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Modeling the Phenomenon Versus Modeling the Data Set

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This paper investigates students' mathematical modeling activity in data-rich modeling tasks. It aims at gaining insight into how students develop meaning when modeling data-rich situations and the mathematical models produced. A tendency to model a particular dataset, rather than the phenomenon that the dataset is a particular instance of, has been observed previously. Students concentrate on fitting mathematical objects such as functions to data, rather than using domain knowledge about the situation being modeled, mapping this to the data so as to capture the phenomenon as a whole. In other instances, students find functions that simply linearly interpolate the data and do not consider key features of the phenomenon, particularly when they have access to technological tools. The extent to which students' reasoning indicated awareness of their taking either approach was investigated in a qualitative study with Year 10/11 students. How the approach taken affected the processes students engage in whilst modeling was also investigated. The paper contributes to our currently limited literature on research into this issue and how it affects the outcome of students learning to model in classrooms at this level of schooling.

Keywords: data-rich situations; modeling the dataset; modeling the phenomenon; technology use; modeling purpose of use; modeling processes

Introduction

Mathematical modeling is clearly part of mathematics and is now recognized as a fundamental mathematical practice for all students in many different educational jurisdictions worldwide (see e.g. Barquero & Jessen, 2020; National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010; Villarreal et al., 2018; Vorhölter et al., 2014) and is increasingly seen as necessary in mathematics teacher education (e.g. Association of Mathematics Teacher Education, 2017; Borromeo Ferri, 2018; Gilbert & Suh, 2021; Villarreal et al., 2018). It is essential that teachers are able to select, implement and assess modeling tasks in a meaningful manner that is commensurate with the hallmarks of mathematical modeling.

To ensure they can do so, they need to know how students act when working with different types of modeling tasks. One such task type is data-rich modeling tasks.

This paper aims to investigate secondary school students' mathematical modeling activity in data-rich modeling tasks. In particular, it aims at gaining insight into how students develop meaning when modeling data-rich situations and the mathematical models produced. A tendency to model a particular dataset (e.g. the daylight hours forecast for a specific year at a particular location at 30 day intervals), rather than the phenomenon (e.g. the daylight hours for that location as they vary over time) for which the dataset is an instance, has been observed in the past (Galbraith, 2007) and similarly in sports contexts (Riede, 2003). We refer to these two approaches as *modeling the dataset* and *modeling the phenomenon,* respectively.

Students, and in some cases their teachers (Riede, 2003), concentrate on fitting mathematical objects such as functions to data becoming a slave to the data rather than using domain knowledge about the situation being modeled (e.g. year length and timing of longest and shortest days of daylight) mapping this to the data so as to capture the phenomenon as a whole (Galbraith, 2007). In other instances, students find functions that simply linearly interpolate data and do not consider key features of the phenomenon (Berry, 2002; Maull & Berry, 2001; Zbiek, 1998) such as performance capacity of weight lifters not growing linearly as weight of the lifter increases as it becomes increasingly more difficult to perform necessary movements (Riede, 2003). It thus is appropriate to ask whether students are aware they are doing this and to investigate the effects these different approaches have on the modeling processes such students engage in when modeling these situations. This is particularly important in classrooms where students are learning to model and develop meta-knowledge about mathematical modeling, that is, what, why, when and how they would mathematically model.

Mathematical Modeling Activity in Data-Rich Tasks

Mathematical modeling

We agree with Caron (2018), who notes that mathematical modeling has to be an aspect of mathematical practice, which is an integral part of doing mathematics (see Stillman et al., 2020), whether that be as an engineer (Gainsburg, 2013), "a practitioner of mathematics, …a mathematician, an economist, a biologist, any citizen, a teacher or a student" (Caron, 2018, p. 548). We also conceive of modeling as real-world problem solving but acknowledge that it does not have to be. In this conception of mathematical modeling, understanding the situation being modeled and implications of the mathematical problem solving for it, are of paramount importance to the modeling enterprise.

Mathematical modeling is a process, as many note (e.g. Caron, 2018; Doerr et al., 2017; Niss et al., 2007; Stohlmann et al., 2016), and can be represented as a cyclic activity (Gainsburg, 2013) that is iterative. A representation of this process as it proceeds from a messy real-world situation (A) to a series of stages (B-G), which are products of intervening actions related to cognitive activities, is presented in Figure 1. The thick arrows represent transitions in activity between stages with the articulation of specific cognitive activity (1-7) undergone by modelers between stages. These are often referred to as the *processes of modeling*. Such a representation is an idealized and simplified version of what occurs during modeling which, in reality, involves much more to-ing and fro-ing between stages. The process of mathematical modeling "is driven by considerations of both the external world and mathematics. The motivation for what to do next is a continuing give-and-take between the two" (Pollak, 1997, p. 101). The double headed arrows indicate reflective metacognitive activity that acts on the cognitive activities (Krüger et al., 2020; Vorhölter, 2018). This can involve looking forwards or backwards with respect to stages in the modeling (Stillman, 2011). The diagram itself can also act as a scaffold for beginning modelers that can be used by teachers during their interactions with students and by students themselves to monitor and evaluate their progress as was done in the study described here. For both teachers facilitating the modeling and the students engaging in the modeling, such diagrams bring "an accompanying managerial understanding of when to appropriately use" (Yoon & Thompson, 2007, p. 204) particular cognitive activities and helps keep track of progress.

[INSERT FIGURE 1]

The mathematical modeling process as a whole builds "a mathematical model out of a real-life situation, and [uses] that model to analyze the situation, understand it, address an issue, solve a problem, make a decision, etc." (Caron, 2018, p. 548). Furthermore, any decision making is not based on "portraying real-world argumentation as a unitary rationalistic phenomenon" (Gainsburg et al., 2016, p. 340) but rather, it is discipline or community of practice specific and thus must be related to the expectations and norms of the audience or community for whom the modeling was carried out, as well as the modeler's own convincing of self and others in the modeling team, during the modeling, that decisions are sound (Gainsburg et al., 2016). A modeling solution must be not only mathematically correct and explicable but also practical, giving reasonable and desirable answers in the real-world context (Pollak, 1997).

Empirical Versus Theoretical Modeling

Mathematical modeling is a complex process and experienced modelers adopt different approaches depending on their perception of the real situation presented to them to model and the problem posed (Thompson & Yoon, 2007). Purposes of model building in such situations include, separately or in combination, measurement, deciding between alternatives, replication, prediction, explanation or manipulation (Thompson & Yoon, 2007). Mathematical modeling can be characterized as empirical modeling or theoretical modeling (Berry, 2002) although a combination of the two can also be used. In an educational context, *empirical modeling* occurs when models are fitted to data that were either given with the problem statement or collected by the modelers; whereas *theoretical modeling* occurs when a model is constructed or developed from a theoretical perspective incorporating key features of the situation or phenomenon being modeled. In both approaches data can be integral, however, because in theoretical modeling data can be used to verify the appropriateness of the model. Empirical modeling is employed when the purpose of the modeling is for prediction more so than understanding the underlying relationships of the phenomena involved. If the modeling is meant to give greater insight into a situation, then theoretical modeling is more appropriate.

Mathematical Modeling Tasks and Approaches in Secondary Schooling

Bautista et al. (2014) characterize modeling activities or tasks as being on "a continuum along 'theory-driven' to 'data-driven' situations" (p. 9). The tasks discussed in this paper are data-rich tasks, which are considered to be towards the data-driven end of the continuum but that does not preclude students taking a theoretical approach to solution. However, the mathematical, digital display and analysis tools available influence both the mathematical structure perceived in a particular real-world situation and how mathematizing is attempted (Brown, 2015; Greefrath, 2020; Zbiek & Conner, 2006).

Zbiek (1998) previously conducted a study with a cohort of 13 prospective secondary school mathematics teachers who were novice modelers engaged in a series of data-rich, descriptive modeling activities constructing models directly from collected or provided data. They had access to curve-fitting software, graphing calculators, and a Computer Algebra System (CAS). Zbiek classified the participants' modeling approaches as: (a) Fitted Function Selector (FFS), where a curve fitter was used to generate all the software's fitted functions for the data and the best goodness of fit values was selected as the appropriate model; (b) Potential Function Generator (PFG), where a curve fitter was used to generate potential best fitting functions from the software's repertoire of functions but the appropriate model was selected by the match between the features of a function and those the modeler perceived as features of the real-world situation; (c) ScatterPlot/Graphing Tool (SCT), where scatterplots of the data were produced but other functions beyond the tool's repertoire of prototypical functions to fit were graphed, and the model selected by a qualitative judgement of scatterplot fit and function characteristics; and (d) Unneeded/Unused Tool (UUT), where the modeler's judgements about relationships between relevant quantities in the situation without technological support was integrated with their knowledge of mathematical entities such as ratios and functions to construct a model based on a small subset of the data. The first three approaches are in the empirical paradigm whilst the last is theoretically based and more towards theoretical modeling. According to Zbiek (1998), approaches were distinguished by the extent of modelers' reliance on the tool for model choice, the extent modeling was influenced by mathematical accuracy, mathematical understanding of relevance and/or real-world considerations.

The study of functions typically characterizes Year 9 -12 curriculum in many countries including Australia, especially in academic pre-tertiary mathematical pathways to university (Caron, 2018; Smith & Morgan, 2016). Perceived pressure not to take time away from mathematical content to be taught can result in compromises by teachers in jurisdictions where mathematical modeling has gained a foothold in curriculum documents and assessment requirements (e.g. Victorian Curriculum and Assessment Authority [VCAA], 2015). According to Caron (2018), a Canadian tertiary mathematics educator, mathematical modeling in the last years of secondary schooling can often be reduced to finding the curve of best fit for a set of data points usually with the support of digital tools. This occurs in Australia as well. Such an approach to modeling, in Caron's opinion, limits experience of modeling processes relying on little development of understanding of the situation being modeled or gaining a deeper understanding of mathematics applied. However, Caron tempers her critique by noting the value in "using real life situations with their intrinsic complexity and authentic data" (p. 560). Agreeing with Maull and Berry (2001), she implores teachers to move away from the empirical paradigm, where manifestations of Zbiek's (1998) first three approaches to modeling using digital tools are likely to occur, to the theoretical paradigm.

What is it then, that students are losing in this approach to data-rich modeling tasks and how might teachers counter this? A theoretical modeling approach requires an engagement with the underlying phenomenon at the level of understanding of the relationships between important variables in a situation. To achieve this, a structuring of the messy real-world situation (Figure 1) based on simplifications might allow the modelers to gain traction on the situation with the mathematical, representational or analysis tools at their disposal. This can involve clarifying and employing physical, physiological, or social theories about the phenomenon or phenomena involved (see, e.g. Jensen et al., 2017). These can be based on researching the particular domain area where the problem resides, recalling experiential knowledge (Caron, 2018) or by consulting with experts in the domain (Vos, 2015). Modelers also form one or more mental models of the situation (Waisel et al., 2008), which can be refined by reflection on technology-generated external visualizations (Brown, 2015) as the modeling continues. However, to successfully model a situation there has to be an integration of the modeler's (a) physical/physiological/social theories about the relations between key variables of the phenomenon in the situation (Czocher & Hardison, 2021), and how these might change in relation to each other, if this is relevant to the situation being modeled; (b) the modeler's mental models of the situation; and (c) the mathematical modeling that the modeler employs. Having experience in being able to bridge gaps among these, and being able to successfully integrate all three aspects, is necessary if students are to develop a rich meta-knowledge about the modeling process as a whole. Teaching practices that lead to the curtailing of the necessity for such integration to structure a messy real-world situation will be a poor substitute for modeling in secondary classrooms. Using curve-fitting software and/or regression in computers or calculators, in a manner that allows modelers to go directly from data to a mathematical model, diminishes the modeling experience for them as they are not involved in decision making related to the structuring and mathematizing of the situation being modeled; that is, the cognitive activities associated with transitions 1 and 2 in Figure 1, surrendering these to the technological tools they use. Another consideration, that might affect the modeling, is the possibility that some variables in the situation are not just controlled by physical/physiological principles; but are subject to human decision making related to factors external to the situation necessitating the testing of different scenarios, rather than just extrapolating from a best fitting function model.

In this paper, we seek to identify relationships between students' perceptions of the nature of their modeling approaches, as they work in small teams on data-rich modeling tasks, and their cognitive activities that affect their learning of mathematical modeling and meta-knowledge about the modeling process in a technology-enabled teaching/learning environment. In particular, we attempt to reveal the extent of their awareness of their modeling approaches leading to modeling the dataset, or part thereof, or modeling of the phenomenon and the perceived, versus actual, generality of their modeling.

Methods

To gain insight into this issue, we present a qualitative study where Year 10 and Year 11 students have been given data-rich modeling tasks in their regular classroom lessons as part of the *Using Mathematics to Solve Real-World Problems: The Role of Enablers Project* (DP17010555)[1] (Geiger et al., 2018). The research questions we will be addressing are:

RQ1 (a) To what extent does student reasoning indicate awareness of modeling a phenomenon, rather than a specific instance of it, when upper secondary students (Years 10/11) are presented with data-rich modeling tasks? (b) Does access to technological tools influence prevalence of either approach?

RQ2 How does the approach taken (i.e. modeling the phenomenon or modeling the dataset) affect the processes students engage in whilst modeling?

Participants and Their Backgrounds

Thirteen students in a Year 10 class from one school, Heathcliff College[2], in a one semester mathematics elective which had a problem solving and modeling focus and 22 students in a Year 11 two semester General Mathematics class at another school, Earnshaw College, participated. Both schools were Years 7-12 secondary schools located in outer and inner metropolitan areas, respectively, of a large Australian city.

The mathematical background of the Year 10 class included the study of linear functions for all students and non-linear functions for some students, plotting a dataset

[1]This research was supported by the Australian Government through Australian Research Council's *Discovery Projects* funding scheme (DP17010555). The views expressed herein are those of the authors, not necessarily those of the Australian Government or Australian Research Council. Chief Investigators are V. Geiger, G. Stillman, J. Brown, and P. Galbraith. M. Niss is a Partner Investigator.

[2] All names used for schools and students are pseudonyms.

and finding a suitable model. The local state curriculum (VCAA, 2016, p. 6) indicates it is expected that mathematics lessons will foster “the ability of students to make choices, interpret, formulate, model and investigate problem situations, select and use technological functions and communicate solutions effectively”.

In Year 11 and 12, modeling is one of the essential mathematical activities described for all mathematics units (VCAA, 2015) including Year 11 General Mathematics. Two aims of the senior curriculum show how “worthwhile and challenging mathematical learning” (p. 6) is intended to be implemented so that students can “apply mathematics to analyze, investigate and model a variety of contexts and solve practical and theoretical problems in situations that range from well-defined and familiar to open-ended and unfamiliar [aim 2] and use technology effectively as a tool for working mathematically [aim 3]” (p. 6). The Year 11 students’ mathematical background included linear relations and equations, linear graphs and models, direct, inverse and joint variation, investigating relationships in datasets between two numerical variables using scatterplots, Pearson’s *r* and linear regression, and interpreting these in context, in addition to their Year 10 background.

The students also engaged in teacher designed or selected real-world/realistic tasks during the semester and two video-recorded modeling sessions within the project. At Heathcliff College, these included the *Up Task,* about the lifting of a house with balloons based on the movie *Up* (Docter, 2009), as an introduction to the modeling cycle diagram in Figure 1 and its cognitive activities of modeling, and a version of the *Filling Up Task* (Blum & Leiss, 2006) before implementation of the first researched modeling task for this class. A further two data-rich tasks, the first comparing the salary spend by American baseball teams and their season wins, and the second comparing the careers of National Basketball Association of North America stars, Michael Jordan and

Lebron James, were used before the implementation of the second modeling task for this class in the project. During a pre-lesson interview, the teacher described these tasks as involving "essentially just two sets of data plotting them against one another, and using the data to try and read it with functions on it."

The Year 11 students had engaged in only two other real-world open modeling tasks, *The Million Dollar Robbery* as an introduction to modeling and *Drought Proofing the Farm,* both conducted by their teacher. The modeling cycle (see Figure 1) was introduced during the introduction to the second task. This task was part of the data collection for the larger research project. Neither are data-rich tasks.

The Tasks

Two tasks are the focus of this paper. The first, *Protective Service Officer* (*PSO) Task,* was about height estimation. The other task was related to sport as sport was a major interest of many in the class. Key details of the tasks are presented next[3].

PSO Task. The Minister for Police announced the recruitment of extra Protective Service Officers to improve public safety, especially during international events, but also to improve public confidence in using the transport network. Police Command has suggested that the PSO's are not good at estimating people's heights but are reasonably good estimators of small lengths, and where this is not the case, it can be easily remedied. The suggestion is that using a small body part length, easily estimated, a rule of thumb could then provide a good height estimate. Your team, as consultant mathematical modelers to Police Command, are to compile a report evaluating various rules of thumb that PSOs could use to provide a good estimate of height of offenders or suspects using shorter lengths. Police Command have provided some sample data. [Data

[3] Fuller versions available at www.mathsmodellingenablers.com

supplied: first name, height, shoe length, knee height, wrist to elbow length, face height, gender, age, ethnicity for 41 adults aged 20-58, one with missing data.]

Weightlifting Task (WLT). In Olympic weightlifting one type of lift is the clean and jerk where the barbell is first lifted to the shoulders (the clean) and then, in a second motion (the jerk), to above the lifter's head. There are male and female categories and then weight classes within, so Women's 58 kg allows female lifters to compete who are up to this weight. Records are kept for each weight class. The world associations of weightlifting use a model that is based on the world records from the beginning of one Olympic year until the end of the next Olympic year. The International Weightlifting Federation (IWF) in 2018 disallowed all previous world records and started afresh. New records exist in some but not all categories. Your team, as modelling consultants to the IWF, is to **provide 'world standards' where no record exists** using your best models. The IWF will only ratify lifts as World Record Lifts that are 1 kg higher than the new world standards which also need to be both marketable and achievable. To increase public interest in weightlifting before the Olympics, a **World Champion, irrespective of gender**, will be crowned for different types of lifts by determining the best weightlifter across all weight categories. Your team is to argue who should be crowned World Champion for Clean and Jerk based on comparing the Actual World Record Lift with the predicted Performance Capacity of the lifter and to **Rank the top ten** for a countdown. [Datasets showed current world record lifts for male and female weight classes with actual weight of lifter in super heavy class only, record holder's name and country, date record set as well as current world records for men and women with name of record holder and actual competition body weight in all classes.]

Data Collection

Data for this paper were collected from implementations of the two modeling tasks. The

PSO Task was used with the Year 10 students as their first video-recorded modeling task and with the Year 11 students as their second. The Year 10 students undertook the *WLT* as their second video-recorded task. In both tasks, students were provided with data, but could also collect more data themselves (via measurement for *PSO Task* or online for *WLT*). Students worked on the tasks in teams of two or three. One student from Heathcliff College was absent during the second task so there was one fewer team. See Table 1 for details of teams and implementations. Teams were either video or audio recorded as they engaged in the lessons led by their teacher.

Data used for analysis to answer the research questions came from scrutinizing the lesson implementation video-recordings of selected teams and their transcripts, together with transcripts of audio-recordings of the other teams, post lesson focus group interviews (of 6-8 students from a mix of teams for each implementation), post lesson video stimulated recall interviews with the teams video-recorded during implementation, and students' written reports of their modeling as they worked on the modeling tasks. In the stimulated recall interview, team members were asked direct questions about whether or not they considered themselves to be modeling the dataset given or modeling the phenomenon more generally at particular times during their solution, stimulated by short clips of the video-record of their team's working during the lesson. These questions were also posed to members of other teams in the focus group interviews.

Analysis

As a first pass through the data, the video files were viewed by both researchers, and all text data in the form of transcripts from video and audio files and the student written modeling reports were read and subject to data filtering by both researchers. Instances where the students' words or actions in images appeared to indicate that they were modeling the phenomenon, or restricting their approach to part or the whole dataset, or collecting data, were selected by each researcher separately. We then cross checked that the selected images and passages of text or mathematical working were the same, discussing and culling or including until they were. An interim case summary (Miles et al., 2014) was produced for each of the groups and their approach to a specific modeling task. These were cross-checked by the researchers individually for completeness and ensuring that the context in which the modeling took place was preserved (Richards, 2005). To conduct thematic analysis, several qualitative, partially ordered meta-matrices (Miles et al., 2014) were constructed for data from the two sites. The meta-matrices assembled descriptive data from different cases in a standard condensed format (Miles et al., 2014) on the basis of inclusion of all relevant data to answer the research question in focus. Initially these were quite large "mega-matrices" (Miles & Huberman, 1994) but they were gradually refined by the researchers transforming the case-level data into short quotations and summarizing phrases and then fracturing and clustering so they became more ordered and contained short, category-grounding phrases in each cell. This was an iterative process of data condensation and category refinement through a series of meta-matrices until emergent themes were distilled through the tactic of making comparisons and contrasting (Miles et al., 2014). At this stage of the analysis, exemplar cases were selected for the reporting of results.

Due to space limitations, only selected rows of case-level displays for partially ordered meta-matrices related to the first research question are shown from Heathcliff College (Table 2a) and Earnshaw College (Table 2b) but this analysis and categorization was done across all teams and tasks. As the tables show, the analytical categories of interest are the approaches taken by teams during modeling (i.e. modeling the dataset or modeling the phenomenon), evidence of awareness of the approach they were taking, the purpose they set for their modeling and the alignment of their personal physical theories, mental models or images and their mathematical modeling.

To understand the possible influence of digital tools (e.g. CAS calculators and laptop computers with Excel spreadsheets) on student approaches to the tasks and the processes engaged in, all data were coded for how digital technology was used (e.g. to make a scatterplot), modeling purpose of use (e.g. to verify mathematical output) using Villareal et al. (2018, p. 333) codes, and modeling approaches of technology use when curve-fitting and regression software were available using Zbiek (1998, p. 198) codes. Additional codes were added as necessary and all coding was subjected to consensual agreement between the two researchers.

Finally, to complete answering the second research question, partially ordered qualitative meta-matrices showing the approach taken (i.e. modeling the dataset or modeling the phenomenon), and the modeling processes engaged in by students at the two sites were constructed. Each set of data for a group was coded for instances of cognitive activities in the transitions in the modeling process shown in Figure 1. Any additional cognitive activities related to modeling processes (e.g. identifying variables) were also coded. These codes were assigned individually by the researchers and then any discrepancies discussed until consensus was reached. See Tables 4a and 4b.

In the coming sections findings from this analysis will be reported, followed by a discussion in light of the literature and the research questions, and then conclusions will be drawn and possible implications for teaching and future research will be identified.

Findings

Selected exemplars from the meta-matrices addressing the themes that were distilled from the data related to students' approaches to the data-rich modeling tasks and their awareness of their approaches are presented to answer RQ1a. Possible influences of the availability of digital technology use on the student data-modeling approaches to the modeling tasks follow to answer RQ1b. Finally, the modeling processes engaged in by students during their modeling of these tasks are presented together with the modelling purposes of use of technology from Table 3 to answer RQ2.

Modeling Part of Dataset not the Phenomenon – fully aware

If the dataset is perceived to be a set of numerical values where contextual meaning is secondary, other purposes than coming to a satisfactory resolution of the real problem presented in a modeling situation can be served quite readily. In the *WLT*, Ned from White Team (Heathcliff College) initially explored ratios and then tried to develop a linear model by hand. After some difficulty with finding a gradient, he decided completing the task with a report in the timeframe was his goal, rather than modeling the weightlifting situation in a realistic manner. He was totally aware in the post task interview that he was not modeling the phenomenon involved, as this response shows:

Ned: I feel like I kind of narrowed myself down, just by viewing it as the whole goal was to find these two [world standards needed], because then I wasn't really having a model of humans doing weight lifting, I was just having a model like based on the averages of numbers to estimate this number [points to world standard needed], like yeah. And that was my mistake.

Initially, he had expected that there would be a "trend between how much they weigh and how much they can lift". To find this, he suggested to his partner, Joe, using ratios "so it is like for each kg they weigh more than 55,…the weight they can lift increases by a set amount". By omitting the last given data point for the superheavy weight lifter, he was able to say that for every 6 kg increase in weight, the lifters lift an extra 9.25 kg. They decided to use Excel on a laptop to make a scatterplot and find a linear trendline. Ned then realized he could use his weight lifted to extra weight of the lifter ratio as the gradient for his own linear model, but could not find the constant by hand techniques, so finally used Joe's constant from Excel at the teacher's suggestion. To complete the task with a report, Ned decided to continue using ratio calculations on his calculator and Joe would use Excel to produce a series of linear models. From this point on, their "modeling" was driven by mathematical expediency, as Ned and Joe explained in the post task interview.

Ned: Obviously there is a limit to what humans can hold, because…if any human can carry any weight, then it wouldn't need robots … But we did understand, and like looking at the thing it was, there probably was a limit, and we probably should have taken that into account. However, it was the quickest way to have a linear.

Joe: I was just thinking like, instead [put] those parts into the Excel thing and then formulate like the trend line, and find the equation and stuff…. I guess we didn't really like put in the consideration for like the human limit that much, I think.

Interviewer: So you really chose it on mathematical grounds and convenience and efficiency at the time?

Joe: Yeah, pretty much.

Interviewer: So the actual real world didn't come a lot into your thinking?

Joe: I guess not.

Interviewer: You weren't really thinking a lot about weight lifting during it?

Ned: Nuh [meaning "no"].

In a second example, when again the dataset was perceived to be a set of numerical values where contextual meaning was secondary, tensions between flawed physical theories of what was occurring in the situation, the modeler's mental image of it, and the results from mathematical working using actual values, was overridden. In the *WLT,* Eric and Ray (Blue team, Heathcliff College) both decided quickly that they should put the data values into a graph and obtain a trend line, as that would mean "you can say roughly what it is going to be" (Ray). Eric used a CAS calculator to plot the men's World Record Lifts against weight categories, which they decided looked reasonably linear, except for the last point, which was the second highest data pair given, as he left out the super heavyweight category. Ray drew a line on the plot of the women's World Record Lifts against weight categories on a spreadsheet screen image saying "this one looks pretty much right on the line, yeah." For this line, he used a point consisting of the mean of the weight categories and the mean of the weight lifted (62.3, 140.3), with the point representing the lowest weight category and its corresponding World Record Lift (49, 120), to find the gradient and then the linear equation using $421/3 = 17/11*187/3 + c$. Their model to find missing World Standards for Women was $y = 17/11x + 44$ where y = weight lifted and x = weight class. They then decided to use linear models with the super heavy category missing as their first models, and the actual weights with all data as a second model, not realizing this was just a different data source, not necessarily giving rise to a different mathematical model.

[INSERT Fig 2]

When Eric plotted the men's actual weights with the amount lifted for World records on his CAS calculator, it was not linear (see Figure 2a) with Ray commenting he didn't like it. He noted that, "This one almost looks like it is going to be like a parabola or something." Eric saw it as "going to come back around", but Ray insisted "it is not going to start coming down", suggesting "it is like exponential" and so "turns really gradually", but apart from the last two points, it was "a pretty straight line".

After being reminded by their teacher of how they could find the equation and look at different graph types to use as models on an Excel spreadsheet, they examined the female data on a spreadsheet and various trend line choices, noting that the data plot “is not linear at all” (Ray), but still chose none. Ray proffered the flawed physical theory “if the weight kept going up, like in terms of muscle mass, they will be able to keep lifting more and more as well”. For this reason, they were happy with the linear model as their first choice and their discarding of the super heavyweight class. Eric even suggested it could be “a symbol of underperformance”, that the actual data was so much lower than a linear model would predict. Although Ray was willing to consider an exponential model, which he then rejected, he insisted a parabola “is definitely not right at all,” despite the better visual fit to the points (see Figure 2b).

As the following exchange shows, there was a mismatch between the boys’ mental model of the situation, the physical theories they held about the physiology of the situation, and the mathematical object (the parabola) that appeared to be the best model:

Eric: I mean it doesn’t fit in with like how reasonable it would be.

Ray: If you keep on getting like, like if this [the parabola] just goes on, then you get like heavier, you get more muscle, you can lift less until it is nothing!

Eric: That would mean there is a point at which it peaks.

Ray: Yes, but I doubt it. It just doesn’t seem right. [softly] I don’t like it, it is bad.

A further exchange with the teacher convinced Eric there might come a point where a person had too much muscle, or was too big, and would have difficulty standing up, let alone lifting weights. Reluctantly, Ray accepted the quadratic model, but said they were only choosing it because it “seems to be more accurate”. Even in the follow up interview, Ray said he was not expecting a parabola because “like the way it is kind of formatted, it is like you get stronger, as you like gain weight then you get stronger; but then you keep getting it [more weight] until you get weaker again. And that just didn’t make a lot of sense to me, like in a literal sense.”

This team's actions do not indicate they were a slave to the data – more there was a misalignment between the situation, Eric's mental model of the situation based on his physical theories about human capacity to lift weight in relation to bodyweight, and the mathematical objects at his disposal to model the situation. All their actions and comments were consistent with modeling the dataset given, although they were willing to exclude real data (i.e. the super heavyweight category) that did not fit with the physical theories they held, especially Ray's.

Modeling Part of Dataset not the Phenomenon – believing model more general

In the *PSO Task*, Una and Tegan (Green team, Heathcliff College) constructed the model, $6S = H$ (where S is shoe length, H is a person's height, both in mm), as a rule of thumb to determine height from a small subset ($n = 7$) of the given dataset ($n = 41$) from the youngest (4 people), middle (2 people) and oldest age groups (1 person), using ratios of average height ÷ average shoe length. This meant their sample was mainly Caucasian, although Una inspected the data later and declared that there was little variation, so they would not need to adjust their model, even if they restricted it to female Caucasians. They indicated in the follow up interview that the model applied generally to the adult population "because they are people" (Tegan). Una qualified this by saying, "Maybe not for children because children are a bit different. The growth rate is much different from ours…from adults." Shoe length had been chosen initially: "because shoe length…it's better to the person's overall height, like the bigger shoe is obviously taller, because you need bigger shoes to help balance the person more as they weigh more" (Una). They sifted through the given data, making choices about which variables they considered significant, based on their physical theories about growth, eliminating face height as irrelevant, but investigating knee height, as it contributed to a person's growth and therefore height. Using ratios of averages again, they constructed the model $H = 3\ k$ (where k is knee height, H is a person's height, both in mm). From a trial of 5 people's heights from the dataset, they concluded that use of shoe length was the better estimate.

Modeling the Phenomenon –Believing Model More General

In this case, the modeler believed that the model being constructed applied generally. In the *WLT*, for example, Evan from Red team at Heathcliff College believed that the model being constructed applied to the phenomenon of human capacity to lift weight depending on bodyweight, rather than the phenomenon of weight lifters' capacity to lift weight relative to their bodyweight in Olympic competition. He therefore neglected the effects of human decision making about qualifying benchmarks to be able to participate in the competition, ensuring that the minimum bodyweight was not zero. Evan attempted several times to find what he called the "best matching line". He used a moveable line on his CAS calculator screen showing a joined plot of the world record lifts for males against their estimated or real weight (Figure 3a) with the line locked to pass through (0, 0) (Figure 3b). He anticipated, before he plotted the points, that these would be roughly linear starting from zero, "because weight sort of starts at zero". He realized that the model he produced was only "accurate" for the lower weight categories, as he had removed the others; because, he said, if he included them then the lower categories did not fit his linear model so well.

[INSERT Fig 3]

He acknowledged a linear model did not fit all the data, "because it curves, and there will be a limit on it [running the tips of the fingers of his closed hand across the points in a curved fashion, Figure 3c], like how much it will weigh. And like it is not going to keep a constant rise like this [running his finger-tips up the first 5 points, Figure 3d], because you can't like, you can't lift it."

Modeling the Phenomenon–Believing Model Applies Generally

Finally, there was only one team from either site who were modeling the phenomenon and believed that was what they were doing. Olive team (Sui, Tara, Veta) from Earnshaw College was looking for a rule that covered everyone for the *PSOs*, not just the dataset. They based their modeling on their own measurements and a small subset of the dataset. Following an initial exploration of any physical theories they knew about body part lengths, such as “your shoe length is the same as your forearm” (Tara), they started with linear regression of the full dataset, as this strategy was elicited by the teacher in a well-orchestrated whole class discussion and she had later dismissed a question from Tara about how to show an error interval in a measurement as being off track. They generated a scatterplot on Tara’s CAS calculator, but did not understand it, as she had plotted knee height versus knee height. So, they returned to looking at their body proportions and the relationship of these to their height. Tara indicated they planned to look at knee height (Kh) and then wrist to elbow (WtE), to estimate overall height (H).

Tara had shown her team her wingspan during the initial whole class discussion as an example of a rule of thumb saying, “That’s how tall you are.” They now looked at the length they needed as consisting of their forearms, which should be the same lengths, similarly with upper arm lengths and then their torso, which Sui pointed out was unknown. They started making estimates such as Arm = 2WtE, and suggested checking by measuring on themselves. Sui suggested doubling the WtE could find the demi-span, which they could also test by measuring, as demi-spans were not in the given dataset.

Tara returned to using the scatterplot and linear regression on the CAS calculator after a visit to their team by the teacher. A comment by the teacher about recording what was in each list triggered, for her, the reason that their attempt at linear regression was unsuccessful. She generated and recorded the linear model and its goodness of fit values and her interpretation of these as: $y = -170.5 + 0.4x$, where y = height and x = knee height, $r^2 = 0.67$ = moderate. For Tara, moderate for "the strength of the data" meant the data "isn't very accurate" but, more telling, was the fact that the team still did not understand what it showed them, as it "didn't make any sense" (Tara, VSR).

From this point on, they "used our own bodies and figured it out from there" (Tara, VSR). Arm = 2WtE led to Demi-span = 3WtE to predict height. Testing on Tara and Veta gave mixed results, but an exchange between Veta and Tara indicated Veta expected that any models they made could not exclude short people like herself, so their idea of stating the degree of error expected as an interval, was reiterated. They then returned to considering knee height with Sui observing, "if you times your knee height by four it kind of gets your height". For Alice, the first person in the dataset, Sui had found the ratio, H/Kh = 3.7. They started to test this on the given dataset, but moved on to wrist to elbow, as the teacher told the class they should be onto their second model. Both 5 and 4 were suggested as multipliers for knee height to estimate height, but Tara wanted what they were doing grounded in reality, so brought them back to relating what they were doing to their wingspans. Sui suggested that Tara's chest was about the same length as her forearm. They then decided to get an idea of the width of torsos by finding 4 x WtE, which corresponded to the length of the arms, and compared this to the heights of Asha, Bindi and Alice in the dataset, finding differences of 540, 560 and 550mm, respectively. The model for height was then $H = 4WtE + c$, where c was the average of widths of torso (i.e. the average difference between the three predicted total length of arms and actual heights in their comparisons). They tested on themselves empirically by measuring, believing the model should work generally. Finally, two rules relating wrist to elbow and height were proposed, one for males and one for females: $H_M = 4WtE + 750$ and $H_F = 4WtE + 550$, as they expected males to have wider torsos, so allowed an extra 200 mm. The rule for females they again tested on themselves empirically. They noted the need to expect variation and so “give or take 100 millimetres” when using the models. In this instance, the modeling was driven mainly by a desire to engage with the underlying phenomenon by understanding the relationships existing between the variables in the situation, informed by wanting to integrate their personal theories about the physiology of the situation with the mathematical objects they used to model it, in a way that made sense to them.

Influence of Digital Tools on Data Modeling Approaches to the Modeling Tasks

To gauge whether there was any influence of digital tools, with curve fitting and regression capabilities, on students’ choice of data modeling approach, we turn attention to Table 3. The two teams, where members were engaged in modeling the phenomenon, both used the CAS calculator. Evan from Red team started immediately by plotting a scatterplot of the weight categories of male weightlifters and their world record lifts on his CAS, expecting that he could use the fitting of a line of best fit by eye by using the moveable line facility to construct a linear model in the *WLT,* and so predict the missing world standards. The availability of the calculator to do the plot enabled construction of an initial model and further model exploration. However, the dataset was small and the scatterplot, fitting of the line, and finding an equation for it could have been done by hand or missing values read from the line. Olive team generated a linear model in their attempt at solving the *PSO* task using linear regression with goodness of fit values; but abandoned this approach for one where they were able to make sense of their modelling using their understandings of the relationships between variables in the context, based on proportional models.

With respect to Zbiek's (1998) modeling approaches of technology use, there were just two examples of use of the Fitted Function Selector approach, where final model choice was selected on the basis of Goodness of Fit values, rather than any knowledge of the real situation. Both teams using a modeling the dataset approach, treated the task as practice in function-fitting rather than as an opportunity to build and validate a model. Five teams used the Potential Function Generator approach for at least part of their modeling with some aspects of the real world being used to select or reject their model(s). Given that the purpose of their modeling was seen by virtually all groups as prediction, the high incidence of Scatterplot/Graphing Tool approach is unsurprising. This was used by teams using either data modeling approach. With respect to RQ1b, having access to technological tools allowed either approach to data-modeling to be enacted and enabled the use of least squares regression, which no student was able to do by-hand, as a by-hand approach was not part of their learning experiences. The tools allowed several models to be considered in graphical form easily within the timeframe allocated, a point several students made in the focus group and VSR interviews.

Modeling Processes Engaged in When Modeling These Data-Rich Situations

Tables 4a and 4b present the approach taken (i.e. modeling the dataset or modeling the phenomenon), and the modeling processes engaged in by students at the two sites whilst modeling these data-rich situations as a partial answer to RQ2. Each set of data for a team was coded for instances of cognitive activities in the transitions in the modeling process shown in Figure 1. Some additional cognitive activities related to modeling processes were also coded (see Table notes). Further details of cognitive activities in these transitions is evident in the modeling purposes of use column of Table 3.

For transition 1, there were only two incidences of simplifying, one in a team fully aware of using the modeling the dataset approach and one by a team fully aware they were using a modeling the phenomenon approach. Both occurred in the *PSO Task*. In transition 2, the making of assumptions was very low and there were no incidences of making assumptions in any other transition. The three occurrences were in the *PSO task* with teams taking a modeling the dataset approach. There remained an imperative for teams to understand relationships between relevant variables in the data with either approach. Structuring of the messy real-world situation was not always evident but when it was, it was based on recalling experiential knowledge. Structuring was identified in modeling of both tasks with teams employing either approach. Working mathematically was evident for all teams, with some verifying output of their models from a mathematical perspective as well, enabled for several teams by access to digital technology (see Table 3). Interpreting mathematical output was evident in 14 of the 19 team attempts at the tasks. In transition 5, comparing and, to a slightly lesser extent, critiquing of different models occurred in most teams but validating, in the sense of reflecting on their models in light of the problem posed in the real situation, and evaluating that the correct model had been constructed for it, was rare.

In transition 6, several teams rejected reasonable models in terms of fit and realistic match to the situation based on being too inaccurate when extrapolating beyond the data (for larger values of the independent variable), whilst simultaneously ignoring similar issues when extrapolating for smaller values. This occurred in both tasks and for both data-modeling approaches. The last transition highlighted difficulty with revisiting the model as several teams did not do so, as their time management did not allow, they did not know how to, or they were not inclined to want to engage further in doing any more mathematics. However, of the teams who did attempt to revisit, several merely revisited the same model using different variables, not a new model, or for the refinement of an existing model. Again, this occurred in both tasks and with both approaches.

Discussion and Conclusion

The aim of this paper was to characterize how students working in small teams act when working with data-rich modeling tasks in order to develop meaning for their modeling and the models they produce. In particular, the differences between modeling the dataset and modeling the phenomenon were explored to add to the sparse reporting and discussion of this in the literature to date (e.g. Galbraith, 2007; Riede, 2003). Riede's examples were all in sports contexts and were the inspiration for the *Weight Lifting Task*. However, the *PSO Task* is from a non-sports context, as was Galbraith's daylight example. Most student teams in this study took a modeling the dataset approach, but the degree they were aware of this differed from a deliberate choice to do so, and acknowledgement of this, to believing they were not doing this but instead, modeling the phenomenon itself.

As the students were learning to model and to develop meta-knowledge about mathematical modeling in technology-active classrooms, it was pleasing to see many teams looking for relationships within the situation guided, to some extent, by their personal physical/physiological theories about the interplay of what they considered relevant and irrelevant variables as proposed previously by Czocher and Hardison (2021). When there was more alignment between these personal theories, espoused mental models (Waisel et al., 2008) of the situation (although only a few incidences of these were revealed in the collected data) and the mathematical modeling being employed, the quality of the modeling was higher. The presence of technology-generated external visualizations such as scatterplots, or sorted displays of categorized derived values such as ratios and averages, allowed students to discuss these mental models as they externalized their thinking and augmented argumentation within their teams about suitability for representing the situation.

With respect to the modeling processes engaged in by students whilst modeling these data-rich situations, the foreshadowing by Caron in her polemic chapter (Caron, 2018) that use of such tasks would lead to a quite limited experience of the processes of modeling and rely on little development of an understanding of the real situation being modeled was not borne out for most teams as is shown in Tables 4a and 4b and exemplified in the findings section. For some transitions, incidence of particular cognitive activities, namely, simplifying, making assumptions, verifying and validating was less than ideal, nevertheless. Lack of particular processes was not unequivocally associated with either approach. However, it might be that the tasks used did not evoke enough examples of modeling the phenomenon for any trends to be seen, so further research in this area is warranted.

With respect to *theory development*, we have been able to identify incidences of both types of approaches to data-rich situations in these modeling tasks. The relatively low incidence of modeling the phenomenon identified here may, however, be an artefact of the data used in the study, or its local context, rather than a real one. Data-rich modeling can, and should, involve considered use of data but also accounting for knowledge of the problem context (Galbraith, 2007; Riede, 2003).

From an *educational practice perspective*, data-rich modeling tasks as used here, unlike other forms of modeling tasks, allow modelers to focus on a dataset as an instance of a phenomenon and/or as the phenomenon itself informed by scrutiny and analysis of this particular instance. Only focusing on the dataset limits concentration on the phenomenon, but it also provides a range of options for initiating modeling. It thus allows informed class discussions to contribute to students' meta-knowledge about mathematical modeling by considering whether students are modeling the dataset or modeling the phenomenon, their awareness of the data-modeling approach taken and their thinking about the generality and area of application of the models under construction. There is also opportunity to discuss as a whole class, or within group facilitation, how a model of the data might not easily translate to an explanation of why the model aligns with the situation being modeled.

Students have access to multiple functions in technology-active environments, often before they know much about the various function types typically in secondary education. Teachers can, and should, provide opportunities for students to make sense of how technological functions of tools such as CAS calculators and spreadsheets with in-built trendline choices can be used sensibly when modeling. With data-rich tasks, it seems eminently sensible to produce plots of some, or all, of the data and reflect on these (Brown, 2015). Depending on one's knowledge of the context under investigation, these plots can confirm anticipated relationships, or suggest relationships, that should then be considered in terms of the context. Given data-rich tasks have reasonable sized datasets, using tools such as regression with some thought to function selection can empower users.

An increased emphasis by teachers on students better communicating their reasoning and decision making as they solve real-world tasks is warranted. A tendency to communicate once finished modeling, or time was running short, restricted opportunities for students to recognize errors, resolve different understandings in the team, and emphasize connection between the mathematical and real worlds (Pollak, 1997). This emphasis may also result in students becoming more aware of their data-modeling approach as part of the meta-knowledge about mathematical modeling students are expected to develop when using data-rich modeling tasks of this nature. It may help them keep focus on the real-world problem and not become stuck in the mathematical world, forgetting the purpose of solving the task.

It is important that students reflect on their modeling and model(s) produced (Maull & Berry, 2001; Yoon & Thompson, 2007) and engage in the cognitive activities associated with the transitions from accepting of the solution and reporting (transition 6) and revising the model and returning to the real-world situation (transition 7). To this end, curricula documents should include increased emphasis on the domain over which a function might be useful. The tendency to reject reasonable models, both in terms of fit and realistically matching the real-world situation, might then be counteracted. However, the Fitted Function Selector approach (Zbiek, 1998) of seemingly blindly accepting a regression model and also selecting best on the basis solely of a statistical measure must be discouraged as not appropriate when modeling. Students and teachers need to be more aware that the closeness of fit of selected data to a function does not necessarily reflect reality. Taking a regression model as a starting point can be appropriate where needed, using functionality such as moveable lines (as used by Evan in Red team), but also recognizing that CAS-enabled technologies generally allow users to also dynamically manipulate a graphical model to reflect real-world considerations and represent plotted data. This suggests that with today's digital technology, a merging of the Potential Function Generator and ScatterPlot/Graphing Tool (Zbiek, 1998) should be the approaches encouraged as they are in keeping with expanding students' meta-knowledge about modeling. The results from this study depend on the teaching and learning context, the actual tasks used, and the work of the teams selectively reported. This must be borne in mind in interpreting them.

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Table 1. Details of implementation of the two tasks at the two schools.

Task Implementation	Participants	Teams
PSO mid-August 2019 100 minute regular lesson at Heathcliff College	13 students in Year 10 elective class, 5 females and 8 males	5 teams of 2 and 1 of 3 - self allocated
PSO early September 2019 73 minute regular lesson at Earnshaw College	22 students in Year 11 General Mathematics class, all female	2 teams of 2 and 6 teams of 3 - teacher allocated
WLT late November 2019 100 minute regular lesson at Heathcliff College	12 students in Year 10 elective class, 5 females and 7 males	3 teams of 2 and 2 teams of 3 – 4 teams same self-allocated as *PSO,* fifth team formed by partner of an absent student combined with a pair

Note. All teams were single gender.

Table 2a. Selected Rows from Partially Ordered Meta-Matrix: Modeling Approaches (Heathcliff College Data).

Team	Approach	Awareness	Purpose	Alignment of Physical Theories/ Mental models or images/ Mathematical Modeling
PSO Task				
Green (Una & Tegan)	Modeling part of dataset – models of shoe length and knee height produced from *ratios of averages* based on data from 7 people in given data set – says shoe length "best" based on 5 trials from data of people in given data set.	Believe *model applies more generally in adult population* "because they are people" (Tegan, VSR). "Maybe not for children because children are a bit different. The growth rate is much different from ours. I don't mean ours, from adults" (Una, VSR)	Finding a rule that would allow PSOs to *predict* the heights of the people more accurately (Tegan, Report).	Physical Theory: "Shoe length is it's kind of, really it's better to the person's overall height, like the bigger shoe is obviously taller because you need bigger shoes to help balance the person more as they weigh more" (Una, VSR). Alignment between Physical Theory and Mathematical Modeling.
Blue (Eric & Ray)	At start just thought of data only as "numerical values" (VSR) then thought about them as lengths on their own bodies (Ray, Imp & VSR). Make 5 models for overall height based on *summation of all measurements*, *summation of multiples of selected combination of measurements* (so a proportional model) and use of *ratios of single measurements* (wrist to elbow and face height). They eventually use *averaging with proportional combination* model to generate Ht = Av FH + 2 x (Av KH + Av WtE) based on 40 in the dataset. Earlier models based on a few pieces of data ("focused more on those closest to the top") rejected on basis of lack of precision. Did not think to test on anyone outside of dataset (VSR).	Expected to be able to find a *perfect formula for the data set* (Ray, VSR). Eric says they are assuming that the measurements hadn't changed ("they have not grown since the measurements were taken") but then Ray says it wouldn't matter because they "would all be taken at the same time, regardless". (Imp)	Finding a perfect model to *predict* the heights of the people in the dataset (Ray, VSR).	Mental Image: picturing lengths mapped to make up person's height ("I kind of just looked at my own body and tried to figure out any patterns I could")– rejected shoe length as "a horizontal measurement, not a vertical one" (Ray, VSR & Imp); expecting "we could find something that was perfect" (Ray, VSR). Physiological Theory: Gender, ethnicity. age dismissed as "not a direct measurement, like something that would contribute to height, in my mind" (Ray, VSR). In some of the modeling, alignment between Mental Image, Physiological Theory & Mathematical Modeling.
White (Ned & Joe)	Modeling part of the dataset (36). Looking for a correlation between their height and measurements given. Looking for a pattern (Ned, Imp). Using four "imaginary people that have the averages of those" (Ned, Imp) gender x ethnicity. Hand sort and enter into Excel. Find *averages* for all "mini measurements" for shoe length, knee height, wrist to elbow, face height (including age) then *ratios* from height/mini measurement (not	Believes use of model is unrealistic method "'cause I don't like look at someone and then figure out how long or short their arms are. So I don't think anyone else would've." (Ned, FG)	*Prediction* – making a height "estimative tool" for PSOs "to be able to identify the height of people and suspects" (Ned, Report & Imp).	Physical theories: "If there was someone really young they could still have time to grow. But then I realized that each of the ages were up to 20 and that's usually when people stop growing." "And then the gender, I know that from science, that biologically women usually are smaller". Sister same height but shoe size "drastically different". Friends different

	age) producing *a table of multipliers*. "PSOs are now able to identify the criminals height by multiplying their measurement by the numbers provided categorized by their nationality and gender" (Ned, report).			races and ethnicity – different heights but trends "my friends who are more Asian based are usually smaller. Our shoe sizes don't differ that much." Age irrelevant as you "can't grow after the age of 26" (Ned, Imp). Alignment between Physical Theories and Mathematical Modeling.
Weight Lifting Task				
White (Ned & Joe)	Modeling part of dataset – ignore super heavy weight category "because it was so much more drastically different" (Ned, FG) and the effect it had was negligible. He did not think at the time to test the effect of this (Ned, FG). Ned anticipated there will be a *relationship* between "how much they weigh and how much they can lift" (Imp). He suggested converting the data into "a ratio so it is like for each kg they weight more than 55,…the weight they can lift increases by a set amount". By looking at *differences and averaging* by hand, he states: "For each 6kg increase in weight [above 55 kg] they weight [sic can lift] 9.25kg more" for male data (Ned, Imp). Wanting to continue a ratio approach by hand himself, he asked Joe to use Excel to produce *linear models*. "We put the data onto the computer, made a graph of the data and then we formed a trend line from the data" (Ned, Report). *Choice* of model *based on mathematical expediency as linear is considered easiest and quickest* (Ned, VSR). Ned uses his ratios as gradients in linear models which are very close to the computer generated versions. Uses computer models to generate missing world record lifts, compares predictions to actual lifts to select a World champion based on the lifters whose points were above the trendline being good performers, distinguishing between these by size of difference between actual & predicted lifts.	*Fully aware restricting to dataset given*. "I wasn't really having a model of humans doing weight lifting, I was just having a model, like based on the averages of numbers, to estimate this number [points to world standard needed]" (Ned, VSR). Both totally aware in post task interview they were not modeling the phenomenon. "Obviously, there is a limit to what humans can hold because if there wasn't, if any human can carry any weight then it wouldn't need robots"… "There probably was a limit and we probably should have taken that into account. ...quickest way to have a linear" (Ned, VSR). Reason super heavyweight class omitted trying to make a linear graph & using point would have "curved it more" and "messed up everything". Claims better to use linear model with small dataset (Ned, FG).	*Prediction* – "you need to make a model to *predict* this [pointing to World Standards needed] (Ned, Imp). "Filling the empty parts of the table" (Ned, Report). "I feel like I kind of narrowed myself down just by viewing it as the whole goal was to find these two [world standards] (Ned, VSR). Seeking "a trend between how much they weigh and how much they can lift" (Ned Report).	Physiological Theories: Humans are limited by their physiology in how much they can lift and it would not "go forever" so it was acceptable to leave out the upper end (Ned, FG). Females have different physiology so expect their relationship could be different from that for males (Both, Imp). Misalignment between Physiological Theories and their Mathematical Modelling.

Note. FG focus group; Imp lesson implementation; VSR Video stimulated recall; FH face height, KH knee height, Ht height; SL shoe length; WtE wrist to elbow length,.

Table 2b. Selected Rows from Partially Ordered Meta-Matrix: Modeling Approaches (Earnshaw College Data) to *PSO Task*

Team	Approach	Awareness	Purpose	Alignment Physical Theories/ Mental models/ Mathematical Modeling
Silver (Deja, Eve)	Modeling *full data set*. Write problem in form that can be mathematised: 'the PSOs are struggling to be able to estimate the height of people potentially envolving [sic] crimes]'. Use Knee height versus height and wrist to elbow versus height with *scatterplot and Lin Reg and record only a, b, r and* r^2 but seem unaware of how this could be used to predict height. Then did some *nonsensical calculations (height – shoe (max)) saying they are all means, min. and max* (Silver Imp, Report).	Appear to *think* it is an *artificial set of data* that they are modeling – "These people, they are real? Isn't Ken the Barbie doll?" "Who would name their kid Guy?" (Silver, Imp)	*Prediction* - "trying to find the height" (Eve, Imp). Deja struggles to understand purpose given height of dataset is known (Imp). (Mentions this throughout).	Physical Theory: "Like roughly on the age, kind of they are the same height because people are roughly around the same. It was like the same age, kind of like the same race and gender, were roughly the same height." (Imp). Flawed mental model of situation: "I did… maximum height take away the maximum shoe height to find the average" (Deja). "It gave you like a knee length average of people's height" (Deja). Misalignment of some of their modelling & mental model.
Pink (Iona, Jen, Kay)	Modeling part of dataset (40 – omit Ken). Need multiple rules as might be only able to get a specific estimate at a given time. No point just saying 'Shoe length is 'best'. Consider finding rules for a particular subset (e.g., males or Asian) to provide a more accurate rule if known, "if the regression was stronger when we only did boys then you could be more specific" (Kay, Imp). *Generate several linear models for several variables stating reliability using goodness of fit values*, e.g., height = 871.3707 + 1.6260894 *knee height. R = 0.7090863 "Oh, that is pretty strong" (Kay, Imp). Test one model on Alice & another on Bob. Report includes only values, although during modeling Kay notes "I just tested knee height [male specific model] on Bob and it works". Use r & r^2 to compare strength, report only list of 5 models with no discrimination to impractical degree of accuracy but stating in VSR would round in practice "because they are on the spot, on the job".	Iona and Kay say *realized* it was just a *specific set of data they were modeling* (VSR). In FG, Kay argues against rest of FG saying *solution to problem is realistic* & includes estimating from CCTV footage. In Imp realize they *need to test their models beyond data set* – on themselves if have time. "It will be interesting when we get our lines of best fit and test it on each other" (Kay). "Yes, see if it works" (Iona). "Yeah, see if it gives our heights" (Kay). "We tested it but the person we tested it on, was included in our making that model" (Kay, Imp). Should have found a model for girls rather than boys so testing on themselves was an option (Kay, FG).	*Prediction* -To generate a list of rules and statements about their reliability. "We used this [the dataset]. We figure out a formula to get to that height" (Iona, Imp). "The PSO lacks good height estimation skills. Utilizing their strengths of estimating short lengths, a series of reliable rules of thumb must be evaluated to provide a good height estimate" (Kay Report – similar in other reports).	"Different things (body parts) are proportionate to different. …I felt like they weren't all connected" - couldn't just add up randomly (Iona VSR). "Gender: you would expect all the boys to be on one side and all the girls on the other one. You can use all of them (i.e. variables). That's why they are all there." (Kay, Imp). "Depend on what smaller values they are able to get" (Kay, VSR). "It was kind of not to just have one that would like kind of work for all, to have …ways to work out a few of them" (Iona, VSR). "You can see the gender of someone … rule can be a bit more specific" (Kay, VSR). Mathematical Modelling aligned to mental model of situation.

Note. Imp - Audio and video recorded lesson implementation, VSR - Video post lesson video stimulated recall interview, FG - focus group interview post lesson.

Table 3. Coding of Purposes of Use of Technology.

Digital Technology Use	Modeling Purposes of Use Code[a]	Modeling Approaches of technology use code[b]	Teams[c]
Home screen calculations on basic function calculator on laptop/CAS calculator	Making choices about features considered significant during pre-mathematization facilitated by digital technology use	N/C	PSO2019 Green(HC); PSO2019 Blue(HC); PSO2019 Orange(HC); PSO2019 Red(HC); PSO2019 Mauve(EC); WLT2019 White(HC); WLT2019 Green(HC)
	Using technology to verify mathematical model output	UUT	PSO2019 Red(HC); PSO2019 Mauve(EC); PSO2019 Purple(EC) use solve to compare predicted with actual height; PSO2019 Pink (EC); **WLT 2019 Red(HC)**; WLT2019 White (HC)
	Using digital technology to Work Mathematically with model to carry out simple calculations such as summation, finding averages and ratios	N/C	PSO2019 Blue (HC); PSO2019 Orange (HC)(+/x only); PSO2019 Red(HC)**;** PSO2019 White(HC); PSO2019 Teal(EC); **PSO Olive(EC)**; WLT2019 Blue(HC); WLT2019 Red(HC) [not Evan]; WLT2019 White(HC)
Using Excel Spreadsheet with Function formulae such as AVG	Using digital technology in building models – constructing the formula (e.g. Ht = SL x AvHt/AvSL) or ratio	SGT	PSO2019 Green(HC); PSO2019 Red(HC); PSO2019 White(HC); PSO2019 Purple(EC); PSO2019 Pink(EC)
Use of scatterplot	To identify error [interchanging variables]	N/C	PSO2019 Pink(EC)
	To identify or get a sense of relationship	SGT	WLT2019 Green(HC); **WLT Red(HC)**
	To confirm expected ‘linear’ trend [visually] - ‘looks pretty much like a line’ holding a pencil up to CAS	SGT	WLT2019 Blue(HC).
	Compared graph of model with data points, determine best has ‘more dots closer to line’	SGT	WLT2019 Green(HC); WLT2019 Orange(HC)
Use of Scatterplot and Lin Reg to generate line of best fit	Using digital technology in building models – constructing the formula	SGT	PSO2019 Grey(EC); PSO2019 Mauve (EC); PSO2019 Gold(EC); PSO2019 Silver (EC) [model not recorded]; PSO2019 Purple (EC); PSO2019 Pink(EC); WTL2019 Red(HC)[not Evan]; WLT2019 White(HC); WLT2019 Green(HC); WLT2019 Orange(HC); **PSO2019 Olive(HC)** abandoned for more sense making model grounded in reality

Generation of GoF Values	Recorded/ generated but not used for model selection or comparison	PFG	PSO2019 Grey(EC); PSO2019 Gold(EC) as model with smaller r value (KH 0.71 cf 0.77 for WtE) gave closer predictions; PSO2019 Pink(EC); PSO2019 Teal(EC); **PSO Olive(EC)**
	Used to select the "best" model	FFS	PSO2019 Mauve(EC); PSO2019 Purple(EC)
Sorting data with spreadsheet initially	Sifting through data of a modelling situation facilitated by technology as part of pre-mathematization	N/C	PSO2019 Green(HC)
Sorting data with spreadsheet after first models produced	Using sorted data to refine models produced for the entire dataset for subpopulations of datasets (e.g., based on gender/ethnicity)	SGT	PSO2019 Red(HC)
Using technological output (e.g. spreadsheet, calculator calculations, graphical displays)	Using technological output to facilitate group work by supporting argumentation between members of group	SGT	PSO2019 Red(HC); **PSO Olive(EC)**; WLT2019 Blue(HC); **WLT2019 Red(HC);** WLT2019 Green(HC)
Tracing along the trend line	Using technological output to facilitate group work by supporting argumentation between members of group – "Just explaining it (Lyn, VSR) During implementation – traced slope "as your height gets bigger, the knee height …	SGT	WTL2019 White(HC); WLT2019 Blue(HC); PSO2019 Mauve (EC)
Using moveable line to find by-eye location of best linear model	Using digital technology in building models - comparing fit of model by eye	SGT	**WLT2019 Red(HC)[Evan]**
Using Excel spreadsheet to store and display manually sorted data	Using digital technology in building models – ratio	UUT	PSO2019 White(HC)
Use of technology enabling use of larger data set	Having access to technology allowed choice of size of sample to investigate (not limited by more time consuming by-hand methods)	N/C	PSO2019 Gold(EC); PSO2019 Silver(EC); **PSO2019 Olive(EC)**
Using internet to find procedural descriptions of how to carry out a technological technique (e.g., sorting, using a moveable line, how to form an equation)		N/C	PSO2019 Green(HC); WLT2019 Green(HC)

Note. AVG Average command; EC Earnshaw College; HC Heathcliff College; FFT fitted Function Selector; GoF Goodness of Fit; Ht height; N/C no code; SGT Scatterplot/Graphing Tool; SL shoe length; UUT Unneeded/Unused Tool; VSR Video post lesson video stimulated recall interview.
[a]Initial codes from Villareal et al. (2018, p. 333); [b]Codes from Zbiek (1998, p, 198); [c]Use of bold indicating teams displaying modeling the phenomenon as approach

Table 4a. Partially Ordered Meta-Matrix: Modeling Processes (Heathcliff College Data).

Team	Approach	Cognitive activities in Transition 1	2	3	4	5	6	7
PSO Task								
Green (Una & Tegan)	MD	U, St, S, IntC	IdV, F, M, J	WM, Ver	IntM	Cp, Cq	Com, J (prec.)	RV (same model form but other Var)
Blue (Eric & Ray)	MD	U, St, IntC	F, M	WM	IntM	Cp, Cq, Val	Com, J (prec. – max error)	RV (other models & to refine proportional combination model)
Orange (May, Clare, Ruby)	MD	U, St, IntC	F, M	WM		Cp	Com	
Red (Tom, Pat)	MD	U, St, IntC	F, M	WM	IntM	Cp, Cq	Com, J (consistency)	RV to refine models accounting for ethnicity
White (Ned & Joe)	MD	U, St, IntC, N, S	A, IdV, F, M, J	WM, Ver	IntM		Com	
Brown (Evan & Zald)	MD	U, Int C	F, M	WM, Ver	Int M	Cp, Cq, Val	Com, J (prec.)	RV (same model form but other Var)
Weight Lifting Task								
Green(Una & Tegan)		U, St, IntC	F, M	WM	IntM	Cp, Cq	Com, J (fit)	RV(other models linear&logarithmic)
Blue (Eric & Ray)	MD	U, St, IntC	F, M	WM	IntM	Cp, Cq	Com	
Orange (May, Clare, Ruby)	MD	U	F, M	WM		Cp	Com	
Red (Evan, Tom, Pat)	MD/MP($_{\text{Evan}}$)	U, St, IntC	F, M	WM	IntM	Cp, Cq	Com	RV (same model form but other Var) Evan does not RV
White (Ned & Joe)	MD	U	F, M	WM	IntM	Cp	Com	

Note. A assuming; Com communicating; Cp comparing: Cq critiquing; F formulating; IdV identifying variables; IntC interpreting context; IntM interpreting mathematical output; J justifying; M mathematizing; MD modelling dataset or part thereof; MP modelling phenomenon; N noticing; RV revisiting; S simplifying; St structuring; U understanding; Val validating; Var Variable(s); Ver verifying; WM working mathematically; prec. precision.

Table 4b. Partially Ordered Meta-Matrix: Modeling Processes (Earnshaw College Data).

		Cognitive activities in Transition						
Team	Approach	1	2	3	4	5	6	7
PSO Task								
Gold (Ann, Bria, Carol)	MD	U, St,	F, M	WM	IntM	Cp, Cq	Com	
Silver (Deja, Eve)	MD	U, St					Com	
Purple (Fay, Gen, Honi)	MD	U, St, Int C	F, M	WM	IntM	Cp, Cq	Com	
Pink (Iona, Jen, Kay)	MD	U	A, F, M	WM		Cp	Com	
Mauve (Lyn, Macy, Leita)	MD	U, Int C	F, M (ratio model)	WM	IntM	Cp, Cq	Com	RV (look at refining to Caucasian Female model – reject as inaccurate and low r; test several variables)
Grey (Noni, Ona, Polly)	MD	U, Int C	F, M	WM	IntM	Cp, Cq	Com, J (prec.)	RV (reject an incorrect model, same model -using other variables)
Teal (Quin, Rosa)	MD	U, Int C	A	WM		Cp, Cq	Com	
Olive (Sui, Tara, Veta)	MP	U, St, S,	F, M	WM	IntM	Cp, D	Com	RV (other models)

Note. A assuming; Com communicating; Cp comparing; Cq critiquing; D deciding; F formulating; IntC interpreting context; IntM interpreting mathematical output; J justifying; M mathematizing; MD modelling dataset or part thereof; MP modelling phenomenon; RV revisiting; S simplifying; St structuring; U understanding; WM working mathematically; prec. precision.

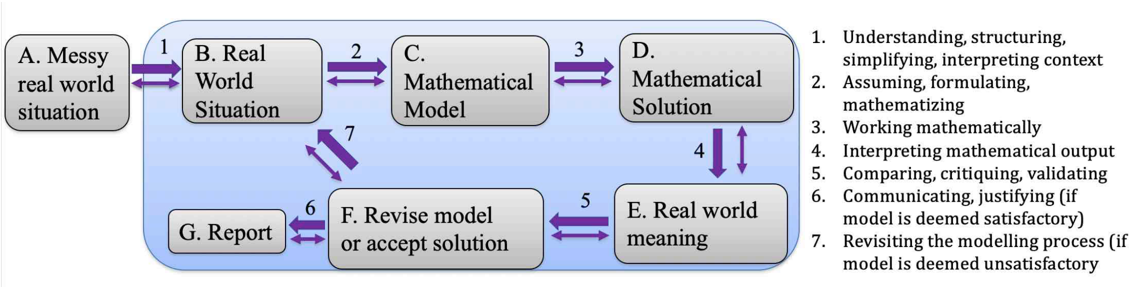

Figure 1. Representation of the mathematical modeling process (Stillman, 2011).

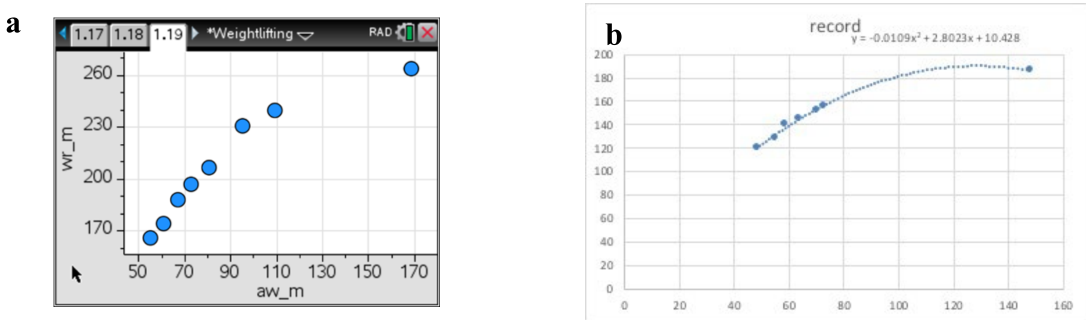

Figure 2. **a** Eric's plot of weight lifted vs actual weight of male record holders. **b** Ray's quadratic model for weight lifted vs actual weight of females.

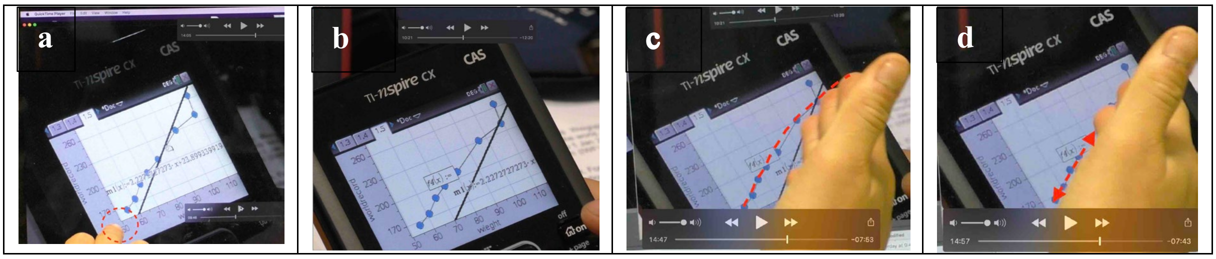

Figure 3. a & **b** Evan fitting a "best matching line" passing through (0, 0). (Note: Last data point is incorrectly entered.) **c & d** acknowledging top weight categories do not fit his linear model produced with the lower weight categories.

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