THE INFLUENCE OF TEACHER PERCEPTIONS AND TEACHING APPROACHES ON SENIOR SECONDARY MATHEMATICS STUDENTS' USE OF CAS CALCULATORS

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The focus of this paper is to examine the extent to which teacher perceptions and approaches to CAS calculator use influence whether students utilise this technology in senior secondary mathematics. Using an embedded multiple case study approach, data were collected from two Year 11 and two Year 12 mathematics classes and their respective teachers. While the results presented highlight the key role of the teacher in fostering a CAS environment, other contextual factors also need to be taken into consideration.

INTRODUCTION

Within the past few decades, researchers in the field of mathematics education have recognised the potential for information and communications technology (ICT) to transform the teaching and learning of mathematics (Goos, Galbraith, Renshaw, & Geiger, 2000). The effective use of ICT has also been seen as a key capability within the Australian curriculum and is incorporated across all learning areas, including mathematics (Australian Curriculum, Assessment and Reporting Authority, 2015). However, while the use of technology has presented many advantages, "the vision of using information and communication technology (ICT) to transform the teaching and learning process ... [has been] far from becoming a reality" (Rodríguez, Nussbaum, & Dombrovskaia, 2012, p. 81), with concern that ICT integration in school mathematics has fallen behind the promising expectations of previous decades (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010).

Computer Algebra System (CAS) devices are an example of one such technology that have faced various obstacles in its integration within the school mathematics domain. "In spite of the long history of work with CAS in educational settings, the impact of technology on school mathematics has to date been marginal, and the incorporation of CAS in classrooms has been even slower" (Heid, Thomas, & Zbiek, 2012, p. 599). Student attitudes, time restrictions, and the technical skill required to operate CAS technology are just some of the factors that have made CAS integration within school mathematics difficult to achieve successfully (Barkatsas, Kasimatis, & Gialamas, 2009; Drijvers, 2002; Schmidt, 2010).

Goos et al. (2000) also noted that the role of the teacher was crucial in developing a technology-rich learning environment. However, as highlighted in a study by Teo (2011) involving Singaporean teachers, "perceived usefulness, attitudes towards use,

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and facilitating conditions [all] have direct influences on behavioural intention to use technology" (p. 2437). With CAS calculators forming an integral part of the Victorian senior secondary mathematics curriculum in Australia, and the significant role a teacher may play in their successful integration, the main focus of this paper will be to report on how teacher perceptions and approaches regarding CAS calculator use may influence students' utilisation of CAS technology in mathematics.

THEORETICAL FRAMEWORKS

Within this study, two theoretical frameworks were used to investigate and describe teachers' use of CAS-based systems within the mathematics education domain. These frameworks are summarised briefly below.

Technological Pedagogical Content Knowledge (TPCK)

Mishra and Koehler (2006) introduced the theoretical framework Technological Pedagogical Content Knowledge (TPCK) to describe the relationship between teaching and technology. In order to successfully integrate technology as part of teaching practice, teachers need to have three essential forms of knowledge: Technology Knowledge (knowledge of how to use technology), Pedagogical Knowledge (knowledge of teaching practice) and Content Knowledge (knowledge of the subject matter). The interaction between each of these three forms of knowledge is crucial to the development of a teachers' TPCK. As summarised by Koehler, Mishra and Yahya (2007): "at the heart of TPCK is the dynamic transactional relationship between content, pedagogy and technology. Good teaching with technology requires understanding the mutually reinforcing relationships between all three elements" (p. 741).

MSPE Framework

In their three-year longitudinal study, Goos et al. (2000) identified four metaphors to describe the interaction between teachers and technology (the MSPE Framework). Technology plays the role of *Master* if a teachers' implementation of technology is limited by their technological knowledge and skills. Technology plays the role of *Servant* when its only purpose is to support preferred teaching practices (e.g., using a calculator solely for its speed and efficiency to replace pen and paper techniques). Technology is used as a *Partner* when it aids in the implementation of a teaching practice that gives students more control over their learning, such as sharing or mediating mathematical discussions. Finally, technology is used as an *Extension of self* when its "powerful and creative use ... forms as natural a part of a teacher's repertoire as do fundamental pedagogical and mathematical skills" (Goos et al., 2000, p. 308).

METHODOLOGY

The findings presented within this paper form part of a larger mixed methods study which utilised a quantitative phase followed by a qualitative phase. The population investigated for this study were Year 11 and Year 12 mathematics students and their teachers. Local schools in the region of Melbourne, Victoria, Australia were invited to participate via a letter explaining the nature and intention of the research. Within the time frame allocated, six schools agreed to participate in the study and the findings of the quantitative phase involving a questionnaire can be found in a prior paper (Orellana & Barkatsas, 2015).

The participants for the qualitative phase, presented in this paper, came from two of the initial six schools who participated in the quantitative phase of the study. Findings from the quantitative phase aided in the selection of participants through examination of key differences between schools on variables such as technology confidence, years of CAS experience and attitudes towards using CAS calculators in mathematics. Within each participating school, one Year 11 and one Year 12 mathematics class (taught by the same teacher) were selected with the aid of the school's mathematics coordinator.

An embedded multiple case study approach was used for data collection incorporating classroom observations and interviews with students and teachers. Observations were non-participant and overt while also adopting a semi-structural approach to allow for greater flexibility and responsiveness to naturally occurring events (Flick, 2006). Interviews with participants also allowed for greater flexibility by being semi-structured and one-on-one, without diverging too far from the research aims (Berg, 1995). For the interview process, students were selected with the aid of the classroom teacher and restricted to those with parental consent. In total, 20 students were interviewed along with the two respective teachers from each school (henceforth labelled Teacher A and Teacher B).

The collected data were initially analysed using a preliminary exploratory analysis in order to obtain a general sense of the data (Creswell, 2005). Once complete, the data were analysed using a thematic analysis procedure as outlined in Braun and Clarke (2006) in order to identify key patterns or themes within the observational fieldnotes and interview transcripts. While the larger study, of which this qualitative phase forms a part, focuses on both teachers' and students' use of CAS technology, this paper will report on the findings with respect to Teacher A and Teacher B.

RESULTS

Teacher A

Teacher A was a female, secondary mathematics teacher who had been teaching in schools for over 30 years. Teaching students from Years 9 to 12, she was introduced to the CAS calculator between the years 2006-2007, when the CAS became a compulsory element of the Victorian Certificate of Education (VCE – the final two years of secondary schooling) study design. Having had no prior experience with this technology, initially her use of CAS was limited due to the fact that she was not involved in teaching VCE mathematics: "Back then [I] didn't use it as much as what I do now because I wasn't involved in [years] 11 and 12". However, with professional development and support from her colleagues, along with self-teaching, Teacher A became proficient with CAS and used it more frequently in her teaching practice.

With respect to her views on CAS calculators in mathematics, Teacher A is fairly positive about using this technology and believes it has aided her students' achievement as well as providing a means to explore mathematical concepts. However, she has also encountered various obstacles along the way, such as trouble resolving complex technological errors, over-reliance on CAS by students, and changing students' mindsets "to get them to realise that they don't have to do everything by hand". The approach adopted by Teacher A when teaching with the CAS calculator involved a more student-centred approach by using open-ended questions to get students thinking about the mathematics being taught. The calculator was used as a means to 'explore' mathematics and to aid in the development of conceptual understanding.

The Year 11 and Year 12 classes taught by Teacher A were Mathematical Methods (CAS) classes, a subject focused on calculus, algebra, functions, and probability. Students were first introduced to the CAS calculator in Year 9 and were expected to bring this technology to every class. From the observational data, and from student interviews, it was found that students in both classes encountered difficulties with the CAS calculator including syntax errors, problems with settings, and interpreting output. Students in the Year 11 classroom also struggled to understand when CAS calculator use would be more efficient than by hand methods, which may also be why the latter techniques were preferred by students regardless of whether the task was considered technology-rich or technology-free. In contrast, the Year 12 classroom used the CAS calculator more frequently. However, at times, students were too reliant on the technology, using it for questions that would have been faster by hand.

Teacher B

Teacher B was also a female, secondary mathematics teacher with over 30 years of teaching experience. Teacher B focused exclusively on VCE mathematics, teaching only Year 11 and Year 12. With no experience using the CAS calculator prior to its implementation, she learned to use the CAS through guest speakers and practising in her own time with the instruction manual. She uses CAS everyday as part of her teaching practice and incorporates it into every mathematics class.

Teacher B displayed a very positive attitude towards using CAS calculators in mathematics and was enthusiastic about learning as much as she could about this technology. She did not encounter many difficulties with the CAS and many students turned to her when they experienced any problems with hardware or software: "I fix the calculators for them". Teacher B believed that her students benefited greatly as a result of having CAS technology available, particularly lower achieving students, with improved student results, attitudes, and confidence. However, while she noted that the CAS calculator could help lower achieving students "pass exams", she also acknowledged that it may not necessarily improve their mathematical understanding:

> For those students who do not really know the value of the numbers, actually it's just useless for them ... I would say the calculator actually is spoiling them ... they don't understand how ... they get the answer because the calculator don't show them step by step.

With respect to her teaching style, Teacher B's approach focused on the development of conceptual knowledge before introducing CAS as an efficient way of obtaining solutions. However, depending on the level of achievement of her students, she has also introduced the CAS calculator as soon as possible for students who appeared to be struggling with mathematics. Unlike Teacher A, CAS was not used as a means to investigate mathematical constructs, but rather as a faster, alternative means of solving mathematical problems.

The Year 11 and Year 12 classes taught by Teacher B were General Mathematics and Further Mathematics classes respectively. These subjects place more emphasis on topics such as geometry, data analysis, and business related mathematics rather than calculus or algebra. Students were first introduced to the CAS calculator in Year 11 and were expected to bring this technology to every class. From the observational data, and from student interviews, it was found that students in the Year 11 class displayed moderate CAS calculator use. Some students used the CAS frequently, others preferred by hand methods, and others did not know how to use it. However, students were more inclined to use the CAS when they were shown how efficient it was. Additionally, students in the Year 11 class experienced difficulties knowing how and when to use the CAS calculator, requiring a lot of teacher support and guidance. In contrast, the students in the Year 12 classroom used CAS frequently to complete mathematical work, making seamless transitions between by hand and CAS techniques. CAS use came naturally to these Year 12 students as they understood both how and when to use this technology.

DISCUSSION AND CONCLUSIONS

From the results presented above, it was evident that both teachers had various characteristics in common, such as their years of mathematics teaching experience and their lack of familiarity with the CAS when it was first introduced. However, one difference between the two teachers was their experience with VCE level mathematics. Whereas Teacher B had more familiarity with Year 11 and 12 (teaching only these year levels), Teacher A had only recently become involved with VCE mathematics and thus had not used the CAS calculator as frequently when it was initially introduced. Thus, it could be argued that Teacher B had a greater understanding of how to use the CAS calculator as part of VCE mathematics teaching (TPCK) with more years of experience in this regard. Her high level of CAS knowledge (the 'T' of TPCK) also allowed her to solve a majority of her students' problems when they arose in class, whether they were technical in nature or due to incorrect notation.

Although less experienced with VCE mathematics, Teacher A was confident in her mathematics teaching knowledge (PCK) to take a more constructive and open-ended approach with technology, using the CAS calculator to explore mathematical concepts in whole class discussions. This type of instruction also reflected her belief that one of the many advantages of CAS lie in its capacity to allow students to see patterns and to think about mathematical constructs. Using the metaphors developed by Goos et al. (2000), Teacher A could be described as working with the CAS as a *partner* using this technology "creatively in an endeavour to increase the power students collectively exercise over their learning" (p. 307). For example, when introducing students to the inverse function, Teacher A asked students to take out their CAS calculator and input three functions: $f_1(x) = f(x)$, $f_2(x) = g(x)$ (the inverse), and $f_3(x) = x$. She then prompted a discussion with students, asking questions such as "What do you see?" to allow them to discover what the inverse represents. These types of discussions took up a large portion of Teacher A's classes.

In contrast, Teacher B used a more traditional and structured mode of instruction, explaining concepts on the board before moving onto CAS-based examples and questions. CAS, in this context, was used as an efficient and alternative way to solve mathematical problems (as a *servant*), encouraged as a means to save time in examinations rather than changing the nature of classroom activities (Goos et al., 2000). For example, when introducing the topic of graphs and relations, Teacher B briefly went through the key points and definitions before moving into an example where CAS could be used, outlining step-by-step instructions on the board: "Menu → Stat → Stat calc→ 2 → x list x, y list y → select $y = a + bx$ or $y = mx + c$ ". Teacher B announced to students that if they had forgotten how to find the equation of a straight line given two points, then the CAS was a "useful" way to obtain a solution. Explanations and discussions did not take up as much of the lesson as those of Teacher A, and the main focus was to complete set questions from the textbook or from provided worksheets. Thus, while there was a preference for students to develop conceptual understanding prior to learning CAS procedures, there was also an emphasis on performance and procedural knowledge. This may be why lower achieving students in her classes were, at times, introduced to CAS-based procedures earlier than their peers, despite Teacher B preferring the development of conceptual understanding prior to this.

Analysis of the classroom results revealed various differences between Teacher A's and Teacher B's participating classes. While in Year 11 the results were quite similar for both classes, students in Teacher B's classes appeared to use the CAS more frequently and efficiently than students in Teacher A's classes. In particular, Teacher B's Year 12 class encountered fewer difficulties when working with the CAS calculator and had higher levels of CAS knowledge, knowing both when and how to use this technology. Having endeavoured to learn as much as she could about the calculator, Teacher B's familiarity and confidence with the CAS appeared to have positively influenced her students. Even though students had fewer years of CAS experience compared to students in Teacher A's classes, students in Teacher B's classes were able to quickly overcome any CAS-related issues with the aid of their teacher, and appeared to be developing greater confidence with this technology, consequentially influencing their use of CAS in the classroom.

In comparison, students in Teacher A's classroom displayed a lack of CAS knowledge, encountering more difficulties when using the CAS calculator. Common errors included issues with syntax and settings, which were also evident in Year 12, although to a lesser extent. Knowing when to use the CAS was also an area students struggled to understand. For example, not using CAS for technology-rich questions, or using CAS to solve $V(t) = 0$ for $V(t) = 10^3 (90 - t)^3$. Teacher A's approach to addressing these difficulties reflected her teaching style. If students encountered errors, Teacher A preferred to use prompts to help students discover where they went wrong rather than provide them an answer. She would also use discussions to find out the different solutions that students had to solving a mathematical problem with the CAS calculator rather than giving them step-by-step instructions: "How can I use my calculator to get the rule for the inverse function?" Thus, the results, in a sense, have been counterintuitive as this approach has been suggested in prior research by Drijvers (2002), who proposed using CAS obstacles as opportunities for learning:

> Instead of trying to ignore the obstacles encountered, I suggest to make them the subject of classroom discussion ... such an approach turns the obstacles of computer algebra use into opportunities for learning, and enriches mathematical discourse in the classroom. (p. 228)

In summary, the results presented in this paper showed that the participating teachers in this study found the CAS calculator to be useful for different reasons and have incorporated this technology into their teaching based on these beliefs. However, it is difficult to determine the extent to which the teachers' perceptions and teaching approaches in this study influenced students' use of CAS calculators considering the potential contextual factors involved (e.g., mathematics subject) and the limited sample size. Further research is needed to compare and contrast the views of teachers and students with respect to CAS technologies to determine how they may potentially influence each other.

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