comments when asked what they found difficult in addressing the open-ended aspect of tasks, for example, *I was often unsure if my mathematics working was going in the right direction [TI10];* and, *I struggled with the open-endedness* [TI10]. While these are all difficulties typically encountered when individuals attempt to solve complex problems (Makar & Fielding Wells, 2011), students concern about success can be exacerbated when assessment performance is a factor. This can be of particular concern when students believe mathematics is a subject in which uncertainty should not be present *-my favourite thing about maths...is that it does have a right and wrong answer... I don't like that uncertainty, I really do like things black and white* [SH01].

Utility value: Student perceptions of the utility value of the tasks and the mathematics inherent in the tasks were not highly prevalent in their responses, however, two aspects related to the utility of the mathematics tasks were identified: the potential use of tasks as a future teacher, and the use of task to enhance ITEs' personal mathematical knowledge, for example, *Opportunities for me to use different strategies to find the volume and area of 2-D shapes and 3-D shapes* [JT32]).

Relative cost of undertaking the task: Many of the students expressed surprise at the effort and time required to engage with challenging tasks. For some this was in the application of aspects of mathematics, for example, *I found the geometry extremely hard* [LO26]), while for others it was the lack of direction they felt when faced with an open-ended task, for example, *I wasn't sure how much information to provide* [KI16].

DISCUSSION AND IMPLICATIONS

In utilising the EVT framework (Eccles & Wigfield, 2002) for analytical purposes, we could see that while ITEs largely expected to achieve success on the challenging tasks (*expectancy*), their motivation to engage with the tasks varied considerably according to their own perceptions of mathematics and mathematical learning (*subjective task valuing*). As such, *subjective task valuing* (*interest, attainment, utility and relative cost*) is of interest in this study and is predominantly addressed in the discussion.

On being questioned, ITE's equated *interest* predominantly with the life-related context of each task but to a lesser extent with the specific mathematical content and degree of mathematical sophistication. Some of the context interest was derived from the *situational interest* of the context but more so existing *individual interests* (Hidi & Renninger, 2006) which are unlikely to be known to the task designer. This suggests to the task designer that there are multiple aspects of the task that may have potential to pique interest but that the degree of challenge and the mathematics inherent are potentially the most critical to consider in designing and scaffolding tasks.

Some ITEs indicated they enjoyed the challenge and the opportunities for reasoning and creativity associated with open-ended questions. Others, however, argued the absence of explicit direction as to the mathematics and mathematical processes to be used was problematic. These latter responses suggest existing views of mathematics as predominantly memorisation and application of procedures and thus the tasks presented conflicted with these views. As such, the benefit of undertaking the tasks

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(*purpose*) and the usefulness of the mathematical approaches (*utility*) was not evident to many of the ITEs (Ainley et al., 2006). A further factor impacting on motivation relates to *utility value* as defined by Eccles and Wigfield (2002) – while some ITEs valued the activities for future classroom use, others could not see how the tasks could be used either in terms of context or mathematical content. ITEs may not readily make connections between the underlying intentions of challenging tasks and this may need to be made explicit.

If ITEs are to develop competence and a positive disposition towards implementing challenging problems in their future classrooms, then these aspects of challenging mathematics problems that appear to serve a demotivating influence must be addressed. Accordingly, teacher education programs must: 1) challenge the notion that the nature of mathematics is purely procedural; 2) provide scaffolded opportunities for ITEs to experience open-ended problems as a learner; and, 3) make explicit the utility value and purpose of challenging tasks to ITEs.

This study has provided evidence that the implementation of challenging mathematical tasks with ITEs is complex, suggesting that tasks must be carefully designed, scaffolded and their use justified with the ITE cohort if motivation to engage deeply with such tasks is to be cultivated and maintained. A major issue in achieving this goal, however is impoverished attitudes and beliefs about the nature of mathematics as a discipline held by some ITEs. These attitudes and beliefs must be addressed if ITE's are to be motivated to implement challenging mathematical tasks in their own practice.

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