EXPLORING THE WHOLE NUMBER KNOWLEDGE OF CHILDREN IN GRADE 1 TO GRADE 4: INSIGHTS AND IMPLICATIONS

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This paper provides a snapshot of the whole number knowledge of nearly 2000 Australian primary school children that was gained through use of a one-to-one assessment interview. The assessment data was related to a corresponding growth point framework that describes learning trajectories in counting, place value, addition and subtraction strategies, and multiplication and division strategies. The findings highlight the broad distribution of growth points in each domain for each grade level, and the wide distance between the lowest and highest growth points in each grade level and each domain. This demonstrates the complexity of classroom teaching and highlights the challenge of meeting each student's learning needs. Particular issues related to children's learning emerged from the data, and these suggest important themes for teacher professional learning and for refining mathematics curriculum. These include: (1) interpreting 2-digit and 3-digit numbers; (2) using reasoning strategies as opposed to counting-based strategies in addition and subtraction; (3) and strategies for solving partially modelled and abstract problems in multiplication and division.

INTRODUCTION

Education is well established as a significant factor in breaking the cycle of poverty for marginalised people throughout the world (Zappalà, 2003). Education provides knowledge that ultimately empowers people to access further education, employment and active citizenship. Sadly, educational outcomes for Australian children living in low Socio-Economic Status (SES) communities and Aboriginal and Torres Strait Islander communities are lower than for children not living in these communities (Commonwealth of Australia, 2008; Zevenbergen & Nieske, 2008). Thus, the current Australian Federal Government has both a continued and renewed emphasis on closing the education gap between these groups of Australians. One initiative launched by the Federal Government is a series of Pilot Projects that seek insight about how to close the literacy and numeracy gap for Australian children. This paper reports on one pilot, *Bridging the Numeracy Gap* (Gervasoni, Parish, Hadden, Turkenburg, Bevan, Livesey, & Croswell, 2011) that is a collaboration between 44 school communities, the Catholic Education Offices (CEOs) in three dioceses in south-eastern Australia (Ballarat, Sandhurst and Sale), the CEO Western Australia, and Australian Catholic University.

Key approaches for improving mathematics learning in this Pilot were: one-to-one interview-based mathematics assessment using the *Early Numeracy Interview* and associated framework of growth points (Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002; Gervasoni, Turkenburg, & Hadden, 2007) using this data to guide instruction and curriculum development at individual, class and whole school levels (Gervasoni & Sullivan, 2007); and using the *Extending Mathematical Understanding* Program (Gervasoni, 2004) in the second year of schooling (six-year-olds) to provide intensive specialised instruction for children who were mathematically vulnerable.

Gervasoni (2011). Exploring the whole number knowledge of children in grade 1 to grade 4: Insights and implications. In T. Dooley, D. Corcoran & M. Ryan (Eds.), *Mathematics Teaching Matters.* (Proceedings of the 4th conference on research in mathematics education, pp. 168-178). Dublin: St Patrick's College Drumcondra.

This paper reports on one aspect of this project; using a one-to-one interview-based assessment and an associated framework of growth points to gain insight about primary school children's whole number knowledge. Based on the insights gained about this knowledge, the implications for classroom instruction and teacher professional learning will be discussed with the view to enhancing mathematics learning for all.

USING FRAMEWORKS AND INTERVIEWS TO IDENTIFY CHILDREN'S WHOLE NUMBER KNOWLEDGE

Clinical assessment interviews are now widely used by teachers in Australia and New Zealand as a means of assessing children's mathematical knowledge. This is due to the experience of three large scale projects that informed assessment and curriculum policy formation in Victoria, NSW and New Zealand: *Count Me In Too* (Gould, 2000) in NSW, the Victorian *Early Numeracy Research Project* (Clarke, Cheeseman, Gervasoni, et al; 2002) and the *Numeracy Development Project* (Higgins, Parsons, & Hyland, 2003) in New Zealand.

A common feature of each of these projects was the use of a one-to-one assessment interview and an associated research-based framework to describe progressions in mathematics learning (Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge & Gould, 2005). Teachers participating in each project indicated that the benefits of the assessment interview, though time-consuming and expensive, were considerable in terms of creating an understanding of what children know and can do, and for subsequently planning instruction. Indeed, an important feature of clinical interviews is that they enable the teacher to observe children as they solve problems to determine the strategies they used and any misconceptions (Gervasoni & Sullivan, 2007). They also enable teachers to probe children's mathematical understanding through thoughtful questioning (Wright, Martland, & Stafford, 2000) and observational listening (Mitchell & Horne, 2011).

The insights gained through this type of assessment inform teachers about the particular instructional needs of each student more powerfully than scores from traditional pencil and paper tests, the disadvantages of which are well established (e.g., Clements and Ellerton, 1995). Bobis et al. (2005) concluded that one-to-one assessment interviews and associated frameworks assisted to move the focus of professional learning in mathematics from the notion of children carefully reproducing taught procedures to an emphasis on children's thinking. This is an important outcome at a time when it is broadly accepted that the traditional focus on taught procedures for calculating can negatively impact on children's number sense (Clarke, Clarke, & Horne, 2006) and may impede children's development of powerful mental reasoning strategies for calculating (Narode, Board, & Davenport, 1993). Thus, because of the deep insight about children's mathematical knowledge gained through the use of one-to-one assessment interviews, the *Early Numeracy Interview* was chosen as the assessment tool for the *Bridging the Numeracy Gap* (BTNG) project. It was anticipated that the data obtained would be insightful for teachers and that the cohort data would provide a rich snapshot of children's whole number knowledge for principals and system leaders.

THE EARLY NUMERACY INTERVIEW AND GROWTH POINTS

One feature of the BTNG project was the assessment of nearly 2000 Preparatory to Grade 4 (5-9-year-old) children's whole number knowledge at the beginning of each year using the *Early Numeracy Interview* (Department of Education Employment and Training, 2001). *The interview* was refined during the project and renamed the *Mathematics Assessment Interview (MAI)* in 2010 (Gervasoni et al., 2011). The data examined in this paper was drawn from the interviews and the associated framework of growth points, so it is important that both are clearly understood. An outline is provided below.

The *Early Numeracy Interview* (Department of Education Employment and Training, 2001), developed as part of the *Early Numeracy Research Project* (ENRP, Clarke et al., 2002), is a clinical interview with an associated research-based framework of growth points that describe key stages in the learning of nine mathematics domains. The principles underlying the construction of the growth points were to: describe the development of mathematical knowledge and understanding in the first three years of school in a form and language that was useful for teachers; reflect the findings of relevant international and local research in mathematics (e.g., Steffe, von Glasersfeld, Richards, & Cobb, 1983; Fuson, 1992; Mulligan, 1998; Wright, Martland, & Stafford, 2000; Gould, 2000); reflect, where possible, the structure of mathematics; allow the mathematical knowledge of individuals and groups to be described; and enable a consideration of children who may be mathematically vulnerable.

The growth points form a framework for describing development in nine domains, including four whole number domains that are the focus of this research: Counting, Place Value, Addition and Subtraction, and Multiplication and Division. The processes for validating the growth points, the interview items and the comparative achievement of children in project and reference schools are described in full in Clarke et al. (2002).

To illustrate the nature of the growth points, the following are the growth points for Addition and Subtraction. These emphasise the strategies children use to solve problems.

1. Counts all to find the total of two collections.
2. Counts on from one number to find the total of two collections.
3. Given subtraction situations, chooses appropriately from strategies including count back, count-down to & count up from.
4. Uses basic strategies for solving addition and subtraction problems (doubles, commutativity, adding 10, tens facts, other known facts).
5. Uses derived strategies for solving addition and subtraction problems (near doubles, adding 9, build to next ten, fact families, intuitive strategies).
6. Extending and applying. Given a range of tasks (including multi-digit numbers), can use basic, derived and intuitive strategies as appropriate.

Each growth point represents substantial expansion in knowledge along paths to mathematical understanding (Clarke, 2001). They enable teachers to: identify any children who may be vulnerable in a given domain; identify the zone of proximal development for each child in each domain so instruction may be customised and precise; and identify the diversity of mathematical knowledge in a class. The whole number tasks in the interview take between 15-25 minutes for each student and are administered by the classroom teacher. There are about 40 tasks in total, and given success with a task, the teacher continues with the next tasks in a domain (e.g., Place Value) for as long as the child is successful. Teachers reported that the *Early Numeracy Interview* (ENI) provided them with insights about children's mathematical knowledge that might otherwise remain hidden (Clarke, 2001). This was an important reason for using this assessment instrument during the *Bridging the Numeracy Gap* project.

GAINING INSIGHT ABOUT CHILDREN'S WHOLE NUMBER KNOWLEDGE

The data examined in this paper were collected in 44 school communities associated with the Catholic Education Offices of Ballarat, Sandhurst and Sale in Victoria (south-eastern Australia), and the Catholic Education Office of Western Australia. Participants included nearly 2000 Grade 1 to Grade 4 (6-9-year-old) children who were assessed at the beginning of 2011 by their classroom teachers using the *Mathematics Assessment Interview* (MAI).

The detailed interview record sheets were independently coded by Australian Catholic University (ACU) research staff to determine the growth points children reached in the domains of Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies (see Appendix A). Independent coders were used to increase the validity and reliability of the data. The growth points for each student were entered into a database and analysed to determine the percentage of children on each growth point in each whole number domain and grade level. This enabled a rich snapshot of children's whole number knowledge to be obtained, and also enabled the researchers to: (1) measure children's growth in mathematical understanding over the 3-year period of the project; (2) inform the refinement of the interview; and (3) measure the impact of the EMU intervention program (Gervasoni, 2004) for participating children in 2009 and 2010. Creating a snapshot of children's whole number knowledge at the beginning of 2011 is the focus of this paper.

INSIGHTS ABOUT CHILDREN'S WHOLE NUMBER KNOWLEDGE

Examination of the 2011 MAI growth point distributions for nearly 2000 children provides a rich picture of the current whole number knowledge of children across the first five years of schooling, and enables associated insights and issues to be identified. The following section explores the findings for the domains of Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies.

Counting Knowledge

The 2011 Counting growth point distributions for children in Grade 1 to Grade 4 are shown in Figure 1. The data highlights that there is a wide distribution of growth points in each grade. For example, the growth points for Grade 1 (6-year-old) children range from GP0 to GP5, and from GP1 to GP6 for Grade 4 (9-year-old) children. This demonstrates the complexity and challenge of classroom teaching, and the need for activities and instruction to be customised for individuals. Apparent also is the growth in knowledge that occurs from one grade to the next; the median Counting growth point increased by one growth point in each grade level.

Examination of the Counting growth point distribution for the Grade 4 children provides some insight about the growth of children's knowledge across the first four years of primary school. By the beginning of Grade 4 (fifth year at primary school), about 50% of children were working towards Growth Point 6 – extending and applying their counting knowledge. Thus, assisting children to make this growth point transition needs to be a key focus for Grade 4 teachers. In contrast, about 10% of Grade 4 children could not yet count forwards and backwards by ones beyond 110, and another 20% could not yet skip count by 2s, 5s and 10s from zero. It is anticipated that these children (30%) would struggle with accessing some aspects of the Grade 4 curriculum and may benefit from customised instruction.

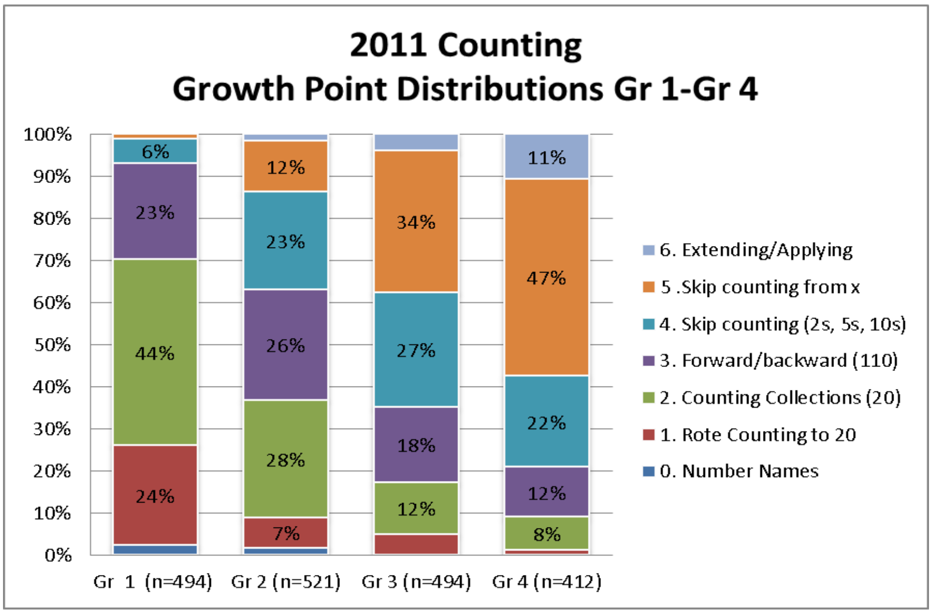

Figure 1. Counting growth point distributions for children in Grades 1– 4.

Place Value

The Place Value knowledge assessed by the MAI includes children’s abilities to read, write, order and interpret numbers. The growth point distributions are shown in Figure 2.

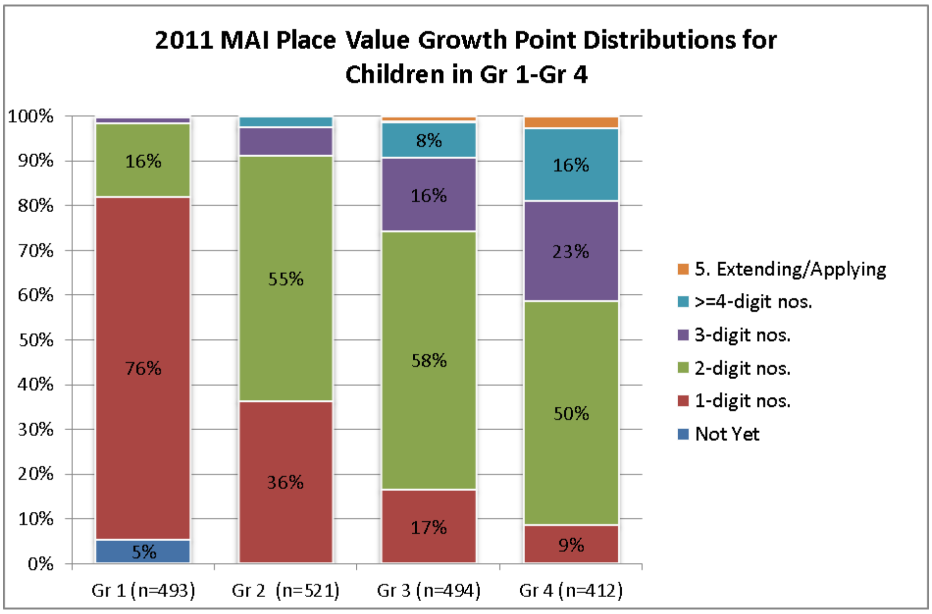

Figure 2. Place Value growth point distributions for children in Grades 1– 4.

Figure 2 also indicates a wide distribution of growth points in each grade, but less growth is apparent from grade to grade compared with Counting. Indeed the median growth point remains GP2 (2-digit numbers) from Grade 2 to Grade 4. Reaching GP2 and then GP3 (3-digit numbers) are significant issues for children in these grade levels.

By Grade 4, 50% of children remained on GP2 in Place Value, and almost 10% were still on GP1. This means they could read, write, order and interpret 2-digit and 1-digit numbers respectively. Thus the focus for 60% of Gr 4 children is learning to interpret 2-digit and 3-digit numbers. This implied struggle with understanding 3-digit numbers well into Gr 4 is important for teachers to recognise, as some may assume that children who can read and write these numbers can also appreciate them as quantities. This is not so. Indeed, it was the interpretation of quantities rather than the ability to read, write and order numerals that posed difficulty for children in this cohort (Gervasoni et al., 2011). Highlighting the complexity of planning instruction for Grade 4 children is the fact that 40% can already understand 3-digit numbers, while 15% have reached GP4. This wide variation in the range of numbers that children understand has implications for their approaches to calculating and solving number problems, and for the instruction planned by teachers.

Addition and Subtraction Strategies

A key focus for learning and teaching in addition and subtraction is the development and use of reasoning strategies for calculating as opposed to counting-based strategies. However, the 2011 growth point distributions for Addition and Subtraction Strategies (Figure 3) indicate that many children in all grades relied on less efficient counting-based strategies for calculating.

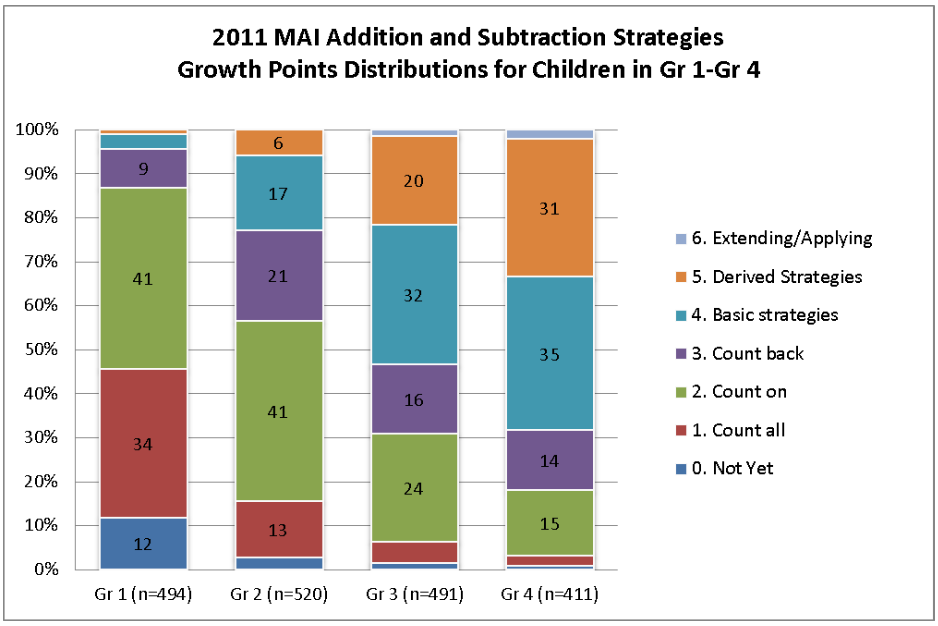

Figure 3. Addition & Subtraction growth point distributions for children in Grades 1– 4.

The data in Figure 3 indicate that 96% of Grade 1 children, 75 % of Grade 2 children, 46% of Grade 3 children and 30% of Grade 4 children used counting-based strategies for calculations involving 1-digit numbers, such as 4+4 and 10-3. The fact that so many Grade 4 children remain reliant on counting-based strategies for calculating, and that almost no Grade 4 students could solve mental calculations involving 2-digit and 3-digit numbers (GP6), is at odds with the tasks typically found in Grade 4 text books that involve calculations with much larger numbers. The data in Figure 3 also highlights the wide distribution of growth points in each grade, and also the growth in knowledge from one grade to the next.

Multiplication and Division Strategies

In the Multiplication and Division domain, the key issue for learning and teaching is children's ability to perform calculations without models being present to assist the calculating, first in partial modelling situations and also when models are completely removed. Figure 4 shows the 2011 growth point distributions for Multiplication and Division Strategies.

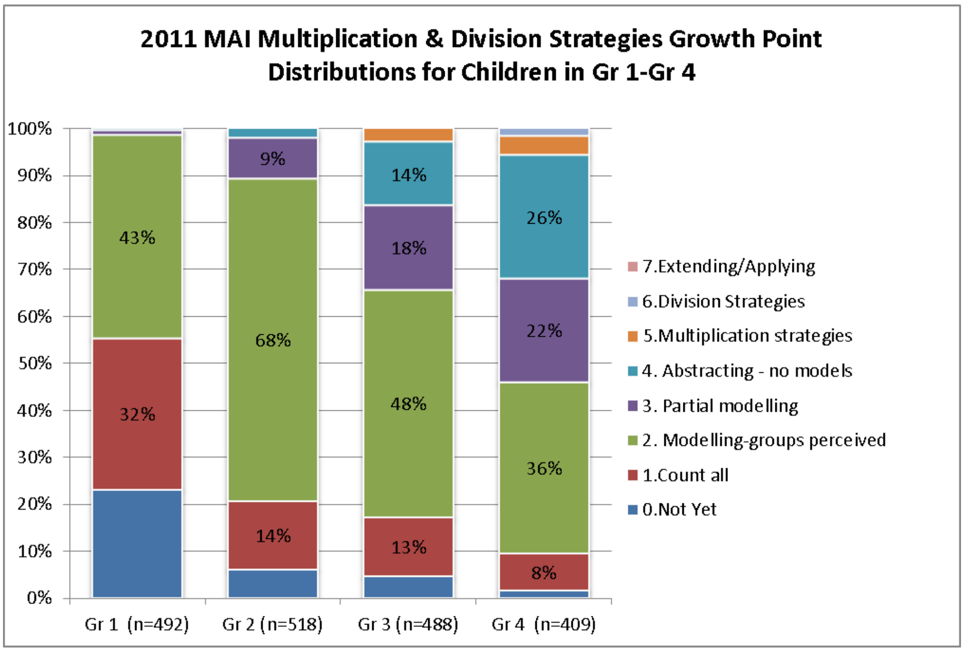

Figure 4. Multiplication and Division growth point distributions for Grade1– 4 children.

An outstanding feature of the growth point distributions shown in Figure 4 is the large number of children in each grade on GP2 (models all objects to solve multiplicative and sharing situations). In Grade 3 only 35% of children were able to solve multiplicative problems in partial modelling or abstract situations. This increased to 50% of Grade 4 children. Overall, it is likely that children need more experience with an expanded range of multiplicative problems presented in partial modelling situations in order for their multiplicative reasoning to develop. Other features of the data are the wide distribution of growth points in each grade, and the similarity in growth point distributions for the lower 20% of children in Grade 2 and Grade 3. This suggests that instruction for these children may not be sufficiently meeting their needs. Also, two-thirds of Grade 4 children were not able to solve simple multiplication problems without partial models being provided. This finding is contrary to the expectations inherent in many mathematics programs and text books.

DISCUSSION

The findings presented in the previous section highlight several important issues related to the instructional needs of children. First is the broad distribution of growth points in each domain and each grade level, and the wide distance between the lowest and highest growth points in each grade level and each domain. This highlights the complexity of classroom teaching and of meeting each student's learning needs. The data indicate that children in each grade have diverse learning needs, and this calls for customised instructional responses from teachers. It is likely that teachers need to make individual decisions about the instructional approach for each student, and that there is no 'formula' that will meet all children's instructional needs. Thus it is essential that classroom teachers are (1) able to identify children's current knowledge in order to plan suitable learning opportunities for them, and (2) have the ability to refine the curriculum and customise learning opportunities for individuals according to the range of knowledge represented in a teaching group. Principals and teachers participating in the BTNG Pilot (Gervasoni et al., 2011) noted that classroom teachers who used MAI data to refine the curriculum and design instruction for individual children, increased the capacity of their school communities to provide effective instruction for all.

The second point to note is that progress through the growth points is clearly obvious from one grade level to the next in each domain. However, it is also clear that progress from one growth point to the next is more challenging for some growth points and in some domains, than for others. For example, progress from GP2 to GP3 in Place Value took considerable time for many children. Knowledge about these challenging points in children's learning and about how to assist children to reach them is necessary to enable teachers to be most effective, and may be a useful focus for professional learning programs.

A third issue highlighted by the growth point distributions is the significant number of children on very low and very high growth points in most grade levels and domains. These children may be particularly vulnerable if teachers are not able to cater for their particular instructional needs learning in the classroom. Of particular interest are the children on the lowest growth points who are not thriving in the context of the regular classroom. These children are easily identified by the growth point framework and may benefit from a specialised program.

A fourth point emerging from the data is that few children reached the highest growth points in each domain, even in Grade 4. This provides confidence that the MAI may be useful throughout the primary school, and certainly beyond Grade 4. Indeed, some schools have used the MAI with children in Grades 5-7 and few children have reached the ceiling in each domain. However, these data also emphasise the importance of creating high quality learning environments that enable all to thrive and reach these higher growth points.

Finally, the analyses suggest that several growth points represent challenging aspects of whole number learning. These include: (1) interpreting 2-digit and 3-digit numbers; (2) using reasoning strategies as opposed to counting-based strategies in addition and subtraction; (3) and solving partially modelled and abstract problems in multiplication and division. It is recommended that professional learning opportunities for teachers focus on these challenging growth points and the associated powerful pedagogical actions and tools that will assist students to learn successfully.

CONCLUSION

The findings presented in this paper suggest that there is no single ‘formula’ for describing children’s whole number knowledge or the instructional needs of children in a particular grade. Meeting the diverse learning needs of children is a challenge, and requires teachers to be knowledgeable about how to identify each child’s current mathematical knowledge and customise instruction accordingly. This calls for rich assessment tools capable of revealing the extent of children’s knowledge in a range of domains, and an associated framework of growth points capable of guiding teachers’ curriculum and instructional decision-making. Growth points help teachers to identify children’s zone of proximal development in mathematics so as to create appropriate learning opportunities, and in order to adjust activities to increase engagement and remove features that are creating barriers to learning. Thus, reference to a framework of growth points helps to ensure that instruction for children is closely aligned to their initial and on-going assessment, and is at the ‘cutting edge’ of each child’s knowledge (Wright et al, 2000).

Assisting children to learn mathematics is complex, but teachers who are equipped with the pedagogical knowledge and actions necessary for responding to the diverse needs of individuals are able to provide children with the type of learning opportunities and experiences that enable them to thrive mathematically. This must be the goal of all school communities.

ACKNOWLEDGEMENTS

The research reported in this paper was funded by the Australian Government as part of the *Bridging the Numeracy Gap in Low SES and Indigenous Communities Project*. The author acknowledges gratefully the contribution of teachers, parents and students in the 44 school communities involved in the research, and of her inspirational colleagues in the research team, Kate Bevan, Melissa Croswell, Teresa Hadden, Carole Livesey, Linda Parish, and Kathie Turkenburg.

REFERENCES

Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, R., Young-Loveridge, J. & Gould, P. (2005). Supporting Teachers in the Development of Young Children’s Mathematical Thinking: Three Large Scale Cases. *Mathematics Education Research Journal*. 16(3), 27–57.

Clarke, D. (2001). Understanding, assessing and developing young children's mathematical thinking: Research as powerful tool for professional growth. In J. Bobis, B. Perry & M. Mitchelmore (Eds.), *Numeracy and beyond: Proceedings of the 24th Annual Conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 9-26). Sydney: MERGA.

Clarke, D., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, M., McDonough, A., Montgomery, P., Roche, A., Sullivan, P., Clarke, B., & Rowley, G. (2002). *Early Numeracy Research Project Final Report*. Melbourne: Australian Catholic University.

Clarke, B., Clarke, D., & Horne, M. (2006). A longitudinal study of mental computation strategies. In Novotná, J., Moraová, H., Krátká, M, & Stehlíková, N. (Eds). *Proceedings of the 30th Conference of the International Group for Psychology of Mathematics Education, Vol. 2*, pp. 329-336. Prague: PME.

Clements, M. A., & Ellerton, N. (1995). Assessing the effectiveness of pencil-and-paper tests for school mathematics. In B. Atweh & S. Flavel (Eds.), *Galtha: MERGA 18: (Proceedings of the 18th Annual Conference of the Mathematics Education Research Group of Australasia*, pp. 184–188). Darwin, UNT.

Commonwealth of Australia (2008). *National Numeracy Review Report*. Retrieved 18 March 2010, from COAG Web site: http://www.coag.gov.au/reports/docs/national_numeracy_review.pdf.

Department of Education Employment and Training. (2001). *Early numeracy interview booklet*. Melbourne: Department of Education, Employment and Training.

Fuson, K. (1992). Research on whole number addition and subtraction. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 243-275). New York: Macmillan.

Gervasoni, A., Parish, L., Hadden,T., Turkenburg, K., Bevan, K., Livesey, C., & Croswell, M. (2011). Insights about children's understanding of 2-digit and 3-digit numbers. In J. Clark, B. Kissane, J. Mousley, T. Spencer & S. Thornton (Eds.), *Mathematics: Traditions and [New] Practices (Proceedings of the 23rd biennial conference of The Australian Association of Mathematics Teachers and the 34th annual conference of the Mathematics Education Research Group of Australasia, Vol 1*, pp. 315-323). Alice Springs: MERGA/AAMT.

Gervasoni, A., & Sullivan, P. (2007). Assessing and teaching children who have difficulty learning arithmetic. *Educational & Child Psychology*, 24(2), 40-53.

Gervasoni, A., Hadden, T., & Turkenburg, K. (2007). Exploring the number knowledge of children to inform the development of a professional learning plan for teachers in the Ballarat diocese as a means of building community capacity. In J. Watson & K. Beswick (Eds), *Mathematics: Essential Research, Essential Practice Hobart: MERGA (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia*, 305-314. Hobart: MERGA.

Gervasoni, A. (2004). *Exploring an intervention strategy for six ans seven year old children who are vulnerable in learning school mathematics.* Unpublished PhD thesis, La Trobe University, Bundoora.

Gould, P. (2000). Count Me In Too: Creating a choir in the swamp. In Improving numeracy learning: *What does the research tell us? (Proceedings of the ACER Research Conference* 2000, pp. 23–26). Melbourne: Australian Council for Educational Research.

Higgins, J., Parsons, R., & Hyland, M. (2003). The Numeracy Development Project: Policy to practice. In J. Livingstone (Ed.), *New Zealand annual review of education*. (pp 157–174). Wellington: Victoria University of Wellington.

Mitchell, A. & Horne, M. (2011). Listening to children's explanations of fraction pair tasks: when more than an answer and an initial explanation are needed. In J. Clark, B. Kissane, J. Mousley, T. Spencer & S. Thornton (Eds.), *Mathematics: Traditions and [New] Practices (Proceedings of the 23rd biennial conference of The Australian Association of Mathematics Teachers and the 34th annual conference of the Mathematics Education Research Group of Australasia, Vol 1*, pp.515-522). Alice Springs: MERGA/AAMT.

Mulligan, J. (1998). A research-based framework for assessing early multiplication and division. In C. Kanes, M. Goos & E. Warren (Eds.), *Teaching mathematics in new times: Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia (Vol. 2, pp. 404-411)*. Brisbane: MERGA.

Narode, R., Board, J., & Davenport, L. (1993). Algorithms supplant understanding: Case studies of primary students' strategies for double-digit addition and subtraction. In J. R. Becker & B. J. Preece (Eds.), *Proceedings of the 15th Annual Meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 254–260). San Jose: Center for Mathematics and Computer Science Education, San Jose State University.

Steffe, L., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: Philosophy, theory, and application*. New York: Praeger.

Wright, R., Martland, J., & Stafford, A. (2000). *Early Numeracy: Assessment for teaching and intervention*. London: Paul Chapman Publishing.

Zappalà, G. (2003). *Barriers to Participation Financial, Educational and Technological. A report into the barriers to societal participation among low-income Australians.* Camperdown, NSW: The Smith Family. http://www.thesmithfamily.com.au/webdata/resources/files/TSF_barrieries_2003.pdf Retrieved 18 March 2010.

Zevenbergen, R., & Nieske, R. (2008). *Reforming mathematics classrooms: a case of remote Indigenous education. Working Paper 2,* Brisbane: Griffith Institute for Educational Research, Griffith University.

APPENDIX A: ENRP 2010 REFINED ASSESSMENT FRAMEWORK

Growth Points for the Number Domains

Notes:

- *Growth points are not necessarily hierarchical, but involve increasingly complex reasoning and understanding.*
- *It must be emphasised that conclusions drawn in relation to placing students at levels within this framework are based on a 30-minute (approx.) interview only. On-going assessment by the teacher during class will provide important further information for this purpose.*
- *Student understanding may be reported as a "0". This should not be taken as an indication of "no knowledge" or "no understanding", but rather as an indication of a lack of evidence of "1".*
- *The Growth Points below were revised in 2010 as part of the Bridging the Numeracy Gap Project.*

A. COUNTING

0. Not apparent.
   Not yet able to say sequence of number names to 20.
1. Rote counting
   Rote counts the number sequence to at least 20, but is not yet able to reliably count a collection of that size.
2. Counting collections
   Confidently counts a collection of around 20 objects.
3. Counting by 1s (forward/backward, including variable starting points; before/after)
   A count forward and backwards from various starting points between 1 and 100; knows numbers before and after a given number.
4. Counting from 0 by 2s, 5s, and 10s
   Can count from 0 by 2s, 5s, and 10s to a given target.
5. Counting from *x* (where *x* >0) by 2s, 5s, and 10s
   Given non-zero starting points, counts by 2s, 5s,10s to given target.
6. Extending and applying counting skills
   Counts from non-zero starting points by any 1-digit number, and applies counting skills in practical tasks.

C. STRATEGIES FOR ADDITION & SUBTRACTION

0. Not apparent
   Not yet able to combine & count 2 collections of objects.
1. Count all (two collections)
   Counts all to find the total of two collections.
2. Count on
   Counts on from one number to find the total of two collections.
3. Count back/count down to/count up from
   In subtraction contexts, chooses suitably from count-back, count-down-to & count-up-from strategies.
4. Basic strategies (doubles, commutativity, adding 10, tens facts, other known facts)
   Given an addition or subtraction problem, strategies such as doubles, commutativity, adding 10, tens facts, and other known facts are evident.
5. Derived strategies (near doubles, adding 9, build to next ten, fact families, intuitive strategies)
   Given an addition or subtraction problem, strategies such as near doubles, adding 9, build to next ten, fact families and intuitive strategies are evident.
6. Extending and applying addition and subtraction using basic, derived and intuitive strategies
   Given a range of tasks (including multi-digit numbers), can solve them mentally, using the appropriate strategies and a clear understanding of key concepts.

B. PLACE VALUE

0. Not apparent
   Not yet able to read, write, interpret and order single digit numbers.
1. Reading, writing, interpreting, and ordering single digit numbers
   Can read, write, interpret and order single digit numbers.
2. Reading, writing, interpreting, and ordering two-digit numbers
   Can read, write, interpret and order two-digit numbers.
3. Reading, writing, interpreting, and ordering three-digit numbers
   Can read, write, interpret and order 3-digit numbers.
4. Reading, writing, interpreting, and ordering numbers beyond 1000
   Can read, write, interpret & order numbers beyond 1000.
5. Extending and applying place value knowledge
   Can extend and apply knowledge of place value in solving problems.

D. STRATEGIES FOR MULTIPLICATION & DIVISION

0. Not apparent
   Not yet able to create and count the total of several small groups.
1. Counting group items as ones
   To find the total in a multiple group situation, refers to individual items only.
2. Modelling multiplication & division
   Models all objects to solve multiplicative and sharing situations.
3. Partial modelling multiplication & division (some objects perceived)
   Solves multiplication and division problems where objects are partially modelled or perceived.
4. Abstracting multiplication and division
   Solves multiplication and division problems where objects are not all modelled or perceived.
5. Basic, derived and intuitive multiplication strategies
   Can solve a range of multiplication problems using strategies such as commutativity, skip counting and building up from known facts.
6. Basic, derived and intuitive strategies for division
   Can solve a range of division problems using strategies such as fact families and building up from known facts.
7. Extending and applying multiplication and division
   Can solve a range of multiplication and division problems (including multi-digit numbers) in practical context.

Gervasoni (2011). Exploring the whole number knowledge of children in grade 1 to grade 4: Insights and implications. In T. Dooley, D. Corcoran & M. Ryan (Eds.), *Mathematics Teaching Matters.* (Proceedings of the 4th conference on research in mathematics education, pp. 168-178). Dublin: St Patrick's College Drumcondra.

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