The professional learning leadership of school mathematics leaders after participation in a large-scale primary school mathematics improvement project : An activity theory perspective

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THE PROFESSIONAL LEARNING LEADERSHIP OF SCHOOL MATHEMATICS LEADERS AFTER PARTICIPATION IN A LARGE-SCALE PRIMARY SCHOOL MATHEMATICS IMPROVEMENT PROJECT: AN ACTIVITY THEORY PERSPECTIVE

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DECLARATION OF AUTHORSHIP AND SOURCES

This thesis contains no material that has been extracted in whole or in part from a thesis that I have submitted towards the award of any other degree or diploma in any other tertiary institution.

No other person’s work was used without due acknowledgment in the main text of the thesis.

All research procedures reported in the thesis received the approval of the relevant Ethics/Safety Committees (where required).

Written permission has been granted to use the actual names of people referenced in the pages of this thesis.

Matthew (Matt) Ross Sexton

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2023

ABSTRACT

THE PROFESSIONAL LEARNING LEADERSHIP OF SCHOOL MATHEMATICS LEADERS AFTER PARTICIPATION IN A LARGE-SCALE PRIMARY SCHOOL MATHEMATICS IMPROVEMENT PROJECT: AN ACTIVITY THEORY PERSPECTIVE

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This thesis examines how mathematics leaders, working as middle leaders in three separate primary schools in Melbourne, Australia, contributed to project sustainability in the years that followed participation in a large-scale school mathematics professional development project. Informed by a cultural-historical activity theory (CHAT) perspective, the thesis presents how those mathematics leaders contributed to the sustainability of project-initiated reforms through their post-project professional learning leadership activity.

Project sustainability is a complex issue in school settings, often proving to be challenging activity for practitioners and researchers alike. One important element of enduring school improvement requires a focus on sustaining change, yet that proves to be the significant challenge. A reason for that challenge is the lack of knowledge, derived from research studies, that examines the sustainability of projects beyond the time of participation in them. Another added layer of complexity is that middle leadership, like that enacted by the mathematics leaders, remains an under-researched area of educational leadership. Even fewer studies have drawn attention to how mathematics leaders contribute to project sustainability.

This thesis asked how the mathematics leaders contributed to project sustainability as middle leaders of professional learning in their school sites. Acknowledging that middle leadership has been theorised as a form of practice, CHAT was used as the approach to understand the collective facets of the mathematics leaders' activity. A focus was taken on how their object-oriented activity developed and transformed over time. CHAT concepts associated with the activity system were used as analytical tools to understand how the mathematics leaders' professional learning leadership activity progressed in response to contradictions that surfaced within their activity.

Drawing on the work of CHAT researchers, a research process was specifically designed as a series of actions intended to realise the objective of the thesis. Those actions included conducting CHAT workshops with the mathematics leaders, clarifying the historicity of their activity system, producing and reporting findings of the historical activity system as a member-checking strategy, conducting a detailed inquiry into post-project activity system, and producing and reporting findings of the post-project activity system for member-checking.

As a means of generating data to respond to the research question, the data methods of interviews, observations, and document retrieval were employed. Data were analysed using deductive and inductive analysis. Concepts from the CHAT and literature were used to create coding schemes to support those data analysis approaches.

The findings of this thesis are presented through three discussion chapters that offer a temporal sequence of activity experienced by the mathematics leaders from the time of their leadership activity during project participation to the years following participation. The first of the findings chapters discusses the historical trajectory of the mathematics leaders' professional learning leadership, beginning with the descriptions of the historically accumulated contradiction that gave rise to the decision for the mathematics leaders' schools to participate in the project. There is an explanatory focus on the motive objects of activity pursued by the mathematics leaders during project participation. The claim is presented that the mathematics leaders shifted the direction of their leadership from managerial motive objects to ones that focused more on leadership as project participation progressed.

The next chapter discusses findings that reveal that at completion of project participation, the mathematics leaders were relieved that their principals established commitment rules intended to mediate project sustainability. The historical contradiction, which had faded in prominence during the project, resurfaced and manifested as a critical conflict realised as several problems of practice. That gave rise to several post-project problems of practice that realised struggle for the mathematics leaders. That struggle was compounded by feelings of responsibility for project sustainability in their schools. The claim is made that mathematics leaders responded to their struggle and feelings of responsibility through care and creativity which initiated enactment of their form of resourceful practice.

The final discussion chapter presents further evidence that the mathematics leaders enacted their resourceful practice that became their contribution to project sustainability. Through their attempts to resolve the post-project practice problems, the mathematics leaders reconfigured the motive objects of their activity, seeing them privilege relational trust building for and about mathematics teaching. Through that motive object reconfiguration, the mathematics leaders’ activity was multi-motivational activity and realised through new leadership actions. Evidence of a newly surfaced contradiction is reported, and its existence is explained. The claim is presented that that contradiction surfaced due to the relational motive object that the mathematics leaders privileged, revealing the enabling yet constraining potential of the relational dimension of their professional learning leadership activity.

This thesis contributes to knowledge about project sustainability, claiming that as middle leaders in their schools enacting their resourceful practice, the mathematics leaders acted as agents of project sustainability. The findings add further information of mathematics leaders as middle leaders who play a crucial role within the school leadership factor of project sustainability. The implication of the thesis is that professional development designers and facilitators must attend to the factors of project sustainability and pay attention to the vitality of relationality that penetrates the motive objects of activity enacted by mathematics leaders through their professional learning leadership.

DEDICATION

I dedicate this thesis to my mother and father, who instilled in me a love of learning for as long as I can remember.

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I have many to thank for the support offered to me during my engagement in the academic work presented in this thesis. I express my heartfelt appreciation for the dedication, interest, and care demonstrated by my supervisors, colleagues, family, and friends.

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I am incredibly grateful for the research mentoring from Professor Joce Nuttall and Professor Suzy Edwards. They significantly influenced the conceptualisation of the story of the mathematics leaders I share in my thesis. I appreciate their patience and guidance during the final stages of my PhD journey. I aspire to think and work like Joce and Suzy.

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PEER-REVIEWED PUBLICATIONS AND CONFERENCE PRESENTATIONS

Peer-Reviewed Publications Related to Thesis Work

Sexton, M., & Nuttall, J. (2021). Leadership of strengths-based approaches for early years mathematics education: Using CHAT as a framework for educational leaders professional learning leadership. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.), *Excellence in mathematics education: Foundations and pathways* (Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia, pp. 92-95). MERGA.

Sexton, M. (2019). Object-motives of mathematics leaders' professional learning leadership during participation in a mathematics project. In G. Hine, S. Blackley, & A. Cooke (Eds.), *Mathematics education research: Impacting practice* (Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia, pp. 466-473). MERGA.

Sexton, M., & Lamb, J. (2017). Using activity theory to understand a mathematics leaders' motivations and use of mathematical knowledge for teaching. In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning!* (Proceedings of the 40th annual conference of the Mathematics Education Research Group of Australasia, pp. 466-473). MERGA.

Sexton, M., & Downton, A. (2014a). School mathematics leaders' perceptions of successes and challenges of their leadership role within a mathematics improvement project. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia, pp. 581-588). MERGA.

Sexton, M., & Downton, A. (2014b). School mathematics leaders' beliefs about their role when participating in a school mathematics project. *Australian Primary Mathematics Classroom, 19*(3), 3-5.

Research Conference and Keynote Speaker Presentations Related to Thesis Work

2021 MERGA43: Mathematics Education Research Group of Australasia 43rd Annual Conference (Virtual): Research Symposium Presentation

2019 MERGA42: Mathematics Education Research Group of Australasia 42nd Annual Conference (Perth, Australia): Research Conference Presentation

2017 MERGA40: Mathematics Education Research Group of Australasia 40th Annual Conference (Melbourne, Australia): Research Conference Presentation

2016 MAV53: Mathematical Association of Victoria 53rd Annual Conference (Melbourne, Australia): Keynote Speaker Presentation

2014 MERGA37: Mathematics Education Research Group of Australasia 37th Annual Conference (Sydney, Australia): Research Conference Presentation

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LIST OF ABBREVIATIONS

ACU	Australian Catholic University
BSA	Between-Session-Activity
CEOM	Catholic Education Office Melbourne
CHAT	Cultural-Historical Activity Theory
CTLM	Contemporary Teaching and Learning of Mathematics
ESM	Eight-Step Model
ET	Emergency Teacher (casual relief teacher)
MCK	Mathematical Content Knowledge
MLTC	Mathematics: Learning That Counts (initiative)
NAPLAN	National Assessment Program – Literacy and Numeracy
PCK	Pedagogical Content Knowledge
PD	Professional Development
PLT	Professional Learning Team
SAM	School Advisor Mathematics
SIM	School Improvement Mathematics
SML	School Mathematics Leader

CHAPTER 1: THE INTRODUCTION

1.1 Introduction to the Thesis

My thesis examines how three mathematics leaders, working as middle leaders in primary schools in Melbourne, Australia, contributed to project sustainability in the years that followed participation in a large-scale school mathematics professional development project. Informed by a cultural-historical activity theory (CHAT) perspective, I describe how those mathematics leaders contributed to the sustainability of project-initiated reforms through their post-project professional learning leadership activity.

The major claim of my thesis is that the mathematics leaders, as middle leaders in their school sites, enacted a form of resourceful practice as their contribution to project sustainability. Within this thesis, I present evidence of that resourceful practice, and how the mathematics leaders played an essential role in contributing to project sustainability.

1.1.1 Writing in First Person

I positioned this thesis as the product of my research reasoning. Acknowledging that this is a presentation of that reasoning, I chose to write in first person. I made that decision recognising that my academic work was an act of forming my personhood (Stetsenko, 2012) as a mathematics education researcher. Choosing to write in first person was an essential means of enacting my personhood and expressing the claims contained within the pages of my thesis (Hyland, 2002). This thesis is the realisation of my research reasoning, which became far more academic as I engaged with the rigours of doctoral study. In the final chapter, I revisit that personhood formation as a simultaneous process of development of my research activity and the development of myself as a person entering the mathematics education research community.

1.2 Overview of Chapter 1

I start this chapter with the genesis of my thesis and how I became interested in mathematics leadership. I also share my interest in CHAT as a methodological approach to research design. Recognising the importance of culture and history in CHAT (Roth, 2012), I provide the historical and cultural context of the study, paying attention to the large-scale school mathematics professional development project in which the three mathematics leaders participated.

After that background information, I discuss the problematic I chose to investigate, and I clarify the unit of analysis. I present the research purpose and the associated aims of my study. The research question that guided my research activity is then posed. I continue the chapter by stating the significance of my study, and the chapter concludes with articulation of the structure of my thesis.

1.3 Genesis of the Thesis

The impetus for my study stems from my professional history as a primary school teacher, numeracy coordinator, and mathematics educator. At the beginning of the fifth year of my teaching career in 2001, my principal offered me the role of numeracy coordinator at the school at which I worked. I enacted that numeracy coordinator role from 2001 to 2007 whilst undertaking classroom teaching responsibilities. Theorising that historical aspect of my career, I was a middle leader within my school's leadership system (Grootenboer, 2018). As a middle leader of mathematics, I had opportunities to design and facilitate professional learning opportunities for colleagues.

To support my middle leadership work, I started a Master of Education (MEd) degree in 2003 at Australian Catholic University (ACU), specialising in mathematics education. During my MEd program, I studied the leadership of mathematics curriculum in primary schools. That initiated my academic interest in mathematics leadership.

In 2007, I completed that MEd degree, and at the end of that year, I successfully garnered the role of School Advisor Mathematics (SAM) at the Catholic Education Office, Melbourne (CEOM). In 2008 and 2009, my work in mathematics curriculum leadership continued as I supported mathematics leaders and executive leadership teams (principals, deputy principals, and other curriculum leaders, e.g., literacy leaders) in primary schools participating in the *Contemporary Teaching and Learning of Mathematics* (CTLM) project. CTLM, which I discuss later in this chapter, was a joint initiative between CEOM and the mathematics education team at ACU (Melbourne Campus). The CTLM project started in February 2008 and concluded in December 2012.

As a SAM, my interest in mathematics leadership developed due to my work with mathematics leaders, where I was given insights into the successes and complexities associated with that leadership role. Through my work as a SAM in the CTLM project, I developed professional relationships with the ACU mathematics education team, specifically Professor Doug Clarke.

Towards the end of 2009, Professor Clarke approached me with an offer to join the ACU staff as a mathematics educator. I accepted that role, and in January 2010, I started my tenured position at ACU. Along with university teaching duties, I continued to work in the CTLM project but as an ACU staff member. My role changed to that of a professional learning provider, where I presented workshops for teachers and leaders and taught demonstration lessons in CTLM schools. During my first two years of service at ACU, the informal research reasoning for my thesis began. I recall informing colleagues that I wanted to produce a thesis informed by a robust theoretical framework.

In 2011, I attended the 34th Mathematics Education Research Group of Australasia (MERGA) conference in Alice Springs, Northern Territory. I was introduced to CHAT at that conference by Associate Professor Mary Coupland (University of Technology Sydney), when she shared her understanding of how CHAT can used in mathematics education research. I was driven to know more about how that theory could be used to frame research designs.

In 2012, Professor Joce Nuttall (who became my doctoral supervisor) joined ACU, and my interest in CHAT was extended. Through discussions and research meetings with Professor Nuttall, I adopted a CHAT perspective with my work as a mathematics educator. CHAT started to influence my worldview, and when CTLM was ending (December 2012), I decided to focus my study on mathematics leadership using a CHAT lens.

This information about the genesis of this thesis is essential. My professional history motivated my desire to investigate the research topic. My historical experiences have shaped my personhood as a mathematics education researcher, with the pinnacle thus far being the production of my thesis. Maxwell (1998) stated that the researcher's history and experiences are important to research design, so, therefore, I explain in Chapter 4 (Research Design) how my professional history affected my positionality as a researcher (Greene, 2014).

I now turn to an explanation of the context of my study.

1.4 Context of the Study

In this section, I present the historical and cultural context of the study for which I was granted permission to conduct by the Human Research Ethics Committee (HREC) at ACU (Appendix A) and the Policy and Research Committee at CEOM (Appendix B) in 2014. History plays an essential role in CHAT, so when using it as a research framework, it is important to acknowledge the historical aspects of the activity under investigation (Roth, 2012).

The activity I investigated in this study concerned how mathematics leaders, as middle leaders within their school's leadership system, contributed to project sustainability through their post-project professional learning leadership. Consequently, as a means of honouring CHAT's methodological implications, I present historical information about the context of my study.

1.4.1 Location and History of the Study Sites

This study was situated in Melbourne, Australia. It took place in three primary schools that were once involved in the CEOM mathematics initiative, *Mathematics: Learning That Counts* (MLTC). The three mathematics leaders and the staff in their schools (principal, executive leadership team members, and classroom teachers) participated in the *Contemporary Teaching and Learning of Mathematics* (CTLM) project in 2011 and 2012. CTLM was just one opportunity CEOM offered Catholic primary schools to engage in their MLTC initiative. However, the historical context of this study goes back to 2007, when CEOM launched their plan to focus on improving mathematics education in its Archdiocesan schools.

1.4.2 Mathematics: Learning That Counts Initiative

In 2007, CEOM released its five-year plan to improve mathematics education in its primary and secondary schools. This plan was realised through the MLTC initiative that reportedly cost $22.5 million from 2007 to 2012 inclusive ("Maths First for Melb Catholic Schools", 2007).

The MLTC initiative focused on four strategic directions: Educational Leadership; Quality Teaching; Continuous School Improvement and Accountability; and Differentiated Support (Catholic Education Melbourne, n.d.). Each direction was intended to support Catholic schools in improving the mathematics learning outcomes for all students attending schools within the Archdiocese. There was one major goal associated with each direction.

Regarding the Educational Leadership strategic direction, on which my thesis is focused, the goal was to strengthen the leadership capacity at all levels within schools so that leaders in the Archdiocesan schools could initiate and manage continuous improvement of mathematics education beyond the life of the MLTC initiative.

A critical aspect of the MLTC initiative included CEOM forming partnerships with universities and mathematics education academics. The partnering universities were Monash University, Melbourne University, and ACU. Along with university partnerships, CEOM partnered with the national mathematics teacher organisation, the Australian Association of Mathematics Teachers (Catholic Education Melbourne, n.d.).

CEOM paid tenures to partner universities, and the mathematics educators in those universities were expected to design, in consultation with CEOM, projects that met the strategic directions of their mathematics initiative. ACU developed its response to the partnership through the CTLM project under the direction of Professor Doug Clarke. CLTM was specifically designed for classroom teachers and school leaders in primary schools.

Once endorsement was mandated by CEOM, school leadership teams were required to engage in the *School Improvement Mathematics* (SIM) program. SIM acted as an onboarding mechanism to prepare for professional development offered through participation in the CTLM project. Any school interested in participating in CTLM must have identified mathematics as an area of improvement through school review processes.

1.4.3 School Improvement Mathematics Program

The SIM program was co-designed and co-facilitated by CEOM and ACU staff members. Involvement in the program allowed school leadership teams to reflect on their commitment to focus professional development and associated resources (human, physical, and monetary) on mathematics education in their school for at least three years. The first year involved participation in the SIM program, and the following two years involved participation in CTLM. After the SIM program, school leaders decided whether to participate in the CTLM project.

During SIM, the executive leaders (principal, deputy principal, Learning and Teaching Leader) were required to engage in self-evaluation exercises and reflect on ways that mathematics education could be improved in their school communities. An element that did not feature as part of that SIM program was the analysis of tensions in mathematics education. Opportunities to identify reasons for tensions and plan leadership actions to address them were missing from the program (J. Walker, personal communication, October 31, 2014). Instead, school leaders only considered their reflections and used them to create a *Mathematics Improvement Plan* (MIP), which focused on articulating goals, strategies, and evidence of improvement success. Aspects of the MIP included parent and school community links, mathematics teaching and learning, and mathematics leadership.

SIM participation was dependent on participation in CTLM. An aspect of SIM participation required principals to create a new leadership role in their school's leadership system. This leadership role was given the title of *School Mathematics Leader*, a change in role name from *numeracy coordinator*, which was commonly used in Archdiocesan schools at that time (Clarke et al., 2005).

If schools participated in CTLM, conditions to support the School Mathematics Leader were expected to be established. Conditions included providing at least six hours of time allocation (equivalent to one day) for the School Mathematics Leader role, preparing budgets for purchasing mathematics education resources, allocating time release for teachers to work with the School Mathematics Leader, and providing extra time allocation for the School Mathematics Leader to meet with CEOM and ACU staff members when school visits took place.

1.4.4 Contemporary Teaching and Learning of Mathematics Project

The CTLM project was led by Professor Doug Clarke (ACU) as CTLM Director, with Dr Ann Downton as the CTLM Coordinator. Mr Gerard Lewis (CEOM) was the CTLM Lead Project Manager. The CTLM project was the first for CEOM due to its large scale and the joint involvement of CEOM staff and ACU mathematics educators. The ACU staff designed and facilitated professional learning opportunities for classroom teachers, whilst CEOM provided funding for the project. Although some research findings were published (e.g., Clarke et al., 2013a; McDonough et al., 2010; McDonough & Sexton, 2011), CTLM was primarily a mathematics education professional development project.

A total of four cohorts (which were called *intakes*) of schools participated in CTLM from 2008 to 2012. The first two intakes of schools included 11 schools (Intake 1) and 21 schools

(Intake 2), respectively. The third intake included 14 schools; the final intake, Intake 4, involved 24 schools. At the end of 2012, close to one-quarter (~70 schools) of Archdiocesan primary schools had participated in CTLM. Any school participating became known as a *CTLM School* within the Melbourne Archdiocese.

1.4.4.1 CTLM Aims and Structure

Several aims guided the CTLM project. The primary goal sought to improve classroom teachers' mathematical content knowledge (MCK) and pedagogical content knowledge (PCK). A secondary goal was focused on the improvement of students' mathematics learning. Some examples of research findings associated with these goals were published by ACU staff: improvement of student learning outcomes (Wilkie & Clarke, 2014); enactment of teachers' reflective practice (McDonough et al., 2010); and measurement of teachers' PCK (Roche & Clarke, 2011). Another aim of CTLM was to strengthen leadership practice in CTLM schools, especially that of the School Mathematics Leader.

The CTLM project was characterised by professional learning sessions facilitated mainly by ACU mathematics educators. Within CTLM schools, those sessions were often called *CTLM days*, which took place away from CTLM schools, in venues close to the ACU Melbourne Campus (1.5 km from Melbourne's CBD). Each year, there were five to six CTLM days for teachers and school leaders. The first year was devoted to the MCK and PCK associated with *Number*, *Algebra,* and *Working Mathematically*. In the second year, the MCK and PCK focus shifted to *Measurement*, *Geometry*, *Probability*, and *Statistics*.

All teachers were required to participate in the CTLM days, with the principal and other school leaders (e.g., deputy principal, literacy leader) expected to participate. After each CTLM day, the teachers were given tasks to complete in their classrooms. Those tasks, called *Between-Session-Activities* (BSAs), usually included using a mathematics task or completion of a professional reading activity. School Mathematics Leaders were asked to engage teachers in work associated with the BSAs back in their schools.

Further opportunities for professional learning occurred through 'on-site' modes within each CTLM school. Those modes included facilitating demonstration mathematics lessons by ACU mathematics educators (Clarke et al., 2013a). The CEOM staff members who visited schools and provided on-site professional learning opportunities, primarily for the School Mathematics Leaders, were the *School Advisors Mathematics* (SAMs).

1.4.4.2 School Advisors Mathematics

CEOM decided that schools needed further assistance if all strategic directions of the MLTC initiative were to be met. This assistance was realised by establishing a new role within CEOM's staff structure. In 2007, CEOM hired suitably qualified numeracy coordinators who worked in their Archdiocesan schools to undertake the role of School Advisor Mathematics (SAM).

Eight SAMs were initially hired, with two SAMs working in each of the four regions of the Melbourne Archdiocese. The SAMs worked only in CTLM schools from 2008 to 2012 inclusive. During CTLM, more SAMs were employed to support the growing number of schools that agreed to participate in the project. The SAMs visited each CTLM school approximately twice each school term (~ eight school visits per year). Each visit was scheduled for the length of the school day (J. Walker, personal communication, October 31, 2014). The primary role of the SAMs was to provide in-school support and to mentor the School Mathematics Leaders.

The SAMs' support was focused on developing the School Mathematics Leaders' capacity to implement the CTLM-advocated mathematics teaching reforms. The SAMs provided feedback and advice concerning the developmental work to achieve the goals within the school's MIP. SAMs also mentored School Mathematics Leaders about designing and facilitating site-based professional learning for teachers.

1.4.5 School Mathematics Leaders

At least 70 Catholic primary school staff members undertook the role of School Mathematics Leader during the lifespan of CTLM (2008-2012 inclusive). In most cases, one staff member from each school enacted the role. In rare situations, two staff members undertook the School Mathematics Leadership role, often separating the role into Prep to Grade 2 and Grade 3 to 6 ('Prep' being the name given to the first year of school in Victoria). This separation was due to how the CTLM project was structured, with professional learning sessions usually divided into 'Prep to 2' and 'Grade 3 to 6' sessions.

Data I generated and analysed, then reported to CEOM (Clarke et al., 2013b), showed that most School Mathematics Leaders in Intake 4 also had classroom teaching responsibilities. This meant they were middle leaders within their school's leadership system as they enacted leading and teaching responsibilities in their schools (Grootenboer, 2018).

1.4.5.1 School Mathematics Leader Responsibilities During CTLM

Through consultation with ACU, CEOM approved a role description (ACU & CEOM, 2011) for the School Mathematics Leader (Appendix C). This document was created as a means of articulating the purpose and responsibilities of the role. The primary aim was to facilitate the maximum benefit the school could derive from CTLM participation. The role description was developed around three main sections: *leadership*, *organisation and management*, and *consultation with others/liaison*. Each section listed specific actions for the School Mathematics Leader to enact their mathematics leadership role.

Actions articulated in the leadership section included the: promotion of a school culture that valued mathematics learning for students and teachers; the formation of a mathematics team to support the School Mathematics Leader; and the provision of support for teachers as they embedded practices, ideas and principles highlighted within CTLM.

Recognising that the School Mathematics Leader role included organisation and management work, the role description also stated that leaders were required to purchase, organise and conduct audits of mathematics equipment and resources for classroom teacher use; facilitate and monitor the implementation of the Mathematics Improvement Plan; and ensure that the daily one-hour mathematics lesson was taught across the school.

The final section of the role description was focused on actions related to consultation with stakeholders, including executive leadership team members, teaching staff, ACU and CEOM personnel, and parents/caregivers. Some of the descriptors included in this section included: informing the school community of developments in mathematics education, regular correspondence with ACU and CEOM staff members, and communication with school staff members.

1.4.5.2 Focus on Professional Learning Leadership

The leadership section focused on designing and facilitating professional learning for teachers. Six of the nine descriptors in that section about leadership related to the School Mathematics Leader engaging classroom teachers in site-based professional learning.

Professional learning leadership actions articulated in the role description included: conducting regular (fortnightly) mathematics professional learning team (PLT) meetings that focused on the implementation of mathematics teaching reforms (e.g., BSA completion, use of teaching strategies in classrooms, analysing student assessment data, engagement in professional reading); supporting teachers by mentoring and modelling teaching approaches aligned with advice in the CTLM professional learning sessions; and, building the capacity of staff to become reflective practitioners and effective teachers of mathematics. Those descriptors were quite broad, and the SAMs supported School Mathematics Leaders in interpreting how to enact those leadership actions.

Along with a change in title and status of the role, School Mathematics Leaders were expected to focus their leadership work on facilitating professional learning for teachers. For many leaders, that focus on professional learning leadership was a new requirement, not previously highlighted with the numeracy coordinator role in the Archdiocesan schools (J. Walker, personal communication, October 31, 2014).

1.4.6 Expectations Beyond CTLM Participation

Once participation in the project ceased, CEOM expected that the CTLM schools would continue with the project-initiated reforms. It was also expected that the School Mathematics Leader would continue to lead mathematics professional learning in their schools. CEOM did not mandate school requirements, nor were there any directives to abide by those expectations. When CTLM finished, the SAMs continued to work for CEOM; however, they were not permitted to support schools that had previously participated in CTLM (J. Walker, personal communication, October 31, 2014).

With CEOM only setting expectations and not requirements, schools were left with sustaining the project-initiated reforms following CTLM participation. The impact of that meant leadership teams were required to make decisions, create conditions, and allocate resources (human, physical, temporal, and monetary) to facilitate project sustainability in their schools. It is this situation that provides the problematic that I chose to investigate in my study.

I now turn to the articulation of that problematic, realised as the research problem.

1.5 Research Problem

A research problem within an educational setting is a general education issue or concern that narrows a research topic. This research topic holds the research problem investigated through a study (Creswell, 2012). My thesis topic is the middle leadership of mathematics as enacted by mathematics leaders in primary schools.

The research problem of my study concerns a lack of theoretical knowledge about how mathematics leaders, as middle leaders in their schools, contribute to project sustainability through their professional learning leadership activity in the years following participation in a large-scale mathematics professional development project.

1.6 Unit of Analysis

A unit of analysis is the aspect of the phenomenon studied during the research period (Gall et al., 2007). When using CHAT, the basic unit of analysis is the activity itself (Engeström, 2001, 2015; Nardi, 1996). Roth (2012) stated that activity is the minimal unit of analysis that allows researchers to interpret the "sense making by the research participants" (p. 89) engaged in the research problem under focus within a study. Kuutti (1996) claimed that the history of the context must also be included in the basic unit of analysis when using CHAT.

Using CHAT, I focused on the motive objects of activity (Kaptelinin, 2005; Leont'ev, 1978). Therefore, the unit of analysis for this study is the professional learning leadership activity of the School Mathematics Leaders. The context of that activity in which those mathematics leaders practised their professional learning leadership activity is analysed, focusing on the cultural tools, rules, and the division of labour, which are positioned as mediators of activity within a CHAT framework (Engeström, 2015; Nuttall et al., 2015; Roth, 2012). The historical dimension of the context is honoured by focusing on the temporal enactment of activity, paying attention to the School Mathematics Leaders' activity during participation in CTLM and the years following project participation.

1.7 Research Purpose

Considering the research problem and the unit of analysis, I now present the purpose of my study. The purpose is to generate knowledge about how School Mathematics Leaders, as middle leaders in their primary schools, contribute to project sustainability through their professional learning leadership activity.

1.7.1 Research Aims

The research aims for my study are:

- to generate theory about how mathematics leaders, as middle leaders in schools, contribute to project sustainability through their professional learning leadership;
- to contribute knowledge about mathematics leadership in primary schools and its position as a form of middle leadership; and
- to demonstrate my interpretation and use of CHAT as a framework to design a research study in response to the research problem.

Throughout this thesis, I reference my understanding and use of CHAT and present this to show how I used it to design and enact a study. In doing so, I hope to support the reader of my thesis, who may or may not be familiar with CHAT, in understanding how it can be used as a research tool.

At this point, it is essential to articulate that I did not use CHAT in an interventionist way to affect change within the activity system under investigation (Engeström, 1996). Instead, I used concepts from CHAT to understand the historical and cultural context of professional learning leadership activity of the School Mathematics Leaders and to generate knowledge about their contribution to project sustainability.

I now turn to the research question of my thesis.

1.8 Research Question

In this chapter so far, I have articulated the research context, research problem, the unit of analysis, and the research purpose and its associated aims. Considering these aspects, I present the research question guiding my inquiry.

The research question for my study is:

As middle leaders of site-based professional learning, how do School Mathematics Leaders contribute to the sustainability of mathematics teaching reforms in the years that follow participation in a large-scale school mathematics professional development project?

1.9 Significance of the Study

I have identified several reasons for the significance of my study. This study is significant, and the research problem is worthy of investigation, primarily due to a lack of knowledge about mathematics leaders' professional learning leadership in primary schools. It is also important because of the paucity of project sustainability and how middle leaders, such as the School Mathematics Leaders, contribute to the sustainability of project reforms.

Firstly, most literature about educational leadership has tended to emphasise the role of the principal in school sites (Carter, 2016; Cranston, 2009; Grootenboer et al., 2019; Gurr, 2019). Middle leadership is an emerging field of educational leadership research having primarily been neglected by researchers, and as a result, this leadership construct remains under-theorised (De Nobile, 2018; Grootenboer et al., 2017; Hammersley-Fletcher & Kirkham, 2007; Li et al., 2021; Turner, 2007). As a form of middle leadership, literature concerning school mathematics leadership is even more sparse (Driscoll, 2017; Sexton & Downton, 2014). As a mathematics education research community, we do not have enough theory about the middle leadership role of the mathematics leader in primary schools.

Secondly, and continuing this idea of professional learning leadership, the mathematics leader is deemed the most immediate source of professional learning for classroom teachers in primary schools (Millet & Johnson, 2004). They are regarded as having the most significant influence in changing teachers' mathematics teaching practices (Grootenboer et al., 2015; Jorgensen, 2016; Millet & Johnson, 2004). Despite this recognition, very little is known about the leadership of professional learning enacted by School Mathematics Leaders. Even though mathematics leaders are seen as an important influence on teacher knowledge and practices in mathematics education, there is little discussion in the literature about how they enact their professional learning leadership activity in schools.

Finally, concerns exist about the sustainability of mathematics professional development projects like the CTLM project, with the ability to sustain such projects remaining a critical issue (Bobis, 2011; Goos et al., 2018; Smit et al., 2019; Zehetmeier, 2014). This concern is warranted, considering there is a significant lack of studies that follow up on teacher professional development projects and how the effects of improvement are sustained after the life of those projects (Smit et al., 2019; Zehetmeier, 2015). Most of what we know about the impact of projects on teachers' professional development tends to come from studies undertaken during or shortly after the life of the project (Chapman, 2012; Coburn et al., 2012; Zehetmeier, 2015, 2017). Knowledge about the sustained impact of projects on teachers' professional learning is also sparse (Datnow, 2006). This is particularly concerning considering that when participation in projects end, external expert support for school staff, which was once available to them during the project, tends to discontinue (Heirdsfield et al., 2010).

1.10 Structure of the Thesis

This thesis is presented in eight chapters. In this first chapter, I have provided information about the genesis of my thesis. I have explained the historical and cultural context of the CTLM project with some information about the genesis of the School Mathematics Leader position in Melbourne Archdiocesan primary schools. Important aspects of the study were also presented, including the research problem, the unit of analysis, the research purpose and aims, and the research question. I also articulated my understanding of the significance of this thesis.

In Chapter 2, I present the literature review. I begin that chapter by sharing the methodology that I enacted when engaging with the background of the research problem of my study, as evidenced in the substantive and relevant literature. Insights from relevant literature about project sustainability in school settings, middle leadership, mathematics leadership, and mathematics professional learning are discussed. I identify a gap in the literature, stating that there are very few studies that make known how mathematics leaders contribute to project sustainability as middle leaders of professional learning in school settings.

Chapter 3 discusses CHAT, the theoretical framework I chose for my study. In this chapter, I provide insights into CHAT's history through a brief overview of its origins. I then present contemporary views of CHAT, focusing on my understanding of its fundamental principles and concepts.

Chapter 4 is dedicated to the research design. I explain the reasons for a qualitative research approach and articulate the ontological and epistemological foundations of the research design. I describe the methodology that I created for my study used as a means of operationalising CHAT. A discussion about the data methods and my approach to data analysis is articulated. My positionality with the research design and its influence on my research work is also explained, along with ethical considerations that informed my study.

Three chapters are used to respond to the research question. I wanted to tell a cohesive story about the School Mathematics Leaders' activity and how they contributed to project sustainability in their schools. Therefore, I made the methodological decision to present and discuss findings simultaneously in Chapters 5, 6, and 7. I appreciate that this is different from the traditional presentation of a thesis. I made this decision due to the complexity of the research problem, my use of CHAT, and the data used to respond to the research question.

In Chapter 5, as a means of honouring the CHAT’s methodological implications, the history of the School Mathematics Leader activity system is presented. I describe how the historically accumulated contradiction of the diminished priority of mathematics education motivated the decision for the School Mathematics Leaders' schools to participate in CTLM. I present evidence of the motive objects at which the School Mathematics Leaders directed their professional learning leadership activity during participation in that project. I claim that the School Mathematics Leaders shifted the focus of motive objects as participation in CTLM endured. I finish that chapter describing how the mathematics leaders were concerned about the sustainability of the project-initiated reforms beyond the life of CTLM.

Chapter 6 presents evidence of the resurfaced diminished priority of mathematics education contradiction within the activity system of the School Mathematics Leaders in the years following CTLM participation. I describe the struggle that the mathematics leaders experienced through the practice problems that realised for them that resurfaced contradiction. I claim that in response to those practice problems, the School Mathematics Leaders felt responsible for the sustainability of the CTLM project. I conclude Chapter 6 by suggesting that their struggle experience and their response through care and creativity was the impetus for the School Mathematics Leaders' enactment of a form of resourceful practice.

In Chapter 7, I share that the School Mathematics Leaders' resourceful practice was further realised through a reconfiguration of motive objects that they pursued through their post-project professional learning leadership. Three motive objects are identified, and I explain the post-project leadership actions that they enacted to realise their resourceful practice as their way of contributing to project sustainability. I conclude that chapter by explaining the presence of a new contradiction that surfaced for the School Mathematics Leaders.

I conclude my thesis with Chapter 8, presenting my response to the research question. As agents of project sustainability, I argue that the School Mathematics Leaders enacted a form of resourceful practice motivated by multiple motive objects. I then state my study's contributions and the implications of the findings for mathematics leaders, school leaders such as principals, and professional development project designers. I offer further research opportunities for the mathematics education research community.

I now turn to the literature review that provided me with the historical and cultural context of the research problem.

CHAPTER 2: THE LITERATURE REVIEW

2.1 Introduction to Chapter 2

In Chapter 1, I presented a summary of the historical and cultural context of the School Mathematics Leaders' leadership activity that provided the background to my study. I outlined an overview of the CTLM project and the general experiences of the mathematics leaders, principals, and classroom teachers leading up to their participation in the project. I also included general details of what participation in CTLM entailed.

I have presented brief references to relevant literature associated with the problematic thus far. That problem concerns the lack of theoretical knowledge about how mathematics leaders contribute to project sustainability as middle leaders of professional learning in their schools. In my study, I contextualised that problematic by focusing on the professional learning leadership of the School Mathematics Leaders working in Melbourne Archdiocesan schools, understanding how they contributed to project sustainability in the years following participation in CTLM.

In this chapter, I start by articulating the methodology I enacted when searching for literature sources. I also present my understanding of pertinent literature findings associated with the research problem. This chapter finishes with an articulation of the gap within the field, and the research question that guided my research reasoning during the study.

2.2 Literature Review Methodology

As a means of familiarising myself with the historical and cultural context of the problematic, and as a way of identifying sensitising concepts (Van den Hoonaard, 2008) associated with that issue, I conducted a review of relevant literature. I took advice from

Maxwell (2006) who advised that doctoral students focus on the criterion of *relevance* as much as possible when selecting and using sources to inform the literature review.

As I accessed and read literature, I thought about ways that the authors' work informed my research reasoning about the problematic of my study. In that sense, I was not aiming for coverage (Maxwell, 2006), but rather drawing out insights from sources that drew my attention to the: problematic; possibilities for the research design; and sensitising concepts for use when analysing data. That was supported by enactment of searches for theoretical and empirical research literature sources associated with the fields of project sustainability, middle leadership, mathematics curriculum leadership, and mathematics professional learning.

I predominantly used the databases of *Journal Storage* (JSTOR), *Education Resources Information Center (*ERIC), and *Informit.* I also used *Google Scholar* to support searches. I accessed and read journal articles, books, book chapters, theses, and conference proceedings. I sought sources that were peer-reviewed. That did prove challenging due to a lack of research literature concerning the problematic of mathematics leadership of project sustainability in primary school settings. In most cases, I drew on studies set in primary/elementary school settings, but I realised that I needed some support from sources set in secondary schools (e.g., the Bennett et al. (2007) foundational review of secondary middle leadership was used).

I created a list of search terms that I determined to be relevant to the research topic. Examples of the search terms are presented in Table 1.

Table 1

Example List of Search Terms for the Literature Review

Search terms
“Project sustainability” AND “math*”
“Project sustainability” AND “primary school*” (included “elementary school*”)
“Project sustainability” AND “leader*”
“Project sustainability” AND “middle leader*”
“Project sustainability” AND “professional learn*” (included “professional development”)
“Math* leader*” AND “middle leader*”
“Math* leader*” AND “professional learn*” (included “professional development”)
“Math*” AND ““professional learn*” (included “professional development”)

The terms captured in Table 1 are not exhaustive, but they acted as ones that held prominence during literature searches. With the focus on project sustainability and mathematics leaders’ activity as contribution to that, those terms featured when I searched the databases.

Along with the use of those pre-determined terms, I found literature using a search strategy known as *ancestry chasing* (Li & Wang, 2018). I used that approach to mine for sources using the citations in reference lists within the literature I found. That proved to be a helpful strategy in locating sources that provided the historical and cultural context of the problematic. Other sources were also accessed on advice from my supervisors.

I found recent literature concerning middle leadership and mathematics leadership was predominately published within Australasian contexts, the location of my study. As the problematic was set within Australia, the literature I sourced and used tended to be offered by Australian authors with support from international sources written in English. I also found it was important that I updated the literature review as the years progressed during the study period.

This was due to the increased frequency of published research about both middle leadership and mathematics leadership, particularly from Australia in the middle of the 2010s and onwards.

Guided by the search terms, the literature review is divided into four sections with the research question presented at the end of this chapter as a response to the gap in the literature.

I start the literature review, with a focus on relevant literature about project sustainability.

2.3 Project Sustainability in School Settings

The sustainability of teaching reforms initiated through participation in projects is a complex issue in school settings, often proving to be challenging activity for school staff and researchers alike (Bobis, 2011; Chapman, 2012; Coburn et al., 2012; Datnow et al., 2005; Hargreaves & Fink, 2003; Warren & Miller, 2016; Zehetmeier & Krainer, 2011). One important element of enduring school improvement requires a focus on sustaining change (King, 2011), yet that proves to be the significant challenge for schools and researchers who live and study the phenomenon (Datnow et al., 2005; Hargreaves & Fink, 2003; Zehetmeier, 2017). A reason for that challenge is the lack of knowledge, derived from research studies, that examines the sustainability of projects beyond the time of participation in them (Coburn et al., 2012; Fishman et al., 2011; King, 2011; Saito et al., 2012; Smit et al., 2019; Zehetmeier, 2015, 2017)

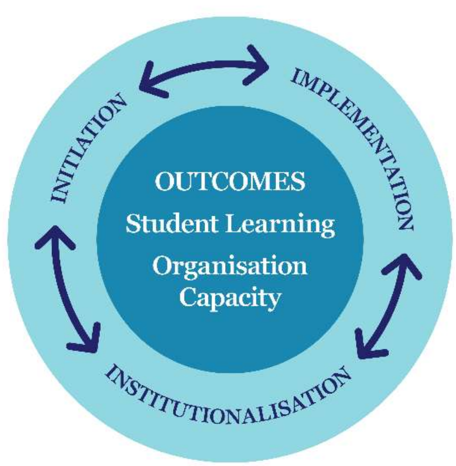
Historically, within the literature concerning reform sustainability, the term *institutionalisation* was used (e.g., Anderson & Stiegelbauer, 1994; Huberman & Miles, 1984; Miles, 1983; Zehetmeier & Krainer, 2011). That was the case until the early 2000s when there was a shift to the term *sustainability* (Datnow et al., 2005). Within earlier literature, institutionalisation was understood as the final phase of the reform process, with Miles (1983) offering one of the first models to explain how change became sustained in school settings.

Miles (1983) claimed that institutionalisation began when principals made commitments to participate in the reform initiative, providing both pressure and support for teachers to implement the reform. When teachers worked on the implementation phase, commitment to the reform led to a phase of stabilisation, and if that phase continued, institutionalisation was realised. That was particularly the case when the project reform lost its initial novelty and became a durable aspect of practice within the school setting (Huberman & Miles, 1984). According to Miles (1983), school leaders needed to set in place structures and rules that enabled and governed reform efforts. Miles' process to institutionalisation was presented as a linear one.

Fullan (2008) wrote about project sustainability within the area of educational change, drawing on some work of Miles (1983). In his synthesis of research, Fullan claimed that the process to institutionalisation was iterative, characterised by three main phases: *initiation*, *implementation*, and *institutionalisation*. Figure 1 represents Fullan's ideas of the phases in relation to the outcomes of reform innovation. The use of the two-way arrows shows that the change facilitated by reform efforts was not linear but rather, an iterative process.

Figure 1

Overview of the Change Process (Fullan, 2008)



Fullan (2008) claimed those phases captured the activity enacted by the school community when first undertaking a project (initiation), then adopting and trialling teaching and learning strategies as featured within the project (implementation), and finally sustaining the reforms beyond project participation (institutionalisation). The outcomes of any school reform are focused on the improvement of student learning and the organisational capacity to learn and engage in further change processes (Fullan, 2008).

The first phase of Fullan's (2008) process, initiation, related to the practices of decision-making that leads up to the school's adoption and engagement with the reform and its changes. Fullan (2008) attributed the terms of *mobilisation* and *adoption* to that phase of the change process. The implementation phase concerned the attempts at putting into practice the ideas shared through the reform action. Fullan claimed that that phase may take two to three years for schools to adopt teaching practices advocated for by project reform teams.

With the final phase, institutionalisation, Fullan (2008) used other terms including *continuation*, *incorporation*, and *routinisation* to describe that aspect of the change process. At that phase, the school community embeds the change "as an ongoing part of the system or disappears by way of decision to discard or through attrition" (p. 65). That articulation of institutionalisation echoes that of Anderson and Stiegelbauer (1994) who defined institutionalisation as a phase after implementation where the reform becomes part of the school's organisational structure, enacted beyond that phase when reform action is trialled.

Fullan (2008) emphasised that all three stages were important for the success of change efforts, and that *change agents* (people enacting the change) need to pay attention to each stage. That was done to ensure that the change effort was sustainable and meaningful. The importance of leadership in sustaining the change within the institutionalisation phase was emphasised by Fullan, however, that focused on the principal and other executive leaders with generalised references to other leaders within the school setting.

The term institutionalisation appears to have lost its frequency of use within the field. In the early 2000s, Hargreaves and Fink (2003) claimed that institutionalisation shifted to the idea being as one that is understood as “the even more complex problem of sustainability” (p. 694). Datnow (2005) claimed, however, that the two terms are linked because if any reform is determined to be sustained, it must be institutionalised, and for any reform to be institutionalised, it must be sustained. The generally agreed upon term used now in mathematics education literature is that of *sustainability* (e.g., Bobis, 2011; Chapman, 2012; Coburn et al., 2012; Goos et al., 2018; Saito et al., 2012; Tirosh et al., 2015; Zehetmeier, 2015).

Zehetmeier (2015) purported that as a term, sustainability has its origins within the fields of ecology and economy. Within recent years, the term has been adopted by those working and studying within the field of education (Tirosh et al., 2015; Zehetmeier, 2015). It is used to describe the efforts and activity associated with the continuation of reforms that were initiated through project participation.

At this point, it is important that I clarify that I am not focusing on the scaling-up of projects. I appreciate that scaling-up and sustainability are related concepts (Bobis, 2011; Coburn et al., 2012; Tirosh et al., 2015). However, for the purposes of my thesis, I understand scaling-up to mean the spread of a project’s impact to school communities that did not initially participate in the project (Tirosh et al., 2015; Weißenrieder et al., 2015). My focus is on how the School Mathematics Leaders collectively sustained the CTLM-initiated reforms within their schools in the years following project participation. Therefore, I am focusing on project sustainability only within my thesis.

As a way of understanding the concept of project sustainability, I now turn to a discussion of definitions of the term.

2.3.1 Contemporary Definitions of Project Sustainability

Various definitions for project sustainability have been offered by researchers within the field (Zehetmeier, 2015). The variety of definitions provide insights into the nature and function of project sustainability in both general and nuanced ways. Generally, project sustainability in school settings refers to the *ability* of a project or intervention to sustain the impact on student learning outcomes and school improvement (about which the project is focused), after the initial funding has ended and when access to human and physical resources has been withdrawn (Clements et al., 2015; Coburn et al., 2012; Fishman et al., 2011; Hargreaves & Fink, 2003; Tirosh et al., 2015; Zehetmeier, 2015).

Other nuanced characteristics are offered by mathematics education researchers, providing more further understanding of project sustainability. Clements et al. (2015) claimed that project sustainability entails the maintenance of and fidelity to the beliefs and values that underpin the project design. Continuing with the project reforms in ways that align with and demonstrate fidelity to the intent and content, as evidenced in its pedagogical principles and approaches, features as further definitions of project sustainability in some mathematics education sources (Clements et al., 2015; Coburn et al., 2012; Ell & Irwin, 2006; Fishman et al., 2011).

Researchers have extended upon the notion of fidelity to the intent and content of projects suggesting that project sustainability can include adaptations in response to the conditions that may surface for teachers and school communities in the years following project participation (Coburn et al., 2012; Tirosh et al., 2015; Zehetmeier, 2014). Tirosh et al. (2015) recognised the school environment as one that is always shifting, and therefore, reported that project sustainability extends beyond mere reform maintenance. For Tirosh et al., project sustainability requires development, adaptation, and integration of the project content in ways that serve the changes within the school environment in the years following project participation.

Sustaining instructional practices, that were trialled and implemented during project participation, requires more than “continuing to do the same thing” (Coburn et al., 2012, p. 165). Project sustainability is also defined as a process of self-renewal as schools and staff members respond in autonomous ways to the shifting conditions in schools, where new processes and products (e.g., teaching approaches, student learning outcomes) are generated and enacted from those practices initiated during the life of the project (Tirosh et al., 2015; Zehetmeier, 2014).

Another aspect of definitions for project sustainability includes references to its temporal nature (Zehetmeier, 2014). Clements et al. (2015) reported that sustainability is understood as the continuation of the project for “the length of time” (p. 428) that the reform efforts remain faithful to its implementation, whereas Taylor (2006) used the phrase “over the years” (p. 335) when describing time associated with sustainability. Ell and Irwin (2006) claimed that sustainability was an ongoing process, whilst Zehetmeier (2015) said that it was a lasting continuation of benefits and effects. Hargreaves and Fink (2003) stated that project sustainability is more than just a matter of persistence with project reforms over time or understanding it as the durability of reforms.

The temporal nature of project sustainability appears to be contentious, with researchers having questioned the time that determines when projects have been sustained beyond project participation (Tirosh et al., 2015; Zehetmeier & Krainer, 2011). Tirosh et al. (2015) reported that project sustainability can be determined at a time that was *distant* from when the project concluded. They did caution, however, that that discernment was a subjective judgement. Zehetmeier (2015) claimed that effects of project participation that are both short-term and long-term in nature can be considered as sustainable. Zehetmeier and Krainer (2011), in their summary of previous project sustainability research claimed that the commonality of references to time was that the focus was on "durable continuation" (p. 879). They did add that there is a lack of clarity on the timeframe associated with project sustainability, and that researchers need to define those timeframes when investigating the sustainability of projects.

As seen with my review of authors' definitions, project sustainability is understood and defined as a multi-faceted phenomenon. For the purposes of my thesis, I define project sustainability as:

> In the years following project participation, project sustainability is the durable continuation of reforms which remain faithful to the project's intent, content, beliefs, and values whilst being open to adaptation and development in response to the shifts and changes that characterise the school environment.

Now that I have defined project sustainability, I turn attention to how researchers have framed its enactment through factors that enable and constrain project sustainability. Before I do, it is important that I clarify the difference that I noticed about those project sustainability factors and the concept of *project diffusion* and its characteristics.

2.3.1 Project Diffusion

As a prelude to a discussion of the factors of project sustainability, I want to highlight how I am separating those from other knowledge that concerns project diffusion. Through my reading, I noted literature about project diffusion, and how that concept is understood as the potential for the uptake and influence of the project in relation to its aims (Zehetmeier & Krainer, 2011). I interpret project diffusion as an aspect of project design that requires attention *before* reforms are initiated and implemented through project participation (Anderson & Stiegelbauer, 1994; Fullan, 2008; Huberman & Miles; Miles, 1983). Project diffusion supports the potential for project sustainability when it is built into the design of projects (Zehetmeier, 2015). Several characteristics of project diffusion have been offered, with the notion that they require attention from project designers when professional development projects are conceptualised and organised.

An example of a project diffusion characteristic is *compatibility and need* which captures the perceptions of the project participants where they see the reform meeting their desires, beliefs, and values for improvement (Zehetmeier, 2014). Another is *quality and practicality* that concerns the degree to which the project reforms are applicable to teachers' practice and readily accepted as part of the change process (Zehetmeier & Krainer, 2011). Other project diffusion characteristics include *relative advantage* which is about the participants' views that the project provides benefits which in some cases may differ from the project aims identified by the project designers (Zehetmeier, 2015); *observability* concerns the degree to which the implemented changes mediated by project participation are visible to the school community and its stakeholders, such as families, principals, and the wider community (Zehetmeier, 2014, 2015); *complexity and clarity* concerns participants' perceptions of the degree of difficulty in understanding the intent and content of the project and enacting those through their reform efforts (Zehetmeier & Krainer, 2011); and *trialability* is the potential for project participants to trial, experiment with, and test the application of reform practices in classroom and school settings (Zehetmeier, 2014).

Although I recognise that project diffusion is crucial in project design, it is important to state that I am focusing on factors that mediate the continuation of projects *after* the intervention has finished. Therefore, I now elaborate on those project sustainability factors.

2.3.2 Factors of Project Sustainability

A review of the literature led to my reasoning that positions factors of project sustainability as a variety of important mediational means that contribute to the long-term success and viability of the project once participation within the intervention has ceased (Bobis, 2011; Coburn, 2003; Saito et al., 2012; Zehetmeier & Krainer, 2011). Factors have been also referred to as *mechanisms* within some literature sources (Bobis, 2011; Coburn, 2003). Project sustainability factors have been used to understand how interventions have continued operating beyond the initiation and implementation phases of the project (Fullan, 2008; Saito et al., 2012; Zehetmeier, 2015).

Saito et al. (2012) claimed that authors writing within the field have categorised project sustainability factors using two classifications: *internal factors* and *external factors*. Internal factors are considered conditions *within* the school site that have the potential to enable or constrain project sustainability (e.g., Coburn et al., 2012; Datnow et al., 2005; Goos et al., 2018), whilst external factors are those conditions that are mediated by stakeholders *outside* of the school setting like district and sector leadership, local community, or larger governing bodies (e.g., Datnow, 2005; Datnow et al., 2005; Saito et al., 2012)

When studying project sustainability, it is crucial that researchers pay attention to those factors and how they have the potential to enable and constrain sustainability efforts beyond project participation (Saito et al., 2012; Zehetmeier, 2014, 2017). The reason for that attention lies in how the factors are seen as various and complex mediators of project sustainability (Zehetmeier & Krainer, 2011). Bobis (2011) warned that one factor alone cannot be understood as the most influential mechanism of project sustainability. That is due to the interactional nature of the factors and their function in influencing how reforms may or may not be sustained over time.

I now move to a discussion of those factors of project sustainability, using the classifications of internal and external factors as suggested by Saito et al. (2012).

2.3.3 Internal Factors

Researchers writing within the field of project sustainability have offered insights into several factors that are considered *internal*. An internal factor is one that resides within the school context (Saito et al., 2012). Internal factors that were most salient within the project sustainability literature were *school leadership*, *staff turnover*, *school-based professional learning*, and *access to project resources*.

2.3.3.1 School Leadership

Leadership within the school setting features as an important factor that has potential to enable the sustainability of mathematics education reforms beyond project participation (Bobis, 2011; Coburn et al., 2012; Datnow et al., 2005; Goos et al., 2018; Saito et al., 2012; Tirosh et al., 2015; Warren & Miller, 2016; Zehetmeier, 2015; Zehetmeier & Krainer, 2011). The literature, however, has focused overwhelmingly on the leadership of the principal and their role in sustaining teaching reforms.

Datnow et al. (2005) reported that those in formal positions of authority play an essential role in contributing to project sustainability, citing the principal and the district leader (superintendent) as those formal positions. Goos et al. (2018), reporting on Australian data generated with regional leaders (an Australian equivalent to superintendent), claimed that those leaders saw the principal as central in providing support for school communities to engage in the activity of project sustainability. A school environment, under the leadership of the principal who promotes the importance of sustaining project reforms, is more likely to influence teachers' work that maintains the use of teaching practices implemented during project participation (Tirosh et al., 2015; Warren & Miller, 2016; Zehetmeier, 2014, 2015).

Due to their position as senior leader with the authority to make and lead school improvement decisions (Datnow, 2005; Datnow et al., 2005; Saito et al., 2012), the principal is positioned to offer support, incentives, and pressure that influences teachers to continue enactment of project reforms (Anderson & Stiegelbauer, 1994). Principals are the leaders within schools who create the conditions and organisational structures required to support project sustainability (King, 2011). Warren and Miller (2016), reporting about the success of a state-wide professional development project in Queensland, Australia, claimed that principals who created conditions for teachers' professional development when project support is withdrawn, were more likely to maintain the momentum of teaching that was practised during project participation.

Several of those conditions have been reported in the project sustainability literature concerning mathematics education and beyond. King (2011) reported on the role that the principal played in organising the condition of time for teachers to collaboratively plan when project participation came to an end. Drawing on interview data from a qualitative case study of five schools in Ireland, King found that principal support was vital in setting up structures that enabled teachers to continue their professional learning. That teacher learning was focused on reflecting upon and consolidating the practices trialled during the project. Such conditions are dependent on fiscal resources, and principals who factor in the use of school budgets to support those teacher professional learning opportunities are more likely to sustain teaching reforms in their schools (Bobis, 2011; Datnow et al., 2005; Warren & Miller, 2016).

A lack of school leadership support for teachers can impede sustainability efforts in schools (Warren & Miller, 2016). When there is a lack of principal leadership support after project participation has ceased, teachers may end up reproducing pedagogical practices that were enacted prior to engagement in the reform (Tirosh et al., 2015; Zehetmeier, 2015). Lack of principal leadership support can also extend to funding withdrawal which has potential to surface challenges for reform efforts to continue (Datnow et al., 2005). When fiscal resources used to establish new structures to work and learn collaboratively are taken away, opportunities for teachers to engage in further learning tend to diminish, impacting the project sustainability efforts (Thomas & Ward, 2006).

In rare cases in the literature, as part of the school leadership factor, the role of leaders not part of the school executive leadership team was reported. Bobis (2011), writing about the sustainability of a large-scale project (*Count Me in Too* [CMIT] numeracy program) in New

South Wales, Australia, reported that a school-based facilitator model repeatedly featured as a factor that mediated sustainability of reforms in schools. That model provided the means for "leaders in mathematics" (Bobis, 2011, p. 47) to act as facilitators within that model.

Datnow et al. (2005) referred to the importance of teacher leadership positions in schools that supported the continuation of reform efforts. They referred to that leadership role as *reform coordinator* (p. 206) who worked in a full-time capacity or was enacted by a lead teacher who also had classroom teaching responsibilities. Datnow et al. reported that those lead teachers were less effective in mediating project sustainability due to conflicts with time management between teaching and leading the post-project participation reforms.

Generally, the school leadership factor of project sustainability focused on the role of the principal as the senior leader of the school. That focus on principals also featured in the literature about the next factor about staff turnover.

2.3.3.2 Staff Turnover

Staff turnover, as a factor of project sustainability, is understood as the rate at which staff members who participated in the professional development project leave the school for reasons that could include employment at other schools, the transfer or swap of staff from one school to another, or retirement from the profession (Anderson & Stiegelbauer, 1994; Taylor, 2006; Zehetmeier, 2015). Turnover of staff includes changes at the teacher level (Coburn, 2003; Huberman & Miles, 1984; King, 2011; Pritchard & McDiarmid, 2006; Thomas & Ward, 2006; Zehetmeier, 20015); the principal level (Coburn, 2003; Saito et al., 2012; Sindelar et al., 2006; Thomas, 2006); and it extends to shifts in staffing at the district level (Thomas, 2006). Taylor (2006) reported that teacher turnover influences project sustainability more than principal and district level turnover. However, Datnow et al. (2005) claimed that turnover within key leadership positions at the principal or district level leads also has potential to compromise project sustainability efforts.

Staff turnover at any level tends to act as a constraining factor of project sustainability because it disrupts the continuity of the project reform caused by a loss of institutional knowledge and project experience (King, 2011; Pritchard & McDiarmid, 2006; Taylor, 2006; Thomas & Ward, 2006; Zehetmeier, 2015). Elaborating on teacher turnover, Taylor (2006) claimed that that factor caused difficulty with reform efforts. That was because teachers who have little to no historical experiences of the project reform require constant professional learning about the fundamental ideas that underpin the project.

The factor of staff turnover constrains project sustainability efforts when new teachers who are added to the staff roster do not have historical experiences with the project intent and content (Thomas & Ward, 2006). In response to that, reform continuation efforts tend to focus on revision of project content rather than extension and innovation of learning achieved through project participation (Saito et al., 2012; Taylor, 2006). Teacher motivation to continue with project reforms can diminish (Saito et al., 2012) because there are fewer teachers and leaders within the school community who can engage in the shared understanding of the reform teaching practices. That is because staff members, who originally contributed to that shared understanding through project participation, were no longer part of the school community (Pritchard & McDiarmid, 2006; Sindelar et al., 2006).

2.3.3.3 School-based Professional Learning Opportunities

Continuation of teachers' professional learning that was initiated and implemented through project participation is another factor that can facilitate project sustainability (Bobis, 2011; Kaur, 2015; Pritchard & McDiarmid, 2006; Saito et al., 2012; Sindelar et al., 2006; Warren & Miller, 2016; Taylor, 2006; Zehetmeier, 2015). Continued professional development that provides opportunities to maintain learning of the project intent and content can happen within teachers' schools (Pritchard & McDiarmid, 2006; Saito et al., 2012; Zehetmeier, 2015), or through formal opportunities outside of the school site (Kaur, 2015; Warren & Miller, 2016).

As a sustainability activity, Warren and Miller (2016) suggested that it was essential that teachers engage in off-campus professional development sessions with mathematics educators after project participation. They highlighted that those sessions had potential for teachers to work with "experts in the field of mathematics" (p. 118). Part of that work included trialling and reflecting on new teaching practices associated with the project guided by the support of mathematics educators. That, in turn, can provide ways for innovation upon the intent and content of the projects (Coburn et al., 2012; Tirosh et al., 2015; Zehetmeier, 2014).

Kaur (2015), writing about a secondary school project in Singapore, reported that teachers who had participated in project acted as *experts* for *novice* teachers who had not engaged with the reform. By participating in collaborative professional learning after project participation and by sharing understanding of the pedagogical intent and content of the project's resources, aspects of the mathematics teaching reforms were sustained. That activity was dependent, however, on the maintenance of staff who had participated in the project (the "expert" teachers), as well as the continuation of access to project resources that were introduced to the teachers during project participation.

2.3.3.4 Project Resource Access

The accessibility of project resources beyond participation in the reform is considered another internal factor of project sustainability (Bobis, 2011; Fishman et al., 2011; Hargreaves & Fink, 2003; Smit et al., 2019; Warren & Miller, 2016). Through project participation, teachers and leaders are introduced to resources and research-informed materials to support the trialability and implementation of the pedagogical practices that align with the project’s intent and content (Warren & Miller, 2016; Zehetmeier, 2015). As a way of sustaining the reforms beyond project participation, those resources provide continued access to the information and knowledge that are captured within the resource purposes (Saito et al., 2012). Therefore, access to project resources can support sustainability efforts but they must remain easy to use when teaching mathematics when project participation has concluded (Ell & Irwin, 2006).

Lack of access to resources beyond project participation can leave principals and teachers frustrated about ways to sustain the developmental work with reforms (Thomas & Ward, 2006). Teachers and leaders must, however, understand the pedagogical potential of the project resources if they are to be used beyond project participation (Bobis, 211; Warren & Miller, 2016). It is essential that teachers understand the pedagogical potential of resources that can impact student learning and appreciate the pedagogical advice on how to use those resources when there is no further access to the project lead staff (Smit et al., 2019). That is vital if the resources’ potential is to be realised fully as a means of mediating project sustainability beyond participate in the reform and as a way of impacting students' mathematics learning outcomes (Bobis, 2011; Zehetmeier, 2015).

I now move to a discussion of several external factors of project sustainability.

2.3.4 External Factors

External factors of project sustainability are those about which the school does not have a direct line of influence (Saito et al., 2012). Those external factors mediate what happens within school sites. External factors that surfaced in the project sustainability literature were *district leadership*, *teacher networks*, and *high stakes testing programs*.

2.3.4.1 District Leadership

Datnow (2005), in her systemic review of educational reform in the US between 1983 and 2003, offered insights into the role that district leadership plays as a factor of project sustainability. Datnow claimed the district leadership support is crucial for reform sustainability. The reason for this is that district leaders can offer schools, that once participated in projects, opportunities to engage in further professional development. Districts can also stimulate the creation of learning partnerships and provide further financial resources if possible (Datnow et al., 2005).

District leaders also can mediate project sustainability by providing access to external experts to support schools with their sustainability efforts (Datnow et al. 2005). As part of ongoing relationship building, principals leading schools that are engaged in project sustainability work are advised to inform district leaders of success (Datnow, 2005). That has potential to mediate further district leadership that can support school communities in sustaining teaching reforms. That is important considering that schools can experience insufficient assistance from district leaders concerning project sustainability in school sites (Datnow et al., 2005; Saito et al., 2012).

2.3.4.2 Teacher Networks

The literature has presented some insights into the role of the network factor focusing on teacher networks. Opportunities for teachers to participate in networks with other teachers who shared the experience of project participation presented are important for project sustainability (Bobis, 2011; Coburn et al., 2012; Zehetmeier, 2015). Slavin (2004), writing about project sustainability outside of mathematics education field, reported that when teachers participate in networks with others who shared the same project experience reforms are more likely to be sustained. Zehetmeier (2015, 2017) extended further on the mediating potential of teacher networks by claiming that networks, as understood as peers providing each other social and learning support, afforded the time for teachers to collaborate on purposeful ways to sustain reforms.

Coburn et al. (2012) studied the factor of networks by focusing on teachers' social relations and the connections of those with the sustainability of project reforms. Their three-year study within four elementary schools in the same district using the methods of interviews, observations, and document retrieval provided important insight into that teacher network factor. Coburn et al. found that social relations influenced teachers to continue with reforms two years beyond the end of participation in an intervention. They claimed that that sustainability work was also influenced by quality interactions between teachers about mathematics teaching and learning. That interaction quality was mediated by high expertise with mathematics teaching that was shared between the teachers within the network.

Bobis (2011) reported on the benefits of an online program that provided collegial support for teachers following project participation. Having teachers meet to network in real time can be challenging. Bobis claimed that online networking allowed for the development of virtual collaboration between teachers *within* their schools. However, those same teachers were reluctant to use that online space to share knowledge and experiences of practice with teachers outside of their settings. Bobis found that there were rare uses of the discussion component on that online site that acted as evidence of cross-school collaboration.

2.3.4.3 High Stakes Testing Programs

Few studies presented the factor of high stakes testing programs as one of project sustainability. The place of testing regimes can have both an enabling and constraining influence on how teachers continue with reforms initiated and trialled during project participation (Datnow, 2005; Datnow et al., 2005; Sindelar et al., 2006). Datnow (2005) found that teachers, in her study situated in the USA, tended to favour teaching activity that prepared students for state tests when the testing times approached. That meant that the pedagogical approaches highlighted within the reform were sidelined at those testing periods.

If there are improvements in testing scores, then teaching approaches practised through the reform may remain part of teachers' practice (Datnow, 2005; Datnow et al., 2005). In schools where testing data suggest low achievement, then those school communities tend to not continue with reform efforts working on the assumption that the intent and content of the project did not work (Datnow, 2005). Efforts to sustain project-initiated reforms may not continue if the high stakes tests focus on student performance in academic areas that do not neatly align with the intent and content of the project reforms (Sindelar et al., 2006).

2.3.5 Project Design and Project Sustainability Factors

Although it is a limited knowledge area due to its neglect as a research topic, several authors have called for attention to be paid to sustainability factors. Along with planning for project diffusion (e.g., Zehetmeier & Krainer, 2011) by building its characteristics into project design, Timperley et al. (2007) and Warren and Miller (2016) claimed that project sustainability also needs to form part of project design. Sustainability of the project must be planned with the intention of supporting school communities to take responsibility for the sustainability *during* project participation, and especially *beyond* the life of the project.

Careful consideration of the project sustainability factors, in terms of their potential to enable and hinder efforts with sustaining reforms, is required when project designers create their plans for mathematics professional development projects (Zehetmeier & Krainer, 2011). That appears to be important when thinking that different projects will have different aims, content, settings, and participants (Zehetmeier, 2014).

Even though it is recognised that it is important to consider project diffusion and project sustainability factors when designing mathematics education professional development projects, knowledge about project sustainability and its factors is severely limited (Coburn et al., 2012; Tirosh et al., 2015; Zehetmeier, 2015; Zehetmeier & Krainer, 2011). Calls have been made for researchers within the field for further studies into project sustainability (e.g., Chapman, 2012; Smit et al., 2019; Zehetmeier, 2014) and its factors that can enable and constrain the sustainability of reforms beyond project participation (Coburn et al., 2012; Saito et al., 2012)

2.3.6 Need for Further Research About Project Sustainability

Project sustainability of professional development programs is a complex issue, mainly due to the lack of knowledge about the phenomenon within mathematics education literature sources (Bobis, 2011; Datnow et al., 2005; Zehetmeier & Krainer, 2011). Researchers tend to study the impact of those projects during the initiation and implementation phases of projects, and report on data that are generated during those phases (Chapman, 2012; Coburn, 2003; Coburn et al., 2012; Smit et al., 2019; Zehetmeier, 2015, 2017). Little is known about project sustainability enacted by school communities after project participation because the financial resources tend to be depleted and the project researchers have moved onto studies investigating other research issues and problems (Fishman et al., 2011)

Coburn et al. (2012), drawing on their three-year longitudinal study of teachers working in four urban elementary schools in the USA, claimed there was a lack of studies that took an in-depth analysis of one or more of project sustainability factors. They called for further studies that examined the timing, reasons for, and actions of how the factors of sustainability specifically support the continuation of reforms initiated through project participation. Bobis (2011) claimed that there is not one factor of sustainability that is greater than another needs to be considered in light of that call from Coburn et al. (2012).

Study into the specificity of project sustainability factors, that develop knowledge into how they are enacted within the reform efforts of school stakeholders, is required if practitioners and researchers alike are to mutually understand the potential of those factors that enable project sustainability (Saito et al., 2012; Zehetmeier, 2014). Further studies into the long-term effects of professional development projects are needed if understanding of the impact of such projects on student learning outcomes and organisational learning is to be achieved (Bobis, 2011; Smit et al., 2019; Zehetmeier, 2014).

When a researcher designs and enacts a study into project sustainability, Zehetmeier (2014) advised the use of qualitative research approaches that incorporate the use of triangulated data. Insights into the nature and influence of project sustainability factors can be generated when the methods of interviews and observations are included in the research design (Tirosh et al., 2015; Zehetmeier & Krainer, 2011). The use of interviews and observations together can avoid methodological issues related to the reliance of self-reports which can impact on the validation of findings (Tirosh et al., 2015). Zehetmeier (2015) also recommended the use of document retrieval as further method of data generation.

When analysing data, Zehetmeier (2015) incorporated the use of both deductive and inductive approaches in his sustainability studies. Those approaches were enacted as a means of comparing results from the different data sources. Zehetmeier used both analytical approaches as a way of understanding the project sustainability factors that already existed within the literature.

The call for further studies into project sustainability is crucial, especially in the case of the problematic that I am investigating concerning the contribution of the School Mathematics Leaders.

I now move to a discussion of literature concerning middle leadership in school settings.

2.4 Middle Leadership in Primary School Settings

Educational leadership research literature has a long history that has focused on the effectiveness of schools and the potential for those institutions to improve learning and teaching (Grootenboer et al., 2019). Educational leaders are important in school communities for several reasons that include developing expertise in teaching and learning; strengthening collaborative cultures; building teacher and organisation capacity; and, increasing teachers' sense of professionalism and empowerment (Cobb et al., 2018; Loucks-Horsley et al., 2010; Timperley, 2008). Timperley (2008) suggested that designated educational leaders in schools have an important role in developing expectations for improved student learning outcomes and promoting and engaging teachers in opportunities for professional learning.

The focus of educational leadership research on school improvement and effectiveness, however, has tended to showcase the leadership activity of principals (Carter, 2016; Grootenboer et al., 2017). Evidence of that has already been presented with the focus of the principal within the school leadership factor of project sustainability (e.g., Datnow et al., 2005; Goos et al., 2018; Saito et al., 2012). That focus on the principal is not surprising considering that it is ultimately the responsibility of the principal to lead the direction and the function of the school as the senior leader (Cranston, 2009; De Nobile, 2019; Grootenboer, 2018).

In the 21st century, with the emphasis on school improvement, accountability, and the need for more effective ways of measuring student progress, other forms of leadership other than principalship have become increasingly important (Bennett et al., 2007; De Nobile, 2018; Grootenboer, 2018). With the increasing complexities of school environments, distributed leadership as a form of leading that spreads leadership across the school setting, surfaced as a way of supporting the work of learning, teaching, and improvement that once only sat with principals (Camburn et al., 2003; Grootenboer, 2018; Lárusdóttir & O'Connor, 2017).

In recent years, researchers have investigated the nature and function of another form of leadership, distinct from principal leadership, that has potential to support teaching and learning

(Bennett et al., 2003 Carter, 2016; Grootenboer, 2018). The construct of middle leadership has surfaced as a form of leading within the school leadership system that has significant potential in influencing school improvement and teacher professional development (Grootenboer, 2018; Lipscombe et al., 2021; Turner, 2007).

Middle leadership has been reported as rising from distributed leadership activity (Grootenboer, 2018). It has, however, been perceived as conceptually narrower than distributed leadership. Heng and Marsh (2009) claimed the reason is that middle leadership focuses primarily on the leadership roles of middle management or subject leader positions. More recent studies, however, have positioned middle leadership positions to include more than just subject leadership (e.g., De Nobile, 2018; Lipscombe et al., 2021).

De Nobile (2019) claimed that another shift has been in terminology that has seen the term middle management (as referenced by Heng and Marsh, 2009) move to middle leadership. De Nobile reported that that was done as a means of capturing the shift within the practices of middle leaders. That change has seen a move from managerial and administrative work to activity that is deemed more influential and linked to school strategic goals for improvement.

2.4.1 Defining Middle Leadership

As a way of foregrounding the discussion about middle leadership, I present definitions of middle leading as found in relevant literature sources. Due to its recent focus and its growing interest within the field of educational leadership literature (Cranston, 2009; De Nobile, 2018; Grootenboer et al., 2019), researchers have offered various definitions of middle leadership. Further definitions have extended understanding of middle leadership as a type of position held by such leaders (e.g., Lipscombe et al., 2021); a form of practice (e.g., Grootenboer, 2018); and how the middle leader role is enacted through roles and responsibilities (e.g., De Nobile, 2018). More recent work has also defined the middle leader as one who cultivates relational trust with teachers in school settings (Edwards-Groves et al., 2016).

There has been some contention about defining middle leadership in the literature. Those shifts have surfaced a problem in that a definitive meaning for middle leadership in school settings is proving to be elusive (De Nobile, 2018; De Nobile & Ridden, 2014; Gurr, 2019; Lipscombe et al., 2021). That is mostly influenced by disagreements about the place of classroom teaching responsibilities (Lipscombe et al., 2021).

In the United Kingdom, Camburn et al. (2003) used the term middle leader to describe a role related to subject leadership in schools. They contended that middle leadership consists of a formal role within the school where a teacher undertakes the management and pedagogical responsibilities of a subject area. Hammersley-Fletcher and Kirkham (2007) also defined the middle leader as a teacher who holds responsibility for an aspect of business related to the functioning of the school, usually a curriculum area. They, however, believed that the middle leader does not need to hold any formal position. Hammersley-Fletcher and Kirkham also reported that the middle leader does not necessarily have to be a member of the school executive leadership team.

There appears to be some sustained contention within the Australian literature concerning the definition of middle leader as well. Some authors have claimed that a middle leadership role is one with a designated formal position (or responsibility of leadership) that includes classroom teaching responsibilities (Edwards-Groves et al., 2016; Grootenboer, 2018; Gurr & Drysdale, 2012). Lipscombe et al. (2021), in response to their recent review of middle leadership literature, proposed that there is no requirement for the middle leader to engage in classroom teaching responsibilities. They did, however, stipulate that for a role to be deemed a middle leadership one it must be appointed formally by the school, be assigned responsibilities for which they are accountable, work between the executive leaders and classroom teachers, and that leadership actions need to influence student and teacher learning in positive ways (Lipscombe et al., 2021).

2.4.1.1 Different from Teacher Leadership

There are differences between middle leadership and teacher leadership. Although like teacher leadership, in that teaching staff undertake the work of leading, middle leadership is viewed within the literature as being different (Lipscombe et al., 2021). Heng and Marsh (2009) stated that middle leadership is aligned with understandings of teacher leadership but there is a conceptual difference between the two leadership forms. They claimed that the difference relates to formal recognition of the middle leadership role within the school's leadership system.

Grootenboer et al. (2015) stated that middle leadership is not the same construct as teacher leadership, claiming that the latter was "more generic and ubiquitous" (p. 509). Teacher leadership concerns informal enactment of leading whilst middle leaders are recognised formally by a position within the school leadership system (Camburn et al., 2003; De Nobile, 2018; Edwards-Groves et al., 2016; Grootenboer, 2018; Heng & Marsh, 2009; Lipscombe et al., 2021).

2.4.3 Middle Leadership Positions in Schools

With recognition that middle leadership is linked to a formal position of leadership within the school leadership system (e.g., Edwards-Groves et al., 2016; Grootenboer, 2018), middle leadership researchers have identified a variety of formal positions that fulfil the definition of middle leader.

As a way of summarising those insights, I present the following table. Table 2 presents information concerning the title of middle leadership positions as identified in select literature sources.

Table 2

Middle Leadership Positions in Schools Found in Literature Sources

Middle leadership position	Literature sources
Assistant principal	De Nobile (2018); De Nobile and Ridden (2014)
Curriculum coordinator / Subject leader	Bennett et al. (2007); De Nobile (2019); Lipscombe et al. (2021)
Religious Education coordinator	De Nobile (2018); De Nobile and Ridden (2014)
Pastoral care coordinator	De Nobile (2018); Irvine and Brundrett (2019)
Department heads	Bennett et al. (2007); Bryant et al. (2020); De Nobile (2019); Irvine and Brundrett (2019); Shaked and Schechter (2017)
Specialist coordinator (e.g., ICT or Special Needs/Wellbeing)	Bryant et al. (2020); De Nobile (2019); Irvine and Brundrett (2019); Shaked and Schechter (2017)
Year level coordinators	Bryant et al. (2020); De Nobile (2019); Irvine and Brundrett (2019);
Pedagogical leaders	Bryant et al. (2020); Lipscombe et al. (2021); Shaked and Schechter (2017)

As seen in Table 2, researchers within the field of middle leadership have presented positions which are ones that are not the principal. The principal would be deemed the executive or senior leader of the school (Grootenboer, 2018). Department heads tend to feature as a middle leadership role in secondary school settings (e.g., Bennett et al., 2007). Lipscombe et al. (2021) claimed that since the foundational review of middle leadership in secondary schools by Bennett et al. (2007), middle leadership positions have diversified beyond that of subject or curriculum leader. Interestingly, mathematics leaders have not been specifically identified as a middle leadership position. Grootenboer and Larkin (2019) reported, however, their work with a middle leader of a year level group who took on responsibility for mathematics leadership in a study in which they participated.

Middle leadership positions in schools have been identified in the literature, as have understandings of how middle leadership is practised in school along with their positionality as leaders within those educative settings.

2.4.4 Practice and Positionality of Middle Leaders

In recent times, middle leadership in schools has been investigated as a practice rather than a study into the characteristics or traits of individual leaders (Grice et al., 2023; Grootenboer, 2018). Researchers have presented insights into middle leadership by focusing on it as a form of practice. Middle leadership has been presented as a dynamic and context-specific process that involves a range of practices and activities (Bennett et al., 2003; De Nobile, 2018; Edwards-Groves et al., 2019). Recent Australian studies have drawn on the use of the *theory of practice architecture* (TPA) to investigate middle leadership, using the verb "leading" when discussing the concept, referring to it as "middle leading" (e.g., Edwards-Groves et al., Grice, 2019; Grootenboer et al., 2017; Lipscombe et al., 2021).

Framing middle leadership as a practice acknowledges that it is not fixed or static activity but rather it is enacted in and between spaces of influence and change (Bennett et al., 2003; Lipscombe et al., 2021). Middle leading activity has also been understood to be a form of practice that is shaped by the context in which it occurs (Edwards-Groves et al., 2019; Grootenboer et al., 2017). In middle leading studies that use TPA as the theoretical lens, there are references to *conditions* that have potential to shape middle leading practice in school sites (e.g., Edwards-Groves et al., 2016; Grice, 2019; Grootenboer, 2018; Lipscombe et al., 2021).

According to Edwards-Groves et al. (2016), conditions have the potential to enable the work of middle leading, and those same conditions can also constrain middle leading practice.

As a leadership role within its own and with its use of “middle” in its naming convention, middle leading is seen as being operationalised *between* the executive leaders (e.g., principal, deputy principal) and the classroom teachers within school sites (Ainsworth et al., 2022; Bennett et al., 2003; Bryant et al., 2020; Edwards-Groves et al., 2019; Grice, 2019; Grootenboer, 2018). That positionality of middle leaders means that they engage practice that has them *working up* influencing the executive leaders in their schools, and they engage in practice as they *work across* influencing the teaching peers (Bryant et al., 2020; Edwards-Groves et al., 2016; Grootenboer, 2018).

That positionality can also be described as the middle leaders working both vertically up to the executive and horizontally across with teaching colleagues within their school’s leadership system (Bryant et al., 2020). Due to their positionality, middle leaders are afforded opportunities to enact their activity as practising members of both groups within the school; as members of the leadership team and members of the teaching staff (Bennett et al., 2003; Edwards-Groves et al., 2019; Grootenboer, 2018). Edwards-Groves et al. (2016) claimed that middle leaders do not occupy a “peculiar space” (p. 372), but they are indeed practising members of both school groups. Lárusdóttir and O’Connor (2017) have reported though that being *separate from* whilst simultaneously *being part of* the teaching staff can cause middle leaders challenges, usually associated with isolation.

Middle leaders' positionality, however, does provide opportunities to practise leadership in and around classrooms (Grootenboer & Larkin, 2019). Their positionality has them close to teachers and their work (Bryant et al., 2020; Grootenboer et al., 2017). This, in turn, mediates conditions for them draw on school policy and improvement directions and use them to influence the activity of classroom teaching (Bennett et al., 2003; Edwards-Groves et al., 2019; Grice, 2019; Grootenboer, 2018). That means that the middle leaders can have more influence on classroom teaching than that which can be achieved by the principal (Edwards-Groves et al., 2019).

As middle leaders engage in the practice of their middle leading, it is also understood that they undertake several roles in their school sites.

2.4.5 Roles of Middle Leaders: Leading and Managing

Middle leading practice has been characterised by roles and activities interpreted as leading and managing (De Nobile, 2018; Lipscombe et al., 2021). De Nobile (2018), in this literature review that proposed a model of middle leadership in school settings, offered ways of thinking about those activities in term of leading and managing. Managing roles are those activities that are concerned with "managerial work" focused predominately on administration and organisation. Leading roles are understood as "leadership work" that influence and shift ways that others in the school behave and think (De Nobile, 2018, p. 403). Leading and managing are deemed important to middle leading practice.

Examples of leadership work that is practised by middle leaders has been offered by several authors. That has focused mostly on the leadership of teachers' professional learning in school sites (Bennett et al., 2007; Carter, 2016; De Nobile, 2018; Edwards-Grove et al., 2016; Grootenboer & Larkin, 2019; Grootenboer et al., 2017). That leadership work has been focused on opening spaces for teachers to come together to collaborate on developmental work focused on the improvement of teaching (Edwards-Groves et al., 2019). Acting as a mediator of teacher agency within those collaborative spaces has also featured in the literature (Lipscombe et al., 2020; Lipscombe et al., 2021).

Managerial work that is enacted by middle leaders includes attention to operational matters of the school functioning (Carter, 2016) that see the middle leader developing procedures and protocols so that school operational tasks can be completed (De Nobile & Ridden, 2014). Other aspects of management include the creation of databases for recording keeping (De Nobile, 2018). Managing human and physical resources forms another aspect of the managerial work enacted by middle leaders (De Nobile, 2018; De Nobile & Ridden, 2014; Grootenboer & Larkin, 2019). Part of their work is also focused on financial management (Irvine & Brundrett, 2019).

Lipscombe et al. (2021), through their recent review of middle leadership literature, claimed that there are tensions between leading and managing, with managerial work taking precedence in their practice. They claimed that this was due to a division of labour in school sites that saw principals delegating tasks to middle leaders (Lipscombe et al., 2020). An increased focus on school improvement and the accountability associated with that has influenced this work on managerial aspects of middle leading practice (Grice, 2019). Lárusdóttir and O'Connor (2017) claimed that middle leaders in their study positioned themselves more as managers than leaders and did not connect with their work as being influential.

Middle leaders, working on practice that is both influential and managerial between principals and classroom teachers require the support of the executive leader in their school sites.

2.4.6 Importance of Principal Support

Principal support is vital for middle leaders because it helps them to effectively carry out their responsibilities as they enact their work directed at the goals of the school (De Nobile, 2018; Hammersley-Fletcher & Kirkham, 2007; Lipscombe et al., 2021). That is especially important if principals want the middle leader to engage in activity that is more leadership-focused and influential in nature, rather than engagement in too much managerial work (Edwards-Groves et al., 2016; Lipscombe et al., 2021).

Middle leaders' practice is impacted by and benefits from the support of principals (Lipscombe et al., 2021). Principals can influence middle leaders' practice because of their senior leadership position with the school (Gurr, 2019). The potential influence of middle leading practice depends on support from principals and other executive leaders in schools (Lipscombe et al., 2020).

As senior leaders, principals play an important role in supporting middle leaders by firstly sustaining those middle leading roles within their schools (De Nobile, 2018). Principals can mediate support when they establish role clarity about middle leaders' work that captures it as practice focused on improvement within the school setting (Bryant et al., 2020; Farchi & Tubin, 2019; Gurr, 2019). It can be challenging for middle leaders when there is a lack of role clarity (Drysdale et al., 2016; Farchi & Tubin, 2019).

The improvement focus, that is important for role clarity, is concerned with the ways that middle leaders are usually charged with leading teacher professional learning with the support of the principal (Farchi & Tubin, 2019). Middle leaders have high potential to influence teachers' professional development when they have the support of the principal to do so (De Nobile, 2018;

Edwards-Groves et al., 2019; Gurr & Drysdale, 2012). The principal influences that teacher development by establishing structures that create the conditions for middle leaders to engage in the developmental work facilitated through their leadership of site-based professional learning (Bryant et al., 2020; Edwards-Groves et al., 2019).

An important structure that principals have authority with concerns appropriate time allocation for the middle leader to enact their role (Bennett et al., 2003; De Nobile, 2018; Grootenboer, 2018; Gurr, 2019). Middle leaders need time as a condition that enables them to enact their leadership and classroom teaching responsibilities (Gurr, 2019). Principals sourcing time for middle leaders becomes especially important when the middle leader needs to collaborate with teaching peers as a way of influencing professional learning (Lipscombe et al., 2020). Grootenboer (2018) reported that although having enough time to undertake the middle leadership role is important, it is the quality of time and how the middle leader manages that time that is more critical.

Principals provide important support that enables conditions for middle leaders to enact their work. One way for middle leaders to enable conditions for themselves is through the development of relational trust.

2.4.7 Relational Trust

Relational trust has been positioned as a key resource that school leaders use to improve practices within school settings (Bryk & Schneider, 2003; Cranston, 2011). It is deemed a necessity when leadership of school-based development is required, within which relational trust is fostered and maintained through trusting relationships between stakeholders (Edwards-Groves

& Grootenboer, 2021). Relational trust is required when school leaders wish to secure and then influence change in school sites (Lipscombe et al., 2020).

There have been attempts to define relational trust within the educational leadership literature. Using surveys with school stakeholders over a four-year period, Bryk and Schneider (2003) identified relational trust as relationship between school stakeholders, mediated by confidence in the intent and actions of others. They identified that relational trust, realised through relationship, was characterised by *respect* (belief that others will act with good intent and acknowledgement of others' knowledge and skills), *competence* (recognition that others have the required knowledge and skills, and access to resources required to enact their role), and *regard* (belief that others care about self and others' well-being). Cranston (2011) highlighted the importance of recognising that relational trust is a dynamic and evolving process, rather being in a static or fixed state. Trust can be built or eroded over time based on the actions and behaviours of individuals and groups within the school community.

2.4.7.1 Relational Trust and Middle Leadership

Research about the place of relational trust in educational leadership literature has focused primarily on how the principal builds trust as the executive leader (Cranston, 2011). The focus has been on how principals nurture relational trust, needed to engage staff, students, and families in the core business of the school concerned with learning and teaching (Bryk & Schneider, 2003). That focus on the principal has meant that the nature and role of relational trust within the work of middle leaders has been neglected (Edwards-Groves & Grootenboer, 2021). With their leadership practised close to classrooms, it is important that we have knowledge about how relational trust it is realised within and by the work of middle leaders (Edwards-Groves et al., 2016). The neglect of research focus on relational trust and middle leadership could be explained by the fact that middle leadership as a construct in educational leadership research is relatively new when compared to research about principal leadership (Bennett et al., 2007; Grootenboer, 2018; Lipscombe et al., 2021).

In response to the lack of knowledge about how relational trust is realised within middle leading practice (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2018), there has been growing interest in understanding how that trust is evidenced within the leadership activity of middle leaders. Most of the recent developments of work in relational trust and middle leading has been led by Christine Edwards-Groves (Australia), Peter Grootenboer (Australia), and Karin Rönnerman (Sweden). Their research work has been foundational in understanding relational trust and its nature and function within middle leadership in educative settings.

Edwards-Groves et al. (2016) claimed that an understanding of relational trust is required if knowledge of practices in schools is sought. They added that relational trust is a requirement in any school where site-based professional learning takes place. As evidenced in previous literature sources (e.g., Bryk & Schneider, 2003; Cranston, 2011), relational trust plays a critical role in enabling sustainable change led by school leaders. Fostering of relational trust is required for the purpose of opening and preserving communicative spaces within which teachers' professional learning takes place (Edwards-Groves & Grootenboer, 2021). Middle leaders are best positioned to foster relational trust due to their enactment of leadership between the principal and executive leadership team and classroom teachers (Edwards-Groves et al., 2016). The middle leader's positionality mediates that relationality for professional learning (Grootenboer et al., 2019).

Using research tools available through practice architecture theory, Edwards-Groves, Grootenboer, and Rönnerman proposed relational trust as a multi-dimensional construct enacted within middle leading (Edwards-Groves et al., 2016). That research has been important considering that Turner (2007) claimed that the relational aspects of middle leadership has been under-theorised. Edwards-Groves et al. supported their relational trust research using data generated from accounts provided by middle leaders, principals (and other executive leaders), classroom teachers and district leaders working in or associated with primary and secondary school settings.

The relational construct, offered by Edwards-Groves and colleagues, is characterised by five distinct, yet interrelated dimensions (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2021). Those dimensions have been named in specific ways: *interpersonal trust* (demonstrations of empathy, respect, and confidence building with teaching colleagues); *interactional trust* (launching and sustaining professional learning spaces that are characterised by collaboration, open dialogue and decision-making); *intersubjective trust* (demonstrations of collegiality through participation in opportunities for the development of shared language and understanding); *intellectual trust* (enactment of appropriate levels of content and pedagogical knowledge associated with the developmental work focus); and, *pragmatic trust* (demonstrations of leadership that leads development that is understood to be achievable, relevant, and practical).

I now turn to elaborations of the relational trust dimensions and how they have been conceptualised by Edwards-Groves et al (2016) and Edwards-Groves and Grootenboer (2021).

2.4.7.2 Interpersonal Trust

Interpersonal trust is fostered when the middle leader enacts an approach to leadership that is characterised by mutual respect for the people with whom they work (Edwards-Groves & Grootenboer, 2021). That mutual respect is realised when the middle leader nurtures dispositions that support the development of respectful relationships between their teachers and other staff. Those dispositions have been identified as empathy and care, along with trustworthiness (Edwards-Groves et al., 2016). Interpersonal trust is nurtured when middle leaders demonstrate genuine care for and interest in the colleagues with whom they work and for the developmental work that is of focus within professional learning leadership.

The interpersonal dimension of relational trust is demonstrated through the ways that middle leaders relate to their colleagues through respectful, non-judgemental interactions (Edwards-Groves et al., 2016). It is further realised when the middle leader enacts that care and empathy with the purpose of instilling confidence about the developmental work. Interpersonal trust is further fostered when the middle leader is responsive to the professional learning needs of teachers and the ways that teachers approach their own learning (Edwards-Groves & Grootenboer, 2021). That dimension of trust is also enacted through middle leaders' activity when they demonstrate their own enthusiasm for the developmental work which is leveraged to instil a sense of belonging and purpose amongst teaching peers (Edwards-Groves et al., 2016).

2.4.7.3 Interactional Trust

Successful teacher professional learning is mediated when middle leaders pay attention to opportunities that develop interactional trust (Edward-Groves & Grootenboer, 2021). That dimension concerns establishing and maintaining secure spaces for dialogue that is focused on the development work agenda. Interactional trust is nurtured through middle leadership efforts that create democratic and safe communicative spaces that allow for collaborative and meaningful professional dialogue about which the professional learning is focused. When middle leaders open those spaces up in ways that keep conversations deliberate and strategic, opportunities to foster interactional trust with teachers can surface (Edwards-Groves et al., 2016).

Interactional trust between middle leaders and teachers can be realised when opportunities are provided ways for ideas to be shared freely, and when alternative positions and viewpoints are sought as a way of developing deeper meaning for the developmental work that takes place through the professional learning (Edwards-Groves et al., 2016). An important aspect of interactional trust is concerned with dialogic spaces where the middle leader communicates effectively, allows for others to share the leading practice, and supports the identification of solutions to issues with their peers (Edwards-Groves & Grootenboer, 2021). Interactional trust is fostered when the middle leader creates conditions for teachers to develop understanding of the nature and purpose of the professional learning through deliberate, sense-making conversations (Edwards-Groves et al., 2016).

2.4.7.4 Intersubjective Trust

The dimension of intersubjective trust has potential to be fostered when middle leaders engage with teachers in ways that build relationships through collegiality and cooperation (Edwards-Groves et al., 2016). Middle leaders demonstrate intersubjective trust when they invest in the developmental work alongside their teachers whose professional learning they lead, developing a sense of ‘withness’ and collaboration (Edwards-Groves et al., 2016). That dimension is understood to be developed when the middle leader creates a sense of community amongst teachers, sharing a common goal about and purpose for the professional learning (Edwards-Groves & Grootenboer, 2021).

Intersubjective trust is also fostered by middle leaders when they provide opportunities for the development of shared language and understanding of the professional learning focus (Edwards-Groves & Grootenboer, 2021). When middle leaders create conditions for democratic, open, and collaborative spaces, that shared language and understanding has potential to be realised. As a means of creating those spaces, middle leaders can establish rules for participation by establishing shared norms with colleagues that make the professional learning collaborative and cooperative (Edwards-Groves & Grootenboer, 2021).

2.4.7.5 Intellectual Trust

Middle leaders demonstrate intellectual trust when they present themselves in ways that are perceived to be knowledgeable and confident in their ability to lead the developmental work through professional learning (Edwards-Groves & Grootenboer, 2021). Intellectual trust is nurtured through middle leading practice when the middle leader enacts the professional knowledge associated with their curriculum area or the developmental work focus (Edwards-Groves et al., 2016). That enactment of content and pedagogical content knowledge can support the middle leader in extending developmental work in ways where teachers are challenged to develop their practice and pedagogical reasoning.

Intellectual trust is conveyed through the middle leader's enactment of their own expertise and wisdom. That form of relational trust is also nurtured when they can recognise and draw on that wisdom and expertise from teachers when engaged in professional learning situations

(Edwards-Groves & Grootenboer, 2021). When the middle leader presents themselves as one who understands the practice and knowledge associated with the developmental work focus, then opportunities for building intellectual trust can be fostered (Edwards-Groves et al., 2016). For professional learning success, intellectual trust must be conveyed by the middle leader within the developmental spaces where they interact with teachers (Edwards-Groves & Grootenboer, 2021).

2.4.7.6 Pragmatic Trust

The final of the relational trust dimensions identified by Edwards-Groves et al. (2016) is that of pragmatic trust. This form of trust is fostered by middle leaders when they situate their leadership work in contexts that are relevant to the teachers' professional learning (Edwards-Groves & Grootenboer, 2021). When a middle leader engages teachers in tasks, that are intended to mediate teacher professional learning, they have opportunities to build pragmatic trust when the tasks are practical and relevant to teachers' work in classrooms. Nurturing pragmatic trust can also occur when the middle leader offers practical ways of working on the developmental focus that drives the teachers' professional learning.

Middle leaders enact pragmatic trust when they set learning agendas that meet the teachers' professional learning needs with whom they work (Edwards-Groves et al., 2016). It is also nurtured when the middle leader sets reasonable timelines for the change focus, and that cycles of change and the improvement actions associated with those change cycles are sensible and achievable. Middle leaders can develop pragmatic trust with their teachers when strategic decision-making processes are shared, and the reasons for the change process are articulated and justified (Edwards-Groves & Grootenboer, 2021). When developing pragmatic trust, it is important that the middle leader makes explicit what is being worked on, explains reasons for the developmental work, facilitates professional learning that is situated in the practicalities of teachers' work in classrooms, and sets a professional learning agenda that is timely and achievable (Edwards Groves et al., 2016; Edwards-Groves & Grootenboer, 2021).

The dimensions of relational trust proposed by Edwards-Groves and colleagues are interconnected and interdependent. Building and maintaining relational trust is an ongoing process that requires attention and effort on the part of middle leaders. When middle leaders can cultivate relational trust in these dimensions, they create conditions for collaborative developmental work with their colleagues (Edwards Groves et al., 2016; Edwards-Groves & Grootenboer, 2021).

I now move to a discussion of literature concerning mathematics leadership in primary school settings.

2.5 Mathematics Leadership in Primary School Settings

There is limited available research literature that focuses on the role of the mathematics leader in Australian primary school contexts (Cheeseman & Clarke, 2006; Downton et al., 2022; Roche et al., 2020). There is a need for further research in the leadership of mathematics in schools, particularly concerned with mathematics teaching and learning reforms (Lamb, 2010). In 2010, a special issue edition of the *Mathematics Teacher Education and Development* journal (MERGA) was devoted to educational leadership and mathematics learning in Australasian schools. Despite stories that detailed leaders and teachers working closely to improve teacher and student knowledge, the focus tended to be on principal leadership. That echoes the prominence of principal leadership in educational leadership research (Carter, 2016; Edwards-Groves & Grootenboer, 2021; Grootenboer et al., 2017; Lipscombe et al., 2021) and as the focus of the leadership factor of project sustainability (e.g., Datnow et al., 2005; Goos et al., 2018; King, 2011; Saito et al., 2012; Tirosh et al., 2015; Zehetmeier, 2017; Zehetmeier & Krainer, 2011).

Since 2010, the mathematics education research community in Australasia has started to pay attention somewhat to mathematics leadership as a form of middle leadership. That growing literature within Australia has been provided by members of the MERGA community (e.g., Cheeseman & Clarke, 2005; Copping, 2022; Downton et al., 2022; Driscoll, 2017, 2021; Faragher et al., 2014; Sexton & Downton, 2014; Vale et al., 2021). Some other research literature is available from the United States (Bolyard & Baker, 2021; Fennell et al., 2013; Jackson et al., 2015); the United Kingdom (e.g., Millett & Johnson, 2004); and New Zealand (e.g., Higgins & Bonne, 2011; Higgins et al., 2007).

Within those different sources, researchers have used different titles for the mathematics leadership role. I now present information about those titles for this curriculum leadership role.

2.5.1 Defining Mathematics Leadership

Authors writing within the field of mathematics leadership have offered several definitions for that form of leadership. A theme across the definitions is that the mathematics leadership role is understood as a formal position within the school leadership system, with the leader charged with improving mathematics learning within the school (Bolyard & Baker, 2021; Copping, 2022; Downton et al., 2022; Fennell et al., 2013; Jackson et al., 2015; Jorgensen, 2016; Millett & Johnson, 2004; Sexton & Downton, 2014). That learning development focuses on teacher professional learning (Downton et al., 2022; Fennell et al., 2013; Jorgensen, 2016; Sexton & Downton, 2014), students' mathematics learning (Copping, 2022), and the whole school system's learning of mathematics education (Bolyard & Baker, 2021; Jackson et al., 2015)

Millett and Johnson (2004) claimed the mathematics leader was the *primary source* of teacher professional development in primary schools. Jorgensen (2016) reported that it was the mathematics leader who played the role of *boundary crosser* as they support the enactment of mathematics teaching in classrooms as captured in schools' vision statements for mathematics education. Mathematics leaders have been described as teachers with formal responsibilities for improving students' mathematics learning through the leadership of school processes focused on the leadership of teacher teams (Copping, 2022; Vale et al., 2021). Mathematics leaders reportedly support teachers with planning and curriculum development (Downton et al., 2022; Driscoll, 2017; Sexton & Downton, 2014; Vale et al., 2021); leading the development of teaching and assessment practices (Vale et al., 2021); and affecting the resourcing of materials required for mathematics teaching and learning (Corbin et al., 2003; Copping, 2022; Jorgensen, 2016).

Several authors have specifically referred to the mathematics leadership role as that of *change agent* (Corbin et al., 2003; Millet & Johnson, 2000, 2004; Jorgensen, 2016). Those authors used that term to focus on mathematics leaders' change leadership in terms of shifting teachers' and students' practices for and about mathematics education. As they enact that change agent role, the mathematics leader acts as a mediator between the change initiative and the teachers who are expected to implement the changes as highlighted within the reform (Millett & Johnson, 2004).

Some studies have suggested that another defining characteristic of the mathematics leadership role is that of relationship builder around mathematics education (Bolyard & Baker, 2021; Corbin et al., 2003; Driscoll, 2022; Fennell et al., 2013; Higgins et al., 2007). The image of the mathematics leader as a nurturer and negotiator of relationships, specifically with teachers, has been offered through general claims drawing on the complexity of relationships associated with mathematics education in primary school spaces (Driscoll, 2022; Fennell et al., 2013).

I now turn to a discussion where I identify the titles that have been associated with the mathematics leadership role within the literature.

2.5.2 Mathematics Leadership Titles in Primary School Settings

In the previous section of this chapter, I provided the titles used within literature concerning middle leadership. As was evident, the titles for that leadership position were varied. As mathematics leadership is the focus of my study, I also note the position titles used in the literature for that school leadership position. Those titles and the sources in which they are found are presented in Table 3.

Table 3

Mathematics Leader Titles Found in Literature Sources

Mathematics leadership role title	Literature sources
Elementary/Primary mathematics leader	Copping (2012); Fennell et al. (2013); Jackson et al. (2015)
Elementary mathematics specialist	Bolyard and Baker (2021)
Mathematics middle leader	Roche et al. (2020)
Mathematics/numeracy coordinator	Cheeseman and Clarke (2006); Clarke et al. (2005); Corbin et al. (2003); Millet and Johnson (2004); Thomas and Ward (2006)
Mathematics subject leader	Cheeseman and Clarke (2006); Millet and Johnson (2004)
Numeracy lead teacher	Higgins and Bonne (2011); Pritchard and McDiarmid (2006); Thomas and Ward (2006)
School mathematics leader	Downton et al. (2022); Driscoll (2017); Sexton and Downton (2014)
Teacher leader	Gaffney and Faragher (2010)

A review of literature shows that there are also various names of the mathematics leadership role, and in one source (e.g., Cheeseman & Clarke, 2006) two different titles were used to name the leadership position. It is recognised that this is not exhaustive, but the data within the table shows that for the role, authors tend to use the term *mathematics* or *numeracy* as part of the title. Mathematics tends to be used in the USA (Bolyard & Baker, 2011; Fennell et al., 2013; Jackson et al., 2015) and Australia (Downton et al., 2022; Driscoll, 2017. Roche et al., 2020), and in recent times, the term *numeracy* tends to be included in the title of the role in the United Kingdom and New Zealand (Higgins & Bonne, 2011; Pritchard & McDiarmid, 2006; Thomas & Ward, 2006). The title *School Mathematics Leader* is also a recent title for the role within Australasian literature (Downton et al., 2022; Driscoll, 2017; Sexton & Downton, 2014).

Recognising that this study was about the work of mathematics leaders in Victorian Catholic school settings (as articulated in the previous chapter), and that in the CTLM project the name of *School Mathematics Leader* (Sexton & Downton, 2014) was assigned to the position, the title of School Mathematics Leader is used throughout the thesis.

2.5.3 Distinguishing Mathematics Leaders from Mathematics Coaches

Before I engage further with the review of literature about School Mathematics Leaders, it is important that I outline that I am not focusing on *mathematics coaches* (e.g., Anstey & Clarke, 2010; Campbell & Malkus, 2014; Gibbons & Cobb, 2017). Although mathematics leaders and mathematics coaches engage in similar professional learning leadership in that they work alongside teachers with the aim of developing mathematics instruction, and that that work with teachers endures for periods of time beyond one-day workshop style professional development

(Campbell & Mallkus, 2014), there are important distinctions between these two forms of leadership.

Mathematics coaches tend to be employed by educational districts and/or sectors and therefore may work in a selection of schools across a district or region (Anstey & Clarke, 2010; Gibbons & Cobb, 2017). That means that mathematics coaches may come from outside of the locale in which the schools are positioned, meaning they are less versed in the cultural context in which the teachers work. Mathematics coaches may not be so familiar with work that teachers enact through their day-to-day teaching in school settings (Gibbons & Cobb, 2017).

The most important distinction is that the mathematics coach tends not to be regarded as a member of the school staff, as they are not employed as staff members within the school. That is unlike the mathematics leader who holds a middle leadership position within the school leadership system and has a tenured position within the school staffing roster (Bolyard & Baker, 2022; Corbin et al., 2003; Copping, 2022; Driscoll, 2017; Millett & Johnson, 2004; Sexton & Downton, 2014). As a member of staff, the mathematics leader is expected to also undertake other duties and responsibilities like the other staff employed at the school. If the mathematics leader has leadership duties along with classroom teaching responsibilities, then they would meet the definition of middle leader as identified by Edwards-Groves et al. (2019) and Grootenboer (2018).

To differentiate that further in the context of my study, an example of a mathematics coach would be the School Advisor Mathematics (SAM) who supported the activity of the School Mathematics Leaders during participation in CTLM. The SAMs were not tenured staff members within the schools in which they worked, as they were employed by CEOM as the governing body of the Melbourne Archdiocesan schools.

With that delineation between mathematics leaders and mathematics coaches, I now turn to titles for the mathematics leadership position in primary school settings as found in literature within the field.

2.5.4 Mathematics Leaders as Middle Leaders

Within the middle leadership literature, the role of the School Mathematics Leader tends to be ignored as a formal middle leadership position (De Nobile, 2018; Lipscombe et al., 2021). If we take the Lipscombe et al. (2021) definition of a middle leader, then the School Mathematics Leader would fulfill the criteria established by those authors. The School Mathematics Leader role is a formal one within the school leadership system (Driscoll, 2017; Sexton & Downton, 2014); they are positioned between the school executive leaders and classroom teachers (Jorgensen, 2016); and their role has them as the primary source of professional development for teachers (Millett & Johnson, 2004).

Drawing on those sources, there is evidence that determines the School Mathematics Leader as a middle leader position within school sites.

2.5.5 Roles of School Mathematics Leaders: Leading and Managing

The mathematics leader in primary schools has opportunities to mediate change and influence the teaching practices of classroom teachers (Driscoll, 2017; Lamb, 2010; Millett & Johnson, 2004). That, however, depends on how the role is organised and understood within the school setting (Higgins & Bonne, 2009). If we take the classifications provided by De Nobile (2018), concerning the roles of middle leading practice as being leadership work and managerial work, then School Mathematics Leaders' work, as a form of middle leadership, can be understood in a similar way. Much of what is known about the leading and managing work of mathematics leaders comes from research of their activity during participation in professional development projects (e.g., Cheeseman & Clarke 2005; Driscoll, 2017; Vale et al., 2021).

During projects, School Mathematics Leaders engage in several roles that have seen them enact activity that would be deemed leadership work. Some of that leadership work has focused on the provision of site-based professional learning (Cheeseman & Clarke, 2005; Copping, 2022; Driscoll, 2017; Fennell et al., 2013; Gaffney & Faragher, 2010; Millet & Johnson, 2004; Sexton & Downton, 2014; Vale et al., 2021). It has been reported that the purpose of that professional learning concerning influencing teachers' pedagogical content knowledge (Copping, 2022; Cheeseman & Clarke, 2006; Driscoll, 2017; Downton et al., 2022), as well as teachers' mathematical content knowledge (Downton et al., 2022).

Gaffney and Faragher (2010) highlighted the need for mathematics leaders to plan and facilitate quality professional learning as part of their leadership work. That would be important considering that the situations in which the mathematics curriculum leader works are highly influential on how they can provide and facilitate professional learning for classroom teachers (Driscoll, 2017; McNamara & Corbin, 2001; Millett & Johnson, 2004).

As part of that professional learning leadership, School Mathematics Leaders have used analysis of assessment data, with focus on analysing student work (Vale et al., 2021). That assessment focus has supported teachers' understanding of assessment strategies (Jorgensen, 2016), and extended to mathematics leaders' self-reporting of leadership of moderation practices as identified by Vale et al. (2021).

That happened through leadership work that focused on developing teacher planning practices (Cheeseman & Clarke, 2005; Sexton & Downton, 2014), and creating opportunities for teachers to engage in dialogue through staff meetings and other collaborative settings like informal conversations (Cheeseman & Clarke, 2005; Higgins & Bonne, 2011; Sexton & Downton, 2014). Mathematics leaders also engaged in teachers in dialogue activity by providing in-classroom support for teachers through co-teaching experiences (Cheeseman & Clarke, 2005; Driscoll, 2017; Higgins & Bonne, 2011).

Developing a shared understanding for mathematics teaching with staff was also work enacted by mathematics leaders during projects (Higgins & Bonne, 2011; Sexton & Downton, 2014). Roche et al. (2020) reported findings from survey data generated with primary and secondary mathematics leaders, claiming that School Mathematics Leaders hold a vision for mathematics. They positioned that vision as aspirations espoused by those leaders of mathematics (Roche et al., 2020). It was unclear, however, how those mathematics leaders used their visions within their mathematics leadership activity.

Mathematics leaders have also engaged with management aspects of their leadership. Cheeseman and Clarke (2005) identified that the mathematics leaders who engaged in the Early Numeracy Research Project (ENRP), enacted several managerial tasks that were important to the role. Cheeseman and Clarke grouped those into themes of: organisation and management (focused largely on the administration tasks that featured as part of the role, e.g., information dissemination for classroom teachers); management of teaching and learning resources (included the creation, purchase and organisation of resources, as well as leading parents who volunteered their time to create such teaching materials); and, documentation of leadership work (concerned the compiling information for school review processes and completing documentation that related specifically to the project).

Other managerial work enacted by School Mathematics Leaders has included organisation and preparation of documentation used within the school such as timetables (Higgins & Bonne, 2011) and the distribution of documents to staff (Cheeseman & Clarke, 2005; Higgins et al., 2007). The management of mathematics resources for has featured as an enduring aspect of managerial work enacted by School Mathematics Leaders (Cheeseman & Clarke, 2005, 2006; Corbin et al., 2003; Driscoll, 2017; Higgins et al., 2007; Millett & Johnson, 2004; Vale et al., 2021). As the accountability measure became more prominent in the work of teaching in the early 2000s, the organisation of routines for assessment data collection and management formed part of the managerial work of mathematics leaders, with that working continuing in recent times (Cheeseman & Clarke, 2006; Copping, 2022; Corbin et al., 2003).

2.5.6 Tensions Associated with Mathematics Leadership Activity

The leadership of the mathematics education in primary schools is complex in nature (Cheeseman & Clarke, 2005; Millett & Johnson, 2004). That is due to the multi-faceted nature of the role (Cheeseman & Clarke, 2006), especially considering that those leaders can be responsible for administrative tasks, liaison and communication with school members and external personnel, and the provision of professional learning for classroom teachers (Cheeseman & Clarke, 2005; Higgins et al., 2007; Millett & Johnson, 2004). Another contributing factor is that a multiplicity of roles is typical for those teachers who undertake the mathematics curriculum role (Higgins & Bonne, 2009, 2011).

Several tensions associated with the leadership activity has been presented in some literature. Corbin et al. (2003), when working with mathematics curriculum leaders in an improvement project in the United Kingdom, noted that those leaders experienced challenges with the role related to the management of resources, offering appropriate subject and pedagogy advice to classroom teachers, managing data related to student learning, and managing staff members. Cheeseman and Clarke (2006) reported that after participation in a school improvement project, the further provision of professional learning for classroom teachers can be daunting for the mathematics curriculum leader. That could be because the supports offered through the improvement project are no longer available to those leaders (Heirdsfield et al. 2010). Mathematics curriculum leaders who hold the role with a lack of enthusiasm and who are not supported by their principals and community tend to relinquish the role (Millett & Johnson, 2000).

Lack of time allocation to undertake the mathematics leadership role has been reported in the literature (Cheeseman & Clarke, 2005; Gaffney et al., 2014; Millett & Johnson, 2000). That has been mostly attributed to the added responsibility of classroom teaching requirements (Gaffney et al., 2014; Millett & Johnson, 2000). In other cases, it was due to a lack of funding provided for the role by the principal (Gaffney et al., 2014). Another reason for the lack of time that School Mathematics Leaders are afforded to undertake their leadership work could be the multiple roles that they are required to fulfill within their school sites (Higgins & Bonne, 2011).

It is agreed that the mathematics leader undertakes a formal position within the school leadership system, and that the focus of their work concerns the leadership of mathematics learning for all within the school system. Mathematics leaders have also been defined as the *primary source* of teacher professional development (Downton et al., 2022; Driscoll, 2017; Millett & Johnson, 2004), a *boundary crosser* in that they mediate the school vision for mathematics education with the reality of classroom practice (Jorgensen, 2016), and a *change agent* as they support innovations in mathematics teaching and learning through their leadership activity (Corbin et al., 2003, Millett & Johnson, 2000, 2004; Jorgensen, 2016). Images of the mathematics leader as *relationship negotiator* have also recently surfaced within the literature (Bolyard & Baker, 2021; Driscoll, 2022; Fennell et al., 2013).

Currently, clarity of knowledge that elaborates upon and extends those claims is needed by the mathematics education research community, particularly if the mathematics leader is positioned as the most immediate source of professional development and learning in schools (Downton et al., 2022; Driscoll, 2017; Millett & Johnson, 2004). If we consider the factor of continued professional learning as a mediator of mathematics project sustainability (Bobis, 2011; Kaur, 2015; Pritchard & McDiarmid, 2006; Saito et al., 2012; Warren & Miller, 2016; Zehetmeier, 2015), and if the mathematics leader is required to facilitate that professional learning, then further need for knowledge is imperative.

I now turn to a discussion of insights about teachers' mathematics professional learning, recognising they are positioned as middle leaders of teachers' mathematics professional learning.

2.6 Mathematics Professional Learning for Teachers

Teachers engage in professional development every day within and outside of their classrooms which is important considering the expectation that teachers will engage in ongoing professional learning (Borko, 2004; Cobb et al., 2018; Kaur, 2015; Loucks-Horsley et al., 2010; Opfer & Pedder, 2011). Professional development can happen through activities that see them reflecting on and discussing their practice with others (Gibbons & Cobb, 2017), engaging in processes that support meaning-making about student assessment data, participating in conferences and workshops, and engaging in dialogue with colleagues and students' families (Rösken-Winter et al., 2015). Professional development for teachers is associated with in-service programs, designed to meet the needs of teachers' changing roles, and has shifted in focus to professional development that occurs within context of teachers' school sites (Cobb & Jackson, 2015; Gibbons & Cobb, 2017; Kaur, 2015; Loucks-Horsley et al., 2010; Opfer & Pedder, 2011; Rösken-Winter et al., 2015; Sowder, 2007).

2.6.1 Defining Professional Learning

It appears that the terms *professional development* and *professional learning* are used interchangeably within the research literature (Anderson et al., 2008). Professional development tends to be the term used within the US literature sources (e.g., Borko, 2004; Cobb et al., 2018; Loucks-Horsley et al., 2010; Sowder, 2007) whilst outside of that territory, professional learning seems to be the preferred term (e.g., Bobis et al., 2020; Opfer & Pedder, 2011). However, Anderson et al. (2008) reported that a distinction needs to be made between the terms.

Professional development as a term refers to formal programs, including planned and focused activities, which build upon teacher professional learning (Anderson et al., 2008; Bobis et al., 2020; Faragher et al., 2014). In contrast, professional learning is seen as the growth in teacher practices and expertise which can be achieved through participation in those formal programs that constitute professional development. Professional learning can also occur during classroom experiences, professional reading, and activities like further study (Anderson et al., 2008; Bobis et al., 2020). Faragher et al. (2014) reported their distinction between the two terms, claiming that professional development refers to formal programs that require teachers to be removed from their regular work settings to engage in the learning, whilst professional learning captures the sustained process of professional growth that is achieved through regular work activity.

For the purposes of my thesis, I use the term professional learning because I am focusing on the activity of the School Mathematics Leaders within their school sites.

2.6.2 Characteristics of Mathematics Professional Learning

Borko (2004) viewed teacher professional development consisting of four distinct elements, which make up what she describes as a "professional learning system" (p. 4). Those components are the professional development program; the classroom teachers, who act as learners within the learning system; the facilitator, who guides the construction of knowledge and practices associated with the intended learning (in the case of my thesis, that would be the School Mathematics Leaders); and the context within which the professional development occurs (that would be the school setting in my study). Teachers' orientations to participate in professional learning within the school setting is influenced by what the school does in terms of facilitating access to that professional learning whilst providing support and encouragement (Opfer & Pedder, 2011).

In their review of literature concerning teacher professional learning, Darling-Hammond et al. (2009) stated that the nature and function of effective professional learning is characterised by four features. Those authors believed those characteristics act as basic principles that schools are advised to consider when designing professional learning for classroom teachers. The characteristics identified by Darling-Hammond et al. (2009) were: 1) intensive, on-going professional learning that is connected to practice; 2) a focus on student learning and addresses specific curriculum content and it associated pedagogies; 3) an alignment with school improvement priorities and goals; and 4) supportive structures for building strong working relationships between teachers.

Timperley (2008) in her review of the literature of professional learning found similar themes but also noted that for professional learning to be effective it requires: multiple opportunities for teachers to learn new information in environments that offer both trust and challenge; and the use of student assessment data to identify the learning needs of teachers. Timperley highlighted that educational leaders in schools are important for effective professional learning because they develop expectations for improving student outcomes and provide further opportunities for professional learning.

The principles that underpin effective professional learning for teachers are also important for mathematics professional learning. Those principles are also noted by authors in the mathematics education field (e.g., Clarke, 1994; Koellner et el., 2011; Sowder, 2007), however, some distinctions are evident within the literature associated with mathematics professional learning for teachers.

Garet et al. (2001) surveyed 1027 mathematics and science teachers and found that teachers reported significant positive effects in changes in knowledge, skills, and classroom practice when professional learning activities were focused on: content knowledge; opportunities for active learning; and coherence with other learning activities that align with teachers’ own goals. Specific to mathematics teachers, Clarke (1994) stated that professional learning should aim to address the needs and interests of the teachers involved in the learning, and there should be some aspect of choice given to the teachers. That point was reiterated by Fullan (2005) who nominated that element as essential as teachers must be involved in identifying what they need to learn professionally so that greater ownership of change in practices can occur.

Clarke (1994) included a principle for professional learning that relates to facilitators modelling mathematics teaching strategies, where classroom approaches and tasks are used with teachers. That modelling to teachers supports the development of a shared vision for task enactment in classrooms. According to Clarke, time for planning, reflection and feedback should also be a feature of professional learning so that the "wisdom of practice" (p. 44) is shared between the teachers engaged in the learning activities.

Lack of reflection on practices can impede teachers' development in sustaining mathematics practices that are highlighted in professional learning sessions (Pritchard & McDiarmid, 2006). When time and resources are not committed to opportunities for professional learning, including those that relate to teachers' mathematics planning opportunities, quality mathematics teaching is impeded (Lamb, 2010). Clarke also (1994) advocated for the promotion of goal setting by teachers for future improvement professional learning sessions as another important principle associated with mathematics professional learning.

2.6.3 Forms of Professional Learning in Mathematics

Knapp (2003) suggested that opportunities for professional learning can occur through several forms. These forms were grouped into the following themes: professional learning through formalised structures and activities (e.g., workshops, seminars, PD sessions; large scale development projects); site-based professional learning through teaching practice itself (when teachers investigate and draw conclusions about their daily work); professional learning that occurs in settings outside of practice (e.g., partnerships or networks with other educative organisations); and, professional learning through informal settings (e.g., reading journals, conversations with colleagues).

One of the forms of professional learning, according to Knapp (2003), is the learning through formalised means such as projects. In the Australasian context, there have been several successful, large-scale mathematics professional development projects which have included the *Early Numeracy Research Project* (ENRP) in Victoria (Clarke et al., 2002), the *Count Me In Too* (CMIT) project in New South Wales, and the *Numeracy Development Projects* (NDP) in New Zealand (Bobis et al., 2005). Another large-scale project in Victoria was the *Contemporary Teaching and Learning of Mathematics* project (Clarke et al., 2013a). Those projects have had some impacts on teacher learning as measured by instruments developed by the project researchers (Bobis et al., 2005; Clarke et al., 2013a). Bobis et al. (2020) claimed that there has been very little evidence of the sustainability of those projects within the Australasian literature and called for more studies into ways that such projects are sustained.

Other forms of professional learning can take place within the school setting through site-based professional learning. That focus on school-based professional learning for teachers has become more prominent in recent times, with Boylan (2018) claiming that the landscape of teacher professional learning has shifted to spaces that align more with teachers' actual work in classrooms. Boylan stated that that shift was understood as an international tendency that has emphasised the role of learning interactions amongst peers within the school. Some school-based activities involve observations in classrooms (e.g., McDonald, 2010; Olson, 2005).

Elmore (2004) contended that there were few opportunities for teachers to engage with observing, and being observed, when teaching in classrooms. However, Clarke et al. (2013a) found that demonstration lessons in mathematics, where teachers observed university lecturers teaching in primary school classrooms through a professional learning project, provided opportunities for teachers to reflect on observed practices. Peer observations, that includes the use of lesson study and co-teaching situations as site-based professional learning opportunities, can develop teacher knowledge of effective mathematics teaching (Gibbons & Cobb, 2017; Sanders, 2009; Saito et al., 2012).

With the movement to more opportunities for school-based professional learning, the leadership of that mathematics professional learning is required. To date, however, there are limited studies concerned with the leadership of mathematics professional learning within school sites (Boylan, 2018). Some evidence, however, is available about school-based mathematics professional learning that includes the use of mathematics coaches' use of productive coaching activities (Gibbons & Cobb, 2017). Those school-based professional learning situated in teacher classrooms is seen as an important form of teacher learning (Eden, 2018; Jackson & Cobb, 2013; Cobb et al., 2018; Saito et al., 2012).

Gibbons and Cobb (2017) found that teachers can be supported by the mathematics coach through professional learning activities focused on engagement in mathematics as a content area, using student work samples as formative assessment, using videos to analyse teaching practice, and engaging lesson study to support co-teaching and the modelling of effective practice. It is important to note though that when mathematics leaders use demonstration lessons, it can be a stressful experience for those leaders due to lack of proficiency in teaching mathematics or confidence in teaching in-front of peers (Millett & Johnson, 2004). Part of that situation also involves the role of trust in mathematics professional learning.

2.6.3.1 Trust and Mathematics Professional Learning

The role of trust in mathematics professional learning situations has generated some interest of late (e.g., Eden, 2018; Zheng et al., 2016). Drawing on evidence from a questionnaire used with 35 primary teachers in China, generating insights into the relationship between principal leadership practices and trust in mathematics professional learning communities, Zheng et al. (2016) found that when principals develop trust in those situations, teachers feel that they can participate in a shared sense of purpose for the developmental work. They warned that the use of only questionnaires was not enough to understand the role of trust in mathematics professional learning situations.

Eden (2018), writing from a CHAT perspective about three New Zealand primary school teachers' experience of trust in mathematics professional learning, claimed that as trust developed, teachers were more likely to engage in important discussions focused on student learning. Those discussions then mediated the development of further trust within the collaborative group setting. Eden claimed that moving from a focus on teachers' feelings to a shared object of children's learning can support the development of trust for risk-taking that is required in mathematics professional learning spaces.

2.6.4 Professional Learning and Mathematics Knowledge for Teaching

Recognising that professional learning for teachers should focus on improving subject knowledge and pedagogical knowledge with the purpose of impacting on students' mathematics learning outcomes (Borko, 2004; Darling-Hammond et al., 2009; Timperley, 2008), Koellner et al. (2011) argued that it was critical that mathematics professional learning attend to the development of teachers' *mathematical knowledge for teaching* (e.g., Ball et al., 2008). Mathematical knowledge for teaching has been described as the specialised knowledge that teachers need to engage with the work of teaching mathematics (Ball et al., 2008; Hill et al., 2005). By attending to that knowledge, as well as fostering a professional learning community and adapting the professional learning support to local interests and teacher needs, Koellner et al. (2011) believed that opportunities exist for ensuring that the teacher professional learning was sustainable.

Research interest in teacher knowledge can be historically traced back to the foundational work of Shulman (1986) that brought attention to the specialised knowledge that teachers require for effective teaching. Shulman highlighted the importance of subject matter knowledge (SMK) that was required for teaching and understood another form of knowledge that teachers used when teaching that SMK. Shulman's work led to understanding of a new construct in educational research known as *pedagogical content knowledge* (Stein & Nelson, 2003).

Shulman (1986) introduced that term of pedagogical content knowledge to capture the special kind of teacher knowledge that connects content with pedagogy. At that time, Shulman argued for a change in thinking that the only knowledge that teachers possessed concerned the two distinct categories of subject matter knowledge and pedagogical knowledge. Schulman (1986, 1987) elaborated on the intersection of content knowledge and pedagogical knowledge, as a way of making sense of the knowledge that teachers use to organise, represent and adapt topics, problems and issues for a diverse range of learner needs and interest, and present these for use in classrooms.

To characterise the professional knowledge that was required for teaching, Shulman (1987) presented categories for that professional knowledge. Seven major categories were developed that are: *general pedagogical content knowledge* (including knowledge of classroom management strategies); *knowledge of learners*; *knowledge of educational contexts* (including understanding classroom contexts, cultures, and school governance); *knowledge of educational purposes and values* (including historical and philosophical backgrounds to education); *content knowledge; curriculum knowledge*; and, *pedagogical content knowledge* (the unique professional knowledge owned by teachers) (Shulman, 1987). The first four categories were developed to address the general domains of teacher knowledge and were not the real focus of Shulman's work. The final three categories that are listed above were the ones which were the content-specific dimensions. It was these categories that Shulman used to distinguish the important knowledge that teachers require for effective teaching.

In the years since the work of Shulman (1986, 1987), the education research community has recognised that not only does this type of knowledge exist, but it also plays an important role in teaching and learning practices in school settings (Hill et al., 2008). To that end, Deborah Ball and colleagues in the US have spent much time extending on the ideas of Shulman (1986) by researching further the knowledge that teachers of mathematics require to teach effectively (e.g., Ball & Bass, 2000; Ball et al., 2008; Hill et al., 2004).

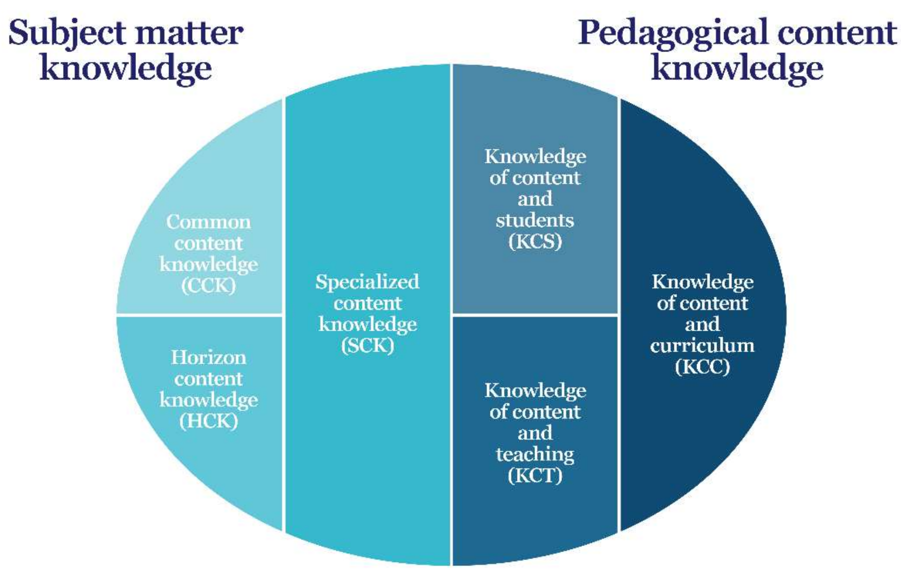
Ball and her colleagues contended that when teachers are engaged in the complex task of teaching mathematics, there is a demand for "a kind of deeply detailed knowledge of mathematics and the ability to use it" (p. 84). It is that focus on using pedagogical content knowledge in practice that drove Ball and her colleagues to develop the construct of *mathematics knowledge for teaching* (Ball et al., 2008; Hill et al., 2005).

Mathematical knowledge for teaching, as the knowledge required by teachers to carry out the work of teaching mathematics, supports teachers in the following practices: explanations of mathematical terms and concepts; interpretative skills that make sense of students' explanations, solutions and strategy uses; accurate and appropriate use of representations; amendment of limited treatment of mathematical ideas that might be found in teacher resource materials; and the provision of appropriate examples used for mathematical concepts, algorithms and proofs (Ball et al., 2008; Hill et al., 2005).

Ball et al. (2008) noted that the categories put forth by Shulman were theoretical and lacked empirical data. In response, they formulated a framework to establish categorisations that could be used for research purposes. That framework is presented below in Figure 2.

Figure 2

Representation of Mathematical Knowledge for Teaching (Ball et al., 2008)



Ball et al. (2008) developed that model to represent their refinement of Shulman's (1987) categories of teacher knowledge. The six categories are grouped using the terms *subject matter knowledge* (includes common content knowledge, horizon content knowledge, and specialised content knowledge) and *pedagogical content knowledge* (knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum). Those various components were described by Ball et al. (2008) as:

- *Common content knowledge*: knowledge of mathematics that is commonly shared by both teachers and other users of mathematics.
- *Horizon content knowledge*: an awareness of the topics that are found across the curriculum and knowing how these mathematics topics are related and developed across the span of the curriculum.
- *Specialised content knowledge*: knowledge about mathematics that is beyond what is expected of most educated adults.
- *Knowledge of content and students*: knowledge that combines knowledge about mathematics and knowledge of what and how students learn mathematics.
- *Knowledge of content and teaching*: knowledge that combines knowledge about mathematics and knowledge of and for teaching.
- *Knowledge of content and curriculum*: knowledge that combines knowledge about mathematics and knowledge of how mathematics knowledge is represented in curriculum and program materials.

Ball et al. (2008) stated that the different categories of mathematical knowledge for teaching are closely related but warned that the different categories should not be seen as discrete entities. Sullivan et al. (2009) called for mathematics professional learning to focus on all six components of mathematical knowledge for teaching (Ball et al., 2008), so that teachers develop capacities to develop tasks for use in mathematics learning experiences.

Hurrell (2013) critiqued the Ball et al. (2008) framework, claiming that the categories were too broad and lacked clear boundaries. As a result, Hurrell believed that that as a knowledge form, it was difficult to operationalise in the classroom and measure in research settings. Another critique has been that the framework represents an individualistic and decontextualised view of teacher knowledge, ignoring power and identity in shaping teachers' experiences and mathematics teaching practices (Gutiérrez, 2013). Despite criticisms, mathematical knowledge for teaching has been of use by Australasian researchers who have investigated teachers' professional learning (Beswick et al., 2016).

2.6.5 Reasons for Developing Teachers' Mathematical Knowledge for Teaching

Several reasons exist for the development of teachers' mathematical knowledge for teaching. The first and most important reason lies in the claim made by Hill et al. (2005) that if teachers engage in professional learning opportunities that focus on the development of mathematical knowledge for teaching, positive impacts on student learning outcomes can occur. Another important reason is reported by Ball et al. (2008) who claimed that teachers who do not understand the content that they are teaching are less likely to support student learning of that content. Those authors also contended that teachers need to know mathematics in useful ways like making sense of student thinking or understanding the best representations to use for mathematical concepts.

One goal for deepening teachers' knowledge of mathematics subject knowledge, as well as developing understandings of how students think mathematically, is to improve the instructional practices that teachers can use in the classroom (Borko, 2004). Clarke (2008) stated that the teacher's knowledge about mathematics teaching and learning is crucial for interpreting information that is found in curriculum frameworks that are required for use when teaching in the classroom. For teachers to teach mathematics effectively, which includes the provision of effective feedback to students (Sullivan, 2008), teachers need all knowledge types captured in the Ball et al. (2008) framework. Improved mathematical knowledge of teachers assists them when teaching, including the use of a variety of representations to support student learning and encouraging more purposeful discourse in classroom discussions (Warren, 2009).

Of particular interest is that the development of teachers' mathematical knowledge impacts on teacher affect, specifically confidence in teaching mathematics in classrooms. Warren (2009) observed that when teachers were engaged in learning about mathematics content knowledge, greater confidence to experiment in classrooms also developed. The knowledge that teachers possess is closely connected to their confidence levels and beliefs that they hold about how mathematics is taught and learned (Beswick et al., 2011).

2.7 Response to the Literature Review

The response to the literature is a concern about the limited research literature that constrains understanding within three of the four areas that formed content of the literature review. Project sustainability is an under-researched area within the mathematics education field

(Coburn et al., 2012; Tirosh et al., 2015; Zehetmeier, 2015; Zehetmeier & Krainer, 2011), and with what is known about the factor of school leadership, the focus is mostly on the principal and their post-project activity (Coburn et al., 2012; Datnow et al., 2005; Goos et al., 2018; Saito et al., 2012; Tirosh et al., 2015; Warren & Miller, 2016; Zehetmeier, 2015; Zehetmeier & Krainer, 2011).

The activity of the mathematics leader in primary school settings is largely neglected within the project sustainability literature with only brief references to how they lead the sustainability of reforms (e.g., Bobis, 2011; Datnow et al., 2005). That could be rationalised due to the under-theorised educational leadership area of middle leadership (Carter, 2016; De Nobile, 2018; Grootenboer et al., 2017), which I have argued for the place of mathematics leadership as a form of middle leadership in school settings.

Therefore, as a means of contributing to the research field, I have identified a gap in the literature. The research question was posed to guide my inquiry into knowing more about how the School Mathematics Leaders contributed to project sustainability as middle leaders through their professional learning. The research question was used to drive my inquiry, research design, and the presentation of my findings to contribute knowledge about mathematics leaders as a project sustainability factor of school leadership.

As a means of contributing to the field of mathematics education, addressing the gap that I have identified throughout this chapter, I have posed the following research question:

> *As middle leaders of site-based professional learning, how do School Mathematics Leaders contribute to the sustainability of mathematics teaching reforms in the years that followed participation in a large-scale school mathematics professional development project?*

2.8 Chapter Summary

In this chapter, I presented the methodology that I used to access relevant literature sources concerning the problematic of a lack of knowledge about how School Mathematics Leaders contribute to project sustainability through the middle leading of their professional learning leadership. I disclosed examples of search terms I employed, along with the names of the main databases I used to access literature sources. I articulated how that supported me in finding relevant sources to inform the literature review.

I stated that the topic of project sustainability is an under-researched area within the mathematics education field. The historical background of the topic was understood as institutionalisation that shifted to sustainability at the turn of the 21st century. Of late, factors of project sustainability have been developed as a way of understanding how project reforms are sustained in school settings. Those factors have been classified as internal and external factors. I presented insights about internal factors that are considered conditions within the school that include school leadership, staff turnover, school-based professional learning, and access to project resources. The external factors I discussed were district leadership, teacher networks, and high stakes testing regimes. I claimed that the research focus concerning the school leadership factors has been centred mostly on the leadership of principals, neglecting the work of middle leaders like School Mathematics Leaders.

I moved the literature review to insights that are available about middle leadership in school settings. This field was also presented as an emerging area of education leadership research that has been neglected in favour of studies that focus on the leadership of principals. The theorisation of middle leadership as a form of practice was presented, along with descriptions on the positionality of middle leaders in school settings. I also presented the roles and responsibilities of middle leaders using the classification of leadership work and managerial work. Insights from recent research activity about how relational trust is realised within and through the work of middle leaders concluded that section.

I turned to a discussion of mathematics leaders in school settings, again reporting that the field is an under-researched area. I positioned mathematics leadership as a form of middle leadership despite its neglect in the middle leadership literature. I supported that claim drawing on definitions of middle leading as offered by researchers with that field. Using the classification of middle leadership as leading and managing practice, I presented information about the activity of mathematics leaders revealing that they also engage in leadership and managerial work in their school sites. That section finished my claim that concerns the need to have further clarity of knowledge about how the mathematics leaders enacts their role as the primary source of professional development in schools.

The final main section of this chapter focused on mathematics professional learning. I clarified the difference between professional development and professional learning, suggesting that professional learning is the growth in teachers' practices and expertise as a result of engaging in the opportunities that professional development can provide. I presented my understanding of the characteristics of mathematics professional learning, claiming that there needs to be a focus on both mathematical content and pedagogical content knowledge development. The forms of professional learning were also presented. I drew and explained research work on a form of mathematical knowledge for teaching due to its prominent use in Australasian research about professional learning in mathematics.

In the next chapter, I articulate my understanding of the theoretical framework of my study. I explain how my attention was drawn to the middle leadership literature that positioned it as a form of practice. I provide information about how I saw the potential for using CHAT as the lens with which to investigate how the School Mathematics Leaders contributed to project sustainability after participation in CTLM.

CHAPTER 3: THE THEORETICAL FRAMEWORK

3.1 Introduction to Chapter 3

In Chapter 1, I stated that this thesis is not only about the School Mathematics Leaders' contribution to project sustainability, but it is also about CHAT. In the previous chapter, I proposed that project sustainability is influenced by several internal and external factors, with school leadership being one that can influence the sustainability of project-initiated reforms. Principals, however, tend to be the focus of discussion about that leadership factor, neglecting the contributions of activity enacted by middle leaders like School Mathematics Leaders. I also highlighted that middle leading, project sustainability, and mathematics leadership are complex activities, and that further knowledge about these aspects of educational leadership is required.

I also presented that literature concerning middle leadership has positioned it as a form of practice (e.g., Edwards-Groves et al., 2016), moving away from research that highlights characteristics or traits of those school leaders (Grice et al., 2023; Grootenboer, 2018). As such, middle leadership tends to be studied using practice-based theories. With this focus on practice, which is a form of activity (Nicolini, 2012), a further reason surfaced for my use of CHAT.

In this chapter, I detail my understanding of CHAT and present reasons for its choice to study the complexity of the middle leadership activity of the School Mathematics Leaders' contribution to project sustainability. I begin by presenting information about CHAT's origins and important historical thinking that has influenced it as a theory suitable for educational research.

As a theoretical framework, CHAT can be traced back to philosophy of the ancient Greek philosopher, Heraclitus of Ephesus (Roth et al., 2013). Therefore, I provide my understanding of aspects of his philosophical stance. I then share an overview of contributions from Karl Marx, focusing on his thinking about human labour processes as a form of practical activity. I present pertinent work from Soviet Russian psychology of the early twentieth century, most notably offered by Lev Vygotsky, and his collaborator, Aleksei Leont'ev. I acknowledge that whole theses could be written about the works of these thinkers. However, due to limitations, I present only relevant aspects of their works that have influenced CHAT as it is understood today.

Following the discussion about the historicity of CHAT, I present my understanding of how it is conceptualised as a theoretical framework today. I pay attention to concepts researchers can employ when using CHAT to inform research activity, drawing mainly on the work of the contemporary CHAT theorist, Yrjö Engeström. I also share my understanding of resourceful practice, a relatively new CHAT concept offered by another contemporary theorist in the field, Anne Edwards.

I draw this chapter to a close by discussing how I came to view the research problem through a CHAT lens. This discussion foregrounds the next chapter, Chapter 4, which articulates my research design and how I specifically operationalised CHAT, using concepts afforded to researchers when they use that theoretical framework.

I now present a brief historical overview of CHAT and its origins.

3.2 Brief Historical Overview of CHAT Origins

Generally, CHAT is used as an approach to understand both the individual and collective facets of human activity, focusing on how object-oriented activity develops and transforms society (Cong-Lem, 2022; Roth, 2012). CHAT positions not only the primacy of object-oriented activity, but it looks to understand how human development progresses in response to contradictions within the practices that people enact (Nuttall et al., 2015).

As a theory, it is claimed that CHAT was initiated in the 1930s by the work of Lev Vygotsky in Soviet Russia, who sought to develop a psychology based on Marxist thought (Engeström, 1999a; 2000; Engeström et al., 1999; Roth & Lee, 2007; Roth, 2012). Due to this, CHAT has origins that can be traced back to German philosophy of the eighteenth and nineteenth centuries, notably in the work of Karl Marx (Jonassen & Rohrer-Murphy, 1999; Kaptelinin, 2005; Kuutti, 1996; Roth & Lee, 2007).

Among his other significant philosophical contributions, Marx significantly influenced how we think about the concept of activity (Jonassen & Rohrer-Murphy, 1999; Kuutti, 1996). Marx grounded his thinking in a worldview that would later be known as *dialectical materialism* (Nuttall & Brennan, 2016; Roth & Lee, 2007), a type of realist ontology. Dialectical materialism concerns "understanding and modelling the apparently ever-changing world" (Roth, 2012, p. 94). This core tenet of dialectical materialism can be traced back to the writings of Heraclitus of Ephesus, a pre-Socratic philosopher believed to have lived between c. 544 BCE to c. 484 BCE (Bowe, 2005).

3.3 Heraclitus of Ephesus

The Greek philosopher, Heraclitus of Ephesus, is acknowledged as being the first Western philosopher to observe the world (*kosmos* for Heraclitus) in critical ways and critique how people came to know the world they live in (Graham, 2008; Hussey, 1999). Known for writing paradoxically, Heraclitus's ideas of the *doctrine of flux*, the *unity of opposites*, and *experience as* *the source of knowledge* became important underpinnings of ideas in the contemporary understanding of CHAT.

Those three ideas are only a few that formed Heraclitus' philosophical stance, and I have chosen them because of their relevance to CHAT.

3.3.1 Doctrine of Flux

Heraclitus has been viewed as a philosopher of change who interpreted the world in a state of constant flux (Bowe, 2005; Colvin, 2007; Graham, 2008). He is best known for the idea of the *flux state*, which is the notion that all things exist in a state of change and flow (Bowe, 2005; Colvin, 2007; Roth, 2012; Wood, 2015). The example of the flowing river was used by Heraclitus to highlight that flux doctrine. For Heraclitus, one cannot step into the same river twice because its waters are ever flowing; therefore, that river cannot be the same even though it looks like a fixed, stable object. When one steps into the flowing water, one appreciates the river's apparent stability due to its ever-constant motion (Shaw, 2018).

Another comparable example that Heraclitus used considered the barley cocktail (Colvin, 2007). In ancient Greece, the cocktail that Heraclitus referenced was supposedly made from barley, honey, and wine. The only way to enjoy the drink was to stir it continuously. Without constant movement, the drink would split into its components and thus no longer be the cocktail it was intended to be (Colvin, 2007; Shaw, 2018).

Heraclitus made a point using those two examples: flux gave the river and the barley cocktail, two material objects, their characteristics and identity (Colvin, 2007). Without flux, those objects were no longer; the river was just a ditch with two slopes, and the drink was just a mixture of disparate ingredients that repelled each other (Colvin, 2007). For Heraclitus, the world was in an ever-changing state of flux, and what appeared to be stable parts of it were, in fact, in constant motion and a constant state of change (Bowe, 2005; Shaw, 2018). In its constant state of flux, the world was in the space of continuous production, reproduction, and transformation (Roth et al., 2013). The flux doctrine, according to Heraclitus, was due to the unity of opposites.

3.3.2 Unity of Opposites

Heraclitus saw the universe as a product of the unity of opposites or unity in opposition (Bowe, 2005; Graham, 2008). If we again take the famed river example, his doctrine of flux was highlighted through the unity of two opposites: motion and stability (Colvin, 2007). It was due to the changing waters in the river, constantly in motion, that the river was a constant and stable object in the world (Graham, 2008). Without flowing and shifting waters, the river was no longer a river; it was just two slopes and a ditch (Colvin. 2007). Here, we see Heraclitus using the two ideas of motion and stability, two opposite forces, and uniting them to bring structure and order. The unity of opposites provided the means of making sense of the ever-changing world.

Paradoxically, however, the union of opposites happened through the struggle of the opposites (Wood, 2015). All opposites were unified through a struggle, war, strife, or tension. Heraclitean thought privileged that it was the tension of the opposites that brought structure to the world (Graham, 2008). War and strife brought structure, yet that structure brought on war and strife. For Heraclitus, tension unified opposites, and that unity made the universe work (Bowe, 2005; Graham, 2008).

The unity of opposites needed to be understood as more than just a tension of two forces in opposition. The opposition was essential to the unity (Hussey, 1999), and what was revealed through that was the interconnectedness of the contradictory state of opposites (Graham, 2015). What was important to Heraclitus was that pairs of opposites were interdependent and had the potential to transform one another (Hussey, 1999). That thinking offered a way of seeing the world as one of becoming, where opposites were united yet interdependent. Heraclitus believed opposites had transformative potential through a process of becoming. For this reason, Heraclitus is viewed as one of the first dialectical philosophers in Western philosophy.

3.3.3 Experience as the Source of Knowledge

Heraclitus claimed that it was experience that was necessary for knowledge (Graham, 2008). He believed that direct observation, specifically experience within the world, was the source of knowledge needed for true understanding of the *logos* (Cankaya, 2017). According to Heraclitus, logos was the source of order and harmony that governed the universe and, interestingly, was unchanging despite his doctrine of flux (Graham, 2008). Understanding the logos was possible, but it was only through humans' experiences of the world. Through that experience, understanding could be achieved. In this way, knowledge was gained through and from firsthand experience of acting in the world (Hussey, 1999).

For Heraclitus, direct observation involved engaging the senses of sight and hearing (Cankaya, 2017). Heraclitus believed knowledge came from experiencing things firsthand through those senses rather than relying on preconceptions or abstract reasoning (Hussey, 1999). Heraclitus considered that one's own perception and experience were the only reliable sources of knowledge and that those sources were far superior to hearsay or second-hand information. For that reason, the sense of sight was considered far more reliable than hearing. Truth that arose from knowledge was not always obvious or easily accessible; understanding goes beyond that accumulation of information, generated through direct observation, experience, and action in the material world (Curd, 1991; Cankaya, 2017).

3.4 Karl Marx

Karl Marx was a German philosopher and political theorist most famous for his ideas about capitalism and communism. He was born in Trier, Germany, in 1818 and died in London, England, in 1883. Marx's most famous work was *The Manifesto of the Communist Party*, which he co-wrote with Friedrich Engels. *The Communist Manifesto* (as it was subsequently titled) was a political pamphlet that laid out the principles of communism, a system of government in which the means of production were owned and controlled by the community rather than by private individuals.

Marx's ideas about communism were heavily influenced by his critique of capitalism, which he saw as a system that exploited the working class and led to economic inequality. He also wrote the book *Das Kapital* (also known as *Capital,* first published in 1867 and republished many times since, with a 2013 publication used in this thesis). Capital is considered one of the most important texts in the field of political economy. Marx (2013) offered a critique of capitalist production, including how human activity shapes and is shaped by that production.

Before Marx's academic contributions, much of Western thought focused on knowledge and virtue as the product of a life lived in contemplation (Nicolini, 2012). That idealist ontological approach ignored the concept of activity enacted in the material world. Within the pages of Capital, Marx (2013) offered new ways of thinking about work as a form of activity, unified through action and motivation, enacted in purposeful ways to achieve valued goals and outcomes. Marx's contributions concerning human practice have since been used to inform CHAT as a framework for understanding activity (Engeström, 2015; Nicolini, 2012).

I now describe important tenets of Marxist thought that align with CHAT as a theoretical framework.

3.4.1 Marx and Dialectical Materialism

As an ontology, dialectical materialism was developed from the writing of Marx and his collaborator, Engels. Dialectical materialism explains that there is only one material world, which positions that world in a continuous process of change (Roth, 2012), an acknowledgement of Heraclitus' doctrine of flux. Like Heraclitus, Marx recognised that that continuous change process was characterised by production, reproduction, and transformation (Roth et al., 2013).

Another of Marx's contributions positioned as foundational to dialectical materialism concerned human consciousness. For Marx, consciousness (the mind) was seen as inseparable from the material world, and that consciousness was the product of human action within that world (Marx, 1970). Links to this understanding of consciousness can be traced back to Heraclitus' who privileged the idea that experience, enacted in the material world, provided the source of knowledge (Cankaya, 2017). Marx's notion of the mind as an inseparable entity from lived experiences in the material world and the belief that consciousness was the product of human activity challenged the predominant idealist thought at the time. That idealist thought took a dualistic view of the mind and the body as two separated entities (Nicolini, 2012).

That understanding of consciousness, as being inseparable from the conditions within the material world in which humans exist, was explained further by Marx in collaboration with Engels (1970):

> Life is not determined by consciousness but consciousness by life. In the first method of approach, the starting point is consciousness taken as the living individual; in the second method, which conforms to real life, it is the real living individuals themselves, and consciousness is considered solely as *their* consciousness (Marx & Engels, 1970, p. 47).

Consciousness, as captured in the preceding quote, was recognised as an entity that was not independent of individuals and their lives, nor was it fixed or predetermined. Marx positioned consciousness as ever-changing, constantly evolving due to humans going about in the world, engaging in practical activity and producing the means of their existence (Marx, 1970). An essential element of Marx's thought concerned practical activity that he viewed as a form of labour.

Practical activity was considered by Marx as the means through which humans went about producing the means of their existence (Engeström, 2015; Marx, 2013; Nicolini, 2012). According to Marx, as humans enacted that practical activity, they changed the material world in which the activity took place. As they created what was needed, humans transformed the world, and with a recognition of the dialectic, Marx also claimed that humans were transformed through that change process at the same time (Lektorsky, 2009; Marx, 1970). That thinking gave rise to privileging the role of history and how human activity was not only shaped by the historical context in which it took place, but activity played a role in shaping history itself.

Dialectical materialism also adopted another aspect of Marx's writings that concerned the role of contradictions within human activity (Engeström, 2015; Marx, 1970; Mussachia, 1977; Roth, 2012). Contradictions were positioned by Marx to be inherent in all social and natural aspects of the world, and he viewed things in those worlds in opposition or conflict (Mussachia, 1977). Those conflicts were understood to have driven change throughout history and the development of forms of practical activity enacted by people. According to Marx (1970), contradictions arose between different forces, classes, or ideas, through which struggle and conflict ensued. The resolution, however, ultimately happened through a synthesis of those opposing forces that surfaced the tension in the first place. Here, another of Marx's ideas can be traced back to Heraclitus and his concept of the unity of opposites (Bowe, 2005; Graham, 2008).

When writing, Marx wrote about the material world and its contradictions and how those tensions form the basis of social and historical development (Marx, 2013). The contradictions between the forces of production (the means of producing goods and services) and the relations of production (the social relations between people in the process of production) were seen as the central driving force of historical change. Marx and Engels (1970) saw the resolution of contradictions as a critical factor in the development of society and the progress of human history. They believed resolving contradictions would result in a new, higher stage of social development, characterised by a higher level of productive forces and a more advanced form of social relations.

3.4.2 Marx and Activity

Although he wrote predominantly about critiques of capitalism, the investigation of its contradictions, and the laws governing its evolution (Marx, 2013), Marx also offered important ideas about activity, specifically human activity as labour processes. With his work situated within a materialist ontology, Marx wrote about "practical activity" (Marx & Engels, 1970, p. 48) to refer to human labour or work as a process of material production.

When articulating the complexities of human labour, Marx (2013) presented work as the union of both effort and motivation directed at the realisation of a goal (Nicolini, 2012). According to Marx, to understand the complex nature of work, attention must be given to six distinct yet interrelated elements comprising of the worker (subject); the material (object) on which the worker exerts their activity; the instruments (tools) which act as the means to carry out the work; actions that the worker enacts that; the goal (outcome) that provides the motivation for the worker to undertake the actions; and, the product of the work which is the new material that is created through the labour process (Marx, 2013; Nicolini, 2012). Marx offered the use of instruments and the importance of purposeful activity to understand human labour processes.

3.4.2.1 Use of Instruments

Marx (2013) saw tools as a crucial aspect of human labour and a defining characteristic of human activity. Marx wrote the following as a way of articulating his understanding of the role of instruments within labour processes: "The use and fabrication of instruments of labour, although existing in the germ in certain species of animals, is specifically characteristic of the human labour process, and Franklin therefore defines man (sic) as a tool-making animal" (Marx, 2013, p. 122).

For Marx, the conscious use of instruments (which would become *tools* within CHAT) was imperative in the development of human society and the capacity for humans to transform the world in which they lived. He argued that the use of tools allowed humans to increase the efficiency and productivity of their labour and, therefore, create more complex forms of society. He saw the development of tools closely tied to the development of human consciousness; as humans learned to reflect and consciously change their activities, they also changed and developed themselves (Engeström, 2015; Marx, 2013; Nicolini, 2012). That development occurred within human practical activity through a relationship between the material on which the worker exerted their activity and the historically evolving use and creation of instruments (or tools).

Regarding the relationship between humans and their tools, Marx (2013) believed that the tools were not simply passive instruments but rather active components of the labour process carrying historical remnants and cultural significance within them. Marx argued that the tools used by workers actively shaped and were shaped by the needs and wants of society. This thinking meant that tools were constantly engaged in the process of production, reproduction, and transformation over time.

3.4.2.2 Purposeful Human Activity

Another of Marx's ideas concerning human activity was its purposeful nature (Marx, 2013). For Marx, the material and the goal at which the worker (subject) directed their labour, and thus sought to achieve, gave activity its purpose (Nicolini, 2012). Marx differentiated between animal and human labour, drawing on the weaving of spiders and hive-making by bees to support that differentiation.

Marx (2013) believed that animals engaged in *simple* labour, characterised by the instinctual repetition of certain activities, such as a spider building a web. In contrast, he saw human labour as *purposeful* and directed towards producing goods and services that satisfy human needs and desires. The key difference between animal and human labour was the ability of humans to reflect on and consciously change their activities and, in doing so, transform themselves and the world around them. This notion of purposeful activity with its projection onto the material world was captured by Marx when he wrote: "At the end of every labour-process, we get a result that already existed in the imagination of the labourer at its commencement"

(Marx, 2013, p. 121). Through that stance, Marx positioned the idea of object-orientedness within human activity.

As part of his description of purposeful activity, Marx (1970) saw *objectification* as a central aspect of human activity. Marx believed that objectification was a key feature of human labour, and that the objective of work possesses both internal and external qualities: internal, in that practical activity had its purpose in the mind of the subject before the enactment of the activity, and external, in that the intention of the activity was realised and achieved when undertaken by the subject in the material world (Nicolini, 2012).

Those foundational ideas of Marxist thought, later incorporated as tenets of dialectical materialism, were adopted and used in the work of Russian psychologists at the turn of the twentieth century. Their aim was to conceptualise a form of Marxist social psychology.

3.5 Lev Vygotsky

Lev Semionovich Vygotsky, a Soviet-Russian psychologist, used the principles of Marxist thought to develop a unique form of social psychology (Roth, 2012). Drawing on Marxist ideas related to work, Vygotsky offered the world new ways of understanding how humans engage in activity. Vygotsky built on Marx's ideas by acknowledging that consciousness and activity no longer belonged solely to the individual (as adopted by Cartesian, idealist thinkers). He proposed that activity must be understood as a cultural and historical phenomenon (Nicolini, 2012).

Vygotsky (1978) contended that "every form of behaviour presupposes a direct reaction to the task set before the organism" (p. 39). As a result of that thinking, Vygotsky proposed the idea of the "complex mediated act" (p. 40), which explained human behaviour when engaged in activity. Vygotsky took the Marxist concepts of the object (the work itself), the subject (the worker), and the instruments (tools) concerning labour processes (Marx, 2013, p. 121) and used them to offer new ways of understanding activity by offering the concept of *mediated action*.

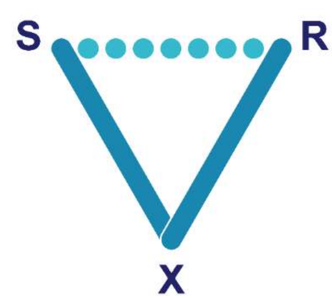
3.5.1 Vygotsky and Activity

Vygotsky's thinking about activity can be captured within the idea of mediated action. Vygotsky (1978) claimed that that the behaviourist presupposition could be modelled through the simple formula of S→R, where 'S' represents the stimulus and 'R' represents the response. However, when the structure of a sign, understood as a tool in the way Marx thought about instrument use (Marx, 2013), operations must be considered. For Vygotsky, an intermediate link between the stimulus and response was required if human behaviour was to be truly understood. Vygotsky reframed the simple stimulus-response process, the dominant behaviourist idea at the time, by replacing it with that complex, mediated act (Havnes, 2010; Nuttall et al., 2015).

Vygotsky (1978) purported that the intermediate link was a "second order stimulus" (p. 39) that he called a *sign*, which fulfilled a specific purpose. The sign created a new relationship or link between the stimulus and the response. That was no passive act according to Vygotsky. Instead, the individual was recognised as being engaged in creating that new link. A model of Vygotsky's (1978) thinking is represented in Figure 3.

Figure 3

Representation of a Mediated Act (Vygotsky, 1978)



In Vygotsky's (1978) model of the mediated act, that presupposition was represented by the 'S' (the stimulus), and the response was represented by the 'R'. The use of the mediating artefact was represented by the 'X' to show that all activity is mediated. It was through mediation that a relationship existed between the stimulus and the response (Vygotsky, 1978). It was this thinking that led to what Engeström (2001) called the "first generation of activity theory" (p. 134). The generations of CHAT are elaborated on further in this chapter.

Vygotsky used the notion of the mediated act to explain that learning was a mediated process. When an individual interacted with a sign (or instrument or mediating artefact) or with other people in the environment, that individual constructed their own meaning(s) throughout the process (Huang & Lin, 2012). That was the unit of analysis for Vygotsky in understanding how a subject's object-oriented action was mediated by tools and signs (Engeström, 2001; Engeström & Sannino, 2021). Vygotsky died before he had the chance to realise his ideas fully, and one of his collaborators, Leont'ev, extended Vygotsky's ideas of activity, moving from individual to collective activity.

I return to further ideas offered by Vygotsky when I explain the generations of CHAT (Engeström, 2001, 2015) later in this chapter.

3.6 Aleksei Leont'ev

Like Vygotsky, Aleksei Leont'ev worked on developing a Marxist psychological approach, working alongside Vygotsky from 1924 to 1930. Leont'ev offered significant extensions of Vygotsky's thinking. Several important contributions by Leont'ev included the concept of the motive of activity (Leont'ev, 1978). For Leont'ev, the motive was the driving force for all activity. By focusing on the motive of activity, understanding the difference between human activity and non-human activity was made clear (Kaptelinin, 2005; Kuutti, 1996; Leont'ev, 1978).

3.6.1 Leont'ev and Activity

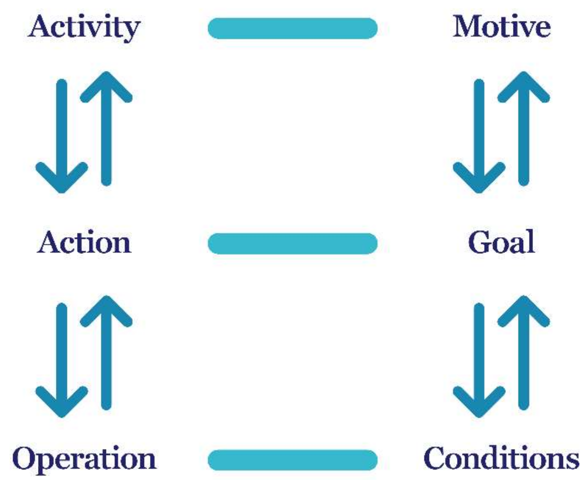
For Leont'ev (1978), activity was more than a performance. His thinking about this went back to the motive (object) of activity. Leont'ev claimed that all activity was an act motivated by a specific need that was biologically or culturally constructed. The need was transformed into an *object of activity* when it oriented and directed the activity enacted by an individual or collective subject (Kaptelinin, 2005). It was that motive that provided the purpose for the activity (Kaptelinin et al., 1995). Therefore, for Leont'ev (1978), there was no such thing as object-less activity. Object-orientedness was considered an integral part of activity, and understanding objects was important if activity was to be understood. Activity, according to Leont'ev, was the most basic unit of analysis, and that activity was determined by its motive.

Leont'ev's expansion on the work of Vygotsky (1978) included thinking about the hierarchical structure of activity (Kaptelinin et al., 1995). Leont'ev (1978), who accounted for individual action within social activities, developed the idea of hierarchical levels of human activity. Leont'ev believed that activity was composed of three levels: *activity*, *actions*, and *operations*.

For Leont'ev, activities were longer-term formations where their objects were transformed into outcomes through a process that involved several phases (Kuutti, 1996). Figure 4 represents Leont'ev's thinking about the hierarchical structure of activity.

Figure 4

Hierarchical Nature of Activity



When participating in an activity, Leont'ev proposed that a subject performed actions that had an immediate goal (Kuutti, 1996). Activity was achieved through a series of actions, with the same action contributing to different activities (Wilson, 2006). For an individual to be considered skilled in an activity, fluency with operations must have taken place. In doing so, the scope of action had the potential to become broader (Kuutti, 1996). Therefore, according to Leont'ev, an activity driven by motive(s) was performed through specific actions directed toward goals. Actions were enacted through certain operations regulated by conditions (Kaptelinin et al., 1995; Kuutti, 1996; Leont'ev, 1978).

Leont'ev claimed that an important aspect related to activities was that they had a double nature (Kuutti, 1996). Every activity had an internal and external side (Kuutti, 1996), with the subject and the object existing as part of a dialectical relationship (Wilson, 2006). This aspect drew on Vygotsky's (1978) ideas of internalisation and externalisation, where the subject transformed the object into an outcome, whilst the properties of the object "penetrate into the subject and transform him or her" (Kuutti, 1996, p. 32). For Leont'ev (1978), it was through *internalisation* that processes were generalised, verbalised, and ultimately developed further, allowing for the possibility of external activity.

3.6.2 Multi-Motivated Activity

Another contribution of Leont'ev (1978) was that of *multi-motivated* activity. To make sense of the complex and often contradictory nature of motives, Leont'ev claimed that an activity could be motivated by several of them at the same time. Complexity occurred due to the desires, wants, and aspirations that the subject attached to motives and how those changed through the course of the activity (Miettinen, 2005). Therefore, according to Leont'ev, activity became multi-motivational when it simultaneously responded to two or more motives.

With activity understood as multi-motivational, Leont'ev (1978) claimed that humans were required to juggle those motives often in simultaneous and complex ways (Nuttall et al., 2019). Leont'ev claimed that the subject experienced complexity through the competitive and contradictory nature of two or more multiple motives driving their activity. When the subject experienced a contradiction between the two (or more) competing motives, Leont'ev claimed that a *need state* surfaced. That need state may be experienced as a discomfort by the subject (Kaptelinin & Nardi, 2006) as they encountered a conflict concerning which motive to honour through a specific action at a specific time (Kaptelinin, 2005). That conflict occurred because the subject believed only one motive could be privileged and pursued at a time. According to Leont'ev (1978), the resolution of the need state happened when the subject enacted a form of action, directing that at a specific motive.

Leont'ev (1978) proposed that as a way of managing the conflict between motives and supporting subsequent action on them, the subject develops and uses a *hierarchy of motives*. Leont'ev's hierarchy was not like Maslow's work concerning the hierarchy of needs. The hierarchy of motives needed to be understood as "a conflict resolution mechanism" (Kaptelinin, 2005, p.14). The hierarchy of motives was recognised as a way to support the subject in prioritising the competing motive objects and, thus, managing and resolving the need state. Leont'ev proposed that the subject selects an action to enact by comparing the motive objects within the hierarchy, and the motive with the highest position is prioritised (Kaptelinin, 2005). When that occurs, the subject carries out actions honouring that privileged motive.

Leont'ev’s (1978) concepts of multi-motivated activity, the need state, and the hierarchy of motives were important in making sense of activity. They were essential to understanding how the subject interacted within their environment when faced with a conflict between motives and how the subject and the activity developed together over time.

I move the discussion to contemporary understandings of CHAT, showing how CHAT theorists have adopted several concepts from the historical thinking I have presented thus far.

3.7 A Contemporary Overview of CHAT

From its historical origins in the work of Heraclitus, Marx, Vygotsky, and Leont’ev, contemporary views of CHAT see it as a variety of approaches guided by broad principles. More focally, CHAT has been defined by two features (Sannino et al., 2009). The first is that CHAT is a practice-based theory, and the second is that the theory is both historically and future-orientated. Throughout its history, CHAT has demonstrated consistent viability since the 1930s when Leont’ev framed its basic principles and formulated the structure of activity (Sannino et al., 2009).

Due to this framing, CHAT has the potential to support researchers when analysing, interpreting, and understanding human activity and the mediational means of that activity (Engeström, 2015; Hashim & Jones, 2007). As a theory in use today, CHAT draws the researcher's attention to the interactions between human activity and consciousness within the context in which the activity exists (Engeström, 2015; Jonassen & Rohrer-Murphy, 1999).

I now discuss CHAT's contemporary definition and purpose, including several principles underpinning it. I also present my understanding of the generations of the theory as purported by Engeström (1996, 2001, 2015) and how historical contributions have supported the development of CHAT as a theoretical framework (Engeström, 2001; Roth, 2012).

3.7.1 Purposes of CHAT

Since the 1970s, CHAT has been used by Western researchers as a multidisciplinary framework to explore domains of human activity, including activities associated with work enacted within workplace contexts (Engeström, 2000, 2001; Nardi, 1996). From that time, the use of CHAT has become more prevalent in Western literature (Roth & Lee, 2007), where researchers have developed the theory by extending the foundational work of Vygotsky and Leont’ev (Engeström, 2001; 2015; Havnes, 2010; Roth & Lee, 2007). Roth and Lee (2007) described CHAT “as an integrative road map for educational research and practice” (p. 188) and saw its potential in systematically researching complex work activity within educative settings.

I understand the purposes of CHAT in the following ways:

Understanding the nature of human activity: CHAT views human activity as a dynamic and complex process that shapes and is shaped by the context in which it occurs. As a way of investigating that activity, attention is paid to the motives pursued by the subject (individual or collective), along with the mediational means (rules, division of labour, and cultural tools) that enable and constrain the achievement of those motives (Engeström, 2015; Kaptelinin, 2005; Nuttall et al., 2015; Roth, 2012)

Analysing the development of activity: CHAT is interested in the way that activities change and develop over time, as well as the ways in which they are influenced by historical and cultural factors (Nuttall et al., 2015; Yamazumi, 2007). Development is understood in CHAT from a Marxist viewpoint, so another purpose of the theory is to understand how activity evolves in response to historically accumulated contradictions that surface in practices within the activity (Engeström, 2015; Engeström & Sannino, 2011; Nuttall et al., 2015; Roth, 2012).

Improving educational and work practices: CHAT is often applied in the fields of education and workplaces, with the goal of improving practices and creating more effective and engaging learning and work environments (Engeström, 2015; Roth & Lee, 2007). This is done by taking a systematic approach to analysing those practices using concepts that CHAT offers, with the intention of highlighting the complex nature of activity (Engeström, 2001, 2015; Nuttall et al., 2015; Roth & Lee, 2007).

3.7.2 Principles of CHAT

The contemporary use of CHAT is guided by several principles that support its characterisations as a practice-based theory (Engeström, 2001; Sannino et al., 2009). CHAT privileges considerations of the subject, their activity, the mediational means of that activity, and the cultural and historical contexts in which the activity occurs. This is important to highlight, considering that within the field of middle leadership, recent thinking understands middle leading in relation to practice within the contexts in which the leadership is enacted (Grice et al., 2023; Grootenboer, 2018).

I now turn to my interpretation of those principles. In my thesis, I understand the contemporary CHAT principles as follows:

> *Mediation*: Human activity is always mediated activity. Mediational means (e.g., cultural tools and *signs*, rules, and division of labour) are used by the subject to represent and make sense of the world. Those mediational means shape and are shaped by historical and cultural factors.
>
> *Object-orientedness*: Human activity is directed towards pursuing culturally valued motives embedded within the social and cultural context in which the activity occurs.
>
> *Collective activity*: Human activity is always collective and social, and it is organised around those motives, enabled and constrained by mediators of activity.
>
> *Development*: Human activity and development are interconnected, with each stage of development building on the previous one and leading to new forms of activity and consciousness.
>
> *Contradiction*: Human activity is characterised by internal contradictions arising from the tension between the current situation and the desired motives that drive activity. Contradictions drive the process of learning, which leads to development.

Historicity: Human activity is shaped by the historical trajectory that led to its current cultural form and its enactment.

I describe now how those principles are realised through discussion of the generations of CHAT.

3.8 Generations of CHAT

CHAT has been reformulated and re-theorised over the years of its use. There appears to be conflicting information about the concept of *generations* of the theory within CHAT literature. In this section, I draw mostly on Engeström's writing from 1996 and 2001 when presenting my interpretation of CHAT's generations. I use the work of Engeström because he is considered one of the most influential CHAT theorists in recent times (Blunden, 2010).

According to Engeström (2001), CHAT has undergone several evolutions he described as generations. Three main generations characterise the theory (Engeström, 1996, 2001) as it has been used and adapted as a theoretical framework in Western research since the 1970s. Interestingly since then, Engeström and Sannino (2021) proposed a fourth generation with the purpose of resolving societal problems (e.g., homelessness, climate change, pandemic response). That fourth generation involves collective subjects within heterogeneous activity systems in local, regional, national, and global contexts to work towards new collective motive objects of activity. Due to the scope of my study and the limitations of the thesis, I focus only on the first three CHAT generations.

Engeström (1996) claimed that the first generation of CHAT focused on the work of Vygotsky during the 1920s and early 1930s. That generation centred on the ideas associated with mediated action in which an individual exerts control over their actions using mediating artefacts that were both physical and psychological in nature. Limitations of Vygotsky's thinking would be later addressed by his collaborator, Leont'ev (Engeström, 1996, 2001).

Leont'ev's contribution to CHAT, conceived as the second generation of the theory (Engeström, 1996), advanced the work of Vygotsky, moving from individual to collective activity. That generation focused more on the object-oriented nature of activity (Roth, 2007), the nature of collective activity and its mediational means, and the hierarchical levels of activity (Engeström, 1996, 2001).

The third evolution of CHAT has been mainly attributed to Engeström (1999b; 2001; 2015). This third generation is used to understand multiple perspectives of activity through the interaction of two or more activity systems, leading to Engeström's theory of expansive learning (Engeström, 2001).

In the following sections, I present overviews of the three CHAT generations, drawing out foundational ideas that underpin each.

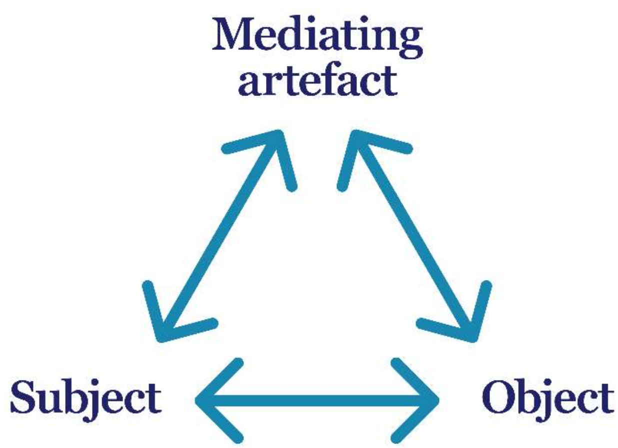
3.8.1 First Generation of CHAT

According to Engeström (1996, 2001, 2015), Vygotsky's idea of mediation plays a significant role in contributing to the theory of CHAT. Vygotsky believed that humans act as agents who act towards a particular object. Through those actions, humans use mediating artefacts found in the environment, such as tools, signs, and instruments (Nussbaumer, 2012). In other words, human activity is driven by needs where a person strives to achieve specific purposes, and this object-directed activity is mediated using one or more artefacts.

Cole and Engeström (1993) asserted that the triangular model captured the basic structure of human cognition using mediating artefacts. These foundational ideas related to human activity are understood to form the first generation of CHAT (Cole & Engeström, 1993; Engeström, 2001; Havnes, 2010). This earliest generation of CHAT was captured by Vygotsky's (1978) representation of mediated activity (Figure 3) and later reformulated to its familiar form associated with the first generation (Engeström, 2001), as seen in Figure 5.

Figure 5

Model of Activity: First Generation (Engeström, 2001)



First-generation CHAT represents human activity at an individual level. Activity is realised through a mediating role between the subject and the object (Kaptelinin et al.,1995). In this generation of CHAT, we see that it assumes that mediating artefacts facilitate interaction between the subject and the object. Cole and Engeström (1993) differentiated between two types of functions within the first generation of CHAT.

The first type is known as *natural* (or unmediated) functions which are those along the base of the triangular model. In a natural function, the subject engages with activity toward the object unassisted by artefacts. The second type of function, known as *cultural* (or mediated) functions, is where a subject and an object are mediated by an "auxiliary means" (Cole &

Engeström, 1993, p. 5). The auxiliary means is depicted at the "vertex" of the triangular model and represented by the mediating artefact.

3.8.1.1 Mediated Action

In first-generation CHAT, mediating artefacts can be physical tools (e.g., a hammer) or symbolic tools such as language and symbols which, during activity, are created or transformed (Vygotsky, 1978). This generation honours the claims of Vygotsky about two types of mediating artefacts that humans use when engaged in activity: *tools* and *signs*. Tools are used with the intention of manipulating physical objects. An example of a tool is a hammer; it is a material artefact used, along with other tools, to change a wood plank into a table. Signs differ from tools in that humans use them to influence themselves or other human beings (Kaptelinin et al., 1995). An example of a sign is a concept; it is a psychological artefact used to change or adapt how humans think. Tools and signs are considered cultural means, but they differ in how they orientate the activity that the subject enacts (Miettinen et al., 2012). Tools are practically and externally oriented, whilst signs are internally oriented (Nuttall et al., 2019).

Mediated action was a fundamental concept within the work of Vygotsky and featured predominantly in that generation of CHAT (Cole & Engeström, 1999; Cong-Lem, 2022; Vygotsky, 1978). Mediation captures the relationship that exists between the subject and the object, motivating psychological development through activity (Miettinen et al., 2012; Nuttall et al., 2019).

3.8.1.2 Limitations of First Generation

The first generation had limitations in explaining human activity (Cole & Engeström, 1993; Engeström, 2001). The major limitation was that the unit of analysis was focused only on the activity of the individual. It was the work of Leont'ev that gave rise to the second generation of CHAT, which recognised and captured that individual activity takes place within a collective system of activity (Engeström, 2001; Leont'ev, 1978; Roth, 2007). It was Leont'ev who came to call the collective nature of human activity as an *activity system* (Cole & Engeström, 1993).

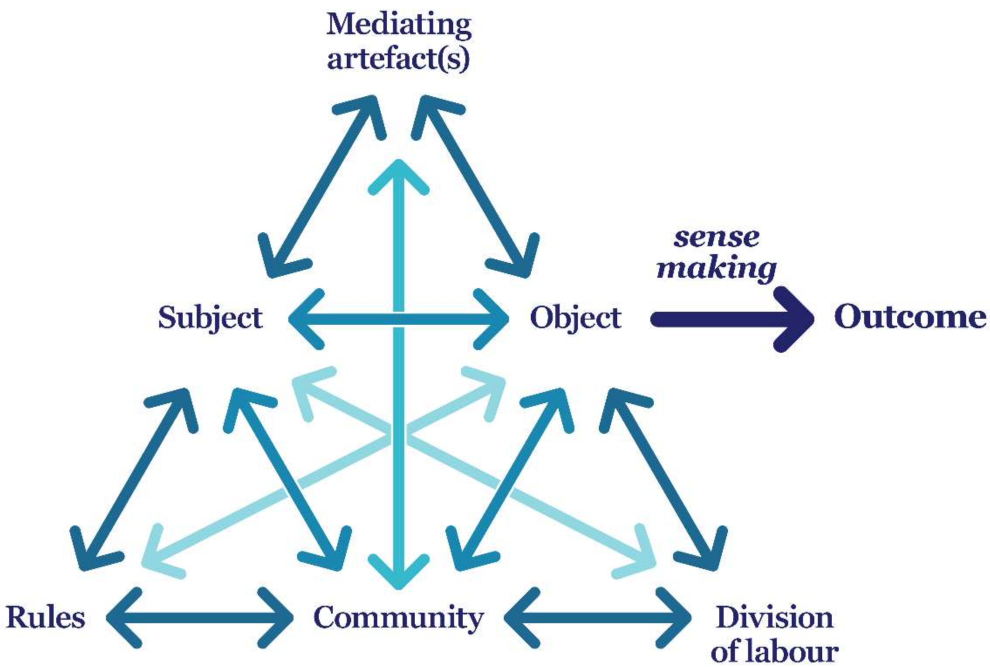
3.8.2 Second Generation of CHAT

In contrast to the first generation of CHAT, the second generation has the activity system as the basic unit of analysis (Cole & Engeström, 1993). That idea of Leont'ev (1978) about the activity system was captured in the well-known, triangular representation (Figure 6). Leont'ev never diagrammatically captured his thinking using that representation (Engeström, 2001). Instead, Engeström (2015) was the first to create the activity system diagram in 1987, capturing Leont'ev's (1978) concept of collaborative human activity.

The triangular representation is central to CHAT's second generation and is sometimes called the "activity triangle" (Roth & Lee, 2007, p. 197). It is regarded as the best-known representation of the activity system concept (Engeström, 2015; Roth, 2012). Engeström (2015) used that triangular model (Figure 6) to represent how the community mediates the subject's relationship with the context in which activity is enacted (Blunden, 2010; Engeström, 2015). The two-way arrows represent the dialectical relationship between the mediational elements within the activity system. It is a convention that is also used to capture the idea that those mediational elements co-evolve, and that they shape and are shaped by each other over time.

Figure 6

Model of an Activity System: Second Generation (Engeström, 2001)



The second generation of CHAT supports understanding of the activity system through the study of the integral mediating artefacts that are inseparable from human functioning. The other mediational elements of activity, proposed by Leont'ev through his notion of collective human activity (Engeström, 1999a), are also represented in the activity "triangle" (Roth & Lee, 2007). Those are included because, in this generation of CHAT, there are other means of mediation beyond just mediating artefacts. Those additional mediational means have the potential to enable and constrain activity that is enacted within the system. Cole and Engeström (1993) stated that the activity system diagram provides a "conceptual map" (p. 8), showing how human cognition is distributed within an activity system. This distribution is represented by the social (*community*, *division of labour*, and *rules*) and material and conceptual mediational elements (mediating artefacts) that have the potential to enable and constrain activity within that activity system.

3.8.2.1 Components of Activity Systems

Engeström (1999, 2001, 2015) defined the mediational elements of activity as six components (or means) that exist within an activity system: *subject*, *object*, *mediating artefact*, *community*, *rules*, and *division of labour*. Within an activity system, an individual and their activity are embedded within and obtain meaning from a community of people whose activity is directed towards the same object (Cole & Engeström, 1993; Engeström, 2015). The fundamental characteristic of the activity system is its object-orientedness (Engeström, 2015; Kaptelinin, 2005; Leont'ev, 1978), which draws on previous thinking that understands human activity as being practical and objective activity (Leont'ev, 1978; Marx, 2013; Vygotsky, 1978). It is the object, also known as the *motive object* (e.g., Nuttall et al., 2015) that brings meaning to the activity enacted within the activity system (Kaptelinin, 2005).

The activity system is the unit of analysis that includes the role of the individual or collective subject, as well as the impact that the wider social system has on the activity of the subject (Havnes, 2010). According to Engeström (1999b), the integral mediational elements of an activity system are the subject, object, and community. Those are unified through the mediators within the activity system through its mediating artefact(s), rules, and division of labour. It is those mediators that affect the mental and practical processes of the subject and community of the activity system (Bellamy, 1996). Crucially, those mediators are historically formed yet are open to future development (Kuutti, 1996).

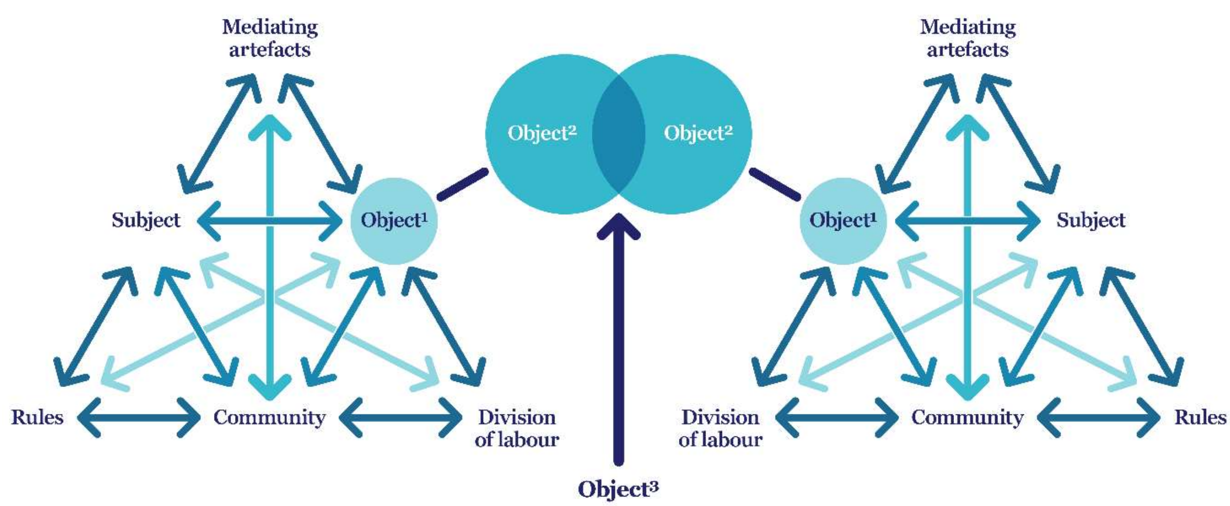
The second generation affords the CHAT researcher several concepts drawn from the mediational elements of the activity system. As I used this version of CHAT in my research design (Chapter 4), I return to several concepts associated with the activity system later in this chapter.

3.8.3 Third Generation of CHAT

The third generation of CHAT is built upon the notion of multiple activity systems interacting through a *shared* motive object (Engeström, 1996, 2001). The unit of analysis within that CHAT generation moves from a single activity system to a unit that focuses on the interaction of at least two or more activity systems. Generally, third generation CHAT is oriented in business, management, and workplace areas of research (Engeström & Sannino, 2021), as it is used to make sense of organisational activity and how networks of activity systems interact (Engeström, 2001). Those ideas of CHAT's third generation are captured in Figure 7. In that figure, the minimum number of activity systems are represented and are seen interacting as a network of systems around the shared motive object of activity.

Figure 7

Model of Interacting Activity Systems: Third Generation (Engeström, 2001)



As seen above, the third generation of CHAT employs the triangular model, usually depicting two (or more) activity systems connected through shared objects of activity. This represents how at least two distinct activity systems come together around shared motive objects (represented with Object^2 and Object^3). The contradictions within and between each activity system are identified and seen as opportunities to develop the activity across the interacting systems (Engeström, 2001).

Object^2 is understood as an object that holds meaning for the activity systems involved in the interaction, and Object^3 is a transformation of that object into one that becomes jointly constructed. Third-generation CHAT understands that Object^3 is pursued by the subjects within and across the interacting activity systems, thus instigating networking between those systems (Engeström, 2001, 2015). By focusing on that shared object (or objects), an understanding of multiple perspectives and how the subjects interact within and across the activity systems can be achieved (Engeström, 1999a). For this reason, third generation CHAT clarifies cooperative work across multiple activity systems.

Each generation of CHAT has its own purpose in researching human activity. First-generation CHAT can be used to make sense of individual activity directed at the achievement of motive objects focusing on mediated action using mediating artefacts. Second-generation CHAT can be used in interpreting object-oriented collective activity and the mediational means of that activity beyond just mediating artefacts. It aims to understand the complexity of interrelations between the subject and the community where the activity occurs. Finally, third generation CHAT allows researchers to interpret the interactivity and collaborative work of two or more activity systems. It is also helpful in understanding networking through analysis of the shared object pursued by subjects within those multiple activity systems.

3.8.4 Reason for Using Second Generation of CHAT

I decided to use second-generation CHAT (Engeström, 1996, 2001) to understand how the School Mathematics Leaders contributed to project sustainability. My reason for this is that first-generation CHAT does not consider the mediational means that enable and constrain activity beyond the use of mediating artefacts. First-generation CHAT also does not consider social relationships inherent in human activity (Leont'ev, 1978).

I did not choose third generation of CHAT due to its unit of analysis being the formation of a shared (or partially shared) motive object. I also focused just on the activity system of the School Mathematics Leaders for pragmatic reasons due to the scope of the thesis. For that reason, there was no study of the interaction between two or more activity systems which is what third generation CHAT focuses on (Engeström, 2001; Engeström & Sannino, 2021).

I made the methodological decision to focus solely on the leadership activity of the School Mathematics Leaders. Using the second generation of CHAT allowed me to use several important concepts to systematically study the complexities of the mathematics leaders' leadership activity that contributed to project sustainability.

I now turn to a discussion of those concepts and how I interpret them, drawing on support from the CHAT literature field.

3.9 Definitions of Key Concepts in CHAT

CHAT offers the qualitative researcher several concepts that support the interpretation and understanding of a problematic (Nuttall et al., 2015). With my decision to frame the research design using CHAT, it is essential that I demonstrate my understanding of those concepts. It is also important that I clarify them for the reader because those concepts are understood differently when used outside of the CHAT framework.

The concepts I discuss in the following section are drawn from the second generation of CHAT, focusing on the relevant ones I used in my study. After describing each concept, I provide insights into my understanding of how they sensitised and influenced my research reasoning within the context of the School Mathematics Leaders' contribution to project sustainability.

3.9.1 Subject

The subject in an activity system is an individual or group of people engaged in an activity (Engeström, 2015; Jonassen & Rohrer-Murphy, 1999; Lektorsky, 2009; Roth, 2004). It is the subject who acts on the object within the activity system (Engeström, 2015), so the subject's perspective is used to interpret activity. Within CHAT, activity does not exist without a subject, and the subject within the activity system does not exist without a motive object of activity (Leont’ev, 1978).

The concept of subject in CHAT differs significantly from how the concept is perceived in a Piagetian sense, where a person constructs knowledge individually (Roth & Radford, 2011). This main difference concerns the dialectical relationship that exists between the subject and the activity they enact. In CHAT, the subject shapes and is shaped by the context in which their activity occurs. The subject and the motive object are understood to be connected through a dialectical relationship.

Within CHAT, the subject within an activity system may be a *collective subject* (Kaptelinin, 1995; Lektorsky, 2009). The collective subject refers to a group of individuals working on the same motive object(s). The people, who form that collective subject, do not necessarily have to engage in labour processes in the same context or setting. The collective subject may work across different sites. However, they are positioned as a collective subject because they share that common motive object of activity: their actions are coordinated towards the enactment of the activity, directed toward the same motive object (Lektorsky, 2009).

In my study, being sensitised to the concept of the collective subject, I considered the three School Mathematics Leaders as the collective subject. Although they were working in different schools, the mathematics leaders collectively enacted their professional learning leadership to contribute to project sustainability in each of their schools.

3.9.2 Motive Object

As the subject engages in activity, they pursue a motive object. According to Leont'ev (1978), the motive object in an activity system is the motive of the subject, and the motive for the activity is embedded in the object of the activity (Engeström, 2000). However, Engeström (1999b) elaborated that the motive object is more than just motivation. In CHAT, motivation is understood differently from ways in dominant psychological viewpoints that tend to position motivation as an individual and internal force of will.

Along with providing the direction for activity, the motive object of activity also relates to production (Engeström, 2015). That production can be either physical or mental in nature (Jonassen & Rohrer-Murphy, 1999). Therefore, the motive object of an activity system is known in two ways: as the motive object held by the subject, providing the reasons for the subject's behaviours (Kaptelinin, 2005); and it can be a mental or physical product (Engeström, 1999b; Jonassen & Rohrer-Murphy, 1999), or an ideal object (Kaptelinin & Nardi, 2006) desired by the subject.

In CHAT, the motive object is considered the entity at which activity is directed (Cong-Lem, 2022), and it acts as the driving force that motivates activity (Kaptelinin, 2005; Miettinen et al., 2012). Motive objects can also be conceptualised as the tasks or undertakings the subject enacts as they engage in activity (Nuttall et al., 2015). Throughout the course of the activity, the subject transforms the motive object, which in turn, also transforms the subject (Nardi, 1996). Motive objects are transformed through human labour processes (Engeström & Blackler, 2005) into a use-value that is deemed culturally important (Miettinen et al., 2012).

Motive objects are also understood in CHAT as possessing a temporal nature. That means that they have the potential to move and be temporarily pursued by the subject within the activity system (Engeström & Blackler, 2005). Temporary motive objects are important in CHAT because they provide a temporal focus of activity and support understanding of reasons why new or revisited tasks and undertakings are enacted by the subject within the activity system.

An example of a temporary motive object might be the introduction of a new mathematics curriculum. In CHAT terms, that curriculum (with its concepts, structure, and pedagogical advice) would be a mediating artefact or tool a mathematics leader uses to mediate the development of teachers' pedagogical content knowledge. As a new tool, however, understanding the content and structure of the curriculum becomes a temporary motive object as it becomes the focus of the work enacted by the mathematics leader. The curriculum documentation continues to be a motive object of activity until an understanding of it is achieved. After that, the documentation would move within the activity system and take on its mediational role as a cultural tool.

In its contemporary use, CHAT adopts the Leont'ev (1978) concept of multi-motivated activity. The subject may pursue more than one motive object with the activity system. Multi-motivated activity highlights the complexity and dynamic nature of human activity (Kaptelinin, 2005). Furthermore, multi-motivated activity is not a fixed state, but it is in constant change and evolution. This is because motive objects have the potential to change and shift over time (Engeström & Blackler, 2005; Kaptelinin, 2005; Miettinen, 2005). That means that the need state and hierarchy of motives, important concepts offered by Leont'ev (1978), form part of the contemporary CHAT understanding of the concept of motive object.

In my study, I used the concept of the motive object to make sense of the 'what' and 'why' of the School Mathematics Leaders' professional learning activity. I focused on the undertakings and tasks they enacted and looked for evidence of multiple motive objects. I also looked for evidence of temporary motive objects and their movement within the School Mathematics Leaders' activity system.

3.9.3 Outcome

It seems fitting now to describe the concept of outcome due to its connection to the motive object of activity. All activity is directed toward motive objects leading to an outcome desired by the subject (Nuttall et al., 2015). As a CHAT concept, the outcome is captured in the triangular model with the arrow pointing from the motive object(s) to the outcome, as seen in Figure 6. That represents the idea that following the motive object of activity is the outcome (Engeström, 2015), which is deemed necessary by the subject and perceived as culturally valuable (Nuttall et al., 2015). In the case of my study, this would concern the culture of mathematics education, the mathematics professional learning of the classroom teachers, and the mathematics learning of classroom students.

Recognising that there are always one or more motive objects at which the subject directs their activity (Engeström, 2015; Leont'ev, 1978; Kaptelinin, 2005), within any activity system, there may be one or more outcomes. The outcome of an activity system adds further meaning to the ‘what’ and the ‘why’ of the activity as the outcome is materialised because of the transformation of the motive object(s) (Engeström, 2015). Through achieving the outcome, the sustainability and development of culture ensue (Engeström & Kerosuo, 2007).

In my study, I sought to make sense of project sustainability as an outcome of the School Mathematics Leaders’ professional learning leadership activity, whilst also being open to their perspectives of the outcome of the enactment of their professional learning leadership within their activity system.

3.9.4 Mediating Artefact (Cultural Tool)

Leading up to this thesis, I have used several terms to capture the concept of the mediating artefact. Havnes (2010) claimed that terms are used synonymously for this concept, including sign, meditational means, and, recently, *cultural tool*. For the purposes of my thesis, I use the term cultural tool when referring to the mediating artefact to align my thinking with previous CHAT researchers (e.g., Miettinen, 2006; Nuttall et al., 2015).

The concept of cultural tool is essential in CHAT as it draws on the foundational work of Vygotsky (1978). Cultural tools are what the subject uses as they direct activity towards the motive objects pursued within the activity system. They hold significance in CHAT as they are deemed as "integral and inseparable components of human functioning" (Engeström, 1999a, p. 29).

Activity within any system is mediated using both technical and psychological tools (Engeström, 2015). All tools are culturally constructed and transmitted, and when used within activity, tools are transformed whilst carrying with them a particular culture, the remnants of historical development (Kaptelinin et al., 1995). Recognising the positioning of CHAT within dialectical materialism, cultural tools transform activity, and in turn, the tools are also altered by the activity (Jonassen & Rohrer-Murphy, 1999).

Another way that cultural tools alter activity is through activity development via the introduction of new tools into an activity system. Development within an activity occurs when new cultural tools are introduced and then subsequently adopted and used by the subject to mediate motive objects of activity (Miettinen, 2006; Vygotsky, 1978). When faced with a problem within activity, a new cultural tool that is adopted and used can potentially develop practice enacted by the subject (Miettinen, 2006; Nuttall et al., 2019). In CHAT terms, this *adoption* and use of cultural tools to solve problems is known as *remediation* (Miettinen, 2006). Adopting new tools can lead to changes in the activity system, including shifts in the division of labour and the distribution of power and authority (Cole & Engeström, 1993). Those then surface new forms of activity.

Tool *adaptation* refers to the process by which the subject modifies or creates tools to mediate the motive object in ways to better suit the activity in which they are engaged (Miettinen et al., 2012). Tool adaption can happen with externally oriented, physical tools (such as tools for manufacturing) and internally oriented, abstract tools (such as signs, language, and concepts).

Adaptation can involve minor adjustments to existing tools, the creation of new tools from existing ones, and the attribution of new meanings of purpose for the cultural tools already in use within the activity system (Miettinen et al., 2012; Nuttall & Brennan, 2016).

Tool adaptation is an important aspect of human activity, as it allows the subject to adapt to changing circumstances and improve their ability to achieve motive objects of activity (Nuttall et al., 2019). The subject has agentic potential to transform themselves, the contexts, and their activity when they adapt their cultural tools (Vygotsky, 1978), which can then drive cultural change (Nuttall & Brennan, 2016). In CHAT, that change is understood to be a dialectical process in which the subject not only adapts and shapes the cultural tools, but those tools simultaneously shape the subject.

In my study, I focused on understanding the significance and use of cultural tools that the School Mathematics Leaders used in their professional learning leadership activity, recognising that cultural tools are physical and psychological (Vygotsky, 1978). I was also sensitised to the ideas of tool adoption and adaptation, and how tools developed the activity of the School Mathematics Leaders over time.

3.9.5 Community

With Leont'ev's (1978) thinking about the individual mediated action to include the collective nature of activity, Engeström's (2015) addition of the concept of community acts as an important concept within CHAT. According to Engeström (1999b, 2015), the community is the element of the activity system composed of the subject and the other people within the activity system. Individuals are brought together because of the motive object(s) within the activity system.

Cole and Engeström (1993) stated that the community is a mediational element of any activity system because the subject does not act in isolation. The subject is understood to act within a social space. That stance is essential considering the contribution of thought offered by Leont'ev (1978) about the unity of activity; human activity does not exist outside of communal and social relationships. Bellamy (1996) noted that the subject's relationship with the community is mediated by the interaction and collection of the cultural tools, rules, and division of labour that exist within the activity system. That places the analysis of the activity within the social and cultural contexts of the environment.

In my study, I recognised the community of the School Mathematics Leaders' activity system as including the classroom teachers and principals (and other executive leaders) with whom the mathematics leaders worked due to their positionality as middle leaders in their schools (e.g., Grootenboer, 2018). It was also important that I paid attention to other aspects of community from neighbouring activity systems, such as the CTLM project team members (during CTLM and post-participation), CEOM, and others with whom the School Mathematics Leaders interacted.

3.9.6 Rules

The rules of an activity system are the laws or accepted practices followed by the subject and community within that system (Bellamy, 1996). Rules can also be described as the sets of conditions that signify ways and reasons for the subject and community's behaviour within the activity system (Hashim & Jones, 2007). Rules are the implicit and explicit norms, conventions, and social relations which mediate between the community and the subject (Kuutti, 1996). Those rules can include explicit formal laws and regulations, as well as implicit rules that the subject and community might take for granted. They can also include the implicit and explicit expectations that govern interactions between people within the activity system.

Rules also refer to the social and cultural norms, practices, and expectations that govern interactions within the activity system (Engeström, 2015). Rules mediate and shape how the subject interacts with the community, and again recognising the dialectical process, those rules shape the subject within the activity system (Kuutti, 1996). The concept of rules is understood as being dynamic and constantly evolving, shaped by the historical and cultural context in which they exist. Like cultural tools, the rules with the activity system have the potential to be adapted. Rules are open to challenge and change by the subject and community to achieve the motive objects and outcomes of activity. Rule bending, as a form of rule adaptation, can open new possibilities for the subject when they are faced with tensions (or *contradictions* in CHAT terms, which are discussed later) within the activity system in which they enact their activity (Edwards et al., 2010).

I interpret the concept of rules in relation to the routines that governed the School Mathematics Leaders' activity. I was conscious of the implicit and explicit nature of rules and how they had the potential to enable and constrain the activity of the School Mathematics Leaders. Like cultural tools, I was also sensitised to rule-bending as adaptation of rules and how that developed the activity of the mathematics leaders.

3.9.7 Division of Labour

The division of labour is the CHAT concept that describes the organisation of the community within the activity system. That organisation is realised through the actions and interactions between the subject and community as regulated by the division of labour within the system of activity (Engeström, 2015; Havnes, 2010). The division of labour is like that of rules in that it is explicitly and implicitly organised, with the division of tasks and actions taking on a mediational role in transforming the motive object into the outcome desired by the subject (Kuutti, 1996). Owing to this, the subject and community's relationship with the motive object is mediated through the division of labour (Bellamy, 1996). As with the other mediational elements in the activity system, the division of labour has the potential to enable and constrain actions.

A vertical division of labour concerns the distribution of power, status, and responsibility associated with the activity within the activity system (Cole & Engeström, 1993). The division of labour also concerns how the activity is distributed among the community members, including the tasks each individual undertakes within the activity system (Bellamy, 1996; Cole & Engeström, 1993). This is understood as a horizontal distribution of tasks.

The concept of division of labour is important when considering the School Mathematics Leaders as middle leaders within their school sites. I came to understand the concept of division of labour as "who does what and why" and how responsibility was realised within the School Mathematics Leaders' activity system. With their middle leading positioned between the principal (and other executive leaders) and classroom teachers and practised between those two groups (e.g., Edwards-Groves et al., 2016; Grootenboer, 2018), how tasks were distributed required my attention in the study. The vertical distribution of power, status, and responsibility was also important, considering their role as middle leaders in their schools.

3.9.8 Contradiction

Contradiction is essential in dialectical materialism and Marxist thought, where it is privileged as a necessary aspect of the change and progress process in the natural and social world (Mussachia, 1977). CHAT adopts Marx's stance on contradictions, positioning them as historically accumulated and recognises their potential to trigger resolutions through dialectical processes (Nicolini, 2012). Within a CHAT framework, contradictions are viewed beyond how they might be interpreted as a logical contradiction in a thought, or a spoken or written statement (Mussachia, 1977).

A significant advance of using CHAT is that it supports researchers in studying the interrelationships and the dynamic interaction between the different mediational elements of the activity system (Cole & Engeström, 1993). A way of examining those interrelationships and interactions is through a focus on contradictions. By examining interrelationships between the elements of the activity system, contradictions (tensions) can be found and resolved, and thus, expand activity within the system (Engeström, 1999a).

Contradictions exist in all activity systems because they are not static, stable systems. They are understood to surface due to tensions between the systems' mediational elements. Activity systems are in constant states of flux, and due to this, an activity system is characterised by internal contradictions (Cole & Engeström, 1993). Roth (2004) used *dilemmas*, *disturbances*, and *discoordinations* to describe contradictions.

Transition and change are features of any activity system, with reorganisations constantly occurring within activity systems (Cole & Engeström, 1993). In this chapter, I have presented this notion of transition and change regarding tool adaption and rule-bending. Of particular interest when investigating an activity system are contradictions that exist within and between the mediational elements of the system.

For example, contradictions may be faced by the subject of the activity system when the division of labour that governs the activity system interferes with the attainment of the motive object or when conditions privilege one component over another of the system (Huang & Lin, 2012). Another example might be a rule that constrains the enactment of undertakings and tasks that are enacted with the purpose of achieving the motive object(s). Those contradictions can bring about change and development, initiating the possibility of expansive transformation in the system (Engeström, 1999a). However, Roth (2012) noted that contradictions within an activity system cannot be removed because contradictions are characteristic (constitutive) of the activity system itself. This brings further reason for understanding the activity system in a constant state of flux (Cole & Engeström, 1993; Roth, 2012), where the subject negotiates meaning to make sense of the object within the activity system (Engeström, 2015).

Seeing that contradictions are the driving force for transformation within any activity system (Engeström & Sannino, 2010), the cause of tensions can be recognised when the subject and community identify them (Engeström, 2000). According to Engeström (2000), shared visions for working with contradictions can be achieved when those involved in the activity spend time analysing the contradictions collaboratively.

3.9.8.1 Manifestations of Contradictions

According to Engeström and Sannino (2011), contradictions are characterised as being systematic and historically accumulated, and due to this, they can only be identified as *manifestations*. Contradictions are manifested through the subject or community's words and actions. This is the only way that contradictions can be accessed through empirical study. Engeström and Sannino identified four manifestations that allow researchers access into contradictions that are surfaced within the activity system under investigation. The four manifestations, as reported by Engeström and Sannino (2011), are:

Dilemmas: Those are situations that arise when the subject (or others in the activity system) is faced with a choice between two incompatible options, and decisions and judgements for action are replete with hesitation. This manifestation is realised through phrases like "on the one hand…but then on the other hand" or "I am torn between doing this…and doing that".

Conflicts: Those manifestations occur when people engaged in activity dispute each other's opinions and actions and express that opposition through resistant and argumentative behaviours. Other expressions of conflicts are realised through actions characterised by questioning, disagreeing, and engaging in criticism.

Critical conflicts: Those manifest when the subject is in paralysing situations where they experience doubts and confusion about the motive objects of activity, usually because they are contradictory but mutually dependent. The subject struggles, often experiencing isolation or guilt, because meaning-making is hampered, leaving them indecisive about which actions to take.

Double binds: Those manifestations surface for the subject or community when there is a pressing need to act, yet simultaneously a perceived impossibility of action exists. Subjects ask rhetorical questions (e.g., "What do I do here?"). Furthermore, they often experience feelings of helplessness or desperation. Due to reaching a crisis point, the subject is forced to act in ways they perceive that they cannot enact.

In my study, it was important that I honoured the historically accumulated nature of contradictions and how the School Mathematics Leaders worked with them as a means of developing their activity. I also attended to the idea of manifestations as the realisation of the contradictions within the mathematics leaders' activity system. Through my reading of *resourceful practice* (e.g., Edwards, 2005), the role of contradictions became important in understanding how the activity is characterised as resourceful. This provided another methodological reason for my attention to contradictions.

I now turn to a discussion of resourceful practice and several of its characterisations, focusing on ones that were relevant to my study.

3.10 Resourceful Practice

In this section, I turn to a CHAT concept outside of the representation of the activity system. I do this because later in the thesis, I adopt that concept to explain several of my essential claims. I specifically draw on work within the last 20 years by Anne Edwards, a CHAT researcher, who developed the notion of *resourceful practice* (e.g., Edwards, 2005, 2010a) as a contemporary concept within the field of CHAT. That concept has explanatory power in understanding how the subject (individual or collective), when faced with contradictions or tensions, creatively and flexibly uses resources as a means of resolving the problems of practice with which they are confronted (Edwards, 2010a).

Resourceful practice positions resources as both physical and psychological, honouring them as forms of cultural tools (Engeström, 2015; Vygotsky, 1978) where the subject understands the transformative potential of tool use to develop their activity. Resources are also understood in terms of human resources, where people and their expertise from neighbouring activity systems can be accessed and used as a means of addressing contradictions. Those physical, psychological, and human resources can support the subject in driving their activity forward as they seek to resolve those contradictions within their activity system (Edwards, 2011). By engaging in resourceful practice, development within the activity system occurs as new forms of activity are enacted in response to contradiction resolution.

Activity development happens when new or different actions are undertaken, brought on by the adaptation of already established mediational means or through the adoption of new cultural tools, rules, or divisions of labour (Edwards, 2010b). This adaptation and adoption of mediators happen as the subject directs their activity towards the new or shifted motive objects of activity. This is part of the objectification of *what matters* for the subject as they direct their activity at those shifted motive objects, working on the problem space, which is the resolution of contradictions within the activity system (Edwards & Thompson, 2013).

Resourceful practice emphasises the active role of the subject engaged in the activity using resources available to them. Those resources are within their own activity system and can be resourced from systems outside their immediate activity system (Edwards, 2005). Through that active role shaping their activity, the subject is also shaped through that dialectical relationship between the subject and motive objects of activity.

Edwards (2010a) identified several characterisations of resourceful practice. For the purposes of my thesis, I focus on three characteristics. I do this because of their relevance in supporting my explanation of the School Mathematics Leaders' contributions to project sustainability.

3.10.1 Adapting Tools

One characteristic of resourceful practice is that of tool adaptation. The concept of tool adaptation sits within resourceful practice due to its Vygotskian roots. Vygotsky (1978) claimed that through the process of tool adaptation, humans have the capacity to not only change cultural tools but also transform themselves as well as the circumstances in which they find themselves. That adaptation of tools becomes a part of resourceful practice as means of resourcefully and creatively using tools in different ways to resolve practice problems confronting the subject (Edwards, 2010a).

Edwards and colleagues (Hannan et al., 2011) reported that resourceful leaders are ones who trial new tools and adapt them as required when working on complex problems of practice. That requires knowledge of resources that are already available and changing them in ways that better suit the direction of activity. Through that tool adaptation process, new meaning is attributed to the cultural tools, as they are used with different purposes, thus mediating new motive object of what matters to those resourceful leaders (Edwards & Thompson, 2013).

3.10.2 Rule Bending

Within resourceful practice, the concept of *rule-bending* is featured (Edwards, 2010a; Edwards et al., 2010; Edwards & Thompson, 2013). The subject engages in rule-bending when they creatively and resourcefully adapt the explicit and implicit rules that govern the activity system. The subject engages in rule-bending to solve problems and achieve the motive objects of activity in more efficient, effective, and new ways (Edwards, 2010a).

The bending of rules to better meet the purposes of achieving motive objects is seen as a resourceful method of adaptation and creativity (Edwards & Thompson, 2013). Rule-bending can be realised in several ways. It can be done by modifying, altering, or breaking the explicit or implicit rules that have been historically followed to achieve motive objects.

3.10.3 Accessing Distributed Expertise

Resourceful practice is not only an individual process but one that is collective in nature. People engage in resourceful practice as members of communities within and networks across interacting activity systems (Edwards, 2010a). They use the resources within their own community, as well as access and use resources from others outside of their activity system. Resources from different practices used in different activity systems are employed to understand and enact new forms of activity (Edwards, 2011).

Resources, within this concept of resourceful practice, go beyond material ones and include the expertise of others from neighbouring activity systems (Edwards, 2010b). That is considered *distributed expertise* by Edwards (2011). When the subject accesses distributed expertise, they acknowledge and engage with what others may offer as resources. Those resources are then adopted as new mediational means. The subject also recognises what they have to offer to others across other activity systems (Edwards, 2010b).

When accessing distributed expertise, the subject recognises the *resource potential* that those others bring to support the resolution of contradictions. Accessing distributed expertise could include the interaction between subjects to understand the professional motive objects pursued by subject(s) in neighbouring activity systems. In turn, it acts as a resource that mediates collaboration between activity systems to work on complex problems (Edwards & Thompson, 2013). Through that process, the subjects within the interacting activity systems appreciate what matters for the other. They become resources for each other in their attempts to resolve the complexity of their practice problems.

I now present how I viewed the research problem of my study through a CHAT lens.

3.11 Viewing the Present Study Through a CHAT Lens

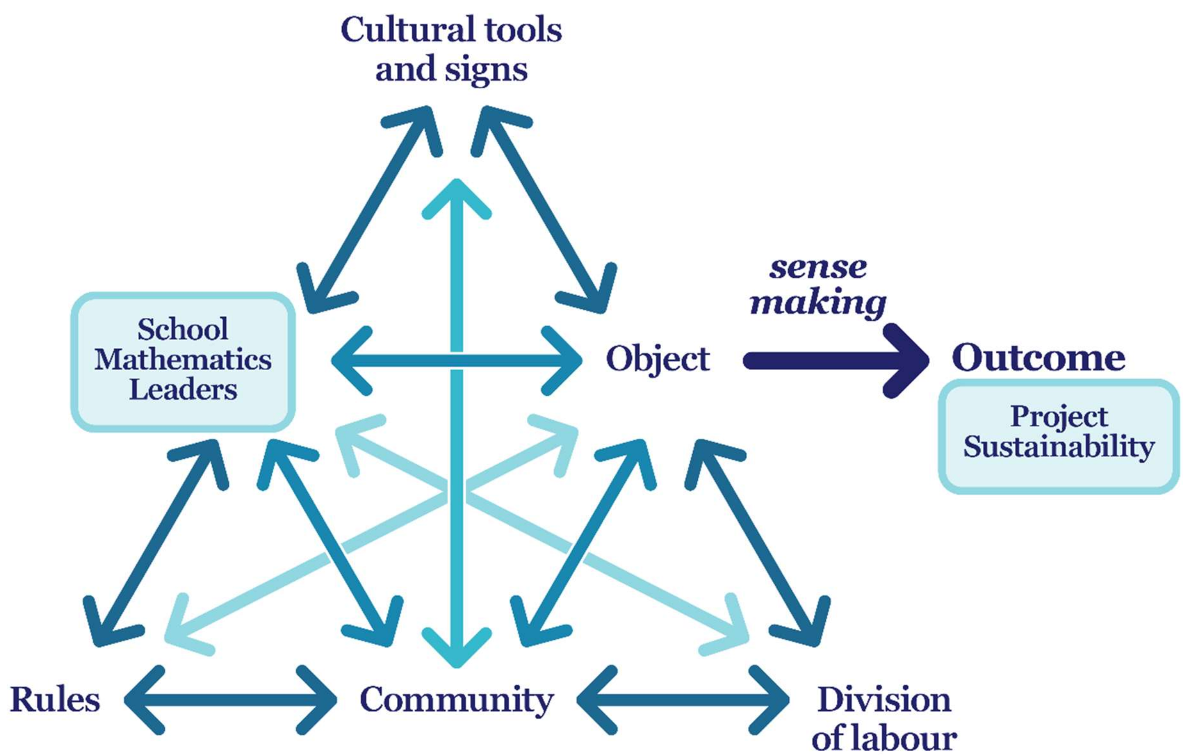
In the previous chapter, I provided a review of relevant literature. A highlighted aspect was the complexities of project sustainability and the practice of mathematics leadership in schools. I argued for mathematics leadership as a form of middle leading and how it could be deemed a middle leadership role within the school leadership system (Camburn et al., 2003; De Nobile, 2018; Hammersley-Fletcher & Kirkham, 2007; Lipscombe et al., 2021). I highlighted that recent research has focused on middle leadership as a form of practice (e.g., Grootenboer, 2018). Knowledge of middle leading has moved away from understanding it in terms of the characteristics and traits of middle leaders (Grice et al., 2023). Therefore, as a way of supporting my contribution to literature, it was important that I also take a practice approach to understand the activity of the School Mathematics Leaders as middle leaders and how their activity contributes to the complex nature of project sustainability (Bobis, 2011; Datnow et al., 2005. Zehetmeier & Krainer, 2011).

In this chapter, I have presented my understanding of CHAT as a theoretical framework showcasing how it is a practice-based theory that supports an understanding of learning and development that comes out of the culturally and historically mediated practical activities (Engeström, 2015; Roth, 2012; Sannino et al., 2009; Yamazumi, 2007). I presented CHAT as a means of understanding practice, with the concept of the motive object acting as a powerful analytical tool to support understanding of the what and the why activity takes place (Kaptelinin, 2005). I provided reasons for employing the second generation of CHAT and how it is used to understand practice as an activity system (Engeström, 2015; Roth, 2012). I also shared my understanding of the concepts afforded to the researcher who chooses to use CHAT as a methodological tool.

As a way of demonstrating my initial research reasoning about the use of CHAT to study how the School Mathematics Leaders contributed to project sustainability through their professional learning leadership, I present the following diagram (Figure 8) using the triangular model (Engeström, 2015; Roth, 2012; Roth & Lee, 2007)

Figure 8

Using the Second-Generation CHAT Activity System to View the Study



As can be seen in Figure 8, the triangular model of the activity system (Engeström, 2015; Roth, 2012; Roth & Lee, 2007) as a representation of second-generation CHAT has been used to represent the activity of the School Mathematics Leaders. I positioned the School Mathematics Leaders as the collective subject (Lektorsky, 2009) within the activity system, and project sustainability has been represented as the outcome of their professional learning leadership activity. In that way, with the descriptions of my understanding of the CHAT concepts available to me, I was charged with identifying and analysing the multiple relationships that existed within and between the mediational elements of the activity system (Engeström, 2015; Foot, 2014; Roth, 2012). Using the activity system model also became important in visually representing my focal theory (articulated in Chapter 8).

To support the development of my focal theory, I focused my research reasoning on understanding the enabling and constraining nature of the mediational element of the School Mathematics Leaders' professional learning leadership. CHAT, with its principles, allowed me to appreciate the historicity of the School Mathematics Leaders' activity, thus fully understanding their post-project leadership (Foot, 2014; Roth, 2012). Therefore, I needed to pay attention to the activity that the mathematics leaders enacted during their CTLM participation and their leadership activity after that project.

This background to CHAT described in this chapter demonstrates the potential the theory has for generating and analysing empirical data required to respond to the problematic of my thesis and answer the research question. CHAT supported the planning and development of the research design articulated in Chapter 4 that I developed to systematically study the School Mathematics Leaders' professional learning leadership activity.

3.12 Chapter Summary

In this chapter, I presented CHAT as a practice-based theory used to make sense of the nature and function of activity. I provided the historical background of the theory, starting with relevant contributions of thought from Heraclitus and Marx, with Marx's thinking about activity positioned within the ontology of dialectical materialism. I followed by describing the pertinent work of Vygotsky and Leont'ev. That was followed by descriptions of three generations of

CHAT, showing how concepts have been taken up within those generations. That included articulating the purposes of those CHAT generations in studying activity.

I explained my choice of using second-generation CHAT to study the School Mathematics Leaders' professional learning leadership activity. I articulated my understanding of the purposes and principles of CHAT. That was followed by a discussion of the concepts associated with the activity system as a feature of CHAT's second generation. I focused on those concepts that support investigations into activity systems. I also described how I was sensitised by those concepts, articulating how I saw them positioned within the activity of the School Mathematics Leaders.

I presented information about resourceful practice. I conveyed that this was a form of activity understood in contemporary uses of CHAT. I explained three characteristics of resourceful practice due to their relevance to my study.

I described how the use of second-generation CHAT, and its triangular model, focused my investigation into the mathematics leaders' activity system. I shared how the model drew my attention to the interactivity of the activity system components. I noted that attention needed to be paid to the historicity of the School Mathematics Leaders activity system.

In the next chapter, I articulate how I operationalised concepts from the second-generation CHAT, drawing on my understanding of that practice-based theory. I explain the research reasoning that informed the research design of my study.

CHAPTER 4: THE RESEARCH DESIGN

4.1 Introduction to Chapter 4

The purpose of this study was to generate knowledge about how mathematics leaders contribute to project sustainability through their post-project professional learning leadership activity. That was realised by focusing on the motive objects of activity to understand the how and why of the School Mathematics Leaders' activity following participation in a large-scale mathematics education professional development project. Having reviewed background literature within the areas of project sustainability, middle leadership, mathematics curriculum leadership, and mathematics professional learning (Chapter 2), and then having presented information about CHAT as a theoretical framework (Chapter 3), I now focus on the research design that I conceptualised and enacted in my study.

I do not use the title *methodology* for this chapter. Instead, I use *research design* (Creswell, 2013) to capture the entirety of my research activity: my research reasoning and actions. Research design as a term captures the conceptualisation (ontological and epistemological considerations) and enactment (methodological actions and analyses practices) of my research activity.

In this chapter, I detail how I operationalised CHAT. I present how I combined the use of a CHAT-informed model (Mwanza, 2001, 2002) with advice from other researchers within the CHAT field (Engeström, 2001; Mwanza & Engeström, 2003, 2005; Uden et al., 2008) to create a *research process* that mediated my research activity. I identify and explain the methods of data generation and disclose my data analysis approach. This chapter concludes with the identification of the ethical considerations of my study.

4.1.1 Research Design as a Mediator of My Research Activity

Within Chapter 3, I explained how I viewed my study through a CHAT lens, positioning the post-project professional learning leadership of the School Mathematics Leaders as an activity system (Engeström, 2015). I reflected more on the concept of activity system and arrived at the realisation that my own research work was indeed an activity system unto itself. This chapter, therefore, became an essential mediating tool for my research activity. It captured the rules, cultural tools, and division of labour that mediated the motive objects of my research activity.

4.2 Revisiting the Research Question

The research question should integrate with the overall research design (Trede & Higgs, 2009). Recognising that, I revisit the research question to foreground the research design discussion and acknowledge influence of the research question on my research reasoning (Punch, 2005; Trede & Higgs, 2009).

The research question for my study was:

As middle leaders of site-based professional learning, how do School Mathematics Leaders contribute to the sustainability of mathematics teaching reforms in the years that followed participation in a large-scale school mathematics professional development project?

4.2.1 Impact of the Research Question on Research Design

To fully appreciate the research question, I reflected on its impact on my research reasoning and actions. I accepted that I needed to spend time with the School Mathematics Leaders in their school sites. That became a rule ("spend time in the field") for my research activity. Recognising the importance of that, I needed to gain insights into the mathematics leaders' enactment of their leadership activity. That meant I needed opportunities to discuss professional learning leadership with the School Mathematics Leaders and observe their enactment of their activity. Observations and discussions needed to be cultural tools that I required to mediate the motive objects of my research activity. I recognised the need to know more about the cultural tools the mathematics leaders used within their leadership, so retrieval of documents and resources was required.

Considering those points, I needed to study the lived experiences of the School Mathematics Leaders' activity through a process of exploration (Creswell, 2012). Recognising that, along with acknowledgement of advice offered by previous mathematics education researchers who have studied project sustainability (e.g., Tirosh et al., 2015; Zehetmeier, 2014), I realised that I required the affordances of qualitative research, especially the mediational means that are offered through that form of research.

4.3 Qualitative Research

Qualitative research can be used to inquire into a problem where limited literature is available about the problematic under investigation. It can be used when there is a need to develop or refine theories associated with that problem (Creswell, 2012, 2013). As highlighted in Chapter 2, there needs to be more research literature about project sustainability (e.g., Bobis, 2011; Coburn et al., 2012; Saito et al., 2012; Smit et al., 2019; Zehetmeier, 2015) and the middle leading activity of mathematics leadership in primary schools. Considering those insights from Creswell, I saw a further reason for designing a qualitative research study.

Qualitative research is used to generate insights into the quality of meaning and provides researchers with opportunities to explain "the meaning individuals or groups ascribe to a social or human problem" (Creswell, 2013, p. 44). Qualitative research, therefore, requires the use of approaches which facilitate inquiry and that honour the perspective of the people engaged in the problematic under investigation (Denzin & Lincoln, 2011a; Creswell, 2013; Merriam, 2009; Punch, 2005; Snape & Spencer, 2003). Qualitative research also provides ways to generate and analyse data that support the interpretation and presentation of the perspectives of participants engaged with the problematic (Creswell, 2013; Merriam, 2009; Punch, 2005).

In Chapter 3, I discussed CHAT as a framework for understanding workplace activity from the subject's perspective within the activity system (Engeström, 1999, 2001; Nardi, 1996; Nuttall & Brennan, 2016; Roth, 2007). That feature of CHAT aligns with qualitative research because exploration of the subjects' (participants) perspectives can be sought through the generation of detailed descriptions of their lived experiences, an essential characteristic of qualitative research (Creswell, 2013; Merriam, 2009; Snape & Spencer, 2003).

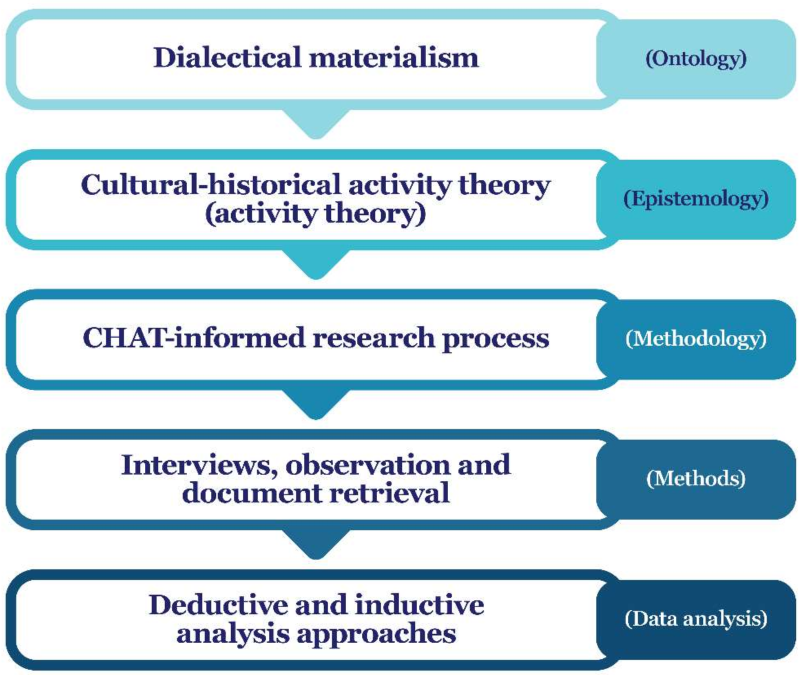
4.4 Conceptualisation of the Research Design

My research design articulates the philosophical assumptions and research guidelines (rules, tools, and division of labour within a CHAT perspective) that link theoretical underpinnings with the strategies used to generate data that support my focal theory in response to the research question (Creswell, 2013; Crotty, 1998; Denzin & Lincoln, 2011a; Merriam, 2009). The theoretical underpinnings and assumptions that I considered when conceptualising the research design included ontology, epistemology, and methodology (Crotty, 1998; Denzin & Lincoln, 2011b).

I considered my own philosophical assumptions and their influence on the research design of my study. Therefore, I aligned the components to create a cohesive research design (Crotty, 1998; Denzin & Lincoln, 2011a; Lewis, 2003). As a way of representing my research reasoning, I present the following figure (Figure 9) based on the work of Crotty (1998). The arrows show how each component within the research design was aligned. The arrows are not intended to represent a hierarchy but rather show the influence of each component on another.

Figure 9

Representation of the Research Design



Dialectical materialism is the ontology in which this study is situated. Within that ontology are epistemological beliefs informed by CHAT. The methodology for my study was the research process that I created, informed by the work of researchers working within the CHAT field (Engeström, 2001; Mwanza, 2001, 2002; Mwanza & Engeström, 2003, 2005; Uden et al., 2008). Finally, the methods chosen to generate data were aligned: interviews, observation, and document retrieval. Recognising that dialectical materialism as the ontology and CHAT as the epistemology, I used deductive and inductive analysis approaches to generate findings.

4.4.1 Ontology

Ontology concerns the nature of reality and its characteristics (Denzin & Lincoln, 2011b). CHAT has its own epistemological framework and ways of viewing the world. As explained in Chapter 3, CHAT has its immediate roots in twentieth-century Russian social psychology, but its tradition lies in eighteenth and nineteenth-century German philosophy (Jonassen & Rohrer-Murphy, 1999) and even goes as far back as the thinking of Heraclitus. German philosophy during the eighteenth and nineteenth centuries emphasised reality in both developmental and historical ways, including the active and constructive role of humans within that reality (Kuutti, 1996).

Dialectical materialism privileges the idea that reality exists outside of and is independent of the perception of humans (Nuttall & Brennan, 2016). Having history in Marxist thought, dialectical materialism acknowledges an external reality that makes a material difference to how humans act in and experience that reality (Denzin & Lincoln, 2013). As a type of realist ontology, dialectical materialism is interested in "understanding and modelling the apparently ever-changing world" (Roth, 2012, p. 94) and understanding that present human activity has manifested in response to situations, especially contradictions, in that ever-changing world (Nuttall & Brennan, 2016). Only that ever-changing material world is real, and human consciousness is manifested through and from enactment within it (Snape & Spencer, 2003).

4.4.2 Epistemology

Epistemology is focused on how humans know the world or how they come to know what they know (Creswell, 2013). It focuses on explaining how human beings come to understand reality (Denzin & Lincoln, 2013). In Chapter 3, I presented my understanding of CHAT and how it relates to my study, so I will highlight only the key features as they relate to the epistemology of my research design.

Within CHAT, epistemological beliefs about knowledge are viewed as an interrelationship between knowledge and activity. CHAT espouses the view of consciousness and activity as two entities that are "dynamically interrelated" (Jonassen & Rohrer-Murphy, 1999, p. 62). Kaptelinin et al. (1995) described this interrelatedness as a principle of unity and inseparability of the human mind and activity.

When thinking about epistemology through a CHAT perspective, knowledge is understood within the context of meaningful, object-orientated, and historically and culturally mediated activity between humans and the material world (Engeström, 2015; Kaptelinin, 2005; Kaptelinin et al., 1995; Nuttall & Brennan, 2016; Roth, 2012). Knowledge is viewed as being historically mediated because present human activity and human consciousness, the enactment of knowledge, is an accumulation of past forms of activity (Engeström, 2015; Nuttall & Brennan, 2016). CHAT also sees knowledge as culturally mediated because Vygotsky (1978) claimed that all activity was mediated action. To this assertion, human beings and knowledge cannot be understood without mediators, the cultural means by which humans come to act and therefore know (Engeström, 2015). In this sense, knowledge is created from human activity, and it is not a precursor to activity which the idealistic views of human knowledge claimed (Jonassen & Rohrer-Murphy, 1999). Within a CHAT perspective, knowledge creation is a process enacted through and from meaningful, object-focused activity mediated by historical and cultural means.

4.4.3 Methodology

Methodology is centred on recognising the best way of acquiring knowledge about the world (Denzin & Lincoln, 2011b). The methodology of my research design articulates the research process I enacted throughout my study (Creswell, 2013). When planning this, I considered the most appropriate cultural tools, rules, and division of labour to best meet the motive objects of my research activity.

4.4.3.1 Reason for Not Using Case Study

I considered using a case study as a potential methodology for my research design. I considered its various types, but this proved a struggle because case study has been named in different ways, with titles including *methodology*, *method*, and *approach* (Creswell, 2013; Denzin & Lincoln, 2011b; Stake, 1995). Further tensions were faced when I noted that case studies had been a popular choice for qualitative researchers (Hyett et al., 2014; Yazan, 2015). Historically, they have provided CHAT researchers with opportunities to examine subject-object activity relationships (Yamagata-Lynch, 2010).

I explored the interpretations of case study provided by Stake (1995), Merriam (2009), and Yin (2003). Those methodologists were chosen due to their prominence in the case study field (Hyett et al., 2014; Yazan, 2015). I noted that both the Stake and Merriam interpretations of case study were situated within constructivist paradigms, whilst Yin's work aligned with post-positivism. I struggled to reconcile CHAT's ontological and epistemological orientations with those underpinning case study methodology.

In response to that struggle, I created a research process specifically for my study. I drew on previous work of CHAT theorists (Engeström, 2003, 2005; Mwanza, 2001, 2002; Mwanza-

Simwami, 2011; Uden et al., 2008), using Mwanza's *Eight-Step Model* (ESM) that she developed to operationalise CHAT in her studies. The ESM proved to be a helpful cultural tool that I incorporated into my research process.

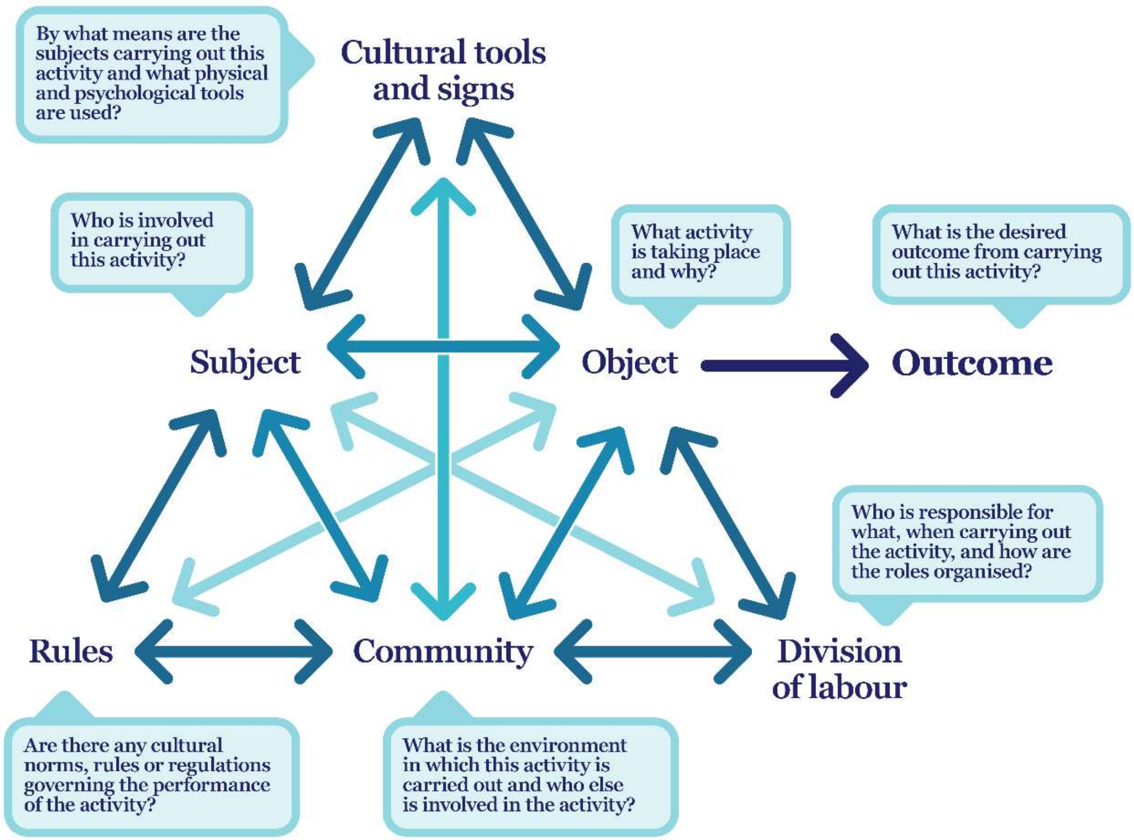
4.4.3.2 The Eight-Step Model

The ESM was initially developed to create a CHAT-based methodology to investigate the activity of computer system developers (Mwanza, 2001). It was devised to operationalise Engeström's (2015) concept of the activity system. The ESM has since been used as a methodological tool to study other workplace activities (Mwanza-Simwami, 2011; Mwanza & Engeström, 2003, 2005; Uden et al., 2008).

The ESM is a series of open-ended questions that address the components of the activity system (Engeström, 2015). Mwanza and Engeström (2003) suggested that researchers use those eight questions to interrogate and analyse activity system components. I situated those questions within Engeström's well-known triangular representation to show how the ESM questions relate to the activity system (Figure 10).

Figure 10

Representation of the Eight-Step-Model Mapped to Activity System Triangle



Mwanza-Simwami (2011) reported that mapping the ESM supports the CHAT researcher in understanding: the structure of the activity under investigation; the motive objects pursued by the subject; the interrelated nature of the components of the system; and the role that cultural tools, rules, and division of labour play as mediators of motive objects. Using the ESM (Mwanza, 2001, 2002; Mwanza-Simwami, 2011), I created a CHAT-informed research process to generate data in response to the research question.

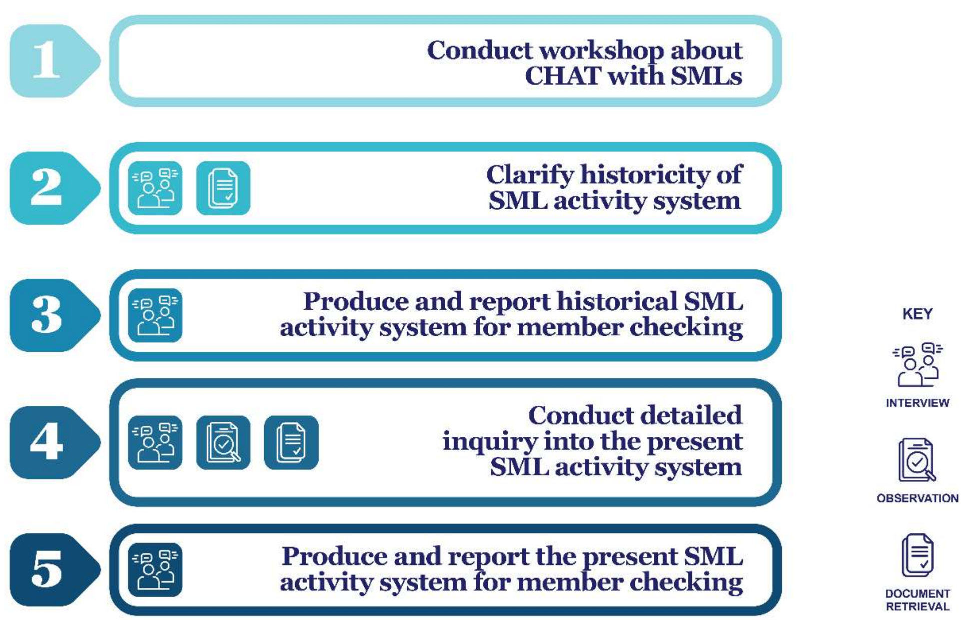
4.4.3.3 Research Process

Drawing on the work of CHAT researchers (Engeström, 2001; Mwanza, 2001, 2002; Mwanza & Engeström, 2003, 2005; Uden et al., 2008), I developed a research process as a series of actions intended to realise the motive objects of my research activity. Those actions (represented in Figure 11) were: *conduct workshop about CHAT with SMLs*; *clarify historicity of SML activity system* (Engeström, 2001; Mwanza, 2001, 2002; Uden et al., 2008); *produce and report historical SML activity system for member-checking* (Engeström, 2001; Mwanza; Uden et al., 2008); *conduct detailed inquiry into the present SML activity system* (Mwanza, 2001); and, *produce and report the present SML activity system for member-checking* (Engeström, 2001; Mwanza, 2001, 2002; Uden et al., 2008).

When creating the research process, I needed to consider appropriate methods. I recognised that it was vital that they were chosen purposefully and that they aligned with the research design (Creswell, 2013; Patton, 2002; Punch, 2005). The methods were selected and matched to each action of the research process. The key in Figure 11 shows images representing each of the methods matched to the actions of the research process.

Figure 11

Research Process Within the Research Design



4.4.3.3.1 Conduct Workshop about CHAT with SMLs I wanted to be as transparent as possible with my research activity, so I decided to share information about CHAT with each School Mathematics Leader. Before enacting that research action, I ensured that each mathematics leader was informed of my study by sharing an information letter with them (Appendix D) and securing consent for participation (Appendix E).

Before any data generation occurred, I met with each School Mathematics Leader in their school site and engaged in a dialogical space called a "CHAT workshop". I also did this as a means of building trust with each School Mathematics Leader, honouring that critical feature of qualitative research (Creswell, 2013; Greene, 2014). I shared the Engeström (2015) triangular representation of the activity system with each leader and how CHAT can make sense of activity by focusing on motive objects. I also discussed how all activity is influenced by mediational elements such as cultural tools, rules, and division of labour. I shared the role of contradictions in activity systems and their importance in CHAT analysis. I then shared how I initially translated the use of CHAT, using Figure 8 as a prompt, sharing my understanding of those CHAT concepts with the School Mathematics Leaders.

After presenting that information, the School Mathematics Leaders were invited to ask questions about my study. I shared with them the research question and how this related to the research problem. At the end of the CHAT workshop, I asked for a pseudonym from each mathematics leader. Those pseudonyms were used during data generation, data analysis, and in the discussion of the findings (Chapters 5, 6, and 7).

4.4.3.3.2 Clarify the Historicity of the SML Activity System. As described in Chapter 3, it is essential that the CHAT researcher honours and explores the historical aspects of the activity system under investigation (Engeström, 2015; Nuttall & Brennan, 2016; Roth, 2012). Therefore, the second action of the research process concerned clarifying the historical activity system (Mwanza, 2001; Uden et al., 2008). That required identifying the historical motive objects of the professional learning leadership activity of the School Mathematics Leaders, which they pursued during participation in CTLM.

During that action, I employed the interview method, using questions based on the ESM (Mwanza, 2011, 2002) to generate data. Those questions were included in the "during CTLM leadership" interview protocol (Appendix F). As a means of building further trust with the School Mathematics Leaders, I presented them with that protocol, sent via email two weeks prior to the school visit.

At those interviews, which were electronically recorded and transcribed, the School Mathematics Leaders were asked to share the historical aspects of their professional learning leadership enacted during CTLM. I asked for access to documents and resources that they used in their professional learning leadership at the time of CTLM participation (if they were readily available, recognising that those interviews took place more than two years after participation in CTLM).

4.4.3.3.3 Produce and Report Historical Activity System for Member Checking. That subsequent action of the research process included defining the mediational elements of the historical activity system (Mwanza, 2001; Uden et al., 2008). Data generated through the preliminary interviews and document analysis were analysed using deductive and inductive approaches (Section 4.7). Key concepts from CHAT and the ESM questions (Mwanza, 2001) were used to analyse data. That was done to map the activity system using the triangular model

(Engeström, 2015), representing the School Mathematics Leaders' historical professional learning activity system. Those mapping exercises (Mwanza, 2001, 2002) allowed me to interpret the mediational elements of the activity system, mainly focusing on the motive objects of the mathematics leaders' historical activity during CTLM.

I also built in the member-checking validation strategy (Creswell, 2013; Merriam, 2009; Punch, 2005) into that action of the research process. Member checking happened when I took my preliminary analysis of the historical activity system back to the School Mathematics Leaders for their feedback (Creswell, 2013). That was done by visiting each school site again, asking the mathematics leaders about their perceptions of the initial data analysis. Those conversations were electronically recorded and transcribed. Data generated from those interviews were later used to refine the analysis so that it reflected more accurately perspectives of the School Mathematics Leaders about the historicity of their professional learning leadership.

4.4.3.3.4 Conduct a Detailed Inquiry into the Present SML Activity System. That action supported the generation of data about the School Mathematics Leaders' post-project professional learning leadership activity using the methods of observation, interview, and document retrieval (Section 4.6). The ESM questions were again used to focus the data generation (Mwanza, 2001). Those questions informed the “post-CTLM leadership” interview protocol (Appendix G) and guided my observations of the School Mathematics Leaders as they enacted their professional learning leadership (Appendix H).

Examples of questions that I used during that research action of the methodology were:

- What motive objects are pursued by the SMLs? What is it that they are working on?
- Which motive objects are given priority?

- What cultural tools, including knowledge, do the SMLs use to work on those motive objects, and how are these used?

- What are the rules that mediate the motive pursued by the SML? How do those rules work?

- How does the division of labour affect the way the SMLs work on the motive objects?

- What tensions are evident in the SMLs' work?

- What has changed in activity since CTLM, and why have those changes occurred?

During that action, I interviewed and observed the School Mathematics Leaders in their school sites for just over 3.5 years. That prolonged period acted as another validation strategy (Creswell, 2013) as I learned more about the situation in which the mathematics leaders worked, and that prolonged time supported me in staying attuned to what was relevant to the School Mathematics Leaders and my study (Creswell, 2013; Merriam, 2009). It also allowed me to notice aspects of activity that endured and shifted over time.

That period in the field provided opportunities to learn from the School Mathematics Leaders through 21 school site visits. The intention was to spend the equivalent amount of time with each mathematics leader. That did not happen due to the busyness of the School Mathematics Leaders' schedules, and with my ethical commitment to honour their time, I adjusted my schedule to accommodate. With that noted, one leader was visited nine times (total time in the field: ~12.5 hours), seven visits for the second leader (total time in the field: ~12 hours), and five visits for the third mathematics leader (total time in the field: ~7.5 hours). It was planned that at each visit, a professional learning opportunity facilitated by the School Mathematics Leader was observed. That happened for the first and second leaders. However, with the final mathematics leader, she was observed only four times during the five site visits to her school. As a way of mitigating any bias due to the differences in time within the field, I used some of the time with the third leader to check and test my interpretation of the data generated with the first and second mathematics leaders (whilst also being open to generating new insights with that third leader).

Each professional learning opportunity observation lasted from a minimum time of 45 minutes to a maximum of 90 minutes, with the mean time of each session equal to 60 minutes. I discuss how I enacted the observations later in this chapter (Section 4.6). It was intended that each School Mathematics Leader was interviewed before and after each observed professional learning session. This again only sometimes happened, with the most frequent reason being the busyness of the leaders' workday. When the "before interviews" occurred, they usually took between 5 to 12 minutes. These interviews acted as a time for the School Mathematics Leader to discuss their professional learning intentions. The "after interviews" were extended, and they acted as a time for the School Mathematics Leader to reflect on their leadership activity. Documents that the School Mathematics Leaders used were also retrieved during this research action.

At the conclusion of each school site visit, I wrote analytical memos (Merriam, 2009) which included a summary of the school site with preliminary data analysis and interpretation (Appendix I). I discuss using analytical memos later in this chapter (Section 4.7).

4.4.3.3.5 Produce and Report the Present Activity System for Member Checking. That action concerned data analysis that answered the research question fully. I followed a similar process to the action for data generation about the historical activity system. However, the analysis focused on the School Mathematics Leaders' post-project professional learning leadership activity.

I aimed to identify the School Mathematics Leaders' motive objects and the mediators of their post-project professional learning leadership activity. I again used deductive and inductive analysis approaches, drawing on key concepts from CHAT, the ESM questions (Mwanza, 2001) and sensitising concepts from background literature to code and analyse the data (Van den Hoonaard, 2008). I again mapped aspects of preliminary data analysis to the triangular representation of the activity system (Engeström, 2015) for member checking by the mathematics leaders (Creswell, 2013).

The outcome was then reported to the School Mathematics Leaders through an interview for member checking. That was done to check that my analysis stayed true to their experiences as middle leaders of mathematics. Those interviews were electronically recorded, transcribed, and analysed (Creswell, 2013; Merriam, 2009).

I chose to analyse the data using a deductive analysis approach using CHAT concepts, including the sensitising concepts from background literature (Van den Hoonaard, 2008). I also decided to use an inductive approach because it is a feature of qualitative studies (Creswell, 2013; Merriam, 2009). I was interested in the "what else" of the School Mathematics Leaders' post-project professional learning leadership activity.

Contradictions are essential to analyse within a CHAT perspective because they help the researcher understand the activity system's functions (Engeström, 2001; Mwanza, 2002). I looked for evidence of contradictions within the School Mathematics Leaders' activity system

(Engeström, 2015; Mwanza, 2001; Uden et al., 2008), including manifestations of contradictions (Engeström & Sannino, 2011).

The research process that I developed for my study placed the School Mathematics Leaders as participants in a prominent position as the people experiencing the phenomenon first-hand. For that reason, I carefully considered the School Mathematics Leaders who would participate. Consideration included an analysis of the research question and the creation of participant selection criteria.

4.5 Participants

I valued the vital role that the School Mathematics Leaders played in my study. Therefore, when enacting my research design, I needed to pay attention to the choice of participants (Creswell, 2013; Merriam, 2009; Patton, 2002). Creswell (2013) reported a sampling strategy known as *purposeful sampling* for use in qualitative research. Merriam (2009) suggested that sampling is helpful when looking for participant diversity. For my study, I interpreted the idea of purposeful sampling by carefully considering the research question. Through that consideration, I developed participant selection criteria, enacting a type of sampling known as *criterion sampling* (Creswell, 2013).

4.5.1 Participant Selection Criteria

The criteria for participant selection were determined through an analysis of the research question. Figure 12 presents that analysis which led to the development of the participant selection criteria. That provided the means to enact criterion sampling (Creswell, 2013).

Figure 12

Analysis of Research Question to Generate Selection Criteria

From the analysis of the research question, I established three selection criteria:

1) participated in the CTLM project (preferably in Intake 4, as explained in Chapter 1);
2) undertaken the School Mathematics Leader role during and after participation in the CTLM project; and,
3) responsibility for the leadership of mathematics professional learning for teachers during the CTLM project and during the research period of this study (data generation phase).

Those criteria were used to decide which school principals would be selected and contacted, abiding by ACU and CEOM ethics protocols.

4.5.2 Participants' Background Information

Three participants, who fulfilled the selection criteria, chose to participate in my study. During the data generation period (December 2014 to February 2018), those School Mathematics

Leaders undertook the mathematics leadership role in their respective schools from nine to 13 years. That meant that they had undertaken the leadership of mathematics in their schools prior to participation in CTLM.

As there were only 24 schools in Intake 4 of CTLM, I was conscious of not disclosing information that might jeopardise the anonymity of the three School Mathematics Leaders. Therefore, I was mindful of the amount of information I disclosed as means to reduce the likelihood of the leaders being identified. To mediate that, I decided to position the three participants as a collective subject (Lektorsky, 2009).

4.5.3 Participants as a Collective Subject

CHAT provides the lens with which to study the activity from the perspective of the subject within the activity system, where that subject might be an individual or a collective (Kaptelinin, 1995; Lektorsky, 2009). As discussed in Chapter 3, CHAT allows the researcher to study activity that may take the form of individual or collective activity (Lektorsky, 2009). Even though the School Mathematics Leaders worked in different schools, I positioned their professional learning leadership as collective activity because each leader worked on a common task: contributing to project sustainability through professional learning leadership activity.

By positioning their professional learning leadership as collective activity, I was afforded the opportunity to position all three leaders as a collective subject. This is because there is always a collective subject engaged in collective activity (Lektorsky, 2009). Therefore, the focus was on the School Mathematics Leaders as a collective rather than as individuals, working in the activity system of post-project professional learning leadership. That was one way for me to mitigate the disparity of time spent in the field with each mathematics leader. Focusing on the

School Mathematics Leaders as a collective subject meant that as I analysed the data, I generated themes that captured the shared experiences of the mathematics leaders in their work of leading professional learning. To support the discussion of findings, however, the pseudonyms that each participant chose during the first action of the research process were used (Rachel, Cindy, and Penny).

4.5.4 Previous Relationships with Participants

Previous relationships between researchers and participants are becoming more common in qualitative research studies in education (Greene, 2014). That was undoubtedly the situation for my study. As described in Chapter 1, my work history included participation in the CTLM project as a member of the CEOM team as a SAM (2008-2009 inclusive) and as part of the ACU staff team (2010-2012 inclusive). That allowed me opportunities to generate knowledge about the School Mathematics Leader role in CTLM schools, as well as knowledge of the three participants before my study began.

I had contact with the three School Mathematics Leaders several times throughout their participation in CTLM. Through that, I developed professional relationships with each of the mathematics leaders. I knew the participants by name as I had worked with them in workshops during the CTLM professional development days.

Although I had prior information about the School Mathematics Leader role and had professional relationships with the three leaders, my study was not insider research (Greene, 2014; Merton, 1972). I did not consider the information that I had about the role and the School Mathematics Leaders as intimate knowledge, a vital characteristic of insider research (Merton, 1972). Despite my previous career as a numeracy coordinator and having worked in the CTLM project, I was not part of the community engaged in the collective activity of sustaining project-initiated mathematics teaching reforms through professional learning leadership.

Instead, I acknowledged that my study was outsider research. I did, however, have information about the role, the three participants, and previous experience as a numeracy coordinator in a primary school. Possessing this *a priori* information and experience meant that I appreciated my research role as one that I called an *attuned outsider*. I was an outsider, but I was empathetic (attuned) to the phenomenon of mathematics professional learning leadership because of my previous work experiences. I was cognisant that my role as the attuned outsider influenced my positionality within my research design.

4.5.5 Positionality: The Attuned Outsider and Being Reflexive

I decided to include a section in this chapter about my positionality, which I interpreted as my place as a researcher in relation to the School Mathematics Leaders. I did this because qualitative researchers rarely address positionality (Greene, 2014). I decided to include my research reasoning about my positionality in this section about the participants before information about the methods and data analysis because those aspects of research design are influenced by the researcher's positionality (Creswell, 2013; Greene, 2014). That idea was important to me because I understood that, as a qualitative researcher, I was the primary instrument of generating and analysing data with the School Mathematics as participants (Creswell, 2013; Janesick, 2003; Punch, 2005; Merriam, 2009).

Data generation and its analysis are influenced by researcher positionality, which is determined by how the researcher stands in relation to the participants (Greene, 2014). Even though I had no contact with the School Mathematics Leaders for nearly two years after the conclusion of CTLM, I understood that I held a particular position with them as someone who worked for ACU. My position as the attuned outsider required me to be conscious of the ways that I interacted with the mathematics leaders, how I interacted with the data, and how I came to author this thesis.

In that sense, the role of the attuned outsider supported me in my enactment of reflexivity, which I interpreted as making known the experiences, values, and biases that I brought to my research activity (Creswell, 2013). I paid attention to the way that I positioned myself and the way that the School Mathematics Leaders may have positioned me.

Regarding reflexive practice, I took on the advice of Hellawell (2006), who advised that doctoral students include an extended methodology chapter articulating how they enacted reflexive practice. I acted on this advice and saw this chapter as an important way of articulating my values as a researcher. I noticed that my work roles affected my relationships with the School Mathematics Leaders during the initial phases of the research process.

When beginning the data generation, I noted that all three mathematics leaders tended to seek approval from me concerning their responses during interviews and their actions during professional learning opportunities that I observed. During initial interviews, I noted that I had to remind them that the information that they had to share about their leadership was important and that I was learning from them. That was vital learning for me as a novice researcher as I connected with the role of power relationships within qualitative research and the importance of presenting myself as a co-investigator with participants (Breen, 2007).

I was mindful not to present myself as an expert even though I continued to work as a mathematics educator at ACU. I was conscious that I might have been positioned in a specific way by the School Mathematics Leaders, so I often stated that my role in the research process was that of co-investigator: learning from them about what it meant to lead mathematics professional learning after participation in a project like CTLM. I ensured that I asked clarifying questions when the leaders spoke about actions of their professional learning leadership activity as a means of enacting my role as co-investigator. However, I noted in my research journal that by the third visit, each School Mathematics Leader discontinued asking if their responses and actions were appropriate and that their requests for my approval had reduced.

4.5.5.2 Valuing the School Mathematics Leaders as Experts

As the attuned outsider, I paid attention to positioning the three participants as the experts enacting the work of professional learning leadership that contributed to project sustainability. When I first shared this with them, I documented that I was met with some doubt by all three School Mathematics Leaders. To let them know of the value that I held them as experts, I changed aspects of interview protocols to include phrases such as "As someone doing this important work in schools…" and "As a School Mathematics Leader living and breathing this important work…"

I made efforts to remind the School Mathematics Leaders that they were in unique positions as only they could tell the story of their professional learning leadership. In that sense, I needed to show that I valued them as experts in the field and that, as a novice researcher, I certainly was not the expert.

4.5.5.3 Enacting Empathy and Trust with the School Mathematics Leaders

As the attuned outsider, I needed to enact empathy for the School Mathematics Leaders. This empathy was necessary when building trust with them so that they would allow me to enter their world of mathematics professional learning leadership. I needed to build trust so that I could have opportunities to generate data with them that represented the reality of their leadership activity. When engaging in interviews, there were times that I acknowledged aspects of their activity that the mathematics leaders themselves deemed challenging. In those cases, I made efforts to share my empathy, ensuring that there were opportunities for them to elaborate without me being too obtrusive or making assumptions about their leadership activity.

It took time to develop trust with the School Mathematics Leaders. I recall that one of the participants was reluctant to provide insights into the tensions of her leadership activity with me. After reflecting on that situation using my research journal notes, listening to interview recordings, and debriefing with my supervisors, I realised that it may have been due to the School Mathematics Leader's loyalty toward her school community (e.g., principal and classroom teachers).

To gain access to possible tensions, I reposed questions that were less obtrusive about potential contradictions (e.g., "Let's say that the school budget was no issue for your role here at your school, what would you do in your mathematics professional learning leadership then?"). By rephrasing the questions, I could generate data about the contradictions associated with the role, thus positioning myself in a way where I could access information whilst enacting empathy at the same time.

4.5.5.4 Using Research Journals

The use of research journals proved helpful when thinking about positionality and reflexivity (Altrichter & Holly, 2005; Greene, 2014). I kept four prominent research journals during my study; one for each of the three School Mathematics Leaders and one journal where I recorded reflections, questions, and connections that I made as part of the data analysis. The information within those research journals was used in supervisory meetings. During those meetings, I often put forth ideas or questions about my data interpretation, using the journals to mediate that aspect of my research activity. My supervisors often asked for evidence from data sources to support my interpretations. The research journals proved a helpful mediating tool in drawing attention to those data (quotes and initial analyses), supporting me in mitigating any biases or unfounded claims.

I now turn to a discussion of the data generation aspect of my research design.

4.6 Data Generation and Methods

The methods within a study act as the means for measuring and observing social situations via data generation and analysis, culminating in the reporting of findings (Creswell, 2013; Merriam, 2009; Punch, 2005). Punch (2005) stated that qualitative researchers "study spoken and written representations and records of human experience, using multiple methods and multiple sources of data" (p. 168). Due to this study being qualitative in nature, methods were used to generate meaning about the lived experiences of the School Mathematics Leaders. Methods were chosen that allowed access to those experiences (Creswell, 2013; Merriam, 2009; Punch, 2005).

Before I justify the data methods I used in my study, I turn attention to my use of the term data generation.

4.6.1 Data Generation as a Term

I use the term *generation* (and its associated forms, e.g., generate, generating) when I discuss the place of data in my research activity. That is because of the theoretical influence of the place of data within qualitative research design (Garnham, 2008). As a result of choosing

CHAT as the theoretical framework for my study, it became the lens through which I viewed the whole research design, especially the place of data.

Garnham (2008) stated that for some qualitative researchers, data are not viewed "to be out there just waiting to be collected" (p. 193). I espouse that view, so I use the term *data generation* (over 'data collection') because generation captures the idea of data as products of active research interactions. I viewed those data products as the result of interactions between my research reasoning and the data sources (e.g., participants, documents, etc.) using methods to create those data products. That notion of data production aligns with a CHAT perspective with the mind interacting with the material world within a dialectic interaction.

Within CHAT, knowledge is viewed as the product of the dynamic and dialectic interaction of consciousness and human activity enacted in the material world (Leont'ev, 1978). Thus, knowledge is produced from human activity through the mediation of physical and psychological tools (Jonassen & Rohrer-Murphy, 1999; Vygotsky, 1978). When thinking about my research design, I came to view data as a form of knowledge and understanding data as being generated through a dynamic, active research process. Therefore, terms like 'data collection' and 'collecting data' became redundant to me.

4.6.2 Methods of Data Generation

Throughout my study, data were generated using a range of methods. The three methods I used were interviews, observation, and document retrieval. The reason for their lies in ensuring that multiple sources of evidence were used to respond to the research question (Patton, 2002; Punch, 2005). Using those multiple sources, rich descriptions (Merriam, 2009) of the motive objects of activity and the mediational means of that activity enacted by the School Mathematics Leaders were produced.

4.6.2.1 Interviews

Interviews are an essential data generation method for qualitative research (Creswell, 2012, 2013; Merriam, 2009; Patton, 2002; Punch, 2005). Interviews happen when the researcher asks participants questions to elicit information about their viewpoint of the phenomenon under investigation (Creswell, 2013; Merriam, 2009). Interviews are considered the most effective data generation method during a qualitative study (Merriam, 2009). They play an important role in generating information about participants, such as their emotions and motivations, which may not be generated through observational methods (Creswell, 2013; Patton, 2002).

Patton (2002) acknowledged three main approaches for generating qualitative data: the informal conversational interview, the general interview guide approach, and the standardised opened-ended interview (p. 342). A combination of interview types can be used within a study (Merriam, 2009; Patton, 2002). However, each interview design differs based on how the interview questions are generated and standardised.

The informal conversational interview (Patton, 2002) or the unstructured/informal interview (Merriam, 2009) generates questions spontaneously during the enactment of the interview. That type of interview is mainly connected with ongoing participant observation fieldwork (Patton, 2002). The general interview guide approach involves exploring several issues to be discussed, outlining them to form a guide used during the interview. Carefully constructed questions are required when using a standardised open-ended interview (Patton, 2002). That is done to ensure that variation is minimised when interviewing several participants.

I used a combination of all three interview types to support the achievement of the motive objects of my research activity. That combination allowed for flexibility in probing and determining which topics to pursue further with the School Mathematics Leaders (Patton, 2002). Enacting advice provided by methodological authors (Creswell, 2013; Merriam, 2009; Patton, 2002; Punch, 2005), interview protocols were also created to mediate the facilitation of interviews (Appendices F and G). The questions used in the interview protocols were based on ones found in the ESM (Mwanza, 2001, 2002). Examples of questions used in the interview protocol were:

- What are you working on with your maths leadership, and why are you doing that?
- What resources do you use, and how do you use them?
- What information or knowledge helps you with your maths leadership?
- How do routines help your professional learning leadership?
- What are the challenges that you face in your maths leadership?

Interviews with the School Mathematics Leaders took place at the start of the study, before and after most observation sessions during the study period, and towards the end. By conducting interviews prior to and after observation sessions, the observed behaviours and actions were used as reference points during the interviews (Merriam, 2009). I did this as another way of honouring the mathematics leaders' position as the expert within the research process, seeking their interpretations of those actions rather than making assumptions. Other interviews were organised to generate further data or to confirm insights from analysis (Creswell, 2013; Merriam, 2009; Patton, 2002). All interviews were electronically recorded and transcribed to support data analysis (Creswell, 2013; Punch, 2005).

4.6.2.2 Observation

Like interviews, observation is another important method of data generation in qualitative studies (Creswell, 2013; Merriam, 2009; Patton, 2002; Punch, 2005). Observations conducted during fieldwork provide opportunities for the researcher to understand the phenomenon in greater detail (Creswell, 2013; Merriam, 2009). Observations occurred in the schools where the School Mathematics Leaders practised their leadership activity.

Patton (2002) stated several advantages when choosing observation as a data generation method. Firstly, observations allow the researcher to understand the context in which the participants interact. That was essential because context is central to developing a holistic perspective of the phenomenon studied when using CHAT (Engeström, 2015; Foot, 2014; Kuutti, 1996; Yamagata-Lynch, 2010). Secondly, observations allow the researcher to be open and oriented to the inquiry rather than relying on preconceived notions or expectations of the setting. Finally, when observation is chosen as a method, opportunities exist for the researcher to see the routines within in the setting that might be taken for granted by the participants.

Those points offered by Patton (2002) were necessary because understanding context and routines are central concepts to CHAT where those routines can be interpreted as the rules and division of labour (Bellamy, 1996; Engeström, 2015) that mediate the motive objects pursued within the activity system (Engeström, 1999b, 2001, 2015; Mwanza, 2001). Therefore, the choice of observation aligned with the epistemological background of my research design.

I followed advice from Merriam (2009) who recommended recording field notes that are "highly descriptive" (p. 130). This was done using a fieldnote observation book for each School Mathematics Leader, where I handwrote notes at each observed professional learning opportunity. I positioned myself to the side of each leader's workspace during observations. This was done as a means of not being too obtrusive and as a way of demonstrating my role as co-investigator.

Observation notes were recorded in five-minute intervals, focusing on what the School Mathematics Leader said and did during the professional learning opportunity. Attention was paid to cultural tools, including references to psychological ones, such as concepts and knowledge. Each observed professional learning opportunity was audio-recorded, but attention during transcriptions was paid only to the School Mathematics Leaders' sayings following ethics guidelines.

After each observation, handwritten notes were typed into my observation template as soon as possible (Creswell, 2013; Merriam, 2009). The audio recordings were used to check the accuracy of notes, particularly checking what the School Mathematics Leaders said. The typed observation notes supported my writing of analytical memos (Merriam, 2009), acting as school visit summaries that included preliminary data analysis and interpretation (Appendix I).

4.6.2.3 Reasons for not Using Shadowing

It was suggested at my doctoral candidature presentation that I use shadowing as the observation method. Shadowing requires the researcher to closely follow and observe the participant from when the working day starts until the participant leaves for home (Czarniawska, 2008; McDonald, 2005). Like other observation forms, the researcher records field notes to capture those behaviours and explain why they occur (Bartkowiak-Theron & Sappey, 2012).

The researcher can decide whether to shadow the participant over consecutive days. However, there is the expectation that the shadowing will occur over an extended period. The benefit of shadowing includes access to the participants' day-to-day experiences as they negotiate the complexities of their workspace. McDonald (2005) stated that CHAT could be operationalised particularly well when the researcher uses the shadowing method.

Shadowing, however, was not employed as the observation method in my study. That happened because the School Mathematics Leaders refused permission to be observed so closely. Two concerns they expressed related to confidentiality issues and feelings of intimidation (Czarniawska, 2008; McDonald, 2005). Even after attempts to address the concerns through discussions and rapport building (Bartkowiak-Theron & Sappey, 2012), the mathematics leaders refused to be observed in that way. In yet another way to honour my positionality, I made the ethical and methodological decision to engage in observations of the mathematics leaders' professional learning leadership at times that they nominated.

4.6.2.4 Document Retrieval

Merriam (2009) used the term *documents* to refer to written, visual, and physical materials relevant to a study. Documents are a rich data source (Patton, 2002; Punch, 2005), and documentary data are often collected with observations and interviews with qualitative research (Punch, 2005). Patton (2002) asserted that before using observations, researchers should organise access to important documents used within the setting by the participants.

It is necessary that relevant documents are sourced if used in a qualitative study (Creswell, 2013; Merriam, 2009). The retrieval of documents was important for this study because, within a CHAT perspective, these can be viewed as cultural tools that mediate activity within the activity system (Engeström, 2015). It was important to gain access to documents so that I could analyse them for meaning in how the School Mathematics Leaders used them. Whenever the School

Mathematics Leaders used documents, I sought permission to obtain electronic copies or take photographs using my smartphone.

4.7 Data Analysis

There is no best way to analyse data in a qualitative study (Janesick, 2003; Rapley, 2011; Saladaña, 2013). However, any data analysis aims to communicate understanding and convey information in clear ways (Baptiste, 2001; Merriam, 2009; Saladaña, 2013). Part of that communication involves making clear decisions about data analysis methods that lead to the reporting of findings (Attride-Stirling, 2001; Baptiste, 2001; Braun & Clarke, 2006).

In this section, I state my approaches to data analysis and the intellectual processes that I enacted when working with the data. By disclosing that information, I aim to present the rigorous nature of my analysis (Attride-Stirling, 2001), and thus be better positioned to defend the focal theory of my thesis (Baptiste, 2001; Saladaña, 2013).

4.7.1 Use of Term 'Generation'

Although I have already discussed data generation, I want to articulate my use of the verb "generate" about data analysis. Again, I use the verb 'to generate' (including its associated words, e.g., generating, generation, etc.). When reading qualitative research literature about data analysis, I was confronted by the term "emerge" (and its derivates, e.g., emergent), due to my theoretical assumptions and active role in the research process.

Braun and Clarke (2006) and Ely et al. (1997) wrote about that active role within qualitative research. They claimed that themes do not merely emerge from data, nor are they discovered by the researcher. Researchers actively generate concepts and claims from categories and codes that are produced when analysing data, and then used to support theory building

(Braun & Clarke, 2006). Therefore, I use terms associated with the verb “generate” and avoid phrases that infer a diminished role enacted by the researcher during data analysis (e.g., "themes emerged").

4.7.2 Analysis Method

Viewing my own research work as an activity system required an understanding of data analysis as the rules that mediated my research activity. At one point in my study, however, data analysis shifted from a rule to a temporary motive object for some time (Engeström & Blackler, 2005). For data analysis to become routinised, I had to focus intently on understanding how to analyse data. To work on this temporary motive object, I drew on an analysis process by Saladaña (2013). A representation is presented in Figure 13.

Figure 13

Approach to Data Analysis and Generation of Codes to Claim

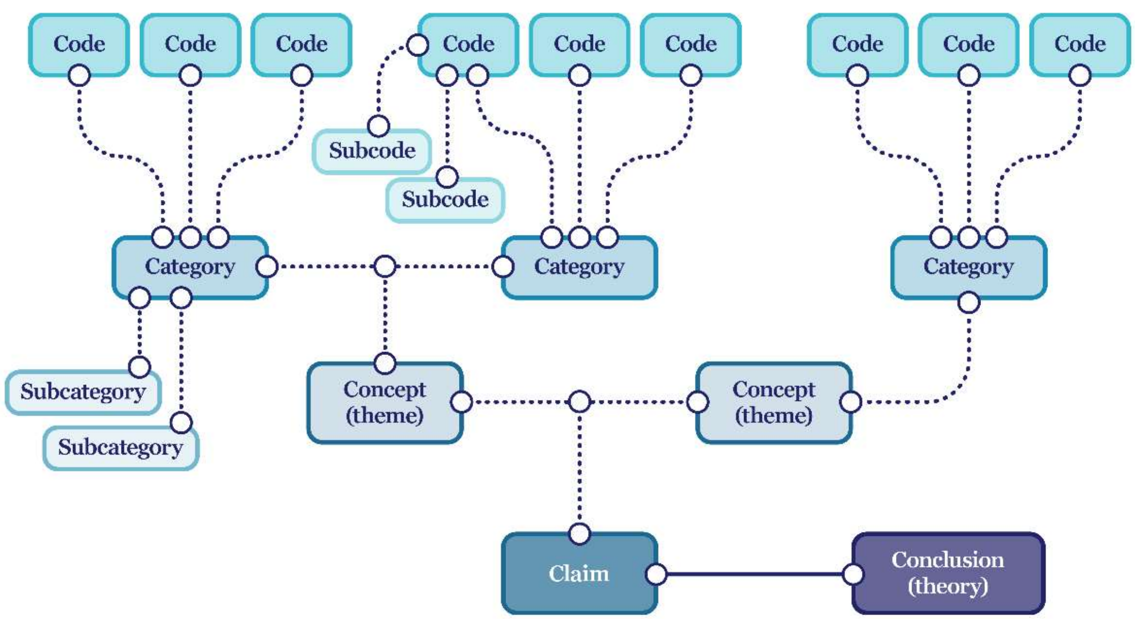


Figure 13 represents the general approach to data analysis which started with the raw data (top section of Figure 13). Data were reduced by generating codes using both deductive and inductive approaches. I interpreted coding as a process where I placed names, tags, or labels on specific data pieces (Baptiste, 2001; Braun & Clarke, 2006; Creswell, 2012; Rapley, 2011; Saladaña, 2013). I engaged in coding as a way of initially analysing, segmenting, and labelling the dataset (Creswell, 2012; Saladaña, 2013).

Once all data were deductively and inductively coded, I enacted a process of condensing, combining, reducing, and removing codes. That was done to generate categories (Saladaña, 2013). Some sub-coding was used to build categories and subsequent sub-categories. If there was a lack of saturation with a code or if a code did not relate to others already existing in the set, I removed that code. Similar codes were combined or condensed to generate a category during this process (Saladaña, 2013).

In turn, categories were used to construct concepts that were thought about and treated as themes. That was done using a thematic approach as highlighted by Braun and Clarke (2006). As I positioned the School Mathematics Leaders as the collective subject (Lektorsky, 2009), I looked to generate themes that captured collective activity through their professional learning leadership. Those conceptual themes were generated using CHAT and sensitising concepts from the literature (Table 4). I viewed those themes as the partial outcomes of my analytical reflection on the coding and categorising processes (Saladaña, 2013). The subsequent distillation process of those themes was then used to generate the focal theory of my thesis (Saladaña, 2013).

Saladaña (2013) warned that working from codes to conclusions is never streamlined or neat. I found this to be true. That analysis method was a continuous and interactive process, which I captured using the dashed lines in Figure 13. I worked back and forth from specific raw data pieces (codes) to the focal theory to ensure that I was generating theory appropriately.

Whilst I acknowledge that that analysis method was not as seamless as I have suggested, it does capture the general approach to my deductive and inductive data analysis approach.

4.7.3 Using Deductive and Inductive Analytical Approaches

A characteristic of qualitative research is the concurrent generation and analyses of data where inductive analysis approaches are mainly used to analyse those data (Baxter & Jack, 2008; Creswell, 2013; Lewis, 2003; Merriam, 2009; Patton, 2002; Punch, 2005). Although inductive data analysis approaches are primarily used, qualitative researchers do not work in "theory-free" spaces (Braun & Clarke, 2006). There is scope for using deductive data analysis processes within a qualitative study (Braun & Clarke, 2006; Snape & Spencer, 2003). Taking on that advice, I used deductive and inductive approaches to data analysis.

4.7.3.1 Deductive Analysis

A deductive analysis approach may be employed when the qualitative researcher uses a theoretical framework that informs the research design, which in turn, informs the production of coding schemes used for data analysis (Braun & Clarke, 2006; Potter & Levine-Donnerstein, 1999; Saladaña, 2013). That theory-informed coding scheme is used to search for evidence of the enactment of that theory within and across the dataset (Snape & Spencer, 2003). That deductive approach has been described as an explicit analyst-driven approach to data analysis (Braun & Clarke, 2006).

With CHAT as the theoretical perspective for my study, I was afforded concepts (e.g., motive object, cultural tools, rules, etc.), which proved helpful in the production of a coding scheme (Appendix J). The ESM (Mwanza, 2001, 2002) questions were also used to code the dataset deductively. Those questions helped generate evidence of the enactment of CHAT concepts and as a way of starting to map the School Mathematics Leaders' professional learning leadership activity system (Mwanza-Simwami, 2011). The preliminary mapping of the activity system was supported by creating a data wall (Section 4.7.6).

The coding scheme included concepts from project sustainability, middle leadership, and mathematics leadership. Those concepts were chosen primarily due to the frequency of their occurrence within the literature sources. To frame the coding, I looked for examples of those concepts within the dataset. Table 4 presents concepts that were used in that process.

Table 4

Examples Background Literature Concepts Used for Deductive Analysis

Project sustainability	Middle leadership	Mathematics leadership
Principal support	Relational trust	Characteristics
Sector/district support	Positionality	Principal support
Staff turnover	Principal support	Time
Professional learning	Conditions for leadership	Professional learning
Resourcing (external & internal)	Role purpose / clarity	Managing activity
	Role responsibilities	Leading activity

Those concepts, along with those from CHAT, were used as part of the overall coding scheme where I searched the dataset for examples, generating evidence of the enactment of those concepts within the School Mathematics Leaders' professional learning leadership activity. That searching was a process of reading, re-reading, and coding using those concepts as nodes that were set up in an NVivo project (Section 4.7.4). That allowed me to also tag specific data using further inductive coding.

4.7.3.2 Inductive Analysis

When an inductive analysis approach is used, the researcher is open to themes, and patterns that can be generated from the dataset (Braun & Clarke, 2006; Janesick, 2003; Potter & Levine-Donnerstein, 1999). The qualitative researcher analyses the data by generating codes, which are then used to construct categories, which in turn are reduced to form themes that support theory building (Creswell, 2013; Janesick, 2003; Merriam, 2009; Potter & Levine-Donnerstein, 1999; Saladaña, 2013). Taking on that advice, I combined the use of an inductive approach with a deductive approach to the data analysis.

As explained, I used deductive coding to generate specific examples of CHAT and literature concepts within the dataset. The decision to use inductive coding became evident as I immersed myself in the data through close reading and paying attention to hunches (Rapley, 2011). I recorded notes about frequent references by the School Mathematics Leaders to relational and affective aspects of their work. Although I was aware of relational trust as an important concept associated with middle leading (e.g., Edwards-Groves et al., 2016), the saturation of relational trust was unexpected in the work of the mathematics leaders. That was a major decision considering that CHAT, as a theory, is yet to fully understand and appreciate the role that affect plays in activity (Roth, 2007). Therefore, by paying attention to hunches and being open (Braun & Clarke, 2005), the inductive approach proved important to data analysis.

As part of the inductive analysis approach, I was open to generating other codes through that close reading of the data (Rapley, 2011). On advice from one of my supervisors, I asked questions of the data that included: What are the leaders talking about? What else are the leaders talking about? What are they not talking about? Those questions proved crucial in helping me understand the School Mathematics Leaders' work and tuned me into ways of working from the codes to eventually the focal theory of my thesis.

Using deductive and inductive analysis required ways of working with (analysing and interpreting) and storing the data that made the project manageable. I decided to invest time and energy into using NVivo™ as a tool to support me further in analysing data.

4.7.4 Using NVivo to Support Data Analysis

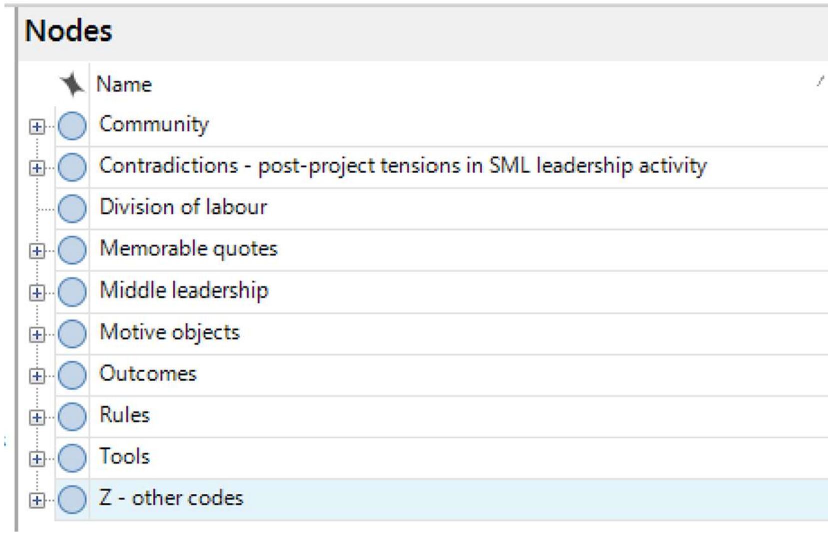
I used the tool NVivo™, a computer-assisted qualitative data analysis software (CAQDAS) program, to support the analysis and management of data (Creswell, 2013). I used that program as another cultural tool of my research activity. That was not part of my initial plan, but due to the large data set, I decided to use NVivo™ to manage the dataset. I explain the cleaning and management of data in Section 4.9.

After ensuring that I had cleaned the data, I uploaded them into two separate NVivo™ projects; the first was about the School Mathematics Leaders' activity during CTLM (DP), and the other project was for the mathematics leaders' post-project leadership activity (PP). The first project contained the data sources of interviews and documents, while the second contained interview, document, and observation data.

With each NVivo™ project, I used concepts from CHAT and background literature to set up *nodes* for the raw data. Before data analysis, I worked deductively by creating a set of nodes in NVivo™ that matched each mediational element of the activity system (Engeström, 2015; Mwanza, 2001; Mwanza & Engeström, 2005). That created the frame in each NVivo™ project (Figure 14).

Figure 14

Frame for Coding Data in NVivo Showing CHAT Concepts as Nodes

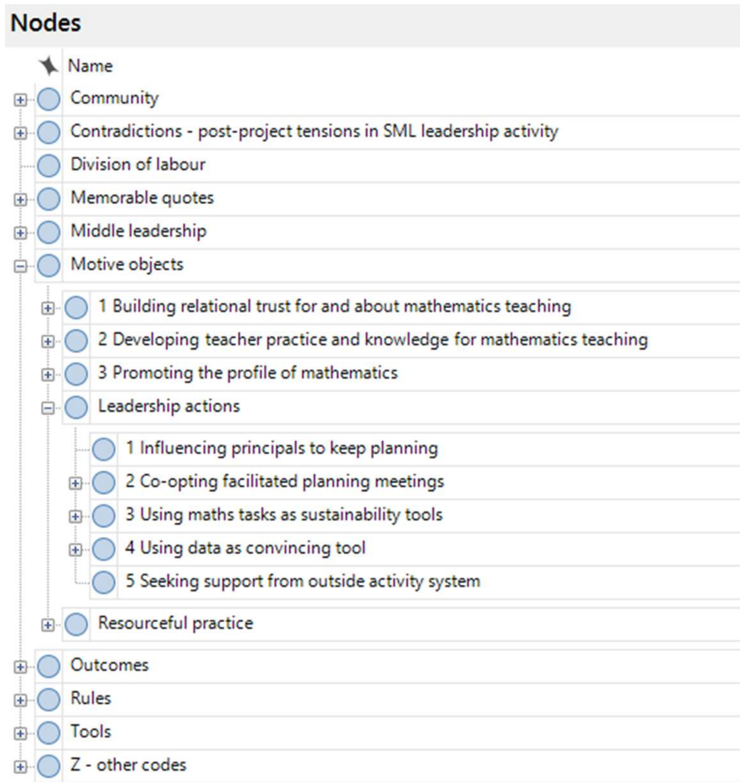


As I read the data sources closely, I highlighted and tagged the raw data to the node in the NVivo™ project using the coding scheme. I created other nodes before and during data analysis as I engaged in the deductive and inductive approaches. Those nodes were informed by sensitising concepts from background literature and inductive codes. Those were added to the frame as I worked with the data (Figure 15).

Using the NVivo frame, I coded the data using deductive and inductive analysis approaches. I used inductive and deductive codes, as seen in Figure 15. For example, the CHAT concept of motive object is used as an NVivo™ node that contains inductive codes and categories that I generated from analysing the data sources.

Figure 15

Frame for Coding Data in NVivo Showing Inductive Codes



The nodes in the NVivo™ project were then used named and renamed as the data analysis endured. Those node names changed as I engaged in conceptualisation processes as I sought to understand how the School Mathematics Leaders contributed to project sustainability.

4.7.5 Visualising Data Analysis Using a Data Wall

A critical practice I enacted to support my sensemaking with the data analysis was using a data wall (Figure 16). That supported my visualisation as I used NVivo™ to analyse data. That visualisation strategy supported me in generating themes through the data analysis process. I used a wall in my working space, and as I constructed themes, I added them to the data wall. Part of that process also included questions that I asked of the data.

Figure 16

Data Wall as a Visualisation Strategy to Support Data Analysis

The data wall was set up at the start of the data analysis phase using the Engeström (2015) triangular model to map the School Mathematics Leaders' activity system. That mapping was my way of enacting the advice of Braun and Clarke (2006), who advised that researchers create *thematic maps*. As I generated codes and categories by interrogating the data, I wrote them on Post-it® notes and mapped them onto the data wall. I then cross-checked that with the coding in NVivo™ for consistency and refined it to fewer categories and themes as the data analysis process progressed.

I also used the data wall to record questions I asked of the data based on those within the ESM (Mwanza, 2001, 2002). An example of a question was “For what purpose are those tools used by the SMLs?” The data wall supported the writing of analytical memos (Appendix I), forming part of my data analysis approach (Merriam, 2009; Saladaña, 2013).

4.7.6 Analytical Memos

To support the interpretation of data, I used another tool of analytical memos (Merriam, 2009; Saladaña, 2013). They were used in conjunction with my research journals. Analytical memos (Appendix I) were generally written after each school visit, drawing on data I generated through interviews and observations. The analytical memos acted as a school visit summary, with raw data examples within the memos to illustrate and test preliminary codes. Some codes were used to generate subsequent categories and themes. References to CHAT and background literature were also used to make sense of the data where I undertook initial data analysis, moving to interpretation of data drawing on the theoretical and background literature concepts.

4.8 Validation

With any qualitative research study, the researcher seeks to deeply understand the problematic under investigation. In doing so, questions about the accuracy of findings surface. Creswell (2013) used the term *validation* to capture the process that the qualitative researcher uses to document and influence the accuracy of findings. I recognise that other terms can be used to discuss this aspect of the qualitative study, but because I viewed validation as a process, I chose to use Creswell's term.

Validation is an important aspect of any qualitative research because reality is viewed as multidimensional and in a state of ever-change (Merriam, 2009). That assumption of an ever-changing world is most pertinent. As already stated, my study is situated in a dialectical materialist ontology which understands that human activity manifests in response to an ever-changing world (Nuttall & Brennan, 2016; Roth, 2012). That reference to reality is important in this section because validation concerns how well the researcher's conclusions match the reality of the problematic under investigation (Creswell, 2013).

As a means of matching findings to reality, qualitative researchers are encouraged to use a range of validation strategies (Creswell, 2013). There are eight strategies of validation offered by Creswell. However, he suggested that a minimum of two be used to validate findings. Within my study, I built in four of Creswell's validation strategies into the research design for my study. I have already mentioned several of those strategies as I discussed the research process that forms part of my research design.

4.8.1 Prolonged Engagement with Data Generation

Prolonged engagement with data generation is a strategy of validation that sees the researcher use practices that build relationships and trust with the participants and that understand the situation in which the participants engage with the problematic (Creswell, 2013). I interpreted that advice within my study as staying in contact with the School Mathematics Leaders for just over 3.5 years, generating data through school visits. That prolonged time enabled me to notice changes and enduring aspects of the mathematics leaders' professional learning leadership. It also afforded opportunities to engage in reasoning about the data through preliminary interpretations and testing those interpretations in subsequent visits. I also maintained relationships with the School Mathematics Leaders during the data generation period by sending emails and checking in to maintain that trust building I initiated with them through the data generation phase of my study.

4.8.2 Triangulation

Triangulation as a validation strategy is enacted when the researcher uses several methods of data generation and when themes are supported using evidence from across those multiple sources (Creswell, 2013). Taking on that advice from Creswell, as well as that offered by Zehetmeier (2014) about project sustainability research, I honoured that validation strategy by using three methods to generate data in the field with the School Mathematics Leaders: interviews, observations, and document retrieval. The use of NVivo™ supported the management of data as well as my work in corroborating themes generated from those three sources.

4.8.3 Member Checking

Member checking is the practice of seeking participants' views of the researcher's analyses and interpretation of data. That is done for confirmation of accuracy. It involves sharing data, analyses, and conclusions with the participants, and asking for them to check the credibility of findings (Creswell, 2013). I engaged that strategy by building opportunities for the School Mathematics Leaders to read and offer feedback on preliminary interpretations of their professional learning activity system mapping. Member checking as a practice occurred in two of the five research actions within the research process articulated in the methodology section of this chapter.

4.8.4 Rich, Thick Descriptions

The use of rich, thick descriptions, as a feature of qualitative research, requires the researcher to use data that provide detailed descriptions of the findings (Creswell, 2013). That is done to support the reader in transferring information about the findings to other contexts and test the transferability of those findings. I translated the use of that validation strategy into my study by making efforts to describe the activity of the School Mathematics Leaders and generating themes using CHAT and sensitising concepts from the background literature. I also tried to use strong quotes from interview data and observation records to illustrate themes that supported the claims and focal theory of my thesis.

It is important that, at this point, I share the convention I used when drawing on evidence from the data, presented in those rich, thick descriptions. To ensure that I was sharing the story of the collective activity (Lektorsky, 2009) of the School Mathematics Leaders, I made attempts to use data from each leader in equitable yet purposeful ways. That meant that I required a way of tracking data I use in the following chapters. An example of the convention is in Figure 17.

Figure 17

Example of Convention for Tracking Data Use

> Relationship plays a big part in leadership, even more so with maths leadership. There is something about mathematics, trust and relationships that allows me to know how my teachers really feel about maths and their teaching of it, especially the teachers with maths anxiety (Cindy, PPI, 02.11.17).

As seen in Figure 17, data used in my thesis were tracked using the convention of: School Mathematics Leader name, code of data source, and data generation date. That information was contained within brackets after each data excerpt.

The example in Figure 17 shows that that quote was provided by Cindy, through a post-project interview (PPI), on 02 November 2017. The convention PPO (post-project observation) was used if the data source was from an observation source. The abbreviation of DPI was used to denote data generated through interviews about the School Mathematics Leaders' activity during CTLM (during project).

4.9 Data Management

Data management included the processes I took in cleaning and managing the data from their generation in the field to their use in the discussion of findings in my thesis. That required attention to data cleaning and storage in NVivo™.

I engaged in data cleaning processes prior to uploading files into NVivo™ as part of data preparation (Davis, 2010). I re-read transcripts of interviews and observation notes, looking for errors and missing data that may have been evident in the transcription process. That required re-listening to recorded interviews and audio recordings of the professional learning opportunities I observed. When doing that, I matched those audio recordings to the data presented in the transcriptions.

When errors or missing data were evident, amendments were enacted at that time in the Word™ files that I created for the interview and observation data. Data were also cleaned by changing names and references to people or places that could identify the School Mathematics Leaders, people with whom they worked, and the location and school sites of their work. I kept dates of amendments through data cleaning in my research journal, which served as a log of emendations (Davis, 2010).

The data files were stored first in electronic folders; one set up for each School Mathematics Leader on a password-protected desktop computer hard drive. Those files were then imported into two different NVivo™ projects: one for the data relating to leadership during the CTLM project (DP) and another for post-project leadership (PP). Those two NVivo™ projects were also stored in that password-protected computer.

Separate folders were created to store the data files for each mathematics leader in each NVivo™ project. Again, all three folders were named using the pseudonyms attributed to the School Mathematics Leader. An example of this is provided in Figure 18.

Figure 18

File Naming and Storage in NVivo

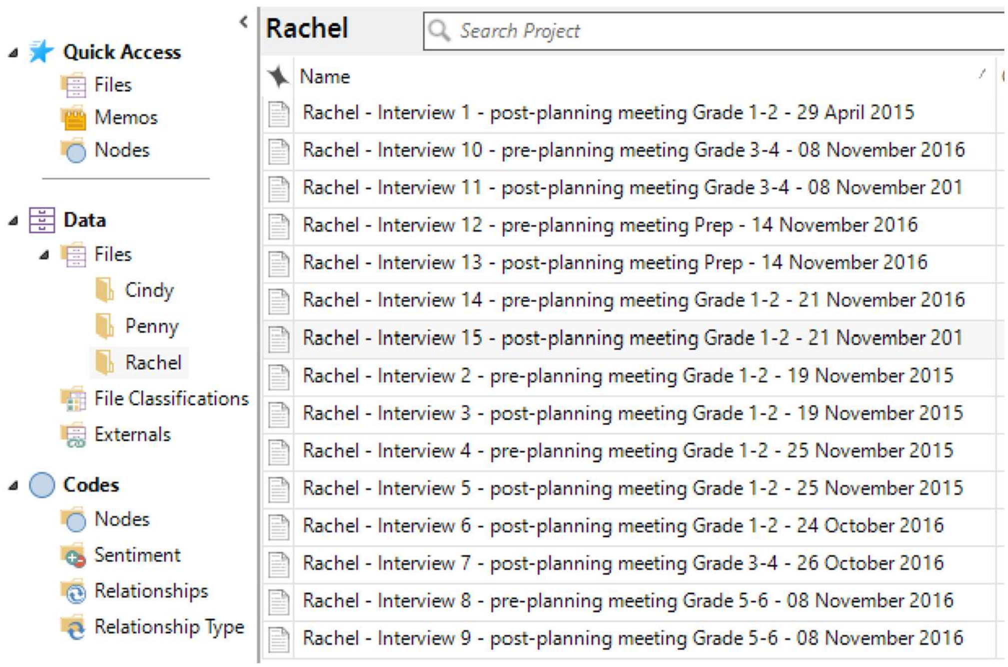


Figure 18 shows an example of how the data were named and stored in NVivo™. That example shows Rachel's interview data, the interview number, information about the context, and the date that the interview took place. That convention was used for all three School Mathematics Leaders and the data I generated with them in the field.

4.10 Ethical Considerations

It is important that during all phases of the research process, ethical considerations are dealt with in imperative and sensitive ways (Creswell, 2013; Punch, 2005). I have already presented how my positionality and role as the attuned outsider influenced the rules within my research activity. Recognition of that meant that I needed to concern myself with ways of enacting the research design that protected the School Mathematics Leaders as participants and respected their wishes and time (Piper & Simons, 2011). The first of the ethical considerations that I actioned meant ensuring that my study was undertaken in ethical ways, adhering to guidelines upheld by the Human Research Ethics Committee (HREC) at Australian Catholic University and the Policy and Research Committee at CEOM.

In the following section, I share how I acted in ethical ways during my study.

4.10.1 Permission to Conduct Study

I observed the processes of the ACU and CEOM for conducting research with human participants. Permission to conduct my study was granted by the HREC at ACU on 04 April 2014 (Application ID: 2014 52Q; Appendix A), and the Policy and Research Committee at CEOM granted permission on 26 May 2014 (Project ID: 2022 Sexton; Appendix B).

4.10.2 Informed Consent

Informed consent was sought from the three participants after they received information regarding participation in my study at the CHAT workshop. The participants were informed about the following aspects: the research purpose and aims; their role within the study; storage of data; benefits and possible harm; and procedures of the study, including time constraints, sites of data generation, and options for ceasing participation (Creswell, 2013; Henn et al., 2006; Piper & Simon, 2011; Punch, 2005). That information was presented to the participants through information letters and consent forms adhering to protocols set by the HREC at ACU (Appendices D and E).

4.10.3 Avoiding Coercion

I was conscious of my previous relationship with the participants when I sought their permission to participate. Enacting advice by Henn et al. (2006), I provided the opportunity to consent freely, making clear in written and verbal communication that the participants were not obligated to participate. That information was provided through the information letter and consent form in written form. It was elaborated on further in the first (Conduct workshop on CHAT with SMLs) and second actions of the research process (Clarify historicity of SML activity system). Avoiding coercion was important considering my positionality with the School Mathematics Leaders and my previous professional relationships with them through their participation in CTLM.

4.10.4 Data Generation Focusing on School Mathematics Leaders Only

I used CHAT to position the School Mathematics Leaders as the collective subject of the activity system. Therefore, the data I generated concentrated only on discussions about and enactment of their professional learning leadership activity. During observations of professional learning opportunities, data were occasionally generated about the teachers' utterances or actions. They were only used if they supported my interpretation of the School Mathematics Leaders' activity in those situations. That act of using data from the teachers in that way was done to comply with the research guidelines set by the HREC.

I prepared an information letter for teachers who had questions about my presence in their professional learning opportunities facilitated by the School Mathematics Leaders. That document (Appendix K) was a modified version of the information letter that I used with the mathematics leaders. As it turned out, I did not need to use that document at any time during the data generation phase of my study.

4.10.5 Respecting Participants' Requests and Time

I was cognisant of the important role that the School Mathematics Leaders played in supporting me to achieve the motive objects of my research activity. For that reason, I respected the mathematics leaders' requests and time wherever possible. I enacted that respect in two distinct ways.

The first way was through the modification of the method of shadowing, which I had intended to use. I changed that method to observations during times that they nominated. The second way that I respected the participants was when I worked with their timetables. There were times when I wanted to generate data with them on specific days. That often changed because of situations that occurred at the schools and the busyness of their work as mathematics leaders. That meant I did not always conduct the school site visits at times that I had planned.

As the attuned outsider, I made attempts to act in ethical ways that respected the School Mathematics Leaders' wishes and act in empathetic ways when it came to responding to changes in times of planned school visits.

4.10.6 Confidentiality and Anonymity

Confidentiality and anonymity were aspects of ethical consideration that was important to me. I recognised the potential for disclosure of information about the participants and their school locations. My study was context-specific, as indicated by the participant selection criteria, and the term *School Mathematics Leader* was unique to Melbourne Archdiocesan primary schools during the initial period of my study. I do recognise, however, that other researchers have also referred to that title for the mathematics leadership role in Victorian government primary schools (e.g., Driscoll, 2017).

When considering the ethics of qualitative research, I was mindful that confidentiality and anonymity required separate treatment by me as the researcher (Piper & Simons, 2011). That was important because these two terms are often treated and used interchangeably when, in fact, they have distinct meanings (Henn et a., 2006). Confidentiality is considered the attempt by the researcher to delete identifiers about participants from research records, whilst anonymity is focused on assurances that keep the participants nameless within the study and in subsequent publications (Henn et al., 2006; Piper & Simons, 2011)

To maintain confidentiality, I deleted the names of the School Mathematics Leaders in all data sources including any identifiers about them such as their school and suburb name (Christians, 2011; Henn et al., 2006). I also deleted names that the School Mathematics Leaders used during interview and observations, such as the principal and teacher names. School logos on documents using by the mathematics leaders were also removed or hidden. It was also crucial that, when speaking with colleagues from ACU and CEOM, I spoke in general terms and did not provide any information that might identify the School Mathematics Leaders or their school communities in any way. That was important considering that those colleagues may have worked with the mathematics leaders during CTLM.

Enactment of anonymisation was done by using the participant-selected pseudonyms when managing and analysing data. It was also enacted when discussing the findings within my thesis (Christians, 2011; Henn et al., 2006; Piper & Simons, 2011). The pseudonyms were also used in peer-reviewed publications I wrote during my study (e.g., Sexton, 2019) and at the times I presented at mathematics education research conferences (e.g., MERGA42 conference in Perth, 2019).

4.11 Chapter Summary

In this chapter, I have articulated the research design for my study that draws on concepts made available to qualitative researchers who use CHAT as a theoretical framework. I positioned the research design as the cultural tools, rules, and division of labour that acted as mediators of the motive objects of my own research activity.

I presented the ontological, epistemological, and methodological considerations that shaped the overall research design, demonstrating my research reasoning abiding by qualitative research practices and principles. I shared the research process I developed to operationalise CHAT as a practice-based theory. I positioned CHAT concepts in ways that supported the study of the phenomenon of the professional learning leadership activity of School Mathematics Leaders.

Importantly, I articulated my positionality as a qualitative researcher as the attuned outsider and its impact on my research activity. I explained the data generation and analysis methods, making my approaches to deductive and inductive data analysis explicit. I also stated the ethical considerations that were further rules of my research activity.

The outcomes of the analysis and interpretation of data are discussed in the following three chapters, which I use to respond to the research question.

CHAPTER 5: THE HISTORICAL PROFESSIONAL LEARNING LEADERSHIP ACTIVITY SYSTEM OF THE SCHOOL MATHEMATICS LEADERS

5.1 Introduction to Findings and Discussion Chapters

The purpose of my study was to generate knowledge about how School Mathematics Leaders, as middle leaders in their primary schools, contributed to project sustainability through their professional learning leadership activity.

The research question addressed in my thesis was:

As middle leaders of site-based professional learning, how do School Mathematics Leaders contribute to the sustainability of mathematics teaching reforms in the years that followed participation in a large-scale school mathematics professional development project?

In the following three chapters, I respond to my research question via three central claims, presented as a sequence of three findings and discussion chapters. As articulated in Chapter 1, the methodological decision was made to present the chapters in that way to describe the story of the School Mathematics Leaders' contribution to project sustainability cohesively.

The following three chapters not only outline my argument but correspond to the temporal sequence of activity experienced by the School Mathematics Leaders from the time of their leadership activity during CTLM to the years following participation in that project. Chapter 5 begins that historical trajectory by describing the historically accumulated contradiction that gave rise to the decision for the mathematics leaders' schools to participate in CTLM. I then explain the motive objects of activity the School Mathematics Leaders pursued during participation in CTLM. That description demonstrates my claim that the pressing post-project contradiction experienced by the School Mathematics Leaders through problems of practice (described in Chapter 6) had its genesis in historical practices at their school sites before participation in CTLM. It also serves my other claim that as participation in CTLM endured, the School Mathematics Leaders shifted the direction of their leadership from managerial motive objects to ones that focused more on leadership.

In Chapter 6, I describe how the School Mathematics Leaders were initially relieved that their principals established commitment rules intended to mediate project sustainability following participation in the CTLM project. I explain that after CTLM ended, that historical contradiction resurfaced and was manifested as a critical conflict realised as several problems of practice. That gave rise to the post-project struggle experienced by the School Mathematics Leaders. I describe how that struggle was compounded by feelings of responsibility for project sustainability in their schools. I claim that the School Mathematics Leaders responded to their struggle and feelings of responsibility through care and creativity, and by doing so, they initiated their form of resourceful practice (Edwards, 2005; 2010a).

In Chapter 7, I present further evidence that the School Mathematics Leaders enacted a type of resourceful practice that became their contribution to project sustainability. I describe how, in their attempts to resolve that resurfaced contradiction and its practice problems, the mathematics leaders reconfigured the motive objects of their activity, seeing them privilege relational trust building for and about mathematics teaching. I propose that the School Mathematics Leaders' post-project leadership was multi-motivational activity (Leont'ev, 1978; Kaptelinin, 2005) and that they enacted new leadership actions as means of achieving those new motive objects of activity. I conclude Chapter 7 with evidence of a newly surfaced contradiction. I reason that that contradiction was due to the relational motive object that the School

Mathematics Leaders privileged, thus revealing the enabling yet constraining potential of the relational dimension of their professional learning leadership activity.

My overall thesis, the general focal theory in summary, is that the School Mathematics Leaders enacted a form of resourceful practice as their contribution to project sustainability, which was directed at multiple motive objects of activity as they negotiated complex post-project problems of practice.

I now move to the description of the historicity of the School Mathematics Leaders' professional learning leadership as a means of honouring history as an essential principle and methodological implication of CHAT (Roth, 2012).

5. 2 Introduction to Chapter 5

In Chapter 4, I presented the research process I developed, where I built in the opportunity to generate data about the historicity of the School Mathematics Leaders' activity system (Roth, 2012). That was done to make greater sense of the post-project professional learning leadership activity of the School Mathematics Leaders, the focus of my study. That initial emphasis on historicity allowed me to show how the mathematics leaders' activity developed over time.

In Chapter 5, I articulate why the School Mathematics Leaders' schools decided to participate in the CTLM project. I interpret the mathematics leaders' perspectives, claiming that participation was motivated by a historically accumulated contradiction of the diminished priority of mathematics education in their schools. I then focus on identifying and interpreting the motive objects of activity pursued by the School Mathematics Leaders during participation in CTLM. I do this because when using CHAT, a focus on motive objects supports understanding of the activity under investigation (Kaptelinin, 2005; Leont'ev, 1978).

I explain that by the time participation in CTLM ended, changes in the motive objects pursued by the School Mathematics Leaders occurred. Those shifts show that the mathematics leaders changed the direction of their activity from managerial motive objects to ones that became more focused on leadership.

I now explain the historical activity system of the School Mathematics Leaders, turning attention to their perspectives about why their schools participated in CTLM.

5.3 Reason for Participating in the CTLM Project

I explained the historical context of the CTLM project in Chapter 1, where I provided information about the SIM program. During SIM, part of the onboarding for project participation required the school leaders to engage in identification of goals for improvement. A missing element of that program was the identification of reasons for tensions in mathematics education practices in the School Mathematics Leaders' schools. In CHAT terms, there was no prior study of the historically accumulated contradictions, nor were those contradictions analysed and used as an opportunity to develop practice (Engeström & Sannino, 2010).

I present the School Mathematics Leaders' perspectives about their understanding of the reason for participation in CTLM. I start by describing the contradiction as a means of honouring the dialectical materialist positioning of contradictions (Mussachia, 1977; Roth, 2012) as catalysts for change and development within activity systems (Engeström & Sannino, 2010; Nuttall & Brennan, 2016; Roth, 2012).

According to the School Mathematics Leaders, who, prior to CTLM participation, were known as *numeracy coordinators* (Cheeseman & Clarke, 2006; Clarke et al., 2005), mathematics competed for priority with literacy education in their schools' improvement agendas.

Mathematics was perceived as not holding the equivalent importance in the way that literacy education was privileged. Penny exemplified this by sharing: "Before CTLM, maths was the little, poorer sister of literacy!" (Penny, DPI, 25.03.15).

That diminished positioning of mathematics in the School Mathematics Leaders' schools was realised through several means: routines were in place that constrained opportunities for the mathematics leaders to lead mathematics professional learning; time allocation for the mathematics leaders to enact their role was diminished; and there was limited access to mathematics planning and teaching resources.

One routine, or rule within CHAT terms (Bellamy, 1996; Engeström, 2015), was a prioritisation of professional learning team (PLT) meetings dedicated to literacy education. That rule constrained opportunities for the mathematics leaders, surfacing a struggle to secure opportunities to engage teachers in professional learning.

That was exemplified by Rachel, who provided insights into the struggle that the School Mathematics Leaders had in securing space to lead professional learning in school meeting schedules:

> Before CTLM, I really struggled to have maths on the agenda. It was really only literacy, and we would focus on maths once a term in staff meetings. Each week there were the literacy PLT meetings, and they had to stay. There was just no time for maths PLTs (meetings) (Rachel, DPI, 23.04.15)

The School Mathematics Leaders claimed that before CTLM participation, teachers in their schools were expected to participate in weekly literacy education PLT meetings. Due to the frequency of those meetings, which took place after school, there were limited opportunities for the mathematics leaders to facilitate professional learning.

The priority and frequency of literacy education professional learning also caused another tension. That was heightened due to the focus on numeracy in the national standardised testing scheme of the *National Assessment Program – Literacy and Numeracy* (NAPLAN). NAPLAN was introduced in Australian schools in 2008, and primary school students in Year 3 and Year 5 were expected to participate each year. The mathematics leaders were conflicted about the lack of opportunities for teachers' mathematics professional learning and the potential impact of that on students' performance in the numeracy tests.

Penny provided an example of that tension. She shared her frustrations about the lack of priority given to mathematics professional learning and her limited opportunities to lead professional learning in her school. That was caused by knowledge of the importance of numeracy in the NAPLAN tests for students:

> The thing that was most frustrating at the time was that NAPLAN has a focus on maths, right? So, our students were tested on their maths skills but there were hardly any times for me to work with teachers and their professional learning in maths so we could improve the test scores. No wonder the NAPLAN scores were not good. It was all about literacy, and there I was fighting for a staff meeting here and there for maths. Before CTLM, there was really only a focus on maths professional learning when NAPLAN was coming up, and then there was this rush to focus on maths (Penny, DPI, 25.03.15).

Before participation in CTLM, the School Mathematics Leaders experienced another struggle that concerned disparity in the time allocation for their leadership role. The mathematics leaders shared that there were mandates from CEOM concerning the provision of literacy education in the Archdiocesan schools. One rule concerned the time allocation for the literacy leader, partially supported by funds provided by CEOM.

According to the School Mathematics Leaders, literacy leaders were provided time to lead literacy education for at least 0.5-time allocation (equivalent to 2.5 days per week) funded by

CEOM. The mathematics leaders claimed that their principals supplemented that time allocation using school funds. That increased that time allocation to 1.0 time allocation for the literacy leaders in each of their schools. That left little funding for allocating time release for the mathematics leaders. Prior to participation in CTLM, Penny and Rachel claimed that they had two hours per week time allocation for their role, whilst Cindy claimed that she had one hour per week time allocation.

That caused tension, further iterating for the School Mathematics Leaders the lack of appreciation for mathematics education in their schools. Rachel shared the following comment that captured frustrations about that diminished time allocation:

> At that time, before CTLM, it was frustrating because I just didn't have the time to think about planning any professional learning, really. I had my classroom teaching that I had to do, and then I had about two hours release time to do the maths leadership stuff. Most of that time was spent reading emails or I might have photocopied a professional reading (text) that I popped on the staffroom table for teachers to read. I remember I kept wishing that CEOM would set a rule like they had with the literacy leader. They (literacy leaders) got funding for their role (Rachel, DPI, 23.04.15).

That lack of time allocation impacted the division of labour within the School Mathematics Leaders' activity system. Before CTLM, that meant they could not lead professional learning in the ways they wanted. The lack of time allocation and their classroom teaching responsibilities at that time meant an impact on the organisation and division of labour (Cole & Engeström, 1993; Engeström, 2015; Havnes, 2010) of their mathematics leadership.

Another aspect of the contradiction concerned a lack of access to mathematics teaching resources. According to the School Mathematics Leaders, that was due to limited school budgeting, which was significantly lower than that for literacy education. The impact was that the resources to support mathematics planning and teaching practices (e.g., teacher reference texts and concrete materials) were sparse. I have interpreted that as a lack of access to cultural tools (Vygotsky, 1978; Engeström, 2015) that mediated both the teachers' work in teaching mathematics and the School Mathematics Leaders' activity in supporting teachers in understanding the use of those resources.

Cindy captured that situation about the lack of cultural tools prior to CTLM participation:

> I remember back in the day before CTLM, I would be looking around to see what resources we had. Not that we had that many, by the way. Limited resources and a limited understanding of what you could do with the resources that were at the school was something I remember back then. I would find some (teaching materials) in the back of a cupboard somewhere, and all I could do was put them in the staff room, hoping that the teachers would see them and ask about how to use them (Cindy, DPI, 23.04.15).

Those perspectives from the School Mathematics Leaders highlight the existence of the contradiction that I have interpreted as the diminished priority of mathematics in their schools. That could be understood as a manifestation experienced as a *double bind* (Engeström & Sannino, 2011), noting that there was a desire to enact their leadership. However, the mathematics leaders were helpless in that situation. That was due to the tensions within the rules, division of labour, and cultural tools within their activity system before CTLM participation. They expressed that they wanted to enact their role in ways like leading professional learning and having more time allocation to do that work. However, the mediational means constrained them from that activity. That contradiction was situated in historical practices associated with mathematics as a curriculum area. For the mathematics leaders, these formed why their schools participated in CTLM.

As it turned out, however, the presence of that contradiction faded in prominence for the School Mathematics Leaders when participation in CTLM began. That was because CTLM brought with it the introduction of new rules (requirements and expectations for school-based mathematics professional learning and mandated time release of at least one day in each CTLM school for the mathematics leader), a new division of labour (School Mathematics Leaders leading fortnightly PLT meetings for teachers with funds provided by CEOM for those meetings to take place during school hours), and cultural tools (new resources and materials, increased fund allocation for mathematics, and knowledge of mathematics teaching practices through the CTLM professional development sessions).

I now move to the discussion about the School Mathematics Leaders' professional learning leadership at the time of their participation in CTLM.

5.4 Overview of Motive Objects of Leadership Activity During CTLM

As I made the methodological decision to position the School Mathematics Leaders as the collective subject (Lektorsky, 2009), I analysed data looking for commonalities in the work focus of the mathematics leaders during CTLM. I used the concept of motive object as an analytical tool to understand the what and the why of their activity (Kaptelinin, 2005). I claim that the School Mathematics Leaders pursued four salient motive objects of activity during participation in CTLM. Those motive objects were complying with CTLM project team requests and expectations; managing human and physical resources; establishing mathematics professional learning routines; and developing shared visions for mathematics teaching.

I focused on motive objects as the actions, tasks, and undertakings (Nuttall et al., 2015) that were enacted at specific times during CTLM. Those motive objects were determined by matching the frequency with which the School Mathematics Leaders discussed aspects of their leadership activity. That temporal aspect proved influential when determining shifts in the motive objects of activity.

I turn to descriptions of the four motive objects. Recognising that a motive object is something at which actions are directed (Kaptelinin, 2005) and that actions are initiated by the motive object (Leont'ev, 1978), I also describe the actions that the School Mathematics Leaders undertook as they pursued those motive objects.

5.5 Complying with CTLM Project Team Requests and Expectations

The first motive object worked on by the School Mathematics Leaders was one that I interpret as compliance. That compliance motive object required attention quite early in their professional learning leadership activity. At the beginning of CTLM, the mathematics leaders were introduced to members of the CTLM project team, which included staff from ACU and CEOM, with primary contact with the SAMs via school visits.

Penny offered evidence supporting my claim about the surfacing of that compliance motive object. She discussed complying with demands set by the CTLM project team, recalling that this was the start of the work as a School Mathematics Leader within the project:

> Back in the days of CTLM, at the start, I was just making sure that I was doing what I was told by the SAMs and the ACU staff. It was all new, and there were all of these expectations of what needed to be done. There was this real focus on setting it all up so we could focus on maths. My role as the maths leader at the beginning was really making sure I was following what was expected (Penny, DPI, 25.03.15).

For the first time, the School Mathematics Leaders were required to work with other mathematics educators external to their school. That meant that others, outside of their own activity system, placed requests and set expectations for them. In response, the School Mathematics Leaders engaged in activity that complied with those demands.

Their response was realised through several actions which I now discuss.

5.5.1 Changing Professional Learning Team Meeting Frequency

The School Mathematics Leaders' first action with the compliance motive object involved changing the frequency of PLT meetings. Each School Mathematics Leader reported scheduling fortnightly PLT meetings once participation in CTLM began. They positioned that scheduling as part of the agreement to project participation. All three mathematics leaders recalled meeting with their principals within the first month of CTLM participation to negotiate the scheduling of fortnightly PLT meetings.

The following comment from Rachel highlights the action that concerned changes to the PLT meeting schedules:

> At the start of CTLM, well, by agreeing to do CTLM, there were agreements that had to come along with it, and one of them was fortnightly PLTs (professional learning team meetings). So, I did focus on those fortnightly PLTs with the staff on maths because that was part of the agreement. I also did that because I wanted to follow what they (CTLM project team members) wanted (Rachel, DPI, 23.04.15).

According to the School Mathematics Leaders, their principals readily accommodated those schedule changes. They claimed this was due to their principals' understanding the importance of the school-based professional learning opportunities that complemented the external professional development facilitated by the ACU team members during the 'CTLM days'. Penny highlighted this as she explained: "The changes to the timetable were easier to make because we were part of CTLM. He (principal) was onboard for the changes because it was expected by ACU and the SAMs” (Penny, DPI, 25.03.15).

Changing the PLT meeting frequency emerged in response to expectations for CTLM participation. By working on that compliance motive object, changing the scheduling of the PLT meetings, the new rule of "We have fortnightly mathematics PLT meetings" surfaced within the

School Mathematics Leaders' activity system. That change in the frequency of PLT meetings was one way that influenced the fading of the prominence of the contradiction of diminished priority of mathematics.

5.5.2 Purchasing Mathematics Resources

Another early leadership action enacted by the School Mathematics Leaders concerned purchasing new mathematics resources which were advocated for by the CTLM project team. That was primarily due to a lack of access to such resources, as a realisation of that contradiction concerning the diminished priority of mathematics. Through CTLM participation, the mathematics leaders were introduced to resources not previously used in their schools.

Participation in the CTLM professional development sessions, facilitated by the ACU staff, introduced to the mathematics leaders and teachers new teaching resources, mostly concrete materials (e.g., bead strings for counting and place value), along with teacher resource texts. Other resources, such as planning templates and several websites, were also introduced to support planning and teaching practices.

All three mathematics leaders claimed they were provided with a document containing a suggested list of mathematics resources, shared with them at the first CTLM days in 2011. Even though it was a *suggested* list, the School Mathematics Leaders attributed meaning that saw them feel obligated to purchase the mathematics resources contained within that list.

The following comment from Penny captured that sense of obligation:

> I remember thinking that if I was going to do professional learning here at school, I had to make sure that I bought the resources that ACU said that we should have if we were going to be in CTLM. I mean, there was hardly any here before CTLM, anyway. I remember I also bought materials and things like that that the SAMs said that we should also buy because we were in CTLM (Penny, DPI, 25.03.15).

According to the School Mathematics Leaders, their schools were given further funding by CEOM as part of their participation in CTLM. The introduction of the suggested list of resources document, the lack of mathematics education resources in their schools, and the extra funds further influenced the mathematics leaders to purchase new resources. That is an example of tool adoption (Miettinen, 2006; Nuttall et al., 2019) in that, through the introduction of new cultural tools, they saw them as necessary in resolving an aspect of the contradiction they faced concerning the lack of mathematics resources in their schools. They also adopted them as tools because they saw value in those resources to support mathematics teaching practices.

That was particularly highlighted by Cindy: "I remember spending part of my leadership time making sure we had the right maths equipment and books if we were going to be in CTLM and do the teaching we wanted to do" (Cindy, DPI, 17.11.15). The reference to 'the right maths equipment' reveals the obligation to purchase materials, highlighting their activity directed at the compliance motive object.

5.5.3 Organising School Visits by CTLM Project Team Members

Through CTLM participation, schools were required to implement structures for school visits facilitated by the CTLM project team. The School Mathematics Leaders worked on that compliance motive object by organising those visits and observing the requests and specifications from the CTLM project team members. The ACU staff team taught demonstration lessons in the schools (Clarke et al., 2013a), whilst the SAMs led PLT meetings through their school visits.

The School Mathematics Leaders were required to comply by meeting deadlines concerning the management of the school visits communicated and organised with the ACU and

SAM staff members via electronic means. That action saw a digitisation of their leadership activity due to increased email use.

Cindy shared insights into the increased digitisation of the School Mathematics Leaders' work during participation in CTLM:

> Oh, the emails! So, there'd be backwards and forwards emails, and there'd be certain deadlines and timelines that I needed to meet regarding planning for when they (ACU staff members) were going to visit. This was also for when we had our SAMs come visit. They were coming out twice a term. So, there was that responsibility of meeting their expectations for the professional learning sessions that I was organising at our school as part of my role (Cindy, DPI, 17.11.15).

Penny also spoke of increased work email correspondence:

> Back in the early days of CTLM, I would be on the email a lot to ACU and the SAMs, checking to make sure I was doing what I was supposed to be doing, especially with the school visits and making sure I made the dates and the timelines for the visits (Penny, DPI, 25.03.15).

Rachel confirmed the organisation of school visits as an aspect of that compliance motive object. She enacted that by ensuring she met the SAMs' expectations about organising meeting rooms at her school. Rachel explained: "I had to be doing things for the SAMs coming to visit the school. I had to set things up for their visits that they wanted to have ready, like a meeting space" (Rachel, DPI, 23.04.15).

5.5.4 Following Directives from CTLM Project Team Members

Another action of compliance concerned enacting the directives for professional learning planning and facilitation as set by both the SAMs and ACU staff members. The requests and expectations came from the SAMs via their school visits, whilst directives from the ACU team were communicated to the mathematics leaders through the ‘CTLM days’, mainly via the Between Session Activities (BSAs).

5.5.4.1 Directives from SAMs

The School Mathematics Leaders claimed that the school visits that occurred early in the CTLM project were ones by the SAMs. During those visits, the SAMs directed the focus of professional learning sessions and offered advice on facilitating site-based professional learning opportunities. The mathematics leaders claimed they were expected to facilitate professional learning sessions through the newly scheduled fortnightly PLT meetings and felt compelled to act according to that advice from the SAMs.

When asked to elaborate on the school visits, Cindy spoke about the interactions she had with the SAMs and the impact that had in terms of compliance:

> I led them (professional learning meetings) except for when the SAMs came in. But I felt that it was my job then to follow up with what they set for me to keep going with the staff with the professional learning. At the start of CTLM, they would give me advice and then tell me to follow up on things that they thought I needed to follow up on after the meetings that they ran. I did what they basically told me to do (Cindy, DPI, 17.11.15).

The School Mathematics Leaders interpreted the advice from the SAMs as directives for their professional learning leadership activity. There was a sense of obligation as they positioned that advice as directives with which they ought to do with their professional learning.

Rachel elaborated more on that interpretation, but she positioned expectations from the SAMs in a favourable way:

> The SAMs led the meetings when they visited, and then of course, once I'd seen it in action, I had the expectations set for me from the SAMs. Then the rule was that I would follow what they would tell me to do, so that was really good (Rachel, DPI, 23.04.15).

Those recounts show that the SAMs influenced the professional learning leadership activity of the School Mathematics Leaders early in the CTLM project. That was realised through actions of compliance by positioning advice from the SAMs as directives that influenced their leadership activity.

The mathematics leaders shared that during the SAM visits, the SAMs were responsible for leading the professional learning opportunities. After each SAM visit, the mathematics leaders were expected to comply with the set directives. By complying with the SAM directives, a new division of labour (Cole & Engeström, 1993) surfaced in the School Mathematics Leaders' activity system.

That new division of labour meant that when SAMs visited, the School Mathematics Leaders acted as participants in the professional learning opportunity. That division of labour, however, acted as a time for the mathematics leaders to become more familiar with the project expectations. It was also an opportunity to learn about professional learning leadership, using the examples from the SAMs' practice, as evidenced in Rachel's comment.

Another new rule became a mediator of the School Mathematics Leaders' activity at that time. That new rule concerned mathematics leaders 'following up' on the SAMs' advice. That rule influenced the focus of the professional learning opportunities led by the School Mathematics Leaders when the SAMs were not present in their schools.

5.5.4.2 Directives from ACU Staff Members

The Between Session Activities (BSAs), provided by the ACU staff members, played a specific role for the School Mathematics Leaders at the start of CTLM. All three mathematics leaders positioned the use of the BSAs as a directive from the ACU staff. As a result, they incorporated the BSAs into their professional learning leadership activity, thus becoming a new cultural tool within their activity system. The BSAs, however, as I interpret them, were used in the mathematics leaders' professional learning leadership activity because they saw them as an obligation.

The BSAs, which included lesson ideas, mathematics tasks and games, and professional reading texts, were shared with all mathematics leaders and teachers after each CTLM professional development day. The following comment from Penny exemplified how the School Mathematics Leaders used the BSAs as a form of compliance:

> I do remember, though, feeling like I had to use those Between Session Activities, those tasks that ACU gave us at the end of the CTLM days. I felt like I had to use them in my meetings. I remember that feeling of being told what to do (Penny, DPI, 25.03.15).

For Cindy, using the BSAs with her teachers happened early in CTLM because of the expectations for their use:

> I'd make sure the teachers followed up with and did the Between Session Activities at and between the PLT meetings because ACU expected that the maths leaders would use them in their PLT meetings. So, I had a focus on using the Between Session Activities in my PLTs, especially at the start of CTLM (Cindy, DPI, 17.11.15).

Rachel provided another example of how the School Mathematics Leaders positioned the BSAs as a directive. Like Cindy, Rachel used them because she understood the expectations of their use. Rachel, however, shared the usefulness of the BSAs in supporting her in knowing more about what her teachers noticed about their mathematics teaching:

> Well, I mean, the good thing was when we had the PD days in the city. We would then get told about the Between Session Activities. So, even though I knew I was expected to follow up with those activities by ACU, I found them useful. It was a really good follow through, and then we would have a PLT (meeting) about those BSAs and see what people found out about their teaching (Rachel, DPI, 23.04.15).

Those examples show how the BSAs were used in the leadership action of following directives from ACU staff members, mediating that compliance motive object. The School Mathematics Leaders used the BSAs in compliance with expectations. They adopted the BSAs as new cultural tools within their activity system but with a sense of obligation. Rachel, however, attributed meaning to those tools (Miettinen, 2006) in ways that provided opportunities for teachers to inquire into their mathematics teaching practice.

5.5.5 Complying as a Form of Managing Activity

The School Mathematics Leaders prioritised the compliance motive object at the start of CTLM due to the need to set up conditions for participation in the project. That is not surprising considering the organisation that is required to establish structures that facilitate participation in a large-scale professional development project such as CTLM. Through their work on complying with project team requests, the mathematics leaders mainly focused on activity related to managing. Responsibilities focused on managing form part of the practice of middle leaders (De Nobile, 2018; Lipscombe et al., 2021), which includes the organisation of operational implementation (Irvine & Brundrett, 2019).

This finding concerning the work focus on compliance provides new insight into how middle leaders first undertake their leadership activity within the parameters of large-scale professional development projects. The recount of their work suggests that at the onset of project participation, middle leaders, like the School Mathematics Leaders, engage in leadership activity characterised by the management aspect of middle leadership. That management focus was realised through compliance with requests from project facilitators.

That focus on managing activity was further enacted by the School Mathematics Leaders as participation continued into the first half of the first year of CTLM participation. In the following section, I describe how a new motive object focused on the management of human and physical resources surfaced at that time of project participation.

5.6 Managing Human and Physical Resources

The second of the motive objects of the School Mathematics Leaders' professional learning leadership activity concerned the management of human and physical resources. That motive object surfaced in prominence when the frequency of the CTLM school visits increased. It surfaced for the mathematics leaders about three to four months into the first year of their participation in CTLM.

As that new management motive object took prominence, the School Mathematics Leaders' attention to the compliance motive object and its associated leadership actions began to fade into the background of their activity. Rachel shared an important insight that captured the development of the School Mathematics Leaders' practice as participation in CTLM endured:

> After those first few months of CTLM though, I didn't take much notice of making sure I was doing what I was told. I started to just do things automatically. It was like it was 'under my belt' if that makes sense. I got used to it all so that I could focus on other things that came up as we did more of CTLM. (Rachel, DPI, 23.04.15).

They talked about how familiarity with the expectations from ACU and SAM staff meant that it required less attention than it did at the start of the CTLM project. The introduction of the school visits, however, meant that managing human and physical surfaced as a new motive object. That required enactment of management-focused actions realised through managing people and mathematics resources in their schools.

5.6.1 Managing Casual Relief Staff

Participation in CTLM facilitated opportunities for the ACU and SAM staff members to visit the School Mathematics Leaders' schools. Those visits meant that the mathematics leaders hired and timetabled the placement of casual relief teachers (referred to as *emergency teachers* or *ETs* by the mathematics leaders). That leadership action allowed the teachers in the School

Mathematics Leaders' schools to be released from teaching responsibilities, allowing them to participate in those school-based professional learning opportunities.

During CTLM, site-based professional learning took the form of demonstration lessons facilitated by the ACU staff team (Clarke et al., 2013a) and the facilitation of PLT meetings that the SAMs led. With those opportunities occurring during school hours, teachers had to be released from their teaching obligations. Therefore, a new leadership action concerning the hiring and organisation of casual relief teachers surfaced for the School Mathematics Leaders.

Typical of the leadership actions enacted by the mathematics leaders was exemplified by Penny. When asked to discuss the focus of her leadership activity during that early time in CTLM, Penny recalled: "Organising emergency teachers, because then in 2011, there was a lot of that with my work because we had the SAMs and ACU people come out for school visits often" (Penny, DPI, 25.03.15).

Cindy recalled the enactment of similar actions, ensuring that teachers would be available to participate in the demonstration lessons and the subsequent debriefing sessions that formed an essential aspect of the demonstration lessons:

Cindy shared:

> I remember making sure I booked the ETs and then working out exactly what the plan was for the professional learning session, mostly focusing on making sure everyone was covered with each session when they (ACU) came in and ran the PD (Cindy, DPI, 17.11.15).

By working on the management of casual relief teachers, the leaders created the conditions for teachers' professional learning in their school sites. Another aspect of that management was a focus on creating timetables that coordinated the facilitation of school visits.

5.6.2 Managing School Visits by Creating Timetables

Managing the human and physical resources motive object was worked on by the School Mathematics Leaders' activity when they undertook actions associated with creating timetables for school visits. Those documents were created for their teachers and the causal relief teachers. All three leaders created and used timetables as a cultural tool that mediated their activity towards that management motive object.

The School Mathematics Leaders reported that a significant amount of their leadership activity was given to those timetables used during school visits. Rachel highlighted the management work associated with that leadership action:

> I had a lot of timetabling, which was a big part of my role at the start, too, because if they (the SAMs) were coming in on a Tuesday, I then had to make sure that the (Grade) 1/2 level (teachers) were available between 9:00 and 10:30. I had to make sure that the ETs (emergency teachers, a.k.a., casual relief teachers) knew which grades they would be in. I had to make sure that I had grades covered by an ET so that the teachers could attend the professional learning provided by the SAMs (Rachel, DPI, 23.04.15).

Cindy shared a similar recount:

> There was that responsibility of meeting SAM and ACU team expectations for the professional learning sessions that I was organising at our school as part of my leadership role at the start of CTLM and during it. There was always a timetable to put together (Cindy, DPI, 17.11.15).

That comment from Cindy revealed that by focusing her work on that motive object of managing human resources, she was also complying with the expectations of the CTLM project team members, the previous motive object of activity. That is evidence of activity directed at more than one motive object, revealing that the School Mathematics Leaders' activity started to become multi-motivational (Kaptelinin, 2005; Leont'ev, 1978).

Penny also created timetables to organise the school visits by the SAMs:

> You know, I was just trying to coordinate all the timetables that were involved in putting together to facilitate for the PLTs (professional learning team meetings) that they (SAMs) would run so that teachers could be available to participate in those PLTs with the SAMs (Penny, DPI, 25.03.15).

Those recounts from the School Mathematics Leaders suggest that managerial aspects of the middle leadership role were of focus early in their professional learning activity during CTLM. By undertaking that action of creating timetables, it enabled conditions for teachers to engage in the school-based professional learning opportunities facilitated by the CTLM project team. That action also saw a new cultural tool of school visit timetables enter their activity system.

5.6.3 Managing Mathematics Resources

Along with the management of human resources, there was also a work focus directed at the management of physical resources. That concerned auditing mathematics teaching equipment and managing school budgets allocated to mathematics during the CTLM project. Previously, when the School Mathematics Leaders worked on the compliance motive object, they purchased new mathematics resources. That purchasing of materials then surfaced the need to audit and manage those materials as a leadership action.

For Penny, a new action involving the managing of newly purchased mathematics resources surfaced for her activity. A product of that action was the creation of an area that she named the “maths resource room” in her school. Penny recounted: "As part of CTLM, I would say one of my initiatives, I thought we needed, was to have a common area, a maths resource room" (Penny, DPI. 25.03.15).

Penny elaborated further, revealing that the purpose of the mathematics resource room was to develop teacher knowledge of the mathematics resources:

> My aim at the time was that anything that I ordered since we started working in the CTLM project needed to be barcoded and catalogued because before CTLM, what maths equipment, the small amount that was there that we did have, was not catalogued. As the School Mathematics Leader, I wanted the teachers to know that they had the resources to teach the activities and the ideas that they were learning at the CTLM days (Penny, DPI. 25.03.15).

Rachel also enacted this leadership action. When prompted to explain her reasoning, Rachel shared: "The more the teachers knew and the better they felt, especially when they had the right equipment to teach maths, the better it was (the school's participation in the CTLM project). The kids were going to learn more, too." (Rachel, DPI, 23.04.15).

Rachel recognised that her school's participation in CTLM was enhanced when her teachers had access to appropriate mathematics resources. Rachel believed that by focusing on the management of those resources, then the students' mathematics learning would be positively affected. Rachel attributed that influence on student learning was due to the teachers' developing knowledge of effective mathematics teaching through participation in CTLM. It also meant for Rachel that when teachers had access to mathematics resources, then their affective responses to participation in the project were more favourable.

The School Mathematics Leaders reported that there were decisions by the executive leadership teams to increase the mathematics budget to support CTLM participation. That mediated opportunities for them to purchase new mathematics resources. With that increase, the action of managing the new budget became prominent. Cindy shared her experience of that by recalling: “I could probably have a guess, it was like $500 or probably less than that (before CTLM), but it then went up to $3,000 per year, so that was a big jump. I had to manage that!” (Cindy, DPI, 17.11.15).

Cindy then articulated the impact of that increased budget on her leadership activity:

> Because, of course, from CTLM (team members), there was a whole load of suggested resources, so I think we had most of them, but I ended up buying a few more and buying some more! I think everyone (her staff) had a copy of Peter Sullivan's open-ended questions book (Cindy, DPI, 17.11.15).

That recount from Cindy highlighted the management of physical and monetary resources as a critical component of the School Mathematics Leaders' role during CTLM. That confirms findings from previous studies about this management element of middle leadership activity (Bennett et al., 2003; Irvine & Brundrett, 2019; Lipscombe et al., 2021), as well as the mathematics leadership literature (Cheeseman & Clarke, 2005; Driscoll, 2017; Millett & Johnson, 2004), where there is a work focus on managing those cultural tools for mathematics education.

At that time of their leadership in CTLM, the School Mathematics Leaders' activity was still characterised by managerial aspects of their middle leadership. That was realised through leadership actions focused on managing causal relief teachers, organising school visits, and auditing newly purchased mathematics resources.

However, that management emphasis started to shift when a new motive object became prominent. A new undertaking focused on establishing mathematics professional learning routines surfaced in the School Mathematics Leaders' leadership activity.

5.7 Establishing Mathematics Professional Learning Routines

Directing leadership activity on establishing mathematics professional learning routines became prominent as the next motive object pursued by the School Mathematics Leaders. That motive object surfaced in the second half of the first year of CTLM participation. At that time, the mathematics leaders were more familiar with project demands, directing their activity towards the previous compliance and management motive objects. They claimed that in July

2011, the frequency of professional learning opportunities increased. That included the off-school site “CTLM days” and the school visits by the ACU staff and SAMs.

At that time, the School Mathematics Leaders also reported that *facilitated planning meetings* became part of the CTLM participation. Those facilitated planning meetings became a mandated practice by CEOM, and principals set up the structures for the meetings in their schools. Those planning meetings were site-based opportunities for the School Mathematics Leaders to support teachers in their work of planning and developing mathematics teaching documentation. It is common practice in Australian schools for teachers to come together regularly and collaboratively plan for mathematics teaching (Davidson, 2016). Facilitated planning meetings took place during school hours, and teachers were released from their classroom responsibilities to plan for mathematics teaching with the School Mathematics Leaders. When the SAMs visited the schools when facilitated planning meetings occurred, they led those planning meetings.

With that increase in opportunities to lead professional learning, the School Mathematics Leaders were confronted with a tension they had not noticed previously. That tension concerned teachers' participation and engagement in CTLM professional learning and facilitated planning meeting opportunities. Working with that tension meant that the mathematics leaders worked on a new motive object that concerned the establishment of professional learning routines.

That new work focus saw the School Mathematics Leaders temporarily make the rules of the activity system a motive object of activity. Those rules, which shifted to become motive objects (Engeström & Blackler, 2005), included focusing their leadership activity on expecting teachers' attendance at meetings; encouraging greater teacher participation in those meetings;

making the rules for professional learning teams more explicit; and establishing new meeting structures.

The tension concerning their teachers' participation in professional learning required new leadership actions focused on influencing teacher behaviour in professional learning settings. That new motive object saw a shift from previous leadership activity that was primarily managerial in nature. The shift from managerial to leadership activity also saw the School Mathematics Leaders engage in middle leadership activity that became more relational. The relational entered their developmental work as they set up dialogical spaces that enabled the development of interactional and intersubjective trust (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2021).

5.7.1 Establishing Attendance Expectations

The first action that the School Mathematics Leaders took with the motive object of establishing professional learning routines was setting expectations for teacher attendance. The mathematics leaders claimed that prior to CTLM participation, there were no attendance expectations for such meetings. Setting attendance expectations became a focus to bring change to their professional learning leadership activity system.

Rachel exemplified the enactment of that leadership action of establishing attendance expectations. At her school, prior to CTLM, there were implicit rules in place that allowed teachers to run errands or use meeting times as opportunities to prepare for the following day's teaching:

> I remember working on setting expectations with the teachers. If you were not in your classroom teaching, you were expected to be present at the meetings. If it was a PLT or a facilitated planning meeting, you had to be taking part of it. So, that was a big

> expectation that changed, I guess, in about the middle of the first year of CTLM (Rachel, DPI, 23.04.15).

Penny shared an example of why she enacted that leadership action about expecting attendance at PLT meetings:

> So, once a fortnight, the expectation became that all teachers would be expected to attend a numeracy PLT meeting, and they had to be there. No more, 'Oh, sorry I can't attend. I have an appointment.' Nope, no more of that! (Penny, DPI. 25.03.15).

The School Mathematics Leaders enacted that leadership action because tension surfaced within their leadership activity system. That tension was a conflict between the rules of the system and their teachers' enactment of those rules. The School Mathematics Leaders focused their activity on establishing a rule for attendance, and in doing so, that became a routine for school-based professional learning. Rachel highlighted that by saying: "That expectation just became part of our school life later on" (Rachel, DPI, 23.04.15). That is evidence of how tensions within the activity system can drive change within the system (Engeström, 2015; Engeström & Sannino, 2011; Roth, 2012).

After the School Mathematics Leaders set attendance expectations, they enacted another influencing action. That action was a consequence of expecting teacher attendance in professional learning opportunities. Generating greater participation in meetings became another undertaking that helped the mathematics leaders establish professional learning routines in their schools.

5.7.2 Establishing Greater Participation in Professional Learning Opportunities

The School Mathematics Leaders engaged in establishing greater participation in professional learning opportunities in response to another tension they noticed. Even though their teachers' attendance increased as more frequent professional learning opportunities surfaced due to CTLM participation, the teachers were not participating in them at levels desired by the School Mathematics Leaders.

Cindy recounted the time when she worked on that aspect of the establishment of mathematics professional learning routines: "The main expectation I guess, was to get involved. They (teachers) couldn't just sit there anymore. There were always activities to do and all things to discuss. It wasn't ‘sit back time’ anymore” (Cindy, DPI, 17.11.15).

She reiterated further:

> So, one expectation was that you're going to contribute to the maths PLTs (professional learning team meetings). It wasn't a 'sit and listen' (time), and Cindy or the SAMs are going to present, and you just sit there. No 'chalk and talk', I guess. It became time for all staff to contribute to PLTs. I started focusing on that more (Cindy, DPI, 17.11.15).

Cindy noticed teacher passivity in the meetings at her school during CTLM. All three School Mathematics Leaders shared experiences of that tension. To address it, the rules for professional learning became a temporary motive object (Engeström & Blackler, 2005) of their professional learning leadership activity.

That leadership action was realised through efforts to influence teacher behaviours. Rachel was concerned about the behaviours of several teachers, primarily due to their responses to the demonstration lessons facilitated by the ACU staff.

Rachel shared:

> Whenever ACU (staff) were coming out, people (teachers) were knocking down the door to have their classroom be the one where they'd all (ACU staff) come in and they (teachers) were quite happy. There was probably just a couple who weren't, and that was basically why we early on worked together on a professional learning team protocol. So, one of the things it said in there (the protocol) was "engage positively in the meeting." That was a really important component. Because people were agreeing to not to just to be turning up and having places on seats. They were agreeing that they were actually going to engage. (Rachel, DPI, 23.04.15).

Rachel introduced a new cultural tool, a *PLT protocol*, to resolve that tension. Figure 19 shows the protocol that Rachel introduced and later adapted for her use. I explain that tool adaptation later in this chapter.

Figure 19

Example of a Professional Learning Team (PLT) Protocol

School information hidden

Whenever we meet as a Professional Learning Team in Mathematics, we will:

Staff will:

Come prepared by:

- arriving on time
- turning mobile phones to silent
- bringing requested items as per agenda.

Show respect towards each other by:

- listening to each person when they are speaking
- taking turns in a group discussion to allow all members opportunity to contribute
- valuing the opinions of others by responding appropriately using eye contact, positive body language and tone of voice
- being encouraging towards each other
- being open and responsive towards different ideas and opinions and receptive to honest feedback.

Engage positively in the meeting by:

- staying focused on the process and product, and resisting distraction
- being willing to contribute and share ideas with the group
- having an open attitude to suggestions and ideas
- being prepared to compromise and reach a realistic consensus.

The presenter will:

Demonstrate a clear purpose by:

- acknowledging apologies and providing an agenda/focus with realistic goals that can be achieved within the time frame
- chairing the meeting and keeping it on track
- finishing the meeting on time
- allowing time for meaningful and open discussion when there is a need, allowing staff to question or ask for clarification and offer suggestions.

As seen in the protocol (produced from a screenshot of an electronic file), expectations for teacher behaviours were stated to generate teacher participation in the professional learning opportunities at her school. Each mathematical leader claimed they used a protocol of this type, and its use was highlighted at the start of meetings as a reminder for expected behaviours.

The School Mathematics Leaders claimed that the SAMs offered them the protocol. They subsequently adopted it as a cultural tool to mediate their motive object of establishing professional learning behaviours. That is an example of how introducing and adopting a cultural tool can remediate activity (Miettinen, 2006; Nuttall et al., 2019).

With greater teacher participation in meetings worked on by the School Mathematics Leaders, they shifted their leadership focus to establishing the expectations of behaviours captured in those PLT protocols. That saw them position those as rules for professional learning with their activity system. That leadership action then became another temporary motive object of activity (Engeström & Blackler, 2005).

5.7.3 Establishing Rules for Mathematics Professional Learning Opportunities

As a way of further realising the motive object of establishing professional learning routines, the School Mathematics Leaders undertook the leadership action that saw them work on making the rules for teacher learning more explicit. The leadership action of *rule setting* became a temporary motive object (Engeström & Blackler, 2005), which simultaneously made the mediational element of rules more explicit within their activity system (Bellamy, 1996; Hashim & Jones, 2007; Kuutti, 1996).

Penny revealed part of the reason for that temporary motive object of rule setting. The School Mathematics Leaders saw that action as necessary to their commitment to CTLM participation:

> I set the rules that they (teachers) had to come to the meetings, and they had to be prepared to participate, and talk about their work in their classrooms. They were to participate and let others speak. If they had to bring something to the meeting, they had to bring it. I had to set the rules if we were going to do CTLM properly (Penny, DPI. 25.03.15).

Rachel elaborated further on why she introduced and used that PLT protocol, revealing it to remediate ways that teachers engaged with each other:

> Even though we might all think, "Hang on. That's actually not very polite what you're doing, just sitting there like that." But that person could say, "Well, I didn't know what was expected." Whereas, if it says here, you will “Stay focused on the process and product”, well it actually says here, so that (behaviour) actually is counterproductive. So that's why that (PLT protocol) was really good because that actually made it really clear (Rachel, DPI, 23.04.15).

According to the School Mathematics Leaders, the PLT protocol provided by the SAMs was generic and acted as an example on which they based their own protocols. The SAMs encouraged the mathematics leaders to include specific statements as rules, thus working with that tension of teacher passivity. That is evidence of the School Mathematics Leaders adapting tools to suit better the activity in which they were engaged (Miettinen et al., 2012).

Cindy highlighted that when she shared: "That was part of the SAMs' work with me around the middle of Year 1 of CTLM. They suggested to me, "Let's do some work on protocols", and they helped me put one together just for our school." (Cindy, DPI, 17.11.15). Cindy further elaborated that introducing that cultural tool was not only about establishing rules but also about building a sense of collegiality amongst teachers. Cindy shared: "That (the protocol) became an agreement for the whole school because of being in CTLM. I used that to just lift our expectations of working together more, and that we were all in it together." (Cindy, DPI, 17.11.15).

That is evidence of a shift in the School Mathematics Leaders' work that surfaced relational trust within their professional learning leadership. I have interpreted that the protocol had another meaning for them in that it was used to build intersubjective trust. The protocol was a cultural tool to develop collegiality and that sense of *withness* that middle leaders work on when that dimension of relational trust enters professional learning spaces (Edwards-Groves et al., 2016; Edwards & Grootenboer, 2021).

Establishing rules for professional learning opportunities was not only about influencing teacher behaviours. The School Mathematics Leaders started to foster conditions for developing professional learning cultures. That was one way the mathematics leaders paid attention to the relational dimension of teacher professional learning (Edwards-Groves et al., 2016; Lipscombe et al., 2020; Turner, 2007).

Looking at the work of the School Mathematics Leaders' activity from a middle leadership perspective, there is evidence that they also worked on building interactional trust with their teachers (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2021). By establishing rules, interactional trust building became a focus of their work through efforts to form respectful and communicative professional learning spaces for their teachers. As seen in the PLT protocol example (Figure 19), that was done by expecting teachers to favour equity of voice, turn-taking, and acting in cordial ways. The leaders sought to make cooperation and collegiality more explicit. Therefore, as part of their work as School Mathematics Leaders, they made the rules of the activity system more explicit (Bellamy, 1996; Hashim & Jones, 2007), enabling conditions for professional learning, that also mediated the developmental of relational trust within those settings (Edwards-Groves et al., 2016).

That action led to another enacted by the School Mathematics Leaders that saw them establish in new structures to their PLT meetings.

5.7.4 Establishing New Meeting Structures

The following leadership action that the School Mathematics Leaders enacted when pursuing the motive object of establishing professional learning routines concerned changes to meeting structures. The enactment of the previous leadership actions led them to review how PLT meetings allowed them to work in those professional learning spaces. That was realised when the mathematics leaders added new components with specific purposes to their PLT meetings.

New meeting structures facilitated opportunities for teachers to discuss aspects of their mathematics teaching. That action of adjusting meeting structures created greater dialogic spaces for professional learning, showing further work on interactional trust development (Edwards-Groves et al., 2016). As mentioned by Rachel, opportunities for teachers to share their work facilitated by CTLM participation provided space for teachers' voice: "It gave people a chance, I guess, to speak up about what they were finding was hard and what they thought needed to change." (Rachel, DCTLM, 23.04.15).

New meeting structures allowed the mathematics leaders to generate knowledge of their teachers, specifically their professional learning strengths and needs. New “practice sharing” components in PLT meetings held multiple purposes for the mathematics leaders beyond active participation. Those components facilitated further opportunities for relational trust building.

Cindy highlighted this as she explained her introduction of a meeting component that encouraged not only greater teacher participation but also fostered relationships with her teachers, specifically interpersonal trust (Edwards-Groves et al., 2016). That was achieved by incorporating a component for sense-making about effective mathematics teaching practice.

Cindy explained: "So, I had a new part to my PLT (professional learning team meeting), which was "share your ah-ha moment". It could have been your ah-ha moment, or it could have been someone else's ah-ha moment." (Cindy, DPI, 17.11.15). Cindy further remarked about the reason for its introduction: "It was a celebration of their work, so that was now a part of how the PLT (meeting) was run, and it was also to help them know that they were valued." (Cindy, DPI, 17.11.15). Interpersonal trust was developed through that meeting component, as the School Mathematics Leaders demonstrated interest in their colleagues' work, and teachers were positioned as valued members of the professional learning team as their efforts and successes were acknowledged (Edwards-Groves & Grootenboer, 2021).

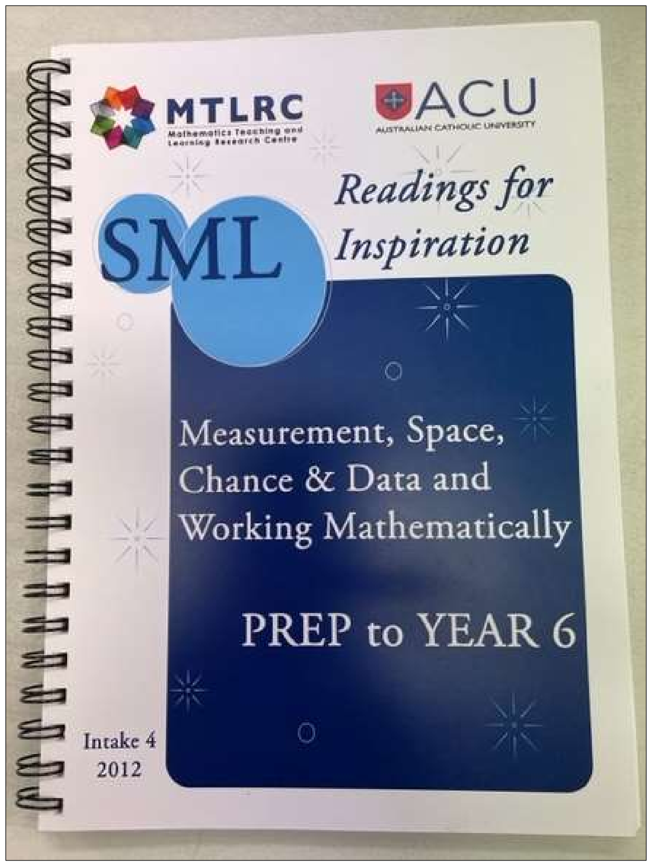
Cindy elaborated further on the new component's multiple purposes, which saw her also position that component as a way of demonstrating care and support for teachers. That provides evidence that the School Mathematics Leaders used new meeting components to nurture intersubjective trust, building that sense of journey with teachers as project participation endured (Edwards-Groves et al., 2016):

> The meetings was (sic) about actually, about working side by side and I'd often say, and you know that's exactly the wording I used to say, "I'm working with you side by side. I want to know your successes." So, I introduced that component to not only get them more active and not just sit there, but also to let them know that I was there for them (Cindy, DPI, 17.11.15).

Including time to read mathematics education texts also became a part of meeting structures for the School Mathematics Leaders. As leaders, they were given a "School Mathematics Leader Booklet of Readings", a resource provided by the ACU team (Figure 20).

Figure 20

School Mathematics Leader Booklet of Readings



That booklet consisted of articles from teacher professional journals, which the School Mathematics Leaders were encouraged to use during their site-based professional learning opportunities. As a new cultural tool, those articles mediated the inclusion of a "professional reading" component into the mathematics leaders' meeting structures.

Typical of reports from the mathematics leaders about this action was provided by Penny. The professional reading component not only held the purpose of encouraging greater teacher participation in meetings, but for Penny, it facilitated dialogical spaces for collaborative decision-making concerning implications for practice:

> I can honestly say that through CTLM, that's where I ensured that every single agenda that went out had a 'professional reading discussion' as a component. I brought in that (component) to get people more active in the PLTs (professional learning team meetings) because they would just sit there. I wanted to set the rule that we all read a reading, which I gave the teachers from the SML (School Mathematics Leader) Readings Booklet, and then we all had to discuss which ideas we would put into practice in our classrooms. We would make the decision as a team, and we would agree as a team (Penny, DCTLMI. 25.03.15).

I interpret that that professional reading component served the purpose of building the pragmatic trust dimension of relational trust (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2021). As part of reading professional texts, the School Mathematics Leaders engaged teachers in discussions of implications for teaching, as seen in Penny's comment. By identifying and committing to action for classroom practice, the School Mathematics Leaders focused on developmental work that was practical, achievable, and relevant to teachers (Edwards-Groves & Grootenboer, 2021).

The School Mathematics Leaders' use of professional reading texts also nurtured the relational trust dimension of intellectual trust (Edwards-Groves et al., 2016). The mathematics leaders supplemented the development of their own professional knowledge concerning effective mathematics teaching. That was evidenced by Cindy and Penny when they talked about the importance of professional reading through the following comments. They highlighted how intellectual trust (Edwards-Groves & Grootenboer, 2021) nurturing was realised in their work during CTLM.

Cindy revealed the importance of the mathematics leaders demonstrating engagement in their own professional reading, and setting examples for professional learning:

> I would always read the professional reading first before the meeting. I did that so that the teachers knew that I was learning, too, and it was important that I talked about that. I wanted to model that professional reading is an important part of professional learning (Cindy, PPI, 17.11.15).

Penny's comment added further evidence of how the incorporation of professional reading in meetings was a means of building trust as a leader who had the knowledge that was required for the role:

> I used the professional reading as a way of providing the teachers the evidence-base that showed what we were doing was important. There was also that element of the teachers knowing that I knew my stuff about maths. I think that's so important that the teachers trust that you know what you're going on about (Penny, PPI, 25.03.15).

The inclusion of new meeting components acted as new cultural tools used by the School Mathematics Leaders that mediated the motive object of establishing professional learning routines. That was enacted through a leadership action that saw them establish new meeting structures. Establishing those new meeting structures served multiple purposes and brought in new rules within the School Mathematics Leaders' professional learning activity system. Teachers were required to engage in dialogue by sharing thoughts and work successes and contribute to decision-making, as seen in the example of committing to actions arising from discussions from professional reading texts.

That motive object also provided opportunities for the School Mathematics Leaders to nurture relational trust with their teachers. That adds knowledge to how middle leaders develop relational trust (Edwards-Groves et al., 2016) and answers the call for understanding about how middle leaders nurture relational trust that contributes to working cultures amongst teachers

(Turner, 2007). That work saw a shift in the School Mathematics Leaders' leadership activity from managerial-focused work to activity that was becoming focused on influencing teachers' learning and practice.

I now turn to descriptions of the final motive object, a more distinctively leadership-focused space, that the School Mathematics Leaders collectively pursued during CTLM.

5.8 Developing Shared Understanding of Effective Mathematics Teaching

As participation in CTLM endured into the second year, the School Mathematics Leaders recognised the need to engage teachers in more sense-making processes. The final stages of CTLM participation brought with it mixed emotions for the mathematics leaders. Their responses to those emotions surfaced that final motive object of activity.

The School Mathematics Leaders shared insights about their emotional experiences when CTLM participation drew to a close. They shared that they felt excitement and pride concerning the developmental work that they mediated as middle leaders of mathematics. Those feelings were attached to recognition of how their school communities took up the CTLM mathematics teaching reforms, along with the development of teachers' use of practices associated with those reforms. Along with positive affective responses, however, the School Mathematics Leaders experienced worry and concern about life after CTLM. That concern motivated a new direction for their leadership activity as they pursued a motive object focused on developing shared understanding of effective mathematics teaching with their teachers.

Evidence that supports my interpretation of that motive object came from Cindy, who claimed she engaged in the intentional activity of engaging her teachers in sense-making processes about CTLM participation. Cindy recalled her emotional response that motivated her work on developing shared understanding, highlighting how motive objects are laden with affect (Nuttall et al., 2015):

> All of a sudden, towards the end of CTLM, we started talking about the maths key ideas and common misconceptions in our planning meetings. It was exciting but then I remember feeling a little sick that as we were making these connections, CTLM was ending. I remember working with the teachers and developing those shared understandings. We were finally getting on the same page, so I focused on building those shared understanding, so we didn't lose them (Cindy, DPI, 17.11.15).

Cindy revealed how the School Mathematics Leaders worked on that motive object by generating further dialogical spaces with teachers. They worked on creating conditions that enabled space for them to develop a shared understanding of mathematics teaching practice. That motive object focused on making sense of their CTLM experiences, supporting teachers to see the connections between the pedagogical purpose of the mathematics teaching reforms and the impact of them on classroom practice. I interpret that as further facilitation of intersubjective trust (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2021) as the mathematics leaders and teachers collectively engaged in the development of shared language and meaning-making about mathematics teaching practice.

The following comment from Rachel confirmed my interpretation of the School Mathematics Leaders' activity directed at the motive object of building shared understanding, and how they engaged in ways of supporting teachers' sense-making:

> I really wanted to help the teachers to see connections with what we learned in CTLM and why that was really important. I wanted us all to see where and how it all fitted in with what they did in the classroom with the kids. I focused on that a little more in the planning meetings towards the end of CTLM because I could see it was part of the bigger picture where we were leading to, and it was important that I helped the teachers see it, too (Rachel, DPI, 26.03.15).

Rachel alluded to the role of her vision for mathematics teaching and learning. I draw this from her reference to the 'bigger picture' as she discussed the focus of her work on developing shared understandings of mathematics teaching practice. The importance of mathematics leaders having visions for mathematics teaching has been reported in the literature (e.g., Roche et al., 2020). However, this has been mostly positioned as a characteristic of mathematics leadership. In the case of the School Mathematics Leaders, however, a vision for mathematics teaching became a cultural tool (Engeström, 2015; Vygotsky, 1978) used by them as they worked on the motive object of shared understanding development.

I now move to a description of the leadership actions that the School Mathematics Leaders enacted as they worked on the motive object of developing a shared understanding of mathematics teaching. That motive object again allowed the School Mathematics Leaders to work on the dimensions of relational trust (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2021).

5.8.1 Using Planning Meetings to Build Shared Understanding of Practice

Participation in CTLM provided opportunities for the schools to build in facilitated mathematics planning meetings. That new routine was established and supported by the human and monetary resources available through CTLM participation. Access to human resources included the SAMs who acted as facilitators in those planning meetings when they visited the School Mathematics Leaders' schools. Monetary resources allowed the teachers to be released from their teaching responsibilities so that they could engage in those planning meetings. The primary purpose of those meetings was to allow the mathematics leaders and teachers to plan collaboratively for mathematics teaching (with support from the SAMs on scheduled visits to the

School Mathematics Leaders' schools). Towards the end of CTLM, the mathematics leaders found opportunities for them to work on their motive object of building shared understandings of practice.

As CTLM ended, the School Mathematics Leaders claimed that during the facilitated planning meetings, they not only supported teachers in preparing planning documentation, but they used part of those meetings as opportunities for teachers to articulate beliefs about effective mathematics teaching. By opening those spaces, and facilitating dialogue about shared understandings, the School Mathematics Leaders worked on developing both intersubjective and interactional trust with their teachers (Edwards-Groves & Grootenboer, 2021).

Rachel highlighted how the School Mathematics Leaders worked on the motive object of building shared understandings as well as development of relational trust. She also provided further evidence of how that time was emotionally freighted for the mathematics leaders:

> Towards the end of CTLM, I remember thinking, "this is it! What happens next?" I was feeling really proud of what we had done, especially with the work in the shared planning and getting that sense of that shared responsibility, but I remember feeling worried about CTLM ending. That's when I decided to spend time with the teachers on building a shared understanding of what we learned about teaching maths best to kids. Just so we were all on the same page, you know? (Rachel, DPI, 26.03.15).

The School Mathematics Leaders used the planning meetings as interactional spaces, establishing a new rule that expected teachers to discuss their understandings of effective ways to teach mathematics. I interpret that work of developing shared understanding as opportunities that also built the teachers' mathematical knowledge for teaching (Ball et al., 2008). That new rule mediated not only those aspects of their developmental work but also fostered interactional and intersubjective trust with teachers (Edwards-Groves et al., 2016). The opening of space to make sense of effective teaching through collaborative discussion was a leadership action that aligned with interactional trust building (Edwards-Groves & Grootenboer, 2021).

The reason for my interpretation of the simultaneous activity directed at not only their motive object of developing shared understanding but also fostering relational trust (Edwards-Groves et al., 2016) lies in using the term "same page" by the School Mathematics Leaders. That featured in the interviews as the mathematics leaders recalled the focus of their leadership towards the end of CTLM.

Penny discussed the importance of shared understanding development at the end of CTLM, also referring to "same page", revealing the reason why that surfaced as a motive object of activity:

> Towards the end of the CTLM, though, I spent time trying to build those shared understandings of how to teach maths well. I was trying to get us all on the same page because there was going to be no more support from the SAMs or ACU. I remember feeling worried (Penny, DCTLMI, 25.03.15)

The School Mathematics claimed that they recorded those shared understandings during those sense-making opportunities. They created documents that articulated the shared understanding of practice. By engaging in that action, the mathematics leaders created a cultural tool that captured the collective understanding of practice.

5.8.2 Creating Collective Commitment Documentation

The School Mathematics Leaders undertook the leadership action of creating *collective commitment* documentation to record the shared understanding of effective mathematics teaching. I borrowed that term from Rose and Norwich (2014), who wrote from a CHAT perspective about group collaboration. Collective commitment draws together individual and collective decision-making and intended action processes within collaborative settings. The intention for such is to commit to goals and outcomes decided upon by the collective (Rose & Norwich, 2014). An example of one of those collective commitment documents is presented in Figure 21 (an image produced from a screenshot of an electronic file).

Figure 21

Example of a Collective Commitment Document

At **Identifier removed** we believe that the purpose of mathematics education is to:

- Develop our students', parents', and teachers' knowledge, skills and attitudes for and about mathematics and mathematics teaching and learning
- Support our students and families in understanding the important role that mathematics plays for future employability, life choices, and quality of life
- Prepare our students for their future pursuits beyond their primary education at our school
- Develop our student and family capacity to understand mathematics as the study of patterns and relationships

When teachers teach mathematics at **Identifier removed** they:

- Use tasks that have multiple entry and exit points
- Promote positive attitudes towards mathematics
- Ask questions to engage, challenge, and assess but not to give answers
- Plan lessons with curriculum requirements and student needs in mind
- Plan and use tasks that consider student interest and relevance
- Use data and knowledge of individual student learning to help plan tasks and activities
- Develop, use, and extend activities and tasks that require students to respond using visuals, language, and symbols
- Set clear learning intentions and use them to support learning during lessons
- Use mathematical terminology, terms, and their definitions with students and expect students to do the same
- Use warm up activities to stimulate, engage and tune in students to the learning intention and lesson focus
- Use a variety of assessment tools and strategies to inform teaching and to provide feedback to students
- Seek for and use "teachable moments" when monitoring student interaction during activities and lessons
- Include cognitive closures to inspire future learning and check for student understanding and wonderings
- Use a range of manipulatives, concrete materials, and representations appropriate to the mathematics content area
- Anticipate student responses to tasks during planning and teaching and respond appropriately to these using questioning

Teachers will do this by:

- Engaging in professional learning opportunities that build on and extend mathematical content knowledge (MCK) and pedagogical content knowledge (PCK)
- Having deep understanding of mathematics content and curriculum knowledge for the year level that they teach as well as knowledge that extends prior to and beyond that year level
- Modelling positive attitudes to mathematics for students and families, as well as modelling growth mindset for learning mathematics

As seen in that figure, the collective commitment, referred to as a *Mathematics Teaching and Learning Statement*, stated the beliefs concerning mathematics education practices deemed necessary by the teachers in the School Mathematics Leaders' schools. The document contains a series of leading statements, focused firstly on beliefs, and then pedagogical actions. According to the School Mathematics Leaders, the content of the document was a summary of the PCK that was of focus during the CLTM project. They claimed there was support from the SAMs who provided feedback, and in some cases, modified the use of terms and phrases without changing the content.

Cindy highlighted the leadership action concerning the collective commitment documentation. She recalled the need to bring the teachers together ("getting on the same page") in relation to documenting understanding of effective mathematics teaching developed through participation in CTLM:

> All of a sudden, towards the end of CTLM, we started talking about the maths key ideas and common misconceptions in our planning meetings. It was exciting but then I remember feeling a little sick that as we were making these connections, CTLM was ending. I remember working with the teachers and developing those shared understandings. We were finally getting on the same page, so I focused on building those shared understanding. We spent time typing them up and putting them in the planning room for us, so we didn't lose them (shared understandings) (Cindy, DPI, 17.11.15).

That leadership action was another way that the School Mathematics Leaders realised the vision for mathematics teaching with their teachers. That is an example of the notion of vision moving from an ideal concept situated as a characterisation of mathematics leaders (Roche et al., 2020) to a cultural tool intended to guide the teachers' mathematics teaching practice. The School Mathematics Leaders reported that their focus on that motive object of developing shared understanding was due to their understanding of the project ending and knowledge of no further access to resources and support from ACU staff or the SAMs (Heirdsfield et al., 2010).

Penny captured the theme of concern experienced at the end of CTLM. She explained the reason for developing documentation about shared understanding of practice as a means of maintaining the implementation of the teaching reforms. Her comment highlighted the School Mathematics Leaders' feelings about project sustainability when the supports offered through project participation are withdrawn:

> I was feeling very nervous and unsure when we came to the end of CTLM. I think everyone was feeling it, you know? That feeling of not knowing how to keep the momentum going was a pretty stressful. We did all that work and then having all that support just taken away. I was worried (Penny, DCTLMI, 25.03.15).

As seen in the School Mathematics Leaders' statements, the foreboding end of participation in the project caused a temporal shift in the motive objects of activity. It is also evident that the mathematics leaders experienced feelings of worry and concern about the end of participation in CTLM. They expressed concerns about what the sustainability of the project would look like beyond 2012, highlighting that they were aware of no further access to project supports that they had for two years (Heirdsfield et al., 2010).

I now turn to a discussion of those concerns and the influence of them as participation in CTLM ended in November 2012.

5.9 Shifts in Motive Objects: From Managing to Leading

I have described the motive objects of activity pursued by the School Mathematics Leaders and how they shifted during the time of CTLM participation. I explained that the School Mathematics Leaders engaged in an ongoing sequence of professional learning leadership activity focused on a series of related motive objects. Using evidence, I explained the what and the why of their activity, focusing on the motive objects of their activity (Kaptelinin, 2005).

The shifts in motive objects occurred due to influences from outside their immediate activity system (complying with ACU and SAM staff requests), the need to manage human and physical resources (accommodating school visits and managing mathematics concrete materials), noticing of disturbances and then realising the need for new rules to govern teachers' professional learning behaviour, and need to document shared understanding of mathematics teaching brought on by a temporal factor that influenced affective responses about completion of project participation. Through those shifts in motive objects, the School Mathematics Leaders moved their leadership activity from managerial-focused undertakings to ones that were more influential and focused on leadership.

Evidence of that shift and development of the School Mathematics Leaders' professional learning activity was shared by Rachel. Her comment highlighted how shifts in motive objects influenced shifts in the overall activity developed over time:

> I don't think I was leading in that first year of CTLM. I think I was just learning my role. I think I was more coordinating and managing, really. So, I would say, probably, when you say "leading", it was in the second part of CTLM that I would say I was doing that (Rachel, DCTLMI, 26.03.15)

In discussions where they reflected on their work in CTLM, the School Mathematics Leaders talked about their growth as mathematics leaders. Like Rachel, Penny shared the following comment, highlighting what the impact of CTLM participation meant for her:

> When we were involved at CTLM, I grew as a maths leader. I know I moved from being the coordinator of the maths materials to focusing more on the teachers' learning (Penny, DPI, 25.03.15).

The evidence I have used thus far illustrates that the professional learning leadership enacted by the School Mathematics Leaders during CTLM was multi-motivational activity (Kaptelinin, 2005; Leont'ev, 1978). They directed their activity at motive objects that shifted during CTLM participation. As they did that, their leadership developed from managerial-focused work to leadership activity more influential on teacher practice. The School Mathematics Leaders also described changes that happened to them as middle leaders of mathematics as they worked on that sequence of motive objects of activity. That is evidence of how activity develops and transforms over time as motive objects are achieved and how new motive objects surface within the activity system (Engeström, 2015).

I interpret this as evidence of the School Mathematics Leaders experiencing the CHAT concept of the *person-practice dialectic* (Edwards, 2017). That concept concerns the simultaneous development of practice and the person (in this case the collective subject of the mathematics leaders) when engaged in activity. During CTLM, there was a dynamic interplay between the School Mathematics Leaders and the social contexts they worked in, where they shaped aspects of their mathematics leadership in response to situations in which they worked. That interplay was realised as the mathematics leaders shifted their activity from compliance and management motive objects to more influential and relational ones. As the School Mathematics Leaders engaged in ways of resolving tensions that surfaced for them during CTLM, not only did their leadership activity develop, but they did as mathematics leaders in their schools.

5.10 Chapter Summary

In this chapter, I honoured the methodological implications of using CHAT by drawing on evidence to explain the professional learning leadership activity of the School Mathematics

Leaders during CTLM participation. I shared the perspectives of the mathematics leaders concerning the reason for their schools' decision to participate in the project. I claimed that there existed a contradiction of the diminished priority of mathematics education, which impacted the enactment of their mathematics leadership prior to CLTM participation. I reasoned that that contradiction was experienced as a double bind due to the over-prioritisation of literacy education in their schools. That was realised through diminished scheduling of mathematics professional learning opportunities and limited time allocation for their leadership role. The contradiction was also realised through a lack of access to cultural tools.

I drew attention to the direction of the School Mathematics Leaders' activity by focusing on their motive objects of activity because of their explanatory potential in understanding activity. Having positioned the mathematics leaders as a collective subject within the activity system, I identified and explained four salient motive objects at which the School Mathematics Leaders directed their professional learning leadership during CTLM.

I provided my interpretation of evidence to explain that those motive objects shifted in prominence as project participation endured. Through CTLM participation, the School Mathematics Leaders moved their activity from motive objects focused on compliance and management to ones that influenced teachers' knowledge and practices for mathematics teaching.

Drawing on evidence, I claim that mathematics leaders enact multi-motivational leadership activity when engaged in a large-scale professional development project. They start with a focus on activity associated with compliance and management, but as the project participation endures, shifts to leadership activity that influence practice and relationships happen. This occurs when the mathematics leaders seek to resolve surface tensions within their activity system, using cultural tools introduced to them by project team members.

I have presented evidence that when project participation ends, it is an emotionally freighted time for mathematics leaders as they face uncertainty concerning project sustainability. I also claim that through project participation, where they work on achieving motive objects and resolving tensions in their leadership, mathematics leaders have opportunities to simultaneously develop their leadership practice and themselves as mathematics leaders.

In the next chapter, I share my interpretation of the School Mathematics Leaders' experiences *after* CTLM, and how they responded to the situations they found themselves in when CTLM had ended.

CHAPTER 6: THE SCHOOL MATHEMATICS LEADERS' STRUGGLE WITH POST-PROJECT PROBLEMS OF PRACTICE AND THEIR RESPONSE

6.1 Introduction to Chapter 6

In the previous chapter, I presented the historical professional learning leadership activity that the School Mathematics Leaders enacted during participation in the CTLM project, focusing primarily on their motive objects of activity (Engeström, 2015; Kaptelinin, 2005; Leont'ev, 1978). I highlighted that, as participation in CTLM endured, the mathematics leaders shifted their activity from managerial motive objects to ones focused more on leadership. Towards the end of project participation, the School Mathematics Leaders engaged in undertakings that sought to influence the development of collective commitments (Rose & Norwich, 2014) for mathematics teaching, as well as opportunities for fostering relational trust (Edwards-Groves et al., 2016). Those shifts in motive objects brought on change and development (Engeström & Sannino, 2011; Roth, 2012) within the mathematics leaders' activity and themselves as mathematics leaders. That provided evidence of the enactment of the person-practice dialectic (Edwards, 2017) within the School Mathematics Leaders' activity.

At the end of Chapter 5, I described how the School Mathematics Leaders recalled that they were worried about the end of CTLM participation. That was situated chiefly in concerns about no longer having access to the support and resources available during CTLM (Heirdsfield et al., 2010). Even though they developed themselves and their leadership practice, they were worried about the uncertainty of the impending change.

In this chapter, I move forward in time as I draw on data analysis concerning the School Mathematics Leaders' professional learning leadership activity in the years following participation in CTLM. I turn to a discussion about the struggle that they experienced in those years. I explain that this was due to the *resurfacing* of the historically accumulated contradiction that motivated their schools' participation in CTLM in the first place. I describe the manifestation (Engeström & Sannino, 2011) of that contradiction, along with realisations of that manifestation as *practice problems*, also explaining their influence on the School Mathematics Leaders' activity.

6.2 Overview of Chapter 6

Through this chapter, I present the case that as middle leaders (e.g., Edwards-Groves et al., 2019; Lipscombe et al., 2021), the School Mathematics Leaders engaged in struggle as they contributed to project sustainability. That was due to the resurfacing of the diminished priority of mathematics contradiction, which had faded in prominence during participation in CTLM. I explain the reasons why that contradiction resurfaced.

I continue to position the School Mathematics Leaders as the collective subject (Lektorsky, 2009), presenting themes of their shared experiences and enactment of their post-project leadership activity. I start by presenting evidence of the commitments to continue with the CTLM-initiated reforms that the mathematics leaders' principals endorsed at the end of CTLM. I explain how those rules remained in place in the years following project participation, but I share a tension that surfaced within those rules.

I present evidence of the resurfaced contradiction as a manifestation (Engeström & Sannino, 2011) that was further realised as post-project problems of practice that confronted the School Mathematics Leaders and their efforts to contribute to project sustainability. The discussion continues by highlighting their response to those practice problems, and I interpret that drawing on the concept of responsibilization (e.g., Nuttall et al., 2022). I finish this chapter by explaining that the School Mathematics Leaders acted out that responsibilization, as a form of division of labour (Engeström, 2015), through an approach characterised by care and creativity.

I now turn to a discussion of findings concerning the commitments made to continue with the CTLM-initiated reforms.

6.3 A Commitment to Continue with Project-Initiated Change

According to the School Mathematics Leaders, the commitment to continue with the CTLM-initiated mathematics teaching reforms was established in the month after participation in the project (December 2012). At that time, the formal professional development sessions had ended, and the mathematics leaders recalled that leadership discussions focused on project sustainability ensued in their schools. Those discussions focused on ways of maintaining the reform efforts beyond the life of CTLM. The School Mathematics Leaders claimed that in those discussions with their executive leadership teams, they shared the uncertainty brought on by the completion of CTLM participation.

The School Mathematics Leaders reported that they pressed for the continuation of the reforms. They stated that when CTLM participation finished, they felt that they had just started the development of their professional learning leadership, including the work focused on influencing mathematics teaching practice.

The following comment from Penny supports that claim about the School Mathematics Leaders' engagement in decision-making conversations, and how they felt about their work at the end of CTLM:

> I remember back when we were heading into 2013. I think that was just after CTLM, and I remember that I pushed for the CTLM work to continue because I felt like I was just getting into it. I remember saying to the principal that if I was feeling that, then the teachers must be, too (Penny, PPI, 25.03.15).

That involvement in decision-making at the end of CTLM confirms that as middle leaders, the School Mathematics Leaders were practising members of the leadership team (Edwards-Groves et al., 2016; Irvine & Brundrett, 2019; Lipscombe et al., 2020) at that time. Part of being a practising member of the executive leadership team meant that they engaged in activity that influenced the principal and the direction of their schools (Grootenboer, 2018). The School Mathematics Leaders engaged in practices that convinced their principals to continue with the CTLM-initiated reforms.

I now describe how the School Mathematics Leaders' post-professional learning leadership activity was mediated by the introduction of commitments to continue with those reforms. Through a CHAT perspective, I position them as rules that governed the mathematics leaders' activity system (Bellamy, 1996; Engeström, 2015; Kuutti, 1996).

6.3.1 Rule of Continuing the Mathematics Teaching Reforms

The first of the commitments concerned the decision to continue with the mathematics teaching reform efforts initiated through CTLM participation. The School Mathematics Leaders confirmed their involvement in planning discussions with their executive leadership teams, focused on identifying ways to sustain the project-initiated changes in mathematics teaching. That is further evidence of the sustainability factor of the principal, that in their authority as the senior leader, they play an important role in enabling reform efforts when they endorse commitments to them (e.g., Anderson & Stiegelbauer, 1994; Coburn et al., 2012; Datnow et al., 2005). It also highlights that the middle leader also plays an essential role within that leadership factor of project sustainability. In my study, that was enacted when the principals made commitments to continue beyond the life of project participation (Bobis, 2011; Saito et al., 2012; Warren & Miller, 2016; Zehetmeier, 2017), influenced by the School Mathematics Leaders and their perspectives on the need to sustain the mathematics teaching reforms. Through that principal leadership action, the rule of commitment to the project continuation surfaced within the School Mathematics Leaders' activity system (Bellamy, 1996; Engeström, 2015). It was not just the principals making decisions to sustain the reform, but it included the influence of the School Mathematics Leaders as middle leaders with that decision-making.

Penny highlighted the roles of the principal and the School Mathematics Leaders, as forms of school leadership, acting as project sustainability factors. The rule of commitment to continue with CTLM-initiated reforms meant that, for the mathematics leaders, their position as a middle leadership one was also maintained:

> I remember the conversations at the end of CTLM about how we would continue with what we learned and keep some things in place. The principal made the decision to continue with CTLM. I think that was more important for me as SML because I couldn't have done the role without his blessing (Penny, PPI, 25.03.15).

In their work as middle leaders, as practising members of the executive leadership teams in their schools, the School Mathematics Leaders were invited to partake in post-project planning discussions. They demonstrated, however, an understanding of the limitations of their authority (De Nobile, 2018) and that their middle leadership was dependent on principal support (Lipscombe et al., 2020). They all acknowledged that the principal mandated the decision to continue the reforms.

Rachel shared further insights about the limited authority of middle leaders and the importance of the principals’ decision to commit to project sustainability:

> I am so glad that the principal decided that we would keep it going because there's no way that we could without him backing it at that 'principal level'. You need principal support because I couldn't keep it going without it (Rachel, PPI, 25.11.15).

Viewing that commitment to continue with the CTLM-initiated changes through a CHAT lens sees the role of the principals in establishing a rule within the School Mathematics Leaders' activity system (Engeström, 2015). That rule became one that was established with the intention to enable conditions to continue with the project reforms despite not having access to the external support that was offered through project participation (Heirdsfield et al., 2010).

The establishment of that rule also influenced the School Mathematics Leaders' affective responses to project sustainability. As evidenced in Rachel's comment, they were grateful that the principals had authorised the continuation of the mathematics teaching reforms. That sense of gratitude felt by the mathematics leaders deepened when their principals established another commitment: a pledge to keep the role of School Mathematics Leader as a formal leadership role within their schools' leadership system.

6.3.2 Rule of Maintaining Mathematics Leadership Position

As another commitment to continue with the project reforms, the principals agreed to maintain the School Mathematics Leader as a middle leadership position. That commitment remained in the schools for the data generation phase (up to May 2018).

According to the School Mathematics Leaders, they had heard that when participation in CTLM ended, the role was disestablished from leadership rosters in most schools within and beyond their local areas. The commitment to maintain the position made the School Mathematics Leaders feel fortunate and relieved. They attributed that to the knowledge they claimed they had about other CTLM schools and the disestablishment of the School Mathematics Leader role.

Cindy exemplified that feeling and knowledge:

> It is great that the mathematics leader role is still here at our school, and I have to thank the principal for that. I know that at some other schools that did CTLM, the mathematics leader role just went almost straight away after CTLM finished (Cindy, PPT, 06.11.14).

Rachel provided further evidence of the gratitude experienced by the School Mathematics Leaders about having the support of the principal in maintaining the position:

> I am so pleased that we have that expectation that we will continue with what we started in the CTLM days. Just even the principal keeping the role of SML as a leadership position in the school has been important (Rachel, PPI, 19.11.15).

The School Mathematics Leaders credited the principals for establishing that rule about maintaining the mathematics middle leadership position. Penny confirmed that by saying: “He (the principal) has allowed for the School Mathematics Leader role to continue at our school, and with me in the role to help that continuation of what we started. I am thankful for that” (Penny, PPI, 25.03.15).

Through a CHAT lens, the commitment to maintaining the mathematics leader position was another rule that entered the School Mathematics Leaders’ post-project activity system (Bellamy, 1996; Engeström, 2015). That again confirms how principals are positioned with authority to support the work of middle leading (Bennett et al., 2003; Bryant et al., 2020; De Nobile, 2018; Grootenboer, 2018; Gurr, 2019; Lipscombe et al., 2021). By establishing that rule, the principals enabled the potential for the mathematics leaders' professional learning activity to continue beyond the life of the CTLM project.

That evidence adds further knowledge about the leadership factor associated with project sustainability (e.g., Datnow et al., 2005). If middle leadership positions are established during project participation, it is crucial that the principal mandates post-project rules that maintain the middle leadership position. In the case of my study, the principal maintaining the School Mathematics Leader position plays a part in sustaining reforms initiated through mathematics project participation.

6.3.3 Importance of Principal Support in Project Sustainability Decision-Making

The evidence I have presented supports the place of principals in mandating school decisions to continue the work of project sustainability. It is recognised that the principal plays a vital role in the continuation of reforms initiated through participation in mathematics education projects (e.g., Bobis, 2011; Goos et al., 2018; Saito et al., 2012; Tirosh et al., 2015; Zehetmeier, 2015). However, the School Mathematics Leader, as a middle leader in the school leadership system, also plays a part in influencing decisions to continue with project sustainability.

From the perspective of the School Mathematics Leaders, the principal facilitated the initial condition for them as middle leaders to continue with the project-initiated reforms (Bryant et al., 2020; Grootenboer, 2018). That was realised by establishing two critical commitments: one to continue with CTLM-initiated reforms and the other to maintain the position of School Mathematics Leader. Within CHAT terms, I have interpreted that as establishing rules that could mediate the School Mathematics Leaders' post-project activity system (Engeström, 2015). As the executive leader, the principals enacted a vertical division of labour (Cole & Engeström, 1993), exercising their power and authority to mandate the rules to set up conditions for project sustainability.

The principals, with their authority as the executive leader, mandated explicit rules with the intention of sustaining project reforms. In a CHAT sense, the power enacted by the principal in establishing those rules had the influence to promote that change (Foot, 2014). The relations between the School Mathematics Leaders, as the collective subject, and the community within the activity system were mediated by that division of labour (Engeström, 2015; Foot, 2014; Kuutti, 1996). The positionality of the principal determined the establishment of rules as the executive leader influenced somewhat by the School Mathematics Leaders as middle leaders within their schools' leadership system (Edwards-Groves et al., 2016; Grootenboer, 2018).

The School Mathematics Leaders themselves desired to continue with the changes initiated through participation in CTLM. They felt that it was only at the end of CTLM participation that they were coming to understand their leadership. They understood that their principals were responsible for making decisions concerning school direction about reforms. The principal, as part of the community of their activity system (Engeström, 2015), with their decision to continue the changes in mathematics teaching practice, mediated the mathematics leaders' activity through a vertical division of power (Foot, 2014). However, the School Mathematics Leaders were not passive in that decision-making process. Instead, as middle leaders in their schools, they convinced their principals to continue with the project reforms through those discussions as a practising member of the executive leadership team (Edwards-Groves et al., 2016; Grootenboer, 2018).

These findings confirm the role of principals as a leadership factor in supporting project sustainability (Bobis, 2011; Goos et al., 2018; Saito et al., 2012; Tirosh et al., 2015). That was realised when the principals mandated two important post-project rules within the School Mathematics Leaders' activity system: "We will continue with CTLM" and "We will maintain the School Mathematics Leader role at our school."

I claim, however, that the school leadership factor of project sustainability should include the middle leader's role in influencing decisions that influence principals in mandating and setting up conditions to engage in project sustainability. In my study, the School Mathematics Leaders played an essential role in the decision-making process that established those post-project rules for continuing the CTLM-initiated reforms.

I interpret the School Mathematics Leaders' affective responses (gratitude and relief) to the establishment of those rules as an awareness of their positionality as middle leaders working between the principal and the teachers in their schools (Edwards-Groves et al., 2019; Grootenboer, 2018). They recognised that authority, which I have interpreted through the CHAT concept of a division of labour (Cole & Engeström, 1996), in that principals had the ultimate power in establishing the project sustainability rules.

With the project commitments established, those rules alone did not fulfil the intention of mediating the School Mathematics Leaders' post-project leadership activity. The establishment of those rules surfaced significant tension for the mathematics leaders following participation in CTLM. The contradiction of the diminished priority of mathematics resurfaced in the years following project participation, through a new manifestation, bringing with it several practice problems. That facilitated struggle for the School Mathematics Leaders and their post-project leadership activity.

I now turn to a discussion about that resurfaced contradiction and how it confronted the School Mathematics Leaders in their efforts to contribute to project sustainability.

6.4 Resurfacing of the Historically Accumulated Contradiction

I acknowledge the ontological positioning of CHAT within dialectical materialism (Nuttall & Brennan, 2016; Roth, 2012), and therefore, I understand the importance of contradictions within that ontology (Mussachia, 1977). I again turn to an explanation of the contradiction that the School Mathematics Leaders faced, but this time, within their post-project activity system. That is important considering that CHAT theorists are sensitised to contradictions as historically accumulated tensions that have the potential for change and development within activity systems (Engeström, 2015; Engeström & Sannino, 2011; Roth, 2012).

I present evidence of that resurfaced contradiction, focusing on its manifestation (Engeström & Sannino, 2011). I illustrate how that that manifestation was realised through post-project practice problems for the School Mathematics Leaders, and how they engaged in struggle in response to the tensions they faced (Roth, 2012).

6.4.1 Resurfacing of the Diminished Priority of Mathematics Contradiction

The School Mathematics Leaders claimed that they were pleased about with the establishment of the project sustainability commitment rules, however, frustrations and disappointment soon set in for them. Even though the principals had intended that those rules mediate project sustainability, the mathematics leaders struggled to reconcile them due to the resurfacing contradiction of diminished status of mathematics.

I claim that a conflict ensued between the commitment rules of project sustainability with mediational elements within the School Mathematics Leaders' activity system, including other rules, the community, and the division of labour. That set up a contradictory space for the School

Mathematics Leaders, giving rise to the struggle they faced in their efforts to contribute to project sustainability.

The following explanation supports my claim that CTLM participation brought only a temporary resolution to that historically accumulated contradiction in the School Mathematics Leaders' activity system. As middle leaders in their schools, the mathematics leaders knew about the content of their schools' yearly improvement goals, documented in *annual action plans* (AAPs). Those AAPs articulated targets to improve students' mathematics learning outcomes. In CHAT terms, I interpret those plans as a conceptual tool (Nuttall et al., 2019).

For the School Mathematics Leaders, however, there was tension between the knowledge captured within that conceptual tool of the AAPs. That tension existed between the rules of project sustainability and a mismatch between those rules and conditions that the mathematics leaders found themselves within their schools in the years following CTLM participation.

Cindy exemplified that as she struggled to make sense of the clash between the mathematics improvement goals for students in the AAP and the attention that literacy education received over mathematics:

> But you can't ignore the fact that if we say numeracy is a focus in our school on our annual action plan, then surely, we need to have some attention on it: attention on mathematics in the same way as literacy! We would never say, "Oh, we have done literacy!" But somehow, it's okay to say, "Oh, we have done maths because we did CTLM." (Cindy, PPI, 06.12.16).

Rachel shared a similar story to Cindy, where the contradiction was experienced as competition between curriculum areas and how that resulted in a resurfaced lack of focus on mathematics. Rachel's comment also revealed the resurfacing historical accumulation of the contradiction of the diminished priority of mathematics:

> It is getting messy nowadays, which did not happen during CTLM. It is a bit like the days before CTLM when you think you're working with a team, and then you are not because something has come up. I feel like the lack of focus is back. That was the reason why we went and did CTLM. It was that lack of focus on maths (Rachel, PPI, 19.11.15).

Penny’s comment revealed her experience of the tension between the commitment rules and the shifted status of mathematics education in the years following CTLM participation:

> But, given that we went through CTLM it's almost like maths was the flavour of the month and that's no longer that. As I said, I'm thankful for the continuation of the role and we have said that we will continue with what we started in CTLM, but the reality is that there's this thing that maths has had its turn (Penny, PPI, 25.03.15).

Those comments highlight that through engagement in the professional development that participation in CTLM offered for two years, the focus on mathematics education shifted its status and raised its profile as a school curriculum area. My interpretation, however, suggests that CTLM participation offered only a temporary disruption to the historically enduring contradiction of the diminished priority of mathematics. Phrases like “we have done maths,” “the lack of focus is back”, and “maths was the flavour of the month” support my interpretation. Those phrases also reveal the struggle that the School Mathematics Leaders experienced after CTLM. For them, the contradiction resurfaced when the CTLM project support was rescinded, and the attention given to mathematics abated after project participation.

That diminished priority of mathematics as an enduring contradiction and references to mathematics having been "done" and "had its turn" in the school improvement agenda is not new for Melbourne Archdiocesan primary schools. Clarke et al. (2005) reported that many schools did not sustain commitment post-involvement in the initiative after participation in a previous CEOM primary mathematics initiative (that concluded in 2004). Those authors concluded that it was due to the consideration that mathematics had been "done" rather than positioning it as a continual focus of school improvement.

Based on my interpretation of the School Mathematics Leaders' struggle, I believe that they did not hold the view that mathematics had had its turn or been done. Instead, they were buoyed by the commitment rules established by their principals to sustain the CTLM-initiated changes and desired to contribute to project sustainability. The struggle for the mathematics leaders was the conflict that they experienced between the establishment of those commitment rules for project sustainability, the knowledge of goals of improvement articulated in AAP documentation, and the desire of the School Mathematics Leaders to continue with reforms.

I interpret that those disturbances resurfaced the contradiction of the diminished priority of mathematics. The resurfacing of that contradiction was experienced as a manifestation (Engeström & Sannino, 2011) and realised through several practice problems. Those all coalesced for the School Mathematics Leaders, meaning they were engaged in struggle as they sought to contribute to project sustainability.

6.4.2 Manifestation of the Diminished Priority of Mathematics as Practice Problems

I now present evidence of how the School Mathematics Leaders experienced the contradiction of the diminished priority of mathematics. Taking on advice from Engeström and Sannino (2011), I describe the manifestation of that historically enduring contradiction. For my thesis, I further interpreted that manifestation through several practice problems that had evolved from changes or resurfaced in the years following CTLM participation.

Interestingly, as a term, *practice problem* was used often in the education research literature that I sourced, but it remained undefined by authors. That led me to believe it is a taken-for-granted term. I experienced a struggle about including citations to support a definition that I could use to capture those problems faced by the School Mathematics Leaders in the years following CTLM.

I reconciled that struggle by adapting a term used by Booth et al. (2016). Those authors claimed that consequences that surface in workplace conditions due to adverse changes could be called a “practical problem” (p. 52). Booth et al. added information describing that those practical problems are ones that also impose consequences or costs for workers in those situations. That definition suitably captured the realisations of that diminished priority of mathematics contradiction, especially the consequences that surfaced in the struggle for the School Mathematics Leaders.

I now present evidence of five practice problems that the School Mathematics Leaders encountered as the contradiction of the diminished priority of mathematics through a discussion of each practice problem.

6.4.2.1 Diminished Sector Leadership Support

The first practice problem that had consequences for the School Mathematics Leaders' leadership activity, concerned diminished sector leadership support. As soon as participation in CTLM concluded, the leaders reported that support from the CEOM ceased. That changed conditions for them because they had no further access to the SAMs who supported their implementation of reforms during the project. The lack of CEOM support was also realised through an absence of directives, rules within a CHAT perspective (Engeström, 2015), about ways to negotiate project sustainability. The provision of extra funding from CEOM, used by principals to subsidise time allocation for the School Mathematics Leader role, also discontinued at the end of CTLM.

6.4.2.1.1 Lack of Support and Directives from CEOM. The School Mathematics Leaders claimed that the lack of support and directives from CEOM constrained aspects of their post-project professional learning leadership. The rule of "no further support from SAMs" surfaced when they most needed that assistance. Rachel highlighted that and explained the importance of access to advice from mathematics education specialists when sustaining project reforms:

> I had the SAMs then. I could talk things through with them like issues and questions, but I cannot contact them for help now because we were in CTLM. I get told that I cannot have their help. It would be nice to have someone because I have more questions now than I did back then (Rachel, 25.11.15).

The lack of directives from CEOM, as rules from the neighbouring activity system (Engeström, 2015), added to the struggle of that practice problem. The School Mathematics Leaders wanted the influence of CEOM, which they often referred to as “non-negotiables”. In CHAT terms, I interpret that as the mathematics leaders desiring rules to govern actions within their activity system.

For the School Mathematics Leaders, they positioned those CEOM-imposed rules as ones that had the potential to influence the rules within their own activity system. I interpret that as the mathematics leaders knowing their own positionality as middle leaders and the authority that came with their position in their school's leadership system (Bennett et al., 2007; De Nobile, 2019; Grootenboer, 2018). The mathematics leaders appeared to believe that those CEOM rules could enable conditions for project sustainability and force the principals to align conditions with the commitment rules established at the end of CTLM.

Penny referenced the idea of non-negotiables, wishing that CEOM had set directives that would force principals to support project sustainability:

> There are no non-negotiables with maths now from CEOM because the project is now over. During CTLM, there were the non-negotiables, especially with maths PLTs each fortnight. CEOM needs to keep them so that principals can follow them so we can keep going with CTLM (Penny, PPI, 02.12.16).

The School Mathematics Leaders reported frustrations with the lack of post-project directives from CEOM. That is further evidence that the School Mathematics Leaders were conscious of their positionality within their schools' leadership system, recognising that although they desired to, the division of labour within their activity system constrained the establishment of explicit rules for sustaining the reforms. I interpret that as a vertical division of labour because of the distribution of power (Cole & Engeström, 1993), which in this case was limited authority due to their middle leadership position (De Nobile, 2019).

6.4.2.1.2 Withdrawal of Funding for Mathematics from CEOM. Diminished sector leadership also manifested as a post-project practice problem for the School Mathematics Leaders through the reduction of CEOM funding. Rachel captured the experience of that tension and its consequences, revealing the strain that the reduced mathematics funding had on school budgets: "We had a lot of funding from CEOM during CTLM, but now the school needs to find that funding for maths which I know is a struggle" (Rachel, PPI, 19.11.15).

A further enduring impact of the reduced CEOM funding was the lack of money for resourcing the time allocation for the School Mathematics Leader position. That had the School Mathematics Leaders wishing that mandates were issued from CEOM for permanent funding at a school level for the mathematics leadership position. All three mathematics leaders knew that it was a strain on school budgets and that with the rule of commitment to maintain the leadership position, there was a financial obligation to fund it in the years after CTLM participation.

Penny exemplified that when she talked about the issue of reduced funding:

> The directive isn't there from CEOM like it was with things like the funding for the maths leader role. I would say that that's something that I really think that CEOM needs to look at if we want to keep going with what we started in CTLM (Penny, PPI, 16.05.18).

One factor that has the potential to enable or constrain project sustainability in schools is that of external sector or district leadership (e.g., Coburn et al., 2012; Datnow et al., 2005; Jackson et al., 2015). In the case of the School Mathematics Leaders, their experience of the abated external sector leadership after CTLM constrained their professional learning leadership. They revealed, however, that they understood the importance of sector leadership support and the influence that CEOM could have had in facilitating project sustainability efforts. Within a CHAT lens, I interpret that as the mathematics leaders wanting CEOM to establish further rules (Bellamy, 1996; Engeström, 2015) and provide the cultural tool of money to mediate the project sustainability commitment rules established by their principals.

The School Mathematics Leaders understood that, as the sector leader, CEOM had the authority to influence the principals in setting up conditions that enabled the mathematics teaching reforms to continue. Those wishes for CEOM directives were important to the mathematics leaders because they experienced changes in the levels of support provided by their principals after CTLM had finished.

6.4.2.2 Changed Principal Leadership Support

The changed leadership support from principals in the years following CTLM participation also contributed to the School Mathematics Leaders' struggle to continue with project sustainability. That was compounded by the knowledge that even though their principals were supportive, by establishing the rules of sustaining project reforms and maintaining the mathematics leadership role, further actions enacted by their principals conflicted with those rules.

Evidence from Cindy supports this claim when she shared her thoughts about the changes to the frequency and prevalence of principal support: "There's not the same amount of support from the principal. The support is nowhere near what it was like in the CTLM days. I still need support now more than ever because it's hard keeping CTLM going" (Cindy, PPI, 02.11.17). Similar comments from the School Mathematics Leaders highlighted that they understood the importance of principals providing support for middle leaders to lead site-based professional learning (Edwards-Groves et al., 2019; Gurr, 2019; Lipscombe et al., 2021).

The School Mathematics Leaders attributed changes in principal support to the previous practice problem of the diminished support from CEOM as their sector governing body. They believed that the reduced support from CEOM influenced their principals' abated levels of support. Penny specifically showcased this as she rationalised that the changed principal support was due to the withdrawn support from CEOM: "I would say that one of the reasons for the lack of leadership support here at the school is because we don't have any support for maths from CEOM because CTLM is now done, a long time ago" (Penny, PPI, 02.12.16).

The change in support that was most noticeable for the School Mathematics Leaders was the shift in the frequency of feedback from their principals.

6.4.2.2.1 Reduced Principal Feedback. Changes in principal support confused the School Mathematics Leaders due to a clash between the principal's project sustainability commitment rules and the reduction in principal feedback. That confusion was articulated by Rachel, who shared her uncertainty about the decreased feedback frequency on her leadership activity. She was unclear if this was due to her principal's confidence in her mathematics leadership or whether it was due to the reduced priority of mathematics in her school: "But, I must say that the leadership feedback and support has changed from the CTLM days. I'm not sure if it's because there's trust there or whether it's because maths has had its turn" (Rachel, PPI, 26.10.16).

Penny also experienced a reduction in principal feedback. She experienced frustration due to the increased demands that came with sustaining the project-initiated changes:

> The feedback was far greater then compared to what the feedback is now. And, I wouldn't say that I was doing more then. In fact, I think I'm doing things at a far more intense level now as I work to keep what we learned in CTLM going nowadays (Penny, PPI, 25.03.15).

Penny later revealed that she connected the reduced principal feedback with the diminished principal engagement in the professional learning opportunities that she facilitated following CTLM participation:

> During CTLM, the principal was attending the PLT meetings, so he could see what was happening at the meetings, what I was doing, and how I was facilitating things. And I would get comments at the end of it like, "Fantastic Penny, that's great", or "I like the way that you do this, that, and the other." But I'm not getting that feedback now because no one's (from the executive leadership team) coming to my meetings apart from the actual teachers (Penny, PPI, 25.03.15).

The reduced engagement of principals in the professional learning opportunities led by the School Mathematics Leaders was common. That reduction in engagement could have occurred due to another of the practice problems I share later: the reduction in formal professional learning meetings post-CTLM participation.

6.4.2.2.2 Minimised Principal Engagement in Professional Learning. During CTLM, the mathematics leaders claimed that their principals were active contributors during professional learning opportunities that they led, primarily facilitated through the fortnight PLT meetings. For the School Mathematics Leaders, that meant their principals had first-hand knowledge concerning the progress of the implementation of the mathematics teaching reforms. In the years following CTLM participation, principal engagement in professional learning opportunities was reduced. That meant that the School Mathematics Leaders were required to inform principals of developments in project sustainability which they did not have to do during CTLM.

Cindy yearned for greater principal engagement in professional learning meetings in the years following CTLM:

> It would be nice to have him (the principal) more interested or be part of the maths meetings I run like it was in CTLM. When we were in CTLM, he was involved much more and knew what we were doing, like the changes we were making (Cindy, PPI, 06.11.14).

The impact of the reduced principal engagement meant that the School Mathematics Leaders had to find time to report to their principals about how the project sustainability was progressing. Rachel captured that saying: "I have to go and talk with him because they do not attend my meetings now that CTLM is over. During CTLM, they knew more about what was happening because they were there all of the time” (Rachel, PPI, 26.10.16).

Those comments from the School Mathematics Leaders reveal that if leadership is considered an internal factor of project sustainability (Bobis, 2011; Goos et al., 2018; Saito et al., 2012; Tirosh et al., 2015), then continued engagement in site-based professional learning would be an essential leadership action for principals. The mathematics leaders revealed that principal engagement in their professional learning opportunities could be ways of informing principals of project sustainability. That could also provide contexts for the mathematics leaders to receive feedback on their professional learning leadership.

6.4.2.2.3 Reduced School-Allocated Budgets for Mathematics. Changed principal support was also experienced as reduced budgets for the mathematics curriculum. The impact of CEOM withdrawing funding also meant a reduction in school-allocated budgets in the School Mathematics Leaders' schools. That highlighted yet again the tension between the rules associated with commitment to project sustainability and the diminished priority of mathematics. Cindy was grappled with that tension: "This is confusing because on the one hand there is this, "Let's continue with CTLM" but on the other, it is, "But we will not give the funding to it." I just don't get it." (Cindy, PPI, 06.11.14).

Rachel reported that reduced school-allocated budgets impacted opportunities to facilitate professional learning with teachers. That required more of her attention on managing those funds post-CTLM:

> It is definitely not the same as it was in the CTLM days. I get that it has to change, but not having the money for maths from the leadership team is challenging, especially with releasing teachers from their classrooms so that I can work with them. I have to think about how to use the little money I now get for maths (Rachel, PPI, 24.10.16).

Principal support is vital in mediating the middle leadership of site-based professional learning (De Nobile, 2018; Gurr, 2019). Principals can enable or constrain the work of middle leading like that enacted by the School Mathematics Leaders (Lipscombe et al., 2020; Lipscombe et al., 2021). In the years during CTLM participation, the School Mathematics Leaders claimed that they had that executive leadership support. After CTLM, they were encouraged when their principals mandated rules to commit to project sustainability. However, in the years that followed CTLM, the principals' actions conflicted with those commitment rules. The principals' actions did not match the rule they established.

That caused conflict for the School Mathematics Leaders, compounding their struggle with that resurfaced contradiction of the diminished priority of mathematics. Through a CHAT lens, the principal, as a key member of community within their activity system (Engeström, 2015), played a role in that practice problem. That, however, mainly constrained the School Mathematics Leaders' activity compared to the support they received during CTLM.

6.4.2.3 Shifted Staffing in Schools

Another practice problem that confronted the School Mathematics Leaders was associated with shifts in teaching staff rosters in their schools. Change in staffing can influence the sustainability of reforms. That, realised through the factor of staff turnover, has the potential to enable or constrain sustainability efforts (e.g., Coburn, 2003; Huberman & Miles, 1984; Saito et al., 2012; Taylor, 2006). The mathematics leaders experienced that practice problem of staff turnover in two ways: when teachers who participated in CTLM moved to teach in other schools (external transfer) and when teachers moved within the school to teach different year levels (internal transfer).

6.4.2.3.1 External Transfer of Staff. Staff turnover as *external transfer* caused challenges for the School Mathematics Leaders. When teachers who participated in CTLM gained employment at other schools, they took with them the practice and knowledge developed during and after participation in CTLM. That tension was compounded when the replacement teachers, coming on board as staff members in the mathematics leaders’ schools, had little or no historical experiences with CTLM. That meant that the School Mathematics Leaders had to spend time supporting those new teachers in understanding the mathematics practices and mathematical knowledge for teaching (Ball et al., 2008). That caused struggle because, at the same time, the mathematics leaders realised that they needed to build on and extend the practices and knowledge of the existing staff in their schools who did participate in CTLM.

The consequence of that practice problem was exemplified by Rachel, revealing the struggle that the School Mathematics Leaders experienced with staff turnover through external transfer. For Rachel, that invoked feelings of worry mediated by her knowledge of teachers' historical professional development experiences:

> The teachers that haven't done CTLM tend to come from different points of view. I worry that those people often feel left behind in a planning meeting, particularly teachers who have not been through CTLM. It is a challenge to keep everyone on the same page. I am aware of that I have to help the teachers who didn't do CTLM and then make sure that I keep going with the teachers who did do CTLM (Rachel, PPI, 25.11.15).

That situation meant that the School Mathematics Leaders needed to accommodate a wide range of teachers' professional learning needs while maintaining and building upon a shared understanding of effective mathematics teaching. Challenges ensued for the mathematics leaders as they were required to constantly visit and revisit the foundational aspects of the CTLM-endorsed practices and principles of effective mathematics teaching (Taylor, 2006).

An unexpected issue intensified that struggle with the *internal transfer* of staff when teachers moved year levels to different grades within their school.

6.4.2.3.2 Internal Transfer of Staff. It is a typical routine in Australian schools for classroom teachers to move year levels after teaching a specific grade for several years. That routine was enacted within the School Mathematics Leaders' schools. During CTLM, with its professional development structure partitioned into Year Prep to Year 2 (junior primary) and

Year 3 to Year 6 (middle and senior primary), teachers focused on the MCK and PCK suitable for those years of schooling.

For several teachers who remained working in their schools post-CTLM but had moved internally to different years levels, the application of mathematics teaching practices and its associated mathematical knowledge for teaching (Ball et al., 2008) was a challenge. That surfaced further struggle for the School Mathematics Leaders.

Cindy captured that practice problem and its impact on her leadership:

> The number one feedback from teachers in CTLM was that they actually felt empowered in their level during the project. But since CTLM, some of them have moved levels, and I know that they have found it quite difficult, and they haven't necessarily been able to apply their knowledge into their teaching in the new level. I know my teachers need support there, so I am going over a lot of CTLM ideas and helping them see how to apply their knowledge (Cindy, PPI, 06.11.14).

That evolved as a post-project problem of practice for the School Mathematics Leaders because instead of having CTLM-trained teachers, who might support their leadership by sharing and using their practice and mathematical knowledge for teaching (Ball et al., 2008) with others, the mathematics leaders had to focus their attention on those colleagues in ways like the newly recruited teachers with little or no experience of CTLM. Again, that meant foundational ideas associated with effective mathematics teaching were focused on rather than extending and transforming those practices and principles (Fullan, 2008; Taylor, 2006).

That practice problem raises an essential point for professional development project designers and how the project's content supports teachers in transferring the MCK and PCK, mathematical knowledge for teaching (Ball et al., 2008), when they move from one year level to the next.

6.4.2.4 Diminished Role Clarity and Status

Diminished role clarity and status manifested as further evidence of the contradiction of the diminished priority of mathematics. During CTLM, the School Mathematics Leaders claimed they had a clearly defined responsibility, focused on implementing the CTLM mathematics teaching reforms. Following CTLM, however, the mathematics leaders claimed that the clarity of their role diminished, confirming that role clarity can be challenging for middle leaders (Drysdale et al., 2016; Farchi & Tubin, 2019).

The diminished role clarity was experienced emotionally by the School Mathematics Leaders through feelings of self-doubt and underappreciation, harkening back to their numeracy coordination role in the years before CTLM participation. The mathematics leaders' diminished role clarity and status were characterised by a multiplicity of roles, reduced time allocation for their leadership role, and diminished involvement in decision-making processes.

6.4.2.4.1 Multiplicity of Roles. In the years following CTLM, the School Mathematics Leaders managed and led areas of their schools' improvement agendas that went beyond the mathematics curriculum. They struggled with the multiplicity of roles, a characteristic of the mathematics leadership role in primary school settings (Higgins & Bonne, 2011). The extra roles also impacted their time to engage in their mathematics leadership activity.

The School Mathematics Leaders reported that at least two roles were added to their work duties that were not part of their leadership during CTLM. Rachel evidenced that situation by sharing:

> I am still the School Maths Leader, but I am also the Maths Intervention Teacher, the Acting Deputy Principal, and I am the one who fills in when we have a teacher away. It wasn't like this in CTLM days (Rachel, PPI, 08.11.16).

The multiplicity of roles post-CTLM impacted the time the School Mathematics Leaders had to plan, facilitate, and monitor post-project professional learning. It meant that within their activity system, the division of labour (Cole & Engeström, 1993) associated with the undertaking of responsibilities, another rule surfaced about reduced time to work of leading mathematics professional learning for teachers.

6.4.2.4.2 Reduced Time Allocation. With the multiplicity of roles and the reduced budgets to fund the leadership role, there was reduced time for the School Mathematics Leaders to focus on professional learning in the ways that were enabled to do during CTLM. That reduction in time made them feel like they were not influencing teachers' professional learning in the focused ways they wanted.

Penny captured the experience of that practice problem, alluding to the idea of juggling simultaneous roles that affected the focus of her work:

> I know that the things that I was doing in maths as a leader, perhaps even last year or the year before, are not being done at the same level because the time for the maths leader role has been cut back form what it was in CTLM. I'm not getting to do now what I was able to do when we were in CTLM and I had the clear focus on maths leadership (Penny, PPI, 25.03.15).

That practice problem further intensified the struggle experienced by the mathematics leaders as the clarity of their role shifted, conflicting again with the principals' establishment of the project commitment and continuation of the mathematics leadership role rules. That was further impacted when the School Mathematics Leaders experienced exclusion from executive leadership team decision-making processes.

6.4.2.4.3 Diminished Involvement in School Decision-Making Processes. In the years following CTLM, the School Mathematics Leaders claimed that their participation in executive leadership team discussions about school directions abated. Even though they practised their middle leadership as a member of the executive leadership team (Lipscombe et al., 2020) during CTLM and for some time shortly after when they influenced their principals to establish rules of commitment to project sustainability, they noticed that as the years endured beyond CTLM participation, they were no longer privy to such leadership decision-making processes. The mathematics leaders reported exclusion from those executive leadership meetings, realising for them further evidence of the diminished priority of mathematics through their diminished role clarity.

That exclusion impacted how the mathematics leaders saw their own positionality within the leadership systems of their schools. The following comment from Penny supports that claim, revealing that that shift signified a change in the status of the mathematics leadership role after CTLM: “It definitely has changed. I have no voice in certain things now with the upper leadership. I'm being left out of certain leadership decisions and conversations, and that never happened during CTLM” (Penny, PPI, 02.12.16).

Rachel highlighted the School Mathematics Leaders’ struggle when she disclosed her doubts in response to no longer participating in executive leadership decision-making processes: “I am always asking myself questions: Where do I go next year? What do I do? How do I keep CTLM going in our school? I worry that I am questioning myself and overthinking too much” (Rachel, PPI, 08.11.16).

For the School Mathematics Leaders, the practice problem of abated participation in executive leadership decision-making processes was highly imbued with affect, brought on by no longer engaging as a member of the school leadership team. That was a change from CTLM when being part of the decision-making processes, providing direction for their own work as middle leaders of mathematics.

6.4.2.5 Reduced Scheduling of Mathematics Professional Learning Opportunities

The final practice problem that complicated the School Mathematics Leaders' struggle was a reduction of formal mathematics professional learning opportunities. The frequency of opportunities to lead professional learning in formal settings was reduced significantly after CTLM. In CHAT terms, that reduced scheduling of opportunities became another rule that conflicted with the rules of commitment to project sustainability. That rule also mediated a horizontal shift in the division of labour (Cole & Engeström, 1993) through a reduction of work tasks concerning the planning and facilitation of formal mathematics professional learning opportunities.

The impact of that practice problem was first realised with the removal of the fortnightly PLT meetings from meeting schedules. With no directives from CEOM that mandated those meetings beyond CTLM participation, the School Mathematics Leaders had to negotiate the scheduling of mathematics professional learning meetings with principals. That was reminiscent of their work prior to CTLM participation when they were numeracy coordinators in their schools.

Penny evidenced that as she recounted her frustrations with the diminished regularly scheduled formal meetings:

> We just don't have the PLT meetings like we did when in CTLM. Our PLTs used to be numeracy and literacy, and that was a fortnightly seeing each team (of teachers), but now, I am very lucky to have two PLT meetings a term (Penny, PPI, 25.03.15).

Rachel shared her frustrations with minimised formal meetings with teachers: “I will not have the time to meet with the teachers in a PD setting this again year. It's frustrating because, during CTLM, I would be working with teachers right up into the middle of December” (Rachel, PPI, 16.11.15).

The struggle to secure opportunities to lead professional learning for teachers became more pronounced as the years endured. It became more difficult for the School Mathematics Leaders to secure meetings in the professional learning schedules in the years following CTLM participation. Cindy evidenced that as she shared her struggle: “I've had to fight for two maths meetings a term at the moment. This term and last term, I only managed to get two meetings, not through a lack of wanting more and trying to get more” (Cindy, PPI, 23.10.18).

The rule of reduced formal meetings heightened frustrations for the School Mathematics Leaders because they noticed that it again conflicted with the rule of commitment to project sustainability established by their principals. The shifts in the community through staff turnover meant that they wanted to lead formal professional learning opportunities for teachers. Those meetings that they could secure, however, were mainly focused on reviewing shared understanding of practice and knowledge for mathematics teaching (Ball et al., 2008) because of the practice problem of staff turnover (Taylor, 2006).

I interpret that those five practice problems were a *critical conflict* manifestation (Engeström & Sannino, 2011) of the contradiction of the diminished priority of mathematics education. My reasoning for that lies in that the practice problems had the School Mathematics Leaders in situations where they were paralysed to act, and the conditions associated with those practice problems constrained their activity to contribute to project sustainability. Their emotional responses to those practice problems had them engaged in feelings of isolation and self-doubt (Engeström & Sannino, 2011), providing further evidence of the critical conflict that they experienced following participation in CTLM.

6.4.3 Reason for the Resurfaced Contradiction

The contradiction of the diminished priority of mathematics manifested as a critical conflict through those practice problems that were not present during participation in CTLM. Following CTLM, the School Mathematics Leaders experienced a struggle between their desire to contribute to project sustainability and abide by the rules of commitment established by their principals, with the changes that transpired after participation in CTLM ceased. I interpret that the reason for that is that participation in CTLM brought only a temporary resolution to that historically accumulated contradiction of the diminished status of mathematics.

Participation in the CTLM project facilitated opportunities for the School Mathematics Leaders' schools to focus on an improvement agenda for mathematics education. That afforded access to human, physical, temporal, and monetary resources that mediated several changes to the leadership and teaching of mathematics. However, that only resolved the contradiction briefly (the two years of participation in CTLM). I interpret that as *contradiction fade* because CTLM participation brought on a temporary disruption to the tensions experienced in the School Mathematics Leaders’ schools.

That may be reasoned through the notion that the CTLM project attempted to focus on an overall change in the mathematics education system in Archdiocesan primary schools rather than supporting the CTLM schools in noticing, addressing, and changing the specific practices within their school settings (Sannino, 2010). In the case of my study, that would have been shifting practices that enabled more effective mathematics teaching, along with the School Mathematics Leaders, noticing, addressing, and changing conditions that enabled middle leadership to take place beyond the project's life. That is my interpretation of why the contradiction resurfaced and presented itself as practice problems after CTLM participation.

In the years following CTLM, changes ensued within the School Mathematics Leaders' activity system. That saw a change in rules (diminished external and internal leadership support and reduction of formal mathematics professional learning opportunities), the community (staff turnover and changed principal support), and the division of labour (diminished role clarity, multiplicity of roles, and exclusion from executive leadership decision-making processes) which clashed with the School Mathematics Leaders’ motivations to sustain the mathematics teaching reforms. That became emotionally freighted for them as middle leaders of site-based professional learning (De Nobile, 2018; Edwards-Groves et al., 2016) experienced through feelings of isolation, self-doubt, and frustration. That provides further reason to interpret the practice problems that the School Mathematics Leaders faced as realisations of the critical conflict manifestation of that contradiction (Engeström & Sannino, 2011).

Those emotional reactions and their experiences of those practice problems had the potential to influence the School Mathematics Leaders’ decision to discontinue their post-project professional learning leadership activity. Instead, the mathematics leaders worked with the struggle they faced. They responded to that struggle with a strong sense of responsibility to continue with the CTLM-initiated reforms through a commitment to care. They each decided to endeavour with the work of project sustainability.

I now move the discussion to how the School Mathematics Leaders felt responsible for project sustainability whilst facing those post-project practice problems.

6.5 Responsibilization for Project Sustainability

The resurfacing of the contradiction of diminished priority of mathematics manifested as a critical conflict (Engeström & Sannino, 2011) through practice problems that heightened the School Mathematics Leaders' experiences of the struggle they sought to contribute to project sustainability. They found themselves working within an activity system where the mediational elements of that activity changed from those that enabled their leadership during CTLM participation. After CTLM, shifts occurred that constrained their leadership activity.

As it turned out, however, the School Mathematics Leaders endeavoured with the work associated with project sustainability. Facing the struggle with the critical conflict (Engeström & Sannino, 2011) they faced, realised through those practice problems, they were motivated by deep feelings of responsibility. The mathematics leaders felt responsible for the work of continuing with the reforms.

The idea of educational leaders feeling responsible for educational change has been reported by Nuttall et al. (2022). They used the concept of *responsibilization* to interpret how educational leaders in Australian early learning settings experienced the responsibility for solving complex problems in the workplace. That experience was compounded by burdens associated with that responsibility.

As a concept, responsibilization was offered by Shamir (2008) as an explanation for the allocation of responsibility from governing bodies to individuals for the outcomes of actions and decisions. Shamir reported that responsibilization is "namely expecting and assuming the reflexive moral capacities of various social actors – is the practical link that connects the ideal typical scheme of governance to actual practices on the ground" (Shamir, 2008, p. 8). For the purposes of my thesis, I have interpreted responsibilization as a shift of responsibility for significant social and economic problems (effective mathematics teaching through project sustainability) from governing bodies (CEOM and the school principals) to individuals (the School Mathematics Leaders) who are led to believe that personal responsibility is required to address those significant issues.

That concept of responsibilization is one that I have used to interpret the experience of the School Mathematics Leaders as they responded to the struggle they faced with the resurfaced contradiction and its practice problems. The mathematics leaders felt it was their responsibility to continue with project sustainability in the face of the diminished CEOM and school leadership support after CTLM participation.

As middle leaders of mathematics, they were no longer offered the levels of support at the governance (i.e., CEOM) and executive school (i.e., principal) levels that they once had during CTLM. Even though there was initial support from the principal through the establishment of the commitment rules, the actual "practices on the ground" (Shamir, 2008, p. 8) for sustaining the project reforms required the School Mathematics Leaders to take on the responsibility for it themselves. For them, responsibilization was interpreted as an implicit division and organisation of the task associated with project sustainability. In CHAT terms, that responsibilization is positioned as a division of labour (Engeström, 2015; Havnes, 2010) within their post-project activity system.

The following comment from Rachel supports my use of responsibilization as a means of interpreting that division of labour:

> I think what we're doing is right when it comes to maths, so I just keep going. I have a passion for maths, and I care about the kids, and I care about the teachers. I want them all to do well. So, I look for ways to make it work (Rachel, PPI, 08.11.16).

The experience of responsibilization was not fleeting for the School Mathematics Leaders either. It endured well beyond the time when CTLM participation ceased. Responsibilization, as a form of division of labour, was especially highlighted by Cindy six years after participation in CLTM:

> I guess I'll just keep plugging away because I'm not giving up. I care about it all: the maths, the kids, the teachers. You know after all these years, I still feel it's my job to keep going with what we started in CTLM (Cindy, PPI, 23.10.18).

Through their experiences of responsibilization, along with a passion for mathematics education and a sense of care for members of their school community, the School Mathematics Leaders believed it was their duty to find ways of contributing to project sustainability. That was experienced in the face of the struggle that they felt through the post-project practice problems. Therefore, as part of their initial contribution to project sustainability, the School Mathematics Leaders engaged in processes associated with responsibilization. They did that, as evidenced in the comments from Rachel and Cindy, in caring and creative ways.

I now turn to my interpretation of that response, suggesting that they did so through an approach characterised by care and creativity.

6.5.1 Responding Through Care and Creativity

Through their experience of responsibilization, the School Mathematics Leaders responded through care and creativity. The School Mathematics Leaders cared for the work that was achieved during the CTLM project, and as middle leaders practising their mathematics leadership activity in proximity to classrooms (Edwards-Groves et al., 2016; Grootenboer, 2018; Grootenboer et al., 2019), that care extended to the teachers and the students who worked and learned in those classrooms.

Cindy articulated how the School Mathematics Leaders' care for teachers, along with their passion for mathematics education, mediated their response to the responsibilization for project sustainability: "I have a passion for maths, and that's what keeps me going with all of these mixed messages about continuing CTLM. I care about the teachers, and I care about them becoming good maths teachers" (Cindy, PPI, 06.12.16).

Penny provided further evidence of their response to responsibilization, sharing the importance of care for students and their mathematics learning, as well as an appreciation for CTLM-initiated changes in their schools:

> So, it puts me into a bit of a situation thinking, "As a leader, this is what I know to be right, and these are the things that a leader needs to be doing, but lack of time and maths priority is stopping me." But I keep going because I care for the students and the teachers. I care about maths and what we started in CTLM, so I try to be creative (Penny, PPI, 25.03.15).

Seeking ways to be caring and creative in their leadership characterised the School Mathematics Leaders' response to responsibilization. I interpret that as the mathematics leaders initiating a prioritisation of *what mattered* (Edwards & Thompson, 2013) for them. Through their experience of the struggle associated with that resurfaced contradiction and its practice problems, they focused on what was important with project sustainability. For the School Mathematics Leaders, it mattered that they were passionate about mathematics, that they cared for the teachers and students, and that they honoured the work achieved through participation in CTLM.

By seeking ways to respond through care and creativity, I claim that the School Mathematics Leaders initiated their form of resourceful practice (Edwards, 2005; 2010). Instead of succumbing to the practice problems and the burden of responsibilization, they chose to focus on what mattered (Edwards & Thompson, 2013).

6.6 Chapter Summary

In this chapter, I explained a shift in the School Mathematics Leaders' activity system from that which was enacted during CTLM. I reported that the principals in their schools mandated a commitment to the continuation of mathematics teaching reforms and the maintenance of the School Mathematics Leader role. Those rules proved to be an essential way of initiating project sustainability. I continued to position the mathematics leaders as a collective subject as I reported how they were buoyed by establishing the project sustainability commitment rules shortly after CTLM participation ceased.

I claimed that participation in CTLM only brought a temporary disruption to the historically accumulated contradiction of the diminished priority of mathematics education. I supported that claim by sharing evidence of five practice problems that the School Mathematics Leaders faced after CTLM: diminished sector leadership support, changed principal leadership support, shifted staffing, diminished role clarity and status, and reduced scheduling of mathematics professional learning opportunities. I explained that those practice problems were a critical conflict manifestation of that enduring contradiction within their activity system. That was due to the way that the practice problems significantly constrained the activity of the mathematics leaders and left them experiencing doubts about ways to contribute to project sustainability. That intensified the struggle for the School Mathematics Leaders in the years following CTLM participation.

I drew on the evidence to suggest that the School Mathematics Leaders experienced a form of responsibilization as they engaged in the struggle with that critical conflict manifestation and its practice problems. I claimed that the mathematics leaders themselves felt responsible for sustaining the reforms initiated in CTLM when facing the diminished support of CEOM as their governing body and the changed principal leadership support. I provided evidence showing that the School Mathematics Leaders responded to responsibilization through a caring and creative approach that supported them in identifying what mattered to them.

In this chapter, I presented findings that partially respond to the research question. To initialise their contribution to project sustainability, the School Mathematics Leaders initiated a form of resourceful practice that saw them focus on what mattered when facing practice problems that brought on tensions with their post-project activity system. Focusing on what mattered was mediated by responsibilization for project sustainability, which saw them respond through care and creativity.

In the next chapter, I focus more on the School Mathematics Leaders' resourceful practice as their contribution to project sustainability. I do that by describing more of their response to responsibilization that saw the mathematics leaders reconfigure the motive objects of their activity and establish a new rule of relationality within their activity system. I explain how the mathematics leaders then engaged in new leadership actions that saw them work in more relational and creative ways, characterised by attributes of resourceful practice (Edwards, 2005; 2010).

CHAPTER 7: THE RESOURCEFUL PRACTICE OF THE SCHOOL MATHEMATICS LEADERS AS THEIR CONTRIBUTION TO PROJECT SUSTAINABILITY

7.1 Introduction to Chapter 7

In Chapter 6, I claimed that CTLM participation temporarily disrupted the contradiction that the School Mathematics Leaders believed was the reason for their schools' participation in the project. I presented a discussion of findings that suggested that even though commitment rules were established by their principals to support project sustainability, those rules were not enough to inhibit the practice problems that resurfaced that enduring contradiction that led to their schools' decision to participate in CTLM in the first place. That meant that the School Mathematics Leaders were engaged in struggle due to changes within their post-project activity system.

The struggle of the School Mathematics Leaders was compounded by their experience of responsibilization, where they felt it was their responsibility to continue with reforms without the support of CEOM and the shifted support from their principals. I claimed that in response to that struggle, the School Mathematics Leaders looked for ways of responding through care and creativity. They cared for mathematics education, the teachers and students in their schools, and the reforms that had taken place through CTLM participation. I also suggested that School Mathematics Leaders initiated enactment of their form of resourceful practice by focusing on what mattered in response to their responsibilization experience.

In this chapter, I elaborate further on how that resourceful practice realised the School Mathematics Leaders' contribution to project sustainability. I present my interpretation of data showing that by engaging in struggle through care and creativity, the School Mathematics

Leaders developed new forms of practice mediated by the creative and flexible use of resources (Edwards, 2010a).

My interpretation of the School Mathematics Leaders' resourceful practice is explained by proposing that it was initially enacted by reconfiguring the motive objects of activity (Kaptelinin, 2005; Leont'ev, 1978). I propose that that was done in response to the critical conflict that they faced, realised through those post-project practice problems. I present evidence that the School Mathematics Leaders worked on multiple motive objects with the building of relational trust for and about mathematics teaching as the primary motive object of activity pursued through their professional leadership. I claim that privileging the relational motive object enabled the mathematics leaders to work on another motive object: the development of teachers' mathematics teaching practices and mathematical content knowledge. I share evidence of a managerial undertaking that concerned the promotion of mathematics as a sustained school improvement area.

I move to a discussion about how by privileging that relational motive object, another rule surfaced within the School Mathematics Leaders' post-project activity system. I claim that a rule of relationality became one that mediated the mathematics leaders' activity, especially their relational motive object. Several new leadership actions that supported the achievement of their motive objects of activity are also explained. I contend that those leadership actions not only align with what is known about resourceful practice (Edwards, 2005; 2010a), but they are unique to the post-project leadership activity of the School Mathematics Leaders.

Towards the conclusion of this chapter, I describe the surfacing of a new contradiction that confronted the School Mathematics Leaders following participation in CTLM. I explain how they struggled to work on both the relational and development motive objects of activity when tensions in teachers' mathematics practices became known to them. That was especially heightened when issues of teachers' mathematical knowledge were exposed. I argue that that contradiction remained unresolved for the School Mathematics Leaders due to the privileging of their relational motive object, suggesting that that direction of their work simultaneously enabled and constrained their efforts in contributing to project sustainability. I end this chapter by claiming that by engaging in their form of resourceful practice (Edwards, 2005, 2012), the School Mathematics Leaders played a pivotal leadership role in contributing to project sustainability.

I turn now to a discussion about how the School Mathematics Leaders reconfigured the motive objects of activity.

7.2 Reconfiguration of Motive Objects

In the years following CTLM, when facing those post-project practice problems, the School Mathematics Leaders sought ways of acting creatively and resourcefully to contribute to project sustainability. They directed their professional learning leadership activity at three main motive objects, revealing again that the School Mathematics Leaders' activity was multi-motivational (Leont'ev, 1978). I use the terms *relational*, *developmental*, and *managerial* (Nuttall et al., 2018) to name those multiple motive objects and to describe them as the undertakings that contributed to project sustainability.

I interpret that that reconfiguration of motive objects occurred in response to the struggle that the School Mathematics Leaders experienced due to the practice problems they faced and the responsibilization they experienced. That response was imbued with a passion for mathematics education, care for the community members within their activity system, and appreciation for the historical work undertaken during CTLM. By reconfiguring their motive objects of activity, the School Mathematics Leaders brought to bear what mattered for them in their mathematics leadership work (Edwards & Thompson, 2013).

7.2.1 Building Relational Trust for and About Mathematics Teaching

Towards the end of CTLM participation, I reported that the School Mathematics Leaders shifted the direction of their professional learning leadership activity to motive objects that focused more on leadership. In doing so, I interpret that the School Mathematics Leaders worked on dimensions of relational trust (Edwards-Groves et al., 2016). I claimed that relational trust dimensions were developed as the mathematics leaders collaborated with their teachers in dialogical spaces, creating agreed-upon behaviours for effective mathematics professional learning as well as effective mathematics teaching practices through shared understanding development (Edwards-Groves & Grootenboer, 2021).

As their way of engaging in the work of project sustainability, the School Mathematics Leaders gave even further prominence to the relational dimension of their professional learning leadership activity. That greater attention to the relational entered their mathematics professional learning spaces in more specific and salient ways. With their experience of responsibilization (Nuttall et al., 2019; Shamir, 2008) for project sustainability and their response through care and creativity, the mathematics leaders recognised the primacy of relationships for and about mathematics teaching. In the face of the practice problems that confronted them, especially staff turnover, the diminished sector and principal leadership support, and the reduced opportunities for mathematics professional learning, developing relational trust for and about mathematics teaching became even more critical for the School Mathematics Leaders.

Through their experience of the other practice problem of diminished role clarity, realised through the multiplicity of roles (Higgins & Bonne, 2011), the School Mathematics Leaders experienced other forms of middle leadership in their schools. Although that practice problem diminished their mathematics leadership role clarity, they were afforded the opportunity to compare elements of those various roles. According to the School Mathematics Leaders, the relational dimension of mathematics leadership was recognised as imperative, holding an even more privileged space than the other leadership roles they were required to undertake. Building relational trust for and about mathematics teaching became paramount in their efforts to contribute to project sustainability.

Cindy exemplified that as she discussed the place of relationship within mathematics leadership, highlighting the importance of demonstrations of empathy:

> Relationship plays a big part in leadership, even more so with maths leadership. There is something about mathematics, trust and relationships that allows me to know how my teachers really feel about maths and their teaching of it, especially the teachers with maths anxiety (Cindy, PPI, 02.11.17).

Recognition of differences in teachers' affect within professional learning settings and demonstrating empathy for those differences is considered important work for the middle leader, understood as an element of interpersonal trust (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2021). For the School Mathematics Leaders, middle leadership of project sustainability was concerned with demonstrating empathy mediated by knowledge of teachers, specifically their peers' affective responses to mathematics education. That comment from Cindy highlights how interpersonal trust was enacted within the mathematics leaders' post-project leadership activity.

The following comment from Rachel further confirms my claim about the importance placed on relational trust building through the School Mathematics Leaders' leadership:

> The relational part of the role is really important with being a maths leader and working with teachers in maths, and I do think it's different in maths. You're working with people with different understanding of maths, so relationship plays more of a role (Rachel, PPI, 26.10.16).

That relational motive object they pursued meant that the School Mathematics Leaders worked on building different dimensions of relational trust with their teachers, specifically interpersonal trust (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2021). With the enactment of their leadership activity happening alongside teachers, they were also afforded opportunities to develop knowledge of their colleagues (Edwards-Groves et al., 2019), specifically, knowledge of teachers' dispositions for and about mathematics education.

Penny provided insights into the importance of demonstrating empathy, revealing again that building interpersonal trust (Edwards-Groves et al., 2016) was an essential aspect of this relational motive object:

> I think the way that you approach the content is what you need to be mindful of the teachers. And look, I'm by no means a mathematician, but I fear I sometimes feel that staff can already be a bit tense about the mathematical component. There's lots of research to indicate that. I am always respectful that my maths journey is not the same as theirs, and I have to show empathy and keep those relationships at the front of what I am doing (Penny, PPI, 25.03.15).

The School Mathematics Leaders shared insights into how their positionality as middle leaders enabled the space for them to work on relationships with their teaching peers. Working in proximity to the teaching and learning in classrooms (Bennett et al., 2003; Bryant et al., 2020;

Grice, 2019; Grootenboer, 2018), the School Mathematics Leaders were able to build relationships with teachers and do that through mathematics education.

One practice problem specifically gave reason for the surfacing of that relational motive object. Through internal and external transfer, staff turnover highlighted to the School Mathematics Leaders the need to build relationships with their peers for and about mathematics teaching. Building that relational trust enabled opportunities for the School Mathematics Leaders to work with teachers and generate knowledge about their professional learning needs.

The School Mathematics Leaders reported teachers' struggle in transferring the MCK and PCK from year level to year level. That, in turn, created a practice problem for the mathematics leaders. Their response was to direct their activity at the relational motive object, where trust building became central to their work.

Cindy shared the School Mathematics Leaders' experience with several teachers in their schools. As she recalled a story of working with a colleague who moved year levels, Cindy highlighted how relational trust building enabled access to teachers' professional needs:

> I feel for her because she's found it difficult to apply what she learned in CTLM since moving year levels. I have worked really hard on building that relationship with her so that she trusts me that I won't judge her and that she feels safe to share with me what she is struggling with. If the teachers don't trust you like that, they won't share with you what they are going through which means that I can't really help them in ways that they need (Cindy, PPI, 06.11.14).

Rachel shared further insight into the impact of staff turnover on the project sustainability work. She highlighted the influence of the external staff turnover practice problem:

> The teachers that haven't done CTLM tend to come from different points of view. I worry that those teachers who have not been through CTLM are often feel left behind in a planning meeting. I have to care about those people which I guess is why relationships

> are important so they can let me know that they are feeling overwhelmed without me judging them (Rachel, 25.11.15).

In the years following CTLM participation, the School Mathematics Leaders found themselves working more and more with teachers who had not participated in the project. They recognised that there was something significant about mathematics leadership and the vitality of building relationships with that curriculum area they led. As evidenced by their comments, the School Mathematics Leaders positioned the importance of relationships in their leadership activity. That appeared to be mediated by their knowledge of their teachers. Relational trust building realised through interpersonal trust as care for teachers through a non-judgemental approach became an essential undertaking of that relational motive object of activity (Nuttall et al., 2015).

There was a sense of care that the School Mathematics Leaders held for their teachers with whom they worked. The relational motive object of activity focused on relationship building with teachers, which enabled opportunities to generate knowledge of their teachers' dispositions and their professional learning needs.

Penny highlighted this relationship building through mathematics education as a realisation of that relational motive object:

> I am always thinking about them to be able to do the best that they can, so they'll all be great practitioners. I'm trying to build that relationship with them so that it allows them to see that they can be great maths teachers. I guess you can say that I am building relationships through maths. Relationships with them, and relationships with maths. Better people through maths; all of us better together through maths (Penny, PPI, 25.03.15).

This finding confirms the importance of relational trust within the developmental work of middle leaders (Edwards-Groves et al., 2016), and it especially highlighted the vitality of the relational dimension of the mathematics leadership activity enacted by School Mathematics Leaders. Those findings add further knowledge about the central place of relational trust in middle leading activity (Cranston, 2009; Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2021; Lipscombe et al., 2020), suggesting that it is realised having an even more important place in the middle leading activity of mathematics leadership.

This finding also adds further information to mathematics education literature about how the mathematics leader enacts the role of relationship builder (e.g., Bolyard & Baker, 2021; Corbin et al., 2003; Driscoll, 2022; Higgins et al., 2007). For the School Mathematics Leaders, relationship building for and about mathematics teaching held a privileged space within their leadership, as a clear motive object of activity (Kaptelinin, 2005; Leont'ev, 1978).

7.2.2 Developing Teachers' Mathematical Knowledge for Teaching

With the School Mathematics Leaders focusing their activity on building relational trust for and about mathematics teaching, they created conditions that enabled them to work on another motive object of activity. I use the term *developmental* for this next motive object because of its focus on developing teachers' knowledge for mathematics teaching (Ball et al., 2008) and practices associated with its enactment in classrooms.

Recognising that leadership of teacher professional learning is the core work of middle leading (e.g., De Nobile, 2018; Edwards-Groves et al., 2019) and that as the mathematics leader, they are the source of teachers' professional development in school settings (Driscoll, 2017; Millett & Johnson, 2004; Sexton & Downton, 2014), it is not surprising that a developmental motive object surfaced as one within the School Mathematics Leaders' professional learning leadership activity. However, the articulations from the mathematics leaders provide insights into the what and the why (Kaptelinin, 2005) of that developmental motive object.

The School Mathematics Leaders claimed that they worked on developing PCK, as part of the Ball et al. (2008) mathematical knowledge for teaching, to support teachers in their work of impacting the students' experiences of mathematics in classrooms. That development of teacher knowledge for mathematics teaching was mediated by the shared understandings that were developed at the end of CTLM. Those collective commitments (Rose & Norwich, 2014) mediated that work, with the School Mathematics Leaders using that shared understanding as an important mathematics leadership tool that mediated that developmental motive object.

Evidence of the School Mathematics Leaders' use of their collective commitments as a mediating tool on that developmental motive object was provided by Rachel:

> I work on helping the teachers see that there are ways to create good mathematical experiences and lessons for the students and that they need to do that as a team. It is important for developing those shared understandings of practice as a team. The other great thing is that it is aligned with what we believe as a school. We need to continue that vision that we set up in CTLM (Rachel, PPI, 25.11.15).

Cindy also highlighted how the historical tool of the collective commitment, which she also called a "vision" like Rachel, was used in the School Mathematics Leaders' activity as a mediating tool of project sustainability:

> I keep that focus on going back over CTLM ideas, really focusing on helping the teachers understand what a good teacher of mathematics looks like, the key components of a maths lesson, and how to differentiate tasks for the kids. I make that part of our shared thinking in facilitated planning. I focus on that vision for teaching maths with the teachers (Cindy, PPI, 06.12.16).

The School Mathematics Leaders, when discussing the content of their developmental work about improving teachers' mathematical knowledge for teaching (Ball et al., 2008), made frequent references to PCK development. The comments from Rachel and Cindy provided insights into that PCK focus with references to lesson structures, planning, and differentiation of mathematics teaching. Those are aspects of knowledge captured by Ball et al. as elements within the PCK domain of mathematical knowledge for teaching.

The School Mathematics Leaders' references to developing teachers' knowledge of differentiation featured considerably in their discussions as well as in the observed enactment of their leadership during facilitated planning meetings (that routine endured beyond CTLM, and I elaborate further on that later in this chapter). Differentiation focused on mathematical task adjustments where the School Mathematics Leaders used their own knowledge of differentiation strategies as a mediating tool for that developmental motive object.

That was explicitly highlighted by Penny when she was asked what she mainly focused on when developing teacher knowledge for teaching since the time of CTLM finishing:

> I work a lot on differentiation with the teachers like how to individualise and differentiate the activities and tasks, using what I know about enabling and extending prompts. Differentiation is a big issue for the teachers, so I tend to focus on that quite a bit (Penny, PPI, 02.12.16).

Developing teachers' knowledge of ways to differentiate tasks to meet diverse students' learning needs was worked on by the School Mathematics Leaders because they claimed that this was an area of teacher learning that required attention. Literature within the field suggests that that area of PCK is challenging for teachers' mathematics teaching (e.g., Russo et al., 2021). The School Mathematics Leaders reported that their own knowledge of enabling and extending prompts (Sullivan et al., 2009), which they were exposed to during CTLM, mediated their work on supporting the development of their teachers' PCK related to mathematics teaching differentiation.

Another prominent aspect of teacher knowledge that the School Mathematics Leaders worked on through the developmental motive object of activity was what I have interpreted as *task selection* and *task implementation*. Task selection and task implementation would again be considered aspects of PCK, categorised as *knowledge of content and teaching* (KCT) by Ball et al. (2008). Task selection was associated with knowledge of mathematical tasks, activities, and games that have a specific mathematical focus and learning intention. Task implementation concerned knowledge that the School Mathematics Leaders used to support their teachers' understanding of effectively employing tasks in classrooms with students, using appropriate teaching strategies, representations, and assessment approaches. Differentiation strategies that adjusted task difficulty to meet the range of student learning needs would be considered an aspect of task implementation.

That developmental motive object was focused on building the teachers' awareness of specific tasks associated with mathematics content areas and then supporting their understanding of how to implement the tasks in mathematics lessons. The mathematics leaders' work on the developmental motive object was mediated by their own knowledge of effective tasks and the pedagogical approaches and strategies that support the successful use of those tasks when teaching students.

Rachel provided insights into motive object, discussing why she was observed sharing specific tasks and their implementation in a facilitated planning meeting with her Grade 2 teachers:

> I spend a bit of time helping the teachers become aware of good tasks to use when teaching. I think they need to have a set of tasks and activities that they know are good in helping the kids learn the maths. But there's more to it than just choosing tasks, so I also help them know how to use the task or the activity in the maths lesson. I mean, you can

> have a great task but if you can't use it properly, it doesn't stay a great task (Rachel, PPI, 29.04.15).

The School Mathematics Leaders not only developed their teachers' knowledge of task selection but also focused on their implementation. Other aspects of task implementation worked on as the mathematics leaders pursued that developmental motive object of activity included developing teachers' use of appropriate representations for mathematical ideas ("We do a lot of empty number line work as I know that they are a really helpful model" Rachel, PPI, 24.10.16), understanding of learning trajectories in specific domains of mathematics ("It is important that we all know how students learn place value best and the stages that they go through when understanding place value" Penny, PPI, 16.05.18), common misconceptions that students can form in those areas ("I spend time on misconceptions with the teachers so that they are aware of them when they teach mathematics" Rachel, PPI, 08.11.16), and knowing how to use curriculum documentation to inform teaching and assessment ("I spend time on understanding the curriculum and helping teachers see how tasks match content descriptors, and how they match the big ideas of maths in the curriculum" Cindy, PPI, 06.12.16).

The School Mathematics Leaders claimed that the focus of their work on that developmental motive object was informed by the professional learning needs of their teachers. With their positionality as middle leaders being close to teachers and their work (Edwards-Groves et al., 2016; Grice, 2019), they had insights into the area of professional needs concerning mathematics teaching. By directing their activity at the relational motive object, investing time in building relationships for and about mathematics teaching, the mathematics leaders were afforded further opportunities to generate those insights into teachers' needs. That knowledge of teachers' professional learning needs became an important mathematics leadership tool, with that knowledge acting as a mediating cultural tool (Vygotsky, 1978).

Using knowledge of teachers' professional learning needs was shared by Penny:

> I see my role as taking the time to address the learning needs of the teachers. I know their needs and what they need to learn to be better teachers of maths. That gives me focus in my leadership role (Penny, PPI, 25.03.15).

Of particular interest was the content of the School Mathematics Leaders' discussions about the developmental motive object of activity. Their attention was on developing teachers' practical knowledge for teaching mathematics (e.g., PCK), with very few mentions of the development of teachers' MCK.

When asked about the reason for the focus on PCK, Penny shared the following:

> Most times, I work on their pedagogical knowledge. They (teachers) always ask more about the pedagogy than content knowledge. I definitely use more of what I know about pedagogy than what I know about maths content because I work more on the teaching maths in my professional learning (Penny, PPI, 16.05.18).

Cindy, however, provided a further reason for the developmental emphasis on PCK. With the importance that they placed on relational trust building, the development of PCK was perceived as being safer to work on in comparison to MCK:

> It is easier to work on a teacher's pedagogy than their content knowledge. It's less threatening. I am just trying to build that relationship because that trust has been really important. It is so easy to lose that relationship if you focus too much on (mathematical) content knowledge (Cindy, PPI, 06.11.14)

Recognising the mathematics leader as the primary source of professional development for classroom teachers (Millett & Johnson, 2004) and that the work of teachers is to teach, it would seem appropriate for PCK to be the focus of the developmental motive object. However, if the content of mathematics professional learning needs to address both PCK and MCK (Darling-

Hammond et al., 2009; Garet et al., 2001; Koellner et al., 2011; Timperley, 2008), it appears that the School Mathematics Leaders neglected somewhat the place of MCK within the developmental motive object of activity.

Later in this chapter, I share that the hesitancy to work on teachers' MCK surfaced a contradiction for the School Mathematics Leaders. I explain that contradiction and provide reasons for its surfacing, partly due to their privileging of the relational motive object of activity pursued through their professional learning leadership for project sustainability.

7.2.3 Promoting the Profile of Mathematics

The final motive object that formed part of the School Mathematics Leaders' reconfiguration of motive objects concerned the promotion of mathematics within the school improvement agenda. With the contradiction of the diminished priority of mathematics having resurfaced, and with the care and passion that they held for mathematics education, they directed activity towards promoting the status of mathematics. That was worked on by managing the profile of mathematics in their school communities.

That motive object involved undertakings that sought to manage and persuade principals to maintain mathematics as a prominent place within the school improvement agenda. It also extended to ways that they convinced their teachers of the importance of continuing with project sustainability.

Evidence of that managerial motive object as part of the School Mathematics Leaders' post-project activity was shared by Penny. At the time when she shared that comment, Penny had been managing the place of mathematics due to competition from other curriculum focus areas:

> I think I give maths the status since CTLM finished. This year, the focus has been the RE (religious education) curriculum renewal. I know part of my maths leader role is to keep managing maths, looking for ways for maths to have the status it had in CTLM and what it still needs. I keep cheering on maths (laughs) (Penny, PPI, 02.12.16).

The focus of that managerial motive object, as realised through work that saw the promotion of mathematics as a curriculum area, was also highlighted by Cindy. As evidenced by the date of the comment, the further the time from the conclusion of CTLM participation, the more the School Mathematics Leaders were required to engage in that managerial motive object:

> I said to the principal, "I understand there's other things that have had priority, but when is mathematics going to get a guernsey again?" I see it as part of my role to fly the 'maths flag' and keep maths on the agenda (Cindy, PPI, 06.12.16).

The School Mathematics Leaders' promotion of the mathematics profile did not feature with the same frequency as the relational and developmental motive objects. References to managing the profile of mathematics were made, mainly when the School Mathematics Leaders discussed the struggle that they encountered with their contributions to project sustainability. Rachel provided evidence for that when she shared: "You know, sometimes, the maths role is about cheering on maths and making sure it's not forgotten. Sometimes, I have to manage the principal with reminders, so it (mathematics) stays on the agenda" (Rachel, PPI, 14.11.16).

I interpret the surfacing of that managerial motive object as a response to the School Mathematics Leaders' experiences of responsibilization for project sustainability. They realised that the promotion of the mathematics curriculum meant they had to manage its profile and engage in work that managed their principals.

I now discuss that the reconfiguration of motive objects, that privileged relational trust building for and about mathematics, mediated the surfacing of a new rule within the School Mathematics Leaders' activity system.

7.3 Surfacing of the Rule of Relationality

With the reconfiguration of motive objects of activity and understanding the importance of relational trust within their middle leading (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2021), the School Mathematics Leaders prioritised the relational motive object of activity. By privileging relational trust building for and about mathematics, a new rule surfaced within their activity system. That new rule was one focused on keeping everyone safe around mathematics education.

As a way of mediating relational trust building, an implicit rule came forth that enabled the School Mathematics Leaders to enact the care that motivated their response to their experience of responsibilization for project sustainability. A rule of relationality became an essential mediator of their leadership activity. That rule influenced how the School Mathematics Leaders interacted with their teachers.

As I have claimed thus far, the primacy of relationships with colleagues became central to the School Mathematics Leaders. Cindy captured the importance of collegial relationships, highlighting the presence of that relationality rule: “I always make sure that people are okay, and we have trust before I push. I have to make sure we are confident in our relationships” (Cindy, PPI, 02.11.17).

Rachel also provided evidence of the existence of that relationality rule and the vitality of relationships within their professional learning leadership after CTLM. That became especially important when it came to teachers’ misconceptions, again revealing the fragility that the mathematics leaders held for the development of teachers’ MCK:

> If there is a teacher's misconception, you have got to tread carefully because you can't make anyone feel bad. You can't risk it if you do not handle it properly because there goes the relationship and probably the way that person feels about maths (Rachel, PPI, 29.04.15).

Penny reiterated the importance of the relationships in response to the practice problem of the reduced formal professional learning meetings, capturing the care for mathematics that the School Mathematics Leaders had for mathematics as a curriculum area:

> They say often, "Penny, can you please come help with this?" or "Have you got a minute?" So, of course, I'm always going to be help and be there. I care, so I make myself available especially because we don't have the meetings like we used to have. I care about them (teachers), and I care about the students' maths learning. You have to care about maths, too, and use that to help your leadership (Penny, PPI, 25.03.15)

I have interpreted that the rule of relationality, "*We keep everyone safe around mathematics*", became an implicit mediational element of the School Mathematics Leaders' professional learning leadership. I believe that the mathematics leaders were not cognisant of that rule, yet their leadership activity was influenced by it as they worked in their post-project activity system. That rule surfaced from their care for teachers and the relational motive object they pursued through their leadership activity.

In that way, the School Mathematics Leaders developed relational trust that enabled the conditions that they saw as crucial for contributing to project sustainability through their professional learning leadership (Edwards-Groves et al., 2016). The rule of relationality mediated their work with project sustainability, but as it would turn out, that rule also constrained achievement of the developmental motive object of their activity. I discuss that later in this chapter.

I now turn attention to the leadership actions the School Mathematics Leaders enacted as they directed their activity towards their reconfigured post-project motive objects.

7.4 Post-Project Leadership Actions

In the previous sections, I described how the School Mathematics Leaders sought to resolve the contradiction of the diminished status of mathematics, and its manifestation as a critical conflict (Engeström & Sannino, 2011) realised through its practice problems. I claimed that they did this by reconfiguring their motive objects where they privileged the relational motive of their professional learning leadership activity. I reasoned that a rule of relationality surfaced to mediate their undertakings concerned with that prioritised motive object.

I focus now on leadership actions using the idea of Leont'ev (1978), who noted the hierarchical levels of activity and how activity comprises of actions. I present and explain five salient leadership actions that the School Mathematics Leaders enacted as they worked towards achieving their reconfigured motive objects of activity. I share my understanding of their leadership actions as ways of working resourcefully, further characterising their contribution to project sustainability.

7.4.1 Influencing Principals to Maintain Facilitated Planning Meetings

The first leadership action that the School Mathematics Leaders enacted concerned influencing their principals to keep the facilitated planning meetings as a routine within their schools. That action was a way of pursuing the managerial motive object of activity concerning the promotion of mathematics, creating a space for mathematics to continue as a school improvement agenda item. That is an example of the School Mathematics Leaders acting resourcefully by accessing distributed expertise (that of their principal) and recognising their authority within the division of labour of their activity system (Cole & Engeström, 1993). By managing and influencing their principals, they opened spaces to engage in undertakings concerned with the developmental motive object of their post-project leadership activity.

The mathematics leaders often mentioned knowing that change processes take time. They saw the facilitated planning meetings as a means of continuing to work on that change that was initiated through CTLM participation. They situated those facilitated planning meetings as cooperative and dialogical spaces (Edwards-Groves & Grootenboer, 2021) for and about mathematics teaching. However, as middle leaders aware of their positionality and their limited formal authority (Ainsworth et al., 2022; Bennett et al., 2007; De Nobile, 2019), they understood that they required principal agreement to maintain those meetings as routines in their schools.

Rachel exemplified that knowledge about change processes that the School Mathematics Leaders claimed they possessed. She also shared how influencing the principal meant keeping them informed of the importance of those facilitated planning meetings:

> But of course, you've got to keep the planning meetings going for the changes we started to become part of the common practice and shared practice. I keep the principal informed about how the planning meetings are important because change takes time, and I want them to continue (Rachel, PPI, 29.04.15).

An aspect of influencing principals included engaging in convincing practices, where the School Mathematics Leaders sought to persuade the principal to maintain the facilitated planning meetings introduced during CTLM. That was one of those creative leadership actions that supported the mathematics leaders to focus on what mattered for them in their leadership activity (Edwards & Thompson, 2013).

That was evidenced by Penny, who spoke about why persuading the principal was crucial in opening up spaces for her to work with the teachers. She shared how it was also a time to monitor teachers' professional learning, as well as teachers' efforts in sustaining the mathematics teaching reforms:

> I spend time sharing, especially with the principal, that we should keep the facilitated planning meetings here. I tell him that we should keep them, so I have the ability to go into planning and to really be working with the teachers, looking at their planners and how they are going. I can see how they are going with the things we learned in CTLM (Penny, PPI, 02.12.16).

That leadership action of influencing principals to maintain the facilitated planning meetings was a way of working resourcefully. The School Mathematics Leaders looked for ways to align those meetings with the project commitment rules that the principals established at the end of CTLM. By doing so, they again demonstrated understanding of their positionality but acted resourcefully to keep the planning meetings as routines.

Understanding their position "within the middle" (Edwards-Groves et al., 2016; Lipscombe et al., 2020), the mathematics leaders recognised the need to work vertically (Bryant et al., 2020), persuading their principals to agree to the maintenance of the facilitated planning meetings beyond CTLM. In doing so, they enabled conditions and spaces for them to practise their leadership (Lipscombe et al., 2020) and created opportunities to work with their teachers.

The School Mathematics Leaders, however, extended their resourceful practice with those facilitated planning meetings. They knew the importance of the accountability aspect associated with teachers' planning. However, when they secured them as a routine in their schools with their principals' approval, the School Mathematics Leaders resourcefully co-opted them as space for them to practise their professional learning leadership.

7.4.2 Co-opting Facilitated Planning Meetings as Professional Learning Opportunities

With the School Mathematics Leaders influencing principals to maintain the facilitated planning meetings, they created space for them to work resourcefully by enacting another leadership action. As a way of resolving the practice problem of reduced mathematics professional learning opportunities, the mathematics leaders co-opted the facilitated planning meetings as spaces to lead teachers' professional learning. Interestingly, the facilitated planning meetings were the spaces in which I observed the School Mathematics Leaders enacting their post-project professional learning leadership with every school visit, highlighting the prevalence of that leadership action in their work associated with project sustainability.

Supporting teachers in planning meetings has been documented as an aspect of mathematics leadership work in primary schools (e.g., Cheeseman & Clarke, 2006; Driscoll, 2017; Vale et al., 2021). In the case of the School Mathematics Leaders, however, they resourcefully repositioned those facilitated planning meetings by attributing new meaning to them.

The School Mathematics Leaders repositioned the facilitated planning meetings to create the time and space that was constrained by the post-project rule of reduced mathematics professional learning opportunities. The mathematics leaders understood that those meetings were provided by their school principals to support teachers in meeting accountability and compliance measures required for school program documentation. The School Mathematics Leaders, however, co-opted the meetings as opportunities to pursue the developmental motive object of activity, focused on developing teachers' mathematical knowledge for teaching (Ball et al., 2008).

The School Mathematics Leaders worked resourcefully by shifting the meaning of those meetings. By co-opting the facilitated planning meetings as professional learning opportunities, they shifted the rules associated with those meetings. That bending of the rules (Edwards, 2010a) meant that the School Mathematics Leaders not only used the facilitated planning meeting to observe the accountability rules associated with teachers' planning responsibilities, but they repurposed those meetings as professional learning opportunities.

By enacting that leadership action, the School Mathematics Leaders created the space to work on the interactional and intersubjective dimensions of relational trust (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2021). Rachel demonstrated this by using the facilitated planning meeting as spaces that enabled cooperation and collaboration (interactional trust) whilst communicating a sense of togetherness (intersubjective trust) as she sought to develop her teachers' understanding of the newly introduced *Victorian Curriculum: Mathematics* documentation (VCAA, 2016):

> I think this (facilitated planning) has really supported the teachers and their learning, especially for the teachers who didn't do CTLM or are struggling to use what they learnt in the new year levels. Rather than say, "Okay, here's the maths curriculum, off you go," it is so much better to say, "Let's talk about what's in the curriculum together." The facilitated planning has been good for me to work on that teacher knowledge as well as help them with their planning, of course (Rachel, PPI, 08.11.16).

Co-option of the facilitated planning meetings as professional learning opportunities created the space to also work on the developmental motive object of activity by developing teachers' understanding of shared practice. Penny highlighted that through the following:

> Since CTLM, the facilitated planning meetings are not only for teachers to plan, but it's my way of also leading professional development for the teachers to keep that shared understanding of how we teach maths here at our school. I also do it because we have lost a fair few teachers who did CTLM and have gone onto other schools. So, the planning

> meetings are PD for them, too. It's also a way that we can keep CTLM going (Penny, PPI, 25.03.15).

Evidence of rule-bending and how that was enacted to negotiate the practice problem of reduced formal mathematics professional learning opportunities was also exemplified by Cindy. She revealed co-option of the facilitated planning meetings in two ways: working on teachers' professional learning and sustaining mathematics teaching reforms.

Cindy shared:

> This is the reason why, since CTLM stopped, I use the facilitated planning meetings as PD with teachers. I mean, we don't have the PD in maths like we used to in the 'CTLM days', so the planning meetings are a way for me to get around that and so that we can keep going on with what we started in CTLM (Cindy, PPI, 06.11.14).

I interpret that leadership action of co-opting facilitated planning meetings through repurposing and attributing new meanings to those meetings as resourcefully and creatively responding to the critical conflict they faced through the practice problems of reduced formal professional learning opportunities and staff turnover. By co-opting the facilitated planning meetings, the School Mathematics Leaders opened up possibilities (Edwards et al., 2010) to work on the relational and developmental motive objects activity constrained by those practice problems. That leadership action is evidence that the School Mathematics Leaders engaged in resourceful practice as they enacted rule-bending as a form of professional decision-making to achieve the motive objects of activity (Edwards, 2010).

The co-option of facilitated planning meetings, as a leadership action of the School Mathematics Leaders' resourceful practice, facilitated another action that contributed to project sustainability. I now turn to how the School Mathematics Leaders repurposed resources introduced to them during CTLM.

7.4.3 Repurposing CTLM Resources as Sustainability Tools

The following leadership action that realised the School Mathematics Leaders' contribution to project sustainability was repurposing and attributing new meanings for cultural tools within their professional learning leadership activity system. Having attributed new meaning to the facilitated planning meetings by co-opting them as professional learning opportunities, they assigned new purposes for the resources they were introduced to during CTLM. Those resources included mathematics tasks and activities for classroom teaching, provided by the project team during CTLM, as well as units of work (planning documentation) initially created in the facilitated planning meetings during CTLM participation.

That repurposing of resources saw the School Mathematics Leaders use them as mathematics leadership tools for multiple purposes: to work on the development motive object by influencing teachers' practices and knowledge for teaching, and to highlight the importance of sustaining the CTLM-initiated mathematics teaching reforms with their teachers. I interpret that as the School Mathematics Leaders using those CTLM resources as project sustainability tools.

Evidence of that claim about repurposing CTLM tasks as tools for project sustainability was exemplified by Penny. She highlighted how the School Mathematics Leaders attributed multiple purposes for the tasks beyond their original use during CTLM participation:

> The tasks we learned in CTLM are excellent. I still get teachers planning with them and thinking about how to use them in their teaching. Actually, by using those tasks from CTLM, we are keeping CTLM going here and what we started in CTLM keeps going. It's important that those who went through CTLM use them so that teachers who didn't do CTLM get to know about them, and they use them in their teaching, too (Penny, PPI, 02.12.16).

The following data, an excerpt from an observation record, supports the leadership action of repurposing CTLM tasks. The data were generated as Cindy led a facilitated planning meeting that focused on setting goals for students' mathematics learning for the following year. A discussion about concerns with students' fluency took place. After one teacher said that they were not aware of fluency tasks that went beyond worksheet-based activities, Cindy responded in the following way:

> Cindy shares with the teachers a task she likes to use that develops students' basic fact knowledge. She tells the teachers that this was a CTLM activity and one that everyone could use. Cindy says that she uses it as an open-ended task where multiple responses are encouraged. She gives an example of using the task where the students are told that the answer to a basic fact is 12, and then students write expressions with the result of 12.
>
> Cindy says that she calls the task “the one-minute fact challenge” but adds it was not called that in CTLM. She then describes how the activity is used, where students have one minute to record as many expressions as possible that match the nominated result. Cindy then adds that it is important that there is a discussion of student responses after the minute has passed.
>
> Cindy asks one of the teachers if she remembers the task from CTLM. The teacher says that she cannot remember. Cindy says, "Okay. Well, it is a good fluency task from CTLM.” She adds that for teachers who did not do CTLM then this is an activity they can use. Cindy then says, "It would be good if we all used this so we can keep some of the ideas were learnt in CTLM going into next year."
>
> (Cindy, PPO, 06.12.16)

Those data highlight how the School Mathematics Leaders used CTLM tasks with several purposes following project participation. They used the tasks as tools that not only mediated their developmental motive object of activity, that is, improving teachers' knowledge of mathematics tasks and supporting planning decisions, but they used them in ways of sustaining aspects of the CTLM-initiated teaching reforms. For the School Mathematics Leaders, by repurposing the CTLM tasks, they understood the historical significance of them, and the potential they had to act as tools of project sustainability.

The mathematics leaders claimed that during CTLM, they created units of work to help teachers implement the teaching reforms as highlighted in the project. The School Mathematics

Leaders and teachers planned those units of work using knowledge of tasks, practices, and principles as highlighted in the CTLM professional development sessions. As part of the planning routine at the time of CTLM participation, the School Mathematics Leader encouraged teachers to save the planning documentation on the schools' intranet servers. In the years that followed participation in the CTLM project, the mathematics leaders claimed that planning documentation was adjusted as teachers revisited mathematics topics each year.

Just as they did with the CTLM tasks repositioning them as project sustainability tools, the School Mathematics Leaders attributed new meaning to the units of work (planning documentation). They worked creatively again by reusing that planning documentation and adjusting the content when necessary to address the learning needs of the different cohorts of students each year. As they did that work, the mathematics leaders influenced teachers' PCK as the documentation articulated information about task selection and implementation.

Rachel provided an example of that repurposing as she led a facilitated planning meeting with her Year 2 teachers. The focus of Rachel's developmental work was supporting the teachers' planning for geometry teaching with attention on isometric transformations. The planning meeting took place in one of the teacher's classrooms, and Rachel used the interactive whiteboard (IWB) that displayed a unit of work dated 2012.

Discussions about tasks to use in the mathematics lessons occurred as Rachel drew the teachers' attention to the tasks within the 2012 planning documentation. Rachel reminded the teachers of their planning work during CTLM and reiterated that the planning documentation would help sustain teaching reforms:

> Rachel then asks as she points to the IWB, "What tasks do you think the children can do in the unit of work using this planner to work from?" Two teachers start to refer to some activities that they have used in the past. Rachel says that those tasks were ones that everyone learnt about during CTLM and that they are "good" tasks to use. The two teachers who offered the tasks say that they agree.
>
> Rachel then says that she wants to "map out" the tasks for the teachers to use with their students. Rachel says she knows of another task regarding the use of pattern blocks where the students perform transformations of designs with the blocks. Students are then expected to describe the transformations using "slide, "turn", and "flip" as terms.
>
> Rachel says that task was in the unit of work as she asks a teacher to scroll down. She asks if the teachers remember that task from CTLM. There is some agreement with one teacher saying that she cannot remember. Rachel adds, "We did a lot of work on these planners in CTLM, so it would be good to use them and keep the good things we learned in CTLM going."
>
> (Rachel, PPO, 19.11.15)

Those data examples provide insight into how the CTLM resources (tasks and planning documentation) became enduring cultural tools used by the School Mathematics Leaders in their professional learning leadership. Those cultural tools had become embedded as part of the mathematics planning culture in their schools, like how they had resourcefully attributed new meaning to the facilitated planning meetings. They found new purposes for those tasks and planning documentation.

Within a CHAT lens, I interpret that those resources, as cultural tools, travelled with the School Mathematics Leaders into their post-project professional learning leadership. However, they moved with shifts in meaning. There is evidence that suggests that as a way of sustaining the CTLM-initiated teaching reforms, the School Mathematics Leaders adapted those cultural tools (Nuttall & Brennan, 2016; Vygotsky, 1978) in creative ways that mediated both their developmental motive object (i.e., influencing teachers' practices and knowledge for mathematics through co-planning of units of work) and their contribution to project sustainability. That was achieved through tool adaptation by attributing new meaning and purpose for those CTLM tasks and the units of work (planning documentation) as tools of project sustainability.

That adaptation of tools by attributing new meaning and purpose to cultural tools used in facilitated planning meetings did not only happen with the CTLM resources. The School Mathematics Leaders extended their resourceful practice to include new meaning for student assessment data. I now turn to a description of how they used student assessment data sources to persuade teachers to continue with the mathematics teaching reforms initiated during CTLM.

7.4.4 Using Student Assessment Data as a Convincing Tool

Another leadership action I interpret as working resourcefully to sustain mathematics teaching reforms concerned the School Mathematics Leaders' repurposing of student assessment data. During CTLM, the mathematics leaders and teachers learned to use assessment data to inform mathematics planning decisions. That continued as a routine in the School Mathematics Leaders' activity system, but as a way of contributing to project sustainability, they attributed new meaning to student assessment data. They used the data as a mathematics leadership tool to convince teachers to sustain the mathematics teaching reforms in their schools.

When working in the facilitated planning meetings, the School Mathematics Leaders were observed using student assessment data in ways that went beyond informing discussions about student learning needs and supporting planning decisions. The data were used to convince teachers of the need to continue with the reforms and motivate them to use the mathematics tasks, and the teaching practices focused on in the facilitated planning meetings.

The following observation record excerpt details Penny's enactment of that leadership action. Penny was working with the Grade 1 teachers in a facilitated planning meeting, and they had just discussed initial analyses of data generated using an assessment interview:

> Penny says that she is surprised by some of the data from the assessment interviews that the teachers used with a selection of students in their grades.
>
> She asks the teachers what they think of the data. A teacher mentions that she is also surprised, adding that some students did not perform as expected. Penny nods and says that is the reason for her surprise, too.
>
> Penny then says, "I think this data is telling us that we need to keep going with what we learned in CTLM. We need to make sure that we are helping the kids as much as possible. Don't you agree?"
>
> (Penny, PPO, 02.12.16)

In the follow-up interview, Penny justified her actions in the planning meeting, revealing the meaning she attributed to the cultural tool of student assessment data: "I have kept the teachers using their assessment data to inform their planning, but now I use it to show the teachers that we have to keep going with CTLM" (Penny, PPI, 02.12.16). I interpret this as how the School Mathematics Leaders worked resourcefully using student assessment data as a convincing tool.

It was not only school-based assessment data that was an adapted cultural tool used by the School Mathematics Leaders. They attributed new meaning to the NAPLAN data, using it again to convince teachers about the importance of project sustainability. Rachel shared her use of NAPLAN as a way of providing feedback to teachers, as well as using the data to convince them to continue with the CTLM-initiated reforms in their classroom teaching:

> It's good to use the NAPLAN data with teachers to show them that by continuing with what we started with CTLM, we have kept going, and we saw improvements in the

> NAPLAN data. That's why we have to keep going, too. The NAPLAN data is good for that (Rachel, PPI, 19.11.15).

Attributing multiple meanings to student assessment data was another way for the School Mathematics Leaders to work resourcefully as they contributed to project sustainability. That was realised by using the data as a convincing tool, persuading teachers to continue with the mathematics teaching reforms in classrooms.

7.4.5 Seeking Support from External Mathematics Educators

The final collective leadership action that the School Mathematics Leaders enacted through their post-project professional learning leadership concerned seeking support outside their own activity system. With the practice problem of the reduced sector and principal leadership support, the mathematics leaders looked to build relationships with and seek support from mathematics educators outside their school sites.

The School Mathematics Leaders revealed the importance of having access to external support from people who worked in mathematics education. They claimed they valued the support and advice from others who understood the work of leading mathematics. By seeking support outside of their school sites, they sought knowledge from external sources to affirm their work focused on project sustainability. Rachel evidenced that when she shared: "You need to know you can have someone to bounce those ideas off. It is like reassurance" (Rachel, PPI, 25.11.15).

Cindy emphasised that as she explained the need for the School Mathematics Leaders to access others' knowledge, confirming the need for mathematics leadership advice:

> Having an outside person from the school who is into maths is so helpful for me as the maths leader. That person acts as a 'sounding board' because I know I cannot access the

> CEOM staff, and the principal support has really dropped off with maths. You need that outside person who 'gets it' for advice on ways to continue what we started (Cindy, PPI, 23.10.18).

I interpret that action of seeking support outside of the activity system as one that the School Mathematics Leaders used to not only resolve that practice problem of reduced leadership support but also to alleviate the effect of isolation and self-doubt they experienced. The mathematics leaders realised that with the practice problem of withdrawn leadership support, they had to look outside their school settings for advice.

That emotionally motivated leadership action, focusing on building knowledge of others outside of their schools who had the potential to offer support, contributes further evidence of the affective and relational nature of the School Mathematics Leaders' professional learning leadership activity.

I now move the discussion to my interpretation of the School Mathematics Leaders' contribution to project sustainability as a form of resourceful practice (Edwards, 2005; 2010a).

7.5 Resourceful Practice That Contributed to Project Sustainability

I have presented how the School Mathematics Leaders, in their struggle to negotiate the practice problems faced in the years following CTLM, contributed to project sustainability. Drawing on evidence from their descriptions and observations of activity, I have reason to believe their contribution was a form of resourceful practice (Edwards, 2005; 2010a). As their contribution to project sustainability, they worked creatively and resourcefully to solve the complex practice problems they faced. In doing so, the School Mathematics Leaders developed, adapted, and integrated the CTLM project's intent and content in response to the shifting environment in which they practised their leadership activity (Coburn et al., 2012; Tirosh et al., 2015; Zehetmeier, 2014). Through their resourceful practice, the School Mathematics Leaders contributed to a durable continuation of the mathematics teaching reforms in their school sites (Zehetmeier & Krainer, 2011).

In this section, I use evidence of the mathematics leaders' post-project activity to explain how the School Mathematics Leaders contributed to project sustainability by enacting a form of resourceful practice (Edwards, 2005; 2010a).

7.5.1 Focusing on What Matters by Reconfiguring Motive Objects

Edwards and Thompson (2013) offered the concept of *resourceful leadership* to understand the creativity of organisational leadership within and across activity systems. An element of that creativity concerns identifying what matters when leaders face contradictions within their activity system. That process requires a reset of motive objects focused on resolving tensions that are brought to the attention of the subject (in my case, the School Mathematics Leaders). Creativity is realised through the objectification of the “what matters'” as newly valued motive objects surface to work on contradiction resolution. Those newly created motive objects are usually imbued with affect and relational dimensions.

After participation in CTLM, the School Mathematics Leaders experienced responsibilization (Nuttall et al., 2019) for project sustainability. As I have claimed, their response was an approach imbued with care and creativity. Drawing on the evidence I have presented about their post-project leadership activity, the School Mathematics Leaders engaged in processes of identifying what mattered to them, their mathematics leadership, and the sustainability of the CTLM project in their schools.

Even though commitment rules were established to support sustaining the mathematics teaching reforms, they did not mediate that activity. Instead, those rules resurfaced the historically accumulated contradiction that had faded in prominence during CTLM participation. I believe that the School Mathematics Leaders, through their care for mathematics education, teachers and students, and the work achieved through project participation, reconfigured their motive objects of activity to focus on what mattered for them (Edwards & Thompson, 2013). They acted resourcefully by refocusing their activity on what they cared about within their professional learning leadership.

What mattered for the School Mathematics Leaders was objectified by resetting their motive objects, creatively shifting from those motive object pursued during CTLM. That was due to the changed conditions they faced following CTLM participation and their struggle with those post-project practice problems. I have highlighted in this chapter that a focus on the relational dimension of their leadership, by building relational trust for and about mathematics teaching, formed a significant aspect of that reconfiguration, creating conditions that enabled them to engage the work of project sustainability in their schools.

7.5.2 Rule Bending by Co-option of Planning Meeting as Professional Learning Opportunities

Another characteristic of resourceful practice concerns rule-bending. According to Edwards (Edwards, 2010a; Edwards et al., 2010), rule-bending is a way for the subject to work resourcefully and creatively by adapting explicit and implicit rules that govern the activity system. Bending the rules in ways that shift them from constraining to enabling activity is understood as an element of enacting resourceful practice (Edwards, 2010a). The School

Mathematics Leaders engaged in a form of rule-bending through their post-project leadership activity.

During CTLM, the facilitated planning meetings were established as a routine that allowed space for teachers to plan collaboratively with the product of those meetings being the creation of mathematics planning documentation. In the years that followed CTLM, facilitated planning meetings remained due to the mathematics leaders' action of influencing principals to maintain them as a routine. However, the rules about that planning routine were modified (bent) by the School Mathematics Leaders. They attributed new meaning and purpose for the facilitated planning meetings to become professional learning opportunities. Continuing professional learning beyond project participation is considered an important factor that mediates project sustainability (Kaur, 2015; Pritchard & McDiarmid, 2006; Saito et al., 2012; Warren & Miller, 2016; Zehetmeier, 2015). Instead of the rule for those planning meetings being only about teachers meeting accountability and compliance measures associated with planning documentation, the School Mathematics shifted the routine to include the rule of "We use facilitated planning meetings as mathematics professional learning time." The mathematics leaders created spaces for professional learning as a way of contributing to project sustainability.

That resourceful bending of the established rule about the facilitated planning meetings mediated the achievement of what mattered (Edwards & Thompson, 2013) for the School Mathematics Leaders. The relational motive object was worked on as the planning meetings provided opportunities for the mathematics leaders to work with teachers in interactional spaces (Edwards-Groves et al., 2016). Bending of the rules also provided the space to work on the developmental motive object by influencing teachers' mathematical knowledge for teaching

(Ball et al., 2008), primarily through development of knowledge concerning task selection and task implementation. In doing so, the mathematics leaders resolved the practice problem of reduced mathematics professional learning meetings that manifested the resurfacing of the diminished priority of mathematics contradiction.

7.5.3 Adapting Tools: Attributing New Meaning for CTLM Tasks and Student Assessment Data

When engaging in resourceful practice, the subject seeks to address contradictions within the system by refocusing on what matters (Edwards & Thompson, 2103), and tools are adapted to work on those motive objects to resolve complex problems of practice (Hannan et al., 2011). Adapting cultural tools is another way of acting resourcefully, where tools within the system are attributed new meanings in terms of their purpose (Edwards, 2010a; Vygotsky, 1978). Through their post-project professional learning activity, the School Mathematics Leaders adapted tools in two specific ways.

The first way that the School Mathematics Leaders adapted tools in their post-project leadership concerned the CTLM resources, the mathematical tasks and planning documentation. In the years following CTLM, the mathematics leaders used them as mathematics leadership tools to mediate the developmental motive object focused on influencing teachers' PCK. When they attributed new meanings to them, those CTLM materials functioned as tools of project sustainability.

In the years after participation in the project, the School Mathematics Leaders saw the potential of those CTLM resources to remain within their activity system. What is important about that significant shift is that the mathematics leaders retained those project resources and adapted their meanings and purpose. The mathematics leaders used those tools to remind teachers of the historical use of the materials and retained them for use as tools for project sustainability.

This finding adds another critical feature of resources that schools are introduced to through project participation. One factor of project sustainability is the availability of resources beyond participation in the project (Coburn et al., 2012; Fishman et al., 2011; Saito et al., 2012; Thomas & Ward., 2006; Warren & Miller, 2016). My finding, however, suggests that mathematics leaders do more with those resources than reuse them in the years following project participation. The School Mathematics Leaders resourcefully engaged in *tool retention* and *tool adaptation* (Miettinen et al., 2012) by repurposing CTLM resources as project sustainability tools. They took the intent and content of those project tools and adapted them as post-project mathematics leadership tools.

The second way that the School Mathematics Leaders engaged in tool adaptation, as an element of their resourceful practice, concerned attributing a further meaning to student assessment data. During CTLM, the mathematics leaders used student assessment data to inform teachers' planning decisions. However, through the facilitated planning meetings in the years after CTLM, the mathematics leaders not only continued using data in that way, but they also used the assessment data to persuade teachers of the need to continue with CTLM-initiated teaching reforms. That repurposing saw the School Mathematics Leaders resourcefully adapt the cultural tool of assessment data as a convincing tool to support their contribution to project sustainability.

7.5.4 Accessing Distributed Expertise from Other Mathematics Educators

Accessing distributed expertise is another characteristic of resourceful practice (Edwards, 2011). When faced with complex problems of practice, resourceful practitioners recognise the importance of acknowledging and engaging in the resource potential of human resources from neighbouring activity systems. That is achieved by accessing distributed expertise (Edwards, 2010a, 2011) which happens when the subject identifies the potential that others might bring from other activity systems to understand better what matters for them (Edwards & Thompson, 2013). That distributive expertise is adopted as a cultural tool to resolve practice problems within the activity system.

The School Mathematics Leaders engaged in that aspect of resourceful practice when looking for support from mathematics educators outside their school sites. Due to the diminished sector and principal leadership support, the mathematics leaders recognised the need to access distributed expertise of human resources beyond their own activity system (Edwards, 2011). That was not only a way of looking for assurance in their leadership activity that sustained the mathematics teaching reforms, but it was also about connecting with other mathematics educators. The School Mathematics Leaders created the conditions for project sustainability by seeking expertise from their own network, engaging with others outside of their activity system who understood the nature and complexity of mathematics leadership. They created their own network to support the project sustainability in their schools (e.g., Bobis, 2011; Coburn et al., 2012; Zehetmeier, 2015). Accessing distributed expertise as a form of resourceful practice also highlights the importance of mathematics leaders having a network of other mathematics leaders as a resource to support their activity with project sustainability (Gaffney & Faragher, 2010).

I have presented my interpretation of evidence that suggests that as their way of contributing to project sustainability, the School Mathematics Leaders enacted a form of resourceful practice (Edwards, 2005; 2010). That resourceful practice was motivated by multiple motive objects with building of relational trust as the privileged motive object of activity. Mostly, their resourceful practice enabled the mathematics leaders to resolve the critical conflict manifestation (Engeström & Sannino, 2011) of the resurfaced contradiction. They resourcefully and creatively looked for ways to negotiate the post-project practice problems they faced.

Literature concerning mathematics leaders has defined them as agents of change (e.g., Corbin et al., 2003; Millet & Johnson, 2000, 2004; Jorgensen, 2016). Through my interpretation of their contribution to project sustainability through enactment of resourceful practice, I propose that mathematics leaders also be defined as *agents of project sustainability*. I claim that as middle leaders within their school system, the School Mathematics Leaders played a key role in sustaining the mathematics teaching reforms as they engaged in the struggle that they faced in the years following project participation. I interpret that as agents of project sustainability, they should have a more prominent place within the school leadership factor of sustainability that has traditionally been attributed to principal leadership (e.g., Datnow et al., 2005; Goos et al., 2018; King, 2011; Saito et al., 2012; Tirosh et al., 2015; Zehetmeier, 2017; Zehetmeier & Krainer, 2011).

I now turn to a discussion of another post-project contradiction that endured and remained unresolved for the School Mathematics Leaders as they enacted their resourceful practice in the years following CTLM participation.

7.6 An Enduring Contradiction

I have presented evidence so far that supports my claim that the School Mathematics Leaders acted as agents of project sustainability as they engaged a form of resourceful practice as their contribution to project sustainability. As they struggled to negotiate the practice problems associated with the enduring contradiction of the diminished priority of mathematics, the School Mathematics Leaders reconfigured their motive objects of activity, focusing on what mattered (Edwards & Thompson, 2013), and enacted new post-project leadership actions. That saw them engage in new forms of activity. CHAT acknowledges, however, that as contradictions are worked on through activity, other tensions surface due to the dialectical nature of contradictions and the flux state of activity (Engeström, 2015; Roth, 2012).

I move to a discussion of the School Mathematics Leaders' experiences of another contradiction that surfaced in the years following CTLM participation. I interpret that contradiction as a tension between their reconfigured motive objects of activity. I focus on that new contradiction to not end the narrative of the School Mathematics Leaders' contribution to project sustainability in a negative manner. Instead, I present this due to the prominence of contradictions within a dialectical materialist ontology (Mussachia, 1977; Nuttall & Brennan, 2016; Roth, 2012), and how they can drive and expand activity forward when acknowledged and worked on (Engeström, 2015). I also discuss that new contradiction in a hopeful way because the mathematics leaders revealed that they were aware that resolution of that contradiction was needed next for the development of their leadership activity and for themselves as School Mathematics Leaders. I draw mainly on the concepts of the need state and hierarchy of motives (Leont'ev, 1978) as I explain that new contradiction.

7.6.1 Struggle of Addressing Issues in Mathematics Teaching Practice

I claim in this thesis that the School Mathematics Leaders' post-project professional learning leadership activity was multi-motivational (Leont'ev, 1978), presenting evidence that they worked on relational, developmental, and managerial motive objects of activity. For the most part, despite the struggle they experienced negotiating complex practice problems, the mathematics leaders contributed to project sustainability by working in creative and resourceful ways, as evidenced in my explanation of their enactment of resourceful practice. Tensions, however, surfaced for them as middle leaders of mathematics when they had to address issues in mathematics teaching practice, especially with MCK issues.

The School Mathematics Leaders revealed that struggle, disclosing their uneasiness and uncertainty about maintaining relationships with their teaching peers while resolving issues in practice. According to the mathematics leaders, issues surfaced when they became aware of teachers' practices that did not align with the shared understandings captured in their collective commitment documentation (Rose & Norwich, 2014). My interpretation of that contradiction and its influence on the School Mathematics Leaders' activity is that their work on the relational motive object not only enabled them to work on their developmental motive object, but it also constrained that activity concerned with developing their teachers' mathematical knowledge for teaching (Ball et al., 2008). In other words, the School Mathematics Leaders struggled to work on the relational and developmental motive objects simultaneously when issues in mathematics teaching surfaced.

Rachel’s comment, when she recounted an aspect of mathematics leadership that was challenging, supports my interpretation of the existence of that contradiction:

> Look, I still do find the difficult conversations hard when you have to talk to someone about an issue in their teaching. I know they (conversations) need to happen, but my relationship with the teachers is so important. I let a lot of things go and hope that it all sorts itself out, or I might address it as a general thing to everyone in the planning meetings (Rachel, PPI, 26.10.16).

Rachel's comment captured the theme of the comments from the School Mathematics Leaders about that aspect of their leadership. I have further interpreted that that struggle surfaced in response to the importance of building relational trust for and about mathematics teaching motive object and the relationality rule within their activity system.

Penny highlighted how the need to maintain harmonious relationships was privileged, with the importance of remaining in favour with colleagues, when she noticed conflicts between teachers' practice and the collective commitments:

> I consider those difficult conversations to be one of the main things of my School Maths Leader role. I cannot be judgmental. I need to make opportunities for those discussions, so teachers can learn from it and ultimately become better teachers. That is the aim, but they are more challenging than I am making out. In fact, I would rather avoid them, so I tend to wait it out and watch to see if the issues get resolved. I cannot have the teachers hating me (Penny, PPI, 25.03.15).

That comment from Penny confirms that the School Mathematics Leaders experienced that tension in their work, confirming knowledge in the literature about that middle leadership issue. With their position as practising members of the leadership team and the teaching staff, middle leaders experience challenges in resolving issues with teachers and their practice (Bennett et al., 2003; Irvine & Brundrett, 2019). For the School Mathematics Leaders, ways of managing their teachers' performance concerning the enactment of effective teaching practices proved to be a tension in their post-project leadership activity (Lipscombe et al., 2021; Turner, 2007).

When issues in mathematics teaching practice came to the attention of the School Mathematics Leaders, they experienced struggle through feelings of apprehension. They knew they had to be cautious when attempting to address issues because of the primacy of the relationships they built with teachers. The School Mathematics Leaders upheld their relationship with the teachers by *avoiding conflict,* evidenced in Penny's comment: "I would rather avoid them. I tend to wait it out and watch to see if the issues get resolved" (Penny, PPI, 25.03.15).

I have reasoned that the conflict avoidance action was enacted due to their close working conditions with teachers as middle leaders (Hammersley-Fletcher & Kirkham, 2007). That was a general action response when the School Mathematics Leaders found themselves in that conflict between the relational and developmental motive objects of activity.

Cindy also revealed the enactment of avoiding conflict. She reiterated the privileging of relationships with staff:

> I don't like those challenging conversations. I know it's something that I need to do when something comes up. I am very careful, probably too careful, and I probably don't really address the problem. I kind of skirt around the edges a bit (laughs). I always feel funny afterwards doing that, always asking myself if I handled it correctly. You just can't damage the relationship with your staff (Cindy, PPI, 06.12.16)

The School Mathematics Leaders disclosed that they were aware that addressing issues in teaching practice was contradictory for them. Relational responses to issues were generally favoured, with the maintenance of "good" relationships with teachers prioritised. The mathematics leaders appeared conscious that they knew they were required to act in developmental ways. However, due to the primacy of relationships, their actions tended to protect relational trust by avoiding conflict.

The struggle that the School Mathematics Leaders experienced in addressing tensions in the teachers' level of PCK provided uneasiness for them as middle leaders of mathematics. That, as evidenced by the data, revealed the conflict they experienced due to the privileging of the relational motive object of their activity. For the mathematics leaders, however, an even deeper struggle was experienced when they encountered issues with teachers' MCK.

7.6.2 Deeper Struggle of Addressing Issues in Mathematics Knowledge

The mathematics leaders' developmental motive object of activity tended to focus on improving teachers' PCK of mathematical knowledge for teaching (Ball et al., 2008), neglecting to some extent the development of teachers' MCK. I interpret the reason for that PCK focus lies in the School Mathematics Leaders' perceptions about the fragility of relationships and the development of teachers' MCK. The School Mathematics Leaders shared the struggle they experienced with addressing issues in teachers' PCK, but they revealed an even deeper struggle when issues in teachers' MCK were exposed.

When asked about possible challenges with addressing MCK issues, the School Mathematics Leaders discussed the challenge of engaging in conversations with their peers about adult misconceptions with MCK. Penny highlighted that when she shared:

> It is a challenge though, such a challenge, to have those conversations with adults about their mathematical misconceptions. It is so hard to talk to an adult about that. It could go wrong very quickly with teachers' maths knowledge, and that's why it's a challenge (Penny, PPI, 25.03.15).

With their motive object of activity focused on relational trust building, the School Mathematics Leaders tended to worry that it would be jeopardised when they needed to address mathematical misconceptions that their teachers held. They claimed they had access to knowledge of those teacher misconceptions through the facilitated planning meetings that they had co-opted as mathematics professional learning opportunities.

Like Penny, Rachel revealed the challenge of attending to misconceptions and how avoiding conflict was the response to such situations:

> You have to be careful. I love working with kids' misconceptions because that's on a different level and that's much easier to deal with. If the teacher has the misconception, then that's a different story. I tend to not say anything, especially not in a planning meeting. I *might* talk about it as if a kid has the misconception, like, "Ah, I've seen the kids in your year level have this misconception..." and then I will explain the misconception, hoping the teacher who has it will take notice (Rachel, PPI, 29.04.15).

The primacy of relationships with teachers was often highlighted when the School Mathematics Leaders discussed the struggle to address teachers' MCK issues. Their comments provided further evidence for my reasoning that they perceived their relationships in fragile positions when those MCK issues surfaced. The mathematics leaders did not want to jeopardise relationships with their teachers within those situations, protecting the relational trust for and about mathematics teaching.

The School Mathematics Leaders talked about the knowledge of several teachers having teacher mathematics anxiety. That phenomenon can be experienced by primary school in-service teachers (e.g., Ramirez et al., 2018). Due to their work on building relationships with teachers, the mathematics leaders claimed they generated such knowledge of teachers' anxiety for mathematics and mathematics teaching.

Cindy shared that knowledge that the School Mathematics Leaders held about teachers' mathematics anxiety. She recounted an experience of walking by the Year 5/6 classroom on her way to the staffroom. Cindy recalled that she heard the teacher talking about decimal fractions and noticed that the decimal fraction of 4.5 was recorded on the teacher's whiteboard. She recalled that the statement, "It is an odd number", was written in the teacher's handwriting under the decimal notation. Cindy said she stopped and stood by the classroom door, in the hallway and out of sight, as she listened to the teacher continue with the lesson. Cindy claimed that the teacher discussed with students that decimal fractions are understood as being odd or even. Then the teacher called on students to offer other examples of odd and even decimal fractions.

When asked what she did in response, Cindy said that she was aware that the mathematics was incorrect, but then offered the following:

> I mean, how do you say to someone that the maths on their whiteboard is incorrect? How do I tell a teacher that I do not think they understand the maths they are teaching? I have to have a coffee with them at recess time and I know that she's got maths anxiety already. It is really challenging. I am still learning how to have those conversations with my staff because relationship is so important (Cindy, PPI, 06.11.14).

Cindy claimed that she was unsure of what action to take, reiterating her knowledge of the teachers' mathematics anxiety and the importance of maintaining relationships with her teachers. Cindy was not alone in situations like that, as it was evident in discussions with the School Mathematics Leaders that they faced tensions in addressing issues in teachers' PCK, with the struggle experienced at deeper levels when issues in teachers' MCK surfaced.

I now turn to an explanation for that tension in the School Mathematics Leaders' professional learning leadership, using the concept of the hierarchy of motives (Leont'ev, 1978).

7.6.3 Reason for the Contradiction: Struggle within the Hierarchy of Motive Objects

The prominence of the relational motive object of activity and rule of relationality within the School Mathematics Leaders' activity system created conditions for the mathematics leaders to enact their professional learning leadership. Although they worked creatively and resourcefully to contribute to project sustainability, the mathematics leaders struggled to do this when tensions surfaced in teachers' PCK and MCK.

I have claimed that the School Mathematics Leaders reconfigured the motive objects of their post-project leadership activity to identify what mattered to them (Edwards & Thompson, 2013). The outcome of that reconfiguration process was that their post-project leadership became multi-motivational activity (Leont'ev, 1978), meaning that they pursued multiple motive objects as they engaged in their resourceful practice (Edwards, 2005, 2010). With that multi-motivational activity, the School Mathematics Leaders were required to juggle multiple motive objects simultaneously (Nuttall et al., 2019).

That juggling of motive objects became even more complex when the School Mathematics Leaders were confronted with issues concerning the limitations in teachers' PCK and MCK. Drawing on my interpretation of their comments, I believe that the complexity was due to School Mathematics Leaders' experience of a need state (Leont'ev, 1978). They experienced a contradiction between the relational motive object and the developmental motive object, and they felt uneasiness and discomfort at those times (Kaptelinin & Nardi, 2006). That was evidenced by Cindy when she shared: "I always feel funny afterwards" (Cindy, PPI, 06.12.16).

I have interpreted this contradiction as a *double bind* manifestation (Engeström & Sannino, 2011). My reasoning for this is that the School Mathematics Leaders knew that they had to act in developmental ways on the issues in PCK and MCK, yet with their privileging of relational trust, it meant that they engaged in acts of conflict avoidance. They instead sought to maintain relationships with their peers. The mathematics leaders were unsure about ways of acting when PCK and MCK tensions surfaced within their activity system due to the contradiction between the rule of relationality and the relational and developmental motive objects of activity.

The comment from Penny about not wanting the teachers to hate her and other references from the mathematics leaders about actions of avoiding conflict provide reasons for my claim. Another interpretation of the situation is that the School Mathematics Leaders believed there was only one motive object upon which to act when issues in teachers' PCK and MCK surfaced. Due to the prominence they gave to the relational dimension of their leadership, maintaining relationships and collegiality was paramount to them as a leader and especially as a colleague. The rule of relationality, "We keep everyone safe around mathematics", enabled conditions for the School Mathematics Leaders' professional learning leadership, but at the same time, it also constrained their activity. That happened when they were required to engage in developmental work that addressed the tensions in PCK and MCK.

Another reason that I offer to explain this situation is that the School Mathematics Leaders had not yet developed a hierarchy of motives (Leont'ev, 1978) that was at the stage of acting as a "conflict resolution mechanism" (Kaptelinin, 2005, p. 14). I interpret that the relational motive object had the highest position within their hierarchy of motives. That means that, at times of conflict concerning issues with PCK and MCK, the School Mathematics Leaders tended to the relational motive object of activity, first and foremost. They desired to maintain relationships with teachers more, rather than work on the development motive object. I interpret that the School Mathematics Leaders perceived that their relational motive object and their developmental motive object were mutually exclusive when tensions in teachers' PCK and MCK surfaced for them.

However, that contradiction within professional learning leadership activity was a potential space for leadership development, acknowledged by the School Mathematics Leaders themselves.

7.6.4 Recognising the Contradiction: Opportunities to Grow Leadership Activity

The need to resolve conflict within their hierarchy of motive objects (Kaptelinin, 2005; Leont'ev, 1978) appeared to be an aspect of the School Mathematics Leaders' leadership activity of which they were cognisant. When conversations turned to areas of middle leadership development, they identified the need to learn practices that allowed them to address issues with PCK and MCK whilst maintaining relationships with the teachers. The interviews revealed that the School Mathematics Leaders were aware that conflict resolution concerning PCK and MCK development was an aspect of their professional learning leadership activity that had the potential to be expanded.

Rachel exemplified that when asked to discuss areas of development within her professional learning leadership:

> I don’t like having to have difficult conversations. I don’t feel right about them. You come away from it and you think, “I don’t know how I went there.” I know that for myself, I need to work on that part of my leadership (Rachel, PPI, 26.10.16).

Cindy confirmed that as she shared: “I’m missing the leadership element of when I have to have those difficult conversations and knowing how to have them” (Cindy, PPI, 24.04.18).

It is important to include these data in the discussion concerning the contradiction that the School Mathematics Leaders faced and how they understood themselves as mathematics leaders in relation to that contradiction. I have used the concept of the practice-person dialectic (Edwards, 2017) to describe the simultaneous development of the School Mathematics Leaders and their leadership activity. That concept can be used to interpret the situation the mathematics leaders faced with that contradiction I have just described. They became self-aware of the aspects of their leadership activity that worked well, were cognisant of areas for leadership development, and understood what was needed to develop their practice and themselves as School Mathematics Leaders.

That situation of the contradiction concerning tensions in teachers' PCK and MCK, along with the School Mathematics Leaders' conscious awareness of the need to resolve that contradiction, stimulates possibilities for mathematics leadership development. Koh et al. (2011) claimed that middle leaders must take ownership of their professional development. This study highlights the potential focus of that professional development for mathematics leaders concerning ways of attending to relational and developmental motive objects simultaneously when issues in teachers' PCK and MCK surface.

7.6.5 Possibility for Introduction of a New Cultural Tool

An aspect of the mathematics leadership professional development could be the introduction of a new cultural tool to remediate (Miettinen, 2006) the School Mathematics Leaders' activity. That remediation would have a specific focus on ways of addressing the contradiction within their hierarchy of motives (Leont'ev, 1978). The remediation would aim to transform the hierarchy so it could be used as a conflict resolution tool (Kaptelinin, 2005).

The outcome would mean that School Mathematics Leaders do not see motive objects of activity as mutually exclusive but rather understand ways of acting in both relational and developmental ways when issues surface in their leadership activity. That would mean acting in ways that draw attention to the development of PCK and MCK, whilst maintaining the relational trust for and about mathematics that enables the conditions for professional learning leadership activity.

As evidenced in their comments, the School Mathematics Leaders used adjectives such as "challenging" and "difficult" to describe conversations about issues in teachers' PCK and MCK. Those adjectives provide insight into the struggle that they experienced with those conversations. Within a CHAT framework, learning is developed and perceived as “expansive” when contradictions are noticed and resolved (Engeström, 2015). Therefore, there is merit in shifting the name of those conversations to *expansive discussions*. Expansive captures the idea of learning and development, as positioned within CHAT, and discussions intimate the relational aspect of their developmental purpose. Those expansive discussions could be highlighted as a cultural tool mediating both the relational and development motive objects of activity.

Expansive discussions could be introduced as a new cultural tool to reframe attention on those tensions in PCK and MCK. An important aspect of the introduction of that new cultural tool would be allowing it to be a temporary motive object (Engeström & Blackler, 2005). Part of the introduction and subsequent use of expansive discussions would require attention to the establishment of new rules and a division of labour that would mediate the use of those expansive discussions as a mathematics leadership tool.

7.7 Chapter Summary

In this chapter, I provided the complete response to the research question. I did this by presenting interpretations of evidence that support my claim that the School Mathematics Leaders contributed to project sustainability by enacting a form of resourceful practice. I claimed that the School Mathematics Leaders responded through a caring and creative approach in response to the responsibilization they experienced for sustaining the mathematics teaching reforms. Through this chapter, I supported that claim by using evidence that the mathematics leaders reconfigured the motive objects of activity in the years following CTLM participation. That motive object reconfiguration focused on what mattered to them, surfacing multiple motive objects of activity that focused on the relational, developmental, and managerial dimensions of their mathematics leadership. In doing so, the School Mathematics Leaders creatively responded to their struggle experienced through the post-project practice problems that surfaced in the years following CTLM.

With their positionality and practice as middle leaders, I reported that the School Mathematics Leaders privileged the building of relational trust for and about mathematics teaching. I proposed that by prioritising relationships, the mathematics leaders created conditions that enabled them to work on the developmental and managerial motive objects of activity. I used evidence to show that their developmental motive object of activity was realised through work on PCK development, focusing on task selection and task implementation with attention to differentiation. The managerial motive object saw the School Mathematics Leaders manage principals to keep mathematics as a school improvement area and promote mathematics with their peers. I drew on evidence to support my claim that the rule of relationality was established within the School Mathematics Leaders' activity system that mediated relational trust building, further creating conditions that enabled them to engage in their form of resourceful practice.

I moved to a discussion of the post-project leadership actions that realised the School Mathematics Leaders’ resourceful practice. As their contribution to project sustainability, the mathematics leaders worked resourcefully to resolve the practice problems they faced in several important ways that included: influencing their principals to maintain the facilitated planning meetings; co-opting those facilitated planning meetings as professional learning opportunities; repurposing CTLM resources as tools of project sustainability; using student assessment data as convincing tools to persuade teachers to continue with the project-initiated reforms; and accessing expertise from mathematics educators working in other settings. I then drew on characteristics of resourceful practice to argue that by reconfiguring the motive objects to focus on what mattered, engaging in rule-bending, adapting cultural tools, and accessing distributed expertise from neighbouring activity systems, the School Mathematics Leaders enacted a form of resourceful practice as understood in CHAT terms.

I drew attention to a post-project contradiction that surfaced for the School Mathematics Leaders in the years following CTLM participation. I interpreted that contradiction as a double bind manifestation in that the mathematics leaders experienced tension with acting in relational and developmental ways when issues in PCK and MCK surfaced for them. I claimed that this was due to the privileging of their relational motive object and the relationality rule that enabled their resourceful practice but also constrained that leadership activity. I further interpreted that the School Mathematics Leaders had not yet developed their hierarchy of motives of their multi-motivational activity to the level that it was used as a conflict resolution mechanism. To support that, I offered the concept of expansive discussions as a cultural tool that could remediate the School Mathematics Leaders' activity and facilitate the development of their hierarchy of motives as a conflict resolution tool.

In the next chapter, I conclude my thesis. I include a summary of my study, and I restate the research question that guided my inquiry. I offer the claims presented in the previous three chapters, and I state the focal theory of my thesis that acts as my response to the research question. I present the contribution of my findings and the implications of that knowledge for the field of mathematics leadership in school settings. I state the limitations of the study, and I offer ideas for further research opportunities related to my findings. I conclude the thesis by describing how the person-practice dialectic was realised through my research activity.

CHAPTER 8: THE CONCLUSION

8.1 Introduction to Chapter 8

In this chapter, I draw the thesis to a close by firstly summarising my study. I revisit the research question and declare my claims drawing on information from the three previous chapters. I then present the focal theory of my thesis that acts as my response to the research question. I articulate the contribution of my findings to knowledge about project sustainability through my theorisation of the School Mathematics Leaders' activity.

The discussion then moves to one focused on the implications of my study. I present my interpretation of the impact of findings for stakeholders at the school level (mathematics leaders and principals), the district/sector level, and for professional development project designers who intend to include mathematics leaders in their projects. I present the study's limitations and offer recommendations for further research. I include a statement of my intention to pursue mathematics leadership as a line of inquiry for my immediate research activity. My thesis finishes with a statement about how I experienced the person-practice dialectic, and how that has influenced my personhood as I enter the mathematics education research community.

8.2 Summary of the Study

In my study, I examined the problematic of project sustainability, focusing on the activity of three mathematics leaders working as middle leaders in their schools. The purpose of my study was to generate knowledge about how School Mathematics Leaders, as middle leaders in their primary schools, contributed to project sustainability through their professional learning leadership activity. The unit of analysis was their leadership activity (Kuutti, 1996; Roth, 2012), paying attention to how that leadership activity contributed to project sustainability. That topic of investigation surfaced in response to paucity within the literature concerning project sustainability and how middle leading activity, like that enacted by mathematics leaders, contributes to the continuation of mathematics teaching reforms. I explored that problematic through the context of the CTLM project (e.g., Clarke et al., 2013a) and the leadership of three School Mathematics Leaders who participated in CTLM in 2011 and 2012.

In response to scarcity of information about the research problem and acknowledging that the literature concerning middle leadership has framed it a form of practice (e.g., Edwards-Groves et al., 2016; Grootenboer, 2018), I positioned the School Mathematics Leaders' professional learning leadership as a form of activity. I recognised the potentiality of CHAT as an appropriate theoretical framework to study the activity of those three mathematics leaders.

As the means of making sense of the School Mathematics Leaders' contribution to project sustainability, I framed the research design drawing on CHAT and its concepts associated with activity systems, the feature of CHAT's second generation (Engeström, 2001). Other CHAT concepts were used as analytical tools such as resourceful practice (Edwards, 2005, 2010a). Attention was paid to the motive objects of activity, in recognition of their analytical potential to understand activity (Kaptelinin, 2005; Leont'ev, 1978). I honoured the methodological implications of CHAT (Roth, 2012) by examining the School Mathematics Leaders' leadership during CTLM, as well as their leadership activity in the years following CTLM participation.

Due to the lack of knowledge concerning the problematic, and as a way of operationalising CHAT, I designed a qualitative research study. That enabled exploration of the research problem (Creswell, 2012). I created a research process as a methodology specifically for my study, drawing on previous work of researchers who have used CHAT as a methodological tool (Mwanza-Simwami, 2011; Mwanza & Engeström, 2003, 2005; Uden et al., 2008). Having positioned the School Mathematics Leaders as the collective subject (Kaptelinin, 1995;

Lektorsky, 2009), the data generation methods of interview, observation, and document retrieval were used at different phases of that research process. The three sources of data generation, a prolonged data generation period (December 2014 to May 2018), along with opportunities for member-checking were built into the research process, serving as validation strategies for the research design (Creswell, 2013).

Data were analysed using deductive and inductive approaches. Concepts from CHAT and the background literature supported deductive analysis as I searched for evidence of enactment of those concepts within the dataset. Inductive analysis also proved important in paying attention to the “what else” of the School Mathematics Leaders’ contribution to project sustainability.

I used three chapters to discuss the findings through rich, thick descriptions of the School Mathematics Leaders’ activity. A chapter was devoted to the historicity of the School Mathematics Leaders’ professional learning leadership, focusing on how their activity during CTLM shifted from managerial undertakings to motive objects focused more on influential leadership (Chapter 5). The next chapter focused on the mathematics leaders’ experiences of post-project practice problems and their response to them through experiences of responsibilization for project sustainability (Chapter 6). The final findings chapter (Chapter 7) focused on how the School Mathematics Leaders contributed to project sustainability through a form of resourceful practice, realised through a reconfiguration of motive objects and enactment of several new leadership actions. A post-project contradiction was also identified and explained.

The findings of my thesis offer important contributions to the field. They highlight the essential activity of the School Mathematics Leaders, as middle leaders in their schools, and how their enactment of resourceful practice through their leadership activity contributed to project sustainability.

I now revisit the research question. I declare three claims, drawing on information within those three chapters, and I propose the focal theory of my thesis.

8.3 Claims and Focal Theory

As a way of inquiring into the problematic, I posed research question of:

As middle leaders of site-based professional learning, how do School Mathematics Leaders contribute to the sustainability of mathematics teaching reforms in the years that followed participation in a large-scale school mathematics professional development project?

In response to the research question, I declare three claims. I do that before I present the final presentation of findings, positioned as the focal theory of my thesis. The first claim relates to the historical enactment of the School Mathematics Leaders' professional learning leadership activity during the CTLM project (Chapter 5).

Claim 1: The School Mathematics Leaders believed that their schools participated in CTLM due to an enduring contradiction of the diminished priority of mathematics that concerned a prioritisation of literacy education. As participation in CTLM endured, the mathematics leaders shifted their motive objects of activity from ones focused on management-related undertakings to ones that were more focused on influencing teacher practice. That suggests that the School Mathematics Leaders' leadership during project participation was multi-motivational which began with activity focused on compliance and management. As participation in CTLM progressed, the mathematics leaders' activity shifted to leadership that enabled conditions for teachers' professional learning and the development of collective commitments for mathematics teaching in their schools. The conclusion of project participation was an emotionally freighted time for the School Mathematics Leaders, experienced mostly through uncertainty concerning project sustainability. That uncertainty prevailed despite an awareness that their leadership and they themselves as middle leaders of mathematics had developed through participation in the CTLM project.

The next claim concerns the School Mathematics Leaders' leadership activity after CTLM finished. It highlights the practice problems that they faced, and how that surfaced struggle for them in the years following CTLM participation. The School Mathematics Leaders' response to that struggle initiated their enactment of a form of resourceful practice.

Claim 2: After participation in CTLM, the principals in the School Mathematics Leaders' schools played an important role initiating project sustainability. That was done when the principals used their authority to establish rules that mediated a commitment to continuation of the project reforms and the maintenance of the mathematics leadership position as one within their schools' leadership system. Participation in CTLM, however, only brought a temporary disruption to the contradiction of the diminished priority of mathematics. Several complex practice problems surfaced for the School Mathematics Leaders that acted as a critical conflict manifestation of that enduring contradiction. Those practice problems surfaced due to several changes within the School Mathematics Leaders' activity system, mediated by withdrawal of CEOM support, changes in principal leadership support, diminished role clarity, staff turnover, and reduced opportunities to lead mathematics professional learning. The School Mathematics Leaders experienced struggle with those problems of practice, intensified through experiences of responsibilization where they felt it was their responsibility to continue with the mathematics teaching reforms. In response, the School Mathematics Leaders focused on what mattered for them and their mathematics leadership, and they sought to act through a caring and creative approach to project sustainability. That response which focused on what mattered initiated their form of resourceful practice.

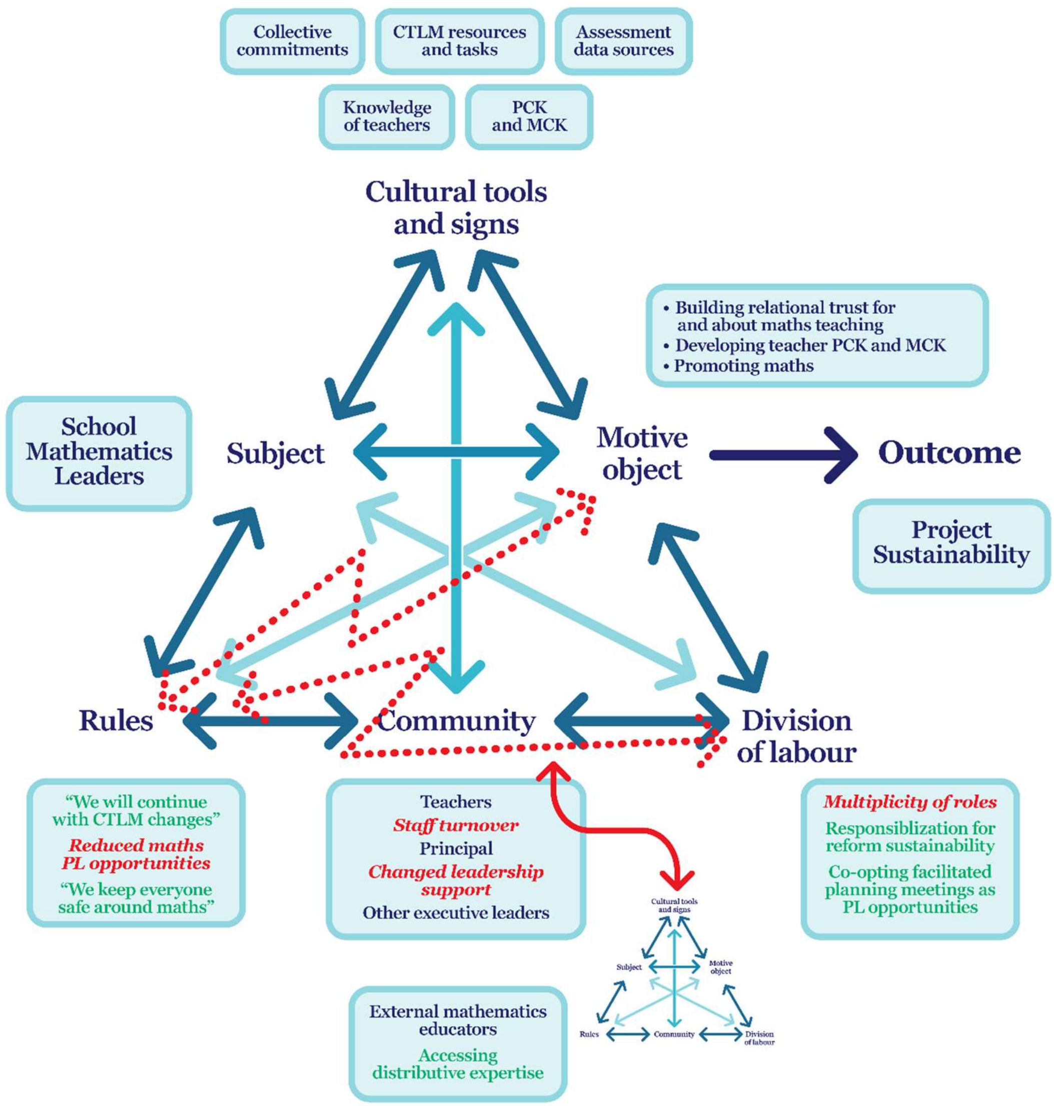
The final claim captures fully the School Mathematics Leaders' resourceful practice as their contribution to project sustainability, and the enduring contradiction of their post-project leadership activity.

Claim 3: As a way of working with the complex practice problems that surfaced within their activity system following CTLM participation, the School Mathematics Leaders' resourceful practice was further realised as they reconfigured the motive objects of activity. That reconfiguration saw the surfacing of a motive object that sought to build relational trust for and about mathematics teaching with teachers. By working on that relational motive object, the School Mathematics Leaders were afforded opportunities to pursue the developmental motive object concerned with developing teachers' PCK and MCK. That motive object mostly focused on improving teachers' knowledge of task selection and task implementation, with a specific focus on differentiation. A managerial motive object was also pursued that sought to manage their principals that promoted mathematics as a persistent school improvement agenda item. With their work on multiple motive objects, the School Mathematics Leaders' leadership remained multi-motivational activity in the years following CTLM participation. To support achievement of the motive objects of activity, a rule of relationality surfaced within the mathematics leaders' activity system. That relationality rule mediated conditions for them to enact their leadership, specifically that which was related to their relational motive object. The School Mathematics Leaders' contribution to project sustainability through resourceful practice manifested through several new leadership actions. Those actions further realised the resourcefulness and creativity of their project sustainability efforts. The enactment of those leadership actions enabled them to focus on: what mattered through the reconfiguration of motive objects and their influencing of principals to maintain facilitated planning meetings; bending the rules with those planning meeting by repurposing them as professional learning meetings; adapting and attributing new meaning for project resources and student assessment data as mathematics leadership tools; and accessing distributed expertise from mathematics educators outside of their activity system. By engaging in that resourceful practice, the School Mathematics Leaders acted as agents of project sustainability. Their project sustainability efforts, however, were not fully realised due to one enduring contradiction that surfaced due to the relationality that the School Mathematics Leaders privileged through their resourceful practice. That contradiction, manifested as a double bind, brought on struggle concerning ways of working on the development and relational motive objects of activity when issues in teachers' PCK and MCK surfaced within the School Mathematics Leaders' leadership activity.

Having declared the claims of the thesis, I now draw attention to the focal theory that acts as my response to the research question. As a means of representing that focal theory, I use the triangular model (Figure 22) of the activity system (Engeström, 2015; Roth, 2012). I do that as a way of mapping the School Mathematics Leaders' resourceful practice as their contribution to project sustainability. I use regions at each mediational element of the activity system (e.g., rules, division of labour), including the examples of each mediator as it relates to the School Mathematics Leaders' resourceful practice. The red font in each region represents the post-project practice problems faced by the mathematics leaders. The convention of the zig-zagged double headed arrows is used to show the existence of contradictions (Engeström, 2015). The smaller image of the activity system represents how the School Mathematics Leaders accessed distributed expertise from neighbouring systems belonging to mathematics educators outside of their school sites.

Figure 22

Mapping Resourceful Practice as Contribution to Project Sustainability



I offer the focal theory of my thesis. Through this thesis, I asked how School Mathematics Leaders, as middle leaders in their schools, contribute to project sustainability through their site-based professional learning leadership in the years following participation in a large-scale mathematics professional development project. Drawing on the claims that I have declared, I present my final response to the research question.

The focal theory of my thesis is:

As their contribution to project sustainability, the School Mathematics Leaders enact a form of resourceful practice, directed at multiple motive objects of activity. That resourceful practice privileges the relationality of their leadership activity as they seek to contribute to project sustainability. The School Mathematics Leaders' prioritisation of relationality enables yet constrains their sustainability efforts as they negotiate complex post-project practice problems, mediating that activity using repurposed and adapted mathematics leadership tools. The School Mathematics Leaders, as middle leaders in their schools, act as agents of project sustainability through their professional learning leadership activity.

8.4 Contributions and Implications

I identified several aims for my study that guided by research activity. The first aim focused on the generation of theory about how mathematics leaders contribute to project sustainability. The second aim concerned the contribution of knowledge about mathematics leadership as a form of middle leadership, and the final aim was about my demonstration of using CHAT to inform a research design and use it to interpret the work of the mathematics leaders as they contributed to project sustainability. In this section, I will address the first two aims as contributions to knowledge about mathematics leadership as a factor of project sustainability and as a form of middle leadership in primary school settings.

A significant contribution of my thesis concerns the knowledge about how mathematics leaders play a crucial role in project sustainability in primary school settings. By focusing my research about mathematics leaders within the context of project sustainability, I responded to calls from researchers who have claimed that further knowledge is required about how schools deal with the complexity of the sustainability of project reforms (e.g., Coburn, 2003; Coburn et al., 2012; King, 2011), specifically within mathematics education (Bobis, 2011; Goos et al., 2018; Saito et al., 2012; Smit et al., 2019; Tirosh et al., 2015; Zehetmeier, 2014). The findings within my thesis have contributed to understandings of mathematics project sustainability through the story of the three School Mathematics Leaders. Through my focus on the activity of the mathematics leaders as middle leaders in their schools, I have added further knowledge about the school leadership factor of project sustainability, an element of sustainability that has not been investigated in detail (Coburn et al., 2012).

My findings, however, offer new information about that project sustainability factor of school leadership, in that it is not just the principal whose work should be acknowledged (e.g., Datnow et al., 2005; Goos et al., 2018; Saito et al., 2012; Tirosh et al., 2015; Zehetmeier, 2017). I have identified that principals, with their authority as executive school leader (De Nobile, 2019; Grootenboer, 2018), do play an important role in enabling conditions for project sustainability by setting rules of commitment and maintaining the mathematics leadership role. However, I have found that mathematics leaders, as middle leaders, working in the space between the principal and teachers (e.g., Grootenboer, 2018; Lipscombe et al., 2020), play an essential role in how they influence the continuation of mathematics teaching reforms through their caring and creative commitment to their community (specifically teachers), the developmental work achieved through project participation, and a passion for mathematics as a school curriculum area.

I have responded to calls from researchers concerning the need for knowledge about what happens after participation in mathematics professional development interventions (Chapman, 2012; Smit et al., 2019; Zehetmeier, 2017). My response focused on the experiences of the School Mathematics Leaders, revealing that several problems of practice surfaced for them as middle leaders of mathematics when CTLM participation ended. I theorised those practice problems drawing on the CHAT concept of contradiction, identifying them as a critical conflict manifestation within the mathematics leaders' post-project activity system (Engeström & Sannino, 2011). I also highlighted the potential for mathematics leaders to experience responsibilization (Nuttall et al., 2022; Shamir, 2008) for project sustainability when sector support is withdrawn and when principal leadership does not go far enough in addressing those post-project problems of practice that may face mathematics leaders.

By using CHAT to systematically study the post-project professional learning leadership activity of the School Mathematics Leaders, I contributed further knowledge about what happens after project participation with my focus on the motive objects of their activity (Kaptelinin, 2005; Leont'ev, 1978). I used evidence to claim that the mathematics leaders' post-project leadership is multi-motivational (Leont'ev, 1978), as they direct that activity towards relational, developmental, and managerial motive objects. My findings confirm the role of relational trust within the activity of middle leaders (Edwards-Groves et al., 2016; Edwards-Groves & Grootenboer, 2018), but I have extended this further with evidence that building relational trust for and about mathematics teaching is a privileged motive object of mathematics leaders' post-project professional learning leadership activity. I have offered further understanding of relational trust in middle leadership in that it takes on an even more vital role within mathematics leadership due to knowledge that School Mathematics Leaders claim to hold about their teachers' historical experiences of and dispositions for mathematics and mathematics teaching.

Another contribution of my study lies in the detailed accounts of the collective post-project professional learning leadership activity of the three School Mathematics Leaders. With my use of the CHAT concept of resourceful practice (Edwards, 2005, 2010), I conceptualised the mathematics leaders' contribution to project sustainability as one characterised by creativity and resourcefulness. I have offered insights into how the School Mathematics Leaders engaged their resourceful practice to resolve as the practice problems that they faced in the years following CTLM participation through leadership actions that saw them focus on what mattered (Edwards & Thompson, 2013) through the reconfiguration of motive objects, rule-bending (Edwards, 2010a) by co-opting facilitated planning meetings as professional learning opportunities, adapting cultural tools (Edwards, 2010a; Hannan et al., 2013; Miettinen et al., 2012; Vygotsky, 1978) of CTLM resources and student assessment data as project sustainability tools to convince teachers for the need to sustain the project-initiated mathematics teaching reforms, and accessing distributed expertise from external mathematics educators (Edwards, 2011).

The findings presented in my thesis offer significant contributions because they respond to paucity in the literature within the fields of project sustainability, middle leadership in primary schools specifically the role of relational trust, and the activity of mathematics leaders as agents of project sustainability.

The implication of my findings, highlighting the resourceful practice enacted by the School Mathematics Leaders, is that project sustainability factors must be used as project design tools by mathematics project designers. Those factors need to be contextualised in relation to the leadership activity of the School Mathematics Leaders to support them in their activity as agents of project sustainability. The factors cannot be left untreated and remain the responsibility of mathematics leaders to decide upon which ones to attend when project participation has ceased. Mathematics professional development project designers and facilitators must bring them to the attention of School Mathematics Leaders before and during the life of the project. Those factors could act as the mediational means to support enactment of resourceful practice by mathematics leaders beyond project participation, as ways of minimising opportunities that might realise experiences of responsibilization by the School Mathematics Leaders.

This implies that when designing for project sustainability, mathematics professional developers and participants should pay attention to the importance of relationality and how that is realised through the activity of School Mathematics Leaders. Project intent and content needs to explore the nature and opportunities for resourceful practice, and its potential for enactment during and beyond project participation. The content about resourceful practice could focus on developing School Mathematics Leaders' use of leadership actions including the repurposing of project resources as sustainability tools, rule-bending that supports further opportunities for ongoing school-based professional learning, and the enduring access to expert advice from mathematics educators beyond the school sites in which the School Mathematics Leaders practise their post-project professional learning leadership activity.

8.5 Limitations of the Study

I did not intend to present claims within the thesis with the purpose of making them generalisable across all situations of mathematics leaders' work focused on sustaining mathematics teaching reforms following participation in professional development projects. My aim was to generate knowledge about how the School Mathematics Leaders involved in my study contributed to project sustainability through their post-project professional learning leadership activity. Through that process, I generated focal theory that offers a way to understand the complexities of the struggle that the School Mathematics Leaders faced as they contributed to project sustainability through enactment of their form of resourceful practice.

Through enactment of and reflection upon my research design, I acknowledge that there were several limitations. The first limitation is that this research design was significantly influenced by a CHAT perspective. I endeavoured to use CHAT, specifically second-generation (Engeström, 2001), as a framework to systematically study the professional learning leadership activity of the School Mathematics Leaders. That, in turn, mediated one of the aims of my study. I appreciate that due to CHAT's influence on the research design, criticisms could be made that this provided a narrow perspective of the research problem. The use of second-generation CHAT requires a close study of the historical and cultural context in which the activity takes place (Engeström, 2001, 2015). With that comes some limitation in generalising to other contexts in which mathematics leaders might work. Despite this, CHAT provided for me the necessary mediational means that facilitated the construction of my response to the research question.

Another limitation of my study, linked to the previous one considering the context-specificity of CHAT, is that the data generation took place within a distinctive historical and cultural context. Data were only generated in three Melbourne Archdiocesan schools, and the study was tightly context-bound, in that the schools in which the School Mathematics Leaders worked had once participated in the CTLM project. I acknowledge that there could be questions about generalisability of the findings considering the large-scale nature of CTLM. The project and its requirements for school staff participation, along with the additional support provided to the School Mathematics Leaders by CEOM and ACU staff were significant features of that project. Despite this specific historical and cultural context, I believe that the claim that I have generated using the concept of resourceful practice (Edwards, 2005, 2010) to conceptualise the School Mathematics Leaders' contribution to project sustainability has relevance to practitioners and researchers outside of that geographical, historical, and cultural context.

I am aware of the focus that was only on the School Mathematics Leaders' activity. My study concentrated on interpreting the research problem from their lived experiences as they sought to contribute to project sustainability. Therefore, the data that I generated and used to answer the research question are from their perspective as middle leaders of site-based professional learning. The stories of the principals and the classroom teachers are not told within my thesis. I appreciate that this could be offered as a critique of my research activity, as I know that the issue of project sustainability is complex (Bobis, 2011; Datnow et al., 2005; Hargreaves & Fink, 2003; Zehetmeier & Krainer, 2011).

The final limitation is that I did not investigate the impact of the project sustainability on students' mathematics learning outcomes. I appreciate that there are questions about the impact of middle leadership on student learning outcomes. I also did not study the effects of the School Mathematics Leaders' professional learning leadership activity on their classroom teachers' practice in terms of sustaining teaching reforms. This was beyond the scope of the thesis, but I acknowledge the studies of sustainability of projects can be investigated in terms of its effects on student mathematics learning and how that is mediated through teachers' practice in classrooms, mediated by continuing professional learning beyond project participation.

8.6 Further Research Opportunities

The focus of my research was on the School Mathematics Leaders' leadership activity, and therefore, offered new perspective on how they as middle leaders contributed to project sustainability. The findings of my thesis highlight the crucial role that the School Mathematics Leaders played as agents of project sustainability through the enactment of their resourceful practice. My findings have contributed to the limited literature concerning the sustainability of projects and have advanced insights into the work of mathematics leaders as middle leaders within school settings. There are, however, further questions raised by my study that offer further research opportunities into mathematics leadership activity.

An element of the research design that could not be accommodated due to the scope of study was an investigation into how the teachers interpreted the resourceful practice of the School Mathematics Leaders. There is potential to study how the teachers enacted their professional learning concerning task selection and implementation that was the major focus of the mathematics leaders' developmental motive object worked on through the facilitated planning meetings. With the School Mathematics Leaders repurposing the CTLM tasks as project sustainability tools, it would be important to investigate how classroom teachers interpreted the use of those tasks and if they shared the same understanding of their repurposing. Understanding the influence of the School Mathematics Leaders and their sustainability efforts on teachers' practices in the classroom could provide a fuller picture of the impact of the mathematics leaders' resourceful practice.

In Chapter 7, I offered the idea of the expansive discussion as a cultural tool that School Mathematics Leaders could use to resolve tensions in teachers' PCK and MCK. Further research opportunities lie in the development of those discussions as a project sustainability and mathematics leadership tool. Studies could explore the structure, content, and rules of concerning expansive discussions, working alongside mathematics leaders in their design and use. The aim of that research could include understanding how the expansive discussions mediate opportunities the development of a hierarchy of motive objects that acts as a conflict resolution tool (Kaptelinin, 2005). Generating knowledge about how expansive discussions remediate the School Mathematics Leaders' activity, where both relational and development motive objects can be simultaneously worked on, could provide important insights for mathematics leaders and researchers alike.

For my own immediate study of mathematics leaders and their leadership activity as agents of project sustainability, I plan to investigate two aspects related to my thesis. I am currently involved in two projects with mathematics leaders in Victoria and New South Wales. The sustainability of the projects has been identified as an aim for the project designers and for the mathematics leaders participating in the projects. Planning for mathematics leadership activity forms part of the intent and content of those projects, and under my influence as an academic lead, mathematics leaders design and lead school-based projects that address tensions in mathematics teaching practices enacted in their schools. One purpose of the projects is that beyond participation, the mathematics leaders adopt and adapt conceptual tools that mediate their leadership of project sustainability.

The first aspect of the projects that I am interested in pursuing is investigating the potential for mathematics leaders to design school-based professional learning projects that are influenced by knowledge of project diffusion and its characteristics (Zehetmeier, 2014, 2015; Zehetmeier &

Krainer, 2011). I suspect that those characteristics can be adapted for use by mathematics leaders and used as concepts to support the design of their professional learning projects with sustainability of changes built into the project design. I am interested in researching how the mathematics leaders interpreted project diffusion and its characteristics and how they were adopted as cultural tools that facilitated the design of their school-based projects.

The second research idea that I am interested in with those two projects concerns a deeper investigation into the relational dimension of the mathematics leaders' activity as middle leaders of mathematics. My findings suggest that relational trust plays a crucial role in mathematics leadership. I have found inspiration in the work of Edwards-Groves and Grootenboer (2021), concerning relational trust and its interconnected dimensions. I am interested in studying in greater detail how relational trust is realised within mathematics leadership, testing how the relational trust dimensions penetrate the motive objects of mathematics leaders' activity. I am also keen to study how relational trust might be used as a mathematics leadership tool that mediates conditions for mathematics professional learning facilitated by School Mathematics Leaders.

8.7 Concluding Remarks

As I draw the thesis to conclusion, having articulated my focal theory along with the contributions, implications, limitations, and further research opportunities, it is important that I pause and reflect. The opportunity I had to enact this study and discuss the findings was due to the generosity of the three School Mathematics Leaders from whom I learned a great deal. I intend to honour the stories of their leadership activity through publications and further research as a means of lifting the profile of mathematics leadership and their important contribution as agents of project sustainability.

I see this research topic of mathematics leadership being an area of further research for myself as I work on understanding more about the complexities of this form of leadership as middle leading activity. Of particular interest for myself is understanding how I have generated theory that explains that, through enactment of a specialised form of resourceful practice, the School Mathematics Leaders contributed to project sustainability, and they need to be recognised as essential players within the sustainability factor of school leadership.

In Chapter 1, I intimated that this thesis was part of the expression of my personhood (Stetsenko, 2012) as a researcher entering the field of mathematics education, alluding to the concept of the person-practice dialectic (Edwards, 2017). As I saw this concept enacted in the leadership activity of the School Mathematics Leaders, I connected with its realisation within my own research activity. When I started the journey with my doctoral study, I struggled to conceptualise the work understanding that I needed to draw together the lived experiences of the School Mathematics Leaders with CHAT and the background literature.

As I draw this chapter to a close, I am reminded that as my research activity developed so did I as a person. To look back at the pages of my thesis, I am heartened by how much my research activity has developed over time, and how I have developed as a mathematics education researcher. Like the School Mathematics Leaders who engaged in struggle with the contradictions and practice problems they faced, I have also sought ways to work resourcefully and creatively, drawing on historical practices (e.g., literature, data generation and analysis methods) from researchers who have gone before me. I have come to appreciate the place of the person-practice dialectic, the transformative and dialectical nature of motive objects, and the historicity of education research influencing my personhood as newly inducted member of the mathematics education research community. I look forward to what the future brings for me and my research work.

The conclusions that I have drawn from my research activity confirm that the School Mathematics Leaders, as middle leaders in their schools, played an essential role in contributing to project sustainability. The enactment of their resourceful practice, realising their multi-motivational activity as agents of project sustainability whilst grappling with complex post-project problems of practice, admirable, aspirational, and worthy of recognition.

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APPENDICES

Appendix A: Human Research Ethics Committee (ACU) Permission

ACU
AUSTRALIAN CATHOLIC UNIVERSITY

Australian Catholic University
Human Research Ethics Committee
Project Approval Certificate

Chief Investigator(s)/Supervisor(s):	Assoc Prof. Janeen Lamb
Co-Investigator(s):	Professor Suzy Edwards
Student Researcher(s):	Mr Matt Sexton
Project title:	The work of school mathematics leaders who support classroom teachers through professional learning following participation in a school mathematics improvement project: An activity theory perspective.
Project approval date:	05 April 2014
Project approval end date:	31 December 2019
Human Research Ethics Committee (HREC) Register Number:	2014 52Q

This is to certify that the above application has been reviewed by the Australian Catholic University Human Research Ethics Committee (ACU HREC). The application has been approved for the period given above.

Continued approval of this research project is contingent upon the submission of an annual progress report which is due on/before each anniversary of the project approval. A final report is due upon completion of the project. A report proforma can be downloaded from the website (link below).

Researchers are responsible for ensuring that all conditions of approval are adhered to and that any modifications to the protocol, including changes to personnel, are approved prior to implementation. In addition, the ACU HREC must be notified of any reportable matters including, but not limited to, incidents, complaints and unexpected issues.

Researchers are also responsible for ensuring that they adhere to the requirements of the *National Statement on Ethical Conduct in Human Research*, the *Australian Code for the Responsible Conduct of Research* and the University's *Research Code of Conduct*.

Any queries relating to this application should be directed to the Research Ethics and Integrity Office (Res.Ethics@acu.edu.au).

Kind regards,

Tanya Quesnel

Research Ethics & Integrity Officer
On behalf of the ACU HREC Chair, Associate Professor Michael Baker

Research Ethics and Integrity | Research Services, Office of the Deputy Vice-Chancellor (Research)
Australian Catholic University
T: +61 2 9739 2646
E: Res.Ethics@acu.edu.au
W: ACU Research Ethics and Integrity

Appendix B: Policy and Research Committee (CEOM) Permission

Catholic Education Office
Archdiocese of Melbourne

GE14/0009 Project #2022 Sexton
26 May 2014

Mr Matthew Sexton
Australian Catholic University
115 Victoria Parade
FITZROY VIC 3065

Dear Mr Matthew Sexton

I am writing with regard to your research application received on 12/05/2014 concerning your forthcoming project titled *School mathematics leaders support classroom teachers through professional learning following participation in a school mathematics improvement project*. You have asked approval to involve a Catholic school in the Archdiocese of Melbourne, as you wish to involve students.

I am pleased to advise that your research proposal is approved in principle subject to the eight standard conditions outlined below.

1. The decision as to whether or not research can proceed in a school rests with the school's principal, so you will need to obtain approval directly from the principal of the school that you wish to involve. You should provide the principal with an outline of your research proposal and indicate what will be asked of the school. A copy of this letter of approval, and a copy of notification of approval from the organisation's/university's Ethics Committee, should also be provided.

2. A copy of the approval notification from your institution's Ethics Committee must be forwarded to this Office, together with any modifications to your research protocol requested by the Committee. You may not start any research in Catholic Schools until this step has been completed.

3. A *Working with Children* (WWC) check – or registration with the Victorian Institute of Teaching (VIT) – is necessary for all researchers visiting schools. Appropriate documentation must be shown to the principal before starting the research in the school.

4. No student is to participate in the research study unless s/he is willing to do so and informed consent is given in writing by a parent/guardian.

1 of 2

James Goold House, 228 Victoria Parade, East Melbourne VIC 3002 Tel: (+61 3) 9267 0228 Fax: (+61 3) 9415 9325
Correspondence: PO Box 3, East Melbourne VIC 8002 Email: director@ceomelb.catholic.edu.au www.ceomelb.catholic.edu.au
ABN 85 176 448 204

5. Any substantial modifications to the research proposal, or additional research involving use of the data collected, will require a further research approval submission to this Office.

6. Data relating to individuals or the school are to remain confidential.

7. Since participating schools have an interest in research findings, you should consider ways in which the results of the study could be made available for the benefit of the school community.

8. At the conclusion of the study, a copy or summary of the research findings should be forwarded to the Catholic Education Office Melbourne. It would be appreciated if you could submit your report in an ***electronic format*** using the email address provided below.

I wish you well with your research study. If you have any queries concerning this matter, please contact Ms Shani Prendergast of this Office.

The email address is apr@ceomelb.catholic.edu.au.

Yours sincerely

[redacted]

Anna Rados
MANAGER ANALYSIS, POLICY & RESEARCH

Appendix C: School Mathematics Leader Role Description (ACU & CEOM, 2011)

School Mathematics Leader's Role within CTLM (ACU & CEOM, 2011)

The overall aim of the role is to facilitate the maximum benefit the school can derive from Contemporary Teaching and Learning in 2011 and 2012, in terms of enhanced teaching and learning in mathematics. The following are suggested components of the role for the information of Principals and CTLM SMLs.

Leadership

- Promote a culture that values mathematics learning for both students and teachers.
- Build a shared vision relating to the teaching and learning of mathematics across the school.
- Oversee professional learning to support the implementation of CTLM.
- Facilitate team discussions and planning to encourage shared beliefs and understandings relating to teaching and learning of mathematics.
- Conduct regular (fortnightly) Mathematics Professional Learning Team meetings that focus on issues, tasks relating to the CTLM project (e.g., BSAs, planning, teaching and learning strategies, student assessment data, and professional readings).
- Support teachers by mentoring and modelling teaching approaches emerging from the professional learning program.
- Build the capacity of maths team to enable them to support the implementation of CTLM.
- Build the capacity of staff to become reflective practitioners and effective teachers of mathematics.
- Provide ongoing encouragement and support to teachers to keep up the enthusiasm, and affirm and celebrate success.

Organisation and management

- Facilitate the implementation of the Mathematics Improvement Plan and Mathematics Annual Action Plan.
- Facilitate on-going review of mathematics policy, and teaching and learning practices.
- Assist with processes to maximise attendance and engagement of all staff at full day professional learning days and PLT meetings.
- Ensure all aspects of the program are successfully and effectively organised and implemented, including supporting teachers to complete between-session activities (BSAs) and recommended readings following each of the PL days.
- Ensure the on-going monitoring and assessment program is planned and monitored.
- Protect the one-hour daily mathematics session and mathematics learning environment from unnecessary interruptions.
- Conduct a regular audit on mathematics equipment and resources, to ensure they are appropriate and adequate.
- Purchase, maintain, organise and allocate resources for classroom teachers.
- Disseminate parent permission letters for student interviews, photos etc., and monitoring return.
- Generate Excel class lists of parent approvals.
- Facilitate and timetable teacher release to observe lessons by ACU or CEOM staff and pre-discussion and post -lesson debriefing sessions.

- Complete the school visit template and email to ACU two weeks prior to school visit.
- In Intake 4 schools, liaise with relevant ACU staff and SAMs in advertising and setting up the parent nights.
- Ensure students in classrooms in which school visits are conducted are wearing nametags (large text, first name only). We ask that those who cannot be photographed have a large red dot on their nametag.
- Outline the purpose of school visits in terms of teacher and student learning.
- Circulate school visit observation sheets to teachers one week prior to the visit.

Consultation with others/liaison

- Act as a channel of communication between CTLM teachers, ACU research team, Catholic Education Office (Melbourne), SAMs, parent community and the Principal, as follows:
- Regular contact with School Advisers Mathematics.
- Regular contact with Ann Downton (ACU CTLM co-ordinator) regarding school visits, PL program.
- Regular contact with school community via newsletter regarding CTLM project, and parent mathematics evenings.
- Intranet communication with staff re PLT minutes, reminders of CTLM tasks, classroom visits from ACU, CEOM staff.

Appendix D: Information Letter for School Mathematics Leaders

PARTICIPANT INFORMATION LETTER

PROJECT TITLE:
The professional learning leadership work of School Mathematics Leaders in CTLM schools

PRINCIPAL INVESTIGATOR: Dr Janeen Lamb
STUDENT RESEARCHER: Matt Sexton
STUDENT'S DEGREE: Doctor of Philosophy

Dear School Mathematics Leader,

You are invited to participate in the research study described below.

What is the study about?
This study investigates the leadership activities of School Mathematics Leaders (SMLs) who lead professional learning for classroom teachers. The focus of the study will be how SMLs enact leadership of professional learning now that the *Contemporary Teaching and Learning of Mathematics* (CTLM) project has finished. This study aims to identify the achievements and the tensions of the SML role, and how contexts and conditions affect the mathematics professional learning opportunities that are provided by SMLs who work in Catholic primary schools that participated in the CTLM project.

Who is undertaking the study?
This study is being conducted by Matt Sexton and it will form the basis for the degree of Doctor of Philosophy at Australian Catholic University, under the supervision of Dr Janeen Lamb and Associate Professor Suzy Edwards.

Are there any risks associated with participating in this study?
There are no foreseeable risks associated with this study. Although it is not expected, there is a slight risk that some participants may experience some distress in the process of self-reflection. If distress is experienced, counselling will be offered and made available to you.

What will I be asked to do?
Your participation in this study will be during the 2014 school year. If you are interested in participating in this study, you will be invited to attend an information seminar (approx. 60 minutes duration). The information seminar will be conducted away from your school setting. During this seminar, Matt Sexton will describe the details of the study and you will have time to ask questions. There will be no obligation to agree to be a part of this study. If you are interested in participating, you will be requested to sign a consent form at the conclusion of this seminar.

After agreeing to be a participant in this study, you will:

- participate in one (1) workshop, outside of your school setting which will be facilitated by Matt focusing on the study's framework and data collection procedures (60 minutes);
- participate in eight (8) interviews over the span of 4 to 6 months that will be digitally recorded (60 minutes per interview);

- provide permission to be observed by Matt as you conduct your professional leadership work activities for five (5) hours a day over separate four (4) days. This should not be too onerous because Matt will take an observer role recording field notes as you conduct your leadership work. Photographs of documents/artefacts that you use in your work will be taken during these observation times;
- provide access to documents/artefacts that you use in your work;
- keep a professional learning record, which is anticipated to take 3-5 minutes to complete per day for a period of two (2) weeks when Matt is not observing your leadership work; and,
- collaborate with Matt to co-analyse some of the data that are collected. This will take place during some of the interviews stated above.

What are the benefits of the research study?

You will be provided with a framework that you will use to reflect upon your work practices related to the leadership of professional learning in your school setting. The experiences of engaging with Matt to identify the aspects of your leadership activities before, during, and after your school's participation in the CTLM project may contribute to a greater awareness of your own leadership capabilities. By working with the Matt, you have the opportunity to identify achievements as well as tensions and contradictions in your own work activities. This may also provide you with deeper understandings of the contexts and conditions in which you are required to work, possibly leading to ways of developing your practices further and devising ways of working with tensions in your leadership role. This study also has the opportunity of highlighting the important work that mathematics curriculum leaders like you do in primary schools. It is hoped that this study will increase the profile of the mathematics curriculum leadership role in primary schools, and provide insights into the achievements and challenges associated with the role, particularly after involvement in a school mathematics project like CTLM. This study is also important because there is a lack of information that describes the vital work that mathematics curriculum leaders do in primary schools. Moreover, it provides an opportunity for other school mathematics leaders, policy makers, education sectors, and researchers to know more about the achievements, tensions, contexts and conditions that face mathematics curriculum leaders as they lead mathematics teaching and learning in primary schools.

Can I withdraw from the study?

Participation in this study is completely voluntary. You are not under any obligation to participate. If you agree to participate, you can withdraw from the study at any time without adverse consequences. If you do withdraw from the study, any data that were collected concerning your work will also be withdrawn and not reported.

Will anyone else know the results of the study?

All electronic data and associated results will be stored in encrypted files during and after the completion of this study. All paper documentation and associated results will be stored in lockable filing cabinets to which only the researchers will have access. Your confidentiality, and that of your school, will be protected throughout and after the study by the use of pseudonyms. These pseudonyms will be used by the researchers when transcribing audio files, writing of results for the thesis, and when publishing findings in journals and conference proceedings. The results of this study will be available through the ACU digital theses. Results will also be published in education journals and conference proceedings, and may be provided to other researchers in a form that does not identify you in any way.

Will I be able to find out the results of the study?
By participating in this study, you will work with Matt by reflecting on your own school mathematics leadership. The process that Matt will use involves you in collaborative ways. The results of the study related to your leadership activities will be known to you because Matt will co-analyse the data with you. A report will be made available to you that will be written for your school and for the Catholic Education Office Melbourne (CEOM).

Who do I contact if I have questions about the study?
Please contact Matt by phone on 03 9553 3283 or email matthew.sexton@acu.edu.au if you have any questions about the study.

What if I have a complaint or any concerns?
The study has been reviewed by the Human Research Ethics Committee at Australian Catholic University (approval number 2014 52Q). If you have any complaints or concerns about the conduct of the study, you may write to the Chair of the Human Research Ethics Committee care of the Office of the Deputy Vice Chancellor (Research).

Manager, Ethics
c/o Office of the Deputy Vice Chancellor (Research)
Australian Catholic University
North Sydney Campus
PO Box 968
NORTH SYDNEY, NSW, 2059
Ph: 02 9739 2519
Fax: 02 9739 2870
Email: res.ethics@acu.edu.au

Any complaint or concern will be treated in confidence and fully investigated. You will be informed of the outcome.

I want to participate! How do I sign up?
Please email Matt to say that you have read this information letter and that you are interested in attending the information seminar about the study. It is appreciated if this email is sent to Matt by 30 June 2014. Matt will contact you to inquire into times that you are best suited to attend the seminar. Attempts will be made to accommodate the times you provide. If you are still interested in participating in the study once you attend the information seminar, you will be asked to complete consent forms by signing both copies of the form. You will keep the copy labelled "Copy for Participant to Keep" and Matt will collect the copy labelled "Copy for Researcher".

Yours sincerely,

[redacted]
Dr Janeen Lamb
(Principal investigator)

[redacted]
Mr Matt Sexton
(Student researcher)
matthew.sexton@acu.edu.au

Appendix E: Consent Forms

AUSTRALIAN CATHOLIC UNIVERSITY

CONSENT FORM

Copy for Participant to Keep

TITLE OF PROJECT:
The professional learning leadership work of School Mathematics Leaders in CTLM schools

PRINCIPAL INVESTIGATOR: Dr Janeen Lamb
STUDENT RESEARCHER: Mr Matt Sexton

I .... *(the participant)* have read and understood the information provided in the Letter to Participants and the information shared at the study's information seminar. Any questions I have asked have been answered to my satisfaction. I agree to participate in this study that investigates my leadership of the professional learning that I provide for the classroom teachers in my school. I understand that I will:

- attend a workshop about the study's framework and data collection methods (60 minutes)
- participate in eight (8) interviews over the span of 4-6 months that will be digitally recorded (approx. 60 minutes per interview)
- be observed as I conduct my professional learning leadership activities while field notes are recorded and photographs of documents/artefacts are taken (4 separate days, 5 hours per day)
- provide access to documents/artefacts that I use in my work as a School Mathematics Leader
- keep a professional learning record that logs the professional learning opportunities I provide my teachers (anticipated to take 3 – 5 minutes per day for a period of two weeks)

I realise that I can withdraw my consent at any time without adverse consequences. If I withdraw, I understand that any data collected about my work will be withdrawn and not be used by the researcher(s). I agree that research data collected for the study may be published or may be provided to other researchers in a form that does not identify me in any way.

NAME OF PARTICIPANT: ....

SIGNATURE: .... DATE: ....

NAME OF PRINCIPAL INVESTIGATOR: DR JANEEN LAMB

SIGNATURE: [redacted] DATE:

NAME OF STUDENT RESEARCHER: MR MATT SEXTON

SIGNATURE: [redacted] DATE:

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Appendix F: Interview Protocol (During CTLM Leadership)

Interview protocol

Methodological action of Research Process

Clarify the historicity of the School Mathematics Leader (SML) activity system.

Leadership of professional learning during participation in the CTLM project

Q): Did you bring any documents and/or artefacts that you used in your leadership of professional learning ***during*** *participation in CTLM?*

Please feel free to refer to any of these documents or artefacts during the interview if they help with your responses. With your permission, I might take some photos of these documents when you use them to respond to the questions or if it is possible may I take some copies?

History of the SML activity system: During CTLM

The questions I pose should help with recalling that information but at any time, please feel free to ask me any clarifying questions.

Q): So, during your school's participation in CTLM, how much release time was provided so you could fulfil your role of School Mathematics Leader?

Q): Were you required to enact any other roles in the school besides this role? If so, what roles were they?

Ask if other roles were enacted at that time

Q): How did these roles impact on your School Mathematics Leadership role?

Q). Can you think back to that time during CTLM? What were you trying to achieve in regard to your mathematics professional learning leadership for your classroom teachers during those years of CTLM?

Q). Do you remember what types of resources, documents, tools or knowledge (artefacts) you used to help you achieve those things that you wanted to achieve? If you have brought some along, can you tell me about them? How did this (artefact) come about?

Q). What were the principles or rules that were set up that guided your approach to mathematics professional learning for your teachers during CTLM?

Q). How did you organise yourselves around mathematics professional development during CTLM, specifically the PD that you led?

Q). Who was involved in the mathematics professional development? Who supported you when you were planning and facilitating PD for your teachers?

Q). Do you remember any specific PD activities that you enacted during CTLM?

Q). What was not quite working with maths PD back then? What did you need to develop and why?

Q). Is there anything else you want to tell me about the maths PD that you led or your mathematics leadership during CTLM?

Closing questions

Q): In general, were there the significant changes to your professional learning leadership from before participation in CTLM and during participation in that project?

Q): What kind of impact did participation in CTLM have on your leadership of mathematics professional learning?

Appendix G: Interview Protocol (Post-CTLM Leadership)

Interview protocol

Methodological action of Research Process

Conduct a Detailed Inquiry into the Present SML Activity System

Use the following questions with the SML before and after observation of their activity.

Before observation

Thank you for your time today. Before we go into the session, can we talk a little bit about what you plan to achieve?

Q): What are you working on today with the teachers and why are you focusing on that?

Q): What are you hoping to achieve?

Q): What resources will you be using today? Where did they come from?

Q): How will the teachers work today? What will they be expected to do?

Q): What else is important for me to know about before the PL session starts?

After observation

Thank you for allowing me to observe you in action. I would like to chat about some aspects of your leadership work.

Q): What are your reactions to the session that you just led?

Q): How did you think you achieved what you wanted to? Why do you think that way?

Q): How did the resources help today? Were they unhelpful in any way?

Q): In the session, I noticed that you (share observation). Can you tell me more about that and why you did it?

Q): Was there anything that you did today that was like your leadership during CTLM? Is there anything different?

Q): What will you work on next and why?

Closing question

Q): What else is important for me to know about today's session that I have not yet asked? Is there anything else you want to tell me?

Appendix H: Observation Protocol

School Mathematics Leader Observation Protocol

Methodological phase:

Conduct a detailed inquiry into the present School Mathematics Leader (SML) activity system.

Data that captures the actions and verbal responses of the School Mathematics Leader will be recorded using this observation protocol. This will be done using handwritten notes (with short-hand conventions) in the observation journal. Data generated from non-participants recorded as utterances with no references to any names (e.g., classroom teachers, school students, or other staff members) will be written. Convention of "Teacher 1" and "Teacher 2" will be used. Those data may be reported to provide further context for SML leadership activity. I will provide the teacher with a one-page information letter of the study for their convenience if there are questions about the observations.

Before observation:
Thank you for allowing me to observe your leadership work today. I will observe you as you undertake your leadership. I will be as unobtrusive as I can so I will sit to the side. I will be writing notes as you work. After today's observation, I will type my notes and then when I see you next time, I will ask that you check the accuracy of my observations, comparing them with your recollections..

Before we begin the observation today, I would like to remind you that participating in my study is voluntary. At any point today, if you would like me to not take any notes or cease observing you, just tell me to do so. I will audio-record the meeting so that I can cross-check my observation notes with what you said during that time. I will not use your name or any teachers' names. Do you have any questions about the observation before we begin today?

After the observation, I am hoping to talk about your leadership activity. I may ask questions for my own understanding of your important work. Is this okay with you?

During observation:

- Use observation journal handwriting observations.
- Generate data in five-minute intervals. Record time intervals.
- Highlight aspects that require discussion in the 'after observation' interview using asterisk (*)
- Record information including: date, time, SML pseudonym, focus of meeting, meeting space and setup, teacher grouping (year level)
- Record notes as recount of SML activity in professional learning situation
- Take observations notes, paying attention to:
    - What is the SML working on in that meeting?
    - What does the SML say and do in the meeting?
    - What tools are being used and what type are used to mediate the SMLs' work?
    - What knowledge does the SML use? (PCK, MCK, knowledge of colleagues/students?)
    - How does the SML respond to teachers?
    - Who does what in the meeting? How is the work organised?
    - What rules appear to be in place?

After observation:
Thank you for allowing me to observe your work today. Could we talk about that leadership work we just saw? Firstly, how do you think it went? What are your reactions to that meeting? Did you achieve what you wanted to?

(Use questions that clarify observation notes)

Day of observation after school visit:

- Type handwritten notes onto Word file.
- Write notes as recount of SML activity.
- Cross-check notes with audio recording.
- Prepare observation notes for member-checking at next school visit.

Appendix I: Analytical Memo Example

Name of participant: Cindy

School visit date: 02 November 2017

Observation focus: PLT meeting (sharing of insights from NAPLAN data 2017 focusing on Year 3 and Year 5 trend data)

Arrival time: 2:35 PM **Departure time:** 5 PM

Motive object

Pre-meeting interview, Cindy said that her work today was focused on the following:

- Supporting teachers in reading and interpreting NAPLAN data results
- Setting goals for teaching for remainder of school year

Analysis of observation:

- Working on development of relational trust (Edwards-Groves et al., 2016)
- Using the data to convince the teachers of the need to continue with the CTLM practices; persuading teachers to see that NAPLAN data need improvement.

Tools – artefacts mediating professional learning activity:

Physical tools	Psychological tools
• Computer, projector • PowerPoint software	• NAPLAN data • Knowledge of children's mathematical performance • Assessment data of children's mathematical performance • PCK - advocated in the CTLM project (use of open-ended tasks; enabling and extending prompts)

Observation summary and analysis

Observation (summary)	Motive object
• Even though Cindy said PLT meeting focus meeting (motive object) was to support teachers in interpreting the NAPLAN data, teachers were not taught interpretative skills. • Teachers listened to Cindy as she pointed to dataset and displays. Cindy referenced the need to support students in higher achievement. • Cindy often told the teachers that she understands that NAPLAN data are only one source of assessment data, and that teaching mathematics is challenging. Cindy reminded the teachers in the meeting that "it's only NAPLAN data". • Cindy kept referring to the importance of needing to improve the NAPLAN data and asked how "we" can do that; wanted collaborative decision-making. • Cindy referred to keeping going with what was learned in CTLM, and asking question like "Do you we need to keep going back to what we learned in CTLM?" • Cindy referenced open-ended tasks and use of enabling and extending prompts (Sullivan, 2009) • Frequent references to "we" and that "we are in this together", and "it's everyone's responsibility".	Cindy's stated motive object before interview: Supporting teachers' interpretation of NAPLAN data and setting goals for teaching for rest of year. **My analysis** • Working on **relational trust** building specifically **intersubjective trust** - references to "journey" and "everyone together", and **interpersonal trust** - demonstrating empathy and managing teacher responses to NAPLAN data (Edwards-Groves et al., 2016); • Using NAPLAN data to convince teachers to sustain teaching reforms initiated in CTLM – using assessment data as persuasion tool.

Relational activity of School Mathematics Leaders

Observation evidence	Deductive code	Category
When Cindy mentioned that it was alarming that high-achieving students were making "low progress" from Year 3 in 2015 to Year 5 in 2017, she quickly highlighted other students who made "high progress". Cindy reminded the teachers in the meeting that "it's only NAPLAN data". Cindy used those data in a way to show that their teaching is having effect on some students; she used the phrase "this is worth celebrating" with the teachers. Cindy made frequent references to "being in this together" and that it was "everyone's responsibility" for the NAPLAN data. Cindy invited teachers to discuss questions and future actions; she invited teachers to talk about addressing the needs of high-achieving students. Cindy referenced CTLM ideas, specifically open-ended tasks, and differentiation prompts; she said the data are reason to continue with the practices learned in CTLM. Cindy invited input from teachers for agreement with her interpretation of NAPLAN data and the need to continue with CTLM practices that she said she knew "worked".	**Interpersonal trust** (Edwards-Groves et al., 2016) • Managing emotional responses to NAPLAN data. • Maintaining teacher affect and monitoring ways of not coming across as judgemental. • Preserving teacher self-esteem and self-worth. **Intersubjective trust** (Edwards-Groves et al., 2016) • Reminding teachers of team approach and a sense of "witness". **Interactional trust** (Edwards-Groves et al., 2016) • Opening spaces for dialogue; inviting questions from teachers • Creating opportunities for teachers to share by inviting discussion and input. **Intellectual trust** (Edwards-Groves et al., 2016) • Demonstrating knowledge of tasks (selecting tasks and implementation) through using of extending prompts.	**Relational trust building** - the motive object of activity The relational motive object is becoming more obvious in the work of the SMLs. Edwards-Groves et al. (2016) recently wrote about relational trust and middle leading. Relational trust supports conditions that enable middle leaders to engage in developmental work. There are five dimensions of relational trust. They are proving helpful with interpretation of the SMLs' activity. My interpretation of Cindy's activity in the PLT meeting suggests that she works on developing all five dimensions of relational trust in dynamic ways. **Leadership action:** Using NAPLAN data as a convincing tool to persuade teachers for the need to continue with the mathematics reforms.
Cindy told the teachers that she was aware of how busy they were at that time of the school year; she also reminded them that she knows what it was like now that she had classroom responsibilities whilst enacting the SML role.	**Interpersonal trust** (Edwards-Groves et al., 2016) • Enacting empathy; understanding of teachers' workloads.	
Cindy used part of the meeting asking the teachers to identify a goal about using open-ended tasks and extending prompts; she sat with a group of teachers and supported them drawing on knowledge of open-ended tasks that she uses and how to build in extending prompts that they could use with students. Cindy references Peter Sullivan when discussing extending prompts.	**Pragmatic trust** (Edwards-Groves et al., 2016) • Goal setting and making the work of extending student learning practical and achievable. **Intellectual trust** (Edwards-Groves et al., 2016) • Demonstrating knowledge of tasks (selecting tasks and implementation) through using of extending prompts.	

Appendix J: Example of Coding Scheme (CHAT concepts)

Deductive coding scheme – CHAT concepts

Code	Definition/description	What to look for:	Example of data
Motive object	What is being worked on or wants to be achieved: • desire and a goal pursued by the subject (SML) • want that drives the activity; understanding an activity system requires understanding its object. • "thing-acted-upon	***What is being worked on by the SMLs?*** Tasks and undertakings Phrases: - working on, working at, spending time on,	Listening to them and being empathetic are important in my maths leadership. I have to be mindful of that. I think if you work on that, then you find people will engage in the professional learning." (Rachel, PPI, 29.04.15)
Tools	Physical (material) and psychological (conceptual) artefacts that are used by the SML including knowledge types employed to achieve the motive objects	***What is being used by the SMLs?*** Material tools (computers, pens, concrete materials) Conceptual tools (curriculum documentation, planning documentation) Knowledge (MCK & PCK), knowledge of teachers	I know that there's a need for a couple teachers in our school that missed out on the CTLM to learn more about what we do here. I know they do not have the same understanding of maths teaching as those that did do CTLM with us" (Cindy, PPI, 06.11.14).
Rules	Routines and norms in place, both explicit and implicit, that mediate the motive object; regulate the actions of SMLs; rules can be from the broader cultural, economic, and political context; reflect professional norms	***What governs what the SMLs do and cannot do?*** References to: - how they do things in their schools - what routines are in place? - rules that enable and constrain	"I care about the teachers, and I care about them becoming good maths teachers. That care has to be in place." (Cindy, PPI, 06.12.16)
Division of labour	Responsibilities for work done within the activity system; what is being done by whom toward the motive object, including the horizontal division of tasks and the vertical division of power, positionality	***Who does what and why in the SMLs' activity?*** ***How is the SMLs' leadership activity organised?*** Distribution of tasks? How the SMLs lead? References to authority and power (and lack of)	"I did that task for them so that for the teachers will use those materials in their classrooms." (Penny, PPI, 25.03.15)
Community	People who mediate the motive object; people who share an interest in and involvement with the same motive object as the subject	***With whom do the SMLs work?*** People in the school (teachers, principals, others?)	"It is really helpful still having teachers who were in CTLM here at the school. It keeps the work we did going." (Rachel, PPI, 14.11.16)
Contradiction	Tensions, problems, breakdowns in the activity system Manifestations: Dilemmas Conflicts Critical conflicts Double binds	***What is disrupting the SMLs work? What history does that have?*** Choice between two incompatible options; "on the one hand…but then on the other hand"; references to being torn. Disputes between SML and community; arguments; blocking behaviours	I sometimes question whether what I do is seen as something valuable nowadays. I ask myself, "don't you care about me and my work?" (Penny, PPI, 25.03.15)

Appendix K: Information Letter for Classroom Teachers

AUSTRALIAN CATHOLIC UNIVERSITY

Information letter for Classroom Teachers

Mr Matt Sexton
ACU (Melbourne Campus)
November 2014

Dear Classroom Teacher,

My name is Matt Sexton. I am completing a Doctor of Philosophy (PhD) degree at ACU, and the focus of my study is mathematics leadership. I am interested in how your School Mathematics Leader leads professional learning since CTLM. You may see me at your school and in professional learning situations with you and your School Mathematics Leader.

My presence in your school has been approved by your principal, ACU, and CEOM. I have ethics approval to collect data but only that which is about your School Mathematics Leader.

I understand that I will be in situations where you are present and where you may offer your ideas.

Please know that I will not collect any information about you or your ideas. I am not allowed to do that and I am not permitted to disclose anyone's identity when I collect data and write about it in my thesis. Any teacher will be referred to as 'Teacher 1' or 'Teacher 2' if I need to that as a way of telling the story of your School Mathematics Leader.

As I have shared, the focus of my study is about the leadership work of your School Mathematics Leader. If you have any questions about my study, I would be happy to talk to you about them.

Thank you for your time.

[redacted]

Matt Sexton (ACU, Melbourne)

Appendix L: Example of Data Coding

Motive object	Leadership actions	Interview data excerpt
Complying with CTLM project team requests and expectations	Changing professional learning team meeting frequency	*At the start of CTLM, well, by agreeing to do CTLM, there were agreements that had to come along with it, and one of them was fortnightly PLTs (professional learning team meetings). So, I did focus on those fortnightly PLTs with the staff on maths because that was part of the agreement. I also did that because I wanted to follow what they (CTLM project team members) wanted* (Rachel, DCTLMI, 23.04.15).
	Purchasing mathematics resources advocated by CTLM project team members	*I remember thinking that if I was going to do professional learning here at school, I had to make sure that I bought the resources that ACU said that we should have if we were going to be in CTLM. I remember I also bought materials and things like that that the SAMs said that we should also buy because we were in CTLM* (Penny, DCTLMI, 25.03.15).
	Organising school visits by CTLM project team members	*So, there'd be backwards and forwards emails, and there'd be certain deadlines and timelines that I needed to meet regarding planning for when they (ACU staff members) were going to visit. This was also for when we had our SAMs come visit. They were coming out twice a term. So, there was that responsibility of meeting their expectations for the professional learning sessions that I was organising at our school as part of my role* (Cindy, DCTLMI, 17.11.15).
	Following directives from CTLM project team members	*I do remember though feeling like I had to use those Between-Session-Activities, those tasks that ACU gave us at the end of the CTLM days. I felt like I had to use them in my meetings. I remember that feeling of being told what to do* (Penny, DCTLMI, 25.03.15).