

**Changing self-limiting mindsets of young mathematically gifted students
to assist talent development**

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BEd, MEd

A thesis submitted in fulfilment of the requirement of the degree of
Doctor of Philosophy

National School of Education, Ballarat
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Australian Catholic University

Date of Submission

7 October 2018

Declaration

This thesis contains no material published elsewhere or extracted in whole or in part from a thesis by which I have qualified for or been awarded another degree or diploma.

No parts of this thesis have been submitted towards the award of any other degree or diploma in any other tertiary institution.

No other person's work has been used without due acknowledgment in the main text of the thesis.

All research procedures reported in the thesis received the approval of the relevant Ethics/Safety Committees.

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Abstract

The aim of this study was to explore the impact of classroom teachers receiving professional learning about students who are *mathematically gifted*, but who may display *self-limiting mindset tendencies*. There has been an emerging emphasis on affective impacts in education in general (Duckworth & Gross, 2014; Dweck, 2015), and in mathematics learning specifically (Boaler, 2016; Williams, 2014), on nurturing positive, non-cognitive learner dispositions, or mindsets. However, there seems to be little research, and limited discussion in the literature, about the effect of mathematically gifted students' mindsets – of how they perceive themselves as learners of mathematics – and the impact this has on their ongoing mathematics learning, and transforming their gifts into talents (Gagné, 2003). The development of positive learner mindsets in students who are mathematically gifted could have profound implications for these students as individuals, as well as for the future of society as a whole, as their gifts continue to be realised, enhanced and transformed into talents.

The research design adopted for the study was a case study with a narrative analysis. The case was the phenomenon of mathematically gifted students who display self-limiting mindset tendencies, with three students at three different levels of primary school identified for the study. The case study, a descriptive research design, was used to observe and describe the effect of teacher professional learning on the mathematics learning and mindsets of these three students, over a three to four-month period. Data were collected from parent and teacher questionnaires, pre- and post-professional learning interviews with students and teachers, and observations of mathematics classroom lesson involvement. A narrative analysis process was adopted, with direct interpretation from data being the dominant approach, as the findings were to be a *description* of happenings rather than a *frequency* of happenings (Stake, 1995). The narrative analytic procedure used was based around the seven criteria for narrative case study first proposed by Dollard (1935), and revised by Polkinghorne (1995).

Analyses and interpretations of data from this study show evidence of the targeted teacher professional learning having a positive impact on the three case study students' mindsets about successful mathematics learning, and on their approaches to mathematics learning, especially their approaches to challenging tasks. It seems targeted professional learning

may be valuable for teachers to develop an understanding of how support for mathematically gifted students is essential, and what it entails.

Generalisations from a qualitative case study are limited because, by definition, it is a bounded system specific to a small number of individuals in a particular environment (Stake, 1995). However, if, as the findings of this study show, mindsets of mathematically gifted students can be nurtured (and changed if necessary) the implications could be profound if this does, indeed, enable extraordinary capabilities, or gifts, to be realised, enhanced and transformed into talents (Gagné, 2003).

This research may also provide a valuable addition, or a 'link in the chain' to the current knowledge base of mathematically gifted students, and how educators can best support their successful on-going learning. It hopefully provides further highlights, and uncovers new understandings of classroom support required for mathematically gifted students.

Acknowledgements

I would like to give my sincere thanks to the students and educators who willingly gave their time and attention to being involved in this study. This is their story, a story of striving, learning and sharing experiences to help others who may be on a similar journey.

Thank you to Associate Professor Gloria Stillman for being an extraordinarily supportive and encouraging supervisor throughout the whole process of this study. Your support has gone well above and beyond any expectations of a supervisor, and this has not gone unnoticed!

Thank you to my co-supervisor, Dr Jill Brown. Your attention to detail and insightful suggestions have helped to bring the thesis to its final form. I also extend my sincere gratitude to Associate Professor Ann Gervasoni who shared much of this journey with me, providing support, encouragement, and many, many suggestions. Thank you, too, to Professor Peter Sullivan who was willing to provide support and advice.

Finally, thank you to my long-suffering family, especially my husband, Stephen. Thank you for your support, your understanding of the space I have needed, at times, to be emotionally and mentally absent, if not physically, and for believing in me and encouraging me to continue during those times when life became complicated.

Without the support, encouragement and friendship of these people, this thesis may have remained unfinished, and a story I feel so passionate about left untold.

Thank you.

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Glossary of terms

Gifted	Possessing exceptional inherent, general aptitudes or innate capabilities; cognitively developmentally advanced.
Gifted child(ren)	Giftedness, as ‘cognitive developmental advancement’, can be observed from very early childhood; it is not something based on schooling or academic achievement (see Tolan, 1996).
Gifted student	A gifted child as a learner (typically at school).
Gifted underachiever	A gifted student who displays a large differential between potential and performance.
Highly capable	Possessing notable inherent, general aptitudes or innate capabilities. Used synonymously with gifted in this study.
Mathematical creativity	Divergent thinking and independent applications in the exploration of mathematics problems.
Mathematically gifted	Capable of constructing robust mathematical concepts with fewer learning experiences than the majority (90%) of age peers.
Mindset	A perception or belief an individual holds about him or herself; self-perception, self-theory. <i>Learner mindsets</i> <i>Positive learner mindset:</i> an incremental belief about learning – “There is always more to learn; I can always learn more regardless of how ‘smart’ I am.” Potentially leads to <i>self-actualisation</i> . <i>Negative learner mindset:</i> an entity belief about learning – “I am either ‘smart’ or ‘not smart’, which will affect how much I am able to learn. Potentially a <i>self-limiting</i> mindset.
Scaffolding	Temporary supports used by the teacher, e.g., tailored examples or suggestions or targeted questions, to assist students with a task until they can work independently.
Self-actualising	Maximising an individual’s potential, performing the best that he/she is capable of doing.

Self-limiting

Limiting the development of an individual's full potential.

Talented

Possessing accomplished or mastered abilities.

Chapter 1 - Introduction

Nobody tried to figure out anymore how Angeline knew all the stuff she knew.... They called her “a genius.” And even though it didn’t really explain anything, everybody considered it a satisfactory explanation. “She’s a genius” they’d be told, and somehow that would explain it. And that way, nobody ever had to really try to understand.

(Someday Angeline, by Louis Sachar, pp.5-6)

1.1 Background to, and Motivation for, the Study



Ben, 1985

(picture used with permission)

Ben was the first student I taught who showed evidence of mathematical giftedness – a student who seemed to readily construct mathematical concepts with minimal learning experiences required. It was my first year as a graduate teacher in a Reception class (first year of formal schooling in South Australia), in 1985. Thankfully, my university teacher education had nurtured a philosophy of teaching the learner and not just the curriculum, but I felt I had inadequate knowledge and skills to draw on to fully support Ben’s needs.

I felt that I did very little, but his parents seemed to appreciate the little I was able to do, especially when, in subsequent primary school years, they felt that his abilities were largely overlooked, or underappreciated. I was one of the first people they contacted when he was accepted on a full academic scholarship to a prestigious college for his secondary education. Ben eventually became an aviation engineer, and I like to think my attempts at engaging him mathematically in that Reception class had at least some positive impact on his future success.

Since those very early days of my career, I have discovered that although Ben may not have been a typical five-year-old, he certainly wasn’t unique. My own son, Brian, ten years later (1995), started school as a very shy, very compliant, very timid but extremely capable child academically. He constructed new knowledge easily and made generalisations readily, often giving the impression he just ‘knew’ something without having to ‘learn’ it.



Brian: First day of school, 1995

(picture used with permission)

However, he did exactly as he was told at school – no less, but no more either, so I felt his actual capabilities were often left unrecognised. My attitude in those days was that I did not want to be a pushy parent, and that his abilities would eventually be noticed and appreciated. He did not have a happy time at school, though, and although he willingly discussed participation in extra-curricular mathematics competitions, he very rarely enjoyed mathematics as a subject. These days, however, he uses mathematics every day, as a computer programmer. I have regrets about what I did not do for my own son back in those early days, and decided I wanted to study young gifted students (primary school aged). With my experience in mathematics education research I chose to narrow this broad topic to focus specifically on young *mathematically gifted* students.

1.2 The Perceived Problem

Melanie was another mathematically highly capable five-year-old I taught many years ago in Reception. The following year, in Grade 1, Melanie's teacher had a very traditional direct instruction approach to teaching, culminating each week in a Friday test. Each Friday Melanie would turn up at my classroom door, together with a small group of other Grade 1 students, to share their impressive test results. The little group, who all got full marks for their test, would visit each classroom in the school, the office, and the Principal, and the expectation was that we would all hand out stickers or stamps and high accolades for a job well done. My dilemma was, that in looking at Melanie's work, I knew she could have received full marks for each of these tests the previous year in Reception, and it bothered me that she did not seem to be learning anything new in Grade 1.

More than twenty years later, when I first began contemplating a PhD focus on mathematical giftedness, I attended a lecture on *mindsets* by American psychologist Carol Dweck, who at that time was relatively unheard-of in Australia (in education circles at least). For the first time I considered that, while Melanie may not have seemed to be learning anything new academically in Grade 1, she may have been learning something else. She may have been learning that 'smart' students do not need to apply effort, because success is easy and straightforward if you are clever. She may also have been learning that she was valued because she got full marks. If so, according to Dweck, this was to her detriment: Dweck's research had identified that students with these core beliefs about learning develop, what she called, *fixed mindsets* (Dweck, 2006). She described students with fixed mindsets as those who tend to strive for external recognition of their successes, and consequently avoid what they perceive as 'failure' at all costs. This results in risk

aversion, including avoiding academic challenges, and often leaves students plateauing well below their full capabilities (Dweck, 2006).

1.3 Formulating the focus of the study

With my interest piqued, I began a review of the literature on mathematical giftedness, and discovered that in the past ten to fifteen years international studies have addressed issues such as identification of mathematical giftedness (e.g., Bicknell, 2009a; Borovik & Gardiner, 2006; Budak, 2012; Mholo, 2017; Mönks & Pflüger, 2005; Trinter, Moon, & Brighton, 2015; Rosario, 2008); neurological differences in the mathematically gifted (e.g., Dehaene, Piazza, Pinel & Cohen, 2003; Geake, 2009a); providing suitable programs for mathematically gifted students (e.g., Bicknell, 2009a; Chesserman, 2010; Chessor & Whitton, 2007/2008; Diezmann, Lowrie, Bicknell, Faragher, & Putt, 2004; Mönks & Pflüger, 2005); best systems approaches to teaching the mathematically gifted – for example, differentiation (Kronburg & Plunkett, 2008; Yuen et al., 2016), acceleration (Colangelo, Assouline & Gross, 2004; Hannah, James, Montelle & Noakes, 2011), and extracurricular opportunities (Bicknell, 2008; Brandl, 2011; Leder, 2008). Recent research has focused on encouraging mathematically gifted students to explore mathematics creatively (e.g., Jonsen & Sheffield, 2012; Leikin & Lev, 2013; Mholo, 2017; Sheffield, 2017; Singer & Voica, 2016). However, there seemed to be little research, and limited discussion in the literature, about the effect of mathematically gifted students' *mindsets* – of how they perceive themselves as learners of mathematics – and the impact this may have on their ongoing mathematics learning.

Dweck published her book *Mindset: The New Psychology of Success* (2006) after many years of research on motivation and theories of self, commencing in the mid-1980s (see Dweck & Leggett, 1988), continuing through the 1990s (see Mueller & Dweck, 1998), and into the 21st Century (see Grant & Dweck, 2003). Dweck's theory on mindsets recognises two theories of intelligence: 1) an entity theory, which is a belief that an individual is born with a certain level of intelligence that will determine how much they can learn; this is what Dweck calls a *fixed mindset*; and 2) an incremental theory of intelligence, which is a belief that recognises effort, dedication, perseverance and hard work as determining success, not how intelligent an individual is; she calls this a *growth mindset* (Dweck, 2006; 2015). Since this book, and the idea of developing growth mindsets, has gained popularity in education circles (Boaler, 2016), there has been emerging emphasis on affective impacts in education in general, on nurturing positive, non-cognitive learner dispositions (Hannula, 2015;

Ingram, 2017). There seems to have been a resurgence of discussion about traits such as *optimism, resilience, confidence* and *perseverance* (Williams, 2014). Other terms have been introduced to the literature such as *grit* (Duckworth, Peterson, Matthews & Kelly, 2007) and *drive* (Pink, 2009). Grit describes traits such as curiosity, conscientiousness, passion and internal motivation, that are characteristics of successful learners (Duckworth et al., 2007); and drive describes the trait that motivates a person to pursue mastery of something, both those with and those without recognised ‘gifts’ (Pink, 2009). Further abstractions that have been shown to play an important role in the ongoing success of children and adults alike are *creativity* (Robinson & Aronica, 2015), the process of having original ideas that have value; and a sense of *flow* (Csikszentmihalyi, 1992; Nakamura & Csikszentmihalyi, 2009), an optimal state of learning where all sense of time, self, and the world around is lost as all energies are focused on completing the task at hand.

In this research, I have brought an abstraction to the fore which is an amalgam of the above dispositions and traits, the idea of a *self-actualising mindset*. A *self-actualising* (Maslow, 1968; Betts & Niehart, 1988) *mindset* is a belief that academic challenges that require independent thinking, risk-taking and creativity are opportunities to learn resilience and the rewards of effort and perseverance. With this belief, students such as Melanie, Ben and Brian, are more likely to become self-actualising, that is, striving to do the best that they are capable of doing, and maximising their potential (Maslow, 1968). The opposite of a self-actualising mindset is a *self-limiting mindset*. With a lack of sufficient challenge that requires independent thinking or risk-taking or creativity, mathematically gifted students may be at particular risk of developing a skewed view of themselves as successful learners of mathematics. When a student such as Melanie, who has always been highly successful in her Friday tests, is one day faced with a difficult task that cannot be solved easily, her conclusion may be that she has reached her innate level of ability so may as well give up (Dweck, 2006). She may consequently miss out on developing resilience in the face of difficulty (Williams, 2003a), on learning about the rewards of perseverance (Williams, 2014), or on entering a state of flow in the desire to solve a specific problem (Nakamura & Csikszentmihalyi, 2009), and she may avoid any risk-taking that is essential for creative exploration (Robinson & Aronica, 2015). This could possibly be one cause of *underachievement*, a serious and complex issue with many students who are gifted (Betts & Niehart, 1988; Siegle, 2013), and I wondered if this could be addressed successfully, or

better still, prevented, through teacher professional learning on issues associated with giftedness and mindsets.

1.4 Refining the focus of the study

With a focus on *mathematically gifted students with self-limiting mindsets* now determined, I decided to explore the role of professional learning for the classroom teacher. Regardless of whether a student has been formally identified as ‘gifted’ or not, and regardless of whether a school has established, or has access to, a ‘gifted program’, students who are mathematically gifted will still need to be considered in the planning of everyday classroom mathematics programs (Peters & Jolly, 2017; Singer, Sheffield, Freiman & Brandl, 2016). The classroom teacher has a responsibility to understand and cater for the learning needs of all students in their class (ACARA, 2014). This is essential for students who are mathematically gifted even though (or maybe because) these students are already successful in mathematics at their current year level standards (Diezmann et al., 2004). All students need to know that ongoing success requires effort, and that difficulties, even failures, provide an opportunity for further learning through perseverance, applying greater effort, and/or trying different strategies (Sullivan et al., 2013).

Research (e.g., Duckworth et al., 2007; Seligman, Reivich, Jaycox, & Gillham 1995; Williams, 2014) indicates that if we want to encourage future creativity and success, we need to nurture students who are willing to take risks, to persevere in the face of difficulty, and value and thrive on constructive feedback in the learning process. The development of these kinds of positive mindsets in students who are mathematically gifted could have profound implications for them as individuals, as well as for the future of society as a whole, as their gifts continue to be realised and enhanced and transformed into talents (Gagné, 2003).

The complexity of the issue is that teachers need to be aware of mathematically gifted students, whilst at the same time considering mathematically struggling students who require extra assistance, as well as the broad band of ‘average ability’ students in-between. Emerging issues for this study, then, were 1) to explore professional learning for classroom teachers that would encourage an approach to mathematics learning and teaching that would sufficiently engage and challenge students who are mathematically gifted, and yet still be accessible to all students in the classroom; 2) to discover the teacher’s role in encouraging and supporting successful on-going learning for mathematically gifted

students (how much, and what sort of support is required); and 3) to see if students who already display self-limiting mindset beliefs are, with teacher support, able to change their dispositions to more positive, self-actualising mindsets, and what effect this may have on their ongoing mathematics learning.

1.5 The Aims and Scope of the Study

1.5.1 Aims of the Study

The aim of the study is to explore the impact of a targeted teacher professional learning experience, and subsequent teaching practice, on the mindsets and mathematics learning of students who are mathematically gifted, but who are displaying self-limiting mindset beliefs. The aim is two-fold, 1) to contribute to the current literature on mathematical giftedness and mindsets, and 2) to refine and develop a sound, research-based professional learning program for pre-service teachers and classroom teachers of mathematically gifted learners.

The teacher professional learning used for the study was about classroom support for mathematically gifted students, developed through teachers' understanding and appreciation of:

- 1) the characteristics and needs of mathematically gifted learners,
- 2) the need to generate a work ethic within a mathematics classroom culture that values high expectations of personal challenge, effort and creativity, and
- 3) different learner dispositions, and how to address self-limiting mindsets and develop positive, self-actualising mindsets.

The professional learning content focused on how to support the learning of mathematically gifted students within a regular, inclusive classroom structure, using whole-class mathematics tasks, with differentiation if necessary. It has been suggested that accommodating for students who are mathematically gifted within an inclusive classroom can have benefits for *all* students in the class (Sheffield, 2009) – raising expectations, allowing for rich discussions, debunking the misconception that those who are good at mathematics do not have to work hard, et cetera (Rosario, 2008). It was expected that effective teaching strategies for supporting mathematically gifted students would be beneficial teaching strategies for *all* students in the classroom.

I wanted to understand the experiences of students, and their teachers, through observing, describing and interpreting their experiences. This resulted in the decision to choose a

qualitative case study, as defined by Merriam (1988). Merriam believes that “research focussed on discovery, insight, and understanding from the perspectives of those being studied offers the greatest promise of making significant contributions to the knowledge base and practice of education” (p. 3). My ‘case’ was *the phenomenon of mathematically gifted students with self-limiting mindsets*, and the design was a multiple, or collective, case study (Merriam, 1998), with purposefully selected participants within the case (Merriam, 2009). According to Merriam (1998), “The inclusion of multiple cases is a common strategy for enhancing the external validity or generalisability of your findings” (p. 40). I chose to work with three individual students, and their teachers, at three different levels of primary school – early years (5-7 years old), middle primary (8-10 years old) and upper primary (11-12 years old), for a period of approximately six months. This was so I could compare data, in part, to see if different issues arose at different stages of Primary School education.

To analyse the data, I chose a *narrative analysis* (Polkinghorne, 1995). Narrative analysis is a re-telling of collected data as a narrative, embedded with analytical interpretations (Merriam, 1988; Polkinghorne, 1995), and is not to be confused with *narrative inquiry*, which is an analysis of participants’ stories, or narratives. My findings are presented, firstly, as three individual ‘stories’ (analyses of the three students’ experiences), followed by a cross-case analysis (Merriam, 1998) that explores commonalities and similarities between these stories.

1.5.2 Research Questions

The case study sought to address the question:

- *What impact does targeted teacher professional learning about classroom support for mathematically gifted students with self-limiting mindsets, have on the mindsets and mathematics learning of these students?*

I provided professional learning for the classroom teachers, developed from current literature on mathematics education, giftedness and mindsets (see Chapter 2 and Chapter 5). This was followed by a period of approximately three months where the teachers enacted their newfound understandings in their classrooms. Pre- and post-professional learning sessions (and subsequent teaching period) interviews with the teachers and students, and observations of mathematics lessons, provided the foundational data set for analysis. Interviewing and observing both the students and their teachers helped form an

overall picture of the impact the professional learning had on teacher approaches, and the learning and mindsets of the three mathematically gifted students.

The subsidiary questions, derived from the overarching question, which framed the research were:

- How do students approach challenging mathematics tasks before and after their teachers receive professional learning (and a subsequent three-month teaching period)?
- How do students who are mathematically gifted view themselves as mathematics learners (i.e., what are their mindsets) before and after their teachers receive professional learning (and a subsequent three-month teaching period)?
- What do the teachers do during the post-professional learning period to support the mathematics learning of students who are mathematically gifted but with self-limiting mindset tendencies?

1.5.3 The Scope of the Study

There are three main areas that need to be acknowledged in the scope of the research:

Firstly, the nature of a case study means there is a limited scope (Merriam, 2009). The purpose of qualitative research is to describe, understand, and interpret a specific situation, rather than generalise back to other populations (Merriam, 1988), so definitive conclusions are limited. However, the depth of rich description and interpretation within a case study may provide particular features that may be recognisable, and helpful, to others in similar situations (Lincoln & Guba, 1985), what Bassey (1999) calls *fuzzy generalisations*. The purpose, and therefore scope, of this research was to describe, analyse and interpret the experiences of three students, and their teachers. The results will provide the reader ‘fuzzy generalisations’ only, but with sufficient detail to make individual judgements about applicability to other contexts.

Secondly, the purpose of this research was not to propose a definitive process for identifying mathematical giftedness, however, to select participants for the study an identification process was required. Identification of students who are mathematically gifted is not a simple linear process. It requires a multi-faceted approach, from formal methods of testing to informal methods of observation and conversation (McAlpine, 2004; Moon, 2006; Reis, 2004). For this study I employed teacher nominations, a parent questionnaire, previous mathematics assessment data, and an assessment task I designed specifically to identify Krutetskii’s (1976) hallmarks of mathematical ability. This process

is described fully in Chapter 4, and, although appropriate for my selection of participants, is not expected to be a definitive process for identifying all mathematically gifted students. Thirdly, the purpose of this research was not to develop a definitive method for identifying mindset tendencies, but, to select participants with a self-limiting mindset, an identification process was required. I employed an adaptation of a method used by Mueller and Dweck (1998) whereby children were asked to work on a set of problems designed to contain a choice of tasks that identifies students' self-theories of what a successful learner is required to do (Dweck, 2010a). This method is described and explained in full in Chapter 4, but, once again, while appropriate for my selection of participants, it is not expected to be a definitive process for identifying all students with self-limiting mindsets.

1.5.4 Defining 'mathematical giftedness'

For this study I have adopted, and adapted, Gagné's (1985; 2009) *Differentiated Model of Giftedness and Talent* (DMGT) to define giftedness in general terms. The DMGT differentiates between 'gifts' and 'talents', and is paramount in understanding findings of this research. Gagné's model recognises that *gifts* are inherent, general aptitudes or innate capabilities, which are only realised as *talents* (mastered abilities) through optimal 'intrapersonal factors' and 'environmental influences' (see Figure 1.1 for a simplified illustration of the DMGT). The DMGT suggests that a child who is gifted in mathematics, for example, does not automatically become successful, let alone outstanding or recognised as talented in mathematics, without optimal environmental influences and interpersonal factors. 'Environmental influences' required for this to happen may include school provision of appropriate mathematics learning experiences; 'intrapersonal factors' required may include a positive learner mindset. It is therefore, in part, the role of the school and teachers to provide every opportunity for gifted students' potentials to be realised as talents. As stated in the Australian Curriculum, "The school plays a critical role in giving students appropriate opportunity, stimulation and experiences in order to develop their potential and translate their gifts into talents" (ACARA, 2014, para. 1, 8).

As an aside, contrary to Gagné's model (2003), I would suggest that giftedness is not a prerequisite for talent development; that other students, through optimal learning environments, hard work and perseverance can also develop mathematical expertise (Dweck, 2006). Just as being gifted in mathematics does not necessarily lead to

mathematical talent or expertise, achieving mathematical expertise does not necessarily require being mathematically gifted (Brandl, 2011; Øystein, 2011). This, however, does not eliminate the fact that there are students who are mathematically gifted.

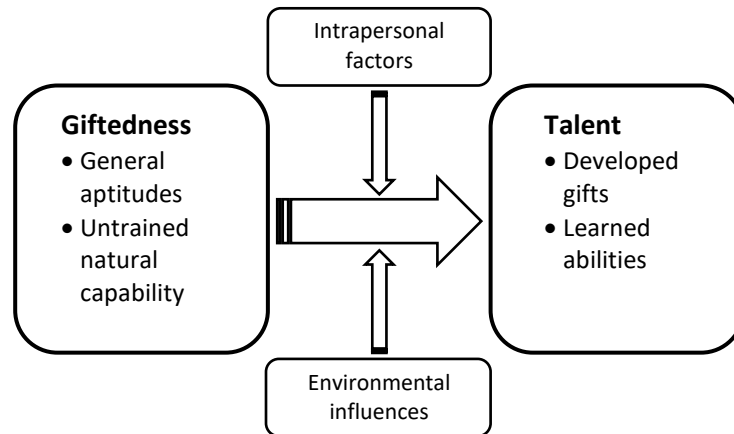


Figure 1.1 Simplified illustration of Gagné’s DMGT (adapted from Gagné, 2009)

Within society, it seems people generally tend to equate giftedness with ‘prodigy’ or ‘genius’ (Silverman, 2013; Winner 1996), the ‘one-in-a-thousand’ profoundly gifted child with a highly exceptional talent. This view is perpetuated by many stories and movies about gifted children (e.g., *Little Man Tate* (1991); *Gifted* (2017)), and, due to the belief of their scarcity, hampers attempts at specific educational provisions for the many students who are capable of working beyond set grade standards.

For the purposes of this study I have defined mathematically gifted as *students who are capable of constructing robust mathematical concepts with fewer learning experiences than the majority (90%) of their aged peers*. This somewhat arbitrary figure of 90% (Bélanger & Gagné, 2006) has been based on Gagné’s (1998) suggested prevalence of giftedness, which equates to 10% of the student population. There is no definitive way of measuring mathematical giftedness (Singer et al., 2016), but this figure suggests there may be, on average, two to three mathematically gifted students in an average class of 25-30 students.

The term ‘gifted’ also tends to be somewhat objectionable to many, and possible reasons for this are discussed in Chapter 2. Indeed, the Principal of the school I worked with specifically requested I not use the term gifted when talking with teachers, parents or students. For this reason, I used the term ‘highly capable’ when talking to teachers and corresponding with parents, and have used ‘highly capable’ interchangeably with ‘gifted’ at times throughout this thesis. I have chosen to intentionally use both terms, as I wanted

to conduct *use-inspired basic research* (Stokes, 1997) that would be beneficial for dissemination of research findings in both academic and professional literature.

Further detailed definitions of mathematics, giftedness and mindsets will be presented in Chapter 2, *An Exploration of the Literature*.

1.6 The Significance of the Study

The discourse about why it is necessary to consider the specific needs of mathematically gifted learners has historically been based around benefits to society and our “globally competitive economy” (Office of the Chief Scientist, 2014), with provision for the gifted even being described as “human capital development” (Ibata-Arens, 2012, p. 3). Mathematicians are a sought-after resource for our modern technological society (Singer et al., 2016; Sheffield, 2017). The Australian government is currently striving for improved mathematics outcomes in Years 11 and 12 (Commonwealth of Australia, 2016), recognising that mathematics ability is required in so many diverse areas – economics, financial marketing, commercial operations, analysis and interpretation of data, real world modelling in the engineering sciences, information technology (computer programming), and new interdisciplinary fields such as bioinformatics (which encompasses modern biology, mathematics, statistics and computer science), just to name a few. If we are inadvertently quashing young gifted mathematicians in the early years of their education by overlooking the effects of self-limiting mindsets, it could be argued that society is potentially missing out on the contribution of the long-term capabilities of these individuals.

However, benefits to *individual students* are what primarily concern me in this study, with benefits to society being secondary to this. In the latest Victorian Government Department of Education’s *Strategy for gifted and talented students and young people* (2014), the benefits to the individual are highlighted: “The chance to realise their potential, pursue a passion and develop a love of learning” (Department of Education and Early Childhood Development, 2014, p. 5). My desire is to enable teachers to encourage their gifted students to develop a love of learning, and to allow them to pursue their passions. If it is possible for teachers to intentionally assist gifted students to develop mindsets that enable their gifts to be transformed into talents, then knowledge of how to do this needs to be recognised and disseminated. Gifted students may be particularly vulnerable in developing self-limiting mindsets if work is not challenging enough, and their worth is recognised in success or

achievement (which may often be relatively easy for them) and not in effort, or perseverance, or resilience in the face of difficulty or ‘failure’ (which are important for ongoing, meaningful mathematics learning) (Williams, 2014). Mueller and Dweck (1998) have shown it *is* possible, to not only cultivate positive mindsets in students, but also to change the mindsets of students who are already displaying self-limiting mindset tendencies. This study, therefore, endeavours to replicate this change, for mathematically gifted students with self-limiting mindsets, by providing regular classroom teachers with strategies based, in part, on findings from Mueller and Dweck’s studies.

The significance of this research lies in the creation of new knowledge gained from the analysis of the impact of targeted teacher professional learning on the mindsets and learning of students who are mathematically gifted but with self-limiting mindset tendencies. The goal is to learn how to generate and maintain a classroom learning environment that supports mathematically gifted students’ ongoing mathematics learning; a classroom environment that cultivates a learner mindset that includes a willingness to persevere, to take risks and to learn from mistakes, to be free to be creative and to value the role of effort in learning. With this sort of environment, it is envisaged that young gifted learners may realise their potential, and hopefully develop an ongoing appreciation for, love of, and willingness to pursue mathematics into higher levels of education and beyond. “Those gifted children most likely to develop their talent to the level of an expert will be those who have high drive and the ability to focus and derive flow from their work” (Singer et al., 2016, p. 6).

1.7 Contribution to Knowledge

The anticipated contribution to knowledge from this study is potentially three-fold. There are contributions to: 1) the field of gifted education, 2) the field of mathematics education, and 3) the field of affect in education. While research abounds in each of these three fields separately, as well as research in the fields of giftedness in mathematics and mindsets in mathematics, a review of the international literature revealed very little in terms of interaction between all *three*. Thus, the outcomes of this study will include insights, for both researchers and practitioners, on the impact of addressing self-limiting mindset issues with mathematically gifted students. In a discourse about recent research on mathematically gifted students, Singer et al. (2016) comment that we do not yet know “how much a growth mindset as described by Dweck (2006) and others might help ... increase a student’s

mathematical performance and passion” (p. 16). This study may go some way towards addressing this question.

1.8 Overview of the Study

This introductory chapter has presented a context for the study. This includes the perceived problem of young mathematically gifted students with self-limiting learner mindsets, and the aim of the study, which is to explore how to address the perceived problem through targeted teacher professional learning. Brief definitions of ‘mindset’ and ‘mathematically gifted’ have been provided, and the significance and proposed contribution of the study explained. The thesis proceeds with a detailed description of the research and its outcomes. Chapter 2 provides an exploration of the literature; making the connections between mathematical giftedness, mathematics as a discipline, mathematics education and mindsets. Chapter 3 explores the theoretical and methodological framework underpinning the study. It also includes an overview of the research design – a description of the research instruments, data collection methods, and data analysis techniques employed – and the ethical considerations and trustworthiness issues required for a qualitative case study. Full details of the selection process for identifying the participants – using teacher, parent, and student data – are described in Chapter 4, with Chapter 5 outlining the targeted teacher professional learning process and suggested teaching strategies. Chapter 6 describes the three case study students – Fred, Sammy and Alex – and tells their individual ‘stories’ within a narrative framework of pre- and post-teacher professional learning comparisons. This is followed by an exploration of the phenomenon of mathematically gifted students with self-limiting mindsets in Chapter 7, which provides a discussion and interpretation of the case, based on a synthesis of the findings of the three narratives in Chapter 6. Chapter 8 sums up the study with a conclusion, including contributions to current knowledge with implications from this study, and recommendations for further research.

Chapter 2 – An Exploration of the Literature

Giftedness, Mathematics Education, Mathematical Giftedness, and Mindsets: Making the Connections

2.1 Chapter Overview

This chapter provides a review of the literature relevant to a study about students who are mathematically gifted, but who demonstrate self-limiting mindset behaviours when viewing themselves as learners of mathematics. It aims to provide background information required to situate this study as a ‘link in the chain’ of current research that is expanding knowledge about mathematically gifted students, and aims to highlight the need for, and establish the importance of, further research in this area.

The chapter is divided into five main sections: 1) a consideration of the history that leads to modern views of ‘giftedness’; 2) an understanding of mathematics as a subject discipline, and the requirements for effective twenty-first century mathematics education; 3) an examination of the characteristics of mathematically gifted children, and current education options for these students; and 4) an exploration of mindsets as a non-cognitive affective domain of learning. Figure 2.1 shows how these first four sections integrate to become the gestalt of the literature exploration. Section 5) reviews research on effective professional learning practices to establish the professional learning design for this study. Each section provides an important part of the whole in research on how to support the learning of students who are mathematically gifted, but who present with self-limiting mindset behaviours. The chapter includes a consideration of the paucity of research on the effect of mindsets on students who are mathematically gifted, thus establishing the contribution to knowledge this study may provide in both theoretical and practical knowledge about how to support the ongoing learning of these students.

2.2 Giftedness

Much stigma surrounds the issue of children being identified as gifted (Silverman, 2013). Intellectual, or academic giftedness has developed negative connotations with both the general public, and many educators (Bégin & Gagné, 1994; Carrington & Bailey, 2000; Geake & Gross, 2008; Silverman, 2013). The way gifted children are stereotypically portrayed in television shows and movies (especially comedies) attests to this (Kendall,

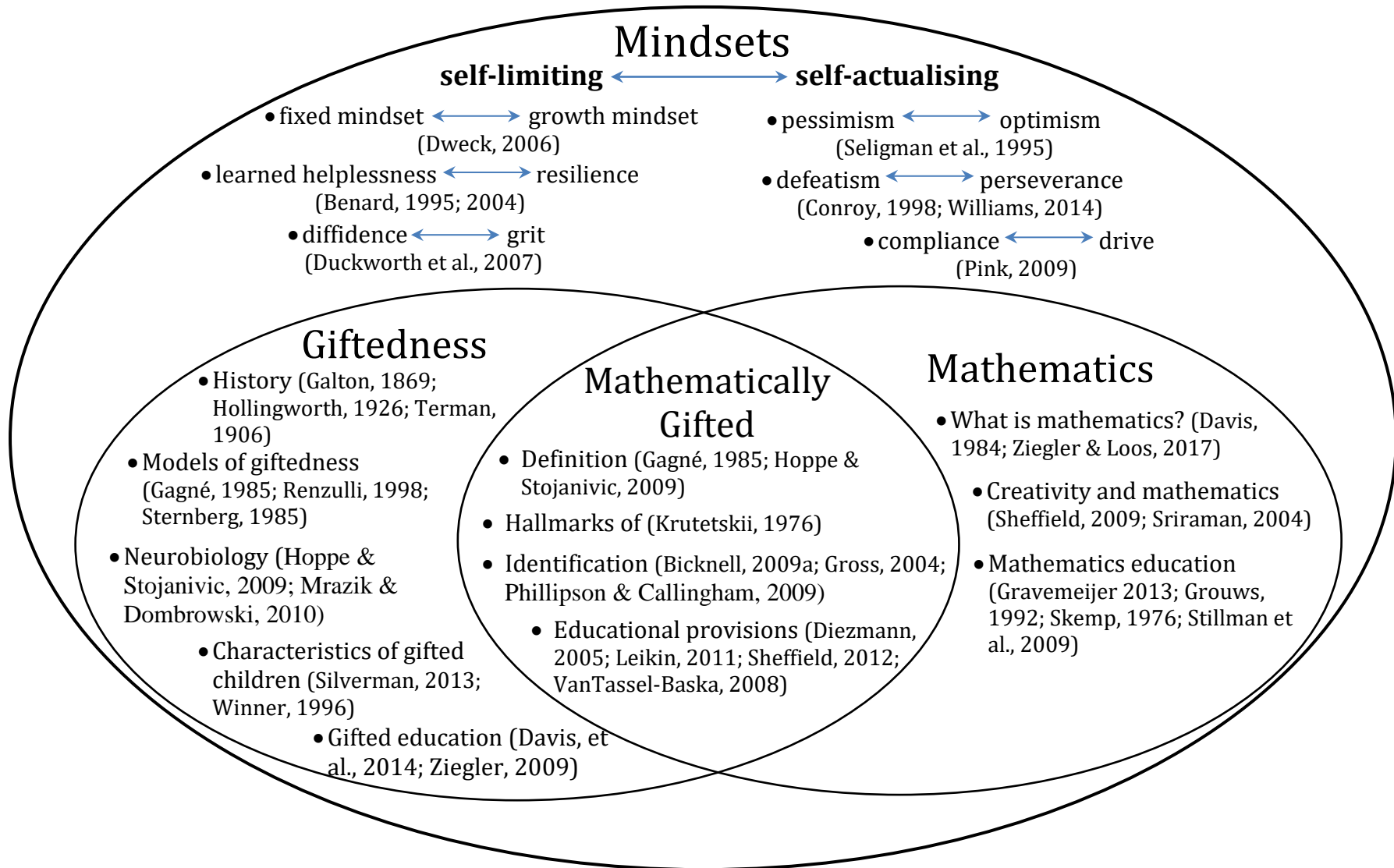


Figure 2.1 Conceptual Framework for Literature Review

1999). This prejudice is not confined to any one country or culture (Geake & Gross, 2008), nor is it shared with precocious behaviours in other areas such as sport, or the arts (Bégin & Gagné, 1994; Geake & Gross, 2008). There are many conjectures put forward for the reasons for this, but the fact remains that discussions regarding provisions for academically gifted children continue to need positive advocates (Silverman, 2013).

The definition of giftedness adopted for any study will influence both the identification of, and curricula needs pertaining to, gifted students, so needs to be articulated clearly. Considering historical definitions of giftedness is necessary for establishing the chosen definitions for this study, which will, in turn, influence any compelling grounds for the need for the study. For example, if giftedness is defined as something measured by outstanding accomplishment, the common question, ‘Why do we need to cater for gifted children, aren’t they going to succeed anyway?’ (Silverman, 2013; Winner, 1996) is possibly a valid argument. However, if giftedness is defined as the possession of an unusually high natural capacity for learning, with a potential that may, or may not, be realised due to various life circumstances (including the way these children are taught) (Gagné, 2003), then it becomes an issue of equity, where we are obligated to provide adequate educational opportunities for gifted students.

With a background in mathematics education, not gifted education, it was essential for me to spend considerable time familiarising myself with the particulars of ‘giftedness’. One of the books I accessed as a teacher, many years ago, on meeting the needs of the gifted child in the regular classroom, stated, “The area of Math ... [was] purposefully excluded from this book. It is assumed that if children are gifted in Math [sic], a text a grade level or two above the regular class’s text will be provided. This effectively meets their needs” (Cochran, 1992, p. 9). There is a significant issue when a publication on giftedness has obviously not adequately considered or addressed the basic foundations of mathematics education. There would be a similar issue with any research on mathematically capable students that has not adequately considered the basic foundations of giftedness.

There is a significant body of research on mathematics education, and a wealth of research on gifted students, but there remains a gap in the dissemination of this research from one field to the other (Leikin, 2011). As Leikin points out,

Analysis of the research literature in the fields of gifted education and mathematics education leads to the conclusion that the studies in these two fields moved in two

tangential rather than intersecting directions ... mathematics education is underrepresented in the field of gifted education, and, vice versa, the research on giftedness and gifted education is underrepresented in the field of mathematics education (p. 168).

For the mathematics component of this research to have an impact within the field of gifted education, the giftedness component needed to be robust, and vice versa. What follows, therefore, is a brief historical background to the concept of giftedness, a synopsis of various understandings and models of giftedness, and a list of recognised characteristics and behaviours of gifted children. This will situate my study within a specific definition of 'gifted', address some of the prejudices that may be based on common fallacies, and establish a basis for the analysis and discussion of a case study of mathematically gifted students.

2.2.1 A Brief History of Giftedness

Despite a rich recorded history of outstanding, eminent humans stretching back as far as ancient Egyptian and Babylonian times, the first scientific exploration of human intelligence is relatively recent (Silverman, 2013). In 1869, Francis Galton, a British anthropologist, published *Hereditary genius: An inquiry into its laws and consequences*. This was the first social scientific attempt to study “genius and greatness in man [sic]” (Galton, 1869), and the first quantitative analysis of human intelligence (Silverman, 2013). Galton was a pioneer of the eugenics movement around the turn of the twentieth century, and was interested in identifying genius in order to ‘better society’ through selective breeding – a very elitist intention. Galton’s definition of genius correlated with ‘man’s [sic] eminence’, and he believed children could inherit the potential to become geniuses (Galton, 1869; Gross, Macleod, Drummond & Merrick, 2001).

In 1905, Alfred Binet, a French psychologist, introduced the idea of ‘mental age’, and, together with physician Théodore Simon, created the first structured intelligence test, the *Binet-Simon Scale*, which measured intelligence as a ratio of a child’s chronological age to their assessed mental age. This produced an intelligence quotient, or IQ. The IQ test was initially intended to discriminate between children who were educable and those who were not, but has maintained popularity even to today, albeit amongst much criticism, in the identification of giftedness, especially in the United States of America (USA) (Gross, Macleod et al., 2001; Silverman, 2013).

In 1906, Lewis Terman, a USA psychologist who was also a prominent eugenicist, published his dissertation, *Genius and stupidity: A study of some of the intellectual processes of seven 'bright' and seven 'stupid' boys* (Terman, 1906), which was based on analysed data from 'mentally superior' children. In 1917, Terman revised the *Binet-Simon* intelligence test, resulting in the *Stanford Revision of the Binet-Simon Scale* (or *Stanford-Binet Scale*), in order to enable identification of children who were 'intellectually superior' (Terman et al., 1917) – yet another very elitist and audacious epithet. His summation was that 'bright' children find learning easy, and are therefore high achievers. He also surmised that children of high intelligence are generally physically and emotionally healthier than children of average intelligence (Gross, Macleod et al., 2001). Terman's data became the catalyst of a popular stereotype, that children of superior intelligence are superior in all areas of growth and development, from physical development, to emotional wellbeing and health, to being more resistant to temptations and juvenile misbehaviour, even though this was never Terman's intent (Whitmore, 1980). This myth prevails still, and when assumed by teachers, results in comments such as, 'If he/she is so gifted, why can't he spell?' 'Why can't she tie her shoes?' 'Why is he always crying?' 'Why is she so immature?' (Silverman, 2013). Interestingly, there is a second, opposing view that believes "gifted children are at greater risk for emotional and social problems, particularly during adolescence" (Neihart, 1998, p. 10). Terman is often considered the 'father' of gifted education due to his lifelong devotion to understanding the gifted (Gowan, 1977; Silverman, 2013), but his studies have inadvertently contributed to two of the modern myths about gifted children: 1) that they (and/or their parents) in some way consider themselves 'superior', and 2) that they are capable of succeeding on their own without any special assistance (Whitmore, 1980).

The actual term *gifted child* first entered psychological literature in the 1920s. It was used by Leta Stetter Hollingworth (1926), another USA psychologist, to delineate children of high intelligence, as measured by the *Stanford-Binet IQ Scale*. Whereas Terman's interest was in *describing and measuring* intelligence, Hollingworth's interest lay in the *psychology* of giftedness, and in planning educational opportunities to support the further development of gifted children (Silverman, 2013). Unfortunately, unlike terms such as *idiot*, *imbecile*, *stupid* and *retarded*, which were once acceptable terms to describe children of lower intelligence but have now been replaced with less stigmatising terms, the term *gifted* has never been successfully ameliorated, even though it has maintained negative connotations

of superiority and elitism (see, for example, Boaler, 2015). This may be one reason for the lack of focus on gifted education in schools (Peters & Jolly, 2017).

2.2.2 Some Contemporary Models of Giftedness

From the 1970s onwards, questions began to arise about the validity of intelligence tests to define giftedness for a number of reasons, not the least of which were that a) intelligence itself is a difficult-to-define construct, and b) standard IQ tests are very culturally biased (Hampshire, Highfield, Parkin & Owen, 2012; Whitmore, 1980). In 1978, Joseph Renzulli, a USA educational psychologist, suggested, as an adjunct to IQ measures, that there was an interaction between three traits in gifted individuals: above average general intelligence (high IQ); high levels of task commitment; and high levels of creativity. He called this a *Three-ring Conception of Giftedness* (Renzulli, 1998), and suggested that a person cannot be identified as gifted without an element of all three traits. “Gifted and talented children are those possessing or capable of developing this composite set of traits and applying them to any potentially valuable area of human performance” (Renzulli, 1978, p. 261). He makes a distinction between the ‘gifted’ and the ‘potentially gifted’: his definition relies on high achievement, so an underachieving gifted student would be, by definition, only ‘potentially gifted’.

In 1985, Robert Sternberg formulated a *Triarchic Theory of Intelligence*, which also takes into consideration three sub-theories: componential intelligence (associated with analytical giftedness, or IQ), experiential intelligence (associated with synthetic giftedness, or creativity), and contextual intelligence (associated with practical giftedness, or ‘street smarts’) (Sternberg, 1985). Sternberg deemed that a gifted person may show high intelligence in one or more of these three intelligence domains. A person’s success depends on how well these are balanced against each other, but success is not classified as a requirement for an identification of giftedness. His three loci of intellectual giftedness include dimensions that take into consideration creativity, and the value of a person’s culture, things that standard IQ tests do not measure, however, Sternberg’s model of giftedness is also based on observable intelligences, and does not consider the phenomenon of the underachieving or ‘hidden’ gifted child.

Around this same time, in 1985, François Gagné, a Canadian professor of psychology, introduced a developmental theory he called the *Differentiated Model of Giftedness and Talent (DMGT)* (Gagné, 1985; 2009). This model expressly differentiates between gifts

and talents. Gifts are unusually high *natural abilities* significantly beyond age peers, and talents are unusually high *levels of achievement* or performance, that are systematically developed from gifts. Gagné's conviction is that gifted children do not automatically become productive achievers; that is, some gifted children may not become talented, or 'intellectually successful' (Gagné, 1991). Talented children are easy to spot. They are the ones achieving great things. Many talented children are also gifted, however not all gifted children are talented (Gagné, 2003; Gross, Macleod et al., 2001). According to Gagné's model, gifts may remain latent for a variety of environmental or intrapersonal reasons, including, for example, poverty, cultural or language barriers, learning disabilities or health issues, geographical isolation, certain temperament or personality factors, or poor learning opportunities or experiences.

The *Differentiated Model of Giftedness and Talent* has gained wide acceptance in Australian education (Gross, Macleod et al., 2001). It is the key model used in discussions about gifted and talented students in the Australian Curriculum document (ACARA, 2014), with the document recognising that, "the school plays a critical role in giving students appropriate opportunity, stimulation and experiences in order to develop their potential *and translate their gifts into talents*" (ACARA, 2014, para 8, italics added). However, although the term 'gifted and talented' was introduced to differentiate between the 'gift' and the 'talent', popular usage often seems to simply replace the word 'gifted' with the phrase 'gifted and talented', with the terms used synonymously and/or interchangeably.

Since 1985, Gagné's *Differentiated Model of Giftedness and Talent (DMGT)* has been revised (see Gagné, 1995, 2004, 2009), and expanded. The *Developmental Model for Natural Abilities (DMNA)* (Gagné, 2015) addresses the non-behavioural influences, or biological 'basements', of outstanding natural abilities, or gifts, and, the most recent revision, the *Comprehensive Model of Talent Development (CMTD)* (Gagné, 2015), merges the *DMGT* with the *DMNA*. The limitations of this study do not extend to the issue of where or how gifts develop in the first place (the nature versus nurture argument), so the *DMNA/CMTD* expansions do not add anything further for consideration.

In 1991, The Columbus Group – a group of USA practitioners, parents, and theorists – gathered together to construct a phenomenological, rather than utilitarian, definition of giftedness (Columbus Group, 1991; Silverman, 2013). Their definition of '*asynchronous development*' refers to uneven intellectual, physical, and emotional development, which combine to create inner experiences and awareness that are qualitatively different from the

norm (Columbus Group, 1991). A gifted child may not fit the cultural expectations of how a child of his or her chronological age 'should' think, feel or act (Morelock, 1992). Asynchronous development places gifted people outside normal development patterns from birth to adulthood, and as such views giftedness as 'cognitive developmental advancement' that can be observed from very early childhood, and not something based on school, or academic achievement (Wardman, 2018; Silverman, 1997). Tolan, a member of the Columbus Group, adopted a cheetah metaphor to describe this concept of giftedness, which could be viewed as a direct reproach to some definitions of giftedness, such as Renzulli's *Three-ring Conception*, by addressing the notion of 'potential giftedness'.

The cheetah is the fastest animal on earth. When we think of cheetahs we are likely to think first of their speed. It's flashy. It is impressive. It's unique. And it makes identification incredibly easy. Since cheetahs are the only animals that can run 70 mph, if you clock an animal running 70 mph, IT'S A CHEETAH! But cheetahs are not always running ... IS IT STILL A CHEETAH? ... if the cheetah is only six weeks old, it can't yet run 70 mph. IS IT, THEN, ONLY A *POTENTIAL* CHEETAH? (excerpt from *Is it a Cheetah?* Tolan, 1996, p. 1, 2, bold type from the original)

Asynchronous development focuses on the "psychological milieu of the individual, it highlights the complexity of the individual's thought processes, [and] the intensity of sensation, emotion, and imagination" (Silverman, 2013, p. 43). This definition adds a further psychological element to giftedness, which is worth considering when working with young children, as it may explain some of the idiosyncratic vulnerabilities of gifted students, where "advanced cognitive abilities and heightened intensity combine to create inner experiences and awareness ... [that] render them particularly vulnerable" (Columbus Group, 1991, para 1).

Specific definitions of giftedness also influence the estimated prevalence of gifted students. Whereas some definitions situate giftedness as two standard deviations from the mean, that is, limited to a prevalence of 1%-2% of the general population (e.g., Silverman, 2013), other definitions broaden this estimate to 10% of the population being the minimum threshold within various, hierarchical, levels of giftedness (e.g., Gagné, 1998), and up to 20% of the population (e.g., Renzulli, 1986).

The history of definitions and models of giftedness is important in considering the perceptions people have developed of giftedness. Definitions have ranged from

associations of superiority and elitism (e.g., Galton, 1869 & Terman, 1906), to sounding like some type of medical condition (e.g., *asynchronous development*, or ‘AS’, as it is referred to colloquially). This may give some indication as to why the term ‘gifted’ seems to have negative connotations within society, and is something that continues to need to be addressed if gifted children are to be viewed as students warranting special consideration within the education system.

2.2.3 Neurobiology and giftedness

“There is substantive evidence that gifted individuals have atypical brains and atypical brain functioning” (Mrazik & Dombrowski, 2010, p. 230). According to Mrazik and Dombrowski (2010) the issue of *defining* giftedness has far outweighed the issue of exploring possible underlying *neurobiological* aspects of giftedness. This may be due to neurobiological explorations of the living brain being a relatively new possibility. Prior to the advent of brain imaging technologies such as computerised tomography (CT scans) and functional magnetic resonance imaging (fMRI scans), the brain could only be examined postmortem (Sousa, 2003). Since the late twentieth century, however, research on the *anatomy* of the human brain has been expanded to include research on the *functioning* of the human brain. Processes such as *neural myelination* and *neural efficiency* can now be observed, measured and compared (Hoppe & Stojanovic, 2009; Mrazik & Dombrowski, 2010). Consequent research has shown that gifted individuals show increased neural efficiency – the ability to form strong neural circuits with minimal repetition in the learning process (Hoppe & Stojanovic, 2009; Sousa, 2003). Miller (1994) suggested that this increased neural efficiency may be a result of stronger neural myelination in gifted individuals. That is, the myelin sheath (that surrounds each neural axon) is produced faster and thicker in gifted children, which consequently enables faster impulse transmission with less electrical activity (or learning experiences) in gifted individuals than in non-gifted individuals (see also Hoppe & Stojanovic, 2009). Zhang, Gan and Wang (2015) have explored neural efficiency of adolescent mathematics learners, and found that, for mathematically gifted students, neural responses to novel tasks rapidly changed from an “effortful processing mode to a more automated processing mode” (pp. 504-505), with this rapid decrease in brain activity being further indication of a faster than normal learning process. This relatively recent discovery of a physiological, neurobiological explanation for giftedness may provide objective evidence of a previously subjective view of giftedness, and help differentiate between ‘giftedness’ and ‘high achievement’.

2.2.4 Characteristics of Gifted Children

Due to the previously unquantifiable nature of giftedness, many myths, stereotypes and misconceptions about characteristics of gifted children have taken root in Western society, producing many negative connotations (Silverman, 2013; Winner, 1996). It is important to address these misconceptions, and this section attempts to defend the realities by confronting some of the common myths, and identifying some truths.

Some Myths

Myth #1: All children are gifted.

Due to strong egalitarian ideals that developed in education in the early 1980s, the idea of singling out gifted students became an anathema for many (Gross, Macleod et al., 2001; Valpied, 2005). To say one child is more intelligent than another became politically incorrect. This idea has continued, in some circles, through to current times, generating equally vehement responses on both sides of the argument of whether or not all children are gifted, with some researchers (e.g., Boaler, 2016; Gladwell, 2008) and parents agreeing wholeheartedly, and other researchers (e.g., Gross, Macleod et al., 2001; Silverman, 2013; Winner, 1996) and parents of gifted students arguing against such pronouncements. Animosity toward giftedness is most likely born from associations with elitism: that gifted children are somehow better than other children (cf. Galton, 1869, & Terman et al. 1917). Gifted children are not better, or worse, than any other children, but they are qualitatively different in the way they learn (Gross, Macleod et al., 2001; Mrazik & Dombrowski, 2010). Just as students with intellectual *disabilities* require ‘special treatment’ in education, so too do students with intellectual *hyper-abilities* (Gross, Macleod et al., 2001).

Even with perfect teaching methods, individual differences, in the sense of different levels of ability, will not be obliterated. Everyone will be able, but there will still be no equality in this respect. In any given province of knowledge some will be relatively more capable, others relatively less so. (Krutetskii, 1976, p. 5)

Myth #2: Children become gifted by being pushed by their parents.

Some people assert that children are ‘made’ gifted by overzealous parents who push their children to learn school-type skills such as reading and arithmetic at an early age (Winner, 1996). With giftedness, though, there is a distinction between *what* is learned, and *how* it is learned. Gifted children are those children who learn at a faster rate (Gross, Macleod et al., 2001; Munro, 2012; Sousa, 2003), which means they require minimal experiences to develop neural networks, or schema, in constructing meaningful concepts (Geake, 2009a;

Hoppe & Stojanovic, 2009; Newman, 2008). They demonstrate these developmental differences, and exceptional reasoning abilities, from a very young age (Csikszentmihalyi & Robinson, 2014; Silverman, 2013). Indeed, it is often the gifted child who is the one pushing the parents (Routledge, Gute, Gute, Nakamura & Csikszentmihalyi, 2014; Winner, 1996).

Myth #3: Gifted students are fast workers.

There is a difference between a fast *learner* and a fast *worker*. A common characteristic of giftedness is perfectionism (Callard-Szulgit, 2012; Siegle, 2013; Silverman, 2013; Whitmore, 1980), and perfectionism often requires careful attention to detail that can be quite slow and laborious. This perfectionism needs to be harnessed as a quality for excellence, not as laborious work on reproducing content that is already known, or as a crippling unattainable ideal (Silverman, 2013). Some gifted students can also be unmotivated, or procrastinators, or not finish their work for a variety of reasons (Siegle, 2013). These students may remain unidentified gifted, and consequently unsupported, if giftedness is not adequately defined.

Myth #4: Gifted students become behaviour problems in the classroom if they are bored.

In western society, the ratio of extraverts to introverts is estimated to be close to 1:1, or 50% for each (Myers, McCaulley, Quenk & Hammer, 1998). However, it is believed that greater than 50% of the gifted population is introverted (Gallagher, 1990; Silverman, 2013). Moreover, introversion increases with intelligence, with the figure of highly gifted people being introverted, being placed at more than 75% (Silverman 1994). Extraverts are generally more vocal than introverts. The extraverted gifted child may indeed exhibit problematic, outspoken behaviour in the classroom if they are bored, but the introverted gifted child will rarely voice dissent, and is more likely to be quietly, and miserably, bored. The more gifted a child, the more likely they are to be introverted, and the less likely they are to exhibit disruptive behaviour in the classroom. The introverted gifted student is more likely to be overlooked (Silverman, 2031). Gifted students' needs must be acknowledged for the sake of their learning, not to simply curb behaviour problems.

Some Truths

Truth #1: Gifted children not only learn differently, they also feel differently.

Gifted children often experience an intensity of feelings beyond the norm (Dabrowski, 1972). They may display profound moral awareness, idealism, and compassion, and a heightened sense of responsibility (Gross, Macleod et al., 2001). This emotional intensity

was described as an “overexcitability”, or “hypersensitivity”, identified by Dabrowski (1972), in his *Theory of Positive Disintegration*, as correlating strongly with giftedness. Hypersensitivities can contribute significantly to an individual’s drive; they can represent the “kind of endowment that feeds, nourishes, enriches, empowers, and amplifies talent” (Piechowski & Colangelo, 1984, p. 87). However, they can also have a negative impact as these hypersensitivities have the potential to become emotionally crippling (Piechowski, 1997).

Truth #2: Gifted children do not automatically become successful.

This truth depends on the definition of giftedness used. If the definition includes a measure of success (e.g., Renzulli, 1978; Sternberg, 1985), then, by definition, gifted children *are* successful. However, if the definition of giftedness is based on inherent differences (e.g., Gagné, 1985; Mrazik & Dombrowski, 2010), there are many factors, such as personality, motivation, the family environment, opportunities (including schooling), and chance, that will influence the outcome of a child’s giftedness, and ultimate success or otherwise (Gagné, 2003; Richotte, Rubenstein & Murry, 2015; Winner, 1996).

Truth #3: Gifted children come from all socio-economic and cultural backgrounds.

Gifted children are represented in all cultural, linguistic, racial and socioeconomic groups (Frasier & Passow, 1994; Gross, 2004; McAlpine, 2004). However, many low socio-economic, and culturally different gifted children remain unidentified due to biased perceptions of giftedness (Gross, 2004). Gifted students at poor schools can lag more than two years behind those at wealthier schools (Wai & Worrell, 2016). This is not the fault of the student, but it is a responsibility of the school system that this be rectified (Gagné & Schader, 2006).

Truth #4: Gifted students can also have learning disabilities

Gifted children with learning disabilities (Baum, 1990; Reis, Neu & McGuire, 1995) are generally called ‘twice-exceptional’ in gifted literature (which is abbreviated as ‘2e’ colloquially) (Dare & Nowicki, 2015; National Education Association, 2006). Twice-exceptional refers to intellectually gifted children who also have some form of disability that interferes with their ability to learn effectively in a traditional environment – disabilities such as dyslexia, visual or auditory processing disorders, autism, and attention deficit hyperactive disorder. The prevalence of twice-exceptionality is hampered by both low awareness about twice-exceptionality and diverse definitions of giftedness (Dare & Nowicki, 2015), but it is estimated that the percentage of students with disabilities who

may also be academically gifted, would be comparable to the percentage of the general population who are gifted (National Education Association, 2006).

Truth #5: Giftedness is more than a high test score.

What distinguishes gifted individuals from others in the general population are the “more complex, adaptive, malleable, and efficient networks within their brains” (Hodge, 2012, p. 58). Therefore, for gifted students it seems that testing requiring specific correct responses, such as in an IQ test, for example, can be fraught with difficulties. “Gifted students are skilled at seeing things in different ways. They come up with possibilities that other people don’t see ... On tests, particularly multiple-choice tests, coming up with a unique interpretation is a real weakness” (Paris, 2009, p. 20). Gifted students may see many layers of meaning in a question, and may find arguments to justify several, or even all, optional answers in multiple-choice tests (Geake, 2008). As a result, gifted children, especially highly gifted children, may actually not score well on these tests.

Truth #6: Some other common characteristics of gifted children.

Apart from being able to learn easily with minimal repetition, common cognitive traits of gifted children include characteristics such as: being very observant; extremely curious; having an intense interest (in many things, or one or two specific things); having an excellent memory; a long attention span; excellent reasoning and problem solving skills; well-developed powers of abstraction, conceptualisation, synthesis and generalization; fluent and flexible thinking; elaborate and original thinking; unusual and/or vivid imagination; and high abilities at fluid analogising (Davis, Rimm & Siegle, 2011; Geake, 2008; Gross, Sleaf & Pretorious, 2001; Silverman, 2013). It is not just *cognitive* traits that are common with gifted individuals. Research has also identified *affective* traits that are common with gifted children. They may experience extreme and complex emotions, for example, extreme shyness, extreme fears; they may have a highly developed sense of justice – concerned about fairness and perceived injustices; they may have a mature, well-developed sense of humour; they may show unusually high empathy, or no empathy at all (Dabrowski & Piechowski, 1977; Gross, Macleod et al., 2001; Piechowski, 1997; Silverman, 1997). Individual characteristics are important to consider in case study research, and understanding common characteristics can help with the analysis of these observations.

2.2.5 Giftedness: Definition Adopted for this Study

Gagné's *DMGT* (2009), with the clear distinction between the *gift* and the *talent*, underpins the primary definition of giftedness adopted for this study. This allows for the plausibility of gifted students who may be underachieving, due to numerous factors, with the prospect that, if these factors can be recognised and understood, there is a possibility that they may be intentionally addressed and remedied. The *DMGT* also allows for the notion that children do not necessarily need to be gifted to develop specific talents. Talents may be developed through the provision of effective learning experiences coupled with enough time, hard work, perseverance, and motivation (Ericsson, Krampe & Tesch-Römer, 1993; Gladwell, 2008). This is important when considering supporting the learning of mathematically gifted students within regular classrooms – the benefits of appropriate support have the potential to impact all students, not just gifted students (Rosario, 2008).

I have chosen to adopt Gagné's threshold of 10% for the prevalence of giftedness (Gagné, 2009), which, in mathematical terms, represent students who are capable of constructing robust mathematical concepts with fewer learning experiences than 90% of their age peers. This percentage is 'fuzzy', and unmeasurable in distinct terms, but translates to an average of two to three students in a class of 25-30 being mathematically gifted, suggesting every teacher and every school needs to be cognizant of the needs of mathematically gifted students who may require adapted learning experiences in mathematics classes (Peters & Engerrand, 2016).

As the term 'gifted' remains controversial for many teachers and parents (Gatto-Walden, 2016), especially for 'mildly gifted' children (Gagné, 2009), the exclusive use of this term has the potential to limit dissemination of results on gifted research to the practitioner. I have, therefore, also used the term 'mathematically highly capable' interchangeably with 'mathematically gifted' throughout the study to suit specific purposes.

2.3 Mathematics and Mathematics Education

Unfortunately, not only does the word *gifted* have negative connotations for many, so too does the word *mathematics* (Goldin, 2002). Understanding mathematics, and the requirements of effective mathematics education in the twenty-first century is important, as mathematics is an often-misconstrued subject that can engender different perspectives and responses from different people (Davis, 1984; Ziegler & Loos, 2017). According to Davis (1984), for many, perceptions of mathematics and mathematics education are based

on personal experiences that have oftentimes been limited, where prior experiences of mathematics at school may have involved simply the learning of skills and procedures (see also Ernest et al., 2016). Lockhart's *Mathematician's Lament* (2002; 2009) grieved deeply the treatment of mathematics in education in the last century. In order to understand the learning requirements of effective mathematics education (as defined by this study), and the characteristics of mathematical giftedness, understanding the discipline of mathematics (Ziegler & Loos, 2017), and its implementation as a subject in the 21st Century primary school classroom (Gravemeijer, 2013), needs to be clarified.

2.3.1 What is Mathematics (as a discipline)?

According to Davis (1984), mathematics is an extremely complex construct culminating in logical abstraction of space, form, magnitude and quantity. A construction of meaningful mathematical concepts and relationships is not simply a matter of learning and remembering mathematical facts, skills and rules. Mathematics is a creative venture (Sriraman, 2004). It is about problem solving (English & Gainsburg, 2016; Polya, 1957). It is about generalising, extending, creating and deriving ways of approaching new problems (Davis, 1984). Basic mathematics concepts of number, space, measurement, chance, and data are the starting points. Skills of calculation, of operating with numbers, measuring, describing shapes, drawing graphs, et cetera, are *tools* that enable us to work mathematically; they are means to an end, not an end in, and of, themselves (Sullivan, 2011). For many, experiences of mathematics at school have involved simply the learning of skills, with very little, if any, opportunity to utilise these skills in meaningful and creative ways (Lockhart, 2009). Mathematics is much more than calculating answers, and mathematics education must focus on mathematics as a whole. Rachlin (1998) poetically likened mathematics to an invisible man. The invisible man, by definition, is invisible. If he is doused in flour, he does not actually become visible, but the flour enables him to be found.

And what of mathematics? Are the symbols of mathematics really mathematics, or are they merely the flour that we throw about to help us find mathematics? Too often mathematics is taught as if the flour is what is important. Yet all the symbols – even the manipulatives, calculators, and computer software – are only flour. (Rachlin, 1998, p. 470)

For this study, I assume a holistic view of mathematics. Mathematics encompasses domains of number and algebra, measurement and calculus, geometry and space, statistics, probability, and logic. Within these domains, the focus of primary, or elementary, mathematics education needs to include, in conjunction with mathematical skill development, the ability to recognise and define mathematical problems (Stillman et al., 2009); the ability to generate multiple solutions, or multiple paths towards solutions (Sullivan et al., 2013); the ability to reason (Lithner, 2017); the ability to justify conclusions (Brown, 2008); and the ability to communicate results (Stillman et al., 2009). Approaches to mathematics education that allow for investigation, problem-solving, perseverance, and sustained effort, are necessary for children to begin to learn and appreciate mathematics as a discipline.

2.3.2 Mathematics and Creativity

Problem-solving in mathematics is still only part of what is required for mathematics into the future. “Today, not only do we not know all the answers, we do not know the questions that students will face in the future ... students must also learn to ask questions that add depth and interest to the mathematics” (Sheffield, 2009, pp. 87-88). Being able to ask mathematical questions to extend and deepen an original problem, to think about mathematical problems in original or innovative ways, and to pose new and unique problems to explore, moves students from being problem-solvers, to also being problem-posers, that is, to being mathematically creative (Hershkowitz, Tabach & Dreyfus, 2017; Silver, 1997), and “Mathematical creativity ensures the learning, and growth, of the field of mathematics as a whole” (Sriraman 2004, p. 19).

‘Creativity’ is not often associated with the traditional image of school mathematics, but from a holistic view of mathematics, mathematical creativity is an extension of mathematical reasoning, strategizing, problem solving and problem posing (Sheffield, 2017). Mathematical creativity requires more than mathematical knowledge that enables students to score well on mathematics pen and paper tests. Mathematical creativity takes students beyond knowing mathematical concepts and procedures, to knowing how mathematics is created and used to explore new concepts, and to solve problems in original and innovative ways (Sheffield, 2006); it requires imagination, insight and intuition (Sheffield, 2009). For students to be creative in mathematics, “they should be able to pose mathematical questions that extend and deepen the original problem as well as solve the problem in a variety of ways ... [realising] that instead of finding a solution to a

mathematical problem being the end of the problem, it is often just the beginning of the most interesting, and rewarding, mathematics” (Sheffield, 2009, p. 88).

Liljedahl and Sriraman (2006) described mathematical creativity as the ability to produce original work, which extends the current body of mathematical knowledge, or opens avenues of new questions for other mathematicians. Mathematical creativity of primary school students will not be the same as the creativity of professional mathematicians, but students are, nonetheless, capable of creative thinking, “At the K–12 level, one normally does not expect works of extraordinary creativity; however, it is certainly feasible for students to offer new insights” (Sriraman, 2005, p. 23). Assmus and Fritzlär (2018) argue that primary school students (especially mathematically gifted students) are capable of not only solving and posing problems, but also of creating new mathematical objects. While mathematical creativity is not limited to the mathematically gifted student (Hershkovitz, Peled & Littler, 2009; Sheffield, 2017) – indeed, one of the major objectives of school mathematics education should be the development of mathematical creativity in *all* students (Barbeau & Taylor, 2009; Craft, 2005; Reiss & Törner, 2007; Silver, 1997) – mathematical creativity needs to be included as an intentional key component in challenging and supporting the ongoing learning of mathematically gifted students (Leikin, Berman & Koichu, 2009; Sheffield, 2017).

These understandings of holistic mathematics, and mathematical creativity, have significant implications for the teaching and learning of school mathematics. Effective teaching and learning of mathematics into the twenty-first century requires much more than the teaching and learning of rules, procedures and algorithms (Gravemeijer, 2013). Encouraging and scaffolding mathematical creativity needs to be, more than ever, a critical part of the school mathematics curriculum as, “innovation and entrepreneurship provide a way forward for solving the global challenges of the 21st century” (Volkman et al., 2009, p. 7). Creativity and innovation is directly related to economic prosperity and success (Zhao, 2012), whereas there is increasing evidence that educational practices that help students achieve by scoring highly on international tests such as the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS), may actually reduce creativity and hamper entrepreneurial qualities (Zhao, 2012). International testing, or getting high grades, becomes ‘competition’ rather than ‘learning’, and the issues with competition have been long recognised as potentially limiting student learning:

Competition motivates the person to perfect or to do over and over things that are already partly learned ... creativity must involve making unique tries. In competitive situations failure is an imminent danger and the child cannot afford to improvise or try the unusual. He is more likely to do the thing which is known to be correct. (Freehill, 1961, pp. 169-170)

2.3.3 Mathematics Education in the 21st Century

Mathematics education research became a discipline in its own right in the 1960s/1970s (Schoenfeld, 2016), and as a result, mathematics education has changed significantly over the past few decades (Australian Association of Mathematics Teachers (AAMT), 2009; Schoenfeld, 2016), albeit slowly (Hiebert, 2013). By the 1980s and early 1990s, issues such as the culture of the mathematics classroom, teacher beliefs being consequential to student learning, and research on affect in mathematics education began to receive recognition (see Grouws (1992) *Handbook of Research on Mathematics Teaching and Learning*). Research was beginning to inform practice in productive ways, and the classroom was becoming a site for meaningful research (Schoenfeld, 2016). Learning mathematics was being redefined to mean learning the processes of mathematical thinking and reasoning as well as learning mathematical content (Schoenfeld, 2016), usurping earlier behaviourist approaches of teaching students simply how to calculate accurately and efficiently (Hiebert & Carpenter, 1992; Skemp, 1976). From the early 2000s the role of mathematical discourse and communication in relation to mathematical cognition has been explored (e.g., Sfard, 2008), which has led to ideas such as *embodied cognition* (Nemirovsky & Ferrara, 2009) and *collaborative learning* (Hmelo-Silver, Chinn, Chan & O'Donnell, 2013) also receiving attention in mathematics education research. Teaching and learning higher-order metacognitive skills, and mathematical processes such as dialoguing, have become, or are becoming, essential elements of school mathematics (Kilpatrick, Swafford & Findell, 2001; Stillman et al., 2009), classrooms are becoming what Schoenfeld (2016) describes as, “learning environments for cognitive apprenticeship” (p. 509).

Twenty-first century approaches to mathematics teaching and learning, that is, engaging students in learning experiences that facilitate students' own construction of mathematical concepts, requires an active, hands-on approach, through investigation, exploration, dialogue, reflection and self-evaluation of meaningful mathematical tasks (Clarke et al., 2002; Kilpatrick, Swafford & Findell, 2001; Sheffield, 2009; Stillman et al., 2009; Sullivan et al., 2013). The classroom teacher's role in contemporary mathematics classrooms

extends beyond knowing, and ensuring coverage of, a mandated national and/or state standards-based mathematics curriculum. They are required to know and understand the mathematics content their students are learning beyond, and deeper than, the level required for the particular grade ‘standard’ (Clarke et al., 2002). They also need sound *pedagogical content knowledge* (Hurrell, 2013; Shulman, 1986) in order to understand the different paths students may employ to explore a mathematical problem (both correct and incorrect), so they can direct student learning (Stillman et al., 2009). The teacher is a ‘more knowledgeable other’ (Vygotsky, 1978), a facilitator, and orchestrator, of student mathematics learning.

The teacher’s role, then, is to provide appropriate and meaningful tasks that will enable deep mathematics learning (Callingham, 2008; Rosario, 2008; Siemon, Virgona & Corneille, 2001; Sullivan et al., 2013). Meaningful mathematics problems are tasks that provide opportunities for mathematical thinking, reasoning, creativity and dialogue (Hershkowitz et al., 2017). They are often set in contexts that prove to be intriguing, and draw the learner into the mathematics rather than imposing the mathematics learning upon the student (Stillman et al., 2009). These tasks normally require a significant amount of time to complete, and allow opportunities for students to learn how to work collaboratively, to ask mathematical questions, and to communicate and justify their thinking. Meaningful mathematics tasks may take the form of open questions – mathematical questions that require a higher level of thinking than simply remembering facts or reproducing learned procedures, and where there is often more than one acceptable solution, and many possible paths to those solutions (Sullivan & Lilburn, 2004). They may take the form of mathematical investigations or mathematical modelling, which provide students with opportunities to explore real-life scenarios (Geiger, Stillman, Brown, Galbraith, & Niss, 2018; Lesh & Doerr, 2003), and contribute to the development of “powerful mathematical processes such as constructing, describing, explaining, predicting, and representing, together with quantifying, coordinating, and organizing data” (Bahmaei, 2011, p. 3). Meaningful mathematics may also take the form of mathematics games that provide opportunities to address and reason through important mathematics concepts in challenging, but accessible and enjoyable ways (Clarke & Roche, 2010). The teacher’s role, within these tasks, is to also be aware of each student’s zone of proximal development (Vygotsky, 1978) in order to know what questions to ask to delve deeper into students’

thinking, and to build towards appropriate learning targets (Clarke et al., 2002; Clements & Sarama, 2009; Muir, 2008; Storeygard, Hamm & Fosnot, 2010).

The challenge of providing mathematics teaching that goes beyond curriculum content is known to be a concern for many classroom teachers (Stacey, 2010). When faced with a seemingly overcrowded mathematics curriculum (Australian Primary Principals Association, 2014), the temptation is often to rush through important mathematical concepts in order to cover all components, at the expense of developing deep mathematical understandings, or being able to investigate and explore intriguing mathematical discoveries (Clarke & Roche, 2010). An interesting notion about education that began to emerge in Britain around the turn of the twenty-first century, *The Slow School Movement* (Holt, 2002), is a reaction to “the pressure to proceed from one targeted standard to another as fast as possible, to absorb and demonstrate specific knowledge with conveyor-belt precision” (Holt, 2002, p. 265). This pressure was influenced by the standards-based reforms of the late twentieth century, and governmental mandated standards-based tests. *The Slow School Movement* is a philosophy that seeks to redress this by promoting time for deep learning experiences with real outcomes, time for curiosity, passion and reflection to be at the heart of learning experiences, and time for dynamic, collaborative, democratic and supportive relationships for learning (Holt, 2012).

[There are] personal attributes that standardized tests cannot measure — attributes crucial to the cultivation of the virtues and the formation of moral agents: creativity, critical thinking, resilience, motivation, persistence, humour, reliability, enthusiasm, civic-mindedness, self-awareness, self-discipline, empathy, leadership, and compassion. But these are as remote from the activity of fast schools as is gastronomic pleasure from fast food. (Holt, 2002, p. 268)

This philosophy of slow education sounds like a requirement for learning mathematics holistically and creatively, and the key element in each of *The Slow School Movement* principles is *time*. Students need time to investigate and explore, time to think and reason, time to discuss ideas and problem-solving strategies, time to justify and record solutions, time to think in order to be creative and innovative. Sriraman (2004) recognised this as an important part of success: “It is no coincidence that in the history of science, there are significant contributions from clergymen such as Pascal and Mendel because they had the means and leisure to ‘think’” (Sriraman, 2004, p. 23). In considering mathematically gifted students this idea of *slowing* education may feel counter-intuitive, but if we truly value

creativity, innovation, discovery and encouraging invention (Csikszentmihalyi, 1996), through development of the ‘whole student’, maybe this is something worth considering.

Mathematics education in the 21st century needs to be viewed as ‘learning to think and reason mathematically’ rather than ‘learning mathematical rules, procedures and algorithms’ (English & Gainsburg, 2016). Teaching students to think and reason mathematically involves providing learning experiences that enable them to construct new mathematical concepts, and how to apply them, but it also involves providing time and support for students to explore mathematical ideas further, to scaffold creativity (Williams, 2016), and to develop the skills needed to record, prove and share their ideas with others. Mathematics education in the 21st century needs to encourage risk-taking and student curiosity, to enhance creativity. Twenty-first century education “requires a significant shift in our mindset about education from employment-oriented to entrepreneurial-oriented ... [that] affords students autonomy, voice, and choice in what they learn” Zhao, 2012, p. 60).

This discourse on mathematics and mathematics education determines what constitutes meaningful, relevant teacher support in the education of all students, but is particularly relevant to mathematically gifted students. Research on best practices in mathematics teaching and learning continues to be an important focus, in part, as a response to the urgent international challenge regarding the decline in student interest and participation in higher education mathematics studies (Forgasz, 2006; Makar & Fielding-Wells, 2018; Sheffield, 2008; Tytler et al., 2008). It stands to reason that mathematically gifted individuals should make up a significant percentage of students pursuing mathematical studies post compulsory education, so providing for these students is of particular significance in this current climate. Understanding mathematics is also important for defining what constitutes mathematical giftedness. The next focus is, therefore, on integrating the previous two sections – giftedness and mathematics.

2.4 Mathematically Gifted Students

All children are capable of being taught [mathematics]; every normal, mentally healthy pupil is capable of ... mastering the school material within the limits of the curriculum ... But it in no way follows that all pupils can be taught with the same ease. (Krutetskii, 1976, pp. 3-4)

Definitions of giftedness in general, and of mathematics as both a discipline and as a school subject, will, in tandem, determine the definition of ‘mathematical giftedness’ to be used

in this study. Extrapolating Krutetskii's (1976) hallmarks of giftedness, and exploring current educational options for mathematically gifted students (e.g., Singer et al., 2016) may also help identify any gaps in research and current practice that will be important to address. Many of the commonly perceived traits of mathematical ability, such as rapidity of work, and memory of facts and learned procedures, are based on a limited understanding of mathematics, and have been refuted for years (cf. Krutetskii, 1976), and yet they persist. So, who are the mathematically gifted, how are they identified, and how can their unique learning requirements be best supported?

2.4.1 Definition of mathematical giftedness for this study

Based on Gagné's (1985) *Differentiated Model of Giftedness and Talent*, for this study I have defined children who are mathematically gifted as those who *are capable of constructing robust mathematical concepts with fewer learning experiences than 90% of their aged peers*. These are students who possess unusually high natural (or instinctual) aptitudes for constructing and understanding mathematical concepts (Hoppe & Stojanović, 2009; Mrazik & Dombrowski, 2010), and who therefore differ substantively from their peers in the way they view, understand and learn mathematics (Diezmann & Watters, 2002). This definition, coupled with the previous description of mathematics as a discipline, will contextualise both the methods of identification of, and the unique academic requirements for, mathematically gifted students. The aim is to identify optimal teaching practices for ensuring talent development, or realised potentials, from these inherent gifts.

2.4.2. Hallmarks of mathematical giftedness

In conjunction with general characteristics of giftedness (cf. section 2.3.4) students who are mathematically gifted may also show mathematically specific characteristics. Vadim Krutetskii's (1976) work with mathematically gifted students in the former Soviet Union in the 1950s is considered to be landmark work in the study of mathematical abilities in school children, and has influenced many subsequent researchers of mathematical giftedness (Rosario, 2008). According to Krutetskii, children who are mathematically gifted tend to generalise easily, extend, create, and invent new methods (or strategies) for solving mathematical problems, and naturally strive "for the cleanest, simplest, shortest and thus most 'elegant' path to the goal" (Krutetskii, 1976, p. 187). Generalising, extending, hypothesising, creating new methods, or exploring different strategies for problem-solving, are integral parts of mathematics education for all students, as outlined previously, but

mathematically gifted students are the ones who may be intuitively embracing these processes in mathematics classes. They are the students with high abilities at fluid analogising (Geake, 2009a), a cognitive process that enables quick recognition of similarities between problem types, which curtails the need for multiple, scaffolded learning experiences. They may develop their own ‘short-cuts’, or formulas, based on prior knowledge and experiences (Krutetskii, 1976). They are the students with a seemingly innate number sense (Starr, Libertus & Brannon, 2013), who may develop innovative strategies, which can be discussed, explored, tested and possibly adopted by others in the class, if the teacher is attuned to the intricacies of the mathematics being used. They are the students who are asking mathematical questions, often about abstract ideas or concepts, as they are solving mathematical problems (Heinze, 2005). They are the students who have an innate disposition to think about life through a mathematical lens (Brown & Stillman, 2017), or what Krutetskii (1976) describes as a mathematical “cast of mind” (p. 187), or “turn of mind” (p. 199).

A suggested list of characteristics or traits of mathematically gifted students, which distinguishes them from their peers, is summarised here. The list is neither complete, nor all-inclusive for all mathematically gifted children. Researchers (e.g., Benbow & Minor, 1990; Borovik & Gardiner, 2006; Diezmann & Watters, 2002; Ficici & Siegle, 2008; Leikin, 2007; Rachlin, 1998; Sheffield, 2000; Tosto et al., 2014; Wiczerkowski, Cropley & Prado, 2000) suggest mathematically gifted students:

- have a seemingly innate number sense; display “swiftness in reasoning, in mental orientation” (Krutetskii, 1976, p. 196); tend to grasp new concepts quickly, and may appear to ‘know’ the mathematics without having to ‘learn’ it (Sheffield, 2000);
- show “logical thinking, systematic, sequential thought” (Krutetskii, 1976, p. 196); may produce unique, insightful solutions or methods for solutions (Diezmann & Watters, 2002);
- love to explore patterns and puzzles (Wiczerkowski, Cropley & Prado, 2000); readily recognise, create and extend patterns (Diezmann & Watters, 2002); will generalise from patterns and relationships, often unprompted, and may develop unique relations (Borovik & Gardiner, 2006; Rachlin, 1998);
- show “[a]n ability for mathematical abstraction and for rapid and broad generalisation of mathematical material” (Krutetskii, 1976, p. 196); show evidence of fluid analogising (Geake, 2008); readily make connections, recognising when the

approach to one problem can be used for other similar problems (Borovik & Gardiner, 2006; Leikin, 2007);

- are flexible in their thinking – demonstrate “[a] free and easy transfer from a direct to a reverse train of thought” (Krutetskii, 1976, p. 198); can move forwards and backwards through the problem-solving process with ease (Borovik & Gardiner, 2006; Rachlin, 1998); intuitively use proportional reasoning (VanTassel-Baska, Johnson & Avery, 2002);
- strive for mathematical clarity in explaining reasoning, with “[a] distinctive tendency for ‘economy of thought’” (Krutetskii, 1976, p. 198); often have unique ways of looking at, and attempting to solve, mathematical problems (Borovik & Gardiner, 2006);
- may have complex types of reasoning skills, and contract the problem-solving process, condensing several steps into one thought, with “[a] tendency to rapid abbreviation, ‘curtailment’ of reasoning in problem solving” (Krutetskii, 1976, p. 198).
- think logically and symbolically with quantitative and spatial relations (Diezmann & Watters, 2002); adopt and use mathematical symbols with confidence, and move quickly from concrete to abstract (Benbow & Minor, 1990);
- often display a high spatial ability (Tosto et al., 2014); some have such a powerful ability for visualisation that they can instantly “see” a solution, after which they may have difficulty expressing their solution in a logical sequence of steps (Benbow & Minor, 1990);
- ask insightful mathematical questions (Ficici & Siegle, 2008); and
- are often mathematically curious, and will get involved in tasks both mentally and physically (Ficici & Siegle, 2008).

A heightened aptitude for mathematics is often evident before children start school – evidenced by a curiosity with numbers and/or shapes, a precocious number-sense, a love of patterns and puzzles, highly intricate constructions with building blocks, et cetera – so an appreciation of these students’ needs is important from the very earliest years of schooling (Bicknell, 2009b; Diezmann & Watters, 2002; Gross, 2004; Sheffield, 1999; Winner, 1996).

2.4.3 Identification of mathematically gifted students

Unfortunately, translating mathematically gifted characteristics into a reliable form of identification is not straightforward. There is no specific assessment that can measure the hallmarks of mathematical giftedness, or determine a ‘mathematical cast of mind’. Identification requires a multi-faceted approach, including a recognition and understanding of common characteristics or traits of intellectually gifted children, and a recognition and understanding of the intricacies of mathematics as a discipline.

Many of the aforementioned characteristics or traits of mathematical giftedness, are first noticed by parents prior to school entry (Gross, 2004), with parents shown to be able to recognise early signs of both number understanding and spatial reasoning (Bicknell, 2009b). As such, anecdotal reports, and/or written questionnaires from parents, can become a useful part of an identification process (McAlpine, 2004). Indeed, studies have shown parents to be quite accurate in their assessment of their child’s abilities, and professionals, including teachers, have been urged to trust parents’ descriptions of advanced behaviours, regardless of whether or not those behaviours have been evident at school (Feldhusen, 1998; Robinson, 2008; Winner, 1996).

Classroom teachers may also be a reliable source as identifiers of mathematically gifted behaviours that they have observed over time, although teachers are more likely to focus on numeric reasoning and problem-solving abilities than on spatial reasoning (Bicknell, 2009b; Gross, 2004). Teachers are said to be more likely to underestimate, or discount, exceptional abilities than parents (Gross, 2004; Hodge & Kemp, 2006), but their input can add to the whole picture within a multi-faceted identification process.

Students themselves can also provide a piece of the identification puzzle through self or peer nomination (McAlpine, 2004). Students have been shown to be able to recognise exceptional mathematical abilities when comparing themselves with peers at school (Gross, 2004), primarily in association with numeric reasoning and computational abilities (Bicknell, 2009b). Consequently, peer nomination, student questionnaires and semi-formal interviews with students are other possible useful components of an identification process.

Standardised mathematics tests are generally designed to determine how well students have mastered mathematics content. They are usually a measure of mathematical achievement, rather than a measure of a student’s particular way of thinking and reasoning about mathematics. However, certain teacher implemented mathematics assessments designed

to determine not only a student's ability to problem-solve, but also how they approach that problem-solving, may be useful as a component of a multi-faceted identification process (McAlpine, 2004). One-on-one interview assessments that take into consideration student thinking processes as well as correct answers are more suited to this than pencil and paper tests that typically do not (Hodges, Rose & Hicks, 2012). It also must be recognised that any assessment requires a high enough ceiling, with sufficiently difficult items, in order to assess gifted students' full capabilities.

Formal identification of giftedness requires assessment by a psychologist, or other qualified professional, preferably working together with parents and educators (Borland, 2008; Gross, 2004). However, formal assessments generate controversy due to the issue of 'cut-offs', whereby one student with a particular score is identified as gifted, whilst another student, whose score is 'one less,' is identified as not gifted (Davis et al., 2011). Thankfully, formal identification need not be a first step in supporting the learning of mathematically gifted students (Hodge, 2012). Indeed, some researchers and educators have advocated an inquiry-based, problem-solving and problem-posing approach to teaching mathematics *as* an identification model (Iversen & Larson, 2006; Niederer & Irwin, 2001; Rosario, 2008; Voica & Singer, 2014). "A general lifting of expectations for all children and an invitation to children to produce a range of responses to challenging educational opportunities has facilitated the emergence of hidden potential, especially in children from disadvantaged and/or minority families and communities" (Hodge, 2012, p. 64). In fact, with an effective approach to holistic mathematics education, formal identification of mathematical giftedness may not be necessary at all, unless there are specific concerns about a student's progress or well-being, or if there are specific entry requirements for gifted programs on offer (Haylock & Thangata, 2007). However, it *is* imperative that school systems and individual teachers be made aware of, and know how to recognise, and understand, common characteristics and dispositions that may be evident in gifted children. This is especially critical for dispositions such as asynchrony and overexcitabilities that may appear counter to intellectual giftedness (Valpied, 2005), as well as unique problem-solving and/or problem-posing approaches gifted students may adopt when completing mathematical tasks (Heinze, 2005; Span & Overtom-Corsmit, 1986), especially if these approaches are highly creative and not apparently apropos to the task.

For any research on mathematically gifted students, however, some form of identification will be required. The identification process used for this study was a multi-faceted approach

based on recognising aforementioned characteristics and behaviours, as noted by parents and teachers, and as observed by the researcher through classroom participation and student responses to tasks specifically designed to identify ease of learning, and generalisation, of new concepts. This process is described in detail in Chapter 4.

2.4.4 Educational options and provisions for mathematically gifted students

Researched educational options for gifted students include: a) acceleration – which may take the form of early school entrance, grade skipping, grade telescoping, curriculum compacting, or subject-matter acceleration; b) independent study – through home-schooling or distance learning; c) enrichment – through in-class differentiation, or extra-curricular programs; d) like-ability grouping – such as streaming (where students are grouped for all classes), or setting (where students are grouped by subject), or tracking (which could include both streaming or setting), or through special select-entry schools. Each of these options have researched advantages and disadvantages, which depend on the intended purpose of the provision, based on the perceived purpose of school education (Gavin & Adelson, 2008; VanTassel-Baska, 2008).

Acceleration has a focus on covering the curriculum (or subject matter) quicker than normal, and success is measured by significant gains made in achievement in a contracted period of time (Gavin & Adelson, 2008). This is a reflection of a standards-based education culture that portrays education primarily as a curriculum to be successfully completed, and the quicker this happens, and the higher the ‘grade’ achieved, the more successful the student is (Beresford, 2014). This, unfortunately, tends to feed into the elitist perception of giftedness, and “because giftedness in mathematics is characterised by higher levels of thinking ... the special needs of these pupils are not met simply by moving them more quickly through the standard curriculum” (Haylock & Thangata, 2007, p. 85). Acceleration can lead to the mistaken perception that mathematically gifted students’ needs are being met without any further intervention.

Independent study as home-schooling is beyond the scope of this study, but is dependent on the availability of parents or care-givers who are willing and able to guide their child’s learning. Extra-curricular programs are also dependent on availability and affordability, something not all mathematically gifted children have access to.

In-class differentiation is the provision of mathematical learning tasks that can be adapted to each individual student’s, or group of students’, zone of proximal development

(Vtgotsky, 1978), that is, their optimal learning zone. Instructional techniques that include inquiry and problem-based learning, and focus on mathematical thinking and reasoning, rather than just learning skills, are essential for this type of differentiation (Doyle, 2006; English, 2004; Makar, 2011; VanTassel-Baska, 2008). In-class differentiation is theoretically accessible for all school students, however research has shown that differentiation for gifted students may not be common, consistent, or substantial enough in primary school settings (Gavin & Adelson, 2008; Kanevsky, 2011).

There seems to be some agreement in both gifted research and mathematics education research about the benefits of like-ability, or homogenous grouping, for mathematically gifted students, in the top 1%-2% of the population, but not for mathematically gifted students in the top 3%-10%. Homogenous ability grouping in mathematics education research, has generally been found to have a detrimental effect on the majority of students (Bartholomew, 2003; Boaler, William & Brown, 2000; Clarke & Clarke, 2008; Linchevski & Kutscher, 1998; Slavin, 1990; Zevenbergen, 2003). However, researchers of giftedness (e.g., Gross, 2004; Silverman, 2013; Tannenbaum, 1983) propose that gifted students two standard deviations above the mean (the top 1%-2%), require advanced material, gifted peers, fast-paced instruction and specialist trained teachers, the provision of which can be accomplished most effectively in like-ability groupings, preferably full-time (Gross, 2004). Even strong proponents of heterogeneous groupings in mathematics education agree that there is some demonstrated benefit in homogenous ability grouping with mathematically gifted students (Clarke & Clarke, 2008), but this advantage needs to be considered carefully. For some, these benefits have been shown to be only slight, and non-significant in a statistical sense (Clarke & Clarke, 2008). Others disagree with any benefit, especially regarding mathematically gifted girls (Boaler, 1997). The *disadvantages* of homogenous grouping of gifted students, though, are significant. With ability groupings being associated with 'slow' and 'fast' learners, we can end up confusing students' *pace* of learning with their *capacity* to learn (Wheelock, 1992), and like-ability groupings can lead to the mistaken perception that individual differences no longer matter (Boaler et al., 2000; Clarke & Clarke, 2008). The reality is that ability grouping, streaming and tracking simply create heterogeneous groups with a narrower range (Anderson, 2016). Furthermore, structured grouping of mathematically gifted students is not equitable where two students of similar ability can be deemed, respectively, 'gifted' and 'not gifted' by an arbitrary and/or ambiguous 'cut-off' for inclusion in a specialised gifted group (Sheffield, 2008). According

to Sheffield (2008), “no measure of mathematical ability such as a general IQ test or a score on a mathematics achievement test should be used to exclude students from services that would help them develop their mathematical promise” (p. 2). This is particularly pertinent when considering that ability grouping has been shown to often reflect social advantage rather than innate abilities (Zevenbergen, 2003).

VanTassel-Baska (2009) decries the segregation of gifted programs, “Gifted education must be seen as a connected part of the schooling process for all” (p. 267) and “integrating programs for the gifted into the total fabric of the school program is central to the work of the field [of gifted education]” (p. 266). However, she also recognises the dangers of integration being interpreted as simply inclusion:

The myth of separatism must be revealed in all of its dangerous aspects so that the antidote is not worse – for example, solving the problem of the gifted by dumping them into regular classrooms without trained teachers or adapted materials or cluster grouping. The true antidote to separatism for gifted and high achieving learners lies not in inclusion as it is now rigidly interpreted but rather in flexibility in placement and learning opportunities. (VanTassel-Baska, 2009, p, 268)

Krutetskii (1976) similarly warned against separatism:

One of the greatest misconceptions is that notion that special attention to developing gifted children conflicts with the goal of all-round development of every child’s abilities ... [M]athematics teachers should work systematically at developing the mathematical abilities of all pupils, at cultivating their interests in and inclinations for mathematics, and at the same time should give special attention to pupils who show above-average abilities in mathematics by organizing special work with them to develop these abilities further. (Krutetskii, 1976, pp. 5, 7)

Instructional strategies that capitalise on challenging tasks that can be readily implemented in the regular, heterogenous classroom (Sullivan et al., 2013), in conjunction with understanding, and accommodating for, characteristics of mathematically gifted students, seem to be an appropriate way of providing ongoing learning opportunities for many mathematically gifted students (Diezmann, 2005). Diezmann (2005) has presented a “range of strategies that have been successfully used in regular classrooms to provide the cognitive challenge needed by mathematically gifted students” (p. 50). These strategies are:

- 1) Problematising tasks – by providing extended challenges to the whole-class task, such as increasing or changing the quantities in the problem, inserting certain constraints, requiring multiple solutions or solution methods, or developing generalisations (see also Sullivan et al., 2013). Problematising tasks shift regular tasks to just out of the reach of gifted students, and into their zone of proximal development, whilst maintaining a connection to the regular whole-class task (see also Williams, 2004);
- 2) Mathematical investigations, which may be open-ended tasks that require students to apply and create mathematical knowledge through posing and solving novel problems, or opportunities to explore a variety of mathematically oriented topics such as famous mathematicians, mathematical discoveries, or mathematics in other cultures (Diezmann, Thornton & Watters, 2003);
- 3) Extending manipulative use – including physical and/or symbolic representations, mathematical modelling tasks, and various technologies – which help to organise mathematical thinking, and can be helpful for gifted students in supporting higher-level thinking;
- 4) Modifying whole-class mathematical games to provide further challenge, such as developing strategies to maximise winning outcomes.

Diezmann and Watters (2002) explore the importance of challenging tasks for mathematically gifted children, and ways this challenge can be increased to optimal levels for varying abilities. Sullivan et al. (2013), as part of their research project, *Encouraging Persistence Maintaining Challenge*, explore the success of developing a classroom culture that uses challenging mathematical tasks to develop mathematical reasoning, and a willingness to persevere through difficulties, for *all* students. This idea of using and modifying challenging tasks, therefore, seems like an optimal basis for developing a teaching and learning framework for mathematically gifted students in regular classrooms.

2.4.5 Provisions and support for teachers of mathematically gifted students

When considering the educational requirements of mathematically gifted students, there are many references to strategies for identification and provision for the students (see sections 2.3.3 and 2.3.4). The construction of a theoretical model to be used as a basis for ‘good teaching practice’ as a practical support for *teachers*, is another viable and necessary strategy to consider, to ensure the effectiveness of nurturing the gifted student (Koshy, 2000). As VanTassel-Baska (2009) alludes to, identification and provision of programming do not automatically translate into effective teaching practice. Teachers require

professional learning and support in understanding the mathematically gifted student, and in understanding what constitutes rich, challenging mathematical learning experiences that can be used in the regular classroom, and accessed by all students (Diezmann, 2005). Teacher professional learning, then, is a vitally important aspect of a teaching and learning framework to be developed for supporting the learning of mathematically gifted students in regular classrooms (Geake & Gross, 2008).

There have been various inquiries into the education of gifted students, such as, *National excellence: A case for developing America's talent* (Ross, 1993); *The Munich study of giftedness* (Heller, 2001); and *Inquiry into the education of gifted and talented students* (Parliament of Victoria, 2012). Despite these inquiries unanimously supporting the need for developing teacher beliefs and understandings about gifted education and gifted students' special learning needs, there continues to be a widely recognised lack of both pre-service and in-service teacher education on giftedness (Fraser-Seeto, 2013). This has resulted in education systems that fail many gifted students through no fault of the teachers (Fraser-Seeto, 2013; Zeigler, Stoeger, Harder & Balestrini, 2013).

One desire of this current study, therefore, is that it may provide a research basis for developing an undergraduate unit (for pre-service teachers), and a professional learning program (for in-service teachers) that provides support for teachers by addressing issues of giftedness via the medium of mathematics education. Further elements of potential teacher professional learning, pertaining to this study, will be discussed shortly, in section 2.6.

2.5 Mindsets

Mindset (mīnd'sēt') *n.* a habitual or characteristic mental attitude, belief or disposition that predetermines a person's responses to, and interpretations of, a given situation. (Dictionary.com)

Not all factors that maximise a student's mathematical capabilities are based on mathematics teaching and learning experiences. There are also non-cognitive, affective/social-emotional factors that are inherent within all aspects of student learning. These include self-concept, attitudes and beliefs (Bernard, 2006; Ernest, 1989; Hannula, Morselli, Erktin, Vollstedt & Zhang, 2017) that develop into learner *mindsets* that may either enhance, or limit an individual's potential (Dweck, 2006; Hannula et al., 2017; Pieronkiewicz, 2014; Sheffield, 2008). "Belief in one's ability to succeed and belief in the importance of mathematical success are ... important. Lack of such beliefs ... are

acknowledged as a significant barrier to learning for students” (Sheffield, 2008, p. 2). Student mindsets, whether positive or negative, have been shown to predict learning trajectories (Hannula et al., 2017). Indeed, some believe a student’s disposition towards learning has a higher impact on their success, or otherwise, than their innate abilities (see Goleman, 2006). Beliefs are very powerful. Schoenfeld’s (1985) research on mathematical problem solving discovered that students’ belief systems about themselves as learners of mathematics “shaped their mathematical behaviour in fundamental ways” (Schoenfeld, 2016). Dweck (2006) has shown that students who believe they are good at something, as well as those who believe they are not, will be resistant to the rigours of learning. Providing challenging experiences may be an obvious requirement for mathematically gifted students, but students not only need to be provided with suitably challenging tasks, they also need to be supported in their beliefs about learning, and, subsequently about themselves as ongoing learners of mathematics (Hannula, 2015). If mathematics learning has always been relatively easy, a student’s learner mindset may be that success is simply judged by outperforming others (e.g., with high grades), or by achieving success with little effort (Ames & Archer, 1988). This has the potential to affect their willingness to persevere, to develop resilience, and to be optimistic when faced with difficult tasks (Dweck, 2006). Each of these traits have been shown to be significant factors in successful learning (see Benard, 1995; Duckworth et al., 2007; Seligman, 1991; Sullivan et al., 2013; Williams, 2014). Students with this fundamental belief (mindset), derived from previous experiences, may find that challenging tasks, with a lack of immediate success, may generate feelings of inadequacy and low self-worth. Self-preservation may then step in, whereby they avoid any future tasks that make them feel this way (Dweck, 2006). However, Dweck (2006) has also shown that with the right kind of intervention, students’ mindsets can be changed from a belief about *performance* being the goal of learning, to a belief that *mastery of new knowledge* is the goal of learning. “With a mastery goal ... the process of learning itself is valued, and the attainment of mastery is seen as dependent on effort” (Ames & Archer, 1988, p. 260). Fostering positive, mastery-oriented learner mindsets is required to support gifted students in viewing mistakes as part of the learning process, not something to be avoided (Mofield, Parker Peters & Charkraborti-Ghosh, 2016).

2.5.1 Positive learner mindsets

One goal of education in the 21st Century needs to be to develop students who understand learning as an on-going, life-long process (OECD, 2008). Where routine jobs of the 20th

Century are being replaced by robots and software (Gravemeijer, 2013), and where certain jobs of the near future are even not considered possibilities yet (Robinson, 2006), cultivating a lifelong learner mindset is essential. This section considers different views on the positive mindset characteristics of *growth mindset* (Dweck, 2006), *optimism* (Seligman, 1991), *resilience* (Benard, 1995), *grit* (Duckworth et al., 2007), *perseverance* (Conroy, 1998), and *drive* (Pink, 2009). It also considers the impact of a positive mindset outlook as opposed to negative mindset tendencies such as fixed mindset (Dweck, 2006), pessimism (Seligman, 1991), learned helplessness (Benard, 1995), diffidence, defeatism and compliance (Pink, 2009).

Growth mindset

Carol Dweck's work on mindsets arose from investigating goal orientation theory – the effect of 'performance orientation goals' versus 'mastery orientation goals' on human motivation and reasons to achieve (Dweck, 2006; Nicholls, 1975). Goal orientation in education determines whether a student's goal is to *demonstrate* their ability (a performance, or competence goal) or to *develop* their ability (a mastery, or learning goal) (Nicholls, 1975). Dweck describes students with performance oriented goals as having a *fixed mindset*, and students with mastery oriented goals as having a *growth mindset* (Dweck, 2006). Table 2.1 is from Dweck (1986) illustrating key differences.

A study by Ames and Archer (1988) showed that "Students who perceived an emphasis on mastery goals in the classroom reported using more effective strategies, preferred challenging tasks, had a more positive attitude toward the class, and had a stronger belief that success follows from one's effort" (p. 260). With evidence of the benefits of mastery orientation, Dweck (2006) was interested in exploring why some students tend towards a mastery goal (i.e., with a focus on learning), whereas others tend towards a performance goal (with a focus on achievement).

From more than thirty years of research, beginning with a co-authored paper, Dweck and Reppucci (1973), Dweck found that students who believe their brains are malleable, and can change and grow when working through challenging problems, develop a mastery, or what she calls a *growth*, mindset. Students who believe their intelligence is a fixed trait that determines how much they can or cannot learn, develop a performance, or what Dweck calls a *fixed*, mindset. According to Dweck (2010a), "In a fixed mindset, people believe their basic qualities, like their intelligence or talent, are simply fixed traits. They spend their

Table 2.1

Performance v Mastery Goal Orientation Differences (Dweck, 1986)

	Performance	Mastery
Goal or purpose	To look smart To avoid looking dumb To outperform peers	To increase competence To learn, understand, master
Types of tasks students choose	Tasks that are easy for the student but difficult for others	Tasks that are challenging and promote learning
Student response when encountering challenging work or failure	Helpless response Self-denigration Lowered problem-solving ability	Mastery-oriented response Persistence in trying various problem-solving strategies
Impact of “You’re smart” message	Increase persistence	Remain mastery-oriented
Impact of “You won’t do well” message	Helpless response	Remain mastery-oriented
Effort expended	Low	High
Example of how teacher statements can encourage goal type	This activity will evaluate how well you can do (some task).	This activity will help you learn some important things that you will need to know for your profession

time documenting their intelligence or talent instead of developing them. They also believe that talent alone creates success – without effort” (para. 3). Alternatively, “In a growth mindset, people believe that their most basic abilities can be developed through dedication and hard work – brains and talent are just the starting point. This view creates a love of learning and a resilience that is essential for great accomplishment” (para. 4). Students with fixed mindset tendencies become excessively concerned with how smart they appear; they will choose tasks that will confirm their intelligence, and consequently avoid tasks that might prove difficult. Students with growth mindset tendencies recognise that knowledge and abilities can always be enhanced, and will consequently be confident in choosing challenging tasks that can activate and stretch their brains.

Children adopt a fixed or a growth mindset for a variety of reasons, but Dweck (2007a) showed that one main reason seems to be based on the way they have been praised. If their *performance* has been praised they are more likely to develop a performance oriented goal, or fixed mindset; if their *effort* has been praised they are more likely to develop a mastery

oriented goal, or growth mindset (Dweck, 2007a; Mueller & Dweck, 1998). Dweck also discovered that with intentional intervention, students can switch from fixed mindset tendencies to growth mindset tendencies (Dweck, 2006).

Dweck's (2006) research on growth mindset has reportedly had an unprecedented impact on education reform in recent years (Boaler, 2013). However, it may also have done some disservice to gifted education. Growth mindset discourse talks about intelligence as being malleable, or 'non-fixed' – that is, intelligence can be increased through effort and learning. It is difficult to ascertain whether Dweck is stating this increase in intelligence as fact, or whether she is saying that people simply need to *believe* this to be true to develop a growth mindset: "They [people with a growth mindset] do not believe necessarily that everyone has the same intelligence or that anyone can be Einstein, but they do believe that everyone can learn and become smarter" (Dweck, 2008, para. 3). However, other proponents of growth mindset certainly seem to have taken this aspect of intelligence as an irrefutable reality (e.g., Boaler, 2016). Boaler (2016) talks about the 'myth of mathematical giftedness', and suggests that the belief that some people have a genetic disposition for learning mathematics differently is a damaging fallacy: "When we have gifted programs in schools we tell students that some of the students are genetically different; this message is not only very damaging but also incorrect" (Boaler, 2016, p. 94). This is confused somewhat by an earlier statement, however, where she claims, "We have a great deal of evidence that although people are *born with brain differences*, such differences are eclipsed by the experiences people have" (Boaler, 2016, p. 94, italics added). Gladwell (2008) is another proponent of exceptional ability not being determined by gifts or innate capabilities, but by sustained and intentional hard work and practice in a given field – 10 000 hours to be precise – even for people like Einstein and Mozart. This belief, if taken at face value, suggests that no pre-school child could be gifted, as they will not have lived long enough to generate 10 000 hours of practice, which could be quite damaging to the development of gifted education. These beliefs, however, may be coming from an ambiguity of language and a lack of definition of specific terms. For example, learning most certainly equates with becoming more knowledgeable and smarter, but is this the same as increasing intelligence? 'Intelligence', in the context of Dweck's definition of growth mindset, may be better described as 'knowledge', or 'ability'. Instead of, "No matter who you are, you can change your intelligence a lot" (Dweck, 2008, para. 3), Dweck may be really saying 'No matter who you are, you can always learn more and develop your

knowledge and ability further.’ The definition of ‘gifted’ as used by both Boaler (2016) and Gladwell (2008) assumes achievement. Gladwell (2008) is talking about the nature of expertise and success, in becoming a “world-class expert” (p. 40), equating ‘giftedness’ with mastery and eminence. Boaler refutes the existence of giftedness by quoting studies that “have followed people who had been labelled as gifted in their early teens [and] show that they go on to average lives and jobs” (Boaler, 2016, p. 94). She states, “There is no such thing as a ‘math gift’, as many people believe. No one is born knowing math, and no one is born lacking the ability to learn math” (2016, p. 5). While the latter part of this statement is true, she does not address the issue of *how* people learn mathematics – why it is that some students require multiple experiences to construct meaningful schema to support a new mathematics concept, whereas others require only one or two experiences to construct robust concepts and understanding (Hoppe & Stojanovic, 2009; Mrazik & Dombrowski, 2010).

What *is* important to take from the growth mindset discourse, however, is that “Even geniuses work hard” (Dweck, 2010b, p. 8). Giftedness is not the determinant for success or mastery of a talent; a focus on processes such as effort applied, strategies used, choices made, and perseverance in the face of challenge, is the key to success in life (not just in school), even for gifted individuals (Dweck, 2010b; Gagné, 1995). Too many gifted students, who have excelled in school with minimal effort, come to believe that it is the no-effort academic achievement that defines them as smart or gifted, and begin to see challenges, mistakes and the need to exert effort as threats to their concept of self, rather than as opportunities to learn. This can have a devastating effect on promising young people, as the following excerpt displays:

I graduated at the top of my class in high school. I got straight As ... But when I got to Harvard, everyone around me was just as smart or smarter. My grades fell, and suddenly I was no longer exceeding expectations. All that external validation that I’d become accustomed to suddenly stopped. And I crumbled. I felt lost. I learned that I hadn’t formed an identity beyond making people proud of me. So I left school for a while and took a hard look at my life. I learned to cope with failure. I learned that it was OK to rely on other people and ask for help. Eventually I went back and graduated. I’m still not exactly sure who I am. But I’m working on it. (excerpt from Stanton, 2015)

Developing a growth mindset is important for all students, but for gifted students it is essential. Too often gifted students receive praise for their intelligence and academic successes, leaving them particularly fearful of challenges, and vulnerable to feelings of failure when success is not immediate, subsequently minimising the potential for developing their gifts into successful talents. “Great accomplishment, and even what we call genius, is typically the result of years of passion and dedication and not something that flows naturally from a gift” (Dweck, 2007b, p. 43). An educational goal needs to be to ensure students recognise learning as *developing* abilities (even extra-ordinary abilities can be further developed) to be the goal of their schooling, rather than simply *demonstrating* ability by getting good marks or grades.

Optimism

An optimistic mindset, is closely related to a growth mindset. It fosters a positive outlook on learning whereby ‘failure’ is perceived as the result of circumstances (which can be changed), and is therefore “temporary, specific and impersonal” (Seligman et al., 1995, p. 163). A person with a pessimistic mindset, much like a fixed mindset, tends to perceive failure as the result of ‘who you are and what you can or cannot do’, and therefore “permanent, personal and pervasive” (Seligman et al., 1995, p.163). According to Seligman (1991), optimism or pessimism is determined by the way people view causes, about *why* they succeed and *why* they fail, and these views become ‘habits of thinking’. Habits of thinking about causes develop in childhood, and will become lifelong theories about why a person may succeed or fail, and what, if anything, can be done to turn failure into success. By adolescence, a child’s theory of causes crystallises: they will have developed either basic optimistic or basic pessimistic habits of thinking, or mindsets (Seligman et al., 1995). Unfortunately, a pessimistic mindset about causes often develops from well-intentioned significant adults attempting to booster a child’s self-esteem in the face of difficulties. For example, telling the child they have done a wonderful job when the child knows full well that the task did not work out properly; stepping in and ‘rescuing’ the child from their mistakes to make them feel better; or by not acknowledging failures, and therefore not attributing mistakes to any specific cause. A pessimistic mindset, however, is a trait that psychologists have discovered can be changed through intervention, and, even more significantly, something that children may be able to be “immunized against” (Seligman et al., 1995, p. 15). The thinking that success is not instantaneous, that effort needs to be sustained, often over a long period of time, can be modelled to young children (González

& Eli, 2017). Telling young children that their efforts will often not succeed on the first attempt, nor will their finished product look as polished as an older, or a more experienced child's work, is acknowledging the truth. Allowing students to 'fail' tasks enables them to learn that failures, and the deflated self-esteem that may accompany them, are rarely catastrophic. Stepping in and fixing mistakes sends the message that 'if at first you don't succeed, give up and let someone else do it for you' – a learned helplessness (Diezmann & Watters, 1995). By acknowledging a child's difficulties and validating their disappointment, there is an opportunity to model an optimistic mindset about cause. Optimism is not cultured by superficial positive thinking or self-talk; it acknowledges difficulty, disappointment and frustration, but views failure as a setback, as an obstacle to overcome. Neither is optimism synonymous with confidence. Williams (2014) found that confident students could be either optimistic or non-optimistic, and that "Confident non-optimistic students ... possessed a certainty in their high mathematical performances rather than a certainty that by *really thinking*, they could learn more" (p. 29).

Resilience

Optimism is a type of resilience. Resilience is defined as the ability to cope or 'bounce back' after encountering negative events, difficult situations, challenges or adversity: it is the capacity to respond adaptively to difficult circumstances and still thrive (Benard, 2004; Haertel, Walberg & Wang, 1997). As with growth mindsets, research into resilience in school children started with a question (see Benard, 1995) – why do some children manifest resilience and adaptation in the face of risk, adversity or stress, while others crumble and become helpless and emotionally vulnerable, and can resiliency be fostered? Benard's research focused on resilience in children born into high-risk conditions such as severe poverty, or to parents with mental illness or criminal behaviours, or into war-torn communities. He clearly showed that certain characteristics of family, school and community may indeed alter, or even reverse expected negative outcomes and enable students to circumvent life stressors and manifest resilience (Benard, 1995). These characteristics included establishing positive and high expectations and providing opportunities for meaningful participation (Benard, 1995). Learning environments, then, have an important role to play in promoting student resilience: "Student resilience and wellbeing are essential for both academic and social development and this is optimised by the provision of safe, supportive and respectful learning environments. Schools share this

responsibility with the whole community” (Department of Education and Training (Australian Government), 2016).

Research has been conducted on the role of resilience in gifted students (Bland, Sowa & Callahan, 1994; Kitano & Lewis, 2005; Reis, Colbert & Hébert, 2004), and on the role of resilience in students’ mathematics learning (Williams, 2003a). Bland et al. (1994) recognised that the development of resilience in gifted students is curtailed when the classroom learning environment lacks sufficient challenge. This, in turn, may result in chronic underachievement, “A relationship seems to exist between inappropriate or too easy content in elementary school and underachievement in middle or high school” (Reis et al., 2004, p. 111), which has both academic, and social and emotional implications for gifted students (Kitano & Lewis, 2005). If gifted students have had limited experiences of real challenge or failure, they will have had limited opportunity to develop appropriate resilient responses in failure situations (i.e., learn to cope with failure), and will therefore find such situations, when they do arise, confusing, isolating, and possibly even frightening (King, 2004). Williams (2003b) surmised that building resilience in students has the potential to improve mathematical performance, and “teachers need to explore ways to simultaneously increase student understanding of mathematics and student resilience” (p. 378). Williams used Seligman’s (1995) indicators of optimism (viewing causes of failure as being temporary, specific and external) to develop and measure student resilience in working with unfamiliar challenging mathematics problems. It was shown that the practice of using challenging mathematics tasks to develop student resilience is dependent on those tasks being *appropriately challenging* for each student, otherwise the effect could be the exact opposite.

Grit

Duckworth et al. (2007) took the concepts of optimism and resilience a step further, recognising that success depends on more than just a positive response to failure or adversity. They recognized that success also depends on having deep commitment to a task or goal over a prolonged period of time, with an unswerving dedication to achieving that long-term goal. They call this quality ‘grit’. As with growth mindset, optimism and resilience, Duckworth (2016) believes that grit can be developed in students through intentional intervention. She has shown that by teaching students about deliberate, effortful practice with things they cannot yet do in order to succeed, develops not only a growth mindset and resilience, but also grit. Understanding that deliberate practice is not easy, that

it can sometimes be confusing, frustrating and/or tedious, changes students' grit levels by changing their beliefs about the sources of success. One of Duckworth's most surprising findings on grit, is that grit and innate ability are often inversely related, especially in a standards based education system. In terms of school achievement, once a student reaches the desired proficiency level (regardless of how much, or how little, time and effort was required) they feel they can stop trying. To maximise learning, and encourage students to do as well as they can, there needs to be a no-limit ceiling or threshold approach to student outcomes (Duckworth, 2016).

Perseverance

Perseverance is another non-cognitive characteristic required for success. It is similar to grit, but with a focus on each task at hand rather than one specific long term goal. Perseverance has a dynamic element to it. Conroy (1998) differentiates between *persistence* (trying again and again without giving up) and *perseverance* (making adjustments when an approach does not work, and then trying again), and talks about recognising when adjustments to approaches or strategies need to be made. "Part of the skill of the power of perseverance is to make those adjustments as you persist" (Conroy, 1998, p. 30). Perseverance has been shown to be a characteristic of optimism (Williams, 2014), but there is no point in persisting on a task that is not working, and expecting to get different results, without making adjustments. In the context of mathematical problem solving, Thom and Pirie (2002) describe perseverance as "the student's *sense* (i.e., intuitive and experiential) in knowing when to continue with, and not to give up too soon on a chosen strategy ... [and] knowing when to abandon a particular strategy or action in the search of a more effective or useful one" (p. 2). Significantly, Williams (2014) showed that this differentiation between persistence and perseverance illuminates and underpins elements of *creative* problem-solving in mathematics.

Drive

Pink (2009) discovered that, as humans, we possess a deep need to direct our own lives, to learn and create new things, and to constantly improve both ourselves and our world. He summarises these needs as *autonomy*, *mastery* and *purpose*, and explains that they motivate us, and drive our behaviour, in much the same way as our biological needs for food, water and reproduction. "Humans have an innate inner drive to be autonomous, self-determined, and connected to one another. And when that drive is liberated, people achieve more and live richer lives" (Pink, 2009, p. 71). This drive affects both performance and satisfaction.

While Pink's writing is aimed primarily at businesses and organisations, there are clear applications for education (Gillard, Gillard & Pratt, 2015), and many parallels to the mindset descriptors listed previously. Indeed, Pink refers to Seligman and Dweck, together with Csikszentmihaly, as "self-determination theory (SDT) scholars ... leading the positive psychology movement" (Pink, 2009, pp. 71-72).

Autonomy, according to Pink, is self-direction that leads to meaningful engagement (as opposed to compliance). By providing students with frequent and authentic opportunities to engage in their own inquiries (Makar, 2011), to spend as much time as they need to satisfactorily answer their own questions (Holt, 2012), and to choose how they demonstrate understanding, encourages students to own their own learning. Compliance may lead to students to do things the way we want them to, but autonomy leads to meaningful engagement, and "only engagement can produce mastery" (Pink, 2009, p. 109).

Mastery, or the pursuit of excellence, comes from an intrinsic desire or motivation, as opposed to the desire to achieve external benefits such as school grades. Extrinsic rewards have been proven to actually decrease motivation (Deci & Flaste, 1996; Pink, 2009). "In environments where extrinsic rewards are most salient, many people work only to the point that triggers the reward – and no further" (Pink, 2009, p. 56). Once the 'reward' has been achieved, for example, an A+ grade, why continue applying more effort?

Purpose, the third element Pink (2009) describes as one of our biggest motivators, is the need to not only constantly improve ourselves, but also to have meaning in life outside ourselves (to improve our world). Purpose provides a context for both autonomy and mastery. In education students' performance and satisfaction in learning will be heightened when they can see that the learning has meaning and relevance.

Successful people generally have positive mindsets. Whether they be optimistic, resilient, driven, or any one, or combination of, those traits listed above, a positive mindset seems to be a catalyst that can turn potential into realised results. Can the same be said for successful learners?

2.5.2 Effects of a positive learner mindset

When students are optimistic, resilient, show grit, and believe they can always learn more (growth mindset), when they see relevance and meaning, and can 'own' their own learning (drive), they are more likely to be willing to persevere. Through perseverance they ultimately enjoy the learning process – it feels good to learn and grow and strive to learn

more. However, Hattie and Yates (2014), in their commentary on how research into human learning can inform teaching practice, warn that learners are only motivated by knowledge gaps in the learning process if they can perceive means by which those gaps can be navigated. They are discouraged by seemingly insurmountable knowledge chasms, or when there is not sufficient prior knowledge to build upon. If a task is appropriately challenging, however, where students experience success after some initial struggle, the completion of the problem becomes an intrinsic reward, subsequently motivating them to explore further. Having ample opportunity and time to pursue mastery, the “desire to get better and better at something” (Pink, 2009, p. 109), will improve learning experiences. Learning becomes pleasurable, and students, given the right conditions – a safe environment, a true sense of ownership of their work, and a positive learner mindset – will want to be engaged in appropriately challenging work, and may consequently be able to self-differentiate (Anderson, 2016) and ultimately become autonomous learners (Betts, 2004). Autonomous learners are:

Self-confident, self-accepting, hold an incremental view of ability, are optimistic, intrinsically motivated, ambitious & excited ... are willing to fail and learn from it ... [They exhibit] appropriate social skills, work independently ... seek challenge, are strongly self-directed, follow strong areas of passion, are good self-regulators ... resilient, producers of knowledge, [and] possess understanding and acceptance of self. (Neihart & Betts, 2010, p. 2)

If a major goal of education is the development of students as independent, self-directed learners – to “help students develop as autonomous learners, with the appropriate skills, concepts, and attitudes necessary for their journeys” (Betts, 2004, p. 190) – a positive mindset is a must. A positive learner mindset enables the learner to become self-actualising, that is, able to realise their full potential (Maslow, 1968).

2.5.3 Mathematical Giftedness and Mindsets

Gifted students are not necessarily more susceptible than their peers to experiencing negative mindsets, but mathematically gifted students are possibly more at risk of not being provided appropriately challenging learning environments. “Of all the concerns raised by parents in the literature, the failure of schools to provide learning at an appropriate level and pace is the most frequently cited and most often contributes to affective difficulties for gifted students” (Wardman, 2018, p. 77). Underachievement of gifted students is a

concerning issue, with some researchers proposing “that as high as 50% of gifted students underachieve at some point” (Siegle, 2013, p. 1). Others would suggest this may be an underestimation, as underachievement does not necessarily equate to low grades at school, with ‘grades’ generally limited to year-level standards and expectations, not individual capabilities (Peters & Jolly, 2017). High achieving, *seemingly* successful students (and their teachers and parents), may have developed a satisfaction with high school grades, and still be underachieving compared to their true capabilities (Neihart & Betts, 2010). The issue of underachievement is, indeed, a serious one, and a major area of concern in gifted education (Siegle, 2013). Indeed, Renzulli and Park (2000) have suggested that up to 20% of high school dropouts may be gifted.

There have been many reasons suggested for gifted underachievement. For example, low levels of self-confidence (Gallagher, 1990), an inability to persevere (Gallagher, 1990), unhealthy perfectionism (Siegle, 2013), deliberately hiding talents, possibly due to peer influence (Reis & McCoach, 2000), cultural diversity (Reis & McCoach, 2000), and gender expectations (Colangelo, Kerr, Christensen & Maxey, 1993; Weiss, 1972). Compliant gifted students may do exactly what they are told to do, but no more, just in case it is not the right thing to do (Pink, 2009). It is possible that many of these reasons may, in fact, have self-limiting learner mindsets at the core (Siegle, 2013). However, there seems to be little research on the effect of addressing negative learner mindsets in gifted students. Singer et al. (2016) in a *Topical Survey of recent Research and Activities for Mathematically Gifted Students*, mention, in passing, the possible effect of a growth mindset, with some recommendations for teaching practice:

We may not know ... just how much a growth mindset as described by Dweck (2006) and others might help ... [but recommend] teachers pose problems that allow all students, including the most talented, to struggle; expect coherent explanations and critiques of unique and creative solutions; give formative and summative assessments that provide opportunities for students to reason, create problems, generalize patterns, solve problems in unique ways, and connect various aspects of mathematics; and generally act as a role model who is comfortable with making mistakes and demonstrating the joy of solving difficult problems (Singer et al., 2016, p. 16-17).

There is no mention of specific research on mathematical giftedness and mindsets. The development of negative, or self-limiting learner mindsets in students who are mathematically gifted could have adverse consequences for the students themselves, and

for society as a whole, if their full capabilities are not realised, encouraged and supported (Lassig, 2009). Conversely, the development of positive learner mindsets in students who are mathematically gifted could have profound implications for both the student, and the future of society, as gifts transform into talents that continue to be enhanced, and possibly become catalysts for new innovations. However, while a positive learner mindset may open the doors for ongoing, life-long autonomous learning and invention, it is vital to understand that this does not eliminate the need for a supportive teacher, guide and advocate for the student. Neihart and Betts (2010) recognise that autonomous learners require “more support, not less” (p. 2). They require support for risk-taking, and help to cope with psychological issues associated with the ‘rocky path to success’, and they require time and space for their learning (Neihart & Betts, 2010). With this in mind, understanding and recognising the issue of learner mindsets is something educators of gifted students need be aware of. It is an issue deserving of further research.

2.6 Teacher Professional Learning

The aim of this study was to explore the impact of teacher professional learning on the mindsets of mathematically gifted students with self-limiting mindsets. One intention was to use the findings to refine and develop a sound, research-based professional learning program on mathematical giftedness for classroom teachers. It was important, therefore, to consider researched elements of effective teacher professional learning.

Effective teacher professional learning has been linked to substantial benefits for students, including both academic and affective improvements (Cordingley, 2015), which must be the main purpose of continuing teacher professional learning. It is also linked to improved teacher understandings and confidence in implementing new teaching ideas and strategies (Cordingley, 2015). However the “effective” elements of professional learning must be identified for this effectiveness to be realised. According to Cordingley’s (2015) review of research findings, effective teacher professional learning includes specialist expertise and coaching, whole-school support and professional dialogue (including a common language and shared understanding), enquiry oriented learning (as opposed to a prescribed program), learning how to analyse teaching approaches, including understanding what works and what doesn’t and why (reflective practices), and a focus on specific students’ learning requirements as a motivation for continued improvement of teaching and learning practices.

Within professional learning models, Koellner and Jacobs (2015) suggest there are different formats that fall on a *continuum of adaptability*, ranging from *highly adaptive* to *highly specified*. Highly adaptive models are based on “general and evolving guidelines” (p. 51), and highly specified models focus on “specific content, activities and materials” (p. 51). Professional learning programs, according to Koellner and Jacobs, may fall anywhere along this continuum with varying levels of adaptability and specificity. In research, the biggest difference between the two extremes is that specified models can be easily investigated in standard quantitative studies, whereas primarily adaptive models are best explored through qualitative design-based research. The flexible nature of adaptive models has been shown to be more effective (Cordingley, 2015), as it allows for modifications pertinent to individual circumstances, but makes comparisons between individual contexts more challenging.

According to Guskey (2014), effective professional learning begins with identifying *the primary goal* of the professional learning, that is, clarifying which student outcomes we want to see improved (cf. Cordingley, 2015, the focus on specific students’ learning requirements). Once desired outcomes have been established, different ways to disseminate new knowledge and practices, to achieve those outcomes, can be considered. According to Guskey (2014), “The effectiveness of any professional learning activity, regardless of its content, structure, or format, depends mainly on how well it is planned” (p. 12). Methods of ongoing support for teachers who will be implementing these new practices is crucial (Cordingley, 2015; Peters & Jolly, 2017). Ongoing support is provided by the specialist “coach” (Cordingley, 2015), and from school leadership backing (Guskey, 2014; Peters & Jolly, 2017).

Regarding professional learning on giftedness, Lassig (2009) asserts that to implement an effective professional learning program it is important to firstly “be aware of teachers’ beliefs” (p. 7) so that approaches may be targeted. A shared understanding and common language about mathematical giftedness, is essential for a whole-school approach (Cordingley, 2015; Lassig, 2009).

Knowledge of mathematically gifted students with self-limiting mindsets needs to be adaptable to individual circumstances and requirements, and teachers require guidelines, not prescribed, specified content (Koellner & Jacobs (2015). A focus on a coaching/mentoring role (Cordingley, 2015), involving collaborative planning with teachers that can be tailored and relevant to their own classroom context (Cordingley, 2015;

Guskey, 2014), would best suit such an approach. A whole-school component to the professional learning would also add to the ongoing effectiveness by encouraging professional dialogue among peers from a common basis and understood terminology.

2.7 Conclusion

The purpose of this literature review was to understand and define the nature of giftedness, mathematics education, mathematical giftedness, and learner mindsets, and to explore existing theories and research on connections between these fields, in order to establish a context for further study. The classroom teacher has a responsibility to understand and support the learning needs of all students in his/her class. This is essential for students who are mathematically gifted even if, and possibly even *because*, they are already successful in mathematics at school within a standards-based, grade-level structure. There appears to be a need to explore further the phenomenon of students who are mathematically gifted but who have self-limiting mindsets which make them resistant to experiencing effort, and/or resistant to being challenged in their mathematics classes. Research questions that could be explored are:

- How can teachers best provide support for the learning of mathematically gifted students to ensure they develop and maintain positive, self-actualising mindsets that allow their innate capabilities to be transformed into mathematical talents?
- Why and when do self-limiting mindsets develop? Can they be prevented?
- Can self-limiting mindsets be ameliorated through targeted teacher professional learning? What form might that professional learning take?
- Does ensuring a positive learner mindset motivate gifted students to continue to learn mathematics through to higher levels of education?
- Does ensuring a positive learner mindset in mathematically gifted students stimulate them to become the innovators and trailblazers that are increasingly required in the current technological age?

Further research on any of these questions will provide valuable additions to the current knowledge base of mathematically gifted students, and how educators can best support their successful on-going learning. This study will focus on exploring whether targeted teacher professional learning (about mathematically gifted students and mindsets) can have a

positive impact on students who are mathematically gifted but with self-limiting learner mindset tendencies, by asking the question:

- *What impact does targeted teacher professional learning about classroom support for mathematically gifted students with self-limiting mindsets, have on the mindsets and mathematics learning of these students?*

This is explored through the subsidiary questions:

- How do students approach challenging mathematics tasks before and after their teachers receive professional learning?
- How do students who are mathematically gifted view themselves as mathematics learners before and after their teachers receive professional learning?

Exploring *how* teachers can best provide support for mathematically gifted students to ensure their innate mathematical capabilities are transformed into mathematical abilities, or talents is also explored, in part, through the third subsidiary research question:

- What do the teachers do during the post-professional learning teaching period to support the mathematics learning of students who are mathematically gifted but with self-limiting mindset tendencies?

The outcomes of the study will contribute to the research literature on mathematically gifted students by exploring the effect of mindsets on gifted students' mathematics learning, with a specific focus on the impact of providing teachers with knowledge about how to recognise and address self-limiting mindsets. The outcomes will include a discourse on the possible development of a research-based pre-service teacher unit of work and/or in-service teacher professional learning program, aimed at supporting the learning of mathematically gifted students in regular classrooms.

The next chapter, Chapter 3, explores different research paradigms, and sets the study within an appropriate theoretical framework to effectively explore mathematical giftedness and mindsets. It will justify the methodology, the methods of data collection, and process of analysis chosen, and will address the issues of trustworthiness and ethical considerations required for a study such as this. This will situate the research design in context – the selection of students (Chapter 4) and the teacher professional learning process (Chapter 5) – prior to the analysis chapters (Chapter 6 and Chapter 7), and the concluding outcomes chapter (Chapter 8).

Chapter 3 – The Theoretical Perspectives that Shaped the Study, and the Research Design

Ontology, Epistemology & Methodology → Research Design

3.1 Chapter Overview

The previous chapter explored the literature about, and defined the nature of, giftedness, mathematics, mathematical giftedness, and learner mindsets, and provided a basis for ongoing research into these areas. This chapter will consider different research perspectives, and justify the chosen theoretical philosophical stance for this current study. Recognising, understanding and explaining perspective is a vital foundation for any research as it describes the fundamental lens through which the researcher is viewing a particular study. It is a flawed and dangerous assumption that any one person's view or understanding of an entity, such as education, is a common (let alone universal) perspective. This is akin to Westerners being blinded to cultural biases such as 'democracy' and 'individualism', which many may believe to be basic human rights, but are not globally held values any more than Eastern values of 'totalitarianism' or 'communism' (Ball, Dagger & O'Neill, 2014). This chapter explains the 'cultural' paradigm that guides this research into mathematics education and giftedness, situating the study within ontological, epistemological and methodological philosophies. It then goes on to outline and justify the data collection and analysis methods chosen, including trustworthiness issues, and ethical considerations required, based on this theoretical perspective.

3.2 Defining Ontology, Epistemology and Methodology

A *paradigm*, as its simplest definition, is "a basic set of beliefs that guides action" (Guba, 1990, p. 17). Beliefs about education are often as invisible as cultural biases, so highlighting differences in fundamental beliefs, or paradigms, is important in situating and explaining the research approach.

Ontology is concerned with the belief of what is *real* and therefore *what can be known*. Traditional scientific research is based on a reality that is absolute, a reality that can be seen and measured through experimental research: research that can be replicated many times over and readily validated. However, the emergence of sociological research, including educational research, which may also deal with immaterial phenomena or concepts,

required a re-conceptualisation of this fundamental belief of ‘knowing’ (Creswell, 2007). Where scientific research is comfortably grounded in an absolutist, or *realist*, view of the physical world, sociological research adopted a *relativist* view, whereby reality depends on context, where there may, in fact, be many different parallel realities. Whilst the realist researcher believes there is an external objective world that can be known through research, the relativist researcher believes that we can only know the sociological world through our own subjective observations of it – what is ‘real’ is one person’s perspective only: there is no absolute reality, it depends on each individual’s experiences (Creswell, 2007). The realist view focuses on knowledge acquisition; the relativist view focuses on meaning making. Objects within the physical world may be based on the laws of nature, they are generally constant and tangible, but concepts within the sociological realm, such as best-practice education, learning and mindsets, are neither tangible nor necessarily constant (Bogdan & Biklen, 2007; Wolcott, 1994). What can be known in the sociological world is influenced by both the individuals being researched and the interpretations of the researcher (Stake, 2005). As my research is an exploration of individual experiences, with a focus on making meaning of these individuals’ experiences in a mathematics learning context, I have assumed a *relativist* ontology.

Epistemology is concerned with how we come to know the things that are real. Ontology, which determines the research perspective for *what can be known*, directly influences the epistemological stance, *how it can be known*. Epistemology recognises that there is a relationship between the researcher and what is being researched that can either be objectively derived or discovered on the one hand (positivism), or subjectively interpreted and described on the other (interpretivism) (Guba, 1990).

Based on my review of the literature this research takes the stance that in education, learners are not passive receivers, nor simply information processors (Piaget, 1950). Learning is about making meaning of new information, not just memorising it, and learning is influenced by intrapersonal and environmental factors unique to each individual (Vygotsky, 1978). Researching meaning-making (learning) and dispositions (mindsets) is not primarily about ‘discovery’, but about observation, interpretation, and a description of individuals’ experiences. We can only know the reality of these individuals’ experiences through sociological observation, not through objective scientific experimentation. Epistemologically, then, my research is based on an interpretivist view of knowing. The Epistemology underpinning the research approach will impact the type of research

questions used to frame the study, which will subsequently determine the methodological approach.

Methodology is the theory about which methods of data collection are most appropriate and valid in order to generate and justify new or particular knowledge from the research. From a realist view, the methodology would be primarily quantitative (measurable), whereas from a relativist view, the methodology would be primarily qualitative (descriptive and/or interpretive). Methodology is inextricably linked to beliefs of ontology and epistemology – *what can be known* (ontology), and *how it can be known* (epistemology) dictates *how it can be found out* (methodology). The ontological and epistemological approaches for my research would indicate a qualitative method: I wanted to understand the experiences of students who are mathematically gifted but who have self-limiting mindsets. This required accumulating deep insights for descriptive and interpretive analysis, and a qualitative methodology could provide this. Merriam (1988) believes that “research focussed on discovery, insight, and understanding from the perspectives of those being studied offers the greatest promise of making significant contributions to the knowledge base and practice of education” (p. 3).

3.3 Situating the Theoretical Perspective for the Study

In educational research and practice the belief base, or paradigm, explains the researcher’s selection of theory about how humans learn. There are many basic learning theories – for example, behaviourism, cognitivism, constructivism, humanism (Slavin, 2012) – each with its own foundational views of learner identities embedded within the researcher’s own ontological and epistemological beliefs.

My research assumes Piaget’s belief that learning is an active, constructive process; that the learner is a ‘knowledge creator’ with both cognitive and affective influences; that education is about enabling optimal learning for all students, addressing individual potentialities, and maintaining dignity for all. Evidence for this view of learning has increased in recent decades (Geake, 2009b) with the advent of neuroscientific technologies, such as functional magnetic resonance imaging (fMRI), that can map brain function and track the construction of neural connections and subsequent pathways as the learner makes sense of something new.

An emotionally charged brain change when understanding is suddenly and consciously recognised...can be seen by a sudden jump in the intensity of the EEG

signal. This jump is presumably indicating the dynamics of neuronal systems which rely on the anatomical interconnectivity between the subcortical limbic regions and the cortex. (Geake, 2009b, p. 118)

This belief places my study within a *constructivist* paradigm, which has at its foundation a relativist understanding of what can be known, and an interpretivist belief of how this knowledge can be described within context (Slavin, 2012). There are three main strands within the constructivist paradigm; cognitive constructivism, social constructivism, and radical constructivism.

Cognitive constructivism is derived from the work of Piaget (Slavin, 2012). He asserted that humans cannot be ‘given’ information, but must ‘construct’ their own knowledge which is built through experience. Experience enables them to create cognitive schemas (neural networks) which are changed, enlarged, and made more sophisticated through two complimentary processes: assimilation, when new information can be assimilated into already developed schemas, and accommodation, when new information does not fit within any already known constructs and therefore needs to be accommodated by the formation of new schemas. Cognitive constructivism has the learner at the centre, with the teacher providing a rich environment for the learner to engage with. According to Piaget (1950), the learner progresses through discrete stages of maturation, which must be reached prior to construction of new knowledge being possible. These stages can be observed by the teacher who then adapts the learning environment accordingly.

Social constructivism, based on Vygotsky’s theories of learning (Vygotsky, 1978; Slavin, 2012), shares many of Piaget’s ideas about how children learn, but places more emphasis on the role of others in influencing the learning process: teachers, parents, other students, and cultural/community settings. Social interactions are at the centre of an individual’s learning. The learner is continually learning, making sense of new information based on prior knowledge and through the support of more knowledgeable others. According to Vygotsky, learning induces development, with the ‘zone of proximal development’ (ZPD) being the level at which optimal learning takes place, the zone where new knowledge is constructed most effectively.

Radical constructivism (von Glasersfeld, 1995) also asserts that knowledge is not passively received but actively constructed by the learner, but revolves around the idea that each individual constructs their own notion of reality from their own individual experiences, and

learns through personal reflection. The teacher's role is to primarily provide situations that cause cognitive conflict, "a situation in which the students' [current] network of explanatory concepts clearly turns out to be unsatisfactory" (von Glasersfeld, 1995, p.10), in order to trigger the students' own thoughts to be modified.

A *social constructivist* belief of learning is best aligned with the perspective underpinning this study. Within a social constructivist paradigm both the teacher and other students play an important role in an individual's learning (Vygotsky, 1978). The teacher creates a context for learning a particular concept, ensuring students have opportunities to engage in activities that require problem solving, collaboration, discussions with others and justification of findings, to facilitate learning. The concept of a zone of proximal development and teachers being aware of students' zones of proximal development, is integral to this, in targeting optimal learning experiences and ensuring appropriate scaffolding. This view of learning presumes that learning requires struggle and effort on the part of the learner (Dweck, 2006; González & Eli, 2017), with design, management and support from a more knowledgeable other managing a suitable level of productive struggle for individual learners (Lithner, 2017).

3.3.1 Social Constructivist belief and Learning and Teaching Mathematics

With social constructivism, the actively involved teacher and the shared experiences of other students is an important aspect in the accurate construction of mathematical knowledge and understanding. Mathematics and mathematics learning, as described in Chapter 2, is more than a knowledge of rules, procedures and formulae that can be used to generate correct answers (Davis, 1984). Mathematical knowledge and understanding follows a learning trajectory where new concepts rely on previously learned concepts so that new information can be assimilated, and, if necessary, accommodated into schemas (Piaget, 1950; Hiebert & Carpenter, 1992), and this understanding is constructed and refined in social contexts. Constructs and schemas evolve and change as meaningful mathematics experiences are directed and evaluated by an actively involved teacher, with students working as individuals or in groups, with rich discussions throughout (Vygotsky, 1978).

3.3.2 Social Constructivist Belief and Giftedness

A social constructivist view of education also aligns well with the model of gift development adapted for this study (see Chapter 2). Gagné's (2009) *Differentiated Model*

of Giftedness and Talent highlights the role of others in the transformation of gifts into talents. Social interactions – parents, family, peers, teachers, mentors, community and culture – all play a significant role in the transforming of innate capabilities (gifts) into realised accomplishments (talents) (Gagné, 1995; 2003). Mathematical giftedness does not automatically presume mathematical talent; giftedness only transforms into talent through optimal intrapersonal (e.g., mindset) and environmental (e.g., school and teaching) catalysts. In other words, mathematically gifted students require ongoing interaction with, and support from, others in order to realise their learning potentials.

3.3.3 Social Constructivist Belief and Qualitative Research

A social constructivist view of researching individual students' learning and mindset development lends itself to a qualitative approach. Changes in learning dispositions and mindsets of individuals can be observed, described and interpreted, but cannot be quantitatively measured. A qualitative approach is interpretive, with “the central endeavour in the context of the interpretive paradigm [being] to understand the subjective world of human experience” (Cohen, Manion & Morrison, 2011, p. 17). With a social constructivist paradigm, the researcher seeks to understand experiences of individuals, and the consequent meanings of these experiences, within and through interactions with others. “These meanings are varied and multiple, leading the researcher to look for the complexity of views rather than narrow the meanings into a few categories or ideas” (Cresswell, 2007, p. 20). A qualitative study is an in-depth study in which meaning and understanding of the phenomenon of interest are sought, with the researcher being actively involved in data collection and analysis, with findings being inductively derived from the data (Merriam, 2009).

3.4 Case Study Methodology

Case study is one of the most frequently used qualitative research methodologies in educational research (Yazan, 2015). It is defined as an “intensive, holistic description and analysis of a single entity, phenomenon, or social unit” (Merriam, 1998, p. 27). The story, or stories, of people and/or programs of interest, are explored with the purpose of addressing a certain issue, or issues, surrounding the case. The researcher's role is to build a clear picture of the case under study through detailed exploration and “thick description”, and to provide integrated interpretations of observed situations and contexts in respect to the issue/s being explored (Stake, 1995).

Case study, by definition, requires a specific case to be studied and analysed. It is the “unit of analysis”, requiring an in-depth description and analytical interpretation, that defines a case study rather than the topic of investigation (Merriam, 2009, p. 41). For a study to be a case study the unit of analysis must be a bounded system, that is, an object selected for study around which there are boundaries, either natural or imposed by the researcher. Without boundaries, the study would lose focus and become too unwieldy or vague; without boundaries, the phenomenon to be studied does not qualify as a case (Stake, 1995). The selection of the case, or the “unit of analysis”, is a vital step in conducting a case study that is purposive and not random (Merriam, 2009). The unit of analysis of this research is the phenomenon of mathematically gifted students with self-limiting mindset tendencies, with purposefully selected samples to be studied for a period of around six months. This bounds the study as a case study.

A case study may be an in-depth study of an *individual case*, which may be either intrinsic, if the researcher has a personal interest about the case, or instrumental, where the researcher has an interest in a broader phenomenon that they believe may be addressed by studying a particular case. Alternatively, a case study may be a *multiple* or *collective case study*, studying several cases within the same project (Merriam, 1998; Stake, 1995). A multiple case study is more likely to be instrumental, with its purpose being to understand a ‘puzzlement’ (research question), with the belief that general insights into the puzzlement may come about through studying a particular case or cases (Stake, 1995). A multiple case study compares and contrasts different sample cases in different contexts (Merriam, 1998). The data can then be analysed *within* the individual samples, *between* the individual samples, and *across* all individual samples (Baxter & Jack, 2008).

The approach deemed most appropriate for this study was a multiple case study as defined by Merriam (1998). It is a case study, with multiple purposively selected samples of the case – three students of different ages and stages of primary school– used to explore the case. It is an instrumental case study, with the view that researching the phenomenon of mathematically gifted students exhibiting self-limiting mindset tendencies in three purposely selected students may provide insights for the reader, and facilitate more understandings of mathematically gifted students exhibiting self-limiting mindset tendencies.

One of the strengths of case study is that analysis embeds the case within its own particular context, or rather, does not remove the case from its context. It “helps us understand and

explain the meaning of social phenomena with as little disruption to the natural setting as possible” (Merriam, 1998, p. 5). To acquire this perspective, the researcher can become part of the context as a *participant observer* (Kawulich, 2005; Merriam, 1998), which enables the researcher to interact with the students and teachers, to engage in tasks, ask questions, and generate and/or participate in discussions. This enables them to document individual thinking processes as well as observe what participants are doing.

A case study can provide deep, rich and significant insights into a particular case (Merriam, 1998), but one criticism of case study is that insights gained cannot necessarily be generalised to a wider population. By definition, a case study needs a defined boundary, and generalisability of findings beyond the bounded case is limited. However, generalisability in education (in research terms of ‘absolute truth’) will always be fraught with problems as educationalists are dealing with individual students, with individual personalities and learning profiles. With a case study, we may be able to provide what Bassey (1999) calls ‘fuzzy generalisations’ such as, ‘it is possible, or likely (or unlikely), that what was found in this particular case may be found in similar situations elsewhere’: a study of one particular case may suggest to the *reader* approaches to try in other, similar situations. With a background to a situation, illuminated by the case, the possible reasons for a particular problem may be explained, as well as what happened and why. The case may then shed light on a phenomenon that confirms what is already known, or extend current practice by providing a new way of viewing the phenomenon, or by adding to accumulated knowledge of the phenomenon, or provide completely new meanings (Merriam, 1998). Most qualitative researchers would call this *transferability* (see a more detailed explanation of this in section 3.8). Educationalists and researchers may discover that previously held beliefs or theories are either validated, or brought into question by the case study (Merriam, 1998) which may, in turn, produce new understandings, questions, or propositions, which can then be pursued in ongoing research.

Merriam’s case study approach is situated within a constructivist epistemology and is particularistic, descriptive and heuristic (Yazan, 2015). It is therefore an appropriate choice for this research: particularistic (focusing on a particular phenomenon), descriptive (providing a rich, thick description of the phenomenon) and heuristic (enabling the reader to develop their own understanding of the phenomenon).

3.5 Theoretical Perspective and Methodology Overview

The following tables (Table 3.1 and Table 3.2) provide an overview of the theoretical perspective, and the methodology, that directed the approach to the research design and data collection for this study:

Table 3.1

Theoretical Perspective Summary

Theoretical Perspective		
<i>Paradigm</i>	<i>General application</i>	<i>Specific application</i>
<p>Ontology The study of ‘reality’ – the theory of what is real and therefore what can be known.</p>	<p>Sociological research has a <i>relativist</i> view of what can be known. Reality depends on context: there is no absolute reality, it depends on each individual’s experiences.</p>	<p>Mathematical knowledge is constructed in social contexts; constructs evolve and change as individuals and groups work through different experiences.</p>
<p>Epistemology The study of ‘knowing’ – the theory of how we come to know the things that are real.</p>	<p>An <i>interpretivist</i> view of how we can come to know things is primarily about observation, description and interpretation of individuals’ experiences.</p> <p><i>Constructivism</i> is the belief that knowledge is constructed by the individual via the development of neural schemas (Piaget, 1950); the brain constantly assimilating and/or accommodating new information with existing knowledge, developing ever increasingly complex networks of neural pathways that interconnect, in order to make sense of life experiences. In <i>Social Constructivism</i> (Vygotsky, 1978) other people, e.g., teachers, play a critical role in this learning process.</p>	<p>Mathematics learning is an active, constructive process, with the learner as a ‘knowledge creator’ with both cognitive and affective influences. Researching shifts in learning dispositions and mindsets of individuals cannot be quantitatively measured, but can be observed, described and interpreted.</p> <p>Mathematical understanding is more than a knowledge of rules, procedures and formulae that can be used to generate correct answers. Mathematical understanding follows a learning trajectory, where new concepts rely on previously learned concepts so that this new information can be assimilated, and, if necessary, accommodated into existing schemas. This understanding is constructed and refined in social contexts with the support of a ‘more knowledgeable other’.</p>

(continued)

Table 3.1 Continued

Theoretical Perspective		
<i>Paradigm</i>	<i>General application</i>	<i>Specific application</i>
	<i>Differentiation of giftedness and talent</i> (Gagné, 2003) is a belief that there is a distinction between a gift (innate capability) and a talent (realised accomplishment). Social interactions – parents, family, peers, teachers, mentors, community, culture – all play a significant role in the transformation of gifts into talents	<i>Mathematical giftedness</i> is an innate aptitude or high-capacity for understanding mathematical concepts. <i>Mathematical talent</i> is a mastery of mathematical knowledge. Mathematical giftedness does not automatically transform into mathematical talent; giftedness only transforms into talent through intrapersonal and environmental catalysts. Mathematically gifted students require ongoing support from, and interaction with, others to realise their learning potentials.

Table 3.2

Methodology Summary

Methodology The theory of how we can find out new knowledge	Case study is one of the most frequently used qualitative research methodologies in educational research (Yazan, 2015). Case study is defined as an “intensive, holistic description and analysis of a single entity, phenomenon, or social unit” (Merriam, 1998, p. 27), with the purpose of addressing certain issues surrounding the particular case.	Individual students’ mathematics learning, dispositions, and/or mindsets can be observed, described and interpreted, which lends itself to a qualitative research approach. A multiple case study (Merriam, 1998) was deemed most appropriate for this study. An in-depth study of three students to explore the phenomenon of mathematically gifted students with self-limiting mindset tendencies, in regular mathematics lessons.
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Based on this theoretical perspective and view of research, the following study design and approach to data collection and analysis were formulated for this study.

3.6 Research Design and Data Collection

The research design adopted for this study was a case study with a narrative analysis. The case was the phenomenon of mathematically gifted students who display negative mindset tendencies. The case study, a descriptive research design, was used to observe and describe the effect of teacher professional learning on the mathematics learning and mindsets of three mathematically gifted students with negative mindset tendencies, over a three- to four-month period. The data collection was in two parts: 1) collecting initial data prior to

the teachers receiving professional learning, and 2) collecting post data approximately three months after the teachers' professional learning experience. The analysis was also in two parts: a) analysing the individual students' experiences to identify and describe any changes between initial and post-professional learning data, and b) a comparative analysis of the students' experiences to identify possible causal factors for the changes. Narrative analysis produces storied accounts of events and happenings throughout the study period, with an evaluation that is retrospective, linking events to account for how a final outcome may have come about. "The search is for data that will reveal uniqueness of the [case] and provide an understanding of its idiosyncrasy and particular complexity" (Polkinghorne, 1995, p. 15). Analysis of several stories, through the identification of thematic threads, aims to produce meaningful outcomes of the research to assist in identifying effective teacher support for the case of mathematically gifted students with self-limiting mindset tendencies.

The study sought to determine whether targeted teacher knowledge could have an impact on the students' perceptions of themselves as learners of mathematics, and whether these perceptions, in turn, had an impact on their approaches to mathematics tasks. It sought to determine which teaching approaches appeared to have a positive impact, in order to develop a research-based teacher professional learning program for classroom teachers of primary school aged mathematically gifted students.

The overarching research question for the study was:

- *What impact does targeted teacher professional learning about classroom support for mathematically gifted students with self-limiting mindsets, have on the mindsets and mathematics learning of these students?*

The subsidiary questions, derived from this overarching question, which framed the data collection are:

- How do students approach challenging mathematics tasks before and after their teachers receive professional learning, and a subsequent teaching period?
- How do students who are mathematically gifted view themselves as mathematics learners before and after their teachers receive professional learning, and a subsequent teaching period?
- What do the teachers do during the post-professional learning teaching period to challenge the mindsets of students who are mathematically gifted but with self-limiting mindset tendencies?

3.6.1 Setting and Participants

The study was conducted in a local primary school in regional Victoria. Ultimately, I planned to identify and observe three students, purposively sampled (Maxwell, 1997), who were mathematically gifted, but exhibiting self-limiting mindset tendencies – one from early primary, one from middle primary, and one from upper primary.

The Principal and teachers within the school were invited to participate in a focus group conversation, and parents of children identified as having above average *growth point profiles*, as measured by a mathematics one-to-one assessment interview (see section 3.6.2), and/or teacher recommendation of exceptional mathematics ability were also invited to participate in the study by completing a written questionnaire. Staff from the school also participated in professional learning session(s) about mathematical giftedness and mindsets, with the three selected case study students' teachers also receiving additional targeted, research-based professional learning.

The School

Following identification of a likely school in which to conduct my study, through a teacher's suggestion, I met with the Principal and the Assistant Head of the school to explain my proposed research and its purpose. Confirming that it was highly likely I would be able to select the purposive sample I needed from the students within the school, I invited their school community to participate in the study. The Principal provided consent to invite staff, parents and students to participate (see section 3.9 for a full description of ethical considerations). It is important to note that the aim of my study was not to find the most prodigiously gifted students, nor was it necessary to identify all gifted students. It was, therefore, not necessary to approach more than one school as three mathematically gifted students with fixed mindset tendencies were identified.

The school is a major co-educational school in a regional Victorian city. It caters for over 1,500 students from age six-months in a purpose built Early Education Centre, to primary school aged children in the Junior School, through to Year 12 in the Senior School. It was the Junior School Principal I approached, but I also gained consent from the school's Headmaster. The Junior School consists of approximately 500 students in 20 single-grade classes. It has a strong emphasis on guided inquiry-based approaches to teaching and learning, and, at that time, they were working through the authorisation process for an International Baccalaureate Primary Years Programme (PYP). The PYP provides schools with a Program of Inquiry framework to develop "a rigorous and challenging

curriculum that is engaging, relevant, challenging and significant for young learners.” (International Baccalaureate, n.d.). This authorisation was subsequently awarded, but after my data collection had been completed.

The Students

Students from three different grade levels were chosen for the study to enable a comparison between different stages of learning. Grade 1 was chosen from the early years as students had completed a full foundational year at school but were still beginners within the education system. Grade 3 was chosen as these students represent those transitioning from the lower grades to the middle primary grades. Finally, Grade 5 students were chosen as they represent those beginning the transition to secondary school. It was decided to select a Grade 5 student rather than a Grade 6 student so that follow-up for up to a year after the study was possible, if deemed necessary.

To select the three case study students, a broader selection of students needed to be considered initially. Students from Grade 1, Grade 3 and Grade 5, with above average results in mathematics assessments and/or a teacher nomination of exceptional mathematics ability, were selected for further assessment by myself, as the researcher, to independently assess mathematical abilities and to identify mindset dispositions.

One of the limitations of working in this particular school was that there was not an economically diverse student population to select from. Being a private, relatively high fee-paying school, most students would be from middle to upper socioeconomic status families. However, with the final selection of three students, a cross section of representations could still be considered – for example, gender, cultural background, family make up et cetera. The selection process of the three case study students is described in full in Chapter 4.

The Teacher participants

Each of the school’s Grade 1, Grade 3 and Grade 5 classroom teachers (nine teachers in all, four Grade 1 teachers, two Grade 3 teachers, and three Grade 5 teachers) were invited to participate in four aspects of the study, to 1) nominate mathematically ‘highly capable’ students to be considered for the case study (the Principal did not want me to use the term ‘gifted’), 2) participate in semi-structured interviews and classroom observations, 3) participate in targeted professional learning, and 4) participate in a focus group conversation. All teachers consented to participating in the focus group conversation, and all but one Grade 5 teacher consented to being involved in all other aspects of the study.

The school Principal and the Assistant Head were also invited to be involved in the nomination process and in the focus group conversation.

Parents

Parent consent was sought for the students nominated as being mathematically highly capable by the teachers and/or Principal and Assistant Head. These parents were also asked to complete a written questionnaire.

3.6.2 Data Collection Methods

Multiple data sources were used to generate a comprehensive and accurate description of the case, through rich thick descriptions, to build up an in-depth picture of the case (Baxter & Jack, 2008; Creswell, 2007). Six forms of data were collected for this study: a focus group discussion with teachers and school leaders, archival records, interviews, direct observation, participant observation, and physical artefacts. The following provides a description of, and purpose for, each of the data collection methods used in this study.

Focus Group Conversation

Focus group conversation is a method used for enabling deep discussion (Merriam, 2009). It is a process designed to take participants from superficial comments and ideas to more analytical perceptions of an issue or question in a relatively short period of time. Group interaction is used intentionally to develop ideas and thoughts. People are encouraged to talk to one another: asking questions, exchanging anecdotes and commenting on each other's points of view. Focus group conversations are particularly useful for exploring not only how people think, but also why they think the way they think. The conversation is initiated by the researcher who then takes on a moderator role, and may choose to intervene at times in order to urge discussions to continue (Merriam, 2009). The focus group conversation for this study focused on teacher perceptions of giftedness, specifically mathematical giftedness, prior to nominations of students and any professional learning. This 1) helped target the professional learning, 2) provided a starting point for further interviews with the teachers of selected students, and 3) helped teachers focus on which considerations were important when nominating students as mathematically highly capable. A group exercise was used to initiate the discussion (Kitzinger, 1995). Participants were provided with a Yes card and a No card. Statements of common beliefs about students who are generally highly capable, and mathematically highly capable, were presented to the group with teachers given five seconds to vote either Yes (generally agree) or No

(generally disagree) with each statement, prior to any discussion. For example, *Children who are mathematically highly capable will develop behaviour problems if they become bored in maths classes*, and *Children who are highly capable tend to have pushy parents* (see Appendix 1 for a full list of statements). This ‘vote’ became the catalyst for discussion about teachers’ beliefs and practices as each was given the opportunity to explain and justify their individual responses (see Appendix 1 for the ‘rules’ of this process). Data were collected in the form of a transcribed audio-recording of the conversation and researcher journal notes taken immediately post conversation. Participants were also given a short, written questionnaire to fill in anonymously, giving each one the opportunity to record private comments after the group session if they so desired (see Appendix 1). See Section 4.2.1 for a discussion on the outcomes of this focus group conversation.

Teacher nominations

Identification of children who are mathematically gifted is not a simple linear process (see Section 2.4.3). It requires a multifaceted process, from formal methods of testing to informal methods of observation and conversation (McAlpine, 2004; Moon, 2006; Reis, 2004). Classroom teachers are the ones who have observed students closest within a school context and their opinions, therefore, can provide useful benchmarks. Teacher nominations, then, were used to form part of the identification process for identifying mathematically gifted students for this study. Data were collected via a nomination form (see Appendix 1). See Section 4.2.1 for full details of this nomination process.

Written questionnaires

Questionnaires are useful as a means of collecting information from a wide sample of participants (McAlpine, 2004). They can be completed in participants’ own time allowing participants such as working parents/caregivers to participate outside set school hours.

Parents/caregivers have been shown to be reasonably accurate identifiers of exceptional mathematical ability (Niederer, Irwin, Irwin & Reilly, 2003). Therefore, another part of the identification process included a written questionnaire, used to collect data about parent/caregiver perspectives of their child’s mathematical disposition, especially in the early pre-school years.

Following the focus group conversation teacher participants were given a short, written questionnaire to fill in anonymously, giving each one the opportunity to record private comments after the group session had been completed.

A combination of open questions and Likert-type scaled responses were included in both questionnaires (see Appendix 1).

Archival records

Archival records are also a useful source of case study data. School records of previous years' mathematics assessment were accessed as another source of identification of students' mathematical abilities. Both the *Mathematics Assessment Interview (MAI)* (Gervasoni et al., 2011) and *Progressive Achievement Tests in Mathematics (PAT-Maths)* were used by the school.

The *Mathematics Assessment Interview (MAI)* was routinely used by the school for all students in Prep to Grade 3. The *MAI* is a one-to-one task-based clinical interview conducted at the beginning of the school year in many schools throughout Australia. It is a mathematics assessment developed as part of the *Early Numeracy Research Project* (1999-2001) (Clarke et al., 2002) that corresponds to a research-based learning trajectory in various mathematics domains. It was originally called the *Early Numeracy Interview* (Department of Education Employment and Training, 2001), and later revised, extended and renamed the *Mathematics Assessment Interview* as part of the *Bridging the Numeracy Gap Pilot Project* (2009-2011) (Gervasoni et al., 2011). The structure of the *MAI* provides insight into the mathematical reasoning abilities of individual students in addition to their ability to calculate correct answers. It is used as a formative assessment to determine each student's zone of proximal development (Vygotsky, 1978) to guide targeted teaching, and as a measure of a student's growth in mathematical knowledge and understanding from year to year. The tasks are designed to map students' progress along the research-based learning trajectory, with what are called *growth points* (or key 'stepping stones') along that trajectory (Gervasoni, 2002), with the assessment providing a *growth point profile* for each student. *Growth points* are a research-based framework of the trajectory of early mathematics learning developed as part of a project seeking identification of processes for supporting and enhancing numeracy learning in the early years of school (Clarke et al., 2002). A student's *growth point profile* determines the zone of proximal development for that student's learning, and appropriate instruction can then be designed by the classroom teacher. As the *MAI*, and associated *growth point profile*, provides information about a student's mathematical thought processes and aptitude as well as mathematical achievement, it is a very useful source of information about students who may be gifted, but underachieving. The *MAI* has also been designed with a high ceiling to allow

assessment of students who may already be working ahead of their chronological peers, or beyond their current curriculum grade. All *MAI* record sheets of all nominated students were accessed and analysed as part of the selection process.

The school also used the *Progressive Achievement Tests in Mathematics (PAT-Maths)* assessment, for Grade 4 to Grade 6 students. *PAT-Maths* is a series of standardised tests used in Australia and New Zealand, designed to provide norm-referenced information to teachers about their students' skills and understandings. However, standardised grade-level, multiple-choice tests such as *PAT-Maths* have been shown to be not the most appropriate means for assessing high capability or giftedness for a variety of reasons (see Niederer et al., 2003), but these data were also collected for comparison, to paint a broad picture of students' mathematical practice.

Archival mathematics assessment records, together with teacher nominations and parent questionnaire responses, were selected as appropriate data to identify three students who were mathematically highly capable. It is important to note that the aim of the study was not to identify *all* mathematically gifted students, nor necessarily the most highly mathematically gifted students. This identification process was deemed appropriate for this study, but it is not suggested that this same process is necessarily adequate for a comprehensive identification of all mathematically gifted students.

Clinical task-based mathematics interview

A task-based clinical interview was designed specifically by the researcher for this study to further explore students' abilities to perform novel and creative mathematics tasks (as opposed to assessing previously learned mathematics content), and to assess for mindset tendencies. It assessed three specific areas:

- 1) A student's ability to reason proportionally. Being able to reason proportionally has been shown to be a good indicator of mathematical ability (Lamon, 1999), so a ratio task was included, recognising that ratio is not formally taught in primary school so children would need to employ their own intuitive mathematical understanding and strategies;
- 2) A student's ability to learn something new. Mathematically gifted children have been shown to be able to learn quickly with minimal repetition through their ability to generalise and assimilate new concepts readily (Krutetskii, 1976); and

- 3) A student’s mindset about themselves as a mathematics learner – using choice of difficulty and/or responses in an open task to indicate either a *growth* or *fixed mindset*, based on a task used by Mueller and Dweck (1998).

The interview questions were designed based on the above criteria. For example, for proportional reasoning, “A shop sells lollies at three for 10c, or 35c for a packet of 10. Which is better value?” (Year 3); learning something new and generalisability, the structure of a Japanese abacus was explained (see Figure 3.2), with students then given simple calculations (depending on year level) to see if they could apply this new knowledge practically; and the open task was *Adding Corners* (adapted from Downton, Knight, Clarke and Lewis, 2006) (see Appendix 2 for full interview scripts).

The interview was piloted with several students from different schools, and refined to evoke the most beneficial responses. Two versions were developed – the first version would enable the selection of the three case study students to observe in the classroom observations; the second version was for the follow-up interview to assess for any changes in approaches to the mathematics tasks, particularly in respect to mindsets, of the three case study students. Grade 1, Grade 3 and Grade 5 adaptations for each version were developed. Each task-based mathematics interview was accompanied by a detailed record sheet (see Appendix 2), and all interviews were audio-recorded and transcribed for analysis.

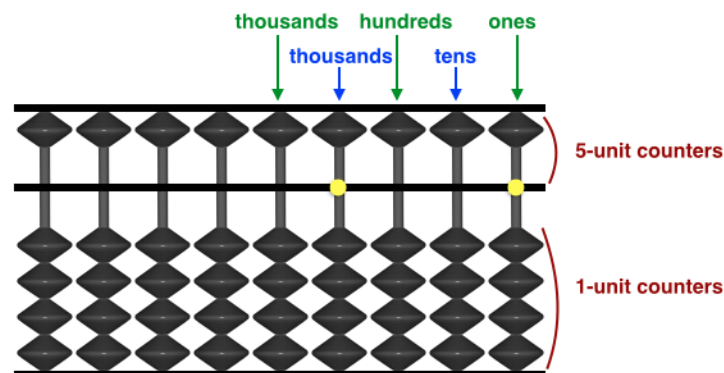


Figure 3.2 Japanese abacus. Image retrieved from <http://mathandmultimedia.com/2014/10/17/japanese-abacus>

Semi-structured interviews

A semi-structured interview is a qualitative method of data collection that combines a set of pre-determined open questions with the opportunity for the interviewer and interviewee to explore particular thoughts or responses further (Merriam, 2009). The use of pre-

determined questions provides uniformity, while the openness of the format allows for individual differences in responses.

Semi-structured interviews were used for collecting data from both teachers and students. All teacher-nominated students were interviewed with a semi-structured interview (see Appendix 1), in conjunction with the task-based mathematics interview, as part of the selection process, to determine perceptions of themselves as learners of mathematics. Teacher semi-structured interviews (see Appendix 1) were also carried out with the classroom teachers to refine the final selection of three students. Further semi-structured interviews (see Appendix 1) were also conducted with both the case study students and their teachers as part of the follow-up process to assess for any changes in self-perception and/or mathematics learning.

All semi-structured interviews were audio-recorded and transcribed. This was to maximise trustworthiness of the research process and data interpretation and analysis (see section 3.8).

Participant observation notes and artefacts

Participant observation allows the researcher to become part of the participants' environment and enables the researcher to personally capture the experience of the participants being observed, "[the researcher] can interpret it (the experience), recognise its contexts, puzzle the many meanings while still there, and pass along an experiential, naturalistic account for readers to participate themselves in some similar reflection" (Stake, 1995, p. 44). The researcher is able to not only observe participants' actions but also interact in a way that enables them to ask questions, to clarify responses and/or choices made, to 'get inside the participant's head'. The participants themselves are fully aware of the researcher's role, that "participation is definitely secondary to the role of information gatherer" (Merriam, 2009, p. 124).

My role as participant observer in this study enabled me to delve deeper into students' mathematical thinking by asking questions such as, 'What were you thinking when...?' 'Why did you do it that way?' 'How do you know that is correct?' I was also able to observe and document reactions to a specific challenge by asking questions such as, 'Could you do that a different way?' 'What if the situation was changed to ... how would you solve it now?' These types of questions help to provide insight into students' thinking and mindsets.

All conversations as part of these observations were audio-recorded using a SmartPen (a ballpoint pen with an embedded computer and digital audio recorder) with the teachers' and students' knowledge and consent, and supplemented by researcher journal notes, once again to maximise trustworthiness of the research process and data interpretation and analysis.

Researcher journal

The researcher's journal is used for keeping field notes, for jotting down thoughts, ideas, reflections, memos, and/or for initiating early analysis through interpretation. These notes can be journaled at all stages of the research process, not just limited to when the researcher is in direct contact with participants. They may be "jotted or scratched notes, taken at the time of observations or discussions and consisting of highlights that can be remembered for later development [or they may be remembered] mental notes when it may [have been] inappropriate to take notes" (Thorpe, 2008, p. 98). They may include a record of perspectives, feelings and decisions made, regarding problems, issues, or ideas encountered as part of the data collection process (Merriam, 2009). This all becomes important data for the reflective components of qualitative research, and also the trustworthiness of the interpretation and analysis. A researcher journal was kept for this study as another component of the overall data collection. It included documented observations (from all interviews and classroom visits), remembered notes from any informal (non-recorded) conversations with staff, comments and reflections on any issues that may arise, personal speculations and/or reactions to events or conversations, and any other thoughts, ideas or musings.

Physical artefacts

Work samples from observed classroom lessons were collected to form part of the data about student approaches to mathematics tasks, and as possible evidence of mindset tendencies. This formed part of the total evidence used for credible data analysis.

3.6.3 Outline of Data Collection Phases

The data collection process involved four phases (see Figure 3.3). The *first phase* was the process of identification of mathematical capability and mindset tendencies in order to select three suitable students for the case study. The *second phase* included observations of mathematics lessons with the three selected students in their regular mathematics classes as part of the benchmark for ascertaining change over time. Research-based, targeted

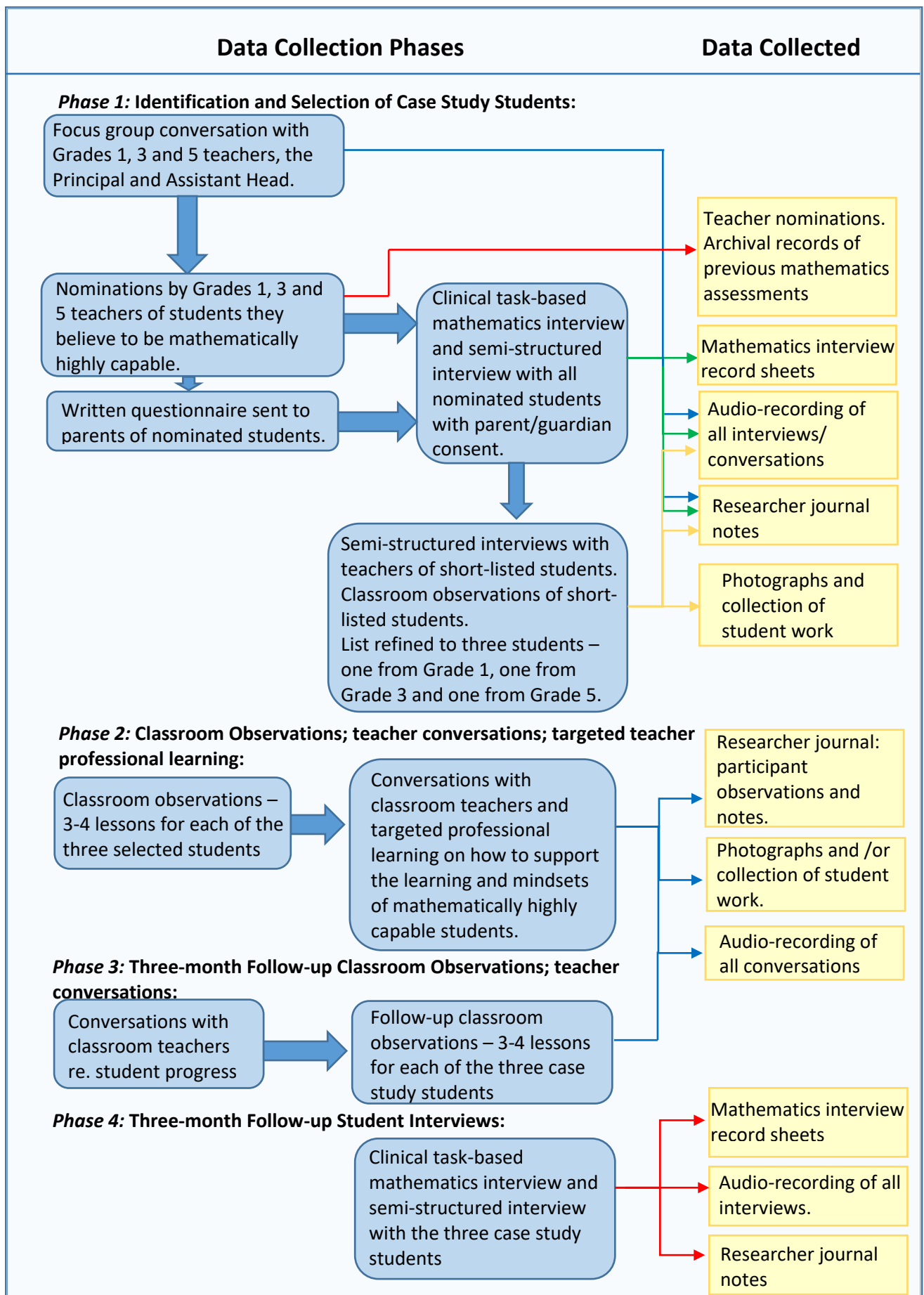


Figure 3.3 Outline of Data Collection Phases and Data Collected

professional learning was then provided for the classroom teachers, focusing on characteristics of mathematically gifted students, an understanding of mathematical challenge in learning, and recognising and modifying self-limiting mindset behaviours. This was followed by an intervening period of approximately three months when the teachers were to implement practices reflecting their new understandings within their regular mathematics lessons. The *third phase* followed the intervening period, and involved further discussions with each of the classroom teachers, and follow-up observations of each student in regular mathematics lessons. The fourth phase comprised of a follow-up assessment and semi-structured interview with the three case study students. A complete description of these data collection phases is provided in Chapter 4. A timeline of the data collection phases is shown in Table 3.2.

Table 3.2

Timeline of Data Collection

	Data Collection Phase	Outcome
March 2014	Information/consent and student nomination forms sent out to teachers Collaboration with Principal and Assistant Head to refine teacher nominations. Parent information/consent forms sent out together with written questionnaire Previous mathematics assessments for nominated students accessed and independently analysed Focus Group Conversation with the nine Grade 1, 3 and 5 teachers conducted	Consent forms and student nominations collected Student participants selected Parent consent forms and questionnaires collected MAI record sheets and <i>PATMaths</i> results collected Focus Group conversation audio-recorded
April 2014	Task-based and semi-structured interviews conducted with selected students	Task-based interview record sheets completed for analysis All interviews audio-recorded Semi-structured interviews transcribed
May-June 2014	Interviews and parent written questionnaires analysed, in order to select case-study students Pre-professional learning classroom observations (as participant observer), and teacher semi-structured interviews to refine the selection to 3 case-study students	Classroom conversations audio-recorded; Researcher journal notes taken (both during and directly following the classroom observations); Semi-structured teacher interviews audio-recorded and transcribed

Table 3.2 (cont'd)

	Data Collection Phase	Outcome
May- June 2014 (cont'd)	Whole-school Professional Learning session conducted with an emphasis on ways to plan for, and differentiate, appropriate mathematics tasks to support the learning of highly capable mathematics students Individual teacher Professional Learning sessions with each of the teachers of the 3 case-study students, including how to scaffold and support student learning and how to promote a change in mindset, specifically for the selected students, but also based on a whole-class approach.	Discussions from these sessions audio-recorded and transcribed. Summary document of suggestions for supporting the learning of students who are mathematically gifted or highly capable provided for teachers (see Appendix 4)
July- Sept 2014	Email support for classroom teachers throughout the post-professional learning period	Specialist coaching/mentoring; some suitable professional readings suggested
Oct 2014	Post interviews with classroom teachers	Interviews audio-recorded and transcribed
Oct- Nov 2014	Post classroom observations conducted Post task-based and semi-structured interviews conducted with the 3 case-study students	Classroom conversations audio-recorded Researcher journal notes taken during and directly after observations Task-based interview record sheets completed for analysis All interviews audio-recorded

3.7 Data Analysis

Most case study research relies on both coded data and direct interpretation from observations, with one or the other being the dominant approach to analysis (Stake, 1995). For this research, an approach to analysis was essential that could chronicle any changes in students' dispositions, or mindsets, and approaches to mathematics tasks, in a way that the researcher could consider the effect of the targeted teacher professional learning. This would not necessarily involve frequency of happenings, but rather a description of happenings. Individual reality is complex, and classroom reality is not only complex but also multidimensional. Merriam's case study approach is defined as a holistic description and analysis of a phenomenon, and I wanted an approach to data analysis where the complexity of the individual and the classroom was not broken down and divided into elements, but considered as a unified whole.

3.7.1 Narrative Analysis

Clandinin and Connelly (2000) describe narrative analysis as a synthesising of the data rather than a separation of it into its constituent parts, which reduces the risk of detracting from the meaning of the whole, as can happen when coding raw data into themes as a reductionist method of analysis (see also Lichtman, 2010). The purpose of narrative analysis is to provide a “dynamic framework in which the range of disconnected data elements are made to cohere in an interesting and explanatory way” (Polkinghorne, 1995, p. 20). It is not to simply produce a description or reproduction of observations, but to provide “storied accounts of educational lives” (Clandinin & Connelly, 2000, p. 4). “Education and educational studies are a form of experience, [and] narrative is the best way of representing and understanding experience” (Clandinin & Connelly, 2000, p. 18).

When appearing as a whole rather than elements, narratives are not abstract, remote, or inaccessible. Instead, they can rather be perceived as familiar, informative, and relevant for those who hear about or read them ... contributing, we hope, to provoking, inspiring, and initiating discussions and dialogues, something that is crucial for reflection on practice and its development. (Moen, 2006, p. 65)

The setting for this research, the mathematics classroom, is an integral part of the case study participants’ experiences. The mathematics classroom encapsulates students’ approaches to mathematics tasks, students’ interactions with their teacher, their peers, and any other adults present, which included the researcher, and students’ feelings about themselves as learners within that classroom environment. The concept of narrative research seemed fitting for my purposes.

Polkinghorne (1995) makes a careful distinction between two forms of narrative research approach. On the one hand, there is narrative *inquiry* which is the analysis of narrative data (that is, participants’ stories); on the other hand, there is narrative *analysis* which constitutes a gathering of events and happenings as data and uses narrative analytic procedures to produce storied accounts from these data. The narrative analysis approach was most appropriate for my multiple case study, producing ‘storied analyses’ of student and teacher experiences over a period of time. In this approach, student and teacher experiences are collected from not only interviews, but also researcher-as-participant observations, which adds an objective perspective to participants’ versions of events (Lichtman, 2010). Polkinghorne (1995) describes narrative analysis as a retrospective explanation of events

organised along a before-after continuum, with past events linked together “to account for how a final outcome may have come about” (p. 16).

The purpose of narrative analysis within a multiple case study is to produce stories of each of the individual samples within the case study (Merriam, 1998), in order to provide an understanding of the commonalities and similarities in each of the individual stories (Polkinghorne, 1995). The stories are an attempt to describe, interpret, and understand individual participants’ actions, as well as comparing and contrasting them to others’ actions within similar experiences and environments.

Qualitative analytic procedures involve direct interpretations of individual events and actions within context, rather than, as in quantitative analysis, through a collection of instances waiting until something can be said from the aggregate of these instances (Stake, 1995). Narrative analysis relates these individual events and actions to one another through the construction of a story, with the final story both fitting the data and bringing forth an order and meaningfulness, through interpretation, which is not apparent in the data themselves (Polkinghorne, 1995). “The researcher should present the characters with enough detail that they appear as unique individuals in a particular situation” (Polkinghorne, 1995, p. 17), with the purpose of answering the question, “How is it that this outcome came about; what events and actions contributed to this solution?” (Polkinghorne, 1995, p. 18). The narrative analytic procedure used for the individual stories of this study was based around the seven criteria for narrative case study first proposed by Dollard (1935), and revised by Polkinghorne (1995). These criteria are:

- 1) Setting the individual within a cultural/family context;
- 2) Considering the individual as an ‘embodied person’, including physical and emotional development;
- 3) Being mindful of the setting, or context, of the study, including the physical space, and significant people that may affect the actions being studied;
- 4) Describing the individual’s choices and actions in response to specific events;
- 5) Considering the changing behaviours of the individual throughout the study;
- 6) Determining the bounded time period of the study – establishing a beginning, middle, and end to a ‘story’ that is evaluating the individual in a particular situation, dealing with specific issues, within a set period of time; and

- 7) Generating a plausible and understandable narrative of the individual’s responses and actions through configuring the disparate data elements into a meaningful explanation.

This is the analytical approach employed in this study, and presented in Chapter 6. Each of the three case study students’ experiences are addressed as an individual ‘story’, considering the questions, ‘How do the students’ approaches to challenging mathematics tasks change, and how do the students’ view of themselves as mathematics learners change, throughout the course of the study?’ and ‘What may have contributed to these changes?’

In a case study with multiple samples, in this study presented as three individual stories, themes of commonalities and similarities may emerge from these stories, which can be analysed further. This is the emphasis of Chapter 7, using what Bruner (1985) described as a *paradigmatic analysis*. In a paradigmatic analysis, the researcher seeks to identify common *themes* intuitively derived from stories collected as, or generated from research data. The researcher is looking “for various kinds of responses, actions, and understandings that appear across the stories” (Polkinghorne, 1995, p. 14), looking for possible covariance among these themes. This triangulation of the individual stories further strengthens the discussion of findings on what may have contributed to student changes, with interpretations coming from deep thinking on the reflected stories from within, between and across the individual samples (Baxter & Jack, 2008). Figure 3.4 shows an outline of the data analysis process, from narrative analyses, to identification of common themes, leading to a paradigmatic analysis, with cross-unit comparisons interpreted within the context of findings from the literature review. An example of the spreadsheets used to record evidence from the various sources, to analyse and interpret that data, is included in Appendix 5.

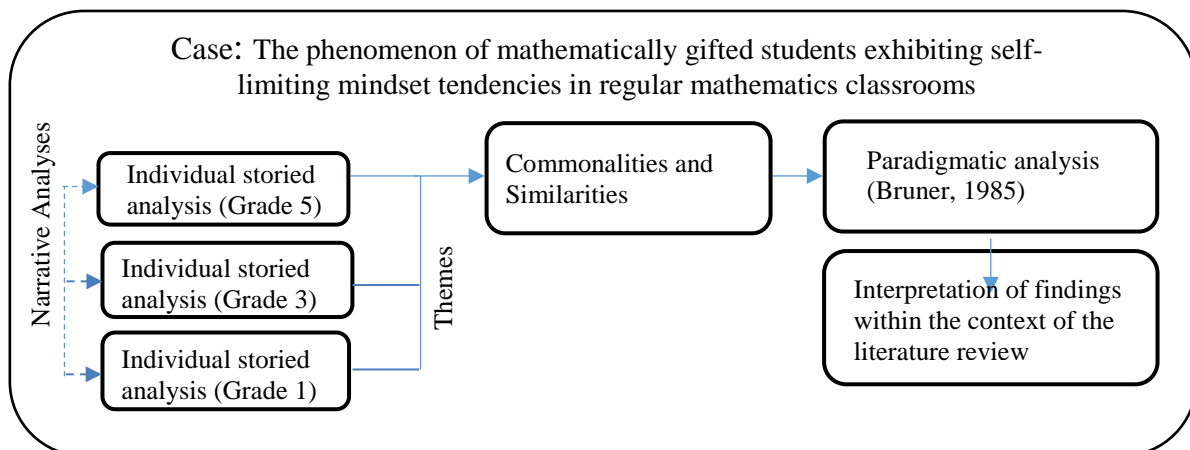


Figure 3.4. Outline of Data Analysis Process

3.8 Trustworthiness Issues

Trustworthiness of research findings is a critical issue. If research is not perceived as trustworthy, then potential consumers of the research will not see it as useful for any change or reform, and it will fall short in making any contribution to the development of knowledge in its particular discipline. The conventional criteria for traditional positivist research are validity, reliability and objectivity – scientific research is all about experimental design whereby the research needs to be replicable to the extent that results are always the same. However, the standards for rigour in sociological qualitative research differ from those in traditional scientific research (Lincoln & Guba, 1985; Merriam, 2009). With qualitative research results are based on individual responses that would never be able to be completely replicable because of the nature of human behaviour. Wolcott (1994) speaks of the “absurdity of validity” (p. 364) in qualitative research, insisting that what qualitative researchers are looking for is “something else, a quality that points more to identifying critical elements and wringing plausible interpretations from them” (p. 366). Wolcott recognises this “something else” as *understanding*: that is, being able to “interpret and explain...seeking to understand a social world we are continuously in the process of constructing” (pp. 367-368). This is consistent with a constructivist paradigm. Criteria for assessing rigour in interpretive-constructivist research, then, are relational, they recognise the relationship between the researcher and the participants in the research (Lincoln, 1995). Lincoln and Guba (1985) established a new set of criteria for ensuring research trustworthiness that resemble the criteria for traditional research, but are more suited to qualitative inquiry. They introduced the concepts of *credibility* (for internal validity), *transferability* (for external validity), *dependability* (for reliability), and *confirmability* (for objectivity), and it is these criteria that I have addressed in relation to my study.

Credibility

Credibility deals with the question “How congruent are the findings with reality?” (Merriam, 2009, p. 213). Connelly and Clandinin (1990) talk about the notion that a study “rings true” to the reader; it is believable. Lincoln and Guba (1985) argue that ensuring credibility, or this verisimilitude, is one of the most important factors in establishing trustworthiness.

Credibility can be established through the complementarity of multiple methods of data collection and/or sources of data (Clarke, Emanuelsson, Jablonka & Mok, 2006). Methods may include individual interviews, observations, group discussions; sources may include

observations at different times, interview data from different people, follow-up interviews with the same people (Merriam, 2009). Supporting data may also be obtained from literature that provides a background to, and may help explain, the attitudes and behaviour of, those being studied (Shenton, 2004). Complementary accounts (Clarke et al. 2006) from these different methods of collection and sources of data, strengthens the analysis and any interpretations made by the researcher, and leaves the reader with this sense of verisimilitude.

Another method for maximising credibility is the use of a researcher journal. This journal becomes a “reflective commentary” (Shenton, 2004, p. 68) used to record initial impressions of each data collection session, patterns that may begin to emerge, and any theories or conjectures generated. This commentary helps maintain the researcher’s original impressions and plays a key role in the interpretation and analysis of data, and in informing the study’s final results. Audio-recording all interviews and discussions also helps maintain credibility, as part of the original data collected is retained verbatim and can be checked by the researcher against selected sections of the interview used to support the narrative.

To address the issue of credibility in this study multiple sources of data were to be collected – interviews, both task-based and semi-structured, written questionnaires, classroom observations, and researcher journal notes. Data from these sources were then triangulated to establish verification of events, and minimise the possibility of any researcher subjectivity or bias. All interviews and classroom observations were audio-recorded, and memos – thoughts, questions, and interpretations – added to the transcriptions even as the data were being collected. With this resource, extensive references to quotations could be used in the resultant narratives (Riessman, 2008), providing a chain of evidence where interpretations could be traced back to original sources, to confirm and verify judgments (Mertens, 2005).

Transferability

Generalisation, or external validity, of a case study is limited because, by definition, it is a bounded system specific to a small number of individuals in a particular environment. “In qualitative research, a single case ... is selected precisely because the researcher wishes to understand the particular in depth, not to find out what is generally true of the many” (Merriam, 2009, p. 224). Lincoln and Guba (1985) advise that the best way to ensure the possibility of transferability is through rich, thick description, with Stake (1995) stating that thick description not only includes the detailed description of the events, but also a detailed

description of interpretations of those events (see also Merriam, 2009). These thick descriptions may allow the *reader* of the study to make connections to their own similar positions (transference). Lincoln and Guba (1985) suggest that with this notion of transferability “the burden of proof lies less with the original investigator than with the person seeking to make an application elsewhere” (p. 298), but, the ‘burden’ of providing sufficiently descriptive data to make transferability possible does lie with the researcher.

Thick description is only possible through comprehensive data collection. Data collection for this research was comprehensive, as outlined above, and the choice of a storied narrative analysis ensured that a thick description of the phenomenon, within the students’ mathematics learning environments, could be captured. This maximised the possibility of transferability for any readers of this research.

Dependability

In traditional scientific research, the issue of reliability is paramount, providing sufficient detail of methods and procedures to ensure findings can be replicated. With qualitative research, however, reliability is problematic because human behaviour and social situations are not static (Merriam, 2009). As such, Lincoln and Guba (1985) suggest that an alternative for qualitative research should be an emphasis on dependability. Dependability refers to the consistency and thoroughness of the researcher’s methods. Lincoln and Guba argue that, in practice, a demonstration of credibility goes some distance in ensuring dependability, and that dependability may be achieved through the use of similar ‘overlapping methods’ as used to ensure credibility. However, to address the issue of dependability further, the researcher should report in detail all processes within the study: data collection methods, selection of participants, how categories, themes or patterns were derived, and how decisions and interpretations were made throughout the study. This in-depth reporting allows the researcher to describe the extent to which proper research practices have been followed, and enables future researchers to replicate the work for a similar study. The researcher should also clearly articulate any assumptions and theories behind the study (Lincoln & Guba, 1985).

For this study, I have provided a detailed description of the research design and its implementation, and have explained the processes of data collection and analysis. I will fully describe the participant selection process (Chapter 4), the teacher professional learning (Chapter 5), and the analysis process (Chapters 6 and 7), and I will reflect on the

effectiveness, or otherwise, of each of these in order to maintain and maximise dependability.

Confirmability

Whereas objectivity in the sciences is relatively straight forward, objectivity in qualitative research is also somewhat problematic. The relativist researcher believes that the world can only be known through our own subjective observations of it; that what can be known in the sociological world is influenced by both the individuals being researched and the interpretations of the researcher. Data may be recorded objectively, but they are simultaneously being interpreted subjectively (Lincoln & Guba, 1985). The concept of objectivity, then, appears moot, and the concept of confirmability seems more appropriate in qualitative research. Confirmability is concerned with providing sufficient evidence that interpretation and analysis of participants' input could be verified by the participants as a true reflection of the participants' input and not altered due to researcher bias. Confirmability does not deny the researcher's subjective interpretations, but it does require the researcher to account for any biases by being transparent about them, and addressing any issues appropriately (Jensen, 2008).

One of the best ways to ensure confirmability is through an audit trail. An audit trail "describes in detail how data were collected, how categories were derived, and how decisions were made throughout the inquiry" (Merriam, 2009, p. 223). This detailed trail can be constructed through the researcher journal. Participant checks are another way to confirm or verify accurate reporting of data. This involves taking preliminary analysis back to the participants for their feedback on the accuracy, or otherwise, of the researchers' interpretations of their experiences.

For this study, I have described in detail all aspects of the data collection and analysis processes, including decision making, and have provided draft copies of each student's 'story' to their respective teachers for feedback before finalising each narrative. This has provided a suitable 'audit trail' to ensure confirmability.

3.9 Ethical Considerations

In all research, "we have to trust that the study was carried out with integrity and that it involves the ethical stance of the researcher" (Merriam, 2009, p. 229). In a nutshell, to be ethical is to 'do good and avoid evil'. Both these edicts require action: there is as much personal responsibility in the *avoiding evil* (or 'doing no harm') as there is in the *doing*

good. It is the researcher's responsibility to be aware of issues that may do harm; ignorance is no excuse, especially in research with children.

Ethics approval for data collection for this study was granted by the Australian Catholic University Human Research Ethics Committee (Registration number 2013 116V). The application of ethical principles for this study involved consideration of key aspects such as informed consent, confidentiality, and honest, open and accessible findings, as well as an awareness of the overarching 'do no harm' throughout the entire data collection process.

Consent was sought from the principal, teachers, and parents via information letters approved by the ACU Human Research Ethics Committee (see Appendix 3), and students signed a child-friendly assent form approved by the Human Research Ethics Committee (see Appendix 3). Consent forms were returned to the researcher via a secured box located in the Assistant Head's office at the school to maintain confidentiality of participants. Student assent forms were signed by each student at the beginning of the initial interview process and handed directly to the researcher. A carefully chosen explanation of a study is required when inviting student participation, in a medium young children will understand. A brief child-friendly explanation of the reasons for the study, what their participation would entail, how long it would be for, and how it would benefit them was drafted and read out to the students by the researcher prior to their signing the assent form (see Appendix 3).

Obtaining informed consent for all involved in the study (students, parents, teachers, and assent from students) is a first step, but *maintaining* privacy, anonymity, and confidentiality also needs to be built into the project design and dissemination (Clandinin & Connelly, 2000; Merriam, 2009). Special consideration needs to be given to student assent to ensure voluntary participation, as children may feel obliged to consent to, and continue with, participation if their teachers and parents have supported it. Students (and their parents) need to be aware that they have the right to withdraw consent at any time throughout the study if they so wish, without having to give any reason, and without penalty. For this study, ensuring students, parents and teachers knew they had a right to question anything about the research, and to withdraw at any stage without reason was outlined in the information letter and student explanation about the study, and re-iterated verbally both at the beginning of the data collection phase and again at the follow-up data collection phase.

Rapport needs to be built to establish participants' trust and to ensure a comfortable working environment, but researchers must also be aware of the difference between voluntary participation and indebted participation. It is important to maintain a researcher/participant relationship that does not bleed into a friendship relationship which may influence the participants' desire to 'please at all costs'. Being cognizant of maintaining an appropriate researcher/participant relationship is important (Ritchie & Rigano, 2001). My role as researcher and as participant observer in the classroom was outlined to both the teacher and the student participants both prior to collecting any data, and reiterated on each return visit. Permission was verbally sought for the audio-recording of all interviews, photographing any student work, and collecting any work samples or artefacts, even though these had been outlined in the information letter (for teachers) and explanation (for students), which also reaffirmed that my role as researcher, collecting data for my study, was my purpose for being there.

Techniques suitable to the different ages of student participants (Grade 1, Grade 3 and Grade 5) were intentionally considered in the designing of tasks, data collection methods and general communication. This ensured continued well-being of student participants throughout the data collection process.

Disclosing the purpose for, and dissemination of, the study's results is another ethical consideration. Information regarding possible publications and presentations from this study were outlined in the information letters sent to all staff and parents (see Appendix 3).

Sharing results with participants also helps ensure reliability of the ethical conduct of the research. Due to teacher involvement in the research, negotiations were carried out with participating teachers about whether they would like to contribute to any writing, and what their role would be in this. Teachers and parents were given the option to receive the results summary at the end of the study, to be indicated on the consent form.

Another ethical consideration relates to security of data that has been collected. All data collected for this study will remain stored for the mandatory period, in accordance with the Human Research Ethics Committee guidelines. Hard copies of data, such as student work samples, written questionnaire responses, researcher journal, and assessment record sheets, are kept in a filing cabinet in a locked office. Soft copies of electronic data, such as audio-recordings, photographs, and interview transcripts, are stored on a secure, password

protected computer. All identifying markers have been removed from any artefacts used in this thesis and in the dissemination of any related research articles and presentations.

3.10 Chapter Conclusion

In this chapter, different research paradigms were considered, and an explanation of why this study was situated within a theoretical perspective of social constructivism has been described. Social constructivism, with a *relativist* view of what can be known, and an *interpretivist* view of how it can be known, lends itself to a qualitative research methodology. As such, a multiple case study methodology was selected for this study, and the methods of data collection and narrative analysis have been outlined. Strategies used to maximise trustworthiness of the findings, and ethical issues associated with the study, have also been discussed.

The following chapter (Chapter 4) will describe in detail how initial data were collected and used to identify and select the three mathematically gifted students, and subsequently their teachers, for the study. Chapter 5 will describe the targeted teacher professional development and how this was implemented. Both these chapters further strengthen the trustworthiness and credibility of the study by providing transparent details of the research. Chapter 6 will revisit, and describe in detail, the individual narrative analysis process, and provide the narrative analyses of the individual case study students. Chapter 7 will further detail the synthesised analysis of these ‘stories’ by triangulating the individual analyses, through identification of themes, commonalities and similarities, and linking these to relevant findings from the literature review (Chapter 2).

Chapter 4 – The Selection

Mathematically gifted students with self-limiting mindset tendencies

Angeline was a genius...but she couldn't remember on which side to put the fork and on which side to put the spoon.

(from *Someday Angeline*, by Louis Sachar, 1983, p.24)

4.1 Chapter Overview

Chapter 3 situated this study within a theoretical perspective of qualitative case study research; it also addressed the issues of trustworthiness within qualitative data collection and analysis. To ensure trustworthiness and credibility of qualitative research it is essential to address the issue of dependability, and one way of ensuring this is to report in detail all processes within the study, including selection of participants and data collection methods (Lincoln and Guba, 1985). This chapter unpacks and details the process of Phase 1, the selection phase, of my study (see Figure 3.3), to ensure that the method for selecting students is well understood to maximise trustworthiness of the research. This depth of understanding enables the reader to have confidence in any subsequent findings.

The research question driving my study was, *What impact does targeted teacher professional learning about classroom support for mathematically gifted students with self-limiting mindsets, have on the mindsets and mathematics learning of these students?*

As outlined in Chapter 3, to collect data to investigate this question, I needed to select students who were mathematically gifted, but with self-limiting mindset tendencies when viewing themselves as learners of mathematics. Therefore, an identification process for selecting mathematically gifted students was needed, as was a method for identifying mindset behaviours. This chapter details and describes the methods, the instruments, and the analyses used to select three case study students. It also describes and interprets students' approaches to mathematics learning prior to the targeted teacher professional learning. This adds to the baseline platform for observing and analysing any changes in student dispositions and/or approaches to mathematics learning following the professional learning.

4.2 Identification of Mathematical Capability

As indicated in Chapter 2, identification of children who are mathematically gifted is not a simple linear process. Giftedness is not always obvious, so high achievement measured by mathematics tests is not a reliable measure of innate mathematical capability. Identification for this study, therefore, required a combination of processes, from formal methods of testing to informal methods of observation and conversations with people who knew the children well (McAlpine, 2004; Moon, 2006; Reis, 2004). The identification process included: 1) teacher nominations, 2) a parent questionnaire, 3) previous mathematics assessment data, 4) classroom observations, 5) student and teacher semi-structured interviews, and 6) a specifically designed clinical task-based mathematics assessment interview. As there is no specific formal test for identifying ‘mathematical giftedness’ (Singer et al., 2016), an assessment interview was designed expressly for this study to identify students’ ability to a) learn a new mathematics concept easily, b) generalise new knowledge readily, and c) reason using intuitive strategies to efficiently solve an unfamiliar mathematics problem beyond the scope of regular primary school curriculum content. Each of these abilities are hallmarks of Krutetskii’s (1976) observations of mathematically gifted students.

4.2.1 Teacher Nomination

Prior to soliciting teacher nominations of ‘highly capable’ students, I facilitated a focus group conversation with the nine Grade 1, Grade 3 and Grade 5 teachers, the Principal and the Associate Principal (cf. Section 3.6.2). The purpose of this discussion was two-fold: to collect data on teacher perspectives of giftedness and talent, and to possibly expand and challenge teachers’ thinking around some of the common understandings of, and myths about, giftedness before being asked to nominate students. Participants were provided with a Yes card and a No card. Statements of common beliefs about students who are gifted (both generally and mathematically), were presented to the group, and participants given five seconds to vote either Yes (generally agree) or No (generally disagree) with each statement. This ‘vote’ became the catalyst for discussion about the teachers’ beliefs and practices as each was given the opportunity to explain and justify their individual responses. Table 4.1 outlines an illustrative selection of comments made.

Table 4.1

Focus group discussion responses: Introducing the Topic of Mathematically Highly Capable Students

Statement	Teacher responses		Selected Comments
	Yes	No	
<i>Children who are highly capable mathematically are privileged.</i>	8	3	<ul style="list-style-type: none"> • I think if we're looking forward then I think someone is more fortunate, in a preferred position if they are highly capable than if they're not. I don't think there are many down sides to being highly capable. (Principal) • If you've got the skill you're lucky, you can apply it with money and I suppose real life, I think it will benefit you. (Gr3 teacher) • I just think it's an advantage if they are capable because I think that they will probably continue that success with their learning in maths. But on the flip side, if they're highly capable and they're in an environment that is not really catering to their needs then perhaps it's not a great thing either because it probably leads to their frustration. (Gr5 teacher) • I said no, because I thought if you're looking at the whole child, it's great if they're highly capable, but they might have other areas of their development that is sort of sorely lacking. (Gr1 teacher)
<i>Children who are highly capable mathematically will develop behaviour problems if they become bored in maths classes.</i>	7	4	<ul style="list-style-type: none"> • I think through my experience children who are highly capable, if they find the content boring and are not challenged enough, I've probably had many students that fit under that bracket and they become disengaged and that might lead to behaviour problems. (Gr3 teacher) • But not just external causing trouble to the rest of the class, but just the behaviour in themselves where they become disengaged, just not engaged in themselves as opposed to interrupting other people. The behaviour's not just distracting other people but their behaviour in what they, I guess their drive, those sorts of behaviours, internal behaviours. (Gr1 teacher) • I put it down to self-regulation and self-management, because I have experienced students who are highly capable, and knowing their personalities and their behaviours in class they won't necessarily misbehave if they're not being challenged because they understand, and they've got really high self-regulation and know the right thing to do at the right time. (Gr5 teacher) • I'd say there's equally examples of kids that won't develop behaviour problems. They might be sitting there bored, they might not be engaged, they might not achieve their best, but they might also just sit, they might sit there and not be a "behavioural problem". (Gr5 teacher)

(continued)

Table 4.1 Continued

Statement	Teacher responses		Comments
	Yes	No	
<i>Children who are highly capable tend to have pushy parents.</i>	6	5	<ul style="list-style-type: none"> You said that they <i>tend</i> to be pushy, like yes you get some that are pushy, but if we're trying to generalise then I'd say no. (Gr3 teacher) I was thinking about my own experiences where I have parents that come and talk to you and say 'Oh but we're not being pushy parents...', but in actual fact some of their requests...yeah, could be interpreted by some as being pushy. (AP) I think we notice the pushy parents. I think if you did an analysis and looked at all the data ... yeah, this one's pushy, this one no, this one maybe, haven't met this one ... I think you tend to think of examples of pushy parents straight away that have, in maths you've had a capable student and you maybe can't think of examples of the other parents, might not necessarily come to mind. (Gr5 teacher) I know quite a few parents out there and I would say some are pretty pushy but others subtly pushy. (Gr1 teacher) High achieving rather than highly capable? I don't think the highly capable kids have necessarily pushy parents, but I think some of our kids that learned their times tables by the time they're six, the reason they can do that is because their parents have been setting the bar high and pushing, and they'll keep on doing that. (Principal) There's a difference between achievement and capability, I think capable is more potential, what potentially could they do as opposed to what are they achieving? (Gr3 teacher)
<i>Children who are highly capable mathematically are fast finishers.</i>	5	6	<ul style="list-style-type: none"> It depends on what the task is...if you give them something that is open-ended they could be busy on it for quite a while. (Gr1 teacher) I've found that often the tasks that you set, even the open-ended ones, those really capable maths students will finish before the others, and I find it really challenging to know where to take them next. (Gr5 teacher) I'm thinking about it in terms of when we talk about an open-ended task my understanding was that it was supposed to take them as far as they can go at that particular time, and I'm wondering whether sometimes it's our own ability or perception of ability that perhaps limits that. (Gr1 teacher)

(continued)

Table 4.1 Continued

Statement	Teacher responses		Comments
	Yes	No	
<i>Children who are highly capable mathematically need as much support in the classroom as children who struggle mathematically.</i>	10	1	<ul style="list-style-type: none"> • Maybe trying to be devil's advocate...but I think talking about a need of support perhaps those kids that are highly capable it would be a great thing to support them and to take them further, it's something we aim for, but the kids that are struggling at the other end really need that support if they're going to have the skills to get by...I think if you have two kids in front of you that are both asking for your help and you know that one's particularly strong in maths and one at the other end is not getting it, and I have to make a choice, I'm going to help the kid who's further behind. (Gr5 teacher) • I took out mathematically and I just looked at my class and I think as much as possible I want to share around my support as equally as I can. I do identify that sometimes there are stronger needs in the classroom, but I do as much as possible try and balance that out because I think every child has just as much right to have an equal portion of my time. (Gr3 teacher) • I think that the kids who are highly capable mathematically, a trait that goes with that is just continually reflecting and thinking, and they're just analysing everything, the ones who are really capable...the kids who are always reflecting and wondering and assessing and reviewing in their heads, if we're not catering to their needs I think they end up in strife. They end up stressed, and disengaged, and causing trouble, and we can see really negative outcomes. (Principal) • I've observed in my classroom some very capable maths students who when they actually come to a task that they're not quite as capable in, or maybe they've got a misunderstanding around something, they don't cope with that really well and they wonder why it's not taking them the same period of time it usually does to complete the task ... and I had a student have a total meltdown because he was so used to finishing quickly. (Gr5 teacher)
<i>Any further comments about supporting and catering for children who are highly capable mathematically?</i>			<ul style="list-style-type: none"> • I feel that perhaps, and this is a generalisation, that in primary school settings professional learning is often focussed on helping teachers develop an understanding of enabling prompts for children who are at risk. I don't know whether professional learning for primary schools regularly taps into, or best serves the notion of, extending prompts for children that are highly capable. (AP) • I think everything that we learn, that we get told that we're trying to meet these benchmarks it's a minimum... (Gr3 teacher) • I see the main benefit of enriching students' mathematical experiences at school, particularly the more capable kids, is to be about creating excitement in the children rather than getting to Year 8 or getting them to Year 9 or whatever, which I think is not something that's necessary or overly valuable. (Principal)

The first thing to note here was that there was not one statement that had a unanimous initial response of ‘yes’ or ‘no’. The subsequent discussions, however, indicated that beliefs were not necessarily completely polarised, and some teachers’ thoughts and ideas were subsequently challenged by others’ perspectives. For example, by far the majority of teachers believed that children who are highly capable mathematically are privileged, or lucky, because “they’d be able to continually succeed in mathematics at school, and apply their skills in real life”. Others, however, challenged this notion suggesting that ability in one area does not define the capabilities of any one student completely, concurring with Winner (1996). The notion that highly capable students tend to develop behaviour problems if they become bored in mathematics classes was predominantly considered to be true until a couple of people pointed out that maybe it is only the ones displaying behaviour problems we tend to notice, a point also noted by Silverman (2013). Someone else countered with the thought that ‘behaviour problems’ do not necessarily exhibit as external disruptions, but may in fact also include disengagement and other negative internal behaviours. The question of ‘pushy parents’ elicited an interesting qualification between highly *capable* students and high *achieving* students, which was an important distinction in my definition of mathematical giftedness (Neihart & Betts, 2010). The issue of ‘fast finishers’ was surprising (and pleasing) to me: the majority, albeit by a small proportion, said they did not believe children who are highly capable mathematically were necessarily fast finishers (Siegle, 2013; Silverman, 2013). The school, as a whole, had been focusing on open tasks and investigations which may have influenced opinions about this aspect of mathematical ability. One Grade 5 teacher still admitted to the challenge she had with some students finishing before others and not knowing where to take them next. I found this to be a very brave statement – admitting her struggle. It is always possible (and indeed probable), that in a setting such as this, individual responses are internally censored because of professional expectations (Kitzinger, 1995). A very clear majority of teachers agreed, in theory, that children who are highly capable mathematically need as much support in the classroom as children who struggle mathematically. One teacher, who admitted to playing the ‘devil’s advocate’, took a pragmatic approach to this statement, however, differentiating between the ideal and the reality in a classroom setting, stating that the student who is struggling would, most likely, claim his attention first. This was alluded to by several others in further comments where it was mentioned that by and large, the focus of professional learning, in primary settings at least, is on enabling students at risk, and meeting minimum

benchmarks. It was good to hear the issues of preventing disengagement, and recognising some highly capable students' stresses when challenged, being discussed, though.

Overall, following this focus group conversation I was confident that teacher nominations would focus on more than student achievement and observations such as 'fast finishers'. There were issues that arose in this setting, such as the difference between highly capable students and high achieving students, and concerns of highly capable students having 'meltdowns' when faced with the unexpected (Dabrowski & Piechowski, 1977) that were able to be discussed without researcher bias because I was not the one to raise the issues. The Principal's comment about enrichment being about "creating excitement" rather than simply accelerating students' learning, was particularly encouraging in confirming this school as a suitable environment for my study.

There were no students at the school who were recognised as having been formally identified as 'gifted'. In fact, the Principal was quite averse to the use of the term gifted because of the competitive nature of some parents within the school. He specifically asked that I not use the term 'gifted' when recruiting participants, but was very happy for me to talk about 'highly capable' students. Therefore, the Grade 1, Grade 3 and Grade 5 classroom teachers were each given a nomination form on which they were asked to list any children in their class whom they ascertained to be highly capable mathematically (see Appendix 1). They were asked to rate the extent of each nominated student's mathematical capability by circling a number on a five-point Likert-type scale, from 1) average, to 3) very capable, to 5) highly capable, to 7) extremely capable, and to provide a brief example of the type of work the students do that gave them this impression. Thirty students were nominated, with 27 of these receiving parent consent to participate in the study – ten students from Grade 1, seven students from Grade 3 and ten students from Grade 5. Table 4.2 summarises teachers' nominations and responses.

The purpose of the 'teacher comments' on the nomination form was to look for indicators of mathematical capabilities that included the types of 'hallmarks of mathematical ability' outlined in Krutetskii's work: able to readily grasp the structure of a problem; tend to generalise easily; able to develop chains of reasoning; use symbols and language accurately and effectively; able to think flexibly – backwards and forwards, switching between strategies; are efficient problem solvers (Krutetskii, 1976).

Table 4.2

Teacher Nominations of Mathematically Highly Capable Students

	Student name (pseudonyms)	Teacher nomination of capacity (on a scale of 1-7)	Teacher Comments (brief description/example of student's exceptional capability)
Grade 1 Students (10)	Bronte (f)	4.5	<i>MAI</i> data from Prep; solid mathematical strategies
	Frank (m)	5	<i>MAI</i> data from Prep; adventurous with his thinking
	Alex (m)	6	Extremely capable and he is <i>aware</i> of his mathematical ability
	Jack (m)	6	Used data from Prep <i>MAI</i> and end of year grades; has demonstrated extended abilities in class
	Brett (m)	5	Used data from Prep <i>MAI</i> and end of year grades; has demonstrated extended abilities in class
	Hazel (f)	6	Used data from Prep <i>MAI</i> and end of year grades; has demonstrated extended abilities in class
	Hamish (m)	5	From <i>MAI</i> data and general observations
	Tom (m)	5	From <i>MAI</i> data and general observations; interesting thought processes, thinks outside the square
	Elsa (f)	5	From <i>MAI</i> data and general observations
Rose (f)	4	From <i>MAI</i> data and general observations	
Grade 3 Students (7)	Jack (m)	5	Mental computation
	Sammy (f)	6	Great at explaining strategies; uses a range of strategies
	Janet (f)	6	Great at explaining strategies; uses a range of strategies
	Emma (f)	5	Very good number sense
	Hamish (m)	4.5	Good place value concepts
	Annabelle (f)	5	Very good number sense
	John (m)	5	[Nominated by Principal & AP]
Grade 5 Students (10)	Murray (m)	6	A very driven maths student. He likes to know what he's done is correct.
	James (m)	6	A capable maths student who loves algorithms. He is quite fixed in his approach to maths.
	Fred (m)	7	Loves maths and looks for challenges all the time. Very high performing student with excellent problem solving.
	Bob (m)	6	Achieves high results in maths. He is very quiet and hesitant to share his thinking, so could easily go under the radar.
	Amy (f)	7	Very capable. Excellent at problem solving. Often has difficulty explaining her mathematical reasoning.
	Gary (m)	5	Enthusiasm, determination to solve more complex problems. Makes discoveries independently. Stanine 6 – <i>PATMaths 3</i>
	Robert (m)	6	Very quick to grasp new complex concepts. Excellent existing knowledge. Stanine 8 – <i>PATMaths 3</i>
	Jim (m)	4	[Nominated by Principal & AP] Stanine 7 – <i>PATMaths 3</i>
	Lucy (f)	5	[Nominated by Principal & AP]
Bruce (m)	5	[Nominated by Principal & AP]	

Note: *MAI* = *Mathematics Assessment Interview* (conducted at the beginning of the school year in Grade Prep to Grade 3); *PATMaths* = *Progressive Achievement Tests in Mathematics* (*PATMaths 3* is Test 3, conducted at the beginning of Grade 4)

Within the nominations there were a few specific descriptions of students' exceptional mathematical abilities: "interesting thought process, thinks outside the square", "excellent at problem solving" and "very quick to grasp new complex concepts". The majority, however, were described in general, non-quantifiable terms, for example, "has demonstrated exceptional abilities in class" and has "good number sense". One Grade 5 student was described as having "enthusiasm, determination to solve more complex problems; makes discoveries independently" and one Grade 1 student was "very adventurous with his thinking", which may be indicators of high capability, but could equally be indicators of personality types. All but one of the Grade 1 students had been nominated, in part, as a result of *Mathematics Assessment Interview (MAI)* data. MAI data, even though based on a formal testing event, give some insight into students' ability to think and reason mathematically due to the nature of the interview (see Section 3.6.2). One Grade 5 teacher used *Progressive Achievement Test in Mathematics (PATMaths)* (Australian Council for Education Research (ACER), 2014) stanines data as part of her nomination criteria. The *PATMaths* assessment, unlike the MAI, is a timed test that relies solely on correct answers (the reasoning behind the answers is not considered), and is the type of assessment that has been shown to be less reliable for identifying mathematical giftedness (Niederer et al., 2003). Interestingly, this teacher's nomination of capacity on the Likert-type scale did not completely reflect the *PATMaths* scores – Jim assessed at Stanine 7 and Gary at Stanine 6, but she classified them both as 3 (highly capable). This is an indication that assessment results alone may only give part of the picture of student capabilities. Teacher observations also provide vital and valuable insights (Niederer et al., 2003).

4.2.2 Parent Written Questionnaire

Parents of each of the nominated students were sent information letters requesting consent for themselves and their children to participate in the study. Twenty-seven of 30 students nominated were granted parent/care-giver consent. The parents were also invited to complete a written questionnaire, focusing particularly on their children's pre-school and kindergarten dispositions that may have been an indication of mathematical ability. Again, the criteria I was looking for were the types of hallmarks suggested by Krutetskii (1976), such as evidence of curiosity and/or ability with activities such as puzzles or building blocks, or with spatial awareness or early number sense, rather than specific formal mathematical skills. Table 4.3 is an indicative selection of comments made by parents.

Table 4.3

Parent/Caregiver Questionnaire Responses

What can you remember about your child and his/her maths-type abilities before they started school? [This includes activities such as building and designing structures (blocks, Lego etc.), jigsaws, recognising landmarks/directions (spatial abilities), as well as activities with number.]

- Started when [Alex] was two. He would count the car parks and mailboxes. He would read out the prices at the checkout supermarket. Checkout used to encourage him to read out prices before scanning next item.
 - Very good at jigsaws and simple equations, but not exceptional. Enjoys card games but is not as gifted at them as some of her brothers.
 - [Rose] has always been an inquisitive and curious child. She enjoyed puzzles and problem-solving activities from an early age. She seemed to have good spatial awareness skills. At times we were amazed by her memory and her ability to do simple calculations.
 - Could follow instructions for Lego before school. Very good at puzzles, blocks. Very good at counting up to 20 and beyond at about 2 ½ years. Excellent directional memory – places, roads.
 - Always enjoyed building blocks, playing with Lego, jigsaws. Could tell time to ½ hour in kinder. Mainly good with numbers – finding patterns, simple addition.
 - Loved construction of Lego, able to follow instruction booklets easily, then redesign own structures and vehicles.
 - Always good at building Lego. Always good at following visual instructions, e.g., making paper planes.
 - [Janet] enjoyed doing jigsaws, she liked building with small wooden blocks and built complex structures and towers. She liked to count things like train carriages and things from the environment. She liked to count up to high numbers.
 - [John] could do jigsaws from a very early age (18 months). He could also order blocks in different colours.
 - Always attracted to puzzles, building Lego, counting in different increments.
 - Very interested and patient with jigsaw completion. Good at number recognition, ability to count. Very good at landmark recognition and direction.
 - Always enjoyed ‘creative’ construction with Lego not often following the instructions. Loved designing boats/planes/rockets. Great picture memory of landmarks, people, places.
 - [Fred] was always good at building blocks e.g., Lego, and at doing jigsaws. He loves any sport and was always good at keeping the score and adding it up.
 - [Bruce] enjoys lots of Lego and mental arithmetic. He would sit for hours working on Lego to get it right, doing jigsaws etc.
-

(continued)

Table 4.3 Continued

Did the kinder teacher ever talk to you about your child's curiosity and/or ability with maths-type activities (as outlined above)?

- Comments were made about her curiosity and interest in learning but we can't remember specifics.
 - Solid mathematical skills, numbers and counting, in 4y.o. kinder, and curious about maths, shapes, puzzles.
 - He mentioned [Frank]'s ability and enthusiasm for building blocks and his curiosity about lots of new ideas and willingness to explore new concepts.
 - At times, teachers were concerned that he was too involved with individual activities (Lego) that required him to be re-directed in order for him to join in other group activities.
 - Yes, he would navigate towards the problem tables rather than the art and craft tables for activities.
 - [Murray] was good at puzzles and often assisted others to complete them.
 - "Has a thirst for knowledge and learning. Very confident." [Kinder teacher's comment].
 - [Fred] was always good at number games.
 - [Bruce] is a naturally curious child who asks endless questions and was very articulate from an early age. He expected answers that were quite complex early on. Asking many questions until he was satisfied that he understood the answer.
 - They talked about his attentiveness and curiosity and how he was able to concentrate for a long time when working on his maths activities. He also chose maths activities as a preference.
-

The majority of parents described their children's mathematical *dispositions* rather than mathematics or number *skills*. One parent mentioned "times tables" before his/her son started school, another stated, "Since she was little we started teach [sic] her some simple numbers and since Prep we started teaching her subtraction addition and multiplication table". Both these parents were Chinese immigrants, so this may possibly be indicative of cultural expectations.

Winner (1996) suggests visual-spatial activities such as building with blocks and doing jigsaws are common indicators of high mathematical capability. Indeed 18 out of the 27 parents (67%) who completed the questionnaire commented on their child's interest and ability in playing with blocks (Lego and regular wooden blocks) and/or jigsaw puzzles prior to school age, although this may also have been influenced by these being listed in the questions. However, the interest, which started very young, as early as 18 months, was

sustained, “he would sit for hours working on Lego to get it right, doing jigsaws etc.”, and involved both following instructions and “creative construction” design. Inquisitiveness and curiosity with numbers were recognised by Krutetskii (1976) as being hallmarks of what he termed a “mathematical cast of mind” (p. 187). Several parents mentioned their child’s interest in numbers, including simple mental calculations, from a young age. Spatial ability – being able to comprehend and mentally manipulate images and shapes in space: a function required for solving puzzles, figuring out maps, in construction et cetera – is another common characteristic of mathematically gifted children (Gardner, 1999; Krutetskii, 1976; McAlpine, 2004). “Spatial ability predicts performance in mathematics and eventual expertise in science, technology and engineering” (Tosto et al., 2014, p. 462). There were a significant number of parent comments about spatial awareness abilities, including ability with jigsaw puzzles, ‘reading’ visual instructions (e.g., in Lego booklets and paper plane instructions), directional memory (places and roads), “very good at landmark recognition and direction”, as well as mentioning Kindergarten teachers remarking on noticing these types of abilities in their children too.

Parents were also asked to rate their perceptions of their child’s current mathematical ability by circling a number on a Likert-type scale from 1 (low) to 3 (average) to 5 (high) to 7 (very high). Out of 27 parent questionnaire responses three (11%) circled 7 (very high), seven (26%) circled 6, 15 (56%) circled 5 (high), and two (7%) circled less than five. This, together with indicators such as *growth point* profiles, task-based interview responses and other observations, seems to affirm findings that parents are generally good at recognising giftedness in their children (Hodge & Kemp, 2006; Jacobs, 1971; Silverman, Chitwood & Waters, 1986), and do not necessarily overestimate their child’s abilities, as is often assumed by teachers (Plunkett, 2000).

4.2.3 Archival Records – Mathematics Assessment Interview Data

The selected school for this study had used the *Mathematics Assessment Interview (MAI)* and associated *growth point* framework (see Section 3.6.2) sporadically for a number of years, but it was only in the previous two years that it had been used routinely with all Prep, Grade 1 and Grade 2 students. The year of this study was the first year they also assessed all Grade 3 students. The school chose to assess student knowledge in the four whole number domains only – Counting, Place Value, Addition and Subtraction strategies, and Multiplication and Division strategies – resulting in a 4-digit *growth point profile* for each student.

For example, a growth point profile of 4242 indicates a student has reached:

- *growth point 4* in the Counting domain, so can meaningfully skip count by 2s, 5s and 10s to determine how many in a collection but cannot yet reliably skip count from a non-zero starting point;
- *growth point 2* in the Place Value domain, so can understand and interpret 2-digit numbers as both a collection of items bundled in groups of 10 and as a position on a number line in relation to other numbers, but does not yet fully understand the structure of 3-digit numbers;
- *growth point 4* in Addition and Subtraction strategies, so is no longer relying on counting strategies to solve simple addition and subtraction problems but uses basic strategies like doubles, commutativity and adding 10, but not yet fully transferring this knowledge to derived strategies such as near-doubles, adding 9, or building to 10; and
- *growth point 2* in Multiplication and Division strategies, so can solve multiplicative problems using the group structure rather than counting by ones, but only if all objects are modelled.

Figure 4.1 shows the *MAI* growth point spread of each number domain for a large cohort of Grade 1, Grade 2 and Grade 3 students from the *Bridging the Numeracy Gap (BTNG) Pilot Project* (Gervasoni et al., 2013), where *MAI* data for approximately 2000 students from 32 Victorian and Western Australian schools were coded and graphed to show growth point distributions for analysis for the *BTNG* project. This database was useful for situating the nominated students for my study within a quantifiable position based on the distribution of mathematical reasoning ability, as an approximation for innate mathematical capability. Mathematically ‘gifted’ students would be those within the top 1-2% of the general population of same-grade peers, and mathematically ‘highly capable’ within the top 2-10% of the general population of same-grade peers (see Chapter 1).

Table 4.4 shows the growth point profiles that were available for the nominated students. The profiles in brackets at the top of the table show median growth point profiles for each grade level (see Figure 4.1).

From the *MAI* data, Alex and Jack were two notable Grade 1 students having reached growth point 4 in place value. This means that Alex and Jack were already able to successfully read, write, order and interpret 4-digit numbers; they could not only read and write these numbers, they could also understand the structure of our place value system in order to manipulate and re-name 4-digit numbers to calculate 10 more/10 less, 100 more/

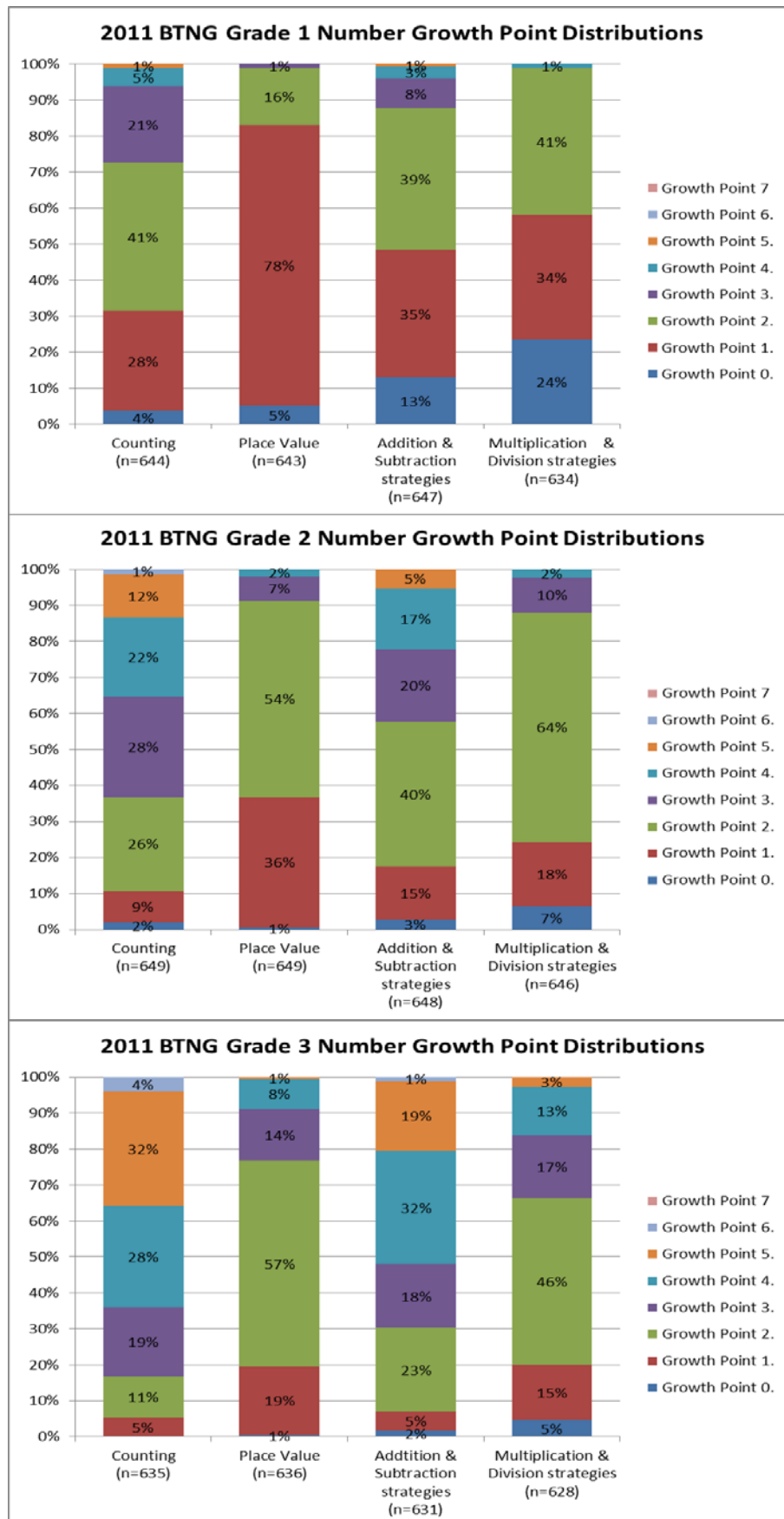


Figure 4.1 Bridging the Numeracy Gap Pilot Project Grade 1, Grade 2 and Grade 3 number growth point distributions

Table 4.4

Mathematics Assessment Interview (MAI) growth point profiles (GPs) for nominated students – most notable students in each grade highlighted

		MAI GPs Prep	MAI GPs Gr1 (2121)	MAI GPs Gr2 (3222)	MAI GPs Gr3 (4242)
Grade 1	Bronte	2122	4242	-	-
	Frank	3132	5352	-	-
	Alex	5222	6454	-	-
	Jack	5242	5454	-	-
	Brett	2221	4242	-	-
	Hazel	1111	5342	-	-
	Hamish	1100	2332	-	-
	Tom	2212	5232	-	-
	Elsa	2122	5242	-	-
	Rose	2120	3242	-	-
Grade 3	Jackson	n/a	n/a	5244	6555
	Sammy	2122	2222	4353	6454
	Janet	2131	4232	4242	4344
	Emma	n/a	2222	5343	5454
	Hamish	2132	5353	6354	6464
	Annabelle	2122	5252	5252	6454
	John	n/a	3330	4352	5453
Grade 5	Murray	n/a	n/a	n/a	n/a (6565 – Gr4)
	James	n/a	n/a	n/a	n/a
	Fred	n/a	n/a	6452	6462
	Bob	n/a	n/a	n/a	n/a
	Amy	n/a	n/a	6354	6455
	Gary	n/a	n/a	n/a	n/a
	Bruce	n/a	n/a	4343	5454
	Robert	n/a	n/a	n/a	n/a
	Jim	n/a	4232	4342	5353
	Lucy	n/a	n/a	n/a	n/a

Note: Profiles in brackets at the top of the table show median growth point profiles for each grade; n/a = not available. Some students (e.g., Jackson) did not start formal schooling at this school. Early years MAI data were not available for most Grade 5 students.

100 less (e.g., 100 less than 3027), and could understand a 4-digit number as a place on a number line relative to other quantities. This was at the beginning of Grade 1 where the focus in the curriculum is on learning about 2-digit numbers. Indeed, only 1% of the 643 Grade 1 students in the *BTNG* project had reached growth point 3 in place value, and none were at growth point 4. Alex and Jack were also able to understand different constructs of multiplicative thinking such as ‘times as many’, arrays, and partitive and quotitive division,

and work abstractly with multiplicative problems without relying on any physical representations. Only 1% of Grade 1 students in the *BTNG* project were above growth point 2 in multiplications and division strategies. These results would suggest that both Alex and Jack may be in the top 1-2% of the general population of same-grade peers in number understanding, at least, and could therefore be considered as possibly mathematically gifted. Both Alex and Jack were therefore considered as possible Grade 1 case study participants.

Jackson was the Grade 3 student with the highest overall number profile in Grade 3. Apart from Jackson there were several other Grade 3 students with similar number profiles at the beginning of Grade 3. The *MAI* growth point framework is designed to measure student growth over a period of time as well as an indication of student knowledge at the beginning of each school year. It is anticipated that on average students will make one growth point increase in each domain per year, with the exception of place value, where the expectation is two growth points in three years (Clarke et al., 2002). It is therefore significant that although neither Sammy nor Emma had notable growth point profiles in Grade 1 (both 2222), their profiles in Grade 3 were 6454 and 5454 respectively. Growth, in two years, of three and four growth points in the counting and addition and subtraction domains, and two growth points in the place value domain is an indication of an above-average rate of progress in learning new number concepts. Jackson, Sammy and Emma were therefore considered as possible Grade 3 case study participants, although most other Grade 3 students had similar point profiles.

There was limited *MAI* data available for the Grade 5 cohort. Fred and Amy had both reached growth point 6 in counting by the beginning of Grade 2 (the top 1% of the *BTNG* cohort), and Fred had also reached growth point 4 in place value by the beginning of Grade 2 (the top 2% of the *BTNG* cohort). Fred and Amy were both possible candidates as Grade 5 case study participants from the *MAI* data, but further data were to be considered before making a final decision.

4.2.4 Task-based One-on-one Mathematics Interview

As a second stage of identification, I interviewed each of the 27 nominated students with parent consent using the task-based mathematics interview designed specifically for this study (see Appendix 2, and Section 3.6.2). A mathematical disposition, or ‘cast of mind’ (Krutetskii, 1976) is not something that can be simply measured by a number in an assessment. It is something that needs to be observed (Niederer et al., 2003). This clinical

task-based interview was designed to be able to observe a student's approach to mathematical tasks, using Krutetskii's hallmarks of mathematical abilities as a guide.

Children who are mathematically gifted tend to generalise easily, extend, create and invent new methods of solving mathematical problems, and they naturally strive "for the cleanest, simplest, shortest and thus most 'elegant' path to the goal." (Krutetskii, 1976, p. 187)

The interview assessed three specific areas: 1) the student's ability to *reason* mathematically. The first task was a ratio task, used to assess how the student went about reasoning proportionally with an unfamiliar type of task, requiring the use of intuitive strategies rather than learned approaches; 2) the student's ability to *learn something new*, and to *generalise*. The second task taught students a new way of working with numbers using a Chinese abacus, to see how they went about learning something new, and how they could *generalise* this knowledge to use the abacus for simple calculations; and 3) the student's *mindset* about mathematics learning. The third task was an open-ended task, *Adding Corners* (adapted from Downton, Knight, Clarke & Lewis, 2006). It was used to see how willing students were to *think creatively* about maths, and whether or not they displayed *self-limiting mindset* tendencies based on work by Muller and Dweck (1998). There were three versions of the interview: for Grade 1, Grade 3 and Grade 5. All interviews were audio-recorded, results written on a specifically designed record sheet, and work samples collected and/or photographed. Responses were transferred to a spreadsheet together with parent statements and teacher nominations to allow for close analysis. The following pages highlight some of the students' data (see Table 4.5, Table 4.6 and Table 4.7).

Table 4.5

Sample of Grade 1 student responses to the task-based one-on-one mathematics interview questions

Gr 1	Proportional reasoning (3 lollies for 10c)	Learning something new/generalising (abacus)	Mindset indications (open task)
Alex	<p>First answer was quick and confident, but incorrect. When asked to explain his answer he realised that the problem may not be as straight forward as he first thought, he had just 'added a zero' to 'times by 10'.</p> <p>Was subsequently able to simultaneously count by threes and keep count of the number of 'threes' counted (pre-proportional reasoning).</p> <p>Able to mentally manipulate large numbers: $5698=5060+500+48$ ("no that doesn't work, I still need another 90") in Adding Corners task.</p>	<p>Very quick to pick up the counting idea with the abacus (although counting backwards was counting the beads he moved instead of beads that were left).</p> <p>Could show all numbers easily (added beads to what he had rather than starting again each time).</p> <p>Had difficulty naming numbers – kept losing place value structure of each rod (called 70 50+2 so 78 became 60), even though he could name 10s-of-thousands and 100s-of-thousands rods.</p>	<p>Adding Corners solution: $5968=2500+2500+698$</p> <p>Second solution: $5968=98+2500+3100$</p> <p>It was challenging because "only about one or two other people in the school could do this".</p> <p>Wasn't happy with his first solution [even though he had persevered with some very complex mental calculations] because it took too long, and "there were too many crossings out." Very happy with the second solution that was quick, easy, and with no errors made.</p> <p><i>[self-limiting mindset]</i></p>
Jack	<p>Able to correctly answer both proportional reasoning questions using pre-proportional reasoning (e.g., wrote 10 10 10 10 while simultaneously counting by 3s in his head).</p>	<p>Picked up counting with the abacus very quickly; fascinated with how it worked.</p> <p>Could name all numbers correctly – no problems and very quick.</p> <p>Could show numbers, but was relying on counting by ones rather than using the 5s structure which sometimes confused him.</p>	<p>Adding Corners solution: $100=40+10+50$</p> <p>Second solution: $100=200-50-50$</p> <p>Very excited about the opportunity to be really creative, "can you do take away?" Chose $100 = 200-50-50$. However, stated that he chose 100 because it was an even number, "and I like evens more than odds because it's easy stuff"</p> <p><i>[self-limiting and positive mindset]</i></p>

(continued)

Table 4.5 Continued

Gr 1	Proportional reasoning (3 lollies for 10c)	Learning something new/generalising (abacus)	Mindset indications (open task)
Frank	<p>Fascinated with the abacus. Wanted to explore larger calculations – predicted what would happen if he +1000, -100, +50, +5, -1</p> <p>His prediction was incorrect which intrigued him and he was keen to find out where and why he had gone wrong.</p>	<p>Very quick to pick up all aspects of the abacus. Could show and name all numbers. When showing numbers moved and counted beads in groups rather than by ones (e.g., 80=5tens + 3tens).</p>	<p>Adding Corners solution: 56=4+20+32</p> <p>Second solution: 56=3+40+13</p> <p>Very focused on quick work, with no obvious effort required. Very self-assured because of abilities.</p> <p><i>[self-limiting mindset]</i></p>
Bronte	<p>Proportional reasoning very intuitive.</p> <p>How much would 12 lollies cost?:</p> <p>3 and 3 is 6 and double that is 12;</p> <p>3 for 10c so 6 for 20c and another 6 for 20c is 40c.</p> <p>How many lollies for 60c?:</p> <p>I counted on 6 from 12 because if you added 6 more you'd have 60c.</p>	<p>Picked up on counting structure of abacus quickly. Could show all numbers up to 4-digit.</p> <p>Could name 2- and 3-digit numbers (with some prompting about place value of each rod), but could not name 5903 due to lack of knowledge about place value conventions (is only at GP2 in place value).</p>	<p>Adding Corners solution: 19=6+4+9</p> <p>Second solution: 19=10+1+8</p> <p>Chose 19 because “it would be trickier than 20” [“but 150 might be a bit tricky for me”]. Thought long and hard, using fingers to explore possible solutions. Chose 6+4 “it could have been 5+5, but that’s and easy one”. Very happy with second solution “because the numbers are more spread out.”</p> <p><i>[positive mindset]</i></p>

Table 4.6

Sample of Grade 3 student responses to the task-based one-on-one mathematics interview questions

Gr 3	Proportional reasoning (3 lollies for 10c)	Learning something new/ generalising (abacus)	Growth / Fixed mindset indications (open task)
Emma	<p>Proportional reasoning: 15 lollies would cost 50c.</p> <p>Skip count by 3s and each 3 is 10c, so 10, 20, 30, 40, 50 (pre-proportional reasoning), but couldn't reverse this to calculate how many lollies would cost 80c.</p> <p>Better value reasoning: 3 for 10c is better value. Counted by 3s up to 33. Either way this one (3 for 10c) is a bit less – I couldn't get to the next [10c] so this one must be less.</p>	<p>Took a while to pick up abacus counting, kept getting confused with rods (place value positions) and 5s structure. Once she worked this out could show all numbers and name all numbers but was relying on counting by 10s and 1s.</p> <p>Correct with all addition and subtraction calculations, using a complimentary numbers strategy for more complex additions, e.g.,</p> $73+3 = 73+(5-2)$ $26+5 = 25+(10-5)$ <p>“because if you add 10 and take away 5 you are adding 5.”</p>	<p>Adding Corners solution: $99=22+33+44$</p> <p>Second solution: $99=33+33+33$</p> <p>Spent a lot of time exploring different possibilities, wanting to make it as creative as possible.</p> <p><i>[no evidence of self-limiting mindset]</i></p>
Jackson	<p>Proportional reasoning: 15 lollies would cost 50c.</p> <p>Counted by 3s to 15 and then counted how many 3s using fingers to keep track (pre-proportional reasoning). Could therefore buy 24 lollies for 80c because “3 more 3s from 15 is 24”.</p> <p>Better value reasoning: 10 for 35c is better value than 3 for 10c because 9 is 30c, one lolly could be 5c and you're getting a discount on the 3rd. The reasoning was sound (9 for 30c) but the attempt at a unit value was confused.</p>	<p>Picked up counting on the abacus very quickly, with no hesitation. Had no problems showing or naming numbers, however was a little hesitant with calculations – ended up using an algorithm-type explanation.</p>	<p>Adding Corners solution: $392=152+190+50$</p> <p>Second solution: $392=222+80+90$</p> <p>Chose the number 392, stating “I like to work with numbers under 400, I didn't want to work with numbers too high” (even though he's at GP5 in place value).</p> <p><i>[self-limiting mindset]</i></p>

(continued)

Table 4.6 Continued

Gr 3	Proportional reasoning (3 lollies for 10c)	Learning something new/ generalising (abacus)	Growth / Fixed mindset indications (open task)
Sammy	<p>Proportional reasoning: Evidence of pre-proportional reasoning (see below), but not all answers were correct: 15 lollies would cost \$1.50 (incorrect), but calculated 80c would buy 24 lollies (correct)</p> <div data-bbox="338 523 636 794" style="text-align: center;"> </div>	<p>Picked up abacus counting easily. Could show all numbers but had difficulty naming 5903 (called it 1503), even though she is at GP4 in place value.</p> <p>Calculations with the abacus were correct, but explanations were vague – possibly worked them out because she knew what the answers should be [from mental calculations].</p>	<p>Adding Corners solution: $199=140+50+9$</p> <p>Second solution: $199=165+25+9$</p> <p>Thought 199 was “pretty creative”; second solution was an adjustment of the first solution to eliminate and decade numbers, “Doesn’t have 10s numbers at the end which makes it not as easy.” Was in-between ‘happy’ and ‘very happy’ with her second solution.</p> <p><i>[mindset tendencies unclear]</i></p>
Janet	<p>Proportional reasoning. Counting by 3s keeping track with fingers but lost count, used pencil and paper (pre-proportional reasoning):</p> <p>10c-3 20c-6 30c-9 40c-12 50c-15 60c-18 70c-21 80c-24</p>	<p>Picked up abacus very quickly. Got stuck with 50, but worked it out herself.</p> <p>Very quick with showing numbers, and no problems with naming numbers.</p> <p>Seemed to pick up on the concept of compatible numbers when adding and subtracting with the abacus, but did not show this in the process – worked out the answers mentally and produced the answer on the abacus (using more than two addends may be worth a try).</p>	<p>Adding Corners solution: $921=11+710+200$</p> <p>Second solution: $921=600\frac{1}{2}+300\frac{1}{2}+20$</p> <p>Chose a ‘big number’ to split with Adding Corners saying, “I like big numbers and wanted to do something different [to the two suggested numbers].”</p> <p>Chose to use fractions in her solution (was the only Gr3 to do this).</p> <p><i>[positive mindset]</i></p>

Table 4.7

Sample of Grade 5 student responses to the task-based one-on-one mathematics interview questions

Gr 5	Proportional reasoning (which is more orangey 2:3 or 3:5?)	Learning something new/ generalising (abacus)	Growth / Fixed mindset indications (open task)
Fred	<p>Very comfortable reasoning with fractions.</p> <p>Ratio of 2:3 (A) compared to 3:5 (B):</p> <p>$A=2/5$ $B=3/8$</p> <p>$2/5=4/10$ which is $1/10$ away from $1/2$</p> <p>$3/8$ is one $1/8$ away from $1/2$.</p> <p>So A is more orangey.</p>	<p>Picked up on abacus use quickly. Counting, naming numbers, showing numbers all no problems.</p> <p>Able to then use the abacus structure and compatible number strategies with calculations, showing an ability to generalise new knowledge:</p> <p>$36+25=36+(30-5)=56+(10-5)$;</p> <p>$73+43=73+(100-60)+3$</p>	<p>Adding Corners solution:</p> <p>$137=48+17+72$</p> <p>Second solution:</p> <p>$137=48+16+73$ Number choice was 137 “sometimes I’m happy when I can make it easy”. Said it was creative because it was more than 2-digits.</p> <p>Was very happy with the second solution because no two numbers added up to a number ending in zero (decade number).</p> <p><i>[self-limiting mindset]</i></p>
Amy	<p>Able to calculate the correct answer using a procedure for finding common denominators:</p> <p>$A = 5$ $B = 8$</p> <p>Common number = 40</p> <p>“I made it into a common number and found out how many oranges.”</p> <p>[A = 16 B = 15]</p>	<p>[Was a little familiar with the abacus, her (Chinese) grandfather had taught her about it, but not how to use it]</p> <p>No problems counting to 50, but when counting backwards from 35 was not really ‘reading’ the beads, was just concentrating on the number sequence.</p> <p>Could show all numbers, but when naming numbers called 5908 59 thousand and 8.</p> <p>For addition and subtraction calculations initially tried to use the structure of the abacus, but ended up using the algorithm rules (“borrowed 1 ten”), then replicated this with the abacus.</p>	<p>Solution for Adding Corners task: $52.8=50+2+0.8$</p> <p>Second solution:</p> <p>$52.8=1.4916+7.875648+43.43278$ - came from playing with the digits from 52.8 (using a calculator): $1.4916=0.528+0.825 \times 0.528+0.528$; $7.875648=1.4916 \times 5$. Took a long time trying to figure out the third number (trying to ‘count on’ to find the difference) before realising it was a simple subtraction problem.</p> <p>Showed perseverance and creativity; said “[the solutions] are endless!”</p> <p><i>[positive mindset]</i></p>

(continued)

Table 4.7 Continued

Gr 5	Proportional reasoning (which is more orangey 2:3 or 3:5?)	Learning something new/ generalising (abacus)	Growth / Fixed mindset indications (open task)
Bob	<p>Very comfortable reasoning with fractions.</p> <p>Ratio of 2:3 (A) compared to 3:5 (B):</p> <p>Initially considered an additive solution</p> <p>[A lemon is only one more (2 orange, 3 lemon) and B lemon is two more (3 orange, 5 lemon)], but then considered them as fractions (2/5 compared to 3/8):</p> <p>$2 \times \frac{2}{5} = \frac{4}{5}$</p> <p>$2 \times \frac{3}{8} = \frac{6}{8}$</p> <p>1/5 is smaller than 2/8 (because $4 \times \frac{1}{5} = \frac{4}{5}$ and $4 \times \frac{2}{8} = \frac{8}{8}$)</p> <p>So A is more orangey.</p>	<p>Picked up on counting, showing and naming numbers with the abacus no worries.</p> <p>Chose to use an algorithmic-type strategy for adding multi-digit numbers (which may not have worked with more than two addends), but used a compatible number process for 424-185</p>	<p>Solution for Adding Corners task: $137.75 = 50.25 + 47 + 40\frac{1}{2}$</p> <p>Second solution: $137.75 = 24.05 + 129.8 - 16.1$</p> <p>Was intent on being creative, and making sure the task he set himself was challenging.</p> <p>Chose a 5-digit number to two decimal places and ‘split it’ into decimals, fractions and negative numbers to make it more creative.</p> <p>Was willing to work hard and take time to be creative and challenge himself.</p> <p><i>[positive mindset]</i></p>
Murray	<p>Very comfortable reasoning with fractions.</p> <p>Ratio of 2:3 (A) compared to 3:5 (B):</p> <p>2 parts out of 5 would be more orangey than 3 parts out of 8 because:</p> <p>$\frac{2}{5} \times 2 = \frac{4}{5}$</p> <p>$\frac{3}{8} \times 2 = \frac{6}{8}$</p> <p>$\frac{2}{8} = \frac{1}{4}$</p> <p>1/5 is less than 1/4 so actually B is more orangey because 1/4 is the bigger part ... no, A is right because I was working out which part of lemon was bigger.</p>	<p>Had some difficulty picking up on the abacus initially.</p> <p>Got everything correct but needed to clarify things all the way through (place value of each rod, the ‘5’ bead on each rod etc)</p>	<p>Solution for Adding Corners task: $793.3 = 264.43 \times 3$</p> <p>Second solution: $793.3 = 1000 - 103.35 - 103.35$ ($206.7 \div 2$)</p> <p>Was willing to challenge himself by:</p> <p>choosing a decimal number; attempting to find three equal numbers to go in the corners; persevered with the division even when it went wrong;</p> <p>exploring a complex second solution to challenge himself further:</p> <p>$793.3 = 1000 - 2 \times \frac{206.7}{2}$</p> <p><i>[positive mindset]</i></p>

From these data, it can be seen that mathematical abilities varied. Whereas some students were using intuitive strategies, others were relying more on learned procedures. For example, Fred, in Grade 5, worked out that an orange:lemon ratio of 2:3 was more orangey than an orange:lemon ratio of 3:8 by considering the ratios as part-whole fractions (a ratio of 2:3 means that orange is two fifths of the punch mix, and a ratio of 3:5 means that orange is three eighths of the punch mix). He then compared these fractions using a ‘benchmarking to half’ strategy, considering two fifths as equivalent to four tenths to help, “ $2/5=4/10$ which is $1/10$ away from $1/2$; $3/8$ is one $1/8$ away from $1/2$. And $1/8$ is a bigger gap than $1/10$, so A is more orangey.” This provides evidence of his ability to consider an unfamiliar problem-type, associate it with something he was familiar with (fractions), conceptualise these fractions as the orange component of the punch, and then reason proportionally to determine that the smaller fraction gap meant the greater orange component in the mix. This ratio question, then, provided evidence of students’ ability to think flexibly, extend, create and invent new methods of solving mathematical problems, and solve problems efficiently (Krutetskii, 1978). As a comparison, Amy also considered the ratios as part-whole fractions, “A is fifths, and B is eighths”, but then “...made it into a common number [A=5 and B=8 so the common number is 40], and then found out how many oranges, and A = 16 and B = 15 so A is bigger.” She used a learned procedure to solve the problem, but unlike Fred (and indeed Bob and Murray, also in Grade 5) did not translate her solution back into the original problem (i.e., A is bigger, not A is more orangey). In this instance, the learned procedure has masked her mathematical reasoning ability – she has achieved a correct answer, but there is not sufficient evidence that she really understands the context of the problem. Similarly, with the abacus, Amy reverted to algorithm rules when attempting addition and subtraction calculations rather than generalising the counting rules of the 5s and 10s structure of the abacus. It was not clearly evident, then, how much of Amy’s mathematical abilities were due to an innate capacity to learn and generalise mathematics concepts, and how much was due to an application of rules and procedures she had learned previously.

Most of the nominated students were able to identify the abacus counting structure with minimal, if any, repetition of the initial instruction. Half of the Grade 5 students, and nearly half of the Grade 3 students, could also generalise this new understanding of the abacus structure to carry out simple addition and subtraction problems. This provided further

evidence of students' ability to readily grasp the structure of a problem and to generalise easily (Krutetskii, 1978).

4.3 Identification of mindset behaviours

As well as the mathematical ability of case study participants, I was also looking for students with evidence of limited mindset tendencies as indicated by their responses to an open task, and through observations of general behaviours when completing mathematics tasks. The open-ended task in the one-on-one mathematics interview (*Adding Corners*) was designed to identify possible self-limiting mindset responses based on fixed- and growth-mindset indicators identified in research by Muller and Dweck (1998), where students were given a relatively easy task to complete, and then given a choice to complete a similar task or to tackle a more difficult version of the task. Students with a fixed mindset generally avoid taking risks and will choose easier tasks they know they can complete successfully (see Tables 4.5 – 4.7).

However, evidence from one specific task and observations from one task-based interview was limited, so semi-structured interviews were also used to help elicit further evidence of students' mindset dispositions. The students themselves were interviewed, as well as their classroom teachers.

4.3.1 Student and teacher semi-structured interviews

In the semi-structured interview (see Appendix 1), questions for the students included, "Are you good at maths? How do you know?" or "What do people who are good at maths do that makes you think they are good at maths?", "Do you enjoy maths: What do you/don't you enjoy?", and a written response sheet "I like doing hard maths never, occasionally, sometimes, mostly, always," with a verbal explanation. Responses to these questions, together with the previous observations, could help identify a student's view of what they believed to be hallmarks of successful mathematics learning, as well as how they perceived themselves as learners of mathematics. Table 4.8 shows some examples of what Muller and Dweck (1998) would classify as fixed and growth mindset type responses to these questions, and Table 4.9 shows a sample of student responses.

There is a mixture of both fixed and growth mindset type responses. Fixed, or self-limiting mindset responses are evident where students view getting correct answers quickly as evidence of mathematical ability, and where 'hard maths' is perceived as enjoyable as long as it is easy enough to not 'get stuck'. Growth, or positive learner mindset type responses

are evident where students mention the challenge of ‘hard maths’ being worthwhile and necessary for learning, and where doing something easy is perceived as being ‘a waste of time’. However, what a student says about enjoying a challenge and how they actually respond to a challenge can be quite

Table 4.8

Examples of fixed and growth mindset responses to interview questions

Mindset	Good at maths	Enjoy math	‘Hard’ maths
Fixed mindset response (example)	I know people are good at maths because they can work out the answer quickly and they always get the right answer.	I enjoy maths because I am good at it and I always get the right answers. I don’t enjoy it when I can’t work out the answer.	I don’t like doing hard maths because I might get it wrong.
Growth mindset response (example)	I know people are good at maths because if they work really hard on a difficult problem they can figure it out.	I enjoy maths because I really like to solve challenging problems. I don’t enjoy it when it’s too easy.	I like doing hard maths because it makes me think.

different. For this reason, part of the identification process also included conversations with the classroom teachers and observations of mathematics lessons.

Semi-structured interview questions for the teachers (see Appendix 1) included, for example, “For the students you nominated, what is it that makes you think that they are highly capable mathematically?”, “What can you tell me about [his/her] disposition in maths classes?”, “Do you think [he/she] has a more fixed or growth mindset? What gives you this impression?” [N.B. the school had previously completed extensive professional learning on Dweck’s (2006) fixed and growth mindsets. Fixed and growth mindset were therefore terms I used when talking with staff and students at the school]. In addition, questions were asked about family background to elicit other information about each student as an individual.

Table 4.10 shows a sample of teacher responses to the semi-structured interview questions. The responses to these questions served to further explore the dispositions of each student, as well as their mathematical abilities, from their teachers’ perspective.

Table 4.9

A sample of student responses to semi-structured interview questions

Student (Grade)	How do you know you (or someone else) is good at maths?	Do you enjoy maths? What do you/don't you enjoy?	I like doing hard maths: never, occasionally, sometimes, mostly, always.
Alex (Gr1)	I've been working with them and most of the time, like, wow!, they blew my socks off! I give them a question, like 485 plus 964, and always they estimate and they're very close.	I love, love, love maths! But I don't enjoy it when I get stuck.	Always. I <i>really</i> love challenging myself... so if I challenge myself I believe that I can learn more things... and I love learning.
Jack (Gr1)	Because I can work out like what ten thousand plus ten thousand is and I can work out numbers really quickly like 85+85. But [Hazel] is the best because she normally gets it done first in class.	Yes, I like how you add numbers and take away and multiply...	Always. Because it's easy for me. I didn't like the really hard take away; it was a bit too hard for me and I didn't really know much of the answer, but I guessed and I eventually got it right.
Sammy (Gr3)	[Janet] always finishes her work in time, she's always going 'done', she always gets the right answer and she always wants to do more.	Yeah [but not said very enthusiastically]. I enjoy how hard it is; it's a good challenge.	Mostly. Because of the challenge!! Because if it's really easy, it's like, it doesn't teach me anything. But sometimes it's too hard and gets...sort of...bleugh. If it was too hard I'd kind of get over it and just get really annoyed with the world.
Jackson (Gr3)	Because most of the time, when we have to do the test, I got all of them right in 5 minutes.	Yes, I enjoy pretty much everything about it, and I want to be a scientist and invent things [when I grow up].	Mostly, but not all the time because sometimes I'm just not in the mood.
Fred (Gr5)	[I'm] getting good grades. Also, because when we're doing tasks I can understand it very quickly and some people have trouble to understand and I can like do problems really quickly. [Murray] is better at some stuff...He's just faster at it, and I'm faster at other stuff than he is.	Yes, but I don't like it when it's a test and I get really worried that I won't finish it in time.	Probably sometimes. Well, it depends what it is because if it's one of the like plus, minuses, times and division, if it's not a test and you've got time to do it then probably mostly. If you're doing something really easy it's a waste of time, but sometimes I just don't like doing it [hard maths].

Table 4.9 Continued

Student (Grade)	How do you know you (or someone else) is good at maths?	Do you enjoy maths? What do you/don't you enjoy?	I like doing hard maths: never, occasionally, sometimes, mostly, always.
Amy (Gr5)	They're super good at times tables, or they're super good at fractions. They kind of like just do it really easily. [Fred] can kind of work everything out in his head, and is speedy.	[Not specified]	Between mostly and always. I like [number puzzles] that make you think, and I like that it makes you feel better when you finally work it out... but I really like the easier ones.

4.3.2 Classroom observations

From information collected and the interviews conducted there was one boy in Grade 1 (Alex) and one boy in Grade 5 (Fred) who seemed to clearly meet the mathematical ability and self-limiting mindset criteria for selection. To support the interview data, I also observed both these students in a regular mathematics lesson, as this provided an impression of their work within their normal classroom environment. My role was as a participant observer (Stake, 1995), which enabled me to interact with the students, asking questions, clarifying their responses and delving deeper into their mathematical thinking and reasoning. These observations confirmed the selection of both Grade 1 student, Alex, and Grade 5 student, Fred, as mathematically gifted but with definite self-limiting mindset tendencies.

There were three Grade 3 students who met the criteria, but no one student stood out. I therefore observed mathematics lessons in both Grade 3 classes to see the students working within their regular learning environments, again as a participant observer. I was observing the way they approached mathematics tasks and was also looking for self-limiting mindset behaviours to help identify the most appropriate Grade 3 student for selection. From these observations, there emerged one boy, Jackson, who was very highly capable, but appeared disengaged and produced minimal work in class, and one girl, Sammy, who was very capable, but whose learning appeared to be seriously at-risk due to obvious self-limiting mindset tendencies. Sammy was ultimately chosen because of her teacher's concerns about her self-limiting behaviours, and also because having a girl added to the gender mix of participants added to the diversity of the group.

Table 4.10

A sample of teacher responses to semi-structured interview questions

Student (Grade)	What is it that makes you think [student] is highly capable mathematically?	Do you think [he/she] has a more of a fixed or growth mindset? What gives you this impression?
Alex (Gr1)	Identified last year (Prep) as having a strong maths background [with the MAI]. He's always thinking about lots of stuff.	He does have quite a fixed mindset about if it's a big number that means I'm clever and that means I'm doing something really important and challenging.
Frank (Gr1)	He's strong at all academics. He almost seems to absorb information by osmosis, whether you're teaching [him] or not [he] just seems to get stuff.	Not sure. [Frank] gets very emotional about things. He will cry openly in the classroom, he'll sit there weeping and wailing.
Jackson (Gr3)	There's lots going on in his little brain. There's stuff going on in there and it's amazing, like fireworks.	Not sure. [Jackson's] a funny one, his spoken language, I guess, lacks a little bit. He struggles to explain his understanding and thinking.
Sammy (Gr3)	She's a perfectionist, and she's good at everything.	I have talked about, you know, fixed and growth mindsets, and [Sammy] knows it, but she's just stuck in her ways, and she just shuts down. It's like she's too scared to push herself, ever. And when things go wrong she freaks out. You can see it on her face, and then she gets all defensive and can get a bit cheeky.
Fred (Gr5)	Very capable at problem solving, skills in using algorithms are a lot higher than the other students in the class, and tends to complete maths tasks quickly. Always seeks out the extension, or what can I do next?	He's got a fixed mindset at the moment. He always likes to get the correct answer, and he likes to know the explicit details of the task. So, if I give him an open-ended maths problem, with limited direction at the beginning, he tends to ask lots of questions. He tends to ask for reassurance each step of the way, so he tends to say, "Is this what you want?", "have I done enough?", "Is this the right way?"
Murray (Gr5)	[same as for Fred]	More growth mindset. He's strong at maths, but he's not the strongest student I have, but he works very hard. He has natural ability, but he's prepared to work very hard to maintain that.

4.4 Outcome of the Selection Process

The semi-structured interview questions (both student and teacher), together with the classroom observations, helped finalise the identification of the three students in Grade 5, Grade 3 and Grade 1 who most aptly fit the category of “*students who are mathematically gifted but who demonstrate self-limiting mindset tendencies.*” [NB. Chapter 6 describes each student’s identification in full detail].

Grade 5: Fred

Fred was very highly capable mathematically, with the highest nomination on the Likert-type scales from both teachers and parents. He demonstrated an ability to reason intuitively with an unfamiliar type of mathematics problem, and could learn and generalise new information readily. However, he also evidenced some typical self-limiting mindset qualities, for example:

- He chose an ‘easy’ number to challenge himself with in the *Adding Corners* task. All other Grade 5 students interviewed chose fractions and/or decimal numbers, were prepared to really challenge themselves with a “creative/interesting” solution by spending time on the task, and seemed to enjoy the challenge they set themselves (see Table 4.7).
- He often let me know that he has always been able to do smart things, even when he was younger (see Table 4.7).
- He seemed to have a fear of getting things wrong, and became distressed if he did not know exactly what he was expected to do (from teacher’s comments, Table 4.10).

Grade 3: Sammy

Sammy had demonstrated significant mathematics learning capabilities, especially evident in her mathematics growth over two years as shown in her *MAI* profiles, but no obvious self-limiting mindset tendencies in the interviews. However, in the classroom observation it became obvious that she indeed struggled with self-limiting mindset tendencies that significantly affected both her perceptions of herself as a learner of mathematics, and her mathematics involvement in class. For example:

- She seemed to believe that she needed to give quick answers to questions. If she didn’t know an answer straight away she would regularly say “I don’t know. I’m no good at maths”.
- She avoided more difficult tasks in case she couldn’t do them.

- She became easily and highly distressed when she thought she was wrong, which resulted in stifled learning opportunities for the rest of the lesson.

Sammy therefore clearly met the criteria and was selected for the case study from the Grade 3 cohort.

Grade 1: Alex.

Alex was very highly capable mathematically: he was able to mentally wrestle with some astoundingly large numbers for a beginning Grade 1 student; he was able to reason proportionally by working with two pieces of information simultaneously; he was able to learn quickly and generalise new knowledge. But Alex also presented with some very typical self-limiting mindset qualities. This was evident as:

- It was very important to him that he was smart, and he was keen to show me how clever he was;
- It was very important that he solved problems quickly and with as little effort as possible; errors and crossings out were unacceptable to him (see *Adding Corners* task, Table 4.5);
- His choice of ‘challenging task’ was one that “only one or two other people in the school could do”, not one that was challenging for him personally (*Adding Corners* task, see Table 4.5);
- He showed evidence of being able to reason proportionally, but was also looking for a ‘short cut’ or formula to solve the problem (possibly needing to solve the problem quickly and without effort), which resulted in calculation errors (*Lollies task*, see Table 4.5).

Within the final selection of students there was a cross-section of ages – a seven-year-old, an eight-year-old and a ten-year-old; both genders were represented – two males and one female; and different family background influences were evident – one student from a single parent family, one who is one of three siblings born within three years of each other, and one from a family with a child who is severely disabled. I hoped these different representations would further add to the richness of the data to be collected for the case study.

4.5 Chapter Conclusion

This chapter has outlined the process of selecting three suitable students for the case study, *What impact does targeted teacher professional learning about classroom support for*

mathematically gifted students with self-limiting mindsets, have on the mindsets and mathematics learning of these students? Each of the students has been identified as 1) being mathematically highly capable, and 2) exhibiting self-limiting mindset tendencies in terms of viewing themselves as learners of mathematics. As a result of this selection, the classroom teachers of these students were also each invited to participate in the study – to receive targeted teacher professional learning about mathematical giftedness, optimal mathematics learning in heterogeneous classrooms, and the importance of developing and maintaining positive learner mindsets. The following chapter, Chapter 5, details the targeted teacher professional learning the classroom teachers undertook prior to implementing their new understandings in the classroom over a three-month period. Chapter 6 will present the detailed ‘narrative analyses’ (Polkinghorne, 1995) of the three selected students – Fred, Sammy and Alex – describing their dispositions as mathematics learners before and after their teachers’ professional learning. Chapter 7 will triangulate these stories to highlight possible causal factors of any notable changes, with Chapter 8 concluding the study with recommendations for effective professional learning for teachers of mathematically gifted students.

Chapter 5 – Targeted Teacher Professional Learning

Supporting Giftedness, Mathematics Learning, and Positive Learner Mindsets in the Classroom

5.1 Chapter Overview

The previous chapter detailed the selection process for the case study participants – Phase 1 of the data collection process (see Figure 3.3) – with Fred (Grade 5), Sammy (Grade 3) and Alex (Grade 1) being selected as three mathematically gifted students with self-limiting mindset tendencies. This chapter outlines the details of, and rationale behind, the targeted teacher professional learning. The focus was two-fold:

1. On understanding gifted characteristics and planning for inclusivity for mathematically gifted students in the mathematics classroom; and
2. On exploring how to develop and maintain positive, self-actualising learner mindsets.

The professional learning was carried out concurrently with Phase 2 of the data collection process (see Figure 3.3), which included classroom observations of the three selected students and teacher semi-structured interviews and conversations. Details of the data collected in Phase 2 will be embedded in Chapter 6 – the storied analyses of each student participant – as well as the changes the teachers made in their mathematics instructional approaches post-professional learning.

5.2 Background to the Professional Learning

As detailed in Chapter 4, the selected students for the case study met two criteria – they were mathematically highly capable, and they exhibited self-limiting mindset tendencies when engaged in mathematics learning. This was necessary to answer the research question, *What impact does targeted teacher professional learning about classroom support for mathematically gifted students with self-limiting mindsets, have on the mindsets and mathematics learning of these students?* The planned professional learning, then, required two embedded targets – 1) to explore how to continue to extend and deepen the mathematics learning of mathematically ‘highly capable’ students, and 2) to understand how to promote a change in students from a self-limiting mindset towards a more positive, self-actualising mindset when viewing themselves as learners of mathematics.

The format of the professional learning program was derived from literature findings on effective professional learning programs (see Section 2.6). It started with identifying the primary goal, or desired outcomes, of the professional learning (Cordingley, 2015; Guskey, 2014), which was to increase teacher awareness of the characteristics and needs of mathematically gifted students, especially those with self-limiting mindsets. The most appropriate format to achieve this desired outcome was a specialist coach/mentor approach, with teachers encouraged to follow up with self-directed professional reading (Cordingley, 2015). The approach was highly adaptive rather than specified (Koellner & Jacobs, 2015) due to the individual nature of mathematics teaching and learning for mathematical gifted students. A focus on a mentoring role, to tailor learning experiences relevant to the teachers' own classroom contexts (Cordingley, 2015; Guskey, 2014), was a major part of the professional learning. There was also a whole-school component to ascertain teachers' beliefs about giftedness (Lassig, 2009), to maximise collegiality, and introduce a common language and shared understanding about mathematical giftedness (Cordingley, 2015).

The content of the targeted teacher professional learning was derived from a range of sources (see Chapter 2):

- Literature findings on common characteristics of giftedness and gifted learners in general, for example, asynchronous development (Columbus Group, 1991), overexcitabilities and emotional intensity (Dabrowski, 1972; Piechowski, 1997), neural efficiency (Hoppe & Stojanovic, 2009) and fluid analogising (Geake, 2009b), exceptional reasoning abilities, evident from a young age (Csikszentmihaly & Robinson, 2014), perfectionist tendencies (Silverman, 2013), underachievement (Siegle, 2013), a mature sense of humour (Silverman, 2013), a heightened sense of responsibility with a highly developed sense of justice (Gross, Macleod, Drummond & Merrick, 2001);
- Literature on specific characteristics of mathematically gifted students (e.g., Gardner, 1999; Krutetskii, 1976; Munro, 2012; Sheffield, 1999);
- Literature findings on strategies for effective mathematics teaching and learning in general, for example, approaching mathematics as a creative venture (Liljedahl & Sriraman, 2006) and a problem solving process (Polya, 1957) that can be imaginative and innovative (Sheffield, 2009); teaching mathematics as a mathematical process of

thinking and reasoning (Kilpatrick et al., 2001), using meaningful tasks set in intriguing contexts that draw the student into the mathematics (Stillman et al., 2009);

- Literature on specific strategies for teaching mathematically gifted students through classroom integration, rather than segregation (VanTassel-Baska, 2009), using task modification (Sullivan et al., 2013) mathematical investigations (Diezmann et al., 2003), and including notions from the Slow Education movement (Holt, 2012) as a contrast to the more prevalent acceleration approaches used for gifted students (Hannah et al., 2011; Munro, 2012);
- Literature findings on influences of a positive mindset outlook on learning: self-concept, attitudes and beliefs (Bernard, 2006) that can enhance an individual's potential (Sheffield, 2008), predict learning trajectories (Tough, 2012) and impact student success (Goleman, 2006); dispositions such as a growth mindset (Dweck, 2006), grit (Duckworth et al., 2007), learned optimism (Seligman et al., 1995; Williams, 2014), resilience and social-emotional competence (Bernard, 1995, 2006; Bland et al., 1994; Williams, 2014), perseverance (Conroy 1998; Thom and Pirie, 2002; Williams, 2014) and drive (Pink, 2009);
- My own observations of mathematically highly capable students working in regular mathematics classrooms; and
- Conversations with classroom teachers about their perceptions of mathematically highly capable students' learning needs – both academic and affective – about pedagogies they have found that support mathematics learning.

Chamberlin and Chamberlin (2010) summarised the competencies of teachers of gifted students as: a knowledge of gifted students' needs (including affective), an ability to promote high-level thinking and creativity, an ability to develop a differentiated curriculum, and how to facilitate learner-centred instruction (including a safe and flexible classroom to encourage students' independent research). Leikin (2011) and Holton et al. (2009) also mention the importance of a teacher's relationship with the student, including a need to be aware of gifted students' social processes.

The professional learning focused on mathematically gifted students, but subsequent teaching was to be implemented in heterogeneous classroom environments. The professional learning, therefore, needed to be practically beneficial for teachers working with a range of students, not just a select few. The focus of the professional learning was

not on providing specific mathematics tasks for gifted students, but on exploring how *any* carefully selected mathematics task could be used most effectively to enable learning for a cross-section of students, *including* mathematically gifted students.

The professional learning relied heavily on research-informed approaches for effective teaching and learning of mathematics (see Section 2.3). It incorporated generating a classroom environment whereby:

1. The organisational style of lessons, and the teaching approach, would engage and focus students' mathematical thinking through careful questioning and meaningful assessment (see Clarke et al., 2002; Clarke et al., 2002; Clements & Sarama, 2009; Kilpatrick et al., 2001; McDonough, Clarke & Clarke, 2002; Siemon et al., 2001; Stillman et al., 2009; Sullivan, 2011);
2. Mathematics learning would rely on student-teacher *and* student-student interactions both during, and as a reflection on, the mathematics lesson (Clarke & Roche, 2010; McDonough et al., 2002);
3. There would be high but realistic expectations of all students, with a belief that all students can learn more (Gervasoni, 2002; Krulik & Rudnick, 1980; Siemon et al., 2001), including gifted students (Leikin, 2007; Munro, 2012; Niederer & Irwin, 2001); and
4. There would be an underlying belief that mathematics learning can, and should be, enjoyable, meaningful and creative (McDonough & Clarke, 2003; Sheffield, 2006; Sriraman, 2004; Stacey, 2010; Stillman et al., 2009).

The professional learning coaching/mentoring component also included the type of teacher support required when engaging mathematically gifted students with challenging tasks, including ways to differentiate tasks to maintain challenge in order to promote and value effort and perseverance as important and necessary aspects of learning (Sullivan et al, 2013; Williams, 2003a, 2014; Zevenbergen, 2003).

The focus of student learning dispositions, was addressed in the professional learning through discussions primarily about fixed and growth mindsets (Dweck, 2006). The school had previously completed extensive professional learning on Dweck's work, therefore, while the professional learning focused on positive learning mindsets in general (optimism, resilience, grit, perseverance, et cetera – see Section 2.6), the language used, when talking

with the teachers and students, was described as ‘fixed mindset’ (self-limiting) and ‘growth mindset’ (positive/self-actualising).

The professional learning was centred on a whole-class approach to teaching and learning mathematics, with a specific focus on the teaching and learning of gifted students within the class.

5.3 Implementation of the Teacher Professional Learning

The teacher professional learning was implemented in two formats. Firstly, there was a whole school two-hour professional learning seminar on understanding characteristics of mathematically ‘highly capable’ students and how to support their learning through differentiation. This seminar, and topic, was conducted at the request of the school principal, and was subsequently planned to be used as a vehicle for introducing new ideas, and generating discussions about mathematically gifted students, rather than as specified content as an end in and of itself. This provided an opportunity for collective participation to maximise collegiality and introduce a common language and shared understanding about mathematical giftedness (Cordingley, 2015). The session included how to use rich, open tasks, and how to model exploring these mathematics tasks further to promote self-selected learner differentiation (Betts, 2004) (see Section 5.4.2 for details). It also revised the need for ensuring positive learner mindsets, but as the school had already received extensive professional learning about fixed- and growth-mindsets, this did not need to be a major focus.

Secondly, the main professional learning for the study was conducted through individual meetings with myself and each of the teachers of the selected case study students in a coaching/mentoring role (Cordingley, 2015). Using interpretation of student interviews and classroom observations, coupled with evidence of best practice from the literature, I provided specific suggestions, and collaborative planning with the teachers, that was tailored and relevant to their own classroom context (Cordingley, 2015; Guskey, 2014). This included two-way discussions about specific details of the perceived learning needs of the students, as well as information about, and the rationale behind, the suggested classroom expectations and lesson structure idea (outlined in Section 5.4.1 and 5.4.2 below). I also provided a summary document outlining suggestions for supporting the learning of students who are mathematically gifted or highly capable (see Appendix 4). There were four teachers involved across the three Grades (with two teachers team-teaching

the Grade 1 class), each with different backgrounds, and years of teaching experience. They were encouraged to incorporate their new knowledge of the characteristics and needs of mathematically highly capable students and the understanding of mathematics as a creative venture, with their knowledge of the impact of mindsets on student learning. They were encouraged to generate a classroom culture, and lesson structure, that would enhance the support for their mathematically highly capable students. Each of the teachers brought their own individuality to the suggestions, their own prior experiences, and their own personalities which naturally led to different approaches. Approximately three months after this professional learning, Phases 3 and 4 of the data collection took place (see Figure 3.3), with follow-up classroom observations and student and teacher interviews.

5.4 The Professional Learning Components Detailed

Meeting the learning needs of mathematically gifted students generally relies on *task differentiation*, through extension of the mathematics within the curriculum (see Kanevsky, 2011; Kronburg & Plunkett, 2008), or on *acceleration*, or ‘curriculum compacting’, of the mathematics program (see Hannah et al., 2011; Munro, 2012), or on *segregation*, with specialist gifted classes (Gross, 2004; Silverman, 2013; Tannenbaum, 1983). The professional learning focus for this study focuses on task differentiation, within a regular classroom, rather than acceleration or segregation. This approach is based on:

1. the philosophy of “deeper learning” from the *Slow Education Movement* (Holt, 2002) which values time on task as a crucial component of learning;
2. on Betts’ (2004) description of levels of differentiation, especially the development of autonomous learners;
3. on anecdotal stories of successful mathematicians and innovators of the past (see Section 2.3.3); and
4. on taking heed of the inherent concerns about segregation and separatism in gifted education (Krutetskii, 1976; VanTassel-Baska, 2009).

Differentiation and ‘slow education’, rather than acceleration, allows students time for deep learning experiences with real outcomes; time for curiosity, passion and reflection to be at the heart of learning experiences; time for dynamic, collaborative, democratic and supportive relationships to develop within learning experiences (the *Slow Education Movement* philosophy, Holt, 2002).

As well as providing challenging tasks, that can be differentiated as necessary, another part of the professional learning was to introduce the element of mathematical creativity: a way of ensuring that mathematically gifted students, especially, had an opportunity to become ‘producers of knowledge’ as well as ‘consumers of knowledge’ (Betts, 2004; Sheffield, 2009), and have permission and time to explore their own mathematical curiosities (Krutetskii, 1976). Research shows that creativity is a critical component of child development (Peedom & Bare, 2014), and that opportunities to practice creativity need to be intentionally created, nurtured, fostered and developed (Robinson, 2006). Indeed, it has been suggested that creativity may be an even greater predictor of success later in life than intelligence (Jauk, Benedek, Dunst & Neubauer, 2013), and creativity needs active encouragement if it is to flourish (Peedom & Bare, 2014). As the professional learning was centered on a whole-class approach to teaching and learning mathematics, creative opportunities within mathematics lessons would not be limited to just gifted students, but would be accessible for all students in the class. Indeed, mathematical creativity is most likely not limited to the mathematically gifted, so planning for creativity would be of benefit for all students (Barbeau & Taylor, 2009; Craft, 2005; Reiss & Törner, 2007; Silver, 1997; Sririaman, 2004; Tanenbaum, 1986).

Consequently, the targeted professional learning included suggested *classroom expectations* for mathematics lessons for optimising the learning environment. The focus was on mathematically gifted students, but the understanding was that the practices would benefit all students. This included information about characteristics of giftedness, and suggestions for redressing self-limiting mindset beliefs and developing and maintaining positive learner mindsets. A framework for an inclusive mathematics *lesson structure* that would enable all students to continue to develop mathematical knowledge and skills, and allow for mathematical differentiation, exploration and creativity, was also included in the teachers’ professional learning. The goal was to help teachers develop a classroom culture that values effort, perseverance and grit in mathematics learning, that celebrates students taking on a challenge and thinking creatively, and that helps students to realise that a positive mindset is a key to successful ongoing mathematics learning.

5.4.1 Mathematics Classroom Expectations

Classroom Expectation 1: Mathematics learning requires hard thinking and sustained effort

The first expectation for the mathematics classroom environment was for the teacher to develop a classroom culture where students understand that *mathematics learning requires hard thinking and sustained effort*. This was to optimise ongoing learning for all students, but especially for mathematically gifted students for whom effort and sustained thinking may have been an unfamiliar experience in mathematics lessons. There were three parts to this expectation that required specific attention: sustained effort, task completion time, and questioning.

Sustained effort

Incorporating challenging problem-solving tasks, which require sustained effort, into mathematics classrooms has been shown to provide optimal conditions for learning, where prior knowledge is activated and new knowledge is constructed (Sullivan et al., 2013). It is *challenge* that activates cognition, not problems that can be solved or answered with minimal time, thought or reasoning (Sheffield, 2006; Stillman, et al., 2009). This is especially important for students who may find typical grade-level tasks relatively easy, and who may have developed an understanding that being ‘good at maths’ means work can be completed quickly and correctly with minimal effort. It was therefore important for the teacher to establish a classroom understanding that hard thinking and sustained effort is expected in mathematics lessons for all students, even gifted students. Hard thinking may include making decisions (that can be justified) on how to tackle a problem, taking risks with strategies or approaches (which may or may not work) in an attempt to solve a problem, and/or sustained deliberation and discussions with other students about the structure of a problem (Clarke, Cheeseman, Roche, & van der Schans, 2014). Teaching strategies for building student persistence on challenging asks: Insights emerging from two approaches to teacher professional learning. *Mathematics Teacher Education and Development*, 16(2), 46-70. Clarke et al., 2014). Sustained effort may include struggles and mistakes, which need to be viewed as a normal part of the learning process, indeed as an indication that new learning is being enabled (Dweck, 2006).

Task completion requires time

Mathematics tasks that require hard thinking and sustained effort will take time. Therefore, it was also important for the teacher to establish a classroom understanding that *task completion that activates meaningful learning will require time*; that if a task can be completed quickly and easily then minimal learning will have taken place. This understanding is necessary to counter the common belief that those who are ‘good at maths’ work quickly and are ‘fast finishers’ (Silverman, 2013).

Questioning


As a participant observer in several classrooms during Phase 1 of data collection (the selection process), I asked questions of individual students during mathematics lessons. This was to promote hard thinking in order to delve deeper into a student’s approaches to novel tasks and their mathematical reasoning. However, I discovered that, where, from my perspective, I was thinking, ‘Here’s a question to make you think about ...’, or ‘Have you thought about this ...?’, the student’s perspective often seemed to be, ‘The teacher [researcher] has asked a question and she’s waiting for my answer!’ This appeared quite stressful for some students as they were questions they *could not* answer quickly or easily. I had to explicitly explain to them that the questions I was asking were intended to make them think, I was neither expecting, nor wanting, quick un-reasoned responses. Another important part of the professional learning, then, was to encourage the teachers to ask questions that require well thought out mathematical explanations, of which the answer is the by-product, not simply ask questions that require quick, or single answers. In doing this, they would need to establish a classroom culture whereby students understood that *when the teacher asks a question, she is posing a problem she wants me to think about, she is not testing me on what I do or do not know*.

Classroom expectation 2: Mathematics is a creative process

Exploring further

As an observer in the Grade 5 classroom during the selection phase of the study, I observed a group of highly capable students working on a page of problems they had been given to complete (see Figure 5.1). They found the problems challenging, and were certainly engaged in the tasks, but were more focused on completing all the questions than on savouring the mathematics involved, or taking time to record or confirm any solutions other than the final answer. Indeed, providing students with multiple problems to solve in one

**TIME PROBLEM
SOLVING QUESTIONS**



1. A train is moving at the rate of 1 km in 1 minute 20 seconds. If the train continues at this rate, how far will it travel in one hour?
2. The last Friday of a particular month is on the 25th day of the month. What day of the week is the first day of the month?
3. Suppose five days before the day after tomorrow was Wednesday. What day of the week was yesterday?
4. A work crew of 3 people requires 3 weeks and 2 days to do a certain job. How long would it take a work crew of 4 people to do the same job if each person of both crews works at the same rate as each of the others?
[Note: each week contains six work days.]
5. Suppose today is Tuesday. What day of the week will it be 100 days from now?
6. A car trip took 2½ hours. It travelled at an average speed of 80 km/h. How far did it travel in this time?
7. What are the times when the hour hand and the minute hand are at right angles?

Figure 5.1 Worksheet given to a group of highly capable Grade 5 students.

lesson seemed to actively discourage this, and the students were quite content to defer to the teacher's judgement as to the correctness, or otherwise, of their answers. According to Mason (1984), "A book [or worksheet] full of questions ... run[s] entirely opposite to a classroom atmosphere of enquiry, of asking questions, since much of the force of the question lies in the asking" (p. 220). Giving just one problem to solve, but requiring students to solve it in two or three different ways, or to explore it from different angles, or to explore it further, requires more mathematical reasoning than solving five or six different problems in a similar way (Polya, 1957). Encouraging the teachers to re-think their approach to providing challenge for their highly capable students was another part of the professional learning focus.

The second expectation for a mathematics classroom environment was to develop an understanding of mathematics as a creative process. Mathematics is much more than performing calculations to find answers to questions on a worksheet. Consequently, exploring further needs to become an integral part of learning mathematics. This would enable mathematical exploration for all students, but may especially unleash the minds of mathematically gifted students who tend to view the world through a mathematical lens (Krutetskii, 1976). Not only giving permission to, but also developing an expectation of,

exploring mathematical ideas further, could lead to immense satisfaction for, especially, mathematically highly gifted students (Leikin, 2008; Sheffield, 2006).

Valuing creativity

Developing mathematical creativity is becoming an essential element of mathematics education for all students, moving them from being simply doers and consumers of mathematical proficiency, to being problem solvers, problem posers and mathematics creators (Sheffield, 2009). However, Robinson (2006) warns that global education systems based on standardisation may in fact threaten student creativity, with schooling often undermining creativity with an overemphasis on standards and conformity, rather than nurturing creativity by embracing individuality and imagination (see also Robinson & Aronica, 2015). To this end, classroom teachers were encouraged to explicitly focus on tasks that would be conducive to creativity, by choosing tasks that encouraged different approaches to problem solving, that would engender discussions about solutions and different strategies used, and would inspire students (especially mathematically gifted students, but not exclusively so) to pose their own questions, to extend and deepen the original problem. The aim of this was to encourage students “to recognise that instead of finding a solution to a mathematical problem being the end of the problem, it is often just the beginning of the most interesting, and rewarding, mathematics” (Sheffield, 2009, p. 88).

Classroom expectation 3: Challenge self-limiting mindset statements

The third requirement for classroom expectations, which would run concurrently with the first two requirements of valuing effort, and creativity, was for the teacher to develop a classroom environment that fosters a positive learner mindset and actively challenge self-limiting mindset statements and behaviours.

Developing a positive learner mindset involves, in part, explicitly teaching students about the learning process: that learning involves taking risks; it often involves making mistakes which can subsequently be viewed as ‘learning opportunities’; it involves trying out new ideas; it requires sustained effort; and it takes time (Dweck, 2010b). Each of these approaches need to be valued in the classroom if they are to be recognised as positive aspects of learning (Brookhart, 2010). Another part of developing a positive learner mindset involves being aware of, and, if necessary, expressly challenging self-limiting mindset comments and behaviours. Based on behaviours observed in both the interviews

and classroom lessons, and on literature about mindset behaviours, a chart (see Figure 5.2) was developed for teachers to adapt and use in their classrooms.

Types of self-limiting mindset statements	Re-training for positive mindset self-talk
<ul style="list-style-type: none"> • I'm no good at maths. <i>(if the answer is not obvious, or takes a bit of thinking to work out)</i> 	<ul style="list-style-type: none"> • Hang on ... I'm going to need to think about this a bit more.
<ul style="list-style-type: none"> • This is too challenging/hard for me. <i>(if the task requires thinking, time and effort to complete)</i> 	<ul style="list-style-type: none"> • Remember learning takes effort.
<ul style="list-style-type: none"> • I'm finished! <i>(possibly indicating a need to be first finished)</i> 	<ul style="list-style-type: none"> • Learning is not a race. There is always something more to learn, what can I explore now?
<ul style="list-style-type: none"> • This is easy! / I know how to do this. 	<ul style="list-style-type: none"> • This is easy for me; how can I challenge myself further?
<ul style="list-style-type: none"> • This is taking too long. 	<ul style="list-style-type: none"> • This is a good challenge for me. I'm needing to think long and hard about this problem. I wonder who I can discuss my thoughts with.
<ul style="list-style-type: none"> • I'm making too many mistakes. 	<ul style="list-style-type: none"> • How can I learn from these mistakes? Where have I gone wrong? Why didn't this work? (Mistakes are an integral part of success. The most successfully innovative people in the world are often those who have 'failed' the most.)

Figure 5.2 Chart for challenging self-limiting mindset statements and behaviours

Summary of classroom expectations

Table 5.1 shows a summary of the classroom expectations that was developed as a basis for the teacher professional learning. Together these classroom expectations comprised the first part of the professional learning developed for this study.

Table 5.1

Professional Learning Focus: classroom expectations (whole class)

Classroom Expectation 1: Mathematics learning requires hard thinking and sustained effort

- Establish a classroom understanding that learning requires hard thinking and sustained effort. Hard thinking is what is expected in a mathematics class. Hard thinking is a good thing, not a sign that you are not good at maths. Constantly ask questions like “How are you challenging yourself?”, “What’s next?”, “How can you be creative with this?”
 - Establish a classroom understanding that task completion requires time – if the task is completed quickly and easily then minimal learning will have taken place. It’s better to solve one problem to absolute certainty than solve lots of problems that may or may not be correct.
 - Establish that when I (the teacher) ask a question, I am posing a problem I want you to think about. I don’t want a quick answer (I am not testing you). What I require is a well thought out explanation, the answer is the by-product of this.
-

Classroom expectation 2: Mathematics is a creative process

- Establish an understanding that mathematics is a creative process, there is always more to explore. Model ways that students can explore further. Teach them how to think deeper (if necessary); develop the skill of learning how to learn.
 - Generate a classroom environment that values creativity. Encourage students to run with their own ideas. Thinking beyond the set task, or ‘outside the square’, is something that is valued, specific permission to do this is normally not required.
-

Classroom expectation 3: Challenge self-limiting mindset statements

- Establish a protocol for fostering growth mindsets, and for challenging fixed mindset comments and behaviours, e.g., a classroom chart
-

5.4.2 Mathematics Lesson Structure

The second part of the teacher professional learning was the suggestion of a framework for a conducive mathematics lesson structure. The mathematics lesson structure is an important element of effective mathematics teaching and learning (McDonough & Clarke, 2003; Sullivan et al., 2013). To enable the mathematics learning experiences mentioned above, it was important for teachers to use a lesson structure that supported a problem solving and reasoning approach to teaching and learning mathematics. It had to enable students to continue to develop mathematical knowledge and skills, and to encourage, and allow for, mathematical exploration and creativity.

From the literature on effective mathematics teaching, the learning needs of mathematically gifted students, and the idea of fostering creativity, a generic lesson structure was developed for the teacher professional learning for the study (see Figure 5.3). This was based on

- | |
|--------------------------------------|
| 1. Solve the problem |
| 2. Explain and justify your solution |
| 3. Explore the mathematics further |

Figure 5.3 Teacher professional learning: proposed mathematics lesson structure

Sheffield's (2003) model for solving and posing problems, which includes steps for "investigating the problem; evaluating the findings; communicating the results; and creating new questions to explore" (Singer et al., 2016, p. 20). This helps students deepen their mathematical understanding through justifying or proving the success of their reasoning and strategies, by solving problems in multiple ways, and/or by posing and solving further, related problems (Singer et al., 2016).

1. *Solve the problem.* This was basically the teacher's normal classroom practice (taking into consideration the use of tasks that would encourage mathematical reasoning and discussion as outlined above). The mathematics task may be an investigation, a game, an open-ended question, a computer task, a worksheet, et cetera, which may or may not need to be differentiated for mathematical abilities within the classroom. The expectation was that all students in the class would undertake the set task.
2. *Explain the solution.* The main objective here was to provide learners with an opportunity to develop the ability to explain and communicate mathematical ideas clearly in order to be able to work with others, to share solutions, and to have their contributions validated and valued. The students involved in this study were in a school that already valued 'explain your thinking' verbally in mathematics lessons, but during the classroom observations in the selection phase of the study, it was noticed that many of the highly capable students had great difficulty reporting their solutions in written format. When asked to do this they would write down their thought processes, which were mostly very condensed and disjointed, as either simply lists of numbers and calculations, or descriptions like, "I just multiplied this by this," without any reference to why they had chosen those particular numbers, and/or why they had multiplied them.

For example, in the Grade 5 lesson with the sheet of time problems (see Figure 5.1), the students were all engaged in solving the problems, working together, arguing, justifying ideas, convincing each other of the validity of their solutions and so forth. However, the final answer for each question was more-or-less a mutual agreement, which was then written down as a singular response, before moving on to the next question. When I intervened, and asked how they knew an answer was correct, they had to go through the whole justification and sharing of strategies again, which resulted in some confusion and even uncertainties of the final answer at times. Suggesting they wrote the solution down in such a way that others could follow, though, was met with a very half-hearted response. They were quite happy to defer to the teacher to find out whether their final answers were correct or not.

The ability to communicate findings and to provide not only verbal but also written explanations is an important outcome of mathematics education (Brown, 2008; Knuth & Peressini, 2001; Sheffield, 2003), but writing a mathematical solution and explanation is a separate mathematical skill that needs its own instruction if it is to be learned and developed appropriately (Burns, 2004; Pugalee, 2004; Urquhart, 2009). It is a skill that involves organising thinking through the use of diagrams, models, tables, et cetera, as well as mathematical symbols and words, in order to support mathematical reasoning and problem solutions. It is a skill that may culminate in being able to deduce mathematical proofs in higher mathematics studies (Pugalee, 2004), and is, therefore, a vital skill to develop, especially for students who have the potential to become innovators and creators within mathematical/scientific realms (Krantz, 2007; Sheffield, 2012). Breaking down a solution and recording it in a logical way also justifies each step of students' thinking. It shows how and why they know the solution to be correct, and it may enable others to reproduce and/or generalise from the solution (Brown, 2008). This process of explaining and justifying solutions can be particularly challenging for mathematically gifted students because their thought processes are often intuitive and naturally very efficient, often combining two or more processes into one thought (Krutetskii, 1976), what Geake (2008) refers to as cognitive flexibility or 'fluid analogising'. Breaking these processes down into sequential logical steps is initially very difficult for these students (it may be likened to a native English speaker trying to explain the meaning of words like 'come' or 'when'). It may require substantial effort that gifted students may initially be quite resistant to (especially if

other skills are learnt relatively easily) and, therefore, will require intentional teacher support. The teachers were encouraged to use the type of questions suggested by Sheffield (2006) – ‘who, what, when, where, why and how’ questions – to help students formulate informative, written records.

Sometimes the brightest kid needs small group instruction for a skill the rest of the class already gets ... someone to sit with them and literally go step by step, asking: ‘Wait, what did you do there?’, ‘Hold on. Why did you do that?’, ‘What do you mean by...?’ (Byrd, 2016, para 10)

The focus of every problem-solving task or lesson may not be about writing down a comprehensive solution (often a written record of thought processes is sufficient (Brown, 2008)), but some lessons need a further written ‘proof’ to be a focus in order to develop this important skill.

The expectation was that all students in the class would learn how to explain their strategies and solutions, to communicate their mathematical thinking, both verbally, and as a written record. Comparing and trying to make sense of other people’s explanations of problem solutions (e.g., from an internet search of famous problems like ‘The Monte Hall Problem’ (Selvin, 1975), or ‘Cheryl’s birthday’ (Wong, 2015)) and writing their own explanation of a solution (Can you explain it better?) was one suggestion for addressing the issue of how to, and a need for, recording mathematical solutions.

3. *Explore the mathematics further* puts the onus of challenge, in part, onto students themselves, to allow for even further meaningful differentiation. This suggestion of students exploring the mathematics of a given task further, to be creative, to set their own challenges is based on Betts’ (2004) third level of differentiation – *Learner-Differentiated Curriculum and Instruction*. This level of learning encourages the development of self-discovery, creativity and autonomy; it permits the discovery and development of passion areas that may be unique to each learner (Betts, 2004), and provides intrinsic value to mathematics learning (Brophy, 2008). It was especially for, but not limited to, mathematically gifted students.

To explore the mathematics further, the expectation is that once the set task is completed (or maybe ascertained as being too simple), the question students are encouraged to ask themselves is, ‘What can I do next?’ or ‘What more can I do?’ Instead of students waiting

to be given more, or more complex, work by the teacher (or being allowed to go on the computer once a task is completed, which is another practice that actively discourages students from pursuing challenging tasks), students were to be encouraged to ask this question of themselves, ‘What else can I explore within this task to be creative, to challenge myself?’ This would need to be scaffolded by the teacher initially, but with the understanding that students would ultimately take on this role for themselves. The suggestion was that a chart, similar to Figure 5.4, could be drawn up and displayed, and added to as new ideas for exploring mathematics further were discovered.

Another reason for exploring the notion of learner-differentiation came about during the selection interviews. I discovered that at times the nominated mathematically highly capable students surprised me with things they did not know, or problems they could not solve easily and quickly. For example, one of the Grade 3 girls, who thoroughly enjoyed the *Adding Corners* task, especially when given the option to choose her own number to go in the centre, saying, “I thought I could be more creative if I chose my own number.” She chose to work holistically, manipulating both the corner numbers and the centre number to come up with an interesting and creative solution, “I made every number like the 11 times table, even the answer is like that, too.” Her solution: $22 + 33 + 44 = 99$ (see Table 4.6).

Explore the mathematics further some examples:

- Can I solve this problem a different way?
 - Can I find another solution (for an open-ended task); how many different solutions are there, and how will I know I’ve found them all?
 - What if I try the same problem but make it more complicated (e.g., larger quantities, fractions, more components)?
 - How can I adapt the rules of this game to improve it?
 - What is the best strategy to use to ensure the greatest chance of winning this game?
 - What other components of this investigation look interesting, are worth exploring? (Permission to use computer search engines for investigations may be part of this).
-

Figure 5.4 Suggestions for exploring the mathematics further.

When asked to come up with a second solution she produced $33 + 33 + 33 = 99$, by manipulating her original numbers, because she thought having all addends the same was also “interesting and creative”. I asked if she thought it would be possible to come up with a solution of three identical numbers for *any number* in the middle of the triangle (I was

actually seeing if she would recognise that she could use fractions, e.g., $25 = 8\frac{1}{3} + 8\frac{1}{3} + 8\frac{1}{3}$). Her response was, “No, it wouldn’t work with all numbers because if you had a 5 at the end it wouldn’t work.” So, I asked about 45 – would that work? She tried this, starting with $30 + 10 + 5$ and manipulating the numbers the same way she had with her previous solution, but without immediate success. At this stage, because of time constraints, I offered her a calculator. Her response surprised me. She said, “You’d have to do divided by... It has to be something divided by something,” but there was no recognition that she would need to divide by three. She chose to go back to manipulating her numbers on paper before realising, simultaneously (with a chuckle), that the solution was $15 + 15 + 15$, and that all she really had to do was divide 45 by three. She checked with the calculator to confirm her answer, but still wondered if there might be remainders because of the five on the end. She asked, “Is division times?”

Sometimes challenge comes from unexpected places. We do not expect gifted students to struggle with seemingly simple concepts, yet this Grade 3 student was yet to fully construct an understanding of the inverse property of multiplication and division, and how this related to repeated addition. As a highly capable student I would not have intentionally chosen a task like this for her (effectively 45 divided by three) as a challenging task, that this was a challenge for her was only brought to the fore through her own exploration.

The rationale behind this proposed mathematics lesson structure – solve the problem, explain and justify the solution, explore the mathematics further – comprised the second part of the targeted professional learning.

5.5 Chapter Conclusion

This chapter has outlined and explained the rationale behind the targeted teacher professional learning, and how it was implemented at both a whole-school, and individual teacher level. This was necessary as the teacher professional learning was an integral part of the study, which documents the case study students’ mathematics learning and learner mindsets before and after their teachers’ targeted learning about mathematically gifted students. The following chapter tells the ‘stories’ of the three students through a narrative analysis, detailing and analyzing their ‘before and after’ approaches to mathematics learning (from Phase 1 & 2 to Phase 3 & 4 of the study), as described by themselves, their teachers, and through observations of their approaches to mathematics tasks, both in the regular classroom, and in task-based mathematics and semi-structured interview. The

stories provide thick descriptions and integrated interpretations (Merriam, 2009; Stake, 1995) of the pre- and post-teacher professional learning observations and interviews to identify any changes in the students' apparent mindsets, and in their approaches to challenging mathematics tasks.

Chapter 6 – Narrative Analyses of Fred, Sammy and Alex

The Stories

Individual case studies have always been crucial...as with the best fiction, it's the particulars of people's lives that unveil the universal truths.

(Kean, 2014, p. 342)

6.1 Chapter Overview

The previous two chapters have outlined the selection process for student participants, and the teacher professional learning rationale and implementation for this study. Fred, Sammy and Alex were identified as mathematically highly capable, but with self-limiting mindset behaviours that potentially put them at risk of not realising their full mathematical capacity – an issue that was hoped to be redressed through enhanced teacher knowledge and understanding about this phenomenon. This chapter details the narrative analyses, or storied accounts (Polkinghorne, 1995), of each of these three students. Each account provides an in-depth description and analytical interpretation of data accumulated both pre- and post-teacher professional learning, analysed through a descriptive and interpretive approach, and ‘retold’ along a before-after continuum (Polkinghorne, 1995). Collected data include interviews with the students and their teachers, both pre- and post-professional learning sessions, written questionnaires from parents/care-givers, and researcher participant observations of each student within the setting of their regular mathematics classroom, both pre- and post-professional learning sessions.

Based on the narrative analytic procedure and criteria of Dollard (1935) and Polkinghorne (1995) (see Section 3.7.1), each story introduces the students as individual children – children situated in their own unique family units, with their own unique personalities and interests. This is followed by a description and analysis of the students’ mathematical abilities (situating them as mathematically gifted), and mindset behaviours (identifying them as displaying self-limiting mindset behaviours), to justify their selection as suitable participants for the study. Their dispositions as mathematical learners are then described and analysed, both pre- and post-teacher professional learning, observing how they approach mathematics tasks, and how mindset behaviours may have evolved and changed. Each student’s story is concluded with a description of how his/her teacher(s) implemented the suggested teaching and learning approaches covered in the professional learning, and

how the teaching and/or thinking within mathematics classes may have changed as a result of the professional learning. These holistic descriptions and analyses provide a basis for developing a clear picture of the case under study (Stake, 1995), to help develop further understanding of the experiences of the students in this particular case. It will also connect these individual student's experiences with previous research findings in the literature, through "puzzl[ing] the many meanings ... and pass[ing] along an experiential, naturalistic account for readers to participate themselves in some similar reflection" (Stake, 1995, p. 44).

Interpretations within the narratives focus on analysis of evidence of students' dispositions or mindsets towards mathematics learning, and their approaches to attempting and completing challenging mathematics tasks in their learning environment. For example,

- evidence of positive mindset behaviours as mathematics learners:
 - a willingness to persevere with difficult tasks, rather than giving up quickly;
 - a willingness to be challenged, rather than avoiding risk-taking behaviours;
 - an openness to learn from mistakes, rather than becoming distressed by them.
- evidence of positive mindset approaches in attempting and completing mathematics tasks:
 - thinking deeper about mathematics tasks (e.g., open tasks and investigations), rather than simply focusing on getting 'correct answers' as quickly as possible,
 - developing mathematical reasoning further by explaining solutions and justifying approaches, rather than relying on the teacher to confirm whether an answer is correct or not;
 - developing mathematical creativity through independent thinking and exploration, rather than simply focusing on finishing the set task and waiting for further direction from the teacher.

There is also an underlying focus on evidence of general characteristics and behaviours of 'giftedness', as described in the literature, that may be beneficial to add to the discourse on implications for teachers working with similar students in their mathematics classroom.

The three individual storied accounts will provide an opportunity for discussion about the impact teacher professional learning, and subsequent classroom teacher practice, may have had on the mindsets and mathematics learning of the three gifted students who exhibited self-limiting mindset tendencies prior to this study. When considered as multiple samples

within a case study, they will provide opportunity for even deeper analysis (in Chapter 7), when these accounts are synthesised.

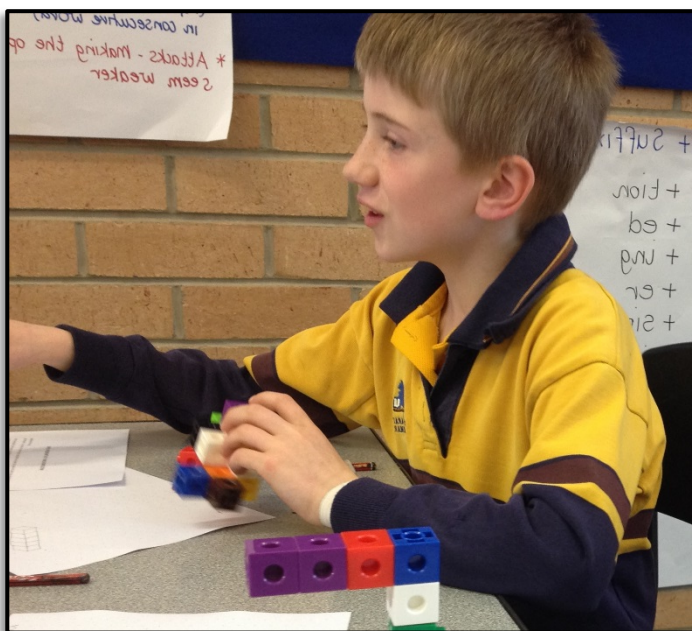
All interviews and classroom observations were audio-recorded, and much of the data presented in these stories consists of direct quotations from the students and the teachers (Riessman, 2008). This, together with the complementarity of multiple sources of data and accounts (Clarke et al. 2006), and the researcher participant observations – which add an objective perspective to participants’ versions of events (Lichtman, 2010) – addresses issues of credibility and trustworthiness of the study, as outlined in Section 3.8.

The stories start with Fred (Grade 5) and his teacher Ms J, then Sammy (Grade 3) and her teacher Ms S, and finally Alex (Grade 1) and his teachers Ms C and Ms K.

NB. As per Figure 3.3 (Chapter 3), Phase 1 of the study (identification of students) was conducted in April-May 2014, Phase 2 (classroom observations, teacher interviews and professional learning) was conducted in June-August 2014, and Phase 3 (follow-up classroom observations and teacher interviews) and Phase 4 (follow-up student interviews) were conducted in October-November 2014.

6.2 Fred – Grade 5

Fred was observed as being a quietly spoken, polite eleven year-old, both one-on-one and in the classroom environment. He is the youngest of three siblings, the oldest of whom has a severe disability; both his parents are medical doctors. As well as being academically capable, Fred is an accomplished tennis player gaining awards in competitive tennis at a State level, and is also very involved in competitive athletics. He plays both the piano and guitar. Fred’s teacher (Ms J) believes Fred is



“Fred”

(photo used with permission)

intrinsically driven in these ventures, rather than being driven by external expectations; she said his parents are very supportive but do not necessarily pressure him. He is absent from

school and/or out of the classroom regularly due to sport and music extra-curricular activities and commitments, and his sporting prowess has also resulted in some serious injuries, including spinal fractures from repetitive serving in tennis, resulting in even more school absences. Ms J's summation was, "[Fred] is very driven and I think he can only live at one pace, and everything he does he's got to do to the very best of his ability."

According to his teacher, and from my own classroom observations and discussions with other school staff members, Fred seems to be well liked by both teachers and fellow students. Earlier in the year his teacher had been concerned that he had, at times, perceived himself as being not very popular. She wondered if he thought some students may be jealous of him because of his high performance in so many areas, and she noted that he had occasionally been caught up in competitiveness with assessment results, which was something she had had to address with him. However, overall, she perceived him as being generally quite sensitive to other students, and a respected class member. [At the end of the Grade 5 year Fred was nominated, and subsequently elected by his peers, to represent them as school captain for the following year, so it would seem his fears of unpopularity were either unfounded, or he had managed to ameliorate that situation throughout the year.]

6.2.1. Identification of Fred's Mathematical Capabilities

Teacher nomination. On a scale of average (1) to very capable (3) to highly capable (5) to extremely capable (7), Ms J rated Fred at number seven, very high, and described him as 'extremely capable' mathematically. Of the ten nominated students in Grade 5 only one other student was rated at number seven by his teacher. On the nomination form, Ms J commented that he "Loves maths and looks for challenges all the time. [He is a] very high performing student with excellent problem solving [sic]."

Parent questionnaire. On a scale of 1 (low) to average (3) to high (5) to 7 (very high) Fred's parents indicated that they thought his mathematical ability was very high (7). They noted that as a young pre-school child he "was always good at building with Lego blocks and doing jigsaws", both common indicators of high visual-spatial ability and mathematical capability (see Tosto et al., 2014; Winner, 1996). According to Fred's parents, his interest in sport also started from a very young age and, "he was always good at keeping scores and adding them up." Fred attended both 3- and 4-year-old kindergarten. His parents reported that his kindergarten teachers commented on his curiosity, and that he "was always good at number games."

Archival records – previous mathematics assessments. From his early years in formal schooling Fred had achieved high growth points in the *Mathematics Assessment Interview* number domains. Based on data from *Bridging the Numeracy Gap Pilot Project (BTNG)* (Gervasoni et al., 2013), Fred was in the top 1%-5% of Grade 2 students in Counting, Place Value, and Addition and Subtraction strategies (with a growth point profile of 6452), and in the top 1%-9% of Grade 3 students (with a growth point profile of 6462). As a comparison, in the *BTNG Project*, the median growth point profiles for Grade 2 and Grade 3 were 3222 and 4242 respectively (see Table 6.1, cf. Figure 4.1).

Table 6.1

Comparison of Fred’s Number Growth Point Profiles with BTNG Students for Gr 2 and Gr 3

MAI Number domain (range of GPs)	Grade 2 GP comparison with BTNG students	Grade 3 GP comparison with BTNG students
Counting (GP0-GP7)	GP6 top 1%	GP6 top 4%
Place Value (GP0-GP6)	GP4 top 2%	GP4 top 9%
Addition & Subtraction Strategies (GP0-GP7)	GP5 top 5%	GP6 top 1%
Multiplication & Division Strategies (GP0-GP8)	GP2 average: 25 th -88 th percentile	GP2 average: 20 th -66 th percentile

Note. There were no archival records for mathematics assessments for Fred in Prep or Grade 1.

It is interesting to note that from Grade 2 to Grade 3 Fred’s number growth point profile did not change significantly. This phenomenon was noticed with many highly capable students in the *BTNG Project* and may be an issue related to a lack of exposure to more complex mathematical concepts (e.g., fractions and decimals), rather than a student’s ability to learn these concepts. This is an issue warranting further research. Similarly, Fred’s growth points in Multiplication & Division Strategies in Grade 2 and Grade 3 do not seem to reflect his overall capability. The reason for this is unknown.

In Grade 4 and Grade 5 *Progressive Achievement Tests in Mathematics (PATMaths)* (Australian Council for Education Research (ACER), 2014) were used by the school for

whole-cohort mathematics data. Fred scored at Stanine 8 (89th-96th percentile) and Stanine 9 (96th+ percentile) in *PATMaths* in Grade 4 and Grade 5 respectively.

Clinical task-based mathematics interview. In the task-based interview designed for the study Fred showed an ability to learn new concepts easily, to generalise new knowledge readily, and to reason using intuitive strategies to efficiently solve an unfamiliar ratio problem which was beyond the scope of normal primary school curriculum content, all hallmarks of Krutetskii's (1976) observations of mathematically gifted students (see Table 6.2).

Table 6.2

Summary of Fred's Clinical Task-based Mathematics Interview Responses

Observed ability	Evidence
Learn new concepts easily	Fred had never seen a Chinese abacus before but once the structure was shown and explained he was able to immediately and accurately count on the abacus up to at least 50 and, after 45 seconds of confusion trying to work out how to transition the decade from 30 to 29, was also able to count backwards from 35 (he did not hesitate when transitioning from 20 to 19 or 10 to 9). He could represent requested numbers up to 4-digits, and he could recognise representations of numbers up to 4-digits with no hesitation.
Generalise and assimilate new knowledge	Fred was able to transfer his new knowledge of abacus counting to using the abacus to complete basic addition and subtraction calculations without further instruction. With one further piece of information about using number partitions and compensation (e.g., (+10-3) to add 7), Fred was also able to correctly complete more complex addition and subtraction problems. Only four of the Grade 5 students interviewed were able to assimilate this new knowledge readily.
Reasons using intuitive strategies	The ratio problem "Which is more orangey, 2:3 or 3:5 (orange:lemon)?" was a problem type that was unfamiliar to Fred. However, he readily associated the task to his understanding of fractions – "A is 2 out of 5 and B is 3 out of 8, so it would be A because it's closer to half ..." [lots of thinking out loud, including some confusion to try and justify and explain his solution] "so...if you simplify this (2/5) it's four-tenths which is only one-tenth away from a half, and three-eighths is one-eighth away from half." Only three of the ten Grade 5 students interviewed used intuitive proportional reasoning strategies to solve this problem.

The archival mathematics assessment data would appear to situate Fred in a ‘mathematically gifted’ category (top 10% of mathematical capabilities in the general population of age peers). The teacher nomination data confirmed Fred’s capability as being observed generally within the mathematics classroom, the parent anecdotal data confirmed his mathematical capability as being evident from a very early age (cf. Silverman, 1997; Winner, 1996), and the task-based mathematics interview data confirmed his mathematical capability as being intuitive rather than just demonstrating learned mathematics skills (see Geake, 2009a; Krutetskii, 1976; Starr et al., 2013). These data, from a multifaceted approach to identification, suggest that Fred satisfies this study’s definition of being mathematically gifted.

6.2.2. Fred’s Mindset and Approaches to Mathematics Learning – Pre-Teacher Professional Learning

Ms J was concerned about Fred’s insecurities and negative responses to mathematical challenges and formal assessments:

He always likes to get the correct answer, and he likes to know the explicit details of the task. So, if I give him an open-ended maths problem, with limited direction at the beginning, he tends to ask lots of questions. He tends to ask for reassurance each step of the way, so he tends to say, ‘Is this what you want?’, ‘Have I done enough?’, ‘Is this the right way?’, so those sorts of questions. He tends to not like my response when I say, ‘Well what do you think?’, ‘How can you prove that this is the correct answer?’, ‘How else could you have solved it?’ He likes to have one way, which is usually a standard algorithm he’s been taught. (Ms J, June 2014).

The school was developing a strong emphasis on inquiry-based approaches to teaching and learning as they were working through the authorisation process for an International Baccalaureate *Primary Years Programme (PYP)*. The *PYP*, “focuses on the development of the whole child as an inquirer, both within and beyond the classroom.” (International Baccalaureate, n.d.). Inquiry-based learning, then – including taking responsibility for your own learning, asking questions, looking for answers and collating information – had been a major focus of Fred’s class in the four months prior to this study. It was also something that Fred really struggled with. According to his teacher,

It made [him] feel quite insecure, and I believe in the first unit of inquiry, and the second, he probably didn’t perform at a level that I thought he was going to (Ms J, June 2014).

Fred also had the opportunity, together with other high achieving students in Grade 5, to explore *Maths Olympiad* sample questions (Australian Problem Solving Mathematical Olympiads, 2015) with the Principal, but he had not performed at all well with these, and that had “really shaken him” (Ms J, June 2014). According to his teacher he got quite distressed with any form of assessment:

With the *AIM* [Victorian (State) Curriculum and Assessment Authority *Achievement Improvement Monitor*] online assessment ... we had tears because he didn't finish it quickly ... it wasn't really about the maths, but because he was used to finishing [assessments] quickly. [His teacher had set him a Grade 6 level test to challenge him, and the test is designed to become harder as correct answers are recorded.] ... He got, again, quite stressed with *NAPLAN* [Australian (National) Curriculum, Assessment and Reporting Authority *National Assessment Program: Literacy and Numeracy*]. He really focused on the fact that there were two questions ... he couldn't work out (Ms J, June 2014).

In talking with me, Fred said that he ‘knew he was good at maths’ because he was “getting good grades [and] also, because when we're doing tasks I can understand it very quickly and some people have trouble to understand [it], and I can like do problems really quickly” (Fred, July 2014). He thought that Tony (in his class) was better at some things than he was because, “He's just faster at it,” although he countered this with, “I'm faster at stuff than he is, but sometimes I get stuff wrong because I'm thinking too fast ... and sometimes he'll get stuff wrong.” He concluded by saying, “... we never really know who is better because we always get A-pluses.” He also stated that if given an option, “like in a test or something, I'd definitely [choose] the easiest, because then I could get it done fast” (Fred, July 2014).

It seemed very important to Fred that he was perceived as being ‘smart’ by completing work quickly, not making mistakes and getting good grades, which are high indicators of a fixed mindset (Dweck, 2006). When asked if he liked hard maths, and given a scale of 1 (never), 2 (occasionally), 3 (sometimes), 4 (mostly) and 5 (always), he chose 3 (sometimes), with his reasoning being, “because if you're doing something really easy it's a waste of time ... but sometimes I just don't like doing it [hard maths] ... I [get] really worried that I won't finish it in time, so that's when I hate doing hard maths” (Fred, July 2014).

When asked, “If you’re having trouble with anything in maths and the teacher was busy, who would you go to for help?” he replied, “I don’t know, I’ve never kinda felt that way before ... Normally when I’m having trouble I just keep on thinking harder and harder and reading over the question again, but I never really need somebody to help” (Fred, July 2014). In observing him in one lesson, where he was required to complete an isometric drawing of a ‘block house’, that he was having considerable trouble with initially, he neither took up the teacher’s offer of group assistance for those wanting isometric drawing help, nor was willing to admit to other students at his table group that he was struggling. Possibly being averse to asking for help may be further evidence of self-limiting mindset thinking.

A student’s choice of task difficulty and/or the type of responses given in an open-ended task can also indicate either positive or self-limiting mindset tendencies. Students with a positive mindset generally relish the opportunity to “learn something new” from challenging tasks, whereas students with a self-limiting mindset generally tend towards easier tasks that would be guaranteed to make them “look smart,” rather than risking more difficult tasks (Dweck, 2010b, p. 7). In the Phase 2 task-based interview, when Fred was given a choice of numbers [$1\frac{2}{3}$, 100.25, or any other number he liked] in the *Adding Corners* task, and encouraged to be creative and challenge himself, to ‘make his brain do some work’, he chose the number 137. His ‘most creative’ solution was $48+73+16$ (see Figure 6.1a). All other Grade 5 students who were interviewed chose to use fractions and/or decimals and/or negative numbers, either in the number they chose to put in the centre, or the numbers they chose for the solution, with Bob choosing to use all of these (see Figure 6.1b). None-the-less, Fred was ‘very happy’ with his solution when asked to indicate how he felt on a smiley chart. He made the comment that, “Sometimes I like being creative, but sometimes I just want to make it easy.” When asked how he challenged himself he said, “Mostly I try to get things done quickly.” Neither Fred’s choice of number to partition, nor his solution to the task, reflected his mathematical ability as evidenced in the identification process. The purpose of the task, as described to him in the interview, was to give him an opportunity to ‘challenge himself and come up with something really interesting’. Other students, interpreted this as, “to find the hardest one” (Bob).

Evidence of Fred’s propensity to focus on speed rather than depth of challenge in mathematics was also observed in the classroom. In one of the classroom mathematics lessons, prior to the teacher professional learning, Fred and a few of his peers decided they didn’t like the high chance component of a *Decimal Path* game the class was playing. One

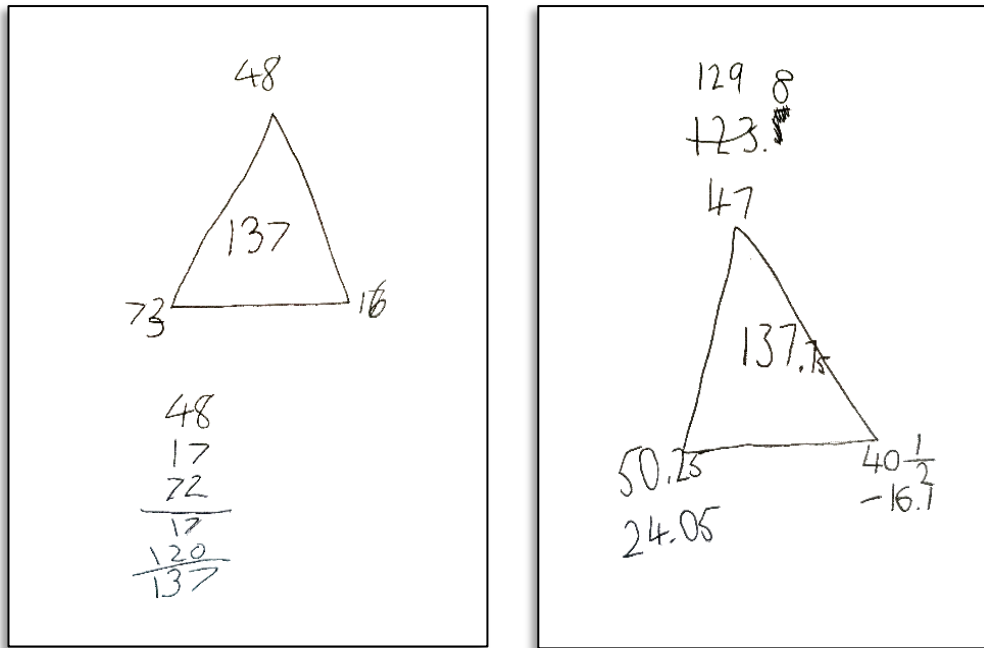


Figure 6.1 Adding Corners task. (a) Fred's solution (left) (b) Bob's solution (right).
 [NB. It was coincidental that both Fred and Bob had the number 137 in the centre of the triangle.]

student commented, "You have an idea in your mind what you want, but then you roll the dice and you have to use that." Another said, "I didn't enjoy it ... [I want to] put a challenge in rather than just chance!" They were consequently given permission to create variations to the game rules, rather than continuing to play the original version of the game with the rest of the class. This provided for more strategising, and subsequently more mathematical thinking and reasoning, with several very good alternative suggestions made. However, Fred was determined to make the game a race, with speed being the ultimate challenge.

James: You could roll the dice and then choose where you want to put each digit.

For example, if you roll 1, 2, 3 you could make 0.123, 0.231, 0.312 et cetera.

Tony: Roll the dice 10 times and *then* order the numbers ... and make the target number between 3 and 3.5 [rather than just 3].

Fred: That would make it easier [making the target number a range rather than a set number]

Tony: It would be easier, but it would make more sense.

Fred: You could see if you can get to 3 as fast as you can.

Tony: My way is harder, has more thinking.

Fred: My version is to have lots of boxes, about 50 [the original game had 16 boxes] and then you have to get to the target number as fast as you can.

(Classroom observation, July 2014)

A focus on speed, needing to understand and complete tasks quickly and easily with minimal effort, choosing low-risk tasks, distress over getting questions wrong, needing to know ‘the right way’ to do something, and focusing on getting high grades, are all indicators of a need to “look smart at all costs” (Dweck, 2010b, p. 7), which Dweck describes as the “cardinal rule” of a “fixed mindset”. Many of Fred’s responses, outlined above, indicate that he had certainly developed these fixed-mindset beliefs and behaviours, and, although his achievements were still high, it is possible these mindset beliefs were limiting his true potential (Betts & Neihart, 1988). It may have been because of his beliefs that Fred struggled emotionally with tasks that he could not complete quickly and easily, or could not complete without sustained effort, and readily admitted that he would choose an easy task that he knew he could do over a task that would challenge him when the stakes were a mark or grade. Fred’s need to finish a mathematics task quickly would seriously limit any desire to make “exciting discoveries” or “independently invent (new) rules” or “devise novel, idiosyncratic ways of solving problems” (Winner, 1996, p. 3), even if he was capable of doing so.

It became apparent, from information from his teacher, many of Fred’s responses, and classroom observations, that providing for his mathematics learning needs would require more than simply planning more challenging tasks for him. Teacher support would also be necessary in promoting a change from a self-limiting fixed mindset view of successful mathematics learners being those who ‘work fast’ and ‘get A-pluses’, to a more positive mindset view of successful mathematics learners being those who revel in a challenge, who persevere, who think creatively in order to overcome difficulties, who are inventive in solving problems, and who extend their mathematical ideas (Krutetskii, 1976; Silver, 1997; Sullivan et al., 2013; Sriraman, 2004).

6.2.3 Grade 5 Teacher’s Approach Post Professional Learning, and Fred’s response

Ms J said she was committed to using real-life contexts as often as possible within mathematics lessons, using personal experiences to interest and engage students, and this was already evident in the lessons observed prior to the professional learning. She gave clear instructions about tasks, and was conscious of providing differentiated versions of tasks, for students with higher or lower abilities, where necessary. She had high expectations of students, and was open to student feedback and suggestions. Classroom organisation allowed for plenty of dialogue about each task, both teacher-student, and

student-student. She was, however, very conscious of the wide gap that existed between the students who struggled with mathematics and those students, like Fred, who excelled. She felt she struggled, particularly, in keeping her highly capable students satisfactorily challenged.

I find as a teacher, to know where to take them next ... so you think that you've built in a high enough sort of extension for them, and then that misses the mark and they finish early. Sometimes it's difficult in the middle of the lesson to then be able to extend that even further. (Ms J, focus group discussion, April 2014)

During the post-professional learning interview, Ms J talked about how she instigated several changes in her practice, based on the suggested approaches. Her changes/adaptations were based on a whole-class approach to teaching and learning mathematics, with a specific awareness of her highly capable students. She stated that she:

- Used more open tasks, and focused on discussions around those tasks that explicitly highlighted that she valued different ideas, strategies and approaches to the tasks, just as much as correct solutions.
- Used more partner work to further generate mathematical reasoning dialogue.
- Provided direction through explaining the mathematical learning focus for the lesson rather than simply giving instructions for the task.
- Was intentionally more consistent in the way she gave students feedback. She used more rubrics so she could include criteria such as how students justified their approach to solving tasks and the strategies they used.
- Was more aware of allowing time to discuss things like, 'What new discoveries have you made?' 'What challenged you?' 'What area do you think you could continue to work on?' 'What did this task show you about your mathematical understanding?'
- Became more aware of how she questioned Fred, and other mathematically highly capable students. "I've become more aware of if I'm setting a task that's pitched at the whole year level, how I can make it a bit harder, and not being really explicit about that, just posing, 'Well, could you try this?' or 'How could you change it?' or 'How could you teach someone else to do it that knew nothing about it?'" (Ms J, November 2014).

At the beginning of these changes, Ms J said Fred did not cope well with not having all the information or being able to find out the information quickly and easily. Over time, however, he discovered that, in some instances at least, "... he found he loved when there

was not explicit teaching, and they had to find out their own way of working within it [the task]” (Ms J, November 2014). He really began to enjoy open tasks once the discussions became more focused on mathematical thinking rather than just the answer obtained, although he still showed some insecurities – “... he was still saying at times, ‘I don’t know whether I’m doing it right or wrong,’ or ‘It mightn’t be the right answer’” (Ms J, November 2014). It was an encouraging change, with Fred seemingly beginning to feel more comfortable with open tasks, but his continued statements expressing concern about whether he might be approaching tasks incorrectly, shows that this change was neither sudden nor absolute within the three months prior to Phase 3 of the study.

Another change his teacher mentioned was an improved ability for expressing his mathematical thinking in writing, and justifying his solutions and strategies.

I often say to him, ‘If you had to explain this to someone who knew nothing about this, how would you tell them what you're thinking?’, and so getting him to really break that down. So that's been an area I think he's developed in.

I think it [making sure there was more discussion time] helped [Fred] in being able to express his mathematical thinking to others, but also to be able to put it in writing.

(Ms J, November 2014)

Criteria such as ‘providing justifications for solutions’ were being included in rubrics for open-tasks that were used as assessment pieces (which was a new approach being adopted by the whole school), and explicitly explained to the students. “Fred really likes to have those really clear instructions ... and he really embraced that [the really clear expectations in the rubric] ... [he] seemed to appreciate that explaining and justifying his solution was valued just as much as getting a correct answer” (Ms J, November 2014).

Fred also seemed to “really appreciate being able to work with his peers, with those who think about maths in a similar way to him” (Ms J, November 2014). Riley, Sampson, White, Wardman and Walker (2015) explain how mathematically gifted students benefit from time spent with like-minded peers, and this is something else that is important for teachers to consider when planning for these students’ learning needs. There were several mathematically highly capable students in Fred’s class, and his teacher felt that the intrinsic nature of open-tasks, where students are expected to discuss their thinking and explore further with others, was satisfying his need to work with similarly capable peers. The school also had a practice of sometimes combining classes, whereby all Grade 5 students, for

example, would engage in the same mathematics investigation. During these times students would have the opportunity to work with other students, not just those in their own class. This enabled opportunities for highly capable students like Fred to participate in even broader discussions with like-minded peers. The Principal was very involved with the students' mathematics learning, too. He occasionally took highly capable students for special classes (e.g., the *Maths Olympiad* experience), and openly invited and welcomed *all* students to share with him any exciting mathematics discoveries and/or solutions and/or dilemmas they may have come across, either at school or at home. This was all being developed as part of the whole-school approach to inquiry-based learning, and formed an important integral component of the teachers' classroom approaches.

6.2.4 Fred's Mindset and Approaches to Mathematics Learning – Post-teacher Professional Learning

From observing Fred in post-teacher professional learning mathematics lessons, it seemed evident to me that he was becoming more willing to think beyond the set task – to be more adventurous and creative in his mathematical thinking. In one lesson, students were given a worksheet, 'Financial Plans and Records', and were required to complete a two-month budget for a proposed after-school rubbish bin collection business, using the example on a worksheet as a guide. Fred and his partner decided, after some discussion about the business practice of 'spending money to make money', that they may be able to increase their profit margin in the second month by advertising. This required more expenditure (photocopying a flyer), but they decided it was a worthwhile expense that could generate more business, and therefore more income. While discussing this with his partner, Fred still seemed a little reticent, voicing a concern about 'getting it right', but did not check with the teacher (as he normally did in the past) before going ahead with their idea. In sharing their final results, others at the table sounded quite indignant when Fred announced that they had made a bigger profit the second month by spending extra on advertising and "drumming up more business". James said, "I didn't know you were allowed to do that!" and Connor said, "I thought we just had to follow this!" (pointing to the example on the worksheet). Fred's reply was, "Well it's not an *exact* question, it's a...an *anything* question." The task was not specifically presented as an 'open task', but Fred and his partner were happy to think beyond the set problem. This contrasted with when Fred previously needed to "know the explicit details of the task" and constantly asked the teacher, "Is this what you want?", "Is this the right way?" before committing himself to the task. This highlights a possible change

in Fred's thinking, indicating a willingness to now 'take a risk' with a task rather than simply focusing on 'getting the right answer'.

Prior to the professional learning his teacher said, "[Fred] struggles with the creative element of any maths task", but during the Phase 3 interview she said that he particularly enjoyed being given the freedom to modify mathematics tasks and games, "building in extra things to make it really difficult, or to make it an *unfair* game [in a unit on probability]."

[Fred] is more willing to do that [be creative], and he gets quite excited, but he likes working with a partner in doing that.

He's not asking as many times, 'Is this right?' 'Is this right?', that he's having a go and thinking about it and justifying his thinking.

He's actually challenged himself to improve in an area he sees as being an area that he finds difficult [spatial tasks]. He actually chose to work at home in creating different models out of Lego so he could practice doing the different views – if he couldn't see the other view what could it look like. The fact that that is an important element of maths helped [Fred], and he worked really hard on that.

I think he's had a good year. His parents are happy with how he's progressed.

(Ms J, November 2014)

In the Phase 4 task-based interview, Fred was again asked to complete the *Adding Corners* task. He only vaguely remembered the task from earlier in the year. This time, when challenged to 'choose a number to go in the centre that's going to give you an opportunity to be creative, and find a solution that is really going to challenge you,' he chose to use factorials and fractions (see Figure 6.2).

I could do kind of a question in the middle, I'd need something to be the square root of ... or I'll just do maybe ... eleven-factorial and ... seven [wrote $\frac{7}{81}$]. That'll be hard though [nervous laugh]. (Fred, November 2014)

He chose to use a calculator to work out eleven-factorial, but pencil and paper to partition the resultant 8-digit number. He considered various fraction denominators to challenge himself further – eighty-firsts, ninths, one-hundred-and-sixty-seconds.

In Fred's work-sample (Figure 6.2) the eleven-factorial partition is correct, but he ended up with the fraction partitioned into $\frac{3}{9} + \frac{15}{162} + \frac{93}{162}$ (because he wasn't comfortable with $\frac{46.5}{81}$) before realising that this was actually $\frac{81}{81}$ instead of only $\frac{7}{81}$. By this stage it was already ten minutes into the lunch break, and I felt I should stop him. He replied, "Oh, that sucks [laugh], I was almost finished." This was in contrast to the Phase 1 interview where his focus had been on finding a correct solution quickly without making any mistakes, and he had admitted to choosing easy numbers so that he could do this. He commented at the beginning of this second attempt, "I could do eleven-factorial [in one corner], and I could do six eighty-oneths [sic], and then I could do one eighty-oneth [sic], but that's too easy. I'll figure it out in a different way."

This time, in contrast to his first interview, Fred was fully engaged in the task for more than 15 minutes, and was willing to keep going if he could. Making a mistake was not distressing or embarrassing for him, fixing it was simply a matter of continuing to work on the problem.

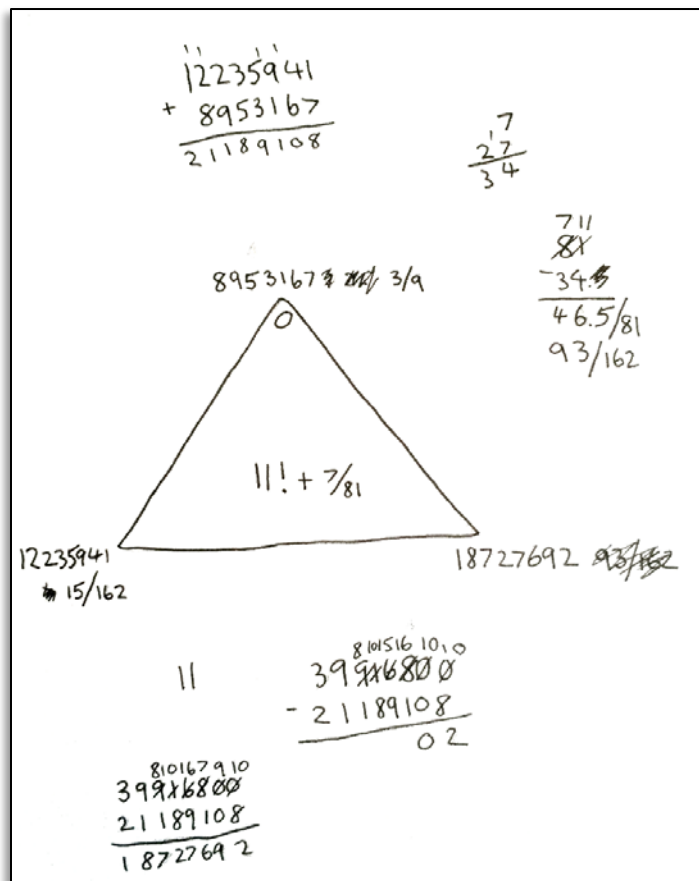


Figure 6.2: Adding Corners task #2. Fred's solution – post-teacher professional learning.

During the Phase 4 semi-structured interview, Fred was asked to recollect the ‘best maths’ he had done in the past term. He talked about a two-week unit he and other Grade 5 mathematically highly capable students had completed with the Principal:

He [the Principal] wrote, ‘How can you order, add, subtract, multiply, divide and incorporate fractions with plus numbers and minus numbers [positive and negative numbers], and then explain why?’ We were in the computer lab and we had things like YouTube ... and we had to figure all of them out by ourselves, and explain why and how and stuff like that. For example, four plus negative four, or four minus negative 4, and dividing and timesing [sic] negative and positive numbers. And we were free, he didn't set out any specific tasks, he just told us to figure them all out for a couple of weeks. (Fred, November 2014)

At the end of the two weeks they were given a test and Fred answered every question correctly, but when asked what he was particularly proud of, he replied:

Completing this big task without teachers telling us how to do it, like discussing it and stuff with my mates and looking at YouTube and learning how all of it works and then figuring it out [by ourselves], and then going to the test and getting all the questions right that we thought at the start were really hard. (Fred, November 2014)

He was still proud of getting all the questions correct, but this time it seemed to be also because he was proud of being able to manage his own learning successfully. I asked him if it bothered him that he could not do the work quickly, that it took a couple of weeks to work out, and he said, “No, because we were all looking forward to our next maths lesson.” Fred’s thinking appears to be changing from simply valuing ‘high grades’ in mathematics, to valuing and appreciating the process of learning mathematics as being beneficial in, and of, itself (cf. Brophy, 2008).

Fred discovered he enjoyed working with others,

If the work is easy and I can do it...I'd rather do it by myself, but if it's something that's reasonably hard I'd probably prefer working with somebody... [When] I work with a friend it's fun because we have two ideas, so we have more ideas on it. (Fred, November 2014)

Overall, thinking back over the previous three months, he said,

It's been hard, but I've learnt quite a bit ... I've probably been pushed more and doing a bit harder work [especially] when I was with [the Principal] I've enjoyed maths this year. (Fred, November 2014)

6.2.5 Fred's Story: Review

Post-teacher professional learning observations of Fred, and comments made by both Fred and his teacher, are a contrast to when Fred had previously wanted to do everything quickly and easily. His focus seemed to have shifted from enjoying mathematics tasks he could complete quickly, to being able to enjoy the mathematics within the productive struggle of trying to solve a problem. There was also a contrast with how Fred had previously become quite distressed if he could not work out what to do, or if he was not sure what was expected of a task, especially with open tasks. He now seemed more willing to take risks with problem solving: to work with others, to discuss the difficulties of the task, and to put forward suggestions. He was, in fact, working more mathematically – in focusing on reasoned arguments and justifications for solutions, and not simply writing an answer to a problem and moving on. These changes came about following a period of intentional strategic changes made by his teacher in whole-class teaching approaches and student expectations within her mathematics lessons.

6.3 Sammy – Grade 3



“Sammy”

(photo used with permission)

At nine years of age, Sammy is the oldest of three siblings. Like Fred, both her parents are medical doctors, and she is an accomplished athlete, competing in State finals as one of the youngest competitors in her gymnastics troupe, and regularly winning awards. She had previously been offered a place in an international training program in Melbourne for gymnasts, but her parents declined this offer. Sammy is often absent from school due to gymnastic commitments and competitions. Her teacher described her as being a perfectionist and very competitive, which may be an influence of her gymnastic

training. Sammy's brother (her youngest sibling) is extremely capable mathematically, and Sammy's teacher (Ms S) wondered if Sammy sometimes compares herself to him and finds her own mathematical ability comparatively lacking. Ms S commented:

[Her brother] is a freak at maths so, I don't know, that sort of sets things really high at home, and I think [Sammy] would be someone that knows how good [her brother] is. She celebrates him in the class and lets me know, but that would be in her mind, and with her competitive nature she'd want to be the best she could possibly be. (Ms S, July 2014)

Sammy generally appeared to be well behaved – with her regular classroom teacher, regular specialist teachers, a pre-service teacher, and when working with me – and she seemed to work well with other students in mathematics group tasks. In one early (Phase 2) classroom observation, however, her teacher was ill, and a relief teacher had been called in last minute. Sammy, together with two other girls, was quite disruptive during the beginning whole-class session, being rude to the relief teacher and non-conforming. This was an unexpected side to Sammy, and her regular classroom teacher was both surprised and horrified when she heard about this. It seems that this was not normal behaviour from Sammy with either her classroom teacher, or her regular specialist teachers, but a side to Sammy that was there, nonetheless.

At times, Sammy appeared to struggle socially. According to her teacher, Sammy's friends were sometimes annoyed by certain personality traits and attitudes exhibited by Sammy. They tolerated these behaviours for a while, but then ostracised her from group social activities, leaving Sammy confused and defensive. Her teacher explained:

She's good at everything except for things like team sports, and things with other kids – she's very competitive, so she likes to be the best. She doesn't handle things too well when things don't go her way. (Ms S, July 2014)

She had a bit of a falling out at the beginning [after the professional learning] with a couple of friends ... and then it just snowballed from there and got worse and worse and worse, and she was just unaware of what she was doing to cause the problem, and she just put a wall up and it was horrible. She went through a really, really tough little time ... and she's just been feeling a bit of a misfit really, socially. (Ms S, October 2014)

It seemed important to Ms S to highlight the social issues Sammy was having; she mentioned them several times, in both the pre- and post-professional learning interviews, and she was concerned about the impact they may have on her involvement in the mathematics lessons I was to observe (in Phase 3), “It will be interesting when you come back in to work with her to see what she’s like, because she has really, really struggled [socially] – big time.” (Ms S, October 2014)

6.3.1 Identification of Sammy’s Mathematical capabilities

Teacher nomination. On a scale of one to seven (from average to extremely capable), Ms S rated her mathematical capability at number six. Her brief description of Sammy’s exceptional capabilities was that she is “great at explaining strategies; uses a range of strategies.”

Parent questionnaire. On a scale of low (1) to average (3) to high (5) to very high (7) Sammy’s parents indicated that they thought her mathematical ability was high (5). Sammy’s parents stated that they could not recall Sammy engaging in any specific mathematics-type activities prior to school, “We had three kids under three years old – [Sammy] was the oldest so we don’t remember much about anything back then” (Sammy’s parents, written questionnaire, March 2014). Sammy attended part-time 0–3-year-old childcare, 3-year-old kindergarten and 4-year-old kindergarten, but, according to her parents, the kindergarten teacher(s) never mentioned anything specific about Sammy’s curiosity and/or ability with maths-type activities. Her parents seemed happy with Sammy’s school progress, “She seems to enjoy maths at school [and] is catered for fairly well.” (Sammy’s parents, written questionnaire, March 2014)

Archival records – previous mathematics assessments. Based on the *Mathematics Assessment Interview (MAI)* data, Sammy’s number profile (2222) at the beginning of her second year of formal schooling (Grade 1) was close to the growth point mean (2121) of Grade 1 students from the *Bridging the Numeracy Gap Pilot Program (BTNG)* (Gervasoni et al, 2013) (see Table 6.3, cf. Figure 4.1).

Table 6.3

Comparison of Sammy's Number Growth Point Profiles with BTNG Students

MAI Number domain (range of GPs)	Grade 1 GP comparison with BTNG students	Grade 2 GP comparison with BTNG students	Grade 3 GP comparison with BTNG students
Counting (GP0-GP7)	GP2 32 nd -73 rd percentile	GP4 top 35%	GP6 top 4%
Place Value (GP0-GP6)	GP2 top 17%	GP3 top 9%	GP4 top 9%
Addition & Subtraction Strategies (GP0-GP7)	GP2 48 th -87 th percentile	GP5 top 5%	GP5 top 20%
Multiplication & Division Strategies (GP0-GP8)	GP2 58 th -99 th percentile	GP3 top 12%	GP4 top 16%

However, it is interesting to note that over the next two years her growth in mathematical understanding in the Counting (+4 growth points), Place Value (+2 growth points) and Addition and Subtraction Strategies (+3 growth points) domains was above average, putting her within the top 4% and 9% of students in Counting and Place Value, respectively, by the beginning of Grade 3. The mean growth in each number domain is one growth point per school year, with Place Value being slightly less than one growth point per year (Clarke et al, 2002). Sammy has demonstrated a growth of two growth points each year in the Counting domain (where the mean is one), one growth point per year in Place Value (where the mean is 0.75), and three growth points in Addition and Subtraction Strategies in Grade 1 (where the mean is one) before seeming to plateau in Addition and Subtraction in Grade 2. Her growth in Multiplication and Division strategies was one growth point per year (which is average growth), although growth point 2 in Grade 1 is slightly above average. This may indicate that Sammy's prior-to-school experiences may not have focused on the school-type mathematics knowledge she would subsequently learn at school. Once at school, Sammy's ability to learn specific mathematics concepts at a faster rate than her age peers became evident.

There is some evidence of number learning plateauing from Grade 2 to Grade 3 in the Addition & Subtraction domain. Indeed, in the *BTNG* data only 1% of Grade 3 students reached growth point 6 in Addition & Subtraction Strategies, which requires mental

calculation of 2- and 3-digit numbers. For Sammy, this was not due to a lack of understanding of 2- and 3-digit numbers, as she had reached growth point 4 in Place Value (i.e., was able to read, write, order and interpret 2-, 3- and 4-digit numbers). This plateauing, also observed with Fred, may be another indication of a lack of exposure to more complex mathematical experiences for students with higher mathematical capabilities.

Clinical task-based mathematics interview. In the task-based interview designed for the study Sammy showed an ability to learn new concepts with minimal difficulty, to generalise new knowledge to a limited extent, and to reason using intuitive strategies to partially solve an unfamiliar ratio problem which was beyond the scope of normal primary school curriculum content. Evidence of these can be seen in Table 6.4.

The archival mathematics assessment data would appear to situate Sammy in the ‘mathematically gifted’ category (top 10% of mathematical capabilities in the general population of age peers) in at least some domains. The teacher nomination, and parent indication of mathematical ability, confirm Sammy’s ability as being observed both generally within the mathematics classroom and at home, and the task-based mathematics interview data suggest that her mathematical capability as being intuitive rather than based on learned mathematics skills and procedures. These data suggest that Sammy satisfies this study’s definition of being mathematically gifted.

6.3.2 Sammy’s Mindset and Approaches to Mathematics Learning – Pre-Teacher Professional Learning

In the initial task-based interview (Phase 1) Sammy presented as a bright, friendly, very self-assured girl. She answered questions quickly and confidently. Even when she was incorrect, she was very quick and confident with her response. However, classroom observations, and a subsequent interview with her teacher (Phase 2), indicated that this ‘confidence’ may actually have been evidence of a self-limiting mindset, whereby she believed she needed to provide quick answers to show her ability, rather than being seen to be having to stop and think, or expending any effort to complete a task (cf. Dweck, 2006).

Table 6.4

Summary of Sammy's Clinical Task-based Mathematics Interview Responses

Observed ability	Evidence
Learns new concepts with minimal difficulty	Sammy could accurately interpret how to count forwards and backwards on the abacus after minimal instruction, she only required prompting when going from 49 to 50. She could show all numbers up to four digits easily and accurately, but had some difficulty naming 4-digit numbers (calling 5903 one thousand, five hundred and three).
Generalises and assimilates new knowledge to a limited extent	<p>Sammy was able to transfer her new knowledge of abacus counting to using the abacus to complete a basic addition calculation without further instruction ($65+32$) but relied on already known mental strategies to calculate $147-25$ rather than the abacus model.</p> <p>She seemed to apply the concept of using number partitions and compensation to add eight (e.g. $+8 = +10-2$) in the explanation, but when trying to add 3 to 73 kept adding 5 and then adding 2 (instead of subtracting 2). She could only work out the answer because she had already calculated it mentally.</p>
Reasons using intuitive strategies	<p>The proportional reasoning question 'If three lollies cost 10c, how many lollies could you buy with 80c?' elicited evidence of pre-proportional reasoning (Lamon, 1999). Sammy set the problem out as:</p> <div data-bbox="745 1227 1256 1529" data-label="Diagram"> <p>The diagram shows a sequence of numbers: 3, 6, 9, 12, 15, 18, 21, 24. Lines connect 3 to 20, 6 to 20, 9 to 20, 12 to 20, 15 to 20, 18 to 20, 21 to 20, and 24 to 20. Below these, lines connect 20 to 40, 20 to 40, 40 to 80, and 40 to 80.</p> </div>
	<p>saying, 'Three for ten cents. Three plus three is six, so that's twenty cents; nine, twelve, twenty cents; fifteen, eighteen, twenty cents ...' [with the rest done in silence], and concluded, 'I could buy 24 lollies with 80c'. This was most likely not a learned procedure, but an intuitive response to the question (see Lamon, 1999; Parish, 2010).</p>

The first classroom observation of Sammy working in a mathematics lesson (May 2014) showed a very different side to Sammy. The lesson was about arrays (visualising multiplication), and the focus of the lesson was ‘writing number sentences to describe arrays’. The introduction session covered what an array is (i.e., arranging counters/dots in equal rows and columns), and the class was then sent off to explore various quantities (12, 15, 18, etc.), using counters to make arrays, and writing number sentences to describe the different arrays they could make with each quantity. Sammy set to work quickly and quietly by writing down a list of ‘number sentences’ (equations) in her book – $12=12\times 1$, $12=1\times 12$, $12=3\times 4$, $12=4\times 3$, $12=2\times 6$, $12=6\times 2$ – and then carefully drawing the corresponding arrays next to each equation (she was not using the counters). She was going through the motions, reproducing work she could already do quite confidently, and she seemed quite content.

As a participant observer, a role that enabled me to interact with the students, to engage in tasks, ask questions, and generate and participate in discussions (Stake, 1995), I decided to intervene to encourage Sammy to explore some other possibilities that she would need to think about mathematically. I drew her attention to an array of 12 she had drawn (Figure 6.3), and told her I could see another number sentence she could write for this array: I could see three rows of three on the top and another row of three along the bottom – $3\times 3+3$ (Figure 6.3). She immediately noticed two rows of three plus another two rows of three – $2\times 3+2\times 3$ (Figure 6.3) – so I left her to see what else she could discover with some of the other arrays she had drawn. She promptly stood up, and eagerly set to work. As I walked away I heard her exclaim to others at her table, “This is so cool!”

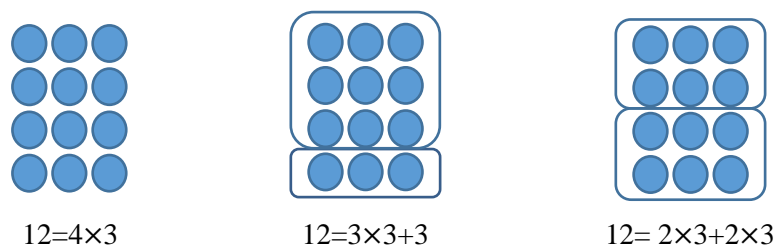


Figure 6.3 Illustration of Sammy’s work samples of a 12 array.

When the teacher called the class back to the mat later in the lesson, Sammy was very eager to share what she had discovered about ‘array busting’, but what she chose to share was that she could write 18 as a number sentence: $3\times 5+3$ (see Figure 6.4). Unfortunately, although the number sentence was correct, and what she drew certainly represented the equation, she had lost sight of the array focus of the lesson. The dilemma was that the rest

of the class was looking puzzled at her partial array representation, and some began to question it. To overcome the awkwardness of the situation I intervened, and asked Sammy to partition a 15 array that was already drawn on the board, in the way she was trying to describe. She was a little confused initially, but ended up describing $15=3\times 3+3+3$ (Figure 6.4).

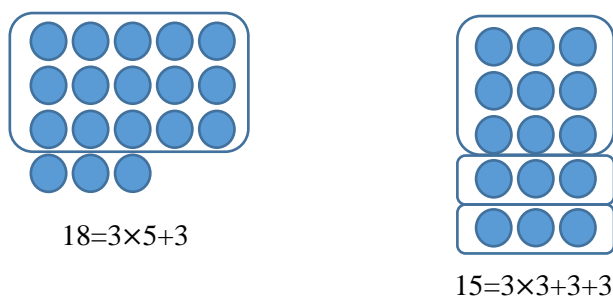


Figure 6.4 Illustration of Sammy's work samples of an 18 and 15 array.

It was not until several minutes later, when the teacher directed the class to the follow-up activity and the students dispersed from the mat, that it became apparent that Sammy was now highly distressed. Initially, she could not explain to us what was wrong because she was sobbing so hard, but eventually pointed to the 18 'array' on the board and choked out, "I can't do it!" After some reassurance from both her teacher and me, she eventually settled back to work, but reverted to writing and drawing basic equations for her arrays – $16=4\times 4$, $16=2\times 8$, $16=8\times 2$ etc. – and remained visibly miserable for the rest of the lesson.

Sammy was fascinated and excited about the possibilities more complex array busting provided, but when she thought she was wrong, she became highly distressed, and any subsequent learning opportunities in that lesson were stifled. This reaction may be evidence of emotional hypersensitivities, a common characteristic of gifted children (Dabrowski, 1972; Silverman, 2013). From an informal discussion with Sammy's classroom teacher it seemed that this kind of reaction, while not an everyday occurrence, was not uncommon for Sammy (I witnessed it twice in the seven lessons I observed), and it was the sort of thing she dwelt on ...

I [Ms S] said, 'Why don't you try something else with that and do the 'explore more challenging things'', and she's like, 'No, I think I'm ok with this,' and she brought up that lesson [the arrays lesson], like from however long ago it was, and I was actually really surprised. But she had brought it up a few times since then, like I've heard about it a few times ... (Ms S, July 2014)

Sammy also tended to be very self-critical. ‘I’m no good at maths’ was a regular utterance when she was asked a question she did not know the answer to immediately. Within 30 minutes of a lesson on ‘How many designs can you make that are one quarter yellow and three quarters red?’ she had uttered, “Let’s do the easy ones first”, “Oh my gosh, you guys are fast! How do you do it [come up with design ideas] so fast?” “I’m not very good at this”, “I’m not good at maths”, “But it’s too hard”, and “I can’t do this”, all while successfully coming up with different fraction designs. This included new mathematical concepts for her, such as non-contiguous three-quarter designs, and non-congruent fractional parts (see Figure 6.5), which she seemed to readily understand, but was constantly wanting to go back to ‘the easy ones’: “Let’s do the easy ones first”; “I’m just doing the easy ones first.” She required minimal scaffolding while completing this task, but still saw it as “too hard.” Based on observations by Mueller and Dweck (1998), constantly wanting to revert to easy work, is a further indication of Sammy’s self-limiting mindset.

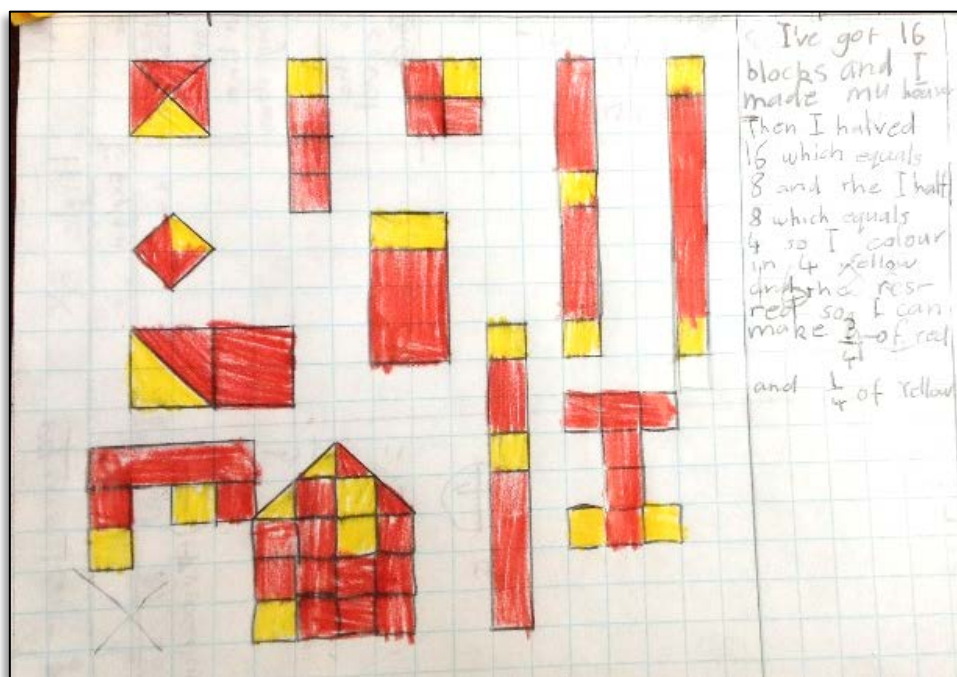


Figure 6.5 Sammy’s $\frac{1}{4}$ yellow $\frac{3}{4}$ red designs.

Another issue for Sammy, alluded to in the ‘one quarter yellow and three quarters red’ task above, and also identified by her teacher, was her need to complete tasks quickly:

There was a maths space thing at the beginning [of the post-professional learning period], and the kids were working independently on it, and some of the kids had finished before her, and so she just thought she was hopeless because she was the

last one to finish, and she just fell into an absolute heap ... She ended up just bursting into tears, she was so upset. (Ms S, October 2014)

Contrary to her demeanour in the initial interview with me (Phase 1), Sammy seemed to struggle considerably with her confidence as a mathematics learner in the classroom. During the Phase 2 classroom observations I noted she rarely led any discussion, and was much more likely to agree with others' comments than make her own. The few times she did suggest her own observation or solution, she would quickly retract her comment, which seemed like a kind of defence response in case she was wrong. For example, in one lesson the class was exploring a fraction wall with the first fractional part of each line coloured in. After others in her group had made several observations like, "It looks a bit like stairs" (Janet), and "It seems to be sloping downwards" (Jackson), Sammy mentioned, "It kind of runs down in a diagonal line," but when nobody responded to her observation this was quickly followed by, "Oh, no, not really..." Another time I asked a question and Sammy said something that I did not hear. When I asked her to repeat herself her response was, "Oh, ah, bleuuuh, I mean, umm ..." When I explained that I had just not heard her, she went on to repeat what she had said, which was a completely valid response. Her lack of confidence in her own mathematical ability was quite extreme. It turned out she thought she was working with me because she was *not* good at maths, even though it had been explained to her that she, and others, were being interviewed because they were very good at maths. Her teacher suggested that,

She wouldn't have heard that, it would have gone in one ear and out the other because she would have thought, oh, I'm doing something different so it must be bad. Her mum tried to make it make sense for her, like, 'Are you sure you're not good at maths? Why do you think you're doing it with [Jackson] and [Janet] as well?' and she said, 'Oh ... because they're really good at maths! But ... oh, I don't really know.' She hadn't thought of that, so she just thought, 'I'm dumb, I'm hopeless at maths, that's why I have to do stuff with you.' (Ms S, October 2014)

Sammy was confirmed as being gifted mathematically. She learnt new concepts quickly, she could transfer new knowledge to novel situations, and she was able to reason abstractly. However, she really struggled when she did not know an answer straight away, or when she had to put in effort, or stop and think about a problem. These things, to her, seemed to be indications that she was 'no good at maths', and she seemed to have developed a skewed perspective of herself as a mathematics learner. When I asked her "How do you know

someone is good at maths?” her reply was, “[They] always finish their work in time. They’re always going ‘done’, and always get the right answer.” (Sammy, May 2014). Indeed, it seemed that it may have been possible that Sammy’s formal assessment (the *MAI*) may not have accurately reflected her capacity with mentally adding and subtracting 2- and 3-digit numbers, and/or multiplying and dividing larger numbers, as these calculations would have required more thinking time than she may have been willing to invest.

It became apparent that providing for Sammy’s mathematics learning needs would require more than simply planning challenging tasks. Teacher support would be necessary in promoting a change from her self-limiting mindset view of successful mathematics learners being those who ‘work fast’ and ‘get the right answers’, to a more positive mindset view of successful mathematics learners being those who persevere through difficulties. She may also require support in dealing with her intense emotions, or hypersensitivities, regarding perceived failures before she could be expected to take risks in approaches to problem solving in challenging mathematics tasks, especially if the approaches proved to be initially wrong and required modification, which is a legitimate stage in the problem-solving process (Polya, 1957).

6.3.3 Grade 3 Teacher’s Approach Post Professional Learning

Post professional learning, Ms S chose to focus primarily on Sammy’s mindset as she was concerned about the effect this was having on Sammy generally, not just in her mathematics learning.

After your visits it was really, really clear that she was fixed mindset, completely. And it wasn’t just in mathematics, and it wasn’t just in academic areas, it was just in life in general. (Ms S, October 2104)

She spent considerable time with Sammy, and Sammy’s parents, explaining fixed and growth mindsets (something that the school had embraced the previous year, so it was not a completely new concept for them). Together they devised a customised chart for Sammy (see Figure 6.6), based on the chart from the framework suggested in the professional learning, to help her change her thinking from a self-limiting (fixed) mindset to a more positive (growth) mindset – particularly in mathematics learning, but also in her outlook on life in general. Whenever Sammy uttered, or alluded to, a negative mindset thought, Ms S would get her to physically go over to the chart and read out a more positive way of expressing what she was feeling. The focus was on getting Sammy to make deliberate

choices in her thoughts (self-talk), and in the language she used when faced with a difficulty.

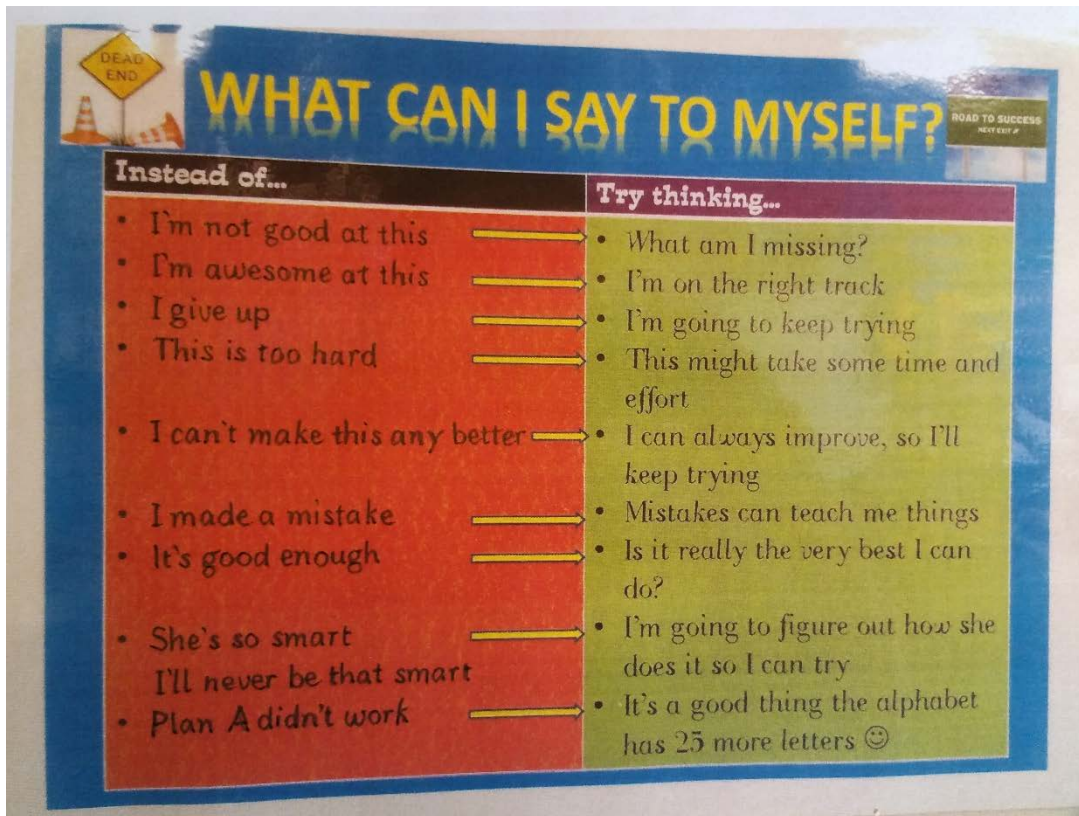


Figure 6.6 Ms S's mindset chart for Sammy.

In mathematics lessons, Ms S was already putting into practice task differentiation for varying abilities within her class, but following the targeted professional learning she was more cognizant of her highly capable students requiring support from her if the tasks were targeted within their zone of proximal development. She was very transparent with them about increasing her expectations, especially with persevering with difficult tasks, and she made a conscious decision to not always back down when Sammy became emotional or distressed, which she recognised was something she tended to do. She sought out information on 'hypersensitivities' (Dabrowski & Piechowski, 1977), and sourced articles on how to help gifted children cope with their intense emotions.

Yeah, when she does that [cries] we [used to] say, 'That's ok, we won't do it.' So now I *was* doing it, 'Keep looking at it...', and she'd get more upset and grumpy, but then she gets through it. It's just that bit that she has to get through. (Ms S, October 2014).

Ms S also introduced to the whole class a mathematics lesson expectation she called ‘Triple eX’ whereby she wanted the students to ‘Explore, Explain and Extend’: “Explore – what can you discover (in completing the task)?; Explain – prove your thinking; and Extend – how can you use what you have learnt to further challenge yourself?” (Classroom observation, August 2014). This was adapted from the lesson structure suggested as part of the teacher professional learning for this research (see Section 5.3.2).

During the post-professional learning period, there was a pre-service teacher working in the classroom and Ms S took this time as an opportunity to focus on her approach to extending her more capable students, and supporting Sammy, especially, in this. She also worked closely with Sammy’s parents, making them aware of the benefits of allowing Sammy to work through a problem rather than stepping in and ‘rescuing’ her.

I said [to Sammy’s mother] ‘I think the problem is that she is really good at everything, and she’s always been good at everything, and she doesn’t know how to fail. It freaks her out completely, and she won’t even get close to it because at the first little thought that something’s going to go wrong she’ll just shut down.’ And I said, ‘So it’s almost like you need to help her in providing opportunities for her to fail ... to let her fail and then realise that’s ok. That’s how you learn.’ (Ms S, October 2014)

In talking with Sammy’s mother, Ms S began to realise the issue of understanding struggle as a normal part of learning may have been something that was being inadvertently undermined at home. When mentioning the idea of providing Sammy with new opportunities for learning...

Her mother said, ‘Oh, yeah, I was thinking about getting her to do these cooking classes ... but I’ll let her have a few goes at home doing it really well first.’ And I was like, ‘No! That’s not the point, the point is to just throw her in!’ I don’t know how much of a difference [I’ve made], but I’ve just given them a whole heap of different papers and articles and whatever. Whatever I could get my hands on to help. (Ms S, October 2014)

She said that Sammy’s parents were very responsive to receiving suggestions of help. Ms S also found the ‘mindset chart’ (see Section 5.3.1, Figure 5.2) very beneficial, and it seemed to have a positive effect on Sammy.

One time in particular that I'm thinking of was a couple of weeks ago and she's like, 'It's too hard! It's too hard!' (How many minutes are there in two weeks, or something like that), so we went over to the little chart and I said, 'Instead of saying 'It's too hard,' try saying, 'This is tough, I'll need to take a while to think about it,' or whatever the thing said. And so anyway she was sort of doing the, 'It's too hard,' but compared to like six months ago where she would just freak out completely, she was aware that that was her fixed mindset, so she sort of persevered with it and worked through it ... Once she got past the first three minutes or so of saying 'It's too hard, it's too hard,' she started thinking about the problem and forgot all that other stuff. But it was just that transition from, 'It's too hard, it's too hard, it's too hard,' to 'Ok, I'm going to give it a go.' But that didn't take as long as it usually would ... Like you could see that she was actually, um, she'd changed. Rather than just doing it because I'm making her, pretty much, she was doing it because [indistinct]. (Ms S, October 2014)

Ms S's approach was primarily focussed on Sammy's mindset, but this focus was a means to an end. She recognised that unless Sammy's view of successful mathematics learning changed – from having no struggles, to being able to strategically work through struggles – she would not be able to expect Sammy to think deeper about challenging mathematics tasks, she would not be able to help Sammy develop confident mathematical arguments or justifications when explaining her solutions, and she would most likely not see Sammy risking independent thinking or creative approaches to mathematics explorations. She recalled one significant lesson just prior to Phase 3 and 4 of the study:

There was another time when she could have absolutely lost it – they [the class] were talking about recording the area of a certain object, so imagine they measured this bench and they recorded that it took 50 large playing cards. [James], the pre-service teacher, was writing '50' and then 'large cards' next to it, and [Sammy's] like, 'And you should put like a little square on the top of it; it's a number 2 and it means squared,' because she was trying to tell him about squared [sic] centimetres. And he was like, 'But is this playing card square?' And she's like, 'Well no ...' and I'd thought, 'Uh oh, things could go pear-shaped here', but no, she didn't lose it or anything ... and then she made the connection that, oh, it's actually centimetres squared [sic] because they were squares, and that was the whole reason behind it!

Usually she would just freak out because he said, 'but that's not a square'... but she was like, 'Oh!', and took it on board and then thought about it. (Ms S, October 2014)

When Sammy did not become distressed when the pre-service teacher questioned her misconception about recording area measurements, she was able to stop and consider where she had gone wrong, and made a significant conceptual connection about 'the unit of measurement' for area being 'a square'.

Ms S suggested a turning point in Sammy's thinking may have been when she recognised she could use trial and error as a legitimate problem-solving strategy, and that 'mistakes' can form part of the problem-solving process.

There was this problem, and anyway it was too hard for her and she couldn't do it. She kept failing at it, and then she'd get more upset, and more upset, and I was like, 'Ok, so you know how you actually made these mistakes, that's ok, that's not a bad thing because they can actually help. What do you notice about these mistakes?' and then she was just like, 'Umm, well I can't do it like this or like that or like that... so it must be this!' So, she actually used trial and error rather than just going mistake, mistake, mistake, mistake. She actually reflected upon what mistakes she had made and eliminated those as options, which helped her finally find a solution to the problem (I think it had something to do with fractions) ... This went on for half an hour, I think, but the whole thing she got out of that was that mistakes aren't always bad, they can actually help you out. So, I think that was probably the turning point, if I was to say that there is one. (Ms S, October 2014)

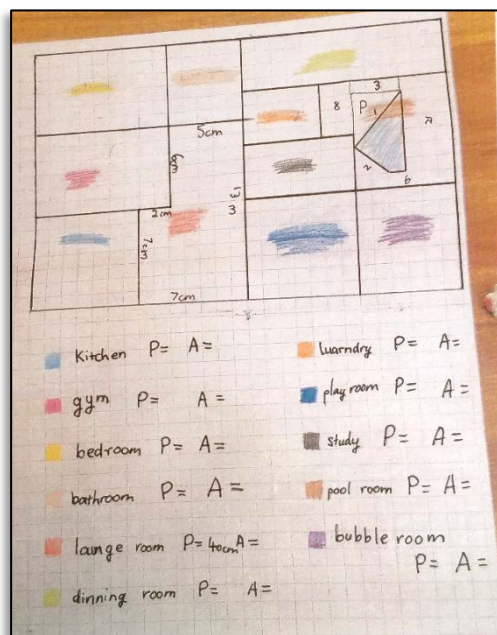
There was another time Ms S recalled, when Sammy was able to successfully stop and reflect on her incorrect solution:

Anyway, it turns out the answer wasn't correct, but she figured out that she had skipped a few little steps, so she actually figured out what she'd done wrong, so that was perfect, and she was quite happy with that. It was the first time I've ever noticed her being happy with herself when getting something wrong ... because she did realise that her mistakes actually helped her to get the answer. Rather than just being 'pointless waste of time mistakes' she actually used those to help her hone in on what she needed to do to find the solution. She actually figured out how to go about solving the problem. (Ms S, October 2014)

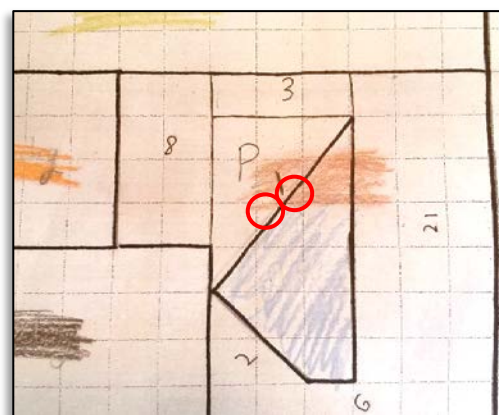
Many of Ms S's recollections during Phase 3 of the study indicate that she was noticing a change in Sammy – a change in her disposition, or mindset, which, in turn, changed her approach to completing mathematics tasks.

6.3.4 Sammy's Mindset and Approaches to Mathematics Learning – Post-teacher Professional Learning

During the Phase 3 classroom observations, I witnessed a child who was more willing to take risks. She still became excited and animated when faced with new ideas to explore, but was now also much more willing to stop and think through things that proved to be hard, or didn't initially make sense to her. This resulted in her being able to explore more complex mathematical ideas. In one lesson Sammy was working on drawing up a house plan – the focus for the lesson was applying area and perimeter to real life settings. The class task was to design and draw a house plan with seven rooms, and calculate the total floor-area and perimeter of each room. Students were using one-centimetre grid paper to draw their designs. To increase the challenge for the more capable students, Ms S requested they design *ten* rooms. Sammy seemed very proud to tell me, “[Ms S] is giving me really hard puzzles to do, and work even harder than this [indicating her house plan]” (Sammy, November 2014). She then decided that ‘ten rooms’ was still too easy for her and raced off to ask her teacher if she could do eleven rooms instead. She decided to include some ‘L’ shaped rooms, and a pool room with an irregular shaped pool (see Figure 6.7).



Sammy's house design floor plan



Close-up of the irregular shaped pool with complimentary corners circled in red

Figure 6.7 Work sample of Sammy's house design floor plan

Drawing up the house plan required some creativity, but no difficult mathematical thinking. The harder mathematical thinking came about when she had to work out the floor area. The 'L' shaped rooms proved to be a little more time-consuming than the regular rectangular rooms, but still not particularly challenging. However, when she came to the pool room she realised she had to calculate the area of the pool and subtract this from the area of the whole room to work out the actual floor area. Her first response was, "I wish I didn't have a triangle [sic] pool in my pool room!" and considered changing it. I questioned her about making it too easy, and she went red in the face and looked possibly a little anxious (but not particularly upset or distressed), so I reminded her that 'not too easy' meant her brain could grow (from Ms S's growth mindset discussions). She returned to her irregular shaped pool and recognised that she could use the grid paper to help her calculate an approximation of the area, indicating that she understood the concept of area, not just how to apply a formula for calculating it. She could see, for example, that a shaded triangle piece in one square could be reflected to almost fill an unshaded triangular piece in another square (see highlighted circles in Figure 6.7), resulting in an approximate square centimetre. This was challenging for Sammy, and time consuming and frustrating, and she was not able to finish the task in the lesson. However, her response to me was, "I want to show you this when it's finished next time you come in!"

In the Phase 3 lessons I observed I did not hear Sammy once mention anything like 'It's too hard,' or 'I'm no good at maths.' When I asked her about this in the Phase 4 interview Sammy described how her teacher had been helping her learn how to not say things like that by drawing up a chart to help her change her mindset, and she drew an example of the teacher's chart for me:

...Like, 'I can't do it', and she has all negative stuff here [indicating the left side of her chart], and then she reversed them here into positives [indicating the right side of her chart] to something like, 'I'll work hard to get the answer, but I might not be able to get it right just now'. (Sammy, November 2014)

I asked if she was stopping and consciously choosing not to say, 'I'm not good at maths' now? She stopped and thought, and seemed quite surprised before exclaiming, "I *don't* think it anymore...It's just kind of worked like magic!" (Sammy, November 2014). This was a delightful moment in the interview for both Sammy and me.

Sammy certainly seemed to be improving, with a more positive mindset towards challenging tasks being evident, however, her teacher recognised it was going to be an ongoing, longer-term issue for her.

I have noticed a little bit of a change. It's not a huge change, and it's pretty hard to go from having a fixed mindset to 'ah' a growth mindset just like that. That's not going to happen ... But I think she's starting to understand her negative thought patterns (Ms S, October 2014).

Sammy was still displaying some intense emotions at times. In Phase 3 lesson observation, I observed Sammy working with Janet (another nominated mathematically highly capable student). In this lesson, taken by the pre-service teacher, the focus was on measuring volume, and the task was 'How can you measure the space inside your shoe?' Students were given two-centimetre multi-link cubes to work with (as the one-centimetre cubes they had could not be linked together). Throughout the course of the lesson, Sammy and Janet moved from constructing a three-dimensional model of Janet's shoe and calculating how many multi-link cubes it took (i.e., the volume with a non-standard unit of measure), to wondering what the volume would be with a standard *one*-centimetre cubic unit of measure. Their initial prediction was to simply double the number of multi-link cubes, but when physically comparing a multi-link cube with two one-centimetre cubes realised that doubling was not going to be sufficient. They were fascinated and intrigued when they realised that it was actually eight one-centimetre cubes that were equivalent to one two-centimetre cube, with Janet identifying this as $2 \times 2 \times 2$. Sammy was, once again, very keen to share their findings with the whole class, and was able to accurately explain that eight one-centimetre cubes fit into one multi-link block, so you would have to multiply the multi-link volume of the shoe by eight, not double it, to calculate the volume in one-centimetre cubes. At this point the Principal came into the class with some school visitors and was very impressed when he heard about Sammy and Janet's mathematical discovery, saying he had Grade 5 students who had trouble with grasping this concept of eight-fold. He publicly praised their effort. Surprisingly, just as in the array lesson at the beginning of the study, when the class was dismissed for lunch we discovered Sammy was once again highly distressed. Instead of being elated by the Principal's affirmation, as we expected, she was upset because he had only mentioned Janet by name. She asked her teacher, "Do you think [the Principal] knew it was me too?" before dissolving into uncontrollable, hiccupping sobs. Again, she could not be consoled, and her teacher said that she was still upset that the

Principal may not know that she could do “that hard work too” even after the weekend. In talking with the Principal about this he could not understand why she was so upset, and seemed to see it as simply an over-reaction, which is a common problem for gifted students with intense emotions (cf. Silverman, 2013).

Sammy’s concept of challenge was another ongoing issue. We (Ms S and I) had been using the term ‘Goldilocks zone’ (MacRae & Furnham, 2014) when talking to students about challenging themselves. We explained the Goldilocks zone as being work that is ‘not too easy, not too hard, but just right’ (i.e., the zone of proximal development). However, it became apparent from Sammy’s responses during the Phase 4 interview that she still seemed to be classifying any task requiring sustained effort as ‘too hard’:

[Sammy]: That’s heaps too hard [pointing to a task she had just completed with minimal support] that is NOT my Goldilocks zone! [said very vehemently, but not angrily]

Researcher: Well you solved the problem...

[Sammy]: Yeah, but it’s not my Goldilocks zone.

Researcher: But you proved it wasn’t too hard, because you did it.

[Sammy]: It *was* too hard! (Sammy, November 2014)

I reiterated that with learning, the Goldilocks zone, the bit that is hard but not too hard, is work that you will not be able to do all by yourself, you will need some help. If you can do it all by yourself without any help, it is ‘too easy’, and you will not learn anything new. Sammy looked genuinely surprised by this, even though it was not the first time it had been explained to her. From this it was decided to no longer use the term Goldilocks zone, but rather focus on reinforcing the importance of effort and productive struggle in the learning process.

I also asked Sammy how her teacher helped her ...

Researcher: When you do work that’s really hard how can [your teacher] help you the best? What does she do that helps you the most?

[Sammy]: She’s just stubborn! [laugh]

Researcher: She’s stubborn? What does that mean?

[Sammy]: ‘You shouldn’t stop, you just keep going.’

Researcher: You shouldn’t stop – that helps you, does it?

[Sammy]: Yeah. [laugh]

Researcher: Do you like it when she does that?

[Sammy]: Ummm ... sometimes. (Sammy, November 2014)

A bit later in the interview I asked, “What could [your teacher] do to help you challenge yourself more?” and her reply was, “... give me even harder problems and be more stubborn” (Sammy, November 2014). It seemed that, as much as Sammy did not always appreciate her teacher’s support, she was beginning to realise the value of sustained effort and perseverance.

I concluded the interview by asking her to recall some ‘maths’ she had done that she was really proud of. She straight away mentioned some work she had done on area and perimeter (just prior to the house plan lesson I had observed). She had previously shown me this particular work when I was in the classroom, so it was obvious she really was very proud of it. When I asked her what she was most proud of, she said, “Well, it was just so hard that time ... yeah, just soooo hard! Yes, it’s just so hard (laugh) I thought I couldn’t do it, but then I did.” (Sammy, November 2014).

6.3.5 Sammy’s Story: Review

The classroom observations in Phase 2 (prior to the teacher professional learning) and then Phase 3 (about three months after the professional learning) provided evidence of Sammy’s mindset becoming more positive, and her willingness to engage in challenging tasks appears to be linked to this. Her teacher summarised the three-month period with this statement:

[Sammy’s] been hard work but I’m seeing little things, like little glimpses of positivity in the fact that she’s sticking at something, or she’s not completely crumbling. She still might get annoyed, or she still might express her frustration, or say ‘I can’t do this,’ or ‘I don’t want to do anymore,’ but she’s starting to stick at things a little bit more. And when she’s questioned she’s not falling in a heap, she’s sort of thinking ‘Okay ...,’ and then looking at something a little more deeply rather than just thinking ‘I’ve failed,’ which has been the biggest positive I’ve noticed. (Ms S, October 2014)

However, these changes did not happen without intensive and sensitive teacher support and encouragement. The experience proved to be a challenging time for both Sammy and her teacher.

That's probably one of the biggest things I've learnt this year. Because she is really great at everything, you wouldn't necessarily look at her and think, 'This kid's struggling,' but she is probably struggling more than anyone in the class, but in a different way. She's been my biggest struggler this year ... It's exhausting. And that's in with all the other things with all the other kids! So, yeah, it's been a big few weeks. (Ms S, October 2014)

By the time of the Phase 4 interview, Sammy was recognising, and admitting, that she *was* 'good at maths.' When I asked her how she knew she was good at maths her response was,

I know I'm good at maths because I did that [pointing to a task she'd just persevered with for over 30 minutes] and I thought it was too hard but I did it! (Sammy, November 2014)

Sammy was now more willing to challenge herself, and was happy and keen to engage in extension work suggested to her, but according to her teacher, "She's still not at the point where she can think of ways to extend herself, so I think that's still a major focus." (Ms S, October 2014). However, Sammy's final statements may be evidence that this will now become possible for her with a little more encouragement and scaffolding from her teacher.

[Sammy]: It's magic – being able to tell yourself those positive things instead of thinking all the negative things.

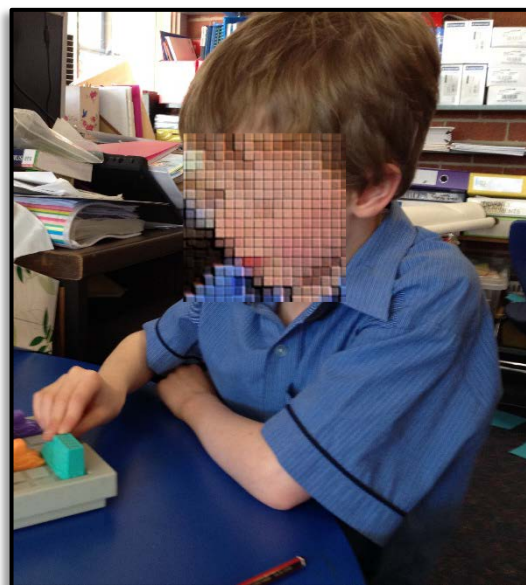
Researcher: I think you've had a very successful year [Sammy]. What do you think?

[Sammy]: Yep, I'm proud of me! (Sammy, November 2014)

As with so many things, prevention is better than a 'cure'. If teachers are aware of the impact of mindsets on students who are mathematically gifted, and make sure these students understand that hard work, effort and perseverance are a normal and expected part of learning, right from the earliest days of schooling, maybe it would be possible to prevent some of the negative mindset issues both Fred and Sammy struggled with. Considering Alex's story, from a Grade 1 student perspective, may help inform this conjecture.

6.4 Alex – Grade 1

Having turned seven in February, Alex was one of the older students in his Grade 1 class, and yet he was also one of the smallest. Indeed, there was only one other student smaller than he was – one of the nominated mathematically highly capable girls. However, although being so slightly built, he was in no way diminutive in character. He appeared confidently outspoken when I first met him for the first (Phase 1) interview. He was able to answer questions and offer further information readily, and was also able to maintain and generate general conversation. His first comment, when I asked him if he was happy to do some mathematics with me,



“Alex”

was, “I love, love, love maths!” (Alex, May 2014). Alex had two teachers, Ms K and Ms C, who both taught part-time in the Grade 1 class, with Ms C taking most of the mathematics lessons. Ms K described Alex’s speech as ‘old man’ talk, which seemed quite apt. For example, to help evoke an image of Alex, when asked how he knew which students in his class were good at maths, he replied,

I've been working with them a couple of times, and most of the time, like, wow! They blew my socks off I give them a question, like four hundred and eighty-five plus nine hundred and sixty-four, and always they estimate and they're very close. And about one time my sock really actually, one sock nearly came off! (Alex, May 2014)

His response to ‘Do you think it's important to be good at maths?’ was,

Ah ... I think so, because if you decide to be a mathematician, and you want to earn lots of money, you do earn lots of money. Then again, if you're not good at maths, and you decide to be something that doesn't involve maths, you couldn't earn such very much money. (Alex, May 2014)

And as for ‘What sort of work do you think mathematicians do?’ he surmised,

Well, they find out new ways to calculate ... and new number facts. They could be helping people learn in a most exciting way. (Alex, May 2014)

Alex lives with his mother, and, according to Ms C, saw his father every second weekend. He has one brother who is three years younger than he is. His father is a tradesman, and his mother a part-time office worker. According to Ms K, family issues had caused some grief for Alex the previous year, but were not affecting him so much at the time of this study. Ms C described Alex and his father as being almost opposites, with his father being “quite a hard, black and white, working tradie type,” and Alex being “a sensitive boy ... even feminine in his way of sometimes presenting himself.” (Ms C, August 2014). Alex’s mother seemed to have had a somewhat chequered past, having admitted to Ms K that she had been concerned about Alex as a baby because of her lifestyle when she was pregnant. Alex’s exceptional mathematical ability seems to be a source of surprise, pride, and a little fear for her.

Because he's shown that he's clever, she [Alex’s mother] hangs her hat on that a little bit. ‘My son, I've got a clever son, I might have had past misdemeanours but I've got a clever son.’ (Ms K, August 2014).

I would describe her as somebody who likes to, maybe for her own security, wants the teacher to know just how clever [Alex] is with mathematics, and that her expectation is that he is extended, and he needs to be challenged, and she wants it done regularly, and she wants to see what the results are. Even though, I think, whatever we do with [Alex] is possibly out of her comfort zone ... She said, ‘I don’t know anything about fractions. I can’t help him, he’s on his own there.’ (Ms C, August 2014).

Alex’s mother also seemed to have high expectations of the school in providing for Alex’s learning needs.

She's very vocal in terms of negative and positive things, ‘oh that was just baby stuff.’ ... At the start of the year she really wanted him pushed ... she had high expectations that he would ... be doing grade 6 maths. You know there's that perception that if you're doing grade 6 maths then you're really clever and that sort of thing, and he's doing ‘baby maths’ because we're in Grade 1. (Ms K, August 2014).

Alex’s teachers gave the impression that his mother was quite outspoken about Alex’s abilities and her expectations of the school, having admitted that her own mathematical understanding of “things like fractions” was limited, so she would not always be able to help him.

According to Ms K, Alex's academic ability was extensive, not limited to mathematics; that he seemed to "absorb information by osmosis, whether you're teaching him or not he just seems to get stuff." (Ms K, August 2014). She also commented,

[Alex] doesn't make a fuss about anything. He's a very quiet worker. He doesn't draw attention to himself ... He's one of the ones you have to be careful doesn't go under the loop [sic], because you can set him a task and he'll do it, so you have to remind yourself to go and see where he's up to or what he's doing. (Ms K, August 2014).

6.4.1 Identification of Alex's Mathematical Capabilities

Teacher nomination. On a scale of average (1) to very capable (3) to highly capable (5) to extremely capable (7) Alex's teachers rated his mathematical capability at number seven, extremely capable. He was the only student from the ten Grade 1 nominated students who was rated this high. The teacher's comment on the nomination form was, "[Alex] is extremely capable, and he is aware of his mathematical ability," with no other description or example of his exceptional mathematical capability. It is not known if this was a value judgement, that they thought it was considered pretentious of him to be aware of his abilities, or whether it was just a statement of fact. Descriptions given for the other students nominated by these same teachers included, 'Adventurous with his thinking' (Frank), 'Demonstrates great reasoning' (David), and 'Solid mathematical strategies' (Brony). Young gifted students are often seen as 'bragging' when they matter-of-factly tell you they are good at something like mathematics. However, for them it may be simply a normal observation, just as a young child may tell you they are a good runner. They may not have yet learnt the social conventions of their culture that dictate which behaviours are acceptable to self-disclose publicly, and which are not (Ruf, 2013; Silverman, 2010).

Parent questionnaire. On a scale of low (1) to average (3) to high (5) to very high (7) Alex's mother indicated that she thought his mathematical capability was high (5). This was lower than expected given her outspoken pride in his mathematical abilities. Alex's mother said that she recognised his mathematical abilities from an early age:

[It] started when [Alex] was two. He would count the car parks and mailboxes. He would read out the prices at the checkout supermarket [sic]. Checkout used to encourage him to read out prices before scanning the next item. (Alex's mother, written questionnaire, March 2014).

Alex attended 3- and 4-year-old kindergarten, but none of the teachers mentioned anything specifically about his curiosity and/or ability with mathematics type activities. According to his mother, Alex also taught himself to read and has an ‘exceptional memory’.

Archival records – previous mathematics assessments. Alex’s Grade 1 number growth point profile, from the *Mathematics Assessment Interview*, places him within the top 1%, at least, of Grade 1 students, based on data from the *Bridging the Numeracy Gap Pilot Program (BTNG)* (Gervasoni et al, 2013) (see Table 6.5). Alex’s number growth point profile was 6454; the median growth point profile for the four number domains for the Grade 1 students in the *Bridging the Numeracy Gap* project was 2121. Indeed, no other Grade 1 student reached growth point six in Counting (with only 1% reaching growth point five), and no other Grade 1 student reached growth point four in Place Value (with only 1% reaching growth point three) (*cf.* Figure 4.1).

Table 6.5

Comparison of Alex’s Grade 1 Number Growth Point Profiles with BTNG Students

MAI Number domain (range of GPs)	Prep Growth Points	Grade 1 GP comparison with BTNG students
Counting (GP0-GP7)	GP5	GP6 top 1%
Place Value (GP0-GP6)	GP2	GP4 top 1%
Addition & Subtraction Strategies (GP0-GP7)	GP2	GP5 top 1%
Multiplication & Division Strategies (GP0-GP8)	GP2	GP4 top 1%

Not only is Alex’s number growth point profile extremely high, his growth in number understanding in his first year of formal schooling was also exceptional – from a growth point profile of 5222 at the beginning of Prep (the first year of formal schooling), to 6454 at the beginning of Grade 1. The mean growth in the number domains is one growth point per school year, with Place Value being slightly less than one growth point per year (Clarke et al, 2002). Alex has demonstrated a growth of two growth points in Place Value, three growth points in Addition & Subtraction, and two growth points in Multiplication & Division. In the Multiplication and Division domain the interviewer noted Alex’s use of

'intuitive strategies'. For example, for the question 'I put 20 biscuits on an oven tray. I put four biscuits in each row on the tray. How many rows of biscuits are there?' his answer indicated that he knew that, " $3 \times 4 = 12$, so, then I would need two more rows of four," (i.e., $3 \times 4 + 2 \times 4 = 5 \times 4$). This shows a beginning understanding of the distributive property of multiplication, certainly not something typically taught in the first year of formal schooling. For the question, 'There are eight stickers in each packet. How many stickers are there in six packets?' he reasoned that 6×8 was equivalent to 3×16 , and he could count by 16s. This again shows a precocious understanding of the structure and properties of multiplication. As a comparison, at the beginning of the year almost all Grade 1 students in the *BTNG* program (99%) required physical objects to be modelled in order to solve multiplicative scenarios (e.g., 'Here are four teddy cars. Each car has two teddies. How many teddies is that altogether?'), with the majority (58%) still relying on counting all objects one-by-one, as opposed to recognising groups and/or skip counting, to determine the total.

Clinical task-based mathematics interview. In the task-based interview designed for the study, Alex showed an ability to learn new concepts easily, to generalise and assimilate new knowledge, and to reason using intuitive strategies – all hallmarks of Krutetskii's (1976) observations of mathematically gifted students (see Table 6.6).

Classroom observation. In the first Grade 1 classroom observation I participated in (Phase 2), Alex was given the task, 'There are 5 packets of seeds with 9 seeds in each pack. How many seeds altogether?' This question was tailored specifically for him by his teacher, to elicit deep mathematical thinking. He immediately started to draw dots in an array. After drawing three rows of nine dots he stated, "I need to do eighteen more." When I asked him how he knew that, he said, "Well, I went five times five is definitely twenty-five, then I added twenty more ... because there's another four [more fives] till nine ... so I knew it was forty-five." Just as with the array lesson with Sammy, I realised Alex was drawing the dots to *represent* the problem, not to solve it. This may be an issue with those gifted students who are compliant, and want to do exactly as they believe the teacher expects them to do. So, I asked if he knew, 'How many seeds in 6 packets of 9 seeds?' His answer of 54 was solved by knowing that "... three nines are twenty-seven ... and then I add on another twenty-seven." I then asked, "What if there were 5 packets of 9 seeds, but only one-third of the seeds were left in each packet?" His immediate response was, "So there are three seeds in each packet? ... That's easy, it's fifteen." When asked to explain his thinking he

Table 6.6

Summary of Alex's Clinical Task-based Mathematics Interview Responses

Observed ability	Evidence
Learns new concepts easily	Alex noticed the structure of the abacus very quickly with no further assistance after the initial instruction. Counting forwards, he was momentarily stuck at 50, but was able to reason through this without further prompting; counting backwards from 34, he hesitated slightly at the first decade transition only (30-29), and then continued to zero with no further hesitation.
Generalises and assimilates new knowledge	Very quick and confident in representing requested numbers on the abacus – realising that he didn't need to start from zero each time – ‘I'll keep these up (154) and just add beads’ to make 2189. Only two other Grade 1 students could represent all numbers successfully. Having been shown the structure of the abacus being in columns – units, tens and hundreds – he was immediately able to recognise this as a place value structure, and could tell me the next three columns would be thousands, tens of thousands and hundreds of thousands.
Reasons using intuitive strategies	Alex was unable to correctly answer the question ‘If 3 lollies cost 10c, how much would 12 lollies cost?’ His response was ‘I knew that four 3s is 12 so it's \$1.20. However, with the Adding Corners task he chose to partition the number 5698. He was able to reason through his response by manipulating this, and other resulting numbers very efficiently, using mental strategies for adding, subtracting and finding the difference.

said, “Five nines equals forty-five, cut in half [sic], which equals fifteen. I knew that ‘cause a third [of nine] was three, I added zero onto it, it became thirty ... which I knew was two-thirds ... so I just added thirty off [from 45], and I got fifteen!” I was having difficulty differentiating the task sufficiently for Alex to provide a deeper mathematical challenge for him. It wasn't until he stated, “I believe you can only half [sic] even numbers,” that I found something for him to explore that he could potentially learn from.

The archival *Mathematics Assessment Interview* data would appear to situate Alex well and truly in the ‘mathematically gifted’ (top 10% of mathematical capabilities in the general population of age peers), in the number domains at least. The teacher nomination data confirmed Alex's capability as being observed generally within the mathematics classroom, even though no specific examples were given, and his mother's anecdotal data confirmed his mathematical capability as being evident from a very early age. The task-based mathematics interview data confirmed his ability to learn new mathematical concepts

easily, and assimilate and apply this new knowledge. Although he was unable to answer the proportional reasoning problem correctly (see Table 6.6), the way he mentally added and subtracted 4-digit numbers was remarkable (and will be described in the next section). Due to the flexibility of his thinking and the speed with which he was able to calculate, his approach appeared to be intuitive rather than employing learned procedures. The *Mathematics Assessment Interview* also shows evidence of the use of intuitive strategies in solving multiplication problems. In the classroom observations, I found it difficult to find a level of number complexity whereby Alex could not easily solve a problem with facts and methods he already knew, indicating that he was certainly working well above the level of his age peers. These data suggest that Alex satisfies this study's definition of being mathematically gifted.

6.4.2 Alex's Mindset and Approaches to Mathematics Learning – Pre-Teacher Professional Learning

Alex, however, exhibited some typical fixed mindset tendencies. His response to the Adding Corners task in the Phase 1 task-based mathematics interview was one indication of this. For this open task, Alex chose a four-digit number to put in the centre of his triangle (see Figure 6.8) – “I thought of a big number; I chose five thousand, six-hundred and ninety-eight” (having been given the Grade 1 options of 85, 150, or ‘any other number you like’). He proceeded to partition this by initially starting with 300 in the first corner and 98 in the second corner, and then proceeded to work out, “Hmm, how many more hundreds will I need to get to five-thousand six-hundred?” He decided the third number might be 5060, which he wrote down in the third corner, but he wasn't sure, “I think it's five thousand and sixty, but I'm not so sure about that,” so mentally began to add the three numbers together to check. He quickly realised that the result would not be enough, so changed the 98 to 198, then the 300 to 500. After a few seconds of further thought he changed the 198 back to 48, which left him with $5060+500+48$ (see numbers highlighted in red in Figure 6.8). At this point he started to get frustrated, “Oh damn, that won't work out either! What'll I do? Think!!”

Up until this stage he had been calculating everything quickly, but now took longer to consider what he had. He muttered “I need another 90” (which was correct), and changed the 48 back to 98. He then said, “Oh, what am I doing here!? ... I'm thinking about crossing that out [the 98] and changing it to zero but that wouldn't be very creative though. How will I do this!?” He decided, “... I'll have to change all the numbers!” and crossed out all the

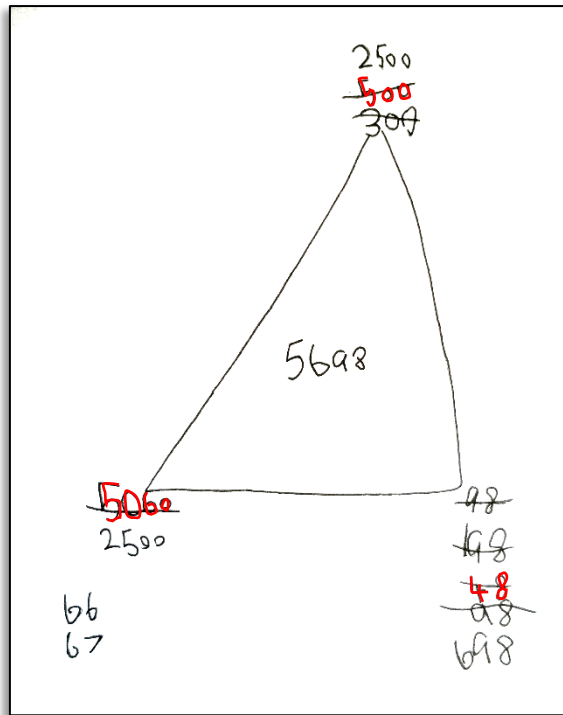


Figure 6.8 Alex's solution to the Adding Corners task (Phase 1).

numbers he had written so far and wrote instead, “twenty-five hundred” and “two thousand five hundred” and “six-hundred and ninety-eight”. Alex’s process of finding a solution for the Adding Corners task showed evidence of his flexibility with number, and the reversibility of mental processes, as described in Krutetskii’s (1976) model of mathematical abilities. Alex indicated, with the Smiley Chart (see Appendix 2), that he was happy with this final solution, but not *very* happy.

He was happy with the fact that he knew $2500 + 2500 = 5000$:

Not many people know what two thousand five hundred plus two thousand five hundred is, not even twenty-five plus twenty-five ... most people don't actually know how to count by twenty-fives ... not even my mum knows how to count by twenty-fives ... I asked her to count by twenty-fives and she was like twenty-five ... forty-seven! (Alex, May 2014)

He was not *very* happy, though, because,

I had to cross out most of the things, I just couldn't get it right, I thought ‘Oh, how do I do this!?’ I got very frustrated, but in the end I finally got it. I knew it was more than ‘ok’ but I wasn't *very* pleased with it, so I just went ‘happy’. (Alex, May 2014)

When asked to come up with a solution for 5698 that he was very happy with, he went straight to $5698 = 98 + 2500 + 3100$; a solution that took him less than 20 seconds to complete (see Figure 6.9)

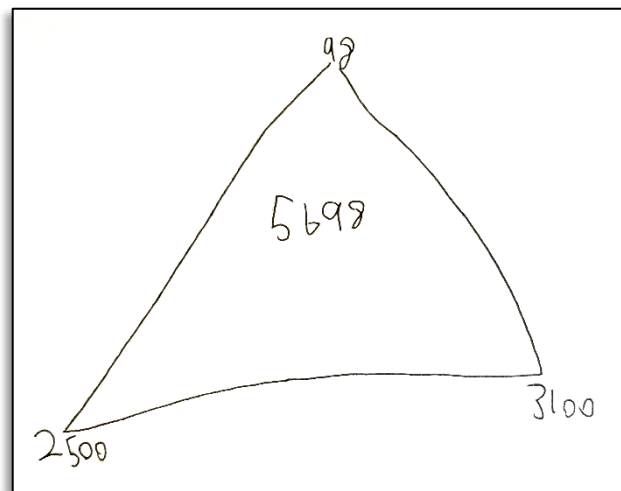


Figure 6.9 Alex's second solution to the Adding Corners task.

His explanation was,

I knew it was supposed to be six-hundred, and I knew that five-hundred plus one-hundred is six-hundred, and I knew that two plus three is five, and since they were thousands [the 2, 3 and 5] they're thousands, and 98 is the tens [sic]. (Alex, May 2014)

Alex was *very* happy with this second solution because he completed it quickly, there were no 'crossings out', and he felt it was something that not many other people would be able to do.

Not many people know ... They go 'one hundred, two hundred, three hundred, four hundred, five hundred, six hundred, seven hundred, eight hundred, nine hundred, one thousand, oh... I don't know what's next ... I have no idea what comes next' [after 1000] ... About one other person in my class could think about this, that's all ... about only two people in the school could do it. (Alex, May 2014)

It seemed that Alex was *not as* happy when the task required time, effort and sustained mathematical thinking and reasoning. He was *very* happy when he could complete the task quickly and easily, with no mistakes, and when it was something he believed would be too hard for others to do. Dweck (2010b) would describe each of these responses as following typical fixed mindset "rules" – being able to effortlessly work out the solution quickly (rule: "don't work hard"); avoiding mistakes by choosing a simpler solution (rule: "don't make

mistakes”); needing to appear smart compared with others (rule: “look smart”). According to Dweck (2010b), “The fixed mindset comes with ‘rules,’ the cardinal rule being: *look smart at all costs*. Not surprisingly, this rule stands in the way of learning.” (p. 7). The concern for Alex was that, although he was currently achieving at a very high standard in mathematics, his belief of what was expected of him at school (being fast and accurate), may potentially prove to be self-limiting, affecting his ongoing long-term mathematics learning, and resulting in underachievement, and/or a diminishing love of mathematics in higher grades.

Ms K suggested that she felt Alex exhibited fixed mindset behaviours as he would constantly ask to work with big numbers. She believed he thought he was smart because he understood big numbers that other Grade 1 students did not yet understand.

He does have quite a fixed mindset about, if it's a big number that means ‘I'm clever’ and that means I'm doing something really important. (Ms K, August 2014)

When Ms C was asked if she thought Alex had fixed mindset tendencies she replied,

He does, because if I do something like fractions, that he's unfamiliar with, his immediate go-to with that is, ‘No, I've never heard of fractions before,’ and he sort of shuts down ... With certain things he's got a lot of self-confidence with mathematics, he can be seen thinking about something and processing it. But sometimes, if there's something he's not sure of he'll back right down to the point of almost being tearful about it. That translates into other work he does as well, but even more so [with maths], because I'd say maths is something he knows that he's good at. (Ms C, August 2014)

Dweck’s (2010b) observations of students when given an opportunity to learn something at a deeper level, was, “students with a fixed mindset were not enthusiastic – they didn’t want to be in a situation where they would not look smart.” (p. 7). This came from working with 12- and 13-year-olds in Grade 7. As Alex was showing evidence of this as a seven-year-old, in only his second year of formal education, this could have implications for special provisions designed for gifted students that are often not offered prior to Grade 3 (Reis et al., 2004; Sheffield, 1999).

Alex had no hesitation in claiming himself to be the ‘best person at maths’ in his class. When asked how his mathematical abilities compared with others in his class, he said, “Um... up the top! ... I'd be right at the top.” When asked if he liked ‘hard maths,’ and given a scale of never, occasionally, sometimes, mostly or always, he circled ‘Yes, always,’

saying, “I really love challenging myself ... so if I challenge myself I believe that I can learn more things ... and I love learning.” He also said, “I like challenging myself, and if I get it right I’m like, ‘Wow! I had no idea what that was, I had a go, and look, I got it right!’” (Alex, May 2014). So, I asked him what would happen if he challenged himself and discovered it was something he *could not* do. He said, “I’d go, ‘Oh, I might learn that next year’ ... leave it till next year,” and went on to explain to me that the only thing he did not know was division.

[Alex]: Mostly I don't know my divided bys [sic]. That would be the only maths thing I don't know, my divided bys.

Researcher: Ok ... so you're happy to wait until next year to learn that?

[Alex]: Yes, I'm happy to wait. Or I could wait till Year 6 if I had to.

(Phase 1 interview, May 2014)

It was common for Alex to tell me that ‘other people can’t do this.’ Every time I asked him what was creative about a solution (in open tasks), he would reply with something like, ‘because other people can’t do it’. For example, in the Adding Corners task, in the Phase 1 task-based mathematics interview, he uttered words to this effect seven times in less than 15 minutes:

- Not many people can count by nines [so I started with nine];
- I know not many people can count by 14s, so I went to 14;
- Not many people know what two-thousand five-hundred plus two-thousand five-hundred is;
- Most people don’t actually know how to count by twenty-fives;
- Not many people know that;
- Everyone else must copy someone [indicating they would not be able to do it by themselves like he could];
- I chose something I think mostly other people can’t do. (Alex, May 2014)

It also seemed important to Alex to let me know that things he may have initially found hard in mathematics he no longer struggled with. For example, when he was asked about some ‘hard maths’ he remembered doing, he recalled learning to count by thirteens:

[Alex]: Counting by thirteens. Yes, that was very hard. That was about when I was three.

Researcher: So, it’s not so hard now?

Alex: Way not! (Phase 1 interview, May 2014)

It appeared that Alex's idea of a mathematical 'challenge' was to be given work that was too hard for other students, but not necessarily difficult for him. He indicated that he was happy to wait until he was older to learn mathematics he did not yet know. Ms K recalled a similar response whereby she felt that when there is something he does not know, he does not feel particularly motivated to learn it.

He has a really good base of lots of things, like with money, like with shapes – he knows lots of different shapes – he has a broad knowledge, but then there'll be something he'll go, 'Oh, that's strange, I don't know that,' but it's like, 'Well that doesn't matter because I know all this other stuff so it's alright.' (Ms K, August 2014).

This could be an indication that Alex simply feels no pressing need to learn things he does not yet know, or it could be a symptom of a mindset issue, where considering the possibility of learning something new exposed him to risks he was not prepared to take (Dweck, 2006). According to his teachers, feelings of inadequacy when having to persevere with unfamiliar tasks were evident with Alex, with tears and/or a refusal to keep trying.

It appeared that providing for Alex's ongoing mathematics learning needs may require more than simply giving him challenging mathematics tasks within his zone of proximal development. For Alex, 'challenging maths' seemed to revolve around being able to work with 'big numbers,' and being able to skip count – by sevens, nines, fourteens, thirteens, sixteens, twenty-fives. This may have been due to a limited understanding of what mathematics actually encompasses, with Alex being so young, or it may have been a reflection of his mother's understanding of mathematics.

6.4.3 Grade 1 Teachers' Approaches Post Professional Learning

The teaching approach, post professional learning, was substantively different in the Grade 1 class from the approaches in the Grade 5 and Grade 3 classes because there were two teachers. One teacher was confident in teaching mathematics; the other felt somewhat inadequate:

I'm scared. This is where I feel like a pre-service teacher, I'm scared about doing that [the professional learning suggestions] the right way ... I don't see maths as a strong point for myself, and whether that ever comes through to the children or not I don't know. I try for it not to, but it's something I've always seen as a deficit in myself. (Ms K, August 2014).

Ms K explained that her ‘teacher training’ (in the 1980s) had very much focused on procedural approaches to teaching mathematics, and although she had tried to change over the years she still found it difficult. She was also concerned about being able to extend the more capable students.

Our training as carers, as nurturers, as training children, has been to pick up that lower end rather than extending forward, and so it can go back, especially with my being a little bit older, I probably haven’t had the training. (Ms K, focus group discussion, April 2014)

She was very pleased with the outlined classroom expectations, with suggestions for how to encourage students to explore mathematics tasks further, and the suggested chart to redress fixed-mindset behaviours (see Section 5.3.1). She said, “This is what I need!” but was still concerned about the differences in teaching approaches and expectations between herself and Ms C.

We are quite different. Good or bad I don't know, but we are quite different. I know that my expectations on certain things are quite different ... and the way we approach things. (Ms K, August 2014).

Ms K’s approach seemed to focus on encouraging Alex to think deeper mathematically about tasks rather than just completing them. For example,

We did a race to 30 where we add on one, two or three, and see who gets to 30 first ... and [Alex] was just working with [Frank] [one of the other Grade 1 nominated mathematically highly capable students], and I said, you guys might like to work it out ... like is there a magic number where you can get to where you can decide who's going to be the winner? And they worked out a number, and I said, well can you change that then? If you know that number, how far back can you go to make sure you have that [magic] number? (Ms K, November 2014).

She said she struggled with how to encourage Alex to explore mathematics tasks further, implying that this was something that she needed to work on further.

If you suggest like what else can we do, or you ask those prompting questions he's like, ‘Why? I've already got the answer.’ He can't see how to explore further, and that's a skill, I guess, where we need to show him how to go further on that. (Ms K, November 2014)

Ms C, on the other hand, felt that many of the ideas and strategies included in the professional learning were approaches she was already using. With the idea of ‘exploring further’ she commented,

I did that with Diamond Dazzle [a game], that we played at the beginning of the term. There wasn't an actual rule about whether you could move backwards or not, but I thought, I'm going to leave it until they ask, and then see ... and I said to them, ‘Well, let's explore that. You tell me whether you think we need to bring that rule in or not.’
(Ms C, August 2014)

When talking about encouraging and nurturing students’ own areas of mathematical interest, she told me how she was already encouraging Alex to explore his own interests in their inquiry units.

We are looking at natural cycles, and so last week for news [Alex] actioned and did his own little research about how a dog's life cycle relates to a human’s, and I said to him, I'm just wondering about a cat, can you maybe go and find something out about that? (Ms C, August 2014)

When talking about scaffolding approaches to challenging tasks, to enable Alex to learn that hard thinking and sustained effort are a normal and expected part of his mathematics learning, Ms C described to me a task that she had organised for the following week, ‘Chocolate Smash’, with the implication that this is exactly what that task was planned to do. Ms C, therefore, seemed quite confident in approaching the three-month teaching period prior to the next Phase of the study.

During the Phase 1 interview, Alex had told me he loves a challenge, and indicated on the response sheet that he ALWAYS likes doing hard maths. However, Ms C gave a different perspective:

His mindset ... is either, ‘I'm fantastically good at maths and I'm the best that there is’, or, like [when] I challenged him on factors of 46, which was completely new to him, the wheels fell off completely, so much so that he was ‘I can't do this, ahhhh!!’ I went [back] to division by 2 and division by 4, and within two minutes he was in his happy place again because it was something he could do ... but then he wasn't even willing to try and look at 46 [again] because that first time he looked at it he didn't comprehend, and it was almost like a mental block shutdown. (Ms C, November 2014)

Ms C subsequently approached the post-professional learning teaching period with a focus on enabling Alex to understand about working within his zone of proximal development, rather than always wanting to return to what she called his 'happy place'.

It's just me being aware of the 'Goldilocks zone' and trying to move him to work within that ... I think, certainly in the lesson with factors, that I was able to take him away from what was a negative place of 'I can't' and move him towards exploring some more factors. (Ms C, November 2014)

Ms C said she did not specifically use the term 'Goldilocks zone' with Alex, "... but I've spoken with him about working in that zone where he's comfortable, where he's challenging himself and going forward" (Ms C, November 2014). However, there is an underlying dichotomy here where Ms C is alluding to this 'zone' as being a zone where he is *comfortable*, but also as a zone where he is *challenging* himself. She seemed sceptical about being able to get Alex to work in a zone where he was *not* comfortable and supporting him in that.

But as soon as he enters that challenge [she showed me a sample of work he had had trouble with] ... he shut down and said, 'I don't want to do this anymore' ... And as much as I tried to say to him, 'Let's see what other ways we can approach it ...,' [he said] 'I don't want to,' and that's when the tears started flowing. At that stage I said, 'Well, just leave it and see what else you can maybe explore,' and he rubbed out what he'd done ... I think that's the biggest problem with challenging him, if he decides in his mind it's too difficult it's basically a blank. (Ms C, November 2014)

This is another indication that there needs to be a mindset shift in students like Alex before they will be capable and/or willing to pursue challenging tasks outside their current comfort-zone (cf. Dweck, 2006).

Ms C used questions to sustain the mathematical thinking of Alex, and other mathematically highly capable students.

Asking these students to prove their work, or try a different additional strategy to solve the word problem, or explore a concept, has also been of benefit to my teaching practice. [The] modelling of persisting [sic] through good questioning with highly capable mathematicians, when these students are showing signs of weariness because they perceive the task as being 'too difficult,' has encouraged me to try and do the same. (Ms C, November 2014)

Neither Ms K nor Ms C used the suggestion of the chart for challenging self-limiting mindset statements and behaviours (See Section 5.3.1, Figure 5.2). The focus seemed to be on challenging Alex with more difficult mathematics tasks, and/or exploring mathematics tasks further, but with little suggestion as to how they supported him in this emotionally. The mindset focus was a key component of the professional learning that was not overtly adopted by the Grade 1 teachers. This may indicate the approach to the professional learning may need to be revised for future implementation.

Another point Ms C mentioned was that she recognised, even with his outstanding capabilities, that there were things in mathematics that Alex did not know that surprised her. She recognised that there were times when he required specific support, such as being encouraged to use manipulatives to assist in the development of new mathematics concepts, which she was not expecting. As well as the incident with exploring factors of 46 (mentioned above), where working with counters and building arrays may have helped him bridge his understanding of the concept to the actual task, she recalled another lesson on time:

The other area I've seen as well ... was in exploring time on the clock, and where our Grade 1 focus is telling time to the hour and half hour. And I suppose I was quite surprised by the fact that [Alex] could just do that. Coming into Grade 1 I would have thought that he would have explored that [telling the time] a bit more, so I've certainly done that, with looking at 5 past, and the meaning behind the 5 past, what does 10 past mean, [using] an orange, cutting it into quarters, for quarter past, quarter to. So he enjoyed that. (Ms C, November 2014)

However, both Ms C and Ms K commented that encouraging Alex to explore mathematics concepts with manipulatives, was somewhat hindered by his mother's view of his mathematics learning.

Ms K: His mother has also got to the point of 'that's baby stuff' [talking about using manipulatives].

Ms C: Yes, she's always saying that...

Ms K: And that's really engrained in him, so it's really hard because it's not just his fault ... She's instilling into him, 'You already know that, so you don't need to see that.' He will reiterate, 'I already know that,' and although you can see there are big chunks of things he hasn't learned or he hasn't explored on a sideways level rather

than on a forward level, he thinks he knows it so doesn't go back over it ... And he's embarrassed if he doesn't get it right, so the fact that he has to use materials would be hard for him. He doesn't like to think that other people think he hasn't got that knowledge, he likes to think he's the smartest, and he gets embarrassed if other people might perceive, and his perception once again, that he doesn't know. (Phase 3 interview, November 2014)

In this statement, Ms K has described evidence of Dweck's (2006) fixed-mindset behaviours in Alex's responses quite succinctly. This appears to provide further evidence that Alex still requires specific support from a knowledgeable other, for example an informed teacher, to help change his mindset, or beliefs, about the things a successful mathematics learner does, which may include using materials to explore, and further understand, new mathematical concepts.

Both Ms K and Ms C had mentioned intense emotional responses from Alex. Ms C described these extreme behaviours as 'meltdowns', and said she really only saw these in mathematics lessons when Alex was challenged, and in social settings in the playground:

When they [other children] have challenged him [with game rules] he's had huge, huge meltdowns, I'd say even bigger than his maths meltdowns, where he was sitting in the corner rocking because they've challenged him out in the playground – when it comes to how games are going to be played with the skipping ropes and whose turn it is. Or some of the girls can be tired of playing the game, and they move on. That's what happened two or three weeks ago. He was mortified and started saying things like, 'I suspect that she's going to say that to that one just so that one can hate me.' ... He sees everything as a conspiracy. (Ms C, November 2014)

Ms K had also mentioned this type of behaviour:

He cries a lot if things don't happen. Relationships, he'll often have to call a conference about something that happened at recess and who's involved – 'I'd like to ask [so-and-so] please, and I'd like to get so-and-so from Mr S's class, if he can come over, and I'd also like to ...' and he'll organise it all ... He likes to have that control of, 'Well this is what has hurt my feelings.' And it seems to be a build-up of things. It's not normally just one thing, it's like, 'Yesterday he did this,' And then there's the catalyst, the camel that breaks the back [sic]. (Ms K, August 2014)

This type of thinking is not uncommon with gifted children. Morelock (1992) tells a story of ten-year-old Greg that parallels, and possibly explains, Alex's behaviour in Ms C's and Ms K's descriptions above.

Greg [in Grade 5] was in trouble at school for getting into a fight with Joe in the playground, and had to go and see the principal ... The boys were each asked to write down their version of events. Greg willingly took a seat at the typewriter and laboriously typed out his story and explanation. An hour and a half later, he handed the pages to his mother: "It all began in third grade..." Greg went on to describe in careful detail how he and Joe had met and embarked upon a rocky friendship ...

Greg listed incidents from 3rd and 4th grades as well as the 5th grade incident that precipitated the immediate problem. For each incident, he detailed each child's behaviours with painful accuracy in an effort to render an objective view of what had happened. Greg's outburst was, according to him, not only a response to the day's happenings, but a reaction to the entire pattern of incidents composing their relationship over the past two years. The argument of the day was simply 'the straw that broke the camel's back'.

Joe, too, wrote out his version of the fight. He wrote simply, 'Greg hit me and then I hit him back and he kept on hitting me'. (Morelock, 1992, p. 12)

Life can be very complex for young children who may view life from a different perspective, and at a much deeper level, to their age peers (Columbus Group, 1991; Gross, 2004).

Greg had an unusually retentive memory and an extraordinary ability to analyse the roles played by both boys in an ongoing series of incidents composing a two year relationship. Joe, a child with more average cognitive abilities, lived each incident as it occurred and forgot it when it was resolved for the day. Apparently Greg and Joe were reacting to very different and individual realities. (Morelock, 1992, p. 12)

This is an issue that needs to be recognised and understood by teachers in order to know how to better support gifted students, especially, in this case, in addressing negative mindset behaviours. These extreme reactions, evident in Alex's behaviour both in social contexts and in challenging mathematics contexts, had been mentioned by both Ms C and Ms K, and by the end of the year neither believed much had changed with respect to Alex's dispositions towards mathematics learning. "I think the mindset is still very much there" (Ms C, November 2014).

There hasn't been a huge shift in his thinking, but it's taken him six, seven years to get to where he is, so a few months won't make much difference ... He [still] does that really simple way first, and really won't go beyond that. (Ms K, November 2014)

This may be further evidence that Alex required his mindset behaviours to be explicitly challenged, using strategies such as the suggestions for challenging self-limiting mindset statements and behaviours with alternate ways of thinking (see Section 5.3.1, and Figure 5.2). The reality could be that Alex was not able to 'shift his thinking' because he was not aware that his way of thinking needed shifting.

There were also some positive reflections on the impact of the professional learning. "...a few months won't make much difference, but it makes a difference in how we're teaching, and how we're questioning and looking at stuff as well, which is good" (Ms K, November 2014).

Ms K mentioned that she was enjoying teaching mathematics more. She especially enjoyed allowing students to explore mathematical concepts for themselves prior to her input, which she said was basically the opposite of her previous teaching approach. I observed one lesson where she had students building tall structures with three dimensional objects (boxes, blocks, etc.), with a focus on noticing the number of sides of different objects, and different properties of the sides – such as flat and curved – and how this affected the placement in their structures, as well as what mathematical name objects might be called based on their properties. She commented that the only lessons she had previously taught on three-dimensional objects were with worksheets. The students were very engaged, Ms K was asking good probing questions, and the lesson seemed a success with much mathematical language and discourse throughout. She said she had previously tried a similar approach with a lesson on capacity, where she took the students outside, "I was just thinking about what we did yesterday in the sandpit. We were talking about capacity and there was a lot of inquiry and working at finding equal containers." (Ms K, November 2014). The approach freed her to be able to challenge her more capable students further:

...but because it [the capacity task] was fairly simple, what's bigger and what's smaller, I asked him [Alex] to look at parts of containers to challenge his thinking – 'What would be one-and-a-half times that container?' 'What would be a third of that?' that sort of thing. And he was able to go and fill up his thing with sand with his partner, so he liked that. [And I was thinking] 'Would he be able to use that

language?’ So that was part of the [challenging] work for him. (Ms K, November 2014).

Ms C said that she had introduced the class to the concept of ‘state of flow’ (Csikszentmihalyi, 1996), something she had been introduced to as part of a whole-school professional learning. “I explained [to the students] it is what they do that makes them just go, ‘Whoa! I’m just so excited about this ... and [Alex] is busy making up a board game for his state of flow.” (Ms K, November 2014) (see Figure 6.10). Whether or not this activity shows students entering a ‘state of flow’, Ms C found it to be a positive way to introduce her students to exploring their own ideas, and opening up opportunities for creativity for all students across all areas of the curriculum.

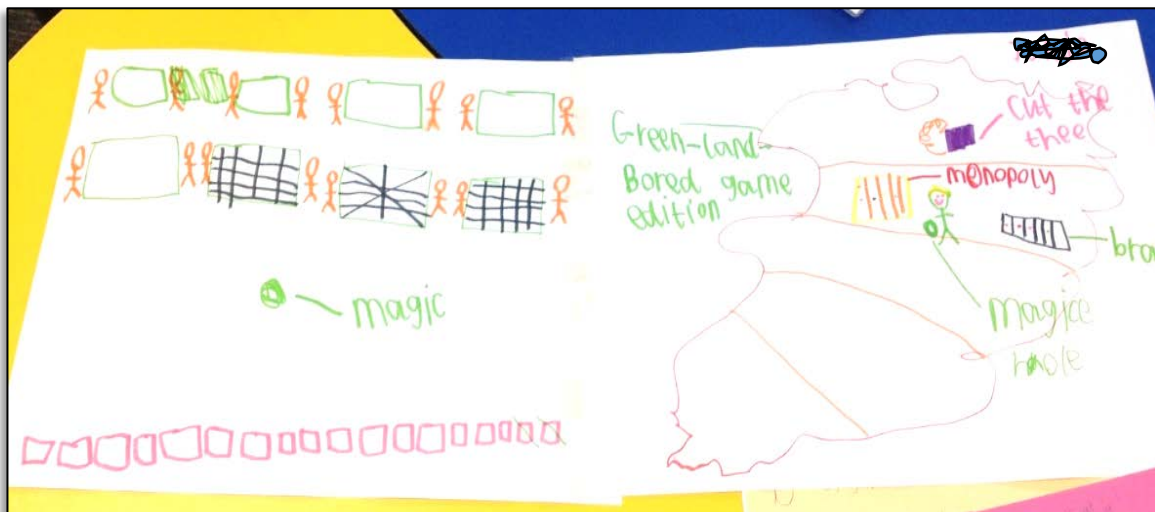


Figure 6.10 Alex’s creativity – the beginnings of his own board-game design.

6.4.4 Alex’s Mindset and Approaches to Mathematics Learning – Post-teacher Professional Learning

The first task given to Alex in the Phase 4 interview was a sliding block puzzle called Rush Hour (©ThinkFun) (see Figure 6.11). This hands-on manipulative activity was used to determine the extent of Alex’s willingness to take a risk with something challenging. Following the model of Muller and Dweck (1998), Alex was first shown how the puzzle worked, using the first card at the ‘Beginner’ level. He was then given three options – to try the same puzzle I had just shown him, but by himself; to try a different puzzle, but at the same ‘Beginner’ level; or try a more difficult puzzle, at the ‘Intermediate’ level. According to Muller and Dweck (1998), students



Figure 6.11 Rush Hour (©ThinkFun) sliding block logic puzzle.

with fixed mindset tendencies are more likely to choose the same activity, or another easy level activity, so that they can be confident in being able to solve the task quickly and easily, to show how ‘smart’ they are. Alternatively, they may choose a more difficult activity to show they are smart, but will change their minds as soon as they believe the task becomes too challenging.

Alex definitely wanted to try a more difficult card from the ‘Intermediate’ level. There was no hesitation in this decision. As he was setting the puzzle up he was quite sure it would still be easy for him, though, saying, “This is going to be easy!” However, just 10 seconds into the puzzle, he groaned and said, “Hmmm, this is actually hard!” After a further 14 seconds he started to become frustrated, exclaiming, “Oh what! What am I doing here!?” but he then sat forward and fully focused on the puzzle. He ended up persevering for nearly six minutes (0:05:57), mostly in silence, but with the occasional expression of exasperation, “Oh, this is so annoying!” “What on earth!?” “Urgh!” There were two times he looked like he might admit defeat, especially just after the four-minute mark when he was becoming visibly distressed – red in the face and head in his hands – uttering, “This is *way* too hard!” but then he kept going. After five and a half minutes there was another big sigh and “I can’t do this!” I was about to intervene, but he did not stop, and within a further 20 seconds he had solved it. When I asked him how he felt, he said, “That’s *such* a relief!”

Alex had persevered with a task that proved to be much more challenging than he had originally anticipated. He had become flustered and frustrated, angry and a little upset, but he kept going. Ms C had mentioned that when Alex becomes visibly distressed she has to back off and give him some easier work, otherwise he will end up in tears (November 2014). From my experience with him, it may be that Ms C is intervening too soon, and he

may not actually end up in tears. Following the professional learning, Ms K had decided to intentionally *not* back off when Alex got upset, but said the results were disastrous. She related an incident (informal discussion, November 2014), that was not mathematics related, but was an example of how she was no longer backing down when Alex was becoming upset when struggling with a task. The task was a phonics/diagraph task where Alex was required to come up with two voiced ‘*th*’ words (i.e., *th* as in *the* or *this*, not as in *thick* or *thank*). There was a list of words as examples – this, then, that, those – but Alex could not think of any other words that were not already in the list. According to Ms K, she continued to encourage him to come up with an answer, even though he started to cry, but he ended up shouting and screaming at her and throwing things around the classroom. She had to be ‘rescued’ by another teacher who came in to investigate the noise. When I spoke to Alex about this incident (it had happened the morning of the Phase 4 interview), he said he had exhausted all ideas he had for finding any words, including looking in the dictionary (Ms K’s suggestion), and he had no idea what to do next. He had not explained this to his teacher, though. This may be an example of expecting students to challenge themselves further without giving them sufficient support and/or the necessary skills to do this. Knowing when and how to ask questions to overcome confusion and uncertainty is one of these skills (Nottingham, 2010). This is another issue that may need to be addressed more explicitly in any future professional learning.

In the Phase 3 classroom observations (November 2014), I observed one lesson that started with a ten-minute whole-class discussion about the attendance at the Australian Football League (AFL) Grand Final. The discussion culminated with students estimating the number of people they thought attended the match, and then comparing the actual number (99 454) with their estimations (e.g., Was it more or less than you estimated? Who was closest?). Ms C said she decided to have this discussion with the whole class to expose everyone to ‘big numbers’, even though she knew most of them were “not yet ready” to do any independent work with these sorts of figures (Ms C, post lesson informal discussion). All students seemed very engaged in the dialogue. After the whole-class discussion, Alex and Frank were given a worksheet to complete, while the rest of the class continued to work as a whole group with Ms C. The focus of the worksheet was to, ‘Explore, and work creatively with the number 99 454.’ The last question on the worksheet was, ‘Design a plan for providing food and drinks for the 99 454 [people].’ Alex decided to build a burger shop called ‘Grild’ (see Figure 6.12). He drew his picture, then said, “Now I just get to colour

the lettuce, tomato, cucumber...” I intervened and asked both Alex and Frank how they were challenging themselves mathematically, as the worksheet had taken them less than five minutes to complete. To increase the challenge, then, Alex decided that each of his burger shops would stock only 454 burgers, and he was “pretty sure” he would need to build 99 shops around the stadium to feed the 99 454 spectators. He wrote $\times 99$ next to his picture (see Figure 6.12)

To test his prediction, he began to add 454s mentally, “So that’s nine hundred and eight burgers in two shops...” before I suggested a calculator might be useful. Calculators were obviously not encouraged as a regular tool to use in the classroom, as Alex did not know where to find one. Once a calculator had been procured, Alex used it to multiply 454 by 99 and realised straight away that he would need to at least double the 99 shops he had predicted, “That would be one hundred and ninety-eight shops.” With some further estimation and trials, he finally discovered that 220 burger shops would provide 99 880 burgers (too many) but 219 burger shops would only provide 99 426 burgers (too few). His ‘solution’ was to figure out how many burgers there would be in “219 and a half shops” [sic] (see Figure 6.12).

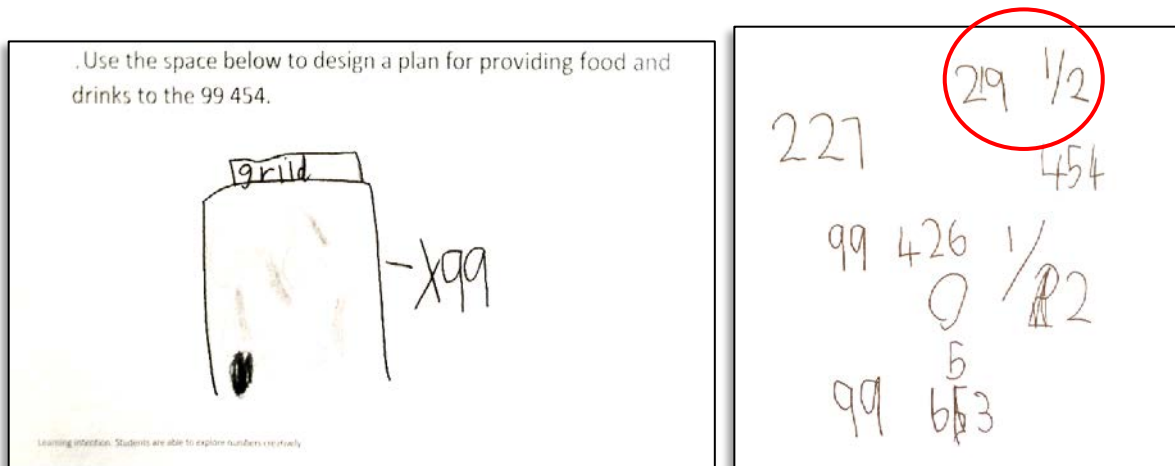


Figure 6.12 Excerpts from the football grand final worksheet. Left, 99 burger shops; Right, 219½ burger shops (circled at top of page).

When trying to work out half of 454 he started to get flustered, “I can’t do this; it’s way too hard!” I intervened once again, and asked him, “If there were four hundred burgers in the shop, how many would be in half a shop?” then, “If there were four hundred and fifty burgers in the shop, how many in half a shop?” before revisiting half of 454, which he was then able to answer quite quickly (227). At this point he was starting to physically wilt – head in his hands and sounding very tired. He was at a loss with what to do with the ‘half a burger shop’ number, “I have no idea. This is way too hard!” and started to get red in the

face and a little teary. I suggested he take a deep breath and clear his brain, then I showed him how far he had come with the problem so far, and pointed out that this was the last little bit to solve. He was then, surprisingly, quite happy to stop and regroup, “Ok, I was thinking ... two hundred and nineteen and a half shops ...” He added the 227 burgers to the total burgers from his 219 shops (99 653), realised it was still more than 99 454, (actually stated straight away that it was 199 more), and said, “No, I can’t do it. I think the only thing that will work will be two hundred and nineteen and maybe a quarter.”

Frank, who had been busy working on his own burger solution, then suggested that some people might have more than one burger. This started a discussion about estimations with food supplies, and that ordering a bit more than possibly necessary was better than not ordering enough (as long as the ‘bit more’ was not way too much more). Alex was then happy to build “219½ burger shops” [sic]. They would produce more than 99 454 burgers, but that was acceptable because some people may want more than one burger, or more people could turn up to the game at the last minute.

Alex had worked on this problem for half-an-hour, with a three minute ‘brain break’. He admitted that he was now “ready for a snooze.” He seemed drained, and not particularly enthusiastic about completing the task. However, during the Phase 4 interview the following week, when I asked him, “What is some maths you’ve done that you were really proud of?” he immediately chose, “The number of people at the grand final” task. He said it was because, “I got it done [when] it was just so hard!” and he was mostly proud for, “completing work that I thought was too hard.” (Alex, November 2014)

There was a difference in Alex’s responses to difficult tasks when his teachers were working with him in the classroom and when I was working with him as a participant observer in the classroom and as a one-to-one interviewer. With his teachers, there were tears and ‘meltdowns’; with me, he persevered through the frustrations. There may have been many possible reasons for this – his teachers were more familiar to him, so he was not afraid to show his emotions with them; the tears were a learned response that he knew would release him from doing hard work; the classroom teachers did not have the time to offer him the support he required to persevere with a hard task, or did not understand the extent, or type of support he required; his expectation when I was there was that the work *was* going to be really hard and it was therefore less threatening if he struggled with it. The one thing that was shown, was that he *was* capable of persevering with hard tasks when he was given support in this.

When I asked Alex how he knew he was ‘good at maths’, he said, “Well, I don’t like to brag, but for all my reports it’s – Prep, Semester 1, A; Prep, Semester 2, A plus; Year 1, Semester 1, A plus...” He then went on to say, “The first time I got an A plus my mum was so excited. She couldn’t stop jumping around.” When I asked him, “What does A plus mean?” he said, “It’s the best you can get, but I don’t know what the A stands for. I know the F stands for fail, though.” (Phase 4 interview, November 2014). Whenever Alex completed a difficult task with me, and I asked him how he felt, his response was always along the lines of, ‘It’s such a relief,’ rather than feeling happy or excited or pleased with himself. I wondered how much of this was tied in with his perception of mathematics success being ‘getting A pluses’ and not ‘failing’.

One of the last questions I asked Alex was, “What would you say to [Ms C] and [Ms K] about maths this year?” He said, “It was great, because I got to do hard work.” When I asked him what he thought next year, Grade 2, would be like, he initially said, “Next year will be even harder maths!” but then stopped and said, “...but actually last year we did hard at the end, and then this year, at the start of the year, it became a yawn-a-thon.” When I asked him what he would do next year if his new teacher gave him some mathematics work that was too easy for him, he said, “Well, I wouldn’t tell them, because I think that would be mean. So, I’d just write [it] down. I’d just take my time, so I’d just wait and be patient.” (Phase 4 interview, November 2014). This correlates with Ms K’s comment about Alex being a quiet worker who does not draw attention to himself. Alex’s summation, though, is that he does not want to draw attention to the *teacher* and the teacher’s misjudgement of his abilities. This may be further evidence of a gifted child’s extreme sensitivities (Dabrowskii & Piechowski, 1977; Silverman, 2010) this time projected onto others (his teacher), which could have a profound impact on the support he receives in the classroom if not recognised.

6.4.5 Alex’s Story: Review

When I first met Alex, he perceived a ‘mathematical challenge’ as hard mathematics that he could do, but other people could not do, rather than as mathematics that was just beyond what he already knew, and would require sustained effort and perseverance to complete (see Sullivan et al., 2013; Williams, 2014). Consequently, he would avoid difficult mathematics tasks that required any effort as much as possible. By the end of the year, according to his teachers, Alex would still get upset with mathematics work he perceived as being too difficult for him, even when they had assessed the challenge as being within

his zone of proximal development. Ms C had spent time with him explicitly talking about working in his zone of proximal development, however, she talked about this ‘zone’ being where work was not too easy, but still comfortable for him. This may have inadvertently reinforced his skewed perception of a mathematical challenge, as learning something new can often be *uncomfortable* initially, especially if this level of challenge is a new experience.

Alex’s teachers seemed to have focussed on helping him realise that ‘hard maths’ was more than just working with ‘big numbers,’ that it also included thinking about problems strategically, and reasoning mathematically, regardless of the quantities being considered. They both talked about applying, from the professional learning, the use of targeted questions to help Alex extend his thinking, and explore mathematics tasks further, with Ms C also including the whole class in more complex mathematics dialogue to extend everyone’s thinking further. They also came to realise that, despite Alex’s precocity in mathematics, he could still benefit from physical and/or virtual manipulatives at times to help make connections with new mathematics concepts, but had not made tools like calculators readily available for students to use in mathematics lessons. Neither of them spoke about intentionally addressing Alex’s negative, self-limiting mindset thoughts or behaviours, and both were sceptical about seeing any changes in Alex’s mindset behaviours as a result of the changes they had implemented post professional learning.

I received an email from Ms C almost a year later, however, telling me she had been following up on Alex’s progress in Grade 2.

[Alex’s] teacher this year [said], “[Alex] is going quite well in maths and is actually really open to new challenges. He understands that he is operating at an advanced level; however, knows that I will still try and challenge him and get him thinking. He is quite open to this and enjoys working with me in these situations. He is always looking for options to extend himself and will often seek my advice on how to do this. He has a very positive outlook and is a brilliant problem solver. He understands to take time and break things into more reasonable chunks. The most challenging of tasks I have for him are given to him in a supported environment where I am working with his little group – so he has me there if he needs me.” (Ms C, email correspondence, September 2015)

This describes a boy who seems well-adjusted in mathematics lessons – who enjoys working at an advanced level, and is open to being challenged further; who looks for ways to extend himself mathematically, and is able to ask for help to do this; and who is developing essential skills required for problem solving. His teacher also seems to understand the importance of supporting her mathematically highly capable group of students when giving them challenging work.

6.5 Chapter Conclusion

This chapter comprises the narrative analyses of Fred, Sammy and Alex – three students who were identified as being mathematically gifted, but who exhibited self-limiting mindset behaviours. Data from interviews with the students and their teachers, a written questionnaire completed by their parents, and classroom observations of their experiences as mathematical learners in regular primary school classrooms, have provided deep insights into each student. These data have been accumulated, analysed through a descriptive and interpretive approach, and ‘retold’ as a narrative along a before-after continuum (Clandinin & Connelly, 2000; Polkinghorne, 1995). Detailed explorations and thick descriptions (Merriam, 1998), from before their teachers received targeted professional learning about mathematically gifted students, to three months after receiving professional learning, have built a picture of each student’s mathematics learning experiences and mindset dispositions. This is within the context of exploring the impact of targeted teacher professional learning, and subsequent changes in teaching practice, on the mindsets and mathematics learning of the students. These interpretations are an analysis *within* individual samples of a multiple case study (Baxter & Jack, 2008; Stake, 1995). The next chapter will explore *between* and *across* these individual samples (Baxter & Jack, 2008), to further explore the phenomenon of students who are mathematically gifted, but with self-limiting mindset behaviours. This will provide further integrated interpretation, for deeper analysis, of the impact of the teacher professional learning on these mathematically gifted students with self-limiting mindset behaviours.

Chapter 7 – Exploring the Phenomenon

Mathematically gifted students with self-limiting mindset behaviours: A synthesis of the data

7.1 Chapter Overview

Having chronicled the individual narrative analyses in Chapter 6, these narratives will now be synthesised. Themes have been identified that relate to the research questions, and commonalities and similarities between the three students' experiences, and the significance of these, analysed and interpreted as deeper analysis of the phenomenon (Polkinghorne, 1995). In qualitative case study, generalisation, or external validity, is limited because the case is a bounded system specific to the individuals being studied (Stake, 1995). Indeed, "the researcher wishes to understand the particular in depth, not to find out what is generally true of the many" (Merriam, 2009, p. 224). However, Lincoln and Guba's (1985) idea of *transferability*, as a form of external validity in qualitative research, ensures trustworthiness, credibility and a contribution to further knowledge. Transferability refers to the degree to which research results and findings can be transferred to other contexts and settings by the reader of the study. The role of the researcher is to provide sufficiently rich, thick descriptions that allow the reader to make connections to their own similar contexts (transference) (Lincoln & Guba, 1985). Stake (1995) describes this as: "[The researcher] can interpret it [the study], recognise its contexts, puzzle the many meanings while still there, and pass along an experiential, naturalistic account for readers to participate themselves in some similar reflection" (p. 44). Connelly and Clandinin (1990) call this verisimilitude – the notion that a study "rings true" to the reader, and therefore establishes credibility.

To continue to build a rich, thick description of the case of mathematically gifted students with self-limiting mindset behaviours, further contextual information is provided in this chapter, through deeper analysis and discussion of the phenomenon. The aim is to synthesise the data from the individual narratives, not to compare the students, but to identify commonalities and similarities that may prove to be idiosyncratic to mathematical giftedness and self-limiting mindsets, and the phenomenon of students who display both these traits. The integration of the individual experiences strengthens the discussion of findings, with interpretations coming from deep thinking on the reflected narratives

(Merriam, 2009), and supported by the literature, to further highlight, and possibly uncover new understandings of the support required for mathematically gifted students in the classroom. Interpretations assume Gagné's (1995) view of giftedness – that gifts, or inherent capabilities, are only realised as talents or developed abilities, through a student's learning experiences, which are impacted by both environmental and intrapersonal factors. Interpretations of mathematics learning are made from a social constructivist view, whereby students construct meaning from relevant mathematical experiences through social interactions and the support of a more knowledgeable other (Vygotsky, 1978), or others, and that optimal learning takes place when those experiences fall within a student's zone of proximal development (Vygotsky, 1978).

This chapter starts with a discussion of behaviours observed in the students compared with gifted characteristics found in the literature (section 7.2). This is to address common misconceptions of giftedness, and to provide a necessary baseline for interpreting some of the students' reactions. Commonalities and similarities observed in the students' approaches to mathematics and mathematics learning (section 7.3), and their mindset behaviours (section 7.4), are then explored, from both before and after the teachers received targeted professional learning. This is followed by a discussion about the similarities and/or differences in the ways the teachers approached their teaching post-professional learning (section 7.5). Each of these sections is divided into subsections that reflect the themes that emerged from the individual narratives that address the research questions, and that compare with literature findings as outlined in Chapter 2.

7.2 Commonalities and Similarities in Gifted Characteristics

It is important, when considering one particular aspect of a child's character, such as outstanding mathematical ability, to not overlook the child as a whole. This section explores the characteristics of the three children involved in this study, to highlight their uniqueness as individuals, to counter some of the gifted stereotypes, and to look for commonalities that may help teachers understand possible gifted tendencies and behaviours in their students.

Section 2.3.4 explored the literature on common characteristics of gifted children, and addressed some of the myths and misconceptions about giftedness that may be held by individuals and/or societies in general. There is a stereotype within Western culture of how an academically gifted student presents – large rimmed glasses, socially awkward, acting

superior or arrogant, fashion challenged, skinny due to lack of physical exercise, and usually male (Eglash, 2002). Caricatures such as the ‘nerd’ costume in Figure 7.1 would be immediately recognisable to most Westerners as a representation of a highly intellectual person. These stereotypes are often meant to be amusing, but “American kids grow up knowing that ‘nerds are bad and jocks are good,’ ... [whereas] in many other countries academically high-achieving children are revered by their peers” (“In Praise of Nerds,” 2008, para 3). Stereotypes can be damaging, but at the same time common characteristics can be useful in both identification, and in knowing how to support gifted children as students. While a case study of three students can neither refute nor confirm any characteristic as being common to all gifted students, the findings of this study are worth highlighting, to alert a reader who may need to question common stereotypes, or who may capture a sense of verisimilitude with other students.



Figure 7.1 ‘Nerd’ costume (HalloweenCostumes.com)

Table 7.1 shows a summary of the notable characteristics of the three case study students, with themes, identified from the literature about gifted stereotypes, that were evidenced in different ways by the students: appearance and disposition (Silverman, 2013; Winner, 1996), classroom behaviour (Silverman, 2013), family background (Routledge et al., 2014; Winner, 1996), global academic ability (Silverman, 2013; Winner, 1996), early mathematical aptitudes (Diezmann & Watters, 2002; Gross, 2004; Krutetskii, 1976; Sheffield, 1999; Winner, 1996), and emotional intensity (Dabrowski & Piechowski, 1977; Gross, Macleod et al., 2001; Piechowski, 1997). The results show that the students often do not display commonly held beliefs about giftedness, such as disruptive behaviour in the classroom, or pushy parents, however, some characteristics, such as emotional intensity, were evident in all three.

7.2.1 Student Characteristics vs Stereotypical Beliefs

Appearance and Dispositions.

All three students were very slightly built, but, in the case of Sammy and Fred this was certainly not due to a ‘lack of physical exercise.’ With both being highly athletic, they certainly challenged the stereotype of academically gifted students being physically weak or feeble or uninterested in sport. None of them wore glasses, or high waisted pants. Alex

Table 7.1

Summary of student characteristics

Theme	Fred	Sammy	Alex
Appearance and dispositions	An athletic 11-year-old; Socially well-adjusted and popular; Knew he was good at mathematics.	An athletic 9-year-old; Struggled socially at times, but not a persistent trait; Believed she was not good at mathematics	Typical looking 7-year-old, but ‘sounded’ gifted with his particular way of talking; Socially more comfortable with girls than boys; Knew he was good at mathematics.
Classroom behaviour and demeanour	Well behaved; sensitive to others in the class; Content as long as he knew what was expected of him.	Generally well behaved; Content, but not always confident.	Well behaved; A quiet, confident worker,
Family background	Both parents doctors; Youngest of three children; Parents supportive but not “pushy”.	Both parents doctors; Eldest of three children; Parents supportive but not “pushy”.	Father a tradesman, mother a part-time office worker; Elder of two children, lived with his mother and brother; Mother very invested in Alex’s mathematics achievement at school, keen to accelerate his mathematics learning.
General academic ability	Highly capable in all areas of schooling.	Highly capable in all areas of schooling.	Strong in all areas, but particularly capable in mathematics.
Early mathematical aptitude	Mathematical aptitude recognised by parents from an early age.	Mathematical aptitude not apparent, or recognised, prior to school.	Mathematical aptitude recognised by his mother from an early age.
Emotional responses	Emotional hypersensitivity evident (Dabrowski, 1972), with intense distress when he did not finish tests, or did not understand what was expected of him.	Emotional hypersensitivity evident, with extreme distress when she thought she was wrong, or when others completed work before she did; and intense distress with social issues.	Emotional hypersensitivity evident, with intense distress when faced with unfamiliar tasks or mathematics difficulties, and extreme emotional outbursts over social issues.

was the only one who ‘sounded’ gifted, with his ‘old-man’ way of talking (see Section 6.4). First impressions of Alex were such that it was actually not surprising to find out that he was academically advanced, however, neither Fred nor Sammy presented as stereotypically ‘gifted’ on initial introductions.

Ms C described Alex as “somewhat feminine in his way of presenting himself [in his weekly news talks] ... and his gestures as well... to the point of sometimes being a little bit embarrassing, where I've got to say, ‘Okay, just tone this back a little’” (Ms C, November 2014). She said, “He doesn’t seem to perceive that others might think that he’s really going overboard,” but was concerned about how the social dynamic would affect him as he got older, when “children will comment on things like his over-the-top theatrics and lacy tights” (Ms C, November 2014). Ms K had also mentioned the lacy tights: “[For the school production] he had come dressed in an unusual shade of green tights with lace around the bottom. He wasn’t at all concerned that it was a different colour to the others’, actually quite pleased that he would stand out” (Ms K, November 2014). She also mentioned the dramatics when he published stories and read them to the whole class: “He does all the voices and is very over the top.” At seven, Alex seems a little eccentric, possibly fitting a ‘gifted stereotype’, and appears quite comfortable being who he is. It will be interesting to see if he continues like this, or whether social pressures will cause him to choose to behave more like others as he grows older (cf. Gross, 2004).

Sammy and Fred, on the other hand, presented as typical nine- and eleven-year-olds, respectively; it was only further mathematical questioning and observations that unveiled their academic capabilities. Fred seemed socially well adjusted (even though he sometimes worried about whether others liked him or not). Sammy, while she struggled at times with social interactions, was not perceived as ‘socially inept.’ Her teacher’s comments gave the impression that Sammy’s social struggles, while intense at times, were not a constant trait – “Like there were a few social issues with her at the beginning of the year, it was happening quite a bit in the first few weeks, but oh, now, once in a blue moon – once or twice a term perhaps” (Ms S, July 2014).

This confirms that gifted students do not necessarily look, or behave differently to other students in a class. Indeed, one of the idiosyncrasies of gifted children is that they have the ability to choose to behave like everyone else, sometimes even choosing to pretend to be not capable (which is seen especially, but not exclusively, in adolescent girls) (Gross, 2004). Students who are twice-exceptional, that is, gifted but with some additional neuro-

atypicality such as autism, are more likely to look or act differently, with a limited ability to pretend otherwise, but these students constitute only a very small percentage of gifted students (National Education Association, 2006). Contrary to popular Western stereotypes, what a student looks like, or behaves like, is not a reliable indicator of giftedness.

Alex and Fred were both aware of their mathematical abilities based on previous grades. Alex was also aware that he knew things that others, including possibly his mother, did not know, and Fred was aware that he could ‘understand and complete problems very quickly’ compared to others. Sammy, however, did not appear to be aware of her mathematical ability at all, indeed, she seemed to be quite concerned about a perceived *lack* of ability (Section 6.3.2).

Fred, regardless of his good grades, admitted to struggling with considerable self-doubt, especially in test situations; Sammy seemed quite unaware of her abilities, and was highly self-critical; and Alex’s comments about ‘knowing more than others’ could just have been a statement of fact, unhindered by social maturity that teaches circumspection in areas such as academic abilities.

Stereotypes about arrogance and superiority in gifted individuals can negatively impact provisions for gifted students, especially in a culture that seemingly wants to ‘cut down the tall poppies’ (Geake & Gross, 2008), and sees provisions for the gifted as being elitist (Gross, Macleod et al., 2001). This study shows that mathematically gifted students are not necessarily arrogant, or feel superior because of their abilities, indeed, they may not even be aware of their abilities, and they are not immune to low self-esteem (Siegle, 2013).

Classroom Behaviour and Dispositions

All three students were well behaved in class, with Alex being described as “a very quiet worker who doesn’t make a fuss about anything” (Ms K), and Fred being described as a dedicated, ‘driven’ boy who was “quite sensitive to other students” (Ms J). Sammy was also described by her teacher as being a content, quiet worker. There was one incident observed where Sammy was rude and disruptive with a casual relieving teacher, but this was certainly not the norm for her. The common belief that ‘gifted students become behaviour problems in the classroom if they are not sufficiently challenged’ certainly did not appear to be an issue with the children in this study, even though Alex complained about mathematics classes sometimes being a “yawn-a-thon.” Seven of the eleven teachers, who participated in the focus group discussion for this study, agreed with the statement,

‘Children who are mathematically highly capable will develop behaviour problems if they become bored in maths classes’ (focus group discussion, April 2014). This stereotype of gifted students is something that many teachers need to be challenged on (see Silverman, 2010).

Fred, Sammy and Alex were all ‘fast workers’ to a certain degree, but only when there were extrinsic ‘rewards’ and/or the work they were doing was relatively easy. Fred admitted that “‘Mostly I try and get things done quickly ... because normally she [Ms J] sets a task and if you finish that quickly you can like play a game or something, and everybody likes playing games, so I would try and finish it as quickly as I can, to play the game and things like that” (Fred, July 2014). Alex was observed rushing through mathematics work quickly when the ‘reward’ was computer time (Classroom observation, August 2014). Sammy was a ‘fast worker’, but in the sense that she seemed to deliberately focus on completing mathematics tasks quickly, possibly due to her belief that if she could *not* do the work quickly, it meant it was something she was not good at. When given challenging tasks, and supported in those tasks, however, they were prepared to spend considerable time on completing them. Being a fast *learner* (gifted) is not the same as being a fast *worker* or fast *finisher* (Gross, Macleod et al., 2001; Munro, 2012; Wheelock, 1992). This shows that planning extra, or extension work for ‘fast finishers’, is not a successful method for supporting the learning of mathematically gifted students (Siegle, 2013; Silverman, 2013), and can, indeed, become a detrimental strategy. Planning rich tasks for *all* students, which include inquiry and problem-based learning, and have the potential for extension, is a more effective approach (Diezmann, 2005; Krutetskii, 1976; Sheffield, 2008; VanTassel-Baska, 2008) and, additionally, may well uncover hidden abilities in students not previously considered exceptionally capable (Niederer & Irwin, 2001).

Family Background

Alex’s family environment seemed to be quite a contrast to that of both Fred and Sammy: Fred and Sammy’s parents were medical doctors, and, while very supportive, did not seem to be overtly involved in their children’s day-to-day mathematics learning. Alex lived with his mother, and only saw his father some weekends. Alex’s mother seemed very eager to not only support, but also accelerate Alex’s mathematics learning, both at school and at home, but admitted to struggling with certain aspects of mathematics herself.

Considering the different family backgrounds of Fred, Sammy and Alex, it is important to realise that parent professions and/or social standing are neither indicators of likely

giftedness nor non-giftedness. Gifted children come from all socio-economic and cultural backgrounds (Gross, 2004; McAlpine, 2004). People are generally not surprised when doctors' children prove to be academically gifted, but too often children from lower-socioeconomic families are overlooked as being possibly gifted (Gross, 2004). It is important for teachers to be aware of this trend in order to avoid this mindset.

Six of the eleven teachers who participated in the focus group discussion (April 2014), believed that 'Children who are highly capable tend to have pushy parents,' with comments such as, "I have parents that come and talk to you and say, 'Oh but we're not being pushy parents,' but in actual fact some of their requests could be interpreted by some as being pushy." and "I know quite a few parents out there and I would say some are pretty pushy, but others subtly pushy." However, another comment questioned the difference between parents of highly *capable* students and high *achieving* students:

High achieving or highly capable? That's one of the question marks for me, because I don't think the highly capable kids have necessarily pushy parents, but I think some of our kids that learned their times tables by the time they're six, the reason they can do that is because their parents have been setting the bar high and pushing, and they'll keep on doing that. (Focus group discussion, April 2014)

Understanding the difference between highly capable and high achieving is another issue that is important to address with teachers, as some of the stereotypes may be coming from different interpretations (Silverman, 2013).

In addressing stereotypical beliefs of students who are mathematically gifted, it is clear, even from this case study of three students, that stereotypes are neither good nor reliable indicators for identification of giftedness. For this study, teachers nominated students who they perceived to be mathematically highly capable. Some of these were subsequently assessed as being high *achievers* (as a result of early mathematics learning experiences and/or extra tutoring), but not clearly on a mathematically *gifted* spectrum (Neihart & Betts, 2010; Silverman, 2013). It is also possible that there were other students who were *not* nominated (particularly twice-exceptional and/or underachieving students) because they did not fit any common 'gifted' stereotype (see Baum, 1990; Dare & Nowicki, 2015; Reis et al., 1995; Valpied, 2005). This is an indication that identification of exceptional mathematical abilities is a complex issue. It also requires an understanding of the complex nature of mathematics and mathematical thinking and learning (Boaler, 2013; Davis, 1984;

Lockhart, 2009; Sriraman, 2004; Sullivan, 2011). While the three students selected for this study were identified as being mathematically highly capable, the task of developing a process for identifying all mathematically gifted students was beyond the scope of this study.

General Academic Ability

Fred and Sammy were both considered by their teachers to be academically highly capable in all areas of schooling, and extremely talented in their respective sports – Fred with tennis and athletics, Sammy with gymnastics – with Fred also being very skilled musically. Alex was described by Ms K as being academically “pretty strong across the board”, but mathematics was considered to be his particular strength. Just like all children, gifted students have strengths and weaknesses. I observed Fred struggling with a spatial task in one mathematics class, and Sammy’s teacher also talked about Sammy really struggling with a ‘space thing’ in one lesson – so it seems that strengths and weaknesses can even manifest within the same discipline.

Whereas some gifted students *are* globally gifted, teachers need to be aware that this is not a prerequisite for the identification of giftedness, especially within such a complex discipline as mathematics. Teachers have a tendency to expect gifted students to work at a high level in all areas, and can be surprised when a gifted student shows evidence that they do not yet know something relatively easy, or are struggling to learn something new. This is not necessarily an indication that that student may not be gifted after all (Winner, 1996).

Early Mathematical Aptitude

Both Fred and Alex’s parents had noticed mathematical aptitudes in their children from an early age (see Gross, 2004), but Sammy’s parents could not remember anything specific from Sammy’s pre-school days that would indicate this. All three attended both 3-year-old and 4-year-old kindergarten, where Fred’s kindergarten teachers had noticed Fred’s curiosity and aptitude with numbers, but neither Sammy’s nor Alex’s kindergarten teachers had communicated anything specific about mathematical dispositions. Fred’s parents rated Fred’s mathematical ability at the same level as his teacher had; Alex and Sammy’s parents each rated them lower than their teachers had. This was particularly surprising for Alex, based on comments made by his teachers about Alex’s mother’s pride in her ‘smart son.’ However, both Alex and Sammy were eldest children, which may have had some bearing on their parents’ perceptions. Alex’s mother had no other school children of her own to compare Alex’s abilities with, and, conversely, Sammy’s parents had a younger son, whose

mathematical ability was described as ‘extreme’ by Ms S (see Section 6.3), to compare Sammy’s ability to. Fred was the youngest in his family, with siblings already having completed primary school.

Parents may be unaware of the extent of their child’s unusual abilities, especially if they have either no other point of reference (as in the case of an oldest or only child), or other extreme points of reference. Previous research, however has found that many parents *are* very attuned to, and accurate about, their child’s precocities (Bicknell, 2009a). Parent perceptions of their child’s abilities are often perceived as being over-rated by teachers, but this has been proven to be an unfounded concern (Bicknell, 2009a; McAlpine, 2004), and teachers need to be encouraged to trust parents’ descriptions of advanced behaviours, regardless of whether or not these behaviours have been evident at school (Feldhusen, 1998; Robinson, 2008; Winner, 1996).

Emotional Responses

Certainly, the most striking commonality between the three students in this study was their tendency towards extreme emotional outbursts; and this observation was not limited to Fred, Sammy and Alex. In the initial interview with Ms K, as she was reflecting on the characteristics, and describing the behaviours of each of the Grade 1 children she and Ms C had nominated as mathematically highly capable, she stopped and exclaimed, “Actually that’s an interesting correlation I hadn’t really thought about, but they would be the three that I would say are emotional, and they’re the three who are really strong academically. And with such different backgrounds that’s quite incredible!” (Ms K, August 2014).

Fred ended up in tears when he did not finish a test as quickly as he thought he should, and became distressed over two questions he could not do in another test. Sammy’s teacher described Sammy as ‘falling into an absolute heap’ and ‘bursting into tears’ when some of the other Grade 3 students finished work in class before she did, and I had observed Sammy’s distress and uncontrollable sobbing twice in classroom observations. Both Alex’s teachers talked about Alex dissolving into tears when faced with unfamiliar tasks, and having ‘meltdowns’ over both mathematics difficulties and social issues. These responses of devastation may seem extreme, but it is not uncommon for gifted students to exhibit intense emotional hypersensitivities (cf. Dabrowski & Piechowski, 1977), where every little setback is felt as earthshattering. These feelings are very real, not imagined, and children with hypersensitivities need to be given strategies to help them cope with the intensity of their emotions. “Emotionally sensitive children seem to respond to each

negative experience as though it were [sic] the end of the world. They cannot help what they feel, but they can learn to put these experiences into a helpful perspective” (Bainbridge, 2014, para 3). According to Bainbridge, an online advocate for gifted children and their parents, understanding hypersensitivity is the first step in helping children cope with their intense emotions. Ms S sought out information on addressing Sammy’s emotional stressors, and, together with Sammy’s parents, used Bainbridge’s suggestion of drawing up an ‘emotional response scale’ – a scale from one to ten, listing what would be perceived as the worst possible thing that could happen (e.g., some life-threatening scenario), through to the most minor thing that could happen (e.g., some slight inconvenience) – and then used this to help her manage her intense feelings by putting them into a more realistic perspective. This is a strategy that may, indeed, have been useful for both Fred and Alex as well, and worth possible inclusion in future professional learning programs.

Together with intense emotions, it is common for gifted children to have a strong sense of justice (Gross, 2004). Ms K’s description of Alex calling for conferences about something that had happened in the playground was a case in point (see Section 6.4.3). Gifted children, with their complex thought processes, good memories, and extreme emotions, naturally tend to assume that everyone else views the world the same way they do, and therefore any action, or reaction, must have a well-planned, logical reason behind it (Morelock, 1992). As Ms C said about Alex, he sees everything as “a conspiracy” against him; Sammy’s social issues may have been grounded in similar concerns; and Fred’s social worries may also have been based on an ‘over-analysing’ of others’ responses to him. It is impossible to fully determine or understand each student’s thought processes, but it is possible to be informed about the commonality of hypersensitivities, and the implications of this in gifted students. All students are dealing with social and emotional issues at times, but for gifted students these issues may be especially magnified, and these social and emotional issues will impact their academic engagement, outcomes and achievement (Department of Education and Training (Australian Government), 2016).

Gifted children may be intellectually ahead of what they are emotionally able to handle (asynchronous development), and may not fit the cultural expectations of how a child of his or her chronological age ‘should’ think, feel or act (Columbus Group, 1991; Silverman, 2013). With this asynchronicity, life can become confusing, scary, at times overwhelming, and can leave children feeling as though there must be something wrong with them

(Bainbridge, 2014). Gifted children may, indeed, be robbed of the carefree simplicity of childhood with their ability to think at a much more complex level to their age peers (Morelock & Morrison, 1996). This was voiced in Ms S's concern about Sammy, "[She] is just so serious, and she's not happy-go-lucky like a normal nine-year-old" (July 2014).

7.2.2 Importance of Considering Student Characteristics vs Stereotypical Beliefs

It is important, when considering one aspect of a child's character, to not overlook the child as a whole. These discussions about gifted characteristics, such as emotional intensity, and stereotypical beliefs about behaviour and parents, need to be included in detailed explorations and thick descriptions of case study integrated interpretations. It is also important to consider these findings for inclusion in teacher professional learning about mathematically gifted students. Common characteristics need to be highlighted; stereotypes need to be challenged. This is an integral part of considering best practice in supporting gifted students' learning, and has therefore been addressed prior to discussing the answers to the research questions.

7.3 Commonalities and Similarities in Mathematics Learning

The first subsidiary question, that sought to address the research question, asked:

- *How do students approach challenging mathematics tasks before and after their teachers receive professional learning and a subsequent teaching period?*

This question sought to uncover and address issues of mathematical challenge (cf. Sullivan et al., 2013) that were highlighted throughout the study. It is reasonable to suggest that mathematically gifted students be provided with appropriately challenging tasks, but student perceptions of challenge may affect their approaches to these tasks. The type of challenge, or indeed, what is deemed to be a challenge, as well as the expectation of what is considered to be a 'completed' task, may all affect student approaches to these tasks. Teachers need to be aware of possible idiosyncrasies of mathematically gifted students. Even though no two children will ever be the same, and there will never be homogeneity in any group of students, even if they are all gifted, it may be possible to identify some commonalities that will help teachers understand how to better support the provision of suitably challenging mathematics learning for this specific cohort of students.

Table 7.2 shows an overview of the students' approaches to mathematics, and mathematics learning, pre- and post-teacher professional learning. Four main themes were identified, based on the literature discussions on mathematics and mathematics education in Sections

2.4 and 2.5, and a fifth theme emerged from a commonality worth noting, which was also paralleled in the interviews with other nominated students. The themes are:

1. The students' perceptions of mathematical challenge (cf. Section 2.5). Diezmann (2005) and Sullivan et al. (2013) advocate the use of whole-class challenging mathematical tasks, with options of extended challenges for mathematically gifted students. This study, however, highlights that it may be more complex than simply providing challenging tasks. Students and teachers need to understand the meaning of challenge (Barbeau & Taylor, 2009), especially gifted students who may not have experienced many significant challenges in their mathematics classes.
2. The students' expectations between employing intuitive mathematical reasoning (e.g., using various mental strategies to partition a three- or four-digit number), or applying learned mathematics procedures (e.g., using defined algorithms) (cf. Section 2.3.3). Learning mathematics is not simply a matter of learning and remembering mathematical facts, skills and procedures, and yet this is what some lessons in school mathematics appear to expect, with some surprising results;
3. Student difficulties in explaining and recording solutions (cf. Section 2.3.3). The ability to communicate findings and to provide verbal and written explanations is an important outcome of 21st century mathematics education (Brown, 2008; Knuth & Peressini, 2001; Sheffield, 2003), especially for students who have the potential to become innovators and creators within mathematical/scientific realms (Krantz, 2007; Sheffield, 2012). This process of explaining and justifying solutions can be particularly challenging for mathematically gifted students because their thought processes are often intuitive and naturally very efficient, often combining two or more processes into one thought (Geake, 2008; Krutetskii, 1976). Exploring how the students (and teachers) in this study addressed this issue may be beneficial for others to consider;
4. Students' approaches to mathematical creativity (cf. Section 2.3.2). Learner-differentiation (Betts, 2004), scaffolding mathematical creativity (Williams, 2016) by encouraging independent further exploration of mathematics tasks students found intriguing, was suggested as one method of providing challenge for students. The findings suggest that that this approach to differentiation may indeed be possible, but would require significant support from a teacher who understands the process.

Table 7.2

Overview and examples of students' approaches to mathematics, and mathematics learning pre- and post-teacher professional learning

Theme	Fred	Sammy	Alex
Perceptions of mathematical challenge	<p><i>Pre-</i> Said he only liked 'hard maths' sometimes, but related hard mathematics to mathematics test questions that he had difficulty with; In class seemed to enjoy challenges and was eager to learn from them, but overall very focused on completing tasks quickly.</p> <p><i>Post-</i> Willing to challenge himself further, even if it meant taking longer to complete a task. Thoroughly enjoyed a classroom mathematics task, with a group of like-minded peers, that took two weeks of self-discovery and learning to complete.</p>	<p><i>Pre-</i> Said she mostly liked 'hard maths', "because of the challenge", but any mathematics task that could not be completed quickly and relatively easily was considered "too hard" and deemed evidence that she was "no good at maths". If others could do a task that she could not do, or if they finished a task before her, she would become distressed.</p> <p><i>Post-</i> Sammy stated, after completing a challenging task that she had persevered with for over 30 minutes, "I know I'm good at maths because I did that, and I thought it was too hard, but I did it!"</p>	<p><i>Pre-</i> Said he loved 'hard maths' because he loved challenging himself, but described a challenging task as something he could do that was too difficult for others. If given a task that he could not do easily he would become frustrated and teary.</p> <p><i>Post-</i> More willing to persevere with difficult tasks, but needed regular reassurance that he was heading in the right direction for a possible solution.</p>
Mathematical reasoning – intuitive vs procedural	<p><i>Pre-</i> Relied almost solely on learned algorithms for mathematical calculations</p> <p><i>Post-</i> Could not solve 87×9 without using the traditional algorithm, but could mentally solve the problem, "We have bags of 87 lollies and Jimmy bought nine bags, how many lollies does he have?"</p>	<p><i>Pre-</i> Used mental strategies to solve most mathematical calculations.</p> <p><i>Post-</i> Made a common error in calculating $100-67$ (giving an answer of 43), but could successfully partition 1007 in the <i>Adding Corners</i> task (see Appendix 2).</p>	<p><i>Pre-</i> Used mental strategies exclusively to solve mathematical calculations.</p> <p><i>Post-</i> Using a learned method calculated $522-367$ as 254, and accepted this answer as correct, but could correctly mentally partition 5023 (which required multi-digit addition and subtraction) in the <i>Adding Corners</i> task.</p>

Table 7.2 Continued

Theme	Fred	Sammy	Alex
Explaining and recording solutions	<i>Pre-</i> Mathematics writing consisted of informally jotting down calculations and thinking processes, and then writing down the final answer as the solution. Verbal explanations reflected a similar non-structured approach.	<i>Pre-</i> When expected to record her mathematical explanations, had great difficulty knowing what to write beyond ‘workings out’ and the answer. Required much direction and scaffolding to verbalise and write just a few sentences.	<i>Pre-</i> Verbal explanations were good; written expectations, as part of classroom practice, focused on generating number sentences, or equations, from mathematics scenarios rather than on a written explanation.
	<i>Post-</i> Ms J noted an improvement in Fred’s ability to express his mathematical thinking in writing, and to explain and justify his processes and strategies.	<i>Post-</i> No specific change noted.	<i>Post-</i> No specific change noted.
Mathematical creativity	<i>Pre-</i> Very anxious to do any task the ‘right’ way. Would not start a task until he was sure he knew what he was meant to do.	<i>Pre-</i> N/A	<i>Pre-</i> N/A
	<i>Post-</i> Learning to be creative with tasks when working with peers, e.g., modifying game rules, considering different approaches to solutions. Discovered a real joy in the freedom of self-discovery in mathematics learning.	<i>Post-</i> Very willing and excited to explore tasks further in collaboration with peers.	<i>Post-</i> No significant evidence of Alex creatively exploring mathematics tasks further.

Note. N/A indicates ‘not applicable’ as there was no evidence of exploring mathematics further.

The fifth theme was a commonality noted among several nominated students, and something worth noting as it relates to preconceptions and stereotypes of mathematical giftedness:

5. The occurrence of surprising mathematical difficulties, considering the students' recognised mathematical capabilities (cf. Section 2.2.2). Often mathematically gifted students are expected to be gifted, and/or to be achieving above and beyond their peers, in all areas of mathematics (Winner, 1996), even though mathematics is such a diverse topic (Boaler, 2016; Davis, 1984). Several nominated students, including Alex and Sammy, displayed mathematical difficulties that were not expected (by their teachers, and by the researcher), showing that it is important to beware of preconceived ideas of what mathematically gifted students may be able to accomplish.

This section explores each of these themes in detail, with analysis and interpretation of the significance of the findings.

7.3.1 Perceptions of mathematical challenge

Pre-teacher professional learning observations and interviews

Fred, Sammy, Alex, and the other students who participated in the initial identification task-based mathematics interview for this study seemed to enjoy the mathematical challenges presented to them in the interview. There was specific feedback from students themselves, such as, "Um, I just wanted to say I really enjoyed doing that work with you today" (unsolicited response from one of the Grade 5 boys following the task-based interview, April 2014). There was feedback from the teachers, with several of them commenting about how happy and excited the students had been when they came back to class after the interview. There was also non-verbal feedback from the dispositions of the students during the interview. All students fully engaged for approximately 40-45 minutes, with postures changing as tasks became more challenging. The older students (Grade 5 and some Grade 3) sat up and leaned forward as they became engrossed in the task at hand; the younger children (Grade 1, and some Grade 3) stood up, and fully engaged in the task with their whole body. If students are regularly given mathematics tasks that they engage in like this, tasks that require effort and perseverance (Sullivan et al., 2013), that develop mathematical understanding through investigation, exploration, dialogue, reflection and evaluation of the tasks (Clarke et al., 2002; Kilpatrick et al., 2001; Sheffield, 2009; Stillman et al., 2009),

society may see an increase, rather than a decline (Forgasz, 2006), in students retaining mathematics studies through to higher education levels.

The results, when students were asked to complete the chart, 'I like doing hard maths – never, occasionally, sometimes, mostly, or always', showed six of the nine Grade 1 students (five boys and one girl) said 'always', including Alex, and all the Grade 3 students, and all but one of the Grade 5 students said 'always' or 'mostly'. Fred was the only Grade 5 student who said 'sometimes', although he did clarify this saying, "Well, it depends what it is, because if ... it's not a test and you've got time to do it, then probably 'mostly'" (Fred, July 2014). Three of the four Grade 1 girls said 'sometimes' or 'occasionally'. The contrast between the responses of the Grade 1 boys and the Grade 1 girls was quite notable; possibly further confirmation of findings from other studies on gender differences in mathematics learning (see Forgasz, 1992; Leder, 2004). However, overall most students indicated that they mostly enjoyed being challenged mathematically, which was also observed in the task-based interview.

The teachers, however, gave different perspectives on how Fred, Sammy and Alex confronted challenging tasks, as did the classroom observations. Fred's teacher, Ms J, wrote on his initial nomination form, "Loves maths and looks for challenges all the time," and commented several times that Fred would push himself to understand something difficult. For example,

[Fred] really focused on the fact that there was [sic.] probably two questions [in the NAPLAN mathematics test] around the visual aspects of mathematics – so location, the flips and turns he had to do in his mind – and he couldn't work that out, so it was really important for [him], at the end of that session, that he got the blocks out, and he made it, and he could work out what the solution was. That was really important for him to know that he could work that answer out. (Ms J, June 2014)

Also,

He was absolutely convinced that he had the winning strategy [for a mathematics game] ... and he was absolutely adamant that he had it, and it was going to work – and then he tested it with different people and it didn't! And he was just so frustrated because he just couldn't work out why. So he was watching what everyone else was doing, and at the end of it, like, it was fantastic the discussion that came out of it. It

was really awesome; and then we didn't discuss it any further, we left it at that, and it drove [Fred] mad [laughter]. (Ms J, June 2014)

Again,

He's found working with three-dimensional shape really difficult, and we've done a lot of work on isometric paper [drawings], and enlarging and reducing an image; and he had to come up with a building design. So, the fact that that is an important element of maths helped [Fred], and he worked really hard on that. He actually worked at home in creating different models out of Lego that he would practice, and then doing the different views, and if he couldn't see the other view what could it look like. So he's actually challenged himself to improve in that area because he sees it as being an area that he finds difficult. (Ms J, November 2014).

Fred's in-class responses to difficult and challenging mathematics scenarios indicate that he seems to be very eager to learn from challenges, going above and beyond what is required or expected of him in a lesson when he comes across something he struggles with. Even though Fred suggested he only enjoyed 'hard maths' *sometimes*, his own perspective of 'hard', or challenging, mathematics seems to be associated with time-pressured and stressful testing situations. The initial task-based interview with me may have been viewed this way. When given the opportunity to challenge himself with the *Adding Corners* task (see Section 6.2.2), his response was, "Mostly I try and get things done quickly... but if it was just a question for fun [as opposed to a test] ... and you had as much time as you can then I'd probably like to challenge myself" (July 2014). In the follow-up task-based interview, when he possibly felt more comfortable with me, he was willing to spend as much time as he could on this same task (see Section 6.2.4, *Adding Corners* task post-interview). Time, especially timed testing, appears to be a critical factor for Fred. If he knew time was not an issue, he was prepared to work at something at a much deeper level, to the extent that he was even prepared to continue working on a task into his lunch break, or at home after school. This is also a critical factor for teachers to be aware of. For students to be willing to engage in, and persevere with, challenging tasks, it is vital to ensure that sufficient time is allocated for these tasks (see Sriraman, 2004). Students need to be aware that speed, and, at times, even completion of a task, is not a limiting factor to mathematical discoveries and/or successful learning (cf. Clarke & Roche, 2010; Holt, 2002).

Sammy's teacher said, "She loves a challenge and she strives for that," but then continued to say that Sammy really only enjoys a challenge that she can complete successfully. If she makes a mistake, or even if she *thinks* she has made a mistake, she gets embarrassed, and then,

Even if we talk about how great it was, and how challenging yourself actually helps you learn, and all of those sorts of things, that's where she cuts it off. That's the only thing she'll think about [getting something wrong] ... It's just that as soon as something like that happens she closes off, and she feels insecure, and like she's really hard on herself. And that's with everything, it's not just with maths. (Ms S, July 2014)

There was another incident Ms S recalled:

She just completely shut off because I said, "Why don't you try this?", because whatever it was we were doing it was too easy for her. I said, "Why don't you try something else with that, and explore more challenging things?" and she's like, "No, I think I'm ok with this." (Ms S, July 2014)

In a 30-minute discussion with Ms S about Sammy, Ms S mentioned Sammy's response to difficulty, or challenge, four times, as, "she just shuts down", "she shuts off", "she cuts it [any discussion] off", and, "she'll put a wall up." She went on to say, "She's very good at lots of things, but as soon as there's a little bump in the road she doesn't like it... It's like she's too scared to push herself. Ever." (Ms S, July 2014). Ms C made similar comments about Alex, "he sort of shuts down"; "if he decides in his mind it's too difficult, it's basically a blank"; "as soon as he enters that challenge ... he shuts down". These comments seem to contrast with the statements made about the children loving a challenge. Is it possible that the word 'challenge' has different meanings in different contexts?

"It's too hard" seemed to be a common cry from Sammy when facing mathematical challenge (see Section 6.3.3). This was something I explicitly addressed with her in the post-task-based interview when she was having difficulty coming up with a second method for solving the equation $522-367=?$ (see Appendix 2). Even when she had completed the task successfully, she still insisted it was too hard for her. I pointed out that *all* of us, even adults, would have to stop and think to work a problem like this out, that it is not something that people would 'just know'. I said to her,

Researcher: Do you realise that your Goldilocks Zone, the bit that's hard but not too hard, means that its work that you won't be able to do all by yourself, you'll need

help? If you can do it all by yourself without any help, it's too easy, you already know how to do it.

Sammy: Oh! (looked quite surprised)

Researcher: So, the fact that Miss S has to help you with stuff doesn't mean that it's too hard, it means that it's in your Goldilocks zone – you can do it, but you need a little bit of help. *That's* the Goldilocks Zone ... So, this wasn't too hard because you did it.

Sammy: Yeah, but it was still really hard. (Phase 4 interview, November 2014)

I realised that Sammy seemed to be thinking that if she needed help it meant the work was too hard. The optimal zone for learning, the zone of proximal development, is where the student requires support and scaffolding in his or her ongoing learning, therefore, 'challenging maths' will entail seeking out and/or accepting help from a 'more knowledgeable other' (Vygotsky, 1978). 'Too hard,' within the context of the zone of proximal development, means that the concepts are beyond a student's current ability to make schematic connections – even with help they still will not be able to understand because of a lack of necessary prior knowledge. It seems that there may be a discrepancy between teachers' beliefs and students' beliefs about what 'too hard' means; what 'working within your zone of proximal development' means. Talking about a Goldilocks Zone, for example, being 'not too easy' and 'not too hard' is not sufficient. We also need to ensure that students understand what we mean by 'not too hard'. 'Really hard' (Sammy's complaint) is not the same as 'too hard'. Other scholars have suggested using terms such as 'zone of confusion' (Sullivan & Davidson, 2014), or the concept of a 'learning pit' (Nottingham, 2010) (see Figure 7.2), when talking with children about learning something new. These may prove to be better analogies for students like Sammy and Alex who may not be familiar or comfortable with the idea of 'struggle' being a normal, and important part of learning. The 'zone of confusion' legitimises the feelings of confusion, or struggle, as being exactly what will be felt when learning something new. Nottingham's (2010) concept of the 'learning pit' not only normalises the struggle, but provides strategies for climbing out of the pit', such as linking to prior-knowledge, making a prediction and testing it out, collaborating with others, asking good questions.

If gifted students have not needed much support in their learning in the past, the feelings associated with struggle may be foreign to them. For Sammy, it was necessary to highlight

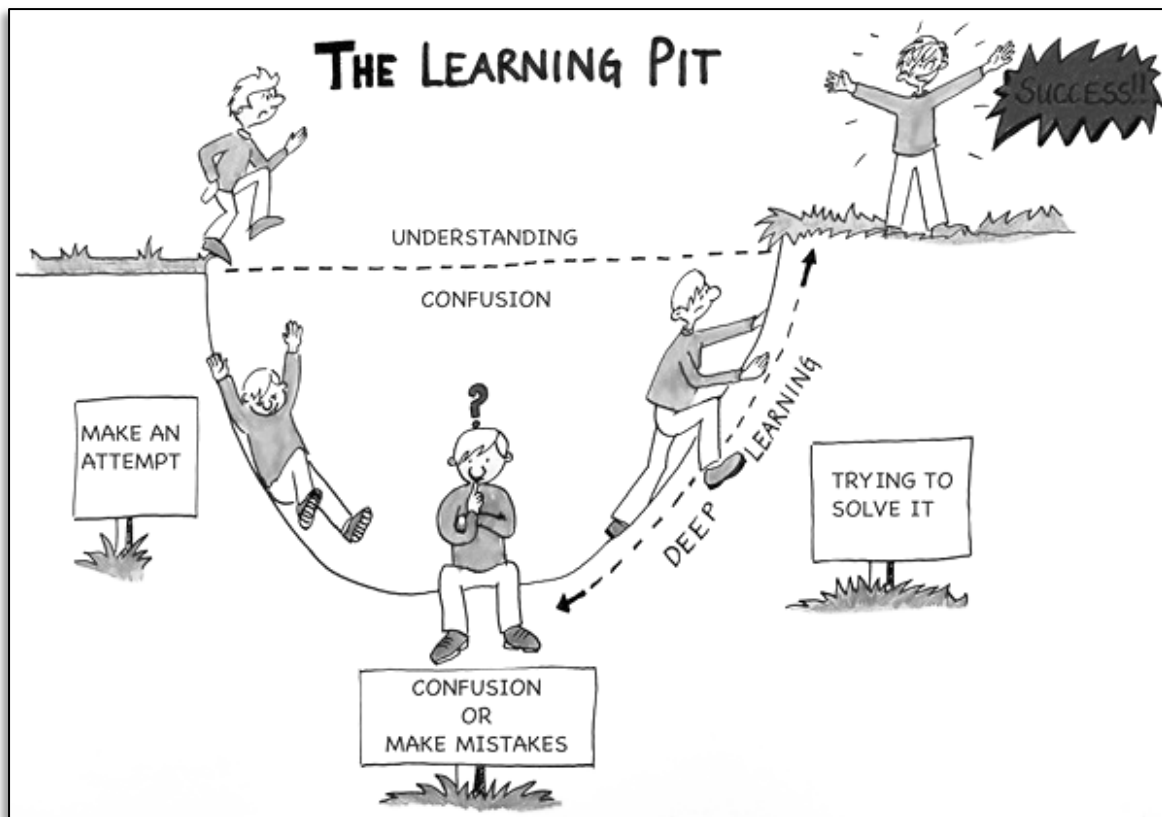


Figure 7.2 The Learning Pit (Nottingham, 2010)

Image retrieved from <http://www.jamesnottingham.co.uk/learning-pit/>

that ‘confusion’ in learning something new, or in attempting a challenging task, and requiring support or needing to ask for help, is normal, and a *good* thing. It will also be necessary to support students like Sammy with these unfamiliar, uncomfortable and possibly threatening feelings, by suggesting strategies to help them work through their confusion – use materials, draw a diagram, put your thoughts on paper, talk with someone else, et cetera – gradually extending the time before stepping in with this scaffolding until they can manage the confusion themselves (Sullivan et al., 2013). It may also be necessary to explicitly talk with mathematically gifted students about asking for help, or asking questions, as an expected part of learning; they may even need to be taught *how* to ask questions. Mathematically gifted students need support in their learning too, it may just be different to the support given to other students in the class (Gross, 2004).

Ms K suggested that she had nominated Alex as mathematically highly capable, in part, because, “I’d say whatever challenge I threw at him he’d take it and he’d run with it, his eyes would light up and he’d explore it” (Ms K, August 2014). Alex had announced at his first meeting with me that, “I *really* love challenging myself. If I challenge myself I believe that I can learn more things, and I love learning” (May 2014). He said he likes to challenge

himself because, “If I get it right I’m like, ‘Wow! I had no idea what that was, I had a go, and look, I got it right!’” However, when asked, “What if you got it wrong? What happens then?” he replied, “I’d go, ‘Oh, I might learn that next year. Leave it ‘til next year ... or I could wait ‘til Year 6 if I had to.’” (Phase 2 semi-structured interview, May 2014). In observing and talking with Alex, it was possible that his interpretation of ‘challenge’, just like Sammy’s, may have been somewhat skewed. For Alex, ‘challenging’ seemed to mean mathematics that was hard for others, but that he could do. When asked to choose a solution that was challenging and creative in the *Adding Corners* task in the Phase 1 task-based interview, he said, “I just chose something that I think mostly other people can’t do” (May 2014). Also like Sammy, in one classroom observation, Alex had completed a task in just a few minutes so I asked him, “What’s next? How could you challenge yourself further?” and he replied, “Oh, I think this is challenging enough for me” (pre-selection classroom observation, June 2014). Once again, it seems the student’s perception of a challenging task is different to what the teacher may consider challenging. It seems teaching students, and especially mathematically gifted students, *about* challenge, what it is, what it feels like, what may be required to tackle a challenge, and the use of a visual like Nottingham’s (2010) *Learning Pit*, is a necessary precursor to providing suitably challenging tasks (Sullivan et al., 2011), and instructional strategies such as problematising and extending manipulative use, as suggested by Diezmann (2005) (see Section 2.5.4).

Ms K had also observed that Alex’s idea of mathematical ‘challenge’ was limited:

Bigger numbers means more challenge with him, don’t they? I mean, if the task is complicated he doesn’t think it is a challenge unless it has big numbers ... If it’s a big number that means ‘I’m clever and that means I’m doing something really important and challenging.’ (Phase 2 interview, August 2014)

To counter this limited view of challenge, during the pre-selection classroom observation, when trying to find a sufficiently difficult task to challenge Alex and Frank (see Section 6.3.1, *Classroom observation*), I differentiated the task by suggesting they explore fractions (that is, smaller, not bigger numbers). Then, when Alex announced that he believed you could only halve even numbers, I asked him to select an odd number of counters to show me why it could not be halved. With the materials in front of him, he realised very quickly that you could, in fact, halve an odd number – the answer would be “something and a half” – but he was still reticent to use the counters, even when he was having difficulty halving the number he had chosen [59]. His first thought was “twenty-four and a half”, but quickly

realised, working mentally, that “twenty-four times two is only forty-eight”, so decided it must be “thirty-four and a half”. When he realised this was incorrect as well, he stated, “I think I chose the wrong number.” I suggested that 59 was, in fact, exactly the *right* number for him to choose because it gave him a challenge. Frank, who had selected 23 counters, which he was able to mentally halve straight away, proceeded to work with Alex on halving 59, and decided it would be “twenty-nine and a half”. At the end of the class, half of 59 had still not been ‘proven’ to be $29\frac{1}{2}$. Both boys were now quite convinced “twenty-nine and a half” was correct, but were not sure how to use the counters to show this. The whole-class lesson was on arrays, and both boys had been drawing arrays to *represent* multiplication problems prior to my suggestion of exploring halves, but neither boy considered organising the counters into an array to *solve* a multiplicative problem.

If mathematically gifted students have not experienced challenges they cannot solve mentally, they may need support in learning how to use materials that other students may already be familiar with, or, they may need support in making connections between the use of materials to represent a known problem, and the use of materials to solve an unknown problem. This affirms the suggestion of Diezmann (2005) to ‘extend manipulative use’ as an intentional instructional strategy for mathematically gifted students. When the class lined up at the door to go to a specialist class, Alex and Frank were still animatedly talking about how to show ‘half of 59’, again confirming that mathematical challenge, in the right context, is a motivator for students to continue to pursue mathematics learning beyond the mathematics lesson (Tytler et al., 2008).

Ms C mentioned the difficulty with challenging Alex because of his negative responses and ‘meltdowns’ (see Section 6.3.2 and 6.3.3). She also told me, “I challenge both [Frank] and [Alex] on higher levels [in Athletics], but not [Brony] because she stresses too much if challenged” (classroom observation, August 2014). It appears that, while students may need to be taught what ‘challenge’ is and how to approach a challenge, what it may feel like, and how to deal with those feelings, it is just as necessary for teachers to know how to support students in how to approach challenges, and how to deal with their feelings. They need to know how to scaffold that support until students are better able to manage their own emotions. Backing down from expecting a student to work through a challenge, or choosing not to challenge a student because of their negative reactions, is going to jeopardise that student’s learning, as challenge and productive struggle are an important

part of optimal learning (cf. Diezmann, 2005; Lithner, 2017; Stillman et al., 2009; Sullivan et al., 2013; Vygotsky, 1978).

It was important to note, from these observations and interpretations, that although most students said they enjoyed a challenge in mathematics, the students' definition of 'challenge' was not always the same as the teachers' or researcher's definition. If students believe that 'challenge' is a task that may be hard, but can still be completed successfully independently, it is maybe not surprising that stress and tears were the result of tasks that could not be completed independently. If a teacher believes a mathematical challenge to be something that a student can learn from (and one would hope this to be the case), it needs to be explicit to the student that they will most likely require assistance, and that, indeed, it is expected that they will ask questions and seek out help.

An important element to a professional learning program about mathematically gifted students, then, would be to ensure teachers understand that *all* mathematics learners require support. If students are working within their zone of proximal development, they will require assistance from a 'more knowledgeable other' (Vygotsky, 1978). In this zone, the work will be challenging (that is, not able to be solved without help, deep thought and/or sustained effort), and 'mistakes' and frustration are inevitable, as integral parts of the learning process. These are elements of learning that teachers need to be aware of, especially for gifted learners who may not have experienced such feelings as often as others, so they can help their students learn how to deal with these feelings.

Post-teacher professional learning observations and interviews

All three students in the post-interviews and/or classroom observations demonstrated changes in approaches to challenging mathematics tasks. For example, Fred's response to the *Adding Corners* task in the Phase 4 task-based interview, provided evidence that he was beginning to think beyond solving challenging problems quickly. Not only did he choose to work with factorials and "eighty-oneths" (see Section 6.2.4), he stated that keeping fractions with a common denominator was "too easy" and wanted to explore alternative possibilities, even though this meant taking longer to find a solution. Sammy was also much happier to spend time on completing challenging tasks, even acknowledging that this was what convinced her that she was, in fact, good at mathematics. She stated that she was most proud of herself when she completed a task that was "so hard", but she persevered (for more than 30 minutes) and eventually completed it (see Section 6.3.5). Alex's responses were similar, being more willing to persevere with challenging tasks, although he still

required constant reassurance during difficult tasks, that he was, indeed, heading in the right direction.

Whether these changes were a result of their teachers' approaches and expectations post-professional learning, or a result of being more familiar with me, having been a participant observer in the classroom, is not clear. However, what is evident is that positive approaches to challenging tasks, including a willingness to sustain effort, and an acceptance that time is a requirement for completing these tasks, is something that can be achieved, in a relatively short period of time, given a conducive learning environment with explicit teacher expectation and support.

7.3.2 Mathematical intuitive reasoning versus mathematical procedures

Another interesting theme that emerged throughout observations of the mathematically gifted students' approaches to mathematics tasks, was the different approaches they took when solving mathematics calculations embedded in a context, versus mathematics calculations solved in isolation of any context. This was evident in the *Solve this problem two different ways* question in the Phase 4 task-based interview in November 2014 (see Appendix 2). Fred was asked to calculate 87×9 , and Sammy and Alex were asked to calculate $522 - 367$. The purpose of the task had been to see if the students could calculate these using different strategies – once they had solved it their initial, preferred way they were asked to solve it another creative way. What was discovered instead, was that while all three of these students had demonstrated their ability to reason with numbers successfully in solving the *Adding Corners* task, their approach to a task written as an isolated calculation was quite different.

Alex, who back in May had been able to mentally partition 5698, and manipulate the resultant three- and four-digit numbers successfully (see Section 6.3.2), chose to use what he called a “Spiderman strategy” (see Figure 7.3), and calculated that $522 - 367$ was 254 (the correct answer is 155). He said that Ms C had taught him the ‘Spiderman strategy’, but Ms C said she had taught this ‘strategy’ [sic] for multi-digit addition, but had never used it for subtraction, and had not called it a ‘Spiderman strategy’ (Phase 4 task-based interview, November 2014). When asked to come up with a second, creative method for solving the same problem, Alex groaned, and sighed and ummed for a while before announcing, “I’ve made up this strategy by myself...” and proceeded to draw his very own “vacuum cleaner strategy” (see Figure 7.3). In both methods (which actually used the same strategy

of subtracting the hundreds, tens and ones separately and then combining the results), Alex was trying to apply the commutativity principle to subtraction, subtracting 20 from 60 instead of 60 from 20 (see Figure 7.3). In his ‘Spiderman strategy’, his recording had also confused the position of the tens and ones.

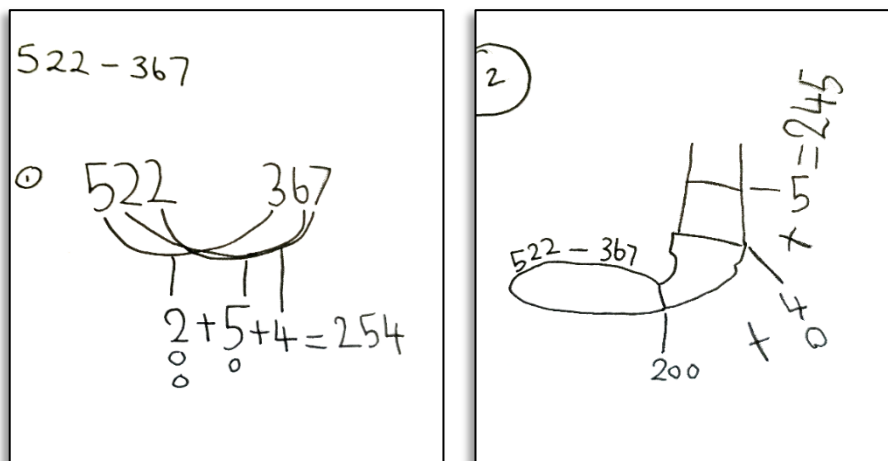


Figure 7.3 522-367 – Alex’s Spiderman strategy (left) and Vacuum Cleaner strategy (right).

The method Alex had been taught worked for multi-digit addition (because $20+60=60+20$), but when he tried to apply it to subtraction, it let him down (because $20-60 \neq 60-20$). However, the bigger concern was that Alex was willing to accept his answer without questioning it. (In hindsight, I realised I should have asked him if the answer would be more or less than 200 to test my theory that his intuitive response would most likely have been correct). This gave the impression that Alex was having difficulty subtracting multi-digit numbers, however, when re-visiting the *Adding Corners* task straight after this question (see Phase 4 task-based interview, Appendix 2), where he needed to add and subtract multi-digit numbers from the centre number to find three addends, he was again able to do this correctly (he chose to partition 5023 and then 705,093). The task required some thinking and mental manipulation, but he did not attempt either a ‘Spiderman’ or ‘vacuum cleaner’ method, and therefore was not struggling with a so-called ‘strategy’ that did not work. It seems that presenting a numerical expression (e.g., $522-367$), elicited a completely different approach from when the same operation was encountered within a problem context [e.g., when busting 705,093 into three addends].

I observed the same thing with Sammy, who also had difficulty with $522-367$, but could successfully partition 1007 in *Adding Corners*. I also observed something similar with Fred. Fred was asked to solve 87×9 any way he liked, and he chose the traditional vertical

multiplication algorithm. When asked to solve it a different way, he stumbled: “um – I haven't done another way for a while – oh, I don't know – I haven't used other strategies for a while – ah – I'll try and make up a method – oh, I don't know – I don't know these other methods that much –” (Phase 4 task-based interview, November 2014). The “other strategies” and “other methods” Fred alluded to, appeared to be alternate algorithms he had seen (such as the ‘lattice’ method) that he could not remember how to do. When I asked him, “In what sort of situation might you have to solve a problem like 87 multiplied by nine?” he replied, “Maybe ... they might come up with a question saying we have 87 bags of lollies and Jimmy bought nine of them, how many lollies does he have?” (which he obviously perceived as nine bags with 87 lollies in each). When asked how he would go about solving this problem if he did not have pen and paper to use, after initially attempting to mentally visualise the algorithm, he eventually said, “I could do nine times *ninety*, and that would be 810, then take away three nines, which is 27. So, take away 27 is ... 783.” With some further questioning about why he changed the 87 to 90, he also suggested, “Oh, yeah, that would also be an easy way of doing it – 87 times 10 which is 870, and then take away 87, which would be ... 783.”

In real life, mathematical calculations are solved in context. In school, we often present students with isolated calculations, supposedly to learn and practise skills that will be needed to solve real life scenarios. However, from the observations of Alex, Sammy and Fred, a student's approach to calculations presented as numerical expressions, and the approach to calculations embedded in a context, may actually be different. Reys and Yang (1998) documented something similar in their research project on sixth- and eighth-grade Taiwanese students, whereby mathematical computation results did not reflect students' number sense, and vice versa. Moreover, if a student as capable as Alex is willing to accept incorrect answers without question, when he was quite capable of solving similar problems successfully *before* being taught ‘how to’ solve them, this may be evidence that teaching mathematical operations in isolation, and/or drill and practice of the four operations, may actually be doing harm. Also, it could be masking student capabilities if a student as capable as Sammy can make basic errors in ‘formal’ calculations that do not appear when she is calculating a problem in context. It could be limiting students to one method of calculation, regardless of whether it is the most efficient strategy or not, if a student as capable as Fred only thinks to apply alternative strategies when prompted to consider a contextual scenario. These observations may be evidence that focusing on practising calculations may not be

assisting students in their application of mathematics in real life scenarios at all. It is certainly an issue worth highlighting with teachers so they can be aware of how they present mathematical problems. It may not mean that they never present mathematical calculations out of context; it may mean that they teach students, as a first step in any isolated calculation, to consider a hypothetical scenario where this calculation may be required. Using and adapting models such as Think Boards or Y-Chart organisers (see Figure 7.4), beyond the early years of schooling, may be one way of encouraging this. There was the issue of both Fred and Sammy appearing to plateau in their number learning from Grade 2 to Grade 3 (see Section 6.2.1 and 6.3.1 respectively). There could be several reasons for this, but maybe one reason is that this is often when students are beginning to learn traditional algorithms, or other taught methods of calculation, that may over-ride their intuitive number sense. This is possibly worth further research.

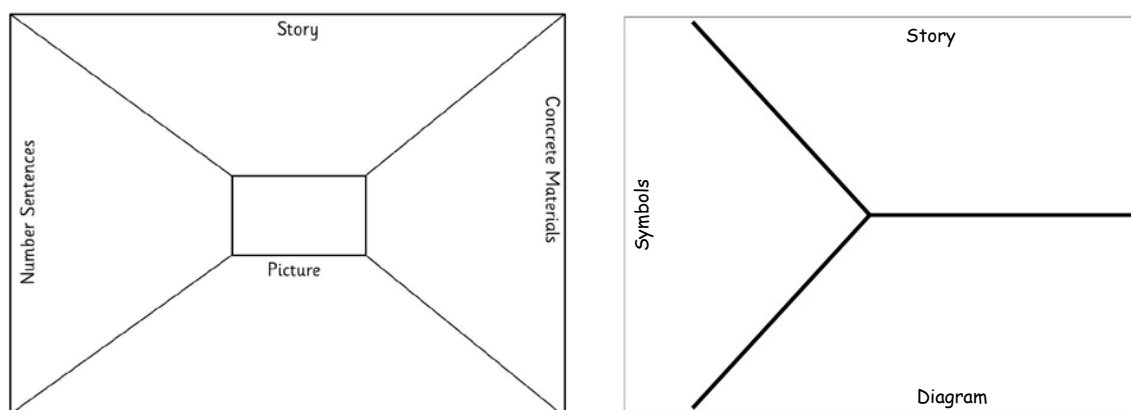


Figure 7.4 A Think board (left) and Y Chart (right) may be used to encourage students to contextualise mathematical calculations.

There also needs to be a distinction made between a *strategy* (e.g., subtracting the 100s then 10s then ones [$522-367 = 200-40-5 = 155$], and a recording *method* (e.g., ‘Spiderman’, empty number line, etc.) (Sullivan, 2011). Strategies are not ‘taught’, they are observed, and can be described or modelled using various written representations. Many students will intuitively develop common strategies such as ‘turn arounds’, or adding 100s first, without explicit instruction, through the development of number sense (McIntosh, Reys, Reys, Bana & Farrell, 1997). These strategies can be described and discussed, and other students may adopt them if they make sense to them; and teachers can plan learning experiences that encourage students to explore alternate and more efficient ways of calculating. However, as soon as teachers try to ‘teach’ these strategies, they are in fact teaching alternate methods, or algorithms.

It was also notable that Alex was already quite procedural in his mathematical thinking, whereas other nominated Grade 1 students were more willing to take time to stop and think and reason through a problem. For example, in the proportional reasoning question in the pre-selection task-based interview – *Three lollies cost 10c, how much would 12 lollies cost?* – Alex answered very quickly, making the comment, “I just added a zero” (which gave him an incorrect answer of \$1.20, which he did not seem to see any need to consider further for reasonableness). Mick, by contrast, when solving the second question – *Three lollies cost 10c, how many lollies can you buy with 60c?* – thought quietly for a long time and then said, “It [60c] is six of the threes, so it’s [he then counted by threes from 12 because he knew 12 was four threes] 18 [lollies for 60c].” Also, Alex seemed almost obsessed with skip counting, showing me how he could count by 100s (beyond 1000), 25s, 13s, 14s, sevens and nines in the first interview (more examples of his ability to skip count came up in the Phase 2 classroom observations), whereas other students were more interested in telling me about mathematical curiosities they had encountered. For example, Henry said, “Do you know how many tens in two hundred and thirty-five? There are twenty-three tens!” Mick announced, “Negative five is less than zero!” and Jack was keen for me to know that, “Even is like two, four, six, eight – ones that have a partner!” – each of these statements were made as random interjections as they were working through the pre-selection task-based interview. Alex’s procedural predilection may have reflected his mother’s input into his early mathematics learning, or something else entirely, but it is important to be aware that some students may come to school already thinking procedurally, which may have an impact on their beliefs about mathematics learning. Teachers need to encourage and celebrate young students’ mathematical curiosities and attempts at intuitive reasoning, and ensure this is maintained and not quashed because of ‘school learning’ (Clarke & Roche, 2010). Interestingly, there was no mention of skip counting in Alex’s post-classroom observations or interview, which may be evidence of a shift in his thinking to mathematics being more than just number skills.

7.3.3 Explaining and recording solutions

Explaining and recording solutions to mathematics problems is an important skill for students to learn and develop (Brown, 2008; Knuth & Peressini, 2001; Sheffield, 2003) as it strengthens mathematical understanding (Pugalee, 2004), and helps students better organise and clarify their solutions (Burns, 2004). Having this skill will also enable students, who have been encouraged to be creative, to substantiate, demonstrate and share

new and innovative ideas from their investigations and explorations (Krantz, 2007). However, what was observed in this study pre-teacher professional learning, was that highly capable students had great difficulty writing formal mathematical solutions. In the Grade 5 classroom, mathematics writing seemed to be limited to students jotting down their thinking processes, basically to free up working memory, and then writing down their final answer as the solution. There seemed to be more expectation for the Grade 3 students to explain and write about their mathematical methods and solutions, but Sammy still found this very difficult, requiring much direction and scaffolding to write just a few sentences. In Grade 1, “How did you work that out?” seemed to be a regular part of teacher/student discussions about solutions, but written expectations focused mainly on generating number sentences (equations) from a mathematics scenario.

One of the components of the suggested lesson structure in the teacher professional learning (see Section 5.4.2 and Figure 5.4) aimed at addressing this issue of written explanations of solutions. Fred’s teacher, Ms J, decided to make this a focus of Fred’s ongoing learning, intentionally building into her repertoire questioning such as, “If you had to explain this to someone who knew nothing about this, how would you tell them what you're thinking?” and including expectations of ‘providing justifications for solutions’ in her marking rubrics. She noted an improvement in Fred’s ability to express his mathematical thinking in writing, and explaining and justifying his processes and strategies as a result of this (see Section 6.2.3).

No further comments were made by either Sammy’s or Alex’s teacher(s) about intentional strategies for developing mathematical explanations or justifications. This continued to be a verbal expectation in the classrooms, but written explanations, especially for Alex did not seem to be a priority. Ms K described one lesson that she had felt had been particularly successful for Alex:

I was just thinking about what we did yesterday in the sandpit, we were talking about capacity and there was a lot of inquiry and working at finding equal containers, but because it was fairly simple, what's bigger and what's smaller, I asked [Alex] to look at parts of containers to challenge his thinking – what would be one and a half times that container, that sort of thing. So he liked that, but I think it's also the recording ... there wasn't that physical thing [of recording his explorations], it was just “What do you think? What would be a third of that?” [and] would he be able to use that language? And he was able to go and fill up his thing with sand, with his partner. So

that was part of the [challenging] work without the stress [of recording]. (Ms K, November 2014)

Ms K seemed to think that the freedom from having to write anything in this outdoor lesson was, in part, what made it successful for Alex. It is important to allow students freedom to explore and think without the constraints of written explanation at times, but with a current emphasis on mental computation in the early years of schooling (see ACARA, 2015), as a reaction to the emphasis on the early learning of written algorithms which proved to be detrimental to children's intuitive number thinking (Kamii, 1994), it may be that the pendulum has swung too far. Both forms of computation, mental and written, have their place in the learning of mathematics, and writing in mathematics also needs to go beyond simply written calculations (Burns, 2004; McIntosh & Dole, 2000; Pugalee, 2004; Urquhart, 2009).

This is particularly pertinent in the dissemination of new and innovative mathematical ideas. Consider the story of Ramanujan, the self-taught Indian mathematician from the early twentieth century. Ramanujan made many astonishing intuitive mathematical discoveries, but had great difficulty recording proofs of his theories (Kanigel, 1991). Proofs are essential as, even though most of Ramanujan's theories have been proven to be correct, and continue to make substantial contributions to mathematics, a few, such as his theorem on prime numbers, were proven to be incorrect (Kanigel, 1991).

We may not be expecting primary school students to be writing mathematical proofs, but learning how to explain and justify thinking processes, and recording these as permanent reports of solutions, is a pre-cursor to this. This seems to be another issue for ongoing teacher professional learning – scaffolding students' ability to both explain and record solutions – especially in regard to supporting mathematically gifted students, who are likely to be those who go on to higher levels of mathematics. It also needs to be realised that for these students, their mathematical thinking is often greatly telescoped (Krutetskii, 1976) and therefore harder to break down into progressive steps without support and maybe some form of scaffolding (Byrd, 2016) (see Section 5.3.2).

7.3.4 Mathematical creativity

Focusing on encouraging mathematical creativity as a form of differentiation, *Learner-Differentiated Curriculum and Instruction* (Betts, 2004), was a significant part of the targeted teacher professional learning for this study, as encouraging and scaffolding

mathematical creativity is becoming a critical part of the school mathematics curriculum in the current global economic climate (Sheffield, 2013; Sriraman, 2017). The idea of using students' own curiosities and interests to explore mathematics tasks further, as a form of differentiation, was presented, by the researcher, at a whole-school professional learning session (see Section 5.3). Mathematical creativity takes students beyond knowing mathematical concepts and procedures, to knowing how mathematics is created and used to explore new concepts, and to solve problems in original and innovative ways (Sheffield, 2006). Unfortunately, standards-based education systems that focus on standardised test scores and grades, give students, and their parents, a conflicting view of educational success. Both Fred and Alex used grades as measures of their mathematical abilities, and all three – Fred, Alex and Sammy – viewed fast thinking to be an indicator of 'someone who is good at maths.' Understanding mathematics as a creative venture, based on problem solving and problem posing, which require time and sustained effort and not just an ability to calculate correct answers quickly as the only objective (Sheffield, 2009; Sriraman, 2004), would require a paradigm shift for them. Such change most likely also requires a paradigm shift for many parents, as attested by Alex's mother, who may also believe that good school grades are the best measure of mathematical ability (see Bicknell, 2009a). It was therefore important to consider the students' creative approaches to mathematics post-teacher professional learning to see if any changes were evident.

The idea of encouraging students to explore their own mathematical curiosities further as a means of differentiation was a new concept not previously considered or used by the teachers in this study prior to the professional learning. During the post-professional learning period, though, both Fred and Sammy were observed creatively exploring mathematical tasks further in classroom activities. When given permission to explore, Sammy was excited and enthusiastic about new discoveries and loved sharing these, with a regular "Wow! I'm going to tell Ms S about this!" The measurement task with the shoe, 'How can you measure the space inside your shoe?' (classroom observation, November 2014, see Section 6.3.4) was an excellent example of how the two girls, Sammy and Janet, were willing to challenge themselves and subsequently discover the basic, but fascinating and counter-intuitive, eight-fold ratio of doubling volume. Fred, having previously been very insecure about even commencing a task that had not been fully detailed, during the post-professional learning period was noted, by Ms J, as discovering that he really enjoyed the freedom of exploring given mathematics tasks independently. For example, he modified

games, directed his own learning, and adopted his own approach to a set task to improve possible outcomes (see Section 6.2.4). Fred also mentioned this himself in the post-semi-structured interview. When asked to recollect the mathematics he had enjoyed the most in the previous weeks, he talked animatedly about being able to self-discover how to operate with negative numbers, with calculations such as ‘four minus negative four’, and multiplying and dividing negative numbers, and how this could be explained mathematically. He was very excited about being able to work with other like-minded peers on this particular task over a period of two weeks. Ms J also recognised the importance of including criteria such as creativity and collaboration in marking schedules, or rubrics. Using an assessment tool this way became a strategy for addressing the required paradigm shift in what is deemed important and valued in students’ mathematics learning, rather than just focusing on an achievement score or grade. The students’ approaches to creative explorations of mathematics within the classroom are inextricably linked to teacher approaches and expectations. However, in observing these developments with Fred and Sammy, it seems that this approach to differentiation is indeed possible. Fred and Sammy still required some scaffolding, with the teacher and/or researcher having to suggest they explore further, and giving some guidance on what they could explore further, but these observations were after only three months post-professional learning, and more time and experience may see these students adopting this approach more independently. A longitudinal study on this could be interesting.

On the other hand, there was little evidence of Alex creatively exploring mathematics tasks further independently. This may have been due to his younger age, or to the approach of his teachers, or to the limited number of lessons observed.

In each of the instances above, Fred and Sammy had been working with other students. Fred commented on how he really enjoyed the process of discussing new ideas with his ‘mates’ and ‘figuring things out’ together. This learning structure aligns with a social constructivist view of learning which places emphasis on the role of others in the learning process (Vygotsky, 1978). It also aligns with the global need for increased collaboration within workplace environments as organisations become more connected internationally (Office of the Chief Scientist, 2014). In a collaborative workplace, ideas can be pooled to make projects more successful, and collaboration cultivates a sense of community within an organisation, which also improves productivity (Becker & Steele, 1995).

7.3.5 Surprising mathematical difficulties

Another commonality worth noting, that became evident through interviews and classroom observations, was the occurrence of some surprising mathematical difficulties encountered by a number of the nominated highly capable students. Mathematically gifted students may learn new mathematical concepts differently due to neural efficiency (Miller, 1994; Zhang et al., 2015), and may be advanced in their mathematical knowledge, but there are still things they do not yet know, or may have difficulty with, and this may be surprising for their teachers. It is easy to assume that gifted students will be good at all things mathematically, but this is usually not the case (Silverman, 2013; Winner, 1996). Teachers also need to understand that even highly gifted students do not just ‘know mathematics’, they have to construct mathematical concepts just as other students do. They may construct those concepts with minimal experience, but we cannot assume that they have had experience of all concepts, nor that the experiences they have had were conceptual, rather than simply procedural, and/or robust enough to prevent the development of misconceptions.

Surprising difficulties were observed with both Alex and Sammy, and with several other mathematically highly capable students nominated for the study. Some examples of these surprising difficulties follow:

- In the pre-selection task-based interview, one of the nominated Grade 3 girls, Emma, decided that she would challenge herself in the *Adding Corners* task by partitioning her centre number into “all the *exact* same number”. She had chosen the number 99 to go in her triangle, and had partitioned it into $22+33+44$ for her first creative and challenging solution. Then, with some trial and error, adjusting and readjusting her original three numbers, she came up with her second solution, $33+33+33$. She was very happy with her results. I made the comment, “I wonder if you could do that with any number you put in the middle?” This was actually an attempt to delve into her thinking to see if she would consider fractions as part of a solution. What transpired, however, was over five minutes of investigation, including Emma’s conclusion that it worked for 156 ($52+52+52$), but “... it wouldn’t work [with all numbers] because if you had a five at the end it wouldn’t work so well.” I challenged her with the number 45, and after watching her for a while, using the same trial and error process as before, suggested, “What if I gave you a calculator, would that help?” (This was taking longer than I had anticipated and I was concerned about running out of time for the rest of the interview).

Her reply was, “Um – maybe...” She took the calculator but did not use it. At one stage she muttered, “If you divide it by something – it would be something divided by something ...,” but it was not until she finally worked out, with her trial and error, that the solution would be $15+15+15$, that she realised, “It would have to be forty-five divided by three!” (pre-selection task-based interview, May 2014). I was certainly surprised that Emma, who was capable of pre-proportional reasoning, and who had learnt the basics of simple abacus calculations readily, was completely stumped when it came to the application of a simple division scenario.

- In a classroom observation (pre-professional learning, August 2014), I watched Sammy and Janet working together exploring equivalent fractions. Sammy noticed that with all fractions equivalent to half, the numerator was, in fact, half the denominator. She wrote down her ‘system’ in a list: $8 \div 2 = 4$, $4 \div 2 = 2$, $16 \div 2 = 8$ and $2 \div 2 = 1$. Janet then intervened, insisting that two divided by two was two, not one. This confused Sammy, so they asked me to clarify the correct answer. I asked them to demonstrate the problem with materials – “If you have two pens, how many groups of two do you have?” – Sammy was now convinced she was correct with her answer of one, but Janet continually separated the pens saying “Two!” Once again, I was surprised with this difficulty. Janet and Sammy were both very highly capable mathematically, but Janet could not conceptualise $2 \div 2$.
- In Sammy’s Phase 4 task-based interview, solving the problem 522-367 in two different ways proved to be difficult for her. She solved it once, but then in solving it a different way arrived at a different answer, so was then asked to check it a third way. She tried simplifying the problem to 520-370 (writing down +2 and -3, to remind herself to reverse what she had done at the end), but then proceeded to subtract 20 from 70 instead of 70 from 20 – trying to apply commutativity to a subtraction problem. When this did not work, she started complaining that she hated “minusing with big numbers,” and “It would be much easier if it was just plus. Can't it just be plus?” I suggested she make up a different problem, one that was still ‘minus’, but one that she knew she could solve (modelling a ‘try starting with an easier problem’ strategy). She came up with 100-67 and quickly solved it mentally as 43 ($100-60=40$; $10-7=3$). Sammy approached 100-67 with a common error of adding the two differences, instead of subtracting and then subtracting again (resulting in an answer of 43 instead of 33). I knew the problem 522-367 was going to be challenging for Sammy, but I had not expected 100-67 to be difficult for her. Sammy’s capability was still evident in this situation, though – not

because she was able to calculate a correct answer with no struggle, but because she was able to figure out where she had gone wrong, “Oh, I get it! I get it! I get it! If I have 100 and minus 60 I will have 40, not 50, and then I have to minus the 7 which will be 33! I get it! I get it!” She could then immediately apply her new understanding back into the original multi-digit subtraction problem. This is evidence of accelerated learning.

- Ms C, in the Phase 3 interview, voiced surprise that, at the beginning of the year, Alex could not tell the time beyond the hour and half hour, the same as the other Grade 1 students, saying, “Coming into Year 1, I would have thought that he would have explored that a bit more.” By the end of Grade 1, Alex certainly knew much more about the structure of a clock and reading the time. A clock he had drawn during a Phase 3 class observation, showed five o'clock (see Figure 7.5), but he then put in a third hand. When asked about his drawing, he explained, “My clock at home has a second hand ... and it's five o'clock and ten seconds” (which he also wrote in digital notation). He knew it was ten seconds because his second hand was pointing to the two, and, “there are 60 seconds in one minute.” When I asked him, “How do you know that [that there are 60 seconds in a minute] by looking at your clock?” he said, “ooo aaw,” and started drawing in minute markers between the numbers. He counted as he drew the marks, “one, two, three, four” then stopped and looked at the numeral one and said, “that's five,” and continued with, “six, seven, eight, nine, and the two is ten.” (classroom observation, November 2014). However, Ms C's comment on this clock picture was, “[He] had the time right, but the hands of his clock were virtually, I got a ruler – virtually the same!” She seemed concerned that Alex had drawn the hands a similar length, and she had asked him to correct this (as seen in Figure 7.5), but did not acknowledge the minute markers nor the second hand details he had included. She seemed distracted by something she was surprised that Alex could not do, and possibly missed a teaching opportunity because of what she expected him to know. The issue with the length of the hands may have been a visual discrimination issue that was age appropriate for him (even Ms C had to get a ruler to check to see if they were the same length or not), requiring discussion that did not rely on simply ‘long hand’ and ‘short hand’ to help discriminate between the minute hand and the hour hand as indicators of

telling the time. Recognising and addressing the difficulties that even gifted students have, could benefit many other students in the class too.

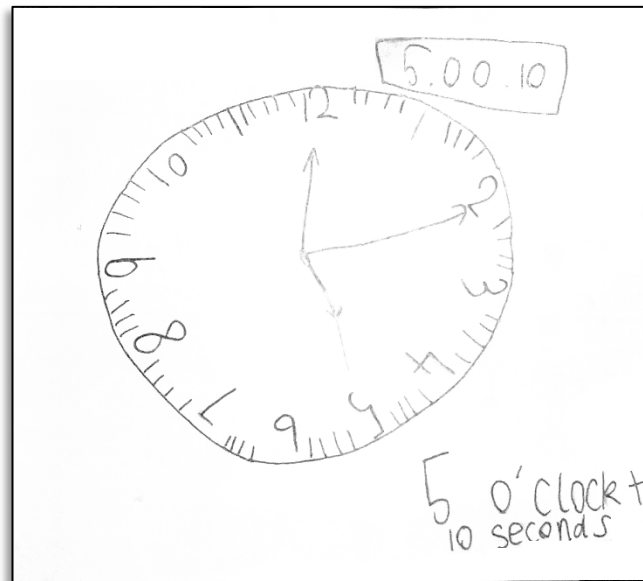


Figure 7.5 Alex's clock drawing
(post-professional learning classroom observation, November 2014)

High ability, and the expectations that may come with students with high ability, can sometimes mask misconceptions and/or gaps in knowledge they may have. Teachers can be quite surprised at the 'unknowns' or the errors highly capable, and definitely gifted students, make in seemingly simple problems when they can solve other quite complex problems. If this is not recognised as 'normal', it may become an issue if teachers are providing students with 'higher grade' mathematics believing this is accelerating or differentiating their learning. Acceleration means to speed up, or telescope the learning, it does not mean to 'bump them up a grade' or skip a whole section of work considered as being 'too easy'. Acceleration may be very effective (Silverman, 2013), but requires a lot of work on the teacher's part to ensure important concepts are being constructed correctly. Sometimes it is not known what will prove to be a challenge for gifted students. Sometimes, what is thought will be a challenge turns out to be quite easy; sometimes, what is thought will be easy turns out to be quite challenging. One of the benefits of allowing students to explore mathematical ideas themselves, things that interest or intrigue them (see Section 5.3.2), is that these explorations may uncover some previously hidden difficulties and/or misconceptions (for example, as revealed when Emma choose to find three numbers the same to put in the corners of her triangle). This seems to be another significant issue to cover in future teacher professional learning.

7.4 Commonalities and Similarities of Mindset Behaviours

Subsidiary research question two asked:

- *How do students who are mathematically gifted view themselves as mathematics learners before and after their teachers receive professional learning, and a subsequent teaching period?*

Fred, Sammy and Alex were selected for the study because each exhibited self-limiting, negative mindset behaviours. Underachievement is a major area of concern in gifted education (Siegle, 2013), and a negative learner mindset may be one causal factor contributing to this. If so, this is a crucial element for teachers to be aware of – how and why it manifests in gifted individuals, and how it may be addressed to effect a positive change. The mindset behaviours of the case study students were not identical, nor was the extent of the changes in their behaviours, as observed post-teacher professional learning. However, in examining the beliefs the students had about themselves as learners of mathematics, which impacted their mindsets, three significant common themes emerged from the narratives: 1) the students' beliefs about what it means to be 'good at maths'; 2) their beliefs about sustained effort, perseverance and making mistakes; and 3) beliefs regarding seeking help with difficulties. This section explores each of these themes, analysing and interpreting the significance of them and the changes observed in students' mindsets from pre- to post-teacher professional learning, based on the literature discussions from Section 2.5. Table 7.3 shows an overview of student mindset beliefs and behaviours pre- and post-teacher professional learning.

Table 7.3

Overview of student mindset beliefs and behaviours pre- and post-teacher professional learning

Theme	Fred	Sammy	Alex
What it means to be 'good at maths'	<i>Pre-</i> Gets good grades (A+), understands things quickly, works quickly, does not make mistakes, does not need to ask for help. I am good at maths.	<i>Pre-</i> Finishes work quickly, never makes mistakes, wants to do more. I am not good at maths.	<i>Pre-</i> Gets good grades, good at quickly estimating correct answers to calculations. I am good at maths.
	<i>Post-</i> Can do harder mathematics (than most others), can work with others and self-direct their learning, can source relevant resources to help. I am good at maths.	<i>Post-</i> Can do hard mathematics tasks and persevere with them. The teacher can help by being "stubborn" (having high expectations). I am good at maths.	<i>Post-</i> Gets good grades (A+), does hard mathematics, perseveres, accepts support from the teacher for difficulties. I am good at maths.
Sustained effort and making mistakes	<i>Pre-</i> Became very distressed if he could not finish some work quickly, especially in an assessment. Admitted that, given the choice, he would choose easier work that he could finish quickly and not make mistakes in. When he did make mistakes, or did not understand a task, would voluntarily follow through on the work at home, persevering until he did understand.	<i>Pre-</i> Became very distressed if others in the class finished work before she did. Also became very distressed if she made a mistake (or thought she had made a mistake) in front of her peers. Reticent to verbally answer questions or make suggestions in a group in case she was incorrect. Chose easier tasks that were 'safe', that she could complete without too much effort, and with no mistakes.	<i>Pre-</i> Believed that mistakes are made if you are 'no good at maths'. He used to make mistakes when he was younger, but not now. Was willing to persevere with a task to a certain extent, however five minutes to complete a task was considered "challenging enough". If expected to persevere beyond what he believed was reasonable would have emotional 'meltdowns'.

(continued)

Table 7.3 Continued

Theme	Fred	Sammy	Alex
Sustained effort and making mistakes (continued)	<p><i>Post-</i> Enjoyed hard tasks that required sustained thinking, said this made them more interesting.</p> <p>Deliberately chose numbers that would be particularly challenging in the post-<i>Adding Corners</i> task, prepared to take time to solve it, and not concerned about mistakes he made, they were just something to be fixed.</p>	<p><i>Post-</i> Discovered mistakes could, in fact, be learning opportunities. More willing to make suggestions in group tasks, and receive responses to her suggestions from others. Was most proud of work she had recently completed that had required considerable sustained effort.</p>	<p><i>Post-</i> Work that he was most proud of was a task he had worked on for half an hour (and had not fully completed due to time), even though he had made a number mistakes that needed to be challenged. Required considerable support and encouragement to keep persevering with this task though.</p>
Asking for help with challenging mathematics tasks	<p><i>Pre-</i> Said he didn't really need to ask for help from anyone, could just keep thinking for himself.</p> <p>In class asked lots of questions to clarify that he was understanding and/or tackling a task correctly, but did not take up the offer of group assistance given in class when he found something challenging.</p> <p><i>Post-</i> Enjoyed discussing difficult tasks with others as having 'more ideas' helped. Recognised the benefit of online resources that could be used for help.</p>	<p><i>Pre-</i> Believed that requiring help was an indication that she was not good at mathematics and/or if she required help the task was too difficult for her.</p> <p><i>Post-</i> Still struggled with the difference between 'really hard' and 'too hard' even though she was willing to persevere with harder tasks. Believed if she had required any assistance the task had been 'too hard' for her.</p>	<p><i>Pre-</i> Rarely asked for help, possibly did not need to with the tasks he was completing.</p> <p><i>Post-</i> Recognised that getting help could support his learning more, but struggled with how to ask for help (specific questions vs 'I can't do this').</p>

7.4.1 Student beliefs about what it means to be ‘good at maths’

In the Phase 1, pre-professional learning interviews, Fred, Alex and Sammy were all in agreement that someone who is ‘good at maths’ works quickly, and rapidly calculates correct answers. Fred and Alex also both mentioned good grades, and Fred recognised ‘understanding new things easily’ as a hallmark of high mathematical ability. Fred and Sammy used ‘not making mistakes’ as one of their benchmarks. Based on their own belief structures, Fred and Alex were aware that they were mathematically highly capable, whereas Sammy believed herself to be ‘no good at maths,’ as she could often not answer mathematical questions quickly, and because she sometimes made mistakes. She did not seem aware that many of the tasks she struggled with were, in fact, more complex versions of the tasks the majority of the class were working on.

One significant change in the Phase 4, post-professional learning interview, was Sammy’s response to the question: ‘If this chart represents all the children in your class, from the *best at maths* to the one who *takes a while to learn new things in maths*, whereabouts would you place yourself?’ She had previously placed herself in the middle, because others finished their work quicker than she did and did not make mistakes. She now placed herself close to the *best at maths*, because “I did that [a challenging task] I thought it was too hard and I did it ... Also, I’ve heard others in the class say something is too hard, but I can do it.” She recognised that if she persevered she could complete work that was too hard for many in her class; speed and ‘no mistakes’ were no longer an articulated factor post-professional learning. Another very significant change for Sammy was that she was no longer constantly saying, “I’m no good at maths,” and she recognised that this was a direct result of intentional positive self-talk that had become automatic over time (see Section 6.3.4). She recognised this as a direct result of her teacher’s help with the mindset chart (see Section 6.3.4), and appreciated her teacher’s “stubbornness” in her expectations of Sammy’s perseverance with difficult tasks. Unlike Sammy, Fred and Alex were both confident mathematicians in the pre-professional learning interviews, each assigning themselves as one of the best at mathematics in their respective classes. There have been significant findings in earlier research on gender differences in mathematics self-efficacy (e.g., Forgasz, 1992; Leder, 2004), and this difference is certainly reflected with these three students. The change in Sammy’s perception post-professional learning may be particularly significant considering these earlier findings.

Fred and Alex had both cited their grades as being significant indicators of mathematical ability, as well as working quickly and, for Fred, not making mistakes. Mathematics assessments, and subsequent grading, have been historically based on timed tests. These are still commonly used in schools, including mandated national assessments such as NAPLAN (ACARA, 2016), so it is not at all surprising that students, and parents, hold these beliefs. However, the hallmark of high-level mathematical thinking and ability is working in depth, not at speed (Boaler, 2016; Krutetskii, 1976), and this needs to be reflected in how we assess and report student achievement. Timed assessments are most likely here to stay, as they are relatively easy to administer to large cohorts of students, and provide numerical data that can be collated and quantitatively analysed for government purposes (ACARA, 2016). This, coupled with parent experiences and perceptions of mathematics achievement, means that teachers, and schools, may need to be very intentional, and very transparent about the intentions of assessing components of high-level mathematical thinking such as independent thinking (rather than simply remembering procedures), sustained effort and perseverance, creativity and ‘risk taking’, mathematical discourse and dialogue (including asking questions), and self-correction (Brookhart, 2010). There is a need to change more than just the students’ beliefs about what it means to be ‘good at maths’; many adult perceptions as a whole also need to be challenged (Boaler, 2016). One aspect of the professional learning addressed this with an example of modifying a marking grid taken from an observed mathematics lesson in the Grade 5 class, *Building a Dream House* (classroom observation, July 2014). Figure 7.6 shows the original grid the teacher used for the task (top) and a modified grid (bottom) showing one way teachers can intentionally assess other components of the learning process that are valued, as well as the mathematics skills being developed.

During the post-professional learning interview, Alex still focused on grades as the best indicator of his mathematical ability, but was more willing to spend sustained time on tasks rather than having to prove himself with speed (see Section 6.4.4 – Rush Hour task). The work he was most proud of was a task he had spent a considerable length of time on, having to persevere through feelings of frustration, and requiring support from the researcher (as participant observer) (see Section 6.4.4 – AFL Grand Final task). Fred did not mention grades at all during the post-professional learning interview, but was still concerned about tests, “In tests I [still] get a bit worried.” He said this had improved, though, because “I

Dream Home marking rubric (teacher’s version)

Skills	High	Medium	Low	N/A
Effective use of isometric paper				
Creative design element				
Ability to calculate building cost				
Ability to calculate painting cost				
Understands the properties of shape, i.e., faces				
Additional Comments:				

Dream Home marking rubric (adapted version)

Assessed Criteria	Excellent	Good	Needs work	N/A
Creative and challenging house design				
Perseverance with isometric drawing				
Efficient organisation of costings				
Skills: <ul style="list-style-type: none"> • Drawing isometric design • Calculating costs • Visualisation skills • Language of geometry (e.g., faces, perspective, oblique /vertical lines, etc.) 				
Willingness to explore further				
Additional Comments:				

Figure 7.6 Grid used for Grade 5 mathematics task Design Your Dream Home. Teacher’s version (top), and researcher’s modified version (bottom).

figured out ... I just don't need to doubt myself, just figure it out and believe it's the right answer" (Fred, November 2014). He also recognised that 'being good at maths' meant that he and his peers were able to work through complex mathematics ideas by themselves, with guidance but not specific instruction from a teacher, by accessing relevant resources and discussing ideas with each other. He seemed to really enjoy discovering this form of self-directed learning. He still tended to check with the teacher that he was doing the right thing, but not as often, and not as anxiously (see Section 6.2.4). Requiring time, perseverance and assistance were not things Fred, Sammy or Alex had previously associated with being 'good at maths', but all had alluded, to various degrees, to these elements when describing themselves as doing their best mathematics in the post-interviews. This may be evidence

of a changing mindset that could be attributed to the way their teachers had begun to notice, and expressly value, these components of their students' mathematics learning.

7.4.2 Student beliefs about sustained effort and making mistakes

At the beginning of the study, Fred, Sammy and Alex had all focused on completing mathematics tasks quickly, with Fred and Sammy also being very insecure about making mistakes. Fred became very distressed if he could not complete timed tests, and admitted that, when given a choice, he would choose easier tasks that he would not make mistakes with, so he could finish quickly. According to his teacher, though, in response to a difficulty and/or mistake, Fred was often determined to revisit the particular task in his own time to figure out where he had gone wrong and remedy this, to ensure he would not get it wrong the next time.

Sammy also became very distressed if others in the class finished tasks quicker than she did, believing this made her "hopeless" at mathematics (see Section 6.3.2). She was also reticent to answer questions, or offer suggestions, in group discussions, in case she was wrong. Any immediate response that was given was often followed through with the proviso, "[but] I don't know, I'm no good at maths." In response to a difficulty or perceived mistake, Sammy reverted to an easier task she felt confident completing without effort, and with no mistakes (see Section 6.3.2).

In the Phase 1 interview, Alex was prepared to challenge himself with a large number in the *Adding Corners* task, and was willing to persevere with this task to a certain extent. However, because it had taken him a long time, and he made mistakes and "had to cross out most of the things" (see Section 6.4.2) meant that he was not particularly happy with the outcome of that solution. He was much happier with a simpler solution that he could complete quickly with no mistakes. In the pre-selection classroom observation, he had been given a 'challenging task', which his teacher had differentiated from the task the other students were completing by using larger numbers. He certainly had to apply some effort to complete this task, but still completed it in under five minutes. His response, on completion, was to huff and puff as though he had run a marathon, and he was keen to have a break from the hard thinking.

Sammy and Alex both alluded to the belief that if you are 'good at maths' you always get correct answers, which confirmed Sammy's beliefs about her own ability (I sometimes make mistakes, therefore I am no good at maths), and affected Alex's choice of task (e.g.,

“This is too hard for me [because I did not get it correct], I think I chose the wrong number” (see section 7.3.1)). Alex mentioned several times how he had made mistakes when he was younger, but not now, and when questioned about mathematics he did not yet know (and therefore may struggle with and not get correct answers), was adamant that this was work that was too hard for him, and he would learn it when he was older. He was happy to wait, even until Grade 6 to do this (see Section 6.4.2).

During the post-professional learning interviews, sustained effort was something all three students were actually proud of. Fred suggested that having to work at something that involved considerable mathematical reasoning and hard thinking was now evidence that he was good at mathematics. In the Phase 4 interview *Adding Corners* task, he chose a challenging number for the centre, and a challenging partitioning of that number. He enjoyed the sustained effort required, was not worried about the time it was taking, nor that he had made a mistake (he seemed to want to correct this because he was enjoying the task, not because he was disturbed by it; he actually laughed about it) (see Section 6.2.4). He alluded to a belief that not being able to complete a task quickly made the mathematics more intriguing, not threatening.

Sammy, during the course of the three months following the teacher professional learning, not only discovered that learning from mistakes can be a positive thing, but also her teacher reported that she observed her being ‘happy’ about making a mistake as part of a particular problem-solving process as it helped clarify where she was going wrong (see Section 6.3.3). As a result, she became more willing to take risks, as she was no longer expecting herself to get the right answer straight away. In the post-professional learning interview, I asked her how she felt when Ms S gave her work that she thought was too hard:

Researcher: Did you feel happy that you were made to work hard?

Sammy: No. (laugh)

Researcher: Did you feel...um...upset?

Sammy: No. (laugh)

Researcher: Did you feel angry?

Sammy: No, not really. I kind of felt a little bit, like a little proud of me, that I finally got an answer, but then I just went ohhhhhhhh (mimicking exhaustion). (Phase 4 interview, November 2014)

In the Phase 3 classroom observations, Sammy was also much more vocal in small group/partner discussions about approaches to challenging mathematics tasks, willing to

make suggestions and consider others' responses to her suggestions without being threatened by this. Sammy's teacher also noticed that Sammy was beginning to realise that mistakes could help her to reach the answer if she used them to "hone in on what she needed to do to find the solution. If they never make mistakes, they'll never learn this important life lesson" (Sammy's teacher, October 2014). In the final interview, Sammy was most proud of work she had done that had required considerable sustained effort (see Section 6.3.4).

Alex, too, was willing to persevere (for more than half an hour) with a mathematics task that required sustained effort and challenge, without ending up in tears (see AFL Grand Final task, Section 6.4.4). However, this required considerable support and encouragement (from the researcher, as participant observer) for him to keep persevering. In the Phase 4 interview, Alex stated that he was most proud of himself for tackling this hard mathematics task. His teachers said they had not observed this for themselves in the classroom, though, saying that he would still end up in tears and 'meltdowns' if he thought the work they had given him was too hard.

Gagné's *Differentiated Model of Giftedness and Talent (DMGT)* (2009) has shown that effort and hard work are two factors that determine a successful development of gifts into talents. Having an innate ability may mean a person can develop talents or mastery earlier, or quicker, than their age peers, but it is still not possible without hard work and sustained effort. Effort is an integral part of the learning process. Indeed, when students experience success after a period of sustained effort, the completion of the problem becomes an intrinsic reward, subsequently motivating them to continue to learn (Pink, 2009). Sustained effort will often include struggles and mistakes, and these need to be viewed as a normal part of the learning process (Dweck, 2006). For students to understand, and believe this, especially gifted students who have not previously experienced the need to persevere with challenging tasks, teachers need to explain, and model the thinking that success is not instantaneous, that effort needs to be sustained, often over a long period of time (González & Eli, 2017). Teachers need to allow their mathematically gifted students to struggle with tasks, and make mistakes, so they can teach them that failures, and the deflated self-esteem that may accompany them, are usually not catastrophic (González & Eli, 2017). However, the students will most likely need considerable support in this process.

Fred, Sammy and Alex had all exhibited hypersensitivities (cf. Dabrowski & Piechowski, 1977), which included extreme emotional reactions to making mistakes, or not being able

to successfully complete mathematics tasks. Alex's teachers' response, especially, was to "back off" on their expectations to minimise these reactions, but rescuing students from their struggles to make them feel better, is not teaching them how to manage their learning. Neither is stepping in and fixing a student's mistakes for them. This sends the message that 'if at first you don't succeed, give up and let someone else do it for you' – a learned helplessness (Diezmann & Watters, 1995). Learning how to manage mistakes, by attributing them to a specific cause that can then be addressed, is important for developing an optimistic outlook on learning (Seligman, 1995). This may be a daunting thing for teachers, especially if students are having extreme emotional reactions, or 'meltdowns', but Sammy's teacher seemed to have managed to successfully moderate Sammy's responses through the use of the 'mindset chart' (see Section 6.3.3).

The idea of an *emotional response scale* (Bainbridge, 2014) is another strategy teachers could use to support students with intense emotional responses. The teacher (or a parent) works with the emotionally intense student to get him or her to list a number of events from the worst possible thing the student believes could happen (possibly something life-threatening), to the most minor thing that could happen (something negative, but reasonably inconsequential). This scale can then be used whenever the student gets distressed, by getting him or her to rate the current event according to the scale. The purpose of the emotional response scale is to give students a strategy to help manage their emotional responses, which can be applied to many different areas in their lives.

There is evidence that there were changes in the students' beliefs about sustained effort and making mistakes in mathematics from pre- to post-professional learning observations and interviews. Their teachers had all described them initially as becoming overtly distressed when faced with mathematical challenges that they could not complete relatively quickly, and/or if they made mistakes, especially in front of their peers. This had also been observed by the researcher with Sammy. However, in the follow-up observations and interviews all three students not only seemed more willing to persevere with challenging tasks that required time, risk-taking and possible mistakes, but all three intimated that it was these tasks that they were most proud of attempting, and that it was completing this type of task that proved that they were good at mathematics. Whether the change for Fred and Alex was a direct result of their teachers' professional learning is not clear, however Sammy's experience with her change in mindset certainly seems to be directly linked with her

teacher's 'growth mindset chart'. Sammy declared, "It's [the chart] just kind of worked like magic!" (Sammy, November 2014).

7.4.3 Seeking help with challenging mathematics tasks

Seeking help with challenging mathematics tasks goes in tandem with recognising sustained effort and learning from mistakes as valuable components of mathematics learning. However, for students who have previously viewed effort and 'not knowing how' to be hallmarks of a lack of capability, seeking out help may be something they have avoided, or they may not have had enough experiences where they have needed to do this. This may be compounded by a common belief that gifted students do not need help, they can manage on their own (Silverman, 2013; Whitmore, 1980).

Prior to the study, Fred was very willing to ask questions to clarify exactly what was required of him in completing a task, but this seemed to be more about reassurance that he was tackling the task correctly. This is different to seeking help with the mathematics involved in the task. He suggested, in his initial interview, that he had never really had to ask for help with mathematics, "I've never kinda felt that way before ... Normally when I'm having trouble I just keep on thinking harder and harder and reading over the question again, but I never really need somebody to help" (Fred, July 2014). During the classroom observation of the lesson on building design and isometric drawing, where Fred was having considerable difficulty, he neither asked for help, nor took up the teacher's offer of group assistance for those wanting help. If teacher assistance is viewed as something only for those who are not good at mathematics, highly capable students may not see these offers as directed at them. The idea of teachers utilising 'masterclasses', as depicted on recent pop-culture cooking shows, but also an old technique from academia where professors offered masterclasses (Stephens, 2006), may be a way of addressing this. The underlying notion is that masterclasses are for further developing abilities; that everyone, no matter how good they are, can always learn more, or improve and refine their abilities.

At the beginning of the study, Sammy seemed to view requiring assistance with a task as an indication that she was not good at mathematics and/or that the task was too difficult for her. At the end of the study, she was still struggling with the difference between 'really hard' (where she required help) and 'too hard,' and, even though she was able to feel proud of work she had completed, her enthusiasm was still somewhat dampened if she had required assistance, "... but you had to help me with that [sounding suddenly quite dejected

after the initial elation of finally solving the 522-367 problem]” (Sammy, November 2014). Because she needed help to solve the problem she did not feel ‘really happy’ about finally solving it, even though the help was mainly reinforcing things she had noticed herself. It was important to point out the parts where her thinking had been correct; there is a difference between being ‘wrong’ and being ‘partly wrong’ or ‘nearly right.’ Very rarely will a student’s response be completely wrong. If students can understand and recognise this, maybe they will not be so hesitant to ask for help – ‘I’ve got this far with my thinking, but I need some help with the next bit.’

Alex, during the post-professional learning period, seemed to understand that getting help with a difficult task could support his learning (cf. the AFL Grand Final task, Section 6.4.4), however, he still seemed to struggle with how to ask for help, with what questions to ask. This was evidenced specifically within the literacy lesson (relayed to me by Ms K) where Alex had had a major emotional meltdown because he could not find two more ‘*th*’ words (see Section 6.4.4). He told me he got upset because he had told his teacher that he could not find any more words and she just told him to keep looking, but he had already looked where she had suggested (in a dictionary). When I asked if he had explained this to his teacher, he said no (Phase 4 interview, November 2014). Is it possible that young children, like Alex, need to be scaffolded in how to ask for specific help? For example, ‘I’ve tried this and this, but it didn’t work. What else could I try?’ rather than just, ‘I can’t do this.’ Teachers need to be aware of this predicament that their gifted students may be facing, especially if their belief has been that gifted students generally do not need help.

Another, possibly related, issue with understanding the role of questioning had become evident with Sammy (and others) in the first Grade 3 Phase 2 classroom observation where the students kept jumping in with quick answers to every question I asked. I realised that I was asking mathematical questions thinking, “Here’s something else for you to consider; what do you think about this?” whereas the students seemed to be hearing, “The teacher has asked a question; I must give an answer.” This incident prompted the suggestion in the teacher professional learning: *Establish a classroom expectation that when I (the teacher) ask a question, I am posing a problem I want you to think about. I don’t want a quick answer, what I require is a well thought out explanation, the answer is the by-product of this* (see Section 5.4.1, Classroom Expectations). The concept and role of questioning – both asking and answering – is something that needs to be specifically addressed in teacher professional learning. It is an important part of the resultant ‘didactical contract’

(Brousseau, 1997; Voigt, 1989) teachers will need to specify in changing classroom expectations of challenging work and hard thinking.

7.5 Commonalities and Similarities of Teacher Approaches following Professional Learning

Subsidiary research question three asked:

- *What did the teachers do during the post-professional learning teaching period to challenge the mindsets of students who are mathematically gifted but with self-limiting mindset tendencies?*

There were three main interconnected components of the targeted teacher professional learning for this study. For the teacher to:

1. establish a classroom culture of understanding that mathematics learning requires effort;
2. challenge self-limiting mindsets and establish and/or maintain positive learner mindsets; and
3. develop a manageable lesson structure that allows for meaningful learning for all students.

These components were not designed exclusively for mathematically gifted students, rather they were designed as a whole-class implementation that enables mathematically gifted students the opportunity to transform their capabilities and gifts into talents (as per Gagné's DMGT, Gagné, 2003).

However, a teaching and learning framework, *per se*, does not change the culture of a classroom. It is the teacher's attitude, understanding, and implementation of the framework that provokes change (Cole, 2012). Just as with the uniqueness of individual students, no two teachers will approach their teaching in the same way. The targeted teacher professional learning for this study provided new information about mathematically gifted students for teachers to work with, but the teachers were at liberty to implement this new knowledge in a way that suited their own approach to teaching, and the individual students within their regular heterogeneous classroom environments. Fred's teacher (Ms J) primarily focused on showing Fred the value of deliberating on how he was working mathematically, not just getting correct answers. Sammy's teacher (Ms S) felt she needed to address Sammy's negative, self-limiting mindset behaviours before she could give her more challenging mathematics tasks. Alex's teachers (Ms K and Ms C) focused on giving Alex

more challenging tasks to provoke a change in his mindset. Within these different approaches there were common themes identified from the narratives:

1. approaches to providing challenging mathematics tasks for highly capable students;
2. approaches to scaffolding students' mindset changes, from self-limiting to more positive self-actualising mindsets; and
3. approaches to scaffolding students' mathematical creativity and encouraging them to explore mathematics tasks further.

Table 7.4 shows an overview of commonalities, similarities and differences of the teachers' approaches to mathematics teaching following the professional learning. A discussion, analysis and interpretation of the significance of these findings follows.

Table 7.4

Overview of teacher approaches to mathematics teaching post-professional learning

Theme	Ms J (Grade 5 – Fred)	Ms S (Grade 3 – Sammy)	Ms C & Ms K (Grade 1 – Alex)
Approach to providing challenging mathematics tasks	<p>Main focus of the post-professional learning period was to encourage her mathematically highly capable students to explain and justify their solutions (i.e., intentionally thinking about how they were working mathematically).</p> <p>Used more open tasks than she previously had, focusing on generating discussions about these, and included criteria such as ‘explaining solutions’ as part of the accompanying rubrics.</p>	<p>Increased expectations with challenging tasks and explained to her mathematically highly capable students that she was intentionally doing this.</p> <p>Focused on her highly capable students learning how to approach a task that required effort, helping them understand that struggle is a normal part of learning, and that ‘mistakes’ can be a beneficial element of this.</p>	<p>Main focus of the post-professional learning period was to continue to provide mathematically highly capable students with suitably challenging tasks.</p> <p>Ms C focused on raising her expectations; used questions to sustain students’ thinking; encouraged them to explain their solutions; encouraged them to use manipulatives when beneficial (and not see them as ‘babyish’)</p> <p>Ms K focused on encouraging students to ‘think deeper’ rather than just complete tasks.</p>
Approach to scaffolding mindset changes, from self-limiting to more positive	<p>Used challenging mathematics tasks to initiate change in perspectives of highly capable students who believed effort indicated a lack of ability.</p>	<p>Main focus of the post-professional learning period was to challenge and change Sammy’s self-limiting mindset behaviours.</p> <p>Drew up a ‘mindset chart’ as a classroom tool to develop positive mindset self-talk and helped Sammy deal with feelings associated with facing a challenge or making mistakes.</p> <p>Sourced articles on how to deal with hypersensitivities and intense emotions in highly capable children.</p> <p>Worked closely with Sammy’s parents.</p>	<p>Ms C encouraged Alex to face up to challenges and not be content to stay in his ‘happy place’, but backed down when he became teary.</p>

Table 7.4 Continued

Theme	Ms J (Grade 5 – Fred)	Ms S (Grade 3 – Sammy)	Ms C & Ms K (Grade 1 – Alex)
Approach to scaffolding mathematical creativity, encouraging students to explore further	<p>Scaffolded by using questions such as “Could you try something else?” or “How could you change it?” to encourage students to explore a task further (rather than giving direct suggestions as she had previously).</p> <p>Used more partner work (than previously) to encourage students to bounce ideas off each other.</p>	<p>Encouraged the whole class to ‘explore’ tasks, modelling and scaffolding how to think about the mathematics rather than just solve the problem.</p> <p>Gave less direct instruction about extension tasks for mathematically highly capable students, giving them opportunity to work more independently.</p> <p>Gave more support to highly capable students when completing challenging tasks, and in helping them verbalise and explain their mathematical thinking with these tasks (had previously underestimated how much support these students may require)</p>	<p>Ms K experimented with allowing students to explore mathematics tasks themselves prior to whole-class instruction, she watched and guided their discoveries with questioning. She enjoyed this new approach to mathematics teaching (for her) but realised she was still learning how to best manage this. She was not sure how to encourage her highly capable students to ‘explore further.’</p> <p>Ms C was experimenting with giving her students opportunities to be creative by allowing them freedom to explore their own passions as a specific activity (not mathematics related unless the students chose this themselves).</p>

7.5.1 Teachers' approaches to providing challenging mathematics tasks

The teachers already differentiated mathematics tasks within their mathematics lessons, with extension ideas or more complex versions of tasks for the more capable students, and prompting ideas or simpler versions of tasks for those students struggling to engage with the whole-class task. This was part of an intentional school-wide approach to mathematics teaching and learning. Ms J was still very concerned that she was not sufficiently meeting the needs of her most highly capable students. Subsequently, during the post professional learning period Ms J's focus, to increase the challenge of tasks for these students, was on the 'explain and justify your solutions' and 'explore the mathematics further' components of the lesson structure framework (see Section 5.3.2). She deliberately encouraged the students to 'make it harder' for themselves without being explicit about what they should do, which is what she had previously tried to do. In general classroom mathematics lessons, she focused on using more open tasks, and generating discussions about those tasks, highlighting the value of exploring different ideas, strategies and approaches, and explaining and justifying solutions (not just providing correct answers). Her feedback for students also intentionally focused on these elements, both informally (in discussions) and formally (including them in marking rubrics for assessment tasks). Ms J also focused on developing Fred's mathematical explanations and written records, using the strategy of asking him, "How could you teach someone else to do it that knew nothing about it?" or "If you had to explain this to someone who knew nothing about this, how would you tell them what you're thinking?" which she felt had helped him improve in this area.

Ms S, in Grade 3, also focused on giving less direct instruction of how to complete an extension task, instructing students, instead, on the mathematical learning focus of the task, thereby deliberately giving them space and permission for different interpretations of, and approaches to, completing the tasks. She also encouraged her highly capable students to explain their thinking more thoroughly by asking, "How could you explain this to someone else who knew nothing about it?" and included 'providing justifications for solutions' in marking rubrics to emphasise its value. Ms S also adopted the idea of 'exploring maths further' from the lesson structure framework, introducing this to the whole class, not just the highly capable students, and spent time scaffolding 'exploring maths' with the whole class. Ms S realised she needed to provide support with challenging tasks, rather than simply expecting more advanced students to be able to work on them independently (cf. Silverman, 2013; Whitmore, 1980). Another of Ms S's focuses, therefore, was to provide

more intentional scaffolding for her highly capable students' thinking within any extension tasks. Her biggest realisation during the study was, just because a student may be “really great at everything,” that did not mean the student would not struggle with their learning (see Section 6.5.3).

Ms C, who took the majority of the Grade 1 mathematics lessons, was quite confident with her approach to providing challenging tasks for her more capable students. She continued her approach throughout the post professional learning period, focusing on higher expectations of her highly capable students with these tasks. She was cognizant of Alex wanting to return to his “happy place” when a task became challenging, and wanted to move him on from that “negative place of ‘I can’t’” towards a more positive approach to challenges. She focused on using questions to sustain the thinking of her highly capable students, and encouraged them to ‘prove their solutions’, and/or ‘try a different strategy.’ In the post-professional learning interview, Ms C was not convinced that much had changed for Alex; she said she still had to “back off” and give him easier work to prevent emotional ‘meltdowns’ when he perceived the task as being too hard.

Ms K focused on encouraging Alex to think deeper mathematically about tasks rather than just completing them. She was very keen to help him explore tasks further, but admitted she struggled with how to scaffold this for him. She did say, however, how much more she was enjoying teaching mathematics, especially in allowing students to explore mathematical concepts for themselves, and watching and guiding their discoveries prior to intentional whole-class teaching, which she said was almost the opposite of her previous approach (see Section 6.4.3).

7.5.2 Teachers' approaches to scaffolding students' mindset change

There were two distinct approaches taken by the teachers in scaffolding student mindset changes. Ms J, Ms C and Ms K, focused on changing Fred's and Alex's mindset by using challenging tasks to initiate a change in mindsets. Ms S adopted the opposite approach with Sammy, focusing on changing her mindset so that she could then approach challenging mathematics tasks more confidently.

Ms J focused on intentionally showing Fred the value of thinking and reasoning mathematically, not just getting correct answers. She wanted him to realise that thinking and reasoning mathematically requires sustained effort, and were opportunities to use and refine mathematical abilities, not an indication that his abilities were not good enough to

solve the problems (preferably quickly and easily). Ms C and Ms K continued to give Alex more challenging work, but did not appear to explicitly address negative mindset issues other than to encourage him to keep trying when the tasks became difficult. Ms C focused on encouraging Alex to face up to challenges and not be content to stay in his ‘happy place,’ but she did not feel comfortable continuing to ‘push’ him once he became teary and ‘shut down’ (see Section 6.4.3).

Ms S drew up a ‘mindset chart’ to help Sammy with her self-talk, sought out information on hypersensitivities and how to deal with them, and worked closely with Sammy’s mother to ensure the strategies she was adopting in the classroom were being reinforced at home. Previously, Ms S admitted, she would back down if Sammy decided a task was too difficult, but now she scaffolded the process of facing challenge, legitimising strategies such as ‘taking a risk’ and trying something, and showing her how she could learn from what does not work. Ms S provided Sammy with considerable intensive teacher support during the post professional learning period; she could do this, in part, because she had a final-year pre-service teacher working in her classroom for six weeks during this time.

The opposite approaches from the teachers in this study – use supported challenge to provoke a mindset change, or, change the mindset so the student is more willing to tackle a challenge – do not provide definitive evidence of which approach may, or may not, be preferable, as a case study of three students is not necessarily designed to do this (Merriam, 2009). What the case study does do is show that the teachers had interpreted and implemented the professional learning in different ways. This could become a more explicit element of future teacher professional learning about mathematically gifted students, so that teachers can make a more considered decision about which approach would best suit their own teaching style and/or would be most appropriate for the age/emotional status of the students they are teaching. Further research may show one approach to be more effective than the other overall, but this is beyond the scope of this study.

7.5.3 Teachers’ approaches to scaffolding students’ mathematical creativity

The third component of the lesson structure framework suggested in the targeted teacher professional learning was ‘exploring the mathematics [of a completed task] further’ (see Section 5.3.2). The concept behind this was based on Holt’s (2002) *Slow School Movement* (see Section 2.3.2), and Betts’ (2004) *Learner-Differentiated Curriculum and Instruction* (see Section 5.3.2). *The Slow School Movement* is a reaction to education being a process of learning and demonstrating specific knowledge, and proceeding from one mandated

standard to another as fast as possible (see Section 2.3.2). The philosophy behind *The Slow School Movement* is that learning should be savoured; that we should be giving students permission and time to be curious, to reflect on and pursue areas of interest, and to be passionate about their own learning and discoveries. That is, to be free to be creative and innovative, as in *Learner-Differentiated Curriculum and Instruction*. This was an area where all four teachers said they found hard to generate any significant changes with, in their students' approaches to mathematics learning.

Ms J (Grade 5) implemented more intentional partner work to generate mathematical discourse, and to encourage students to bounce ideas off one another. Fred seemed to enjoy this and was beginning to gain enough confidence to suggest and try different ideas without asking teacher permission to do so, but this change was happening very slowly. Ms S (Grade 3) introduced the idea of exploring mathematics tasks with her whole class, introducing her 'Triple eX' approach – Explore, Explain and Extend (see Section 6.3.3). This 'exploration' was more an in-depth look at a specific set task than exploring beyond the task, though, with the 'Extend' component being more about creativity. Ms K (Grade 1) admitted that she was not sure how to encourage Alex to 'explore further', and Ms C (Grade 1) did not mention this aspect of the lesson structure framework at all. This concept of free-form, 'slow' learning is something that will most likely require a paradigm shift in the way many teachers, students and parents view school education. This is even evidenced in a school where the principal's view reflects this idea of enrichment rather than acceleration:

When I think of enriching students' mathematical experiences at school, particularly the more capable kids, all my thinking's about ... kids making observations about what's happened, about identifying patterns, and being able to make generalisations and predictions into the future ... I see the main benefit of doing it [enrichment] to be about creating excitement in the children rather than getting to Year 8 or getting them to Year 9 or whatever, which I think is not something that's necessary or overly valuable. (School Principal, focus group discussion, April 2014)

7.6 Chapter Conclusion

This chapter has focused on insights gleaned from cross analysis of the three narratives from Chapter 6, and has provided answers to the subsidiary research questions that framed

the study. The various sections in this chapter discussed commonalities and similarities observed in:

1. the students' gifted characteristics,
2. the students' approaches to mathematics learning,
3. the students' mindset behaviours, and
4. the teachers' approaches to the teaching of their mathematically highly capable students post professional learning.

Discussions, interpretations and implications were based on links to the literature as outlined in Chapter 2. The chapter focused specifically on changes observed in the three mathematically gifted students involved in this study, from pre- to post-teacher professional learning, with interpretations, and some possible implications of these insights, suggested.

The insights gained will inform the conclusions to this study, with the final chapter drawing together the findings, to address the overarching research question:

- *What impact does targeted teacher professional learning about classroom support for mathematically gifted students with self-limiting mindsets, have on the mindsets and mathematics learning of these students?*

The answers to this question will lead to suggestions for future research-based professional learning programs for teachers working with mathematically gifted students. The final chapter also offers recommendations for possible further research.

Chapter 8 – Conclusion

Discussion, Implications and Recommendations

*It's just kind of worked like magic!
She's given me stuff to change my mindset and stuff...
I'm proud of me. (Sammy, November 2014)*

8.1 Chapter Overview

The aim of this study was to explore the impact of classroom teachers receiving professional learning about students who are mathematically *gifted* (Gagné, 2009), but who display *self-limiting mindset tendencies* (Boaler, 2016; Dweck, 2015). The aim was to analyse the impact, in a case study of three diverse-aged primary school students (Fred in Grade 5, Sammy in Grade 3, and Alex in Grade 1), to highlight insights that could be used to add to the literature on mathematical giftedness, and to develop a sound, research-based professional learning program for both pre-service and in-service classroom teachers. The focus was on how to support the learning of mathematically gifted students within regular, heterogeneous classrooms, as most mathematically gifted students will be in regular classes (Australian Association for the Education of the Gifted and Talented (AAEG), 2006), using whole-class mathematics tasks with differentiation (Kanevsky, 2011).

There are other suggested options for gifted students in research literature. For example, gifted withdrawal programs (Silverman, 2013), which are often considered to be the best option for gifted students (Gross, 2004; Silverman, 2013), or acceleration (Gavin & Adelson, 2008), although acceleration is a relatively uncommon practice in Australasian Primary Schools (Diezmann, Stevenson & Fox, 2012). However, this study recognised that, 1) not all schools are in a position to offer specialised classes or programs, especially small rural and isolated schools, and 2) there is an issue of identification and ‘cut-offs’ for specialised programs (Haylock & Thangata, 2007). A student may miss out on inclusion in a gifted program due to a marginally less than acceptable ‘score’ on an entrance requirement, or, with the prevalence of ‘underachieving gifted’ students (Siegle, 2013), a student may miss out because he or she does not present as a high achiever. Many gifted students spend the majority of their time in regular classrooms (AAEG, 2006; Singer et al., 2016), where their distinctive characteristics need to be understood, and their learning supported, by regular classroom teachers. The premise of the study, then, was that not all

gifted students will have access to specialist options, and, even if specialist programs are available, some gifted students may miss out on these due to varying circumstances, and most gifted students will be in regular classes as well. It was also the premise that effective teaching practice for supporting the learning of mathematically gifted students in the classroom would be beneficial for *all* students (Rosario, 2008), making any resultant findings useful for all teachers.

To this point this dissertation has introduced the study (Chapter 1), embedded it in relevant literature (Chapter 2), and situated it within a specific theoretical perspective and resultant methodology (Chapter 3). The research design was outlined (Chapter 3) and described in detail (Chapter 4 and Chapter 5) prior to narrative analyses of individual participants (Chapter 6) and a synthesised analysis of the narratives (Chapter 7), to evaluate the impact of teacher professional learning on mathematically gifted students with self-limiting mindsets. This chapter will conclude the thesis. Firstly, it will provide an overview of the perceived impact of the targeted teacher professional learning about classroom support for mathematically gifted students with self-limiting mindsets. Secondly, the implications of the findings, on how a teacher professional learning program may be used to maximise classroom learning experiences for mathematically gifted students, will be considered. Thirdly, how the findings contribute to the current literature, and suggestions of further research that could be done to strengthen and/or supplement the results of this study, will be discussed. The thesis concludes with a ‘final word’ on the researcher’s hopes for the impact of this research on the future of mathematically gifted students.

8.2 Discussion of the perceived impact of the targeted professional learning

Insights gleaned from the cross analyses of the three narratives in Chapter 7 provided answers to the subsidiary questions that framed the research question, *What impact does targeted teacher professional learning about classroom support for mathematically gifted students with self-limiting mindsets, have on the mathematics learning and mindsets of these students?* The study sought to determine any positive impact targeted teacher knowledge had on students’ approaches to mathematics tasks, and concurrently, any positive impact on the students’ mindsets, that is, their perceptions of themselves as learners of mathematics. It sought to determine which teaching approaches, developed from the professional learning, appeared to determine favourable outcomes, with an understanding that, if the resultant approaches could be collected, collated and refined, a professional

learning program may be developed to provide all classroom teachers with a research-based resource for supporting the learning of mathematically gifted students. This section summarises the changes observed in 1) students' approaches to mathematics tasks, 2) students' mindsets, or perceptions of themselves as mathematics learners, and 3) teachers' beliefs about, and approaches to, teaching mathematically gifted students.

8.2.1 Students' approaches to challenging mathematics tasks

After their teachers had received targeted professional learning, there was evidence of positive changes in Fred, Sammy and Alex's approaches to challenging mathematics tasks. All three appeared to accept that being good at mathematics did not mean that you could necessarily complete tasks quickly (as they had all previously believed), rather, you were good at mathematics if you could persevere with challenging tasks and complete them through hard thinking and sustained effort (González & Eli, 2017; Sullivan et al., 2013; Williams, 2014), grit (Duckworth, 2016) and determination (Sheffield, 2006; Stillman et al., 2009). All three seemed to understand that being good at mathematics did not mean that you did not make mistakes (as they had previously believed), but that, if you were good at mathematics, you could reflect on mistakes, and use what was learnt from them to help find correct solutions to problems (Diezmann & Watters, 1995). This changed understanding affected the way they approached mathematics tasks, but the changes involved more than simply being willing to spend more time and sustained effort on challenging tasks. Students also had to understand mathematics success as a process, learn strategies for overcoming difficulties, learn how to record mathematical solutions, and learn how to think beyond the task.

Understand mathematics success as a process, not just a grade

Standards-based education systems have traditionally focused on standardised test scores and grades, which give students, and their parents, a conflicting view of mathematical success (Bicknell, 2009a; Sheffield, 2006). Teaching mathematical processes of thinking, reasoning, dialoguing and creating are essential elements of school mathematics in the 21st Century education (OECD, 2008; Gravemeijer, 2013; Zhao, 2012). To this end, mastery of knowledge, or the pursuit of excellence, needs to become the goal in mathematics learning, rather than achieving high grades (Gillard et al., 2015). This is especially pertinent for mathematically gifted students as teachers, and parents, need to realise that in a standards-based, graded education system, a mathematically gifted student could receive highest

grades in mathematics but still be underachieving (Neihart & Betts, 2010). Fred and Alex, especially, were very focused on receiving ‘good grades’ and outperforming others (Ames & Archer, 1988). This was particularly obvious with Alex, who was very keen to make his mother happy. A teacher’s role, then, goes beyond providing appropriate and meaningful tasks that will enable deep mathematics learning, to also educating students and parents about expectations of mathematical success that go much deeper than a score or grade.

It was interesting to note that it was only the two boys who focused on grades, and not Sammy. To attribute this to gender, however, would require further investigation into gender differences in the mathematics learning of gifted students (e.g., Leder, 2004) to see whether or not this is an indicative difference; this was beyond the scope of this study.

Learn strategies for overcoming difficulties, including how to ask for help

Successful mathematics learning requires effort (Nottingham, 2010) and perseverance (Williams, 2014), and often support from a more knowledgeable other (Sullivan et al., 2013; Vygotsky, 1976); mathematics learning may involve taking risks and making mistakes (Clarke et al., 2014). These ideas were somewhat new for the three students in the study (based on their responses in the pre-professional learning interviews), but not necessarily for the teachers (e.g., all were providing rich tasks prior to the study, as part of a whole-school approach to teaching and learning mathematics; all hoped their students would persevere with these tasks, and believed that they could learn from mistakes). This shows that teacher beliefs do not automatically translate to student beliefs about their learning. Teachers may need to be explicit about these ideas, and be intentional in showing students that they value much more than correct answers to problems. Using a resource such as an adaptation of Nottingham’s (2010) *Learning Pit* (see Section 7.3.1), could be beneficial for all students, and certainly for mathematically gifted students. This approach normalises productive struggle (Lithner, 2017) as part of dynamic learning, and provides strategies for ‘climbing out of the pit’, such as linking to prior-knowledge, making a prediction and testing it out, collaborating with others, and asking for help.

From the findings of this study, it seems teacher professional learning about teaching mathematically gifted students may require a specific focus on how to assist students in asking and answering questions. All three students had required explicit instruction that asking questions is part of the learning process, and that the teacher is there to scaffold their learning, not just direct and assess learning (González & Eli, 2017). Asking, and answering,

questions needs to be more about communicating current thinking, and discussing further options with someone who can help, than seeking a specific answer (Sheffield, 2009).

All three students, in the pre-professional learning observations, had struggled with asking for help. This struggle may have been a result of not experiencing many challenging tasks that they required help with, or it may have been a result of a mindset belief that only people who are not good at mathematics need to ask questions (Boaler, 2016). Whatever the reason, if students are not experienced with asking for help, they may need to be scaffolded in what questions to ask, and how to ask them, and teachers need to be aware of this. For example, when Alex told the teacher he was stuck on a task and she told him to just keep working at it, he had an extreme emotional reaction. He did not explain to her the strategies he had already tried that did not work for him, nor that he had no idea what to try next. Sammy also struggled with answering questions, believing an expectation that questions needed to be answered quickly and correctly (see Section 6.3.2). During the post-professional learning period, all three students were observed communicating with, and asking questions of their peers, but this was not observed as extending to discussions with the teacher, unless initiated by the teacher herself. This may be something that needs to be included in a refined professional learning for teachers – teaching and modelling how to ask for help. This could benefit all students who tend to simply say, “I don’t know what to do,” and could be better supported if they knew how to ask for specific help.

Learn how to record mathematical solutions

An emphasis on mental calculations, and verbally describing the process, is important in developing sound number sense (English & Gainsburg, 2016; McIntosh et al., 1997), but considering requirements of innovation and creativity in the 21st Century, it is equally important to also develop the skill of *written* records of solutions that can be replicated by others (Pugalee, 2004; Urquhart, 2009). Prior to their teachers receiving professional learning, Fred, particularly, had struggled with recording solutions, and his verbal explanations were sketchy and descriptive rather than deductive and conclusive. Sammy and Alex could verbalise their explanations better than Fred (this seemed to be a common practice, and expectation, within their classrooms), but Sammy still struggled with recording solutions, and Alex’s mathematical recording was probably limited by Grade 1 expectations.

Explaining processes and recording mathematical solutions shows how, and why, the ‘solver’ knows the solution to be correct, and it may enable others to reproduce and/or

generalise from their solutions (Brown, 2008). This is an important skill for all students, but particularly vital for those students who have the potential to become future innovators and creators within mathematical/scientific realms (Krantz, 2007; Sheffield, 2012). This process of explaining and justifying solutions can, however, be notably challenging for mathematically gifted students, because their thought processes are naturally very efficient and they often combine two or more processes into one thought without realising they have done this (Geake, 2008; Krutetskii, 1976).

Fred's teacher (Ms J) had intentionally focused on developing Fred's ability to explain and record his mathematical reasoning, processes and solutions during the post-professional learning period. By the end of the study she believed that his ability was improving. Neither Sammy's nor Alex's teachers mentioned this aspect of the students' learning during the post-professional learning interviews, nor were there any significant changes observed by the researcher.

Teachers need to understand and value the process of explaining and recording mathematics processes and solutions, and they need to recognise the specific challenges mathematically gifted students may face with this, if they are to meaningfully support students in developing this ability.

Learn how to think beyond the task

Being able to ask mathematical questions, to extend and deepen an original problem, to think about mathematical problems in original or innovative ways, and to pose new and unique problems to explore, moves students from being problem-solvers, to also being problem-posers, that is, to being mathematically creative (Sheffield, 2009, 2013; Sriraman, 2004, 2017). According to Sriraman (2004), "Mathematical creativity ensures the learning, and growth, of the field of mathematics as a whole" (p. 19). Sheffield adds that, "students must also learn to ask questions that add depth and interest to the mathematics ... [realising] that instead of finding a solution to a mathematical problem being the end of the problem, it is often just the beginning of the most interesting, and rewarding, mathematics" (Sheffield, 2009, pp. 87-88).

In this study, encouraging and scaffolding learner-differentiation (Betts, 2004) as a form of mathematical creativity, proved to be something that probably required more comprehensive teacher professional learning, and, most likely, longer than three months to evaluate any real benefits of the approach. There was some evidence of independent

thinking and mathematical creativity during the post-professional learning period, though, even if not learner-instigated. For example, in the Grade 5 classroom, a group of students, including Fred, generated new rules to improve a mathematics game, rather than continuing with the rules they were not happy with (Section 6.2.2); in the Grade 3 classroom, Sammy and Janet spent considerable time and dialoguing exploring how many 1cm^3 blocks would be equivalent to a 2cm^3 block (Section 6.3.4); and in Grade 1, Alex and Frank became engrossed in finding ‘half of 59’ (Section 7.3.1).

One element to encouraging and scaffolding mathematical creativity appears to be in allowing students to work together with at least one other like-minded peer (see Mercer, 2013). All three students were observed discussing ideas (Wood, Williams & McNeal, 2006), and taking risks with ideas (Williams, 2014), when working with similarly capable peers (Silverman, 2013), and all could be described as entering a state of flow (Csikszentmihalyi, 1996) when working together on creative approaches. There was evidence of feeling frustrated that time was limited, and wanting to continue conversations, suggestions and trials beyond the dedicated mathematics time and into the next lesson, or out into the playground. For most teachers, this would seem to be an admirable achievement – students who want to continue to explore their learning beyond the requirements of the school classroom. The problem is, it is very difficult to assess experiences like this, or measure them against current educational standards that do not include such criteria. In the long-term, a basic philosophy of the purpose of school mathematics education (see Section 2.3) will either help or hinder this approach to mathematics learning (Holt, 2002).

8.2.2 Students’ mindsets, or perceptions, of themselves as mathematics learners

The changes observed in the students’ approaches to mathematics tasks required more from their teachers than simply providing them with challenging tasks within their zone of proximal development (Vygotsky, 1978), and encouraging them to work hard and persevere with these tasks (Sullivan et al., 2013). A student who has always been ‘good at everything’ may not be accustomed to applying effort, or to struggling with a task, or discovering they have made a mistake. Being thrust into a learning environment that suddenly expects struggle and effort to be a part of the learning process, may be a totally foreign experience for them (Piechowski, 1997). This, together with typical hypersensitivities of gifted children (Dabrowski, 1972), can be quite distressing. This type of response was observed in Fred, Sammy *and* Alex, to varying degrees. However, teacher

support in intentionally helping them understand their emotions and how to manage them, and/or understanding that perseverance through difficulties enables learning (Williams, 2014), and/or that mistakes can be experiences to learn from, not failures (Clarke et al., 2014), appear to have provoked significant changes in the students' mindsets about mathematically capable learners as shown in Table 8.1.

Table 8.1

Summary of students' pre- and post- mindsets, about mathematically capable learners

Pre- professional learning	Student	Post-professional learning	Student
Work fast	Fred Sammy Alex	Work hard (might take a long time) Work on complex problems (may take a long time, may make mistakes)	Fred Sammy Alex
Do not make mistakes	Sammy Alex Fred (in tests)	Recognise mistakes can be a useful part of learning	Fred Sammy
Understand new things quickly	Fred	Understand new things quicker than others	Fred Sammy
Do not need to expend effort	Fred Sammy Alex	Can prove their ability by persevering with hard tasks and completing them	Fred Sammy Alex
Get good grades	Fred Alex	Get good grades	Fred Alex
Do not need help with 'hard maths'	Fred Sammy Alex	Help from the teacher supports learning Can get help from the teacher and/or from other students	Sammy Alex Fred
[No pre- equivalent]	N/A	Can think creatively about mathematical problems	Fred

Not all mindset beliefs had changed, both Fred and Alex still cited 'getting good grades' to be an indication of mathematical ability, rather than mathematical performance, however, within the current education system this is probably a reasonable observation for students to make. Not all students evidenced the same changes in mindsets. Fred and Sammy both indicated that making mistakes can actually be a useful part of learning, but there was no explicit evidence that Alex had developed this same change in mindset. This is not to say that Alex still believed that mistakes were inherently bad – from observations he was more comfortable risking mistakes in challenging tasks – but he did not explicitly state this, as

the other two had, in the post-professional learning interviews. Interestingly all three voiced the belief that mathematically capable learners could enhance their learning by getting help from the teacher and/or other students, even though asking for help still seemed to be difficult for them (see section 8.1.1).

It seems that, whether the primary focus of the teacher was on changing mindsets in order to change approaches to mathematics learning (Ms S), or on increasing challenge and expectations in order to change mindsets (Ms J, Ms C and Ms K), there was a compelling change in students' mindsets about being 'good at maths' during the post-professional learning period. Sammy's teacher, Ms S, and Alex's teacher, Ms C, both voiced concern that 'fixed' mindset attitudes were still evident, however, even though the changes in Sammy's mindset may have been more marked than Ms S was perceiving, her comment is most likely very true:

I have noticed a little bit of a change. It's not a huge change, but it's pretty hard to go from having a fixed mindset to 'aah' growth mindset just like that. It's not going to happen ... It's normal for everyone, I guess, to go into autopilot [at times] and jump back into a fixed mindset. (Ms S, November 2014)

Any change to student behaviour will require on-going support and monitoring. Very rarely will change be sudden or complete, and a fade-out effect following any targeted mediation is common (Bailey et al., 2016). For teachers to address self-limiting mindsets of gifted students most effectively, there needs to be a whole-school approach (Crévola, Hill & Fullan, 2006; Gervasoni et al., 2010) to understanding mindsets *and* understanding gifted students. There needs to be a common consideration, with common expectations across all classrooms. The goal is to enable gifted students to become learners with an incremental view of ability, whereby *any* level of knowledge or ability can be increased through perseverance and sustained effort (Dweck, 2006). This is to enable them to become autonomous learners who are self-confident, optimistic, and resilient (Neihart & Betts, 2010; Williams, 2014). This is, indeed, also an educational goal worthy of *all* students. A whole-school approach to fostering positive mindset thinking, and understanding gifted students, needs to be about prevention and resolution of negative, self-limiting mindsets for these students. As was seen with Alex, very young students may already have skewed beliefs about learning before they begin formal education (Mueller & Dweck, 1998).

8.2.3 Teachers' beliefs about, and approaches to teaching, mathematically gifted students

The teachers in this study had varying approaches to the implementation of the suggested ideas from the professional learning, filtered by their own conceptions about mathematics teaching and learning and teaching experience (Chesserman, 2010; Krijan & Borić, 2012). They also had varying beliefs about mathematical giftedness that were challenged, and this impacted their approaches to teaching mathematically gifted students. Ms K (one of Alex's teachers) voiced a belief that mathematically highly capable students were those children who had been 'hot-housed' by parents or others (focus group discussion, April 2014). Ms J (Fred's teacher), Ms S (Sammy's teacher) and Ms C (Alex's other teacher) all professed surprise at certain mathematical concepts, or knowledge, that their mathematically highly capable students struggled with, or did not yet know, "even though they were highly capable." It is important for teachers to understand that the fundamental characterisation of mathematically gifted students is that they do not just know mathematical concepts, they still need to learn them. They construct understanding through experiences the same as all students, it is just that they may need fewer experiences and may be able to generalise more readily from one concept to understand another (Geake, 2008; Hoppe & Stojanovic, 2009; Krutetskii, 1978). They still require experiences of the concepts to learn them, they may still develop misconceptions through this process of learning, and they still require a 'more knowledgeable other' to guide and support their learning. Teacher professional learning about teaching mathematically gifted students needs to cover this aspect of mathematical learning explicitly due to a common belief that gifted students can work independently (Silverman, 2013; Winner, 1996), and can therefore be left alone to complete advanced tasks.

Using physical materials to help students construct understanding of new concepts may be just as pertinent for mathematically gifted students, at times, as any other student (Diezmann, 2005). Ms C and Ms K both recognised the benefit of using physical materials with Alex when he faced a conceptual 'block', but struggled with how to implement this because of Alex's mother's influence, and her labelling of the use of such materials as being 'baby stuff'. Teacher professional learning about the teaching of mathematically gifted students may require input on how to involve parents in understanding both student needs, and the process of learning mathematics.

Ms J and Ms S also recognised that their mathematically highly capable students needed more support than they had previously given them, but often in different ways to other students (see, e.g., Diezmann & Watters, 2002). Ms J, particularly, recognised the difficulty mathematically highly capable students may have in explaining and justifying solutions, and intentionally addressed this with Fred and other highly capable students in her class. She felt the question, ‘If you had to explain this to someone who knew nothing about it, how would you tell them what you're thinking?’ helped the students break down their processes, enabling them in both expressing their mathematical thinking to others, and in writing down solutions as a record and validation of the mathematics completed.

The teachers were all surprised to learn about the commonality of hypersensitivities in gifted children (Dabrowski, 1972). In hindsight, they realised that this was obvious for many highly capable students they had taught, not just the three selected for this study. Each one realised they tended to back down when students became distressed by hard work, with Ms S (Sammy’s teacher) subsequently voicing how important it is “to know the student and what they are capable of, so you know how hard you can push for that initial success” (Ms S, November 2014). All teachers intentionally tried to avoid backing down when their students became teary, providing them with further support and strategies for building resilience and optimism (Williams, 2014), but Ms C and Ms K (Alex’s teachers) still had difficulty with this, wanting to avoid Alex’s ‘meltdowns’.

All teachers mentioned feeling inadequate in scaffolding students’ mathematical creativity – the ‘explore the maths further’ component of the suggested lesson structure (see Section 5.3.2). Ms J and Ms S seemed to understand the philosophy behind this, but felt they had difficulty in getting the students to embrace the idea. However, Fred and his peers certainly showed some independent thinking in exploring mathematics tasks (see Section 6.2.4), and Sammy and Janet could explore a task further with encouragement (see Section 6.3.4). It may be that three months, especially working with students who have struggled with self-limiting mindsets, is not long enough to expect changes in the students becoming initiators of their own learning. Ms J and Ms S were both keen to continue working on developing this. Neither Ms C nor Ms K commented about encouraging Alex to ‘explore further.’ It is not known if this is something that would require a different approach due to Alex’s young age or not. Despite a history of researched benefits of learning mathematics in and through play (Wager, 2013), there is little mention of mathematical creativity in the literature on

early mathematics learning (cf. English & Mulligan, 2013). Further research on very young students' ability to self-direct further mathematical explorations would be interesting.

Ms S's approach to challenging Sammy's self-limiting mindset was very focused. Her approach was implemented as part of a whole-class focus on growth mindsets, with several mindset charts displayed throughout the classroom. She spoke about drawing up an extra, specifically tailored, chart for Sammy as part of her approach during the post-professional learning period (see Figure 6.9), but it is not known if this chart was used exclusively for Sammy, or whether Ms S found she was able to use it for other students as well. Ms J also had a whole-class focus on developing positive, growth mindsets, however, her approach to challenging Fred's self-limiting mindset in his mathematics learning seemed to be more implied than explicit – embedded within her expectations of effort and perseverance with difficult tasks. There was no overt focus on mindsets in the Grade 1 classroom, and Ms C and Ms K did not seem to specifically target a mindset change with Alex's view of learning mathematics, and what it meant to be good at mathematics. They appeared to expect mindset change to occur incidentally with exposure to, and expectation of, greater challenge in mathematics tasks.

Interestingly, regardless of the different approaches, all three students showed at least some evidence of a more positive mindset about themselves as mathematics learners by Phase 4 of the study. However, Sammy's surprised outburst of, "It's [the mindset chart] just kind of worked like magic!" (see Section 6.3.4) was priceless, and showed that changing a student's 'self-talk' was possible in a relatively short period of time. The obvious pleasure in Sammy's realisation of this most likely added to the impact. The way Ms S used Sammy's mindset chart, in getting Sammy to physically go over to the chart and read out a more positive way of expressing what she was feeling, may be a good model to include in teacher professional learning on how to address self-limiting mindsets with gifted students.

8.2.4 Summary of the perceived impact of the teacher professional learning

Analyses and interpretations of data from this study show evidence of the targeted teacher professional learning having a positive impact on the mathematics learning and mindsets of the three case study students. The teachers approached their support for the students' ongoing learning differently, and the impact was different for the individual students, but the positive outcomes show that professional learning made a difference for both the

teachers (in how they approached their teaching) and the students (in how they viewed themselves as mathematics learners).

From this study, it seems professional learning may be immensely valuable for teachers to develop an understanding of how support for mathematically gifted students is essential, and what it entails. Providing sufficiently challenging work is just the first step in ensuring the learning needs of these students are being met. Teachers require specific professional learning about difficulties mathematically gifted students may encounter, including emotional sensitivities, and how to scaffold students to push through the initial confusion they will feel when tasks are truly challenging, especially if this is a foreign experience for them. Understanding characteristics of giftedness, the nature of mathematics learning for gifted students, how to assist students in the struggles of learning (which all students have, but may be different for gifted students), and how to address self-limiting mindsets (which other students may also have) are all necessary.

8.3 Implications of Findings

Generalisations from a qualitative case study are limited because, by definition, it is a bounded system specific to a small number of individuals in a particular environment (Stake, 1995). However, if, as the findings of this study show, mindsets of mathematically gifted students can be nurtured (and changed if necessary) the implications could be profound. If teaching approaches can foster positive learner mindsets that are optimistic (Seligman et al., 1995), students may be more willing to embrace challenges, and be resilient in the face of these challenges (Benard, 1995; Williams, 2014), show grit (Duckworth et al., 2007), perseverance (Conroy, 1998; Williams, 2014) and drive (Pink, 2009). This may enable their extraordinary capabilities to be realised, enhanced and transformed into talents (Gagné, 2003). These implications benefit the students themselves, and, potentially, the future of society as a whole (Sheffield, 2012). If we want to encourage future creativity, innovation and success, we need to nurture students who are willing to take risks, to persevere in the face of difficulty, and value and thrive on constructive feedback in the learning process (Duckworth et al., 2007; Seligman, 1995; Tough, 2012; Williams, 2014).

These are ‘big picture’ implications, but smaller, more immediate implications can also be drawn from these results in terms of enhancing the literature on both mathematics education and gifted education. This section outlines several of these possible augmentations to the

literature whilst also outlining some of the limitations of the study that need to be acknowledged.

8.3.1 Significance of the Research

In the exploration of the literature (Chapter 2) there appeared to be a dearth of information about students who are gifted but who have self-limiting mindset tendencies which make them resistant to applying effort, and/or to being challenged. There is an abundance of research and literature on the ‘underachieving gifted student’ (Colangelo et al., 2004; Gallagher, 1990; Neihart & Betts, 1988; Reis & McCoach, 2000; Siegle, 2013; Weiss, 1972), which is not surprising with the suggestion that the prevalence of underachievement may be as high as 50% of gifted students at some point in their schooling (Siegle, 2013). However, while prior research and other literature offer suggestions for numerous possible reasons for underachievement (see Section 2.5.3), ‘mindset’ is rarely mentioned, and if so, just in passing. With the recent pervasiveness of mindset literature in so many other areas of education – Boaler (2013) writes about the “The mindset revolution that is reshaping education” (p.1) – this seems surprising.

With general capabilities such as critical and creative thinking, teamwork and communication, and personal and social capabilities being recognised as key dimensions of successful learning (see ACARA, 2013), it is essential that our gifted learners do not miss out because of unfounded beliefs teachers may hold about gifted students. This research, therefore, can provide a significant and valuable addition, or a ‘link in the chain’, to the current knowledge-base of mathematically gifted students, and how educators can best support their successful on-going learning. It provides further highlights, and uncovers new understandings of the support required for mathematically gifted students, especially those who have developed self-limiting mindsets.

8.3.2 Limitations of the study

It is important to explain what this particular research did *not* do, both limitations that were due to design, as well as limitations with the implementation.

This study was about giftedness, specifically mathematical giftedness, but it did not extend to discussing the issue of where or how gifts develop in the first place. It acknowledged the existence of Gagné’s ongoing work in this area – his *Developmental Model for Natural Abilities*, and his *Comprehensive Model of Talent Development* (2015) (see Section 2.2.2)

– but merely assumed mathematical giftedness as being present from a very young age (Columbus Group, 1991).

In terms of identifying mathematically gifted students with self-limiting mindsets, the multifaceted process used for the purpose of this study was highly useful in identifying suitable students for the study. Identification of mathematical giftedness is very complex (see Section 2.5.3), and the method used in this study, given the demographics of the population at the participating school, did not consider identification of giftedness in indigenous, low socioeconomic status, or twice exceptional (gifted students with disabilities) populations. Nor did it fully consider those mathematically gifted students who may be spatially gifted but not gifted in number (cf. Krutetskii's geometric thinkers versus analytic thinkers (Krutetskii, 1976)). However, with some refinement and further development, this identification process could provide a useful tool to help classroom teachers recognise traits of mathematical giftedness in students in primary school classrooms, especially those who may display self-limiting mindset behaviours.

In hindsight, feedback from parents after the post-professional learning period may have provided further valuable insights about changes in the students' mindsets and behaviours. Families play a significant role in children's educational and developmental outcomes (Daniel, 2015; Emerson, Fear, Fox & Sanders, 2012), and, therefore, in promoting the transformation of innate capabilities (gifts) into realised accomplishments (talents) (Gagné, 2003). However, parents need to be cognisant of the issues surrounding both mathematics education (Muir, 2011), and exceptional capabilities in mathematics (Bicknell, 2009a). This is something to consider in any future, similar research, and information about parent partnerships is a recommended addend to a professional learning program.

8.4 Contributions to Knowledge

The contributions to knowledge from this study are three-fold: 1) in the area of gifted education, 2) in the area of mathematics education, and 3) in the area of affect/mindsets in education.

- 1) Contributions to gifted education include the addition of discussion around the impact of mindsets, specifically self-limiting mindsets that may impact gifted students' gifts (innate capabilities) being transformed into talents (realised accomplishments). The issue of underachievement in gifted students is a critical issue currently being researched (Siegle, 2013), and mindsets have been shown in this study to be an

important factor to add to this discussion. This study has shown that if self-limiting mindsets are intentionally addressed, they can be successfully changed (at least in the short-term).

Another contribution to gifted education is in addressing misconceptions of what mathematics learning is, and subsequently what mathematical giftedness is. Mathematics learning is not about remembering mathematical facts, skills and rules. Mathematics learning is about constructing mathematical concepts, thinking and reasoning logically, noticing patterns and generalising, extending, and deriving ways of solving problems. It is also a creative venture of problem posing, investigation and modelling. Mathematical giftedness is not about identifying super-human calculators, but identifying students who can construct robust mathematical concepts easily, have high abilities at fluid analogising so can generalise readily, and may be creative in inventing new approaches or methods for solving and/or posing problems.

- 2) The contributions to mathematics education include elements of effective teaching practice for mathematically gifted students that may indeed benefit all students in the class. A focus on Holt's (2002) *Slow School Movement*, which savours the learning process, and Betts' (2004) *Learner-Differentiated Curriculum and Instruction*, which encourages autonomy and creativity, can add to the discourse on effective approaches to mathematics education for all students.

Another contribution to mathematics education is in addressing misconceptions of what giftedness is, and subsequently what mathematical giftedness is. Giftedness is not about parents pushing, or hot-housing, their children, nor is it about successful, high achieving students who can succeed without teacher support. Giftedness is about students who learn faster, with fewer experiences, than their age peers. Mathematical giftedness, then, is to do with how students learn mathematics concepts, not what they know or can do at a certain age. Mathematically gifted students come from all demographics and cultures, they may or may not be mathematically advanced when they start school (depending on their prior-to-school experiences), they may or may not be high achievers, they may have learning disabilities that mask their giftedness, or they may be underachieving due to a number of reasons. Mathematically gifted students who are high achievers at school may still be underachieving in terms of capability.

- 3) The contributions to the area of affect in education include the suggestion of an amalgam of multiple affective traits such as *growth/fixed mindset*, *optimism/pessimism*, *resilience/learned helplessness*, *perseverance/defeatism*. Coining the phrases *self-actualising mindsets* and *self-limiting mindsets* as general terms to address multiple traits, also serves to describe the effects of these positive and negative affects.

Another contribution to the area of affect in education is in highlighting specific issues gifted students may have as a result of standards-based education systems. Once students have reached grade standards, they may be viewed as successful, and not given appropriately challenging work that is required to develop the positive affective traits listed above.

The interactions between the three areas, gifted education, mathematics education, and affect/mindsets in education, are important. Gifted researchers need to understand about mathematics, and mathematics education researchers need to understand about giftedness, if their research is to be complementary and beneficial to each domain. Both need to understand the effect of student mindsets. Researchers of affect in education would benefit from understanding both giftedness and mathematics, and how students' mindsets may be uniquely impacted in these areas. This study contributes to addressing these interactions, with a particular focus on mathematically gifted students in regular mathematics classrooms. These are the ones who will ultimately benefit from researcher and teacher knowledge and understanding of gifted and mindset issues.

8.5 Recommendations

Having shown evidence of a positive impact of the professional learning in changing self-limiting mindsets of young mathematically gifted students in this study two recommendations are proffered:

- 1) that a professional learning program be developed for in-service teachers that reflects effective continuing professional learning practices (cf. Cordingley 2015), and a course unit for pre-service teachers, to highlight issues associated with mathematically gifted students in regular classrooms, with a specific emphasis on mindsets, and
- 2) that further research be undertaken to: a) analyse the effect of the professional learning program on mathematically gifted students long-term, and b) continually refine the professional learning program by expanding the research to longitudinal data taken from a larger and more diverse cohort of participants.

8.5.1 Develop a professional learning program

The targeted professional learning used for this study had a positive impact on all three students. There were certain elements that worked well, and some that can be refined as a result of the study. Based on the results of this study, the key elements for a professional learning program to help teachers support the learning of mathematically gifted students to assist talent development, are:

- Understanding characteristics of mathematically gifted, or highly capable, students – that ‘giftedness’ is about how a student constructs mathematical concepts (with fewer learning experiences than their age peers), not about mathematical achievement (see Section 2.4)
- Understanding how to identify mathematical giftedness, through a multi-faceted process (e.g., listening to parents, observing how students approach mathematical problem-solving (either in class, or with a purposeful one-on-one interview), accessing archival mathematics assessment data (especially data that measures growth));
- Understanding common characteristics of gifted students, so these can be recognised and appropriately addressed (e.g., drawing up an ‘emotional response scale’ to deal with hypersensitivities) (see Section 7.2.1); understanding that gifted students are found in all cultures and socio-economic strata, and that gifted students can also have learning disabilities that can mask identification (see Section 2.2.4);
- Recognising self-limiting mindset behaviours in mathematically gifted students, and how to intentionally address these (e.g., using a ‘mindset chart’ to encourage positive mindset self-talk) (see Figure 6.6);
- Developing a classroom culture that supports, encourages, and ultimately expects students to explore mathematical investigations further (strengthening creativity), based on Betts’ (2004) *Learner-Differentiated Curriculum and Instruction* (see Section 5.3.2);
- Developing a classroom culture that understands mathematics learning to be a process of higher-order and metacognitive thinking, not merely a process of calculating answers as quickly as possible (using strategies such as the Learning Pit (see Figure 7.2), open questions (see Section 5.3.1), and expecting students to explain and write out mathematical solutions (see Section 7.3.3));
- Developing a classroom culture that understands that *all* mathematics learners require support (cf. zone of proximal development (Vygotsky, 1978), that ‘mistakes’ are an

integral part of the learning process, not something to be avoided or to be ‘rescued’ from, and that knowing how to ask for help is an important part of being a learner;

- Learning how to develop a mathematics lesson structure that enables the above elements to be embedded in mathematics lessons.

The Australian Association for the Education of the Gifted and Talented (AAEGT), in their 2016 National Report, stated,

Of the 37 universities in Australia who offer education at a tertiary level, only 3 presently have a compulsory, stand-alone gifted education unit within their undergraduate programs. To support our educators and provide necessary professional development in gifted education, we need to collaborate as a nation, explicitly incorporating gifted in our curriculum, teaching standards, under-graduate studies and on-going post-graduate professional development. (AAEGT, 2016, para 5)

A recommendation from the Victorian Parliament *Final Report on the Inquiry into the Education of Gifted and Talented Students* (Parliament of Victoria, 2012) was that “All Victorian teachers have a thorough understanding of giftedness and have the support they need to confidently and competently cater for gifted students in their classrooms, in particular through the use of curriculum differentiation” (p. 265).

With this need recognised nationally and locally, it is recommended that a professional learning program be developed into, 1) a specific research-based teacher professional learning program for in-service classroom teachers, and 2) a unit for pre-service teachers as part of undergraduate Early Childhood and Primary Education courses.

The focus of this study is mathematics, but this may serve as a vehicle to educate teachers and prospective teachers about giftedness in general, as well as mathematical giftedness specifically. This research may also serve to partially fill a gap whereby mathematics education research is underrepresented in the field of gifted research, and vice versa (Leikin, 2011).

8.5.2 Further research

With the limitations of this research, particularly as a small-scale case study, further research is recommended to augment the findings. Further research recommendations are:

- On-going, longitudinal research of the impact of a developed research-based professional learning program for teachers of mathematically gifted students, with the aim to produce optimal outcomes for both teachers and students through continual refinement of such a program. Further research may show one approach to be more effective than another (e.g., addressing mindset issues explicitly versus addressing mindset issues implicitly through challenging tasks).
- Further research on learner-differentiation (Betts, 2004) in mathematics, where students are encouraged and expected to explore their own curiosities from within a task, moving from being ‘consumers of knowledge’ to ‘producers of knowledge’ (Neihart & Betts, 2010; Sheffield, 2009; Tannenbaum, 1986). For example, does this approach need to differ for different aged students?
- Research on the prevalence of self-limiting mindsets in mathematically gifted students, coupled with research on gender differences in the prevalence of these mindsets.
- Research on whether positive learner mindsets motivate gifted students to continue to learn mathematics through to higher levels of education.

8.6 Final word

*Wow! They blew my socks off!
One time my sock really actually ... one sock nearly came off!
(Alex, 2014)*

From my interest in gifted students, over the years I have collected numerous books, articles and news stories about these outstanding children. One collection I have is a mini-library of children’s books with gifted protagonists – *Someday Angeline* (Sachar, 2006); *Iggy Peck, Architect* and *Rosie Revere, Engineer* (Beaty, 2007 and 2013 respectively); *On a Beam of Light: A Story of Albert Einstein* (Berne, 2013), *Millicent Min, Girl Genius* (Yee, 2003), *Matilda* (Dahl, 1988), *The Boy Who Loved Math: The Improbable Life of Paul Erdős* (Heiligman, 2003), just to name a few. One of the most notable common themes throughout these books is the depiction that gifted children’s talents, ideas and creativity are stifled at school. For example, from *Someday Angeline* (Sachar, 2006):

Angeline was the only one who raised her hand. Mrs Hardlick looked annoyed, “Somebody else this time,” she said and glared at Angeline. “It’s always the same people.” ... In Mrs Harlick’s mind, Angeline was a genius, which had nothing to do with being smart. It was more like being a freak, like a goat with two heads ... “I figured it

out,” said Angeline. “All I have to do is answer every question wrong, and everybody likes me.” (pp. 10, 12, 126)

Whether this stereotype is true of schooling in general or not, it is, unfortunately, a widely held view. It is a theme that has also been echoed in articles, such as a paper by Jolly (2016), where she laments:

In sport, the notion that, say, backstroke specialist Mitch Larkin or cyclist Anna Meares should hang back or go easy to enable their teammates or other competitors to keep up is, of course, ludicrous. Yet, this is not so far from what we ask of our brightest students. (Jolly, 2016, para 4)

Schools restraining gifted students is also alluded to by students themselves. For example, a newspaper article about a young Australian student, Jacob Bradd, who was accepted into university at the age of 14, quotes him as looking forward to his acceleration to tertiary education by saying, “At university they get you to actually learn things yourself, instead of school where they tell you everything and get you to do it a certain way” (interview statement, cited in McNeilage, 2014, para 17).

However, the good news is, that I have also collected numerous stories, articles and video clips of gifted students who are not only excelling, but already contributing meaningfully to society. For example, 11-year-old American girl, Gitanjali Rao, invented a quick, low-cost test to detect lead-contaminated drinking water, using carbon nanotubes and a mobile phone application. Her invention was in response to observing her parents having to test their drinking water following the Flint (Michigan) water tragedy of 2014-2015 (“Schoolgirl invents low-cost lead detecting device,” 2017).

These stories show us that the capability and performance of school children can be outstanding, even to conceptualising and devising solutions to global problems. If school experience nurtures talent development (Gagné, 1995), and celebrates and supports creativity (Sheffield, 2009), we may see even more of these stories from exceptional students. It has been shown from this research, however, that a significant aspect of educational support may include ensuring that students develop and/or maintain positive mindsets in their early schooling, in order to become self-actualising individuals (Maslow, 1968); autonomous learners who can potentially transform gifts into talents, and possibly change the world:

You don't have to be a professor with multiple degrees to have ideas ... [as a 15-year-old] you can be changing the world.

(Jack Andraka, 2013)

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APPENDIX 1: Focus Group, Interview Questions & Questionnaires

- i. Focus group statements
- ii. Staff questionnaire
- iii. Teacher nomination form
- iv. Parent Questionnaire
- v. Student semi-structured Interview questions – pre- and post-professional learning
- vi. Teacher semi-structured Interview questions – pre- and post-professional learning

Focus Group Conversation

Initiating Discussion

Statement Cards – vote yes/no, then discuss.

- Children who are highly capable mathematically are lucky.
- Children who are mathematically highly capable will develop behaviour problems if they become bored in maths classes.
- Children who are highly capable tend to have pushy parents.
- Children who are mathematically highly capable will be fast finishers.
- Children who are highly capable mathematically need as much support in the classroom as children who struggle mathematically.

Final question

- Any further comments about supporting and catering for children who are highly capable mathematically?

Individual questionnaires

Following the focus group conversation participants were given a short, written questionnaire to fill in anonymously, giving each person the opportunity to record private comments after the group session has been completed: 1. *Principal/School Leader Questionnaire*, and 2. *Teacher Questionnaire*.

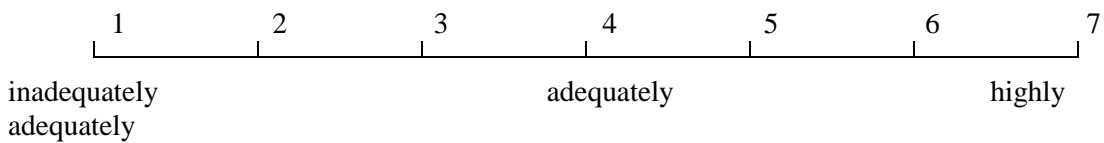
APPENDIX 1: Staff questionnaires

Principal/School Leader Questionnaire

Dear Principal/School Leader,

Thank you for your willingness to participate in this research. Mathematics is an important subject at school and we are constantly trying to find out more about children’s understanding of mathematics so that we can improve teaching and learning opportunities for everyone, including students who are highly capable mathematically. The purpose of this survey is to find out your experiences of, and views towards, your school’s approach to teaching students who are highly capable mathematically. Please answer as many questions as you can. You may use extra paper if you need more space for your responses.

- 1. How well do you believe your school caters for primary-aged students who are highly capable mathematically? (Please circle a number)



- 2. Describe what your school does to support primary-aged students who are highly capable mathematically (at the whole school level and/or classroom level).

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- 3. How do you feel the importance of financially supporting students who are mathematically highly capable compares to the importance of financially supporting students who struggle mathematically?

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APPENDIX 1: Staff questionnaires

What do you think are the dangers (if any) of not supporting mathematically highly capable students?

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4. Please add any further comments you would like to make.

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Thank you again for taking part in this research. Taking the time to complete this survey is very much appreciated. Please return the completed survey via email, or place in the *Research Consent Return* box in [redacted]'s office by **Friday 25 April, 2014**.

Kind Regards,

Linda Parish

APPENDIX 1: Staff questionnaires

5. What do you think are the dangers (if any) of not supporting mathematically highly capable students?

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6. Please add any further comments you would like to make.

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Thank you again for taking part in this research. Taking the time to complete this survey is very much appreciated. Please return the completed survey via email, or place in the *Research Consent Return* box in [redacted]'s, office by **Friday 25 April, 2014**.

Kind regards,

Linda Parish

APPENDIX 1: Teacher nomination form

Teacher Name: Grade (2014): Date:

1. Please list any children in your class who you believe to be highly capable mathematically.
2. Rate what you believe is the extent of their mathematical capability by circling a number on the line.
3. Provide a brief example of the student’s mathematical ability. What type of work do they do that gives you the belief that they are highly capable?

Student Name	Mathematical ability	Brief description/example of student’s exceptional capability

NB. If you would like to nominate more than four children, please photocopy this page.

If you have taught any other children in the past few years who you would consider to be highly or extremely capable mathematically, and these children are still attending Ballarat Grammar junior school, could you please list their names below.

.....

Please complete by and place in the *Research Consent/Return* box in [redacted]’s office. Thank you.

Linda Parish

APPENDIX 1: Parent questionnaire

Dear Parent/Guardian/Caregiver,

Thank you for your willingness to participate in this research. Maths is an important subject at school. I am trying to find out more about children’s understandings of maths so that we can improve teaching and learning opportunities for everyone. The purpose of this survey is to find out your perception of your child’s aptitude for learning maths, and any special abilities they may have. Please answer as many questions as you can. You may use extra paper if you need more space for your responses.

Name of child: Date of birth

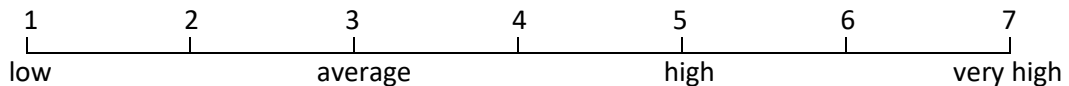
Gender and ages of siblings:

Sibling 1 m / f Age

Sibling 2 m / f Age.....

Sibling 3 m / f Age

1. Please indicate what you believe your child’s mathematical ability is by circling a number below:



2. What can you remember about your child and his/her maths-type abilities before they started school? [This includes activities such as building and designing structures (blocks, Lego etc.), jigsaws, recognising landmarks/directions (spatial abilities), as well as activities with number.]

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3. Did your child go to:

0-3-year-old childcare	YES / NO	Number of hours/week.....
3-year-old kinder	YES / NO	Number of hours/week.....
4-year-old kinder	YES / NO	Number of hours/week.....

4. Did the kinder teacher ever talk to you about your child’s curiosity and/or ability with maths-type activities (as outlined above)?

If YES, please explain: YES / NO

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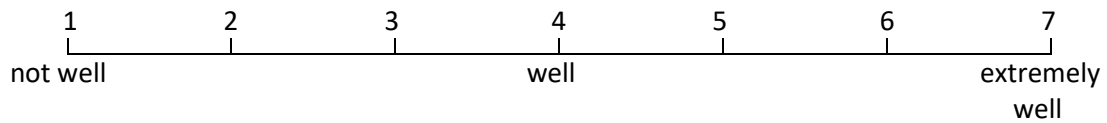
5. Do you believe your child is talented in any particular area/s? (any area/s, not just maths)

If YES, please describe their talent/s:

YES / NO

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6. How well do you feel your child is catered for in maths instruction at school? (please circle a number)



7. If you indicated *well* to *extremely well* in question 6, please describe how your child's needs are being met in maths.

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8. If you indicated less than *well* in question 7, what more do you believe your child needs, to be well catered for in maths?

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Thank you again for taking part in this research. The time taken to complete this survey is very much appreciated. Please return the completed survey and place in the *Research Consent/Return* box in [redacted]'s office next to the main reception by **Monday 24 March**.

Kind regards,
Linda Parish

APPENDIX 1: Student semi-structured interview questions

Student Semi-Structured Interview Questions

[To be completed following the task-based interview. NB. Some of these topics may already have been discussed during the process of completing the tasks.]

1. Tell me about your maths lessons at school. When do you do maths? What sorts of things do you do? Is it easy/hard? What are some things that help you to learn maths?
(McDonough, 2002)
2. Describe any maths you do when you are not at school. (what, when, why, who with?)
3. Do you enjoy maths? What do you/don't you enjoy?
4. Are you good at maths? How do you know? *Hand child the 'best person at maths' sheet.* Where would you place yourself on this chart? Please explain.
5. Who do you think is the *best* at maths in your class? Why?
Is there anyone else who is good at maths in your class?
What do they do that makes you think they are good at maths?
6. If you were having trouble in maths and your teacher was busy, who would you go to for help? Why this particular person?
7. Do you think it is important to be good at maths?
Why, or why not?
8. *Hand child the 'I like doing hard maths...' sheet.*
Which number would you circle?
Please explain.
9. (Depending on previous response) ... Please describe some hard maths you've done that you liked. What did you like? Please describe some hard maths you've done that you didn't like. What didn't you like?

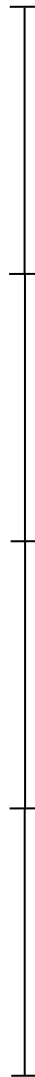
Reference

McDonough, A. (2002). PPELEM: Pupil perceptions of effective learning environments in mathematics. *Australian Primary Mathematics Classroom* 7(3), 14-19.

APPENDIX 1: Student semi-structured interview questions

Thinking about the
people in my class I
think I would go here...

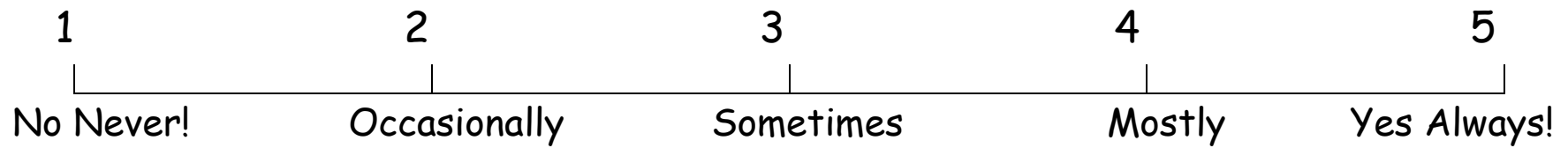
Best person at
maths



Takes a while to
learn new things in
maths

APPENDIX 1: Student semi-structured interview questions

I like doing hard maths ... (please circle one number):



APPENDIX 1: Teacher semi-structured interview questions

Teacher Semi-Structured Interview Questions

1. You nominated X, Y & Z and said you thought they were highly capable mathematically, could you just tell me what it is that makes you think they're highly capable mathematically.
2. I've decided to focus on X, What can you tell me about X's disposition in maths classes? What about other areas apart from maths?
3. Do you think X has a more fixed or growth mindset? What makes you say that?
4. What can you tell me about X's family environment? Parent occupations, siblings etc.
5. What sort of maths assistance, if any, do you think X gets at home? What are parent expectations like?
6. Is there anything more you would like to tell me about X?
7. What about any of the other students you nominated? Is there anything you would like to tell me about any of them?

APPENDIX 2: Mathematics Task-Based Interviews

- i. Interview Scripts – pre- and post-professional learning
- ii. Interview Record Sheets
- iii. Smiley Chart

This clinical task-based interview was designed to determine a student's approach to mathematical tasks, observing for Krutetskii's (1976) hallmarks of mathematical abilities.

It assesses three specific areas:

- 1) A student's ability to reason proportionally. Being able to reason proportionally has been shown to be a good indicator of mathematical ability (Lamon, 1999), so a ratio task has been included, recognising that ratio is not formally taught in primary school so children will need to employ their own intuitive strategies. The first task is a proportional reasoning task to assess how students go about *doing* maths.
- 2) A student's ability to learn something new. Mathematically highly capable children have been shown to be able to learn quickly with minimal repetition through their ability to generalise and assimilate new concepts readily. The second task teaches students a new way of working with numbers using a Chinese abacus, to see how they go about *learning something new* in maths.
- 3) A student's mindset about mathematics learning. Choice of numbers, words, and/or responses in an open task may indicate either a growth or fixed mindset (based on research by Mueller & Dweck, 1998). The third task is an open-ended task to see how willing students are to *think creatively* about maths.

There are three versions: Year 1, Year 3 and Year 5.

Interviews were audio-recorded.

Task-based mathematics assessment interview

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Materials Required

Year 1

- paper and pencil
- 10c piece
- packet of three lollies
- price tag (3 for 10c)
- Abacus (made from icy-pole sticks, skewers and pony beads)
- Sheet with '28' triangle
- Sheet with three triangles - 150, 85 and one '?'.

Year 3

- paper and pencil
- price tags "3 for 10c" and "10 for 35c"
- Abacus (made from icy-pole sticks, skewers and pony beads)
- cards with $65+32$ and $157-25$
- cards with $73+3$ and $26+5$
- Sheet with '156' triangle
- Sheet with three triangles - 502, 199 and one '?'.

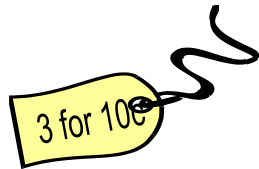
Year 5

- paper and pencil
- orange/lemon punch card
- Abacus (made from icy-pole sticks, skewers and pony beads)
- cards with $65+32$ and $157-25$
- cards with $36+25$ and $73+43$
- card with $424-185$
- Sheet with '502' triangle
- Sheet with three triangles – 100.25 , $1\frac{2}{3}$ and one '?'.

YEAR 1 SCRIPT:

1. Lollies: *proportional reasoning (Parish, 2010)*

*Place a 10c piece and a packet of three lollies on the table.
Show the price tag (3 for 10c).
Provide a piece of paper and pencil.*

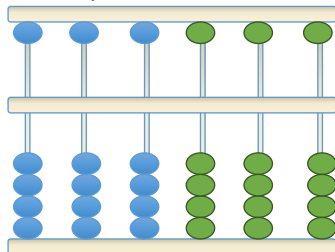


A shop sells a packet of three lollies for ten cents.

- a) How much would 12 lollies cost? How did you work that out?
- b) How many lollies could I buy with 60c? How did you work that out?

2. Abacus: *learning something new*

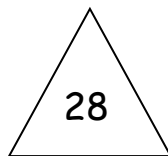
Show the child how the abacus is structured: in place value columns, 'earthly beads' worth 1, 'heavenly beads' worth 5, beads only have value if touching the centre bar.



- a) Can you count to 50 on the abacus? Backwards from 35?
- b) Can you show me 27, 154, 2189 on the abacus?
- c) What are these numbers (*show 78, 285, 5903 on the abacus*)

3. Adding Corners: *open task (adapted from Downton, Knight, Clarke & Lewis (2006))*

a)



Give the child the sheet with the 28 triangle.

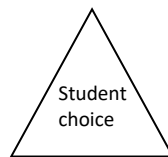
Find three numbers to put in the corners of this triangle that add up to the number in the middle.

See if you can find a really interesting/creative solution.

What do you think is interesting/creative about your solution?

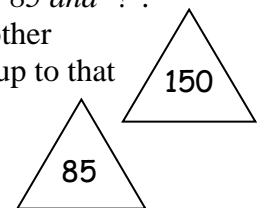
How do you feel about your solution? (*smiley face chart*).

b)



Give the child the sheet with three triangles – 150, 85 and '?'.

Challenge: Choose one of these (85, 150) or any other number you like, and find three numbers that add up to that number. See if you can find a creative solution that makes you feel very pleased (*:-D*) with your thinking and effort.



Why did you choose that number to go in the centre?

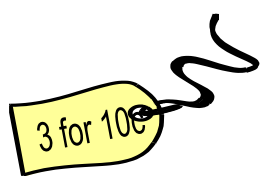
What do you think is interesting about your solution?

How do you feel about your solution? (*smiley face chart*). Explain.

Please find another solution ... Tell me about your choices this time.

YEAR 3 SCRIPT:

1. Lollies: *proportional reasoning* (Parish, 2010)



Provide a sheet of paper and pencil.

a) *Show the student the price tag (3 for 10c).*

A shop sells 3 lollies for ten cents.

- How much would 15 lollies cost? How did you work that out?
- How many lollies could I buy with 80c? How did you work that out?

b) *Show the student the price tag (10 for 35c).*

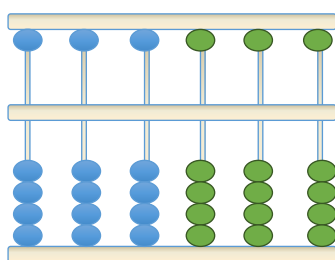
The same shop sells packets of ten lollies for 35c a packet.

Which is the better value, three for ten cents, or ten for 35c?

How did you work that out?

2. Abacus: *learning something new*

Show the child how the abacus is structured: in place value columns, 'earthly beads' worth 1, 'heavenly beads' worth 5, beads only have value if touching the centre bar.



a) Can you count to 50 on the abacus? Backwards from 35?

b) Can you show me 27, 154, 2189 on the abacus?

c) What are these numbers (*show 78, 285, 5903 on the abacus*)

d) *Show cards for 65+32 and 157-25 respectively.*

Can you show me how to solve these on the abacus?

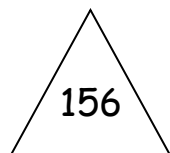
To add some numbers, you need to do some number busting. For example, for 7+8 you need to add 10 and take away 2; for 12+4 you need to add 5 and subtract 1.

e) *Show cards for 73+3 and 26+5 respectively.*

Can you show me how to solve these on the abacus?

3. Adding Corners: *open task* (adapted from Downton, Knight, Clarke & Lewis (2006))

a)



Give the child the sheet with the 156 triangle.

Find three numbers to put in the corners of this triangle that add up to the number in the middle.

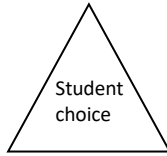
See if you can find a really interesting/creative solution.

What do you think is interesting/creative about your solution?

How do you feel about your solution? (*smiley face chart*).

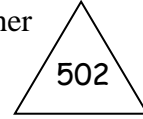
YEAR 3 (cont'd) Adding Corners:

b)



Give the child the sheet with four triangles – 502, 199 and two blank.

Challenge: Choose one of these (199, 502) or any other number you like, and find three numbers that add up to that number. See if you can find a solution that makes you feel very pleased (: -D) with your thinking, effort and creativity.



Why did you choose that number to go in the centre?

What do you think is interesting/creative about your solution?

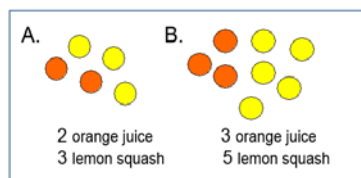
How do you feel about your solution? (*smiley face chart*).

Please find another solution ... Tell me about your choices this time.

YEAR 5 SCRIPT:

1. Oranges and Lemons: *proportional reasoning* (Lamon, 1999)

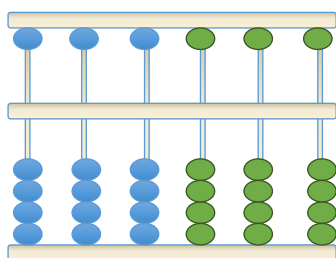
Show the student the orange juice/lemon squash card. Provide a sheet of paper and pencil.



This shows the number of parts of orange juice mixed with lemon squash to make an orange and lemon punch. Which mixture will taste more orangey, A or B?

2. Abacus: *learning something new*

Show the child how the abacus is structured: in place value columns, ‘earthly beads’ worth 1, ‘heavenly beads’ worth 5, beads only have value if touching the centre bar.



a) Can you count to 50 on the abacus? Backwards from 35?

b) Can you show me 27, 154, 2189 on the abacus?

c) What are these numbers? (*show 78, 285, 5903 on the abacus respectively*)

d) Show cards for $65+32$ and $157-25$ respectively.

Can you show me how to solve these on the abacus?

To add and subtract some numbers you need to do some number busting. For example, for $7+8$ you need to add 10 and take away 2; for $17-4$ you need to subtract 5 and add 1.

e) Show cards for $36+25$ and $73+43$ respectively.

Can you show me how to solve these on the abacus?

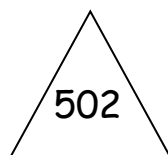
If successful with Q2(e):

f) Show card for $424-185$.

Show me how you would solve this on the abacus.

3. Adding Corners: *open task* (adapted from Downton, Knight, Clarke & Lewis (2006))

a)



Give the student the sheet with the 502 triangle.

Find three numbers to put in the corners of this triangle that add up to the number in the middle.

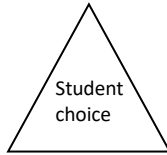
See if you can find a really interesting/creative solution.

What do you think is interesting/creative about your solution?

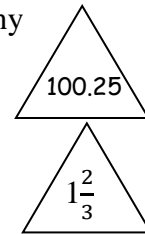
How do you feel about your solution? (*smiley face chart*).

YEAR 5 (cont'd) Adding Corners:

- b) Give the student the sheet with four triangles – 100.25, $1\frac{2}{3}$ and two blank



Challenge: Choose one of these ($1\frac{2}{3}$, 100.25) or any other number you like, and find three numbers that add up to that number. See if you can find a solution that makes you feel very pleased (:-D) with your thinking, effort and creativity.



Why did you choose that number to go in the centre?

What do you think is interesting/creative about your solution?

How do you feel about your solution? (*smiley face chart*).

Please find another solution ... Tell me about your choices this time.

Task-based mathematics assessment – follow-up interview

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1. **Rush Hour:** *challenging fixed/growth mindset*

Show beginner level #1 card of Rush Hour then ask:

Would you like to do that again by yourself, or would you like to challenge yourself with a harder level?



Beginner #1

Beginner #10

Intermediate #11

2. **Solve this problem two different ways:**

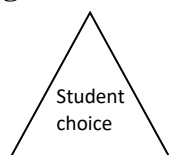
Give the student a sheet of paper with the following equation written at the top:

I would like you to solve this problem any way you like, and then I want you to come up with another creative method for solving the same problem (*relate to 'Goldilocks Zone': what can you do next to challenge yourself?*).

a) Year 1 and Year 3: $522-367$

b) Year 5: 87×9

3. **Adding Corners Task (repeat from initial interview):**



Choose any number you like, and find three numbers that add up to that number. See if you can find a creative solution that challenges you.

Why did you choose that number to go in the centre?

What do you think is interesting about your solution?

How did you challenge yourself?

How do you feel about your solution? (*smiley face chart*). Explain.

APPENDIX 2: Mathematics Task-Based Interview Record sheets

Name: _____ Grade: ____ School: _____ Suburb: _____

Interviewer: _____ Date: _____

YEAR 1

1. Lollies

- a. How much would 12 lollies cost? (40c)

Explanation: _____

- b. How many lollies could I buy with 60c? (18)

Explanation: _____

2. Abacus

- a. Count to 50 Count backwards from 35

Observations (how long to pick up, any prompting (include questions asked), etc.)

- b. Show: 27 154 2189

Observations: _____

- c. Name: 78 285 5903

Observations: _____

3. Adding Corners

- a. 28 Solution: _____

What is interesting/creative about your solution?

How do you feel about your solution?

- b. Number Choice • 85 • 150 • Other _____

Solution #1 _____

Reason for number choice: _____

What is interesting/creative about your solution?

How do you feel? _____

- c. Solution #2 _____

What is interesting/creative about this solution?

How do you feel about this solution?

Any further observations:

APPENDIX 2: Mathematics Task-Based Interview Record Sheets

Name: _____ Grade: ____ School: _____ Suburb: _____

Interviewer: _____ Date: _____

YEAR 3

1. Lollies

- a. How much would 15 lollies cost? (50c)

Explanation: _____

- How many lollies could I buy with 80c? (24)

Explanation: _____

- b. Best value: • 3 for 10c • 10 for 35c (3 for 10c)

Explanation: _____

2. Abacus

- a. Count to 50 Count backwards from 35

Observations (how long to pick up concept, any prompting (include questions asked), etc.):

- b. Show: 27 154 2189

Observations: _____

- c. Name: 78 285 5903

Observations: _____

- d. $65+32$

Method: _____

- $157-25$

Method: _____

- e. $73+3$

Method: _____

- $26+5$

Method: _____

3. Adding Corners

- a. 156 Solution: _____

What is interesting/creative about your solution?

How do you feel about your solution? _____

- b. Number Choice: • 199 • 502 • Other _____

Solution #1: _____

Reason for number choice:

What is interesting/creative about your solution?

How do you feel about your solution? _____

- c. Solution #2 _____

What is interesting/creative about this solution?

How do you feel about this solution? _____

Any further observations:

APPENDIX 2: Mathematics Task-Based Interview Record Sheets

Name: _____ Grade: ____ School: _____ Suburb: _____

Interviewer: _____ Date: _____

YEAR 5

1. Oranges and Lemons

More orangey • A • B

Explanation:

2. Abacus

- a. Count to 50 Count backwards from 35

Observations (how long to pick up concept, any prompting (include questions asked), etc.):

- b. Show: 27 154 2189

Observations: _____

- c. Name: 78 285 5903

Observations: _____

- d. 65+32

Method: _____

157-25

Method: _____

- e. 36+25

Method: _____

73+43

Method: _____

- f. 424-185

Method: _____

3. Adding Corners

- a. 502 Solution: _____

What is interesting/creative about your solution?

How do you feel about your solution? _____

- b. Number Choice: • 1 1/3 • 100.25 • Other _____

Solution #1: _____

Reason for number choice:

What is interesting/creative about your solution?

How do you feel about your solution? _____

- c. Solution #2 _____

What is interesting/creative about this solution?

How do you feel about this solution? _____

Any further observations:

How do you feel about your solution?

:-| It's ok

:-) Happy

:-D Very happy (very pleased with my effort and creativity)

other

APPENDIX 3: Ethics

- i. Ethics Approval
- ii. Information letters and consent/assent forms

Ethics approved by the Australian Catholic University's Human Research Ethics Committee as meeting the requirements of the National Statement on Ethical Conduct in Human Research.

Ethics Register Number 2013 116V,

Date Approved 20/09/2013

Ethics Clearance End Date 31/12/2014

Ethics Clearance Extended Expiry Date: 31/12/2015

APPENDIX 3: Ethics Information Letters and Consent Forms

Ballarat Campus (Aquinas)
1200 Mair St, Ballarat, Vic, 3350
PO Box 650 Ballarat VIC 3353
Telephone: 03 5536 5300
Facsimile: 03 5336 5305
www.acu.edu.au

PRINCIPAL INFORMATION LETTER

PROJECT TITLE: Extending Mathematical Understanding for the Mathematically Highly Capable Student

PRINCIPAL SUPERVISOR: Assoc Prof Gloria Stillman

CO-SUPERVISOR: Dr Ann Gervasoni

STUDENT RESEARCHER: Linda Parish

STUDENT'S DEGREE: Doctor of Philosophy (Education)

Dear Principal,

I am writing to invite your school community to take part in a research project I am undertaking as part of my PhD candidature at Australian Catholic University under the supervision of Associate Professor Gloria Stillman and Dr Ann Gervasoni.

The research aims to provide insight into the experience of learning mathematics at school for young students who are mathematically highly capable, and how teaching approaches are associated with these students continuing to be motivated learners of mathematics. Further, the research will test the hypothesis that catering for mathematically highly capable students within an inclusive classroom is possible and has benefits for all students in the class.

The significance of the research lies in the creation of new knowledge about how to adequately teach students who are mathematically highly capable within regular classrooms. Both teachers and student participants should benefit from this research as we work together to create this knowledge.

The data I seek to collect from your school for the research will be collected through interviews, conversations, and surveys of mathematically highly capable students, their teachers, and their parents, as well as from within the mathematics classroom, using observations of students who are mathematically highly capable working on mathematics tasks that require higher order thinking and effort to solve.

Ultimately, I plan to identify and then observe three mathematically highly capable students and their teachers in a series of maths lessons during the 2014 school year. I ideally plan to choose a student from each level of primary school – early primary, middle primary, and upper primary – and work together with their classroom teachers to identify and plan suitable mathematics lessons that will then be implemented in the classroom by the teacher, and observed by the researcher. These lesson observations will take place over one week on two separate occasions, once early in 2014, and then again later on in the year. It will involve collaboration with the classroom teachers in identifying and planning for suitable mathematics tasks that have the potential to challenge all students in the classroom, with an emphasis on recognising and catering for the needs of mathematically highly capable students.

In order to select the most suitable three students to observe, a larger number of students will need to be initially selected for assessing. I am asking for your permission to access growth point profiles of your 2013 *Mathematics Assessment Interviews* (MAI). Students who are achieving above average mathematically in the MAI data will be identified, and I will request copies of these MAI record sheets which will be independently coded for validation. Conversations with classroom teachers about students' dispositions will provide further evidence of exceptional abilities, and parents of these students will be invited to partake in a short survey to ascertain their perceptions of their child's mathematical abilities. I request that students with above average growth points and/or teacher recommendation and/or parent

perception of exceptional mathematics ability to be assessed using a task-based problem-solving interview, designed and conducted by the researcher, in order to determine student mathematical and mindset dispositions. These students will also be invited to take part in a semi-structured interview to find out their views of mathematics learning. It is anticipated that there will be approximately 12-15 students assessed at this stage, in term 4, 2013 if possible. The three lesson observation student participants will be chosen from this group, firstly on the basis of being mathematically highly capable, and secondly on the basis of possibly having a fixed mindset disposition.

Classroom teachers and school leaders will also be invited to take part in a semi-formal group discussion and short survey about their experiences with mathematically highly capable students in order to paint a picture of teacher perceptions and expectations, and to form a baseline for approaches for subsequent planning and lesson development.

With permission, audio-recordings will be made of assessments, discussions and classroom activities in order to facilitate deeper analysis of these activities.

There are no foreseeable risks associated with this research. Participation is entirely voluntary, and full consent will be sought from teachers and parents and assent will be sought from the students. Any participant is free to withdraw their consent at any time, without giving reasons.

Confidentiality of school, teacher, parent, and student identity will be retained at all times.

Any questions you may have regarding this project should be directed to the Supervisors: Dr Gloria Stillman (Telephone: 03 5336 5329; Email: Gloria.Stillman@acu.edu.au); Dr Ann Gervasoni (Telephone: 03 5336 5395; Email Ann.Gervasoni@acu.edu.au); and/or the Student Researcher: Linda Parish (Telephone: 03 5336 5315; Email Linda.Parish@acu.edu.au), Faculty of Education, Aquinas Campus, 1200 Mair St, Ballarat, 3350.

The study has been approved by the Human Research Ethics Committee at Australian Catholic University (approval number 2013 116V). If you have any complaints or concerns about the conduct of the project, you may write to the Chair of the Human Research Ethics Committee care of the Office of the Deputy Vice Chancellor (Research). Chair, HREC, c/o Office of the Deputy Vice Chancellor (Research), Australian Catholic University, Melbourne Campus, Locked Bag 4115, FITZROY, VIC, 3065. Ph: 03 9953 3150; Fax: 03 9953 3315; Email: res.ethics@acu.edu.au

Any complaint or concern will be treated in confidence and fully investigated. You will be informed of the outcome.

If you give permission for your school to participate in this project, please complete the attached consent form and return it to me (also keep a copy for your own records). I will then provide information letters and informed consent forms for your staff and parents.

I look forward very much to your response.

Yours sincerely



Linda Parish
PhD Student Researcher



Assoc Prof Gloria Stillman
Principal Supervisor

PRINCIPAL CONSENT FORM

TITLE OF PROJECT: Extending Mathematical Understanding for the Mathematically Highly Capable Student

PRINCIPAL SUPERVISOR: Associate Professor Gloria Stillman

CO-SUPERVISOR: Dr Ann Gervasoni

STUDENT RESEARCHER: Linda Parish

I have read and understood the information provided in the accompanying Information Letter. Any questions I have asked have been answered to my satisfaction. I agree for my school to participate in this research project throughout 2013/2014, understanding that consent will be sought from all involved, and that I can withdraw my consent at any time without giving reasons.

I agree to providing the researchers with the school's 2013 MAI growth point data, and to providing copies of selected MAI record sheets for independent coding.

I agree to allowing the researcher to conduct surveys and conversations with staff and parents of the school provided that these people give their consent.

I agree to allowing the researcher to attend the school to interview approximately 12-15 students with a one-to-one problem-solving mathematics task.

I agree to allowing the researcher to work intensively with three classroom teachers to plan, develop and observe a week of mathematics lessons twice throughout the 2014 school year.

I realise that research data collected for the study may be published, or may be provided to other researchers, but in a form that does not identify my school, staff or students in any way.

NAME OF PRINCIPAL:

SCHOOL NAME:

SCHOOL ADDRESS:

PRINCIPAL SIGNATURE: DATE:

I would like to receive a copy of the research findings.

Please return signed consent to:

Linda Parish
Australian Catholic University
PO Box 650
Ballarat VIC 3353

Ballarat Campus (Aquinas)
1200 Mair St, Ballarat, Vic, 3350
PO Box 650 Ballarat VIC 3353
Telephone: 03 5536 5300
Facsimile: 03 5536 5305
www.acu.edu.au

TEACHER INFORMATION LETTER FOR PARTICIPATION IN RESEARCH

RESEARCH TITLE: Extending Mathematical Understanding for the Mathematically Highly Capable Student

PRINCIPAL SUPERVISOR: Assoc Prof Gloria Stillman

CO-SUPERVISOR: Dr Ann Gervasoni

STUDENT RESEARCHER: Linda Parish

STUDENT'S DEGREE: Doctor of Philosophy (Education)

Dear Teacher,

I am writing to invite you to take part in a research project I am undertaking as part of my PhD candidature at Australian Catholic University under the supervision of Associate Professor Gloria Stillman and Dr Ann Gervasoni.

The research aims to provide insight into the experience of learning mathematics at school for young students who are mathematically highly capable, and how teaching approaches are associated with these students continuing to be motivated learners of mathematics. Further, the research will test the hypothesis that catering for mathematically highly capable students within an inclusive classroom is possible and has benefits for all students in the class.

The significance of the research lies in the creation of new knowledge about how to adequately teach students who are mathematically highly capable within regular classrooms. Both teachers and student participants should benefit from this research as we work together to create this knowledge.

Ultimately, I plan to identify and then observe three mathematically highly capable students and their teachers in a series of maths lessons during the 2014 school year. I ideally plan to choose a student from each level of primary school – early primary, middle primary, and upper primary – and work together with their teachers to identify and plan suitable mathematics tasks that will then be implemented in the classroom by the teacher, and observed by the researcher. These lesson observations will take place over one week on two separate occasions, once early in 2014, and again towards the end of the year. It will involve collaboration with the classroom teachers in identifying and planning for suitable mathematics tasks that have the potential to challenge all students in the classroom, with an emphasis on recognising and catering for the needs of mathematically highly capable students.

In order to select the most suitable three students to observe, a larger number of students will need to be initially selected for assessing. Students who are achieving above average mathematically in the Mathematics Assessment Interview (MAI) data will be identified, information provided by classroom teachers about students' mathematical abilities will provide further evidence of exceptional abilities, and parents of these students will be invited to partake in a short survey to ascertain their perceptions of their child's mathematical dispositions. Students with above average growth points and/or teacher recommendation and/or parent perception of exceptional mathematics ability will then be further assessed using a task-based problem-solving interview, designed and conducted by the researcher in order to further determine student mathematical and mindset dispositions. These students will also be invited to take part in a semi-structured interview to find out their views of mathematics learning. It is anticipated that there will be approximately 12-15 students assessed at this stage, in term 4, 2013 if possible. The

three lesson observation student participants will be chosen from this group, firstly on the basis of being mathematically highly capable, and secondly on the basis of possibly having a fixed mindset disposition.

Classroom teachers and school leaders will also be invited to take part in a semi-formal group discussion and short survey about their experiences with mathematically highly capable students in order to paint a picture of teacher perceptions and expectations, and to form a baseline for approaches for subsequent planning and lesson development.

With permission, audio recordings will be made of assessments, discussions and classroom activities in order to facilitate deeper analysis of these activities.

There are no foreseeable risks associated with this research. Participation is entirely voluntary, and full consent will be sought from all involved – teachers, parents and students. Any participant is free to withdraw their consent at any time, without giving reasons.

Confidentiality of school, teacher, parent, and student identity will be retained at all times.

Any questions you may have regarding this project should be directed to the Supervisors: Dr Gloria Stillman (Telephone: 03 5336 5329; Email: Gloria.Stillman@acu.edu.au); Dr Ann Gervasoni (Telephone: 03 5336 5395; Email Ann.Gervasoni@acu.edu.au); and/or the Student Researcher: Linda Parish (Telephone: 03 5336 5315; Email Linda.Parish@acu.edu.au), Faculty of Education, Aquinas Campus, 1200 Mair St, Ballarat, 3350.

The study has been approved by the Human Research Ethics Committee at Australian Catholic University (approval number 2013 116V). If you have any complaints or concerns about the conduct of the project, you may write to the Chair of the Human Research Ethics Committee care of the Office of the Deputy Vice Chancellor (Research). Chair, HREC, c/o Office of the Deputy Vice Chancellor (Research), Australian Catholic University, Melbourne Campus, Locked Bag 4115, FITZROY, VIC, 3065. Ph: 03 9953 3150; Fax: 03 9953 3315; Email: res.ethics@acu.edu.au. Any complaint or concern will be treated in confidence and fully investigated. You will be informed of the outcome.

If you are willing to participate in this project, please complete the attached consent form and return it to me (also keep a copy for your own records).

I look forward very much to your response.

Yours sincerely



Linda Parish
PhD Student Researcher



Assoc Prof Gloria Stillman
Principal Supervisor

APPENDIX 3: Ethics Information Letters and Consent Forms

TEACHER CONSENT FORM

RESEARCH TITLE: Extending Mathematical Understanding for the Mathematically Highly Capable Student

PRINCIPAL SUPERVISOR: Assoc Prof Gloria Stillman

CO-SUPERVISOR: Dr Ann Gervasoni

STUDENT RESEARCHER: Linda Parish

I have read and understood the information provided in the accompanying Information Letter. Any questions I have asked have been answered to my satisfaction. I agree to participate in this research project throughout 2013, realising that I can withdraw my consent at any time. I agree that research data collected for the study may be published or may be provided to other researchers in a form that does not identify my school, staff or students in any way.

I am willing to participate in (please tick as many as appropriate):

- Surveys
- Group conversation (to be audio-recorded)
- Mathematics lesson co-planning and classroom implementation (two weeks throughout 2014)
(to be audio recorded)

TEACHER NAME:

GRADE CURRENTLY TEACHING:

SCHOOL NAME:

SIGNATURE:

DATE

Ballarat Campus (Aquinas)
1200 Mair St, Ballarat, Vic, 3350
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PARENT/CAREGIVER INFORMATION LETTER FOR PARTICIPATION IN RESEARCH

RESEARCH TITLE: Extending Mathematical Understanding for Mathematically Highly Capable Students

PRINCIPAL SUPERVISOR: Assoc Prof Gloria Stillman

CO-SUPERVISOR: Dr Ann Gervasoni

STUDENT RESEARCHER: Linda Parish

STUDENT'S DEGREE: Doctor of Philosophy (Education)

Dear Parent/Caregiver,

I am writing to invite you to take part in a research project I am undertaking as part of my PhD candidature at Australian Catholic University under the supervision of Associate Professor Gloria Stillman and Dr Ann Gervasoni.

Maths is an important part of the school curriculum. This research aims to provide insight into the experience of children learning maths at school, and how teaching approaches are associated with children continuing to be motivated learners of maths.

The significance of the research lies in the creation of new knowledge about how to best teach children maths at school. Both teacher and student participants should benefit from this research as we work together to create this knowledge.

Ultimately, I plan to identify and then observe three children and their teachers in a series of maths lessons. I plan to choose a student from each level of primary school – early primary, middle primary, and upper primary – and work together with their teachers to identify and plan suitable maths tasks that will then be implemented in the classroom by the teacher, and observed by me. These lesson observations will take place on two separate occasions, each over a one week period, once earlier in the 2013 school year, and again towards the end of the year.

In order to select three children a larger number of children will need to be initially selected for participation. Data from your school's Mathematics Assessment Interview (*MAI*) will be analysed, and information provided by classroom teachers will form part of this selection process. As a parent/caregiver you will be invited to partake in a short survey to ascertain your perceptions of your child's mathematical abilities, you may also be asked to be involved in conversations about your child's experiences with maths at school. Approximately 12-15 children will then be selected to be assessed using a one-on-one task-based problem-solving interview, conducted by me, in order to further determine maths dispositions. This interview will take approximately 30 minutes. Children usually enjoy these one-on-one interviews (similar to the *MAI*), as they have an opportunity to show an interested adult what they are capable of doing. There is an element of choice within the interview so that children are never required to work beyond their comfort level. These children will also be asked to take part in a conversation to find out their views about maths learning. The problem-solving interview and the conversation will be audio-recorded.

Three children from this process will be selected for the classroom observations. Lesson observations will include audio recordings in order to facilitate deeper analysis of all activities.

There are no foreseeable risks associated with this research. Participation is entirely voluntary, and full consent has been sought from all involved – your school’s Principal and teachers as well as yourself, and the children will also be required to give their assent. Any participant is free to withdraw their consent at any time, without giving reasons.

Confidentiality of school, teacher, parent, and student identity will be retained at all times.

Any questions you may have regarding this project should be directed to the Supervisors: Dr Gloria Stillman (Telephone: 03 5336 5329; Email: Gloria.Stillman@acu.edu.au); Dr Ann Gervasoni (Telephone: 03 5336 5395; Email Ann.Gervasoni@acu.edu.au); and/or the Student Researcher: Linda Parish (Telephone: 03 5336 5315; Email Linda.Parish@acu.edu.au), Faculty of Education, Aquinas Campus, 1200 Mair St, Ballarat, 3350.

The study has been approved by the Human Research Ethics Committee at Australian Catholic University (approval number 2013 116V). If you have any complaints or concerns about the conduct of the project, you may write to the Chair of the Human Research Ethics Committee care of the Office of the Deputy Vice Chancellor (Research), Chair, HREC, c/o Office of the Deputy Vice Chancellor (Research), Australian Catholic University, Melbourne Campus, Locked Bag 4115, FITZROY, VIC, 3065. Ph: 03 9953 3150; Fax: 03 9953 3315; Email: res.ethics@acu.edu.au. Any complaint or concern will be treated in confidence and fully investigated. You will be informed of the outcome.

If you are willing to participate in this research, please have a chat to your child about their possible involvement, complete the attached consent form and return it to your child’s classroom teacher (also keep a copy for your own records).

I look forward very much to your response.

Yours sincerely,



Linda Parish
PhD Student Researcher



Assoc Prof Gloria Stillman
Principal Supervisor

PARENT/CAREGIVER CONSENT FORM

RESEARCH TITLE: Extending Mathematical Understanding for the Mathematically Highly Capable Student

PRINCIPAL SUPERVISOR: Assoc Prof Gloria Stillman

CO-SUPERVISOR: Dr Ann Gervasoni

STUDENT RESEARCHER: Linda Parish

I have read and understood the information provided in the accompanying Information Letter. Any questions I have asked have been answered to my satisfaction. I agree to participate in this research by completing a short survey, and taking part in conversations with my child's teacher and/or the researcher.

I agree for my child to participate in this research project throughout 2013, realising that I can withdraw my consent at any time. I understand that my child's responses will be audio-taped, but these recordings will be used solely by the researcher for analysis. I understand that data collected for the research may be published, or may be provided to other researchers, in a form that does not identify the school, staff, or my child in any way.

Photo permission (please tick all you give consent to):

- I give permission for photos of my child to be used at teacher professional learning sessions and conferences, understanding that his/her name will not be used.
- I give permission for photos of my child to be included in print, including education journal articles which may be available on the internet, understanding that his/her name will not be used.

*Please note, you can agree for your child to participate in the research but not be photographed. This will not exclude them from being involved.

PARENT/CAREGIVER NAME:

CHILD'S NAME: GRADE:

CHILD'S TEACHER NAME:

PARENT/CAREGIVER SIGNATURE:

DATE

- I would like a copy of the research findings

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ASSENT OF PARTICIPANTS AGED UNDER 18 YEARS**Script to be read to the child by the researcher**

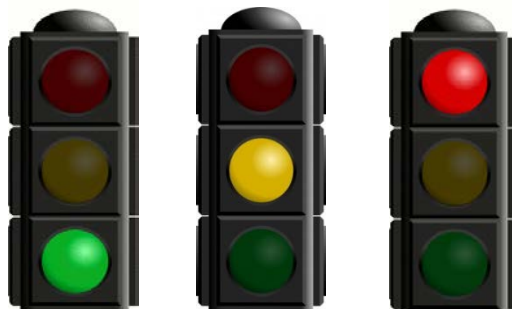
I am doing some research about how children learn maths, and I think you could help me with that. Your [Mum or Dad or Caregiver] has said it's OK for me to talk with you and do some maths tasks with you and ask you some questions about these tasks. Did your [Mum or Dad or caregiver] explain this to you? They also said they were happy for me to audio-record what you say.

You don't have to be part of this research, it's up to you. Even if you say yes now but later change your mind that's OK, just tell me or your teacher [name the teacher, if name is known].

If you say yes to being part of this research, I'll give you some activities to do and I'll record what you say and write down what you do. When I talk to others or write about these things you do, I won't say your name at all. The only people who will know that it was you who answered the questions will be people from my research team and your teacher [name the teacher, if name is known].

Does all that make sense? Do you have any questions you would like to ask me about what I am asking you to do?

What do traffic lights tell us? What do you think these lights might say about you saying 'yes', 'not sure' or 'no' to me asking you some questions and you doing some maths activities with me?



Thanks, from the research team,



Linda Parish
PhD Student Researcher



Assoc Prof Gloria Stillman
Principal Supervisor

STUDENT ASSENT

My name is

I understand that this research is about how children learn maths.

I know I will be interviewed about my answers to maths tasks.

I know I will be audio-taped during the interview.

No-one else other than the researchers and my teacher will know it was me who answered the questions.

I understand that I can say I don't want to be involved in the research any more at any time without having to give a reason for my decision.

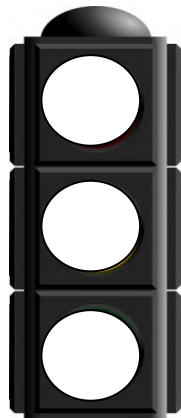
The coloured light on my traffic light lets you know if:

yes I would like to take part in this research [green],

no I would not like to take part in this research [red], or

I am **not sure** if I want to take part in this research [yellow],

Please colour in the light that tells me what you think.



My signature

Date _____ Full Name _____ School _____

APPENDIX 4: Summary Document from Professional Learning sent to Classroom Teachers

Supporting the learning of students who are mathematically gifted or highly capable

Linda Parish

Linda.Parish@acu.edu.au

One criteria for effective mathematics teaching is to accurately target each student's zone of proximal development when planning instruction. If students are working within their zone of proximal development, by definition they are working at a level that requires effort, and they will therefore also require support in their learning. This experience of mathematics learning requiring effort may be something new for highly capable or gifted students, and can feel quite threatening for some. Assisting learners to acknowledge and understand these feelings as part of the learning process is another important role for the teacher.

The hypothesis for my research is that students who are highly capable or mathematically gifted, with the right support, can learn how to explore mathematics concepts further for themselves, to set their own challenges. They need to not only be given support and permission to do this, but to need to realise that this is the normal expectation of them within the learning environment. Task completion, then, may involve three stages: 1) solve the problem; 2) explain the solution; 3) explore the mathematics further.

- 1) *Solve the problem.* This is whatever the normal classroom practice is; the task may be a game, an investigation, an open-ended question, a computer task, a worksheet etc. The 'problem' will be learning and understanding the mathematics concept that is the focus is for that lesson. The task may or may not need to be differentiated for mathematical abilities. The expectation is that *all* students in the class will undertake the task.
- 2) *Explain the solution.* This requires a different set of skills that need to be learned and developed. Again, the expectation is that *all* students in the class will learn how to explain their strategies and solutions in order to justify their answers, orally at first and then written. Written reports are an important part of mathematics; they may not be required for every lesson, but it is a skill that needs to be taught and developed over time. The process of explaining and justifying solutions (how they worked the problem out, why they worked it out that way, and how they know their solution is correct) can actually be quite challenging for mathematically gifted and highly capable students. Because their thought processes are naturally very efficient (often combining two or more processes into one thought), breaking these processes down into sequential logical steps may require substantial effort (that they may initially be quite resistant to), and specific teacher support.
- 3) *Explore the mathematics further.* This is a stage that *not all* students will reach. Once the problem is completed, understood and can be explained, the question to then ask is, "What's next?" Instead of students waiting to be given more work by the teacher, or doing 'busy work' for 'fast finishers', I believe students can learn to ask this question for themselves, "What's next? What else can I do with this task to be creative, to challenge myself?" This will need to be modelled by the teacher initially, but with the understanding that the students will ultimately take on this role for themselves. A chart could be made up and added to as new ways of exploring the maths are discovered:

Explore the mathematics further some examples:

- Can I solve this problem a different way?
 - Can I find another solution (for an open-ended task); how many different solutions are there, and how will I know I've found them all?
 - What if I try the same problem but make it more complicated (e.g., larger quantities, fractions, more components)?
 - How can I adapt the rules of this game to improve it?
 - What is the best strategy to use to ensure the greatest chance of winning this game?
 - What other components of this investigation look interesting, are worth exploring? (Permission to use computer search engines for investigations may be part of this).
-

Comments like “This is easy!”, or “I’ve finished”, or “I already know this”, or “I’m bored”, need to be treated as an indication that the student is not doing what they are meant to be doing. If it’s easy or you already know this – what are you going to do to challenge yourself further? If you’ve finished – what are you going to explore next? If you’re bored – how can you be creative and make it interesting?

Teacher support for mathematically highly capable learners – what it might look like

1. Establishing an understanding that learning requires hard thinking, and that is what we expect. Hard thinking is a *good* thing, not a sign that you are not good at maths.
2. Establishing that when I (the teacher) ask a question I am posing a problem I want them to think about. I don’t want a quick answer (I am not *testing* them). What I require is a well thought out explanation, the answer is the by-product of this.
3. Modelling that there is always more you can explore (teaching them how to think deeper; there is a skill in learning how to learn). This will continue to require support from the teacher, and sometimes a pertinent question is required get students thinking beyond what they currently know and understand. The teacher will need to plan for some possible exploration questions. This requires sound mathematics conceptual knowledge.

For example (from one Year 3 class observation), once the students had showed that they understood that $\frac{1}{4}$ is one of four equal parts (through their own drawings), I wanted to be sure that they also recognised that the remaining $\frac{3}{4}$ does not have to be contiguous, and that the four equal parts can look different, e.g., “Is this shaded section one quarter?” (remember to allow them to struggle with the answer!)



4. Encourage students to run with their own ideas. For example (from one Year 5 classroom observation), a number of students voiced their dislike of the rules of a game they were playing because there was too much chance and not enough opportunity to use their own strategies to enable them to get ahead. While the rest of the class was happy to continue the game as it was, these students were given permission to come up with their own adaptations of the rules to improve the game. They needed to be aware of the maths focus of the game and be able to explain how their rules enabled this maths concept to be learnt by playing the game.
5. Constantly ask questions like “How are you challenging yourself?”, “Are you working in your ‘Goldilocks zone’?”, “What’s next?”, “How can you be creative with this?” In the observation sessions I conducted all students responded particularly well to the words

‘challenge’ and ‘creative’. The Year 3s were voluntarily talking about their Goldilocks zone after only one session.

6. Be aware of, and challenge fixed mindset statements...

Types of Statements	Re-training for growth mindset self-talk
<i>I'm no good at maths.</i> (if the answer is not obvious, or takes a bit of thinking to work out)	<i>Hang on...I need to think about this a bit more.</i>
<i>This is too challenging for me.</i> (if the task requires thinking, time and effort to complete)	<i>Remember learning takes effort. I need to be working in my 'Goldilocks zone'.</i>
<i>I'm finished!</i> (indicating a need to be first finished)	<i>Learning is not a race. There is always something more to learn, what can I explore now?</i>
<i>This is easy! / I know how to do this.</i> (making sure people know they are smart)	<i>This is easy for me, how can I challenge myself further? To learn I need to be working in my 'Goldilocks zone'.</i>
<i>This is taking too long.</i>	<i>This is a good challenge for me. I'm needing to think long and hard about this problem. I wonder who I can discuss my thoughts with.</i>
<i>I'm making too many mistakes.</i>	<i>How can I learn from these mistakes? Where have I gone wrong? Why didn't this work?</i> (mistakes are an integral part of success. The most successfully innovative people in the world are often those who have 'failed' the most)

What's next?

Students who are mathematically gifted or highly capable are possible innovators for the 21st Century. We need to be teaching them how to explore further, to think for themselves, to be creative. We need to give them permission to think outside the box, but still be there to support them in this.

APPENDIX 5: Excerpt from Analysis Spreadsheets

- *How does primary school teaching shape the mindsets of children who are mathematically gifted? [Fixed mindset (FM) Growth mindset (GM)]*
- *How do teachers provide adequate challenge for students who are mathematically gifted? [Teacher Challenge TC]*
- *How do teachers support students who are mathematically gifted to undertake sufficiently challenging tasks within a regular classroom setting? [Teacher Support TS]*
- *How do teachers feel about their ability to cater for students who are mathematically gifted? [Teacher Ability (TA)]*
- *How do parents view their child's mathematical ability and teaching needs? [Parent Views (PV)]*
- *How do students view mathematics and their role as a mathematics learner? [Student Views (SV)]*

	Observations - Initial Assessment Interview - Alex May 2014	Evidence	Further Evidence	Interpretation	Code	Memo
Q1. Lollies	<p>First answer quick and confident (but incorrect). When asked to explain realised that the problem may not be as straight forward as he first thought but seemed to be already influenced by the 'times by 10' thought.</p> <p>Was not able to see the proportionality component of this question even though he was able to reason proportionally (1:3) in his explanation of his answer (to Q1b). He was able to do this mentally up to 12:36, but began to lose count so I suggested he record his thinking to help...</p>	<p>Interview 12/5/14: - Interview record sheet Audio: 00:1:00</p>	<p>Working out sheet#1: 12/36 13/39 14/42... Interview 12/5/14 00:7:30</p>	<p>Expectation that the problem will be solvable quickly and easily - answer given with very little thought. Need to avoid this mindset - teachers need to provide challenging tasks as the norm - develop an expectation that questions require thought, not just a quick answer.</p> <p>This could indicate either 1) no expectation of thought required (as per above), although even when he stopped and reconsidered his answer he ignored the 3 for 10c component, OR 2) even though his mathematical abilities are very advanced for his age there is still an immaturity in his thinking that needs to be considered. Mother has said she wants him to be doing 'Grade 5 maths', but this is problematic if his maturity is asynchronous.</p>	<p>TS</p> <p>PV</p>	<p>Was still unable to recognise a need for proportional reasoning in the second interview in November, even though he demonstrated some amazing calculating skills (10+6)</p> <p>Asynchronous development a common issue with giftedness.</p>
Q2. Abacus	<p>Picked up on the structure of the abacus very quickly with no need of any repetition after the initial instruction. [Also could tell me the place values of thousands, tens of thousands and hundreds of thousands]. Counting forwards got stuck at 50 but was able to reason this out without further prompting (kept asking if he was correct but eventually decided he was for himself). Counting backwards hesitated at the first decade transition only, however was counting the beads he was moving instead of stating the number of beads left. Very quick and confident in showing given numbers on the abacus - even realising that he didn't need to start from zero each time - "I'll keep these up" (154) and just added beads to this to make 2189. When reading the numbers, however, resorted to an additive strategy - for 78 saw the 70 as 50+2 then added another 5+3 (60); for 285 saw the 80 as 50+3 and then added another 5 (258)</p>	<p>Interview 12/5/14 00:08:45</p>	<p>Brony, Harry, Hamish, Sammy, Robert</p>	<p>Very quick to learn a new skill - able to assimilate his prior knowledge of place value quickly and easily for counting, and showing numbers on the abacus. However, this was not automatically translated to 'reading' numbers, which I found surprising but turned out to be reasonably common - 6 out of 27 students interviewed had difficulty reading the numbers but no difficulty making the numbers.</p>	<p>TC - TS -</p>	<p>. Sometimes we don't know what the challenges will be, sometimes something that we don't think will be a challenge for these clever kids turns out to be quite tricky for them. . Counting backwards is too often taught as simply rote. Counting backwards on the abacus for Alex was like counting backwards with the bead string for many adults [a task I often give EMU specialist trainees].</p>
Q3. Adding Corners	<p>a. Splitting 28 - knew there would be lots of solutions. Took 18 secs to write his one creative solution - 9+14+5. When asked how he worked it out he said he knew that 9+14 was 23 and 5 more was 28. This is not actually a description of how he worked it out, more a demonstration that it was a correct solution. I don't know why he started with the 9, or even if he did start with the 9. My suspicion was that he started with 14+14+0 and then split the first 14 into 9+5 because when asked what was creative about his solution he said it was creative because most kids in his class would just go 14+0+14. However, he said he started with 9 because not many people could count by 9s and then went to 14 because not many people can count by 14s. He added the 5 because he knew 9+14 was 23. Was very happy with his solution and creativity.</p>	<p>Interview 12/5/14 00:20:10 - 00:34:15</p>	<p>Further evidence of misconception about the term 'strategy' in Alex's second interview.</p>	<p>When asked to describe his strategy he simply proved that his answer was correct. Do we need to be more explicit in what we mean by 'how did you work that out?' when we are wanting to find out the process their brains used. Do they understand that 'strategy' means what <i>their</i> brains did?</p>	<p>TC</p>	<p>Do teachers need PL on what strategies are? Strategies are a description of a process used by an individual; we don't teach strategies. As soon we try to teach a strategy it becomes a method, or algorithm. There is a difference between 'teaching' a strategy and modelling a strategy with a written representation.</p>

<p>b. Own choice - chose a very big number 5698. Started with 300+98 then had to work out how many more to make 5698. Began skip counting by 100s (from 300), self-corrected ten hundred to one thousand, then decided to count by 500s instead, but didn't seem to be keeping track of how much he had counted. Counted up to 5600, hesitated slightly and then said 'I think that's five thousand and sixty, but I'm not so sure about that.' When checking to see if he was correct discovered quite quickly that it wasn't and changed the 98 to 198. Worked out 498+5060 - 'that would make 58...' Then changed the 300 to 500, thought that was correct, but after a couple of seconds thought changed the 198 to 48. At this point got a bit frustrated 'Oh damn, that won't work out either! What'll I do? Think!' (up until this stage had been mentally calculating everything quickly). Took a bit longer to consider what he had at this stage (25 secs), said at one stage 'I need another 90' (which was correct - had 5060+500+48 (5608) and his target was 5698). Changed the 48 back to 98 and wondered about changing the 5060 to something else ... 'Oh, what am I doing here?' ... 'I'm thinking about crossing that out and changing it to zero but that wouldn't be very creative though. How will I do this!? ... I'll have to change all the numbers.' Crossed out all the numbers he had a started with 'twenty-five hundred' and '2 thousand 5 hundred' ('Not many people know what 2 thousand 5 hundred plus 2 thousand 5 hundred is, not even 25 plus 25 most people don't know how to count by 25s ... not even my mum knows how to count by 25s I asked her to count by 25s and she was like 25...47!'). Felt happy about this solution (had felt very happy about his first attempt with the number 28). Wasn't very happy because 'I had to cross out all the things, I just couldn't get it right...I got very frustrated, I thought Oh, how do I do this!?, but in the end I finally got it. I knew it was more than 'ok' but I wasn't very pleased with it so I went with 'happy'.</p>	<p>Interview 12/5/14 00:22:56</p>	<p>. Check Aaron's interview for skip counting, also classroom observation 1. Check other interviews for reasons they were very happy with their solutions.</p>	<p>. Seems to be big on skip counting (Aaron also wanted me to know that he was good at skip counting by big numbers). Why do they think this is significant? Have they been skip counting in class, or is it something that is encouraged at home? . He continued calculating everything mentally even when he was getting frustrated when it wasn't working out for him. Is he able to record his thinking? Does he have any strategy or method for this? Is this something he needs teacher support with? . I thought he was going to say he wasn't very happy with his final solution because he hadn't been able to solve his original attempt, but he was more concerned about it taking so long and having to cross out all the numbers. He only spent 20secs on his second solution and was very happy with this. Many other students at this point were very happy because they had worked really hard and taken a long time to come up with their solution. . Being happy with quick solutions with no errors, and pointing out that he was able to do work that other students can't do could be an indication of a fixed mindset.</p>	<p>. TS required (written records of thinking) . FM</p>	<p>. No mention of skip counting in second interview or classroom observations in November.</p>
<p>c. When asked to come up with a solution for 5698 that he was very happy with went straight for 98+2500+3100 (20secs). 'I knew it had to be 600 and 500+100 is 600, and I knew that 2+3 is 5, and since they were thousands they're (the 2+3=5) thousands, and 98 in the tens.' Was very happy with this solution. It was creative because 'not many people know - they go 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, oh... I don't know what's next so I'll just count by a different number, and they have no idea what comes next...'. Me: 'It's something you can do' Alex: 'Yes I can!' Me: 'So what makes it creative for you?' Alex: 'It's creative for me because if everyone in the school did this, I knew, like, everyone must copy someone, and except for the really people good at maths, I knew the good people at maths, if they copied someone they would actually be looking at their... So if you got the whole school to do this and they copied someone's they'd actually be a bit cheeky and cheating. So I just chose something that I think mostly other people can't do it. About one other person in my class could think about this, that's all ... about only two people in the school could do it.' Felt very happy about this solution.</p>	<p>Interview 12/5/14 00:30:25</p>	<p></p>	<p>. He came up with this solution quickly, using known facts. I asked him what was creative about his solution when what I meant to ask was what made this a challenge for him. It's creative because he didn't copy someone else.</p>	<p>SV</p>	<p>Ask Alex what he thinks it means to challenge himself.</p>

APPENDIX 6: Papers and Publications

Conference Papers:

Parish, L. (2016). The power of creativity: A case study of a mathematically highly capable grade 5 student. In B. White, M. Chinnappan & S. Trenholm (Eds.). *Opening up mathematics education research* (Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia, pp. 511-518). Adelaide: MERGA

Parish, L. (2015). Challenging the mindset of Sammy: A case study of a grade 3 mathematically highly capable student. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia, pp. 477-484). Sunshine Coast: MERGA.

Parish, L. (2014). Defining mathematical giftedness. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia, pp. 509-516). Sydney: MERGA.

Non-referees Journal Articles:

Parish, L. (2015). The power of creativity: Supporting the learning of highly capable students. *Prime Number*, 30(1), 14-16.